A Non-Walrasian Microeconomic Foundation of the "Profit Principle" of Investment

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Abstract In this chapter, microeconomic foundation of the "profit principle" of investment is discussed from a non-Walrasian/Keynesian perspective. A non-Walrasian "quantity constraint" is introduced in the intertemporal profit maximization problem to consider non-Walrasian/Keynesian excess supply situations. Consequently, we find that it is possible to provide microeconomic foundation for the profit principle in the case of static expectations but it may not in the case of more general types of expectations. We also clarify that Tobin's q can also be defined in non-Walrasian/Keynesian excess supply situations.

Keywords Non-Walrasian analysis · Profit principle · Quantity constraint · Tobin's q

JEL Classification D50 · E12 · E22

1 Introduction

The "profit principle" of investment states that the current investment demand is determined by the current level of income (or the current level of profit) and the current volume of capital stock. Since Kalecki (1935, 1937) and Kaldor (1940) utilized it to formalize the investment function in their business cycle models, it

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has widely been employed as a principle governing corporate fixed investment.[1] For instance, the investment function that obeys the profit principle has been formulated in mathematical refinements and extensions of Kaldor's (1940) business cycle theory (e.g., Chang and Smyth 1971; Varian 1979; Semmler 1986; Asada 1987, 1995; Skott 1989; Murakami 2014, 2015) and in the post-Keynesian theory of economic growth (e.g., Robinson 1962; Malinvaud 1980; Rowthorn 1981; Dutt 1984; Skott 1989; Marglin and Bhaduri 1990; Lavoie 1992). Moreover, it was confirmed by, for instance, Blanchard et al. (1993) and Cummins et al. (2006) that the profit principle fits well with empirical facts. Thus, the profit principle has played an important role in both theoretical and empirical studies.

Unfortunately, however, there have only a few attempts to provide the microeconomic foundation of the profit principle. Grossman (1972) and Skott (1989, Chap. 6) derived the investment function that obeys the profit principle as a solution of the firm's intertemporal profit maximization problem, but they had theoretical flaws in that adjustment costs of installing new capital (e.g., Eisner and Strotz 1963; Lucas 1967; Gould 1968; Treadway 1969; Uzawa 1969) were ignored and that the optimal level of capital and that of investment were not distinguished.[2] Recently, Murakami (2015) was successful in providing a microeconomic foundation of the profit principle in the presence of adjustment costs of investment, but the analysis was confined to the situation where static expectations prevail.

The purpose of this paper is to examine the possibility of microeconomic foundation of the profit principle of investment in general situations. The starting point of our analysis is Murakami's (2015) microeconomic foundation of the profit principle, and we extend the analysis to more general situations. Following Murakami (2015) we introduce not only adjustment costs of investment but also a non-Walrasian "quantity constraint" (e.g., Barro and Grossman 1971; Drèze 1975; Benassy 1975; Grandmont and Laroque 1976; Malinvaud 1977; Hahn 1978; Negishi 1979), which describes the situation where the firm cannot sell all it can produce due to the deficiency of demand for its product, i.e., where Keynes' (1936) principle of effective demand holds true. By so doing, we intend to demonstrate that the profit principle of investment is closely related to non-Walrasian/Keynesian excess supply situations.

This paper is organized as follows. In Sect. 2, we set up a model of optimal decisions on investment and derive the investment function that obeys the profit principle, by following Murakami (2015). In Sect. 3, we attempt to generalize the

[1] The profit principle of investment is often confused with the "acceleration principle" of investment, which was used by Harrod (1936), Samuelson (1939), Hicks (1950) and Goodwin (1951) in their business cycle models, but as Kaldor (1940, p. 79, f. n. 3) pointed out, they are different from each other because the latter asserts that investment demand is determined by the *rate of changes* in income, not by the level of income. In reviewing Hicks (1950), Kaldor (1951, p. 837) also argued that the profit principle is more akin to Keynes' (1936) marginal efficiency theory of investment than the acceleration principle is. Moreover, the acceleration principle is not a theoretical consequence but an empirical law. For these reasons, in this paper, we focus on the microeconomic foundation of the profit principle.

[2] The difference between the concepts of capital and of investment was pointed out by, for example, Lerner (1944) and Haavelmo (1960).

analysis in Sect. 2 by relaxing the assumption of static expectations and verify that the profit principle may not be microeconomically founded without the assumption of static expectations. In Sect. 4, we compare our non-Walrasian microeconomic foundation with several theories of investment: Keynes' (1936) marginal efficiency theory, Tobin's (1969) q theory, the neoclassical optimal capital theory (e.g., Jorgenson 1963, 1965) and the neoclassical adjustment cost theory (e.g., Eisner and Strotz 1963; Lucas 1967; Gould 1968; Treadway 1969; Uzawa 1969). In Sect. 5, we conclude this paper by mentioning the possibility of microeconomic foundation of the "utilization principle" of investment.[3]

2 The Model Under Static Expectations

In this section, we present a model of decisions on investment of a price-taking firm. As specified below, the firm is assumed to incur adjustment costs associated with installing new capital and face with a non-Walrasian "quantity constraint." Since we assume that the firm is a price-taker, we can normalize the price of the firm's product as unity.[4]

[3]We mean by the utilization principle of investment that investment demand is determined by the rate of utilization. Along with the profit principle, use has intensively been made of it in the post-Keynesian analysis (e.g., Steindl 1952, 1979; Rowthorn 1981; Dutt 1984, 2006; Amadeo 1986; Skott 1989; Marglin and Bhaduri 1990; Lavoie 1992; Sasaki 2010; Murakami 2016).

