Robust Tracking Control of Wheeled Mobile Robots with Parameter Uncertainties and only Target's Position Measurement

Lixia Yan and Baoli Ma

Abstract Robust tracking control of wheeled mobile robots (WMRs) is studied in this work. Considering the dynamic model of WMRs with unknown parameters, a robust sliding-mode state feedback controller is proposed, guaranteeing the tracking errors converge to zero asymptotically. Later, combining robust exact differentiators with the proposed state feedback control law leads to a tracking controller, in which only the position of reference robot is included and the tracking errors are driven to the origin asymptotically too. Numerical simulation is carried out to verify the effectiveness of proposed controller.

Keywords Wheeled mobile robots · Robust tracking control · Sliding-mode control · Robust exact differentiator

1 Introduction

To date, the trajectory tracking and path following control of wheeled mobile robots have been widely studied. There are no continuous time-invariant controllers to achieve state stabilization of WMRs due to the limitation of Brockett necessary condition [1]. A trajectory tracking control law based on backstepping method is proposed in [2], within which the tracking errors converge to zero uniformly asymptotically. Using dynamic feedback linearization, a local asymptotical tracking control scheme is shown in [3]. Clearly, sliding-mode control method is also a good way to solve control problem and makes systems robust to uncertainties and

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disturbances. By describing system from cartesian coordinate to polar coordinate, a sliding-mode tracking control law proposed in [4] guaranties the tracking errors ultimately bounded, while a large control may appear near the origin. Considering a universal sliding-mode control scheme for a class of nonlinear systems and transform the model equations of WMRs into a special form, controller proposed in [5] makes the system globally asymptotically stable. By designing a PI-type sliding-mode surface and an adaptive algorithm, the trajectory tracking errors are steered to zero asymptotically [6].

Almost under all situations, the trajectory tracking or path following controllers can be directly used for the tracking control of two WMRs if the position/orientation and linear/angular velocity information of reference robot are completely known by the tracker robot. However, under real circumstance, not all the information of the reference WMR can be known or easily detected, and less communication burden in hardware-layer of controller helps to build a reliable apparatus and decreases error-code rate [7]. Based on above practical considerations, it is desired to solve the tracking control problem of WMRs using only position information of the reference robot, which can be easily obtained even in indoor environment by camera [8] or UWB [9].

In this work, we first refer results in [10] to design estimators of reference robot using only its position information. Later, we introduce a full-state feedback sliding-mode controller which drives the system states converging to the stable sliding surface in finite time despite the model parameter uncertainties. The combination of state feedback control law with estimators contributes to a tracking controller with only position information of reference robot.

The paper is organized as follows. Section 2 contains problem formation, controller design is included in Sect. 3, simulation results and conclusion are presented in Sects. 4 and 5 respectively.

2 Problem Formation

Consider the dynamic model of WMRs described by

$$\begin{cases} \dot{x} = v\cos\theta, \dot{y} = v\sin\theta, \dot{\theta} = \omega \\ m\dot{v} = \frac{\tau_1+\tau_2}{R}, I\dot{\omega}=\frac{L}{R}\left(\tau_1 - \tau_2\right) \end{cases} \tag{1}$$

where (x, y) is the coordinate of mass center, θ denotes the posture angle, v and ω represent linear and angular velocity respectively. $(\dot{v}, \dot{\omega})$ are linear and angular accelerations. τ_1 and τ_2 denote driving torques of the right and left wheels. (m, I, R, L) are mass, inertia around the mass center, wheel diameter, distance between right and left wheel respectively, which are unknown parameters bounded by known bounds, i.e.,

$$\begin{aligned} 0 < m_m \le m \le m_M, 0 < I_m \le I \le I_M \\ 0 < R_m \le R \le R_M, 0 < L_m \le L \le L_M \end{aligned} \tag{2}$$

where $m_m, m_M, I_m, I_M, R_m, R_M, L_m, L_M$ are known positive constants.

