Consensus Control for Multi-agent Networks with Mixed Undirected Interactions

Weili Niu, Deyuan Meng and Xiaolu Ding

Abstract This paper is concerned with the average consensus problem on multi-agent networks with undirected interactions which can be either static or dynamic. Notably, the multi-agent networks involving static and dynamic interactions are represented by graphs with edge weights in the form of real numbers and transfer functions. We propose a distributed consensus control algorithm based on the nearest neighbor rule. It is shown that the connectivity topology condition supplies a necessary and sufficient condition for all agents to achieve average consensus. Numerical simulations are provided to verify the effectiveness of the obtained results.

Keywords Average consensus · Multi-agent networks · Dynamic interactions

1 Introduction

Coordination control for multi-agent networks has attracted considerable attention owing to its wide applications in many areas, such as biological systems [1], vehicle systems [2], complex networks [3], and power networks [4]. An important research topic of multi-agent coordination is consensus since it plays a fundamental role in all related problems. By consensus, it needs all agents to agree on a common quantity [5].

W. Niu
School of Mathematics and Systems Science, Beihang University (BUAA), Beijing 100191, China

W. Niu · D. Meng (✉) · X. Ding
The Seventh Research Division, Beihang University (BUAA), Beijing 100191, China
e-mail: dymeng23@126.com

D. Meng · X. Ding
School of Automation Science and Electrical Engineering, Beihang University (BUAA), Beijing 100191, China

Y. Jia et al. (eds.), *Proceedings of 2016 Chinese Intelligent Systems Conference*, Lecture Notes in Electrical Engineering 404, DOI 10.1007/978-981-10-2338-5_10

In the literature, most results address consensus problems on multi-agent networks with static interactions (see [6–11]). Static interactions between agents denote that the information is directly communicated with each other, and the corresponding adjacency weights are usually represented by real numbers. However, in practice, many systems are subject to dynamic interactions, especially large-scale systems with interconnected storage elements [12]. Dynamic interactions between agents denote that the information is not directly communicated with each other but shared after it is dynamically processed via a system/filter, and the corresponding adjacency weights are represented by dynamic systems or transfer functions. Recently, consensus against dynamic interactions emerges as a hot topic. Consensus problems on multi-agent networks with dynamic interactions represented by positive real systems and stable LTI systems are addressed in [13, 14], respectively. Two consensus problems on directed dynamic multi-agent networks with application to the thermal processes in buildings and undirected dynamic multi-agent networks with application to the power networks are studied in [15]. These new studies have extended the consensus theory to more general multi-agent networks.

In this paper, we study the average consensus problem on multi-agent networks with both static and dynamic interactions. A new consensus algorithm that combines traditional static consensus algorithm and the dynamic consensus algorithm with dynamic weights designed in the form of transfer functions is proposed. We adopt analysis approaches both in the time domain and in the frequency domain. It is shown that the connectivity of the undirected graph plays a crucial role for the mixed multi-agent networks reaching average consensus.

The remainder of this paper is organized as follows. In Sect. 2, we introduce some preliminaries on graph theory and present the problem statement on dynamic consensus. Distributed dynamic consensus results are presented in Sect. 3 and simulation results are provided in Sect. 4 to demonstrate the dynamic average consensus performance. Finally, in Sect. 5, conclusions and future studies are given.

Notations: Throughout this paper, $\mathscr{I}_n = \{1, 2, \ldots, n\}$, $1_n = [1, 1, \ldots, 1]^T \in \mathbb{R}^n$, I and 0 denote the identity matrix and null matrix with appropriate dimensions, respectively, and diag$\{\cdot\}$ represents a block matrix with the off-diagonal elements are all zeros.