[4]The existence of "quantity constraint" may seem incompatible with the assumption of a price-taking firm. Certainly, as Arrow (1959, pp. 45–47) clarified, if a supplier of a commodity cannot sell all he can produce, i.e., if he faces a quantity constraint, he may act as if he were a monopolist, who takes account of the (perceived) inverse demand function of his product in his decision-making. As Negishi (1979) maintained, however, the assumption of a price-taker can be defended even in non-Walrasian excess supply situations, by introducing the assumption of a kinked demand curve (*á la* Sweezy). Indeed, Negishi (1979) stated as follows:

> More important for oligopolistic price rigidity is, therefore, the fact that, as Sweezy stated, any shift in demand will clearly first make itself felt in a change in the quantity sold at the current price. In other words, a shift in demand changes the position of the starting point P at which the kink occurs to the right or left without affecting the price. If the marginal cost is not increasing rapidly, the equilibrium price remains unchanged while shifts in demand are absorbed by changes in the level of output. (pp. 80–81)
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> Although Arrow did not mention it explicitly, such an imperfectly demand curve must be considered to have a kink at the currently realized point or the starting point in the sense of Sweezy. Firstly, perceived demand curves generally have kinks in a non-Walrasian monetary economy where information is not perfect. (p. 87)
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> When demand falls short of supply, the model of competitive suppliers, is therefore, very much like the Sweezy model of oligopoly, at least in some formal aspects. (p. 88)

If the firm has a perceived demand with kinks due to, for instance, lack of information, as Negishi explained, it is rational for the firm to respond to changes in the demand conditions by adjusting the quantity of its output (which corresponds to the output-capital ratio in our case) rather than by varying the price. In this respect, the assumption of a price-taker is compatible with the existence

First, the production technology of the firm is specified. We assume that the production function of the firm is represented as

$$Y(t) = F(K(t), N(t)). \tag{1}$$

In (1), Y, K, and N stand for the firm's quantity of output, stock of capital and labor employment, respectively.

For the analysis, we make the following standard assumption:

Assumption 1 The real valued function $F : \mathbb{R}^2_{++} \to \mathbb{R}_{++}$ is homogeneous of degree one and twice continuously differentiable with

$$F_N(1, n) > 0,\ F_{NN}(1, n) < 0, \quad \text{for every } n \in \mathbb{R}_{++}, \tag{2}$$

$$\lim_{n\to 0+} F(1, n) = 0,\ \lim_{n\to\infty} F(1, n) = \infty,\ \lim_{n\to\infty} F_N(1, n) = 0,\ \lim_{n\to 0+} F_N(1, n) = \infty. \tag{3}$$

Condition (2) means that the marginal productivity of labor is positive but strictly decreasing, while condition (3) is the so-called Inada condition.

Let $n = N/K$ and define $f(n) = F(1, n)$. Then, Assumption 1 implies that $f : \mathbb{R}_{++} \to \mathbb{R}_{++}$ satisfies

$$f'(n) > 0,\ f''(n) < 0, \quad \text{for every } n \in \mathbb{R}_{++}, \tag{4}$$

$$\lim_{n\to 0+} f(n) = 0,\ \lim_{n\to\infty} f(n) = \infty,\ \lim_{n\to\infty} f'(n) = 0,\ \lim_{n\to 0+} f'(n) = \infty. \tag{5}$$

Second, we introduce the concept of adjustment costs associated with increasing stock of capital (e.g., Eisner and Strotz 1963; Lucas 1967; Gould 1968; Treadway 1969; Uzawa 1969). Following Uzawa (1969) in particular, we assume that the effective cost, including the adjustment cost, of capital accumulation Φ is represented as follows:

$$\Phi(t) = \varphi(z(t))K(t). \tag{6}$$

In (6), z stands for the ratio of gross capital accumulation (including replacement investment) to capital stock. In other words, z satisfies the following equation:

$$\dot{K}(t) = [z(t) - \delta]K(t), \tag{7}$$

where δ is a positive constant which represents the rate of depreciation of capital.

As regards the effective cost function of capital accumulation φ, the following standard assumption is made.[5]

(Footnote 4 continued)
of quantity constraint. Thus, in what follows, it is implicitly assumed that the firm faces a kinked demand curve in the Sweezy–Negishi sense.

[5]Grossman (1972) derived the optimum level of *capital stock* from the profit maximization problem and then formalized investment as a discrepancy between the optimum and current levels of capital

Assumption 2 The real valued function $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable with

$$\varphi(0) = 0, \varphi'(z) > 0, \varphi''(z) > 0, \quad \text{for every } z \in \mathbb{R}_+. \tag{8}$$

Condition (8) states that the effective cost function is strictly convex and means that the effective cost of investment is greater as the rate of capital accumulation increases.

