

# A Fatigue-Crack Propagation Analysis Program and Its Application

Yongjun Gao

**Abstract** The theoretical models, calculation algorithms, and an application example of LBB-FATIGUE program are described. Based on the crack propagation analysis method of linear elastic fracture mechanics (LEFM), LBB-FATIGUE program is developed to calculate the propagation length versus the design-transient loads and operational time as well as the lifetime of a fatigue crack in austenite stainless steel pipes within the primary coolant environment of PWR or BWR plants or in the air ( $t > 100$  °C). The main calculation models are as follows: (1) The influence of the primary coolant environment to the fatigue-crack growth rate(FCGR) is considered; (2) according to the superposition principle, the variation range of the total stress intensity factor  $K_I$  is expressed as the sum of tension subpart  $K_I^t$  and bending subpart  $K_I^b$ ; (3) F functions given by Sander's analysis for pure-tension load and pure-bending load are adopted; (4) the design-transient loads are a series of variable amplitude loads which are considered as an equal amplitude load alone, and expressed as variation range of tension force and bending moment; (5) Newton method or the chasing method is used to calculate the variation of the crack propagation length versus design-transient cyclic numbers. In the leak-before-break (LBB) analysis for the primary circuit of CPR1000 nuclear power plant (NPP), the propagation length and the lifetime of a postulated circumferential through-wall crack (TWC) in the sensitive fractural position under design-transient loads are calculated by LBB-FATIGUE program.

**Keywords** LBB-Fatigue Program • Fatigue-Crack Propagation Analysis • LBB Analysis

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Y. Gao (✉)

Nuclear Safety Center, Suzhou Nuclear Power Institute,  
Xihuan Road, No. 1788, Suzhou, Jiangsu Province, China  
e-mail: gaoyongjun@cgnpc.com.cn

## 1 Introduction

In the LBB analysis for the primary circuit of NPP, it is necessary to demonstrate that the operators have adequate time to take protective measures before the fast catastrophic fracture (the instable propagation) takes place in the primary pipes. In order to meet these needs, LBB-FATIGUE program has been developed to estimate the crack subcritical propagation lifetime and the crack length versus the design-transient loads and operational time.

The crack propagation can be divided into the instable propagation and the subcritical propagation [1]. The instable propagation implies that the structure damage will occur if the crack propagates a step further. The subcritical growth rate is relevant to the propagation mechanisms, the load magnitudes, etc. According to the load types and the environmental media, the subcritical propagation mechanisms include the creep, the fatigue, the stress corrosion, and corrosion fatigue [2, 3].

Because the maximum design temperature of the primary circuit of NPP is lower than the creep temperature of the austenite stainless steel (about 450 °C), there is no need to consider the creep failure mechanism in the LBB analysis of the primary circuit. In the meanwhile, the operational limits (such as the limits for the pH value,  $\text{Cl}^-$  and  $\text{SO}_4^{2-}$  concentration) of the primary circuit can ensure that the probability of pipe failure caused by the stress corrosion and the corrosion fatigue is extremely low [2–4]. So, the fatigue is considered as the sole mechanism of the crack subcritical propagation in the LBB analysis of the primary circuit.

The methods for estimating the crack fatigue lifetime include the crack propagation analysis method of LEFM, S–N nominal stress method, and  $\epsilon$ –N local strain method [6, 7]. The crack propagation analysis method of LEFM is based on the classic Paris law to estimate the propagation lifetime of the fatigue crack. At present, the curves of the crack growth rate of a great deal of engineering materials and the solutions of the stress intensity factor of each type of structures with cracks can be obtained, so the crack propagation analysis method of LEFM has been widely applied in the engineering projects.

However, there are four key issues shall be solved in the application of the crack propagation analysis method of LEFM for analyzing the propagation of the fatigue crack in the primary circuit of NPP: (1) How to consider the influences of the coolant environment conditions within the primary circuit to the FCGR? (2) How to calculate the stress intensity factor under the complex tension and bending loads? (3) How to consider the decomposition of the complicated design-transient loads? (4) How to design an effective algorithm for solving the differential equation? In this chapter and Chapter “[A Study on the Tensile Properties of Materials at Elevated Temperature in RCC-M](#)”, the theoretical models and calculation algorithms of LBB-FATIGUE program are described. In Chapter “[Analysis Method of the Temperature for the HeavyReflector](#)”, a calculation example of LBB-FATIGUE program for LBB analysis of primary circuit of CPR1000 NPP is given.

