Adaptive Neural Output Feedback Control for Flexible-Joint Robotic Manipulators

Lingjie Gao, Qiang Chen and Linlin Shi

Abstract In this chapter, an adaptive neural output feedback control scheme is proposed for flexible-joint robotic manipulators. First, the mathematical model of a robotic manipulator is built with considering flexible joints. Then, a Luenberger state observer is employed to estimate the unknown states such that the constriction that all the states should be available for measurements can be relaxed. In order to achieve a satisfactory tracking performance, an adaptive controller is designed by combining neural network control and dynamic surface control techniques to avoid the so-called "explosion of complexity" problem. With the proposed scheme, the tracking error can be guaranteed to converge to a small neighborhood around zero, and simulation results show the effectiveness of the developed method.

Keywords Flexible-joint system · Adaptive neural control · State observer

1 Introduction

Over the past decades, electromechanical servo system has been widely studied, such as robotic manipulators, motor servo system, and so on [1–3]. As a popular automatic equipment, robotic manipulators have attracted considerable attention and been widely applied to the field of industrial automation [4, 5]. Since the existence of the inferior flexibility may affect the practical control precision, the joint flexibility should be taken into account in both modeling and control of manipulators, in which torsional elasticity is introduced [6].

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So far, many significant researches have been presented to handle the issue of joint flexibility to achieve a better tracking performance. In [6], an adaptive state feedback controller for flexible-joint manipulators is presented using the backstepping technique. However, when some states are immeasurable, the above-mentioned researches may not be applied directly. In [7], an extended state observer-based control scheme is developed for trajectory tracking of flexible-joint robotic manipulators with partially known model. Therefore, the control task of the flexible-joint robotic manipulators with immeasurable states and model uncertainties is still a challenging work.

In this paper, an adaptive neural output feedback control scheme is presented for the tracking control of flexible-joint manipulators with immeasurable states and model uncertainties. The unknown states are estimated using a Luenberger state observer, and the output feedback controller is designed based on the estimated states and dynamic surface control technique. Moreover, the model uncertainties of the system are approximated by employing radial basis function (RBF) neural networks. The position tracking error is proven to converge to a small neighborhood around zero via the Lyapunov synthesis.

2   System Description

As shown in Fig. 1, the considered single-link robotic manipulator system taken from [6] is expressed as

$$\begin{cases} I\ddot{q}_1 + K(q_1 - q_2) + MgL\,\sin(q_1) = 0 \\ J\ddot{q}_2 - K(q_1 - q_2) = u \end{cases} \tag{1}$$

where q_1 and q_2 are the angles of the link and motor, respectively, g is the acceleration of gravity, I is the link inertia, J is the inertia of the motor, K is the spring stiffness, M and L are the mass and length of link, respectively, and u is the input torque.

For the purpose of simplifying the controller design, define $x_1 = q_1$, $x_2 = \dot{q}_1 = \dot{x}_1$, $x_3 = q_2$, and $x_4 = \dot{q}_2 = \dot{x}_3$ and the system (1) can be transformed into

Fig. 1 Schematic of flexible-joint manipulator
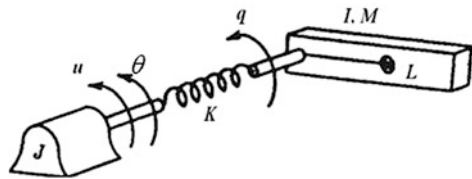
$$\begin{cases} \dot{x}_1 = x_2 + f_1(X) \\ \dot{x}_2 = x_3 + f_2(X) \\ \dot{x}_3 = x_4 + f_3(X) \\ \dot{x}_4 = \frac{1}{J}u + f_4(X) \\ y = x_1 \end{cases} \tag{2}$$

where $X = [x_1, x_2, x_3, x_4]^T \in R^4, f_1(X) = 0, f_2(X) = -\frac{MgL}{I}\sin x_1 - \frac{K}{I}\left(x_1 - x_3 + \frac{I}{K}x_3\right),$ $f_3(X) = 0$, and $f_4(X) = \frac{K}{J}(x_1 - x_3)$, the output y is measurable directly and states $x_t(t = 2, 3, 4)$ are immeasurable.

