Early Mathematics Learning and Development

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Engaging Families as Children's First Mathematics Educators

International Perspectives

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Preface

Families play a central role in children's learning and development. Family involvement and parenting from an early age have been acknowledged to be vital for children's development—a statement that resonates through years of international research and literature. In recent years though, there is a prevalence of research that suggests that parents' engagement in their children's learning varies due to factors such as parents facing multiple disadvantages. Leading this debate, Melhuish et al. (2008), for example, suggest that parents generally engage in their children's development and learning if they have the opportunities and resources to do so. Through this book, we propose that families' engagement in their children's learning is complex than having opportunities and resources. In preparing their children for transition to school, parents have their own views about their child's transition to school and who is responsible for this process (Anders et al. 2012). Hence, creating an awareness of a variety of knowledge and actions for engaging children with early mathematical learning within informal and formal settings is a key focus of this book. A key assumption underpinning this book is that educators and early years professionals can support and partner families in their efforts to engage in their children's learning.

This book is an initial effort to conceptualize a set of research-based suggestions for parents and educators to support children's early mathematical learning. This book is a corner stone of our Australian Research Council Linkage Project *Numeracy@Home* that aims at understanding key variables in families' role in early mathematical learning and how families and early years educators and professionals can be supported in this role and assist children's transition to formal learning at school. The conceptualization of this book started with a symposium of researchers and early years professionals that presented an array of thematic issues that are evident in this book. The organization of this book is discussed in the introductory chapter.

In bringing this book to fruition, we acknowledge the continued support of the Australian Research Council and our Linkage partners—Department of Education and Training (DET) and the Catholic Education Melbourne (CEM). We are particularly thankful to Victoria Hall and Denise Jacobsson (DET), and Emily Black

and Narrelle Struth (CEM) for their thoughtful contributions to the threads of themes in this book. We are also thankful to all the authors for their contributions, and independent reviewers for their time and expertise in reviewing the chapters of this book.

Central to the success of any book compilation of this sort is excellent professional support. For us, this role was undertaken by Wendy May. We are extremely fortunate to have the assistance of Wendy, who expertly manages our expectations, and the authors' and editors' responsibilities in meeting deadlines. For her continued skillful management of this project, we are grateful and thank Wendy for always being there to support us.

Lastly, we would like to acknowledge all the families and educators who have been part of this book's central tenet, research, and evidence. Without them, there would be no book titled *Engaging Families as Children's First Mathematics Educators: An International Perspective*. We are confident that this book presents vital issues to consider and inspiring ideas and approaches for supporting families as their children's first and most important mathematics educators.

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Contents

Part I
Introduction

Chapter 1
Engaging Families as the First Mathematics Educators of Children

Sivanes Phillipson, Peter Sullivan and Ann Gervasoni

Abstract This chapter introduces the premises of this book including the theoretical basis of early learning of mathematics. The chapter describes how research and policies recognise early learning as important and how families (and educators) are considered to be an integral part of the learning process in early childhood experiences. This chapter also outlines the organisation of the book including each contribution in this book.

Keywords Early childhood · Learning · Mathematics · Policies · Family and educator partnerships

Introduction

Mathematics is everywhere—says Count Dracula of the ever popular children educational series Sesame Street. Children are exposed to mathematics from early on in their lives through their interactions with objects, concepts and people in their surroundings. One of the earliest memories that I (the first editor) have of my childhood is a haggling scene between my mother and a vegetable seller in a wet market in the middle of Georgetown, Penang (Malaysia) in the early 1970s. I was about 5 years old then, following my mother on her daily marketing for fresh food. The haggling was over 5 cents—yes, only 5 cents—to reduce the price of a bunch of spring onions from 15 cents to 10 cents. At that time, I was intrigued but confused by my mother's persistent insistence that she would only give the seller

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10 cents though the seller was red in his face through saying 15 cents. I was confused as to why my mother kept saying, with a smile on her face, that it was only 5 cents less—not much of a loss to the seller, when the seller did not think the same. My mother did get her way and got the spring onions for 10 cents. On our walk home, I asked my mother why she insisted on giving only 10 cents when the seller wanted 15 cents. She said to me that she saved 5 cents for the next family meal! For my mother, the 5 cents is part of another meal, but I also learned at that time that 15 cents less 5 cents equated to 10 cents. This early lesson has always stayed with me and shaped my early education, demonstrating that our parents and families are the first mathematics educators that we have in our lives. However, what was poignant from this memory recount is that the everyday happenings involving mathematics may not be a deliberate action but rather something that can stem from an interaction with the environment, reflecting many early and contemporary theories in child development and learning.

One early proponent of child development and learning is Piaget, who believed children made meaning of the objects and concepts in their surroundings. In fact, Piaget's early experimentation with children's learning involved mathematical concepts of number, quantity, and space (as noted by many of the contributors in this book). The key consideration of Piaget's work was around how a child is able to show progressive logical reasoning and mental operations that included conservation of number, quantity and space (1951). Though Piaget conducted the experiments with his own children, it was not clear whether he saw himself as an important part of his children's learning and development. In fact, he saw development as happening in defined stages, facilitated by engagement with particular experiences. In contrast, Piaget's contemporary, Vygotsky, proposed that social interaction with an adult or a "significant other" facilitates higher mental functioning that supports child development and learning (1978). How a child makes sense of mathematical concepts depends upon the interactions he or she has with another person in the environment, just like my experience of early marketing days with my mother. Vygotsky's own experiments began with his own daughter where he was convinced that a child makes better progress in learning and cognitive development through meaningful and purposeful interactions with another who is far more knowledgeable. Vygotsky's early works acknowledged the need for this social support to begin from the birth of the child (2004).

In recent years, the early years of life have been gaining prominence as a crucial time for children's learning and development. Neuroscience research has provided crucial evidence of the importance of early nurturing and support for early learning and later success (Shonkoff and Phillips 2000). Shonkoff and Philips noted the importance of children's interaction with their environment, in particular with their family, as a major factor influencing their learning. This statement assumes that early experiences affect the brain development of young children, and thus the foundation of intelligence, emotional health and educational outcomes (Elliott 2006).

Of late, Shonkoff (2012) postulated that early experiences are biologically embedded and carried over to adulthood, hence highlighting the importance of supporting those who are most vulnerable at the earliest age. Shonkoff argued that for adequate support to be given at an early age, it is crucial for the adults who care and educate children to have the appropriate mind set to support children's development and learning. Shonkoff's theory of change requires a developmental framework that brings together healthy and nurturing early experiences, which are enhanced by positive family and other proximal interactional environments. Along this line of thought, early mathematical and literacy learning have been seen as important developmental milestones, and the role that families play in this early learning has been well acknowledged by research (Melhuish et al. 2008). It is worth noting that there is current debate concerning the extent to which children's early mathematics learning experiences, within family contexts and formal early years care and education settings, should be incidental or intentional (Meaney et al. 2016). The chapters in this volume contribute to this debate with many of the chapter authors arguing that children's mathematical experiences, whether incidental or intentional, must be purposeful and can be enhanced through interactions with their parents and carers.

Recognition of Early Years Development

Recognition of the importance of the early years in human development can be seen in international and Australian policies and frameworks. UNICEF states that,

> Children are the foundation of sustainable development. The early years of life are crucial not only for individual health and physical development, but also for cognitive and social-emotional development. Events in the first few years of life are formative and play a vital role in building human capital, breaking the cycle of poverty, promoting economic productivity, and eliminating social disparities and inequities. (UNICEF 2016)

UNICEF's vision parallels with that of the United Nations where it considers the early childhood period to be "A time of remarkable brain growth, these years lay the foundation for subsequent learning and development" (UNESCO 2016). This statement underlies one of their most important conventions for children's right (United Nations Convention on the Rights of the Child), which advocates quality and equitable early childhood education and care for all children. One of the bases of this convention is to provide equitable opportunities for families to care and educate their children from the early years.

In the Australian context, as illustrative of approaches at a national level, federal and state governments have all taken similar stands concerning early years development and education. In the year 2000, the Australian Federal Government launched the 'Stronger Families and Community Strategy', which emphasised the role of families and communities in providing stronger social support for children. This strategy was created to focus particularly on the needs of families with young children, to strengthen relationships and to enhance the balance between work and family in people's lives (Press and Hayes 2000). Shortly after this initiative the

'OECD Thematic Review of Early Childhood Education and Care Policy, Australian Background Report' was released which also pointed to the importance of parent participation in early childhood in Australia, signifying parental involvement as an important element in ensuring positive outcomes for children (Press and Hayes 2000). This report looked at how parents could influence the positive outcomes of their children's learning by participating in early childhood learning settings.

The 'Best Start Evidence Base Project Report' compiled by the Department of Human Services (DHS) and the Department of Education, Employment and Training (DEET) in Victoria, Australia in 2001, demonstrated at a local level the importance of investment in the children's early years. It highlighted the significance of improved health and wellbeing in leading to the future success of children (DHS and DET 2005). The report authors argued that building the competency and capacity of parents was important for supporting the development of all children. This report took a 'top down approach', by pointing out what it is that parents need to do to contribute to the learning success of their children. In this document, the government suggested that learning opportunities through partnerships between parents and professionals in early childhood settings were key to supporting parents to provide their children with the best start to life.

Furthermore, in December 2008 the 'Melbourne Declaration on Educational Goals for Young Australians' was created and agreed to by all Australian Education Ministers. This declaration highlighted the importance that education plays in creating an equitable, democratic and just society. The overarching goals of the declaration state "Australian schooling promotes equity and excellence; and all Australians become successful learners, confident and creative individuals as well as active and informed citizens" (Ministerial Council on Education, Employment, Training and Youth Affairs 2008). The declaration once again highlighted the importance of partnerships with families to offer support for the wellbeing of young people. To establish the role of families in children's future academic and economic lives, the government began to focus on policy that encouraged stronger relationships between the family and educators.

The Declaration was further amplified through the development of the 'National Early Childhood Development Strategy, Investing in the Early Years'. Seen as a collaboration between the Commonwealth of Australia and the state and territory governments, the Strategy was "to ensure that by 2020 all children have the best start in life to create a better future for them and for the nation" (Council of Australian Governments (COAG) 2009, p. 4), hence providing an equal opportunity platform for all children's development in Australia.

The COAG strategy included the Early Years Learning Framework (EYLF) as well as the Victorian Early Years Learning and Development Framework (VEYLDF). These documents outline how early childhood settings can engage in partnerships with families and include them in the early childhood setting, again highlighting the 'top down' vision for parents' roles in their children's learning. These frameworks were some of the first policy documents produced in Australia to focus on the learning and development of young children in early childhood settings and frame the practice of early childhood practitioners. Whilst the EYLF includes *partnerships with families* as an underpinning principle, the VELDF builds on the principle to make specific reference to a model of partnership as 'family-centred practice' (Rouse 2012, p. 18). This model of partnerships encourages practitioners to engage with families in a collaborative manner. This model also supports the idea that learning and development outcomes are enhanced for young children when there are strong partnerships between families and early childhood professionals.

Established in 2012, the National Quality Framework or more precisely its key quality guidelines, the National Quality Standards capture the EYLF and VELDF model of families' partnership by advocating that educators work closely with families for the benefit of children (Australian Children's Education Care and Quality Authority 2016). Standard 6 in particular provides guidelines for educators and professionals about providing support and building collaborations with parents, families and communities in order to create opportunities for children to thrive in their overall development. Although these standards for educators and professionals describe ways for educators to collaborate with families to support children's learning, research highlights that a disconnection between informal family and formal education settings impacts on children's learning (Hildenbrand et al. 2015). It seems that this disconnect may exist due to educators' and professionals' lack of confidence in working with families. Furthermore, it is noted that the disconnect may come from educators' and professionals' own lack of knowledge about family characteristics and how diverse families engage with their children (Blackmore and Hutchison 2010; Saltmarsh et al. 2015; Stacey 2016). These findings point to a gap in knowledge about how families intentionally or unintentionally engage with children's learning, especially in relation to numeracy and mathematics.

Context and Aim of This Book

This book stems from the *Numeracy@Home* Australian Research Council Linkage Project—that explores the influence of socio-economic background on how families engage in their children's learning. Dandy and Nettelbeck (2002) argued that the way that parents communicate and act upon their expectations in learning and achievement depended upon the opportunities created through their social status. Thomson et al. (2011), reporting on PISA 2009, found that Australian students in lower achievement groups were disproportionally those:

- from the lowest SES quartile (of whom 22 % were not achieving level 2 in numeracy compared with 5 % of the high Socio Economic Status (SES) background students);
- from an Australian Indigenous background (of whom 39 % were not achieving level 2 in numeracy compared with 12 % of non-Indigenous students).

The Australian Early Development Index (Australian Early Development Census 2013) reported that, in 2012, 17 % of the population of 5-year old children (n = 273,896) are at-risk of not attaining expected literacy and numeracy skills. Most of these children were from families experiencing multiple disadvantages situated in low SES suburbs and towns, and include Indigenous children and those with language backgrounds other than English (LBOTE). This effect is such that students from low socio-economic backgrounds tend not to be able to overcome this initial disadvantage.

It is generally acknowledged that children recognised as "at-risk" before formal schooling are developmentally delayed in their learning and carry these risks to adulthood (e.g., Edwards et al. 2009). Although it is important to provide resources to enhance social and cultural capitals (Marks 2006, 2015), this will only be part of the solution if families lack the knowledge, confidence and capacity to take advantage of those resources, and awareness that they can support the learning of their children. Alongside the effect of SES on the availability of resources, the extent of home learning activities and a stimulating environment wields a greater and independent influence on learning (Melhuish et al. 2008). Furthermore, Hayes et al. (2013) reported that Australian parent-child engagement in home learning activities decreased over the early childhood period, and that child gender, maternal ethnicity, education, and family income were significant predictors of the decrease.

Taking into account these dominant research findings, the premise of this book is that many children who start school are confident learners and confident learners of mathematics but some others are not. The book responds to current debate regarding effective approaches for improving educational outcomes for all children especially in light of the acknowledged important contributions of both families and early childhood educators. Whilst educators' contributions are important, a greater focus is needed on the fundamental base of all educational beginnings—families and their engagement with their children in the home. Family involvement in mathematics learning, per se, does not predict how children learn and succeed at school: rather, it is the type and quality of the engagement that matters (Phillipson 2013). As Melhuish et al. (2008) argued, the key is family "provision of opportunities for building intellectual skills" (p. 109) to enable children to be confident learners (Gervasoni and Perry 2015).

The proposition underpinning this book is that it is possible to offer suggestions for professionals and educators working with families of young children about how to engage and support families in the area of mathematics learning, including those families who seem alienated from formal education settings. Specifically, the chapter authors in this book explore key concepts for understanding children's early development of mathematical learning and ways in which families (and educators) can engage with children in ways that will promote early mathematical development. The themes of mathematical concepts and pedagogical practices in this book form a theoretical framework for understanding effective strategies for supporting families in engaging with their children's mathematical learning, including those who are marginalised and experience multiple disadvantages. Thus, the purpose of the book chapters is to explore how families support their children's mathematical learning and their development of positive dispositions for learning. The chapters in this book also address issues around barriers and opportunities within the systems surrounding family engagement in mathematics learning, and offer ways for families and educators to work together to support children's learning and development. The conceptual ideas that arise from the book facilitate the construction of a systematic approach for creating awareness and efficacy for responsive parenting and constructive educator-family relationships focused on children's mathematics learning and future learning success.

Contributions in This Book

This volume, *Engaging Families As The First Mathematics Educators of Children: International Perspectives* explores ways in which professionals and educators might engage with and inspire parents to support the mathematics learning of their young children. The focus of the book is mainly on children's mathematics learning from birth to 5 years of age. Please note that the term parents and families are used interchangeably to reflect that in most cases parents (biological or adoptive) are carers of children but in some cases siblings, grandparents or any other extended family members are children's carers.

Bringing together an international group of expert researchers and scholars, this book presents ways for engaging with and supporting parents, including those who are less aware of their critical role in the development of children's mathematical learning in the their first 5 years. Following this introductory chapter, the first section of the book (Chaps. 2–6) focus on the key mathematical concepts that young children learn and the kinds of pedagogical approaches that parents, families and educators can use to facilitate early mathematical learning and development. The second section of the book (Chaps. 7–10) highlight home learning interactions and experiences that contribute to early mathematical learning. The third section of the book presents (Chaps. 11–14) focus on the importance of establishing family-educator partnerships to optimise early mathematical learning in both informal and formal settings.

Within the first section, Sullivan, Gervasoni, and Phillipson (Chap. 2) begin this book by suggesting to the reader a set of key foci that form a framework for "curriculum" design that parents and educators can use together to facilitate children's early mathematical learning. The authors concede that there is a world of mathematical possibilities for young children's mathematics learning, but argue that suggesting key mathematical concepts and associated experiences for children's early mathematics learning is a helpful guide for many parents and educators. The mathematical foci are presented with the view that children's early experience and learning of mathematical concepts, with the help of the adults around them, can assist the transition of these children into formal schooling. The argument is that, regardless of whether the children's mathematical experiences with adults arise incidentally or intentionally, it will help if the parent or educator is aware of important mathematical learning goals.

Chapter 3 discusses how very young children (infants and toddlers) begin their early mathematical learning through their interactions with their environment. Garvis and Nislev touch upon two pedagogical approaches that can be used to understand the early interactions that infants and toddlers have with the adults around them. Two pedagogical approaches, Variation Theory and the Montessori approach are discussed by the authors to show educators and families how they can engage infants and toddlers in early mathematical learning.

The fourth chapter in the book outlines how parents and educators can encourage pointing to objects and sorting (known as *enumeration*) to their advantage in supporting children's meaning making with numbers. Margolinas, Wozniack, and Rivière argue that their method of assisting children to enumerate can effectively be used to develop early mathematical understanding and learning. They provide the example of using tokens to design enumeration experiences for children's mathematical learning, which they propose should be an intentionally taught rather than an incidental learning process.

In Chap. 5, Björklund and Pramling discuss and analyse the early learning experience of a child, Vidar, through a number of everyday activities in his home environment during his first 6 years of life. The authors suggest that young children have abilities to discern small amounts and changes in quantities. Their abilities to discern come from their capacity to reason and negotiate mathematical objects and relationships that they have in their everyday interactions. How successful is a child like Vidar in learning mathematics is dependent upon the support, understanding and interactions with mathematical concepts integrated in the social interactions with the child.

Whilst the Vidar example comes from Sweden, Mousley illustrates in Chap. 6 (the final chapter in section one), stories of diverse Australian children constructing mathematical concepts and relationships through a range of conversations and stimulating environments where families, peers and educators use everyday contexts to engage children in early mathematical learning. Mousley presents the story of Peter to illustrate the learning of cardinal numbers, the story of Budi for the learning of counting, and the story of Spiros and Allisa to show numeral identification through everyday play situation.

In the second section of the book, the rationale for the importance of early mathematical learning is illuminated by a meta-analysis conducted by Dunst and colleagues in Chap. 7. This chapter includes a synthesis of research about relationships between informal and formal, home and family early numeracy learning experiences and preschool children's mathematics performance. The chapter sets out to show that early learning experiences are important to later learning outcomes. The chapter also suggests that informal learning experiences (such as those found at home) seem to have far reaching impact on children's later mathematical achievement, thus providing further evidence to support the premise of this book.

The authors of Chap. 8, Phillipson, Richards, and Sullivan present some early insights from the *Numeracy@Home* project that illustrate parents' perceptions of their family access to resources (in the form of capitals) and the importance of early learning for formal schooling, especially in relation to mathematical learning. The chapter shares how parents in the study, who live in a financially disadvantaged community, are aspirational and value early mathematical learning as a key to their children's success in schooling. These parents consider that early learning should be a shared responsibility between educators and themselves in preparing children for formal schooling.

Chapter 9 authored by Streit-Lehmann describes an intervention study in Germany that supports families to engage their children in early mathematical learning through games and books. Parents selected games and books to take home from a "treasure chest" in their children's kindergarten so that they could play with their children with the intention to teach them mathematical competencies such as counting and enumerating. The KERZ project described in this chapter shows how educators can support families to successfully engage with children's learning at home.

Whilst chapter nine is set in Germany, Wong in Chap. 10 describes how 174 Hong Kong parents engage in early mathematical learning with their children. Wong explains that how parents interact and use mathematical strategies with their children at home depends on factors such as socioeconomic status (SES), the grade level of the children, and parents' proficiency in and past motivation to learn mathematics. Of particular importance, is how parental expectations of their children's diligence in paying attention to strategies in mathematical learning play a key role in how parents interact with their children's learning at home. Wong suggested that parents used counting forward, using real objects to illustrate mathematics concepts and prompt questions to support their children's mathematical learning. Wong also highlights how SES can influence parents' use of prompt questions.

Section three begins with Chap. 11 that describes the beliefs and attitudes of five preschool teachers in Israel. Tirosh, Tsamir, Levenson, and Barkai discuss how teachers' beliefs and attitude can influence how they encourage families to take part in their children's mathematical learning. The activities teachers can encourage include involvement in homework and participating in intentional play with their children in mathematical activities. Tirosh and colleagues draw our attention to the challenges that teachers have in communicating and building partnerships with parents in their effort to support families develop their children's mathematical learning.

Gervasoni in Chap. 12 discusses how young children's mathematics learning can be supported through families and pre-school educators working together. The author draws on findings from the longitudinal evaluation of the Australian initiative *Let's Count* (Gervasoni and Perry 2015) to suggest that the program itself is successful in attaining its aim in enhancing early mathematical development. Gervasoni also highlights that parents in the program value the educators talking to them about ideas about mathematical activities that heighten parents' own awareness, efficacy and actions in supporting their children's learning.

Interestingly, early swimming training and the pedagogical discourse in teaching swimming is suggested to influence mathematical development. This claim is empirically supported by Jorgensen's study, which is presented in Chap. 13. Jorgensen found that the instructional discourse of swimming programs uses rich language about mathematical concepts, including shape, location, colour, and number, that young children are exposed to when learning how to swim. Hence, the author stresses that aside from physical and health benefits, swimming lessons for young children enhance mathematical development as well.

Furthermore, Edwards, McLean, and Lambert in Chap. 14 of this book and the final chapter in the third section discuss about supported playgroups in schools (SPinS) as contexts for engaging families as the first mathematics educators of young children. This chapter shows how participation in SPinS creates opportunities for families to become aware of the ways in which intentional play can contribute to early mathematical learning. SPinS is shown as both a context where educators and parents can work together to support children's learning and a method whereby parents can observe purposeful engagement in early mathematical learning between educators and children.

In the concluding chapter of this book, the editors Gervasoni, Phillipson and Sullivan present the issues, challenges and recommendations that chapter authors in this book highlighted in building awareness of families' roles and great capacity to support children's early mathematical learning and development. This final chapter presents five themes that arise from the contributions to this book and these five themes support the two core assumptions of this volume—that families are the first educators of children and that mathematical learning starts from birth. The concluding chapter suggests purposeful actions that both educators and parents can take to support children's early mathematical learning.

Conclusion

The broad aim of this book is to increase awareness of and participation in activities that families can implement at home and in partnership with early childhood educators and schools to enhance their children's mathematics learning and positive dispositions to learning. This aim is significant because it addresses the following rationales, namely that:

- there are substantial differences between students in their numeracy knowledge when they start school (Brinkman et al. 2012; Gervasoni and Perry 2015) and that these differences are seldom overcome with time;
- families are as important as formal child care, preschool and school experiences for stimulating the learning of young children (Phillipson 2013); and
- there are substantial differences in awareness by families of their capacity to support their children's mathematics learning and of the strategies they can use for that support (see Melhuish et al. 2008).

Hence, our emphasis on the key relationship between parent-child engagement and the development of numeracy for young children also addresses the differences noted in children's mathematics knowledge and actions when they begin school, thus providing insight for educators about how they might enhance children's development and learning (Hildenbrand et al. 2015).

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Part II
Key Foci and Pedagogical Actions That Support Young Children's Mathematics Learning

Chapter 2
Describing the Mathematical Intentions of Early Learning Childhood Experiences

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Abstract This chapter is written to inform the subsequent design of intentional experiences for young children, especially in family settings. There is clearly a world of mathematical possibilities for young children but it will assist in ensuring that children have experiences that can assist them in interpreting the world mathematically and in adapting to the demands of schooling. Based on analysis of research and critique of similar documents, the chapter presents a set of key foci that can inform the design of suggestions in which parents (and educators) can engage with children.

Keywords Early years mathematics · Measurement · Number · Space and location · Early years curriculum

Introduction: The Purpose of Defining Mathematics Learning Goals

Governments and local communities increasingly recognise that productive family-based experiences and effective pre-school education can position children favourably to participate fully in the learning opportunities that school will offer them subsequently. Most children arrive at school having had a wide variety of educative activities that shape their subsequent learning and dispositions for learning. While formal education settings contribute to children's learning prior to school, family-based experiences are also critical for young children's learning and

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development. This chapter describes, for parents and educators, the potential focus of their engagements with their children around their learning of mathematics. The process followed in the development of the Australian Curriculum: Mathematics (AC:M) was used to develop the descriptions as sets of statements. That process included the articulation of the overall goals and principles informing the subsequent statements, the summary of evidence that informed expectations for learning, and the preparation of statements that describe the mathematical experiences from which it is anticipated that all children would benefit.

Some assumptions that underpin our intentions, and the analysis that follows, are that:

- productive mathematics learning experiences that children have prior to school enhance their effective participation in schooling;
- parents and educators welcome suggestions of the type of intentional experiences in which they can engage with children in ways that enhance children's mathematics learning, thinking and dispositions;
- productive experiences can be the result not only of observation and exploration in response to child initiated mathematics but also of educator and parent initiated activities and
- parents and educators sometimes can be unaware of the impact of their own interactions on their children's learning and enhancing this awareness can improve intentional experiences that can be initiated to provoke learning.

A further assumption is that the first step in the design and analysis of illustrative experiences and engagements is the articulation of mathematical learning goals. In the Numeracy@Home project, an Australian Research Council funded project in partnership with the Victorian Department of Education and Training and Catholic Education Melbourne, this articulation takes the form of a small number of statements. Some principles that guided the development of the statements are that they should:

- be few in number and succinctly written, focusing on the key mathematics learning goals;
- be consistent in form with the content descriptions and processes of the relevant school curricula to facilitate the transition to school learning—in this case this is the Australian Curriculum: Mathematics (ACARA 2015);
- be informative and use language appropriate to a non professional audience;
- inform the design of inclusive experiences appropriate for all children; and
- be focused on children aged 3–5 years (meaning the years just before attending school).

Similar principles were articulated by Clements (2014) in describing standards that apply in the US context. He argued:

> The most important standards for early childhood are standards for programs, for teaching, and for assessment. These should be built on flexible, developmental guidelines for young children's mathematical learning. Guidelines should be based on available research and expert practice, focus on and elaborate the big ideas of mathematics, and represent a range of expectations for child outcomes that are developmentally appropriate (p. 15).

The following section seeks to synthesise and critique some established sources that have described mathematical learning goals for children prior to school and to pose illustrative statements that can guide the design of specific experiences that promote mathematics learning for young children.

The Process and Data Used to Define the Learning Goals

Essentially the analysis reported in this section is intended to inform the creation of learning experiences that are described as tasks in the literature on primary and secondary mathematics teaching and learning. As argued by Hiebert and Wearne (1997), "what students learn is largely defined by the tasks they are given" (p. 395). Anthony and Walshaw (2009), in a research synthesis, concluded that "it is through tasks, more than in any other way, that opportunities to learn are made available to the students" (p. 96). Further, Sullivan et al. (2014) presented results that indicated that teachers welcome suggestions not only of specific tasks but also the structure of lessons in which those tasks might be posed. Sullivan et al. found that teachers appreciated the presentation of theory, the connection of the theory to practice through specific exemplars and the explication of the pedagogies associated with those exemplars. It seems logical that similar thinking can be applied to the design of learning goals and intentional experiences for children prior to school, although the language used to describe the experiences may be different from that used in schools.

We used an adaptation of the theory of didactic situations (TDS) and didactic engineering (DE) developed by Brousseau (1997) in articulating the set of learning goals. DE is a process that incorporates design and implementation and emphasises both the importance of considering the mathematical goals and pedagogical opportunities at the design stage, and the importance of describing the complexity of informal and prompted mathematical experiences at the implementation stage.

The first phase is the preliminary analysis of the mathematical goals of projected learning experiences. The following is intended to represent this preliminary analysis. The subsequent phase is design and a priori analysis though which illustrative experiences that exemplify the mathematical goals are developed. Even though some suggestions of experiences are presented below, the following is intended to elaborate the preliminary analysis rather than define the experiences that will be prepared subsequent to this phase.

The process is that, for different domains within mathematics, we draw on some literature that defines the key ideas, available data and analysis of various sources to articulate the mathematical focus of experiences in early childhood.

The first set of data presented is from the Early Numeracy Research Project (ENRP) (Clarke et al. 2002) in which the first two authors participated. The project involved one on one interviews with trained interviewers using a structured format and supplied materials. The project report presents data from the 868 students who were interviewed six times from the start of the first year of school to the end of the third year of school, although only data from the first interview are presented here. This analysis also includes data from subsequent interviews prescribed to explore emergent mathematical ideas for those children who were not yet able to count 20 small plastic teddies in the initial interview (described in the following as the "detour").

This analysis also includes data from the *Let's Count* Longitudinal Study (Gervasoni and Perry 2015) that assessed the mathematical knowledge of children at the beginning and end of their preschool year. *Let's Count* is an early mathematics program designed to assist educators in early childhood contexts to work in partnership with parents and other family members to promote positive mathematical experiences for young children (3–5 years). The program aims to foster opportunities for children to engage with the mathematics encountered as part of their everyday lives, talk about it, document it, and explore it in ways that are fun and relevant to them.

The children in the *Let's Count* project were assessed using the Mathematics Assessment Interview (MAI) that is a revised version of the instrument used during the ENRP. Overall, 125 children were assessed in December 2012 to form the comparison group, 142 children were assessed in 2013 (March and December), and 172 children in 2014 (March and December). Children were assessed by members of the research team who were experienced in working with young children and received training about using the MAI script and record sheet. The MAI record sheets were analysed independently by research assistants who entered responses and strategies for each task into a database, and also used an established algorithm to code overall performance in each domain to determine the ENRP growth point reached by each child.

Other documents from which we present relevant extracts are:

- the Australian Curriculum: Mathematics (AC:M) (2015) which as used as a form of backward mapping of the respective concepts;
- the Early Years Learning Framework (EYLF) (DET 2015) which "describes the principles, practices and outcomes that support and enhance young children's learning from birth to 5 years of age, as well as their transition to school" (DET 2015, p. 1);
- a curriculum analysis by Clements (2014) focusing on the early years that was intended to complement the standards approach in the United States; and
- the International Baccalaureate Primary Years Program (PYP) (IBO 2012) for children aged 3–5 which is an internationally recognised curriculum standard.

The first two of these documents were written for the Australian context and were extensively consulted with relevant stakeholders. The third source is a widely quoted analysis on early mathematics learning. The fourth was written for an international context and is used in hundreds of schools worldwide. Where other sources are used, they are referenced subsequently.

Analysis and Synthesis of Particular Content Domains

The following presents detailed analyses of the sources and recommendations on the topic of measurement, time, and number. There is also a less detailed description of the domains of shape, location and patterns. It is possible that some experiences related to chance are suitable for young children but there has been limited research on chance and little attempt to describe the foci of chance experiences. In each discussed domain, the intention is to synthesise the respective sources and to distill statements that can be used as the basis of subsequent resources development.

Measurement

Even though many texts on early years numeracy learning do not include this domain, learning about measures and measuring begins from birth and the curriculum in the first years of school builds on these early experiences. McDonough and Sullivan (2011), for example, in re-analysing ENRP data related to the Length items concluded "… the key targets for the learning of length in the first 3 years of school are, respectively, learning to compare, learning to use a unit iteratively, and measuring using formal units" (p. 27). They argued that learning to compare is the first step, and it can be assumed that comparisons are also important in other aspects of measurement such as mass and capacity, although time is conceptualised differently. Interestingly, McDonough and Sullivan found very little relationship between responses to length comparison items and facility with counting, for example, suggesting that learning of measurement is dependent on experiences that are specific to the measurement concepts being learned and not an indicator of some general mathematical capacity.

McDonough and Sullivan (2011) drawing on earlier work by Piaget, argued that there are three key concepts associated with measurement comparisons:

Visual comparisons, which do not require the ends to be aligned in the case of length or about which judgements can be made without testing for mass and capacity;

Conservation (in which size of an object is irrespective of its arrangement) is associated with direct comparisons, even if more than one object is being so compared; and

Transitivity (if John is taller (or heavier) than Sally, and Sally is taller (or heavier) than Ben, then John is taller (or heavier) than Ben) is associated with indirect comparisons in which a third object is needed to make the comparisons.

In the early years, experiences connected to both visual and direct comparisons are desirable for all children whereas in schools students move towards considering transitivity. At the start of the first year of school, the ENRP reports that around half of the students could compare a string and a stick, and another 20 % could say how many paper clips long is a straw. Both of these tasks involve direct comparisons. In the "detour", 60 % of the students could order three candles smallest to largest, and 50 % could order 4 candles. These candle tasks rely on visual comparisons because there is no need to align starting points. Two thirds of students at the start of school could compare the mass of two objects by hand and decide which was heavier. There was no ENRP assessment of capacity.

The *Let's Count* research reported that at the beginning of preschool in 2014, 53 % of 100 children (4 year olds) could compare a string and a stick, and 5 % could say how many paper clips long is a straw. In the "detour", 52 % of 194 children could order 3 candles smallest to largest, and 28 % could order 4 candles.

Both sets of results indicate that while some children have progressed beyond the earliest theoretical indicators, there are others who have not. Given that comparing the string and stick, or comparing which is heavier out of two small containers, seems more or less intuitive, and would arguably develop without any specific or directed experiences at all, it seems possible that it is the language of the task that may explain the numbers of students at the start of school who experienced difficulty.

The Australian Curriculum (AC) Foundation level curriculum includes the following statements, which are written in terms of student actions:

> Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language.

Based on the ENRP and *Let's Count* data, this statement describes important learning for all students, noting that substantial numbers of students can perform such comparisons at the start of school. The statement gives some prominence to language in the expectation that students will explain reasoning, although this could be more explicit. It is arguable that it might be reasonable to expect that all students do more than two way direct comparisons (and so the description could use longest, heaviest, and holds most) in the first year of school.

It is interesting to compare this statement with both the general statements and the details within the PYP documentation (2012) for children aged 3–5 years. The covering statement proposes that "Students will identify and compare attributes of real objects …". The details, under the stem "When constructing meaning, learners …" include the following:

> *Understand that attributes of real objects can be compared and described, for example, longer, shorter, empty, full, heavier, hotter, colder.*

This is a broader set of measures, and in particular the inclusion of "shorter" is useful. The EYLF addresses measurement with the following statement:

> *Children demonstrate an increasing understanding of measurement ... using vocabulary to describe size, length, volume, capacity ….*

This statement does not communicate to the reader that these measures are relative (at least at this level) in that it treats the measures as absolutes in the use of the term "measurement". It is also not clear what is meant by "increasing", "understanding", and "size". Further, the distinction between volume and capacity is nuanced and complex and arguably not appropriate for very young children. The EYLF statement is unlikely to inform the design of productive intentional education measurement experiences.

Clements (2014) in articulating the "big ideas" of measurement wrote:

> Comparing and measuring can be used to specify 'how much' of an attribute (e.g., length) objects possess. Measurement is giving a number to an attribute of objects, such as length, area, capacity or weight (p. 50).

The other measurement comments relate to iteration of a unit, which arguably comes later than the focus of this analysis.

Noting that there is no discussion of learning measurement even though the monograph is titled mathematics, Sousa (2008) includes the following statement of what pre-schoolers should learn:

> Children compare the height of a block tower with the height of a chair or table. They measure each other's height and distance from the desk to a wall. They learn that a block is too long or too short to complete a project (p. 79).

Although this only focuses on length, it emphasises comparisons and provides an interesting prompt as to potential experiences for children.

It is interesting that there are various publications describing early childhood mathematics, like Sousa (2008), that do not include statements related to measurement. For example, Anders et al. (2013) do not include measurement as part of their assessment of child development.

It is noted that the measurement domain is unique in that the concepts being developed are relative in the early years (e.g., longer, heavier) whereas the absolute concepts require use of tools, which are part of subsequent learning. It also seems that the term "attribute" is not appropriate in that objects do not have attributes in isolation.

Comparisons and the use of language seem to be common characteristics of all of the above statements. Indeed, the skills of comparisons (such as aligning ends) seem relatively natural and it can be concluded that nearly all of the experiences of comparisons in the early years should focus on words used to describe the aspects of objects being compared. The notion of visual comparisons (which do not require aligning ends) and direct comparisons (which do) are both important. In summary, the following statement is proposed with the intention of informing subsequent task design, following the stem "children learn maths when they …"

> *Compare objects and describe, in everyday language, which is longer, shorter, heavier, lighter, holds more, hold less.*

The type of formal experiences suggested by Sousa and indicated above are illustrative of what is possible. In addition, other illustrative formal experiences are tasks such as comparing two strings to decide which is longer, or deciding who is taller if one person is standing on a step. In both cases, examples which can be compared visually, followed by experiences that require direct comparison are desirable. It is possible that experiences that require indirect comparisons (such as deciding which is taller: the table in this room; or the table in that room) may help to consolidate the direct comparison experiences.

Time

Although best described as measurement, it seems that learning about time is different from other aspects. The most significant research on children's learning about time was reported by Piaget et al. (1960). In their research drawing on extensive individual task based interviews, they found that children first learn about intuitive time that involves sequencing of events (seriation) and comparing the duration of events. Connected to this is the naming of events such as the names of days, parts of the day, and even "5 min" as code for a short time. As with the other aspects of measurement it seems that the concepts involved are intuitive and the key focus for learning is the relevant language.

In the ENRP interview, children were asked to draw a clock, and prompted to say what clocks are used for, and what are the numbers. There were also questions about specific times on clocks. Clarke et al. (2002) reported "When the children arrive at school at the beginning of their Prep year, 84 % of the children are aware of time, and a further 16 % know some clock times and days, and can relate events to these" (p. 84). While their assessment items address only some aspects of time, at least for such elements many students are aware of time and its use. Gervasoni and Perry (2015) found that 73 % of children at the beginning of preschool were also aware of time and 5 % knew some clock times and days, and could relate events to these.

The AC:M says:

- Compare and order the duration of events using the everyday language of time
- Connect days of the week to familiar events and actions

These statements articulate key time concepts although it may have been helpful to include parts of days, parts of the year (seasons), and even months. The order of the words in the first sentence is not quite right (it would be better as "compare the duration of, and order, events …"). Nevertheless this is a useful description of the various elements of time at this level. The PYP described time concepts as follows:

- Understand that events in daily routines can be described and sequenced, for example, before, after, bedtime, story-time, today, tomorrow

The emphasis on routines is useful although the statement represents some aspects and not others. Some of the terms are about comparisons and some are about events and it would be helpful to delineate these since they represent quite different learning.

The EYLF included:

- Children notice and predict patterns of routines and the passing of time.

This is clearer than other statements in the EYLF although the required concepts are broader than what is described in that statement. The following is an attempt to synthesise this information into statements using the stem "children learn maths … when they …":

- use words that describe points in time, events and routines;
- compare the duration of everyday events using mathematical language; and
- describe and arrange connected events in the usual sequence that they occur.

Illustrative experiences could involve conversations about time, and in particular using descriptive words like day, night, early, late, morning, every day, today, tomorrow, yesterday, sleeps, seasons; and comparative words such as earlier, later, longer, shorter, faster, slower, days, months, before, after; and developing familiarity with clocks and the use of the term o'clock.

Numbers

Learning to use numbers is fundamental to the learning of mathematics. As Sousa (2008) and Clements (2014) explained, children can distinguish between quantities from a very early age. This immediate recognition of numbers, termed subitising (from the Latin for sudden), is described as pre attentive, meaning it does not require conscious activity. Sousa described perceptual subitising as when a number is assigned to a collection without deliberate counting, which for most young children applies to very small numbers (specifically 1, 2, 3, 4).

Sousa (2008) explained that conceptual subitising involves pattern recognition (such as the patterns on dice), as distinct from assigning numbers to objects one by one. Sousa argued that such conceptual pattern recognition is important for more abstract use of numbers. It is also arguable that conceptual subitising happens after other counting concepts and is less central for children prior to school than perceptual subitising.

It goes without saying that the various concepts associated with counting are also critical for learning to use numbers. The ENRP (2002) argued that experiences with counting objects assists with the development of number concepts. A key pre-requisite to learning to count is being able to say the sequence of numbers. While there are many descriptions of the next steps in counting collections, the list by Fuson (1990) is indicative of most lists and proposes that the key counting ideas to be learned, in order, are:

- one to one correspondence (count each object once and only once);
- conservation of number (how many objects there are does not depend on how the objects are laid out); and
- cardinality (the last number counted is the number of the collection).

Margolinas and Wozniak (2014) emphasise similar elements in describing pre-school learning of number in Chap. 10 of this volume.

In the *Let's Count* Study, 22 % of the children at the start of their preschool year could say the numbers names to 20, 79 % could say the numbers to 10, and 22 % could count a collection of around 20 teddies. In the "detour", over 60 % could recognise 2, 0, and 3 dots without counting but accuracy was much lower for higher numbers. Over 60 % could match the symbols for 2 and 3 to sets of dots, and more than 45 % could match 0, 4 and 5. Nearly 50 % could show 6 on their fingers, and 24 % could order numeral cards 1–9. While most children have progressed well on these skills and understandings, there is still about half of the group (aged 3 years 8 months to 4 years 8 months) who have less familiarity at the start of preschool.

In the ENRP interview, 35 % of the children at the start of school could not say the numbers names to 20, and 51 % could count a collection of around 20 teddies. In the "detour", over 80 % could recognise 2, 0, and 3 dots without counting but accuracy was much lower for higher numbers. Around 80 % could match the symbols for 2 and 3 to sets of dots, and more than 60 % could match 0, 4 and 5 to such sets. Nearly 80 % could show 6 on their fingers, and nearly half could order numeral cards 1–9. Eighty per cent could say the number after 4, and 50 % the number before 3. This suggests that specific experiences focused on saying the number names in sequence, working out the totals of collections and connecting these with symbols, and even partitioning numbers (that a 6 can be seen as a 5 and a 1) are not only possible with all pre school children but also desirable. While most students have progressed well on these skills and understandings, there is still close to a fifth of the age group who have developed less familiarity by the start of school. It is argued that purposeful experiences are likely to help those students.

The relevant content descriptions from the Foundation level (the first school year) of the AC:M are:

- Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point
- Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond
- Subitise small collections of objects
- Compare, order and make correspondences between collections, initially to 20, and explain reasoning
- Represent practical situations to model addition and sharing

Given the Let's Count and ENRP data, it seems that these statements are somewhat basic, and the challenge for teachers in the first year of school is to ensure that all students can achieve these while creating opportunities for those students who need greater challenge than these statements infer. It is possible that the statement on subitising lacks specificity in that it presumably does not refer to perceptual subitising (which is innate human ability available to most) and is intended to move students to conceptual subitising.

The PYP, for children aged 3–5 years, has a covering statement that is:

Students will read, write, count, compare and order numbers to 20. They will model number relationships to 10, develop a sense of 1:1 correspondence and conservation of number. They will select and explain an appropriate method for solving a problem.

Also in the PYP, under the stem "when constructing meaning, learners…" are the statements:

- Understand one-to-one correspondence
- Understand that, for a set of objects, the number name of the last object counted describes the quantity of the whole set
- Understand conservation of number
- Understand the relative magnitude of whole numbers
- Recognise groups of zero to five objects without counting (subitising)
- Subitise in real-life situations

The term "understand" is not ideal in that it is hard to know how this could be assessed. It also seems that the list is a more or less random collection of aspects that may assist with early learning. It is also an eclectic set of language. For example, the list uses "conservation" but not "cardinality". The idea of saying sequences is not mentioned.

The EYLF includes two statements:

- Children demonstrate an increasing understanding of … number using vocabulary to describe size … and names for numbers.
- Children use language to communicate thinking about quantities … and to explain mathematical ideas.

While the attention to language and vocabulary is helpful, otherwise this is a limited and unclear statement of experiences for pre school children and is unlikely to inform intentional activities. Clements (2014), in describing learning associated with number for these levels as general statements explained:

> Number can be used to tell us how many, describe order, and measure; they involve numerous relations and can be represented in various ways (p. 16).

> Another key element of object counting readiness is learning standard sequences of number words, learning that is facilitated by discovering patterns (p. 27).

More specifically, for this level, he defined the following aspects of number learning:

> Object counting involves creating a one-to-one correspondence between a number word in a verbal counting sequence and each item of a collection, using some action indicating each action as you say a number word (p. 28).
> Use counting or matching (one-to-one correspondence) to determine the equivalence or order (smaller or larger) of two collections, despite distracting appearances, and uses words equal, more, less, fewer (p. 30).

These statements are helpful in their clarity and specificity. Interestingly, and unusually in the Australian context, Clements (2014) also included:

> Representing collections and numerical relations with written symbols is a key step towards abstract mathematical thinking (p. 29).

We agree that recognising and even writing symbols can be part of the preschool experiences of children. Anders et al. (2013) described an assessment of pre-school children that sought to measure children's knowledge of counting, and recognising numbers. In the first level of their instrument, children are asked to count objects, and identify numerals up to 10. After that, their focus is on operations.

The following presents some statements that could be used to inform advice on experiences for parents, and EC educators, following the prompt "children learn maths … when they …":

Say number names forward in sequence to 10 (and then to 20 and beyond)
Use numbers to describe collections
Describe small quantities without counting, or by counting and matching to compare one collection or part with another
Match number names, symbols and quantities up to 10 and beyond
Show different ways to make or organise a total (with small numbers)

One of the purposes of this chapter is to inform the creation or identification of illustrative experiences. Interestingly, Sousa (2008), in the middle of an outstanding discussion of insights from cognitive science, in describing experiences for pre school children, wrote:

> Children learn about numbers by counting objects and talking about the results. "You gave Billy five cards. How many more does Mary need?" Children count spaces on broad games "You are now on space three. How many more spaces do you need to get to space seven?" They count days until their birthdays. The teacher might say, "Yesterday there were nine days until your birthday. How many days are there now? Children read counting books and recited nursery rhymes with numbers (p. 79).

This excerpt highlights a lack of clarity in identifying aspects in the number domain and emphasises the importance of connecting the mathematical goals with suggested experiences. In the above case, the use of addition does not match with the other statements made by this author.Some examples of illustrative experiences that might be prompted in family contexts are:

- Saying and acting rhymes and reading stories that focus on number
- Playing games that focus on spatial patterns such as dominoes
- Having conversations about comparisons such as "who has more grapes?" Or how many more grapes do you need?

In more formal settings, educators can plan purposeful experiences such as:

- Construction or threading beads projects using number as a describing word (make a three-two pattern)
- Comparison tasks that can be estimated (for example, with one number 2 and the other much larger)

- Comparison tasks that require moving objects (arrange that blocks so that the groups are the same or some that one group has more, etc.)
- Finding a given amount (find me four of something)
- Specific matching tasks that require children to connect representations, including symbols

Shape

Two aspects that lead to the domain of the curriculum described a geometry are shape and location. The *Let's Count* data indicated that two thirds of children could recognise and match similar shapes at the start of pre-school. This was a greater percentage than that identified by the ENRP for school entrants.

The PYP suggests that students aged 3–5 years can "sort, describe and compare 3-D shapes" and "understand that 2D and 3D shapes have characteristics that can be described and compared". Similarly the AC includes the following for the first year of school:

> *Sort, describe and name familiar two-dimensional shapes and three-dimensional objects in the environment.*

We argue that the following statement can be used to inform advice on experiences for parents, and EC educators, following the prompt "children learn maths when they …":

> Play with, name, describe, and organise 2D shapes and 3D objects.

This statement implies the nature of suggested experiences, with the emphasis on handling shapes and objects while talking about what they are doing.

Location (Visualising)

Other experiences leading to geometry are those related to location, including visualising. More than 70 % of children participating in *Let's Count* at the beginning of pre-school demonstrated understanding of the positional words 'beside', 'behind' and 'in front of'. The ENRP found that around two thirds of students could identify shapes in the room that matched ("same shape") with a given rectangle.

In terms of describing experiences, the EYLF proposed:

> *Children demonstrate spatial awareness and orient themselves, moving around and through their environments, confidently and safely*

This statement again lacks the specificity that would inform purposeful experiences. The AC:M also could be more descriptive than it is stating only "Describe position and movement". More helpfully, the PYP described the dual foci of this aspect as:

- explore the paths, regions and boundaries of their immediate environment and their position.
- understand that a common language can be used to describe position and direction, for example, inside, outside, above, below, next to, behind, in front of, up, down

On balance, it seems that the more important aspect for students prior to school is developing familiarity with the language and meaning of location.

The following statement can be used to inform advice on experiences for parents, and EC educators, following the prompt "children learn maths when they …":

> *Use words and ideas to describe where things are, for example, inside, outside, above, below, next to, behind, in front of, up, down, here, there, north, west, middle, across, opposite*

This statement implies the nature of suggested experiences, with the emphasis on conversations about the placement of objects and people with respect to others.

Patterns

More or less all mathematics involves identifying and//or describing patterns in some way. In fact pattern recognition is central to all of the mathematics concepts discussed above. Nevertheless, there can be experiences created that focus the attention of children onto aspects of patterns and structure.

The AC:M includes the following statement on patterns:

- Sort and classify familiar objects and explain the basis for these classifications. Copy, continue and create patterns with objects and drawings.
- The PYP documents include the following statements for pre school children:
- Describes patterns found in everyday situations, for example, sounds, actions, objects, nature
- Describes patterns in various ways, for example, using words, drawings, symbols, materials, actions, numbers
- Copies, extends and creates repeating patterns
- The following statement that can be used to inform advice on experiences for parents, and EC educators, following the prompt "children learns math … when they …":
- Describe, copy, represent and extend patterns found in everyday situations, including sounds, objects, actions and images.

Summary

The analysis presented in this chapter is a first step towards articulating some experiences that can inform parents and educators of some possible foci for mathematics learning of children prior to attending school. The analysis drew on data on achievement of young children on assessments of the mathematics knowledge and also on some similar statements in common use. The resulting statements are presented in Fig. 2.1.

Mathematics for young children

Children learn mathematics when they …

Compare objects and describe, in everyday language, which is longer, shorter, heavier, lighter, or holds more, hold less.

Play with, name, describe, and organise 2D shapes and 3D objects.

Use words and ideas to describe where things are positioned, for example, inside, outside, above, below, next to, behind, in front of, up, down, here, there, north, middle, across, opposite.

Describe, copy, represent and extend patterns found in everyday situations.

Use time words that describe points in time, events and routines (including days, months, seasons and celebrations).

Compare the duration of everyday events using mathematical language and arrange connected events in the usual sequence that they occur.

Say number names forward in sequence to 10 (and eventually to 20 and beyond).

Use numbers to describe and compare collections.

Use, progressively, perceptual and conceptual subitising, counting and matching to compare the number of items in one collection with another.

Show different ways to make a total (at first with models and small numbers).

Match number names, symbols and quantities up to 10.

Fig. 2.1 Resulting statements

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Chapter 3
Mathematics with Infants and Toddlers

Susanne Garvis and Eva Nislev

Abstract Infants and toddlers engage with many interactions with their environment to try and make sense of the world and concepts around them. Early mathematical understanding begins this way and is a social activity that is situated within a context. Communication and interaction are considered key tools that adults can use to support this process. This chapter explores the ways in which infants and toddlers mathematical learning can be enhanced through educational experiences at home. By acknowledging families as the first educators of children and that mathematical learning starts from birth, it provides a theoretical as well as practical understanding of the role of the family in the home context. The chapter concludes with a short discussion about further considerations for the future of infant and toddler mathematical research.

Keywords Infants · Toddlers · Interactions · Families · Communication

Introduction

Infants and toddlers strive on a daily basis to make sense of the concepts they encounter in interactions with other people and the physical environment. Learning early mathematics is social activity and is situated within a context (Lave and Wenger 1991). When children are born into a world of previously defined principles and knowledges, they are active in creating meaning and redefining knowledge. Infants and toddlers experience the importance of expressing and understanding common knowledge. In order to communicate, young children must possess a mutual understanding of the subject being talked about. Encountering

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other people and different ways of understanding a phenomenon focus children's awareness on their own ways of experiencing. Experiencing differences in understanding stimulates the learning process, allowing young children to discern learning as a change in understanding and action (Pramling 1983; Lindahl 1996).

This chapter explores the ways in which infant and toddler mathematical learning can be enhanced through educational experiences at home. Education is defined as making the invisible visible to the child (Pramling Samuelsson and Pramling 2013). Mathematics is defined as a way of describing measurable relations between objects in the surrounding world (Schoenfeld 1994). Basic mathematical skills therefore consist of becoming aware of similarities and differences as well as patterns concerning time, space and quantities. The aim of the chapter is to analyse and discuss strategies for learning that are essential for young children's early mathematical learning. The first section will discuss the role of the family in the home context in support the infant and toddler. The second section will discuss two pedagogical approaches. Practical strategies to assist early mathematical learning will be shared. The chapter will conclude with a short discussion on further considerations for future research on mathematics and infants and toddlers.

Setting the Context

The next section will introduce the concept of families as the first educators of young children. It will also provide a snapshot of mathematical development at home and the importance of communication between adults and children to enhance mathematical learning. Specific examples will also be given related to number learning, patterns and measurement. The importance of play activities inside and outside of the home is also acknowledged.

Families as the First Educators of Young Children

This chapter acknowledges that families are the first educators of young children. For young children, mathematical competency begins at birth (Anthony and Walshaw 2009). Children at any age (including infants and toddlers) can learn (Bruner 1996). This suggests that the family is important for early mathematical experiences with infants and toddlers. Early mathematical activities within families include playing games, reading number books and using money in commercial transactions have a significant impact on mathematical achievement (Guberman 2004; LeFevre et al. 2009). A child's home is thus considered an important place for early mathematics development (LeFevre et al. 2009). As Melhuish (2010, p. 67) states:

> The home learning environment in the pre-school period has association with all aspects of children's cognitive and social development and for much of a child' life is one of the most powerful influences upon development.

The role of families in the lives of young children is supported in a number of recent studies across the world. In a PISA study (Organisation for Economic Co-Operation and Development, OECD 2011) on educational attainment in OECD countries, 15 year olds whose parents read books with them in early childhood had higher results than 15 year olds whose parents did not read books. This finding holds true regardless of the family's socio-economic background. Likewise in the United Kingdom, research has found that by the age of 5, children from the poorest fifth of homes are already nearly a year behind children from middle incomes in development outcomes (Economic and Social Research Council 2012). The family home environment is important for closing this gap. Also recent evidence from the Growing Up in Ireland study (Nixon 2012) indicates children do best when a parenting style is warm and responsive but that also demands appropriate behavior from children. This starts from when the child is born. A warm and responsive parenting style leads to enhanced outcomes in literacy and mathematics.

How Do Families Help with Mathematical Development at Home?

Only a handful of studies have explored children's mathematical development in the home environment (Aubrey et al. 2003). This has included the role of the adult in supporting young children's learning. Clark (2001) has suggested it is possible to pin-point contexts that are more likely to stimulate learning in young children to lay an effective foundation for numeracy. By age five, it has been argued that there are wide differences in the preparedness of children for formal learning and in their grasps on the underlying concepts of numeracy (Aubrey and Godfrey 2003). Wood and Attfield (2005) further posit that the amount of mathematical knowledge children have on entry to school is a strong predictor of their future progress. Communication is the first type of influence on learning and is discussed below.