Third, we assume that the firm perceives the upper limit of demand for its product due to the existing deficiency of demand, i.e., that the firm faces some kind of "quantity constraint" (*á la* the non-Walrasian theory). In the analysis below, the "quantity constraint" is specified as follows:

$$f(n(t)) \leq x, \tag{9}$$

where x is a positive constant. In (9), x stands for the ratio of the firm's perceived upper limit of demand (X) to its capital stock K. From (9), x can be viewed as the *perceived maximum average productivity of capital* by the firm. The index of x reflects the firm's expectation on the future demand condition.

The quantity constraint can be written in the form of

$$F(K(t), N(t)) \leq X, \tag{10}$$

where X is a positive constant. In (10), X stands for the perceived upper limit of *level* of demand.[6] Constraint (9) is similar to (10) but differs from it. The former means that the firm anticipates that its productivity of capital cannot exceed the upper limit x, while the latter that the firm expects that its level of production cannot be larger than the upper limit X. In discussing growing (resp. shrinking) economies, constraint (10) is unnatural one because, in growing (resp. shrinking) economies, it is natural that the firm expects that the demand for its product increases (resp. decreases) in accordance with a rise (resp. decline) in the scale of the economy, which is measured by, for instance, the number of population or aggregate capital stock. However, this problem can be avoided if constraint (9) is adopted. Moreover, it seems a natural assumption that in making decisions on investment, the firm cares more about the (maximum) productivity of capital, which measures the profitability of capital[7] than about the level of demand. For these reasons, we adopt (9) instead of (10) as a "quantity constraint." As we will see below, this constraint plays a significant role in the firm's decision-making on investment.

(Footnote 5 continued)
stock, while we directly derive the optimum investment from the profit maximization problem by introducing the concept of adjustment costs. In his approach, the optimum level of capital stock can be rationalized but investment itself cannot.

[6]This constraint was adopted by Grossman (1972).

[7]As we will see in (14), the ratio x is related to the (expected) rate of profit ρ.

In the rest of this section, we consider the optimal capital accumulation in the case where the firm's expectations on the perceived maximum average productivity of capital x, the real wage w and the rate of interest r are all static, i.e., where their values are constant over time.

Since the price of the firm's product is normalized as unity, the firm's optimal investment plan (made at time 0) can be represented as a solution $\{z(t)\}_{t=0}^{\infty}$ of the following problem (Problem (M)):

$$\max_{\{z(t),n(t)\}_{t=0}^{\infty}} \int_0^{\infty} [f(n(t)) - wn(t) - \varphi(z(t))]K(t)e^{-rt}dt \quad \text{(M)}$$
$$\text{s.t.} \ \ (7) \text{ and } (9),$$

where $K(0)$, x, w and r are given positives.

As Murakami (2015) proved, the case in which the quantity constraint (9) binds can be characterized by the situation in which the marginal productivity of labor corresponding to the perceived maximum average productivity of capital x is larger than the real wage w.

Proposition 1 *Let Assumptions 1 and 2 hold. Assume that the following condition is satisfied*[8]*:*

$$f'(f^{-1}(x)) > w, \tag{11}$$

Then, for every solution to Problem (M), $n(t)$ is equal to the positive constant n^* for all $t \geq 0$ such that

$$f(n^*) = x. \tag{12}$$

Proof See Murakami (2015, p. 29, Proposition 2.1). Since this proposition is a corollary to Proposition 3, see also the proof of Proposition 3. □

Proposition 1 states that if the marginal productivity of labor corresponding to the perceived maximum productivity of capital, which is determined by the firm's expectation on future demand conditions, exceeds the (expected) real wage, the firm's optimum production per unit of capital is reduced to the perceived upper limit of demand per unit of capital. In this respect, condition (11) is the one that characterizes Keynes' (1936) principle of effective demand in the long run.[9]

The expected rate of gross profit $\rho = f(n^*) - wn^*$ can be defined as a function of x and w[10]:

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[8]It can be verified that, under (4) and (5) deduced from Assumption 1, the inverse functions $f^{-1}, f'^{-1} : \mathbb{R}_{++} \to \mathbb{R}_{++}$, exist.

[9]The same condition as (11) can be found in Barro and Grossman (1971, p. 85), which characterizes non-Walrasian excess supply situations. However, their analysis was static in nature.

[10]The expected rate of gross profit ρ is, in principle, identical to Keynes' (1936, chap. 11) marginal efficiency of capital. However, as long as condition (11) holds, it is generally not equal to the marginal productivity of capital.

$$\rho(x, w) = \begin{cases} x - wf^{-1}(x) & \text{if condition (11) holds} \\ f(f'^{-1}(w)) - wf'^{-1}(w) & \text{otherwise} \end{cases}. \tag{13}$$

The partial derivatives of ρ are given by:

$$\rho_x = \begin{cases} 1 - w/f'(f^{-1}(x)) > 0 \\ 0 \end{cases}, \ \rho_w = \begin{cases} -f^{-1}(x) < 0 & \text{if condition (11) holds} \\ -f'^{-1}(w) < 0 & \text{otherwise} \end{cases}. \tag{14}$$

It follows from (14) that the real valued function $\rho : \mathbb{R}^2_{++} \to \mathbb{R}_{++}$ is continuously differentiable. Unlike in the neoclassical theory of investment (e.g., Jorgenson 1963, 1965; Lucas 1967; Gould 1968; Treadway 1969; Uzawa 1969), the (expected) rate of gross profit ρ is affected by the perceived maximum average productivity of capital x, i.e., by the firm's expectation on the future demand condition.