Rewrite (2) in the following form:

$$\dot{X} = AX + F(X) + Bu + Ly \tag{3}$$

where $\quad A = \begin{bmatrix} L_3 & I_3 \\ l_4 & 0_{1\times3} \end{bmatrix}, \quad L_3 = [l_1, l_2, l_3]^T, \quad B = \left[0, 0, 0, \frac{1}{J}\right]^T, \quad$ and $\quad F(X) = [f_1(X), f_2(X), f_3(X), f_4(X)]^T.$

The parameters $l_i(i = 1, 2, 3, 4)$ are chosen such that the characteristic polynomial of matrix A is strictly Hurwitz, and the control objective is to design the controller u such that the output y follows the desired position trajectory y_r.

3 Observer Design

Consider the following observer that estimates the immeasurable state variables X in (3)

$$\dot{\hat{X}} = A\hat{X} + F(\hat{X}) + Bu + Ly \tag{4}$$

where $\hat{X} = [\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4]^T$ and $\hat{x}_i$ are the estimation of x_i,

$$\begin{aligned} F(\hat{X}) &= \left[f_1(\hat{X}), f_2(\hat{X}), f_3(\hat{X}), f_4(\hat{X})\right]^T, & f_1(\hat{X}) &= 0, \\ f_2(\hat{X}) &= -\frac{MgL}{I}\sin\hat{x}_1 - \frac{K}{I}\left(\hat{x}_1 - \hat{x}_3 + \frac{I}{K}\hat{x}_3\right), & f_3(\hat{X}) &= 0, \\ f_4(\hat{X}) &= l_4(x_1 - \hat{x}_1) + \frac{K}{J}(\hat{x}_1 - \hat{x}_3). \end{aligned}$$

Let $e_i = x_i - \hat{x}_i(i = 1, 2, 3, 4)$ be the observer error. From (3) and (4), the observer-error equation can be expressed as follows:

$$\dot{E} = AE + F(X) - F(\hat{X}), \tag{5}$$

where $E = [e_1, e_2, e_3, e_4]^T.$

Construct the following Lyapunov function candidate

$$V_e = E^T P E \tag{6}$$

where $P = P^T > 0$.

In view of (5), the time derivative of V_e is

$$\dot{V}_e = E^T (A^T P + PA)E + 2E^T P \left[F(X) - F(\hat{X}) \right]. \tag{7}$$

According to the mean-value theorem, the term $2E^T P \left[F(X) - F(\hat{X}) \right]$ can be transformed into

$$2E^T P \left[F(X) - F(\hat{X}) \right] = E^T \left[P \frac{\partial F}{\partial x} + \left(\frac{\partial F}{\partial x} \right)^T P \right] E \tag{8}$$

where $\frac{\partial F}{\partial x}$ is a Jacobin matrix with its element at the of ith row and the jth column being $\frac{\partial f_i}{\partial x_j}$.

Substitute (8) into (7) and choose a proper matrix A such that and we have

$$\dot{V}_e \leq E^T \left[PA + A^T P + P \frac{\partial F}{\partial x} + \left(\frac{\partial F}{\partial x} \right)^T P + I \right] E < 0. \tag{9}$$

4 Controller Design and Stability Analysis

In this section, an adaptive neural control scheme is developed for flexible-joint robotic manipulators (2) based on the observer (4) and dynamic surface control technique. The whole control process includes the following steps.

