Stability of Discrete-Time Internal Model Control Against Several Perturbations

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Abstract Internal Model Control (IMC) is a popular control approach that integrates the model of the plant into the controller. In most cases, it is common to have a mismatch between the plant and the model due to noise and disturbance. Actuator constraints may also be another source of instability or performance degradation in IMC. This has lead to the development of IMC structure to sustain its robustness against many different types of uncertainties. This note presents a stability analysis of discrete-time IMC which is subject to saturation and a bounded uncertainty. The stability is guaranteed via one of the discrete counterparts of Popov criterion, namely Jury-Lee criterion (This work was supported by Fundamental Research Grant Scheme (203/PELECT/6071267), Ministry of Education of Malaysia.).

1 Introduction

Internal Model Control (IMC) is an attractive control design strategy for inherently stable plants that utilize the plant model in the formulation of the controller. The advantage of such system was identified and compiled in a series of papers presented by Gracia and Morari [1–3]. While it gives an open-loop framework to check closed-loop stability, it also suffers from performance limitation due to model uncertainties, actuator constraints and non-minimum plant characteristic. IMC structure has received a large amount of criticism due to its weak performance in saturating system. This performance reduction is expected as the original IMC structure was

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never intended to function as an anti-windup scheme. To overcome this issue, several schemes or modifications have been introduced throughout the years [4–6].

In certain applications particularly when the IMC is combined with artificial neural network [7], the transformation of the model into discrete-time setting is required for the analysis of the system. The discrete-time model is also more appropriate when digital controllers are used in the loop. In this note, the focus is on the stability analysis of discrete-time model of IMC which is subject to saturation and a bounded uncertainty. The model is first converted into a Lur'e problem where the stability is guaranteed via one of the discrete counterparts of Popov criterion, namely Jury-Lee criterion. Some examples are included to show that, in the presence of saturation, Jury-Lee criterion provides a higher bound of the uncertainty in which the system remains stable, as compared to the application of the conventional circle criterion.

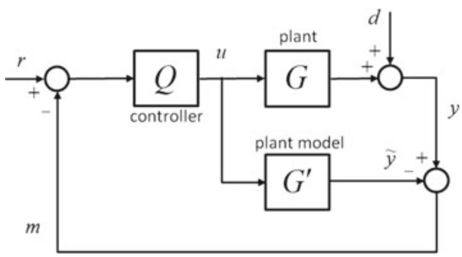
2 IMC Structures

This section presents the basic IMC structure and the structures when it is subject to actuator constraint and a bounded uncertainty which leads to the plant-model mismatch.

2.1 Basic IMC Structure

The basic IMC structure introduced in [1] is shown in Fig. 1 where G, G' and Q denote the plant, model of the plant and the controller respectively. The signal, d refers to the unknown disturbance affecting the system. We can see that if there is no external disturbance or mismatch between the plant and its model, then the system becomes open-loop as there is no feedback, thus eliminating the usual stability problems relating to feedback. However, in practice, it is very unlikely that one can get an exact representation of the plant. Plus, we can never get rid of any disturbances

Fig. 1 Basic structure of IMC



entering the system, and hence there will always be a mismatch between the model and the plant. The design of IMC controller, Q depends on the phase of the plant, G. If G is of minimum phase, it is sufficient to just invert G and augment it with a linear filter that is tuneable for trade-off between robustness and performance. The controller will then become $Q = G^{-1}f$ where f is in the form of $f = \frac{(1-\alpha)z}{(z-\alpha)^n}$ [6, 8]. The filter order n is chosen large enough to make Q proper or strictly proper while α is a tuning parameter which determines the response speed [8]. Increasing α slows down the response speed while decreasing does the opposite. However, if the plant G is of non-minimum phase, the suggested design technique is to factorize the plant as $G = G_+G_-$ where G_+ contains all the non-minimum phase part of G while G_- contains all the minimum phase part of G. The IMC controller is then obtained as $Q = G_-^{-1}$ [8].

2.2 IMC with Saturation

Actuator constraint can be one of the causes of performance degradation which is depicted in Fig. 2a where $\phi(v)$ represents the saturation function with

$$\phi(v) = \begin{cases} 1 & \text{if } v > 1 \\ u & \text{if } -1 \leq v \leq 1 \\ -1 & \text{if } v < -1. \end{cases} \tag{1}$$

Although this structure guarantees closed loop stability in the absence of model mismatch, it suffers from poor nonlinear performance as the controller parameter Q is unaware of its effect on the variable, u as to when and if it has become saturated.

