Pathology in the Market Economy: Self-fulfilling Process to Chronic Slump

Chronic Slump

Akitaka Dohtani

Abstract In this chapter, we construct an extension of Goodwin's nonlinear accelerator model and detect a possible cause that generates a chronic slump. By introducing a nonlinearity expressing a pessimistic outlook for the future economy in our extended model, we demonstrate that a chronic slump cycle arises from the pessimistic outlook through a self-fulfilling prophecy. In the extended model, income on the cycle is locked in a domain lower than the market equilibrium. This implies that private spending in the model economy fluctuates and is continuously insufficient to make use of the available productive capacity that is estimated at the market equilibrium. The periodic attractor gives a partial description of the recent worldwide chronic slump. Our result shows that the extended Goodwin model provides a partial description of the Krugman's view that explains the recent worldwide slump. Moreover, although booms and slumps come in all sizes, our extended model explains how this is possible.

Keywords Chronic slump · Demand side · Self-fulfilling prophecy · Pessimistic outlook · Asymmetric adaptive expectation formation

1 Introduction

Recently, many countries have experienced slumps. The current depression is not as severe as the Great Depression. However, the recent worldwide slump is critical in the sense that it is chronic, and this chronicity implies a difficulty in recovery: the signs of a serious depression have been observed. Which mechanism is responsible

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Dedication: This paper is dedicated to the memory of the late Dr. Tatsuji Owase.

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for generating the recent worldwide chronic slump? Following Krugman's view[1] on the chronic slump, we emphasize the importance of the demand side of the economy. We are interested in the dynamic demand-side model describing Krugman's view. Moreover, the Krugman's view states that the present state of private spending is continuously insufficient to make use of the available productive capacity.[2] Krugman (2008) also asserts that the dynamic notion of the *self-fulfilling prophecy*[3] plays an important role in explaining the chronicity of the recent slump. Regarding the self-fulfilling prophecy, Krugman (2008) focuses on the financial markets. However, we concentrate on the real markets and show that the Krugman's view on the self-fulfilling prophecy holds true for the real markets as well. In a downswing, the self-fulfilling prophecy will often occur in both markets repeatedly. From the perspective of demand-side macroeconomics, we construct a prototype dynamic model expressing a part of the Krugman's view.

The situation that we will describe by the prototype mode is as follows. We will demonstrate that the chronic slump results from a pessimistic outlook on the demand side. We suppose that the Knightian uncertainty[4] arises from a market's loss of confidence, and therefore, the pessimistic outlook spreads.[5] This pessimism often yields the self-fulfilling prophecy. The pessimistic outlook makes the economy inactive. As a result, the inactiveness makes the economic agents believe that the pessimistic outlook is appropriate, and the belief renders the economy even more inactive. This vicious circle (or the self-fulfilling prophecy) continues, triggering an economic avalanche, and a chronic slump emerges. Thus, the pessimistic outlook is a critical barrier to prosperity.

We here make one important remark. The market psychology may change through a "learning", and the pessimistic outlook may change. However, the Knightian uncertainty persists over a long period of time unless the market's loss of confidence is recovered. Consequently, the above vicious circle makes the economic agents believe firmly the validity of pessimistic outlook. Therefore, the pessimistic outlook also persists over a long period of time, and our precondition of argument is robust unless such a loss is recovered.

[1]See Krugman (2008, Chap. 10).

[2]Here, the productive capacity is estimated at the market equilibrium.

[3]For insightful arguments on the self-fulfilling prophesy, see Rosser (1991).

[4]Knightian uncertainty applies to situations where we cannot obtain enough information we need in order to set accurate odds. See Knight (1921). For an important relation between the Knightian uncertainty and market psychology, see also Akerlof and Shiller (2009, Chap. 11).

[5]The role of expectation in business cycles has been discussed by many economists from the Keynesian perspective. See, for example, Matthews (1959, Chap. 3.5). Economists have considered expectation to be fickle, and therefore, the corresponding argument lacks clarity. However, owing to the self-fulfilling prophecy, pessimism becomes inflexible and robust in the long run. Thus, the market psychology of pessimism can be considered as a theoretical subject. Akerlof and Shiller (2009) discuss importance of the market psychology from a much wider viewpoint. By using the Michigan Consumer Sentiment Index, Blanchard (1993) pointed out that the loss of confidence can cause a large economic recession.

We present the analytical details as follows. The prototype model constructed in this paper is based on the classical nonlinear-accelerator business cycle model of Goodwin (1951).[6] The Goodwin model is one of well-known demand-side models of the business cycle in the Keynesian tradition.[7] In the model, the sigmoid type of nonlinearity plays the most important role in generating persistent nonlinear fluctuations. Although the Goodwin model has been criticized for the lack of microfoundation, the Keynesian nonlinear business cycle models like Goodwin's nonlinear accelerator model are useful for explaining actual business cycles. We construct an extension of the Goodwin model and detect a possible cause of a chronic slump.[8] Unlike the Goodwin model, we assume that the household distinguishes between short-run and medium-run consumption plans. In the short-run plan, like the Goodwin model, the household determines its consumption depending linearly on its income. On the other hand, in the medium-run plan, the household determines its consumption in proportion to the expected income, which is adjusted by an adaptive expectation rule. We assume that in the case where the actual income is larger than the expected income, the slope of the adjustment function is smaller than that in the converse case. This implies that the household develops a pessimistic outlook for the future economy, and therefore, in the case where the actual income is larger than the expected income, the latter is adjusted merely by a smaller amount than that in the converse case. We show that the introduction of pessimistic adaptive learning into the Goodwin model does generate a chronic slump in which income and expected income are locked in lower domains than the market equilibrium.

All business cycle models in the Keynesian tradition describe a complete recovery from a slump. However, the economic process in an actual chronic slump is not monotonous in the sense that it gradually descends while repeating partial recoveries and slowdowns. In other words, even in the chronic slump, the market economy persistently fluctuates in a low domain of income. To describe this situation, we must construct a nonlinear business cycle model that possesses a periodic path on which income and expected income are lower than those at the market equilibrium. We show that the above extended Goodwin model has such a periodic path. Thus, the extended Goodwin model constructed will be a business cycle model that analytically expresses the above view held by Krugman on the recent chronic slump. Moreover, as stressed in Krugman (1996, p. 68), an important feature of business cycle is that booms and slumps come in all sizes. Slump cycles are a part of such a feature. The extended Goodwin model also gives a theoretical explanation of the feature.

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[6]Many studies have examined the nonlinear dynamics of the original and extended versions. See, for example, Bothwell (1952), Strotz et al. (1953), Gabisch and Lorenz (1987), Krugman (1996), Owase (1991), and Puu (2003), and many papers on the Goodwin model in Puu and Sushko (2006).

[7]kaldor (1940) constructed another well-known and important nonlinear business cycle model in the Keynesian tradition. The mathematical formulation of the model is given by Chang and Smyth (1971). Our argument holds true for the business cycle model.

[8]For another interesting Keynesian approach, see Varian (1976) and George (1981). This approach employs the catastrophe theory. It also provides the important and useful information on a serious depression. For the catastrophe theory, see Rosser (1991, Chap. 6).

Many models with self-fulfilling features have been proposed.[9] A feature common to these models is that there exist multiple equilibria, which comprise higher and lower equilibria. However, we emphasize that the extended Goodwin model is quite different from these models in the sense that it possesses only a unique equilibrium and all its paths converge to a periodic attractor that is locked in a domain lower than the equilibrium point. From the perspective of Keynesian demand-side economics, we present a new kind of model possessing the self-fulfilling feature.

2 Extension of the Goodwin Model

For constructing the extended Goodwin model, the requirements we impose are as follows:

R.1 The business cycle model is a demand-side model.

R.2 There exists a small periodic path on which income and expected income are constantly lower than their levels at the market equilibrium.

R.1 is the first requirement for following the Krugman's view. In a chronic slump, the market economy does not possess the power of automatic recovery. In this sense, the slump is serious. Therefore, as stated in the Introduction, the economy constantly repeats partial recoveries and slowdowns. To describe such a situation, we require R.2.

R.2 may be stronger than needed. However, R.2 is a convenient requirement for clarifying the meaning of "partial recovery." As stated in the Introduction, R.2 also implies that private spending is continuously insufficient to make use of the available productive capacity.[10] Like the Goodwin model, many business cycle models in the Keynesian tradition possess the power of automatic recovery, and therefore, all the paths fluctuate around the equilibrium point. However, R.2 implies that the model does not possess any power of automatic recovery by itself, and therefore, the income and expected income are lower than their levels at the market equilibrium. The purpose of this section is to construct a prototype dynamic macromodel satisfying R.1 and R.2, which provides an extension of Goodwin's nonlinear business cycle model.

Before constructing the extended Goodwin model, we briefly explain the original nonlinear accelerator model[11] of Goodwin. Throughout this paper, we assume that all functions are continuous. Goodwin's original model is given by

$$\dot{y}_t = \mu\{(c_t + \dot{k}_t) - y_t\}, \tag{2.1a}$$

$$c_t = \alpha y_t + c_0, \text{ and} \tag{2.1b}$$

[9]For the models, see, for example, Krugman (1991) and Murphy et al. (1989).