Step 1:

Define the tracking error $s_1 = y - y_r$ and differentiating s_1 yields

$$\dot{s}_1 = \dot{y} - \dot{y}_r = \hat{x}_2 + e_2 - \dot{y}_r. \tag{10}$$

Consider the Lyapunov function candidate $V_1 = \frac{1}{2} s_1^2$, whose time derivative is

$$\dot{V}_1 = \dot{s}_1 s_1 = s_1 (\hat{x}_2 - \dot{y}_r) + s_1 e_2. \tag{11}$$

Applying Young's inequality $2ab \leq a^2 + b^2$, we can obtain the following inequality

$$s_1 e_2 \le \frac{1}{2} s_1^2 + \frac{1}{2} e_2^2 \le \frac{1}{2} s_1^2 + \frac{1}{2} E^T E. \tag{12}$$

By regarding $\hat{x}_2$ is a virtual control input, we choose $\alpha_1 \triangleq \hat{x}_2$ as intermediate control input as follows:

$$\alpha_1 = -(c_1 + 0.5)s_1 + \dot{y}_r \tag{13}$$

where $c_1 > 0$ is a positive parameter.

To avoid the problem of "explosion of complexity" in [6], we introduce a state variable z_2 and let α_1 pass through a first-order filter with a time constant $\tau_2 > 0$ as follows:

$$\tau_2 \dot{z}_2 + z_2 = \alpha_1, \ z_2(0) = \alpha_1(0). \tag{14}$$

Define the output error of the first-order filter as

$$\chi_2 = z_2 - \alpha_1 \tag{15}$$

From (11) to (15), we can obtain

$$\dot{V}_1 \le s_1 s_2 + s_1 \chi_2 - c_1 s_1^2 + \frac{1}{2} E^T E \tag{16}$$

where $s_2 = \hat{x}_2 - z_2$.

Step 2:

The time derivative of s_2 is

$$\dot{s}_2 = \hat{\dot{x}}_2 - \dot{z}_2 = \hat{x}_3 + l_2(x_1 - \hat{x}_1) - \dot{z}_2 + \hat{f}_2 \tag{17}$$

where the uncertainty $\hat{f}_2$ is approximated by the following neural network

$$\hat{f}_2 = W^* \varphi(Z) + \varepsilon_2 \tag{18}$$

where $Z = \left[\hat{x}_1^T, \hat{x}_3^T\right]^T \in R^2$ is the input vector, $W^* = [w_1, \ldots, w_L]^T \in R^L$ is the ideal weight matrix, and ε_2 is the bounded approximation error satisfying $|\varepsilon_2| \le \varepsilon_{N2}^*$ with ε_{N2}^* being a positive constant, the NN node number $L > 1 \varphi(Z) = [\varphi_1(Z), \ldots, \varphi_5(Z)]$ with $\varphi_i(Z)$ commonly being used as the Gaussian function, which is in the following form

$$\varphi_i(Z) = exp\left[\frac{-\|Z - c_j^2\|}{b_j^2}\right] (i = 1, 2, \ldots, n; j = 1, 2, \ldots, L) \tag{19}$$

where the input of NN $n > 1$, $c_j = [c_{11}, c_{12}, \ldots, c_{1L}]^T$ and b_j are the center of the receptive field and the width of the Gaussian function, respectively.

Construct the following Lyapunov function:

$$V_2 = \frac{1}{2}s_2^2 + \frac{1}{2}\chi_2^2 + \frac{1}{2\gamma_2}\tilde{W}^T\tilde{W} + \frac{1}{2\eta_2}\tilde{\varepsilon}_{N2}^2 \tag{20}$$

where γ_2 and η_2 are positive parameters, $\tilde{W} = W^* - \hat{W}$ and $\tilde{\varepsilon}_{N2} = \varepsilon_{N2}^* - \hat{\varepsilon}_{N2}$, $\hat{W}$ and $\hat{\varepsilon}_{N2}$ are the estimations of W^* and ε_{N2}^*, respectively.

