Chapter 5
FMEA Using Fuzzy Evidential Reasoning and GRA Method

Two most important issues of FMEA are the acquirement of FMEA team members' diversity assessments and the determination of risk priorities of the identified failure modes. First, FMEA team members often demonstrate different opinions and knowledge and produces different types of assessment information because of their different expertise and backgrounds. Second, the traditional FMEA which determines the risk priorities of failure modes by using RPNs has been criticized to have many shortcomings. Therefore, Liu et al. (2011) presented a new risk priority model for FMEA based on fuzzy evidential reasoning (FER) and grey relation analysis (GRA) method to improve the effectiveness of the traditional FMEA. The proposed FMEA can not only capture FMEA team members' diversity opinions under different types of uncertainties and incorporate the importance weights of risk factors into the prioritization of failure modes, but also take advantage of the benefits of fuzzy logic and grey theory without the need of asking experts too much.

5.1 Fuzzy Evidential Reasoning Approach

The evidential reasoning (ER) approach was developed by combing the Dempster–Shaffer (D–S) theory (Shafer 1976) with a distributed modeling framework for dealing with multi-criteria decision-making (MCDM) problems characterized by both quantitative and qualitative attributes with various types of uncertainties (Yang et al. 2006; Guo et al. 2009). Its main advantage is that both precise data and subjective judgments with uncertainty can be consistently modeled under a unified framework. The ER approach provides a novel procedure for aggregating multiple criteria based on the distributed assessment framework and the evidence combination rule of D–S theory.

Extensive research dedicated to the ER approach has been conducted in recent years. Experiences show that a decision maker may not always be confident enough to provide subjective assessments to individual grades only but at times wishes to

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be able to assess beliefs to subsets of adjacent grades. It is to deal with the problem that the interval-grade ER (IER) approach is proposed (Yang and Singh 1994). Another extension to the original ER approach is to take account of vagueness or fuzzy uncertainty, i.e., the assessment grades are no longer clearly distinctive crisp sets, but are defined as dependent fuzzy sets. Yang et al. (2006) proposed the fuzzy ER approach (FER) to extend the original ER individual grades to fuzzy grades to capture fuzziness caused by the fuzzy evaluation grades. Guo et al. (2009) developed a general ER modeling framework and an attribute aggregation process, which is referred to as the fuzzy IER (FIER) algorithm, to deal with both fuzzy and interval-grade assessments. For the MCDM problem with unknown criteria weights, Fu and Chin (2014) proposed a robust ER approach to compare alternatives by measuring their robustness with respect to criteria weights and generate a robust solution in the ER context. Chen et al. (2016) proposed a new fuzzy MCDM method based on intuitionistic fuzzy sets and ER methodology, in which the ER methodology is used to aggregate each decision maker's decision matrix to get the aggregated decision matrix.

In this chapter, the FER approach is used to deal with the diversity and uncertainty of assessment information given by FMEA members, and the involved steps are presented as follows (Liu et al. 2011):

Step 1. Assess risk factors using belief structures

The three risk factors O, S, and D can be evaluated numerically or linguistically. Both of them have been extensively applied and have their merits and demerits. However, there is a high level of uncertainty involved in FMEA since it is a group decision behavior and the assessment information for risk factors mainly based on experts' subjective judgments may be complete or incomplete, precise or imprecise, and certain or uncertain. In addition, most experts are willing to express their opinions by belief degrees (or possibility measures) based on a set of evaluation grades, i.e., $\{\text{Very Low, Low, Moderate, High, and Very High}\}$. As such, in this chapter, we choose linguistic terms for the assessment of risk factors and the individual evaluation grade set is defined as a fuzzy set H_F as follows:

$$\begin{aligned} H_F &= \{H_{11}, H_{22}, H_{33}, H_{44}, H_{55}\} \\ &= \{\text{Very Low, Low, Moderate, High, Very High}\}. \end{aligned}$$

In order to generalize the $\hat{H}_F = \left\{H_{pq}, p = 1, \ldots, 5;\ q = 1, \ldots, 5\right\}$ to fuzzy sets, we assume that a general set of fuzzy individual assessment grades $\left\{H_{pp}\right\}, p = 1, \ldots, 5$ are dependent on each other and only two adjacent fuzzy individual assessment grades may intersect. Based on experts' opinions, we can approximate all the five individual assessment grades by trapezoidal fuzzy numbers for simplifying the discussion and without loss of generality, and their membership function values can be determined according to the historical data and the detailed questionnaire answered by all experts, as shown in Fig. 5.1 and Table 5.1.