[10]See also Footnote 2.

[11]It is well-known that the multiplier-accelerator principle plays an important role in explaining business cycles. See Blanchard (1981).

$$\dot{k}_{t+\theta} = \phi(\dot{y}_t), \tag{2.1c}$$

where c is consumption, y is national income, k is capital stock, $\alpha \in [0, 1)$ is the marginal propensity to consume, μ is the adjustment coefficient, and c_0 is a positive constant. A dot over a variable indicates a derivative with respect to time. Goodwin's original model is given by differential-difference equations. Following Goodwin (1951), we transform this system into a system of differential equations. Equations (2.1a) and (2.1b) yield

$$(1-\alpha)y_t = \dot{k}_t - (1/\mu)\dot{y}_t + c_0. \tag{2.2}$$

The linear approximation of $\dot{k}_{t+\theta}$ is given by $\dot{k}_{t+\theta} \approx \dot{k}_t + \theta\ddot{k}_t$. As is typical, using the linear approximation, we replace (2.1c) with

$$\theta\ddot{k}_t = \phi(\dot{y}_t) - \dot{k}_t. \tag{2.3}$$

Let us define

$$x_t = \dot{y}_t \quad \text{and} \quad w_t = y_t - c_0/(1-\alpha).$$

Then, $x_t = \dot{w}_t$. Moreover, Eq. (2.1b) yields $x_t = \mu\{\dot{k}_t - (1-\alpha)y_t + c_0\}$. Therefore, Eq. (2.3) yields

$$\dot{x}_t = \mu\{\ddot{k}_t - (1-\alpha)\dot{y}_t\} = \mu[\{\phi(x_t) - \dot{k}_t\}/\theta - (1-\alpha)x_t].$$

Equation (2.2) yields $\dot{k}_t = (1-\alpha)y_t + (1/\mu)\dot{y}_t - c_0 = (1-\alpha)w_t + (1/\mu)x_t$. Thus, we obtain the following two-dimensional system of differential equations:

$$\Theta_G : \begin{cases} \dot{x}_t = \frac{\mu}{\theta}[\phi(x_t) - \{(1/\mu) + (1-\alpha)\theta\}x_t - (1-\alpha)w_t], \\ \dot{w}_t = x_t. \end{cases}$$

We call this system the Goodwin model. Throughout this paper, the $\phi-$function is supposed to satisfy the following assumptions:

Assumption 1 $\phi(0) = 0$.

Assumption 2 The $\phi-$function is continuously differentiable.

Clearly, Assumption 1 shows that (0, 0) is the equilibrium point in the Goodwin model. On the other hand, Assumption 2 guarantees the existence and uniqueness of solutions in the Goodwin model.[12]

[12] See Guckenheimer and Holmes (1983, p. 3).

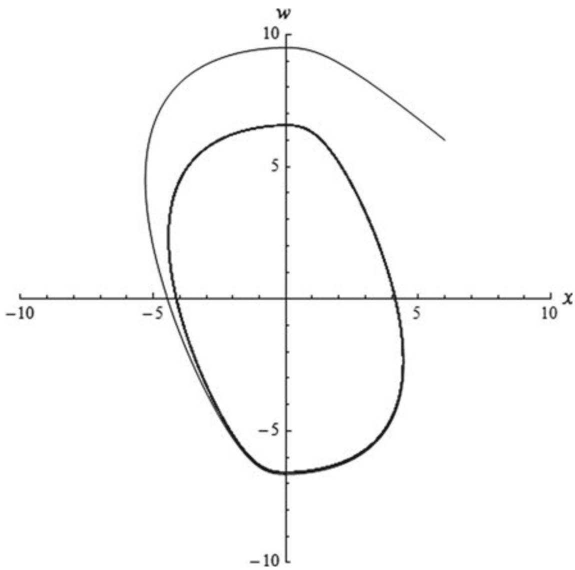


Fig. 1 Typical periodic attractor of the Goodwin model

Goodwin (1951) numerically showed that the $\phi-$function of a sigmoid shape yields a limit cycle. System Θ_G is a system of autonomous differential equations of the Rayleigh type.[13] For the Rayleigh-type equation, many mathematical results exist.[14] Therefore, it is not difficult to prove the existence of a limit cycle in System Θ_G under suitable conditions. Since proving this is not the purpose of the present paper, we merely provide a numerical example, where System Θ_G possesses a limit cycle.

Numerical Example 1 We set $\phi(x) = 2\text{Arctan}(1.5x)$, $\mu = 2$, $\alpha = 0.7$, and $\theta = 1$. Clearly, the $\phi-$function satisfies Assumptions 1 and 2. The $\phi-$function is of a typical sigmoid shape as in Goodwin (1951). Figure 1 shows that System Θ_G possesses a limit cycle. ■

We now extend the Goodwin model. To incorporate the pessimistic outlook for the future economy into the Goodwin model, we consider the long-run consumption plan of the household. Moreover, for this purpose, we replace (2.1b) with

$$c_t = \alpha y_t + \beta y_{et} + c_0, \tag{2.4}$$

where $\beta \in [0, 1]$ and y_e is the expected income. Equation (2.4) states that the consumption plan is decomposed into the short-run plan $(\alpha y_t + c_0)$ and the long-run plan (βy_{et}). Throughout this paper, we assume the following:

[13] System Θ_G can also be transformed into a van der Pol-type equation. See Lorenz (1993, Subsection 5.3.2).

[14] See, for example, Sansone and Conti (1964) and Yanqian (1986).

Assumption 3 $1 > \alpha + \beta$.

As proved later, Assumption 3 is utilized to guarantee the existence and uniqueness of the equilibrium point. The expected income is adjusted by

$$\dot{y}_{et} = \psi(y_t - y_{et}). \tag{2.5}$$

Here, we assume the following:

Assumption 4 $\psi(u)u > 0$ for any $u \neq 0$.

We will later discuss the properties of the $\psi-$function that are closely related to the occurrence of a slump cycle. We call the $\psi-$function the adjustment function and Eq. (2.5) the adjustment equation. Given Assumption 3, we define

$$x_t = \dot{y}_t, \tag{2.6a}$$
$$w_t = y_t - c_0/(1 - \alpha - \beta), \text{ and} \tag{2.6b}$$
$$z_t = y_{et} - c_0/(1 - \alpha - \beta), \tag{2.6c}$$

where w_t denotes the deviation of income from the equilibrium and z_t denotes the deviation of expected income from the equilibrium. Then, $x_t = \dot{w}_t$. In the same way as before, Eq. (2.3) yields

$$\dot{x}_t = \frac{\mu}{\theta}[\phi(x_t) - \{(1/\mu) + (1-\alpha)\theta\}x_t - (1-\alpha)w_t + \beta z_t + \theta\beta\psi(w_t - z_t)].$$

Thus, we obtain the following extension of the Goodwin model:

$$\Theta_{EG}: \begin{cases} \dot{x}_t = \frac{\mu}{\theta}[\phi(x_t) - \{(1/\mu) + (1-\alpha)\theta\}x_t - (1-\alpha)w_t + \beta z_t + \theta\beta\psi(w_t - z_t)], \\ \dot{w}_t = x_t, \\ \dot{z}_t = \psi(w_t - z_t). \end{cases}$$

We call System Θ_{EG} the extended Goodwin model. In the following sections, by introducing a pessimistic outlook into the adjustment function, we consider the dynamic behavior of System Θ_{EG}.

3 Dynamics Resulting from Pessimism

In this section, we demonstrate that the pessimistic outlook held by the household about the future economy causes a chronic slump. The pessimistic outlook is expressed by a nonlinearity incorporated into the adjustment function. Before discussing this, it is convenient to consider the dynamic behavior of System Θ_{EG} with the linear adjustment function. We begin with the verification of simple results on the existence and stability of the equilibrium point. The following lemma is clear.

Lemma 1 *Under Assumptions 1, 3, and 4, System Θ_{EG} possesses a unique equilibrium point (0, 0, 0). In other words, the market equilibrium is uniquely determined and given by* $(y^*, x^*, y_e^*) = (c_0/(1-\alpha-\beta), 0, c_0/(1-\alpha-\beta))$. ■

Proof Direct calculation proves Lemma 1. ■

We now prove the following.