The kinematic equations of the reference robot are as follows:

$$\dot{x}_r = v_r \cos\theta_r, \dot{y}_r = v_r \sin\theta_r, \dot{\theta}_r = \omega_r \tag{3}$$

Assumption 1 *The reference speeds and their first- and second-order derivatives* $(v_r, \omega_r, \dot{v}_r, \dot{\omega}_r, \ddot{v}_r, \ddot{\omega}_r)$ *are bounded by*

$$\begin{cases} v_{rM} \ge v_r \ge v_{rm} > 0, \dot{v}_{rM} \ge |\dot{v}_r|, \ddot{v}_{rM} \ge |\ddot{v}_r| \\ \omega_{rM} \ge |\omega_r|, \dot{\omega}_{rM} \ge |\dot{\omega}_r|, \ddot{\omega}_{rM} \ge |\ddot{\omega}_r| \end{cases} \tag{4}$$

where $v_{rM}, v_{rm}, \omega_{rM}, \dot{v}_{rM}, \dot{\omega}_{rM}, \ddot{v}_{rM}, \ddot{\omega}_{rM}$ *are positive constants.*

Assumption 2 *The exact position* (x_r, y_r) *of the reference robot is known.*

Define the tracking errors as

$$e_x = x - x_r, e_y = y - y_r, e_\theta = \theta - \theta_r \tag{5}$$

With Assumptions 1 and 2, the control task in this paper is to design control law

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} U_1\left(x, y, \theta, v, \omega, x_r, y_r, \Omega\right) \\ U_2\left(x, y, \theta, v, \omega, x_r, y_r, \Omega\right) \end{bmatrix} \tag{6}$$

such that

$$\lim_{t\to\infty} e_x = 0, \lim_{t\to\infty} e_y = 0, \lim_{t\to\infty} e_\theta = 0 \tag{7}$$

where Ω denotes the set of auxiliary variables.

3 Controller Design

In this section, we first give out some preliminary results that refer to [10] and estimate some values of reference robot that are not known exactly. Later, a robust state feedback controller will be introduced. Combining estimating algorithm and state feedback controller leads to the robust tracking controller with only position information of the target.

3.1 Target Observer Design

From Assumptions 1 and 2, we know that (x_r, y_r) is measurable and their derivatives are bounded, so that we can estimate their first, second, and third derivatives by the exact differentiators proposed in [10] as follows:

$$\begin{aligned} &\dot{f}_{0x} = w_{0x}, w_{0x} = -\lambda_0 \left|f_{0x} - x_r\right|^{\frac{3}{4}} \mathrm{sign}\left(f_0 - x_r\right) + f_{1x} \\ &\dot{f}_{1x} = w_{1x}, w_{1x} = -\lambda_1 \left|f_{1x} - w_{0x}\right|^{\frac{2}{3}} \mathrm{sign}\left(f_{1x} - w_{0x}\right) + f_{2x} \\ &\dot{f}_{2x} = w_{2x}, w_{2x} = -\lambda_2 \left|f_{2x} - w_{1x}\right|^{\frac{1}{2}} \mathrm{sign}\left(f_{2x} - w_{1x}\right) + f_{3x} \\ &\dot{f}_{3x} = -\lambda_3 \mathrm{sign}\left(f_{3x} - w_{2x}\right) \end{aligned} \tag{8}$$

$$\begin{aligned} &\dot{f}_{0y} = w_{0y}, w_{0y} = -\lambda_0 \left|f_{0y} - y_r\right|^{\frac{3}{4}} \mathrm{sign}\left(f_{0y} - y_r\right) + f_{1y} \\ &\dot{f}_{1y} = w_{1y}, w_{1y} = -\lambda_1 \left|f_{1y} - w_{0y}\right|^{\frac{2}{3}} \mathrm{sign}\left(f_{1y} - w_{0y}\right) + f_{2y} \\ &\dot{f}_{2y} = w_{2y}, w_{2y} = -\lambda_2 \left|f_{2y} - w_{1y}\right|^{\frac{1}{2}} \mathrm{sign}\left(f_{2y} - w_{1y}\right) + f_{3y} \\ &\dot{f}_{3y} = -\lambda_3 \mathrm{sign}\left(f_{3y} - w_{2y}\right) \end{aligned} \tag{9}$$