Thanks to Proposition 1, the firm's optimal expected profit maximization problem, Problem (M), can be reduced to

$$\max_{\{z(t)\}_{t=0}^{\infty}} \int_0^{\infty} [\rho(x, w) - \varphi(z(t))]K(t)e^{-rt}dt \quad \text{s.t. (7)} \tag{M}$$

In the following, Problem (M), redefined above, is examined.

To solve Problem (M), we impose the following constraint:

$$\int_0^{\infty} [r + \delta - z(t)]dt = \infty. \tag{15}$$

Condition (15) is satisfied if the rate of net capital accumulation $z(t) - \delta$ is less than the rate of interest r. To see what condition (15) implies, suppose, for the time being, that $z(t)$ is a constant z over time. Under this assumption, condition (15) reduces to $z - \delta < r$ and the marginal effect of an increase in z on the discounted present value of the firm's profit at $t = \infty$ is zero because we have

$$\lim_{t \to \infty} \varphi'(z)K(t)e^{-rt} = \lim_{t \to \infty} \varphi'(z)K(0)e^{-(r+\delta-z)t} = 0.$$

In the case where $z(t)$ is constant over time, condition (15) has the same meaning as that of the usual transversality condition.

As Murakami (2015) verified, the solution of Problem (M), $\{z(t)\}_{t=0}^{\infty}$, is constant over time under condition (15).

Proposition 2 *Let Assumptions 1 and 2 hold. Assume that condition (15) holds. Then, if there exists a solution of Problem (M), $z(t)$ is equal to the constant z^* for $t \geq 0$ such that*

$$\frac{\rho(x, w) - \varphi(z^*)}{r + \delta - z^*} = \varphi'(z^*). \tag{16}$$

Proof See Murakami (2015, p. 30, Proposition 2.2). □

Proposition 2 states that, if the firm's expectations on the perceived maximum of productivity of capital (the index of the firm's expectations on demand) x, on the rate of interest r and on the real wage w are static, the optimum rate of capital accumulation z^* is unique and constant. Figure 1 illustrates geometrically how the optimal rate of capital accumulation z^* is determined.

Proposition 2 implies that the optimum rate of capital accumulation z^* can be represented as a function of x, w and r in the following form:

$$z^* = g(x, w, r). \tag{17}$$

Furthermore, it follows from (8), (14)–(16) that the partial derivatives of g are given by

$$\begin{aligned} g_x &= \frac{\rho_x}{\varphi'(z^*)(r+\delta-z^*)} \begin{cases} > 0 & \text{if condition (11) holds} \\ = 0 & \text{otherwise} \end{cases}, \\ g_w &= \frac{\rho_w}{\varphi'(z^*)(r+\delta-z^*)} < 0,\ g_r = -\frac{\varphi''(z^*)}{\varphi'(z^*)(r+\delta-z^*)} < 0. \end{aligned} \tag{18}$$

Condition (18) says that the optimal rate of capital accumulation g is strictly increasing in the index of the expectation on future demand x if the marginal productivity of labor corresponding to x is greater than the given real wage w, while g is inelastic to x otherwise. This implies that investment demand is affected by the firm's expectation on demand provided that the marginal productivity of labor exceeds the real wage. In the sense that the firm's expectation on future demand has an influence on investment demand, the formula given in (16) may be regarded as a natural expression of Keynes' (1936, Chap. 11) theory of investment.

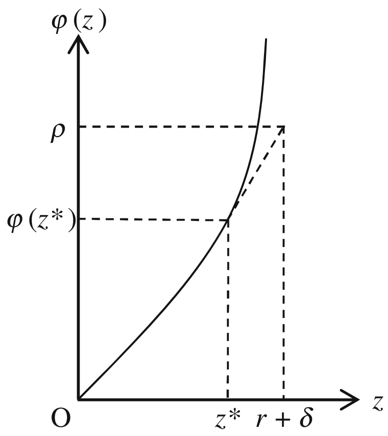

Fig. 1 Optimal rate of capital accumulation

Tobin's (1969) q of investment can also be defined in our context. Since the optimal rate of capital accumulation $z(t)$ is shown to be equal to a unique constant z^* over time, the discounted present value of the firm's expected profit associated with the optimal plan of capital accumulation, or V^*, can be calculated as follows:

$$V^* = K(0)\frac{\rho(x, w) - \varphi(z^*)}{r + \delta - z^*}.$$

Since the replacement cost of capital is $K(0)$ at time 0 because the price is normalized as unity, we know from (16) that Tobin's (1969) q can be defined as follows:

$$q = \frac{V^*}{K(0)} = \frac{\rho(x, w) - \varphi(z^*)}{r + \delta - z^*} = \varphi'(z^*). \tag{19}$$

As the q ratio defined in (19) allows for the case in which the non-Walrasian quantity constraint (9) binds, it may be interpreted as Tobin's q in *non-Walrasian/Keynesian excess supply situations*. This definition of Tobin's q ratio is different from those of Yoshikawa (1980) and Hayashi (1982) in that a non-Walrasian quantity constraint was not taken into account in their definitions and that their definitions dealt only with the case of full employment. Unlike in the interpretations of the q theory by Yoshikawa (1980) and Hayashi (1982), the role of expectation on the future demand condition (represented by x) is taken into account in our interpretation. Since Tobin (1969) defined the q ratio to discuss corporate investment in non-Walrasian/Keynesian excess supply situations, our interpretation of the q theory conforms more to Tobin's (1969) original definition than those in the preceding works.