Lemma 2 *We assume that* $0 < \psi'(0)\theta < \mu\phi'(0) - 1 - \mu(1-\alpha)\theta$. *Then, under Assumptions 1–4, the equilibrium point of System Θ_{EG} is unstable.* ■

Proof See Appendix. ■

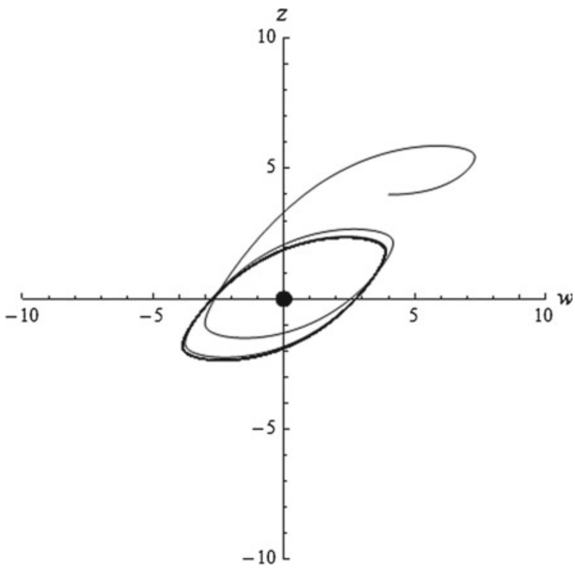
The linear case provides a direct extension of the Goodwin model. In fact, as will be shown in Numerical Example 2, like the Goodwin model, System Θ_{EG} possesses a similar periodic path that surrounds the equilibrium point. The meaning of "similarity" is clarified using the comparison between systems with nonlinear and linear adjustment functions, which will be presented soon.

Numerical Example 2 We consider the linear adjustment function

$$\psi(u) = \eta_L(u; h) = hu\,(h > 0).$$

We set $\alpha = 0.4$, $\mu = 2$, $\theta = 1$, $\beta = 0.46$, $h = 0.45$, and $\phi(x) = 2\text{Arctan}(1.5x)$. It can be easily verified that these parameters satisfy Assumptions 1–4. Figure 2 describes the projection of a typical path of System Θ_{EG} onto the w–z plane, which converges to a periodic path. The black dot emphasizes the equilibrium point in the

Fig. 2 Typical periodic attractor of the extended Goodwin model with a symmetric (linear) adjustment function



$w-z$ plane. The path in Numerical Example 2 represents the usual business cycles in the sense that the path surrounds the equilibrium point. As in Numerical Example 1, the model economy in Numerical Example 2 possesses the power of automatic and complete recovery from the slump, though the recovery is temporal and the economy repeats a pattern of booms and slumps. ■

The nonlinear factor in Numerical Example 2 is merely incorporated into the $\phi-$function. Therefore, the periodic attractor observed in Numerical Example 2 is generated by the sigmoid nonlinearity of the $\phi-$function. System Θ_{EG} with the linear $\psi-$function generates a periodic attractor that is similar to that of the Goodwin model in the sense that the periodic attractor surrounds the equilibrium point.

Next, we incorporate the nonlinearity of our model into the adjustment function. We define

$$\eta_{NL}(u; a_+, a_-) = \begin{cases} a_+ u & u \geq 0, \\ a_- u & u < 0. \end{cases} \tag{3.1}$$

We now make the following assumption:

Assumption 5 $a_- > a_+$.

We use System Θ_{NEG} to denote System Θ_{EG} wherein $\psi(u) = \eta_{NL}(u; a_+, a_-)$ satisfies Assumption 5. Assumption 5 introduces the asymmetric nonlinearity into the adjustment function. As shown later, Assumption 5 is closely related to the emergence of a chronic slump. In this sense, Assumption 5 plays the most important role in our argument. Here, we explain its economic implication. We assume that the representative household is pessimistic about the future economy. We consider the adjustment function under this assumption. When the actual income exceeds the expected income (i.e., $u = y - y_e > 0$), it is expected that the economy will become more prosperous in the future. However, since the household is pessimistic, it does not have hope for further prosperity. Therefore, the upward adjustment of the expected income is excessively small (in other words, the household is hyperopic). Conversely, when the actual income is lower than the expected income (i.e., $u = y - y_e < 0$), it is expected that the economy will worsen even more in the future. Since the household is pessimistic, it will expect further worsening. Therefore, the downward adjustment of the expected income is excessively large (in other words, the household is myopic). Thus, we see that pessimism about the future economy yields the asymmetric nonlinearity of Assumption 5.

We here make one remark. As showed in the Introduction, through the self-fulfilling prophecy, the pessimistic outlook persists over a long period of time unless the market's loss of confidence is recovered. Consequently, the asymmetric nonlinearity persists over a long period of time. Thus, Assumption 5 is robust.

A viewpoint of the behavioral economics about loss aversion is useful in explaining the asymmetric nonlinearity. We here quote the sentences from Kahneman et al. (1991): Responses to increases and to decreases in prices, for example, might not always be mirror images of each other. The possibility of loss-aversion effects suggests, more generally, that treatments of responses to change in economic variables should routinely separate the cases of favorable and unfavorable changes. Our assumption is consistent with the viewpoint of the behavioral economics. We consider the graph of the nonlinear adjustment function $\psi(u) = \eta_{NL}(u; a_+, a_-)$. Changes to the right (resp. left) side of the origin (i.e., the reference point) are favorable (resp. unfavorable) for households. Thus, from the viewpoint of the behavioral economics, we obtain that the adjustment function habitually possesses the (perhaps weak) nonlinearity. In our model, since we assume pessimism about the future economy, the loss aversion will be reinforced and the asymmetric nonlinearity will be stronger.

Before discussing the dynamics of System Θ_{NEG}, we must confirm the existence and uniqueness of solutions. It should be noted here that the $\psi-$function is not differentiable at $u = 0$. However, it can be easily checked that the $\psi-$function satisfies the Lipschitz condition in R^2. Therefore, from Assumptions 2 and 5, the vector field of System Θ_{NEG} also satisfies the Lipschitz condition. This proves the existence and uniqueness of solutions.[15] Thus, under Assumptions 2 and 5, the solutions of System Θ_{NEG} are determined uniquely. The piecewise linear function (3.1) is written in a very simple form to explain the self-fulfilling process to chronic slump. It is not easy to ascertain whether or not the equilibrium point is stable. Therefore, in this paper, we numerically investigate the dynamics of System Θ_{NEG}. Moreover, as shown from the explanation of Assumption 5, we observe that as the a_+-value decreases or the a_--value increases, the pessimism regarding the future economy becomes strong (in other words, the asymmetry of the adjustment function becomes strong).

Clearly, System Θ_{NEG} with an asymmetric adjustment function possesses the mechanism to produce business cycles, much like the Goodwin model. On the other hand, as stated in the Introduction, in System Θ_{NEG}, the household's pessimistic outlook has a direct influence on the adjustment function of expected income, which is closely related to its outlook for the future economy. Assumption 5 describes this influence. Therefore, given the above, the household's consumption decreases through the reduction in its expected income, and the reduction in consumption makes the model economy inactive. Thus, we can expect the resulting economy to become more inactive than in the Goodwin model. To demonstrate that this intuitive observation is correct, we now consider a typical numerical example of System Θ_{NEG}.

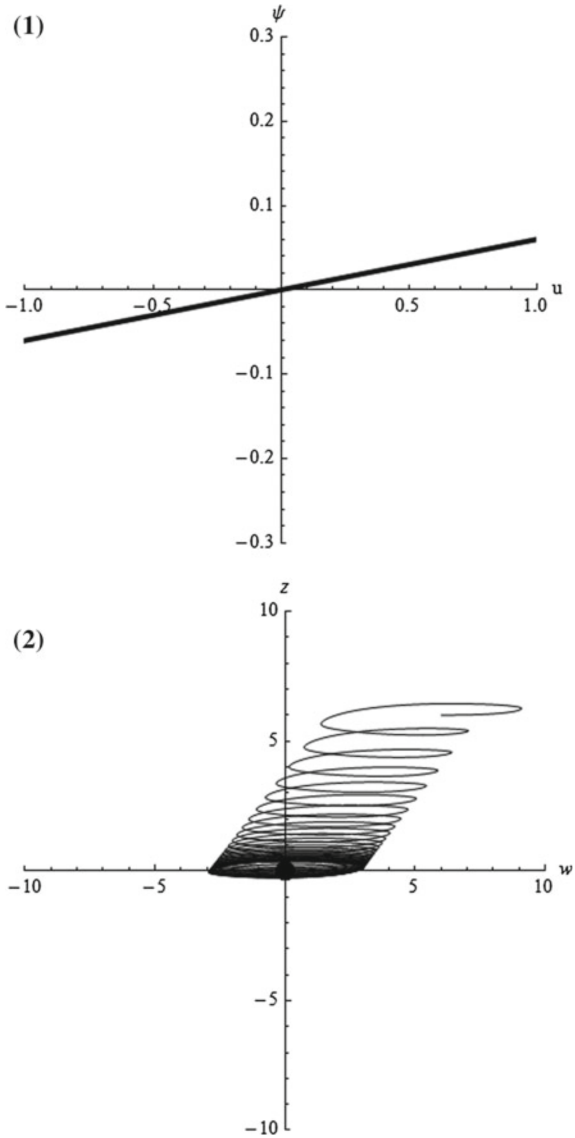
We consider a more pessimistic case than that presented in Numerical Example 2. In Numerical Example 2, we considered the adjustment function $\psi(u) = \eta_L(u; 0.45)$. Therefore, since we consider a more pessimistic case, we set

$$a_+ = 0.1 < 0.45 \quad \text{and} \quad a_- = 0.6 > 0.45. \tag{3.2}$$

We set $\mu = 2$, $\alpha = 0.4$, $\theta = 1$, $\beta = 0.46$, and $\phi(x) = 2\text{Arctan}(1.5x)$. Clearly, these parameters satisfy Assumptions 1–3. It should be noted here that in the symmetric case with $h = a_+ = a_-$, the size of h determines the amplitude of a periodic path that surrounds the equilibrium point. Parts (1) of Figs. 3 and 4 describe the graphs

[15] See Guckenheimer and Holmes (1983, p. 3).