where $\lambda_i > L_r\, (i = 0, 1, 2, 3)$ with $L_r = \max\{|\dot{x}_r|, |\ddot{x}_r|, |\dddot{x}_r|, |\dot{y}_r|, |\ddot{y}_r|, |\dddot{y}_r|\}$. By using (8) and (9), the exact estimation of $(\dot{x}_r, \ddot{x}_r, \dddot{x}_r, \dot{y}_r, \ddot{y}_r, \dddot{y}_r)$ can be obtained by $(w_{0x}, w_{1x}, w_{2x}, w_{oy}, w_{1y}, w_{2y})$ in finite time.

Taking (3) into account and calculating the first- to third-order derivatives of (x_r, y_r), we get

$$\begin{cases} \dot{x}_r = v_r \cos\theta_r, \ddot{x}_r = \dot{v}_r \cos\theta_r - v_r\omega_r \sin\theta_r \\ \dot{y}_r = v_r \sin\theta_r, \ddot{y}_r = \dot{v}_r \sin\theta_r + v_r\omega_r \cos\theta_r \\ \dddot{x}_r = \ddot{v}_r \cos\theta_r - 2\dot{v}_r\omega_r \sin\theta_r - v_r\dot{\omega}_r \sin\theta_r - v_r\omega_r^2 \cos\theta_r \\ \dddot{y}_r = \ddot{v}_r \sin\theta_r + 2\dot{v}_r\omega_r \cos\theta_r + v_r\dot{\omega}_r \cos\theta_r - v_r\omega_r^2 \sin\theta_r \end{cases} \tag{10}$$

which suggests

$$\begin{cases} \theta_r = \arctan 2\left(\dot{y}_r, \dot{x}_r\right), v_r = \sqrt{\dot{x}_r^2 + \dot{y}_r^2}, \dot{v}_r = \ddot{x}_r \cos\theta_r + \ddot{y}_r \sin\theta_r \\ \omega_r = \dfrac{\ddot{y}_r \cos\theta_r - \ddot{x}_r \sin\theta_r}{v_r}, \dot{\omega}_r = \dfrac{\dddot{y}_r \cos\theta_r - \dddot{x}_r \sin\theta_r - 2\dot{v}_r\omega_r}{v_r} \\ \forall \quad v_r > 0 \end{cases} \tag{11}$$

Thus, the estimated values of $(\theta_r, v_r, \omega_r, \dot{v}_r, \dot{\omega}_r)$ can be obtained as

$$\begin{cases} \hat{\theta}_r = \arctan 2\left(w_{0y}, w_{0x}\right) \\ \hat{v}_r = \sqrt{\left(w_{0x}\right)^2 + \left(w_{0y}\right)^2}, \hat{\dot{v}}_r = w_{1x} \cos\hat{\theta}_r + w_{1y} \sin\hat{\theta}_r \\ \hat{\omega}_r = \dfrac{w_{1y} \cos\hat{\theta}_r - w_{1x} \sin\hat{\theta}_r}{\hat{v}_r}, \hat{\dot{\omega}}_r = \dfrac{w_{2y} \cos\hat{\theta}_r - w_{2x} \sin\hat{\theta}_r - 2\hat{\dot{v}}_r\hat{\omega}_r}{\hat{v}_r} \end{cases} \tag{12}$$

Remark 1 As $\hat{v}_r$ appears in denominators of $\left(\hat{\omega}_r, \hat{\dot{\omega}}_r\right)$ and converges to real value in finite time, we adopt the following strategy in control to avoid possible singularity when $\hat{v}_r$ cross zero during transient process.

$$\hat{v}_r = \begin{cases} v_{rm}, \sqrt{\left(w_{0x}\right)^2 + \left(w_{0y}\right)^2} \le v_{rm} \\ \sqrt{\left(w_{0x}\right)^2 + \left(w_{0y}\right)^2}, \sqrt{\left(w_{0x}\right)^2 + \left(w_{0y}\right)^2} > v_{rm} \end{cases} \tag{13}$$