## 2 Theoretical Models

LBB-FATIGUE program adopts the crack propagation analysis method of LEFM to calculate the propagation length versus the design-transient loads and operational time as well as the lifetime of a fatigue crack in austenite stainless steel pipes within the primary coolant environment of PWR or BWR plant or in the air ( $t > 100$  °C). The calculation models of LBB-FATIGUE program are described in this chapter.

### 2.1 Symbol Table

1.  $K_I$ : the total type I stress intensity factor,  $\text{MPa}\sqrt{\text{m}}$
2.  $K_I^t$  and  $K_I^b$ : the type I tension stress intensity factor and the type I bending stress intensity factor,  $\text{MPa}\sqrt{\text{m}}$
3.  $K_I^{\text{max}}$  and  $K_I^{\text{min}}$ : the maximum and minimum values of the type I stress intensity factor in a load cycle,  $\text{MPa}\sqrt{\text{m}}$
4.  $\Delta K_I$ : the variation range of the total type I stress intensity factor in a load cycle,  $\text{MPa}\sqrt{\text{m}}$
5.  $\Delta K_I^t$  and  $\Delta K_I^b$ : the variation range of the type I tension stress intensity factor and the type I bending stress intensity factor in a load cycle,  $\text{MPa}\sqrt{\text{m}}$
6.  $\Delta K_{\text{th}}$ : the threshold value of the variation range of the stress intensity factor,  $\text{MPa}\sqrt{\text{m}}$
7.  $r$  ratio:  $K_I^{\text{min}}/K_I^{\text{max}}$
8.  $F$  and  $M$ : tension force and bending moment,  $N$  and  $N\text{ m}$
9.  $\Delta F$  and  $\Delta M$ : the variation range of the tension force and the bending moment in a load cycle,  $N$  and  $N\text{ m}$
10.  $\sigma_t$  and  $\sigma_b$ : the tension stress and the bending stress,  $N/\text{m}^2$
11.  $\Delta\sigma_t$  and  $\Delta\sigma_b$ : the variation range of the tension stress and the bending stress in a load cycle,  $N/\text{m}^2$
12.  $F_t$  and  $F_b$ : F function for the pure-tension loads and the pure-bending loads
13.  $a$ ,  $a_0$  and  $a_c$ : the half length, the initial half length, and the critical half length of a crack,  $m$
14.  $a_0^i$  and  $a_f^i$ : the crack half length at the beginning and the end of the  $i$ th design transient,  $m$
15.  $R$ ,  $R_o$  and  $R_i$ : the average radius, the outer radius, and the inner radius of the pipe,  $m$
16.  $\theta$ : the half angle of the crack opening
17.  $N$ : the number of the load cycles
18.  $N_c$ : the fatigue-crack propagation lifetime
19.  $N_d^i$ : the cycle number of the  $i$ th design transient
20.  $da/dN$ : FCGR,  $m/\text{cycle}$
21.  $C$  and  $m$ : the parameters in Paris law

22.  $C_0$ : the parameter used to consider the temperature influence
23.  $S$ : the parameter used to consider the influence of  $r$  ratio
24.  $T$ : the temperature, °F
25.  $T_r$ : ascending time of the load in a load cycle, s
26.  $I$ : the inertia moment, kg m<sup>2</sup>

## 2.2 Law of the Fatigue-Crack Propagation

The FCGR is related to the variation range of the stress intensity factor around the crack tip ( $\Delta K_I$ ) [1, 10]. When  $\Delta K_I \leq \Delta K_{th}$ , the crack does not propagate. When  $a \geq a_c$ , the fast instable propagation occurs. When  $\Delta K_I > \Delta K_{th}$  and  $a < a_c$ , the slow stable propagation takes place.

Under the condition of slow stable propagation, the FCGR and  $\Delta K_I$  show linearity relationship in the dual-logarithm coordinates system (i.e., coincide with Paris law [10, 11]):

$$\frac{da}{dN} = C(\Delta K_I)^m \quad (1)$$

here,  $C$  and  $m$  are up to material type and the environment media.

## 2.3 FCGR of the Austenite Stainless Steel Within Primary Coolant Environment of PWR Plant

The fatigue-crack propagation behavior of the austenite stainless steel is influenced by the temperature,  $r$  ratio, and the environment media.