Communication Between Adult and Child

The influence of the adults' communication with the child about mathematical terms is important for preparedness of the child for formal learning. Drake (2009) suggests that talking with children as they play helps to develop their mathematical understandings and language. Several studies specific to mathematics have shown positive links between parents who actively engage children in mathematics in home-based activities and children's math performance (LeFevre et al. 2009). This means that the way adults talk with young children and the learning opportunities they create in everyday home life shape children's development and later learning. Understanding and developing mathematics education, according to van Oers (2013) should focus on developing skills of communicating about number, quantity, space and relations. This can be achieved through a greater understanding of the concept of children's play, where children are encouraged to communicate about specific mathematical concepts.

One approach is with narratives. Infants and toddlers can co-construct their understanding of mathematics through the narratives they co-construct with adults. Narratives provide the adult insight into the child's understanding and level of interest in mathematics.

Lucy and her grandmother are planning a shopping trip. Lucy is 22 months old. Grandma tells her "today we need to buy fruit and vegetables – we need to make a list". Grandma knows that Lucy is confident with the numerals up till five but gets mixed up after that. She is planning to buy between one and ten items to foster Lucy's understanding. As they chat, Grandma writes the list in clear writing (this is indirectly preparing Lucy for the association between the symbols and the verbal number). "We need six oranges, nine carrots, four apples, one cucumber, five potatoes, eight nectarines and seven tomatoes". At the supermarket Grandma gets out the list and together they read each item. Lucy finds the item and together they count the correct quantity into the bags. When they get home, Lucy carefully takes all the items out and shows and tells Grandad, counting each item, touching them as she counts. Grandpa has her sort each group in sizes, for example the longest carrot to the shortest carrot, giving him the opportunity to introduce the concepts of long, longer, short, shortest. As Lucy touches each fruit or vegetable her hands and brain are working together building and reinforcing important mathematical concepts. Grandma gets important feedback as she listens and observes Lucy's recount of their shopping adventure. She learns about Lucy's vocabulary, her ability to discern differences and discovers if any numbers prove troublesome – these all aid her in preparing further adventures and sharing with her parents so they can follow up at home.

Learning objects embedded in narrative play and stories bring several features to the learning process that are essential for goal-oriented learning with young children (Björklund 2014). According to Burton (2001), the narrative features also gives relevance to the acts and tasks within a mathematical activity. Within a narrative framework adults can limit possibilities and exclude alternatives by providing specific focus on critical elements in meaningful situations.

The focus on narrative highlights the importance of communication and interactivity aspects for infant and toddler mathematics. The learning objective bears meaning and it becomes possible for the adult to direct the child's attention to an intended learning object in mathematics. This allows children to be confronted with demands from a situation that requires translation of the experience into mathematical language. The skill requires a level of interpretation in mathematics and then knowing the words to communicate the meaning. Burton (2002, p. 12) highlights that adults, "instead of identifying errors or looking for differences from the mathematics taught or offered in the text", should "create the opportunities for the making of narrative and then look inside the children's stories to try and make sense of their meaning". For example, when children share their narrative understanding, families can understand their level of mathematics by understanding four key areas (Burton 2002):

1. The message of instability (when a child's counting starts to regress, the focus should be on the adult instruction and environment, not on the child);
2. Using counting mistakes as information (when a child makes a mistake with counting, the adult considers what is the real understanding being shown from the child);
3. Avoiding domination of writing (children first share narratives in oral form. Families can support oral sharing of mathematical narratives without the pressure for children to write mathematical symbols); and
4. Inconsistency between knowledge and understanding of structure (when adults rely on textbooks and commercialized early learning program, inconsistencies can appear for children between knowledge and understanding. For example while a child could count to four with an adult on a commercial flash card, the child may not be able to transfer this knowledge to counting four objects within the home. Rather time is needed to explore the meaning of four, and link the concept from one narrative experience to another).

By exploring the mathematical narratives of young children, we also trust and acknowledge that young children are able to show us their own understanding of mathematics. We understand that young children are natural storytellers and practice their learning (including mathematics). Adults are able to support young children in their narrative telling by encouraging the use of stories to explain mathematics and the retelling of stories to help support understanding. One way adults can support young children's learning is by assisting with number learning in narratives. By having a focus within narratives, children can begin to understand counting and numerical concepts.

Assisting with Number Learning

Number knowledge is often seen as an important aspect of mathematical learning because counting and numerical concepts form the basis of quantifying understanding about the wider world (Davies 2003). In a New Zealand study about learning outcomes, children as young as 23 months of age understood some numerical concepts and applied these to situations that has meaning for themselves and others (Lee 2012). In this study, children showed control of their play environment by engaging in counting before taking action.

The earlier work of Young (1995) with 2 and 4 year old children has shown that adult social practices associated with number related verbal activities improved children's performances, first in the presence of others and then in the child's own self-regulated use of number. In the study, parents indicated they began mediating number sequences and one to one correspondence counting to young children from

1 to 2 years of age. This suggests that the parent finds the right opportunities within their daily lives and social practices for numerical meaning. Common home activities included songs, rhymes, using counting books, playing with bricks and toys and money. The role of the family is important for supporting and communicating mathematical understanding with young children including assisting them with more advanced mathematical knowledge like measurement.

Assisting with Measurement

Measurement (for example space and number) has also appeared in some studies with toddlers. The concept of measurement regularly occurs in day-to-day conversations and linked to a child's experience (Pound 2006). This has included engaging toddlers as young as 15 months with play objects that were a variety of sizes (Lee 2012). The most common places for measurement experiences for young children generally occur when playing with sand or water (Lee).

The resources associated with playing with these materials promote experimenting and exploring concepts of measurement in different ways. This includes, for example, tipping sand, pouring sand or water and filling and emptying containers. Lee has noted that toddlers are capable of noticing containers that are not completely full and subsequently choosing to fill them. This comparison of attributes or properties of objects such as size, length, volume develop the toddlers understandings of how we measure the world (Hoorn et al. 2011). The same can be said of how young children develop when they learn sense of patterns.

Assisting with Patterns

Many studies have also focused upon the development of skill and understanding of patterns in young children (Geist 2001; Schwartz 2005). Patterns are connected to algebraic thinking. Papic and Mulligan (2007) have also noted that experience in patterning serves to promote other areas of mathematical learning such as transformation skills. However with infants and toddlers, patterns in the literature often refers to care routines, and the rhythms of stories, poems and songs (Babbington 2003; Pound 2006). Only recently, has young children's pattern making been an object for inquiry (Björklund 2014). Family members can provide opportunities for young children to engage in patterns through modeling. For example, initial patterns may be based on colour. Children can choose to replicate the pattern, and then create their own pattern. Many such mathematical concepts can be learned through play.

Providing Play Opportunities Inside and Outside at Home

Play is an important part of children's lives. In addition to being a vehicle for learning, play is described as a context in which children can demonstrate their own learning and help scaffold the learning of others (Wood 2008). Toddlers engage in play experiences as the primary form of learning (Langston and Abbott 2005). van Oers (1996, p. 71) notes that the potential of play to facilitate children's mathematical thinking depends largely on educators' ability to "seize on the teaching opportunities in an adequate way". In this case this means the role of the family to provide teaching opportunities for infants and toddlers.

Research has found that young children's play often involves mathematical concepts, ideas and explorations (Perry and Dockett 2008). While

> play does not guarantee mathematical development, it offers rich possibilities. Significant benefits are more likely when teachers follow up by engaging children in reflecting on and representing the mathematical ideas that have emerged in their play (NAEYC/NCTM 2002, p. 6).

Importantly, children become "real-world" mathematicians when participating in everyday practices at home (Wood and Attfield 2005). In the context of a family, this means providing mathematical play opportunities that engage children's natural curiosity, allow children to recognise mathematics as a social activity and provide opportunities for the promotion of mathematics that has relevance to the child and family's everyday lives.

The tenet of fun as a positive experience through play also appears important in early mathematical learning. Previous research suggests "we can influence young children's keenness to learn mathematics by making the tasks we do of interest to them…by showing that we really think maths is important and fun" (Clemson and Clemson 1994, p. 19). When families have positive and fun mathematical play experience with infants and toddlers, they can influence children's keenness towards learning mathematics and provide foundational understanding of mathematics before a child starts school.

Outdoor play environments can also provide opportunities for mathematical play for infants and toddlers. Outdoor environments have been situated as the 'place to play' by research studies in New Zealand (Greenfield 2007; Lee 2012) and internationally (Greenman 2005; Herrington 2005; Pica 2006). For example, when a child tips and pours sand, they are discovering space, weight, and may be considering ways to move the sand from one place to another. It is within this period that toddlers are engaged in play experiences as the primary form of learning (Langston and Abbott 2005). Thus, play within families is important for young children mathematical learning, including opportunities for outside play.

This section has provided an explanation of how parents can assist children. It has also recognised parents as the first educators of children. The next section will illuminate two pedagogical approaches; Variation Theory and the Montessori approach. Both provide strategies for enhanced learning.

Pedagogical Approaches

Burton (2002) describes mathematics learning as a socio-culturally negotiated story, the meaning of which is shared between persons interacting in meaningful contexts. "Mathematics is not simply a cognitive matter but involves people in their entirety" (Burton 2002, p. 7). From this perspective, mathematics teaching relies on a supportive environment where children are trusted to take responsibility for their own learning. The role of the family as well as the early childhood setting is to create conditions where meaning-making and negotiation are possible and children's questions and answers are supported.

One way to create conditions for learning is to consider pedagogical approaches that promote learning. Such pedagogical approaches include Variation Theory and the Montessori approach. Both provide examples of how to work with the youngest of children regarding mathematical learning in both the home and formal early childhood environment. While it is not anticipated that families with implement these pedagogical approaches, they are able to develop an understanding of what they are and begin to understand the work of early childhood educators.

Variation Theory

According to Variation Theory of learning (Runesson and Marton 2002), understanding is defined as those aspects of a phenomenon that a person is able to focus on simultaneously. This means it is important for children to experience phenomena in many different ways so that they may understand concepts more thoroughly. When the child understands a phenomenon in a new ways, certain aspects of the concept have been discerned by the child that have not earlier been focused upon. For example, if two objects are classified as 'big' and 'small', what will happen if an objective that is an in-between size is placed beside the objects? Which object is now bigger and which object is smaller? The child must work with the new understanding. When the child's understanding of the phenomenon changes, learning has occurred which will result in the child changing their actions in the surrounding world. According to Marton and Booth (1997) an individual cannot act towards the world in a way different to how one understands phenomena that appear in the world.

Variation Theory is based on three intertwined and interdependent concepts: discernment, simultaneity and variation (Marton et al. 2004). For example, learning the meaning of a specific concept such as 'long' requires the child to discern the features in relation to objects that differ in length. Further it is important to recognise that the relative meaning of 'long' changes when longer objects are added and the reference object may no longer be described a 'long'. Length therefore depends on a specific special feature of the object, which differs from other features such as weight. Thus, there are several aspects that need to be discerned by the child in order to develop a conceptual understanding of a notion (Björklund 2007). While young children show competencies in discerning and exploring different features of phenomena, it is necessary that adults pay attention to what materials or objects are used in exploring and representing mathematical concepts.

Another pedagogical approach is the Montessori approach. Similar to Variation Theory, it also acknowledged the importance of working and the physical environment.

The Montessori Approach

One approach to early mathematical learning is the Montessori approach. Maria Montessori (1870–1952) realised that children learn quite differently to adults—they appear to 'absorb' the world around them. Montessori believed that education begins at birth and the first few years are the most important for the mind to absorb. The baby is born with an active mind and families are challenged with educating the potential within the child.

Working with the absorbent mind are what Montessori called 'sensitive periods' (Montessori and Costelloe 1991). These are special times when the developing brain is particularly receptive to certain stimuli—there is a special sensitivity for something. The easiest to see is the sensitive time for walking. We know that when the child is ready to walk—he will—it is like a driving force within him that seems to propel him upward and forward. Montessori also made an important connection between the hand and the brain (Montessori 1989; Montessori and Costelloe 1991). She wrote that "the hands are the instruments of man's intelligence" the child making discoveries "playing with some thing" (Montessori 1989, p. 27). Current early childhood writing supports the view that sensory stimulation is crucial to the development of the brains circuitry (Berk 2012; Sigelman et al. 2012). It follows then, that the environment the infant is in must give him the freedom to reach out and touch many different objects, each experience forming a new and different impression offering a continuous "feedback loop of hand-to-brain-to-hand" (Lillard and Jessen 2003, p. 50)—adding richness to his development.

The environment and the adults in it provide the greatest influence on the development of young children. While the physical and psychological aspects are always carefully considered the parents and educator can develop an awareness of how to establish the environment in a manner that best aids the growing child (Berk 2012; Briggs et al. 1999; Bruce and Meggitt 2005; Johnston and Nahmad-Williams 2009; Sigelman et al. 2012). The richer the environment—the more impressions the infant and toddler can absorb. It is the absorbent mind that allows every child to grow up and adapt to his own culture so easily (Montessori 1989).

One of the most important elements to consider is maintaining 'order' within the environment. The presence of 'order' within the environment enables the developing child to make sense of his world in a logical manner. He can comfortably predict routines and then more easily and logically begin the task of categorising, classifying and sequencing, predicting and sorting—the foundations or 'indirect preparation' to his later mathematical understanding. The mathematical mind is an exploring mind—it is able to order, sequence and finally abstract, but if the environment is chaotic it is very difficult for the infant to begin these tasks.

Montessori (2002) introduced an area called "practical life" into her curriculum. Practical life activities are designed to enable the developing child to function in his own environment. These activities are all the daily tasks we carry out everyday—washing, pegging, cooking, folding, cleaning and so on. While the toddler is 'doing' practical life he is indirectly learning basic mathematical concepts. As he sorts the socks he distinguishes different shapes, sizes and colours, while folding the cleaning cloths and face washers he is beginning fractions and learning geometry, but more importantly he is extending his concentration, co-ordination, dexterity and independence.

There are many activities utilized in the Montessori early years classroom that can also be applied to engage the infant and toddler around the home. Counting objects with children for example, counting fingers and toes, the steps to the car, or from room to room, counting the forks into the cutlery drawer, eggs in the carton and so on. By using objects we avoid young children 'rote counting' and further provide them with the opportunity to grasp the idea of different quantities,

Table 3.1 Practical strategies for infants and toddlers

Activity	Method
Counting	Count ears and eyes, toes and fingers, cups and bowls, chair legs, pegs, spoons, fruit. As the infant begins to enjoy this activity work up to ten. Ten is the base of our decimal system, if there are more than ten objects, count to ten and make a group and them count from one again (this gives indirect preparation for the decimal system)
Sorting	Start with two different objects and have the child sort into two baskets, once mastered add more different objects up to four. Then make the objects more similar and sort, e.g. all pegs different sizes. You can also sort into colours, blue, yellow and red socks. The objects should be from everyday life, for example, teaspoons, walnuts, socks, pencils, pegs, lids, small travel bottles, large buttons, fruit and vegetables. Have your child arrange in groups and then later in straight lines which you can then count
Ordering	Order objects by size, left to right, Organize in height and width. Later the child can order onto a string and make a necklace
Stacking	Stacking plastic cups or tins from the cupboard is a game children love. Give the infant time and freedom to experiment—to build and crash down, put inside each other and so—remember the hand and brain are working together, they are 'problem solving', an essential ingredient for later mathematics
Matching	This activity is limited only by your imagination. Matching socks, face washers, spoons, cups, containers, pegs and so on. Place the items in a basket so the child can match—begin with just two different items e.g. two different types of teaspoon and build up from there in difficulty
Patterning	Once the infant is older we can introduce the idea of patterns. Start with two different objects and place them in a row alternately e.g. spoon, fork, spoon and so on. As they get confident you can make more challenging—2 spoons and 1 fork, 2 spoons and so on

unconsciously absorbing that numbers are all around them. Table 3.1 offers some practical strategies, which can also enhance a young child's mathematical learning.

Research suggests that much of young children's early mathematical understanding involving thinking, perception and movement, are developed through activities such as counting and pairing (Kilpatrick et al. 2001, p. 159) and are strong predictors for later connections with symbolic mathematics (Starr et al. 2013). The opportunities for indirect preparation for future mathematics success are endless in the family home.

Conclusion

In the discussion about early years mathematics, it is important to also realise the importance of families and educators working together to support the child. As van Oers (2013, p. 271) states:

> The future of mathematical thinking in young children strongly depends on [adults] to recognise mathematical actions in children, to see the mathematical potential of play activities and play objects, and to guide children into the future where they can still participate autonomously and creatively in mathematical communications.

It is the adults' responsibility to be aware of and responsive to supporting a child's mathematical knowledge and skill development. However for this to occur, adults need to have an understanding of mathematical content knowledge, pedagogical knowledge and pedagogical content knowledge (Lee 2012). This means families need to understand the importance of early mathematics experiences such as counting and find learning opportunities within their home that encourage play.

Further research is necessary to explore infant and toddler mathematical learning. This includes learning within early childhood settings and home environments. The focus of further research includes investigating suitable pedagogical approaches, supporting families and educators as well as identifying early mathematic learning in children who are pre-verbal. The research field needs to identify that if mathematical learning start after birth, how can adults identify signs of early mathematical understanding.

Families play an important role in supporting the early mathematical learning of infants and toddlers. They are the first educator of young children and provide numerous experiences to support the young child's learning. This chapter has discussed the importance of understanding the context as well as two pedagogical approaches with strategies. It is hoped that as families play and learn with their children, they can also show children the importance and enjoyment of mathematics. Returning also to the words of Clemson and Clemson (1994, p. 19), "we can influence young children's keenness to learn mathematics by making the tasks we do of interest to them…by showing that we really think math is important and fun". For toddlers and infants, families are vital in early mathematical learning.

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Chapter 4
Enumeration: Counting Difficulties Are Not Always Related to Numbers

Claire Margolinas, Floriane Wozniak and Olivier Rivière

Abstract When parents and educators think about numeracy and how to help children's use of number, they often try to help them to memorise the numbers in order and to ask them to count how many objects are present. This is certainly useful in order to attain the level of numeracy which is important at school, but it is not sufficient. Parents and educators generally pay little attention to the way in which the objects are pointed to in order to give the right number. This chapter will focus on this aspect of numeracy that has been named "enumeration" by researchers in mathematics education (Brousseau in L'enseignement de l'énumération. International Congress on Mathematical Education, 1984. http://guy-brousseau.com/2297/l%E2%80%99enseignement-de-l%E2%80%99enumeration-1984/; Briand in Rech Didact Math 19:41–76, 1999. https://halshs.archives-ouvertes.fr/halshs-00494924).

Keywords Counting · Enumeration · Organisation · Numbers · Theory of didactical situations

What Is and What Is not Enumeration?

In French and in English, "to enumerate something" is "to name things on a list one by one" (Oxford Advanced Learner's Dictionary). It does not thus refer to mathematics or to counting. For instance, on my table I have "pen, my glasses, an eraser" this list is an enumeration, in Brousseau's meaning of this term, if it respects two conditions, which are not explicit in the dictionary's' definition:

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all the objects on my table are present in the list
no object is present in the list more than once.

Another difference between the common meaning of the verb "to enumerate" and the sense we have given to enumeration is that it does not necessarily refer to an oral recitation. For instance, if I point silently to every objects on my table avoiding pointing twice the same object, we will name "enumeration" this silent procedure. Enumeration is involved in counting, as we develop in this chapter, because in order to count a collection of objects, one has to consider every objects once.

Comparing One to One Correspondence and Enumeration

Enumeration is thus a component of counting, it is, in this sense, "pre-numerical" (Briand 1999). The most well-known pre-numerical component of counting: one-to-one correspondence has been described by Piaget (Piaget and Szeminska 1941). One-to-one correspondence is linked to the acquisition of the concept of quantity. Two collections have the same quantity if they can be put in one-to-one correspondence: one egg with one egg holder, for instance, two collections have not the same quantity if this is not possible: for instance if there is no egg for some of the egg holders. It is pre-numerical in the sense that the concept of "number" is base on the construction of quantity (for a detailed description, see Margolinas and Wozniak 2012).

However, even if enumeration and one-to-one correspondence are both pre-numerical knowledge, they are different. In particular, one-to-one correspondence involves necessarily two collections which are matched by the one-to-one procedure between one element of the first collection and one element of the second collection. Enumeration involves only one collection, whose elements are pointed to exactly once.

Why is enumeration important in the context of counting? In order to understand this concept, we remain in the domain of counting and start with a basic example, which is how to count the dots in Fig. 4.1.

In order to count the dots, you have to know the numbers in order up to ten, but it is not sufficient. You have also to be able to coordinate the recitation of the numbers and the pointing of the dots. It is in fact common to observe children who

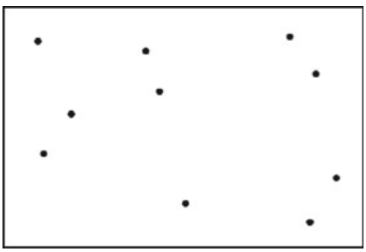


Fig. 4.1 Ten dots which are not so easy to count!

have been trained to rattle off the numbers but who are not pointing the dots at the same pace.

What this Chapter illustrates is that counting the dots requires even more competencies. We refer here to Briand (1999) who explained in detail the steps a child has to undertake in order to count a given collection.

> [the child has]:
> 1- to distinguish two different elements […] (Briand 1999, p. 52, authors' translation)

Why is this important? If you are describing a collection of animals, you can say that there are ten animals, or you can count two cats, three dogs and five birds, or you can name each animal: one ginger cat, one black cat, one terrier, etc. To count a collection requires children to distinguish each element and to consider these elements as parts of a whole. To count the dots, you have to distinguish them, even if they are identical in shape and colour, because they occupy different positions and they are elements of a whole collection of dots.

> [the child has]:
> 2- to choose an element of the collection
> 3- to pronounce a number word (one or the succeeding number word in the list of number words). (Briand 1999, p. 52, authors' translation)

If you start to count with point A (Fig. 4.2), it is possible to find a simple path to count the dots, following a pattern which forms some kind of horizontal parallels, path 1 and path 2 are examples of this procedure.

But if you start to count with point B (Fig. 4.3), it is more difficult to find a path which allows remembering all the dots which have already been counted. For instance you can go up, turn right and count all the dots going around the borders and forget that you have already counted the point of departure. Or you can go around and forget that you have not counted the point in the middle, etc. Of course you can also succeed, but it is more difficult than with the horizontal paths shown in Fig. 4.2.

Starting with any point, you have thus:

> 4- to memorise which elements of the collection have already been chosen
> 5- to conceive the collection of the elements which has not already been chosen

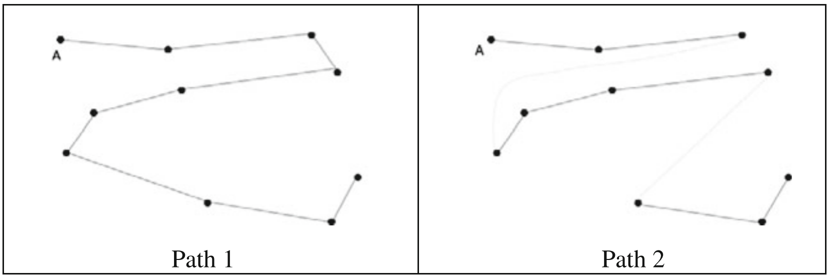


Fig. 4.2 Successful "horizontal" paths

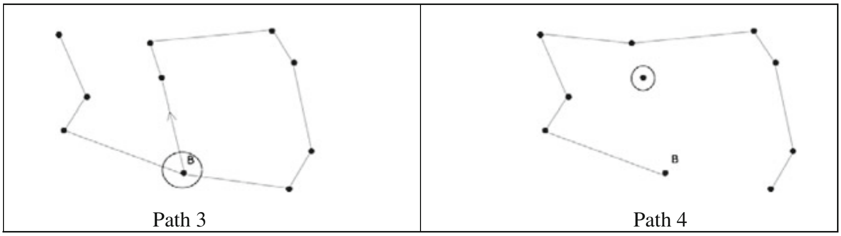


Fig. 4.3 Two ways to fail to count the dots, starting from point B

6- to repeat (for the collection of the elements which has not already been chosen) steps 2-3-4-5 until the collection of elements to be chosen is empty
7- to know that the last element has been chosen
8- to pronounce the last number word." (Briand 1999, pp. 52–53, authors' translation).

If you can draw the path or strike the dots which have been already counted, it may prove easy, but if you cannot do that (which is often the case when you have to count something), you have to find an organised path that you can remember. This is why path 1 and 2 shall be easier to remember, because these paths begin with a straight line from left to right which is similar to the writing. Path 2 is consistent with a "write-like" disposition, whereas path 1 is similar to writing only because it is composed by horizontal lines but different because it is not always oriented from left to right. These paths reflect what Goody (1977) calls the "domestication of the mind" which is the result of the familiarity of writing, even for children who are not yet able to read or write.

In Briand's citation, only lines 3 and 8 refer to numbers which are necessary steps for counting a given set of objects, the other lines do not refer to any number. In fact, if what you want to do is not to count but to point silently exactly one time at every object, you have to follow steps 1, 2, 4, 5, 6, 7. These steps are a way to characterise "enumeration" as knowledge. In order to understand why this is important, consider what happens when you fail to achieve some of those steps.

Step 1: if you do not distinguish elements you might point to the entire collection and give the recitation of the "number song" and stop at some indeterminate number.

Step 2: to choose an element of the collection is not difficult in itself but, as is seen above, it is very important to have an efficient strategy.

Step 4: If you do not remember the elements of the collection you have already chosen, you might count again some object which has already been counted and fail to give the correct final number.

Step 5: If you do not conceive the collection of the elements which have not already been chosen, you might forget to count some object and fail to give the correct final number.

Step 6 and Step 7 emphasise the importance of steps 4 and 5: you have to know that all the elements have been counted when the collection of the elements which has not already been chosen is empty. If you are not aware of this, you will continue to count, not knowing that you have to stop exactly at the last element.

Didactical Engineering and Task Design

Didactical Engineering has been developed in France within the context of Brousseau's Theory of Didactical Situation in Mathematics (Brousseau 1997). Artigue (2009, 2015) describes didactical engineering as similar to the work of the engineer, who is acquainted with the major scientific knowledge and accepts the scientific methods but at the same time is obliged to work with very complex objects, far from the simplified objects which are studied by science. On the other hand, the theoretical framework gains from the results of didactical engineering.

Sometimes it is comforted by the result of the experiments but most of the time, the process involved during the research and experiment of didactical situations lead to important discoveries in the core of theories. The specificity of the French paradigm of research in mathematics education might be in the fact that the design in itself is not viewed as the final goal of the research and that the theoretical developments are most of the time more important than the design in itself (Margolinas and Drijvers 2015).

Enumeration as a Knowledge in Situation

In particular for early knowledge, it is quite impossible to describe the knowledge at stake without referring to real situations. In fact, a formal mathematical definition of enumeration can involve high level mathematics (Briand 1993; Margolinas et al. 2015). This is one of the reasons that we now describe some situations which can be considered as characteristics of enumeration.

These situations have been observed in clinical teaching experiments (Wittman 1995, pp. 367–368):

> […] 'clinical teaching experiments' in which teaching units can be used not only as research tools, but also as objects of study.
> The data collected in these experiments have multiple uses: they tell us something about the teaching/learning processes, individual and social outcomes of learning, children's productive thinking, and children's difficulties. They also help us to evaluate the unit and to revise it in order to make teaching and learning more efficient.

Our research reported here is not strictly based on clinical teaching, since the experiment was conducted in individual interviews. However, to understand the concept of enumeration exactly, we describe what the children produce when they carry out a certain kind of task in a situation where enumeration was required to be used. Thus, in this chapter we give examples of social situations and games designed to enhance children's abilities to enumerate. This illustrates that it is possible to educate children in order to enhance their abilities to enumerate. In the first part of the following text, we describe some situations which are similar to "counting the dots", that is situations where it is not possible to move the object you want to enumerate. In the second part, we consider the enumeration of objects you can move and we show that these situations are very different.

Enumeration of Fixed Objects

When a child is counting, she has to pay a lot of attention to the number words, their order, etc. Moreover, a child cannot count a collection without knowing the number word sequence matches up to the number of elements. If we want to improve enumeration, we have to think about designed games and social situations where enumeration is present without counting. The following is an example of such a game.

Description of the Game of Hidden Objects

The first game we describe has been designed and observed by a team of researchers (Margolinas et al. 2015), based on ideas by Briand (1993) and Berthelot and Salin (1992).

The rationale for the game is this: if you have only dots on a sheet of paper and you ask children to point to every dot one at a time, you cannot verify the work of the child and, what is more important, the child herself is unaware of the result. Thus, if you want to design a game for enumerating the dots, you have to find a way for the validation to be apparent to both adult and child. It is possible now to do so with a digital device, because the device can memorise the dots which have been touched or not and give feedback about this information to the child. However, it seems also important that children understand that this kind of procedure is not due to any sophisticated device, but is related to common activities in real life. What we present now is one solution. We first explain the game and then we analyse the reason for the different materials and phases.

In order to play the game of hidden objects, you need a large sheet of paper, some little objects (e.g., counters) and little cups (e.g., cups for baking little cupcakes), the cups should be larger than the objects in order to hide the object totally when covered by the cup.

On the sheet of paper, you draw some dots. The number of dots is important: if you increase the number of dots, the game is more difficult. For children aged 2–4 year-old, ten dots might be sufficient to be a real challenge, for older children, you can draw up to twenty. For some adults, even thirty might be a challenge. We discuss later other variables of the game, apart from the number of dots. You need the same number of objects and cups plus one cup, and an open box near the sheet of paper. You can set the game on a table or on the floor, so that the child can reach easily all the materials. In order to introduce the game, the adult says something like this:

You will play a special game with all the materials. I will explain how it is played. First we have to set the game, can you help me? We have to put one counter on each dot.
When this is done, you explain the goal of the game:
To win the game you have to take all the counters and put them in the box. But this is really too easy! We will hide together the counters with the cups, this way, the game will be really challenging.

The result of the setting is shown in Fig. 4.4. It is important for the child to be associated with the setting of the game: she thus knows exactly what is hidden by the cups.

The adults thus explain the rules of the game, demonstrating these rules using the extra set of counter and cup in order to show how to play. It is very important not to disturb the game which has already been set, because if the adult himself takes one cup, this cup might be considered by the child as the one to be taken first. The adult explains the rules, setting the cup over the counter and he manipulates the cup during the explanation:

When you play, you take the cup; you thus pick up the counter; you put the counter in the box and immediately after you put back the cup on the dot. You will do that with all the counters. When you think you have gathered all the counters, you say: "I have finished". We will then remove together all the cups. If you have put all the counters in the box, you win the game. If you have forgotten to pick some counters, you lose the game. You have to be careful with something else: during the game, if you take a cup and there is no counter, you also loose. Look: if I take this cup, there is no counter there! It has already been taken, it's here, in the box! In order to win the game, you have to be careful not to forget any counter and not to take off a cup more than one time.

In this game there are: dots, counters and cups, we now explain the role of these elements of the game.

The game's cognitive role is to induce children to enumerate without any other interfering difficulties, in particular there is no counting involved here. However, as stated above, enumerating the dots does not immediately produce facts which can be validated. In order to obtain a validation, there are two main possibilities: to

Fig. 4.4 Game of hidden objects

mark the enumerated dots (e.g., with a pencil) or to find a way to determine which dot has been enumerated without any mark. To mark the elements is an interesting choice, which has been explored with older children by Briand (1999).

In the hidden objects game, the function of the dots is to provide a set of fixed objects. Since pointing at the dots does not result in anything tangible, the counters permit the child and the adult to know that a dot/counter has been enumerated. If the counters were always visible, the enumeration would have been too simple and not similar to the procedure required in order to count a collection. The function of the cups is to hide the presence/absence of the counters. To understand the importance of hiding the counters, if we refer to Fig. 4.3 path 3, if there were no cups, the child would have seen that there was already no counter on dot B. The decision not to enumerate this dot again would not derive from the memorization of the path but from the simple absence of the counter. The cups, which are always at the same place (they are only momentarily displaced when the counter is picked up), hide the fact that the counters have been already taken or not.

In this situation, it is possible to try different procedures: we consider, like Tsamir et al. (2010), that pupils aged 5–6 years are capable of solving problems using several methods. We also consider that it is important for a game not to be too easy and in particular to offer the possibility to win… and to fail! To be able to fail is in fact important for learning mathematics. It is the fact that reality does not always match our anticipation which triggers the will to improve our procedure (Brousseau 1997). Of course, as it is the case with any game, adults have to be supportive, but the child has to reach a new ability to overcome the difficulty. Our observations show that children (from 3 to 10 years old): are not all able to win the first time, even the older ones; already adapt their procedure the second time, either winning or finding a better path, without any intervention of the adult.

Results of Experimentation

Table 4.1 shows the results of the experiment with 44 children. The experiment was conducted by a researcher in a room situated near the children's classroom. The game was explained by the researcher to each child. The children had the opportunity to try a second time if they did not win the game the first time. The number of dots was 11 for children aged 3–5, 15 for children 6–8 and 20 for children 7–11, in order to take into account the ability of older children to recall a greater number of facts. The whole process was video recorded.

The number of children involved in the experiment at each level does not permit a valid statistical analysis, thus we examine only the major trends. The first result is the improvement between the first try and the second try (the winners of the 1st game were not allowed to play another time). The second result, consistent with Briand's findings (1993) is the stability of the proportion of winners across the groups.

Table 4.1 Results of an experiment of the hidden objects' game

Age range	Number of pupils	Total of winners after the 1st game (%)	Total of winners after the 2nd game (%)
3–5	17	29	53
6–8	9	11	67
7–11	18	22	56

We now examine some of the procedures employed by the children in this situation. The problem is to remember the spots already dealt with. Since there is no possibility of distinguishing the cups, you have to remember the positions of the cups taken or not. The number of the cups involved render almost impossible the memorization of the spots one by one. For instance, if you choose a spot in the middle and you take a spot left and now right, etc. it will be extremely difficult to win. Figure 4.5 shows two trials of a child who failed the first time after 9 cups and the second time with the 11th cup.

In order to win, children have to organise the path. One of the easier ways to do so is to draw mentally horizontal or vertical paths. Learning to write includes too the ability to recognise and to imagine these paths (Goody 1977). Another way to win is to mentally separate the collection of objects in subsets, as seems to be the case in Fig. 4.6.

Each subset is easier to enumerate than the whole set because you can memorise a rather linear path.

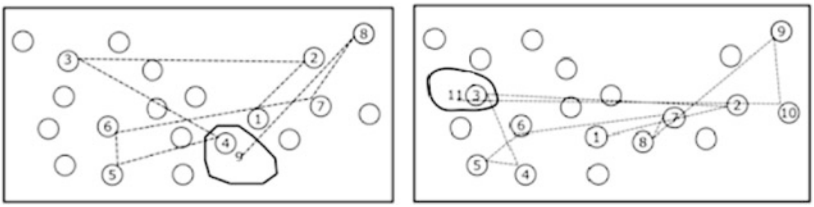
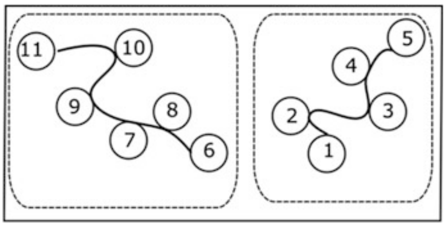


Fig. 4.5 First and second trial of a 9 year-old with 20 hidden objects

Fig. 4.6 First winning game of a 5 year-old with 20 hidden objects



Role of Teachers or Educators

What can thus be the role of an adult who is trying to help the child to build a successful procedure in the game of hidden objects? First of all the adult has to engage the child in the activity to ensure that it is possible to win the game, even if it is a challenging one. We have seen, in fact, that the mere repetition of the game was in itself sufficient to improve their procedure for some children.

Another step might be taken in order to help the children understand that the crucial point in the game is to memorise the path. Questions like, "which path have you taken?" might lead the child to understand that when you pick up the different cups, you might follow a path, and that some paths are better than others.

The number of objects might be adjusted to the possibilities of the child, with her acknowledgment: "do you want to try with fewer objects" or, in the case the child has been successful: "do you want to try with more objects?"

Of course the number of objects is not the only variable: in all our examples, the dots were purposefully set without any obvious order on the page, thus rendering the task quite difficult. For instance, if there were two visible subsets of dots, the procedure shown in Fig. 4.7 would be easier to achieve. Or, if there were some visible horizontal directions for the dots, the procedure shown in Fig. 4.2 would be favoured.

It is important for the adult to observe the child without any initial prompting from his part, in order to better understand which scaffolding might be useful if the child fails to win the game and build a successful procedure. For instance, it is different trying to help a child who has not yet understood that you have to build a path in order to win (Fig. 4.5) than helping a child who has almost successfully built an horizontal path as in Fig. 4.7.

Social and school situations similar to the hidden objects game in terms of enumeration. All the situations where fixed objects have to be all considered one by one require some procedure in order to enumerate all objects.

It is for instance the case in the experimental situation devised by Cornell and Heth (1983), who were investigating "spatial cognition" in the context of "hidden objects". The common features of the situations they studied were: "the unconstrained search for objects in open environments" (p. 94). The second experiment (pp. 99–108) is interesting to reconsider using the concept of enumeration. The experiment involved "32 children in each of the two age-groups [3 year-olds and 5 year-olds]" (p. 101). The children were tested individually in interaction with a

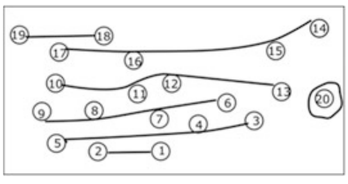


Fig. 4.7 11 year-old horizontal procedure, one cup has been forgotten

tester. Each child was seated on a short rotating chair at the centre of the test area and was shown a puzzle (Fig. 4.8).

The tester started by saying:

> *This is like an Easter egg hunt. You sit here and watch me hide these puzzle pieces. When I have hidden 12 pieces, I will sit down and you get to find them. I'll bet you can find every piece, but you'll have to watch very carefully.*
> *The tester then rotated the chair so that the child faced a predetermined hiding place. There were 24 hiding places equidistant from the chair. (p. 101)*

T is the tester, sitting in the swivel chair, C is the child, standing at start, M is the mother, seated within view outside the testing area. Twelve puzzle pieces have been hidden in different containers, represented by labeled circles. Identical foils are represented as E, envelope. Numbers indicate the order of hiding. The dashed lines depict the path of a 5-year-old boy allowed to gather at will. He committed one error, a repetition at 7 o'clock. (Cornell and Heth 1983, p. 102).

Different conditions were included in the experiment. We focus on the following conditions: in the first, the child had to bring immediately the gathered piece in order to complete the puzzle at the centre of the area. In the second, the child was allowed to collect pieces and complete the puzzle afterwards (see figure above for an example).

The results show that:

Children were surprisingly good at finding all the pieces. […] the analysis of the total number of searches indicated that the older children used less (mean 14.6) than the younger children (mean 16.3). […] At least two kinds of errors could lead to more searches—intrusion (searches in containers where a piece had not been hidden) and repetitions (searches in containers where a piece had been previously found). […] Repetitions were more common […]. Younger children were more likely than older children to search at a place they had already searched (mean repetitions 19 and 10 % respectively). Children who returned to the centre of the test area after finding each piece repeated 18 % of their searches; children who were not so constrained averaged 11 % repetitions. […] Children were more likely to exhibit least-distance searches in the no return condition, but this was primarily evident in the older age group (p. 103).

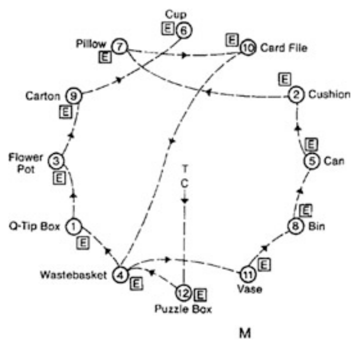


Fig. 4.8 An overhead schematic of the layout of events in the children's laboratory

Using a Monte Carlo simulation, the authors were able to conclude that children use a "spatially organised heuristic, such as searching the container immediately clockwise of the last searched" (pp. 108–109).

This experiment was not designed to observe the spatial strategies of the enumeration puzzle pieces. In fact, repeating the search, which was considered as an "error" by the authors, has no impact on the reconstitution of the puzzle, which was the task given to the children. However, it shows that this kind of task can be analysed in terms of enumeration in a spatial context where no counting is needed. The results also show the possible 'spatial' abilities of children of pre-school age (3–5 year-old).

A great number of social and school situations involve enumeration abilities and procedures, like giving a treat to each seated child, etc. In this evocation, what happens if the children are not seated in a fixed position but are able to move freely or to be ordered to move in a certain way by an adult? This is basically the difference between the distribution of food in a traditional restaurant, where the guests are seated and the distribution of food in a fast-food, where the customers are moving to get their order. This is what we explore in the next section.

Enumeration of Mobile Objects

Let us return to counting. If you count some counters on a table, you take any counter, say one, discard this counter in another part of the table and so on. At the end of the process, all the counters have been displaced from their original place (in Fig. 4.9, from left to right).

This gesture is so natural for adults that they often fail to understand the reason for this procedure. The target of this procedure is to enumerate the collection of counters: consider them all one by one in order to count the right amount of counters. When you can move objects, instead of following a path (which is often an alternate procedure), you can move the object from the place where the objects are initially to a place where you decide to put the objects you have already counted. In order to be successful, the places for the different status (already considered or already counted/not considered or not counted) have to be distinct during the whole process.

Children often learn to count objects imitating adults, which is useful in order to accumulate some ready-made procedures. However, this way of learning does not permit variations in the procedures. For instance, it is easy to observe children who are able to count objects which lay on a table and unable to do so when the same

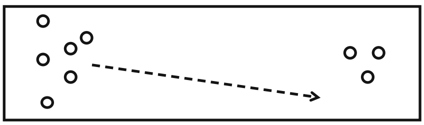


Fig. 4.9 Counting counters on a table

objects are on their legs when seated. Since the same gesture (push to the right) is not available (the objects fall on the ground) they cannot adapt their procedure. It is thus important to devise situations where counting is not at stake but enumeration is the explicit purpose. This is for instance the case in the game of the “marked counters”.

Description of the Game of Marked Counters

For this game you only need some identical counters (15–25 counters) and some identical stickers (the relative sizes have to allow stickers to be placed on the counters). You put a sticker on only one face of some counters (for instance 10 counters). You have thus a collection of counters, some of them have a sticker and some have not. You also need a little box which contains all the counters you will use for the game.

In order to play (with children from 3 year-old), you first give the box to the child, tell her “to open the box and slowly overturn the box on the table”. At this point it is important that the adult does not touch anything: the child has to make all the decisions to move everything (part of the box/counters). You thus ask the child to observe the counters, and tell what she observes, encouraging her to touch the counters. The child may tell you that some of the counters have a sticker and some have not. If it is not the case, you can prompt the child to take one of each sort of counters in hand and observe them. You thus ask the child to sort the counters, if the child does not understand ‘to sort’, you tell her to separate the counters with stickers and the counters without stickers.

Analysis of an Observation

In order to understand the importance of this kind of situation, we show the procedure of 4 year-old Pauline which is characteristic of the evolution of procedures in the enumeration of this kind of collection (Fig. 4.10).

At the beginning Pauline sees that there are some visible stickers, she thus decides to take them. At this point she could have chosen to use the nearby box to place all the stickered counters (some children this age use this procedure) but it was not her choice. We can see that she has started to accumulate some stickered counters near her right hand (Fig. 4.11).

Pauline has now progressed in her procedure and delimited a special space for the stickered counters, pushing them closer to her. She is still taking out the obvious stickered counters (Fig. 4.12).

During her work, she observed that some stickers were not immediately visible (remember that only one face has a sticker). At first, she simply rejected these plain stickered counters in the heap of non-treated counters. But after discovering more

Fig. 4.10 The beginning of Pauline's work

Fig. 4.11 Some stickered counters are separated

Fig. 4.12 Apparition of a place for plain counters

plain counters (in order to be sure you have to examine both faces) she began to create a new place, visible near her left hand: the place for the plain counters.

However, you can see that this may not be completely conceptualised for different reasons. The first, is that she is still not using the different parts of the box (which are both usable to store the counters). The second, is that she has not rigorously separated the three spaces: one space for the non-treated counters, one space for the treated counters with two places: one for the stickered counters, one for the plain counters (Fig. 4.13).

It is not thus surprising to find that at the end she has mixed up some plain counters with non-treated counters which do not obviously appear to be stickered.

She thus failed to sort the whole collection of counters, but during the game she encountered some useful procedures. In this experiment, the researcher did not intervene or say anything to Pauline, but if it had been in another context (for instance an educative context) some intervention of an adult would have been useful.

Fig. 4.13 Pauline has not yet finished her work, some counters have been mixed up

Role of Teachers and Educators

The crucial point here is for the child to understand that different spaces are needed: a space for the initial stock, a space for the stickered counters, and a space for the plain stickers that have been verified on both faces. Questions like: "what are you putting in there?" might help the child to realise that she had an implicit intention underneath her action. If needed, the teacher or educator may insist: "where are you putting the stickered counters?"

However, the most important role of the educator is certainly to help the child to build bridges between different activities having the same characteristics in terms of enumeration. For instance, "where do you put the counters you have already counted? Where do you put the paper you have already written upon?" etc. These phrases can be associated with more standard phrases like "where do you put the objects you have already dealt with?" The different activities can be linked by the adult: "remember when you mixed up the stickered counters with the plain, it is the same in this activity, you have to carefully determine the spaces".

The importance of the observation of children strategies is also high here. For instance, if you ask the child to put the stickered counters in the box, she might not understand the meaning of this action, which may be only a material action and not a procedure. On the contrary, if the child has observed other children using the box, it is interesting for the adult to see if and how she understands this procedure. Some children may put all the counters in the box, misunderstanding the procedure. Prompting from the part of an adult is interesting only if the child can understand the reasons for the suggestion. Involving children in meaningful situations gives the opportunity for the adult to give some suggestions, at the right moment when they are really useful in situation.

Social and School Situations Similar to the Marked Counters Game

Many situations are similar to the marked counters game and we do not try to encompass all of them. However, an historical and social remark might be of interest here (Margolinas et al. 2006). The games and activities which are proposed to children are different from one family to the other, and not always dependent on social conditions.

For instance, some families like to play social games: cards, board games, etc. The children involved in these social games may have many occasions to count in different situations. They may have developed also enumeration strategies in these situations. However, when they are young, children are rarely able to count collections that have more than a dozen elements, and thus to enumerate these collections might not require any sophisticated procedures.

On the other hand, in the beginning of the twentieth century, children were generally involved in domestic tasks. For instance in France, they sometimes had to sort pebbles from lentils, which was a painstaking task which required them to be organised, because the number of lentils was great. Nowadays, some children may have regularly to sort a huge collection of building blocks in order to find the exact colour and shape they want to finish their construction. But you may find educated children who have never had opportunity to sort collections of objects, for instance because they like books so much that they never play with blocks!

This is why it is important for educators and teachers to be aware that enumeration is not a spontaneous behaviour. The procedures will develop only if the children have the occasion to encounter the right situations and the appropriate prompting from educators.

Conclusion

This chapter has been dedicated to the development of some competencies which are necessary during counting but are independent of counting. These competencies should be considered as aspects of numeracy, but are often considered only in relation to a number's construction. Our experiments show that children do not develop these competencies in usual social situations: there is a need for specifically designed situations.

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Chapter 5
Discerning and Supporting the Development of Mathematical Fundamentals in Early Years

Camilla Björklund and Niklas Pramling

Abstract A large body of research shows that young children have abilities to discern small amounts and changes in quantities, and reason about mathematical relationships they encounter in everyday situations. How these early abilities are allowed to develop is contingent on the child's network of social interaction and how mathematical notions and principles are introduced and made sense of in mutual activities. Key insights from educational theories contribute with a basis for how to provide ample opportunities and support for the child to discern important principles (relationships and distinctions) of a mathematical nature, particularly how to communicate with children in a developmental way. In this chapter, we analyse a number of everyday activities with a child in his home environment during his first 6 years of life. These observations allow us to illustrate how mundane activities can provide the basis for gaining access to, and supporting the further development of a child's mathematical abilities in interaction with adults and peer.

Keywords Conceptual development · Communication · Socio-cultural theory · Variation theory · Discerning

Introduction

In this chapter, we analyse a number of everyday activities with a child in his home environment. The observations span a period from when he was 1.9–6.2 years. These observations allow us to illustrate how mundane activities can provide the basis for gaining access to, and supporting the further development of a child's mathematical abilities.

The background of our discussion is cognitivist and developmental research, claiming that children are born with the ability to discern small amounts and

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changes in quantities (Lipton and Spelke 2003; McCrink and Wynn 2004; Starkey et al. 1990; Wynn 1998; Wynn and Chiang 1998). There is a large body of research on infants and children below the age of one on these matters. It is argued that these abilities—which may be referred to as intuitive—constitute the origin of arithmetic abilities. But how these abilities develop are contingent on the challenges and support provided and the expectations held in the environment and the culture (Aunio et al. 2008). Since very young children do not express their abilities or understanding in arithmetic terms, it can be difficult for adults to discover children's intuitive abilities. However, observations of young children's self-initiated activities show that they discern mathematical relationships and explore mathematical concepts in every-day situations, routines and in communication with peers and adults, making coherence and sense on the basis of their earlier experiences (Björklund 2007, 2010).

There are key mathematical principles that children need to discern in order to develop mathematical abilities. These include: the one-to-one principle, abstraction, the cardinal principle, parts-and-whole relationships, and base-ten operations. The development of conceptual understanding in mathematics is broader than merely procedural skills such as counting or reproducing number facts. To become a competent user of numbers and a skilled arithmetic problem solver, the child needs to master the conceptual bases. This development starts in the early years of childhood, as will be illustrated and discussed in this chapter. All empirical examples we present, analyse and discuss in this chapter are based on transcribed activities of the interaction and communication of a focus child and his family members, primarily his mother.

In addition to reporting and analysing these examples, we will present and discuss some key insights from educational theory. These principles will be of two kinds: those that concern how to provide ample opportunities for the child to discern important principles (relationships and distinctions) of a mathematical nature; and those that primarily concern how to communicate with children in order to provide developmental opportunities and support. These principles are based on Variation Theory of Learning (e.g., Marton and Tsui 2004) and Sociocultural Theory (e.g., Daniels et al. 2007), respectively.

A Theoretical Framework for Understanding Learning

Every study, as well as every form of understanding and support of children's learning, presumes some idea of how learning comes about, what triggers development in thinking, and how learning and development may be facilitated. There are many theories on learning, but we will discuss mathematical learning and development in accordance with the Variation Theory of Learning, since this theory offers concepts that are functional in studying the process of learning and conjectures how learning and concept development are informed by providing awareness-raising patterns. Informed by this theory, the learning of mathematics is a matter of discernment of increasingly more aspects of mathematical phenomena, where earlier experiences play a central role for what is perceived in a particular situation. Learning then follows some kind of trajectory, but in a broad sense, where certain aspects are considered critical for developing understanding. This theory has been proven to be powerful as a theoretical framework in empirical studies of learning and concept development among both older and younger children (Björklund 2010; Björklund and Pramling 2014; Ljung-Djärf et al. 2013; Magnusson and Pramling 2011; Reis 2011). These studies have contributed to the development of an understanding of the complexity of concept development and also to a framework for how to support this development through pedagogical activities.

Variation Theory (Marton 2015) conjectures that learning means to see or experience some phenomenon in the world in ways that a person has not previously been able to. Depending on the person's earlier experiences of similar phenomena, and the aspects of the phenomenon that are provided in a particular situation, a certain way of understanding the phenomenon takes shape. Some aspects are considered critical to discern, and are foregrounded in studies of how a person develops his or her understanding. Variation Theory is particularly powerful when studying the development of mathematical concepts, such as number concepts or arithmetical principles, since there are many dimensions to the understanding of a mathematical concept and aspects of the same that are necessary to discern. Take for example the notion "five". In order to understand and use this concept, it is necessary to be aware of the numerical dimension, in that it is answering the question "How many?", an ordinal dimension, meaning that five has a value in relation to other numbers (always before six and after four in the counting rhyme), but also a non-numerical dimension in that numbers are used to label unique phenomena without any numerical values (e.g. phone numbers and registration plates on cars).

These dimensions, and possible others, contain several aspects that are necessary to discern and account for, aspects that the learner may have encountered earlier in other situations or are provided in a particular situation. In order to learn the numerical dimension of numbers, numerical relations have to be discovered, these relations include those that are possible to quantify, and may be grouped and counted. Another numerical aspect has to do with quantities that are possible to compare and subsequently make a series of their numerical values of (a group of two compared with a group of three objects, followed by a group of four, and so on). Variation Theory conjectures that such aspects are necessary but can only be discerned in contrast. The number five for example, has no numerical meaning to learners if they hear the word "five" when seeing a hand, a foot, and cars on the parking lot, apples in a bowl, and children in the playground. However, when five apples are contrasted to a group of four apples, the numerical idea may be possible to (first and foremost) become aware of, and further to explore and develop an understanding of. Only thereafter, according to Variation Theory of Learning (Marton 2015), is it possible to generalise the meaning of five and discern the "fiveness" that can be found when observing a foot, a hand, a group of cars and so on. Studying learning and development in terms of Variation Theory means to focus on those aspects that are present in a situation and those aspects that the learner has not yet discerned; and, further, how the latter aspects may be put into play in interaction and communication.

According to Variation Theory, every concept is possible to differentiate as aspects. Teaching, then, means offering the learner opportunities to discern such aspects that are important but not yet discerned by him or her. Learning is then made possible through a carefully orchestrated act of differentiating the aspects and contrast for discernment. The complexity of conceptual understanding becomes apparent in communication with young children. Most teachers and adults can recall episodes where the child and the adult give very different answers to the same question, such as when counting out loud and pointing at items together with a child, and the child suddenly protests, saying “This is not four, THIS is four”, pointing at another item. The child has then not yet discerned the idea that it does not matter which item you start to count, the total quantity is independent of the order the items are counted. Variation Theory highlights the necessity of paying attention to which aspects that are at the centre of attention to the child, and which are not, as these not yet discerned aspects are those that need to be differentiated and generalised by the child.

Mathematical concepts are complex in their nature, not least because mathematics cannot be described as a physical object to be found. Rather, it is a collection of knowledge, tools, and principles to investigate, understand, and handle relationships in time and space. This leads to a challenge, as mathematics cannot be “seen” physically. Mathematical notions describe relationships between visible objects, but also open opportunities to be explored by very young children who encounter these mathematical relationships in their block play, dividing and sharing fruit, running and climbing in the playground—and actually most activities a child engages in offer experiences that may be described as mathematical. However, to develop mathematical skills and knowledge presupposes paying attention to those experienced mathematical relationships. Teaching in an early childhood education context means to make the invisible visible to the child (Pramling and Pramling Samuelsson 2011). In line with this reasoning, to teach mathematics in early childhood education means directing the learner’s attention to experienced mathematical relationships and supporting further exploration of that relationship. A six-year old may have experiences of patterns as the idea of ordering objects regularly by colour. When encountering a peer’s pattern making in the form of ordering objects by size, his perception of the idea of pattern is expanded and the aspect of regularity is possible to explore if attention is directed to the common relationship of the two different patterns. The latter is crucial in concept development, as it makes possible the discernment of abstract relationships and idea of the broader concept, rather than just adding another example (Björklund and Pramling 2014). Variation Theory here contributes to our understanding of the processes and puts a focus on what is necessary to discern in order to develop a deeper and more complex way of understanding common notions and principles.