3.2 Sliding-Mode Controller

Define the auxiliary position tracking errors

$$e_1 = e_x + l\left(\cos\theta - \cos\theta_r\right), e_2 = e_y + l\left(\sin\theta - \sin\theta_r\right) \tag{14}$$

where constant $l > 0$. Differentiating (14) along state trajectory of (5) results

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = A(\theta)\begin{bmatrix} v \\ \omega \end{bmatrix} - A\left(\theta_r\right)\begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \tag{15}$$

where

$$A(a) \stackrel{\Delta}{=} \begin{bmatrix} \cos a & -l\sin a \\ \sin a & l\cos a \end{bmatrix} \to A^{-1}(a) = \begin{bmatrix} \cos a & \sin a \\ -\frac{\sin a}{l} & \frac{\cos a}{l} \end{bmatrix} \tag{16}$$

Define the stable sliding-mode surfaces

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} \dot{e}_1 + k_1 e_1 \\ \dot{e}_2 + k_1 e_2 \end{bmatrix} = A(\theta)\begin{bmatrix} v \\ \omega \end{bmatrix} - A\left(\theta_r\right)\begin{bmatrix} v_r \\ \omega_r \end{bmatrix} + k_1\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{17}$$

in which k_1 is a positive constant.

Let $\left(\bar{\tau}_1, \bar{\tau}_2\right) = \left(\tau_1 + \tau_2, \tau_1 - \tau_2\right)$ and $\left(p_1, p_2\right) = \left(\frac{1}{mR}, \frac{L}{IR}\right)$, the derivative of (17) becomes

$$\dot{s} = A(\theta)\begin{bmatrix} p_1\bar{\tau}_1 \\ p_2\bar{\tau}_2 \end{bmatrix} + A(\theta)B(\omega)\begin{bmatrix} v \\ \omega \end{bmatrix} - \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \tag{18}$$

where

$$B(a) = \begin{bmatrix} k_1 & -la \\ \frac{a}{l} & k_1 \end{bmatrix}, \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = A\left(\theta_r\right)\begin{bmatrix} \dot{v}_r \\ \dot{\omega}_r \end{bmatrix} + A\left(\theta_r\right)B\left(\omega_r\right)\begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \tag{19}$$

To realize the input-output decoupling, define the new sliding-mode surfaces

$$\bar{s} = \begin{bmatrix} \bar{s}_1 \\ \bar{s}_2 \end{bmatrix} = A^{-1}(\theta)\, s \tag{20}$$

Differentiating $\bar{s}$ leads to

$$\dot{\bar{s}} = \begin{bmatrix} p_1 \bar{\tau}_1 \\ p_2 \bar{\tau}_2 \end{bmatrix} + \begin{bmatrix} \delta_{11} \\ \delta_{21} \end{bmatrix} + \begin{bmatrix} \delta_{12} \\ \delta_{22} \end{bmatrix} \tag{21}$$

where

$$\begin{aligned} \begin{bmatrix} \delta_{11} \\ \delta_{21} \end{bmatrix} &= B(\omega) \begin{bmatrix} v \\ \omega \end{bmatrix} + \dot{A}^{-1}(\theta) A(\theta) \begin{bmatrix} v \\ \omega \end{bmatrix} + \dot{A}^{-1}(\theta) k_1 \begin{bmatrix} e_x + l\cos\theta \\ e_y + l\sin\theta \end{bmatrix} \\ \begin{bmatrix} \delta_{12} \\ \delta_{22} \end{bmatrix} &= -A^{-1}(\theta) \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} - \dot{A}^{-1}(\theta) A\left(\theta_r\right) \begin{bmatrix} v_r \\ \omega_r \end{bmatrix} - \dot{A}^{-1}(\theta) k_1 l \begin{bmatrix} \cos\theta_r \\ \sin\theta_r \end{bmatrix} \end{aligned} \tag{22}$$