In view of (14) and (15), we can obtain

$$\dot{\chi}_2 = \dot{z}_2 - \dot{\alpha}_1 = -\frac{\chi_2}{\tau_2} + B_2(s_1, s_2, \chi_2, y_r, \dot{y}_r, y_r) \tag{21}$$

where the function of B_2 satisfies $|B_2| \leq M_2$ and M_2 is a positive parameter.

Set the virtual input α_2 as

$$\alpha_2 = -l_2 e_1 - \hat{W}\varphi(Z) + \dot{z}_2 - c_2 s_2 - \hat{\varepsilon}_{N2}\tanh\left(\frac{s_2}{\delta_2}\right) \tag{22}$$

where c_2 and δ_2 are positive parameters.

Similarly, introduce a new state variable z_3 and let α_2 pass through a first-order filter with a time constant $\tau_3 > 0$ as follows:

$$\tau_3 \dot{z}_3 + z_3 = \alpha_2, \quad z_3(0) = \alpha_2(0). \tag{23}$$

Define the output error of the first-order filter as

$$\chi_3 = z_3 - \alpha_2. \tag{24}$$

From (14) and (15), we can obtain the derivative of V_2

$$\dot{V}_2 \leq s_2(s_3 + \chi_3) - c_2 s_2^2 - \frac{\chi_2^2}{\tau_2} + \frac{\sigma_2}{\gamma_2}\tilde{W}^T\hat{W} + \chi_2 B_2 + \varepsilon_{N2}^*\left[|s_2| - s_2\tanh\left(\frac{s_2}{\delta_2}\right)\right]$$
$$+ \varepsilon_{N2}^* s_2 \tanh\left(\frac{s_2}{\delta_2}\right) - \hat{\varepsilon}_{N2} s_2 \tanh\frac{s_2}{\delta_2} - \frac{1}{\eta_2}\tilde{\varepsilon}_{N2}\dot{\hat{\varepsilon}}_{N2} \tag{25}$$

where $s_3 = \hat{\chi}_3 - z_3$.

The adaptive laws of $\hat{W}$ and $\hat{\varepsilon}_{N2}$ are given as

$$\begin{cases} \dot{\hat{W}} = r_2 s_2 \varphi_2(Z) - \sigma_2 \hat{W} \\ \dot{\hat{\varepsilon}}_{N2} = \eta_2 s_2 \tanh\left(\frac{s_2}{\delta_2}\right) \end{cases} \tag{26}$$

where r_2, σ_2, and δ_2 are positive parameters.

Then, use the facts

$$0 \le |x| - x \tanh\left(\frac{x}{\delta}\right) \le 0.2785\delta \tag{27}$$

and

$$\frac{\sigma_2}{\gamma_2} \tilde{W}^T \hat{W} = \frac{\sigma_2}{\gamma_2} \tilde{W}^T \left(W^* - \tilde{W}\right) \le -\frac{\sigma_2 \|\tilde{W}^2\|}{2\gamma_2} + \frac{\sigma_2 \|W^{*2}\|}{2\gamma_2} \tag{28}$$

Substitute (26)−(28) into (25), and we have

$$\dot{V}_2 \le s_2(s_3 + \chi_3) - c_2 s_2^2 - \frac{\chi_2^2}{\tau_2} + \chi_2 B_2 + 0.2785\delta_2 \varepsilon_{N2}^* + \frac{\sigma_2 \|W^{*2}\|}{2\gamma_2}. \tag{29}$$

Step 3:

Following the similar procedures of the Step 2, consider the following Lyapunov function candidate

$$V_3 = \frac{1}{2} s_3^2 + \frac{1}{2} \chi_3^2 \tag{30}$$

and set the virtual input α_3 as

$$\alpha_3 = -l_2 e_1 + \dot{z}_3 - c_3 s_3 \tag{31}$$

where c_3 is a positive parameter.