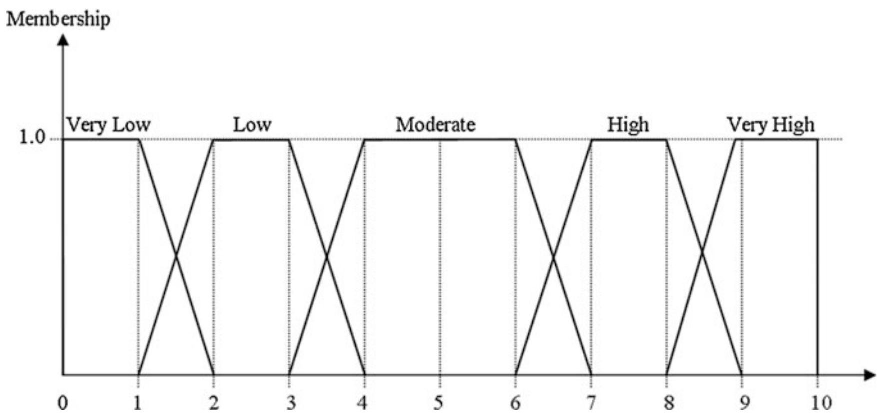

Fig. 5.1 Fuzzy membership function for linguistic terms (Liu et al. 2011)

Table 5.1 Linguistic terms for rating failure modes

Linguistic terms	Fuzzy number
Very low	(0, 0, 1, 2)
Low	(1, 2, 3, 4)
Moderate	(3, 4, 6, 7)
High	(6, 7, 8, 9)
Very high	(8, 9, 10, 10)

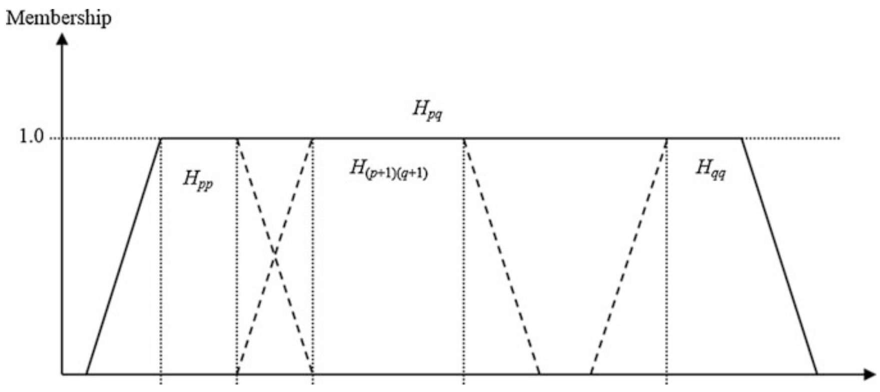

Fig. 5.2 Interval fuzzy grades set (Liu et al. 2011)

Furthermore, we define the interval fuzzy assessment grades sets H_{ij} for $p = 1, \ldots, 4$ and $q = p + 1$ to 5 as trapezoidal fuzzy sets that include fuzzy individual grades H_{pp}, $H_{(p+1)(p+1)}$, …, H_{qq}. If the individual assessment grades are trapezoidal fuzzy sets, every interval grade will be a trapezoidal fuzzy set as shown in Fig. 5.2.

In the real FMEA, the assessment grades of a FMEA team member may represent a vague concept or standard and there may be no clear cut between the meanings of two adjacent grades. In other words, these evaluation grades may not be regarded as crisp sets. Such a problem can be solved with the help of the FER approach, which allows FMEA team members to provide their subjective judgments in the following flexible ways:

- A certain grade such as *Low*, which can be written as $\{(H_{22}, 1.0)\}$. Such an expression is referred to as belief structure in the FER approach.
- A distribution such as *Low* to 0.4 and *Moderate* to 0.6, which means that a failure mode is assessed with respect to the risk factor under consideration to grade *Low* to the degree of 0.4 and to grade *Moderate* to the degree of 0.6. Here, the degrees of 0.4 and 0.6 represent the confidences (also called belief degrees) of the FMEA team member in his/her subjective judgments and the distribution can be equivalently expressed as $\{(H_{22}, 0.4), (H_{33}, 0.6)\}$. When all the confidences are summed to one, the distribution is said to be complete; otherwise, it is said to be incomplete. For example, $\{(H_{22}, 0.4), (H_{33}, 0.5)\}$ is an incomplete distribution or called incomplete assessment, where the missing information of 0.1 is referred to as local ignorance and could be assigned to any grade between *Very Low–Very High* according to the D–S theory (Shafer 1976).
- An interval such as *Low–Moderate*, which means that the grade of a failure mode with respect to the risk factor under evaluation is between *Low* and *Moderate*. This can be written as $\{(H_{23}, 1.0)\}$.
- No judgment, which means the FMEA team member is not willing to or cannot provide an assessment for a failure mode with respect to the risk factor under consideration. In other words, the grade by this FMEA team member could be anywhere between *Very Low* and *Very High* and can be expressed as $\{(H_{15}, 1.0)\}$. Such judgments are referred to as total ignorance.