Fig. 3 Periodic attractor of the extended Goodwin model with a small symmetric adjustment coefficient



of the adjustment functions of the systems with $h = a_+ = a_- = 0.1$ and $h = a_+ = a_- = 0.6$, respectively. Parts (2) of Figs. 3 and 4 describe the projections of typical periodic paths of the systems with $h = a_+ = a_- = 0.1$ and $h = a_+ = a_- = 0.6$ onto the $w-z$ plane, respectively. In the asymmetric case with (3.2) (i.e., in the mixture of these two cases), different dynamic behavior occurs. Figure 5 shows it. The thick black line of Part (1) of Fig. 5 describes the piecewise linear graph of the adjustment function of System Θ_{NEG}. The piecewise linear graph is the mixture of the graphs of

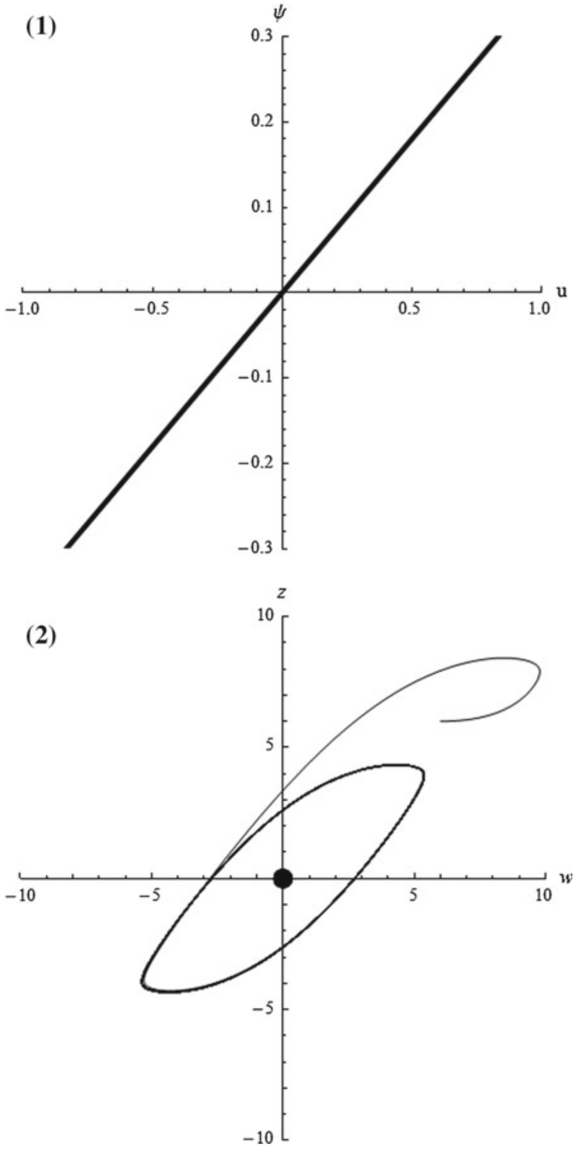


Fig. 4 Periodic attractor of the extended Goodwin model with a large symmetric adjustment coefficient

Parts (1) of Figs. 3 and 4. Part (2) of Fig. 5 describes the projection of a typical path of System Θ_{NEG} onto the $w-z$ plane, which converges to a periodic path. In Parts (2) of Figs. 3, 4 and 5, the black dot emphasizes the equilibrium point in the $w-z$ plane. The black curves in Parts (1) and (2) of Fig. 6 describe the time series of the deviations of income and expected income of the path of Fig. 5. In (1) and (2), thick straight lines emphasize the time series of the equilibrium income and equilibrium expected

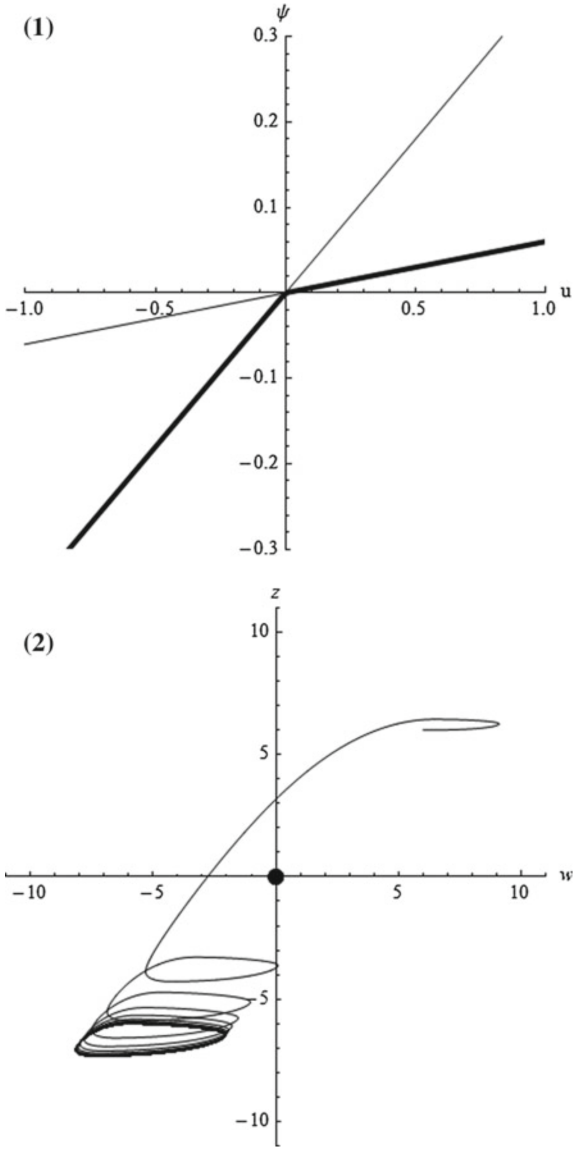


Fig. 5 Periodic attractor of the extended Goodwin model with an asymmetric adjustment coefficient

income paths, respectively. As compared to the periodic path of Figs. 3 and 4, that of Fig. 5 appears in a domain lower than the equilibrium point (0, 0). In this sense, in a chronic slump, private spending is continuously insufficient to make use of the available productive capacity. See the Introduction. Thus, the periodic path of Fig. 5

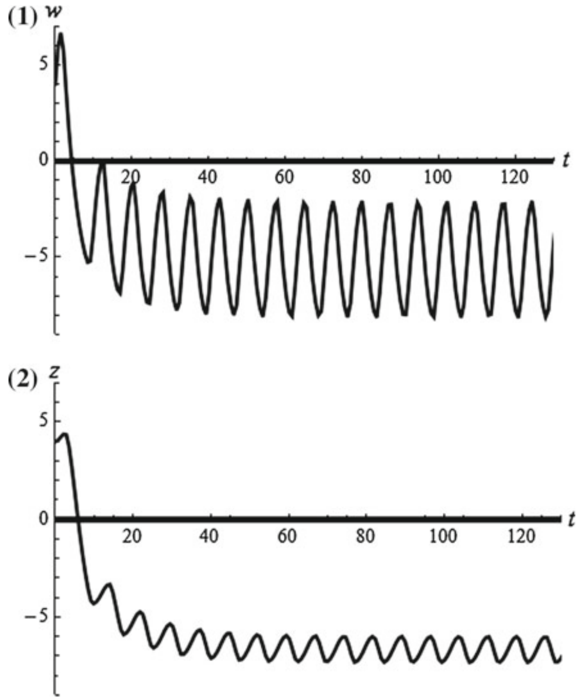


Fig. 6 Time series of the path of Fig. 5

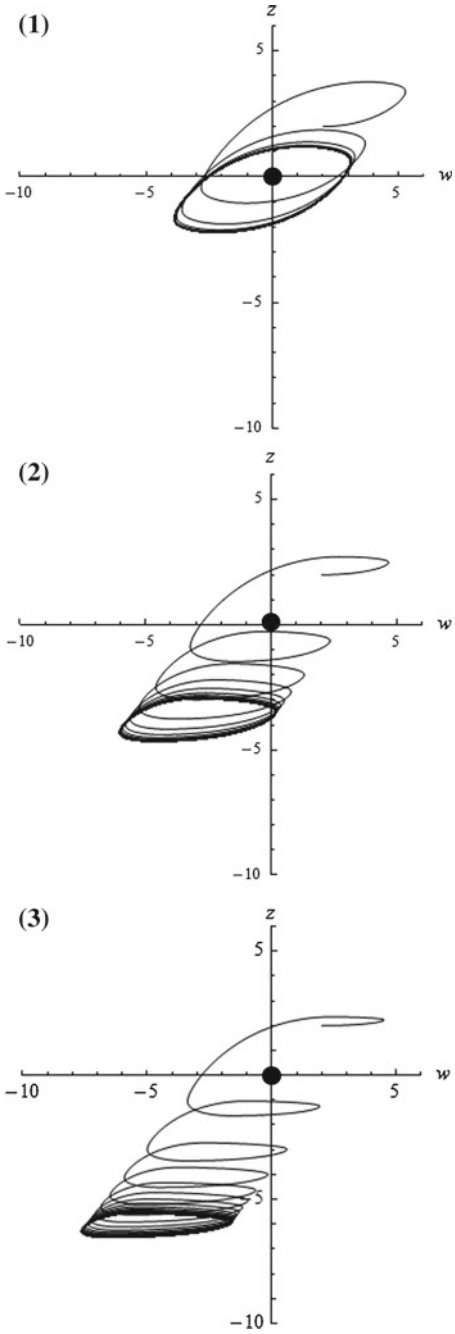
describes the situation in which the economy constantly repeats partial recoveries and slowdowns. We call such a periodic path a *slump cycle*.[16]