Examples of Opportunities for Learning

We are, in the following, continuing the discussion on mathematical learning and development through some examples of a young child's mathematical encounters and exploration in communicative situations. These examples are then put in a broader context of research on how we understand mathematical reasoning today, informed by an extensive body of research on mathematical development.

The child we are following is Vidar, a boy living with his older sister, mother and father in the outskirts of a larger city in Sweden. All examples are authentic observations that were documented by his mother. The observations we analyse were documented during a period from when Vidar was 1.9–6.2 years.

Counting Without Numbers

When studying young children's mathematical development, counting and using numerals is a fairly late achievement. Most two-year-olds know some counting rhyme, "one, two, buckle my shoe; three, four, open the door…", but they are rhymes like any other rhyme, devoid of any numerical meaning (Fuson 1992). Research on cognition and developmental psychology claim that children are born with abilities to discern small quantities and changes in quantities, such as subitising and arithmetical expectations, abilities that are present long before any number words are uttered by the child (Mandler and Shebo 1982; McCrink and Wynn 2004; Wynn 1998). These abilities are innate and intuitive. However, the direction in which these abilities are developing depends on both the physical and social environment; in other words, what the child needs to learn to survive and participate in his or her community and what is expected and considered valuable knowledge (Aunio et al. 2008).

Intuitive attention to quantities is considered by many researchers to be the basis for learning to use arithmetical principles and strategies. Still, these abilities are not easy to recognise in the young child's exploration of the quantifiable world, because their reasoning is often expressed other than verbally. Take, for example, the idea of counting. Counting is a very practical strategy or tool for determining the number of objects in a group. However, there are many aspects of the counting skill that are explored in early childhood in situations that will not necessarily involve number words. Gelman and Gallistel (1978) conceptualise counting as a skill that is constituted by five principles, necessary to grasp if the child is to use counting for problem solving and communication about quantities. Two of these principles do not involve number words, but are equally important: one-to-one correspondence and the principle of abstraction. The first means to make pairs of units from different groups, combining units into new "pair-units". The principle of abstraction is critical for acknowledging phenomena as units that may be part of a larger whole, independent of their nature or features. Consequently, the child may explore these two principles when grouping cats and dogs, then giving each peer one toy. Another example is to match one hand clap to the beat of a drum or symbol on a schema. No number words are needed, but the principle of abstraction and one-to-one correspondence are crucial for learning how number words are representing quantities.

Episode 1

Vidar (1 year, 9 months) sits on the sofa with a picture book in his lap. The book has tactile surfaces. On page one there is a train and underneath the caption reads "1 One train". The second page has two chickens in the picture with the text "2 Two chicken". The following pages, up to the tenth page, present different objects in a similar way. Vidar browses through the book. The fourth page has four flowers in different colours with different numbers of petals. He points with his index finger on the flowers, one by one, saying "Mommy, Daddy, Nea, Dida". [He calls his older sister Nea and himself Dida at this point in time.]

This example is interesting from the point of view that Vidar is reasoning about relationships and uses the principle of one-to-one correspondence in his "reading" of the book. We cannot tell why he connects each flower with the names of family members; it could be an idea of giving each member one flower or naming the flowers with labels of family members that he is familiar with. Nevertheless, he expresses an intuitive awareness of principles that are important for counting procedures (one-to-one, compare with one number word uttered for every counted object).

Three months later, Vidar uses his powerful principle in another situation, but encounters a challenge.

Episode 2

Vidar (2 years, 0 months) sits on the bathroom floor. He is to put on his sock but starts to play with his toes. He grabs his big toe and says "Daddy", then the next toe, saying "Mommy", then "Nea" and "Dida" for the following toes. Vidar looks at his foot quietly for a while. Then he grabs his little toe and says "Kitty" with a big smile.

Vidar uses his strategy of pairing one item with one label (or one-to-one correspondence between different groups of objects). However, the strategy does not end up with the same result as in the earlier episode. The number of toes does not match the number of family members. Vidar discerns the difference in number only when the conflict makes him aware of the numerical dimension, which triggers him to direct attention to the experienced difference and how to deal with the problem. Wynn (1998) has shown that toddlers, even younger than Vidar, express so called arithmetical expectations, meaning that a quantity that is added items is expected to change in number. This is probably one of the critical abilities for developing skills to elaborate deliberately with quantities. As seen in the example with Vidar, he experiences the contrast between the expected pairing activity and the outcome of the pairing, which directs his attention to the numerical relationship between the two groups of units and challenges him to “see” the groups in another way. He then adds a unit, “Kitty”, and is seemingly satisfied when the pairing act evens up.

Numbers as Collections of Units

There is a strong belief among most researchers on mathematical learning that the part-whole relationship is fundamental for developing conceptual knowledge of numbers (Doverborg and Pramling Samuelsson 2000; Dowker 2005; Hunting 2003; Piaget 1952). Such knowledge is further considered important for the use of procedural skills and retrieving number facts for powerful arithmetical strategies. Conceptual knowledge of numbers arises from experience with numbers as a collection of differentiated units. The number five, for example, will be perceived as a whole with parts that may be combined in different ways, still constituting the whole of five. Five may further be seen as a unit of a larger whole. The ability to perceive the parts and the whole simultaneously is, according to Variation Theory, a key condition for learning, since the ability to fluently elaborate with numbers as collections of units, according to Marton and Neuman (1996), provides the child with powerful counting strategies, which children with mathematical difficulties often lack. However, the idea of numbers as collections of units may be expressed in different ways and younger children experience the part-whole relationship even though they may not use number words to express the discerned relationship. In the following example, Vidar encounters dots as units and is challenged to explore the part-whole relationship.

Episode 3

Vidar (3 years, 2 months) has one dice in his hand. He opens a drawer and finds two more. He says “Look, the same”. His mother asks: “How many do you have?” Vidar points with his right index finger on each dice, saying “One, two, three; I have THREE”. He throws the dice on the floor and his mother asks: “How many dots are there on the dice?” Vidar points at all dots on top of all three dices and says “One dot, one dot, one dot, one dot, one dot, one dot. That many.”

This example is classic in the way Vidar, as a 3-year old, deals with number and quantities. He perceives objects of different kinds as countable items, that is, that both dice and dots may be enumerated. Furthermore, he knows what is expected when someone asks “How many?” and gives an answer that would be interpreted as an expression of the cardinality principle (Gelman and Gallistel 1978) when he accentuate the notion three, including all the previously counted items into a whole. The following question, to count all the dots on the dice, challenges his part-whole perception, since the dots are situated on different objects but are still perceived as units of dots. In other words, he differentiates the dots as countable items from the physical objects. His verbal utterance further reveals that number words above three are not his primary choice for enumerating larger quantities. Piaget (1952) would suggest that this is an expression of early arithmetic thinking, when numerating as one, one more, one more and another one. The challenge is to perceive all those "ones" as parts of one collection, simultaneously. Number words may contribute to this change of focus, from "one dots" to "six-including-all-of-the-dots" in the same way Vidar is expressing his understanding of the notion "three". This example reveals that mathematical principles are complex and not easily generalised and transferred to larger quantities. In accordance with Variation Theory, several aspects have to be discerned by the child to develop numerical understanding and the skill of using numbers in problem solving and communication. Those aspects that the child in particular need to discern (but has not yet discovered) are called critical, meaning that they should be in focus for teaching acts and challenged in that the child may broaden his or her understanding. In the episode above, cardinality is one aspect that is central and that Vidar shows knowledge about, but the part-whole aspect becomes the critical one.

Learning to count and calculate are closely related to how the child represents quantities and is able to mentally model different possibilities to solve a problem, without losing important information and relationships between the parts and the whole. In the next episode, Vidar explores number concepts and how numbers relate to each other. The collection of units becomes critical to remain constant in the task he encounters.

Episode 4

Vidar (5 years, 4 months), Linnea (8 years, 8 months) and their mother play a dice game where one is to collect fish. Vidar gets two on the dice, moves two steps and draws an event card. The card states that the player loses a fish on the last spot he was standing (from where the last move was made). Together with his mother he counts backwards: "Two, one, that's where it stood" and places a fish on the board. The game proceeds and when it is his turn again, Vidar gets a three on the dice. He says: "I want that fish" and his mother answers: "If you get to that point again, you can take the fish". Vidar takes two steps, stops at the spot where the fish lies, looks at it and asks: "Can I take it?" Linnea then explains that he must stop on the final counted number. Vidar suddenly moves his piece to a spot three steps from the fish and says: "I wanna start here instead!"

In this episode we can follow Vidar's reasoning in a task that is quite complex in character. He simultaneously keeps a specific position as a target, while he explores possibilities to solve the problem of getting the desirable fish by taking three steps (that is different from two steps). The number of steps has to be held constant in relation to the target spot. This might seem like an easy task to adult reasoning, but the task involves calculation with unknown components, which is quite demanding for a 5-year-old and demands mental representations of numbers as differentiated units. The task involves many aspects that need to be focused on simultaneously, such as the number line and the part-whole relationship of numbers. The discerning act is, however, supported by the context of the board game, which provides props and structure that are meaningful and supportive for the specific task.

Large Numbers

The need to handle large quantities has provided humanity with systems and structures that help coordinating and communicating these quantities. There are different systems in different cultures, but the English and Swedish systems are similar in that they use ten as basis for structuring numbers. In order to handle large quantities, the base-ten system is valuable as a mental structure, meaning that the child does not have to learn individual labels for every number there is. Many preschool children learn the counting rhyme up to twenty or even one hundred. The transition from one ten to the next is challenging, often leading to individual number lines such as "twenty-eight, twenty-nine, twenty-ten …" Once children discern the structure of the counting rhyme, "twenty-nine, thirty, thirty-one …," the structure of numbers and how numbers relate to each other become powerful tools for handling large numbers and quantities, as we can see in the following example.

Episode 5

Mother	It is grandpa's birthday today
Vidar (5 years, 5 months)	Why?
Mother	He was born on this day of the year
Vidar	How many years will he be?
Mother	64
Vidar (after a pause)	Then he's 63. And will become 64

Like many other children Vidar is well aware of the fact that birthdays mean you become one year older. The same is probably true for grandfathers as well. To reason in the way Vidar does, he has to discern numbers as related to other numbers and even though the numbers are large, adding one is possible to figure out as units on a number line.

Young children strive to make sense of phenomena they encounter in their daily activities. Numbers are no exceptions. But there are many aspects that need to be discerned in order to handle the structure and system of large numbers. The base-ten structure is known to be important, and the language may also provide clues to the children. However, the number concepts used by children may be based on a different logic than the conventional meaning expected by adults. The next example shows what Vygotsky (1987, 1998) refers to as pseudo concepts, where children and an adult may use the same words and procedures, but their understanding of these terms are different. A closer analysis of the language used in the counting rhyme reveals the child's logic and what aspect that is focal to him, guiding his reasoning.

Episode 6

Vidar (5 years, 7 months)	Thirty plus thirty is sixty
Mother	Yes, it is
Vidar	And thirty-one plus thirty-one is sixty-two
Mother	That's correct. How did you figure that out?
Vidar	First, one plus one is two and then thirty plus thirty is sixty
Mother	But what is thirty plus thirty-one?
Vidar	That's easy, sixty-one
Mother	Can I ask you one more? What is twenty plus twenty?
Vidar	THAT I don't know

Vidar knows how to add numbers up to ten. He sometimes uses his fingers but is quite fluent in retrieving answers like number facts. He applies this strategy to large numbers as well, and seems to have discerned that ones and tens have to be separated in the act of addition. The Swedish language provides support for this as well, since thirty ("trettio", in Swedish) is a clear combination of three ("tre") and ten ("tio"). The same structure is found in sixty ("sextio", in Swedish), a combination of six ("sex") and ten ("tio"). However, twenty ("tjugo", in Swedish) does not reveal a similar relation to two ("två"). An aspect that becomes critical in this episode is the numerical meaning of large numbers. In other words that thirty as well as twenty refer to quantities. It is interesting though that he has revealed an aspect that refers to the linguistic resemblance and ten-base structure of the number line.

In retrospect, Vidar's experiences of mathematical concepts and principles draw a picture of mathematical development that is quite representative for children of his age. We can follow the earliest exploration of part-whole relationships and discoveries of equality and inequality between sets of objects, via the milestone of relying on the perception of quantities versus the idea of counting, to the emerging control of the base-ten structure in conventional calculating and verbal counting on the number line. Conceptual knowledge as well as procedural skills are intertwined in children's mathematical activities, but procedures may be used in more powerful ways when they are based on a conceptual understanding, such as knowing how to make even sets by adding units or that "adding one" works for small numbers as well as large. These knowledge and skills are prerequisite aspects for developing more advanced and abstract mathematical thinking, also shown by Dowker (2005) and by Gray and Tall (1994). This is exposed, for example, when Vidar explains how he adds thirty-one with thirty-one, and uses the same conceptual idea to other arithmetic tasks. However, he fails to adapt the idea to "twenty", which does not have the same linguistic clues; which reveals that conceptual knowledge is complex and builds upon several aspects of the logical reasoning that is necessary for mastering procedures like addition. To become aware of the conceptual foundations in mathematical principles and concepts, these have to be discerned by the child. Such an occurrence does not in general happen in isolation, rather in encounters with other ways of interpreting notions and different ways of drawing conclusions. The theoretical framework Variation Theory helps interpreting what it is that a child focuses on in a particular situation, and what is not yet discerned by the child and thereby crucial for any teaching attempt to account for.

Communication and How to Support a Child's Mathematical Development

For an adult or another more experienced participant to contribute to the child's mathematical development highlights issues of communication. A first principle is to access the child's understanding. That is, without finding out what the child knows and how he or she understands, it becomes very difficult to support further development. Without relating one's support to the child's understanding, the more experienced will risk giving suggestions and asking questions that the child cannot relate to and make sense of, or that are too simple and therefore do not provoke new insight. The episodes represented give ample examples of how the adult gains access to Vidar's understanding and then not only confirms (supports) this understanding but also challenges him to consider more complex variants (see, for example, Episode 6).

It could be argued that education is at heart a communicative endeavour. However, how we communicate is contingent on what we understand communication to be, that is, what our—generally implicit and un-reflected—notion of communication is. A common-sense notion of communication depicts it as the transmission of information from a sender (knower) to a receiver (learner). Traditional schooling practices where a teacher lectures to children listening could be seen as an institutionalisation of this notion of communication (see, for example, Wells and Arauz 2006, for a discussion), and its related notion of knowledge as having information. However, there is a long and large research literature on learning showing that this notion of knowledge and how to promote it through communication is counter-productive (Pramling 1996; Sommer et al. 2010). Instead, other ways of understanding communication and knowledge have been conceptualised and investigated in educational research. In these accounts, communication is instead seen in terms of the etymology, that is, the origin and development of the term, as “making common” (Barnhart 2000). To make something common is fundamentally different from one individual transmitting information to another who receives and stores it. To make common presumes some negotiative work; that is, to try to coordinate perspectives and understanding among participants. Hence, communication becomes a shared project rather than a one-way process (Pramling and Säljö 2015). The more experienced peer—a teacher or an adult—thus needs to be responsive to the response of the child. Such consecutive, unfolding responsiveness can be seen in the episodes in that the adult adjusts her questions to Vidar’s suggestions. Her participation is not imposed from without but sensitive to what the child expresses and what this indicates in terms of mathematical understanding.

With this changed understanding of communication, also how we understand knowledge can be rethought. Instead of merely seeing knowledge as information acquired and stored in the mind of the individual, knowledge can be understood as membership in a cultural form of knowing (Dewey 1916/2008). Mathematics is a prevalent and powerful cultural form of sense-making and communication. Children are empowered through becoming members of this culture. This implies more than knowing of how to count: it also involves the learner’s notion of self, that is, identity. Developing an identity as mathematical is an important part of becoming mathematically skilled, being able to take on mathematical tasks or take on problems in mathematical terms. A parallel case is the learner’s notion of literacy. Studies have shown (e.g., Dahlgren and Olsson 1985) that whether learners who develop an identity as someone for whom being able to read are of interest and relevance, become more skilled at this form of sense-making and communication than children who do not see the relevance of this skill and instead see it as something they have to learn in order to manage school. Arguably, children developing an identity as someone who is mathematical, that is, can see the use and relevance of using mathematics in everyday life, will have a developmental advantage in this respect. The presented episodes show how mathematics comes into play during the course of various everyday activities, such as dressing (Episode 2), playing games (Episode 4), and conversing about family matters (Episode 5).

The understanding of communication as making common highlights the importance of coordinating perspectives; that is, making sure that participants speak about the same thing, not only terminologically but also conceptually. A common feature of face-to-face communication is the use of local terms—what is called “deictic references” (Ivarsson 2003, p. 387). Some examples of such terms are ‘that’, ‘there’ and ‘this’, often accompanied by pointing. These kinds of words are useful in directing and coordinating attention. Language is our most powerful tool for directing someone’s attention (Tomasello 1999). However, while these deictic terms may work in coordinating the adult’s and the child’s attention to the same objects, these objects may be understood by the two in conceptually distinct ways. For example, if sorting objects, adult and child may agree that those are similar to those, or that there is a pattern among the objects. But what these similarities and what this pattern is may be understood differently among the two. For the adult, these may be similar geometrical shapes while for the child the focus may be on the colour or kind of object. If so, adult and child only appear to share perspectives, while they in effect talk past each other. In terms of Variation Theory, they have different aspects of the phenomenon in the foreground of their attention. This makes it difficult for the adult to provide developmental support, for example, using the principle of variation (see above) to challenge the child further and facilitate his or her discernment.

Conclusion

Mathematical principles and concepts are explored and developed by children long before they encounter formal schooling. This is a fact that Vygotsky (1978, 1987) made clear in the early 1930s. Family and friends are in this respect the child's first mathematics teachers, since it is in the interaction and communication that mathematical notions are made common and the child is enabled to make sense and implement powerful problem solving strategies to daily life challenges.

In discussing the five episodes in terms of communication and teaching, we reason per analogy. In his pioneering theoretical work, Vygotsky (1978), among many other things, wrote about how to promote the development of literacy in children. He argued that this is preferably done during what he referred to as the preschool years, and that "writing must be 'relevant to life'", taught "as a complex cultural activity" (p. 118), not writing "taught as a motor skill" (p. 117). Furthermore, he argued the importance "that writing be *taught* naturally … and that writing should be 'cultivated' rather than 'imposed'" (p. 118, italics in original).

> [T]he best method is one in which children do not learn to read and write but in which both these skills are found in play situations. … Natural methods of teaching reading and writing involve appropriate operations in the child's environment. Reading and writing should become necessary for her in her play (Vygotsky 1978, p. 118).

If applying this line of reasoning to our present concern, that is how to promote children's mathematical skills, we can argue that the episodes we have represented and analysed constitute precisely such conditions. For example, in Episode 4, the focus child, Vidar, his older sister and his mother all engaged in a mutual activity: playing a game. Mathematics emerges as a relevant tool embedded in this play activity. Being able to count is vital to being able to participate in this activity and thus provides an incentive for engaging in such matters. Actualising mathematical distinctions and relations in such a context is very different from traditional schooled instruction. Motivation is inherent in such activities (Lave and Wenger 1991). In the context of play, mathematics is, in Vygotsky's terms, cultivated rather than imposed.

As we have already mentioned, one of many important actions people carry out through speaking is to direct someone's attention (Tomasello 1999). Often participants in a practice do so through employing deictic references. An example of this can be seen in Episode 3, when Vidar exclaims, “Look, the same”. In this way he makes the interlocutor attend to what he himself has noticed. He does so through an expression that functions as a pointing gesture. What is further important in this case is how the interlocutor responds to this verbal gesture. His mother asks him “How many do you have?” In this way, she not only implicitly acknowledges that she notices what he is focused on, she also formulates her response in mathematical terms. In this way, Vidar and his mother came to share not only attention but also perspectives on what they attend to. (Consider, in contrast, if the mother had replied, for example, “That’s nice”, “They’re beautiful”, or “Yes, green ones”.)

In terms of sociocultural theory, what mother and child here do constitute what they speak about in certain terms; they semiotically mediate (Wertsch 2007) what they speak about. Establishing not only joint attention but also mediating what is spoken about in compatible terms have been shown to be pivotal for participants (e.g., mother and child, or child and preschool teacher) to engage in a mutual activity, rather than parallel ones (see Pramling and Pramling Samuelsson 2010, for examples with very young children). Being participants in the same activity provides a frame for the more experienced, such as the parent, to contribute to furthering the child’s understanding through supporting and challenging him or her. This goes hand-in-hand with the Variation Theory framework for understanding learning, as different perspectives and new ways of seeing the world constitute necessary contrast for developing understanding (see Marton 2015). As we have shown in this chapter, common everyday practices such as dressing, conversing about family members or playing games provide entry points into supporting a child’s mathematical development.

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Chapter 6
Number Stories

Judith A. Mousley

Abstract True personal stories are used to introduce some of the research into pre-school children's development of number knowledge and skills. A range of conversations and stimulating environments illustrate how parents, grandparents, peers, and early childhood professionals support the mastery of new number words and concepts as well as mathematical actions, in everyday contexts and play situations. The stories discuss the learning of real children developing knowledge and skills in the pre-school years. They tell about early quantity identification along with some young children's growth of interest in and skills with cardinal and ordinal number and counting; learning about more and less, then very simple addition and subtraction; early recognition and naming of multiplication "arrays"; written numeral identification; and one child's earliest abstract understanding of the idea of infinity. For each of these topics, some research on pre-school learning is outlined. The growth of children's self-concepts as they handle mathematics and the situatedness of learning in varied and everyday, informal learning contexts are supplementary themes of this chapter.

Keywords Pre-school · Number · Counting · Number operations · Abstraction

Introduction

Everyone loves stories. They are the reason most people turn on television sets, download videos, buy e-readers, and love to gossip. Stories shape our realities, help us to understand life, and give us things to think about. Stories persuade, and they move people to action (Aaker 2013).

Stories are one way to reach all parents: to get them to stop, listen, and think about what they could be doing to help their children's mathematical development. I do not mean the sort of number activities that young children do at school, but the

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everyday, informal and incidental number experiences that very young children and parents have in shops, playgrounds, travelling together or eating as a family, playing at home in a bath or an outdoor playhouse—and in a myriad of other places that are not normally considered places of instruction and/or learning. Parent, grandparents and carers need to be aware of their roles in identifying moments for developing children's mathematical understandings.

Here are some stories about early years number and number operation development. They illustrate some points in time and place where pre-school children are coming to understand a range of number concepts including counting and some number operations. Each story is followed by some relevant research undertaken by others and published in academic journals and books, as I aim to present some research findings that will offer insight and elaborate on the potential of each story.

The stories themselves are not imaginary: they are grounded in my experience as a family member and in my research from across many years. Essentially, they are stories from my life: my experience as a researcher as well as my roles as a parent and as a grandparent. They are stories of very young children growing up in a mathematical world, from their earliest months to when they reach school age—the greatest years for learning and development. They are mainly stories of young children learning through play and everyday family experience: stories of children learning mathematically with parents and other family members. Please feel free to use these stories in childcare centre newsletters, in discussions with parents, or in thinking about the development of young children in your care.

Peter and "Two"

I remember well when a few weeks before his second birthday, when my young son Peter showed that he understood "two". Playing with plastic cups and sand, he noticed that two of the cups were red. He sat them side-by-side on the sand, saying "Two!" "Oh, good boy" was my response "Two. Two red cups. Two cups that are red". I was pointing with two hands. "One, two. Fill up the two red cups".

Later that morning, Peter toddled around, pointing with two hands to two shoes, pairs of handles on cupboards, pillows, his Dad's hands, my eyes, and lots of other pairs of things in our house. "Two! Two! Two!" he called out many times, as if surprised and delighted by each individual discovery.

Over the next week or so, I set up many "three" situations: three frozen peas (his favourite vegetable at the time), three blocks, three dried apricots, three Lego people and so on; but without success. But he showed no interest in threes.

The next group size Peter recognised and named was actually four, and again the experience was a delight for him and for us. I was fascinated with his immediate naming of four as "two two" and the fact that his interest in doubles became a long-term one. (At about seven I would hear him drifting off to sleep chanting "Two, four, eight, …, two hundred and fifty six, …".)

As a parent, was I doing what I was meant to be doing at the time he recognised twoness? The positive feedback I was giving him was intuitive, with my repeating his words and reinforcing his correct ideas with pointing and praise being what all parents do with their children. As a teacher at the time, I knew that the frequency of exposure to a new idea helps children's development, and that it is important to present some counter examples ("not two"). However, I was clearly wrong in expecting him to make orderly progress—like the school curriculum does—because he showed no interest in threes. Who would have thought the next group he focused on would be double two?

What else could we have been doing with twos? Months later, I realised I could have pointed out pairs (and later fours) of things in storybooks, to consolidate and strengthen his learning. I had seen how his learning about number had been self-motivated and self-directed, but I also wondered when and how I should get Peter to understand that dissimilar things like an apple and orange, or a toy car and bus, can also illustrate "two-ness". Further, I did not realise at that time that recognising "two-ness" or "four-ness" is quite a different skill from being able to count to two or four. Like all parents, I was wondering at that time, "When do children typically learn these ideas and skills?"

These questions point to the focus of this chapter. My aim is to describe some ways that parents can support the mathematical learning of very young children, right from their earliest months through until they reach school age. What do we know about the mathematical development of young children in respect to number and simple arithmetic concepts? How can children's storybooks and familiar objects be used here, and how can parents draw out the mathematics in everyday events? What roles might some technologies play? How can various types of games be used to promote pre-school children's mathematical development? How can we make the most of young children's passionate interests and hobbies as well as various family routines? What are the big ideas of number content that children can learn, before going to school, that will underpin their mathematical learning and initial achievement at school?

It is not only content (such as counting words and ideas) that is important in the prior-to-school years. In fact, I argue that specific content is not very important compared with the development of processes of thinking. How can young children be helped to develop organisational, problem solving, and reasoning skills as well as creative handling of aspects such as number, pattern, order, and mathematical relations? How can we assist them to develop persistence and resilience with tasks and encourage logical thinking during their everyday activities? Are there ways of encouraging their capacities for explanation, justification, estimation, and reasoning? These were questions that have rarely been the focus of research even though there has been much research on very early number ideas.

Cardinal Number: Some Research

Cardinal number is the “two-ness” of a pair of objects, the “three-ness” of any group of three, the “fifty-ness” of fifty toothpicks, and so on. When we talk about the number in a group, perhaps of five or fifty or one thousand things, we are using the idea of cardinal number. When we count objects “One, two, three, four” then declare there are four things in the group, we use the cardinal principle—that the last number we say names the quantity of the group.

Cardinality is absolutely essential in numerical thinking, the basis of all we do with numbers; so without yet being able to count orally, young Peter had grasped one of the very big ideas in mathematics. Having a passionate interest in young children’s mathematical development at that time (and ever since then), it is not surprising that I remember that first “two” day as if it were yesterday. It seemed that this was his first true idea of number.

It is important to note that this realisation about number was long before Peter started counting things. In fact, what is called subitising, which means seeing objects as groups of a certain number without counting (like adults do reading the dots on dice), has long been recognised as a prerequisite not only to the first counting words and meaningful counting but also to the understanding of all number ideas (see, for example, Douglass 1925; Spelke 2003; Wynn 1990; and especially Kaufman et al. 1949, who coined the term subitizing for the recognition of small group numbers).

Many more recent studies have explored whether babies who are a few months old can distinguish between one and two objects—or later between three and four things (they can, showed Resnick 1992; as well as Rouselle and Noël 2008); whether young children create and use mental models of collections (they do, showed Benson and Baroody 2003; as well as Hannula et al. 2007); whether children from different cultures subitise more readily or more often (some cultures seem to, showed Willis 2000); whether subitising necessarily precedes cardinal and ordinal number understandings (it does, found Hannula et al. 2007); and other questions about very young children’s recognition of a number of objects in a group. Baroody et al. (2006) point out that for the development of any of these concepts, young children at least need to understand object permanence and that different objects are distinct. They also need an ability to compare group sizes, which is commonly well developed by 2.5 years of age (Mix et al. 2002).

None of this research detail is vital to parents or to practitioners in early childcare centres, except that we all need to realise that number ideas do start within the first few months of life, that cardinality underpins all number thinking, that children will not recognise and say “Two” if this has not been modelled for them, and that for children aged under five experience with subitising is just as important as the use of counting. Further, parents, grandparents and carers can all help very young children’s development by using nurturing opportunities to recognise and say “Two shoes”, “Three dogs”, “Four legs”, etc., at appropriate times during everyday activities. Frequency of experience with the mathematical words and ideas of cardinality is critical. So is showing our interest and excitement with smiles and other positive reinforcement as well as encouragement of the use of key words and ideas by young children—especially before they are ready to learn verbal object counting like Budi, below.

Budi's Counting

I was a visitor at Budi's house and he was helping me to set their table for dinner. I asked him how many plates we needed. Three-year-old Budi has been asked this before, so he walked confidently around the table, touching each of the four chairs and saying, "One, do, free, four". His parents beamed: "Budi counts well" they assured me.

I was helping with the cutlery so put out five spoons and ask Budi "How many spoons are here, Budi?" He touched each one, saying, "One, do, free, four", touching but not naming the last one. I spread out three knives, asking, "How many knives are there?" and again he touched each one once, but kept his finger on the last knife for two words: "One, do, free, four". As if to summarise, Budi smiled and touched each group of cutlery, saying, "Four knife, four spoon. Set da table. Dood boy, Budi". He was still beaming, but Budi's parents' smiles had faded somewhat. Later, Budi realised he was a knife short and happily fetched one without being asked. He also put the extra spoon aside without being prompted.

Budi's parents should have remained very proud, because (as I pointed out to them) three-year-old Budi knew a lot about counting. He knew that any sort of objects can be counted, no matter how they are laid out; that the correct sequence of number names is "One, two, three, four" (with evidence of understanding that would make this more than merely a serial recall task); and that the answer to a question of "How many?" is a counting word. He definitely understood one-to-one correspondence, as evidenced by his eventually placing one plate, one spoon, one knife, and one fork in front of each chair—even if he had not yet developed the "one touch, one word" skill yet. He had learnt to mimic most of the right counting actions, including (nearly) touching each object once while saying counting words in sequence.

In fact, during the meal I found out that Budi not only knew the counting words to ten in English but could also sing them in his family's home language, Bahasa Melayu. He was, indeed, "Wise"—the meaning of his name, as his parents proudly told me when I pointed out all that he seemed to understand already. During the meal, I took the opportunity, too, to talk about his sense of geometry, with the sense of "right" and "left" (knifes versus forks on the correct sides on both sides of the table) that he was displaying long before he knew the meaning of the words "right" and "left". Just as important was Budi's persistence and his ability to finish a complex task on his own, as well as his independence.

His mathematical problem solving skills also seemed strong, from his actions when he had more of less cutlery than he needed. He added another knife and put a spoon aside to cope with the need for more then less, not getting frustrated with my putting out the wrong number. So had Budi shown evidence of being mathematically wise, for his age? What does the research show in relation to learning to (a) count and (b) solve problems related to more or less?

Counting: Some Research

With regard to counting, many young children say some number names in order before they turn three (Fuson 1992; Resnick 1992). Further, many researchers (including Clements and Samara 2007; Fuson 1992; Rouselle and Noël 2008) have found that three-year-old children are able to count small numbers of objects functionally with accuracy and understanding. Labinowicz (1985) stresses a counting understanding that children develop gradually: "progressive inclusion" (called "progressive integration" by Steffe et al. 1982). That is, a four includes the three just counted; a five includes the four just counted; and so on. This is "a 'one more than' relation … an elaborate simultaneous relationship between numbers in the sequence" (Labinowicz 1985, p. 60). Such understandings (as well as the "one less than" idea) would be necessary for meaningful, as opposed to rote, counting forwards and backwards.

With "more" and "less", by 21 months, many typical children are able to pick up the same number of balls that they saw dropped into a box (Starkey 1992a)—although not consistently. By about 24 months, many tend to realise that adding objects results in more (Mix et al. 2002). The ideas of less and subtraction are harder to grasp, but follow soon after addition concepts (Hughes 1981). Baroody and Rosu (2006) gave examples of children aged 28–30 months adding successfully, using small numbers; and found that many children aged three succeeded with simple non-verbal addition and subtraction tasks (Baroody et al. 2006). Further, three-year-old children seem to have a good sense of equality ("the same") when they see counters dropped together into two side-by-side containers, and can tell that one container has "more" if an extra counter is dropped into it (Cooper 1984; Ginsburg 1977). However, toddlers' nonverbal addition and subtraction performance appears to drop off dramatically when collections larger than 2–3 are involved (Huttenlocher et al. 1994).

With regard to problem solving, again Budi's ability was not outstanding. Lee (2012) recorded examples of toddlers' problem solving with quantifying sets of objects, using one-to-one correspondence. Three-year-old children examined by Patel and Canobi (2010) generally coped well with simple, object-based addition problems, both using and before the development of counting words.

So it is clear that three-year-old Budi was not very unusual. Budi was demonstrating excellent progress towards all of these understandings and skills, developing a good sense of some number words, which Spelke (2003), claims is a necessary first step in forming numerical concepts. Budi was showing a good knowledge of one-to-one correspondence and simple number operations while doing family tasks like setting the table. Budi also had mastered the abstract idea that number is not linked to specific objects or their arrangement. He remembered where he had started counting, and he recognised that the cardinal number of one set could be more or less than needed to match another set.

Certainly Budi demonstrated a wide range of mathematical skills and knowledge in the not-so-simple task of setting the family table! It will not be long before he is ready to play number games like the next two children we meet.

Spiros and Alissa Play 'Concentration'

"Pair!" claimed four-year-old Spiros, and picked up the two cards.
"No, that's not a pair", claimed his older sister, Alissa. She took the cards and turned them face up. "Look, that's a six, and this is a nine".
"But it same. It's the same".
"No, look at the diamonds. See. One, two, three, four, five six; and this one's one, two, three, four, five, six, seven, eight, nine. That's more—nine. The numbers look the same, but one is upside down".
"Which one? Which one is … Which one is upside down?" asked Spiros.

The term numeral refers to the symbols that represent number names, such as 46 and 29, and the term digit refers to only single numerals (e.g., 2). Recognising digits is a number skill that many children learn prior to school, and indeed it is in the curriculum of many kindergartens because the visual identification of numerals is a gateway to all written number work.

Young children see numerals on letter boxes, buses, television, computer screens, birthday cards, clocks, book pages, calendars, shop signs and labels, car registration plates, phones, and many other everyday objects. They meet many variations in the forms of digits, as well as the rotational symmetry of 6 and 9 that confused little Spyros above.

Then there are the complexities. I remember well a three-year-old grandson—a lover of buses—not understanding that we had to wait for bus number 715 rather than taking the approaching bus 751. "But the numbers are right, Nan!" How does one explain to a child of that age that the order of numerals is important?

Numeral Identification: Some Research

Young children of this age also start to differentiate letters from numerals by recognising different functions, shaped by the activities that take place around different types of print (Tolchinsky 2003).

The work of Durkin (1968) with pre-school children aged 4–5 showed a strong correlation between letter and numeral identification, suggesting that knowledge of one symbol system was positively associated with knowledge of the other, although the results for both may have been affected by other factors such as family nurturing and/or learning expectations and activities in family and pre-school settings.

Gifford (1995) points out that there are many examples of themed play settings for pre-school children's number recognition—such as inside cardboard-box aeroplanes (e.g. seat numbers, pilots' dials with numbers on them), a play post-office (addresses, post codes), and mock grocery shops (price signs, cash register or calculator)—that can be used to expose young children to numerals in context; while Neumann et al. (2013) found a positive correlation between numeral identification in such nurturing contexts and early primary-school numeracy proficiency. As Benson and Baroody (2003) point out, meeting number symbols (including both numerals and words) in play functions as a necessary catalyst for essential understandings of number equivalence and operations.

These days, children also play in such contexts represented on computer and tablet software, like Sandi below.

Sandi and Her Tablet

As a four-year-old, Sandi could already count to twelve meaningfully. She counted confidently, had a good sense of cardinality, and in dice and cards read numbers up to ten confidently. She also read road speed signs, as well as numbers in games when using technology to play her favourite games.

Sandi's family was fairly strict with "screen time": she had 1 hour per day of watching television or playing with any technology that used a screen. (Because of this, Sandi had become quite good at watching the "big hand" hand on the clock sweep 1 h and even half an hour and before the age of four, already she understood that 2 half hours make an hour!)

The screen time limit had caused a few minor arguments between Sandi and her parents, but the rule was all she had known and was applied consistently, so she generally did not fuss. Sandi was playing a game on her Dad's tablet: a game where the aim was to move shapes in order to get three of the same shapes in a row. After the game, names of "previous players", including Sandi, were listed with their scores.

"First! I won", she called to her father.

He looked at the list and corrected her. "No, you are third. Look, your name is after the first one and after the second one. Look. [Pointing to the screen] First, second, third. Yours is the third name. Okay, turn it off now. [Ignored.] Close the cover, Sandi: your time it up".

"But I want to get first. First, not after, after! Third no—first."

Here, Sandi was coming to grips with a different set of counting words and ideas from the children above. Ordinal number is about identifying where one object is placed in relation to others, using a sense of order. Our house is the second one in the street. We eat dinner first, before dessert. The pencils are kept on the third shelf and the cooking implements in the second drawer. We have a corresponding set of words (first, second, etc.) to name ordered places—along with our use of counting numbers to name a place, such as "number three in the line".

Ordinal Number: Some Research

Children aged 2–3 show knowledge of ordinal number in comparing collections that is not related to their counting skills (Gelman and Gallistel 1978; Mix et al. 2002). They are used to parents outlining the order of a day's activities: first we will do the shopping and then next we will go to the park. "First" and "last" are typically understood by the age of four, and the ordinal counting words soon follow. About 29 and 20 % of a large number of Australian children entering school could point to the third and fifth objects in a line, respectively (Clarke et al. 2006).

So learning to count is not just a matter of learning to chant one set of number words in their correct order, but involves in complex set of skills and understandings about both cardinal and ordinal number. Children have little time to consolidate counting knowledge though, because as they are learning to count, they are also starting to operate with numbers. Usually, the first operation is addition (more). As noted above, recognition of "more" starts in babyhood, but the idea of adding groups of numbers purposefully develops later than that.

Elouise Plays 'Snakes and Ladders'

"Roll the dice, Elouise", said Nan, "It's your turn". The family of three-year-old Elouise played a game with her most nights in her "quiet time" before bed, just before her drink of milk, teeth cleaning, and story.

Elouise grabbed the die with glee and rolled it into the shoebox lid—a three. "Three!" she announced. "Three" repeated Nan, nodding then tapping the next few squares on the game board: "One, two, three. Move your dice here, El" [leaving her finger on the right square].

Elouise looked along the line of squares and said, "I want four, four for the ladder".

"Just move it three, Elouise. You didn't roll a four".

Elouise moved her counter to the correct square and then surprised her grandmother by saying "Three and one more. That's four. I want one more."

In bed soon after that, Elouise's mother was reading Jack and the Beanstalk to her. "So Jack gave the man the cow, and he took home the magic beans".

"There were five", said Elouise, "Four: three and one more. Five: four and one more". "Well, maybe …", said her mother, "maybe five is three and two more". "Three and three more!" laughed Elouise. Elouise had been able to hold up three fingers since her third birthday, and now did this on each hand.

Her mother touched each little digit in turn: "One, two, three, four, five, six! Three and three are six. Six beans. Now, off to sleep, my Three Girl. Here are six kisses for a very good girl. Three on this (cheek) and three on this. There: three and three make six".

Here, Louise seemed to be demonstrating some knowledge of number composition and decomposition. She was not just "counting on" from three to four then to five, but was recognising that four is made up of three and one more. Her mother pushed this on to "five is three and two more" then six being made up of three and another three, reinforcing this latter concept with her kisses. "One more" and "two more" are different addition concepts from adding two groups together, but Elouise and her mother slipped comfortably between these different ideas. "Children learn by varying what is done … math content requires repeated experiences with the same numbers … and related similar tasks (Clements and Samara 2007, p. 68).

Addition: Some Research

The basic idea in addition is that two smaller groups are put together make one larger group. Researchers such as Gelman and Gallistel (1978) and Starkey (1992b) have shown how this understanding develops in pre-school children, starting with the idea of adding or taking away one item at a time. Young children soon start to enjoy seeing part-whole relationships as "numbers inside numbers" and then enjoy playing with putting numbers together and breaking them down (Fuson et al. 2001, p. 523). Fuson et al. (1983), Jung (2011) and Fischer (1990) all present some everyday examples of typical development in the prior-to-school years as well as lots of engaging learning activities.

Surprisingly, counting words are not needed for simple addition. When Huttenlocher et al. (1994) put out a small number of counters and covered them before adding a few more, children as young as 2–3 years could make the total number with their own counters. Huttenlocher et al. found that concept of adding more, as well as the related "mental models" (p. 284), develop with non-verbal manipulation of objects before verbal counting, and their findings "strongly support the claim that a mental model underlies the acquisition of exact nonverbal calculation ability" (p. 295). An interesting point is that children in their experiments were reasoning numerically, so it was not only the formation of mental models for number that were developing but also the power to reason mathematically. In fact, Gallistel and Gelman (1992), after previously undertaking similar research with even younger toddlers had argued that this competency provides "the framework—the underlying conceptual scheme—that makes it possible for the young child to understand and assimilate verbal numerical reasoning" (pp. 65–66).

It is also important to consider the implications of the finding of Huttenlocher et al. (1994) that infants in different socio-economic areas performed equally on such non-verbal tasks, even though differences were noted in later verbal arithmetic (e.g., Jordan et al. 1994; Baroody et al. 2006).

Jules Takes Away

"How many sandwiches did you have, Jules?" "Four", four-year-old Jules answered her father confidently. "Okay, and you ate one?" "Yes. All gone". "Good girl. How many are left?" "Three". "Ah, so you had four and you ate one. There are three left. Let's see what happens when you eat another sandwich."

And so the conversation continued over lunch—not all the conversation, of course, but Dad kept coming back to the "take away" idea as Jules' sandwiches were eaten. They both laughed when the answer was "None".

That evening, Dad walked past the bathroom, where Jules was happily playing in the bath and chattering to herself. "Take away. Take away, Fishy. Swim away, swim, swim, swim. Four take away one is three". Dad thought that Jules was probably just repeating what she had learned by rote so walked into the bathroom, but there was Jules with four plastic fish, three floating freely and one being "swum" away by Jules.

Dad stayed with her, playing "swim away" with the other fish too, and again no fish being left delighted Jules. Then Dad blew four soap bubbles for her. He did not have to encourage her to pop one. "Pop" shouted Jules. Four pop one. Pop goes the weasel! Three. Pop, pop, pop! None!" Again, the idea of "none left" had Jules laughing aloud.

Jules seemed to have made a big jump in her learning. It is fairly easy to teach young children subtraction ideas in everyday situations. Food is eaten, birds fly away, objects are hidden, and toy cars drive off. Poems and songs such as "Mother duck said 'Quack, quack, quack, quack', but only four little ducks came back" provide more contexts for modelling subtraction in imaginative contexts. Ten green bottles fall off the wall, and rolling over in a bed causes someone to fall out.

The exciting jump that Jules had made of her own accord, was the realisation that subtraction of one from four was not just about sandwiches being eaten, but an abstract idea (i.e., one that is less dependent on objects) that could be applied not only to fish but also any other objects that might run, swim, fly, jump, fall, roll, pop—and so on—away. This jump, not only made with subtraction, or course, is called abstraction.

Of course, counting down one object at a time is different from taking away several objects at a time, but at least Jules has made a good start on understanding the idea of the "take away" action of subtraction. Jules would have two further subtraction actions to learn in the future—both typically solved by young children "counting on":

Difference: I have two cards, and you have five. How many more do you have?
What do I add? I have two cards, and you have five. How many more do I need?

Subtraction: Some Research

In fact, quite a few researchers, such as Wynn (1992) and Koechlin et al. (1997), have noted that babies as young as five months of age react to objects disappearing unexpectedly. They found that infants stared longer when the number of objects in a familiar picture had been reduced than when an extra object had been added, although researchers have been criticised for projecting the idea of number into such situations (see, for example, Sarama and Clements 2009). But typically three-year-old children are able to solve simple subtraction problems by using familiar objects and contexts, just as Jules did above (see, for example, Fuson 1992; Kilpatrick et al. 2001).

Addition and subtraction, though, are only two of the four arithmetical processes. Multiplication is another.

Building a Police Van

Nan and Pa lived 5 h drive from four-year-old Parisa, but used Skype to chat several times a week. Sometimes they read storybooks to each other, and in fact, Nan had photocopied some of Parisa's favourite books so both could look at the words and pictures while she was reading. This time, though, Pa and Parisa were both building a fire engine with Lego: each in front of a computer with a video camera, but 500 km apart. "Now, Parisa, you have a flat white base there. Put that down first. It's not the narrow one: it's four dots wide. Right, good girl. That's an 8 by 4." "I've got a black one too", said Parisa. "Okay, leave that for later. Leave your black 8 by 4, because you need a longer one now. Use the black 10 by 4. The biggest one. Look [holding the piece up to the camera]. It's your biggest one you need. Count the ten dots long, and it's four dots wide. Okay. That's it. Good. Now push that on top of the white 8 by 4." "It's too big. It hangs off"."Great. Put it in the middle. Leave one row of dots each end. That's where the bumper bar will go. Can you find your bumper bar? It's got lights on it. Watch me while I push my bumper bar on. Okay. Then push the back one on the other end. It's a 4 by 1 too, but there's no lights on it. Front 4 by 1 has lights. Okay. Back one. Okay." "Look, Pa. Can we do the wheels now?" Parisa went ahead adding wheels anyway, and then held the base of her fire truck up to the camera. Together they moved on in this manner to add the body, windscreen, and roof of the police van before adding its lights, and little policemen.

With her relative independence in using Skype, Parisa seemed to be developing confidence and competence with skills that will enable her to make use of new communication technologies as they are developed in future years. In this activity with her grandfather, though, she was also getting one type of experience with multiplication arrays. Neither Pa nor Parisa counted the dots on the 8 by 4 piece, so they were not yet using multiplication, but Parisa was receiving valuable learning about the fact that rectangular shapes can have rows and columns. It is unusual for young children to name rectangular arrays, such as "8 by 4", but Parisa's grandfather was a builder who was used to talking about timber as "90 × 45" (mm) or the like, so naming Lego pieces that way came naturally, especially when he had to describe a shape at a distance. Whether it be by builders, other adults, or children of any age, all learning is shaped by context and purpose, being very much situated in specific social and cultural contexts (Lave and Wenger 1991). In fact, Parisa soon started to use that nomenclature herself:

"Pa, push a wheels on. Got them? Black, four: two by two? Good. Now more wheels. Push them all on, Pa".

Here, Parisa demonstrated that she was just starting to understand the idea of multiple groups, with her comment: "… four: two by two. Perhaps this could have been the start of her understanding of multiplication being a product of two numbers, but it was not necessarily so. In this response, though, she did show a very basic understanding of the way her grandfather was using the rows and columns of Lego dots.

Multiplication: Some Research

The most common idea in early learning of the concept of multiplication is repeated groups or sets of objects. Becker (1993) found that many 3–5 year old children could associate the count "one two" with one toy, "Three, four" with the next, and so on (i.e., "2 to 1 mapping"). Typically, they also can confidently put two teddies in each toy car and model other small sets to solve simple multiplication problems (Carpenter et al. 1997; Clarke et al. 2006).

There is little equivalent research, though, on division—other than on fractioning. Sharing is a division context.

Anne and Ruby Share

A four year old, Anne, loved musk sticks (log-shaped sweets), and their delicious smell and flavour makes her weekly treat—buying her own stick—a highlight of each week. She hands her 10c to the shopkeeper, who gives her a musk stick as well as 5c back "… for your money box, Annie".

One day, Anne's cousin was with them for this treat time, so her mother said, "Buy two, Anne. Buy another one for Jess.

"No, she can have mine. Some".

"Okay. That's good sharing. You can give Jess half then".

In the car on the way home, without being prompted further, Anne broke the musk stick close to half, and held the two pieces together then gave Jess the slightly longer piece. The two happy children licked and sucked their treats with delight.

It seems that despite all the situations where children use sharing, there has been very little research on the development of division concepts in young children. Children are usually two years old before they understand the word "share", but that can just mean giving another child time with a toy or other possession: sharing of access rather than sharing of objects. Equal sharing of discrete (separate) or continuous (whole) objects like Anne used involves quite complex actions and concepts, although three year old children may have a good sense of the ideas of "some", "fair share", and then "half" fairly early in practical contexts when these concept are nurtured (Clements and Samara 2007).

Partitioning and Sharing: Some Research

However, transfer between contexts is not easy. Holmqvist et al. (2012), for example, found that three children (age 4, 5, and 6 years) were unable to imagine the shape of halves of a whole cake before it was cut. They watched the cake being cut into two halves, and counted the two halves, but were still unable to say how many halves would be in a whole apple. It appeared to the researchers that the children's knowledge of half, demonstrated with a cake, was not easily transferred to a different-shaped representation. What is more, young children eventually need to be able to find half of one-dimensional (e.g. string, stick), two-dimensional (paper shapes, bread, pizza), and three-dimensional shapes (cake, apples, drinks)—as well as collections of many discrete objects such as cards, pencils, sweets, and game tokens.

While foods (both solid and liquid) are shared between family members or with friends before early school years, the idea of dividing a set of objects into equal groups is usually practised in the first year of school curricula. Nunes and Bryant (1996) explored the ability of five-year-old children to make equal groups—the key division idea—with most of the children succeeding. However, they found that while three- and four-year-old children can usually follow instructions such as "Put two beans in each match box", the children of that age could not share larger numbers of objects fairly.

The most common division action for young children, however, is sharing of one continuous quantity through breaking or cutting—like the musk stick above. However, in the story below, Ruby's family is sharing out whole donuts, which are separate (discrete) objects. Nanna and Pop arrived with some donuts one afternoon; and after everyone had eaten one, including three-year-old Ruby. There was one left on the bench. Cinnamon donuts were one of her favourites, and it wasn't long before Ruby started eyeing off the last one. To distract her from it, her mother told her it was for Pop's dessert after he'd eaten his dinner. Ruby accepted that and went about playing without another word about the donut.

At the dinner table, as soon as her Pop had put his knife and fork down, Ruby sidled up to him and said, "Now don't forget your donut, Pop". She had not only remembered but had waited patiently until he finished his dinner before mentioning it. Then Ruby said, "I've been thinking, and have had an idea (all said with one hand on hip and pointer finger to her face), why don't you get a knife and cut the donut? Then we can both have one".

As if Pop could resist that logic!

The two stories above both illustrate sharing (division) of objects: first a musk stick (broken in half along its length, to share) and then the donuts (sharing of discrete objects, with a remainder). The two pre-school children solved both types of problems sensibly, even if Ruby's solution was not fair to the rest of the family.

Children observe different ways of fairly equal sharing of food such as pouring drinks into more than one glass, cutting and distribution of a cake, everyone taking two biscuits or being entitled to one scoop of ice-cream, and sharing a packet of nuts between family members. Games are further excellent contexts for learning about sharing equally: every person needs one token for the board game, seven cards are distributed to each person for a particular game of cards, or perhaps eight tokens are placed on the white spaces for each person to set up a draughts game. In art and craft activities, one pot of paste is dipped into or one ball of playdoh is broken up to distribute between a group of children, or the scissors and paper are "handed out". Children participating in such contexts soon get used to the idea that distributed groups are "the same" (equal) in many different respects. They can be encouraged to put two shoes in each locker, make three play-dough eggs for each nest, put two cookies and half an apple on each plate, put an amount of fruit juice in each cup, and so on. Such experiences will ready a child for later use of more formal division ideas and calculations. It is important that children also get involved with having to deal with some remainders and that they hear this word in meaningful situations long before they meet them in formal division contexts at school.

Despite a detailed search, I have been unable to find any research findings on the development of division actions and concepts in pre-school-aged children. This is a PhD topic just waiting for a candidate! However, division has been the focus of detailed research with school-aged children. For example, based on their research in schools, Kouba and Franklin (1995) recommend that teachers in grades K to 4:

(a) give children a rich communicative experience with various multiplication and division situations,
(b) evaluate and reward more than just producing an answer quickly in a prescribed way, and
(c) help children build from their own experiences and understandings many ways to represent and model multiplication and division situations (Kouba and Franklin 1995, p. 576). Kouba and Franklin also suggest many classroom activities that are suitable for early years' schooling, but these are not suitable for pre-schoolers.

Peter and Infinity

I have often questioned the claim by Piaget (1928), in his "stages of cognitive development" theory, that children do not become ready to learn abstract ideas until they are about eleven. While it makes sense to proceed from the concrete to the abstract, and as children get older they certainly handle more complex abstract ideas more capably, I have noted many instances where pre-school children discover for themselves and understand quite abstract ideas. One of those ideas is infinity, which leads to my final—and favourite—story.

We had been travelling for a year, and were north of Mt Isa on the way to "The Gulf". Peter was nearly five, and for his pre-school education we used the excellent correspondence materials available for Queensland's children who were travelling or living in remote areas.

When the novelty wore off games and songs during long-distance travel, we used to fill the time with correspondence kindergarten activities that included number play like "What's the next number?"

Me: What's next after fifty-six?
Peter: Fifty-seven. What's after one hundred?
Dad: A hundred and one. What's after a million?
Peter: A million and one. [Pause] What's the biggest number?
Dad: There is none. [Long pause]
Me: That's right, Pete. You can always say the next number
Peter: Like one thousand hundred billion million and one?
Me: Yes.
[Long silence, with Peter looking teary]
What is wrong?
Peter: [Pause] I can't learn to count. I will never be able to count!
Dad: Don't worry. You will understand one day
Peter: I do understand. I know it. [Pause] It is beautiful!

As noted by Tall (2001), "Young children's thinking about infinity can be fascinating stories of extrapolation and imagination [and to] capture the development of an individual's thinking requires being in the right place at the right time" (p. 7). I have argued elsewhere (Mousley 1999) that out of multiple experiences with specific examples, young children are capable of further (and ultimately more important) levels of abstracted understanding as well as understanding about self in relation to mathematics learning. In very early forms, both of these aims were illustrated by young Peter, above. The former involved a jump from the "game", with many kilometres of specific examples where higher numbers were always possible to his realisation of the abstract idea that counting numbers are infinite. The latter was demonstrated by his personal engagement with a mathematical principle that seemed to this four-year-old to be quite beautiful. As Tall wrote, parents and teachers "should be aware of the surprisingly sophisticated and complex ideas of the young that deserve to be treated on their own terms with respect" (p. 19).

Abstraction: Some Research

Tall's (2001) research was with children who were seven or older—and mainly with his son Nic as he matured and further developed a capacity for abstract thought Abstraction in young children was described well by Schoenfeld (1986), who identified levels of children's number understanding as development that moves from (a) understanding of objects and operations on these, to (b) understanding of symbols and operations on these (arithmetical processes). As numbers get too large to model easily, working with abstract ideas becomes inevitable, but also more powerful and hence more beautiful—as young Peter realised. I have no idea whether he thought of infinity as an enormous number, but his comment that he would "never be able to count" suggested that he had a better understanding of infinity than that.