Obviously, belief structures in the FER approach provide FMEA team members with an easy-to-use and very flexible way to express their opinions and can better quantify risk factors than the conventional RPN methodology. All failure modes with respect to the risk factors can be evaluated using belief structures.

Step 2. Compute the fuzzy group belief assessment matrix

Suppose there are l members (TM_1, …, TM_l) in a FEMA team responsible for the assessment of m failure modes (FM_1, …, FM_m) with respect to n risk factors (RF_1, …, RF_n). Each team member TM_k is given a weight $\lambda_k > 0$ ($k = 1, \ldots, l$) satisfying $\sum_1^l \lambda_k = 1$ to reflect his/her relative importance in the FMEA team. Let $\tilde{w}_j^k = \left(w_{ja}^k, w_{jb}^k, w_{jd}^k\right)$ is the weight of risk factor RF_j given by TM_k to reflect its relative importance in the determination of risk priorities of the failure modes. Since they are not easy to be precisely determined due to the same reason as risk factors, the relative importance weights of risk factors are assessed using the linguistic terms

Table 5.2 Linguistic terms for rating risk factor weights

Linguistic terms	Fuzzy number
Very low (VL)	(0, 0, 0.25)
Low (L)	(0, 0.25, 0.5)
Moderate (M)	(0.25, 0.5, 0.75)
High (H)	(0.5, 0.75, 1)
Very high (VH)	(0.75, 1, 1)

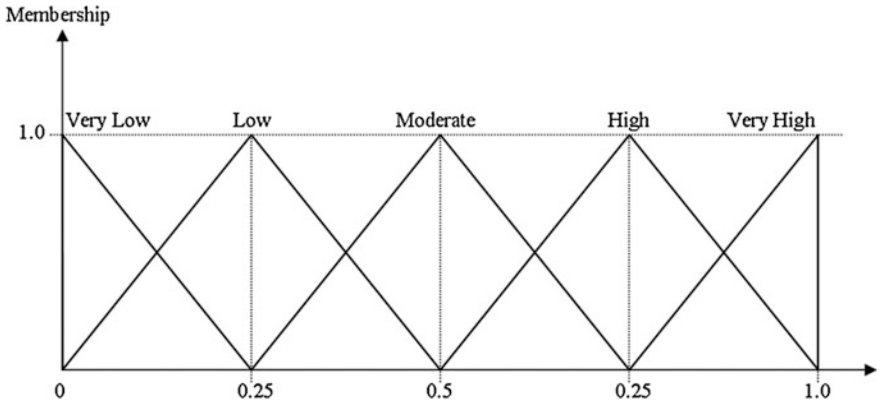

Fig. 5.3 Membership functions of fuzzy weights (Liu et al. 2011)

in Table 5.2, whose membership functions are visualized in Fig. 5.3. The group weight of risk factor RF_j of the l team members is denoted as

$$\tilde{w}_j = \sum_{k=1}^{l} \lambda_k \tilde{w}_j^k = \left(\sum_{k=1}^{l} \lambda_k w_{ja}^k, \sum_{k=1}^{l} \lambda_k w_{jb}^k, \sum_{k=1}^{l} \lambda_k w_{jd}^k \right), \quad j = 1, 2, \ldots, n. \tag{5.1}$$

The group weights of risk factors are first defuzzified using Eq. (5.7) and then normalized by

$$\bar{w}_j = \frac{w_j}{\sum_{j=1}^{n} w_j}, \quad j = 1, 2, \ldots, n, \tag{5.2}$$

where w_j is referred to as the crisp number of the group risk factor weight $\tilde{w}_j$.

