

Stabilization of Inverted Pendulum: A Multirate Output Feedback Based Discrete Time Sliding Mode Control

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Abstract This paper presents the application of multirate output feedback with discrete time sliding mode control (SMC) for controlling the inverted pendulum system. The SMC are well known as completely insensitive to parameter variations and external disturbance. Normally, in order to design controller, the state feedback from the entire state variable is utilized. However not all of the state feedbacks are always available. Therefore, the concept of the multirate output feedback (MROF) in which the output state that are always available at any situation is proposed. The nominal plant model for inverted pendulum system is fourth order with additional of external disturbance. Simulation results verify the proposed controller's performance even though nonlinearity present in the inverted pendulum system.

Keywords Multirate output feedback · Sliding mode control · Inverted pendulum system

1 Introduction

Inverted pendulum system is known as unstable system, highly nonlinear and difficult to control. A lot of investigations have been delivered to control the problem of the inverted pendulum [1, 2]. Sliding Mode Control (SMC) was proposed in the 1960's is a special nonlinear control strategy with discontinuous

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H. Ibrahim et al. (eds.), *9th International Conference on Robotic, Vision, Signal Processing and Power Applications*, Lecture Notes in Electrical Engineering 398,
DOI 10.1007/978-981-10-1721-6_45

property [3]. The special feature of SMC is a switching control law is utilized to drive the state trajectory onto a selected sliding surface in the state space and stay on this sliding surface infinite time. Due to the above property, SMC is attractive to control many highly nonlinear uncertain systems [4–6]. Recently, due to widespread development of computer, the trend of controller has changed from continuous to discrete time as well as sliding mode control. Unlike the continuous time SMC, in discrete sliding mode control (DSMC) the state trajectory cannot stay on sliding surface and cross it in finite time. It will cross the surface again in the following sampling period and produce a zigzag motion called quasi sliding mode band [7]. The DSMC have many advantages compared to continuous SMC such as high accuracy, low power consumption and the others [8].

The state feedbacks are normally used to design control law in SMC. However in practical situation not all of the state always measurable. In this situation, an observer need to be designed where it will provide an estimation of state variable. [9]. However the used of observer makes the system more complex. The Multirate output feedback (MROF) method that only used the output samples to design the controller can be used in place of an observer. The concept of MROF is the control input and output of a system will be sampled at two different sampling rates. The control input is sampled at a slower rate, meanwhile the output is sampled at a faster rate [10]. Several nonlinear plant system such as Hydraulic Manipulator [11], Magnetic Levitation System [12] was controlled by MROF. In this paper, the main objective is to design the multirate output feedback based on discrete SMC and implemented the developed controller to inverted pendulum system to improve the system performance.

2 System Overview

An inverted pendulum system consists of a freely pivoted rod, mounted on a motor driven cart. The aim of this task is to move the cart to a desired position and at same time balancing the inverted pendulum to maintain at up-right position. The continuous time of inverted pendulum system with additional of external disturbance $d(t)$ can be expressed as follow

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + Bd(t) \\ y(t) &= Cx(t)\end{aligned}\tag{1}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{(M_p l_p)^2 g}{P} & -\frac{B_{eq}(I_p + M_p l_p^2)}{P} & -\frac{M_p l_p B_p}{P} \\ 0 & \frac{(M_c + M_p) M_p g l_p}{P} & -\frac{M_p l_p B_{eq}}{P} & -\frac{(M_c + M_p) B_p}{P} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{(I_p + M_p l_p^2)}{P} \\ \frac{M_p l_p}{P} \end{bmatrix} \quad (2)$$

$$P = (M_c + M_p) I_p + M_c M_p l_p^2 \quad x(t) = [x_1 \quad x_2 \quad x_3 \quad x_4] = \begin{bmatrix} x_c & \theta & \dot{x}_c & \dot{\theta} \end{bmatrix}$$

where x_c is the cart position, θ is the pendulum angle, \dot{x}_c is the cart velocity, $\dot{\theta}$ is the pendulum angular velocity, M_p is the pendulum mass, I_p is the pendulum moment of inertia, B_p is the viscous damping coefficient, B_{eq} is the equivalent viscous damping coefficient, l_p is the pendulum length, M_c is the cart mass and g is gravitational constant on earth.