For some cases, a better performance may be obtained when the parameter Q is partitioned in terms of Q_1 and Q_2 such as shown in Fig. 2b. In the absence of saturation (i.e. in the linear case), it simply becomes

$$Q = (I + Q_2)^{-1}Q_1 \tag{2}$$

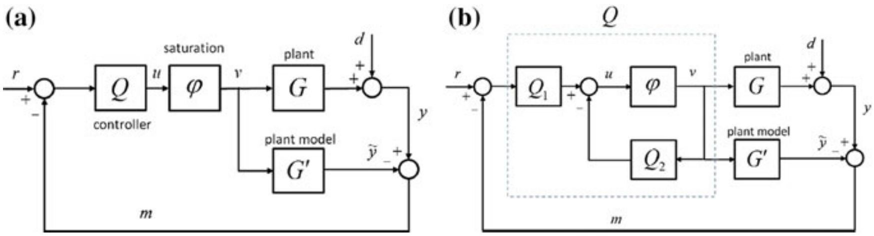


Fig. 2 Development of IMC structure for windup problem. **a** Conventional IMC structure for windup problem. **b** IMC structure with factorization of Q

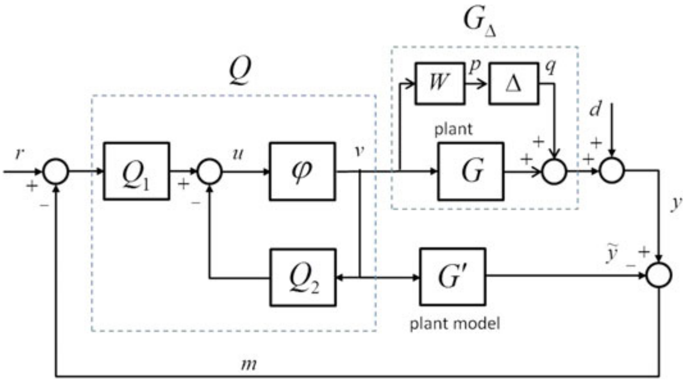


Fig. 3 IMC with saturation and uncertainties

Several methods have been proposed to factorize Q into Q_1 and Q_2 in the literature, and the commonly used ones are as follows:

1. **Method 1** [5]: $Q_1 = Q(\infty)$ and $Q_2 = Q(\infty)Q^{-1} - I$.
2. **Method 2** [4]: $Q_1 = Q$ and $Q_2 = 0$ which reduces to the structure in Fig. 2a.
3. **Method 3** [5]: $Q_1 = \Lambda Q + (1 - \Lambda)Q(\infty)$ with $\Lambda \in [0, 1]$. This reduces to Methods 1 and 2 when $\Lambda = 1$ and $\Lambda = 0$ respectively.
4. **Method 4** [5]: $Q_1 = f_A GQ$ where f_A is a non-causal filter that is chosen based on the criteria that $f_A GQ$ is of minimum phase and $f_A G|_{z=\infty} = I$.

2.3 IMC with Saturation and Uncertainties

Model mismatch in IMC may be due to many sources such as noise and disturbance, particularly during the modeling process. An extension of the anti-windup IMC structure to accommodate the mismatch is shown in Fig. 3 where W, and $\Delta(z)$ represent a known frequency weighting function and an uncertainty with $||\Delta||_\infty \leq \beta$ respectively. In this note, we consider the cases when the system is perturbed by an additive uncertainty (i.e. $W = 1$) and when it is perturbed by an input multiplicative uncertainty (i.e. $W = G$).

3 Stability Analysis

For stability analysis purposes, the IMC structure shown in Fig. 3 is converted into Fig. 4. The corresponding linear block of the system is given by $H(z)$

where

$$H(z) = Q_1(G - G') + Q_2 \sim (A_h, B_h, C_h, 0) \tag{3}$$

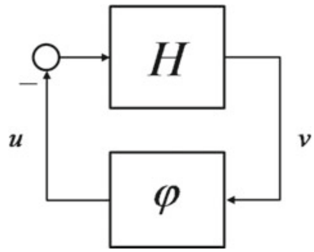


Fig. 4 Equivalent representation of Fig. 3 in Lur'e structure

Note that for additive uncertainty, we get

$$H(z) = Q_1 \varDelta + Q_2, \tag{4}$$

whereas for input multiplicative uncertainty, we get

$$H(z) = Q_1 G \varDelta + Q_2. \tag{5}$$

Since ϕ is static, sector- and slope-bounded such that

$$\phi(0) = 0, \quad \text{and} \quad 0 \leq \frac{\phi(y) - \phi(x)}{y - x} \leq 1; \quad \forall y \neq x \tag{6}$$

the stability of the feedback system can be guaranteed if there exist $R_1, R_2 \geq 0$ such that

$$\text{Re}\,[(1 + R_1(1 - z) + R_2(1 - z^{-1}))(1 + H(z))] > 0 \tag{7}$$

which is derived via one of the Jury-Lee criteria for monotonic slope-restricted non-linearity [9]. Define

$$M_P = \begin{bmatrix} A_h^T P_{11} A_h - P_{11} & -A_h^T P_{11} B_h & A_h^T P_{12} \\ -B_h^T P_{11} A_h & B_h^T P_{12} A_h - P_{22} & -B_h^T P_{12} \\ P_{12}^T A_h & -P_{12}^T B_h & P_{22} \end{bmatrix} \tag{8}$$

$$M_1 = \begin{bmatrix} 0 & 0 & (A_h - I)^T C_h^T R_1^T \\ 0 & -R_1 & -B_h^T C_h^T R_1^T + R_1^T \\ R_1 C_h (A_h - I) & -R_1 C_h B_h + R_1 & -R_1 \end{bmatrix} \tag{9}$$

$$M_2 = \begin{bmatrix} 0 & -(A_h - I)^T C_h^T R_2^T & 0 \\ -R_2 C_h (A_h - I) & -R_2 + R_2 C_h B_h + B_h^T C_h^T R_2 & R_2^T \\ 0 & R_2 & -R_2 \end{bmatrix} \tag{10}$$

$$M_3 = \begin{bmatrix} 0 & 0 & A_h^T C_h^T \\ 0 & 0 & -B_h^T C_h^T \\ C_h A_h & -C_h B_h & -2I \end{bmatrix}. \tag{11}$$