Let $\left\{ \left(H_{pq}, \beta_{pq}^k (\mathrm{FM}_i, \mathrm{RF}_j) \right), p = 1, \ldots, 5; q = 1, \ldots, 5 \right\}$ be the belief structure provided by TM_k on the assessment of FM_i with respect to $\mathrm{RF}_{j,}$ where H_{pp} for p = 1, …, 5 are fuzzy assessment grades defined for risk assessment, H_{pq} for p = 1, …, 4 and $q = p + 1$ to 5 are the intervals fuzzy assessment grades between H_{pp} and H_{qq}, and $\beta_{pq}^k (\mathrm{FM}_i, \mathrm{RF}_j)$ are the belief degrees to which FM_i assessed on

RF_j to the intervals H_{pq}. All the grades H_{pp} for p = 1, …, 5 and the intervals H_{pq} for p = 1, …, 4 and $q = p + 1$ to 5 together form the frame of discernment, which is expressed as $\hat{H}_F = \{H_{pq},\ p = 1,\ldots,5;\ q = 1,\ldots,5\}$, or equivalently

$$\hat{H}_F = \left\{ \begin{matrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} \\ & H_{22} & H_{23} & H_{24} & H_{25} \\ & & H_{33} & H_{34} & H_{35} \\ & & & H_{44} & H_{45} \\ & & & & H_{55} \end{matrix} \right\}. \tag{5.3}$$

The collective assessment of the l team members for each failure mode with respect to each risk factor is also a belief structure, called group or collective belief structure, which is denoted as

$$\begin{aligned} X_{ij} = \{(H_{pq}, \beta_{pq}(\mathrm{FM}_i, \mathrm{RF}_j)),\ p = 1,\ldots,5;\ q = 1,\ldots,5\}, \\ i = 1,2,\ldots,m;\ j = 1,2,\ldots,n, \end{aligned} \tag{5.4}$$

where $\beta_{pq}(\mathrm{FM}_i, \mathrm{RF}_j)$ is referred to as group or collective belief degree and is determined by

$$\begin{aligned} &\beta_{pq}(\mathrm{FM}_i, \mathrm{RF}_j) = \sum_{k=1}^{l} \lambda_k \beta_{pq}^k(\mathrm{FM}_i, \mathrm{RF}_j), \\ &p = 1,2,\ldots,5;\ q = 1,2,\ldots,5;\ i = 1,2,\ldots,m;\ j = 1,2,\ldots,n. \end{aligned} \tag{5.5}$$

That is, a group belief degree is the weighted sum of the individual belief degrees corresponding to the same grade or interval. In addition, the group belief structures for m failure modes with respect to n risk factors form a fuzzy group belief assessment matrix as shown in Eq. (5.6), which differs from the traditional assessment matrix in that it consists of both fuzzy assessment grades and belief structures.

$$\tilde{X} = \begin{bmatrix} \tilde{X}_{11} & \tilde{X}_{12} & \cdots & \tilde{X}_{1n} \\ \tilde{X}_{21} & \tilde{X}_{22} & \cdots & \tilde{X}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{X}_{m1} & \tilde{X}_{m2} & \cdots & \tilde{X}_{mn} \end{bmatrix} \tag{5.6}$$

Step 3. Obtain the crisp group belief assessment matrix

Based on the fuzzy group belief assessment matrix $\tilde{X}$, group belief structures on the assessment of each failure mode with respect to the n risk factors can be aggregated into an overall belief structure using the defuzzification method and the weighted average method successively. Chen and Klein (1997) have proposed an easy defuzzification method for obtaining the crisp number of a fuzzy set, which is shown here in Eq. (5.7).

$$h_{pq} = \frac{\sum_{r=0}^{g}(b_r - c)}{\sum_{r=0}^{g}(b_r - c) - \sum_{r=0}^{g}(a_r - d)}, \quad p = 1, 2, \ldots, 5;\ q = 1, 2, \ldots, 5, \quad (5.7)$$

where g is the number of α-levels and h_{pq} is the defuzzified crisp number of H_{pq}.

Finally, the overall assessment of the failure mode FM_i with respect to the risk factor RF_j is also a crisp number, called overall belief structure, which can be aggregated by the following equation:

$$X_{ij} = \sum_{p=1}^{5}\sum_{q=1}^{5} h_{pq}\beta_{pq}\left(\mathrm{FM}_i, \mathrm{RF}_j\right), \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \quad (5.8)$$

Consequently, the fuzzy group belief assessment matrix $\tilde{X}$ can be defuzzified to get the crisp group belief assessment matrix X, which is shown as follows:

$$X = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & \cdots & X_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ X_{m1} & X_{m2} & \cdots & X_{mn} \end{bmatrix}. \quad (5.9)$$

5.2 The GRA Method

The grey theory, first proposed by Deng (1989), deals with decisions characterized by incomplete information, such as operation, mechanism, structure, and behavior, which are neither deterministic nor totally unknown, but are partially known. It explores system behavior using relation analysis and model construction. The use of grey relation analysis (GRA) within the FMEA framework is practicable and can be accomplished (Chang et al. 2001; Liu et al. 2013, 2015).