As stated before, the periodic paths of the Goodwin model and System Θ_{EG} with the linear adjustment function surround the equilibrium point. On the other hand, Fig. 5 show that in the slump cycle of System Θ_{NEG}, the income and expected income are locked in domains lower than the equilibrium point. Thus, we note that the System Θ_{NEG} (i.e., System Θ_{EG} with the asymmetric adjustment function) satisfies R.1 and R.2. We now observe the dynamic behavior of the expected income. Figure 5 also shows that given the loss of symmetry due to the pessimistic outlook, the expected income is locked in a "narrow" domain lower than the equilibrium point. This shows that the household is convinced of the pessimistic outlook for the future economy. Thus, it becomes difficult that the household escapes from a pessimistic outlook.

We consider the effect of the intensity of pessimism about the future economy (i.e., the degree of asymmetry of the adjustment function) on the location and the amplitude of the emerging slump cycle. We set $a_{-} = 0.4$ and $a_{+} = 0.4i$, where i

[16]Needless to say, the periodic paths of System Θ_{NMG} with asymmetric adjustment functions are not necessarily lower than the equilibrium. Therefore, the notion of a slump cycle is restrictive. In the case where the asymmetry of the adjustment function is sufficiently strong, the periodic path becomes lower than the equilibrium. For this point, see Fig. 7 to be given later. However, such a notion is useful in making our argument clear-cut. The occurrence of slump cycles is the most interesting feature of the present paper.

Fig. 7 Periodic attractor goes away from the equilibrium point as the degree of asymmetry of adjustment coefficient (the intensity of pessimism) becomes large



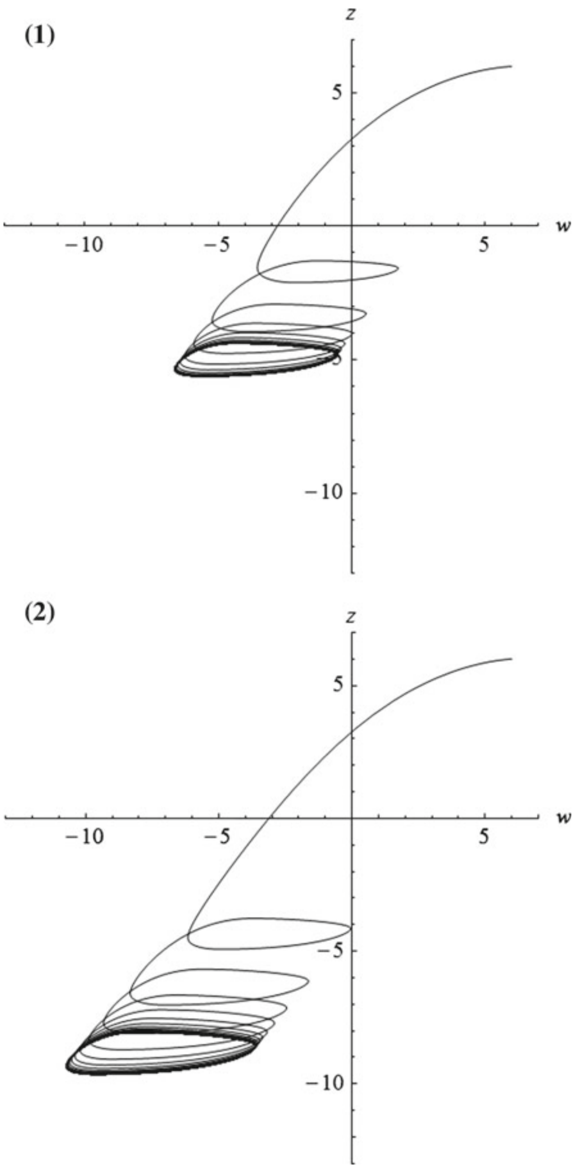


Fig. 8 Periodic attractor goes away from the equilibrium point as the propensity to consume becomes large

represents the intensity of pessimism. Assumption 5 yields $0 < i < 1$. Parts (1)–(3) of Fig. 7 describe the projections of the paths for $i = 0.9$, $i = 0.4$, and $i = 0.2$ onto the $w{-}z$ plane, respectively. The parameters of Fig. 7 apart from $a_{\pm}$ are the same as in Fig. 5. In Fig. 7, the black dots emphasize the equilibrium point in the $w{-}z$ plane. Parts (1)–(3) describe how the attractor changes as the intensity of pessimism (i.e., the degree of asymmetry) increases. Figure 7 shows that the intensity of pessimism has a strong effect on the location of the emerging slump cycle. This indicates that as the intensity of pessimism increases (i.e., the parameter a_- increases or the parameter a_+ decreases), the maximum point of a business cycle decreases. Moreover, the figure reveals that the intensity of pessimism has a weak effect on the amplitude of the emerging slump cycle. As the intensity of pessimism increases, the amplitude of the $z-$value in the slump cycle decreases slightly. This indicates that as the household becomes more pessimistic, the outlook for the future economy (expressed by the expected income) becomes more inflexible in a low domain.

System Θ_{NEG} possesses two propensities to consume: the propensities concerning income and expected income. We here numerically see the relation between the seriousness of slump and the propensity to consume concerning income. See Fig. 8. Parts 1 and 2 of Fig. 8 describe the typical dynamic behavior in the case where we set $\mu = 2$, $\theta = 1$, $\beta = 0.43$, $a_+ = 0.1$, $a_- = 0.5$, and $\phi(x) = 2\text{Arctan}(1.5x)$. In Parts 1 and 2 of Fig. 8, we set $\alpha = 0.4$ and $\alpha = 0.47$, respectively. Figure 8 shows that as the propensity to consume concerning income becomes larger, the slump cycle becomes more severe. In other words, comparing with the case where the propensity is small, the occurrence of pessimism in the converse case makes the slump cycle more severe. Since we can obtain the same result on the propensity to consume concerning expected income, we omit the argument.

4 Chronic Slump and Local Stability

Our main result in Sect. 3 is that the maintaining of pessimism can yield the chronic slump through self-fulfilling prophecy. However, the maintaining of pessimism does not always yield the chronic slump. It should be noted that in Sect. 3 we assumed the instability of equilibrium. This assumption is essential to the occurrence of chronic slump. In this section, we make clear this point.

In order to see it, we need the smooth adjustment function in the sense that ψ' is continuous. The $\psi-$function in Sect. 3 is nonsmooth. Therefore, throughout this section, we consider the following smooth adjustment function:

$$\psi(u) = \psi_{m,n,d}(u) = \varphi_{m,n,d}(u) \cdot u = m\{n - d\text{Arctan}(gu)\} \cdot u, \tag{4.1}$$

where m, n, and h are positive constants. The $\varphi_{m,n,d}-$function represents the adjustment coefficient that depends on $u = y_t - y_{et}$. We work under the assumption:

Assumption 6 $n/d > \pi/2$.