The LMI search for stability analysis of IMC can then be stated as follows:

Proposition 1 *Given the IMC structure as shown in Fig. 3 where G is the discrete-time plant, G' is the model of the plant, W is a known weighting function and Δ is the uncertainty with $||\Delta|| < \beta$. Q_1 and Q_2 are the parameters of the IMC. If there exist P, $R_1 \geq 0$ and $R_2 \geq 0$ such that*

$$M_P + M_1 + M_2 + M_3 < 0 \tag{12}$$

where M_P, M_1, M_2 and M_3 are the matrices as defined in (3), and (8)–(11), then β is the maximum size of the allowable uncertainty for which the system remains stable.

Proof The LMI in (12) can be derived via the application of KYP Lemma [10] on the frequency domain condition in (7).

When R_1 and R_2 are set to zero, then the condition above reduces to the circle criterion. It is also worth noting that the small gain theorem [11] can also provide a stability test where the condition $||H||_\infty ||\phi||_\infty < 1$ must be satisfied. Since $||\phi||_\infty < 1$, the stability of the closed-loop system is guaranteed if and only if $||H||_\infty < 1$. Thus, minimization of $||H||_\infty$ results in the maximization of the size of allowable uncertainty $\Delta(z)$ for which the system remains stable.

4 Applications

Consider a second-order discrete-time plant given by $G(z) = \frac{1}{z^2+0.7z+0.3}$ which is subject to saturation. The parameter Q is constructed via Method 1 above such that

$$Q_1(z) = 0.25; \quad Q_2(z) = \frac{-1.7z - 0.005}{z^2 + 0.7z + 0.3}. \tag{13}$$

The corresponding LTI system for the Lur'e structure is constructed as follows:

$$H_1(z) = 0.25\frac{\beta}{z} - \frac{1.7z + 0.005}{z^2 + 0.7z + 0.3} \tag{14}$$

In this example, the circle criterion and the small gain theorem do not show any stability for any values of β. However, applying the LMI from Proposition 1, we get a maximum β of 3.7307. By simulation, the maximum value before the system goes unstable is 3.8. This is very close to the result obtained from Proposition 1. The step response for this example is shown in Fig. 5 with $\beta = 2$. It is shown that the plant output can track back the input when it is subject to disturbance with different step sizes.

As for the second case where $W = G$, the LTI system is constructed as such that $H_2(z) = 0.25\frac{\beta}{z(z^2+0.7z+0.3)} - \frac{1.7z+0.005}{z^2+0.7z+0.3}$. The maximum value of β obtained from the

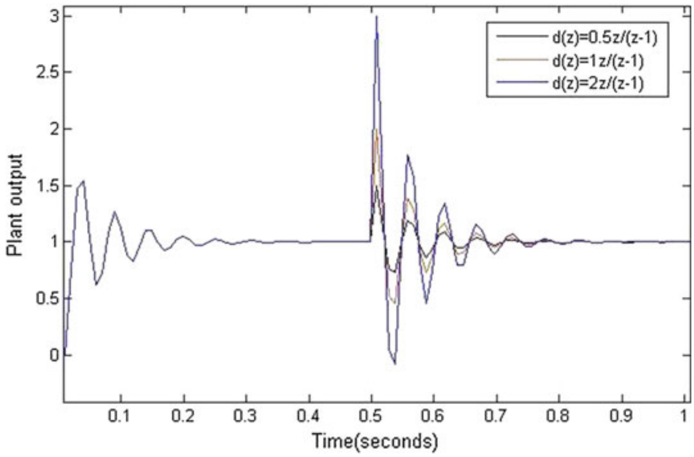


Fig. 5 Step response for the first example when the plant is subject to disturbance with different step sizes

application of Proposition 1 is 1.99. Similar to the previous case, the circle criterion and small gain theorem do not support stability for any values of β.

5 Conclusions

In this note, the stability analysis of discrete-time IMC which is subject to saturation and a bounded uncertainty is presented. The stability condition is derived via the frequency domain Jury-Lee criterion which is then converted into an LMI search. It is also shown from the numerical examples that Jury-Lee criterion can provide stability condition when the circle criterion and small gain theorem fail to do so.

For future work, a more general result can be derived for unstructured uncertainties in the plant model where only their infinity bounds are known. This may be done via integral quadratic constraint method which is a systematic approach to deal with different types of uncertainties including the nonlinearities in the loop.

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