According to (19), the optimal rate of capital accumulation z^* may also be represented as an increasing function of q. Moreover, if $\varphi'(0) = 1$, which is often assumed as a property of the effective cost function φ, condition (16) implies that when $\rho(x, w) - \delta \leq r$, we have $z^* = 0$. This means that when the (expected) rate of profit (net of the rate of depreciation) is less than or equal to the rate of interest, no new investment is carried out. This result is consistent with Tobin's (1969) q theory of investment and Keynes' (1936) marginal efficiency theory of investment.

In what follows, we proceed to derive the investment function that obeys the profit principle of investment.

Since Proposition 1 implies that the firm's current output is represented as $Y(0) = xK(0)$ under condition (11), non-Walrasian/Keynesian excess supply situations can be characterized by

$$f'\left(f^{-1}\left(\frac{Y(0)}{K(0)}\right)\right) > w. \tag{20}$$

On the other hand, when the quantity constraint (9) does not bind, the optimal level of production is so determined that the marginal productivity of labor would be equal to the real wage. So if condition (20) is not met, the following condition is fulfilled:

$$f'\left(f^{-1}\left(\frac{Y(0)}{K(0)}\right)\right) = w. \tag{21}$$

The situations in which the *notional* labor demand is met are characterized by (21).

Condition (18) can thus be replaced by

$$g_{Y/K} \begin{cases} > 0 & \text{when condition (20) holds} \\ = 0 & \text{when condition (21) holds} \end{cases}, \; g_r < 0, g_w < 0. \tag{22}$$

In particular, condition (22) says that investment demand g is positively influenced by the current (average) productivity of capital $Y(0)/K(0)$ in non-Walrasian excess supply situations (in the case of (20)) but not affected by it in the situations where the notional labor demand is fulfilled (in the case of (21)). The negative effects of the rate of interest and of the real wage on investment demand are also confirmed by (22).

Therefore, the gross capital accumulation function and the investment expenditure function, Z and I, can be defined, respectively, as follows:

$$Z(Y(0), K(0), w, r) = g\left(\frac{Y(0)}{K(0)}, w, r\right)K(0), \tag{23}$$

$$I(Y(0), K(0), w, r) = \varphi\left(g\left(\frac{Y(0)}{K(0)}, w, r\right)\right)K(0). \tag{24}$$

It follows from (22) that the partial derivatives of Z and I are given by

$$Z_Y = g_{Y/K} \begin{cases} > 0 \\ = 0 \end{cases}, \; Z_K = g - g_{Y/K}\frac{Y}{K} \begin{cases} \lesseqgtr 0 & \text{when condition (20) holds} \\ \geq 0 & \text{when condition (21) holds} \end{cases},$$
$$Z_w = g_w K, Z_r = g_r K < 0, \tag{25}$$

$$I_Y = \varphi'(g)g_{Y/K} \begin{cases} > 0 \\ = 0 \end{cases}, \; I_K = \varphi(g) - \varphi'(g)g_{Y/K}\frac{Y}{K} \begin{cases} \lesseqgtr 0 & \text{when condition (20) holds} \\ \geq 0 & \text{when condition (21) holds} \end{cases},$$
$$I_w = \varphi'(g)g_w K < 0, I_r = \varphi'(g)g_r K < 0. \tag{26}$$

The investment functions Z and I ((23) and (24)) can be considered to obey the profit principle of investment, because the current investment demand is a function of the current income $Y(0)$ and the current capital stock $K(0)$. Furthermore, since the (current) level of income $Y(0)$ has a positive impact on the current investment demand only in the case of (20), i.e., only in the case where the equality of the quantity constraint (9) holds, the profit principle of investment is closely related to non-Walrasian/Keynesian excess supply situations.

In this section, we have verified that the profit principle of investment can be rationalized by following Murakami (2015), but we have assumed that the firm's expectations (on x, w and r) are static. In the next section, we shall drop the assumption of static expectations and investigate whether the profit principle can also be rationalized under more general types of expectations.

3 The Model Under General Expectations

In this section, we explore the possibility that the profit principle of investment is microeconomically founded in general expectations.

To allow for the case in which the firm's expectations on the perceived maximum of productivity of capital x, on the real wage w and on the rate of interest r vary with time t, they are represented as functions of $t : x(t), w(t), r(t)$.

Under the general expectations, the quantity constraint (9) is replaced with

$$f(n(t)) \leq x(t). \tag{27}$$

Note that constraint (27) includes (10) as a special case.