Then, we can obtain

$$\dot{V}_3 \le s_3(s_4 + \chi_4) - c_3 s_3^2 - \frac{\chi_3^2}{\tau_3} + \chi_3 B_3 \tag{32}$$

where $s_4 = \hat{x}_4 - z_4$, $\chi_3 = z_3 - \alpha_1$, and the function of B_3 satisfies $|B_3| \le M_3$ with M_3 being a positive parameter.

Step 4:

Similarly, the following neural network is utilized to approximate the nonlinear uncertainty $\hat{f}_4$

$$\hat{f}_4 = \theta^* \varphi(Z) + \varepsilon_4 \tag{33}$$

where $\theta^* = [\theta_1, \ldots, \theta_5]^T \in R^5$ is the ideal weight matrix and ε_4 is the bounded approximation error satisfying $|\varepsilon_4| \le \varepsilon_{N4}^*$ with ε_{N4}^* being a positive constant.

Construct the following Lyapunov function candidate

$$V_4 = \frac{1}{2}s_4^2 + \frac{1}{2}\chi_4^2 + \frac{1}{2\gamma_4}\theta^T\theta + \frac{1}{2\eta_4}\tilde{\varepsilon}_{N4}^2 \tag{34}$$

where γ_4 and η_4 are positive parameters, $\tilde{\theta} = \theta^* - \hat{\theta}$ and $\tilde{\varepsilon}_{N4} = \varepsilon_{N4}^* - \hat{\varepsilon}_{N4}$, $\hat{\theta}$ and $\hat{\varepsilon}_{N4}$ are the estimations of θ^* and ε_{N4}^*, respectively.

The control law u is designed as

$$u = -\mathbf{J}\left[l_4 e_1 + \hat{\theta}\varphi(Z) - \dot{z}_4 - c_4 s_4 - \hat{\varepsilon}_{N4}\tanh\left(\frac{s_4}{\delta_4}\right)\right] \tag{35}$$

where c_4 and δ_4 are positive constants, and the adaptive laws of $\hat{\theta}$ and $\hat{\varepsilon}_{N4}$ are

$$\begin{cases} \dot{\hat{\theta}} = r_4 s_4 \varphi(Z) - \sigma_4 \hat{\theta} \\ \dot{\hat{\varepsilon}}_{N4} = \eta_4 s_4 \tanh\left(\frac{s_4}{\delta_4}\right) \end{cases} \tag{36}$$

where r_4 are σ_4 are positive constants.

Differentiating (34) yields

$$\dot{V}_4 \leq -c_4 s_4^2 - \frac{\chi_4^2}{\tau_4} + \chi_4 B_4 + 0.2785\delta_4\varepsilon_{N4}^* + \frac{\sigma_4\|\theta^{*2}\|}{2\gamma_4} \tag{37}$$

where the time constant $\tau_4 > 0$ and the function of B_3 satisfies $|B_3| \leq M_3$ with M_3 being a positive parameter.

Finally, construct the following Lyapunov function candidate

$$\mathrm{V} = V_e + V_1 + V_2 + V_3 + V_4 \tag{38}$$

According to (9), (16), (29), (32), and (37), differentiate (39) with respect to time and we can obtain

$$\dot{V} \leq -\sum_{k=1}^{4}\alpha_0 s_k^2 + 6\eta + 0.2785\left(\delta_2\varepsilon_{N2}^* + \delta_4\varepsilon_{N4}^*\right) + \frac{\sigma_2\|W^{*2}\|}{2\gamma_2} + \frac{\sigma_4\|\theta^{*2}\|}{2\gamma_4} \tag{39}$$

where α_0 and η are positive constants.

Consequently, all the signals of the closed-loop system are semiglobally uniformly bounded, and the output tracking error s_1 converges to a small neighborhood around zero on the condition that the parameters are chosen properly.

5 Simulation Results

In this section, simulation examples are carried out to show the feasibility and superiority of the proposed scheme. The sinusoidal wave $y_r = \sin t$ is adopted as the desired reference signal.