Under the condition of air media, the FCGR of the austenite stainless steel is [11]:

$$\left[ \frac{da}{dN} \right]_{\text{AIR}} = C_{\text{AIR}}(\Delta K_I)^m \quad (2)$$

here, the units of  $da/dN$  and  $\Delta K_I$  are inch/cycle and ksi $\sqrt{\text{in}}$ , respectively. The parameter  $C_{\text{AIR}}$  and  $m$  can be calculated by the following correlations, respectively:

$$C_{\text{AIR}} = C_0 \times S \quad (3)$$

$$m = 3.3 \quad (4)$$

Here,  $C_0$  is used to consider the temperature influence and can be expressed as the function of the temperature.  $S$  is used to consider the influence of  $r$  ratio and can be expressed as the function of  $r$  ratio.

Equation (2) can be rewritten as [12]:

$$\left[\frac{da}{dt}\right]_{\text{AIR}} = C_{\text{AIR}}(\Delta K_I)^m / T_r \quad (5)$$

The following correlation is used to calculate the FCGR of the austenite stainless steel within the primary coolant environment of PWR plants:

$$\left[\frac{da}{dt}\right]_{\text{PWR}} = \left[\frac{da}{dt}\right]_{\text{AIR}} + 1.5 \times 10^{-4} \left[\frac{da}{dt}\right]_{\text{AIR}}^{0.5} \quad (6)$$

Here, the unit of  $da/dt$  is m/s.

## 2.4 FCGR of the Austenite Stainless Steel Within Primary Coolant Environment of BWR Plant

The FCGR of the austenite stainless steel within the BWR coolant environment is [14]:

$$\frac{da}{dN} = 8.17 \times 10^{-12} T_r^{0.5} (\Delta K_I)^{3.0} / (1 - r)^{2.12} \quad (7)$$

here,  $1 \text{ MPa}\sqrt{\text{m}} \leq \Delta K \leq 50 \text{ MPa}\sqrt{\text{m}}$ . If  $T_r < 1 \text{ s}$ , then  $T_r = 1 \text{ s}$ . If it is difficult to give  $T_r$  value, then  $T_r = 1000 \text{ s}$ . If  $r \geq 0$ , then  $\Delta K_I = K_I^{\text{max}} - K_I^{\text{min}}$ . If  $r < 0$ , then  $\Delta K_I = K_I^{\text{max}}$ .

The ratio of the FCGR of the austenite stainless steel in the air, PWR, and BWR coolant environment is 1:2:3.5 for the same crack dimension [12, 13]. So, it is conservative to calculate the FCGR in the PWR coolant environment by using the correlation (7).

## 2.5 FCGR of the Steel in the Air ( $t > 100 \text{ }^\circ\text{C}$ )

The FCGR of the steel (include the stainless steel) in the air ( $t > 100 \text{ }^\circ\text{C}$ ) or in the non-caustic environment can be calculated by the following correlation:

$$\frac{da}{dN} = 5.21 \times 10^{-13} (\Delta K_I)^{3.0} \quad (8)$$

here, the units of  $da/dN$  and  $\Delta K_I$  are mm/cycle and  $N/\text{mm}^{3/2}$ , respectively.

The influences of load ratio  $r$ , the ascending time of the load  $T_r$ , and the environment are not considered in the above correlation. So, if this correlation is used to calculate the growth length of the postulated crack in the PWR primary circuit, the result is not conservative (smaller).

## 2.6 Calculation of the Crack-Opening Half Length

The fatigue-crack propagation lifetime  $N_c$  is the cyclic number propagated from the initial crack half length  $a_0$  to the critical half length  $a_c$ . Under the equal amplitude alternate load,  $N_c$  can be calculated by the following equation:

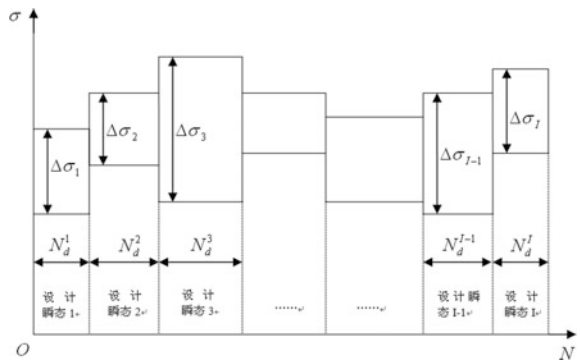
$$N_c = \int_{a_0}^{a_c} \frac{da}{C(\Delta K)^m} \quad (9)$$

The design-transient loads of the pipe are a sequence of the variable amplitude loads, but each type of the design-transient load can be considered as the equal amplitude load shown in Fig. 1).