3 Multirate Output Feedback

The Eq. (1) also can be rewrite as

$$\dot{x}(t) = Ax(t) + Bu(t) + Dd(t) \quad (3)$$

The reference tracking, $z(t) = r(t) - y(t)$ is added to the system. Then the continuous time of inverted pendulum system can be represented as

$$\begin{bmatrix} \dot{z}(t) \\ \dot{x}_1(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & a_{11} & \dots & a_{1n} \\ \vdots & \vdots & & \vdots \\ 0 & a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} z(t) \\ x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} 0 \\ b_1 \\ \vdots \\ b_n \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ d_1 \\ \vdots \\ d_n \end{bmatrix} d(t) + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} r(t) \quad (4)$$

The system in Eq. (4) can be represented as follow

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u(t) + D_p d(t) + R_p r(t) \\ y(t) &= C_p x(t) \end{aligned} \quad (5)$$

when the continuous time system in Eq. (5) is sampled at $\tau = 0.004$ s. The discrete time of the system is given as

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k) + Ff(k) + Hr(k) \\ y(k) &= C_p x(k) \end{aligned} \quad (6)$$

where $u(k) \in R$, $\Phi(k) \in R^{n \times n}$ and $\Gamma(k) \in R^{n \times 1}$. Assume that the (Φ, Γ) is controllable and (Φ, C) is observable. The past N multirate-sampled system outputs can be represented as

$$y_k = \begin{bmatrix} y(k-1)\tau \\ y(k-1)\tau + \eta \\ y(k-1)\tau + 2\eta \\ \vdots \\ y(k\tau - \eta) \end{bmatrix} \tag{7}$$

Then, system in Eq. (7) can be represented in faster sampling time $\eta = 0.001$ s as

$$\begin{aligned} x(k+1) &= \Phi x_\eta(k) + \Gamma_\eta u(k) + F_\eta f(k) + H_\eta r(k) \\ y_{k+1} &= C_0 x(k) + C_u u(k) + C_f f(k) + C_r r(k) \end{aligned} \tag{8}$$

where C_0 , C_u , C_f , and C_r , can be expressed as

$$\begin{aligned} C_0 &= \begin{bmatrix} C_p \\ C_p \Phi_\eta \\ C_p \Phi_\eta^2 \\ \vdots \\ C_p \Phi_\eta^{N-1} \end{bmatrix} & C_u &= \begin{bmatrix} 0 \\ C_p \Gamma_\eta \\ C_p (\Phi_\eta \Gamma_\eta + \Gamma_\eta) \\ \vdots \\ C_p \sum_{i=0}^{N-2} \Phi_\eta^i \Gamma_\eta \end{bmatrix} & C_f &= \begin{bmatrix} 0 \\ C_p F_\eta \\ C_p (\Phi_\eta F_\eta + F_\eta) \\ \vdots \\ C_p \sum_{i=0}^{N-2} \Phi_\eta^i F_\eta \end{bmatrix} \\ C_r &= \begin{bmatrix} 0 \\ C_p H_\eta \\ C_p (\Phi_\eta H_\eta + H_\eta) \\ \vdots \\ C_p \sum_{i=0}^{N-2} \Phi_\eta^i H_\eta \end{bmatrix} \end{aligned} \tag{9}$$

Using Eqs. (8) and (9) the following equation is obtained

$$\begin{bmatrix} x(k) \\ f(k) \end{bmatrix} = \Psi (y_{k+1} - C_u u(k) - C_r r(k)) \tag{10}$$

where $\Psi = ([C_0 \ C_f]^T [C_0 \ C_f])^{-1} [C_0 \ C_f]^T$

Substitute Eq. (8) into Eq. (10),

$$\begin{bmatrix} x(k+1) \\ f(k) \end{bmatrix} = \begin{bmatrix} \Phi_\eta & F_\eta \\ 0 & I \end{bmatrix} (\Psi y_{k+1} - \Psi C_u u(k) - \Psi C_r r(k)) + \begin{bmatrix} \Gamma_\eta \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} H_\eta \\ 0 \end{bmatrix} r(k) \tag{11}$$

Then state $x(k+1)$ can be represented

$$x(k+1) = [\Phi_\eta \quad F_\eta] \Psi y_{k+1} + (\Gamma_\eta - [\Phi_\eta \quad F_\eta] \Psi C_u) u(k) + (H_\eta - [\Phi_\eta \quad F_\eta] \Psi C_r) r(k) \quad (12)$$