The black curve of Fig. 9 describes the graph of the $\varphi_{m,n,d}-$function. In Fig. 9, we set $m = 1$, $n = 0.5$, $d = 0.3$, and $g = 3.4$. The adjustment coefficient function satisfies the following properties:

Lemma 3 *$\varphi'_{m,n,d}(u) < 0$ and $u\varphi''_{m,n,d}(u) > 0$, for any $u \in (-\infty, 0) \cup (0, +\infty)$. $\lim_{t\to+\infty} \varphi_{m,n,d}(u) = m(n - d\pi/2) > 0$ and $\lim_{t\to-\infty} \varphi_{m,n,d}(u) = m(n + d\pi/2)$.* ■

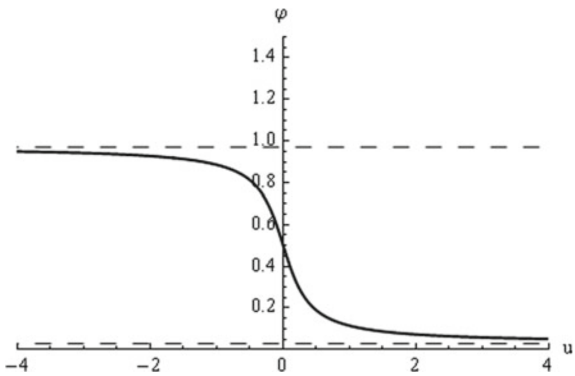


Fig. 9 Adjustment coefficient function

Proof See Appendix. ■

Thus, we consider the asymmetric adjustment coefficient. In the case where the adjustment coefficient function takes the form of (4.1), the adjustment function satisfies the following properties:

Lemma 4 *we have* $\psi_{m,n,d}''(u) > 0$, $\lim_{u\to+\infty}\psi_{m,n,d}'(u) = m(n - d\pi/2) > 0$, $\lim_{u\to-\infty}\psi_{m,n,d}'(u) = m(n + d\pi/2)$, *and* $\psi_{m,n,d}'(u) > 0$, *for any* $u \neq 0$. ■

Proof See Appendix. ■

From Lemma 4, we see that the form of the smooth $\psi_{m,n,d}$−function is almost the same as that of the adjustment function in Sect. 3. The difference between them is in the continuity of the derivative. Figure 10 describes a typical graph of the adjustment function with $m = 1$, $n = 0.5$, $d = 0.3$, and $g = 3.4$.
Moreover, the $\psi_{m,n,d}$−function possesses the following important property:

Lemma 5 *If* $d > s > 0$, *we have* $\psi_{m,n,s}(u) > \psi_{m,n,d}(u)$ *for any* $u \neq 0$.

Proof See Appendix. ■

See Fig. 11. In Fig. 11, we set $m = 1$, $n = 0.87$, $g = 3$, $d = 0$, $d = 0.12$, $d = 0.3$, and $d = 0.45$. Figure 11 describes that the degree of flexion is larger as the value of d is larger. Thus, the property of Lemma 5 shows that as the parameter d is larger, the degree of pessimism becomes larger. Especially, the line with $d = 0$ describes the usual symmetric adjustment function. Thus, the parameter represents the degree of pessimism. Here, we have the following result:

Lemma 6 *The stability of equilibrium does not depend on the degree of d.* ■

Proof See Appendix. ■

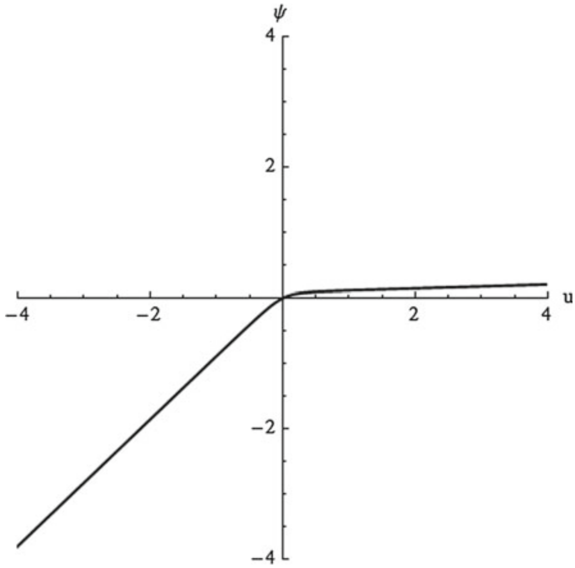


Fig. 10 Adjustment function

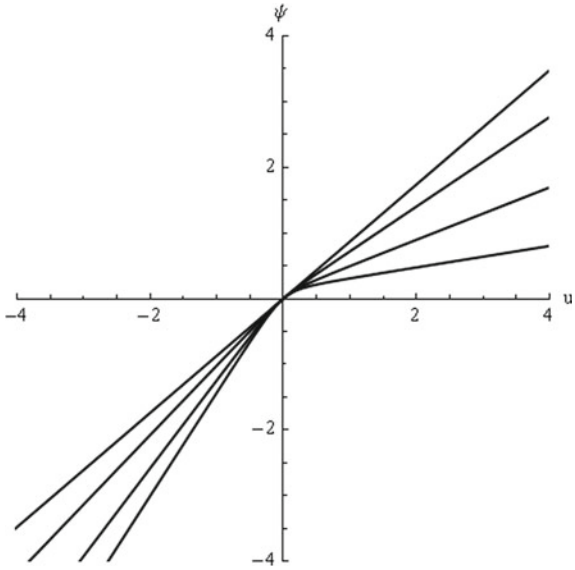


Fig. 11 The degree of flexion (the degree of pessimism) becomes large as the parameter *d* becomes large

Lemma 6 gives us an important message. In order to explain it, in the following, System Θ_{EG} with $d = 0$ and System Θ_{EG} with $d > 0$ are called the nonpessimistic and the pessimistic Goodwin models, respectively. Lemma 6 shows that if the nonpessimistic Goodwin model is locally stable, the pessimistic Goodwin model is locally stable independently of the degree of pessimism.

We numerically show that if the equilibrium is asymptotically stable, the (persistent) chronic slump does not occur. We set $\alpha = 0.4$, $\mu = 2$, $\theta = 1$, $\beta = 0.46$, $m = 1$, $n = 0.5$, $d = 0.3$, $g = 3$, $\phi(x) = \text{Arctan}(qx)$. Under the setting, we have

$$\psi'(0) = \varphi_{m,n,d}(0) = mn = 0.5.$$

Therefore, since $q = \phi'(0)$, we have

$$\lambda_1 + \lambda_2 + \lambda_3 = \left\{ \frac{\mu q - 1 - \mu(1-\alpha)\theta}{\theta} - \psi'(0) \right\} = 2q - 2.7, \tag{4.1a}$$

$$\begin{aligned} \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 &= \frac{\mu\{1 - \alpha - \theta\beta\psi'(0)\} - \{\mu q - 1 - \mu(1-\alpha)\theta\}\psi'(0)}{\theta} \\ &= (1.2 - 0.92\psi') - (2q - 2.2)\psi' = 1.84 - q, \end{aligned} \tag{4.1b}$$

$$\lambda_1\lambda_2\lambda_3 = -\frac{\mu\psi'(0)(1 - \alpha - \beta)}{\theta} = -0.14. \tag{4.1c}$$

For $\lambda_k (k \in \{1, 2, 3\})$, see the proof of Lemma 2 in Appendix. We here define

$$\begin{aligned} -(\lambda_1 + \lambda_2 + \lambda_3)(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1) + \lambda_1\lambda_2\lambda_3 &= (2q - 2.7)(q - 1.84) - 0.14 \\ &= 2q^2 - 6.38q + 4.828 \equiv \Pi(q). \end{aligned}$$

Then, the solutions of $\Pi(q) = 0$ are given by

$$q_+ \equiv \frac{6.38 + \sqrt{6.38^2 - 4 \times 2 \times 4.828}}{4} > 1.84, \tag{4.2a}$$

$$q_- \equiv \frac{6.38 - \sqrt{6.38^2 - 4 \times 2 \times 4.828}}{4} < 2.7/2. \tag{4.2b}$$

We here prove the following lemma:

Lemma 7 *If $q < q_-$, the equilibrium is asymptotically stable. Moreover, if $q_- < q$, the equilibrium is unstable.* ■

Proof See Appendix. ■

Lemma 7 shows that the stability of the equilibrium point depends on q. That is, it depends on the form of the investment function. See Fig. 12 that shows typical paths of the pessimistic Goodwin models. Parts 1 and 2 of Fig. 12 describe paths of the pessimistic Goodwin models with $q = 1.9 > q_+$ and $q = 1.11 < q_-$, respectively. It should be noted here that the adjustment functions of Parts 1 and 2 are the same. Thus, we see that even if the pessimism is not recovered, stabilizing the equilibrium recovers the chronic slump. The recovery from pessimism often requires a lot of time. Therefore, the observation in this section suggests that not only recovering the pessimism but also stabilizing the equilibrium are necessary for the recovery from the chronic slump.