For given expectations $\{x(t), w(t), r(t)\}_{t=0}^{\infty}$, the firm's expected profit maximization problem (Problem (G)) can be formalized as follows:

$$\max_{\{z(t),n(t)\}_{t=0}^{\infty}} \int_0^{\infty} [f(n(t)) - w(t)n(t) - \varphi(z(t))]K(t) \exp\Big(-\int_0^t r(s)ds\Big)dt \quad \text{s.t. (7) and (27),} \tag{G}$$

where $K(0) > 0$; $x(t) > 0$, $w(t) > 0$ and $r(t) > 0$ for all $t \geq 0$.

Let $\lambda(t) \geq 0$ be the Lagrange multiplier concerning (27) and set the Hamiltonian as follows:

$$\begin{aligned} H(K(t), n(t), z(t); \lambda(t), \mu(t)) = {} & [f(n(t)) - w(t)n(t) - \varphi(z(t))]K(t) \\ & + \lambda(t)[x(t) - f(n(t))] + \mu(t)[z(t) - \delta]K(t). \end{aligned}$$

Then, we know from the Kuhn–Tucker condition and the maximum principle that, if a solution of Problem (G) $\{z(t), n(t)\}_{t=0}^{\infty}$ exists, it satisfies (7), (27) and

$$\begin{aligned} & \frac{\partial H(t)}{\partial n(t)} = 0, \\ & \frac{\partial H(t)}{\partial z(t)} = 0, \\ & \lambda(t)[x(t) - f(n(t))] = 0, \\ & \dot{\mu}(t) = r(t)\mu(t) - \frac{\partial H(t)}{\partial K(t)}, \end{aligned}$$

or

$$\left[1-\frac{\lambda(t)}{K(t)}\right]f'(n(t))=w(t), \tag{28}$$

$$\dot{z}(t)=\frac{1}{\varphi''(z(t))}\{[r(t)+\delta-z(t)]\varphi'(z(t))-[f(n(t))-w(t)n(t)-\varphi(z(t))]\}, \tag{29}$$

$$\lambda(t)[x(t)-f(n(t))]=0. \tag{30}$$

One can find from (30) that if $\lambda > 0$, the equality of (9) is fulfilled. In other words, if $\lambda > 0$, the firm's output per unit of capital $f(n)$ is restricted to the perceived upper limit of demand per unit of capital x. As in Sect. 2, we can characterize the situation in which the quantity constraint (27) binds.

Proposition 3 *Let Assumptions 1 and 2 hold. Assume that the following condition is satisfied at t:*

$$f'(f^{-1}(x(t))) > w(t). \tag{31}$$

Then, for every solution of Problem (G), $n(t)$ satisfies the following equation at t:

$$f(n(t)) = x(t). \tag{32}$$

Proof Suppose, for the sake of contradiction, that $f(n(t)) < x(t)$ (note that $n(t)$ must satisfy (27)).

By the implicit function theorem, conditions (4) and (5) imply that the inverse function $f^{-1} : \mathbb{R}_{++} \to \mathbb{R}_{++}$ of f exists and satisfies $df^{-1}/dx > 0$. Then, the above assumption implies

$$n(t) < f^{-1}(x(t)).$$

Because of (4), we have

$$f'(n(t)) > f'(f^{-1}(x(t))). \tag{33}$$

Since we have $\lambda(t) = 0$ by condition (30) and the above assumption, conditions (28) and (33) imply

$$w(t) = f'(n(t)) > f'(f^{-1}(x(t))).$$

But this contradicts (31).

Therefore, condition (32) is fulfilled. □

Proposition 3 is a generalized version of Proposition 1 and condition (31) is a generalized version of (11) and characterizes the situation where Keynes' (1936) principle of effective demand holds.

With the help of Proposition 3, Problem (G) can thus be reformulated as follows:

$$\max_{\{z(t)\}_{t=0}^{\infty}} \int_0^{\infty} [\rho(x(t), w(t)) - \varphi(z(t))]K(t) \exp\Bigl(-\int_0^t r(s)ds\Bigr)dt \quad \text{s.t. } (7), \tag{G}$$

where ρ is defined by (13).

Then, the first order conditions for optimality in Problem (G), (28)–(30), can be reduced to

$$\dot{z}(t) = \frac{1}{\varphi''(z(t))}\{[r(t) + \delta - z(t)]\varphi'(z(t)) - [\rho(x(t), w(t)) - \varphi(z(t))]\}. \tag{34}$$

This fact is summarized in the following proposition:

Proposition 4 *Let Assumptions 1 and 2 hold. Then, if there exists a solution of Problem (G),* $\{z(t)\}_{t=0}^{\infty}$*, it satisfies (34).*

As in Sect. 2, the first order condition for optimality, (34), can also be interpreted *à la* Tobin's (1969) q theory. To verify this fact, assume that the following transversality condition holds:

$$\lim_{t\to\infty} \varphi'(z(t))K(t) \exp\Bigl(-\int_0^t r(s)ds\Bigr) = 0. \tag{35}$$

By (7) and (35), we find that a solution of Problem (G), $\{z(t)\}_{t=0}^{\infty}$, satisfies

$$\begin{aligned}
&\int_0^{\infty} \varphi''(z(t))\dot{z}(t)K(t) \exp\Bigl(-\int_0^t r(s)ds\Bigr)dt \\
&\quad = -\varphi(z(0))K(0) + \int_0^{\infty} \varphi'(z(t))[r(t) + \delta - z(t)]\Bigl(-\int_0^t r(s)ds\Bigr)dt.
\end{aligned} \tag{36}$$