Conclusion

It is clear that curious little children, through varied prior-to-school experiences such as those above, develop number understandings and skills that will be invaluable for understanding mathematical content in the first few years of formal schooling as well as attitudes that will also support their later learning of mathematics.

Fuson et al. (2015) noted that "children are inherently driven to learn because of the feelings of competence that result" (p. 63) but they also seem to need the interest, feedback, and approval of family members and carers. These researchers also debunk the play versus learning dichotomy, noting the importance of "guided play that is initiated by an adult and therefore can support educational goals" (p. 64).

Earlier in this chapter, I listed a number of questions that I hoped to: "What do we know about the mathematical development of young children in respect to number and simple arithmetic concepts? How can children's storybooks and familiar objects be used here, and how can parents draw out the mathematics in everyday events? What roles might some technologies play? How can various types of games be used to promote pre-school children's mathematical development? How can we make the most of young children's passionate interests and hobbies as well as various family routines? What are the big ideas of number content that children can learn, before going to school, that will underpin their mathematical learning and initial achievement at school?" All of these questions have been attended to in part, but certainly not completely. Overall, I have touched on cardinality, subitising and counting, addition and subtraction (but not so well multiplication and division). Hopefully readers will take up the challenge of providing more detailed answers through their own interest and active engagement. In this chapter I have focused only on number ideas, while there are other stories I could tell about my observations of what young children learn informally about time concepts and other measurement ideas and skills, shapes and their properties (geometry), chance (probability), and data representation.

I am not a great believer in positive effects of lecturing parents or early childhood professionals about research implications or about what they should and should not do, but can point out some common themes in the stories above. One is that the adults listened to and observed very young children and thought about the mathematics in a great variety of everyday activities and contexts. Another is that they communicated with the children about that mathematics, asking questions, extending knowledge, and reinforcing learning as it was happening. Importantly, engagement in such experiences and conversations with adults gives children a sense of belonging as well as a sense of self-respect, recognition, achievement, and affirmation. The stories above are true examples of natural curiosity and desire to learn being cherished and temporarily satisfied, along with children's wishes to please parents and other significant people in their social environments. These factors provide motivation for persistence and for mastery of new words, ideas, and actions as well the growth of children's broader knowledge, interests and strengths.

Very young children's strategic mathematical thinking continues to delight me. I wish all children could grow up in nurturing environments like the lucky little ones above who were developing their mathematical understandings with the aid of significant others and of stimulating contexts.

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Part III
Home Interactions and Learning Experiences That Support Early Mathematical Learning

Chapter 7
Meta-Analysis of the Relationship Between Home and Family Experiences and Young Children's Early Numeracy Learning

Carl J. Dunst, Deborah W. Hamby, Helen Wilkie and Kerran Scott Dunst

Abstract This chapter includes analyses of the relationships between informal and formal home and family early numeracy learning experiences and preschool children's mathematics performance. The research synthesis consisted of 13 samples of children between 36 and 84 months of age (Median age = 69 months). The 13 samples comprised more than 5000 children and their parents or other primary caregivers. Results showed that variations in the children's early numeracy experiences are associated with variations in the children's mathematics performance. The various analyses of the relationships between the early numeracy experiences measures and children's mathematics achievement also showed that informal learning opportunities are better predictors of children's mathematics achievement compared to formal teaching activities, and that the types of experiences afforded children as young as three years of age are beneficial in terms of explaining variations in the children's mathematics achievement. Implications for both research and practice are described.

Keywords Numeracy experiences · Informal and formal learning opportunities · Toddlers and preschoolers · Mathematics performance · Meta-analysis

LeFevre (2000), in the introduction to a special issue of the *Canadian Journal of Experimental Psychology* on early literacy and numeracy, noted that

> Although research on the numerical skills of children has a history just as long as that on reading processes…, the quantity of research on early numeracy is much less than that on early literacy (p. 58).

According to Ramani and Siegler (2014), this is the case, to a large degree, because parents and practitioners tend to place more emphasis on early literacy development during the preschool years compared to early numeracy development which evidently translates into fewer mathematics-related experiences for young children

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compared to other types of early learning opportunities. As part of a study by Munn and Schaffer (1993) on the literacy and numeracy experiences of 2 and 3 year old children in early care settings, the investigators found that “the total incidence of literacy events was far greater than the incidence of numeracy events” (p. 70). In a study comparing the frequency of child home literacy and numeracy activities experienced by young children, Skwarchuk et al. (2014) found that parents more often engaged their children in literacy activities than in numeracy activities.

The experiences afforded young children as part of everyday child learning opportunities have been described as activity settings (Alvarez 1994; Farver 1999) or microsystems (Bronfenbrenner 1992) and are considered contexts for interpersonal exchanges that can have either development-enhancing or development-impeding characteristics and consequences. As noted by Bronfenbrenner (1993), “the personal characteristics likely to be most potent in affecting the course…of [child] development…[include] those that set in motion, sustain, and encourage processes of interaction between the [developing] person and two aspects of the proximal environment: first, the people present in the setting; and second, the physical and symbolic features of the setting that invite, permit, or inhibit engagement in sustained, progressively more complex interaction with an activity in the immediate environment” (p. 11). There is evidence that young children’s everyday activities vary considerably in terms of how many and how frequently they participate in different kinds of everyday activities where variations in these experiences have been found to be related to variations in children’s learning and development (e.g., Gauvain 1999; Hart and Risley 1995; Trivette et al. 2004). In a meta-analysis of the relationship between child participation in everyday home and community activities and young children’s literacy and language development, for example, Dunst et al. (2013b) found that variations in children’s participation in home and community activities were related to differences in both literacy and language outcomes.

The purpose of this chapter was to synthesize available evidence on the relationship between young children’s early numeracy experiences and their mathematics performance and achievement. The focus of investigation was the everyday numeracy activities afforded as part of home and family life and whether variations in those experiences were associated with variations in early numeracy learning as has been found to be the case for other child outcomes (e.g., Dunst et al. 2001; Rogoff et al. 2006; Tudge et al. 2003). The research synthesis was guided by an ecological framework (Bronfenbrenner 1992) where the factors associated with variations in child learning and development are considered multiply determined, and children’s early numeracy experiences are but one of a number of environment factors hypothesized to be related to variations in child competence.

Byrnes and Wasik (2009) and Munn and Schaffer (1993), for example, proposed models that include a number of factors that they contend are important for early numeracy learning. The factors that have been hypothesized to influence young children’s early numeracy learning, in addition to everyday numeracy experiences and activities, include parents’ education (Aunio and Niemivirta 2010; Dearing et al. 2012), family socioeconomic backgrounds (Anders et al. 2012; Dearing et al. 2012; Kluczniok et al. 2013), parents attitudes toward mathematics (Skwarchuk

2009; Sonnenschein et al. 2012), parents' expectations for child academic achievement (Galindo and Sheldon 2012; Kluczniok et al. 2013), parents' interactional styles during everyday child activities (e.g., Anderson 1997; Lukie et al. 2014), and children's interests in early numeracy activities (Cheung 2013; Lukie et al. 2014). Although our main interest was the relationships between children's early numeracy experiences and mathematics performance, we also examined the influences of child age, gender, and condition (typically developing vs. at-risk); parent education and beliefs (e.g., attitudes toward mathematics); and family socio-economic status on the study outcomes to place the results in a broader-based ecological context.

The research synthesis described in this chapter specifically focused on the relationship between the different kinds of everyday home and family numeracy activities experienced by young children and the extent to which variations in those experiences accounted for variations in children's mathematics performance and achievement. In each of the studies in the meta-analysis described in this chapter, similar types of multi-item measures of home and family numeracy activities were used to capture children's numeracy experiences. These types of measures have been termed a child's home numeracy environment (HNE) and are considered by many investigators the best indicator of a child's early numeracy experiences and activities (e.g., Lukie et al. 2014; Manolitsis et al. 2013; Niklas and Schneider 2014). The HNE of young children has been described by others as home numeracy experiences (e.g., Kleemans et al. 2013; LeFevre et al. 2009), home numeracy activities (e.g., Cheung 2013; Skwarchuk et al. 2014), and home numeracy practices (e.g., LeFevre et al. 2010). These include, but are not limited to, activities such as songs, poems, or rhymes including numbers; playing with blocks and toys of different sizes and shapes; counting or reciting numbers; playing number games; and identifying or counting coins. Our main interest was investigation of the relationships between the home and family early numeracy experiences of young, preschool age children and mathematics achievement, and therefore the research synthesis focused on studies where HNE measures were used to measure children's numeracy experiences and activities in the early years.

Method

Search Strategy

Studies were located using the following search terms: (infan* OR toddler OR preschool* OR young child*) AND (home OR family) AND (numeracy OR numb* OR math* OR count*) AND (research OR study OR investigat* OR correl*). Follow-up searches were conducted using controlled vocabulary, key word, and natural language searches as alternative search terms were identified from retrieved publications and reports.

ERIC, PsychInfo, Academic Search Complete, CINAHL, ABI/INFORM COMPLETE, PROQUEST Psychology, PROQUEST Education, and MEDLINE were searched to identify studies. These were supplemented by searches in Infotrac, Google Scholar, Google, and WorldCat. The reference sections of retrieved journal articles, book chapters, and other published and unpublished reports and papers were examined to identify additional studies.

The search strategy identified a number of key investigators (e.g., Belinda Blevins-Knabe, Tijs Kleemans, Jo-Anne LeFevre, Sheri-Lynn Skwarchuk) whose names and numeracy publications were used to conduct author and article citation searches to identify research by these same investigators and research by others who cited these investigators' research as part of their own research on early numeracy. This iterative search process was used until no new studies were identified.

Studies were included if investigators employed a multi-item measure of everyday home or family child numeracy experiences, one or more measures of child mathematics performance or achievement were administered, the numeracy experience measure was obtained prior to kindergarten, and the correlations between the two measures were included in the research report. The synthesis was limited to only quantitative studies where the effect sizes (Hedges 2008) between the child, parent, family, and numeracy experience measures and the child mathematics outcome measures could be calculated. Five studies were located that included children whose ages, on average, were 5 years or younger (Baker 2014; Harris et al. 2014; LeFevre et al. 2002; Manolitsis et al. 2013; Melhuish et al. 2008). Inasmuch as only five studies were located that met the inclusion criteria, the child age criterion was relaxed and studies of kindergarten age children were included in the meta-analysis if all of the other inclusion criteria were met. Six additional studies were located that met the relaxed inclusion criteria (Blevins-Knabe and Musun-Miller 1996; Kleemans et al. 2012; Kleemans et al. 2013; LeFevre et al. 2010; Niklas and Schneider 2014; Skwarchuk et al. 2014). The 11 studies were all published in peer reviewed journals. All of the studies except one (Blevins-Knabe and Musun-Miller 1996) were published between 2002 and 2014, and the majority (73 %) were published between 2010 and 2014.

More than 20 other studies were located but excluded for a number of reasons. The reasons for exclusion were the children were neither preschool nor kindergarten age (e.g., Carmichael et al. 2014; Dearing et al. 2012), the data for younger children were not reported separately from that of older study participants (e.g., LeFevre et al. 2009), no home or family numeracy experiences measure was used in the studies (e.g., Aunio and Niemivirta 2010; Kluczniok et al. 2013), incomplete correlations among measures were reported (e.g., Skwarchuk 2009), or the data were not reported in a format necessary to ascertain the relationships between early numeracy experiences and child mathematics performance (e.g., Anders et al. 2012; Galindo and Sheldon 2012). Searches for other research reports on the excluded studies to identify data in the formats necessary to include in the research synthesis proved unsuccessful.

Search Results

Participants

The 11 located studies included 13 samples of children. Table 7.1 lists the studies and shows the background characteristics of children who were study participants. The studies included 5036 children. The sample sizes in the studies ranged between 49 and 2354 (Mean = 387, Median = 100). Child gender was reported in all but two studies. In those studies where child gender was reported, 74 % were boys and 26 % were girls.

The children's ages ranged between 36 and 84 months (Median = 69). Most samples (N = 8) included children who were described as typically developing, three samples included children where about half the study participants lived in

Table 7.1 Background characteristics of the study participants

Study	Sample size	Child gender		Child age (months)			Child condition[a]	Location of study
		Male	Female	Mean	SD	Range		
Baker (2014)	1202	1202	0	67.95	4.06	–	AR	USA
Blevins-Knabe and Musun-Miller (1996) study 2	49	25	24	84.00	–	57–77	TD	USA
Harris et al. (2014) sample 1	50	0	50	36.00	–	–	AR	USA
Harris et al. (2014) sample 2	61	61	0	36.00	–	–	AR	USA
Kleemans et al. (2012)	89	39	50	73.20	–	60–84	TD	Netherlands
Kleemans et al. (2013)	150	–	–	72.70	–	59–84	TD/SLI	Netherlands
LeFevre et al. (2002)	65	32	33	53.00	–	36–67	TD	Canada
LeFevre et al. (2010) sample 1	100	48	52	70.00	–	–	TD	Greece
LeFevre et al. (2010) sample 2	104	53	51	70.00	–	–	TD	Canada
Manolitsis et al. (2013)	82	53	29	64.67	3.26	–	TD	Greece
Melhuish et al. (2008)	2354	–	–	41.00	4.60	–	TD	Great Britain
Niklas and Schneider (2014)	609	283	326	77.00	4.51	63–96	TD	Germany
Skwarchuk et al. (2014)	121	72	49	70.00	3.30	64–78	TD	Canada

[a]*AR* At-risk for family background characteristics (e.g., low educational achievement, single parent household, low socio-economic status), *TD* Typically developing, and *SLI* Speech and language impaired

households where the presence of intrafamily risk factors might adversely affect child learning opportunities (e.g., parents with less than a high school education, single parent household, parent unemployment), and one sample included both typically developing children and children with speech and language impairments.

Parents' education levels were reported in eight studies. The majority of parents had completed some formal education beyond high school. Only three studies included information on the parents' ages.

The studies that were located were conducted in six different countries. Three studies each were conducted in Canada and the United States, and two studies each were conducted in Greece and the Netherlands. One study was conducted in Germany and one was conducted in Great Britain. No studies were located that were conducted in any other country that met the inclusion criteria.

Synthesis Variables

The main focus of analysis was the relationships between home and family numeracy experiences and children's mathematics performance, and whether the relationship between numeracy experiences and child outcomes differed for preschool children compared to kindergarten aged children. We also evaluated the relationships between child gender (N = 7 samples), child condition (N = 12 samples), parent education (N = 6 samples), family socioeconomic status (N = 4 samples), parents' attitudes toward mathematics (N = 5 samples), parents' expectations for child academic achievement (N = 5 samples) and child mathematics performance. The parents' attitudes toward mathematics and numeracy learning and their expectations for children's later academic achievement were both assessed using investigator-developed measures (Kleemans et al. 2012, 2013; LeFevre et al. 2010; Skwarchuk et al. 2014).

Home and Family Numeracy Experiences Measures

The numeracy experiences measures used in the studies are listed in Table 7.2. Except for Baker (2014) and Harris et al. (2014), the home and family experiences measures were all investigator-developed or adapted from measures developed by other researchers. Four of the investigators computed coefficient alpha to ascertain internal consistency of the item content. Alpha was reasonably high given the small number of scale items. Examination of the item pools on the other measures found them more similar than different and therefore there is no reason to believe that reliability estimates would not be similar (see e.g., Dunst et al. 2000).

All but two numeracy experiences measures were completed by the parents who rated the frequency of child participation in different types of home and family activities. The majority of HNE measures included only numeracy activities,

Table 7.2 Scales and instruments used to measure family and home child numeracy experiences

Study	Numeracy measure	Scale characteristics				Type of measure	
		No. of items	Coefficient alpha	Type of scaling	Respondent	Item content	Type of activity
Baker (2014)	HOME (Caldwell and Bradley 1984)	10	0.73	Frequency	Parent	Numeracy/literacy	Informal
Blevins-Knabe and Musun-Miller (1996)	Child numeracy activities	13	–	Frequency	Parent	Numeracy	Informal
Harris et al. (2014)	HOME (Caldwell and Bradley 1984)	7	–	Yes/no	Researcher	Numeracy/other	Informal
Kleemans et al. (2012)	Parent child numeracy activities	4	0.76	Frequency	Parent	Numeracy	Informal
Kleemans et al. (2013)	Parent child numeracy activities	6	0.93	Frequency	Parent	Numeracy	Informal
LeFevre et al. (2002)	Parent teaching activities	5	–	Frequency	Parent	Numeracy/literacy	Formal
LeFevre et al. (2010)	Parent numeracy practices	4	–	Frequency	Parent	Numeracy	Formal
Manolitsis et al. (2013)	Home numeracy questionnaire	5	–	Frequency	Parent	Numeracy	Formal
Melhuish et al. (2008)	Home learning environment	7	–	Frequency	Parent	Numeracy/other	Informal
Niklas and Schneider (2014)	Home numeracy environment	3	–	Frequency	Parent	Numeracy	Informal
Skwarchuk et al. (2014)	Home numeracy practices	4	0.73	Frequency	Parent	Numeracy	Formal
Skwarchuk et al. (2014)	Home numeracy practices	4	0.71	Frequency	Parent	Numeracy	Informal
Skwarchuk et al. (2014)	Home numeracy games	10	0.88	Yes/no	Researcher	Numeracy	Informal

whereas four measures included a combination of numeracy and literacy activities or numeracy and other everyday child experiences. The types of numeracy activities on the different measures included, but were not limited to, counting toys or objects, playing counting games, rehearsing counting rhymes, playing with a child cash register, repeating sequences of numbers, counting coins, ordering objects by size, singing number songs, and comparing sizes (e.g., small vs. big).

The different numeracy experiences measures were categorized as either informal or formal child leaning opportunities depending on the instructions for completing the scale items or the focus of the numeracy experiences. Measures were coded as formal activities if the instructions explicitly asked parents to indicate how often they taught their children different types of numeracy skills (LeFevre et al. 2002, 2010; Manolitsis et al. 2013) or parents were asked to indicate how often they helped or worked with their child to learn different numeracy skills (Skwarchuk et al. 2014). Formal numeracy activities included teaching episodes that had explicit instructional targets (e.g., “I teach my child the names of numbers,” “I ask my child to recite numbers in the correct order”) where parents prompted or elicited correct responses (e.g., LeFevre et al. 2010; Manolitsis et al. 2013; Skwarchuk et al. 2014). Measures were coded as informal activities if parents were asked to indicate how often their children participated in or experienced different types of numeracy-related learning opportunities but without an explicit focus on teaching or instruction (e.g., singing number songs, playing counting games, rehearsing counting rhymes). Informal numeracy activities included everyday experiences that involved parent and child participation in activities involving arithmetic and mathematics elements (e.g., “How often do you and your child play number games?,” “How often do you and your child play with a calculator?”) where there was no intentional attempt to prompt or elicit predetermined instructional targets (Baker 2014; Kleemans et al. 2013; Niklas and Schneider 2014). Formal and informal numeracy experiences differed primarily in terms of the questions posed to parents or researchers. Questions about formal numeracy learning opportunities were asked in terms of how often informants taught specific numeracy skills, whereas questions about informal numeracy learning opportunities were asked in terms of how often a child participated in numeracy-related activities.

Numeracy Outcome Measures

The numeracy and mathematics outcome measures are listed in Table 7.3. Fourteen different scales or measures were used to assess child numeracy outcomes. The numeracy constructs on each outcome measure varied considerably, and included a mix of investigator-developed measures (e.g., LeFevre et al. 2002; Niklas and Schneider 2014), subscales on standardized measures (e.g., Harris et al. 2014; Manolitsis et al. 2013), adapted standardized measures (e.g., Baker 2014; Manolitsis et al. 2013), and standardized mathematics scales (e.g., Blevins-Knabe and Musun-Miller 1996; LeFevre et al. 2010).

Table 7.3 Ages at administration of the home numeracy environment and numeracy achievement measures and the instruments used to assess mathematics-related child competence

Study	Child age (months)		Numeracy outcome measures	
	Numeracy experience measure	Numeracy achievement measure	Scale	Construct
Baker (2014)	68	68	NAEP-adapted (National Center for Education Statistics 2001)	Math concepts composite
Blevins-Knabe and Musun-Miller (1996)	84	84	TEMA-3 (Ginsburg and Baroody 2003)	Math concepts composite
Harris et al. (2014)	36	54	WJ (R) (Woodcock and Johnson 1989)	Applied problems
Kleemans et al. (2012)	73	73	UENT (R) (Van Luit and Van de Rijt 2009)	Math concepts composite
Kleemans et al. (2013)	73	85	Basic calculation skills (Schneider et al. 2002)	Addition
	73	85	Basic calculation skills (Schneider et al. 2002)	Subtraction
LeFevre et al. (2002)	53	53	Children's numeracy skills (Miller et al. 1995)	Rote counting
	53	53	Children's numeracy skills (Miller et al. 1995)	Object counting
	53	53	Children's numeracy skills (Miller et al. 1995)	Number recognition
LeFevre et al. (2010)	70	70	KeyMath (R) (Connolly 2000)	Math concepts composite
Manolitsis et al. (2013)	65	65	TEMA-3 (Ginsburg and Baroody 2003)	Rote counting
	65	65	TEMA-3 (Ginsburg and Baroody 2003)	Math concepts composite
	65	73	TEMA-3 (Ginsburg and Baroody 2003)	Rote counting
	65	73	TEMA-3 (Ginsburg and Baroody 2003)	Math concepts composite
	65	85	Number sets/calculation (Geary et al. 2009)	Math fluency
Melhuish et al. (2008)	41	60	BAS II (Elloitt et al. 1996)	Number concepts composite
	41	84	National standardized test (not reported)	Mathematics composite

(continued)

Table 7.3 (continued)

Study	Child age (months)		Numeracy outcome measures	
	Numeracy experience measure	Numeracy achievement measure	Scale	Construct
Niklas and Schneider (2014)	77	77	Math precursors (Krajewski and Schneider 2009)	Math concepts composite
	77	82	PIPS (Tymms and Albone 2002)	Mathematics composite
	77	89	DEMAT (Krajewski et al. 2002)	Mathematics composite
Skwarchuk et al. (2014)	70	70	Early arithmetic skills (Huttenlocher et al. 1994)	Basic math concepts
	70	70	KeyMath (R) (Connolly 2000)	Number concepts composite

A content analysis of the items on each measure was used to categorize the outcomes for purposes of data analysis as assessing simple, basic, or complex mathematics achievement. An outcome measure was coded as a simple mathematics measure if it assessed rote counting, object counting, number recognition or other numbering or counting abilities. An outcome measure was coded as a basic mathematics measure if it assessed addition, subtraction, or other types of similar binary operations in addition to simple mathematics skills. An outcome measure was coded as a complex mathematics measure if it assessed multiplication, division, and other types of calculations in addition to simple and basic mathematics skills.

Investigators in eight of the studies obtained child outcome measures at the same age at which the numeracy experiences measures were completed. Investigators in five studies obtained child outcome measures at times later than when the numeracy experiences measures were completed. This permitted tests of both the concurrent and predictive relationships between the numeracy experiences measures and the child mathematics outcome measures.

Method of Analysis

The pooled weighted average correlation coefficients between the numeracy experiences measures, parent and family measures, and child mathematics outcome measures were used as the sizes of effects for the relationship between the study variables (Lipsey and Wilson 2001). The Z statistic was used to test the null hypothesis that no relationship existed between the numeracy experiences and the study outcomes (Shadish and Haddock 2009). The 95 % confidence intervals for the average weighted effect sizes were used for substantive interpretation of the relationships among measures. An average pooled weighted correlation coefficient with a small confidence interval indicates that the mean effect size is a reliable population estimate of the relationship between an independent and dependent measure. Cohen's (1988) benchmarks were used for ascertaining if the average weighted correlation effect sizes were small (0.10–0.29), medium (0.30–0.49), or large (0.50–0.69).

Q_{BET} was used to test for differences in the sizes of effect for different comparisons and contrasts (type of numeracy measures, timing of the outcome measures, etc.). Q_{BET} is "analogous to the omnibus F-test for variation in group means in a one-way ANOVA" (Hedges 1994, p. 290). Additional post hoc analyses were done as necessary to clarify the nature of the relationships among measures.

Synthesis Findings

Numeracy Experiences

The average effect sizes, confidence intervals, *Z*-scores, and associated *p*-values for the relationships between the numeracy, parent, and family measures and the child numeracy outcome measures are shown in Table 7.4. The sizes of effects were all small to medium, although the strength of the relationship between the parent, family, and numeracy experiences measures and the study outcomes differed as a function of the predictor variables, $Q_{\mathrm{BET}} = 282.20$, $df = 4$, $p = 0.0000$. Numeracy experiences proved to be the best predictor, where the size of effect differed significantly for all other predictors, $Q_{\mathrm{BET}} = 13.76–179.99$, $df_{\mathrm{s}} = 1$, $p_s = 0.0000$.

The main effect results indicate that although all the predictors accounted for variations in the children's numeracy and mathematics outcomes, children's early numeracy experiences stood out as being a more potent explanatory variable accounting for variations in the child mathematics outcomes. Further examination of the study results indicated that the influences of home and family numeracy

Table 7.4 Average weighted effect sizes and 95 % confidence intervals (CI) for the relationships between the predictor variables and child numeracy outcomes

Predictor variables	Number		Effect sizes (r)		*Z*-score	*p*-value
	Study samples	Effect sizes	Mean	95 % CI		
Home numeracy experiences	13	28	0.46	0.44, 0.47	59.38	0.0000
Expectations for child achievement	5	7	0.33	0.27, 0.39	11.02	0.0000
Parent attitudes toward mathematics	5	6	0.31	0.24, 0.39	8.08	0.0000
Family socioeconomic status	4	10	0.30	0.28, 0.31	36.95	0.0000
Parent education	6	13	0.27	0.25, 0.30	24.33	0.0000

experiences on children's mathematics performance do not appear to share much variance with the parent and family measures. In those studies reporting the correlations between two or more parent and family measures and early numeracy experiences, neither parent education nor socioeconomic status was correlated with the numeracy experiences measures, and the relationships between parents' attitudes towards mathematics and expectations for children's academic achievement and children's early numeracy experiences were significantly correlated in only two studies (Kleemans et al. 2013; LeFevre et al. 2010).

Child Age

As indicated in the introduction to our chapter, our main interest was the influences of the home and family numeracy experiences on the mathematics performance of children in the early years. Whether the relationships between the early numeracy experiences measures and study outcomes differed as a function of child age was determined by computing the sizes of effects separately for studies of preschool-aged children (36–65 months) and studies including kindergarten age children (68–84 months). The average effect size for the younger children was $r = 0.57$, 95 % CI = 0.55, 0.59, Z = 62.91, $p = 0.0000$, and the average effect size for the older children was $r = 0.17$, 95 % CI = 0.14, 0.19, Z = 11.46, $p = 0.0000$.

The between age group comparison showed that the sizes of effects for the two groups differed significantly from one another, $Q_{\text{BET}} = 563.62$, $df = 1$, $p = 0.0000$, where the effect size between the numeracy experiences and outcome measures was three times larger for the younger children compared to the effect size for the older children. This would suggest that numeracy-related experiences and activities of very young children may be especially beneficial for the children's mathematics learning and achievement.

Child Condition and Gender

The children in all but one study (Kleemans et al. 2013) were described by the investigators as either typically developing or at-risk because of intrafamily risk factors (e.g., low parent educational achievement, single parent households, low socioeconomic status). The average effect size for the typically developing children was $r = 0.49$, 95 % CI = 0.47, 0.51, $Z = 60.10$, $p = 0.0000$, and the average effect size for the at-risk children was $r = 0.11$, 95 % CI = 0.06, 16, $Z = 4.04$, $p = 0.0001$. Results showed that the numeracy experiences had more pronounced positive effects on the typically developing compared to the at-risk children, $Q_{\text{BET}} = 179.54$, $df = 1$, $p = 0.0000$.

The extent to which child gender was related to differences in the child mathematic outcomes was determined by grouping the studies into two categories: (1) entirely or mostly boys (N = 4 samples) and (2) entirely or mostly girls (N = 3 samples). The average effect size for boys was $r = 0.11$, 95 CI = 0.07, 0.15, $Z = 5.40$, $p = 0.0000$, and the average effect size for girls was $r = 0.15$, 95 CI = 0.11, 0.20, $Z = 6.97$, $p = 0.0000$. There was no difference in the average effect sizes for boys compared to girls, $Q_{BET} = 2.22$, $df = 1$, $p = 0.1364$.

Type of Numeracy Experiences Measure

The sizes of effects for the relationship between informal and formal numeracy experiences and the study outcomes are shown in Fig. 7.1. The average effect size for the informal numeracy experiences measures and the study outcomes was $r = 0.47$, 95 % CI = 0.46, 0.49, $Z = 58.91$, $p = 0.0000$, and the average effect size for the formal numeracy experiences measures and the study outcomes was $r = 0.28$, 95 % CI = 0.22, 0.33, $Z = 9.97$, $p = 0.0000$. The size of the effect for the informal experiences measures was almost twice as large as that for the formal numeracy measures, $Q_{BET} = 44.60$, $df = 1$, $p = 0.0000$. The findings suggest that exposure to a range of different informal numeracy-related experiences as part of everyday child learning may be more important for young children's mathematics learning compared to the use of instructional practices to teach young children mathematics skills at least prior to formal schooling.

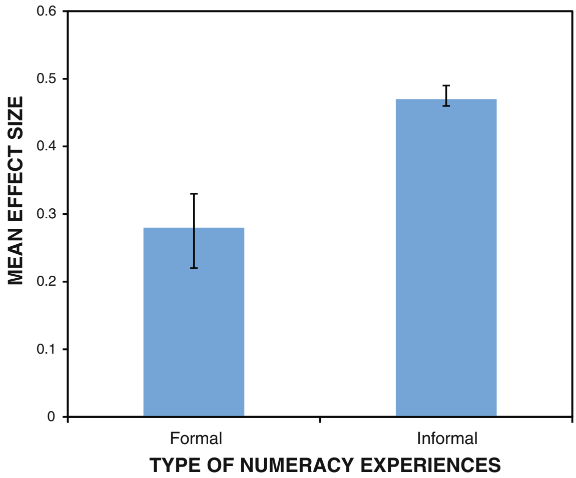

Fig. 7.1 Average effect sizes and 95 % confidence intervals for the relationships between informal and formal early numeracy experiences and children's mathematics achievement

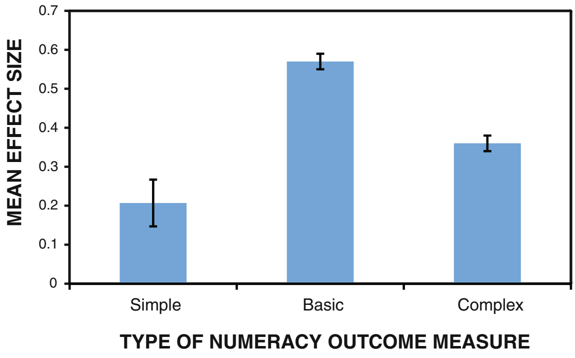

Fig. 7.2 Average effect sizes and 95 % confidence intervals for the relationship between early numeracy experiences and three types of child mathematics skills

Numeracy Outcome Measures

Figure 7.2 shows the relationships between the numeracy experiences measures and the three types of mathematics outcome measures. Although numeracy experiences were significantly related to all three types of outcomes, the sizes of effects differed significantly from one another, $Q_{BET} = 250.09$, $df = 2$, $p = 0.0000$. The average effect size for the basic mathematics measures was the largest, $r = 0.57$, 95 % CI = 0.54, 0.59, $Z = 53.28$, $p = 0.0000$, whereas the average effect size for the simple mathematics measures was the smallest, $r = 0.20$, 95 % CI = 0.15, 0.25, $Z = 7.17$, $p = 0.0000$. Pairwise follow-up tests of the comparisons between the three types of outcomes showed that all of the average effect sizes differed significantly from one another, $Q_{BET} = 29.63–158.35$, $df_s = 1$, $p_s = 0.0000$.

Further analysis found that the number of effect sizes for the relationships between the numeracy experiences measures and type of child outcome differed as a function of child age, $\chi^2 = 10.40$, $df = 2$, $p = 0.0060$. Whereas 92 % of the outcomes for children 36–65 months of age were simple and basic mathematics skills, 94 % of the outcomes for children 68–84 months of age were basic and complex mathematics skills. The pattern of results suggests that the influences of early numeracy experiences on child mathematics achievement differ as a function of both child age and type of mathematics skills.

Timing of the Outcome Measures

The extent to which the timing of the numeracy experiences and mathematics outcomes influenced the relationships between measures was determined by computing the average effect sizes for the numeracy and outcome measures obtained concurrently, 5–12 months apart, or 18–43 months apart (see Table 7.3). The results are shown in Fig. 7.3. The effect sizes for the numeracy experiences and outcome measures obtained concurrently was $r = 0.18$, 95 % CI = 0.14, 0.21,

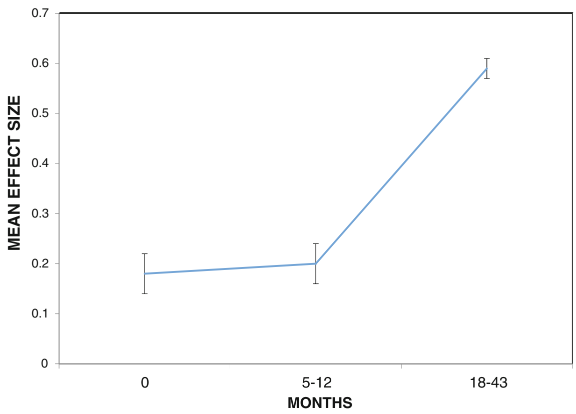

Fig. 7.3 Average effect sizes and 95 % confidence intervals for the relationships between early numeracy experiences and children's mathematics outcomes obtained concurrently and 5–12 and 18+ months apart

$Z = 10.48$, $p = 0.0000$ and for 5–12 months apart the effect size was $r = 0.20$, 95 % CI = 0.15, 0.24, $Z = 8.50$, $p = 0.0000$. Both effect sizes were small, and did not differ significantly from one another, $Q_{\mathrm{BET}} = 0.58$, $df = 1$, $p = 0.4630$.

The average effect size for the numeracy experiences and outcome measures obtained 18 or more months apart was $r = 0.59$, 95 % CI = 0.57, 0.61, $Z = 62.84$, $p = 0.0000$. The effect size was large and differed significantly from both of the other effect sizes, $Q_{\mathrm{BET}} = 244.35$ and 463.65, $df_s = 1$, $p_s = 0.0000$. Further inspection of the number of effect sizes for the relationships between numeracy experiences and timing of the study outcomes found that they differed as a function of child age, $\chi^2 = 8.14$, $df = 2$, $p = 0.0170$. Whereas all of the effect sizes for the relationships between numeracy experiences and the child outcomes obtained 18 or more months later were for children 36–65 months of age, there were no effect sizes for the same comparisons for the children 68–84 months of age. The results indicate that the effects of the numeracy experiences afforded the younger children continued to manifest themselves almost two years, on average, after the HNE measures were first obtained.

Discussion

Results from the meta-analysis described in this chapter permit a number of tentative statements and conclusions about the relationships between young children's early numeracy experiences and their mathematics achievement. We say tentative because of the small number of studies (N = 11) and sample sizes (N = 13), the number of effect sizes for the relationships between the early numeracy experiences and study outcomes (N = 28), and an even smaller number of effect sizes between the parent and family predictor variables and study outcomes (N = 6–13). Accordingly, the small number of effect sizes did not permit more detailed effect size disaggregation. The sizes of effects for the relationships among the variables that were computed, however, may be considered robust estimates given the fact with only a few exceptions, the confidence intervals for the average effect sizes were very small.

As expected, young children's early numeracy experiences were one of a number of factors associated with variations in children's mathematics achievement (Table 7.4). Early numeracy experiences, however, proved to be the best predictor of children's mathematics achievement where the average effect size for numeracy experiences differed significantly from all other parent and family measures. Inasmuch as a number of investigators proposed and tested models where parent and family measures were hypothesized to influence children's early numeracy experiences, where numeracy experiences in turn were expected to be related to children's mathematics achievement (LeFevre et al. 2010; Manolitsis et al. 2013; Niklas and Schneider 2014; Skwarchuk et al. 2014), the main effects results in Table 7.4 may be artifactual. Examination of the results in these modelling studies and the correlation matrices in other investigations (Baker 2014; Kleemans et al. 2012, 2013) showed, with only one exception (LeFevre et al. 2010), that the parent and family variables were not even minimally related to the early numeracy experiences measures as postulated. This indicated that parent or family factors other than those used in the studies in all likelihood account for variations in young children's home numeracy experiences.

The various analyses of the relationships between the early numeracy experiences measures and children's mathematics achievement showed that informal learning opportunities were better predictors of children's mathematics achievement compared to formal teaching activities (Fig. 7.1) and that the types of experiences afforded children as young as three years of age were beneficial in terms of explaining variations in the children's mathematics achievement. It would therefore be of investigative interest to know if early numeracy experiences afforded even much younger children as part of nursery rhymes, songs, games, book reading, and other everyday activities that include numbers and numeric concepts are related to mathematics performance as some contend (Caulfield 2000) and as some research indicates (see e.g., Aubrey et al. 2003; Van de Rijt et al. 2003).

The fact that the numeracy experiences measures were related to variations in the children's mathematics performance some 18+ months later deserves special comment due to the finding that the results were found only for the six samples of children who were 36–65 months of age at the start of the studies (Baker 2014; Harris et al. 2014; LeFevre et al. 2002; Manolitsis et al. 2013; Melhuish et al. 2008). More specifically, the early numeracy experiences of the younger group of children were related to the children's later simple and basic mathematics skills. However, the long term benefits of early numeracy experiences on the children's complex mathematics performance could not be determined simply because these outcomes were not included in the studies of the younger children.

Notwithstanding the tentative nature of the results from the meta-analysis, the findings showed that the everyday numeracy experiences of young children matter in terms of accounting for differences in child learning and development. The results add to a body of evidence demonstrating that variations in children's everyday early learning experiences are related to differences in child outcomes in many domains of functioning (e.g., Dunst et al. 2001; Hart and Risley 1995; Rogoff et al. 2006; Tudge et al. 2003).

The research synthesis described in this chapter was conducted in the same or a very similar manner to meta-analyses we have completed on the relationships between young children's everyday activities and language and literacy outcomes (e.g., Dunst et al. 2013a, b), the relationships between young children's interests in and preferences of certain types of everyday activities (e.g., Dunst et al. 2011; Raab and Dunst 2007; Raab et al. 2013), and the manner in which parents' and other caregivers' responsive interactional styles during child participation in everyday activities have development-enhancing consequences (e.g., Raab et al. 2013). The results from these syntheses showed that environmental experiences (everyday activities) and both child and adult characteristics contributed to differences in child behavioural and developmental outcomes. Notably missing in the studies included in the numeracy meta-analysis were measures of child and parent characteristics that have been hypothesized to be related to early mathematics performance (see e.g., Lukie et al. 2014), and which have been found to be related to other types of child outcomes (e.g., Coleman and Karraker 2003; Dunst and Raab 2012; Renninger et al. 1992; Richter 2004). Studies of early numeracy experiences might therefore contribute to a better understanding of the ecology of early numeracy learning if they included measures of child and parent explanatory variables in addition to home and family numeracy experiences (see especially Dent-Read and Zukow-Goldring 1997; Wachs 2000).

In addition to the implications for future research, the findings from the meta-analysis have at least one major implication for practice. Results indicate that there are benefits of using numeracy experiences and incorporating numeracy-related content into the everyday child learning activities as a way of building a foundation for later mathematics learning and achievement. Vandermaas-Peeler et al. (2012a, b), for example, describe several ways that everyday activities were used to promote young children's early numeracy learning. Doig et al. (2003), as part of a review of numeracy research and practice, identified a host of parent and family activities that were easily used as part of everyday child learning for promoting young children's numeracy skills (see also Copley 2010). Systematic reviews like the one described in this chapter can contribute to an understanding of how and in what manner young children's everyday numeracy activities are in fact important contexts for early and later mathematics achievement.

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Chapter 8
Parental Perceptions of Access to Capitals and Early Mathematical Learning: Some Early Insights from Numeracy@Home Project

Sivanes Phillipson, Gerarda Richards and Peter Sullivan

Abstract This chapter illustrates the perceptions of a small community of parents from a disadvantaged area in Victoria, Australia, on what they think about their family access to resources (in the form of capitals) and the importance of early learning in preparation for formal schooling especially in relation to mathematical learning. A total of 23 parents responded to the Family Educational and Learning Questionnaire, which was administered individually as part of a pilot study for the Numeracy@Home project. The questionnaire surveyed parental perceptions of their children, their access to educational and learning resources and their views on what kind of early learning in mathematical concepts is essential to happen before schooling and who should be responsible for those learning. Two of the parents also voluntarily participated in interviews around their home engagement with their children. Findings indicate that parents in this study are aspirational and value early mathematical learning as key to their children's success in schooling. Parents' engagement contributes to their children's learning and the dynamic learning environment. Parents also advocate that early learning is a shared responsibility between educators and themselves in preparing children for formal schooling.

Keywords Parent perception · Parent engagement · Capitals · Early mathematical learning

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Hey! I'm Peggy. My partner drives trucks for a living so I don't see him much. Ohhh... he tries to come home when he can but I am it for my kids! I have two sons, 16-year-old Craig and two year old Josh, and two daughters, nine-year-old Mandy and six year old Cristy. Josh dropped out of school last year and has done some odd jobs here and there. I try to keep tabs on him but hey, I find it hard...what do you do with a teenager who doesn't want to listen to me? And I worry about the others... especially my girls and little Josh. You know he is real smart, he said mama earliest and can feed himself coz' I showed him how to hold the spoon as early as when he sat down by himself. The other day at playgroup, he picked up a book with a chook on it and asked me to read to him. I am not real good at reading but I can count some. So I tried to read the book. You know it felt weird, I don't do that at home...we don't have books at home. The only "books" we have are the shopping catalogues we get in the junk mail. But reading the book with Josh was fun...the book got me counting and Josh liked it too. Isn't it funny the worm wasn't worm after all – it was a shoelace!

I know people say that I can do more with Josh and maybe I like to but it's not easy for me ...

Introduction

Peggy is one of thousands of parents who limit their engagement in their children's learning for varied reasons that may include multiple disadvantages they have previously experienced. In fact, Peggy could come from anywhere in the world as her profile would fit many parents that research has found to have lesser involvement in their children's learning from an early age. Melhuish et al. (2008), for example, explained that such parents often fail to engage with their children due to lack of opportunities especially access to resources and "knowledge". This "knowledge" centres on parents' own willingness and capacity to have interactions with their children to support learning within a daily home context. These interactions can range from reading to engaging children in everyday home chores. Such interactions are made possible with readily available resources within the home environment, yet most times in disadvantaged family homes these resources are scarce or not used for the purposes of learning.

However, it has been recognised that parents' perceptions surrounding their access to resources can lead to their own lack of confidence (Shonkoff 2012). Shonkoff suggested that parents have lesser engagement with their children's early learning because of stresses of disadvantage, such as those experienced by Peggy and her children. Parents tend to also underestimate their own role in early learning and this is suggested to be related to the family's socioeconomic status. The recent Australian Productivity Commission report highlighted that "family characteristics play a key role in facilitating children's learning and development" (2014, p. 149). The report suggests that parent engagement in early learning is somewhat determined by the family's access to better resources and knowledge, similar to claims made by Melhuish et al. (2008). The underlying assumption under the Productivity Commission's assertion is that parents with better access to resources and knowledge (i.e., those with higher income and education) may be able to provide better learning environments for their children.

Most of the time, parents are not aware of the resources that are available to them and although it is important to provide resources to enhance both social and cultural capitals (Marks 2006), they will only be part of the solution if families do not feel they have the knowledge, confidence and capacity to take advantage of those resources, and awareness that they can support the learning of their children. Alongside the effect of SES on the availability of resources, the extent of home learning activities and stimulating environment wields a greater and independent influence on learning (Anders et al. 2012; Melhuish et al. 2008).

In preparing their children for school, parents may have their own views of how "ready" their child should be and who is responsible for such a preparation (Anders et al. 2012). The extent of parental perceptions is usually influenced by parental expectations and what they perceive as contributing factors to their children's learning (Phillipson 2013). Indeed, parental expectations do play a key role where high expectations can lead to better learning outcomes. Having these expectations displayed early in children's upbringing and learning can influence parental own engagement in their children's learning (Hayes et al. 2013).

An important consideration is whether the nature of the parental perceptions of their potential influence and associated actions are more related to the family background or their aspirations. If, for example, parents have high aspirations but limited awareness of their capacity to enact those aspirations, then this would influence the nature of the support that might be offered to such parents. This key consideration has obvious implications for the relationships between educators and parents. An understanding of the nature of the parents' aspirations and awareness can inform the design of the support offered to parents.

This chapter illustrates the perceptions of a small community of parents from a disadvantaged area in Victoria, Australia, the same community that Peggy and her family come from. These parents responded to the Family Educational and Learning Questionnaire, administered individually to parents, as part of a pilot study for the Numeracy@Home project. The questionnaire surveyed parental perceptions of their children and their access to educational and learning resources, and their views on what kind of early learning in mathematical concepts is essential to happen before schooling and who should be responsible for those learning. Two of the parents also voluntarily participated in interviews that focused on questions surrounding their home engagement with their children. The chapter aims to articulate how parents like Peggy perceive their family access to resources (in the form of capitals) and the importance of early learning in preparation for formal schooling especially in relation to mathematical learning.

Influences on Parental Engagement in Learning

Shonkoff in his (2012) *Theory of Change* framework argued that early experiences are biologically embedded and carried over to adulthood, hence highlighting the importance of supporting those who are most disadvantaged at the earliest ages. Shonkoff's work has highlighted the need to identify healthy and nurturing early experiences, which are enhanced by positive family and other proximal interactional environments. Hence, the conceptual and methodological approach of this chapter utilises the *actiotope model of giftedness* (Ziegler and Phillipson 2012), a systems approach model that "includes an individual and the material, social and transformational environment with which that individual actively interacts" (Ziegler et al. 2013, p. 3).

Whilst initially describing the development of exceptional achievement, the actiotope model can also be used to articulate a conventional developmental trajectory through the identification of transformational environments for learning (Ziegler and Baker 2013). In line with the broader theory of change as proposed by Shonkoff (2012), the actiotope model describes the interactions between the individual (learning capitals) and the environment (educational capitals) as key processes for the development of learning and achievement.

In the actiotope model, the educational capital and learning capital construct the architectural design of the transformational environment. Educational capital comprises all external resources such as tools and knowledge of learning as imparted by parents, teachers and peers, whereas learning capital refers to all internal cognition and affect such as attention span and memory, motivation and learning goals that child and parent exhibit in their learning. Educational capital can be further constructed as *economic* (financial capacity), *cultural* (access to learning centres and schooling), *social educational* (support from parents, teachers and peers), *infrastructural* (facilities found in centres, schools and at home—such as playroom and learning technologies) and *didactic educational* (access to quality pedagogy and training). Learning capital includes *organismic* (mental and physical health), *actional* (intentional actions), *telic* (expected goal to reach), *episodic* (content knowledge such as numeracy) and *attentional* (interest in the subject matter). These capitals constitute the socioeconomic background and social values and cultural beliefs of families that affect children development and learning.

The following sections review relevant research that shows how parental socioeconomic status (SES), social values and cultural beliefs affect children outcome. A review of parental contribution to early mathematical learning is also included to show the interactions between parent and child as a result of aspirations that parents have for their children in learning outcomes.

Parental Socio-economic Status

Both the educational and learning capitals can be described as the resources available to the child, which contribute to their overall learning ability. SES plays a significant role in gaining access to these resources especially in the form of economic and infrastructural capitals. Socioeconomic status (SES) as defined by the Australian Bureau of Statistics (ABS) refers to the social and economic position given to an individual, family or group of people within society (ABS 2011). The ABS further discusses status of advantage and disadvantage in terms of access to materials, social resources and the ability to participate in society. In 2012, the poverty line for a family of 2 adults and 2 children was considered to be at 50 % of the median Australian household income with low SES being between $21,688 and $43,210 per annum (Wilkins 2015).

Mayo and Siraj (2015) conducted a comparison study between working class and lower SES families. Their research aim was to describe how families might support their children academically using the resources available given their economic status. The study found parents of lower SES tended to have poor perceptions of education; they appeared to view education as a matter of process rather than benefit to the child. On the other hand, the middle class families had the opposite perception in which they considered education as vital in leading their child to a strong future. These families were found to provide stimulating home environments with the children in these families succeeding academically above expectations.

Similarly, DeFlorio and Beliakoff (2015) compared low and mid SES families of children entering kindergarten. The study explored SES related differences in the home environment as well as parental perceptions towards learning ability. The research findings suggested that families with middle SES provided more stimulating home environments and held higher beliefs about education compared to lower SES families, contributing to the varying degree of a child's academic ability when entering kindergarten. The mid SES families were found to dedicate more time and resources towards learning at home, and had a clearer understanding of the importance of parent–child interaction in the home and the contribution it had to learning.

Comparably, a prior study conducted by Siegler and Ramani (2008) stressed that the difference between SES in families contributes to children's experience and knowledge prior entering formal education. The authors conducted an experiment that involved an hour a day of playing a board game to minimize the gap in mathematical knowledge between varying families of different SES. The study included fifty-eight preschoolers, aged four to 5 years old. The study found that playing the board game with children from lower SES families increased their numerical knowledge to the point it was indistinguishable from that of children from middle SES families. In other words, it is not so much characteristics of home environments that may be contributing to educational disadvantage as it is the children missing out on particular educational experiences, in this case games. The authors concluded that simple strategies of mathematical teaching at home could minimise the gap in prior to school numerical knowledge and experience for preschoolers of lower SES background.

Overall, these factors can influence the nature of the relationships between educators and families.

Parental Contribution to Mathematical Early Learning

It has been well documented that children's cognitive development and academic achievement can be influenced by their home environment (Melhuish et al. 2008), especially in relation to home support towards school readiness. Promoting school readiness is suggested to be an effective way to raise children's achievement levels (Bleach 2015). However, it is noted that early numeracy has received less attention than early literacy, particularly in terms parental contribution in preparing their children for formal schooling (Anders et al. 2012).

Bleach (2015) aimed to improve the numeracy outcomes of children of lower SES families from infancy to 6 years of age by improving the skills of parents as well as practitioners in supporting children's numeracy attainment. The program ran over a 3-year period in Ireland. It involved parent numeracy workshops and practitioner training. The majority of parents who took part in the research indicated the program increased their involvement with their children's numeracy learning in the home. Importantly, the overall outcome of the program indicated that numeracy skills of the children in the program increased over the course of the 3 years.

Similarly, a study by Anders et al. (2012) involved five hundred and thirty two preschool children in Germany. The research aimed to explore early numeracy experiences of the children through observations as well as questionnaires and interviews. Of particular interest was the home environment. The findings suggested both the quality of the numeracy and literacy stimulation in the home had positive effects on children's overall numeracy skills. It was found that adequate literacy skills are important for early numeracy skills. Significantly, the study highlights the impact home learning environment has on shaping children's cognitive processes at an early age, which in turn influence their mathematical conceptions.

Two studies mentioned earlier in this chapter—DeFlorio and Beliakoff (2015) and Siegler and Ramani (2008) had similar results. The studies stressed the importance of parental involvement at home in relation to mathematical learning at an early age. Though both studies compared families from different SES backgrounds, the result also suggested that values and beliefs that parents held towards mathematical education in general contributed to their involvement in mathematical learning in the home. For example, Siegler and Ramani (2008) found that simply by playing mathematical boards games with lower SES children increased a child's mathematical knowledge to that of a middle SES child prior to entering preschool, strongly demonstrating the impact that educational experiences both in the home environment and elsewhere can have on early numeracy skills. The authors found that parents who did get involved had a belief that their involvement mattered for their children's development.

Parental Values and Beliefs

Parental involvement and practices in a child's education at home can derive from values, beliefs and aspirations that parents hold towards their child's education (Davis-Kean 2005; Phillipson and Phillipson 2007). In other words, the initiatives parents take in engaging with their children are pretty much driven by their own beliefs about the value of education and the aspirations they have for their children as found in a study in England (Siraj-Blatchford 2010). The latter study investigated the personal and social backgrounds of minority group families in England and the quality of learning support and environment in the home. The study consisted of individual case studies of children and their families who succeeded against the odds of disadvantage in their community. Families in this study strongly believed education to be key to their families' economic success and employment opportunities. These parents also had high expectations for their children and had aspirations of their children attending higher education leading to successful future careers. It appears that these expectations originated from values, beliefs and aspirations that parents hold and could be considered as a feature of cultural capital that could affect their actions.

Giallo et al. (2013) looked at the gaps of knowledge between maternal and paternal involvement in the home, however an element of interest they examined was the level of parent self efficacy and how this affected their involvement with playing, learning and activities with their young children. They found there was little difference between mothers and fathers based on employment status. However, what they did find was parents' self efficacy greatly contributed to the level of involvement the parent had with the child in the home environment. If a parent believes that their child is difficult to manage they underestimate their parenting ability, therefore in turn reducing the level of engagement with the child in the home. This is another example how parent involvement is affected by their values, beliefs and aspirations.

As previously discussed, DeFlorio and Beliakoff (2015) found disparities in the home environment as a contributing factor to a child's mathematical knowledge at the beginning of kindergarten. The authors argued the disparities were partly attributed to parents' beliefs about early mathematical learning. The beliefs parents held towards early mathematical learning varied between SES, with parents of lower SES considering kindergarten more important than the home environment for learning mathematical skills, therefore engaged less with their children in the home environment. The middle SES families reported engaging in a larger range of mathematical activities as well as a higher frequency of engagement. These results support the body of knowledge that demonstrates how parents' beliefs affect the levels of involvement they have with their children's learning in the home environment.

Another contributing element that can be considered to influence parental beliefs and aspirations is their perceptions of life context variables and how these perceptions contribute to their involvement (Tekin 2015). The research by Tekin was conducted in Turkey involving 374 families of children in first and second year of primary school. The author explored parental perceptions of personal knowledge and skills for involvement activities and parental perceptions of personal time involvement in activities. In essence, the study explored how parents viewed their own capacities and how their beliefs influenced their active involvement with their children's education. The findings suggested positive parental perceptions leading to the understanding that parents believed they had enough knowledge, skills, energy and time to be actively involved in their children's education. Interestingly, this study found demographic factors such as marital status, education level, employment status and number of children did not influence parental beliefs contributing to parental involvement.

See and Gorard (2015) explored whether parental attitudes and behaviors were associated with educational outcomes of children. Their study involved an extensive review of eight electronic databases with the key variable being parental involvement. Parental involvement was considered to be any strategies or behaviors that contributed to engagement in education in a formal manner; and educational attainment considered to be school readiness, cognitive ability and educational participation variables. The review found that the attitudes and beliefs parents held towards education and how these attitude and beliefs in turn influenced their children's views towards education, as one of the more crucial contributing factor to educational success.

The studies reviewed thus far show that parental values and beliefs are imperative for shaping not only their engagement with their children's learning but also the shaping of children's own attitude towards learning and education. Parental values and beliefs clearly underscore how parents access the appropriate resources for their children and how they actively participate in their children's learning (Phillipson 2013). An awareness of the nature of such influences can inform interventions to support parents.

Methods

This study employed a mixed method approach to survey parents. A 65-item questionnaire and semi-structured interviews were used to collect data on parental perceptions on educational and learning capitals, and early mathematical learning. In this chapter, interview data are used to supplement the findings from the questionnaire.

Participants

The participants of this study were 23 families that have children attending early childhood learning centres, kindergarten or foundation year at a school in Victoria.