Theorem 1 *Suppose that Assumption 1 establishes and the control parameters satisfy $k_1 > 0, \varepsilon_1 > 0, \varepsilon_2 > 0$, the sliding-mode control law*

$$\begin{cases} \bar{\tau}_1 = -\hat{\bar{p}}_1 \delta_{11} - \hat{\bar{p}}_1 \delta_{12} - \operatorname{sign}(\bar{s}) \left(\bar{p}_1' |\delta_{11}| + \bar{p}_1' |\delta_{12}| + \varepsilon_1 \right) \\ \bar{\tau}_2 = -\hat{\bar{p}}_2 \delta_{21} - \hat{\bar{p}}_2 \delta_{22} - \operatorname{sign}(\bar{s}) \left(\bar{p}_2' |\delta_{21}| + \bar{p}_2' |\delta_{22}| + \varepsilon_2 \right) \end{cases} \tag{23}$$

guarantees that $\left(\bar{s}_1, \bar{s}_2\right)$ converge to the origin in finite time, where $\bar{p}_1 = p_1^{-1}$, $\bar{p}_2 = p_2^{-1}$ are unknown positive constants bounded by known constants $\left(\bar{p}_{1M}, \bar{p}_{1m}, \bar{p}_{2M}, \bar{p}_{2m}\right)$, i.e.

$$\bar{p}_{1M} \geq \bar{p}_1 \geq \bar{p}_{1m} > 0, \bar{p}_{2M} \geq \bar{p}_2 \geq \bar{p}_{2m} > 0 \tag{24}$$

and

$$\begin{cases} \hat{\bar{p}}_1 = 0.5\left(\bar{p}_{1m} + \bar{p}_{1M}\right), \hat{\bar{p}}_2 = 0.5\left(\bar{p}_{2m} + \bar{p}_{2M}\right) \\ \bar{p}_1' = \max\left(|\bar{p}_1 - \hat{\bar{p}}_1|\right) = 0.5\left(\bar{p}_{1M} - \bar{p}_{1m}\right) \\ \bar{p}_2' = \max\left(|\bar{p}_2 - \hat{\bar{p}}_2|\right) = 0.5\left(\bar{p}_{2M} - \bar{p}_{2m}\right) \end{cases} \tag{25}$$

Proof Choose $V_1 = 0.5\bar{p}_1 \bar{s}_1^2$ and $V_2 = 0.5\bar{p}_2 s_2^2$ as Lyapunov candidates functions and compute their derivatives along with the trajectory of closed-loop system (21)–(23) as

$$\begin{aligned} \dot{V}_1 &= \bar{s}_1 \left[\delta_{11} \left(\bar{p}_1 - \hat{\bar{p}}_1 \right) + \delta_{12} \left(\bar{p}_1 - \hat{\bar{p}}_1 \right) \right] - |\bar{s}_1| \left(p_1' |\delta_{11}| + p_1' |\delta_{12}| + \varepsilon_1 \right) \\ &\leq |\bar{s}_1| |\delta_{11} \left(\bar{p}_1 - \hat{\bar{p}}_1 \right)| + |\delta_{12} \left(\bar{p}_1 - \hat{\bar{p}}_1 \right)| - |\bar{s}_1| \left(p_1' |\delta_{11}| + p_1' |\delta_{12}| + \varepsilon_1 \right) \\ &\leq -\varepsilon_1 |\bar{s}_1| - |\bar{s}_1| \left(p_1' - |\bar{p}_1 - \hat{\bar{p}}_1| \right) |\delta_{11}| - |\bar{s}_1| \left(p_1' - |\bar{p}_1 - \hat{\bar{p}}_1| \right) |\delta_{12}| \\ &\leq -\varepsilon_1 |\bar{s}_1| \\ \dot{V}_2 &= \bar{s}_2 \left[\delta_{21} \left(\bar{p}_2 - \hat{\bar{p}}_2 \right) + \delta_{22} \left(\bar{p}_2 - \hat{\bar{p}}_2 \right) \right] - |\bar{s}_2| \left(p_2' |\delta_{21}| + p_2' |\delta_{22}| + \varepsilon_2 \right) \\ &\leq |\bar{s}_2| |\delta_{21} \left(\bar{p}_2 - \hat{\bar{p}}_2 \right)| + |\delta_{22} \left(\bar{p}_2 - \hat{\bar{p}}_2 \right)| - |\bar{s}_2| \left(p_2' |\delta_{21}| + p_2' |\delta_{22}| + \varepsilon_2 \right) \\ &\leq -\varepsilon_2 |\bar{s}_2| - |\bar{s}_2| \left(p_2' - |\bar{p}_2 - \hat{\bar{p}}_2| \right) |\delta_{21}| - |\bar{s}_2| \left(p_2' - |\bar{p}_2 - \hat{\bar{p}}_2| \right) |\delta_{22}| \\ &\leq -\varepsilon_2 |\bar{s}_2| \end{aligned} \tag{26}$$