Moreover, it follows from (34) that along a solution of Problem (G), $\{z(t)\}_{t=0}^{\infty}$, we have

$$\begin{aligned}
&\int_0^{\infty} \varphi''(z(t))\dot{z}(t)K(t) \exp\Bigl(-\int_0^t r(s)ds\Bigr)dt \\
&\quad = \int_0^{\infty} \{[r(t) + \delta - z(t)]\varphi'(z(t)) - [\rho(x(t), w(t)) \\
&\quad - \varphi(z(t))]\}K(t) \exp\Bigl(-\int_0^t r(s)ds\Bigr)dt.
\end{aligned} \tag{37}$$

Comparing (36) and (37) with each other, we find that a solution of Problem (G), $\{z(t)\}_{t=0}^{\infty}$, satisfies

$$\int_0^{\infty}[\rho(x(t), w(t)) - \varphi(z(t))]K(t)\exp\Big(-\int_0^t r(s)ds\Big)dt = \varphi'(z(0))K(0). \quad (38)$$

Letting V be the left-hand side of (38), we know that a solution of Problem (G), $\{z(t)\}_{t=0}^{\infty}$, fulfills

$$q = \frac{V}{K(0)} = \varphi'(z(0)). \quad (39)$$

The left hand side of (39) is Tobin's (1969) q because V is the discounted present value of the firm's expected profit and $K(0)$ is the replacement cost of capital at time 0. Condition (38) says that the discounted present value of the firm's expected profit is equal to the marginal cost of installing new capital. Since the firm's expectations are general, the q given by (39) can be interpreted as a generalized version of Tobin's q in *non-Walrasian/Keynesian excess supply situations*.

We are now ready to check if the profit principle of investment can be rationalized even under general expectations. In the below, it is demonstrated that, without the assumption of static expectations, the investment function that obeys the profit principle may not be obtained. Since the profit principle states that the current investment demand or the current rate of capital accumulation $z(0)$ is a function of the current income $Y(0) = x(0)K(0)$ and the current stock of capital $K(0) > 0$, it suffices for our purpose to show that, even if the current capital stock $K(0)$ and the current perceived upper limit of productivity of capital $x(0)$ are specified, the current optimal rate of capital accumulation $z(0)$ is not uniquely determined without the assumption of static expectations. To do so, we assume that the real wage $w(t)$ and the rate of interest $r(t)$ are positive constants w and r, respectively, for all $t \geq 0$ and consider the optimal rate of capital accumulation $z(0)$ for the following two time paths of $\{x(t)\}_{t=0}^{\infty}$; the one is $x_0(t) = x$ for all $t \geq 0$; the other is $x_1(t) = x$ for $t \in [0, t_0]$ or $t \geq t_1$ and $x_1(t) > x$ for $t \in (t_0, t_1)$, where x is a positive constant that satisfies (11) and t_0 and t_1 are positive with $t_0 < t_1$. Let $\{z_i(t), K_i(t)\}_{t=0}^{\infty}$ and V_i be the optimal plans of capital accumulation and capital stock corresponding to $\{x_i(t)\}_{t=0}^{\infty}$ and the discounted present value of the firm's expected profit obtained along $\{z_i(t)\}_{t=0}^{\infty}$, respectively, for $i = 0, 1$. Then, we have

$$\begin{aligned} V_0 &= \int_0^{\infty}[\rho(x_0(t), w) - \varphi(z_0(t))]K_0(t)e^{-rt}dt \\ &< \int_0^{\infty}[\rho(x_1(t), w) - \varphi(z_0(t))]K_0(t)e^{-rt}dt \\ &\leq \int_0^{\infty}[\rho(x_1(t), w) - \varphi(z_1(t))]K_1(t)e^{-rt}dt = V_1 \end{aligned}$$

Because the optimal plan of capital accumulation $\{z_i(t)\}_{t=0}^{\infty}$ satisfies (38), we obtain

$$\varphi'(z_0(0))K_0(0) < \varphi'(z_1(0))K_1(0).$$

Since we have $K_0(0) = K_1(0) = K(0) > 0$, we find from (8) that

$$z_0(0) < z_1(0). \tag{40}$$

Noting that $K_0(0) = K_1(0) = K(0)$ and $x_0(0) = x_1(0) = x$, inequality (40) implies that, even when $K(0)$ and $x(0)$ are specified, $z(0)$ may not uniquely be determined. This consequence suggests that the profit principle may not be rationalized without the assumption of static expectations. Therefore, the profit principle of investment may not necessarily be obtained as the optimal plan of capital accumulation under general expectations.

In this section, we have extended the argument in Sect. 2 to the case where more general expectations prevail and checked if the profit principle of investment can be microeconomically founded. We have revealed that the optimal plan of capital accumulation and Tobin's (1969) q can be derived in the existence of non-Walrasian quantity constraint even under general types of expectations but that the profit principle of investment may not be rationalized without static expectations. This clarifies that the assumption of static expectations plays a critical role in the profit principle of investment.