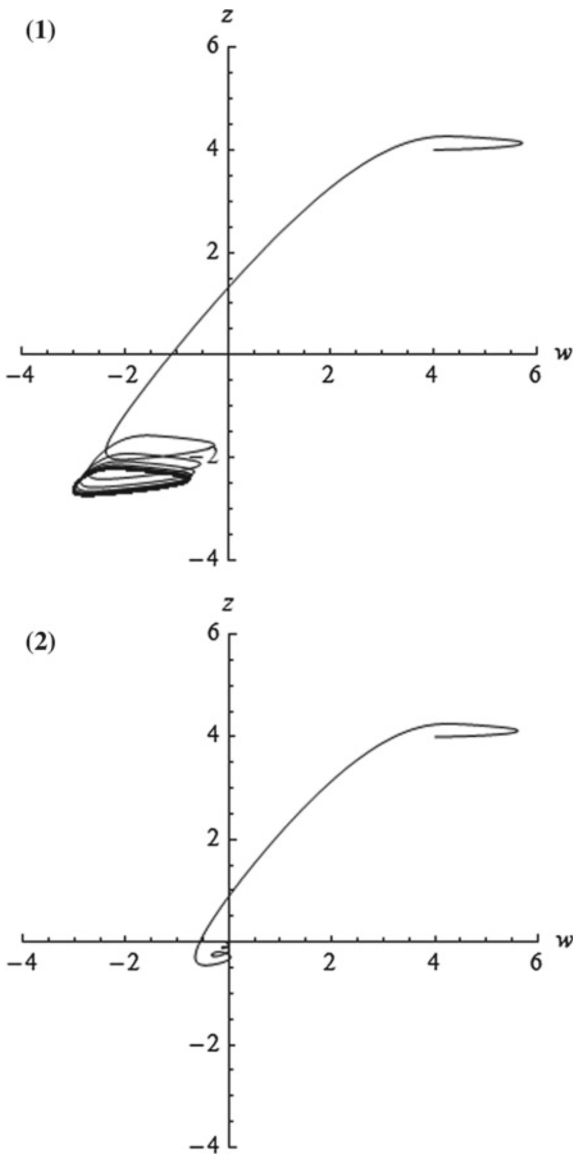


Fig. 12 Paths of the extended Goodwin model: *1* with the unstable equilibrium point; *2* with the stable equilibrium point

Finally, we make one remark. Although stabilizing the equilibrium is necessary for the recovery from the chronic slump, the pessimism makes the recovery slower. Figure 13 describes it. In Fig. 13, we set $\alpha = 0.4$, $\mu = 2$, $\theta = 1$, $\beta = 0.46$, $m = 1$, $n = 0.5$, $q = 1.1$, and $g = 3$. Since $q = 1.1 < q_{-}$, we see from Lemma 7 that the equilibrium point of the extended Goodwin model is asymptotically stable. Dashed curves of Parts 1 and 2 of Fig. 13 describe typical time series of the deviations of

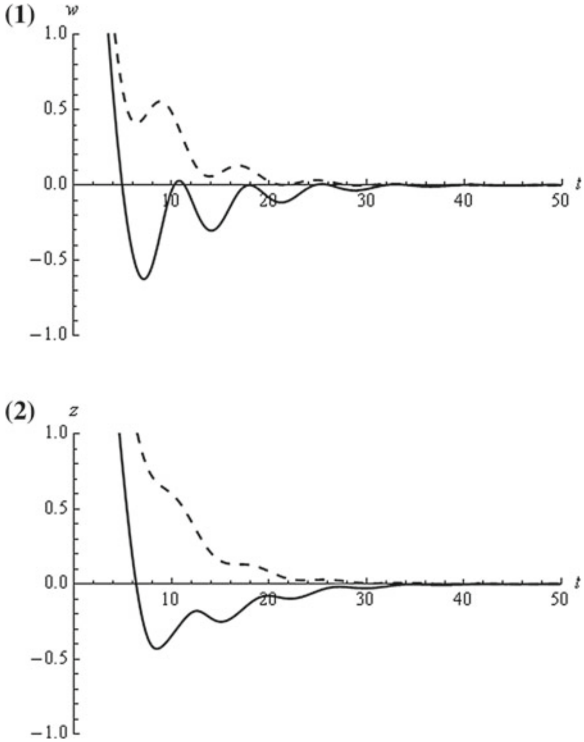


Fig. 13 Paths in the case where the extended Goodwin model with asymmetric adjustment coefficient is stable

income and expected income in the nonpessimistic Goodwin models with $d = 0$. On the other hand, black curves of Parts 1 and 2 of Fig. 13 describe typical time series of the deviations of income and expected income of the pessimistic Goodwin model with $d = 0.3$.

5 Conclusions and Final Remarks

From the Keynesian perspective, we constructed a prototype dynamic model expressing a part of the Krugman's view (Krugman 2008) concerning the recent chronic slump that has spread across the world. We constructed an extension of Goodwin's nonlinear accelerator model, and attempted to show that the pessimistic outlook of the household is an important cause of the chronic slump. Unlike the Goodwin model, the representative household distinguishes between the short-run and long-run consumption plans. The short-run plan is the same as that in the Goodwin model. On the other hand, in the long-run plan, the household determines its consumption in proportion to the expected income, which is adaptively adjusted. We assumed that the household possesses a pessimistic outlook; according to this outlook, the upward adjustment of the expected income is excessively small (in other words, the household is hyperopic). Conversely, the downward adjustment of the expected income is excessively large (in other words, the household is myopic). This assumption introduces an asymmetric nonlinearity into the adjustment function. We also observed that the assumption is related to the result about loss aversion in the behavioral economics. We demonstrated that the asymmetric nonlinearity plays an important role in generating a chronic slump.

An intuitively explanation of our result is as follows. First, we considered the case where the extended Goodwin model is completely unstable. The asymmetric nonlinearity implies that pessimism makes an upturn difficult but makes a downturn easy. Through this mechanism, the model economy spirals downward and falls into a chronic slump. Moreover, in the process, the model economy constantly repeats partial recoveries. But, income and expected income are locked in a domain lower than the market equilibrium. Thus, in the extended Goodwin model, the model economy in the chronic slump cannot continuously achieve the potential ability to produce, which is estimated at the market equilibrium. Thus, we revealed a way in which local instability and a pessimistic outlook cause a chronic slump.

Another important feature of the extended Goodwin model with a pessimistic outlook is that

> *the model economy goes into chronic slump from everywhere, regardless of initial economic conditions.*

The reason is that, in the cases where we numerically investigated, any slump cycle is globally stable. Immediately after the collapse of the bubble economy, the Japanese economy from 1991 through 2002 experienced a chronic slump. The above feature may explain such a transition from a bubble economy to a chronic slump economy. This feature does not appear in the models with multiple equilibria (for example, stable higher, unstable middle, and stable lower equilibria), because, in any model with multiple equilibria, the destination of a path depends on the initial condition of the path.

Next, we considered the case where the extended Goodwin model is stable. We numerically showed that even if the pessimistic outlook is not improved, the economy converges to the equilibrium and therefore, it recovers from the slump, though the recovery time may depend on the strength of stability. Thus, we conclude that the chronic slump results from the instability of equilibrium and the pessimistic outlook about future economy.

From the consideration in this paper, we presented two ways of recovering from the chronic slump: the recovery from the pessimistic outlook and the stabilization of economy. It often takes a long time to recover from the pessimistic outlook. In such a case, it will be effective to carry out a stabilizing policy.

As stated in Introduction, an important feature of business cycle is that booms and slumps come in all sizes. This paper proved that, according to the measure of pessimism, slumps come in all sizes. Moreover, by the same argument as that in this paper, we can prove that, according to the measure of optimism, booms come in all sizes. Thus, the extended Goodwin model also gives a theoretical explanation of the feature.

Acknowledgments The author thanks Toichiro Asada for his helpful comments and suggestions.

Appendix

In this appendix, we prove Lemmas 2–7.

Proof of Lemma 2 We use J to denote the Jacobian matrix of System Θ_{EG}. The characteristic equation of J is given by

$$\begin{aligned}\Lambda(\lambda) &\equiv \det(\lambda I - J)\\ &= \det\begin{bmatrix} \lambda - \dfrac{\mu\phi'(0) - 1 - \mu(1-\alpha)\theta}{\theta} & \dfrac{\mu\{1-\alpha-\theta\beta\psi'(0)\}}{\theta} & \dfrac{\mu(\theta\beta\psi'(0)-\beta)}{\theta} \\ -1 & \lambda & 0 \\ 0 & -\psi'(0) & \lambda + \psi'(0) \end{bmatrix}\\ &= \lambda^3 - \left\{\frac{\mu\phi'(0) - 1 - \mu(1-\alpha)\theta}{\theta} - \psi'(0)\right\}\lambda^2\\ &\quad + \frac{\mu\{1-\alpha-\theta\beta\psi'(0)\} - \{\mu\phi'(0) - 1 - \mu(1-\alpha)\theta\}\psi'(0)}{\theta}\lambda + \frac{\mu\psi'(0)(1-\alpha-\beta)}{\theta}.\end{aligned}$$

We use λ_k $(k \in \{1, 2, 3\})$ to denote the eigenvalue of J. Assumption 3 yields $\Lambda(0) > 0$. Therefore, at least one eigenvalue is negative. Without loss of generality, we suppose that $\lambda_3 < 0$. Assumption 3 gives that $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = -\mu\psi'(0)(1-\alpha-\beta)/\theta < 0$. Thus, we have $\lambda_1 \cdot \lambda_2 > 0$. Therefore, if λ_1 and λ_2 are real numbers, then λ_1 and λ_2 must be simultaneously positive or negative. The assumption of Lemma 2 shows

$$\lambda_1 + \lambda_2 + \lambda_3 = \frac{\mu\phi'(0) - 1 - \mu(1-\alpha)\theta}{\theta} - \psi'(0) > 0. \tag{A.1}$$

Hence, $\lambda_1 > 0$ and $\lambda_2 > 0$. Moreover, we observe from (A.1) that if λ_1 and λ_2 are complex conjugates, then $\mathrm{Re}\,\lambda_1 > 0$ and $\mathrm{Re}\,\lambda_2 > 0$. Thus, we complete the proof.