These parents (5 fathers, 13 mothers, 3 guardians, 1 sibling and 1 grandparent) responded to the Family Educational and Learning Questionnaire as part of a pilot study in validating the questionnaire for the Numeracy@Home project. Two of the parents also voluntarily participated in interviews seeking information on their home engagement with their children.

Family Educational and Learning Questionnaire

The pilot version of the Family Educational and Learning Questionnaire (FELQ) surveyed parental perceptions of their children and their access to educational and learning resources, and their views around what kind of early learning in mathematical concepts is essential to happen before schooling and who should be responsible for those learning.

The questionnaire consisted of 11 items on family background and child's schooling information, 40 educational and learning capital items and 16 items of mathematical knowledge and skills. It is important to note that the participants were invited to participate through personal approach and the instrument was administered by project team members individually with the parents, where items were explained one by one and responses recorded electronically. Where necessary, the instrument was administered by a bilingual project member. Readers can therefore have more confidence in responses than had the participants been enthusiasts completing the survey individually.

The educational and learning capital items were responded on a four-point Likert scale of "strongly disagree", "disagree", "agree", and "strongly agree". One of the items for educational capital is "In my culture, children should work hard to achieve success at kindergarten/school". One item for learning capital is "At home, my child can always ask for help with their learning."

Parents were required to nominate whether mathematical knowledge and skills such as "To say the number words in order from 1 to 10 and backwards" should be learned before school and who should teach them. Parents had a choice of one of these responses for learning to happen or not before school—"no need to learn this before school", "useful to learn this before school" and "essential to learn this before school". Parents also had to choose as to who should teach their children about the mathematical knowledge and skills, whether it is "Early Educators", "Parents and Early Educators" or "Parents and families".

Descriptive and inferential analyses were conducted on the data to obtain an overview of parental perceptions on access to resources and their views about early learning. The responses to the 40 items on educational and learning capitals were reduced to five educational capitals and five learning capitals (based on the actiotope model) by averaging the responses into composite scores, presented in the form of an index on a four-point scale.

Cross-tabulations of responses for parental perception of early mathematical learning and who should be responsible for the learning were computed using counts. Independent sample *t*-tests were conducted to see whether there were differences in parental perceptions for boys and girls.

Semi Structured Interviews

The additional semi-structured interviews were conducted with the two parents using questions to supplement the results from the survey. The interviews consisted of questions on how families perceive learning in the everyday context and the interactions that their children participate in that promote early mathematical learning.

The interviews were transcribed and carefully read through twice by the second author. Thematic analysis was used to discern patterns of parental views on access to resources and their engagement with their children's early learning. Key concepts were coded and sorted accordingly to aspects of capitals and early learning. The coding and sorting of the data were checked by the first author to further ensure the credibility and trustworthiness of the interpretation and results of the interview data. Both parents and their children's names are replaced by pseudonyms when the data are presented in the results.

Results

Most of the parents in this study were secondary school educated (52 %) with six of them vocational trained and four of them being tertiary graduates. The parents were of mixed ethnicity including European, Afghani, African, Chinese, Indonesian, Samoan, Islander and Pakistani backgrounds, speaking diverse languages at home. These parents reported on their children (11 girls and 12 boys) aged from 2 to 8 years old, with 17 of the children's age ranging from 3 to 6 year olds.

Though six of the parents preferred not to answer regarding their family income, it appeared that majority of the remaining parents reported that their family income fell below $50,000 per annum, similar to families reported by Wilkins (2015) that fell within the low SES bracket.

Parental Perceptions of Educational and Learning Capitals

Parental perceptions of their educational capital were constructed as *economic* (my family has sufficient financial capacity), *cultural* (my culture encourages high achievement), *social educational* (my child has support from home and at school), *infrastructural* (my child has all the physical resources they need) and *didactic educational* (teaching approach suits child's needs). Parental views of their children's learning capital were seen *organismic* (my child is physically and mentally

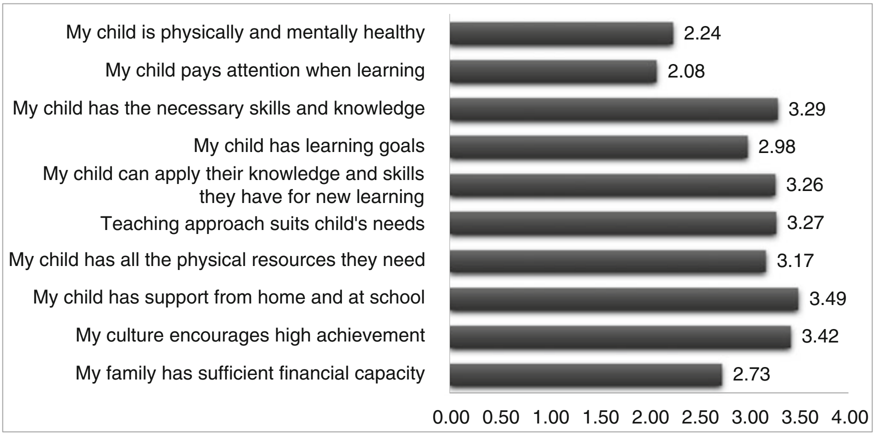

Fig. 8.1 Parental perceptions of educational and learning capitals for their children's schooling, with the *bottom* five statements relating to educational access whereas the *top* five statements relating to their children's learning capacity

healthy), *actional* (my child has the necessary skills and knowledge), *telic* (my child has learning goals), *episodic* (my child can apply their knowledge and skills for new learning) and *attentional* (my child pays attention when learning).

Figure 8.1 shows the distribution of parental perceptions of the ten capitals. Parents had positive responses to all of the capitals except for three. Parents felt that they did not have sufficient financial capacity to support the education of their children and this is not surprising as the parents in this study were from a disadvantaged community with many of them within the low SES bracket. Parents with sons seemed to report lesser financial capacity than parents with daughters. However, this difference is not (statistically) significant. These parents also believed that their children paid less attention when learning and had poorer health. When compared between child genders, girls were considered to be significantly worse off than boys in their health by their parents ($t = 2.44$, $p = 0.03$).

Parental Perceptions of Early Mathematical Learning

Parents responded to items that asked about their children's motor skills development, counting ability, comparing and classifying of length and shapes, telling of time, reading numbers and names (see Table 8.1).

Parents felt that it is useful (39 %) or essential (48 %) for children to be able to catch a ball before going to school. On the other hand, 61 % of the parents thought that it is useful for children to learn how to hammer with 35 % of them thinking it is not a necessary skill to learn before going to school. Many of the parents (61 %) also had a view that both educators and families should be responsible for teaching

Table 8.1 Parental indications of the importance of mathematical learning prior to starting school and who is responsible for the learning

Items	No need to learn this before starting school	Useful to learn this before starting school	Essential to learn this before starting school	Early educators	Parents and early educators	Parents and families
1. To play a DVD or take a photo	8	12	3	3	13	7
2. To catch a ball	3	9	11	1	14	9
3. To hammer a nail	8	14	1	0	14	9
4. To say the number words in order from 1 to 10 and backwards	4	4	15	1	14	8
5. To work out how many objects are in a small collection (e.g., 8 spoons)	4	4	15	0	17	6
6. To compare two small collections to work out which has more (e.g., 5 and 8)	5	7	11	1	17	5
7. To compare two small collections to work out which has more and by how many (e.g., 5 and 8)	5	7	11	1	18	4
8. To compare two objects and work out which is longer	4	8	11	2	17	4
9. To compare two objects and work out which is longer and by how much	6	7	10	1	18	4
10. To know the meaning of most words for parts of the day (e.g., morning, sunset, lunchtime)	5	5	13	1	18	4
11. To work out the time on an (analogue) clock (e.g., it is half past 7)	6	8	9	1	18	4
12. To read and write their name	3	6	14	1	16	6
13. To read numbers from 1 to 10	3	6	14	1	17	5

(continued)

Table 8.1 (continued)

Items	No need to learn this before starting school	Useful to learn this before starting school	Essential to learn this before starting school	Early educators	Parents and early educators	Parents and families
14. To order numbers from 1 to 10	3	10	10	1	20	2
15. To know the names of shapes such as triangles, circles, and squares	5	5	13	1	18	4
16. To compare two objects and work out which is heavier	7	6	10	1	19	3

of these skills though some (39 %) thought that these skills should mainly be families' responsibilities.

When it came to saying number words in order from 1 to 10 and backwards and to work out number of object in a collection, majority of parents (65 %) were of the view that it is essential for children to know to how to count before starting school. The same view was reported for reading numbers 1–10. However, parents were divided between useful to know before school (43 %) and essential to know before school (43 %) in expecting their children to order the numbers 1–10. Furthermore, many of the parents (61–74 %) thought that the teaching of these skills is the responsibility of both early educators and families.

A proportion of parents (43–48 %) consistently rated that it is essential for their children to learn to compare and classify lengths and shapes. Many parents (57 %) also thought it essential for their children to name shapes before starting school. The other proportion were somewhat split in their perceptions that it is useful to know these skills (30–35 %) and that it is not necessary to know these skills (17–26 %) prior to starting school. Nevertheless, most of the parents (74–78 %) felt that both early educators and families should be responsible for the teaching of these skills.

Interestingly, though many parents (57 %) thought their children should learn to tell the time of the day prior to starting school, they did not expect the same for their children to tell time with an analogue clock. Parents were equally divided in their perceptions that it is a skill that a few considered necessary (39 %), others considered useful (35 %) and the minority considered not necessary to learn before starting formal schooling (26 %).

The next section illustrates how two of the parents who participated in the survey thought about their own involvement at home with their children's mathematical learning. The two parents, Nadia and Sally, participated in interviews which provided some key observations on their perceptions around their own role in preparing their children for future success.

Parental Interaction with Their Children

Both Nadia and Sally are passionate about how early mathematical learning is important for their children's readiness for school and later success. Both parents talked about how they are aware of the need for their children to gain fundamental numeracy and literacy knowledge through their everyday interactions and that this involvement existed for them from the time their children were born. Both parents believed that family and social support for education are important for their children to thrive.

Nadia has a 3-year-old son, Wahid, who has only recently begun attending the early learning centre at the participating school. Having arrived as a migrant to Australia within the past 5 years, she believes that her son's education is her responsibility, and hence, has intentionally kept a routine at home where she has introduced many basic mathematical concepts whilst getting Wahid to play and talk with her.

> With Wahid it's on daily basis. We used to sit for twice and three times in a day, so we can talk, because he's– a little bit of delay in his language. So it's both a focus on his language and his learning abilities, because he's good in learning different things and he is interested always, like reading in books and looking pictures, and these kinds of things. So it's both kind of things: his learning and his language. So we can focus on that, both things.

Through her efforts to ensure that her son converses with her more, Nadia has consistently introduced basic number writing, reading, counting and other more advanced mathematical concepts. Some of Nadia's conversations with her son intentionally involve mathematical concepts such shapes, colours and distance, and many of these conversations happen during their daily routine at home or during playing indoors and outdoors.

> His favourite book is "Where is the green sheep?" So we both are doing a "Where is the green sheep?" And then in that book, we are doing, "This is the far sheep. This is the near sheep. This is the up sheep. This is the down sheep."

> When we go to the playground, we talk about different colours and flowers and things like that. If we are in shopping centre, so yes, we talk about– he read the numbers from the boats

Figure 8.2 shows Wahid writing in random numbers from 1 to 10. Nadia has initially modeled the writing of 1–10 on top of the page and asked Wahid to copy it. Wahid was able to grasp his pencil and wrote the numbers he liked on the page and said them out loud as he did so. He managed to write the numbers, though not in the order his mother had written.

Similarly, Sally spends some quality time on a regular basis with her 4 year old son, Dillon to work on mathematical games and activities that gets him counting, adding and subtracting (as shown in Fig. 8.3). She views these activities as unintentional teaching times that gets her son to learn the basic mathematical knowledge and skills to get him ready for schooling.

> I try a couple of times a week, so probably a lot more but it's probably unintentional teaching that's going on like but yeah when we sit down… Probably two or three times a week.

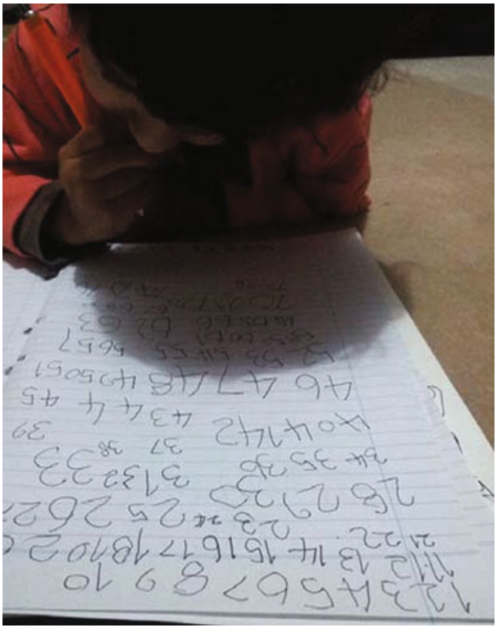

Fig. 8.2 Wahid concentrating on writing his numbers and reading them with his mother's help

Fig. 8.3 Dillon learning to count and add through flash cards in a game played with this mother

However, Sally insisted that she tries her best not to make her involvement with her son's learning too "teacher-like". Her intention is for him to learn about counting and numbers through their family everyday home activities. Whether it is in the kitchen helping Sally cook or in the garden helping his father with the gardening, Sally feels that Dillon gets to learn about addition and subtraction, and measurement.

> It might be more comfortable to grasp the concept or to ask questions or to just do it freely, so he's not pressured. It could be anything, like he could help me cooking, so we could do measuring and stuff. Or how many eggs "I need this many…" you know, things like that. So, he probably doesn't realise he's learning. He just thinks "I'm making cake with Mum" or whatever. And same with, you know, with his Dad in the garden.

Sally also felt that her son's learning does not only happen at home. For her, Dillon learns about mathematical concepts through their outdoor activities and these outdoor activities can be as simple as going to the shops or the park. Sally encourages her son to engage in counting, classifying, and sorting the things he collects during their outdoor activities, and enjoys her son's fascination with a measurement concept like "medium".

> Just when we go to the shops and the park nearly every day or every second day and we find stuff along the way. It's always, kind of, count or observe and see. He likes to collect things; gum nuts and leaves and… But we'll probably end up counting them first, then we'll go 'big' and 'little' or 'small', 'medium' and 'large'. He likes the 'medium'. It's hard to get that 'medium'. You know, things like that.

Hence, it is apparent that both Nadia and Sally engage in activities that encourage their children to explore mathematical ideas on an everyday basis. Some of these activities are intentional such as Nadia's modeling of writing numbers or Sally's number card games. However, a lot of the learning that Wahid and Dillon engage in with their parents stem from their everyday activities which involve everyday things and materials. Most importantly, both Nadia and Sally believe that their children's learning does not begin when they go to primary school, but rather it begins from infancy with parents taking early initiatives and schools extending these initiatives.

Discussion and Conclusion

This chapter explored parental perceptions on educational and learning capitals and their beliefs around early mathematical learning. When parents believe it is important for their children to achieve, they engage with their children in ways that contribute to their achievement regardless of their SES background (See and Gorard 2015). Parents in this study were mainly from a low income bracket and thus claimed poorer access to economic capital. However, they did not claim to have poor social and cultural capitals nor did they claim lack of goals and access to appropriate schooling for their children. Accordingly, most of these parents showed aspirational values for their children's learning of mathematical concepts by indicating that their children should learn counting, measurement, and telling time of the day before attending school. These beliefs and values echo Nadia's and Sally's engagement with their children at home where early mathematical concepts are learned intentionally and non-intentionally. Apparent from these findings is the fact that engagement in early learning can be more effective if parents have an awareness of the impact of such an engagement (Siegler and Ramani 2008).

Of significance, parents in this study thought that it was vital that their roles as their children's first mathematics educators be supported by early years educators to ensure that their children are ready for schooling. A proportion of the parents thought that they were capable of teaching their children early mathematical concepts through their everyday activities, as shown by Nadia and Sally. The teaching of simple counting, classification and measurements are seen as crucial mathematical activities in which families can engage. The belief and awareness that they can be their own children's educators is important for early family engagement in learning (Siraj-Blatchford 2010; Tekin 2015).

Nevertheless, parents in this study felt that early years educators' roles are equally important in ensuring their children's preparation for formal schooling. This perception could stem from parents of lower SES groups being more likely to consider kindergarten as more important than the home environment for learning mathematical skills (DeFlorio and Beliakoff 2015). As Nadia and Sally have shown, a lot of what their children learn about mathematics can happen at home as part of everyday activities. It is a matter of being aware of the impact that parents and families can have on children's learning regardless of the little financial resources they have. What is important is the belief in their children's learning capitals and capacity, and their own aspirations for their children's formal schooling. As previously found (e.g., Anders et al. 2012; Bleach 2015), parent contribution to early learning in promoting school readiness is an effective way to raise children's achievement levels. It also suggests that, rather than being a liability, parents can take collaborative responsibility with educators, and educators can find ways to support parents in this.

However, educators can play a keen role in encouraging early learning at home by linking learning that happens in the kindergarten with home related activities (Powell et al. 2010). When educators involve parents in what happens in the kindergarten or early learning centres, the communication between educators and parents provide clarification about the capitals that parents have access to including, their beliefs pertaining their children's health and knowledge. A good relationship between parents and educators provides opportunities for parents to engage with the relevant resources in their environment and to contribute further to their children's capacity in early mathematical learning. A good relationship between parents and educators also allows for educators to be aware of parents' aspirations for their children's learning, which can be crucial to children's future success (Melhuish et al. 2008; Phillipson and Phillipson 2012).

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Chapter 9
Involving Parents in Games and Picture Books

Julia Streit-Lehmann

Abstract The early years, prior to school, are a significant time period for the mathematical development of children. Parents play an important role in the early learning of their children through creating an inspiring home learning environment in everyday situations. Considering that the first mathematics learning takes place at home and at kindergarten, parents' involvement supports the academic achievements of young children learning not only during their first years but also during the whole school time period, so parents and pre-school-teachers should work together in a trusting relationship. The KERZ project discussed in this chapter shows that opportunities for a successful cooperation between parents and kindergarten staff have positive benefits for children's mathematics learning, at least in the short term. This project is an intervention study with a pre-post-test design investigating the impact on mathematics learning of families with young children regularly playing and reading games and books with mathematical content, like board, dice, and construction games and picture books dealing with counting, enumerating, and Piagetian pre-numerical competencies. These games and books are part of a "treasure chest" located at the kindergarten to be borrowed and used only at home.

Keywords Home learning environment · Kindergarten · Fostering · Playing · Reading · Games · Books

Introduction

This chapter deals with involving parents in a family numeracy and literacy project addressing pre-schoolers in their final year of kindergarten in Germany. The early years, prior to school, are a significant time period for the mathematical development of children. Parents play an important role in the early learning of their children through creating an inspiring home learning environment in everyday

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situations (e.g., Bronfenbrenner 2000; Cross et al. 2009). Considering that the first mathematics learning takes place at home and at kindergarten, parents' involvement supports the academic achievements of young children learning not only during their first years but also during the whole school time period, so parents and pre-school-teachers should work together in a trusting relationship. The KERZ project discussed in this chapter shows that opportunities for a successful cooperation between parents and kindergarten staff have positive benefits for children's mathematics learning, at least in the short term. This project is an intervention study with a pre-post-test design investigating the impact on mathematics learning of families with young children regularly playing and reading games and books with mathematical content, like board, dice, and construction games and picture books dealing with counting, enumerating, and Piagetian pre-numerical competencies. These games and books are part of a "treasure chest" located at the kindergarten to be borrowed and used only at home. The kindergarten teachers support the families' borrowing and returning processes logistically, encourage parents to borrow frequently, and provide recommendations or explanations when needed. In the first part of the chapter I introduce the concepts and aims of the KERZ study and present the first results of the pilot study, focusing on the effects of the intervention for children with a migration background. Next I consider our approach for working with parents. In the last part I explore and analyse suitable materials for playing and reading.

The KERZ Study

KERZ is a combined family literacy and family numeracy project addressing pre-schoolers in their final year of kindergarten (5-year-olds) and their families. Hence, KERZ is an abbreviation for "Kinder (er)zählen", which means "children count" and "children tell". In German these are very similar sounding verbs. KERZ is a joint development/research project conducted by mathematics education researchers from three German universities (i.e., Bielefeld University in the middle of Germany, Bremen University in the north of Germany and University of Education, Karlsruhe in the south), collecting data from different German states and regions. Special attention in this project is given to children from families with migration backgrounds and/or a low socio-economic and educational background, because both groups have been identified as educationally disadvantaged in Germany (Baumert and Schümer 2002). On the one hand, research suggests that a migration background is not necessarily problematic with respect to school mathematics learning and that achievement in mathematics is rather influenced by the socio-economic and educational family background. On the other hand, research by Prediger et al. (2013) on factors for underachievement in high stakes test in mathematics suggests that academic language proficiency in the language of assessment is more relevant than other background factors. Development of these basic communication skills is frequently related to the educational background of the parents/families in regard to their socio-economic status (Schmitman gen. Pothmann 2008).

The primary aim of the associated research study is to investigate how family-based activities related to (informal) early childhood mathematics can support early mathematical learning and to monitor possible long-term effects of the intervention in the first years of school. In their play as well as in their everyday life experiences at home and in kindergarten children develop a foundation of skills, concepts and understandings related to early numeracy (Baroody and Wilkins 1999). Parental involvement in this context includes dialogical family reading of mathematics-related picture books and playing board and dice games that require knowledge and abilities with respect to counting and comparing sets, enumerating, number words and symbols as well as spatial visualization.

A second research interest of the main study is to investigate the potential of such a home learning environment in contrast to kindergarten-based mathematical activities that are supposed to foster number-concept development. Hence, the study will follow a control group design with Group 1 being the treatment group in which children experience early mathematics activities at kindergarten by specially trained kindergarten teachers without parental involvement, and Group 2 being the control group with a focus on the home learning environment and no additional mathematical instruction at kindergarten.

Since the KERZ project is addressing families with a low socio-economic and educational background especially, one key obstacle that needed to be overcome was the lack of resources in the home, i.e., children's books and games suitable to foster early mathematics learning in the family situation. In solving this logistical problem, a strategy developed in the ENTER project[1] by Dagmar Bönig and Jochen Hering at Bremen University, was adopted. A treasure chest (see Fig. 9.1) is provided for the kindergartens involved in the project. This treasure chest contains a number of selected books, games and activities that are made available for the children to borrow and take home for a week. In order to assist non-German speaking families, translations of rules and text-reduced picture books are provided to encourage the families to talk in their native language(s) as well as in German.

While the kindergarten is the place where the materials can be borrowed and returned, it is made very clear to children, parents as well as the kindergarten teachers that these materials are for home use only in order to foster the home learning environment. Two of the treasure chest items, one construction game and one picture book, are described below.

The development of the mathematical competencies of the participating children is monitored by a combination of tests. At measurement point 1 prior to the intervention and at measurement point 2, after the intervention, children complete two mathematics tests: The EMBI-KiGa (Peter-Koop and Grüßing 2011), and the

[1]For details about the ENTER project see http://opus4.kobv.de/opus4-bamberg/frontdoor/index/index/docId/5697. Link checked at 2015-11-10.

Fig. 9.1 Treasure chest provided in the KERZ project

TEDI-MATH (Kaufmann et al. 2009). These are both task-based assessments. In addition at measurement point 1, all participating children complete the CPM (Coloured Progressive Matrices Intelligence Test) (Raven et al. 2010) in order to control this variable with respect to the impact of the intervention. An all-embracing individual intelligence diagnosis is not intended.

At measurement point 3 which is at the end of Grade 1 and one year after the children have begun school, a follow-up test is conducted with the DEMAT 1+ (Krajewski et al. 2002)—a standardised paper and pencil test, based on the curriculum for first graders in German primary schools. Like the TEDI-MATH, the DEMAT 1+ uses percentiles to rank children's mathematical competencies. The percentiles cover the whole range of abilities. For a child to reach the 90th percentile, for example, this means that only 10 % of his/her peers perform better.

The EMBI-KiGa is a semi-standardised one-on-one early numeracy interview based on the "First Year at School Mathematics Interview" developed in the context of the Australian "Early Numeracy Research Project" (Clarke et al. 2006) which had been published as a German adaptation (Peter-Koop and Grüßing 2011). It documents early mathematical competencies of children aged 3–6 years old. The EMBI-KiGa addresses early mathematics skills as identified by Krajewski and Schneider (2009). At an operational level the EMBI-KiGa provides information on two sub-tests. The first subtest involves 11 items related to the first two levels of the model of early mathematical development (see Fig. 9.1), such as comparison, part-whole schema and number-word sequence, while the second subtest explicitly focuses on developing counting skills. The EMBI-KiGa is task-based and supported by manipulatives in order to allow children, who for various reasons might struggle with their language, to demonstrate their developing mathematical understanding through the use of specific materials provided for each task.

The TEDI-MATH, originally developed by French psychologists, is a one-on-one clinical interview which compares the mathematical performance of 4- to 8-year-olds with their age group, standardized in half-year sequences

(Kaufmann et al. 2009). The TEDI-MATH covers counting skills, one-to-one correspondence, number words, part-whole-relations, and initial addition and subtraction skills. Both instruments, EMBI-KiGa and TEDI-MATH, were conducted with the complete cohort of children at both measuring points, before the borrowing from the treasure chest began, and after the intervention period. The intervention took place for 4 months in the pilot study (February to June prior to children's school enrolment in summer) but will take place for 10 months in the planned main study, that means for the entire last kindergarten year prior to the summer holidays.

Data is also collected from the parents before and after the intervention with respect to personal data about family background, education, language and migration background as well as their individual attitudes and beliefs with respect to mathematics and mathematics learning. The questionnaire developed for this purpose addresses parents' knowledge about content and curricula in school mathematics, their understanding of what mathematical competencies (if at all) a child should acquire before school entry and a self-assessment of their own mathematical competencies. All questionnaires are disseminated in the parents' first languages if necessary.

In addition, the pre-school teachers fill out questionnaires concerning individual assessments of the families' situations and language skills (fluency) as well as describing the pedagogical approach of their kindergarten and the explicit details about the operation of the intervention.

The intention for the intervention is that the children in the final year of kindergarten keep the selected materials for a few days (usually one week), use them with their families, return them, then select something new and so on. The borrowing process is monitored by the kindergarten staff, who also assist the children and families with explaining the contents and rules, if needed. In order to document what individual children have borrowed, each week the kindergarten teachers complete a chart, which was provided by the research team and hung up in the kindergarten room.

In addition, once a week the kindergarten teachers get the participating pre-schoolers together to talk about and share their experiences and to create interest in borrowing materials that peers have enjoyed reading and/or playing at home. Examples of the questions used are:

What was the book/game about?
How did you like it?
Who did you read it/play it with?
How often did you read/play it?
Which numbers do you have to know to play the game?

The preschool teachers also arrange parent-teacher conferences and social gatherings in the afternoon to invite parents to learn more about early mathematics learning. They present the treasure chest with its books and games and encourage parents to taking part in a committed way. Information about the frequency of usage of the books and games at home and about exactly which materials every child borrowed is also part of the raw data set.

The participants of the main study will include around 1000 children in 50 kindergartens. Apart from the design of the study described above, some children in control groups will not participate in the borrowing process but they will be nurtured by especially trained kindergarten teachers to improve their mathematical competencies. The comparison between these two groups will help to clarify the question, and provide insight about which approach should be supported by human and financial resources in the future: training and further education in the early mathematics learning of kindergarten teachers, or encouraging, involving, and equipping parents in supporting their children, especially those who are educationally disadvantaged.

The KERZ Pilot Study

The pilot study was conducted at Bielefeld University during 2012–2015 with a sample of 57 children, attending three kindergartens. The kindergartens are located in a town in middle Germany with around 70,000 inhabitants near Bielefeld. The geographical position and the socio-economic level of their catchment areas vary across kindergartens. Kindergarten 1 (Kiga 1) is located in an older suburb, mainly inhabited by middle-class families with a high socio-economic status (SES) without migration background. Kiga 1 is pedagogically focused on psycho-motor development and natural science projects. Kiga 2 and 3 are mainly attended by children with a migration background. Kiga 2 is situated in a public housing estate near the city centre and is mainly attended by children with a Turkish family background. Children with five additional mother tongues were also found in the cohort. Developing language skills and cultural integration are the pedagogical focuses of Kiga 2. Kiga 3 is situated in a public housing estate outside the city in a satellite village. In addition to Turkish-German children and asylum-seekers coming from conflict areas in the Middle East, most children are from families originating in the former Soviet States. The pedagogical focus of Kiga 3 also is developing language skills and integration, on top of artistic projects.

The initial results of the pilot study suggest a relationship between the effectiveness of the KERZ project and the migration background of the children. However, it has to be taken into account that in this sample a family migration background correlated with low socio-economic status and low educational background. In order to refer to the development of the mathematical competencies of the children from the first to the second measuring point (MP1–MP2), a distinction between "strong enhancement", "slight enhancement" and "no enhancement" was made. However, "enhancement" does not only mean an absolute increase in competencies, because an increase could be completely explained by the increasing age and corresponding intellectual development of the children. The term

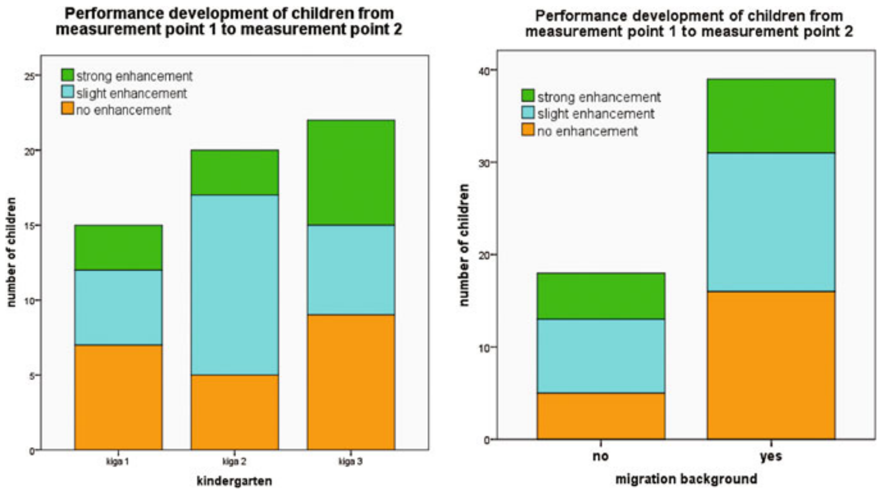

Fig. 9.2 Immediate impact of participation in KERZ on children's mathematical competencies (n = 57)

"enhancement" in this context refers to an enhancement relative to the particular peer group.

The three labels "strong, slight and no enhancement" correspond to the percentile ranks of the TEDI-MATH, and were confirmed by the data of the EMBI-KiGa. The distinction between these labels is quite severe to avoid false-positive interpretations: For example, "strong enhancement" means an increase of at least 50 percentiles of the TEDI-MATH from MP1 to MP2, or alternatively an increase between 25 and 49 percentiles, while moving out of the lowest fifth into the midrange or moving out of the midrange into the highest fifth. In addition, all EMBI-values[2] had to improve to get that label. "Midrange" means in this context the middle three-fifths. When children improved between 25 and 49 percentiles, while staying in the midrange or having stagnating EMBI-values, they were considered to have a "slight enhancement", and it is the same when children improved between 5 and 24 percentiles with all EMBI-values improving.

Both graphics in Fig. 9.2 show the performance development of the participating children (n = 57). The three bars in the graphic on the left correspond to the three kindergartens. Kiga 1 is predominantly attended by children from families with a middle-class SES, without a migration background. About 90 % of all children

[2]The EMBI provides two kinds of information for each child. Firstly a (numerical) point score between 0 and 11 that shows how many of the 11 items of first sub-test (*mathematical precursor skills*) have been solved correctly, secondly with respect to the second sub-test (*counting)* a (ordinal) growth point, that identifies his/her level of counting skills on a range between 0 and 6. These two measures are called "EMBI-values".

attending Kiga 2 and Kiga 3 have a migration background, mostly belonging to families with a low SES.

As indicated by the large blue and green areas in the bar diagram on the very left of Fig. 9.2 in all three kindergartens, more than half of the children clearly improved their mathematical competencies either slightly or strongly.

Eight of the 15 children in Kiga 1 demonstrated an improvement from MP1 to MP2, of these 8 children 3 showed a "strong enhancement". Fifteen of the 20 children in Kiga 2 improved from MP1 to MP2, again 3 of them showed a "strong enhancement". Thirteen of the 22 children in Kiga 3 demonstrated an improvement, while 7 of these 13 children showed "strong enhancement". The orange sections of the diagrams in Fig. 9.2 represent those groups of children that showed "no enhancement". It is important to notice that the group of children who showed no enhancement includes those who already performed highly prior to the intervention and therefore could not demonstrate further substantial gains. In each of the three kindergartens there were a few very highly performing children before the intervention. For example, a child reaching percentile 92 at MP1 and percentile 95 at MP2 is labelled "no enhancement".

The same sample is represented on the right side in Fig. 9.2. Here the children are not grouped by their kindergartens but by their migration background status ("yes" or "no"). The two bars appear in a very similar way: Roughly one-third of both groups do not show any enhancement (orange), and two-thirds show a slight or even strong enhancement. The graph on the right side in Fig. 9.2 shows that around one-third of all children in the sample are not affected by KERZ, independent of a migration background. Thirty-nine of the 57 children in the sample have a migration background. Thirteen of the remaining 18 children without a migration background demonstrated improvement (see blue and green sections in the bar diagrams). Of the 39 children with a migration background 23 children showed improvement. Roughly two-thirds demonstrated "slight" or "strong enhancement", which might be interpreted as a positive result. So, the immediate effects are irrespective of the existence of a migration background, and no significant differences concerning the mean of intelligence scores between the three kindergarten groups have been found.

This picture changes when considering the results of the follow-up test at the end of Grade 1. The analysis of the performances on the DEMAT 1+ clearly shows that mainly children from families without migration background reached percentile ranks higher than those of MP1. Although this finding suggests a sustainable success of the intervention for children without migration backgrounds, children with migration backgrounds mainly reached percentile ranks equal to or even lower than at MP1.

Figure 9.3 shows the results of the follow-up test at the end of Grade 1, one year after the intervention. Only 42 out of the 57 children participated in the follow-up test at the end of Grade 1 as some children had moved away, did not attend school during the testing period or did not start school in the first place. As the right side of Fig. 9.3 shows twenty-nine of the 42 children who participated had a migration background, while 13 children did not. In order to characterize their development four categories were used: “performance like MP2 or better” (green), “performance better than at MP1” (light blue), “performance like MP1 or lower” (red) and “negligible change” (grey). Hence, the green and light blue sections represent positive results, because the children represented by these green and light blue sections showed a better performance than at MP1. The grey sections symbolise the absence of changes, irrespective of the different performance levels (however, this applied to very few children). The red sections represent all of those children who after initial improvement observed from MP1 to MP2, one year later at MP3 showed results equal or even lower than the percentile reached at MP1, which means that the intervention might only had an immediate positive impact but not a sustainable one: These gains are lost. The bars of the left diagram in Fig. 9.3 show large red sections in Kiga 2 and Kiga 3, so many children from these kindergartens could not maintain their achievements. They only reached percentile ranks equal or even lower to their ranks at MP1. This leads to the fact that the two bars on the right side in Fig. 9.3 have lost their similarity observed in Fig. 9.2.

With respect to the transition to school, the data indicates that a sustainable benefit of the intervention was related to family background. Predominantly children without a migration background maintained their progress, while this does not hold true for the children with migration backgrounds (see Fig. 9.3). The two bar diagrams on the right hand side of Fig. 9.3 show that only two of the 13 children without a migration background are represented in the red section (i.e., performance at MP3 $\leq$ performance at MP1), in contrast to 20 out of 29 children with a migration background.

While children from all three kindergartens showed similar engagement and cooperation, the migration status may explain the variance between Kiga 1 (children mainly from middle class families) and the other two kindergartens. Children with migration backgrounds (in our sample mainly from families with low

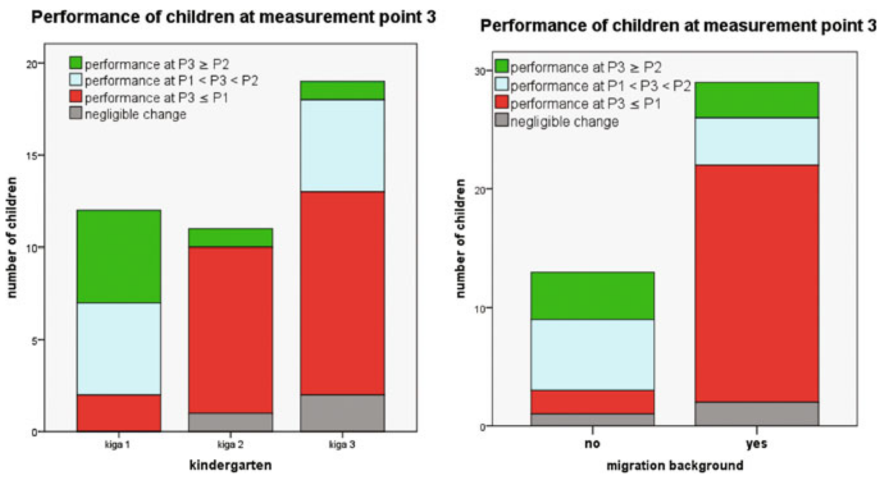

Fig. 9.3 Long-term impact on mathematical competencies (n = 42)

socio-economic and educational background) clearly demonstrated less mathematical achievement at the end of Grade 1 than their peers without a migration background (from predominantly middle-class families). The main study will help to clarify whether this only holds true only for the sample used in the pilot study or more broadly.

Working with Parents—Involving Parents

The German expression "Elternarbeit" (from "Eltern" = parents and "Arbeit" = work) has more than one meaning (Streit-Lehmann 2015). It does not only mean working with parents, which marks a part of that work the pre-school teachers have to do, but also that kind of work parents do at kindergarten or at school. If parents mention this expression they usually mean the work they do by themselves in the context of school or kindergarten, like beautifying the playgrounds and schoolyards, helping the teachers with supervising the children during rambling or museum trips, or baking cakes for pre-school parties. It strongly depends on the socio-economic composition of the catchment area of the institution whether as to whether it is easy or not to engage parents to do that kind of "Elternarbeit". For most teachers, "Elternarbeit" is just the kind of work they have to do to encourage parents to engage in baking, gardening or supervising, that means "engaging parents in school concerns somehow" (Sacher 2014). Usual instruments for this kind of working with parents in Germany are letters to parents and parent-teacher conferences held a few times each year.

In addition, many German teachers, child carers and educators, psychologists and social education workers recognise "Elternarbeit" as a necessary institutional reaction to deficient parental upbringing and educating. There is a wide range of educational programs for parents aiming to foster the parental upbringing and educating competencies. The characteristics of these programs vary with respect to the pedagogical and ideological concepts, the target group, the fee requirements, and the preventive or curative continuity. Sometimes, special groups are aimed at supporting like families with many children, young single mothers, or families from other cultures.

Seeing "Elternarbeit" as working with parents to enable them to accompany and co-create their children's education is another way of understanding this term. In the KERZ project parents are invited to act in this way. There is empirical evidence in the context of PISA studies to suggest that educational success is strongly related to families' financial resources (see Schwarz and Weishaupt 2014). Families' resources might play a role when looking at the availability of suitable toys, games and books at home. The KERZ treasure chest offers free availability plus recommendations and guidance by the preschool teachers which might be able to reach many families.

Like the participating children, their parents also represent a heterogeneous group. The commitment of the parents during the intervention period in the pilot study showed that diversity: Some families participated intensely and used the treasure chest books and games frequently, other families almost did not join in as their low borrowing rates show. The reason for low participations is not automatically low educational awareness or the lack of engagement. Some high-educated families attaching importance to the educational success of their children gave the information that they already owned all or almost all offered books and games, so for them there was no motive to participate in the KERZ project. These families obviously cannot be reached by the KERZ project, but with respect to the aim of the project this may not be relevant. Another group of families experienced a combination of unemployment, language barriers, a complex of health and addiction problems, and the lack of social participation and integration. In these cases the simple invitation of joining in the KERZ project was already an overtaxing.

By creating a personal relationship between parents and preschool teachers the majority of parents can be reached well. Some parents usually do not attend parent-teacher conferences, especially those with grave language barriers, but often there is the opportunity for positive face-to-face encounters in passing while parents bring and pick up their children (for this kind of fast meeting there is an indicative expression in German: "between-door-and-hinge conversation"). The preschool teachers play an important role by making use of these occasions and encouraging parents to use or return the KERZ books and games.

In the pilot study in two of the three intervention kindergartens the parents were invited to join board game afternoons where the parents could get to know and try the games from the treasure chest. The preschool teachers assisted and motivated the parents, answered questions and explained the rules of the games. According to experience that kind of offers to parents is more successful when parents can join them spontaneously, for example while picking up their children in the afternoon and just staying a little bit longer in the kindergarten then. Extra appointments like parent-teacher conferences in the evenings are kept less frequently by parents with language barriers.

To attenuate the consequences of language barriers rules of the games and book texts if existent were translated into the main languages of the participating families. In the KERZ pilot study these languages were Turkish, Russian, Polish, and Arabic. Some parents do not have enough literary language competencies to read together with their children, not even in their mother tongue. In these families elder siblings often assume the role of the reader and read and play with the participating children. In the KERZ project parents were encouraged to deal with the KERZ materials in their mother tongue if they did not speak enough German to deal with German books and games. Enabling parents to play and read together with their children and explore early mathematics experiences is the main goal of the KERZ project—the language used while doing so is secondary.

The KERZ Books and Games

In this part of the chapter a selection of the games and picture books from the KERZ treasure chest are introduced in more detail, using specific criteria to analyse and characterize materials with respect to their didactical properties and mathematical content (Schuler 2013). The collection of all games and books used in the KERZ project originate from a prequel project from Bremen University and will be evaluated during the KERZ project. Some books are picture books without any text, like “Where Is the Cake?” by the Chinese-Dutch illustrator Thé Tjong-Khing. This book tells a lot of simultaneous stories in busy scenes at every spread developing in a surprising way while paging forward. It provides plenty of occasions to raise questions like “What is happening here?”, “What did he do previously?”, “What do you think happens next?”, “What could be the reason for him to act this way?” There are no comments concerning any didactic aims of this book given by Tjong-Khing himself, but the German publishing house characterizes the book as “very suitable for early language promotion” and “inviting to look closely and combine”. While looking at the book children and parents page forward and backward, curious and motivated, because everyone wants to know if the red bottom of the chameleon is caused by the wet paint of the bench or not, or wants to know what happened to the eleventh duckling, talking about what is happening or could happen. The book contains a few arithmetic features. Sets of animals like the eleven ducklings provoke counting and comparing. Gelman and Gallistel’s (1986) abstraction principle leads to the realization that it’s always two, no matter if it’s two rats, two dogs, or two frogs. An educated and prepared reader is needed to bring this to the child’s attention. For the classical Piagetian (1952) pre-numerical competencies it is the same. Figure-ground perception is another mathematical

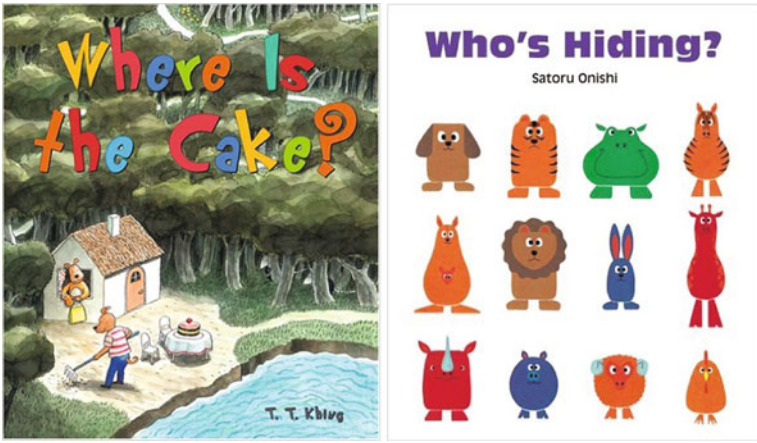

Fig. 9.4 The books “Where Is the Cake?” and “Who’s Hiding?”. By courtesy of Moritz Verlag GmbH

focus of the book. For example, there are three monkeys hiding in the trees and being partially covered by them. Describing the monkeys' positions accurately trains the correct usage of prepositions which is recognised as being important for the development of number concepts (Fig. 9.4).

Another set of books has short texts, such as "Who's Hiding?" by the Japanese artist Satoru Onishi. This book is suitable even for very young children. On each spread eighteen simply drawn and coloured animals are laid out in the same order. A rear view is laid out on the endpapers. Questions alternate on each spread, like "Who is hiding?", "Who's crying?", "Who's angry?", and these are answered by very little but typical changes in the animals, like the rabbit shedding a tear unobtrusively or the blue bear scowling. So, the Piagetian concept of classifying is the main theme of this book, and it invites the children and parents to take a very precise look at each page.

A third group of books contains rhymes to say and sing along with and to foster remembering the content. These texts can also be translated to make it easier for non-German parents to understand all words and the meaning of the text, but the rhythm, rhyme and charm of these texts often become lost in translation. However, it seemed to be important to provide a wide range of different book types to reach as many families as possible (Fig. 9.5). A participating mother fed back after the intervention:

> We really loved "Where Is the Cake?". Such a funny book, full of episodes! We [she and her son] sat down for hours, talking and assuming, and discovering some new details every time we looked into it. (translated)

Another mother annotated:

> Not my cup of tea. Missed the text. Also my son found it boring. (translated)

The KERZ treasure chest contains a selection of board, dice and construction games with different mathematical content. In some Memory games the children match numerals to the corresponding number of spots, animals or items. Some Ludo-like dice games foster the recognition of dice points and the composing and decomposing numbers. In the construction game "Make 'n' Break Junior" multiple levels of difficulty let children improve building skills at their own pace. The game contains 27 wooden cuboid bricks of the same size but in four different colours and 50 task cards. The goal of this game is using the bricks to rebuild the building

Fig. 9.5 The construction game "Make 'n' Break Junior". By courtesy of Ravensburger Spieleverlag GmbH

shown on the card as quickly as possible. The pictures of the buildings on the cards are realistic like photographs and vary with respect to their complexity: Three, four, five or six bricks have to be used to build the buildings correctly. Other options for varying the challenge can be given by additional rules like “Use one hand only!” Necessary mathematical skills are finding out, how many bricks are needed to build the model and figure-ground discrimination. Every element of the picture matches one real brick, bonded by their spatial arrangement, which requires one-to-one correspondence and the understanding of the counting principles described by Gelman and Gallistel (1986). In addition, the three Thurstone skills (see Maier 1999) of visualization, spatial relations, and spatial orientation are needed to identify and build the model. Make ‘n’ Break Junior has a funny and colourful appearance and seemed to be much more attractive to the children than other construction games using bricks without colour or having a boring packaging. This is a relevant finding with respect to the aim of reaching and including as many children as possible. Also a participating mother commented:

> We loved most the games, because they look funny and bring my child to logical thinking without being under pressure to perform. (translated)

Summary and Recommendations

This chapter explored the impact of deliberately involving parents in fostering the early mathematical competencies of their children during their last kindergarten year. First results of the KERZ pilot study have shown positive immediate effects for the majority of participating children, but the intervention is not considered to be successful until there are positive gains in children’s mathematics development between measurement point 2 and the follow-up at measurement point 3 one year after children begin school: The sustained impact of the intervention appears to be strongly associated with family background. The children without a migration background maintained their rankings predominantly but the children with migration backgrounds predominantly did not, although both groups participated in the project with similar levels of engagement. The possible reasons for this result will be investigated in more detail in the planned main study. Maybe the effectiveness of classroom instruction in school mathematics varies for children with and without migration background (who also come from disadvantaged families). In this context our findings confirm the results of a national study focusing on children’s achievements in the subjects German and Mathematics at the end of Grade 4 (Stanat et al. 2012). This study found migration background-related disparities in the areas of reading, comprehension and mathematics in all German states (Haag et al. 2012). When controlling for the variable of socio-economic status, the disadvantages in learning for children with migration background were clearly reduced. The very different types of learning environments in kindergarten and school also have an influence on the involvement of parents. Many educators in kindergartens are aware of the importance of the home learning environment which includes the importance of strong personal relationships and holistic learning situations. Especially allocated educators care for a small group of children, based on family-like structures and maintaining a lively and close communication with the parents. These features of Kindergarten settings connect strongly with the KERZ approach. A mother participating in the Kerz project explained:

> I appreciate educational programs in kindergarten for all children, but I don't want just a preparation for school. Fostering should include everyday competencies like physical skills and emotional skills. (translated)

The situation usually changes in the school enrolment: The children's learning environment becomes more separated from the parents. Parents are not requested anymore to play and read together with their children but to monitor the children's homework. Parental support gets a new focus. Perhaps this could be part of the explanation of the concerning results for children with migration backgrounds at MP3, and it could be an argument for using holistic learning opportunities such as the treasure chest activities in primary school also, not only in kindergarten. Continuing to involve parents in educational processes even after school enrolment, but without the pressure to achieve test scores, could help maintain the positive effects of early fostering as observed at MP2 sustainable. Good relationships between educators and parents have the potential to support positive transitions to school by building bridges between home and school or prior-to-school settings and school (Goff and Dockett 2015), but perhaps for children with migration backgrounds in the KERZ pilot study this potential was not used as effectively as it could be. Another reason that might have had a negative influence on the performance of children from this group is the fact that the DEMAT 1+ is a paper and pencil test that has high demands with respect to reading, which serves as a disadvantage when it comes to testing mathematics skills and understanding for children from non-German language backgrounds.

However, some requirements for the successful involvement of parents emerged from the data that lead to useful recommendations for promoting partnerships between parents and educators that foster young children's mathematics learning. First of all, a personal and trustful relationship between parents and kindergarten staff is very important, not least because "potentially suitable games need a competent educator with regard to didactical and conversational aspects" (Schuler and Wittmann 2009). Kindergarten teachers and child carers are able to motivate and encourage parents to participate and accept suitable offers. In Germany, people with low German language skills are often disadvantaged in social participation and in understanding the characteristics and requirements of the German educational system. This might impair the academic success of their children. Another group of families suffers from a combination of low education, unemployment, and the lack of perspectives and motivation. Although these parents usually have enough time to care intensely for their children and put a lot of effort into their children's learning processes they often have difficulties in spending the required time and energy on their children. Without improving the future perspective for those families it is very difficult to reach them.

Having an appreciation of cultural diversity and not assuming that non-German mother tongues are automatically a complication are two additional factors for a successful cooperation between parents and kindergarten staff. Providing translations of important information including book texts and rules of the games used in the KERZ project seemed to be another helpful approach. Overall, creating and keeping up steady conversation might be the key to inviting, encouraging, and motivating parents to participate in the educational processes of their children.

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Chapter 10
Do Hong Kong Parents Engage in Learning Activities Conducive to Preschool Children's Mathematics Development?

Richard Kwok Shing Wong

Abstract The success of students from Chinese-heritage cultures in international tests of mathematics has led researchers to examine whether the reasons for these students' success lie in their superior mathematics-related cognitive skills, the school environment or the home. This book chapter contributes to the research literature by focusing on the contribution of parents from Chinese-heritage cultures to their children's success in mathematics. Specifically, I examined the use of interaction strategies fostering counting skills within a sample of 174 families with preschool-aged children from Hong Kong, a city that ranked third in the latest PISA results in mathematics. In addition, I also explored whether parents' interactional behaviour was related to factors such as socioeconomic status (SES), class level of the children, parents' proficiency in and past motivation to learn mathematics. The results showed that the three most frequent strategies were counting forward, using real objects to illustrate mathematics concepts and providing prompt questions. SES was only a significant predictor for the use of prompt questions, while children's class level contributed to the strategies of counting backward and using worksheets. Finally, parents' motivation significantly predicted the use of stories to teach number concepts. Implications for future studies are discussed.

Keywords Hong Kong parents · Preschool children · Mathematics learning · Interaction strategies · Chinese-heritage culture

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Introduction

The release of the latest PISA results on mathematics (OECD 2014) was reaffirming to educational authorities, teachers and parents in Asia. The three countries or cities that top the performance chart all share a Chinese heritage: Shanghai-China, Singapore and Hong Kong-China. There are two important aspects of the results. First of all, the results are consistent with extant data showing the superiority of students in Chinese-heritage cultures in tests of mathematics achievements (e.g., Mullis et al. 2008; OECD 2001, 2003; Stevenson et al. 1986, 1990; TIMMS 2011; Wang and Lin 2009). Secondly, consistent with previous findings (Hsu 2007, cited in Anderson et al. 2010), there appeared to be little differences related to socioeconomic status (SES) in the top-performing countries. This presents a sharp contrast to past studies conducted in English-speaking countries which often have found a strong SES effect on children's academic achievement (e.g., Bradley and Corwyn 2002; Chiu and Khoo 2005; Chiu and Zeng 2008; Perry and McConney 2010; Sirin 2005).

Different approaches have been used in order to explain the remarkable mathematics achievement of students in Chinese-heritage cultures. Some studies used the cognitive approach which focuses on how differences in mathematics-related skills contribute to students' mathematics achievement (for information related to the types of skills predicting mathematics achievement, see e.g., Kytala and Lehto 2008; Taub et al. 2008; Zhang et al. 2014). An important skill hypothesised to be important for mathematics achievement concerns counting (see e.g., Ryoo et al. 2014). In terms of pronunciation, a Chinese number tends to contain fewer syllables than the corresponding number in English (e.g., Dehaene 1997). For example, the number 577 has 5 syllables in Chinese compared to 9 syllables in English (compare *wu3 bai3 qi1 shi2 qi1* 五百七十七 vs. five hundred and seventy-seven). However, Miller (1987) found no reliable cross-cultural differences in the way children skip or double count objects. Later cross-cultural comparisons also revealed minimal differences in general skills (e.g., physical size comparison) predicting mathematics achievement (e.g., Rodic et al. 2014).

Other studies used the sociocultural approach which emphasises how agents in the environment, such as teachers and parents, create the context for children's success in mathematics (e.g., Chen 2005; Hung 2007). Gu (2006) found that the school learning environment had more effect on school mathematics achievement in Hong Kong than in Canada in a secondary analysis of the PISA 2003 data. However, the fact that school environment has a major role in students' mathematics achievement does not necessary imply that classroom teaching is "innovative" in Chinese-heritage cultures. Studies focussing on classroom learning revealed that teachers of mathematics in Chinese-heritage cultures tend to adopt a more teacher-centred approach in comparison to their counterparts in other countries (e.g., Leung 1995). There appeared to be much fewer peer interactions and small group discussions, since the teaching in the Chinese classrooms were mostly teacher-led and conducted in the whole-class setting. In addition, direct and explicit instruction and problem practice (within and outside classroom) are essential features of mathematics lessons in these cultures. A lesson typically begins with a revision of the content from the previous lesson, followed by the teacher's direct explanation of a mathematical concept, the presentation of sample problems and further in-class and take-home practice. Since classroom teaching may not contain the main reason for the students' success, other studies turned to the area of parenting practice and proposed that parents in Chinese-heritage cultures create the context for their children's success in mathematics by directly communicating to their children their academic expectations and other important values (e.g., academic pursuit is an important goal in life; success derives more from diligence than intelligence) (e.g., Ho 2000, 2006; Leung 2002; Leung et al. 1998; Phillipson 2009; Phillipson and Phillipson 2012). This view was confirmed, for example, in a series of studies focussing on school-aged children in Hong Kong which showed that parental expectations mediated the relationships between cognitive abilities and achievement in mathematics (Phillipson 2006; Phillipson and Phillipson 2007, 2012).