Next, the GRA method is adopted to rank the failure modes identified in FMEA based on the results of the FER approach. The procedure of GRA is expounded as follows (Liu et al. 2011):

Step 1. Generate the comparative series

An information series with n components or risk factors can be expressed as $X_i' = \left(X_{i1}', X_{i2}', \ldots, X_{in}'\right)$, where X_{in}' denotes the jth risk factor of X_i'. If all information series are comparable, the m information series can be described as the following matrix:

$$X' = \begin{bmatrix} X'_{11} & X'_{12} & \cdots & X'_{1n} \\ X'_{21} & X'_{22} & \cdots & X'_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ X'_{m1} & X'_{m2} & \cdots & X'_{mn} \end{bmatrix}. \tag{5.10}$$

For the application of this matrix in FMEA, the matrix X' is generated based on the crisp group belief assessment matrix X, which is determined by Eq. (5.9).

Step 2. Determine the standard series

Degree of relation can describe the relationship of two series; thus, an objective series called the standard series shall be established and expressed as $X_0 = (X_{01}, X_{02}, \ldots, X_{0n})$. When conducting FMEA, the smaller the score, the less the risk; therefore, the standard series can be the lowest level of all the risk factors:

$$\begin{aligned} X_0 &= (X_{01}, X_{02}, \ldots, X_{0n}) = [H_{11}, H_{11}, \ldots, H_{11}] \\ &= [h_{11}, h_{11}, \ldots, h_{11}] \end{aligned} \tag{5.11}$$

Step 3. Compute the difference between comparative series and standard series

The difference between the comparative and the standard series, D_0, is calculated and reflected in a form of matrix as seen below:

$$D_0 = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \cdots & \Delta_{1n} \\ \Delta_{22} & \Delta_{22} & \cdots & \Delta_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \Delta_{m1} & \Delta_{m2} & \cdots & \Delta_{mn} \end{bmatrix}, \tag{5.12}$$

where $\Delta_{ij} = \left\| X'_{0j} - X_{ij} \right\|$.

Step 4. Calculate the grey relation coefficient

The grey relation coefficient, γ_{ij}, is calculated using Eq. (5.13) for each risk factor of the failure modes identified in the FMEA.

$$\gamma_{ij} = \frac{\Delta_{\min} + \zeta\Delta_{\max}}{\Delta_{ij} + \zeta\Delta_{\max}}, \quad i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n, \tag{5.13}$$

where $\Delta_{\min} = \min_i \min_j (\Delta_{ij})$, $\Delta_{\max} = \max_i \max_j (\Delta_{ij})$, and ζ is an identifier, $\zeta \in (0, 1)$, only affecting the relative value of risk without changing the priority. Generally, ζ can be 0.5 (Deng 1989).

Step 5. Determine the degree of relation

This step is to obtain the degree of grey relation based upon the grey relation coefficients γ_{ij} and the group weights of risk factors $\bar{w}_j$, which is determined by Eq. (5.2). The degree of grey relation is calculated for each failure mode using the following formulation

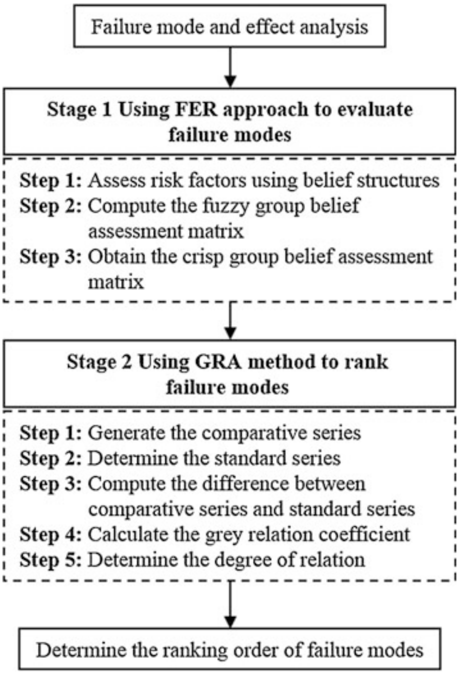

Fig. 5.4 Flowchart of the proposed FMEA model

$$\Gamma_{ij} = \sum_{j=1}^{n} \bar{w}_j \gamma_{ij} \tag{5.14}$$

The degree of relation in FMEA represents the relationship between potential failure modes and the optimal value of risk factors. The higher the degree of relation obtained from Eq. (5.14), the smaller the effect of the failure mode. As a result, all the failure modes can be ranked according to the degree of grey relation of each failure mode.