4 Discussion on Non-Walrasian Microeconomic Foundation of Investment

We have so far explored non-Walrasian microeconomic foundation of the investment function. In particular, we have inquired into the possibility of microeconomic foundation of the profit principle of investment. In this section, we turn to the advantage of non-Walrasian microeconomic foundation of investment. For this purpose, we compare our non-Walrasian microeconomic foundation with the other representative theories on investment: the Keynesian theories (Keynes' marginal efficiency theory and Tobin's q theory) and the neoclassical theories (the neoclassical optimal capital theory and the neoclassical adjustment cost theory).

First, we compare our results with Keynes' (1936) marginal efficiency theory of investment. Keynes' (1936, chap. 11) maintained that the optimal investment is subjected largely to changes in the marginal efficiency of investment.[11] Our results are in favor of Keynes' (1936) argument because the (expected) rate of profit $\rho(x, w)$,

[11] Lerner (1944) argued that the term "marginal efficiency of capital" used in Keynes' (1936) *General Theory* should be renamed "marginal efficiency of investment" because the concepts of optimal capital stock and optimal investment are different from each other.

which can be regarded as identical with the marginal efficiency, has a positive effect on investment demand. In this respect, the results of our analysis may be viewed as an appropriate expression of Keynes' theory of investment.

Second, the relationship between Tobin's (1969) q and our results is examined. As we have seen in Sects. 2 and 3 (especially (39)), our results can provide a support for Tobin's q theory. Since Tobin (1969) originally coined the concept of q on the basis of Keynes' (1936) theory, it is a natural consequence that our results also constitute a foundation of Tobin's q theory. Furthermore, the microeconomic foundation of Tobin's q theory derived from our analysis is more comprehensive than those by Yoshikawa (1980) and Hayashi (1982) in that excess supply (underemployment) situations can also be discussed in ours unlike in Yoshikawa (1980) and Hayashi (1982).

Third, our results are contrasted with the neoclassical optimal capital theory of investment (e.g., Jorgenson 1963, 1965). In this theory, the optimum investment is discussed in the framework of dynamic optimization, but this theory has been criticized for its failure to describe investment as an optimum activity because, in this theory, investment is explained as an activity to fill the gap between the optimum and current levels of capital stock.[12] Since we have introduced the concept of adjustment costs, we can escape from this kind of criticism. What is more, the neoclassical optimal capital theory is, in general, an investment theory in full employment situations (neoclassical situations) and does not discuss investment plans in non-Walrasian excess supply situations. In this respect, our analysis has an advantage over this theory.

Fourth, the differences between the neoclassical adjustment cost theory of investment (e.g., Eisner and Strotz 1963; Lucas 1967; Gould 1968; Treadway 1969; Uzawa 1969) are mentioned. As we have seen in Sects. 2 and 3, full employment situations alone are considered in the neoclassical adjustment cost theory, but our results accommodate both full employment and underemployment (non-Walrasian excess supply) situations and so include the results obtained in the neoclassical adjustment cost theory as a special case because we have allowed for the case where the quantity constraint (9) or (27) binds (if this constraint does not bind, the results obtained in our analysis reduce to those obtained in the neoclassical adjustment cost theory).

Thus, our analysis has advantages as a general investment theory in that it incorporates both Keynesian and neoclassical aspects by imposing the quantity constraint (9) or (27) and takes into consideration the difference between capital and investment by introducing the adjustment cost function. In this sense, our analysis can be said to retain generality.

[12]The difference between the concepts of capital and of investment was pointed out by, for example, Lerner (1944) and Haavelmo (1960).

5 Concluding Remarks

We summarize the analysis in this paper.

In Sect. 2, we have formalized a model of optimal investment that emphasizes adjustment costs of investment and non-Walrasian "quantity constraint," by following Murakami (2015). We have found through the analysis in this section that Tobin's (1969) q can be extended to non-Walrasian/Keynesian excess supply situations and that the profit principle of investment can be derived as the intertemporally optimal plan of capital accumulation. However, the argument in this section has been confined to the case of static expectations.

In Sect. 3, we have generalized the argument in Sect. 2 by allowing for more general types of expectations. We have made clear that the formula of optimal capital accumulation and Tobin's q can also be derived even under the assumption of general expectations but that the profit principle of investment may not necessarily be microeconomically founded without the assumption of static expectations. By so doing, we have verified that the assumption of static expectations is vital for the profit principle.

In Sect. 4, our non-Walrasian microeconomic foundation of investment has been compared with other investment theories: Keynes' marginal efficiency theory, Tobin's q theory, the neoclassical optimal capital theory and the neoclassical adjustment cost theory. It has been confirmed that our microeconomic foundation provides natural and appropriate expressions of Keynes' marginal efficiency theory and of Tobin's q theory, includes the neoclassical adjustment cost theory as a special case and is superior to the neoclassical optimal capital theory.

Before concluding this paper, we shall mention the possibility of microeconomic foundation of the utilization principle of investment. In this paper, for the sake of simplicity, we have not explicitly taken account of (variations of) the rate of utilization. So our analysis does not directly contribute to microeconomic foundation of the utilization principle. However, since the rate of utilization u is usually measured by the output-capital ratio Y/K and the results in Sect. 2 indicate that the optimal rate of capital accumulation z^* is influenced positively by the ratio Y/K, our analysis may partially make a contribution to microeconomic foundation of the utilization principle.

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