The initial conditions of the system are $[x_1, x_2, x_3, x_4]^T = [0, 0, 0, 0]^T$, $[\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4]^T = [0, 0.5, 0.5, 0.5]^T$, $\hat{\theta}_2 = \hat{\theta}_4 = [1, 1, 1, 1, 1, 1, 1]^T$, and $\hat{\varepsilon}_{N2} = \hat{\varepsilon}_{N4} = 0.01$. The parameters of state observer are $L = [12, 40, -6, -32]^T$. The control parameters are chosen as $c_1 = 4.5$, $c_2 = 3.5$, $c_3 = 2.5$, and $c_4 = 9$. The constants of adaptive laws are $r_2 = 0.2$, $r_4 = 0.2$, $\delta_2 = 0.3$, and $\sigma_2 = \sigma_4 = 0.01$. The time constants are $\tau_1 = 0.01$, $\tau_2 = 3$, and $\tau_3 = 2$. The RBF NN parameters are $c_{ij} = [-3 -2 -1 0 1 2 3]$ and $b_j = 0.5$. The system parameters are MgL $= 5$, I $= 1$, J $= 1$, and K $= 40$. The simulation results are shown by Figs. 2, 3, 4, 5, 6, and 7.
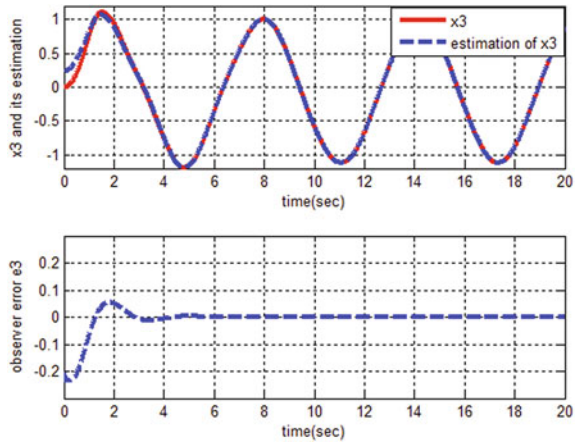

Fig. 2 State x_1 and its estimation $\hat{x}_1$
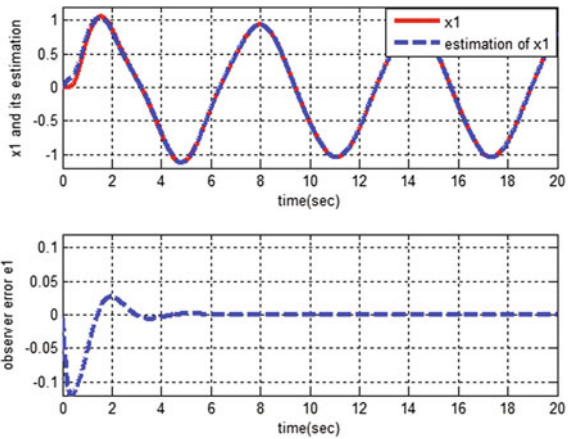
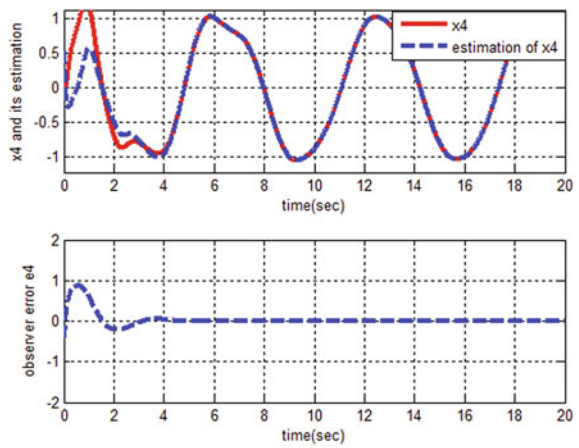