2 Problem Statement

2.1 Preliminaries

We use an undirected graph to model the information exchange among agents. A weighted undirected graph is denoted by a triple $G = (V, E, A)$, where $V = \left\{e_i : i \in \mathscr{I}_n\right\}$ is the vertex set, $E \subseteq \left\{\left(e_i, e_j\right) : e_i, e_j \in V\right\}$ is the edge set, and $A = \left(a_{ij}\right) \in \mathbb{R}^{n\times n}$ is the symmetric nonnegative adjacency weight matrix, which satisfies $a_{ij} > 0 \Leftrightarrow \left(e_i, e_j\right) \in E$ and $a_{ij} = 0$ otherwise. Moreover, $a_{ii} = 0$ is assumed for all $i \in \mathscr{I}_n$. The edge $\left(e_i, e_j\right) \in E$ denotes that e_i and e_j can receive information from each other, and e_i and e_j are neighbors. The index set of neighbors of each agent e_i is denoted by $N_i = \{j : (e_i, e_j) \in E\}$. A path is a finite sequence of edges consisting of distinct vertices $e_{i_0}, e_{i_1}, \ldots, e_{i_j}$ such that $(e_{i_{k-1}}, e_{i_k}) \in E$ for $k = 1, 2, \ldots, j$. An undirected graph is said to be connected if there exists a path between every pair of distinct vertices.

2.2 Problem Description

Consider a mixed multi-agent network with $n + m$ agents, and the interaction topology among these $n + m$ agents is modeled by an undirected graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A}(s))$, where $\mathscr{A}(s)$ is the symmetric adjacency weight matrix with entries in the form of real numbers and transfer functions. According to different interactions among agents, we divided the multi-agent networks into two separate subnetworks: a controlled network with n agents labeled 1 through n, and a controller network with m agents labeled 1 through m. If agents lie in the same networks, the interactions between them are static. Otherwise, if agents lie in different networks, the interactions between them are dynamic. A simple example of such mixed multi-agent networks is shown in Fig. 1, where the controlled and controller network have 6 agents and 4 agents, respectively.

The controlled network is associated with an undirected graph $\mathscr{G}^p = (\mathscr{V}^p, \mathscr{E}^p, \mathscr{A}^p)$, where $\mathscr{V}^p = \{v_i : i \in \mathscr{I}_n\}$, $\mathscr{E}^p \subseteq \{(v_i, v_j) : v_i, v_j \in \mathscr{V}^p\}$, and $\mathscr{A}^p = \left(a_{ij}^p\right) \in \mathbb{R}^{n \times n}$. The dynamics of each agent v_i are given by

$$\dot{x}_i(t) = \sum_{j \in \mathscr{N}_i^p} a_{ij}^p \left[x_j(t) - x_i(t)\right] + u_i^p(t), i \in \mathscr{I}_n \tag{1}$$

where $x_i(t) \in \mathbb{R}$ is the state of agent v_i, $u_i^p(t) \in \mathbb{R}$ is the control input or protocol to be designed according to the dynamic interactions between v_i and its neighbors in controller network, and $\mathscr{N}_i^p = \{j : (v_i, v_j) \in \mathscr{E}^p\}$.

Similarly, we consider the controller network be associated with an undirected graph $\mathscr{G}^c = (\mathscr{V}^c, \mathscr{E}^c, \mathscr{A}^c)$, where $\mathscr{V}^c = \{c_i : i \in \mathscr{I}_m\}$, $\mathscr{E}^c \subseteq \{(c_i, c_j) : c_i, c_j \in \mathscr{V}^c\}$, and $\mathscr{A}^c = \left(a_{ij}^c\right) \in \mathbb{R}^{m \times m}$. The dynamics of each agent c_i are given by

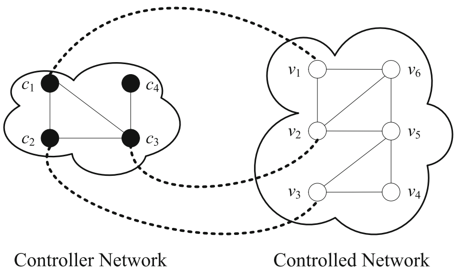

Fig. 1 An example of mixed undirected network, where *solid lines* denote the static interactions, and *dashed lines* denote the dynamic interactions

$$\dot{y}_i(t) = \sum_{j \in \mathscr{N}_i^c} a_{ij}^c \left[y_j(t) - y_i(t)\right] + u_i^c(t), i \in \mathscr{I}_m \tag{2}$$

where $y_i(t) \in \mathbb{R}$ is the state of agent c_i, $u_i^c(t) \in \mathbb{R}$ is the control input or protocol to be designed according to the dynamic interactions between c_i and its neighbors in controlled network, and $\mathscr{N}_i^c = \{j : \left(c_i, c_j\right) \in \mathscr{E}^c\}$.