To sum up, the FMEA model proposed by Liu et al. (2011) based on the FER and the GRA methods can be delineated using the flowchart in Fig. 5.4.

5.3 An Illustrative Example

In this section, we provide a numerical example to illustrate the potential applications of the proposed FMEA and particularly the potentials of using the FER and the GRA method in capturing FMEA team members' diversity opinions and prioritizing failure modes under different types of uncertainties. The FMEA example is adapted from Wang et al. (2009), Liu et al. (2011).

A FMEA team consisting of five cross-functional team members, TM_k $(k = 1, 2, \ldots, 5)$, identifies seven potential failure modes in a system and needs to prioritize them in terms of risk factors such as O, S, and D so that high risky failure modes can be corrected with top priorities. Due to the difficulty in precisely assessing the risk factors and their relative importance weights, the FMEA team members agree to evaluate them using the linguistic terms defined in Tables 5.1 and 5.2. The assessment information of the seven failure modes on each risk factor and the risk factor weights provided by the five team members is presented in Table 5.3, where incomplete assessments and ignorance information are highlighted and shaded. The five team members from different departments are assumed to be of different importance because of their different domain knowledge and expertise. To reflect their differences in performing FEMA, the five team members are assigned the following relative weights: 0.15, 0.20, 30, 0.25, and 0.10.

To carry out a priority analysis, we first use belief structures to express the FMEA team members' individual assessments and synthesize them to construct the fuzzy group belief assessment matrix $\tilde{X} = \left[\tilde{X}_{ij}\right]_{7\times3}$ by Eq. (5.6), as presented in Table 5.4. The group belief structures in the matrix $\tilde{X}$ are then defuzzified and aggregated into overall belief structures using Eqs. (5.7) and (5.8). The results are shown in Table 5.5. During this process, all the fuzzy assessment grades $\hat{H}_F = \left\{H_{pq}, p = 1, \ldots, 5;\ q = 1, \ldots, 5\right\}$ are defuzzified by using Eq. (5.7) to produce a crisp number. The results of the defuzzification are tabulated in Table 5.6.

Next, the data in Table 5.5 are analyzed using the GRA method. The comparative series is generated based on the table using Eqs. (5.9) and (5.10), as seen in the matrix below

$$X' = X = \begin{bmatrix} 0.379 & 0.364 & 0.210 \\ 0.541 & 0.438 & 0.170 \\ 0.604 & 0.531 & 0.130 \\ 0.656 & 0.594 & 0.260 \\ 0.614 & 0.870 & 0.187 \\ 0.376 & 0.234 & 0.377 \\ 0.500 & 0.226 & 0.476 \end{bmatrix}.$$

The standard series is taken to be the lowest level of the linguistic term describing all three risk factors, which is *Very Low*. When the linguistic term *Very Low* is defuzzified, the crisp number obtained is 0.130, this represents the average value, as such the value 0 (lowest possible value) is used to represent the linguistic term *Very Low* in the standard series (Liu et al. 2011). A matrix representing the standard series is generated as shown here

$$X_0 = \begin{bmatrix} H_{11} & H_{11} & H_{11} \\ H_{11} & H_{11} & H_{11} \\ H_{11} & H_{11} & H_{11} \\ H_{11} & H_{11} & H_{11} \\ H_{11} & H_{11} & H_{11} \\ H_{11} & H_{11} & H_{11} \\ H_{11} & H_{11} & H_{11} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The difference between the comparative and the standard series D_0 is then calculated and expressed as a matrix. Since all entries for the matrix representing the standard series were determined to be 0, the difference between the comparative and the standard series would be equal to the comparative series (considering that $\Delta_{ij} = \|X_{0j} - X_{ij}\|$).