For the  $i$ th design transient, it assumes that the initial half length of the crack is  $a_0^i$ , and the final half length is  $a_f^i$  after the crack experiences  $N_d^i$  cycles.  $a_f^i$  can be calculated by using the following equation:

$$N_d^i = \int_{a_0^i}^{a_f^i} \frac{da_i}{C(\Delta K_i)^m} \quad (10)$$

**Fig. 1** The stress variation of the design transients



## 2.7 Calculation of the Variation Range of the Stress Intensity Factor

In the elastic range, according to the superposition principle, the variation range of the total stress intensity factor  $\Delta K_I$  can be expressed as the sum of tension subpart and bending subpart:

$$\begin{aligned}\Delta K_I &= \Delta K_I^t + \Delta K_I^b \\ &= \sqrt{\pi R\theta}[\Delta\sigma_t F_t(\theta) + \Delta\sigma_b F_b(\theta)]\end{aligned}\quad (11)$$

## 2.8 Calculation of the Variation Range of the Stress

The variation range of the tension stress and the bending stress can be calculated by the following equations, respectively:

$$\Delta\sigma_t = \frac{\Delta F}{2\pi R t} \quad (12)$$

$$\Delta\sigma_b = \frac{\Delta M R}{I} \quad (13)$$

## 2.9 Calculation of $F$ Function

$F$  functions of the circumferential TWC under the pure-tension load and the pure-bending load given by Sander's analysis [9] are adopted:

$$F_t(\theta) = 1 + A_t \left(\frac{\theta}{\pi}\right)^{1.5} + B_t \left(\frac{\theta}{\pi}\right)^{2.5} + C_t \left(\frac{\theta}{\pi}\right)^{3.5} \quad (14)$$

$$F_b(\theta) = 1 + A_b \left(\frac{\theta}{\pi}\right)^{1.5} + B_b \left(\frac{\theta}{\pi}\right)^{2.5} + C_b \left(\frac{\theta}{\pi}\right)^{3.5} \quad (15)$$

here, the coefficients (i.e.,  $A_t, B_t, C_t, A_b, B_b,$  and  $C_b$ ) in  $F$  functions are the functions of  $R/t$ .

### 3 Calculation Algorithms

The main algorithms and procedures are as follows:

1. Input data such as the pipe and crack dimensions and the loads.
2. Calculate  $\Delta\sigma_t$  and  $\Delta\sigma_b$  by using Eqs. (12) and (13).
3. Calculate F function by using correlation (14) and (15).
4. Calculate  $C$  and  $m$ , and calculate  $\Delta K_I$  by using Eq. (11).
5. Solve the root  $a_f^i$  of equation  $N_d^i - \int_{a_0}^{a_f^i} \frac{da_i}{C(\Delta K_I)^m} = 0$  by using Newton method or the chasing method.
6. Calculate  $\Delta a^i$  and  $\left(\frac{da}{dt}\right)^i$ .
7. Goes to [2] until  $i > I$  ( $I$  is the number of the design transients).

Newton method is a standard method for solving the root of the equation and will not be described in detail.

The procedures of the chasing method are as follows:

1. Initialize the relevant data.
2.  $a = a + \Delta a$ , and calculate  $C$ ,  $m$  and  $\Delta K_I$ .
3.  $NX = NX + \frac{\Delta a}{C(\Delta K_I)^m}$ .
4.  $\varepsilon_{ABS} = ABS(NX - N_d^i)$  and  $\varepsilon_{REL} = \varepsilon_{ABS}/N_d^i$ .
5. If  $\varepsilon_{ABS} \leq \varepsilon_{ABS}^0$  and  $\varepsilon_{REL} \leq \varepsilon_{REL}^0$ , then the iteration stops, otherwise goes to [2].

$\Delta a$  should be small enough to ensure the convergence criteria ( $\varepsilon_{ABS} \leq \varepsilon_{ABS}^0$  or  $\varepsilon_{REL} \leq \varepsilon_{REL}^0$ ) are satisfied.  $\Delta a$  can be automatically regulated by LBB-FATIGUE program according to the convergence status.

### 4 Application Example

In Ref. [16], it has been demonstrated that the leakage of the primary circuit of CPR1000 NPP can be detected by the leakage detective system before the fast catastrophic fracture takes place. According to the LBB criteria, the allowable time for the operators to take relevant measures shall be smaller than the time for propagating to the critical length after the leakage is detected and before the critical length reaches.

The following assumptions are made in the fatigue-crack propagation analysis:

- (a) The initial length of the TWC is the length corresponding to the minimum detectable leakage rate of the leakage detective system (4 gpm, safety factor 10 is considered). When the length of the TWC is smaller than the initial length, the leakage of the TWC is not detected by non-destructive examination (NDE), the leakage detective system or other checking ways (such as the daily round check).