Let $W_1 = \sqrt{2\bar{p}_1^{-1}V_1} = |\bar{s}_1|$, $W_2 = \sqrt{2\bar{p}_2^{-1}V_2} = |\bar{s}_2|$, we then obtain

$$D^+W_1 = \frac{2\bar{p}_1^{-1}\dot{V}_1}{2\sqrt{2\bar{p}_1^{-1}V_1}} \le -\varepsilon_1\bar{p}_1^{-1}, D^+W_2 = \frac{2\bar{p}_2^{-1}\dot{V}_2}{2\sqrt{2\bar{p}_2^{-1}V_2}} \le -\varepsilon_2\bar{p}_2^{-1} \tag{27}$$

Comparison principle can then be used to obtain the conservative estimation of converging time of $(\bar{s}_1, \bar{s}_2)$ and we get

$$\bar{s}_1(t) = 0, \bar{s}_2(t) = 0, \forall t \ge T_1 = \max\left\{\bar{p}_{1M}\frac{|\bar{s}_1(0)|}{\varepsilon_1}, \bar{p}_{2M}\frac{|\bar{s}_2(0)|}{\varepsilon_2}\right\} \tag{28}$$

According to (21), we know that $(s_1(t), s_2(t)) = (0, 0)$ for $t \ge T_1$. On the sliding surface $(s_1(t), s_2(t)) = (0, 0)$, the auxiliary position tracking errors (e_1, e_2) will converge to zero exponentially.

Next, we show that the overall tracking error system is asymptotically stable because the zero-dynamics subsystem of (15), associated with e_θ, is asymptotically stable. Nulling $(\dot{e}_1, \dot{e}_2)$ in (15) gives rise to

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = A^{-1}(\theta) A(\theta_r) \begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \tag{29}$$

Take out the angular velocity and write the dynamics of $\dot{e}_\theta$ as

$$\dot{e}_\theta = \dot{\theta} - \dot{\theta}_r = \omega - \omega_r = -\frac{v_r}{l}\sin e_\theta + \omega_r(\cos e_\theta - 1) \tag{30}$$

Linearize (30) at $e_\theta = 0$, we obtain

$$\dot{e}_\theta = -\frac{v_r}{l}e_\theta \tag{31}$$

which is exponentially stable under Assumption 1. So the overall closed-loop system is concluded locally asymptotically stable [11] and (7) establishes.

Replacing the unmeasurable variables $(\theta_r, v_r, \omega_r, \dot{v}_r, \dot{\omega}_r)$ with their estimates $(\hat{\theta}_r, \hat{v}_r, \hat{\omega}_r, \hat{\dot{v}}_r, \hat{\dot{\omega}}_r)$ in controller (23) leads to

$$\begin{cases} \bar{\tau}_1 = -\hat{\bar{p}}_1\delta_{11} - \hat{\bar{p}}_1\hat{\delta}_{12} - \text{sign}(\hat{\bar{s}})(\bar{p}_1'|\delta_{11}| + \bar{p}_1'|\hat{\delta}_{12}| + \varepsilon_1) \\ \bar{\tau}_2 = -\hat{\bar{p}}_2\delta_{21} - \hat{\bar{p}}_2\hat{\delta}_{22} - \text{sign}(\hat{\bar{s}})(\bar{p}_2'|\delta_{21}| + \bar{p}_2'|\hat{\delta}_{22}| + \varepsilon_2) \end{cases} \tag{32}$$