The interactions between agents in different networks are achieved by the inputs $u_i^p(t)$ and $u_i^c(t)$ which fulfill the nearest neighbor rules. Let $U_i^p(s) = \mathscr{L}\left[u_i^p(t)\right]$ be the Laplace transform of $u_i^p(t)$, and let $U_i^c(s)$, $X_i(s)$, and $Y_i(s)$ be defined in the same way for $u_i^c(t)$, $x_i(t)$, and $y_i(t)$, respectively. We consider distributed dynamic consensus protocols in the form of

$$U_i^p(s) = \sum_{j \in \mathscr{N}_i^{cp}} g_{ij}^{cp}(s) \left[Y_j(s) - X_i(s)\right], i \in \mathscr{I}_n \tag{3}$$

$$U_i^c(s) = \sum_{j \in \mathscr{N}_i^{pc}} g_{ij}^{pc}(s) \left[X_j(s) - Y_i(s)\right], i \in \mathscr{I}_m \tag{4}$$

where $g_{ij}^{cp}(s)$ and $g_{ij}^{pc}(s)$ are dynamic weights to be designed, which satisfy $g_{ij}^{cp}(s) \neq 0$ if v_i can get dynamic information from c_j and $g_{ij}^{cp}(s) = 0$ otherwise, and $g_{ij}^{pc}(s) \neq 0$ if c_i can get dynamic information from v_j and $g_{ij}^{pc}(s) = 0$ otherwise. Also, in (3) and (4), $\mathscr{N}_i^{cp} = \{j : g_{ij}^{cp}(s) \neq 0\}$ and $\mathscr{N}_i^{pc} = \{j : g_{ij}^{pc}(s) \neq 0\}$.

The problem addressed in this paper is to enable the agents in mixed multi-agent networks to achieve consensus such that

$$\lim_{t \to \infty} \xi(t) = \xi_c, \forall \xi(t) \in \left\{x_1(t), \ldots, x_n(t), y_1(t), \ldots, y_m(t)\right\} \tag{5}$$

where $\xi_c \in \mathbb{R}$ is a constant quantity. In particular, the multi-agent networks achieve the average consensus if $\xi_c = \frac{1}{n+m}\left(\sum_{i=1}^n x_i(0) + \sum_{j=1}^m y_j(0)\right)$, where $x_i(0)$ and $y_j(0)$ are initial states of $x_i(t)$ and $y_j(t)$, respectively.

3 Problem Analysis

3.1 Consensus Analysis

Let $x(t) = \left[x_1(t), x_2(t), \ldots, x_n(t)\right]^T$, and let $u^p(t)$, $y(t)$, $u^c(t)$ be denoted in the same way with $x(t)$. In this case, the consensus algorithm (1) and (2) can be written in a compact form as

$$\dot{x}(t) = -L^p x(t) + u^p(t) \tag{6}$$

$$\dot{y}(t) = -L^c x(t) + u^c(t), \tag{7}$$

where $L^p = \text{diag}\{\sum_{j\in\mathcal{N}_1^p} a_{1j}^p, \sum_{j\in\mathcal{N}_2^p} a_{2j}^p, \ldots, \sum_{j\in\mathcal{N}_n^p} a_{nj}^p\} - \mathscr{A}^p$ and $L^c = \text{diag}\{\sum_{j\in\mathcal{N}_1^c} a_{1j}^c, \sum_{j\in\mathcal{N}_2^c} a_{2j}^c, \ldots, \sum_{j\in\mathcal{N}_m^c} a_{mj}^c\} - \mathscr{A}^c$ are symmetric Laplacian matrix associated with $\mathscr{G}^p$ and $\mathscr{G}^c$, respectively.