The brief review above suggests that parents appear to play a stronger role than teachers or cognitive abilities in shaping the academic achievements of students in Chinese-heritage cultures. Since the role of parents may vary with the developmental status of their children and because cross-cultural differences in mathematics achievement appear to emerge as early as the preschool years (e.g., Ryoo et al. 2014), examining the contribution of parents with preschool children is as important as studying parents with school-aged children. Past studies conducted in Shanghai (e.g., Gao 2010; Zhou 2006; Zhou et al. 2009) suggested that parent-child interactions relating to mathematics, especially counting (e.g., talking about numbers, modelling counting words and counting procedures), may serve as the primary developmental mechanism in promoting young children's number understanding in the early years. These types of interactions focus on facilitating children's attention to number, modelling basic number skills, and helping children to practice and apply new number skills. The study described in this chapter contributes to the literature and extends previous studies by focusing on how parents in Hong Kong facilitate their *preschool children's* mathematics learning before school becomes a primary source for mathematics education. In addition, since parents' attributes, such as their past motivation to learn mathematics and their proficiency in the subject (see e.g., Dandy and Nettelbeck 2002), might influence their behaviour, this chapter also explored whether these parental attributes influence parent-child interactions relating to mathematics. Specifically, I will address the following research questions:

- *What is the style of interaction between parents and preschool children in Chinese-heritage cultures?*

I addressed this question by exploring the types of parent-child interaction strategies relating to mathematics learning. The strategies include: the use of stories to teach number concepts, the use of real objects to illustrate a concept (e.g., counting the number of cookies on a plate. One, two, three. Three cookies), the use of prompt questions while children interact with real objects (e.g., when the child is about to get the chicken wings on a plate, the parents ask how many chicken wings there are on the plate), relating a mathematical concept to real life situations (e.g., ask children to assist buying things according to a shopping list), the use of statistics (e.g., creating a pictogram showing the total number of family members who like apples), counting forward (e.g., 1, 2, 3, 4, 5, 6, 7, 8…), counting backward (e.g., 30, 29, 28, 27….), and using worksheets. These eight strategies were chosen because they are all related to number concepts, especially counting, but differ mainly in the amount of contextualisation. The first five strategies are more contextualised than the last three, and rely less on rote memory/drilling. In addition, the strategies of interest are more specifically related to early mathematics learning than the types of family involvement traditionally described in the literature (e.g., cultural communication, social communication, homework supervision, and cultural activity as used in the PISA studies, see Ho 2006).

- *What predicts parents' interactional behaviour?*

I addressed this question by examining the factors that contribute to parents' use of specific interaction strategies. The factors include: SES, class levels of the children (at entry to preschool vs. at exit from preschool), parents' past mathematics proficiency in school (henceforth parental proficiency), and parents' past motivation to learn mathematics (henceforth parental motivation). SES is chosen because whilst there appear to be little SES-related differences in mathematics learning outcomes among school-aged children, the presence of initial SES differences in math-related parent-child interactions remains a possibility. Children's class level was also used because parents' choice of a particular strategy (e.g., the use of stories vs. using worksheets) might be sensitive to the class level of the children. Parental proficiency and motivation were included because I want to explore whether these variables influence parents' use of interaction strategies. Previous studies tended to focus on motivation of learners rather than their parents (e.g., Chen et al. 1996).

- *From parents' viewpoint, what enables students in Chinese-heritage cultures to have superior performance on international tests of mathematics in comparison to their peers in other cultures?*

I examined this question by exploring whether parents think that the success is due to factors such as students' diligence, presence of high-quality teachers, additional training provided by private tutors, difficult syllabus, parental devotion and expectations. Since two of these factors concern parents (parental devotion and expectations), I also explored whether parents' beliefs about these two factors were related to their interactional behaviour at home.

Hong Kong was chosen as the site for this study for three reasons. First, it has a unique history. Because of its status as a former British colony, new ideas from the West (e.g., the sociocultural approach to learning) were often assumed to reach the city before reaching the rest of China. Second, Hong Kong parents are known to be strong "interventionists". They might require their children to engage in developmentally inappropriate activities with the hope of increasing their children's competitive edge over other children. There were anecdotal reports of Hong Kong pre-schoolers spending too much time on after-school learning activities (HKET 18/07/2014). Finally, conflicting ideas are known to co-exist in the Hong Kong

Chinese culture. In the case of religion, Christianity, Buddhism and Taoism could co-exist within the same family. In the area of education, conflicting learning approaches (e.g., phonics and whole language in the area of language learning) are often adopted in the same preschool language classroom (personal observation). It is unclear whether math-related interaction strategies which differ in terms of contextualisation can co-exist. All these reasons make Hong Kong a fertile ground for a case study of parent-child interaction relating to mathematics.

Methods

Participants

The survey included 174 families whose children (83 females, 88 males) were attending either the first (N = 77) or final (N = 94) year of their preschools (in Hong Kong, preschool education lasts for 3 years and serves children between 3 and 6 years of age). For the younger age group, the mean age of the children was 42.67 months (SD = 4.28 months), ranging from 39 to 46 months. For the older age group, the mean age was 67.91 months (SD = 4.98 months), ranging from 60 to 72 months. Sixty seven of the children were enrolled in a competitive, high cost preschool (monthly school fees = HKD$ 3,200 or USD$ 411), which I characterise as a "high SES" school. The rest of the children were enrolled in a school located in a government-run shopping mall in a district that has one of the lowest median income levels and the highest unemployment rate in Hong Kong (Hong Kong Census and Statistics Department 2013), which I characterize as a "low SES" school. The school fees of the children in the low SES school were highly subsidized by the Hong Kong Government through a fee-subsidizing program (monthly school fees = HKD$ 1,409 or USD$ 181). With respect to the educational level of the parents, in the high SES school, 55.2 % of the fathers and 50.7 % of the mothers had a university degree or above. In the same school, 77.4 % of the families had household income higher than HKD$ 40,000 per month (approximately USD$ 5,155). In the low SES school, only 27.3 % of the fathers and 21.4 % of the mothers had a university degree or above. A total of 71.4 % of the families had monthly household income lower than HKD$ 20,000 (approximately USD$ 2,577). Since school SES is related to children's learning over and above individual SES, even in Chinese populations (Zhao et al. 2012), I used the dichotomised SES data at the school level in subsequent analyses. With respect to the parents' self-reported proficiencies in mathematics and past motivation to learn mathematics, on a scale of 1–7, the mean values were 4.53 (SD = 1.16) and 4.57 (SD = 1.24) respectively. Further analyses revealed SES differences in parents' self-reported proficiency in mathematics, $F(1,\ 172) = 18.02$, $p < 0.01$, with higher SES parents reporting a higher self-reported proficiency in mathematics (4.99 vs. 4.25). The results for self-reported past motivation to learn mathematics was marginally significant, $F(1,\ 172) = 3.84$, $p = 0.05$.

Survey Tool

An investigator-designed questionnaire consisted of four sections seeking information on the families' demographic details (e.g., parental education, income levels), parents' beliefs (e.g., factors contributing to the superior mathematics performance of students in Chinese-heritage cultures), types of interaction strategies aiming to promote preschool children's mathematics learning (e.g., the use of stories, real objects and prompt questions helping young children to learn mathematics) and parents' attributes relating to mathematics (e.g., past proficiency in and motivation to learn mathematics). The items in the questionnaire came from another ongoing study that examines the type of interaction strategies used by Hong Kong preschool teachers when they teach children mathematics. The draft questionnaire was first reviewed by a panel of three experts in the field of early numeracy development, and then piloted on five Hong Kong Chinese families. Based on family feedback, the items were subsequently revised and clarified. The questionnaire was re-administered to the same families two weeks after the first administration. Only one family changed their responses substantially because the mother had given up a full time job temporarily and therefore spent more time with her children.

Analysis

All analyses were conducted using SPSS version 19.0 (SPSS Inc., Chicago IL 2011). Missing data was handled with listwise solution. Raw scores were used in the analyses. All variables were inspected for univariate outliers (>3 SDs from mean), and no outliers were found. To simplify data analyses, I first conducted a series of *t*-tests to examine whether the interaction strategies of interest were related to children's gender. No gender differences were found (all $ps > 0.10$); hence further analyses collapsed data across gender. I then explored whether the interaction strategies employed were sensitive to the class level (at entry to preschool vs. at exit from preschool) of the children. The analyses showed significant class-level differences only for two of the variables: counting backward ($t\ (171) = -5.52$, $p < 0.001$) and the use of worksheets ($t\ (171) = -2.24$, $p < 0.05$), with parents more likely to use worksheets and backward counting with children who were in their final year of preschool education than in the first year.

Results

Table 10.1 shows the means, standard deviations and the correlation coefficients of the variables. On a scale of five (1 = never; 2 = rarely; 3 = sometimes; 4 = usually; 5 = always), parents reported more use of the following strategies than other strategies: counting forward (Mean = 3.70), presenting real objects (Mean = 3.40)

Table 10.1 Means, standard deviations, and correlation coefficients among all variables

	Mean	SD	Correlation									
			1	2	3	4	5	6	7	8	9	10
Parents' proficiency	4.53	1.16	–									
Parents' motivation	4.57	1.24	0.580**	–								
Stories	2.84	0.773	0.170*	0.226**	–							
Real objects	3.40	0.876	0.083	0.089	0.394**	–						
Prompt questions	3.29	0.879	0.205**	0.176*	0.391**	0.564**	–					
Relate to real life	2.99	0.905	0.135	0.086	0.314**	0.314**	0.586**	–				
Statistics	3.04	0.982	0.149	0.158*	0.249**	0.138	0.293**	0.423**	–			
Counting (forward)	3.70	0.850	0.214**	0.156*	0.303**	0.381**	0.427**	0.365**	0.318**	–		
Counting (backward)	2.76	0.974	−0.002	0.114	0.197**	0.045	0.215**	0.328**	0.369**	0.231**	–	
Use of worksheet	2.98	0.943	0.146	0.138	0.395**	0.194*	0.281**	0.376**	0.253**	0.221**	0.379**	–

* indicates $p < .05$, ** indicates $p < .01$

and providing prompt questions (Mean = 3.29). Stories and counting backward did not appear to be common interaction strategies (Mean values = 2.84 and 2.76, respectively). Correlation matrix shows that the correlations among all the interaction strategies were significant except for the correlation between real objects and statistics/counting backward. The significant correlations suggested that parents' use of contextualised strategies (e.g., the use of stories, presenting real objects) and decontextualized strategies were related (e.g., the use of worksheets).

Relations Among Interaction Strategies, SES, Children's Class Level, Parents' Self-reported Proficiency in Mathematics and Their Past Motivation to Learn Mathematics

Next, I examined the extent to which the various predictor variables contributed to parents' interaction strategies. For each interaction strategy, a multiple regression model was run with SES, children's class level, parents' self-reported proficiency in mathematics and their past motivation to learn mathematics as predictors. In total, eight regression models were run. SES and children's class level are coded as dummy variables (0 = low SES, 1 = high SES for the SES variable; 0 = at entry to preschool, 1 = at exit from preschool for the children's class level variable). The results for each regression model are as follows:

Use of stories. The regression model was significant, F (4, 168) = 3.51, $p < 0.01$, accounting for 7.7 % of the variance. Among the four predictors, only parents' past motivation to learn mathematics contributed significantly to the use of stories ($\beta = 0.19$, t (168) = 2.10, $p < 0.05$).

Use of real objects. The regression model was not significant, F (4, 163) = 1.26, $p > 0.05$. None of the predictors were significant (all ps > 0.07).

Prompt questions. The regression model was significant, F (4, 169) = 5.15, $p = 0.001$, accounting for 10.9 % of the variance. Among the four predictors, only SES contributed significantly to the model ($\beta = 0.234$, t (169) = 3.05, $p < 0.01$). The result indicated that the more affluent parents were more likely to ask prompt questions when their children were manipulating objects in comparison to the lower SES parents.

Relate to real life. The regression model was not significant, F (4, 168) = 1.78, $p > 0.05$. None of the predictors were significant (all ps > 0.14).

Statistics. The regression model was not significant, F (4, 166) = 1.65, $p > 0.05$. None of the predictors were significant (all ps > 0.30).

Counting forward. The regression model was significant, F (4, 168) = 2.62, $p < 0.05$, accounting for 5.9 % of the variance. However, none of the four predictors contributed significantly to the model (all ps > 0.11).

Counting backward. The regression model was significant, F (4, 168) = 8.70, $p < 0.001$, accounting for 17.2 % of the variance. Among the four predictors, only class level contributed significantly to the model (β = 0.39, *t* (168) = 5.50, $p < 0.01$). Parents were more likely to do backward counting with their older children than with their younger children.

Use of worksheets. The regression model was significant, *F* (4, 168) = 2.61, $p < 0.05$, accounting for 5.8 % of the variance. Among the four predictors, only class level contributed significantly to the model (β = 0.18, *t* (168) = 2.33, $p < 0.05$).

Reasons Why Students in Chinese-Heritage Cultures Had Superior Performance in International Tests of Mathematics

Next, I was interested in the factors that parents attribute to the success of students in Chinese-heritage cultures in international tests of mathematics. Parents were asked to judge six statements (binary options: agree or disagree): (1) The local mathematics syllabus is difficult (henceforth syllabus); (2) There are lots of high calibre mathematics teachers in Asia (henceforth teachers); (3) Students in Chinese-heritage cultures are diligent (henceforth diligence); (4) Mathematics tutors in after-school classes contribute greatly to students' achievement in mathematics (henceforth tutors); (5) Parents in Chinese-heritage cultures are devoted to helping their children to achieve (henceforth parental devotion); and (6) Parents in Chinese-heritage cultures have high expectations of their children's academic performance (henceforth, parental expectation).

As shown in Fig. 10.1, diligence and parental expectation were the two most important factors (84 and 79.9 % respectively) in why parents think students in Chinese-heritage cultures have superior performance in international mathematics tests compared with students from other cultures. With respect to the role of other potential factors, tutors appeared to be as important as teachers (79 vs. 78.2 %). Finally, 75.6 and 73.6 % of the families believed that the students' success can be attributed to parental devotion and the difficult syllabus respectively.

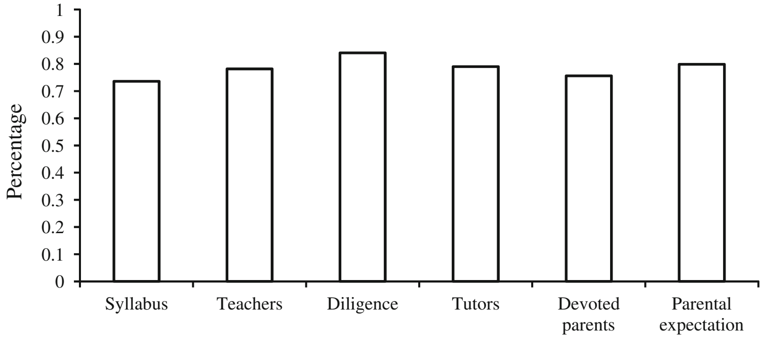

Fig. 10.1 The percentage of parents who supported each statement

In the final step of analyses, I investigated whether parents' responses to two of the statements (statement 5: parental devotion; and statement 6: parental expectation) were related to their use of interaction strategies at home. These two statements were chosen because they are specifically related to parents' attributes. A series of *t*-tests was conducted to explore if the parents who supported these statements used the home interaction strategies differently from those who did not. To simplify data analysis, I categorized the eight interaction strategies into two types (contextualised and decontextualized). The results showed that the parents who supported the statements did not differ from those who did not in their use of contextualised or decontextualized strategies (all $ps > 0.20$).

Discussion

The results from the present study are consistent with those in previous studies conducted in China (e.g., Zhou 2006; Zhou et al. 2009) where counting (forward counting) was the most important home activity relating to mathematics. In addition, in their interaction with children, parents tended to adopt a more direct and explicit approach to help children learn mathematics. Specifically, they were more likely to make use of real objects when illustrating a mathematical concept (e.g., counting the number of cookies on a plate) and provide prompt questions to help children reflect on concepts (e.g., when the child is about to get the chicken wings on a plate, parents will ask how many chicken wings there are on the plate). In addition, although some strategies are more contextualised than others, the majority of the strategies (except for statistics, backward counting and the use of real objects to illustrate a mathematical concept) were significantly correlated with one another. This indicates that the use of the more contextualised strategies does not preclude the use of the more decontextualized strategies (e.g., worksheets). The co-existence of differing strategies is perhaps not surprising and is consistent with the pragmatism deeply rooted in the Chinese culture. A famous saying of Mr Deng Xiaoping, the deceased former leader of the People's Republic of China, once said, "It does not matter whether a cat is black or white. It is a good cat when it can catch mice." Such pragmatism might have led parents to use whatever strategies that they know, irrespective of the theoretical basis behind a particular strategy.

With respect to the factors which might contribute to parents' interaction strategies, SES was only a significant predictor for the strategy of providing prompt questions. The data suggest that in the early years there may be relatively few SES-related differences in the way parents in the Chinese-heritage cultures help their children learn mathematics. This might also explain why the impact of SES was relatively weak in the Chinese-heritage cultures in comparison to other cultures (e.g., Anderson et al. 2010) and why students in Asia performed better than their American counterparts, even after SES is held constant (Zhou et al. 1999; cited in Ryoo et al. 2014). For parental proficiency, it was not a significant predictor in any of the regression analyses. For parental motivation, it was only a significant predictor for the use of stories. The parents who were more motivated to learn mathematics in the past used more stories to engage their children to learn mathematics. Finally, children's class level was only a significant predictor for backward counting and the use of worksheets. The result showed that whilst having high expectations of their children's achievement, the parents were also sensitive to their children's developmental status. In particular, they understand that it is futile to push younger children who cannot count forward to count backward and to require children with weak fine motor skills to work on worksheets.

By considering all these findings together, it appears that factors other than SES and parents' attributes (such as past proficiency in and motivation to learn mathematics) might have stronger influence on children's achievement in mathematics, as the study showed that school SES and parents' motivation have limited influence on their behaviour with preschool children. The factor in the home environment that has a stronger predictive power may well be parental expectation, as has been suggested in extant studies (e.g., Phillipson and Phillipson 2012).

With regards to parents' views of what makes students in Chinese-heritage cultures successful in international tests of mathematics, my findings were largely consistent with the results from previous studies (e.g., Leung 2002). In particular, the parents in the present study believed that diligence was more important than the other factors of interest: parental expectation, teachers, tutors, parental devotion and the difficult syllabus. The greater importance attached to diligence is interesting because it might empower and disempower learners at the same time. Emphasis on diligence is empowering when learners think that diligence can override the effect of intelligence in academic achievement. However, it is disempowering when the roles of learners' enjoyment is downplayed to the extent that all that matters is diligence and it does not matter whether the learning process is enjoyable/engaging or not. The emphasis on diligence and the under-emphasis of enjoyment are exemplified in the following Chinese proverbs: (1) "diligence is good, while play is bad for you." (qin2 you3gong1, xi4 wu2yi4; 勤有功, 戲無益); (2) "with persistence, you can turn a rod into a needle." (zhi3 yao4 you3 heng2xin1, tie3bang4 ye3 ke3 mo2cheng2 zhen1, 只要有恒心, 鐵棒也可磨成針). These deep-rooted values might explain the lack of engaging mathematics activities in the Chinese classrooms reported in previous studies (e.g., Leung 1995) and the less frequent use of stories in parent-child interaction in the present study.

Another important finding in the present study is that the parents appeared to think that parental expectation and after-school mathematics lessons (provided by tutors) were important contributors (nearly as important as teachers!). There are two implications. First, future studies of students' achievement in the Chinese-heritage cultures should control for the role of tutors when evaluating the impact of home and school on children's achievement in mathematics. Second, while high expectations might correspond to better learning outcomes, by accepting parents' academic expectations, learners might have to persevere in classrooms that are less than stimulating and have to spend extra hours after school receiving tuition in mathematics. The perseverance and the extra efforts can be justified in the name of diligence. In the affective dimension, past studies revealed that students in China and Taiwan suffered more mathematics anxiety than their counterparts in the USA (e.g., Ho et al. 2000). Future studies should take into account learners' anxiety and their level of enjoyment during mathematics activities.

Last but not least, while previous studies (e.g., Phillipson and Phillipson 2007, 2012) suggested that parental expectations mediate children's cognitive abilities and their academic achievement, paradoxically, parental expectations were not related to parents' use of either contextualised or decontextualized interaction strategies in the current study. Neither was parental devotion related to their interactional behaviour. These results suggest that parents' behaviour might be decoupled from their beliefs. In other words, what they expect of their children might not be related to the way they interact with their children. If parents' expectations do not affect children's learning outcomes via mathematics-related interaction strategies, future studies should explore the mechanism that governs the relations among parental expectation, children's cognitive abilities and their achievement in mathematics.

Conclusion

The current study has provided new insights into the potential mechanisms underlying the success of students in Chinese-heritage cultures in international tests of mathematics. First, parents set the stage for their preschool children's achievement in mathematics by making use of both contextualised and decontextualised interaction strategies in the home environment. In addition, developmental models for children's development in mathematics may be different across cultures. In particular, in Chinese-heritage cultures, SES might play a lesser role in children's mathematical development. This could be due to the SES variable being overridden by other variables such as parental expectations which are closely linked with important cultural values, e.g., success derives more from diligence than from intelligence. Furthermore, the role of "significant others", such as tutors for mathematics, should be investigated in future studies that focus on the overall sociocultural environment which supports students in Chinese-heritage cultures to learn mathematics.

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Part IV
Family and Educator Partnerships That Support Early Mathematical Learning

Chapter 11
Working with Parents to Promote Preschool Children's Numeracy: Teachers' Attitudes and Beliefs

Dina Tirosh, Pessia Tsamir, Esther Levenson and Ruthi Barkai

Abstract This chapter describes the beliefs and attitudes of five preschool teachers towards involving families in promoting children's numerical competencies, such as saying number words in a sequence to ten. The backgrounds of the children in each class, along with the teachers' educational and social backgrounds, form the context of the study and are important variables when analysing the ways in which each teacher decides to involve families. In addition, the chapter describes various ways that teachers encouraged families to take part in their children's mathematical growth (such as giving the children and parents mathematics homework) and to experience mathematics with their children (such as taking part in play with a mathematical theme). Dilemmas for preschool teacher educators are raised and discussed such as if and how teacher educators may act as mediators between preschool teachers and children's families when promoting early mathematical growth.

Keywords Professional development · Preschool teachers · Case studies · Beliefs · Parent involvement · Numeracy

Introduction

The home environment, including parental involvement, can affect young children's early numeracy skills. Studies have found that young children from disadvantaged homes exhibit lower levels of both number and geometrical knowledge, than children from advantaged homes, (Starkey et al. 2004). Some of the reasons cited for these differences included the home mathematics practices reported by parents such as providing games, toys, and computer software that promote mathematical activities (LeFevre et al. 2009). One study found that when parents involved their children in complex activities such as adding, subtracting, and

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comparing quantities, as opposed to basic activities such as reciting and writing numbers, their children's numeracy skills increased (Skwarchuk 2009).

While parental involvement is commendable, in Israel, it is the preschool teacher who is directly responsible and accountable for promoting children's numeracy. The preschool mathematics curriculum (Israel National Mathematics Preschool Curriculum [INPMC] 2008) specifically states that

> the preschool teacher has an important role in fostering children's mathematical abilities. It is up to her to devote attention both to planned mathematical activities as well as mathematical activities which may spontaneously arise in the class and to pay attention to the mathematical development of the children. (p. 8)

Compulsory public education in Israel begins at age three and the curriculum sets learning standards for children ages 3, 4, and 5. With the education system and preschool teacher taking upon themselves such major roles in the development of young children's numeracy, is it possible to encourage teachers to collaborate with parents with the aim of promoting children's mathematical growth both at home and in school?

While some programs attempt to guide parents without directly involving the school system (e.g., Anderson 1997), this study focuses on the possibility of parental involvement which is mediated by the kindergarten teacher, as opposed to an outside educator. The chapter explores the beliefs and attitudes of five kindergarten teachers working in a low socio-economic neighbourhood in Israel toward parental involvement in their children's mathematical development. These beliefs include views related to the roles of parents in education, the role of the preschool teacher, and the extent to which parents and teachers should communicate and cooperate with regard to children's mathematics education. The chapter also describes different ways teachers attempted to involve parents and some of the dilemmas which arose from those attempts.

Involving Parents in Children's Mathematical Development

Parents may be involved with children's mathematical development without any outside intervention. Ginsburg and Golbeck (2004) found that providing games, books, encouragement, and opportunities for exploration, are ways that parents may positively influence their young children's mathematical development. On the other hand, parents who model fear of mathematics and exert pressure on their children to learn, may negatively influence mathematically development. Parents from low socio-economic backgrounds or from immigrant backgrounds may lack the resources, language, and knowledge to help their young children in mathematics. In addition, low-income parents often believe that the preschool, rather than the home, is responsible for preparing children for school mathematics whereas middle-income families place more emphasis on the home environment (Starkey et al. 1999).

Parents may be encouraged to become involved with their children's mathematical development by a host of interventions not necessarily stemming from the teacher. For example, Anderson (1997) investigated an intervention where parents were given at home a set of materials which included worksheets, blank paper, a book, and multilink blocks. It was found that the materials were used by the parents with the children in various activities, some of which were directly aimed to elicit mathematics and some for which mathematics was an aside. However, Anderson (1997) pointed out that the parents' role of mediator was central to creating a context for mathematical learning. Furthermore, all of the parents in this study were well-educated and middle-class. Taking into consideration that providing parents with materials without instruction of how to use them may not be enough for low-income families, Starkey and Klein (2000) investigated an intervention which included providing such parents with guidance. Parents from low-socioeconomic homes, together with their preschool children, attended a series of classes which taught them how to use various materials and engage their children with mathematics. They were also allowed to borrow materials to take home. While results indicated an improvement in the children's mathematical development, we note that the parent classes were led by preschool teachers who were not the children's preschool teachers in the day-care centre.

At times, it is the teacher who wishes, or is expected, to involve parents in their children's mathematical development. This necessitates taking into consideration additional parameters, one of which is teachers' general beliefs and attitudes regarding parent involvement. To begin with, the relationship between schools and parents may be culturally mediated. For example, in some countries, teachers are considered as part of the family and it is they, and not the parents, who make decisions regarding the child's education (Souto-Manning and Swick 2006). In other countries, parents are expected to be involved, sit on the school board, and are very influential when it comes to setting educational policy. Immigrant populations may not fully comprehend what is expected of them; teachers may be hesitant to involve immigrant families in the child's education or, alternatively, may be frustrated with parents who seem not to be involved. Such tensions may also occur between teachers and parents from low-income backgrounds (Lawson 2003). In addition to cultural and socio-economic factors, Hoover-Dempsey et al. (1987) found that teachers who had a high efficacy had a higher perception of parental support. Power, status, and a sense of responsibility may also affect teachers' decisions to involve or not to involve parents, with some teachers feeling superior to low-income families or feeling their status as the teacher threatened when directed by others to involve parents (Lawson 2003). Some teachers view parent involvement as a blurring of the roles while some may simply resent the extra work load that comes with involving parents (Bouakaz and Persson 2007).

With regard to mathematics, teachers often hold conflicting views. On the one hand, they may have reservations about involving parents because they feel that parents may lack the necessary knowledge and skills to help their children. In one study, parents were told to not interfere with their children's mathematical studies because a new reform mathematics curriculum was being introduced and it was thought that parents' knowledge would be outdated and might even hinder the aim of the new program (Gellert 2005). On the other hand, when given support from administrators and when professional development addresses both teachers and parents, teachers' attitudes may become more positive (Bernier et al. 2003). Another obstacle that teachers may face when attempting to help parents with mathematics is there lack of training to teach adults how to work on mathematics with children (Gal and Stoudt 1995).

For all teachers, including preschool teachers, beliefs regarding mathematics may also affect the extent to which teachers involve parents, as well as the ways parents are involved. Lee and Ginsburg (2009) discuss what they term misconceptions regarding mathematics education for young children. These include: young children are not ready for mathematics, language is more important than mathematics, mathematics should not be taught as a stand-along subject in the preschool, and assessment in mathematics is irrelevant for young children. Ginsburg et al. (2008) found that preschool teachers' beliefs regarding methods of early mathematics education may be related to the socio-economic background of the children. Teachers working with middle-income families believed that it was their task to foster positive attitudes and a positive disposition towards mathematics and children should learn mathematics through self-initiated play. Teachers of children from low socio-economic backgrounds felt more responsible for preparing the children for school mathematics and thus were inclined to employ more direct instructional methods.

Setting

For several years we have been providing professional development for teachers of children ages 3–6 years old. Our program aims to promote mathematical and pedagogical-mathematical knowledge needed for teaching mathematics to young children, along with self-efficacy for teaching mathematics in preschool (Tsamir et al. 2013). The mathematical content of our program is in line with the guidelines set by the INMPC (2008) and includes numerical and geometrical concepts and patterning.

In Israel, children from age 3 to 6 attend municipal preschools located in their immediate neighbourhood, separate from the primary school organization and building. The preschool may accept children of one age group only, or they may accept children of different ages resulting in a heterogeneous class of students with regard to age and development. In the study described here, four classrooms had children ages 4–6 years of age and one preschool had children ages 3–6 years old. All were located in low socio-economic neighbourhoods.

In the particular study we describe in this chapter, four didacticians were involved. The first two authors of this chapter provided the professional development which took place approximately once every two weeks for a period of eight months. The actual meetings took place in the preschool classrooms of the teachers, after school was over, on a rotating basis. Providing intervention for the children was an additional explicit aim of the program. The third and fourth authors provided on-site intervention for the preschool children, along with on-site guidance for the teachers, for about the same period of time. Teachers participating in this program all had a first degree in education which included two one-semester courses focusing on teaching number and geometry in preschool. In addition, the district preschool supervisor had a strong hand in deciding which teachers would participate in the program. Some teachers enthusiastically signed up for the program; others were strongly encouraged by the supervisor to participate in the program. The supervisor was very instrumental in getting the program off the ground, making it clear to everyone how professional development related to teaching mathematics in preschool was essential for the teachers. She personally attended almost all of the program meetings.

As professional development providers, we were first and foremost responsible for promoting the teachers' subject-matter and pedagogical-content knowledge (Shulman 1986), including knowledge of students and tasks (Ball et al. 2008). The teachers were responsible for involving additional supportive adults, such as their assistants, and implementing the tasks in their classrooms. They were free to choose which physical materials they would use when implementing the tasks, when during the day it was appropriate to implement tasks, and if the task should be implemented with individual children, small groups, or the whole class. We discussed such issues during program lessons with teachers, including the advantages and constraints of different options; however, ultimately, it was the teachers' choice. With this setting in mind, our intention at the beginning of this particular study with regard to involving the parents was to allow the teachers sovereignty in deciding if they wanted to involve parents in the children's mathematical education and if so, to what extent and in what ways. This approach was formulated together with the supervisor, who was familiar with the teachers in her district, as well as the parents and home backgrounds of the children. The supervisor made it clear that we were not to be directly involved with the parents. We also took into consideration that for these teachers, teaching mathematics in preschool was not a trivial matter. The mandatory curriculum was in its first years of implementation and it was thought that involving parents in this enterprise may give rise to undo and unwarranted pressure among the parents. Our mandate was to offer suggestions and guide the teachers who decided to involve parents and take our queues from the teachers.

Research Aims and Methodological Approach

The aim of this study is to investigate preschool teachers' perspectives toward involving parents in their children's mathematical education. Specifically we ask (1) What are teachers' beliefs regarding mathematics education in preschool? (2) What are teachers' attitudes towards parental involvement? (3) What are some of the ways that these beliefs and attitudes interact and how is this interaction reflected in the teachers' attempts to involve parents in their children's mathematical development?

This study uses a qualitative approach integrating narrative and naturalistic methods. Data were collected from several sources. During the professional development course the teacher educators sometimes raised the issue of parent involvement and at other times teachers spontaneously shared ways in which they involved parents in mathematical activities. These narratives were recorded. In addition, providing onsite mathematics intervention for the children allowed us to naturally observe ways in which the preschool environment was used to involve parents and how parents interacted with the preschool environment when dropping off or picking up their children. Finally, teachers took advantage of the onsite guidance in order to seek advice from the didactician with regard to parent involvement in preschool mathematics. These impromptu conversations occurred in the teacher's natural environment and added to their personal narratives related to involving parents. During the program, the onsite didacticians and the professional development providers met regularly, as well as at the end of the program, and discussed together issues that concerned the program. These meetings were recorded.

Five Preschool Teachers

For each teacher, we begin by presenting some background related to the specific teacher's kindergarten class, some background information on that specific teacher's education and socio-economic status, and her beliefs regarding her role as the teacher and her beliefs regarding the role of the family in a child's education. Where applicable, we offer descriptions of how teachers included the family in their child's mathematics education. When reading each description, we encourage the reader to attempt to first see the situation from the teacher's point of view and then focus on the nature of the mathematical experiences the teachers afforded the families and the implicit message related to the families through these activities.

Rita—Teachers and Parents Should Work Together

Rita's class included two age-groups of children, 4–5 year olds and 5–6 year olds. Her preschool was located in the same neighbourhood in which she lived and was, for Rita, a second home in every sense of the word. She did not run home after the day was over but often stayed later chatting with parents, writing notes to summarise the day's activities, and helping her assistant to prepare for the next day. She told us that she had attended many professional development courses in the past but none had focused on mathematics. She claimed to know mathematics but was unsure how to teach mathematics in preschool and was glad for the opportunity to take part in the current program.

Rita was well known in the neighbourhood as an experienced but demanding teacher. She demanded a great deal from herself and the children, and in addition, demanded a great deal from the parents in terms of being active in their child's education. Despite knowing that they would have to "work", or perhaps because of this, parents specifically requested that their children attend her preschool. It was also known in the neighbourhood, including among the first grade teachers, that children who attended Rita's preschool, came prepared for first grade, perhaps another reason why parents requested that their children attend her preschool. Not only did Rita proudly tell us about her reputation, but the district supervisor told us the same. As a side note, Rita also expected her assistant to take an active part in the children's mathematical learning.

In general, Rita had a positive attitude towards involving parents and said early on during the program, "I think that we need to involve parents. I saw that it helps tremendously to the children's progress." Rita's involvement of parents included several pathways. First, she prepared worksheets for the children to complete during school hours that involved practicing mathematical skills, such as naming and organizing two-dimensional shapes and counting and matching collections to compare one collection with another. Completed worksheets were displayed on the walls of the classroom or sent home with the children, so that parents could visibly see how their children were progressing mathematically and relaying the message to both parents and children that mathematical productions are to be valued. Along with completed worksheets, Rita also tended to send home additional mathematics worksheets for the children to complete at home. It was expected that parents would be responsible for having the child complete the sheet, offer help and guidance where needed, and that the completed worksheets would be returned to the preschool for Rita to review and save.

In addition to worksheets, Rita had a bulletin board outside of her classroom where she posted the current topics being discussed in class. This was a place where Rita could post which mathematical topics were currently being learned along with other seasonal topics such as the weather and holidays. Parents could read the board and know that for that week, children were learning about, for example, the number three—how it looks, how to count up to three, how to write the number three, different ways of decomposing three, etc. Parents were expected to reinforce these topics at home.

Rita organized evenings approximately once a month, where she would demonstrate and explain what was being done in the classroom. She also organized after-school happenings where the parents and children engaged together in planned activities, including mathematical activities. Rita also made up kits that the parents and children could take home and handed them to the parents, who understood that it was their responsibility to use the kits according to the instructions provided by Rita. Rita would come to us and ask for our opinion with regard to using different materials and how one material or another could better assist the children in learning some concept, but the initiative was hers and the activities were either those she developed on her own or those she adapted from activities we introduced during the professional development course. Besides hosting parent-child activities, Rita held one-to-one meetings with parents, in the morning before class began or after the school day was over, to discuss with the parents their child's progress, including their mathematical progress and what could be done at home to promote further progress. At times, we overheard these conversations and noted that certain ideas learned during the professional development course, such as Gelman and Gallistel's (1978) counting principles, were brought up during the meeting so that parents could continue to work on a specific principle which needed reinforcement. Finally, we note, that although Rita involved the parents in her class to a great extent, she did not lay the responsibility of the child's success or failure on the shoulders' of the parents. In her view, it was up to the parents to back up and strengthen the learning that occurred in the preschool, but she was in charge of the teaching and she was ultimately responsible for the children's success or failure.

Joy—Parents Can Choose If and How to Be Involved

Joy's class, like Rita's, included 4–6 year old children. The assistant in her preschool was considered part of the classroom and was observed engaging with the children in different mathematical activities, according to Joy's instructions. On the one hand, Joy claimed during one of the professional development sessions, "I like mathematics, it is my strong subject. Through mathematics I can do with them a lot." On the other hand, she sometimes complained that we were expecting too much from her. She said at one of the sessions, "Besides mathematics, we have a million other things we need to be doing… I have 16 young children (4–5 year olds). I don't know how many (mathematical activities) I will have time for." This sentiment was mirrored throughout the program. Joy was an enthusiastic participant but accomplished fewer mathematical goals than some of the other participants.

Joy believed that just as she was given the privilege to decide how to implement in her classroom what she learned in the professional development course, parents have the right to decide if and how to engage their children in mathematical activities at home. It was her responsibility to relay to parents what was being learned in the preschool and, like Rita, she had an information board outside the preschool for such a purpose. However, her bulletin board was more like a notice board and was less informative and less instructional than Rita's board. There were worksheets placed in an open and convenient location for parents to take home if they wanted to, but they were not explicitly given to the children or to the parents to take home. Nor did Joy check to see if and what children did at home in terms of their mathematical activities. This was not because Joy did not have time to talk to parents or because she was not interested. Instead, on principle, she did not ask the parents if and what they did with children at home so as not embarrass the parents. On the other hand, she often stayed after the school day was over to meet with parents who requested to talk to her, but it was always at the request of the parents and discussions were informal. Sometimes, parents would wait after school to talk to Joy and hear from her what was going on mathematically in the classroom.

Joy was very cognizant of the low-socioeconomic backgrounds of the families and mentioned to us that several children living in single-parent homes and that several parents were unemployed. She seemed to be aware of the hardships and strains these situations could cause and was thus reluctant to add to the parents' already stressed lives. Just as she would not want anyone to place upon her extra responsibilities, she would not do so to the parents. She was satisfied to hear that parents were able to spend time with their children and did not need to add to the parents' frustration by requesting them to take on responsibilities they would not be able to fulfil. On the other hand, in the case of one child from an immigrant background, she requested the older sister, who was already in school, to help with younger sibling and even gave the sister some tips on how to work with the child at home.

Joy specifically mentioned that it was her responsibility to make sure that the children learned. She related the following story during one of the program sessions:

> There are two very bright children in the class and I feel that one of them is bored. They already know (simple number concepts). He does me a favour in answering my questions. His mother said to me, "Maybe I can buy him workbooks and you can work with him and advance him." So I said to her, "Don't worry. I will find ways to advance him."

Joy took full responsibility for the child's mathematics learning, even to the point of finding appropriate materials for the more advanced children. However, she did say that if the parents requested, she give them some advice. In line with this perspective, she invited the parents to gatherings in the evenings in order to inform them of the curriculum and what was being accomplished during the day. However, she only held two to three such meeting during the year. During the first meeting of the year, not all of the parents came. Yet, those that did attend were interested and showed a willingness to participate in their children's education.

Estie—Parents Should Experience Mathematics Along with Their Children

Estie began the professional development course with a rather negative attitude towards mathematics, believing that mathematics does not have a place in pre-school. As the course progressed, her attitude towards promoting children's mathematical learning improved. She began to seek out and read to the children books during story hour that incorporated counting or geometric shapes, within the context of the story line. For example, she told that us that she specifically went out and bought the *Hungry Caterpillar* (a story about a caterpillar who eats one item on the first day, two items on the second day, etc.) because it exemplifies both cardinal numbers and ordinal numbers. During the eight day Hanuka holiday, when one candle is lit the first night, two the second night, up until eight candles the last night, Estie celebrated in the preschool by coordinating activities centered around the number eight. In short, Estie began to specifically incorporate mathematical activities into the school day. However, as seen from the above descriptions, she felt strongly that the mathematics taught to young children must be relevant to their lives. She did not use worksheets but would promote one-to-one correspondence in real situations, for example, when setting the tables. She also did not assess the children's mathematical knowledge and progress in a systematic manner.

Involving parents in the day to day happenings of the preschool was not part of the weekly routine for Estie. Instead, Estie wanted the parents' involvement to be experiential. She wanted parents to walk into the preschool and be wowed. In accordance with this attitude, Estie created art exhibitions which featured the children's projects, some of which included numbers and numerical themes. Parents were invited to come into the preschool when picking up their children and enjoy the exhibition. She also invited the parents to attend with their children afternoon plays which she produced and enacted based on children's stories. These stories also included elements of mathematics. For example, there is a well-known children's book in Israel called *A Story of Five Balloons*, which tells the story of five children, each of whom was given one balloon, and what happens to each child's balloon. Mathematically, the story could promote saying number names forward in sequence and counting objects up to five, and counting backwards from five to zero. Each time in the story there was a situation which called for counting, the audience of children and parents were invited to join in with saying the number names. On another occasion, Estie invited the parents and children to come in the afternoon after school to celebrate the holiday of trees and plants. Among the various activities, children, along with their parents were requested to fill their baskets with the special holiday fruit. However, even during this occasion, numbers and counting were part of the activity. Children were given cards with a picture of a fruit and a number symbol on each. These cards represented instructions as to how many pieces of that fruit they were to place in their basket. Assisting Estie with these events was Surie, the district coordinator in charge of distributing educational supplies, such as art materials, games, and books, to the preschools. Upon request, she would also assist the preschool teachers in creating an aesthetically pleasing physical environment. Estie often enlisted Suri's advice and expertise when it came to arranging the exhibits and productions in her preschool, ensuring that each exhibition and production was an event parents would remember.

In line with both Estie's belief that mathematics in preschool should be related to the children's everyday lives and that involving parents should be a positive experience to remember, Estie also planned and carried out "Market Day", to which parents were invited to participate. This event occurred in the morning, not after school, but on a day which most parents could come. The preschool was set up to imitate an open-air market with different stalls selling different items and with real scales and weights to weigh the goods. The items had been gathered over time from parents who would send, for example, empty cereal boxes and egg cartons to the preschool, and for the occasion, had brought in real fruit and vegetables. Paper play money was created for the day with numbers from one to ten printed on different bills and little shopping carts were borrowed, also from parents, for the occasion. At the start of the day, each family was given a budget and requested to tell the others what they planned on purchasing that day with their budget. Prices were listed on each item and children had to pay, receive change, and give change in accordance with the prices. One parent was in charge of each stall to help the children with their buying and selling. At the end of the day, Estie had everyone sit around in a circle and tell what they had bought.

It is important to note, that Estie, even before participating in the professional development course, would invite parents to participate with their children in school events, such as holiday celebrations and story plays. Surie also attested to the fact that Estie enjoyed gathering parents for various events, but that these events had little to offer in the way of promoting mathematical concepts. Thus, when we heard from Estie during the professional development course of these upcoming events, we suggested ways of incorporating mathematics into these planned events and ways of promoting additional mathematical concepts into other events that perhaps previously only slightly touched upon mathematical ideas.

Lotty—Parents Should Not Be Involved

Although all five teachers taught in low socio-economic neighbourhoods, Lotty's preschool was located in one of the toughest and poorest neighbourhoods of the city. Several children in the preschool were recognized by the social welfare department as being abused and/or neglected. Violence in the family, to the mothers in particular, was not unknown and to our knowledge, at least one child's father was serving a prison sentence. Lotty lived in the neighbourhood in which her preschool was located and related to us how she sometimes saw 4 and 5-year old children walking around without any adult supervision. One could hear in Lotty's voice that she was quite affected by what she saw and heard. Finally, Lotty was also going through a difficult period in her personal life. She was a single mother who relied on help from her own mother and out of the group of participating teachers, she was the one most often late or absent.

In addition to the difficult social and economic backgrounds of the preschool participants, Lotty's preschool classroom included three age groups, children from three to six years of age. This heterogeneous makeup of the class added to Lotty's difficulties within the class. Lotty was also the youngest teacher of the group and the least experienced. Although the teacher's assistant in her class was experienced and might have been able to help, the assistant resented being an underling to someone less experienced, and according to our observations, the assistant undermined Lotty's authority in her class. Lotty often expressed to us that she felt out of control. The supervisor led us to believe that, in her opinion, Lotty was not one of the more accomplished teachers and thus she had required her to participate in the professional development program. When speaking to Lotty, she hinted that she knew that the supervisor did not have a high opinion of her teaching abilities. To summarise, Lotty had a difficult personal life, the children in her preschool came from difficult homes, and the people who were supposed to help her—her assistant in the preschool and the supervisor in charge of the preschool—did not support her. Essentially, Lotty conveyed a feeling of being stuck, both in the physical sense and the emotional sense, and thus was not in the best position to be an emissary for educational change and involvement.

From the very beginning, Lotty was against involving parents, in any way, in their children's education. The one time she attempted to include parents was, in her terms, a disaster. She described how she wished to celebrate "Family Day" in the preschool with the children and their parents. For days, she worked with the children during the school day, preparing songs, dances, and poems that they could perform for their parents and special activities that children and parents could take part in together. She then invited the parents to come to the preschool with their children in the early evening to partake in the special Family Day program. Only two children showed up with their parents and those parents kept on looking at their watches waiting for the program to end. When she told us this story she cried. In other words, the fact that Lottie did not involve the parents in the children's mathematics education was not because of the mathematics, but because involving parents on any level, was not an option for her. In fact, Lottie had a positive attitude towards learning mathematics and towards the program. She told us, "I gained a lot from the program… practical tools… Today I feel that I know what I want to accomplish. I know what to expect from the children. I also receive feedback from the children. Children who did not know how to say number names in sequence from 1 to 10, who did not recognize the number symbols,… I feel, thankfully, that I succeeded with them." Unlike the other teachers, Lottie did not come early or stay late in order to be available to parents, nor did she communicate with the parents on a daily or weekly basis regarding what was being learned in the classroom. We cannot know what Lottie's attitudes towards parent involvement with their children's mathematics education might have been under different circumstances.

Vera—Parents May Interfere in Their Child's Learning Progress

Vera's preschool was located in a neighbourhood with a large immigrant population from Ethiopia, some of which could not read and write Hebrew. It is important to note that the culture of these immigrants was very different than the local culture and this included their attitude towards education. Most parents had no formal schooling as they were brought up in villages and farms in their native country. With regard to mathematics, the elder of this local community, whom the parents revered, was against teaching mathematics to girls. Vera, however, was strongly committed to promoting the mathematics of all the children in her preschool.

Vera's preschool was the most organised of the five preschools in terms of that every child knew what he or she should be doing at any time during the day. Although Vera relied on her assistant to help out in the school, she, as the teacher, decided what mathematical activity each child would work on and she instructed the assistant exactly what to say, how to help, and also what not to do. During the school day she promoted mathematical learning by implementing mathematical activities that were connected to real life as well as by engaging in activities that were directly related to mathematical concepts and skills. For example, she told us how during snack time, she would sometimes deliberately hand out one less napkin per plate to promote the concept of one-to-one correspondence and its importance. However, she also worked on saying the number names forward to 10 and backward from 10 in sequence, and saying number names in sequence forwards from some arbitrary numbers.

When we at first arrived at the preschool, Vera was able to tell us what the children knew in terms of numeracy and numerical concepts. During the program she looked to us for guidance in scaling up and improving what she was already doing in her classroom. She was very attentive to details and wanted to know what was correct. For example, she discussed with us the symbol for four, and should she present it open (e.g., ④) or closed (e.g., 4) and would it be harmful or beneficial to show the children both possibilities. During the program, she realised how much more there was to learn and believed that it would be too much for the parents, who presumably had little background or experience with preschool mathematics. While Vera was against involving parents in their children's mathematics education from the start, she claimed that as time passed and the professional development course progressed, she was even more convinced of the correctness of her first instinct because she believed that the parents may undo and spoil what was taught in the preschool. This sentiment not only related to children's mathematical knowledge, but to their disposition and emotions as well. During one session she told the following story:

> I have one child who refuses to participate and is quiet whenever I work with him (on numbers). I know what the problem is. The parents tried working with him because they realized that he didn't know (what he was supposed to know) and it was probably too much pressure for the child, also in the group (in class) and also at home.

Vera's reluctance to involve parents in their children's mathematics education did not extend to all other activities. Vera held holiday gatherings and end-of-year parties, as was expected of her. But she did not involve the parents beyond these traditional gatherings. She explicitly said that because she was responsible for the children's mathematical progress, she must be the one to lead the teaching and be in control; she had to be involved in designing the activities, implementing them, and then assessing what each child had learned and what needed extra attention. She also told us that the more she learned with us, the more she realized how difficult it was for her to teach correctly and before she could even think of involving the parents, she had to be sure that she was teaching the mathematical concepts in the correct manner. When it came to the home, she stated that the parents could be involved in other aspects of their children's education such as reading and playing with their children; she let them borrow books and games from the preschool. But, mathematics was to be done only in the preschool classroom because, according to Vera, if the parents were to engage the children with mathematics, she believed she would have extra work in school undoing their mistakes. In light of the level of mathematical pedagogical accuracy she demanded from herself, it is not surprising that she would not entrust the parents with this aspect of their children's learning.

Discussion

This section begins by answering the research questions. It summarises the teachers' beliefs related to preschool mathematics education, their attitudes towards parental involvement, and how these beliefs and attitudes may interact and be reflected in teachers' attempts to involve parents in their children's mathematical development (see Table 11.1). We then discuss some of the dilemmas which arise

Table 11.1 Five teachers' beliefs, attitudes, and ways of involving parents

	Beliefs related to mathematics in preschool	Attitude towards parent involvement	Involving parents in mathematics education
Rita	Preschool mathematics should prepare children for school mathematics; practicing skills is important; explicit assessment is essential	Parents are an integral part of their children's education; the teacher should actively involve parents	Worksheets; personal meetings; monthly parent evening; instructive bulletin boards
Joy	Preschool mathematics should prepare children for school mathematics; practicing skills is important; general assessment is sufficient	It is advantageous to involve parents but it is important to do so without pressure	Worksheets; displays; two-three parents' evenings
Estie	Mathematics should be learned in the context of daily life activities, mathematical assessment unnecessary at this level	Parental involvement should take place in the preschool by engaging children and parents together in educational activities	Market day; mathematical story performances
Lotty	Unknown	It is too difficult and frustrating to involve parents	None
Vera	Preschool mathematics is needed for daily activities and for preparing children for school mathematics, explicit assessment is essential	Parents should not be involved because they may do more harm than good	None

from this study and what it may mean for future professional development programs which attempt to promote preschool mathematics.

As can be seen from Table 11.1, beliefs related to preschool mathematics ranged from the belief that it is the preschool's job to prepare children for school mathematics (Rita, Joy, and Vera) to the belief that at this age, mathematics should be related solely to the children's daily experiences (Estie). For Rita and Joy, this was interpreted in practice by the use of worksheets in promoting and reviewing mathematical skills, as opposed to Estie's preference for injecting mathematics into everyday activities and attempting to bring mathematics alive in the classroom. For Vera, her belief comes out in a more indirect manner, in the tremendous responsibility she feels towards teaching correctly every detail of the mathematics curriculum. Lotty's beliefs on this matter remain elusive. While the teachers did not mention that their beliefs were related to the background of the children, recall that Ginsburg et al. (2008) found that preschool teachers of children from low socio-economic background felt especially responsible for preparing those children for school mathematics. Estie's beliefs are in line with other teachers' beliefs that preschool mathematics should not be taught as a stand-alone subject in the preschool and assessment in mathematics is irrelevant for young children (Lee and Ginsburg 2009).

Teachers' attitudes toward the general involvement of parents also varied. Lotty was against all parental involvement at any level; Vera went along with parental involvement for traditional gatherings. Estie instigated multiple grandiose ways to get parents interested in their children's education; Joy and Rita encouraged parent involvement on both a personal, one-to-one basis, as well as on group levels. One difference between Joy and Rita was their expectations from the parents, with Rita expecting the parents to do their share and Joy more relaxed about parent involvement.

Beliefs related to preschool mathematics interacted with attitudes toward parent involvement and formed several paths of involving parents with mathematics education (see Table 11.1). For Rita and Joy, who believed that the preschool must prepare children for school mathematics and that parental involvement should be encouraged, the resulting interaction led naturally to active involvement of parents in their children's mathematics education such as informative bulletin boards (to a different degree according to their expectations from parents), one-on-one parent-teacher meetings instigated by the teacher (Rita) or by the parents (Joy), and take home math activities (obligated by Rita but voluntary for Joy's class). Vera also believed that preparing the children for school mathematics was her job but that in general, parents should have limited involvement. She rejected parent involvement in their children's mathematical education based on her sense of responsibility. Estie had a vision of preschool mathematics as being set within the context of other daily activities and a general positive attitude towards parental involvement. Taken together, and with gained knowledge and encouragement from her participation in professional development, Estie involved parents in their children's mathematics education by inviting them to events that including mathematical activities. She did not assess the children's mathematical progress, as did

Rita, Joy, and Vera, perhaps because it did not fit in with her views of preschool mathematics and likewise she did not feel it necessary to report to parents on children's mathematical progress. Lotty, who rejected parent involvement at all levels, did not involve the parents in their children's mathematics education.

As professional development providers for preschool teachers, these results lead to additional questions: If parent involvement is not always for the best, then who decides on whether or not to involve parents and if so, how to involve parents? To what extent should professional development providers intervene when it comes to involving parents? As outsiders, can we really know the preschool families?

When investigating teachers' perspectives, it is important to appreciate the context in which the study takes place. The context, in this case, includes the ethnic and socio-economic backgrounds of the children in the school, the teacher's personal life, and additional relevant personnel (teachers' assistants, supervisors, etc.). Several studies (e.g., Lawson 2003) found that children's background (e.g., immigrant families, low-income families, ethnic minorities) affects teachers' involvement of parents in students' mathematics education. In this study, all of the preschools catered to children of low-socioeconomic backgrounds; all had some immigrant families (Vera perhaps more than others); all had children of low-income families (Lotty more so than others). Yet, their attitudes and ways of involving parents in their children's mathematics education differed. As seen above, some of the differences may be attributed to the teachers' beliefs related to preschool mathematics. But some of the differences may stem from other factors. Estie, for example, only began to introduce mathematics into her parent-child events after taking part in the professional development program. Results of the program indicated that teachers not only gained mathematical and pedagogical-content knowledge, but they also gained self-efficacy (Tsamir et al. 2013). Thus, without specifically encouraging parent involvement, professional development that promotes not only knowledge, but also self-efficacy, may allow teachers to feel more confident involving parents. It may also reduce the threat to their own status that some teachers feel when parents are involved (Lawson 2003).

For Vera, it seems that professional development had a different effect. The more she learned, the more she was convinced that parents should not be involved in mathematics education. Perhaps, in her case, dealing with a large immigrant population retaining their old-world beliefs, Vera was rightly worried. Vera, who was committed to advancing the children's education, could not involve parents who believed that girls should not learn mathematics. As Ginsburg and Golbeck (2004) pointed out, parental involvement might also have negative effects on children's mathematical development. Involving those parents would have put an end to mathematics education for some of those children.