If $Q = [\Phi_\eta \quad F_\eta]$ then the state $x(k)$ can be expressed using the output y_k as

$$x(k) = Q \Psi y_k + (\Gamma_\eta - Q \Psi C_u) u(k-1) + (H_\eta - Q \Psi C_r) r(k-1) \quad (13)$$

In the case where the linear sliding surface is used, the forward linear surface $\sigma(k+1)$ can be expressed as

$$\sigma(k+1) = G \Phi x(k) + G \Gamma u(k) + G F f(k) + G H r(k) \quad (14)$$

Since the value of $f(k)$ is unknown, it will be replaced by estimated disturbance of $\tilde{f}(k) = f(k-1)$. It can give the control input as

$$u(k) = -(G \Gamma)^{-1} [G \Phi x(k) + G F \tilde{f}(k) + G H r(k)] + M |\sigma(k)| \operatorname{sgn}(\sigma(k)) \quad (15)$$

Then the control input can be expressed in MROF as

$$\begin{aligned} u(k) = & -(G \Gamma)^{-1} [G \Phi \{Q \Psi y_k + (\Gamma_\eta - Q \Psi C_u) u(k-1) + (H_\eta - Q \Psi C_r) r(k-1)\}] \\ & - (G \Gamma)^{-1} [G F \tilde{f}(k) + G H r(k)] + M |\sigma(k)| \operatorname{sgn}(\sigma(k)) \end{aligned} \quad (16)$$

To ensure the system is in the stable condition, the $(\sigma(k+1) - \sigma(k)) \operatorname{sgn}(\sigma(k)) < 0$ and $(\sigma(k+1) + \sigma(k)) \operatorname{sgn}(\sigma(k)) > 0$ must be satisfied. Therefore, the range of M in Eq. (16) must be between $0 < M < 2$.

4 Result and Discussion

The inverted pendulum system is fourth order system. Thus the observability index of the system is 4, hence the value of N is chosen as 4. In this work, the output is sampled at slow rate of $\tau = 0.004$ s. The inverted pendulum system that discretized in fast sampling time with $\eta = 0.001$ s is shown as below

$$\Phi_\eta = \begin{bmatrix} 1 & -0.001 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.0010 & 0 \\ 0 & 0 & 1.000 & -0.0000 & 0.0010 \\ 0 & 0 & 0.0009 & 0.9881 & 0 \\ 0 & 0 & 0.462 & -0.0513 & 0.9995 \end{bmatrix}$$

$$\Gamma_\eta = \begin{bmatrix} 0 \\ 0 \\ 0.0000 \\ 0.0016 \\ 0.0067 \end{bmatrix}, \quad F_\eta = \begin{bmatrix} 0 \\ 0 \\ 0.0000 \\ 0.0016 \\ 0.0067 \end{bmatrix}, \quad H_\eta = \begin{bmatrix} 0.001 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The MROF has been simulated to demonstrate the ability of cart system to move to desired position and at the same time maintain the inverted pendulum in upright position. The external disturbance is set to $d(k) = 0.05 \cos(2\pi kt/\beta)$. After 1 s the cart is force to move 0.02 m from its initial condition by following the desired input which is the unit step input. The switching surface is designed as $G = [-15.6 \quad -89 \quad 130 \quad -48.4564 \quad 20]$. Thus as comparison, the performance of the MROF is compared with the discrete linear quadratic regulator (DLQR). Thus, the designed gain for the discrete linear quadratic regulator controller are $K = [-43.1696 \quad 125.9181 \quad -40.3888 \quad 13.5309]$. Figure 1 shows the simulation result for cart position and angle of inverted pendulum system with MROF and DLQR controllers. Based on Fig. 1a, although all of the controllers are capable to control the cart position, but the output of each controller gave the different characteristic. Table 1 show the comparison data of both controllers. The output of controllers show that the MROF gives the faster settling time and lower overshoot as compared to DLQR controller. Mean while Fig. 2 shows the control input which is the input voltage of the system.