Using the values obtained from the difference of the standard and the comparative series, the grey relation coefficient, γ_{ij}, is calculated via Eq. (5.13) for each risk factor of the failure modes identified in the FMEA. Take the first failure mode in Table 5.5 for example, the grey relation coefficients for the risk factors O, S, and D are calculated as shown here:

$$\gamma_{11} = \frac{0.130 + 0.5 \times 0.870}{0.379 + 0.5 \times 0.870} = 0.694,$$
$$\gamma_{12} = \frac{0.130 + 0.5 \times 0.870}{0.364 + 0.5 \times 0.870} = 0.707,$$
$$\gamma_{13} = \frac{0.130 + 0.5 \times 0.870}{0.210 + 0.5 \times 0.870} = 0.876.$$

Similarly, the grey relation coefficients for all the failure modes with respect to each risk factor can be calculated in the same way as shown in the matrix below

$$[\gamma_{ij}]_{7\times3} = \begin{bmatrix} 0.694 & 0.707 & 0.876 \\ 0.579 & 0.647 & 0.934 \\ 0.544 & 0.585 & 1.000 \\ 0.518 & 0.549 & 0.813 \\ 0.539 & 0.433 & 0.908 \\ 0.697 & 0.845 & 0.696 \\ 0.604 & 0.851 & 0.620 \end{bmatrix}.$$

On the other side, based upon the information in Table 5.3, the relative importance weights of risk factors are first aggregated using Eq. (5.1) as shown in the last row of Table 5.4. The group weights of risk factors are then defuzzified and normalized using Eqs. (5.7) and (5.2), respectively. The results are provided in the last row of Table 5.5.

Table 5.3 Assessment information on failure modes by the five FMEA team members (Liu et al. 2011)

Risk factors	Team members	Factor weights	Failure modes						
			FM1	FM2	FM3	FM4	FM5	FM6	FM7
O	TM1	M	H_{22}	(H_{33}, 0.50) (H_{44}, 0.50)	H_{44}	H_{44}	H_{44}	H_{22}	H_{33}
	TM2	H	H_{22}	H_{33}	(H_{33}, 0.90)	H_{44}	H_{44}	H_{22}	H_{33}
	TM3	M		H_{33}	H_{33}	H_{44}	H_{34}	H_{22}	H_{33}
	TM4	VH	H_{22}	H_{33}	H_{44}	H_{33}	H_{33}	(H_{22}, 0.20) (H_{35}, 0.80)	H_{33}
	TM5	M	H_{25}	H_{33}	H_{44}	H_{44}	H_{44}	H_{33}	H_{33}
S	TM1	VH	H_{22}	H_{33}	H_{44}	(H_{24}, 0.90)	H_{55}	H_{11}	(H_{11}, 0.70) (H_{22}, 0.30)
	TM2	H	H_{22}	H_{33}	H_{33}	H_{44}	H_{55}	H_{22}	H_{11}
	TM3	VH	(H_{12}, 0.50) (H_{34}, 0.50)	H_{22}	H_{33}	H_{33}	H_{55}	H_{11}	H_{22}
	TM4	H	H_{23}	H_{33}	H_{33}	H_{44}	H_{55}	(H_{13}, 0.80)	H_{22}
	TM5	H	H_{22}		H_{33}	H_{33}	H_{55}	H_{11}	H_{11}
D	TM1	L	H_{11}	H_{13}	H_{11}	H_{22}	H_{11}	H_{22}	H_{33}
	TM2	L	H_{12}	H_{11}	H_{11}	H_{11}	H_{11}	(H_{13}, 0.80) (H_{44}, 0.20)	H_{33}
	TM3	L	H_{11}	H_{11}	H_{11}	H_{22}	H_{11}	H_{22}	H_{23}
	TM4	M	H_{22}	H_{11}	H_{11}	H_{22}	H_{22}		(H_{12}, 0.40) (H_{33}, 0.60)
	TM5	L	(H_{12}, 0.95)	H_{11}	H_{11}	H_{22}	H_{22}	H_{22}	H_{33}

Table 5.4 Group assessments of the FMEA team on failure modes and group weights of risk factors (Liu et al. 2011)