For distributed dynamic consensus protocols (3) and (4), we design dynamic weights in the form of transfer functions as

$$g_{ij}^{cp}(s) = \frac{a_{ij}^{cp}}{s + k_i^p}, i \in \mathscr{I}_n, j \in \mathscr{I}_m; g_{ij}^{pc}(s) = \frac{a_{ij}^{pc}}{s + k_i^c}, i \in \mathscr{I}_m, j \in \mathscr{I}_n, \tag{8}$$

where $k_i^p > 0$, $a_{ij}^{cp} > 0$ if v_i can get dynamic information from c_j and $a_{ij}^{cp} = 0$ otherwise, $k_i^c > 0$, and $a_{ij}^{pc} > 0$ if c_i can get dynamic information from v_j and $a_{ij}^{pc} = 0$ otherwise.

Remark 1 Since the interactions between agents are undirected, it can easily be seen that the dynamic weights $g_{ij}^{cp}(s) = g_{ji}^{pc}(s)$ if there exist dynamic interactions between v_i and c_j, which implies that $k_i^p = k_j^c$, $a_{ij}^{cp} = a_{ji}^{pc} > 0, \forall i \in \mathscr{I}_n, j \in \mathscr{I}_m$.

With dynamic weights in (8), the control inputs $u_i^p(t)$ and $u_i^c(t)$ take the form of

$$\dot{u}_i^p(t) = -k_i^p u_i^p(t) + \sum_{j\in\mathcal{N}_i^{cp}} a_{ij}^{cp} \left[y_j(t) - x_i(t)\right], i \in \mathscr{I}_n \tag{9}$$

$$\dot{u}_i^c(t) = -k_i^c u_i^c(t) + \sum_{j\in\mathcal{N}_i^{pc}} a_{ij}^{pc} \left[x_j(t) - y_i(t)\right], i \in \mathscr{I}_m \tag{10}$$

which can be rewritten as

$$\dot{u}^p(t) = -K^p u^p(t) - D^{cp} x(t) + \mathscr{A}^{cp} y(t) \tag{11}$$

$$\dot{u}^c(t) = -K^c u^c(t) - D^{pc} y(t) + \mathscr{A}^{pc} x(t), \tag{12}$$

where $K^p = \text{diag}\{k_1^p, k_2^p, \ldots, k_n^p\}$, $D^{cp} = \text{diag}\{\sum_{j\in\mathcal{N}_1^{cp}} a_{1j}^{cp}, \sum_{j\in\mathcal{N}_2^{cp}} a_{2j}^{cp}, \ldots, \sum_{j\in\mathcal{N}_n^{cp}} a_{nj}^{cp}\}$, $\mathscr{A}^{cp} = \left(a_{ij}^{cp}\right) \in \mathbb{R}^{n\times m}$, $K^c = \text{diag}\{k_1^c, k_2^c, \ldots, k_m^c\}$, $D^{pc} = \text{diag}\{\sum_{j\in\mathcal{N}_1^{pc}} a_{1j}^{pc}, \sum_{j\in\mathcal{N}_2^{pc}} a_{2j}^{pc}, \ldots, \sum_{j\in\mathcal{N}_m^{pc}} a_{mj}^{pc}\}$, and $\mathscr{A}^{pc} = \left(a_{ij}^{pc}\right) \in \mathbb{R}^{m\times n}$. Clearly, we have $\mathscr{A}^{cp} = (\mathscr{A}^{pc})^T$, $D^{cp} 1_n = \mathscr{A}^{cp} 1_m$ and $D^{pc} 1_m = \mathscr{A}^{pc} 1_n$.