Estie chose a positive experiential way to involve parents. Professional development may help in such situations by providing a platform for sharing ideas related to parent involvement. However, as can be seen from this study, teachers may hold different perceptions regarding preschool mathematics, which may influence whether or not they adopt others' ideas. Beliefs held by teachers are not easily swayed by professional development (Zehetmeier and Krainer 2013).

And then there is Lottie. In addition to the complexities involved in teaching children from low socio-economic backgrounds, Lottie had little support from those around her. Attending professional development did encourage Lottie to introduce more mathematical activities into her preschool, and over the term, her walls were decorated with more and more mathematical productions, such as different ways of representing numbers. However, understandably, she continuously refused to actively involve parents. While it seems obvious that in this case, professional development is not the venue for promoting parent involvement, in an indirect way, it did have an impact. Because Lottie was engaging the children with more mathematics, she was able to assess their progress and, in one particular case was so impressed by a boy's progress, that she related the progress to her supervisor. Unrelated to this, the boy's mother had a meeting with the family's social worker and other officials, including the preschool supervisor. At one point, the mother asked the supervisor from where she knows her child. When the supervisor told the mother how the preschool teacher reported her son's progress in mathematics, she broke down and cried. As professional development providers for preschool teachers, we may not have a mandate to specifically get involved with parents and we may have many questions and dilemmas regarding this issue, but we should keep in mind that indirectly, promoting mathematical knowledge along with self-efficacy can still have an impact on this complex aspect of preschool mathematics education. We began our program with an initial, perhaps naïve, belief that all significant adults in a child's environment should work together towards the promotion of mathematical knowledge. We learned over time that this not always possible.

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Chapter 12
Bringing Families and Preschool Educators Together to Support Young Children's Learning Through Noticing, Exploring and Talking About Mathematics

Ann Gervasoni

Abstract This chapter draws on findings from the longitudinal evaluation of the Australian initiative *Let's Count* (Perry and Gervasoni 2012) to consider how the process of bringing families and pre-school educators together, with a focus on mathematics, enhanced young children's mathematics learning. The data examined is parent and educator interview data that explores the effectiveness of the *Let's Count* approach. The findings, sustained over two separate data collection periods over 2 years, provide clear evidence that *Let's Count* is at least as successful as other mathematics learning programs in terms of children's mathematical knowledge and skills outcomes, and suggest in respect to some mathematical concepts that *Let's Count* may be a superior approach. Themes emerging from interviews with parents highlight that the parents valued the educators talking to them about ideas and suggestions regarding the type of activities that are rich sources of mathematics learning. It many ways these discussions provided parents with prompts, inspiration, encouragement and confidence. The interview data also highlight that sustaining communication between the parents and educators across the year was challenging for some. Recommendations arising from the *Let's Count Longitudinal Evaluation* for future initiatives include: encouraging parents to support their children to notice, explore and discuss the mathematics that is part of everyday experiences; enabling sustained communication opportunities for parents to discuss the mathematics they notice their child using and exploring; and providing suggestions about how to extend this learning.

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Young children's mathematics knowledge and dispositions vary considerably when they begin school, and this suggests that some children are less favourably positioned than others to profit from mathematics teaching at school. This is likely due to their differing experiences and opportunities to engage with mathematical ideas prior to school. While pre-school and other early learning settings contribute to young children's mathematics learning, the most significant influences are children's family interactions and contexts. This raises questions about how families, educators and communities can best approach mathematics learning in the early years so that all children thrive; and also about how best to support children who are less favourably positioned than others when they begin school. This chapter draws on findings from the longitudinal evaluation of the Australian initiative *Let's Count* (Perry and Gervasoni 2012) to consider how the process of bringing families and pre-school educators together, with a focus on mathematics, influences young children's mathematics learning. The implications of the findings for future initiatives and research will also be discussed.

Young Children's Mathematics Learning

Young children are accepted as capable mathematical thinkers and learners (see Balfanz et al. 2003; Clements and Sarama 2002, 2014; Lee and Ginsburg 2007; Sarama and Clements 2002). Previously there was reluctance amongst educators to include mathematics as part of the early childhood curriculum (Perry and Dockett 2008; Sarama and Clements 2002). This meant that often there was insufficient focus on mathematics learning in early years' education, particularly in Western cultures. Lee and Ginsburg (2007) proposed that this lack of attention to young children's mathematics learning "may lead to later school failure, especially for children from poor and minority families, who are less likely to have a home environment in which their academic learning is facilitated" (p. 3). This conclusion is reinforced by early studies that found that children's experiences from conception to age six have the most important influence of any time in the life cycle on brain development, subsequent learning, behaviour and health (e.g., McCain and Mustard 1999), and underpin the interest in governments investing in the early childhood years in order to improve the health, educational achievement, and the social-emotional development of children. Early intervention approaches aimed at the needs of the child and the family can produce improved outcomes for those at greatest risk (Gervasoni 2015; Peter-Koop and Kollhoff 2015; Sarama and Clements 2015; Shonkoff and Phillips 2000).

The recognition that young children benefit from opportunities to explore mathematical ideas through high quality child-centred activities in their homes, communities, and prior-to-school settings is supported by many studies (Anthony and Walshaw 2007; Balfanz et al. 2003; Faragher et al. 2008; Duncan et al. 2007; Gervasoni 2003; Lee and Ginsburg 2007; Perry and Dockett 2005, 2008; Sarama and Clements 2002). For example, Duncan et al. (2007) performed a coordinated analysis of six longitudinal data sets relating changes in early skills to later teacher ratings and test scores of school reading and mathematics achievement. They found that school-entry mathematics, reading, and attention skills were associated with later achievement, but noted that the predictive power of early mathematics skills was particularly impressive. Notably, Duncan et al. cautioned that their findings did not support the adoption of 'drill-and-practice' curricula in early years settings. In contrast, they argued that play-based curricula designed with the developmental needs of children in mind can easily foster the development of academic and attention skills in ways that are engaging and fun. These perspectives are now reflected in national statements on mathematics learning in early childhood (e.g., Australian Association of Mathematics Teachers and Early Childhood Australia 2006; National Association for the Education of Young Children and National Council for Teachers of Mathematics 2002). For example, the Australian Association of Mathematics Teachers and Early Childhood Australia (2006) state that

> all children in their early childhood years are capable of accessing powerful mathematical ideas that are both relevant to their current lives and form a critical foundation to their future mathematical and other learning. Children should be given the opportunity to access these ideas through high quality child-centred activities in their homes, communities, prior-to-school settings and schools (AAMT and ECA 2006, p. 1).

Educative Justice and Mathematics Learning

Children living in communities that are described as 'experiencing multiple disadvantages' by governments are not expected, on average, to perform as well academically as children from more 'advantaged' communities (Caro 2009). This expectation extends to pre-school children (Carmichael et al. 2013; Rimm-Kaufman et al. 2003). Carmichael et al. (2013) concluded that "the socio-economic status of the community in which the family resides was the strongest home microsystem predictor of numeracy performance, explaining 10.5 % of the variance in the home-community microsystem model" (p. 16).

In contrast, there is evidence that many young children, including those living in communities experiencing multiple disadvantages, begin school as capable mathematicians who already exceed many of the first year expectations of mandated mathematics curricula or textbooks (Bobis 2002; Clarke et al. 2006; Ginsburg and Seo 2000; Gervasoni and Perry 2013; Gould 2012; Hunting et al. 2012). For example, Gould (2012) concluded from his study of the results of the mandated *Best Start* assessment in New South Wales (NSW Department of Education and Communities 2013) that the expectation in the Australian Curriculum—Mathematics (Australian Curriculum, Assessment and Reporting Authority 2013) that students can make connections between the number names, numerals and quantities up to 10 by the end of the first year at school "would be a low expectation for at least half of the students in NSW public schools" (p. 109). Gervasoni and Perry (2015) found that this was also true for children living in financially disadvantaged communities who participated in the *Let's Count* initiative commissioned by The Smith family, an Australian children's charity.

There have been many interventions aimed at improving the educational fortunes of children living in communities experiencing multiple disadvantages. There is also considerable debate about whether early intervention programs are able to overcome any educational disadvantage associated with young children living in financial and social disadvantage. Sarama and Clements (2015), who have designed and researched many educational interventions, now question the longer-term efficacy of such interventions and "hypothesise that most present educational contexts are unintentionally and perversely aligned against early interventions" (p. 153). Thus while it appears that early interventions in mathematics can be highly successful in promoting the mathematics learning of young children, systems of schooling can mitigate against the positive effects. Sarama and Clements (2015) argue that schools need to be aligned with the approaches of early interventions in order for their impact to be maintained.

Let's Count

Let's Count is an early mathematics initiative commissioned by The Smith Family (an Australian children's charity) to assist early childhood educators to work in partnership with families living in financially disadvantaged communities to promote positive mathematical experiences for young children (3–5 years). The Smith Family is a children's charity "helping disadvantaged Australian children to get the most out of their education, so they can create better futures for themselves" (The Smith Family 2013). The *Let's Count* approach aims to foster opportunities for children to engage with the mathematics encountered as part of their everyday lives, and talk about it, and explore it in ways that are appealing and relevant to them, and that enable them to learn mathematical ideas in ways that develop positive dispositions to learning and mathematical knowledge and skills (Gervasoni and Perry 2015). The simple mantra of *Let's Count* is *notice, talk about and explore mathematics in everyday activities*. *Let's Count* was piloted by The Smith Family in 2011 in five sites across Australia whose communities were identified as experiencing social and economic disadvantage. In 2012/2013 The Smith Family delivered a revised *Let's Count* program in four additional sites that were also part of a longitudinal evaluation of the program (Gervasoni and Perry 2015).

The *Let's Count* approach initially focused on early childhood educators who participated in two professional learning modules. The theme of Module 1 was noticing, discussing and exploring everyday opportunities for mathematics and opportunities for educators to consider ways in which they might engage with parents to support children's mathematics learning. Module 2 focused on celebrating and extending the mathematics that educators observe children using and learning. Between modules, the educators connected with families to consider ways that together they could encourage children to notice, explore and discuss the mathematics encountered in everyday situations, including through games, stories and songs. The educators used a range of strategies for connecting with parents, depending on which approaches were deemed most effective for their community. Typically, educators either met informally with individual parents to discuss the *Let's Count* ideas or they organised group meetings to initiate the program in their community.

The Let's Count Longitudinal Evaluation

The *Let's Count* Longitudinal Evaluation from 2012 to 2014 took place in two regional Australian cities in 2012 and 2013 and in two additional cities in 2014. In total, parents, children and educators from 21 early years centres took part. The findings of the evaluation provide some important insights about the impact of educators and families coming together around mathematics. One aspect of the evaluation was measuring participating children's mathematical growth across their preschool year and also comparing their knowledge, just prior to beginning school, with a comparison group of 125 children from the same communities whose families had not participated in *Let's Count*. The findings suggest that a family's participation in *Let's Count* was associated with significant progress in their children's mathematical knowledge (Gervasoni and Perry 2015). However, the focus of this chapter is the analysis of interviews with educators and parents about the impact of *Let's Count* for themselves and their children. In 2013, a small number of parents was interviewed by phone about the impact of *Let's Count*, once at the beginning of their involvement in *Let's Count* and again towards the end of the year. On the first occasion, four parents were interviewed from each of the two evaluation sites and centres. At the end of 2013, these parents plus another from each site were interviewed. In 2014, a much larger number of parents were interviewed on three different occasions: the first shortly after they had begun the program (38 parents); the second around July 2014 (36 parents) and the third at the end of 2014 (33 parents). These parents were from centres spread over only three of the four evaluation sites as educators in the fourth site were unable to nominate potential parent interviewees. Again, all interviews were conducted over the telephone by trained interviewers.

Educators at the evaluation sites and centres were also invited to participate in a series of three interviews. Across all sites and centres, there were 41 educators who agreed to be interviewed within 3 weeks of the first workshop. Of these, 35 were interviewed for a second time within 3 weeks of the second workshop. As well, 35 educators were interviewed close to the end of the calendar year in which they were involved with *Let's Count*. These interviews were also undertaken by telephone and then transcribed. All transcripts were analysed and coded to establish the key themes to emerge. Six themes emerged through the analysis of parent interviews and seven themes emerged from the educator interviews. Several of these themes provide particular insight about the impact of parents and educators coming together to promote children's mathematics learning. These themes will be examined in the following section as a means of describing how the process of parents and educators coming together assisted children's learning.

Insights from the Interviews with Educators

The aim of *Let's Count* was for educators to assist parents to help their children learn mathematics in everyday situations through noticing the mathematics that was part of the family's activities, exploring this mathematics and talking about it. Seven themes emerged from the analysis of the educator interview transcripts. Six themes had been noted in the first year of the evaluation while the seventh only arose substantially during the second year. The themes were:

1. Engaging families with mathematical learning and Let's Count;
2. Continuity of mathematical learning between early childhood setting and home;
3. Impact of Let's Count on educator confidence, professional identity and pedagogical practice;
4. Awareness of the potential of everyday tasks for prompting mathematics discussion;
5. Sustainability of *Let's Count* over time;
6. Children's engagement with mathematical learning and mathematical concepts;
7. Importance of mathematical language.

Each theme suggests that *Let's Count* had an impact of educators' pedagogical practice, while also highlighting the challenges educators faced as they navigated how the program might work in their individual settings. The first two themes relate directly to the impact of the educators and parents coming together to focus on children's mathematics learning. These data are discussed in the next section.

Engaging Families with Mathematical Learning and *Let's Count*

The educators who implemented *Let's Count* used a range of strategies to involve families and build their awareness of ways they might engage with their children around mathematics. These ranged from one-off events and sending home maths resources, to more day-to-day strategies incorporating mathematical learning into their everyday dialogue with families. Some illustrative excerpts follow.

The families who are part of the Let's Count program, which is probably about a third of our families I suppose, everybody, every child gets a chance to take this bag home. So it's got so much stuff in it, and play the games with their families and really having the conversations with the families and getting them more confident in the fact that they can help their children with their maths.

Interviewer: How have the families responded to the bags?

Oh they've loved it. Loved it. Yeah, no we've had some fantastic comments. In the back of … I've done up a little book to go home and in the front it has like an introductory letter and then it's got instructions to all the games and then at the back it's got parent comments. And we've had some fantastic comments through there about what the children have learned, what the children have been doing. One little girl went home and measured absolutely everything in the house, including the dog. [Educator A]

It's been really nice to hear parents saying that it's been beautiful to spend that sort of quality family time together and they'd forgotten how much fun it was to play games, board games and things with the children. How they'd gotten out in the environment and looked for things. Like, there was a lot more around to do with maths than what they'd realised. And so it's been really nice. [Educator B]

When they come to the information session, we were going to set up a few things that we might do with the children here and get them to participate in that. One of our educators is actually going to read a book that isn't about maths to the families but showing them how she draws the maths out of it even though it's not a counting or number book. And then we were just going to have some discussions and we was going to talk to them about how things can be done in a play based, fun way, when you're already doing them. Letting them know they don't have to sit down and do maths activities as such to get them ready for school.

I think the main difference that it made was the way we engaged the parents in it and we didn't do a lot, it was just little things like putting notices out, putting little newsletters out about it and also we had a board out the front where we just put a little maths problem on there and the parents could sort of get involved. It was just something they could do on the way home or something they could do on the way in. Like counting buses or plan your trip somewhere, things like that. It was just little problems that we posed on the board. And that sort of got the parents really interested and talking about maths a lot more. And so I think that was probably the main thing but it made us more aware within the centre, even though we do integrate it quite well I think it was being conscious of using the language, the maths language with the children because we do play the games and we do do all the mathematical concepts but we weren't using the language, so I guess that's what we seem to be a bit more aware of. [Educator C]

These excerpts highlight the range of ways in which educators engaged with parents and the reported positive impact for building parents awareness of how they might support their children. It was clear that these strategies focused communication between educators and parents.

Continuity of Mathematical Learning Between Early Childhood Setting and Home

The interviews with educators also highlighted the ways in which *Let's Count* promoted continuity in the mathematics learning between the early childhood settings and the home. Also evident was the importance of established communication strategies among educators and parents. Several excerpts illustrate this theme.

> Oh just one little boy came in today and said ‘I really want to measure my bed’. So we made a measuring tape for him. I said ‘You could use your hands’ and he said ‘No, I want a measuring tape’. So we made a measuring tape. … the information came from his mother first and then we discussed it with the child. The mother came in and said ‘Oh he really wants to measure his bed’ and I went Ok, we can do that, we can work out a way to do that for you. So sometimes … It depends on developing a rapport between the educator and the parent. [Educator D]Well we’ve sent out emails on a regular basis with our parents. And so they’ve been emailing things that have been happening in their home. We also have a feedback journal-type thing that parents can write things up in, in the mornings or and we pose questions to the parents relating to maths. You know, we might just pop on a question, you know, what did you do over the weekend that involved mathematics or that kind of thing. … So they’re able to see what the children are interested in doing here and then maybe continue that on. So I think that’s been a good way of doing it. Rather than having portfolios that go home at the end of the year and they go, ‘Oh I didn’t realise you liked that’. [Educator D]We had one little girl who went into Coles and her mum asked for … No, she got her daughter to ask for a kilo of bacon. And so the lady in Coles actually counted how many pieces of bacon made a kilo of bacon. And there’s like a photo on the [Facebook] page of the little girl and the lady from Coles counting the bacon. So the parents have actually given photo records as well of catching their children doing everyday maths as well. [Educator E]I just think it has been a good thing for us to do and particularly I like the way the parents are really involved and it’s more about them, because that will hopefully continue on for the rest of their child’s schooling and for other children that they may have in their family as well. [Educator F]

These data illustrate the importance of educators using a range of strategies for sustaining engagement with parents about children’s mathematics learning, and the importance of established communication to enhance continuity between learning mathematics at home and in more formal learning settings.

Insights from the Interviews with Parents

Six themes emerged from analysis of the parent interviews. Each theme highlights the positive impact of educators working with parents as part of *Let’s Count* but the final theme acknowledges the challenge of sustaining the program across a year and beyond. The six themes were:

1. Noticing children’s mathematical learning and facilitating that learning in the everyday;
2. Parent–educator communication about mathematics and Let’s Count, with an emphasis on strengths of all involved;
3. Children’s growing confidence, knowledge and enjoyment of/engagement with mathematics;
4. Importance of mathematical language;
5. Positive impacts within families, extending to older and younger siblings’ inclusion in mathematical activities at home; and
6. Sustainability of *Let’s Count* over time.

Four themes address the impact of the educators and parents coming together through *Let's Count.* These themes are explored below.

Noticing Children's Mathematical Learning and Facilitating that Learning in the Everyday

One important finding from the research was that every parent interviewed talked about how their ability to 'notice' mathematical concepts as part of their everyday interactions with their children had increased, along with their abilities to extend those concepts when children showed interest. This is an important outcome of educators and parents interacting through *Let's Count.* Many parents suggested that there was mathematics in everything and that they now appreciated that their role in their child's mathematics learning was to notice, explore and talk about this mathematics. While this noticing of mathematics was not always attributed to their family's involvement in *Let's Count*, in many cases, parents explicitly indicated that this was an influential aspect of the program. The excerpt below shows the impact of noticing the mathematics and opportunities to explore and discuss for one family.

> The major difference I think has been I'm much more aware of how she can learn from everyday things. An example of that was yesterday. My husband brought home a little thermometer, he works in refrigeration, and she wanted to know how it worked. And I was just trying to explain and I couldn't be bothered, and then I thought, 'Oh put it in the fridge'. And then she put it in the fridge and we looked at the degrees and she wanted to put it in the freezer and look at the differences in temperature. Yeah, from that it kind of snowballed into looking at why were there different numbers, what's Fahrenheit, what's Celsius, all that kind of stuff. So I think it was good. At the start of the year I wouldn't have bothered, I wouldn't have even thought about it but it just occurred to me like, this is a good moment for her to explore it. Whereas before I wouldn't have done that and I would have just said, 'I don't know, I can't be bothered teaching you that.' [Parent A]

This example shows that noticing the mathematics, recognising an opportunity to explore, and also a child's curiosity and desire to learn is significant for parents. Parents also noted that they had become for intentional in their mathematical interactions with their children.

> Probably one category would be more intentional so whether it'd be sitting down and playing a game of Uno or a game of dominos where we're focusing on that maths. And he has also developed an interest in dot to dots and stuff like that. So that was what I'd say, more intentional, whilst other opportunities just sort of are spontaneous. So whether it's like he's helping me set the table, well "how many forks are we going to need for our family?" or just things that coincidentally pop up in our everyday lives. Like swimming group this week. He noticed that there was numbers on the side of the pool so he wanted to know what it meant, so we talked about depth and then he went on his own tangent of measuring the depth of the pool in different areas, on his own body. So how high the water would reach. So yeah, I guess it's a whole range of experiences, some are planned for and others have just cropped up coincidentally throughout each day, everyday living really. [Parent B]

Another parent explained the spontaneous mathematics that occurred for her family.

> I guess looking at it, he will say 'What do these two numbers make mummy'. So he's looking at double digit numbers, so say the numbers on our letterbox. He'll say, 'there's a 7 and a 3', although it's 3 and a 7. He just wants to know, 'what do those two numbers make mummy?' and so for him to actually ask me that, I think that's pretty good. And then I'll say 'it doesn't really make 37' but that's what he's asking, is what number is it joined together so I'll say, '37' and he's like 'Oh Ok'. So to be interested and eager to know, that's what surprised me at the moment. [Parent C]

Another outcome for many parents was realising the learning impact of involving their children in everyday activities, rather than 'doing it myself' because it's quicker and easier, or automatic.

> It's made me be more active, to make sure that I keep reminding him about mathematics in everyday stuff. … doing the shopping, making sure I keep them active in it, not just doing it. It's easier to just grab three containers of milk instead of saying to him, 'We need three containers, we've got one, how many should we get'. You know, we're doing that a lot more now instead of just doing it and it's really shown through with him as well. And he's actually showing his brother. [Parent D]

Many parents expressed surprise at the mathematics that their children spontaneously used. Often children knew more mathematics than the parents had noticed previously. It seems in some cases that *Let's Count* heightened parents noticing and awareness of the mathematics that their children knew and used.

> The other night we were having beans for dinner and both the girls, Chiara and Lisa (pseudonyms) sat up in their PJs on the counter and they had to cut … They each had their own board and they had their own little knife and they were asked to cut up beans. So they started to cut. Chiara started to cut the beans up and then she was like 'Oh mum, I'm going to make two piles' and she put all of the medium beans, she called them, in one pile and the small ones in the other. And then she counted there was thirty-five small beans ready for the pot and only five medium ready for the pot. Yeah, and I didn't actually say it. She actually came out with it. She was like, 'These are the medium and this is the small' and I was like, Ok cool! [Parent E]

The process of noticing children's use of mathematics also built some parents confidence about their child's transition to school. Children's successful transition to school and learning school mathematics was a concern for some parents.

> So having that program there has just boosted my confidence enough to say Ok well Lily (pseudonym) is catching on to this very quickly, she's doing all the right things, she's talking about it at home, just in general conversation, not even … Even if I bring it up like … I'm not bringing up 'So how did you do with your mathematics today?'. Like, she's just coming up and saying, 'I did this and this today'. Like it's just a bit of a confidence boost and saying, "Ok maybe she is a bit ready. Maybe she is going to be Ok to go to school." [Parent F]

These illustrations of parents noticing their children's use of mathematics and noticing the opportunities for extending their children's mathematics learning are powerful examples of the role parents play in their children's mathematics learning. The examples highlight children exploring and learning about numbers, shapes and measurement. An important implication for young children's mathematics learning is parents' sense of wonder and surprise at the mathematics their children know and use. It is likely that *Let's Count* tuned parents to noticing their children's mathematics activity and learning when previously this had happened without parents' awareness. The process of noticing the mathematics brings this to the forefront of adults' awareness so that they can discuss and explore the mathematics in everyday situations with their children.

Parent–Educator Communication About Mathematics and Let's Count*, with an Emphasis on Strengths*

One central principle of *Let's Count* is for educators and parents/family members to talk about the mathematics in which the children are involved both at home and in the pre-school or early years centre. Correspondingly, Parent-Educator communication about mathematics and *Let's Count*, with an emphasis on strengths, was a key theme to emerge from the parent interview data. The effectiveness of parent-educator communication, both about children's mathematical knowledge and activities and about *Let's Count*, varied across the interviews. For most parents, the level and intensity of this communication increased across the year in which *Let's Count* was implemented, as did the parents' satisfaction with this communication. In some cases, poorer levels of communication around the mathematics children were doing and *Let's Count* itself, were attributed to parents' own acknowledgement of being time poor or their child not attending the centre regularly. Educators set up various means of communication between themselves and parents, including Facebook pages through which parents could communicate about the mathematics their children were exploring. Some illustrative examples follow.

> If something pertinent to the Let's Count thing crops up then they (educators) will mention it. Probably like on the Facebook page they're seeing what we're doing at home so I guess they're learning more about us as well, through a different way than just chatting. Because I mean, pick ups and drop offs are always so busy, you don't always have that opportunity, so I think that it is giving them a little bit more insight into each child. It's probably giving them a greater awareness of each child's strengths and needs as well because maybe they're getting surprised by some of the stuff that the kids do know, or seeing areas where they could focus on more. [Parent G]

In some cases the busyness of life and work reduced parent's opportunities to talk with the educators about children's mathematics learning. However, despite this busyness, communication strategies such as Look Books were important for building awareness of children's mathematics learning, as was parents' daily 'walk through' of the spaces at the centre.

> Look there has [been some communication] but it was quite a while ago and there hasn't been follow up since. … she gave us all the information and she gave us a little talk about it and then yeah, that was sort of all we knew. There's always so much going on. [Parent H]

> Like, we haven't had another meeting to be updated on what they're doing but they have a look book which is really good. It shows what they've demonstrated … What they've been doing and how they've enhanced it. And even we've just had sort of like parent/teacher chats so yeah, they explained what we've been doing and they've really noticed the difference in how inquisitive Kingston is, so that's really good. [Parent I]

Many parents recognised the importance of *Let's Count* materials and information sessions provided by the educators for building communication and awareness. Parents' recall of these sessions varied, as demonstrated in the excerpts below, but the sessions were clearly important for some parents.

> We had a parent information night earlier in the year and she talked about maths in everyday situations and real life and how you can integrate it at home as well as in the classroom and at kinder. What to look for and things like that. Like, I don't know, like lots of different everyday activities I suppose, is the main thing I got out of it. [Parent J]

> I guess there was a concentrated effort when the program was introduced at kinder and certainly as I said, with some of the things that came home there was that focus. The kinder teacher then provided an information night for families and that's when some of those products were distributed to families and she discussed each item in the bag and how they could work in your everyday life with your child and how you could try different activities, some with family members, children as well as the mums and dads being involved. [Parent K]

> Oh I think it's fabulous, that's exactly how I think kids should learn most things, particularly when it can start at home from such a young age and not just at school in a formal setting. And I think sometimes you don't realise as a parent that you're actually doing it, quite often, much more than you probably think. And I think it's great that education and programs are going in this direction and trying to educate parents too, on how to teach maths and use it every day in a much more holistic approach rather than just 'Let's Count to 10'. [Parent L]

Parents also noticed and valued the time educators were spending on mathematics explorations with their children. One parent explained that she learnt about mathematics learning by observing interactions between the educators and children. This modelling is helpful for some parents.

> The three girls [educators] that I usually spend most of my time with and talking to them and they're all for it. I mean, this is one of those kindergartens that I've come into and had a delight in actually learning myself better ways how to educate kids in learning maths, just by looking around how they make that classroom look like a play area for kids. It's wonderful. And just even them sitting down with the kids and going through it and making it fun and seeing that they're enjoying it too it makes a big thing for kids to learn, if you're happy learning too. [Parent M]

Finally, *Let's Count* has been a catalyst for building communication between educators and parents and also between parents. This is illustrated below.

> Well I definitely think that my relationship with Emma, who is the one who is heading the Let's Count with Jack, like I just talk to her so much more. Like we're engaging so much more. Even with other parents, you know. Because we have this Facebook page as well we're all communicating, we're all uplifting each other. Every day I come in and Emma actually has been amazing. Like, she has done so much in the room. They've got this little mathematics table where they're constantly changing things. They've got scales, they've got estimation, they've got all these types of things and she's so into it that it kind of is … What's the word I'm looking for? Like you take it on board. It's awesome. It's so much fun.

And we talk so much more. Like on an every other day basis she's like 'Oh I did this with the kids' and I'm like 'Oh my gosh, it's awesome!' And she'll mention something that I've done on Facebook and she'll be like 'It was so cute'. The language is open. The communication is open. It's great. [Parent N]

Children's Growing Confidence, Knowledge and Enjoyment of/Engagement with Mathematics

Parents reported their genuine surprise at their children's increased mathematical capabilities and, particularly, their children's confidence in trying out new mathematical ideas. In some cases, this mathematical development exceeded parents' expectations of their children's capabilities at their age. Some parents commented that a child's increased mathematical knowledge and confidence was important for their transition to school. For the most part, the parents attributed these increases to the emphasis on mathematics in both the centre and the home, as a result of their participation in *Let's Count.* Some illustrative excerpts follow.

He is more mathematically literate than he was, which is really good. In particular, when he did his primary school screener to see if he was school-ready, they commented on his mathematics understanding as a really positive thing that he was quite excelling in. … that was really good feedback for us too. And we knew as soon as we heard that we went, 'Oh we know why that's happened, because that's the Let's Count program'. [Parent P]

I loved it after the first couple of months of it. They don't do any structured really teaching at kinder but stuff like this, just to get the kids interested and thinking about numbers is a really good way for them to get comfortable with it without being scared of it. Because sometimes numbers can really intimidate kids if they don't have any background of it when they get to the school level, so I think it's really … Because even just playing around with it is such a good way to get them comfortable with using numbers and the concept of maths. And even just hearing the language and stuff has to be positive for them getting a good head start at school. [Parent Q]

And he'll come home and tell me about it and talk about it. Like, they had a rain gauge and it was measuring the water and he was telling me how many mLs were in the rain gauge. I'm like, 'Oh wow'. So they are interested in all the things that they've been doing at kinder. [Parent R]

She comes out with things every day, basically. Something that really surprised me … Oh, [?] were talking about my birthday and that I'm turning 22 and she said 'Oh mummy, you're turning 22, isn't that two two', as in like 2-2' and I was like 'Yes, that's a number' and then she's just like 'So how do we add …' like 'What do we do to get to that number', like … You know. She was just trying to work out how to get to twenty two, like all different scenarios on how to get to the number 22. [Parent S]

For me personally, with Naomi [pseudonym] things that I've noticed are things like she always says … For setting the dinner table, she's started to do that now and she's like 'How many forks do I need? I need two kids' spoons, two big spoons, that equals four spoons'. She says things like that. She says things like, her sister is there with her and she's like 'I've only got five clips and I need six to make it match together'. Things like the other day we were driving and she's like, 'What's the distance from Newcastle to home?' Just thinking of other examples, we also had this boomerang at home and she's asked me for a measuring tape so she could … .And she lined the tape up from one end to the other and she began to read the numbers. So she was like '5, 6, 7, 8, 9' and then she's like 'It's 300'. Which was a bit whacked but … Not obviously correct but it was about her using the lingo [language] and the skills and all of those things that she's obviously picking up from the program. [Parent T]

Overall these excerpts highlight children's increasing mathematical knowledge and confidence. Perhaps also noticeable is parents increasing confidence in their child's knowledge and the power of everyday experiences and preschool experiences for increasing their knowledge and confidence.

Sustainability of *Let's Count* Over Time

While many of the educators participating in *Let's Count* were quite positive about the steps that they were taking in their centres to maintain the program beyond the year of initial implementation, parents were not so forthcoming. For some parents, communication with educators about the mathematics children were engaging with at the centres and homes seemed to be maintained or even intensified over the year, but some parents felt that the communication faded away or that organisational information was lacking. Often, parents expressed that there was an initial flurry of information when the program commenced followed by a relative lack of input into what parents might do next. This situation, as demonstrated by the excerpts below, is likely to impact the sustainability of the *Let's Count* approach within families.

They [educators] really follow up the kids' interest and let the parents know what's going on … That's one thing I've noticed different over the year. Much more feedback, specific feedback so that we can then follow that up at home and continue it. [Parent V]

At the beginning they would talk about the Let's Count program to me, when the first initial stages, and they would tell me about how he was progressing but nothing since the beginning. [Parent W]

So it's not always me who picks up, sometimes it's my husband, sometimes it's me, sometimes it's my sister-in-law, I think maybe the thing that's lacking is the take home part of it, for me. I don't really know the connection between home and what they're doing at the centre. So that's the thing that is confusing me. I do know they're doing a lot at the centre but as far as what I'm meant to be doing at home, if I'm meant to be doing something, I have no idea what that is. [Parent Y]

These data highlight the challenge of maintaining a focus on *Let's Count* across the year. It seems that some parents are less confident about initiating mathematics activities with their children and are keen for educators to provide suggestions, discussions and ideas. There were also more positive data to report about the sustained impact of *Let's Count.* Two parents, who had children involved in both the 2013 and 2014 *Let's Count* cohorts, spoke in positive terms about the 'start' that they believed *Let's Count* had given their children as they began school and how this start had endured across the first year of school.

The findings from the *Let's Count* Longitudinal Evaluation interview data with parents and educators illustrate the reported positive impact of *Let's Count* for supporting children's mathematics learning. This finding was supported by the assessment of the children's mathematical knowledge collected and analysed as part of *Let's Count.*

Conclusions and Implications for Future Programs that Focus on Young Children's Mathematics Learning at Home

Examination of the themes emerging from analysis of parent and educator interview transcripts and supported by the *Let's Count* longitudinal assessment data related to children's mathematical knowledge and skills (Gervasoni and Perry 2015), suggest that pre-school children learn effectively when their parents and educators notice, explore and talk about the mathematics that is part of their everyday activities.

The themes emerging from parent interview data highlight that parents value the educators talking to them about ideas and suggestions regarding the type of activities that are rich sources of everyday mathematical learning. It many ways these discussions provided parents with prompts, inspiration, encouragement and confidence. At the same time, the interviews also highlight that sustaining communication between the parents and educators across the year was challenging for some. It is evident that the resources and suggestions provided for educators during the *Let's Count* professional learning sessions were important, but also that expanding these resources and also communication strategies for both educators and parents may assist some educators to sustain the program across the year.

The following recommendations about ways in which early years initiatives can support children's mathematics learning emerge from the *Let's Count* Longitudinal evaluation findings.

1. Provoke children to notice, explore and talk about the mathematics that is part of everyday activities;
2. Provide prompts and suggestions for parents and educators about the range of mathematical activities that children encounter as part of everyday life. These include exploring and comparing shapes and patterns, comparing the size of objects through measurement, comparing numbers and groups, organising and discussing collections and data, and discussing the likelihood of events occurring;
3. Create sustained communication opportunities for parents to discuss the mathematics they notice their children using and exploring, and provide suggestions about how to extend this learning; and
4. Provide suggestions and prompts about games, songs and stories that can provoke mathematical interest, discussion and exploration.

Two further issues arising from the *Let's Count* longitudinal assessment data warrant consideration. First, while the *Let's Count* approach provides a broad framework for educators and parents describing the powerful mathematical ideas that young children learn and explore, and while the assessment data highlight that overall, children participating in *Let's Count* made significant mathematical progress across their pre-school year through noticing, discussing and exploring the mathematics that arise in everyday situations, there were some children who did not demonstrate this same progress. It could be that if preschool teachers and parents more intentionally discussed and explored mathematics with the children who less often spontaneously notice, explore and talk about mathematics in everyday experiences, then these children's mathematics learning may be enhanced, and these intentional experiences might position these children to benefit more favourably from instruction when they begin school. This is a profitable area for further research. It is also possible that some parents, children and educators may benefit from a more explicit set of activity suggestions that prompt them to explore the mathematical ideas that they do not spontaneously notice during everyday activities.

The final issue refers to the debate about whether early intervention programs are able to overcome any educational disadvantage associated with young children living in financial and social disadvantage. *Let's Count* is associated with significant mathematics learning and confidence for most children who participate, but are the positive effects sustained when children begin school? Sarama and Clements (2015) hypothesise that most present educational contexts are unintentionally and perversely aligned against early interventions. However, interview data from two parents who had children participating in *Let's Count* in both the 2013 and 2014 cohorts suggest that *Let's Count* assisted their children's transition to school in 2014. Therefore, investigating the impact of *Let's Count* after children's transition to school is another valuable area for research.

Sarama and Clements (2015) argue that schools need to be aligned with an early intervention approach in order for its impact to be maintained. Perhaps an extension of *Let's Count* into the first years of primary school would be one way of better aligning the approach with families' experience of school curricula and thus strengthen the conditions necessary for the successful impacts of *Let's Count* to be maintained for all children.

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Chapter 13
Supporting Early Mathematics Learning: Building Mathematical Capital Through Participating in Early Years Swimming

Robyn Jorgensen

Abstract Much has been written about out of school contexts and their importance and relevance to learning mathematics. This chapter explores the swim environment for under 5s and its potential for learning mathematics. The findings are drawn from a much larger, international study on the potential impact of early years swimming to add capital to young children. The focus here is on adding mathematical capital to under-5s. It was found the there is a very strong case for early years swimming to be of significant benefit to young children. Drawing on both internationally and nationally accredited and recognised psychological testing and observations of lessons, the chapter explores specific results and then offers a potential explanation for how such results may have been achieved by drawing on lesson observations. The results provide interesting and valuable insights into the potential of non-school contexts to add mathematical capital to young children.

Keywords Early years · Swimming · Out-of-school contexts · Mathematical capital · Bourdieu

Introduction

As part of a much larger project that explores the potential of participating in early-years swimming to add capital to under-5s, this chapter discusses the affordances of the early years swimming context for the development of mathematical learning. Drawing on data generated through the project, this chapter initially compares the achievement of young children on an internationally recognised child testing program. The children in this study performed significantly better than the normal population of the standardised testing program. Observations of swimming lessons are offered to help explain these data.

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Within the Australian context, there is a considerable fascination and affinity for water and swimming. Most Australians live within 1 h drive from a body of water—the ocean, river, or lake. From census figures, it is estimated that there are more than 1.1 million pools in Australian homes and there are approximately 7.0 million homes in the nation, thus making for approximately 15 % of homes having their own pool. Each year, another 24,000 pools are built in homes (Smith 2015). In this context, water activities are a major recreational activity—with swimming, boating, fishing and diving some of favourite pastimes. Swimming is a major recreational as well as sporting activity across the nation. In 2009, over half a million children, aged 5–14 participated in swimming as an organized sport. It was, in fact, the most popular sport across all children of school age, beating dancing, soccer, Australian Rules and netball (ABS 2009). Most primary schools offer swimming lessons as part of the standard curriculum offerings. As an organised sport, the nation has a major fascination in swimming in international events such as the Olympics.

While swimming is part of the cultural identity of the nation, it is also a major activity for children under 5. In this age group, the activities are largely focused on water safety, as death by accidental drowning was the largest cause of death in under-5s. While there are no firm figures on how many children participate in early years swimming, the peak body for Australian swimming coaches and Teachers (ASCTA) estimated that approximately 20 % of under-5s participate in swimming lessons, thus making for quite a large number of young Australians who participate in swimming prior to school. The interest in early-years swimming has grown with Australia now boasting 934 swim schools nationwide (RLSA and AustSwim 2010), over 600 of which are registered with Swim Australia. Almost 80 % of swim schools are privately owned and a little less than a quarter are operated by local councils. The remaining swim schools operate under a management group, through a school, are community based or a combination of these, along with many backyard operators whose businesses are not registered.

While largely unregulated, the industry has a number of organizations that contribute to its management, regulation and education. These include ASCTA, Swim Australia,[1] AustSwim and the Royal Life Saving Society—Australia (RLSSA). Even the Australian Taxation Office influences the participation and credentialing of teachers in the industry. The CEO of ASCTA estimated that there are approximately 800 registered swim schools catering for under-5s in Australia and that these schools would represent approximately 550 of the 600 members of ACSTA. These figures, however, do not account for the many swim schools that operate out of back yards that may not have council permits, or are not members of ACSTA. The figures, however, do give some insight into the breadth of uptake across Australia of early years swimming.

[1]Not to be confused with Swimming Australia, the national sporting body responsible for the promotion and development of competitive swimming in Australia at all levels. Swimming Australia has almost 100,000 members and just over 1100 swimming clubs nationwide (www.swimming.org.au).

Different swim schools emphasise different aspects of learn-to-swim. Some may elect to offer the "Swim-and-Survive" program from RLSSA, some adapt this program to incorporate other aspects of swimming. Almost all baby classes emphasise water familiarisation and survival skills. Beyond one year of age, however, swim schools offer any number of a variety of approaches to learn-to-swim. Most swim schools advocate that they invoke in children a respect for the water and aquatic survival skills. Beyond this, the primary focus of some schools is be on the development of technique in young swimmers with the ultimate aim of producing (future) competitive swimmers. Others adopt more of a "general education" approach which incorporates other aspects of learning. What is taught in learn-to-swim and how it is taught may impact on what children take away from their learn-to-swim classes to use in their everyday lives. Children may have very different learning experiences from the types of programs offered by the swim schools. Each of these schools offers new learnings—swimming and other—that may help children in contexts outside swimming.

There have been few studies of the impacts of participating in learn-to-swim for young children. Naturally, the focus on the limited research undertaken has been on how early swimming can enhance some motor abilities such as balance and reaching (Sigmundsson and Hopkins 2010) and motor development in neonatal babies including head holding, steady sitting, and holding items (Jun et al. 2005). Others have looked at the impact of swimming on children suffering respiratory difficulties such as asthma (Wang 2009; Font-Ribera et al. 2011). There has also been some considerable research on how water activities can enhance mobility and aerobic strength for children with physical disabilities (for example, Fragala-Pinkham et al. 2008; Hutzler et al. 2008). However, there has been little research into the impact of swimming lessons on able-bodied students other than a large German study in the late 1970s (Diem 1982) when the learn-to-swim industry was in its infancy. Not only are the conditions in Australia different from those experienced in Europe, but in the three decades ago or so.

It is in this context, where many young children are taking swimming lessons, often from the age of 6 months, and with strong parental involvement, that the potential for swimming to add to the repertoire of skills was researched. This is the first international study of its kind to be undertaken and hence represents a significant analysis of the early years swim context. Many aspects of the potential of swimming to add to young children were investigated including intellectual capital, physical capital, social capital and emotional capital. Using nationally and internationally recognised tests, cohorts of children were researched to monitor their well-being against various milestones. This chapter focuses on the swimming environment for children to add to the mathematical learning of children under 5 years of age.

Adding Mathematical Capital Through Early-Years Swimming

The overall project is framed using Bourdieu's (1983) notion of capital, where through the exchange economy, the learnings from one context can be converted into a capacity to do well in a different context and thus become embodied by the learner as part of their habitus. In this case, the learning from the swim context becomes part of the habitus of the learner. A problem for early childhood education is the fact that habitus, or the dispositions, skills and ways of being a family member assimilated as schemata for learning acquired at home do not always transfer well into the school situation. Some children, as described above, can make effective transitions from home to school when they possess pre-existing schemes for thinking about number, colour, shape and so on, others do not. This study, then, explores the possibility of swimming to add to such schemata that learners may be exchange as capital in other fields, namely schooling. It is argued that early swimming has a part to play in this process. Unlike other activities undertaken by under-5s, swimming can be commenced from a very early age. Some advocates of early years swimming (e.g., Laurie Lawrence, the creator of the Kids Alive learn to swim initiative) advocate that swimming can commence as early as birth. Lawrence has worked with the Australian Government to provide all Australian mothers with a DVD on water familiarisation upon the birth of their child. Part of this initiative is based on the fact that the largest cause of death in under-5s is accidental drowning. In a nation where water activities constitute a significant part of the national Australian identity and recreational activities, water safety is strongly endorsed. As such, not only can it be commenced at a young age, but the possibilities for young children to gain in other areas of learning are being explored in this study.

The study is exploring the possibility that early years swimming may add forms of capital to under-5s that may be of value in other contexts, particularly schooling. Jorgensen (2012) has argued elsewhere that the forms of capital building that are possible through early years swimming could include physical capital, social capital, intellectual (or cognitive) capital and linguistic capital. This chapter draws on two of these capitals—intellectual capital and linguistic capital—to constitute mathematical capital. Mathematical capital, or an aptitude to apply concepts in situations requiring understanding of a mathematical discourse, in the context of this chapter, refers to those aspects of school mathematics that are made possible within the early-years swimming context. Being exposed to aspects of school mathematics in the swim environment may support learning of constructs that become embodied by the learner and that can be then exchanged within the school context. These learnings help to support success within the formal school environment. This may be particularly poignant for learners whose social conditions have traditionally excluded them from many aspects of the mathematical discourse and discursive practices.

Method: Child Testing and Lesson Observations

Three key forms of data collection constitute this study. A large-scale survey was developed with over 3000 responses. This survey has been analysed using a number of techniques and the analysis suggest that parents report their swimming children are performing many of the milestones significantly earlier than would be anticipated on developmental expectations. However, while these data are pleasing in terms of the potential for swimming to enhance the mathematical capital building of young children, it is also a limited methodology due to the potential bias of the parents and the limitations of broad parameters of each milestones. To further clarify any potential capital building made possible through participating in early years swimming, intense child testing was undertaken—some tests were for the physical capital building while others were related to cognition and another for socio-emotional capital. Of interest to this chapter are the outcomes of the cognitive tests related to mathematical capital building.

Child Testing

Drawing on widely-used child testing protocols, a series of tests was selected to be administered to children. It was planned that approximately 200 children would be tested. As the tests require considerable input from the child, language skills needed to be well developed, and an attention span commensurate with the time of the test was required. To this end, children only of 3, 4 and 5 years were tested. Within the sampling profile, consideration was also made of gender (boys and girls), socio-economic status (high, mid and low socio-economic backgrounds) and the swim experience of the participants. The tests employed by the Early Years Swimming (EYS) Project were specifically selected to meet a number of criteria:

- Suitable for our purpose—to assess the physical, cognitive and linguistic development of children.
- Age-appropriate—for assessing 3–5 year olds.
- Could be utilised in one session of 1–2 h per child.
- Mostly administered directly to the child without requiring input from a caregiver (or teacher).
- Could be administered by qualified teachers, but not requiring specialist qualifications (psychology, physiotherapy, occupational therapy, etc.).
- Standardised and norm-based: tests have been administered widely with a pool of previous respondents against which we could assess our participants.
- Provide “age-equivalent” measures.
- Not designed for screening purposes (e.g. for identification of autism)—these tend to focus on deficits and not the achievement of milestones and beyond.

The instruments were selected in order to quickly and accurately determine each child's progress across a number of cognitive and language areas. Of interest to this chapter is the Woodcock-Johnson III test that was used to assess "range of cognitive areas, including: oral language, listening comprehension, maths reasoning, verbal ability, cognitive efficiency" (Jorgensen 2013). Each assessment took approximately 90 min to implement by trained teachers. Parents were usually present but were asked not to contribute to/influence the child's responses. Assessments were conducted on campus or within quiet rooms in swim schools.

Woodcock-Johnson III (WJ III)

The Woodcock-Johnson III (WJ-III) Tests of Achievement is a comprehensive system for measuring general intellectual ability, scholastic aptitude, oral language and achievement. It allows the assessment of a wide range of ages, reportedly 2–90 years. First developed in the United States in the late 1970s, it has been extensively tested, with a wide normative sample in 2001 of over 8000 in the United States. It has since been re-normed with an Australian sample of over 1300 in 2006–2007. Sub-tests from the WJ-III have been used in other large-scale Australian studies, for example, the Child Care Choices Study (Bowes et al. 2009).

At ages 3–5 years, it is difficult to assess cognitive and language skill in one brief sitting. The WJ-III allowed for quick and accurate assessment of each child's progress. Eight test items were selected from the WJ-III Tests of Achievement battery based on appropriateness for the purpose of the study (in assessing cognitive and linguistic levels), suitability for the age group and ease of implementation. Two of these items specifically related to mathematics and are described in Table 13.1.

The results from each of these sub-tests are recorded as "Age Equivalent" scores, sub-test scores can also be amalgamated to allow the formation of five "clusters": Oral Language, Oral Expression, Brief Achievement, Brief Reading and Maths Reasoning. Each of these clusters is designed to provide a highly reliable prediction of future achievement in a minimum amount of testing time. As composites of individual tests, they are more reliable than individual test items (Table 13.2).

Table 13.1 Items selected from Woodcock-Johnson III tests of achievement for EYS child assessments in mathematics

Sub-test item	Brief description
Item 10: applied problems	Mathematics problems need to be solved by the child by listening to the problem and performing simple calculations, eliminating any extraneous information presented. Calculations become increasingly complex
Item 18: quantitative concepts	Understanding of maths concepts and symbols is assessed through counting and identifying numbers, shapes, and sequences. The child may also progress to items where they have to identify a missing number from a series

Table 13.2 Woodcock-Johnson III tests of achievement clusters assessed for EYS

Tests of achievement/clusters	Brief reading	Oral language	Oral expression	Maths reasoning	Brief achievement
Applied problems				💧	💧
Quantitative concepts				💧	

Table 13.3 Overview of ages and gender of swimming children assessed

Age	F	M	Total
Group 1: mean age 40.5 months	30	30	60
Group 2: mean age 48.8 months	36	26	62
Group 3: mean age 60.2 months	29	25	54
Total	95	81	176

As the WJ-III provided age equivalent scores for each item, this standardised test permitted comparison of the child's actual age with the performance on each item and each cluster with a wider population of children. It also provided "Z" scores for each item and cluster.

The data collected for this part of the study were compared against larger populations—the tests were selected on the basis that normative data were available to which comparisons could be made with our swimming children. In most cases, these were Australian norm-referenced populations making it possible to undertake comparisons between the swimming children and a normal population.

One hundred and seventy-seven (n = 177) children were assessed, 95 were female and 82 male. They were aged between 36 and 71 months with the mean age of 49.46 months. For the purposes of our analysis, the children were split into three groups, based on tercile age. The ages were converted to years by taking age in months at time of testing and dividing by 12 and then rounding to the nearest year. The rounding is very important because it means that 0.5 is rounded up and 0.4 is rounded down. The result is a group of years that will be based on children around the whole year but might average slightly lower or higher. The alternative—to select those children aged between 3 and 4 years—would provide an analysis of a mean age closer to half-years (e.g. 3.5 years), making comparisons difficult. Once split into the three terciles, the gender groupings per age were then identified (Table 13.3).

All of the children who took part in child assessments are actively engaged in learn-to-swim classes. They have participated in varying lengths of time, from 6 to 61 months.

The children represent a variety of socioeconomic backgrounds. Parents were asked for the postcode of their residential suburb and data was analysed using the Australian Bureau of Statistics Index of Relative Socio-economic Disadvantage (IRSD). This is a general socio-economic index that summarises a range of information about the economic and social conditions of people and households within an area. A low score indicates relatively greater disadvantage in general, a high score indicates a relative lack of disadvantage.

Of the children assessed for this project, 82 represent residential areas that score in the lowest half of areas on the ABS's Index of Relative Socio-economic Disadvantage (Table 13.4).

The basis for this aspect of the child testing was the Woodcock-Johnson III tests. Using a two-tailed t-test, a number of factors were found to be very highly significant. The Woodcock-Johnson III battery assesses children on a number of items,

Table 13.4 Overview of ages, gender and socioeconomic status of swimming children

Age group	Female				Male			
	Low SES	Med SES	High SES	Total	Low SES	Med SES	High SES	Total
Group 1: mean age 40.5 months	3	14	13	30	11	10	9	30
Group 2: mean age 48.8 months	10	12	14	36	6	10	10	26
Group 3: mean age 60.2 months	10	10	8	28	12	7	6	25
Total	23	36	35	94	29	27	25	81

Table 13.5 Mathematical reasoning cluster

Cluster	Indicative items included in general skill	Mean	Significance	Mean difference
Mathematics reasoning	Simple mathematical calculations and counting and identifying numbers, shapes, and sequences	56.06	0.000	6.597

some of which can be aggregated into clusters which provide quick and accurate measures of performance for general skills.

The mathematical reasoning skills had statistical significance can be seen in Table 13.5.

As a group they were particularly strong in Mathematical Reasoning. They also scored more than 6 months ahead of the normal population on the cluster for mathematical reasoning. These results will now be closer examined by looking an individual subtests and by breaking down the cohort into a number of subgroups (by age, gender and socioeconomic status).

Age Groupings

The 177 children assessed for this research has been broken down into terciles according to age. Sixty children are in the youngest group (Table 13.6).

With a mean age of 40.5 months, they are excelling over the normal population in a number of areas including Applied Problems and Quantitative Concepts at 9

Table 13.6 Performance of the swimming tercile age group 1 on WJIII assessments

Sub-test	Mean	Sig. (2-tailed)	Mean
Applied problems	49.58	0.000**	9.083
Quantitative concepts	44.73	0.001**	4.233

$^{*}p < 0.05$, $^{**}p < 0.01$

Table 13.7 Performance of the swimming tercile age group 2 on WJIII assessments

Sub-test	Mean	Sig. (2-tailed)	Mean difference
Applied problems	57.13	0.000**	8.327
Quantitative concepts	56.57	0.000**	7.771

** $p < 0.01$

Table 13.8 Performance of the swimming tercile age group 3 on WJIII assessments

Sub-test	Mean	Sig. (2-tailed)	Mean difference
Applied problems	65.85	0.000**	5.646
Quantitative concepts	64.10	0.001**	3.896

* $p < 0.05$, ** $p < 0.01$

and 4 months ahead of their same age peers. These results are statistically significant.

Similar results were recorded for the middle tercile. There were 63 children in this group (Table 13.7).

With a mean age of 48.8 months, this group also outperformed the normal population in many statistically significant ways with considerable differences ahead of their same aged peers in the normal population—Applied problems (8.3 months), Quantitative Concepts (7.7 months).

The 54 children in the oldest tercile have a mean age of 60.2 months. Their results are reported as follows (Table 13.8).

The oldest tercile also performed well on both mathematical measures—Applied Problems (5.6) and Quantitative Concepts (3.9). It is noted the differences are not as great in this group, and it is thought that there may be a ceiling effect coming into the data since the selected tests were only for young children (Table 13.9).

These data suggest that young children participating in swimming lessons appear to be achieving better in mathematical domains than the normal population. One of the questions that these data raise is related why the swim environment may be enhancing mathematics learning. In order to better understand the potentialities of the swim environment, the project also included observations of lessons across four different states in Australia.

Table 13.9 Overview of the performance of the swimming cohort by tercile age groups on WJIII maths assessments

Sub-test item	Group 1 mean age: 40.5 months	Group 2 mean age: 48.8 months	Group 3 mean age: 60.2 months
Quantitative concepts	7.398**	3.613*	5.46**
Applied problems	9.083**	8.327**	3.896**

* $p < 0.05$, ** $p < 0.01$

Swim Pedagogy: Fostering a Rich Language of Mathematics

As part of the project, there has been a concurrent investigation of the pedagogical environment in order to understand the ways in which the teaching may, or may not, enhance learning. Not only are the learners being exposed to practices that help build a capacity to swim, but there are hidden aspects to the pedagogic discourse that build other forms of capital. While some of the pedagogic discourse relays convey mathematical concepts and processes, they also may assist in inducting young learners into the ways of schooling. The potential to build forms of capital through the swim pedagogy has been observed across a number of lessons. The research team has been involved in observing many lessons across Australia. Over 12 months, an observations tool has been developed to profile the pedagogy of the swim schools. In developing this tool, a considerable number of lessons were videotaped to ensure the development reliability of the tool, and focus groups conducted with the swim industry to trail and validate the tool. For this chapter, I am not focusing on the specifics of the lesson observation tool, but rather the observations of lessons where it became clear that the pedagogic discourse used by the teachers had considerable potential for enhancing the mathematical capital of the learners.

