Event-Triggered Consensus Control of Nonlinear Multi-agent Systems with External Disturbance

Yang Liu, Jun Gao and Xiaohui Hou

Abstract This paper investigates the consensus problem for a leader-following nonlinear multi-agent system with external disturbance by using the event-triggered control strategy. First, in order to transform the consensus control problem of disturbed system into the H_∞ problem, a controlled output function is defined. Then a distributed event-triggered protocol is designed, and a sufficient condition is given to ensure that the nonlinear multi-agent system can reach consensus with the desired disturbance attenuation ability.

Keywords Consensus · Multi-agent system · Event-triggered control · Nonlinear dynamics · External disturbance

1 Introduction

A multi-agent system consists of multiple independent agents which can act consistently through the transmission among agents. There have been inspiring successes in the areas such as sensor networks, consensus and formation control of vehicles, and cooperative control of robots by using decentralized strategy.

In fact, real multi-agent systems are often equipped with microprocessors that gather information and actuate the agent controller updates with limited on-board

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Y. Jia et al. (eds.), *Proceedings of 2016 Chinese Intelligent Systems Conference*, Lecture Notes in Electrical Engineering 405,
DOI 10.1007/978-981-10-2335-4_43

energy and resources. This brings some problems such as the waste of communication channel and the request for high processing speed of the microprocessors. In view of this, the event-triggered control has been proposed in consensus control of multi-agent system during recent years [1–13]. The event-triggered control indicates that an event condition, which is usually related to the states of the system, triggers the control task execution. Centralized or decentralized consensus protocol based on event-triggered control method is proposed in [2]. In [3], Meng and Chen designed a new event detector to accomplish event-triggered consensus control of multi-agent systems. Meanwhile, event-triggered consensus problem of discrete multi-agent systems with time delay is studied in [4–6]. Obviously, the event-triggered strategy can reduce the resource occupation in multi-agent systems with minimal loss of system behavior. However, the nonlinear system is less investigated in the recent event-triggered consensus research, and the effect of external disturbances is not taken into account.

Motivated by the above work, we consider the event-triggered consensus problem of a disturbed nonlinear multi-agent system. In the following section, a new output $z(t)$ is defined to transform the consensus problem of states into the convergence analysis of $z(t)$ to zero, and then the consensus study of disturbed system is reformulated as the H_∞ control problem of a nonlinear system. To proceed, we design a distributed consensus controller with a new event-triggered condition. Then, sufficient condition is given to ensure that the closed-loop multi-agent system reaches the consensus result with a desired H_∞ performance level.

2 Problem Statements

Consider a leader-following multi-agent system consisting of n following agents and one leader. The ith following agent is modeled by the nonlinear dynamics with unknown external disturbance

$$\dot{x}_i(t) = f(x_i(t), t) + u_i(t) + \omega_i(t) \tag{1}$$

where $t \in [0, \infty)$ is the time variable, $x_i(t)$, $u_i(t)$, $\omega_i(t) \in R^m$ are, respectively, the state, the input, and the external disturbance of agent i, and $f: R^m \times R^+ \to R^m$ is a smooth nonlinear vector-valued function. It is assumed that $\omega_i(t) \in L_2[0, \infty)$, where $L_2[0, \infty)$ is the space of square-integrable vector functions over $[0, \infty)$. This indicates that the energy-limited external disturbance is considered in this paper.

The dynamics of leader is given by

$$\dot{x}_0(t) = f(x_0(t), t) \tag{2}$$

with $x_0(t)$ denoting the state of leader. It is worth pointing out that the nonlinear dynamics is allowed to be unknown, which would not appear in the consensus controller design.

Assumption 1 It is assumed that $f: R^m \times R^+ \to R^m$ satisfies the following Lipschitz condition

$$||f(x, t) - f(y, t)|| \leq \beta ||x - y||, \quad \forall x, y \in R^m, t \geq 0, \tag{3}$$

where $\beta > 0$ is a constant scalar.