- (b) The variation (transient) of the temperature and the pressure of the primary coolant leads to the stress variation and fatigue propagation of the TWC. The influences of other failure mechanisms (such as stress corrosion crack) are not considered.

#### ***4.1 Initial Dimension of the TWC***

The initial length of the TWC is the length corresponding to the minimum detectable leakage rate of the leakage detective system (0.3169 m).

The allowable maximum length of the TWC is the critical length of the leaked TWC (0.343 m, safety margin 2 is considered).

#### ***4.2 Stress Amplitudes and Cyclic Number***

In Ref. [17], the stress amplitudes and cyclic number of each design transient in susceptible fractural locations are given. By using these parameters, the propagation length of the leaked TWC can be conservatively estimated for the sake of comparing with the critical length of the leaked TWC.

It is not convenient to calculate the stress amplitudes of the design transients because there is a great deal of the relevant data, so it is necessary to make some simplifications. Because the FCGR of the TWC is related to the stress amplitudes, stress cyclic number and the ascending time, the following assumptions and considerations are made in the simplifications:

- (a) The similar design transients are merged, and the stress amplitudes of the post-merged design transients adopt the maximum stress amplitude of the pre-merged design transients.
- (b) It is assumed that the allowable time for the operators to take measures (such as the shutdown and repair) is 7 days after the leakage is detected, and the number of the design transients is conservatively estimated according to this assumption (less than once is regarded as once).
- (c) The ascending time of each design transient adopts the conservative value.
- (d) The design transients causing the crack closure are deleted.

#### ***4.3 Calculation Results and Analyses***

The FCGR and the crack length of the TWC in 7 days are calculated by using LBB-FATIGUE program. The main results are as in Table 1.

**Table 1** Results of fatigue propagation analysis

Transient no	Crack half length (m)	Increment of crack half length (m)	Average FCGR (m/s)	$\Delta K_I$ (MPa $\sqrt{m}$ )	C	m
Initial	0.1585	0.00E+00	0.00E+00	90.10	2.49E-10	3.30
1	0.1586	1.09E-04	2.76E-09	90.20	2.49E-10	3.30
3	0.1588	2.70E-04	3.21E-08	84.70	3.55E-10	3.30
8	0.1652	6.40E-03	2.07E-08	46.30	4.12E-10	3.30
12	0.1652	4.30E-06	4.30E-06	56.30	3.97E-10	3.30
15.2	0.1653	4.83E-05	1.34E-08	96.10	2.58E-10	3.30
25	0.1653	5.51E-05	3.67E-06	130.00	3.24E-10	3.30
59	0.1654	1.69E-05	9.39E-10	41.00	2.01E-10	3.30

It can be seen that the final propagation length of the TWC is 0.3308 m (the half length is 0.1654 m) which is smaller than the critical length of the leaked TWC. So, the operators have adequate time to take relevant measures to deal with the primary circuit leakage accident after the leakage is detected and before the critical length reaches.

## 5 Conclusion and Prospction

The theoretical models and calculation methods of LBB-FATIGUE program are described in this paper. LBB-FATIGUE program adopts the crack propagation analysis method of LEFM to calculate the crack propagation length versus the design-transient loads and operational time as well as the lifetime of a fatigue crack in austenite stainless steel pipes within the primary coolant environment of PWR or BWR plants or in the air ( $t > 100$  °C).

An application example of crack propagation analysis for the primary circuit of CPR1000 NPP shows that there is adequate time for the operators to take protective measures to cope with the primary circuit leakage accident after the leakage is detected and before the critical length reaches (so one of the LBB criteria is met).

LBB-FATIGUE program is a useful tool for crack propagation analysis, especially for time-dependent fatigue-crack analysis problems such as the LBB demonstration.

LBB-FATIGUE program should be verified in detail in the future by comparing the calculation results with those of other similar-function credible programs.

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## Author Biography

**Gao Yongjun** Gao Yongjun (1969–) is a senior engineer of researcher’s grade, graduated from the nuclear reactor engineering specialty of Shanghai Jiaotong University in 1991, engaged in incore fuel management calculation in Research Institute of Nuclear Power Operation (RINPO) in 1991–2005, goes in for the independent verification and review of the reactor core reloading design for the NPPs of CGNPC, nuclear safety research, and safety review of domestic NPPs, etc., in Suzhou Nuclear Power Institute (SNPI) from 2005 till now.