where

$$\begin{cases} \begin{bmatrix} \hat{\delta}_{12} \\ \hat{\delta}_{22} \end{bmatrix} = -A^{-1}(\theta) \begin{bmatrix} \hat{\Delta}_1 \\ \hat{\Delta}_2 \end{bmatrix} - \dot{A}^{-1}(\theta) A\left(\hat{\theta}_r\right) \begin{bmatrix} \hat{v}_r \\ \hat{\omega}_r \end{bmatrix} - \dot{A}^{-1}(\theta) k_1 l \begin{bmatrix} \cos\hat{\theta}_r \\ \sin\hat{\theta}_r \end{bmatrix} \\ \begin{bmatrix} \hat{\Delta}_1 \\ \hat{\Delta}_2 \end{bmatrix} = A\left(\hat{\theta}_r\right) \begin{bmatrix} \hat{\dot{v}}_r \\ \hat{\dot{\omega}}_r \end{bmatrix} + A\left(\hat{\theta}_r\right) B\left(\hat{\omega}_r\right) \begin{bmatrix} \hat{v}_r \\ \hat{\omega}_r \end{bmatrix} \\ \begin{bmatrix} \hat{\bar{s}}_1 \\ \hat{\bar{s}}_2 \end{bmatrix} = A^{-1}(\theta) \left[A(\theta) \begin{bmatrix} v \\ \omega \end{bmatrix} - A\left(\hat{\theta}_r\right) \begin{bmatrix} \hat{v}_r \\ \hat{\omega}_r \end{bmatrix} + k_1 \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \end{bmatrix} \right] \\ \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \end{bmatrix} = \begin{bmatrix} x - x_r + l\left(\cos\theta - \cos\hat{\theta}_r\right) \\ y - y_r + l\left(\sin\theta - \sin\hat{\theta}_r\right) \end{bmatrix} \end{cases} \tag{33}$$

Since the estimated variables $\left(\hat{\delta}_{12}, \hat{\delta}_{22}\right)$ converge to real ones in finite time, there exists $T_2 > 0$ such that the performance of controller (32) equals to that of (23) for $t \geq T_2$. Furthermore, we have

$$(\bar{\tau}_1, \bar{\tau}_2) \in L_\infty, 0 \leq t \leq T_2 \tag{34}$$

so that all states are bounded during transient process. Thus, the closed-loop system under control of (32) is also locally asymptotically stable.

4 Simulation Results

The model parameters of tracker robot are chosen from one real-wheeled mobile robot in the authors' laboratory that satisfy

$$\begin{cases} (2-0.2)\,\mathrm{kg} \leq m \leq (2+0.2)\,\mathrm{kg} \\ (0.2-0.02)\,\mathrm{m} \leq L \leq (0.2+0.02)\,\mathrm{m} \\ (0.08-0.008)\,\mathrm{kg}\cdot\mathrm{m}^2 \leq I \leq (0.08+0.008)\,\mathrm{kg}\cdot\mathrm{m}^2 \\ (0.05-0.005)\,\mathrm{m} \leq R \leq (0.05+0.005)\,\mathrm{m} \end{cases} \tag{35}$$

which contribute to the inequalities

$$\begin{cases} 0.121 = \bar{p}_{1M} \geq \bar{p}_1 \geq \bar{p}_{1m} = 0.081 \\ 0.027 = \bar{p}_{2M} \geq \bar{p}_2 \geq \bar{p}_{2m} = 0.015 \end{cases} \tag{36}$$

Let the position of reference robot be generated from an eight-shaped trajectory described by

$$\dot{x}_r = g_r \cos\left(2h_r\right), \dot{y}_r = g_r \sin\left(h_r\right), \dot{h}_r = \Omega_r \tag{37}$$