Definition 1 The multi-agent system (1) is said to asymptotically achieve consensus if all the following agents' states satisfy

$$\lim_{t \to \infty} x_i(t) = x_0(t), \quad i = 1, 2, \ldots, n \tag{4}$$

Directed (or undirected) graphs are used to model the interaction topologies of n following systems (1). Let $G = (V, E, A)$ be a weighted directed graph of order n, where the set of nodes is described by V, the set of directed edges is $E \subseteq V \times V$, and a weighted adjacency matrix $A = [a_{ij}]$ is defined with nonnegative adjacency weights a_{ij}. It is stipulated that the adjacency weights associated with edges are positive, i.e., $(v_i, v_j) \in E \Leftrightarrow a_{ij} > 0$. In particular, node v_i represents the ith agent, and edge (v_i, v_j) represents that information is transferred from the jth agent to the ith one. The Laplacian of a weighted graph G is defined as $L = D - A$, where $D = \text{diag}\{d_1, d_2, \ldots, d_n\}$ is the degree matrix of G, whose diagonal element is $d_i = \sum_{j=1}^{n} a_{ij}$. Further, if a graph has the property that $a_{ij} = a_{ji}$ always holds, then it is called undirected.

Considering the leading role of leader (2), another node v_0 is added to represent the given leader. Then the edge (v_i, v_0) represents that agent i can obtain the leader's state information, and the corresponding weight a_{i0} is positive. To summarize, the set of neighbors of agent i is denoted by $N_i = \{v_j \in \{V \cup v_0\}: (v_i, v_j) \in E\}$. Define $H = L + \Lambda$ as the interaction matrix of the whole leader-following system (1)–(2), where $\Lambda = \text{diag}\{a_{i0}\}_{i=1}^{n}$.

Lemma 1 *Consider a network with undirected communication among the followers. If at least one agent in each connected component of G is connected to the leader, then the symmetric interaction matrix H is positive definite. Otherwise, H has at least one zero eigenvalue.*

Lemma 2 *If the interaction graph of the leader–follower system with directed communication has a spanning tree with the leader as root, then the negative interaction matrix H is Hurwitz stable. Otherwise, H will have at least one zero eigenvalue.*

3 Protocol Design and Consensus Analysis

3.1 Problem Reformulation

In order to quantitatively measure the effect of external disturbance to the consensus performance, we define the following controlled output functions

$$z_i(t) = x_i(t) - \frac{1}{n}\sum_{j=1}^{n} x_j(t), \quad i = 1, 2, \ldots, n,$$

whose compact form is

$$z(t) = (L_c \otimes I_m)x(t), \tag{5}$$

where $z(t) = [z_1^T(t)\, z_2^T(t) \ldots z_n^T(t)]^T$, $x(t) = [x_1^T(t)\, x_2^T(t) \ldots x_n^T(t)]^T$, L_c is a symmetric matrix with diagonal elements (n − 1)/n, and all the other elements −1/n. Meanwhile, the norm of $z(t)$ can reflect the degree of state deviations. Therefore, combing system (1) with the controlled output (5), we transform the consensus control of disturbed system (1) into the H_∞ control problem of the following system:

$$\begin{cases} \dot{x}(t) = F(x(t)) + u(t) + \omega(t) \\ z(t) = (L_c \otimes I_m)x(t) \end{cases} \tag{6}$$

where

$$\begin{aligned} F(x(t)) &= [f^T(x_1(t)) \quad f^T(x_2(t)) \ldots f^T(x_n(t))]^T, \\ u(t) &= [u_1^T(t) \quad u_2^T(t) \ldots u_n^T(t)]^T, \\ \omega(t) &= [\omega_1^T(t) \quad \omega_2^T(t) \ldots \omega_n^T(t)]^T. \end{aligned}$$

Therefore, the present objective is to design an event-triggered protocol such that

$$||T_{z\omega}(s)||_\infty = \sup_{\mu \in R}\bar{\sigma}(T_{z\omega}(j\mu)) = \sup_{0 \neq \omega(t) \in L_2[0,\infty)} \frac{||z(t)||_2}{||\omega(t)||_2} < \gamma, \tag{7}$$

or equivalently, the closed-loop system satisfies

$$\int_0^\infty ||z(t)||^2 dt < \gamma^2 \int_0^\infty ||\omega(t)||^2 dt, \quad \forall \omega(t) \in L_2[0, \infty), \tag{8}$$

where $\gamma > 0$ is a given H_∞ performance index.