3.2 Main Result

Let $z(t) = \left[x^T(t), y^T(t)\right]^T$ and $u(t) = \left[(u^p(t))^T, (u^c(t))^T\right]^T$. By combining (6), (7) and (11), (12), we get

$$\dot{z}(t) = -Lz(t) + u(t) \tag{13}$$

$$\dot{u}(t) = -Ku(t) - Hz(t), \tag{14}$$

where $L = \begin{bmatrix} L^p & 0_{n\times m} \\ 0_{m\times n} & L^c \end{bmatrix}$, $K = \begin{bmatrix} K^p & 0_{n\times m} \\ 0_{m\times n} & K^c \end{bmatrix}$, and $H = \begin{bmatrix} D^{cp} & -\mathscr{A}^{cp} \\ -\mathscr{A}^{pc} & D^{pc} \end{bmatrix}$.

To achieve the primary objective of this paper, we proceed to analyze (13) and (14). Traditional convergence analysis of (13) and (14) generally collapses into checking the Hurwitz property of block matrix $C \triangleq \begin{bmatrix} -L & I \\ -H & -K \end{bmatrix}$. However, it is hard to testify, since the block matrix C does not have nice structure, such as diagonally dominant. We adopt a different analysis approach to addressing this issue, which is motivated by the proof of sufficiency of Theorem 3.1 in [16].

Let $Z(s) = \mathscr{L}[z(t)]$ and $U(s) = \mathscr{L}[u(t)]$ be the Laplace transform of $z(t)$ and $u(t)$, respectively. Taking Laplace transform of (13) and (14) gives that

$$sZ(s) - Z(0) = -LZ(s) + U(s) \tag{15}$$

$$sU(s) = -KU(s) - HZ(s) \tag{16}$$

Substituting (16) into (15) arrives at $Z(s) = \left[sI + L + (sI + K)^{-1}H\right]^{-1} Z(0)$. Now the consensus problem can be transformed into the stability problem of the transfer function matrix $G(s) = \left[sI + L + (sI + K)^{-1}H\right]^{-1}$.

Theorem 1 *For the multi-agent network given by (1) and (2) with undirected graph $\mathscr{G}$, let the control input $U^p(s)$ and $U^c(s)$ be applied with dynamic weights $g_{ij}^{cp}(s)$ and $g_{ij}^{pc}(s)$ satisfying $k_i^p = k_j^c = k, \forall i \in \mathscr{I}_n, j \in \mathscr{I}_m$. Then the multi-agent network achieves average consensus asymptotically if and only if $\mathscr{G}$ is connected.*

The proof of Theorem 1 depends on the Gershgorin's disc theorem [17] and the final value theorem [18], which is omitted here due to the page limitation.

4 Illustrative Simulations

Consider mixed multi-agent networks whose interaction topology among agents is shown in Fig. 1. Note that the undirected graph is connected. Without loss of generality, let the static adjacency weights (solid lines) be taken as 1, and let the dynamic adjacency weights (dashed lines) be taken as $g_{11}^{cp}(s) = g_{11}^{pc}(s) = \frac{1}{s+1}$, $g_{23}^{cp}(s) = g_{32}^{pc}(s) = \frac{3}{s+1}$, $g_{32}^{cp}(s) = g_{23}^{pc}(s) = \frac{2}{s+1}$. The initial states are given by $x(0) = [1, 2, 1.6, 2.5, 2.3, 2.7]^T$ and $y(0) = [1, 2, 3, 2.2]^T$. Simulation results of this mixed multi-agent networks with static and dynamic weights are shown in Fig. 2. It is clear from Fig. 2 that average consensus is achieved for all agents on 2.03. This illustration coincides with the statement of Theorem 1.