Fig. 3 State x_2 and its estimation $\hat{x}_2$
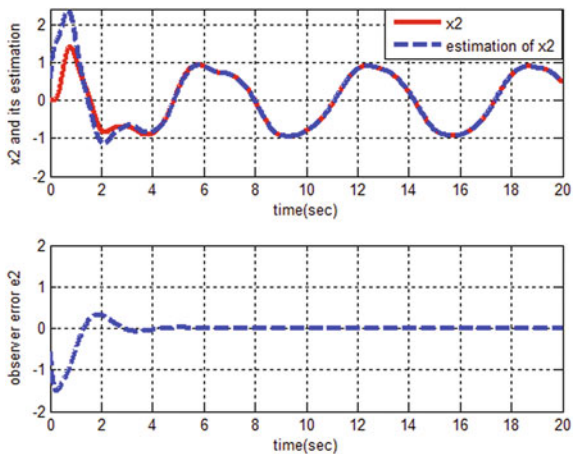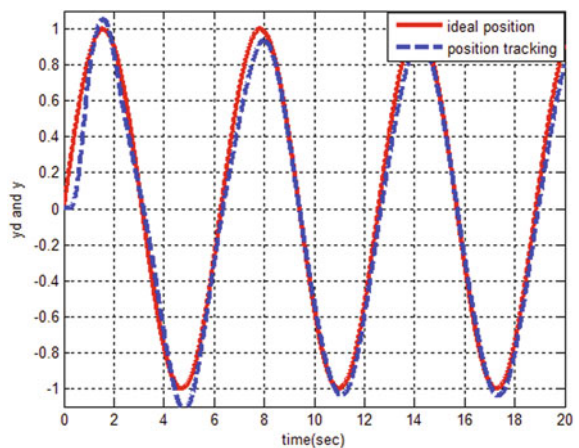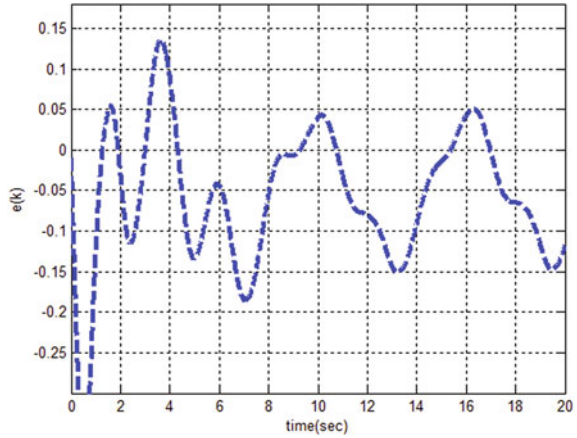
Fig. 4 State x_3 and its estimation $\hat{x}_3$

Fig. 5 State x_4 and its estimation $\hat{x}_4$

Fig. 6 Position tracking performance

Fig. 7 Position tracking error

Figures [2], [3], [4], and [5] show the estimation performance of states $x_i(i = 1, 2, 3, 4)$ and corresponding errors e_i, respectively. We can clearly see that all the estimation states tracks actual states with a small observed errors. The tracking trajectory and tracking error are shown in Figs. [6] and [7], respectively. Obviously, the tracking error can converge to a small neighborhood around zero. From the aforementioned figures, it is clear that the proposed adaptive neural output feedback control scheme can achieve a good tracking performance for the system ([2]) with immeasurable states.

6 Conclusion

In this paper, an adaptive neural output feedback control scheme is investigated for single-link flexible-joint robotic manipulator systems with immeasurable states. The immeasurable states are estimated by constructing a Luenberger state observer, and model uncertainties are approximated by using RBF neural networks. Based on the dynamic surface control scheme, the adaptive controller is designed to avoid the problem of "explosion of complexity." Simulation results illustrate that the proposed method is effective to achieve a good tracking performance.

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