3.2 Event-Triggered Protocol Design

Denote $t_0^i, t_1^i, t_2^i, \ldots$ as the event-triggered time of agent i, where $t_0^i = 0$ for $\forall i$. Then we propose the following distributed event-triggered consensus protocol:

$$u_i(t) = \sum_{j \in N_i} a_{ij}(x_i(t_k^i) - x_j(t_{k_j(t)}^j)) + a_{i0}(x_i(t_k^i) - x_0(t)), \quad t \in [t_k^i, t_k^{i+1}), \tag{9}$$

where $a_{ij} > 0$ is the interaction strength between agents i and j; $a_{i0} > 0$ if and only if the ith agent can obtain the state information of the given leader, or else $a_{i0} = 0$; $k_j(t) = \arg \min_{l \in N: t \geq t_l^j} \{t - t_l^j\}$. It is obvious that $t_{k_j(t)}^j$ is the last event-triggered time of agent j.

Meanwhile, define

$$e_i(t) = x_i(t_k^i) - x_i(t), \quad t \in [t_k^i, t_{k+1}^i)$$

as the measurement error of the ith agent, arising from the event-triggered controller update. The following distributed event-triggered condition is designed to determine the discrete event-triggered time instants

$$||e_i(t)|| = ||\sum_{j \in N_i} a_{ij}(x_i(t) - x_j(t)) + a_{i0}(x_i(t) - x_0(t))||, \tag{10}$$

from which it is immediately derived that

$$||e_i(t)|| \leq ||\sum_{j \in N_i} a_{ij}(x_i(t) - x_j(t)) + a_{i0}(x_i(t) - x_0(t))||, \quad \forall t \in [0, \infty). \tag{11}$$

3.3 Consensus Conditions

To analyze the consensus performance of the closed-loop system, we first define

$$\bar{x}_i(t) = x_i(t) - x_0(t), \quad i = 1, 2, \ldots, n \tag{12}$$

as the disagreement state vector. Obviously, the strict consensus result (4) is realized if and only if

$$\lim_{t \to \infty} \bar{x}_i(t) = 0, \quad i = 1, 2, \ldots, n.$$

Accordingly, the nonzero consensus trajectory is transformed to the origin of $\bar{x}(t) = [\bar{x}_1^T(t) \quad \bar{x}_2^T(t) \cdots \bar{x}_n^T(t)]^T$. Note that the compact form of (12) is

$$\bar{x}(t) = x(t) - 1_n \otimes x_0(t). \tag{13}$$

To proceed, we focus on the derivation of the dynamic equation of $\bar{x}(t)$, by combining the controlled plant and the event-triggered consensus protocol.

First, we have

$$\begin{aligned}\dot{\bar{x}}_i(t) &= \dot{x}_i(t) - \dot{x}_0(t)\\ &= f(x_i(t)) + u_i(t) + \omega_i(t) - f(x_0(t)).\end{aligned} \tag{14}$$

Let $F(x_0(t)) = 1_n \otimes f(x_0(t))$ and $\bar{x}(t) = [\bar{x}_1^T(t) \quad \bar{x}_2^T(t) \cdots \bar{x}_n^T(t)]^T$. Then, from (14) and (9), the closed-loop system in terms of variable $\bar{x}(t)$ is calculated as

$$\begin{aligned}\dot{\bar{x}}(t) &= F(x(t)) - F(x_0(t)) + (L \otimes I_m)(e(t) + x(t)) + (\Lambda \otimes I_m)(e(t) + x(t) + 1_n \otimes x_0(t)) + \omega(t)\\ &= F(x(t)) - F(x_0(t)) + (L \otimes I_m)(e(t) + \bar{x}(t) - 1_n \otimes x_0(t)) + (\Lambda \otimes I_m)(e(t) + \bar{x}(t)) + \omega(t)\\ &= F(x(t)) - F(x_0(t)) + (L \otimes I_m)(e(t) + \bar{x}(t)) + (\Lambda \otimes I_m)(e(t) + \bar{x}(t)) + \omega(t)\\ &= F(x(t)) - F(x_0(t)) + [(L + \Lambda) \otimes I_m](e(t) + \bar{x}(t)) + \omega(t)\\ &= F(x(t)) - F(x_0(t)) + (H \otimes I_m)(e(t) + \bar{x}(t)) + \omega(t),\end{aligned}$$

where the property $L1_n = 0$ is used in the third step.