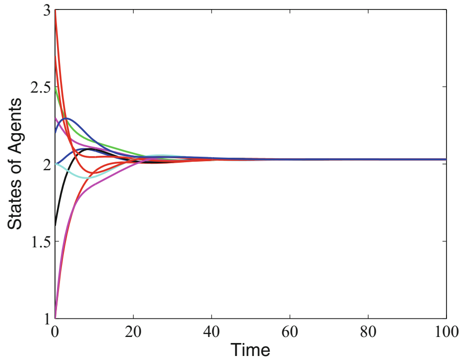

Fig. 2 Average consensus for multi-agent networks with mixed static and dynamic interactions

5 Conclusions

In this paper, average consensus problems on multi-agent networks with static and dynamic interactions have been discussed. We have proposed a new distributed consensus algorithm and have studied under what kind of topology conditions average consensus can be obtained. We adopt analysis approaches both in the time domain and in the frequency domain, which can provide an alternative way to deal with dynamic consensus problems on multi-agent networks. Simulations have been given to validate the effectiveness of our proposed consensus algorithm. Possible future research studies include dealing with multi-agent networks with directed static and dynamic interactions.

Acknowledgments This work was supported in part by the National Basic Research Program of China (973 Program) under Grant 2012CB821200 and Grant 2012CB821201, in part by the National Natural Science Foundation of China (NSFC) under Grant 61473010, Grant 61134005, Grant 61221061, Grant 61327807, Grant 61532006, and Grant 61320106006, in part by the Beijing Natural Science Foundation under Grant 4162036, and in part by the Fundamental Research Funds for the Central Universities.

References

1. Reynolds C (1987) Flocks, herds, and schools: a distributed behavioral model. Comput Graph 21(4):25–34
2. Fax JA, Murray RM (2004) Information flow and cooperative control of vehicle formations. IEEE Trans Autom Control 49(9):1465–1476
3. Ji Z, Hai L, Yu H (2015) Protocols design and uncontrollable topologies construction for multi-agent networks. IEEE Trans Autom Control 60(3):781–786
4. Teixeira A, Sandberg H, Johansson KH (2010) Networked control systems under cyber attacks with applications to power networks. In: Proceedings of the American control conference, pp 3690–3696
5. Olfati-Saber R, Fax A, Murray RM (2007) Consensus and cooperation in networked multi-agent systems. Proc IEEE 95(1):215–233

6. Jadbabaie A, Lin J, Morse AS (2003) Coordination of groups of mobile autonomous agents using nearest neighbor rules. IEEE Trans Autom Control 48(6):998–1001
7. Ren W, Beard RW (2005) Consensus seeking in multiagent systems under dynamically changing interaction topologies. IEEE Trans Autom Control 50(5):655–661
8. Olfati-Saber R, Murray RM (2004) Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans Autom Control 49(9):1520–1533
9. Olfati-Saber R, Murray RM (2003) Consensus protocols for networks of dynamic agents. In: Proceedings of the American control conference, pp 951–956
10. Yucelen T, Egerstedt M (2012) Control of multiagent systems under persistent disturbances. In: Proceedings of the American control conference, pp 5264–5269
11. Ren W, Beard RW (2008) Distributed consensus in multi-vehicle cooperative control: theory and applications. Springer, London
12. Moore K, Vincent T, Lashhab F, Liu C (2011) Dynamic consensus networks with application to the analysis of building thermal processes. In: Proceedings of the IFAC world congress, pp 3078–3083
13. Oh KK, Lashhab F, Moore KL, Vincent TL, Ahn HS (2013) Consensus of positive real systems cascaded with a single integrator. Int J Robust Nonlinear Control 25(3):418–429
14. Wang J, Elia N (2008) Consensus over networks with dynamic channels. In: Proceedings of the American control conference, pp 2637–2642
15. Lashhab F (2012) Dynamic consensus networks: spectral properties, consensus and control. Dissertations and Theses, Colorado School of Mines
16. Cao Y, Ren W, Egerstedt M (2012) Distributed containment control with multiple stationary or dynamic leaders in fixed and switching directed networks. Automatica 48(8):1586–1597
17. Horn RA, Johnson CR (1991) Matrix analysis. Cambridge University Press, Cambridge, UK
18. Chen J, Lundberg KH, Lundberg DE, Bernstein DS (2007) The final value theorem revisited-Infinite limits and irrational functions. IEEE Control Syst Mag 27(3):97–99