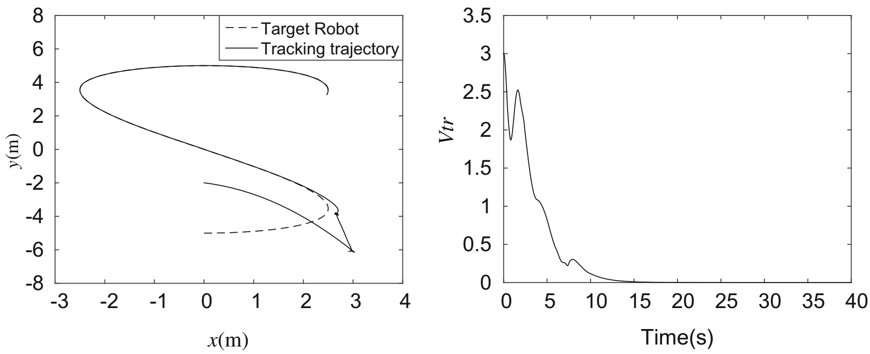

Fig. 1 The tracking trajectory and tracking error (39) under controller (32)

The initial states about robust differentiators are all set to zero, initial states and the rest parameters are

$$\begin{cases} \left[x(0), y(0), \theta(0)\right] = [0, -2, 0] \\ \left[x_r(0), y_r(0), \theta_r(0)\right] = [0, -5, 0] \\ l = 0.1, k_1 = 0.5, \varepsilon_1 = \varepsilon_2 = 0.2 \\ m = 1.8, L = 0.19, I = 0.081, R = 0.05 \\ \lambda_0 = 1.6, \lambda_1 = 1.2, \lambda_2 = 0.4, \lambda_3 = 0.2 \\ g_r = 0.5, \Omega_r = 0.1 \end{cases} \tag{38}$$

Define the tracking error function

$$V_{tr} = \sqrt{e_x^2 + e_y^2 + e_\theta^2} \tag{39}$$

The simulation results are all shown in Fig. 1.

Simulation results show that the robot has successfully catched up with the reference robot under proposed controller (33) and V_{tr} converges to zero asymptotically.

5 Conclusion

A robust sliding-mode controller with only position information of reference robot is obtained by combining sliding-mode control method with robust exact differentiators. Theoretical analysis shows that the overall closed-loop system is locally asymptotically stable. Numerical simulation results verify the efficiency of the propose controller. The proposed controller is robust to model parameters based on the sliding-model technique. The author would like to investigate the multiagent control problem of WMRs with uncertain model parameters and with only position information of neighbors in future work.

References

1. Brockett RW (1983) Asymptotic stability and feedback stabilization. Differ Geom Control Theory
2. Fierro R, Lewis FL (1995) Control of a nonholonomic mobile robot: backstepping kinematics into dynamics. In: Proceedings of the 34th IEEE conference on decision and control, vol 4. pp 3805–3810
3. Oriolo G, De Luca A, Vendittelli M (2002) WMR control via dynamic feedback linearization: design, implementation, and experimental validation. IEEE Trans Control Syst Technol 10(6):835–852
4. Chwa D (2004) Sliding-mode tracking control of nonholonomic wheeled mobile robots in polar coordinates. IEEE Trans Control Syst Technol 12(4):637–644
5. Mu J, Yan X-G, Jiang B (2015) Sliding mode control for a class of nonlinear systems with application to a wheeled mobile robot. In: 54th IEEE conference on decision and control. pp 4746–4751
6. Koubaa Y, Boukattaya M, Dammak T (2015) Adaptive sliding-mode dynamic control for path tracking of nonholonomic wheeled mobile robot. J Autom Syst Eng 9(2):119–131
7. Ren W, Beard RW (2008) Distributed consensus in multi-vehicle cooperative control. Springer
8. Yang JM, Kim JH (1999) Sliding mode control for trajectory tracking of nonholonomic wheeled mobile robots. IEEE Trans Robot Autom 15(3):578–587
9. Yu K, Montillet JP, Rabbachin A et al (2006) UWB location and tracking for wireless embedded networks. Sig Process 86(9):2153–2171
10. Levant A (2003) Higher-order sliding modes, differentiation and output-feedback control. Int J Control 76(9–10):924–941
11. Khalil HK (2011) Nonlinear control system. Prentice Hall