Furthermore, by (5), it is derived that

$$\begin{aligned}z(t) &= (L_c \otimes I_m)x(t)\\ &= (L_c \otimes I_m)(\bar{x}(t) + 1_n \otimes x_0(t))\\ &= (L_c \otimes I_m)\bar{x}(t),\end{aligned} \tag{15}$$

where the fact $L_c 1_n = 0$ is also applied. To sum up, the original consensus study is reformulated as a standard H_∞ control problem

$$\begin{cases}\dot{\bar{x}}(t) = F(x(t)) - F(x_0(t)) + (H \otimes I_m)(e(t) + \bar{x}(t)) + \omega(t)\\ \qquad = F(x(t)) - F(x_0(t)) + B(e(t) + \bar{x}(t)) + \omega(t)\\ z(t) = (L_c \otimes I_m)\bar{x}(t)\\ \qquad = C\bar{x}(t)\end{cases}. \tag{16}$$

Theorem 1 *Under the event-triggered protocol (9)–(10), the disturbed multi-agent system (1) can achieve consensus performance with the desired H_∞ disturbance attenuation ability γ, if there exists a positive definite matrix $P \in R^{nm \times nm}$ and positive scalars α, δ satisfying the following linear matrix inequality*

$$\begin{bmatrix} PB + B^TP + C^TC + \alpha H^TH + 2\delta\beta I & PB & P & 0\\ B^TP & -\alpha I & 0 & 0\\ P & 0 & -\gamma^2 I & 0\\ 0 & 0 & 0 & P - \delta I\end{bmatrix} < 0, \tag{17}$$

where positive constants β, γ are, respectively, the threshold in the event-triggered condition (10) and the given H_∞ index.

Proof First, we investigate the stability property of the closed-loop system, by analyzing system (16) without external disturbance. Define a Lyapunov function

$$V(t) = \bar{x}^T(t)P\bar{x}(t),$$

then its derivative along system (16) with $\omega(t) = 0$ is

$$\begin{aligned} \dot{V}(t) &= 2\bar{x}^T(t)P[F(x(t)) - F(x_0(t))] + \bar{x}^T(t)(PB + B^TP)\bar{x}(t) + 2\bar{x}^T(t)PBe(t) \\ &\le 2\bar{x}^T(t)P[F(x(t)) - F(x_0(t))] + \bar{x}^T(t)(PB + B^TP)\bar{x}(t) + \alpha^{-1}\bar{x}^T(t)PBB^TP\bar{x}(t) \\ &+ \alpha e^T(t)e(t) \end{aligned} \tag{18}$$

Furthermore, it is obtained that

$$\begin{aligned} &2\bar{x}^T(t)P[F(x(t)) - F(x_0(t))] \\ &\quad \le 2\delta \textstyle\sum_{i=1}^n \bar{x}_i^T(t)[f(x_i(t)) - f(x_0(t))] \\ &\quad \le 2\delta \textstyle\sum_{i=1}^n ||\bar{x}_i(t)|| \cdot ||f(x_i(t)) - f(x_0(t))|| \\ &\quad \le 2\delta \textstyle\sum_{i=1}^n ||\bar{x}_i(t)|| \cdot \beta||x_i(t) - x_0(t)|| \\ &\quad = 2\delta\beta \textstyle\sum_{i=1}^n ||\bar{x}_i(t)||^2 \\ &\quad = 2\delta\beta\bar{x}^T(t)\bar{x}(t), \end{aligned} \tag{19}$$

where $\delta > 0$ is a scalar satisfying the matrix inequality

$$P < \delta I. \tag{20}$$

Note that (20) is included in the consensus condition (17). Meanwhile, denote matrix H as the following partitioned form

$$H = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{bmatrix},$$

and from (11), we have

$$e_i^T(t)e_i(t) \le \bar{x}^T(t)H_i^TH_i\bar{x}(t).$$

Then it immediately yields that

$$\sum_{i=1}^{n} e_i^T(t)e_i(t) \leq \sum_{i=1}^{n} \bar{x}^T(t)H_i^T H_i\bar{x}(t),$$