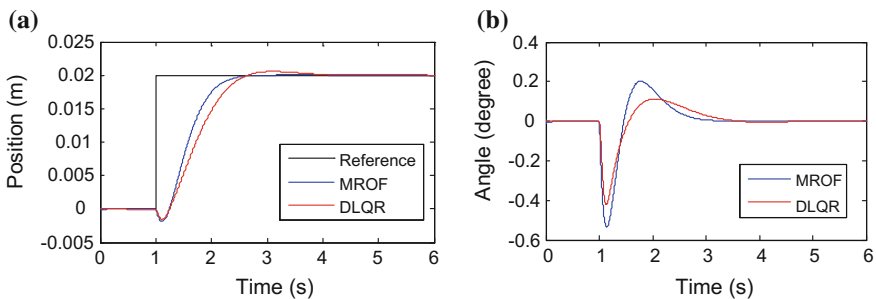
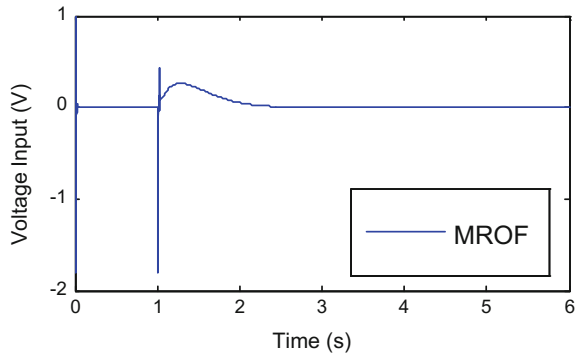


Fig. 1 a Tracking performance of cart position, b Angle of inverted pendulum

Table 1 Performance characteristics of cart position with MROF and DLQR

Characteristic	MROF	DLQR
Rise time (s)	0.96	1.05
Settling Time (s)	2.75	4.02
Percentage of overshoot (%)	0	2.55

Fig. 2 Input voltage



5 Conclusion

In this paper, MROF based DSMC is designed to control inverted system. The MROF is designed using the output feedback and can be used to place an observer. The designed controller is able to deal with existence of external disturbance in the system and guarantee the stability. Only by the usage of output state, the inverted pendulum can maintain at upright, tracked the desired position of cart motor and not depend on the state feedback. This study has shown that developed MROF provide a good pendulum system performance in particular yield solution to nonlinearities problems.

Acknowledgments The authors would like to thank to Ministry of Education (MOE) and University Tun Hussein Onn Malaysia (UTHM) for supporting this research under Research Acculturation Grant Scheme (Vot. no R033) and Short Term Grant (Vot. no 1326).

References

1. Saifizul AA, Zainon Z, Abu Osman NA, Azlan CA, Ungku Ibrahim UFS (2006) Intelligent control for self-erecting inverted pendulum via adaptive neuro-fuzzy inference. *Am J Appl Sci* 3(4):1795–1802
2. Izutsu M, Pan Y, Furuta K (2007) A swing-up/stabilization control by using nonlinear sliding mode for furuta pendulum. In: 16th IEEE international conference on control applications, pp 1191–1196
3. Young KD, Utkin VI, Ozguner U (1999) A control engineer’s guide to sliding mode control. *IEEE Trans. Control Syst Technol* 7(3):328–342

4. Khalid A, Jian X, Yu X (2007) On the discrete-time integral sliding-mode control. *IEEE Trans Autom Control* 52(4):719–725
5. Nai O, Christopher E, Sarah K (2006) Discrete output feedback sliding-mode control with integral action. *Int J Rob Nonlinear*, 21–43
6. Ngadengon R, Sam YM, Osman JHS, Tomari R (2015) Tracking with multirate output feedback (MROF) based discrete sliding mode control. *ARNP J Eng Appl Sci* 10 (20):9902–9907
7. Weibing G, Yufu W, Abdollah H et al (1992) Discrete-time variable structure control systems. *IEEE Trans Ind Electron* 42(2):117–122
8. Bing W (2008) On descritization of sliding mode control systems. Ph.D. Thesis, RMIT
9. Beghi A, Nardo L, Stevanato M (2006) Observer-based discrete-time sliding mode throttle control for drive-by-wire operation of a racing motorcycle engine. *IEEE Trans Act Control Syst Technol* 14(4):767–775
10. Bandyopadhyay B, Janardhanan S (2005) Discrete time sliding mode control: A multirate output feedback approach. Springer, Berlin
11. Waterman D, Nanomi K (2009) Multirate output feedback sliding mode control of a hydraulic manipulator. *ICGST_ARAS J* 9(1):41–49
12. Vitthal S, Pratik N (2010) Design of a discrete-time sliding mode controller for a magnetic levitation system using multirate output feedback. In: American control conference, pp 4289–4294