Proof of Lemma 3 We have

$\varphi'_{m,n,d}(u) = -mgd/(g^2u^2+1) < 0$, $\quad u\varphi''_{m,n,d}(u) = mdg^3u^2/(g^2u^2+1)^2 > 0$, and $\lim_{n\to\pm\infty}\mathrm{Arc}\tan(u) = \pm\pi/2$.

The proof follows directly from this fact. ■

Proof of Lemma 4 We have

$$\psi_{m,n,d}{}'(u) = m\{n - d\,\mathrm{Arc}\tan(gu) - dgu/(g^2u^2+1)\}. \tag{A.2}$$

Therefore, since $\lim_{n\to\pm\infty} \mathrm{Arc}\tan(u) = \pm\pi/2$, we see from (A.2) and Assumption 6 that $\lim_{u\to\pm\infty} \psi_{m,n,d}{}'(u) = m(n \pm d\pi/2) > 0$. Moreover, we have

$$\psi_{m,n,d}{}''(u) = -m\left\{\frac{dg}{g^2u^2+1} + \frac{dg(g^2u^2+1) - 2dg^3u^2}{(g^2u^2+1)^2}\right\} = -\frac{2mdg}{(g^2u^2+1)^2} < 0. \tag{A.3}$$

We now prove $\psi_{m,n,d}{}'(u) > 0$. It follows from (A.3) and Assumption 6 that

$$\psi_{m,n,d}{}'(u) > \lim_{u\to+\infty} \psi_{m,n,d}{}'(u) = m(n - d\pi/2) > 0.$$

Thus, we complete the proof. ■

Proof of Lemma 5 Since $\mathrm{Arc}\tan(u) > 0$ (< 0) for any $u > 0$ (< 0), we have $u\mathrm{Arc}\ \tan(u) > 0$ for any $u \neq 0$. Therefore, we have

$$\psi_{m,n,s}(u) - \psi_{m,n,d}(u) = u\mathrm{Arc}\tan(gu)\cdot m(d-s) > 0 \text{ for any } u \neq 0.$$

This completes the proof. ■

Proof of Lemma 6 Since we have $\psi_{m,n,d}{}'(0) = mn$, $\Lambda(\lambda)$ does not depend on d. This completes the proof. ■

Before proving Lemma 7, we prove the following three sublemmas.

Sublemma 1 Let α, β, and γ be solutions of a cubic equation. We assume $\alpha+\beta+\gamma \geq 0$ and $\alpha\beta\gamma < 0$. Then, one of the real parts of the solutions is positive. ■

Proof of Sublemma 1 Since $\alpha\beta\gamma < 0$, one of α, β, and γ must be a negative real number. Without loss of generality, we assume $\gamma < 0$. If α and β are real numbers, α and β are positive. We assume that α and β are not real numbers. Then, α and β are given as $\alpha = \xi + \omega i$ and $\beta = \xi - \omega i$, where ξ and ω are real numbers and $i = \sqrt{-1}$. From the assumption, we have $0 \leq \alpha+\beta+\gamma = 2\xi + \gamma$. Since $\gamma < 0$, we have $\xi > 0$. This completes the proof. ■

Sublemma 2 Let α, β, and γ be solutions of a cubic equation. We assume that, $\alpha+\beta+\gamma < 0$, $\alpha\beta\gamma < 0$, and $-(\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha\beta\gamma < 0$. Then, one of the real parts of the solutions is positive. ■

Proof of Sublemma 2 Since $\alpha\beta\gamma < 0$, one of α, β, and γ must be a negative real number. Without loss of generality, we assume $\gamma < 0$. We assume that α and β are not real numbers. Then, α and β are given as $\alpha = \xi + \omega i$ and $\beta = \xi - \omega i$, so that

$$0 > -(\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha\beta\gamma = 2[-(2\xi+\gamma)\gamma - (\xi^2+\omega^2)]\xi.$$

Since $\alpha+\beta+\gamma = 2\xi+\gamma < 0$, we have $\xi > 0$. Therefore, if α and β are not real numbers, the proof completes. We next assume that α and β are real numbers. Then

α and β must be simultaneously positive or negative. We assume that α and β are negative. Then, we have

$$\begin{aligned} 0 &> -(\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha\beta\gamma \\ &= -(\alpha^2\beta+\alpha\beta^2+\beta^2\gamma+\beta\gamma^2+\gamma^2\alpha+\gamma\alpha^2+2\alpha\beta\gamma) > 0. \end{aligned}$$

This contradicts to the assumption. Therefore, we see that α and β are positive. This completes the proof. ■

Sublemma 3 Let α, β, and γ be solutions of a cubic equation. A set of necessary and sufficient conditions for all the real parts of the solutions to be negative are given by

$$\begin{aligned} &\alpha+\beta+\gamma<0, \quad \alpha\beta\gamma<0, \quad \alpha\beta+\beta\gamma+\gamma\alpha>0, \text{ and} \\ &-(\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha\beta\gamma>0. \end{aligned}$$ ■

Proof of Sublemma 3 See Gandolfo (1996, Sect. 16.4). ■

We now prove Lemma 7.

Proof of Sublemma 7 Sublemma 3 yields that a set of necessary and sufficient conditions for all the real parts of the solutions to be negative are given by

$$\begin{aligned} &\lambda_1+\lambda_2+\lambda_3 = 2q-2.7<0, \quad \lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1 = 1.84-q>0, \text{ and} \\ &\Pi(q) = 2q^2-6.38q+4.828>0. \end{aligned}$$

Therefore, the necessary and sufficient condition for all the real parts of the solutions to be negative is given by $q < q_-$. This proves the first half. We now prove the latter half. Sublemmas 1 and 2 show that a set of sufficient conditions for one of the real parts of the solutions to be positive are given by

$$\lambda_1+\lambda_2+\lambda_3 = 2q-2.7 \geq 0 \text{ or} \tag{A.4a}$$

$$\lambda_1+\lambda_2+\lambda_3 = 2q-2.7 < 0 \quad \text{and} \quad \Pi(q) = 2q^2-6.38q+4.828<0. \tag{A.4b}$$

Noting $q_- < 2.7/2$, (A.4) implies that $q > 2.7/2$ or $q_- < q < 2.7/2$. Thus, we have a sufficient condition for one of the real parts of the solutions to be positive is given by $q_- < q$. This proves the latter half. ■

References

Akerlof GA, Shiller RJ (2009) Animal spirits: how human psychology drives the economy, and why it matters for global capitalism. Princeton University Press, Princeton

Blanchard O (1981) What is left of the multiplier accelerator? Am Econ Rev 71:150–154

Blanchard O (1993) Consumption and the recession of 1990–1991. Am Econ Rev 83:270–274
Bothwell FE (1952) The method of equivalent linearization. Econometrica 20:269–283
Chang WW, Smyth DJ (1971) The existence and persistence of cycles in a non-linear model: Kaldor's 1940 model re-examined. Rev Econ Stud 38:37–44
Gabisch G, Lorenz H-W (1987) Business cycle theory: a survey of methods and concepts. Springer, Berlin
Gandolfo G (1996) Economic dynamics. third, completely revised and enlarged. Springer, Berlin
George D (1981) Equilibrium and catastrophes in economics. Scott J Polit Econ 28:43–61
Goodwin RM (1951) The nonlinear accelerator and the persistence of business cycles. Econometrica 19:1–17
Guckenheimer J, Holmes P (1983) Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Springer, New York
Kahneman D, Knetsch JL, Thaler RH (1991) Anomalies: the endowment effect, loss aversion, and status quo bias. J Econ Perspect 5:193–206
Kaldor N (1940) A model of the trade cycle. Econ J 50:78–92
Knight FH (1921) Risk, uncertainty and profit. Century Press, New York
Krugman P (1991) History versus expectations. Q J Econ 106:651–667
Krugman P (1996) The self-organizing economy. Blackwell, Oxford
Krugman P (2008) The return of depression economics and the crisis of 2008, 2nd edn. Penguin Books, London
Lorenz H-W (1993) Nonlinear dynamical economics and chaotic motion, 2nd edn. Springer, New York
Matthews RCO (1959) The trade cycle. Cambridge University Press, Cambridge
Murphy KM, Shleifer A, Vishny RW (1989) Industrialization and the big push. J Polit Econ 97:1003–1026
Owase T (1991) Nonlinear dynamical systems and economic fluctuations: a brief historical survey. IEICE Trans 74:1393–1400
Puu T (2003) Attractors, bifurcations and chaos: nonlinear phenomena in economics. Springer, Berlin
Puu T, Sushko I (2006) Business cycle dynamics: models and tools. Springer, Berlin
Rosser JB Jr (1991) From catastrophe to chaos: a general theory of economic discontinuities. Kluwer Academic Publishers, Boston
Sansone G, Conti R (1964) Non-linear Differential Equations. Pergamon Press, Oxford
Strotz RH, McAnulty JC, Naines JB (1953) Goodwin's nonlinear theory of the business cycle: an electro-analog solution. Econometrica 21:390–411
Varian HR (1979) Catastrophe theory and the business cycle. Econ Inq 17:14–28
Yanqian Y (1986) Theory of limit cycles. Translations of mathematical monographs, vol 66. American Mathematical Society, Providence