which results in

$$e^T(t)e(t) \leq \bar{x}^T(t)H^T H\bar{x}(t). \tag{21}$$

Substituting (19) and (21) into (18) leads to

$$\begin{aligned}\dot{V}(t) &\leq 2\delta\beta\bar{x}^T(t)\bar{x}(t) + \bar{x}^T(t)(PB + B^T P)\bar{x}(t) + \alpha^{-1}\bar{x}^T(t)PBB^T P\bar{x}(t) + \alpha\bar{x}^T(t)H^T H\bar{x}(t)\\ &= \bar{x}^T(t)(PB + B^T P + \alpha^{-1}PBB^T P + \alpha H^T H + 2\delta\beta I)\bar{x}(t)\\ &< 0\end{aligned}$$

by applying Schur complement lemma to the matrix inequality (17). This means that the closed-loop multi-agent system is asymptotically stable.

Subsequently, the disturbance attenuation performance of system (16) is given by considering the function

$$J_T = \int_0^T ||z(t)||^2 dt - \gamma^2 \int_0^T ||\omega(t)||^2 dt. \tag{22}$$

To be specific, we have the following result under the zero initial condition:

$$\begin{aligned}J_T = &\int_0^T (z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(t))dt - V(T)\\ = &\int_0^T \{\bar{x}^T(t)C^T C\bar{x}(t) - \gamma^2\omega^T(t)\omega(t) + 2\bar{x}^T(t)P[F(x(t)) - F(x_0(t))] + \bar{x}^T(t)(PB\\ &+ B^T P)\bar{x}(t) + 2\bar{x}^T(t)PBe(t) + 2\bar{x}^T(t)P\omega(t)\}dt - V(T)\\ \leq &\int_0^T \{\bar{x}^T(t)C^T C\bar{x}(t) - \gamma^2\omega^T(t)\omega(t) + 2\delta\beta\bar{x}^T(t)\bar{x}(t) + \bar{x}^T(t)(PB + B^T P)\bar{x}(t)\\ &+ 2\bar{x}^T(t)PBe(t) + 2\bar{x}^T(t)P\omega(t) + \alpha e^T(t)e(t) - \alpha e^T(t)e(t)\}dt - V(T)\\ \leq &\int_0^T \{\bar{x}^T(t)C^T C\bar{x}(t) - \gamma^2\omega^T(t)\omega(t) + 2\delta\beta\bar{x}^T(t)\bar{x}(t) + \bar{x}^T(t)(PB + B^T P)\bar{x}(t)\\ &+ 2\bar{x}^T(t)PBe(t) + 2\bar{x}^T(t)P\omega(t) + \alpha\bar{x}^T(t)H^T H\bar{x}(t) - \alpha e^T(t)e(t)\}dt - V(T)\\ \leq &\ \bar{x}^T(t)\ e^T(t)\ \omega^T(t)]\begin{bmatrix} PB + B^T P + C^T C + \alpha H^T H + 2\delta\beta I & PB & P \\ B^T P & -\alpha I & 0 \\ P & 0 & -\gamma^2 I \end{bmatrix}\\ &\cdot \begin{bmatrix} \bar{x}(t) \\ e(t) \\ \omega(t) \end{bmatrix} - V(T)\\ < &\ 0\end{aligned}$$

where the last step is obtained by using Schur complement lemma to inequality (17). Let $t \to \infty$ in the above result, we have

$$||z(t)||_2^2 < \gamma^2 ||\omega(t)||_2^2,$$

and equivalently, $||T_{z\omega}(s)||_\infty < \gamma$ holds. To summarize, we have proved that if the linear matrix inequality (17) is feasible, then the closed-loop multi-agent system can achieve consensus performance with H_∞ disturbance attenuation level γ. □

4 Conclusions

This paper addresses the event-triggered consensus problem of a nonlinear leader-following multi-agent system with external disturbances. By using the event-triggered control strategy, we have reduced the occupation of system resource and the frequency of the information update. Future work will focus on the leaderless nonlinear multi-agent system with external disturbances.

Acknowledgments This work was supported by the National Basic Research Program of China (973 Prgram, 2012CB821200, 2012CB821201), the NSFC (61473015, 61134005, 61327807, 61520106010).

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