Zevenbergen (2001) has noted the disjunction between the home and school discourses around the mathematical register. The instructional discourse of the learn-to-swim programs uses the rich language of shape, location, colour, number so that young children are exposed to this language from a very early age. Terms such as “get the red ball”, “swim under the rope”, “push through the hoop” are commonplace in the discursive practices employed by the teachers. This enrichment of vocabulary exposes the children to many aspects of the mathematical register but also links the constructs with physical actions so that the children have greater opportunities to learn the school discourse and embed it within a physical-cognitive experience. In this way, there is every chance that the students may have greater success in schools due to their exposure to the patterns of signification (concepts/language) within the learn-to-swim program that augers well with school knowledge. This may be particularly so for those children whose home language is restricted in the use of such terms. Through the instructional discourse, students are exposed to rich iterations of language thus offering potential to extend their linguistic (and mathematical) capital. In this context, the swim environment offers the potential for young children to add to their repertoire of skills, knowledge and dispositions that ultimately may position them favourably for formal schooling. This is now discussed with reference to learning mathematics.

Pedagogic Discourse in Learning Mathematics

Bernstein (1990) pioneered the work on pedagogic discourse where he argued that classroom discourse relays more than just concepts. In this case, not only are swimming skills and dispositions being learnt, but other valuable aspects of the dominant culture are being learned. Most notably for this chapter is the mathematical ideas and processes that are integral parts of Western culture are being embedded implicitly in the discursive practices of the swim pedagogy. Also, other aspects of culture are being relayed—such as turn taking, paying attention to the teacher, not talking while the teacher elicits instructions, or walking in single file to the teaching space. Many of these cultural norms of the swim environment will transition to the formal school context, which will also act as a form of capital for the students.

Of interest in this chapter then, are the possibilities for mathematical learning that is being made possible via pedagogic discourse. This discourse is one that has particular regulatory rules and protocols that are part of discourses to do with engagement and preparation for instruction. For example, as the teacher encourages kicking skill, he/she moves the babies' legs counting one, two, three, four and with each count the leg is moved. This protocol of counting to each kick not only encourages the development of the auditory phenomenon of counting but also the one-to-one correspondence of count-to-kick pattern. Students are exposed to the discourse as they inserted into the teaching/learning environment. As the swim environment is one where there is a high emphasis on safety, teachers work in small classes and are focused on ensuring all children are engaged with the lesson. The engaging environment is one aimed at maximum learning in the time allocated so that students learn important skills regarding attention to the teacher and on-task behaviours.

The protocols associated with the learning environment can be illustrated in the following extract where the teacher is relaying a number of important aspects of the teaching/learning environment. In this extract, the important social skill of turn-taking is being elucidated. At this same time, the importance of waiting until it is the student's turn to undertake an activity, is being learned. By waiting, the teacher is then able to work with, and assess the student's behaviour and undertake any necessary corrections. The importance of being able to take turns is embedded in the interactions.

> Teacher: Jack,[2] go back to the wall, start from the wall and wait your turn, buddy.[3]
>
> This was also noted in an interview:
>
> Teacher: You need to have eyes in the back of your head. As soon as you hear a splash or yell, your immediate reaction is to see what has happened, to see if it is one of your kids.

[2]Pseudonyms are used in this paper to protect the identity of participants and sites.

[3]A term of affection often used by teachers, particularly for young boys.

> You don't get much a chance if they fall over: you have to make sure you know where each of them are at any point in time.

The importance of the pedagogic discourse in coming to learn more than just swimming was observed across a range of patterns and structures within the swim environments. Within the Bourdieuian framework used in this project, what can be seen is that the swim environment is adding new forms of knowing, behaving and communicating (or capitals) to children that, in turn, is internalised into their habitus. These dispositions to the learning environment are likely to position them more favourably with teachers as they display these new learnings. The displays of learning that can be observed in the children need to align with the practices valued in the field if the child is to be seen as displaying valued knowledge. Such displays, in turn, can then be exchanged for other rewards in the learning environment. In the swim environment, these are often certificates that acknowledge what has been learned, and as a consequence, progression into a different class. While the swim environment primarily focuses on skill development leading towards independent swimming, what is of value is the incidental learning that can be readily observed. There were many practices that created potential for mathematics learning that would prepare students for their mathematics learning but also support them in their transition into formal schooling. It is this aspect of the swim environment that is the focus of the remainder of this chapter.

Mathematical Discourse

Jorgensen and Grootenboer (2011) have shown that the pedagogical discourse of the swim context is rich with mathematical signifiers. Throughout the observation of many lessons we found that there were often times when swimming teachers used mathematical language and ideas in their instructions. Most lessons links counting exercises with bodily mechanisms for familiarisation or propulsion through the water, so that the children are exposed to regular counting in threes or up to ten in the very early years. Water familiarisation, which can commence at birth, is a cueing process where the parent pours water of the child's head so that it runs over the face. To cue the child, the parent says "one, two, three, ready". At the cue of 'ready', a very young child, even 3 months of age, will close her/his eyes in anticipation of the water coming over the face. This very early exposure to counting and number brings about a familiarity with the counting patterns. In the lessons, teachers constantly use counting patterns to cue the children for various activities—such as kicking, submersion or floating.

Drawing on the data from Jorgensen and Grootenboer (2011), it can be seen that the pedagogic discourse employs a rich mathematical language. Many terms are used that relate to various aspects of the mathematics curriculum. These include number (one, two three), to measurement (big, fast, slow), to space in the areas of geometry (circle, straight, line, edge) and positions (up, down, underneath, side-by-side, together, backwards, edge). For example:

T: After one-two-three, we are going to push off with our hands like a rocket.

T: I need to see really big arms, big and slow.

T: Clinton, can you follow the big line on the roof" [points to the line painted on the ceiling]?

T: Okay watch me, I am going to have my hands on the edge, toes on the wall, head backwards, looking up at the line on the roof. Watching me, push off the wall, eyes up, glide, like a ferry boat [teacher demonstrates]. Alex, hair in the water first, and push and glide. Hold your body nice and straight and long.

The routine instructions employed by the teachers and constitute a significant component of the lessons observed over the 3 years of data collection and lesson observation. There is much mathematical language and concepts embedded in these interactions. What was also observed was of the strong link between the auditory learning of the words and that the words/concepts are linked with physical actions. There is a strong push in early years learning, through perceptual-motor programs (Stephenson), to link kinaesthetic experiences with linguistic experiences with the view that this partnering of physical actions with concepts further strengthens learning. A growing body of contemporary brain research suggests that there is a linking between the physical embodiment of words and cognition. This research indicates that the physical movement associated with swimming may be helping young children with many of the concepts found in mathematics. If this is indeed the case, then the body movements associated with number, or body movement and position or even colour sorting may enhance the potential for learning and retaining these concepts.

While there is an emphasis on the mathematical signifiers that are integral to learning school mathematics, there is also a need to recognise the role of the 'little' words (e.g., prepositions, adverbs) are important as these often have important meanings in mathematics (e.g., off, up, out). These are often neglected in the study of language and mathematics but are an integral part of learning (and success) in school mathematics. There is a considerable difference between 25 % of 200 and 25 % off 200. Similarly, the enrichment of spatial language terms of near, next, on, below is integral across the spectrum of mathematical experiences. In these cases, the little words have a big influence on mathematical learning. Being exposed to mathematical discourses where prepositions have been used is integral to learning, and it has been found that when students are not able to grasp the use of prepositions, there is considerable scope for error (Zevenbergen et al. 2001). The swim environment offered a range of experiences, with concomitant physical actions, with these signifiers. In the extracts below, the use of these little words can be observed:

T: Sitting on the edge of the pool, rockets up in the air, now on one, two, three, slide in and push off the wall swimming out to me using big arms.

T: Alex, put your rockets up like Benjamin. No, not hands side-by-side, hands one on top of the other. Keep your toes underneath the water, nice long legs, no spaghetti legs, nice long straight legs.

T: Climbing up out of the pool, using your muscles, tummies, hands and one knee. Standing on the edge now. Now, using your hand making a circle with your arm. Going up past your ear and around to your leg, up past your ear and around down to your leg. Big circles, I want big straight arms [teacher manipulates the child's arm to demonstrate].

These extracts are representative of the instructional discourse of the lessons that have been observed consistently throughout this study. The ones used here have come from one lesson and are used here to illustrate the potential of the swim pedagogy to build a rich experience of the mathematical discourse, but to also create strong practical experiences that link action with words. This partnering of action and words may offer richer experiences for learning (and understanding) the mathematical discourse. The swim environment seems to afford particular experiences—both cognitive and physical—that extend and consolidate the learning of a rich mathematical discourse. Furthermore, it was clear during the observations that the children's responses to the mathematical terms were demonstrated their understanding by performing the appropriate action or behaviour.

Conclusion

The findings reported in this chapter indicate that there is considerable potential for the early years swimming context to provide affordances for building mathematical capital among young swimmers. This capital comes in the forms of early number, comparatives (same/different), colour recognition which is an integral part of many of the early sorting and classifying activities that are foundational to number concepts. What is of interest to the early childhood sector, to parents and caregivers, and educational researchers is that the swim environment is one that is not traditionally associated with formal learning, particularly in matters related to education per se. However, what I have shown in this chapter is that participating in early years swimming may offer much more than physical capital and water safety for young children. The data in this project have shown that even for young children from low SES families, there have been achievements in mathematics learning (and other areas) that are significantly better than for the normal population.

In terms of the early years learning of mathematics, it may be prudent to consider avenues for learning that are outside those usually associated with school or formal learning contexts. While this study was focused on swimming, there has been feedback from other sport areas—can ballet enhance learning or fencing? This is difficult to answer. However, what is also clear from this study, is that unlike other activities that young children undertake, swimming can be started at a very early age. Water familiarisation, as advocated by Laurie Lawrence and cited in the beginning of this chapter, can commence as early as the first bath. Formal swimming lessons can commence at times nominated by the swim school but are often between 3 and 6 months. As such, children can start swimming much earlier than any other activity. Unlike other sport or recreational activities where the child's motor skills are considerably advanced, the swim environment supports the child—so even with floppy necks, the water acts as a support (along with the parents' grasp). To this end, the child is often participating in swimming nearly 2 years before they can commence most other physical activities.

What has emerged from this study is that participating in early years swimming has the possibility to enhance young children's mathematical understandings. As noted in the earlier sections of this chapter, there are many differences in schools in what is a largely deregulated industry. Parents have the choice to opt in or out of early years swimming. For those who do, there is a need to be mindful of what constitutes a quality swim environment, and how that environment may (or may not) contribute more broadly to a young child's learning. What has emerged from the study is that much more is learned than just swimming. Quality swim schools have the potential to significantly enhance the mathematical learning of children under 5 years.

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Chapter 14
Fostering Children's Everyday Mathematical Knowledge Through Caregiver Participation in Supported Playgroups in Schools

Susan Edwards, Karen McLean and Pamela Lambert

Abstract In this chapter we discuss supported playgroups in schools (SPinS) as sites for engaging families as the first mathematics educators of young children. We refer to findings from our work to show how caregiver (e.g. parents, grandparents, aunts) participation in SPinS can contribute to an awareness of children's learning of mathematical concepts through play. We discuss the findings from our work in relation to 'everyday mathematics' and make several recommendations aimed at enabling families to engage in everyday mathematics both at SPinS and in the home.

Keywords Supported playgroups · Play · Mathematics in the home · Early childhood

Introduction

Mathematical knowledge for young children is a known predictor of success in later mathematics learning at school (Tudge and Doucet 2008). Research suggests that young children's engagement with mathematical concepts in the home is critical to their later concept formation upon reaching formal education (Skwarchuk 2009). In this chapter we use Ginsburg et al. (2008) concept of everyday mathematics as informed by Vygotsky's ideas about conceptual learning to think about young children's mathematical learning in the home. In particular, we draw on research suggesting that children's participation in play-activities in the home is an important conduit for accessing everyday mathematical concepts. We then reflect on how caregivers (e.g. parents, grandparents, aunts) can be best supported to provide

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children with access to increased opportunities for play in the home by reporting findings from a project we have conducted regarding caregiver participation in supported playgroups.

The supported playgroups we report on in this chapter were co-located on local primary school sites, and are known as Supported Playgroups in Schools (SPinS). Each SPinS was operated by a playgroup co-ordinator in cooperation with two pre-service early childhood educators from a local university. The playgroup coordinator and pre-service early childhood educators designed and implemented play-activities for the children guided by the Learning Outcomes from the Australian Early Years Learning Framework (EYLF) (DEEWR 2009). The EYLF is a curriculum framework used to guide the provision of curriculum for young children aged birth to 5 years in Australian early childhood education settings. These settings include formal kindergarten, long-day care and family-day care. Caregivers indicated that participating in SPinS increased their understanding of how to involve children in play both at SPinS and in the home. We outline recommendations for policy and practice in terms of supporting caregivers and children in the engagement of play-activities intended to increase children's access to everyday mathematical concepts.

Theoretical Framework: From Everyday to Mature Concepts

'Everyday mathematics' is the term Ginsburg et al. (2008) gives to the mathematical engagement of young children in the family home (see also, Ginsburg and Amit 2008). Ginsburg et al. (2008) suggest that everyday mathematical learning is more focused on the mathematical thinking afforded by the objects and events experienced by children in their homes than it is on the explicit learning of mathematics itself. Everyday mathematics is important for young children because it exposes them to the use and purpose of mathematics across a range of areas, including number, geometry and patterning in the context of their daily lives and experiences (Wager and Parks 2014). For example, a 3-year-old boy is taught by an older sibling how to create a pattern for a paper chain they are making to decorate their newly arrived Christmas tree. When the 3-year-old places two orange coloured loops in a row, his older sibling gently explains: "No we are making a pattern. Look you have to go 'blue, orange, blue, orange, blue, orange'. Do you see? You can't put 'orange, orange' that is not the pattern." In another example, a 4-year-old child is invited to set the table for an evening meal and upon being told that five people are eating counts out five sets of implements and correctly places these on the table. In yet another example, a 4 year girl is making birthday cakes in the sandpit. Knowing that she is four and turning five in the coming year she begins to place candles in the cake using pegs to represent the candles. She picks up on peg from the ground and inserts it into the cake. 'One candle' she says. 'I am turning five, so I need four more'. She then counts out four additional pegs as her candles from the nearby peg basket. Other examples involve children assisting with shopping (e.g. counting out eight apples) and helping with directions ('which way do we turn to get to kindergarten?'). Research shows that young children's exposure to everyday mathematics in the home predicts their later levels of mathematical learning (Anders et al. 2012). In particular, the quality of the 'home learning environment' (Melhuish et al. 2008) in provisioning everyday mathematics for young children is considered important. For pre-school aged children this involves interactions with family members that include games and play-activities involving mathematics, such as singing nursery rhymes, playing with blocks and/or Lego, using puzzles, participating in craft, painting and drawing, and sharing books and digital resources that involve counting, shapes and patterning (Sylva et al. 2004). In recent years apps for young children associated with mathematical knowledge have also become an increasingly important home learning activity. These include open-ended apps that encourage children's own mathematical activity (such as drawing, painting) and more structured apps that focus on shape recognition, number and counting (Yelland et al. 2014).

Ginsburg's ideas about everyday mathematics draw explicitly on the work of Vygotsky (1987). Vygotsky argued that everyday concepts are foundational for young children's acquisition of mature concepts. Mature concepts are achieved by children when they are able to blend an everyday concept with what he called a scientific concept. A scientific concept is an accepted knowledge convention, such as a number operation indicated by knowing that two plus three equals five. A child may know this operation at an everyday level as having two chocolates and then asking for three more (or in the case of the birthday candles, one plus four equals five). The child achieves a mature concept of this operation when she also realises that her two chocolates and her next three chocolates may be represented as $2 + 3 = 5$. Here, the everyday blends with the scientific to provide the child with a useable or 'mature' concept for explaining why two chocolates plus three more chocolates provides a total of five. Research shows that everyday concepts are significant for young children's early learning because they set the basis for later connections with scientific concepts upon reaching formal early childhood education settings (Fleer 2011). Research also suggests that everyday mathematical concepts experienced by children are a significant predictor of later mathematical learning (Skwarchuk 2009). Everyday concepts are described as embedded in young children's play, including their 'informal' play such as a pretend play, or more formal play, such as engaging in games or singing songs with adults. Due to the relationship between children's play and their engagement with everyday concepts, play in the home has been promoted in recent years as significant for young children's learning.

Play in the Home

Young children's engagement in play in the family home is increasingly acknowledged as important for their learning. Ginsburg's adaptation of Vygotsky's ideas about concept formation into the idea of 'everyday mathematics' highlights how play is understood to prime children conceptually for later learning. Internationally, 'play in the home' has been harnessed as a means of mediating against social and economic disadvantage (e.g. Desforges 2003). This is because research suggests that children benefit educationally from informal and formal play experienced in the home. Increasing levels of play in the home to mediate against social and economic disadvantage has been achieved primarily through parenting interventions designed to help parents increase the range of play-activities they provide for their children in the home. Well-known approaches include HIPPY and the ABCEDARIAN approach in which parents are provided with sample play-activities they can provide to their children, and are likewise encouraged to focus on the explicit engagement in play with their children for at least 15 minutes per day. Longitudinal research shows that these approaches significantly improve young children's learning outcomes in areas such as literacy and mathematics (Baker et al. 1998; Campbell et al. 2012). Other approaches have focused on making play-activities in the community more accessible to parents. These approaches frequently involve early childhood professionals modelling the use of play-activities to parents in informal settings so as to increase parental awareness about the provision of everyday concepts to children through play. One such approach, from the United Kingdom known as 'Room to Play' was a playroom set up in a local shopping centre (Evangelou et al. 2006). Parents were able to 'drop in' to the playroom with their children at any time and their children invited to participate in play-activities designed by staff. This approach was shown to increase parental interest in children's play at home, and importantly moved away from the notion of parenting interventions designed for 'hard-to-reach' families towards a more a philosophical and practical commitment to realising more accessible services for families instead (Evangelou et al. 2013). In our own research, we have been considering the role of playgroups in creating accessible opportunities for parents to learn about their children's play as a basis for play provision in the home and increasing young children's engagement with everyday concepts prior to participation in formalised early childhood education services.

What are Playgroups?

Playgroups are groups where parents and children gather regularly with their children to participate in shared play-activities. In Australia, families generally access one of two types of playgroups. The first type is known as community playgroup. Community playgroups are typically run by attending parents on a voluntary basis. Parents and children meet in a community facility or family home for approximately two hours per week. During this time, parents provide the participating children with a range of play-activities and support their own and their children's social interactions with others. Community playgroups are attended by families from across the socioeconomic spectrum, although the quality of play-activities offered to children during these sessions is highly variable as it depends on parental knowledge about play and access to resources for play-activities. Supported playgroups are playgroups run by a nominated and paid playgroup coordinator. Generally, supported playgroups are attached to a community services provider and specifically target families considered 'vulnerable' due to factors such as a socioeconomic disadvantage, drug or alcohol use, refugee status or speaking a first language other than English (Jackson 2011). In supported playgroups the play-activities are planned and implemented by the playgroup coordinator. The playgroup coordinator also models play-based interactions with children designed to increase children's exposure to everyday concepts, in areas such as mathematics and literacy learning.

Data from the Longitudinal Study of Australian Children (Hancock et al. 2012) suggests that children who attend community playgroups during their infant and toddler years show increased learning gains in the area of literacy and mathematics than those who do not attend. Research also shows that parental participation in supported playgroups reduces social isolation and increases parents' capacity to engage with their children in play-based activities (Jackson 2013). Playgroups are well established as a sites for parental and child engagement and participation in play. For this reason, community and supported playgroups are increasingly provided on-site by primary schools as a means of broadening community access to play-based learning activities and experiences for children and families. In our own research, we have considered parental perspectives on their participation in supported playgroups located on school sites. This form of playgroup provision is called Supported Playgroups in Schools (SPinS) (McLean et al. 2014).

The SPinS Project

The project on which we report in this chapter involved the provision of SPinS to families in an area of identified socio and economic disadvantage according to Australian Early Development Index (AEDI) and Best Start Atlas data. Five SPinS were co-located on five separate primary school sites. A playgroup coordinator attended each of the SPinS and was supported in the provision of play-activities to the children and families by two pre-service early childhood educators from a local University. The playgroup coordinator and pre-service educators cooperated in the planning and implementation of play-activities for the children and families. The planning and implementation of play-activities was informed by the Learning Outcomes associated with the Australian EYLF (DEEWR 2009). The intention of the SPinS project was to build community connectedness to local schools and to increase the access local children and families had to opportunities for play-based learning prior to moving into formal early childhood education and school based services. For the purpose of this chapter, we report on caregiver perspectives about their participation in the SPinS—paying particular attention to their views regarding the play-activities provided in the playgroup and the extent to which participation in the playgroup influenced the play-activities they provided their children at home. We are interested in caregiver perspectives on the experiences provided in SPinS and whether these influenced the provision of play in the home because of the theoretical and empirical significance placed on children's access to everyday concepts (e.g. everyday mathematics) in the home through play-based activity as a basis for children's later successful mathematical learning.

The SPinS operated over the course of two normal school years. Participants in the SPinS included parents, grandparents and aunts of young children aged from infancy to 4 years. We called the participants 'caregivers' in recognition that not all of the participants were the parents of the attending child. All caregivers were invited to contribute to a pre and post participation focus group interview regarding their participation in SPinS in the first and second year of the project. Of the 50 caregiver participants across all five SPinS, eleven agreed to contribute to the focus group interviews. The focus group interviews employed a semi-structured interview schedule (Krueger 2009) and focused on caregivers' views about the play-activities provided for their children within the SPinS, and the extent to which they perceived their participation in the SPinS as influencing the play-activities they provided their children in the family home. All interviews were audio-recorded, and later transcribed by a professional transcription company. Interview transcripts were inductively analysed by two researchers and checked for coding consistency by a third researcher (Grbich 2013).

Lessons from SPinS: What Do Caregivers Think About SPinS Play-Activities and Children's Play in the Home?

Caregiver views about the play-activities provided for their children within the SPinS suggested value in SPinS as a context for caregiver learning about children's play and the consequent provision of play-activities to children in the home. Caregivers commented on the structured nature of play-activities in the SPinS as provided by the coordinator and the pre-service educators, and the extent to which their participation in SPinS exposed them to play-activities they had not previously considered for their children. Some of these activities were consequently employed by the caregivers in the home, thus increasing the children's access to informal and formal play-activities both in the SpinS and in the family home.

Caregivers commented on the routine informing the conduct of the SPinS and the presentation of play-activities to children so as to maximise play and reduce the level of random activity occurring amongst the children. For example, one caregiver described how the playgroup routine meant that she and her child knew what was going to happen—when they would play, engage in craft activities, have a snack and then participate in shared reading and singing with the rest of the group. This was valued because it meant that the caregiver and child could anticipate what would happen next and so fostered the caregiver's capacity to engage with her child:

> The set-up is good. There's sort of a process. We [parent and child] know that we play and eat and do craft. Then it's pack up, and then it's ok for fruit time. Then that's packed up and it's book [pre-service teachers reading stories to the group] so they start to get to know that it is book and song time. So they [children] know the routine. I think that's really important for kids, routine. (Kate)

Here the caregiver notes value in the routine for herself and her child. Routine is considered 'really important for kids'. From an everyday mathematical perspective, this aspect of SPinS also provides the child with access to concepts associated with time and chronology and so helps the child learn that activities can be sequentially based in terms of 'what comes next'. Other caregivers commented on how the play-activities were presented to the children. They discussed how the pre-service early childhood educators distinguished between play-activities by using blankets to create stations. These blanket stations resulted in more focussed play by the children and reduced the extent to which toys were scattered around the room:

> The girls [pre-service early childhood educators] put down a blanket and there might be baby toys there, blocks there [caregiver pointing to areas around the room] and puppets over there … it's a really clever way of getting the children involved, rather than [having the] toys all over the room. (Tara)

> I noticed that they are sorted into little sections; it focuses their [children's] play. It's a clever way of doing it. Previously it was just a bit random. The children tipped out the toys from the boxes, move to the next box and tip that one out and not play. It's a great improvement to playgroup, putting down those blanket stations. (Candy)

For these caregivers being exposed to the presentation of the play-activities in the form of 'stations' alerted them to how the children's play could become more focused. This was important because it helped the caregivers see that they could focus their children's play without necessarily needing to direct this in a verbal way. Once again, the structuring and presentation of the play-activities also exposed the children to everyday mathematics in terms of categorisation with 'baby toys there, blocks there and puppets over there'. In these examples, the caregivers are learning from the coordinator and pre-service early childhood teachers about structuring routines for children and how to present play-activities for maximum play benefit, while the children are participating in a series of experiences that increases their exposure to everyday mathematics (e.g. time, sequencing and categorisation). This exposure is in addition to the play-based activities children also experienced that are likely to promote their engagement with everyday mathematical concepts such as shape and size through puzzles and block play and number through opportunities for dramatic play (e.g. 'cooking' six pancakes).

Caregivers also reflected on how the play-activities they saw implemented in the SPinS promoted the provision of additional activities in the family home. For some caregivers this was focussed on easily transferable activities, such as the increased singing of rhymes and the conduct of dances learned at SPinS in the home:

> My daughter she is into art a lot, so she loves to paint…and all of my kids they like to draw and do puzzles. I help here. I know she'll hear dances and stuff that she's heard at playgroup and she'll do at home that we haven't done with her before, so she'll pick them up. (Nancy)

> I have [engaged in singing and dancing at home] it's a lot of singing and dancing because the children are all sort of three and under. Ring-a-ring-a-rosy is very big at the moment because they all do it together and they can do it all by themselves together. We've added the second verse [at home] 'the cows in the meadow and then all jump up'. A lot of songs, 'Big Mack Truck' is a good one because of the actions, the children can get involved with the actions and 'Galoop Went the Little Green Frog' is also a very popular one because they can do the 'la-di-da-di-da' [caregiver demonstrates action with hands]. (Deb)

In these examples, the caregivers described their children engaging in activities that they learned at SPinS and had not previously enacted in the home. Singing nursery rhymes is particularly associated with literacy learning (Goswami 2003), and also benefits children's engagement with everyday mathematical concepts where the rhymes include appropriate concepts (e.g. 'up' and 'down' in *Ring-a-ring-a-rosy*; 'little' and 'big' in *Big Mack Truck*; number in *One, two, three, four, five, once I caught a fish alive*) (Aubrey et al. 2003). Other caregivers indicated higher levels of transfer of SPinS play-activities to the home. One caregiver noted how the pre-service educators used natural materials to support the children's play and then sourced her own materials to help the children participate in some craft activities in the home. Craft activities promote opportunities for exploring shape (e.g. how to cut triangles from square paper) and number (e.g. counting and using different craft pieces). Another caregiver particularly valued a session that focused on learning soccer skills and how these skills were practised and shared in the home by the children. Sharing the soccer skills in the home interested the child's father in the SPinS and he consequently expressed interest in attending the next SPinS session:

> A lot of the stuff [things made at SPinS] we take home and then that gives me foundation to move on from. The students [pre-service early childhood educators] would do natural things, it then gave us the opportunity to go back to my house and we'd collect more natural things, find other things to do with those natural resources, just things like that, we could follow on from that original idea. We made teddy bear masks, so we were able to then go back to my place and have teddy bear picnics with our teddy bear masks. (Deb)

> The soccer program was great. You took a ball home and showed what you learned today. You learnt bouncing, kicking and skills like that. It made the children stop and talk about what we did [at SPinS]. Dad was interested and wanted to attend next week. (Tamara)

In these examples the caregivers describe how they transferred play-activities experienced at SPinS into the home for their children. Mathematically, making teddy bear masks for each bear supports one to one correspondence, and playing soccer provides opportunities for counting goals scored. Here the emphasis was on what caregivers were doing with their children in the SPinS context rather than on being directly 'taught' about the value of play and play in the home for their children as is the case in some parenting intervention approaches such as Abecedarian or HIPPY (Baker et al. 1999; Campbell et al. 2002). This is significant for promoting young children's mathematical learning in the early years because these caregivers were learning about children's play through their participation in SPinS, rather than being directed to increase the range and level of play-activities they provide for children in the home per se (see for example, Evangelou and Wild 2014). Instead, the focus on the children's play established by the playgroup coordinator and pre-service educators helped the caregivers to understand how they might provide similar; or even extension-based activities in the home (e.g. making teddy bear masks). One caregiver described SPinS as different from other groups because the focus was not just on parental interaction, but on the children and their play. Another noted how play-activities experienced in SPinS could be repeated at home:

> It's more focused on the children rather than the parent interaction as in a normal mothers' group. It's more focused on the child, it's a kid-focused group. (Deb)

> The kids love to build the high towers and stuff like that and we have the same [blocks] at home, like playgroup, where we have the blocks where we build castles and towers and stuff like that. We often sit down and read a book together … or we'll get on the computer and we'll play some songs and some nursery rhymes on the computer. (Mary)

These suggestions indicate that caregiver participation in SPinS has the potential to increase the range and type of play-activities children experience in the home. This is important for children's early mathematical learning because increased participation in play-activities in the home is likely to increase children's exposure to everyday mathematical concepts. This is particularly the case for the caregiver who described building with blocks at both SPinS and in the home. Here the caregiver also described playing songs and rhymes on the computer and reading books with her child—all activities associated with increased mathematical learning for young children. In the words, of one caregiver: 'playgroup [SPinS] offers different sorts of crafting, like building the horses today, something like that we've never done at home. So just things like that, always getting new ideas I think, which is good' (Marley). Increasing children's access to play-activities in the home as a basis for engaging with everyday mathematical concepts may therefore be considered achievable through appropriately engaging parents and caregivers in learning about how to provide play opportunities for their children. In terms of the SPinS project, caregiver 'engagement' was fostered through the play-activities designed and implemented by the playgroup coordinator and pre-service educators which had the benefit of helping parents to understand how to structure children's participation in play and the later provision of play-activities in the home through repeating SPinS activities, or providing 'new ideas' that they could implement themselves.

Recommendations for Policy and Practice

Our experience researching the SPinS confirms that increased caregiver engagement in young children's play in the home is possible. This may be achieved by providing caregivers with access to a play-based context that models how to implement and structure play-activities for young children. Supported playgroups in particular may be ideally suited to this process because they are not a formal provider of early childhood education for young children, and yet, benefit from the professional expertise of a playgroup coordinator who can design and implement play-activities that children and caregivers experience together (McArthur et al. 2010). Recommendations for policy are that parenting engagement initiatives focus on the *context* of play provision for children and families rather than only the delivery of information about the benefits of play for young children. This is because context enables caregivers to see the provision of play-activities in action, rather than focusing on telling caregivers how important play-activities are in promoting children's access to everyday mathematical concepts. Focusing on context helps to increase the likelihood that children will experience a range of everyday mathematical concepts. Learning Outcome Five 'Children are effective communicators' from the EYLF suggests that it is "essential that the mathematical ideas with which young children interact are relevant and meaningful in the context of their current lives" (DEEWR 2009, p. 38).

In terms of young children's mathematical knowledge, recommendations for practice include the provision of play-activities in supported playgroup situations such as SPinS with a strong focus on everyday mathematic concepts, such as a space, number and geometry through the continued use of puzzles, songs, nursery rhymes and block play (see for example, EYLF Outcome Five: 'Children begin to understand how symbols and pattern systems work', DEEWR 2009, p. 38).This may also include the use of appropriate apps on tablet technologies that connect with more traditional forms of play (e.g. open-ended applications involving children in building with blocks, problem solving and/or drawing). In addition, supported playgroups may be able to capitalise on the structure and provision of play-activities to children as a basis for engaging everyday mathematical knowledge with young children, such as through playgroup routines and the presentation of categorised play-activities (see for example, EYLF Outcome Five: 'Children begin to notice and predict the patterns of regular routines and the passing of time' DEEWR 2009, p. 42).

Conclusion

Young children's mathematical knowledge is an important predictor of their later mathematical success in formal education. Young children's engagement with everyday mathematical concepts in the home provides an important basis for later mathematical learning through the achievement of mature concepts. Children's play in the home is an acknowledged influence on their access to everyday mathematical concepts. Our research suggests that a core aspect of increasing young children's access to play-activities in the home lies in supporting caregivers to understand how to provision children's play and to articulate this to the family home. SPinS appears to be a useful mechanism for engaging caregivers in children's play as it provides a *context* in which families are exposed to planned and structured play-activities for children by a playgroup co-ordinator on a local school site. As caregivers become familiar with the use of play-activities in SPinS they may be influenced in their role as young children's first mathematics educators, in the provision of play-activities to children in the home.

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Part V
Conclusion

Chapter 15
Insights for Engaging Families as the First Mathematics Educators of Children

Ann Gervasoni, Sivanes Phillipson and Peter Sullivan

Abstract This final chapter provides a synthesis of the research and scholarship presented by the 26 contributing authors about the nature and focus of actions that early years educators and professionals can take as part of their work to engage families in supporting the mathematics learning of their very young children. Each chapter contributed to one of three organising themes: Key foci and pedagogical actions that support young children's mathematics learning; Home interactions and learning experiences that support early mathematical learning; and Family and educator partnerships that support early mathematical learning. The authors of each chapter explored and highlighted the critical role that parents play in their children's mathematics learning and collectively provide insight into the factors that support or constrain parents in this role. They also described the impact and effectiveness of interventions that were designed to support parents as the first mathematics educators of children, and made recommendations for future initiatives and research. A list of 11 statements is presented that can be used to guide parents about the type of experiences that support young children's mathematics learning.

Keywords Key foci · Pedagogical actions · Home interactions · Family and educator partnerships

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Introduction

Engaging Families As The First Mathematics Educators of Children: International Perspectives presents an inspiring collection of recent international research and scholarship about the nature and focus of actions that early years educators and professionals can take as part of their work to engage families in supporting the mathematics learning of their very young children. Two key assumptions underpin the focus of each chapter: that families are the first educators of children; and mathematical learning starts from birth. Garvis and Nisley (Chap. 3) argue that these two assumptions provide a theoretical as well as practical understanding of the role of the family in the home context for children's mathematics learning.

Accordingly, the authors of each chapter in this volume explore and highlight the critical role that parents play in their children's mathematics learning and collectively provide insight into the factors that constrain or support parents in this role. They also describe the impact and effectiveness of interventions that were designed to support parents as the first mathematics educators of children, and make recommendations for future initiatives with similar intent. Emerging from the research and scholarship presented in this volume are five themes that align with Shonkoff's theory of change (Shonkoff 2012), which proposes that children's developmental trajectories are dependent upon the purposeful and informal experiences that families and educators provide for them. Hence, this chapter begins with a description of these themes, drawing upon key factors that inform the purposeful and informal experiences that families and educators can provide for children's learning and development in early mathematics.

Everyday Family Experiences in the Home and Local Community Are Rich Contexts for Young Children's Mathematics Learning and Exploration

The first theme in this book is *everyday family contexts in the home and local community are rich contexts for young children's mathematising, mathematics learning and exploration*. Young children are very successful users and learners of mathematics, and parents are very effective first educators. Also, it is clear that some parents underestimate the importance of their role in their young children's mathematics learning and are not fully aware of the affordances of everyday contexts for mathematics learning. Parents living in lower socio economic status (SES) communities seem to be overrepresented in this group. Phillipson et al. (Chap. 8) showed that parents living in a disadvantaged community in Australia, and who were surveyed as part of the Numaracy@Home study, had clear opinions about who should teach their children and argue that it does not entirely fall on themselves.

Strong evidence is also presented in the chapters to demonstrate that children's mathematics knowledge when they begin school is variable. Not all children are well positioned when they begin school to benefit from mathematics instruction, and this is likely due to the type of interactions and activities that they experienced in their first years of life. Duncan et al. (2007) argued that young children's early mathematics knowledge is a strong predictor of children's literacy and numeracy achievement well into primary school, so it follows that providing opportunities for all children to construct this knowledge during their earliest years is important for their later schooling. Many authors also highlighted the role that mathematics plays in ensuring educative justice for children who are typically educationally disadvantaged. Siegler and Ramani (2008) showed that the teaching associated with parents playing board games with their young children for an hour each day resulted in increasing children's mathematics learning to the point that it was equivalent to that of children living in higher SES communities. In this volume, Streit-Lehmann (Chap. 9) and Gervasoni (Chap. 12) also found that parents spending time playing games, reading stories and exploring the mathematics that arises in everyday situations resulted in significant mathematics learning for children living in disadvantaged communities that was equivalent to or exceeded their peers who did not participate.

Although parents recognise their role in their young children's mathematics learning they are not always aware of the opportunities that arise every day for mathematics learning, or are confident about what aspects of mathematics are important for their young children to explore (Phillipson et al. Chap. 8). This finding highlights the importance of parents (and educators supporting parents) being aware of the kinds of mathematical concepts and activities that they can use for intentional teaching and purposeful learning in informal settings (Hildenbrand et al. 2015).

Sullivan et al. (Chap. 2) examined mathematics achievement data for young children and current curriculum advice in order to gain insight about the type of mathematics experiences that are important for young children's mathematics learning. As a result of the analysis, these authors framed 11 statements that can be used by educators to guide parents about the focus and nature of experiences that they might explore with their children. They anticipate that building parents' awareness of the value of these 11 experiences will ensure that more children construct the mathematical knowledge, mathematics discourse and learning dispositions that support their transition to school. Björklund and Pramling (Chap. 5) argue that "mathematics is a prevalent and powerful cultural form of sense-making and communication" (p. 76) and that children are empowered through becoming members of this culture. Björklund and Pramling also argue that this implies young children knowing more than how to count; it also involves the learner's notion of self or identity as mathematical. The authors explain that "Developing an identity as mathematical is an important part of becoming mathematically skilled, being able to take on mathematical tasks or take on problems in mathematical terms." (p. 76)

Early Learning Is a Shared Responsibility Between Parents and Educators

The second theme to emerge is that early learning is a *shared responsibility between parents and educators*. This proposition supports studies that conclude that shared care between parents and educators early in the lives of children have wide implications for children's language and mathematical development (e.g., Drange and Havnes 2015). In this volume, Phillipson et al. (Chap. 8) found in the Numeracy@Home study that parents living in a disadvantaged community were aspirational for their children and valued early mathematical learning as key to their children's later success in schooling. These parents advocated that early learning, in preparing children for formal schooling, is a shared responsibility between educators and themselves. This is an important finding but contrasts with the conclusion of Deflorio and Beliakoff (2015) who found that parents' beliefs about early mathematics learning varied according to SES, with parents from lower SES communities more likely to consider kindergarten to be more important than the home environment for mathematics learning than parents from middle SES communities. Research has also highlighted that some parents with low financial resources may not appreciate the positive influence of their role in their children's learning nor have the knowledge capital about what type of experiences will support children's learning. For example, activities such as playing board games can have positive effects on children's learning. Siegler and Ramani (2008) found that adults playing board games regularly with 4 year old children had a significant effect for children living in financially disadvantaged communities and increased their numerical knowledge to the point that it was indistinguishable from mid SES families. They argued that what was important was not so much the act of playing the board games but the simple strategies of mathematical teaching that emerged from the game playing. They also concluded that the parents who got involved in the game playing believed that their involvement mattered for their children's development. This view was well supported by chapter authors (e.g., Phillipson et al., Gervasoni et al.).

Informal Mathematics Learning in Everyday Situations

The third theme is that mathematics learning occurs in everyday situations. In fact, Dunst et al. (Chap. 7) argued that young children's informal learning opportunities were better predictors of their mathematics achievement compared to formal teaching activities, and that the types of experiences afforded children as young as 3 years of age were beneficial in terms of explaining variations in the children's mathematics achievement. Dunst et al. concluded that children's home numeracy experiences had the largest effect size. This finding echoes the conclusion reached by Hildenbrand et al. (2015) that informal experiences had far reaching implications for later mathematical development. Similarly, Björklund and Pramling (Chap. 5) asserted that common everyday activities such as dressing, talking about family members or playing games provide entry points into supporting a child's mathematical development.

Wong (Chap. 10) found that parents in Hong Kong typically use everyday contexts in the home environment to support their young children's mathematics learning, but also use decontextualised experiences, such as text book activities. In contrast to research in Western countries, Wong found that SES was not a clear factor in predicting Hong Kong parents' interaction strategies with their young children except for the strategy of providing prompt questions in everyday contexts. Wong's findings suggest that in children's early years there may be few SES-related differences in the way that parents in Chinese-heritage cultures help their children learn mathematics. In general, Wong found that SES as a variable was overridden by other variables such as parental expectations, which are closely linked with important Chinese cultural values, e.g., success derives more from diligence and effort than from intelligence. This value aligns with Dweck's (2006) notion of the growth mindset. Wong found that forward counting was the most commonly reported home activity relating to parents engaging in mathematics with their young children. Parents also reported using real objects to illustrate mathematical concepts (e.g., counting the number of cookies on a plate) and providing prompt questions to help children reflect on concepts (e.g., when the child is about to eat a chicken wing from their plate, parents will ask how many chicken wings there are on the plate). Using prompt questions was the only interaction found to be influenced by SES, with more affluent parents reporting use of prompt questions with their children.

Swimming lessons were shown by Jorgensen (Chap. 13) to provide another example of opportunities for young children's informal mathematics learning. Her research indicates that there is considerable potential in early years swimming contexts to provide opportunities for mathematising and building mathematical capital among young swimmers. She argues that this capital comes in the forms of early number knowledge, comparatives (same/different) and colour recognition which is an integral part of early sorting and classifying activities that are foundational to number concepts. It is likely that similar affordances might be found in early years music, dance and gymnastics lessons also. Such alternative affordances seem to mirror the variation theory proposition that pedagogical variation is important for development of learning as suggested by Garvis and Nislev (Chap. 3).

Adults Noticing and Building Upon Young Children's Mathematics Actions

The fourth theme to emerge is the importance for children's learning of adults noticing and building upon young children's mathematical actions. Adults who are present with children can assist them to notice mathematical aspects of their experiences, develop the mathematical discourse associated with their mathematising, and encourage and support children to use mathematics to investigate and talk about their observations and experiences of the world. This noticing extends to adults noticing the mathematics inherent in everyday activities, and children spontaneously noticing and using mathematical ideas when they are playing and engaging in family and community activities. Björklund and Pramling (Chap. 5) claim that one of many important actions people carry out through speaking is to direct someone's attention. This was evident in their description of Episode 3 when young Vidar exclaimed, "Look, the same". Björklund and Pramling point out that this statement functions as a pointing gesture and that his mother's response, "How many do you have?" acknowledges that she notices what he is focused on, but also provides a response in mathematical terms. This responsive action inducts Vidar into the cultural practice and discourse of mathematics. Dunphy (2015) highlights that parents and educators can support children shift from using narrative discourse to mathematics discourse. It is likely that being familiar with the discourse of mathematics assists children and parents during the transition to school.

Garvis and Nisley (Chap. 3) proposed that the future of young children's mathematical thinking strongly depends on adults recognising children's mathematical actions, seeing the mathematical potential of play activities and play objects, and guiding children into the future where they can participate autonomously and creatively in mathematical communications. They argue that adults need to be aware of and responsive to supporting a child's mathematical knowledge and skill development. For this to occur, they stress that the adults need to have an understanding of mathematical content knowledge, pedagogical knowledge and pedagogical content knowledge. This type of technical knowledge falls more into the domain of educators than parents. Therefore, some parents may appreciate the opportunity to build their awareness of what constitutes mathematical knowledge and mathematical activity so as to assist them to notice their children's mathematical activities, and thus find opportunities within their home that encourage mathematical play. Some parents may also value some information about mathematics pedagogies that they can employ at home. The examples from Björklund and Pramling (Chap. 5) about adults acknowledging and building upon children's "pointing" gestures with responses formed in mathematical terms, or suggestions from Gervasoni (Chap. 12) about noticing, talking about and exploring the mathematics that arises in daily activities, can increase parent's knowledge capital about effective pedagogies that support mathematics learning.

Mousely in Chap. 6 also demonstrates the positive effect on children's mathematics learning when adults recognise and build upon their mathematical activity. She presented evidence that very young children can be very interested in mathematics when adults listened to and observed them, and thought about the mathematics present in a variety of everyday activities and contexts. The learning was enhanced when the adults communicated with the children about specific mathematical concepts, asked questions, extended their knowledge, and reinforced their learning as it was happening. These are further examples of effective pedagogies that parents can adopt. Mousely noted that engagement in such mathematical experiences and conversations with adults gave children a sense of belonging as well as a sense of self-respect, recognition, achievement and affirmation. This proposition supports Björklund and Pramling's conclusion as well (Chap. 5). Mousely cautioned that it is very difficult for adults to support children's mathematics development without first finding out what a child knows and how he or she understands. This is why listening and observing children as they engage in activities is such an important pedagogical principle.

Many parents conjure up images of sitting and memorising multiplication tables when they imagine their children learning mathematics. This was evident in data presented by Gervasoni (Chap. 12) and is perhaps why some parents report that it is helpful to watch their child's early years educator draw out the mathematics in a play situation. Some parents find such modelling very useful in strengthening their own interactions with their children. Edwards, McLean and Lambert (Chap. 14) reported a similar finding. Their research suggests that a core aspect of increasing young children's access to play-activities in the home lies in supporting caregivers to understand how to provide play and articulate its importance for all family members in the home. They found that supported playgroups in schools (SPinS) provided a context in which families were involved and exposed to models of planned and structured play-activities for children by a playgroup co-ordinator. They anticipate that as the caregivers became more familiar with the use of play-activities in SPinS they can be influenced in their role as young children's first mathematics educators in the home.

Interventions and Persistence of Positive Effects

The fifth theme focused on interventions aimed at promoting young children's mathematics learning prior to their transition to school. Most interventions discussed by the authors focused on children who lived in financially disadvantaged communities as this group of children is associated with poorer school performance in mathematics. The argument is made from an educative justice perspective that if these children's mathematics learning can be supported strongly in their early years, then this will position them to benefit more effectively from mathematics education at school. However, Sarama and Clements (2015) concluded, after designing and researching the effects of many interventions that despite the best efforts of researchers, the positive effects of these interventions fade when children begin school. They argue that this is due to the culture and processes of schools.

Streit-Lehmann's (Chap. 9) intervention study supports Sarama and Clements' (2015) conclusion. She found that although their intervention initially was very positive for the German children with migration backgrounds, the effects faded one year after the children began school. This intervention provided families with a *Treasure Chest* of games, stories and activities that could promote both literacy and numeracy. The sustained impact of the intervention appeared to be strongly associated with family background. The children without a migration background mostly maintained their rankings but the children with migration backgrounds did not, although families in both groups participated in the project with similar levels of engagement. Streit-Lehmann also found that the children's learning environment becomes more detached from the parents after they begin school. Parents are no longer requested to play and read together with their children but instead are asked to monitor their homework. Parental support thereby gains a new focus. Streit-Lehmann proposes that this lack of continuity in the actions of the parents could partly explain the poorer results following school transition for children with migration backgrounds, but also could be an argument for using holistic learning opportunities such as the intervention's *treasure chest* activities in primary school also, not only in kindergarten. Streit-Lehmann noted also that providing translations of important information, including book texts and rules of the games used in the project, seemed to be another helpful approach for parents. She concluded that strategies for "creating and keeping up steady conversation might be the key to inviting, encouraging, and motivating parents to participate in the mathematics education of their children" (p. 162).

Let's Count was another intervention that aimed to support children's mathematics learning in the years prior to their beginning school. Gervasoni (Chap. 12) described how parents and educators participating in *Let's Count* worked together to support children's learning focused on the mantra—notice, talk about and explore mathematics in everyday situations. Gervasoni and Perry (2015) found that participation in the program was associated with significant mathematics learning for children in comparison to non-participating peers, but their research design did not allow them to follow the children's progress after they began school. However, two parents did comment in interviews that their children in an earlier *Let's Count* group had made a very successful transition to school with respect to mathematics. Common with Streit-Lehmann's recommendation, the focus of *Let's Count* could be very easily extended to the first years of school, and for parents, such alignment of approaches for supporting their children's mathematics learning may reduce the negative impact of starting school on some children's mathematics learning.

Several of the interventions described and explored by the chapter authors focused on providing professional learning for educators about strategies for working in partnership with parents to support children's mathematics learning. Analysis of the interview transcripts for parents and educators participating in *Let's Count* suggest that this program was effective and a positive experience for participants. However, Tirosh et al. (Chap. 11) noted that, as professional development providers for preschool teachers, they did not have a mandate to specifically get involved with parents and had many questions and dilemmas regarding this issue. They maintained that we should keep in mind that, even indirectly, promoting mathematical knowledge along with self-efficacy can still have an impact on preschool mathematics education. These authors began their program with what they described as an initial, perhaps naïve, belief that all significant adults in a child's environment should work together towards the promotion of mathematical knowledge. They learned over time that this was not always possible.

Mathematical Experiences that Promote Young Children's Mathematics Learning

The chapter authors in this book demonstrate that the first years in children's lives are an important time for learning mathematics and that mathematics is learnt and used in children's everyday contexts. Parents and carers are children's first mathematics educators and their interactions with their children in everyday contexts influence children's learning and confidence, supporting the recent findings by Hildenbrand et al. (2015).

Many parents are surprised to learn that their interactions with their children in everyday contexts are so influential for mathematics learning. For some parents, mathematics learning has become synonymous with sitting quietly and memorising number facts and practising calculation procedures. Therefore, it is often a revelation for parents to become aware of the range of mathematical activities that are part of their everyday experiences and integral to the play, noticing and mathematising of their children. Providing opportunities for parents to strengthen this awareness is an important role of early years educators and other professionals, a message that is repeatedly found in this volume.

Research that investigates children's mathematical knowledge and confidence when they begin school has demonstrated that this is highly variable, and that this variation is likely due to children's different opportunities to explore and use mathematical ideas in their early years. It is also likely that many parents may not recognise the affordances of everyday activities for mathematics learning, nor notice opportunities for them to talk about mathematics, use mathematical language or explore mathematical ideas of number, measurement, shape, structure and position. Margolinas et al. (Chap. 4) argue that some mathematical ideas and skills are best learnt by young children when adults model the mathematics or prompt children's noticing and action. For example, the authors demonstrate that enumeration is a complex component of counting that young children seldom learn spontaneously. Their study demonstrates that children learn to enumerate only when they encounter experiences and games that require objects to be sorted and organised, often in response to the prompts of an adult. This finding highlights that mathematics is a culturally based set of knowledge and skills that may require modelling and teaching in order to be learnt.

Accordingly, Sullivan et al. (Chap. 2) present a research-informed framework that aims to build parent and educators' awareness of the type of mathematics and mathematical experiences that promote young children's mathematics learning. We anticipate that this framework will be useful for demonstrating the nature and focus of experiences that are relevant for young children, and that may guide conversations between parents and educators about young children's mathematics learning.

Conclusion and Recommendations

The chapter authors in this book have provided many research informed insights about the power of engaging families as the first mathematics educators of young children. The following statements are drawn from the synthesis of the book chapters. They provide a framework for designing future initiatives and interventions that aim to enhance all young children's mathematics and enhance educative justice for all.

Building and Strengthening Partnerships Between Parents and Educators/Early Years Professionals

1. Educators acknowledge and celebrate parents as the first and primary mathematics educators of young children;
2. Educators recognise that parents value partnerships with educators that focus on their children's mathematics learning;
3. Educators and parents create and maintain steady conversation about the mathematics learning of their children;
4. Educators act to build and strengthen parents appreciation of the positive influence of their role in their children's mathematics learning;
5. Educators act to build parents' awareness of the opportunities that arise every day for mathematics learning;
6. Educators build parents' awareness of the type of experiences that can support and advance children's mathematics learning;
7. Educators offer parents advice, modelling and guidance about the type of mathematics experiences and discourses that are important for young children's mathematics learning;
8. Educators provide parents with examples and access to games, children's literature and other resources that provide contexts for young children to learn mathematics;
9. Educators provide parents with personal models of how to point out, talk about and explore the mathematics arising from children's play and engagement in everyday activities;
10. Educators provide parents with encouragement and access to advice and support when personal engagement is not possible;
11. Educators sustain engagements with parents throughout the period of working with their children.

Effective Mathematics Pedagogies for Parents and Educators

Varying methods of teaching are important in addressing children's diversity and their individual developmental. These include:

1. Learn about children's mathematising by listening to and observing children as they engage in everyday activities;
2. Notice, talk about and explore with children the mathematics that arises in their everyday activities; and
3. Acknowledge and build upon children's "pointing" gestures during play and everyday activities with responses formed in mathematical terms.

These pedagogies guide children into a future where they participate autonomously and creatively in mathematical communications.

Nature and Focus of Experiences that Support Young Children's Mathematics Learning

Everyday activities provide rich contexts for parents to support their young children's mathematics learning. The chapter authors provided many illustrations of parents providing this support. Contexts included:

1. Everyday activities such as dressing, talking about family members or preparing meals;
2. Playing games, telling and reading stories and everyday experiences including shopping and moving throughout the home and local community; and
3. Swimming lessons and other formal early years classes (dance, music, gymnastics etc.)

These types of contexts for learning have been long known to have a positive effect on mathematical development in children (Trawick-Smith et al. 2015). These everyday experiences also provide opportunities for mathematising in ways that align with early years mathematics curricula and school mathematics learning. Sullivan et al. (Chap. 2) provide 11 statements that guide parents about the type of experiences that support young children's mathematics learning. They suggest that young children are learning mathematics when they:

1. Compare objects and describe, in everyday language, which is longer, shorter, heavier, lighter, or holds more, hold less;
2. Play with, name, describe, and organise 2D shapes and 3D objects;
3. Use words and ideas to describe where things are positioned, for example, inside, outside, above, below, next to, behind, in front of, up, down, here, there, north, middle, across, opposite;

4. Describe, copy, represent and extend patterns found in everyday situations;
5. Use time words that describe points in time, events and routines (including days, months, seasons and celebrations);
6. Compare the duration of everyday events using mathematical language and arrange connected events in the usual sequence that they occur;
7. Say number names forward in sequence to 10 (and eventually to 20 and beyond);
8. Use numbers to describe and compare collections;
9. Use, progressively, perceptual and conceptual subitising, counting and matching to compare the number of items in one collection with another;
10. Show different ways to make a total (at first with models and small numbers);
11. Match number names, symbols and quantities up to 10.

When parents notice, talk about and explore these actions with their very young children, they are supporting their children's mathematics learning in ways that also align strongly with school curricula, thus also supporting children's successful transition to learning school mathematics.

An important proposition that underpins the research explored in this book is the importance for young children's mathematics learning of the partnership between parents and educators. Many successful interventions for strengthening such partnerships were described by chapter authors. The findings of Streit-Lehmann (Chap. 9) that the positive effects of such interventions for children living in lower-resourced families substantially fade after they begin school supports the proposition of Sarama and Clements (2015) that school systems are unwittingly structured to mitigate against the positive effects of early years interventions. If teachers in the first years of school continue and extend the focus of early years interventions, and continue to encourage parent engagement as their children's mathematics educators in everyday contexts, then it is likely that a greater continuity between prior to school and school learning will be created for the benefit of all. It is likely that early years interventions that focus on parent engagement must continue through to the first years of school for their positive impacts to be sustained and extended.

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