Failure modes	*O*	*S*	*D*
FM1	{(H_{15}, 0.30), (H_{22}, 0.60), (H_{25}, 0.10)}	{(H_{12}, 0.15), (H_{22}, 0.45), (H_{23}, 0.25), (H_{34}, 0.15)}	{(H_{11}, 0.45), (H_{12}, 0.295), (H_{15}, 0.005), (H_{22}, 0.25)}
FM2	{(H_{33}, 0.975), (H_{44}, 0.075)}	{(H_{15}, 0.10), (H_{22}, 0.30), (H_{33}, 0.60)}	{(H_{11}, 0.85), (H_{13}, 0.15)}
FM3	{(H_{15}, 0.02), (H_{33}, 0.48), (H_{44}, 0.50)}	{(H_{33}, 0.85), (H_{44}, 0.15)}	{(H_{11}, 1.00)}
FM4	{(H_{33}, 0.25), (H_{44}, 0.75)}	{(H_{15}, 0.015), (H_{24}, 0.135), (H_{33}, 0.40), (H_{44}, 0.45)}	{(H_{11}, 0.20), (H_{22}, 0.80)}
FM5	{(H_{33}, 0.25), (H_{34}, 0.30), (H_{44}, 0.45)}	{(H_{55}, 1.00)}	{(H_{11}, 0.65), (H_{22}, 0.35)}
FM6	{(H_{22}, 0.70), (H_{33}, 0.10), (H_{35}, 0.20)}	{(H_{11}, 0.55), (H_{13}, 0.20), (H_{15}, 0.05), (H_{22}, 0.20)}	{(H_{13}, 0.16), (H_{15}, 0.25), (H_{22}, 0.55), (H_{44}, 0.04)}
FM7	{(H_{33}, 1.00)}	{(H_{11}, 0.405), (H_{22}, 0.595)}	{(H_{12}, 0.10), (H_{33}, 0.90)}
Group weights	(0.425, 0.675, 0.8625)	(0.6125, 0.8625, 1)	(0.0625, 0.2875, 0.5375)

Table 5.5 Defuzzified and aggregated assessment information for failure modes and risk priority ranking (Liu et al. 2011)

Failure modes	O	S	D	Γ_{ij}	Ranking
FM1	0.379	0.364	0.210	0.734	6
FM2	0.541	0.438	0.170	0.677	4
FM3	0.604	0.531	0.130	0.649	3
FM4	0.656	0.594	0.260	0.588	2
FM5	0.614	0.870	0.187	0.561	1
FM6	0.376	0.234	0.377	0.763	7
FM7	0.500	0.226	0.476	0.718	5
Weights	0.36	0.45	0.19		

Table 5.6 Defuzzified values for fuzzy assessment grades (Liu et al. 2011)

Assessment grades	Defuzzified values	Assessment grades	Defuzzified values	Assessment grades	Defuzzified values
H_{11}	0.130	H_{22}	0.292	H_{34}	0.567
H_{12}	0.259	H_{23}	0.433	H_{35}	0.606
H_{13}	0.394	H_{24}	0.500	H_{44}	0.708
H_{14}	0.459	H_{25}	0.541	H_{45}	0.741
H_{15}	0.500	H_{33}	0.500	H_{55}	0.870

Substituting the grey relation coefficients and group weights of risk factors into Eq. (5.14) will give the degree of relation for the first failure mode as seen here:

$$\begin{aligned}\Gamma_1 &= [(0.694 \times 0.36) + (0.707 \times 0.45) + (0.876 \times 0.19)] \\ &= 0.734.\end{aligned}$$

In the same way, the degrees of relation are calculated for all the failure modes identified in the FMEA to produce a ranking that determines the priority for attention. The results are shown in Table 5.5. The degrees of relation of the seven failure modes give the priority ranking of the seven failure modes as FM5 $\succ$ FM4 $\succ$ FM3 $\succ$ FM2 $\succ$FM7 $\succ$ FM1 $\succ$ FM6, which is perfectly consistent with the real-world situations of the failures in this study. So, the final conclusion for this example is that FM5 should be given the top priority for correction, followed by FM4, FM3, FM2, FM7, FM1, and FM6.

The potential applications of the proposed FMEA and the detailed computational process of the degree of relation are examined and illustrated with the above numerical example. The results show that the proposed FMEA provides a useful, practical, and flexible way for the risk evaluation in FMEA. In particular, the proposed FMEA model offered a new way for capturing MEA team members' opinions and prioritizing failure modes in FMEA. Compared with the conventional RPN method and its kinds of variants, the risk priority model here proposed has the following advantages: (1) The relative importance weights of risk factors are taken into consideration in the process of prioritization of failure modes, which makes the proposed FMEA more realistic, more practical and more flexible. (2) Risk factors and their relative importance weights are evaluated in a linguistic manner rather than in precise numerical values. This enables the domain experts to express their judgments more realistically and makes the assessment easier to be carried out. (3) The diversity and uncertainty of FMEA team members' assessment information can be well reflected and modeled using belief structures. And it provides an organized method to combine expert knowledge and experience for use in FMEA. (4) Failure modes can be fully ranked and well distinguished from each other unless some of them are assessed to be the same.

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