Chapter 17
Advancing Mathematics Education Research Within a STEM Environment

Lyn D. English

Abstract In presenting the final chapter for this *Research into Mathematics Education in Australasia (RiMEA)* book, I first give consideration to the official curriculum and the operational curriculum as a basis for exploring how we might advance mathematics education research within our Science, Technology, Engineering and Mathematics (STEM) environment. Next, I present an overview of some of the core features of the current national and international spotlight on STEM education. From this basis, I argue that the roles and positioning of mathematics are in danger of being overlooked or diminished within the increased STEM framework. As one approach to lifting the profile of mathematics, I explore problem-solving and modelling across STEM contexts. In utilising findings from the chapter reviews together with my own research, I offer suggestions for (a) developing content and processes through idea-generating problems, (b) promoting in-depth content understanding, and (c) fostering general skills and processes. Next, I address the advancement of modelling across STEM contexts and illustrate this with a problem set within an environmental engineering context. I conclude by offering a few avenues for further research.

Keywords STEM education · Official curriculum · Operational curriculum · Problem solving · General skills and processes · Modelling · 21st century skills · Workplace learning

1 Introduction

Each of the chapters comprising this *Research into Mathematics Education in Australasia (RiMEA)* monograph presents an in-depth and insightful review of the Australasian mathematics education research undertaken over the previous 4 years. Major longstanding issues are addressed together with new concerns emerging from

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L.D. English (✉)
Queensland University of Technology, Brisbane, Australia
e-mail: l.english@qut.edu.au

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the changing political and educational agendas both nationally and internationally. Challenges in the learning and teaching of mathematics from early childhood through to tertiary and professional education are examined, with a number of common threads appearing across the chapters. These include broad concerns about the National Curriculum and how it is enacted, the impact of national and international assessments, how we might close gaps in students' mathematics learning, and ways to advance teacher professional development and leadership. A focus on core content topics appears not as prevalent across the chapters as the foregoing issues.

It is not the intention of this final chapter to review each of the chapters in turn; collectively they present a wealth of research for our current and future mathematics education communities. Rather, I draw upon some of the key findings in the reviews that appeared pertinent in framing my suggestions for advancing Australasian research in mathematics education. In developing this chapter, I was drawn to Way, Bobis, Lamb, and Higgins' application of Remillard and Heck's (2014) model of curriculum policy, design, and enactment (see Chap. 4, this volume). Way et al. consider various components of the model's "official" curriculum and the "operational" curriculum, from which stem many of the issues facing the mathematics education community today. One such issue lies in the escalating focus on advancing Science, Technology, Engineering and Mathematics (STEM) education both nationally and internationally, with numerous reports, policy documents, and media coverage dominating the landscape (e.g., Honey, Pearson, & Schweingruber, 2014; Marginson, Tytler, Freeman, & Roberts, 2013; *National Innovation and Science Agenda*, 2015; National Science and Technology Council, 2013; Office of the Chief Scientist, 2014). Perspectives on what STEM education entails and how it should be implemented vary widely, creating new tensions between the official curriculum and the operational curriculum with respect to each of the disciplines. For example, what is recommended by industry and political leaders might not necessarily align with existing curriculum documents, nor might each of the disciplines be given equitable attention in STEM debates. Furthermore, schools wishing to develop innovative STEM programs might face obstacles from educational authorities bound to the official curriculum. Revisiting Way et al.'s discussion on the official and operational curriculums provides a backdrop for addressing some of the challenges (and indeed opportunities) mathematics education faces in the current STEM climate.

2 The Official Curriculum and Operational Curriculum

The official or mandated curriculum is not as absolute as implied in Remillard and Heck's (2014) model. Rather, the curriculum is in a constant state of flux due to the impact of various political, social, and cultural factors. Way et al. indicate how the official curriculum is often a "political tool", viewed as a means of ensuring the social and economic growth of society as well as for improving student performance (Walshaw & Openshaw, 2011). As noted previously, because the development of the official curriculum is subjected to forces stemming from political cycles of government, recommended changes in curriculum might not necessarily align with what mathematics educators deem important in advancing students' mathematics learning. At the same time, given that curriculum policy is often strongly influenced by perceived declining student performance, national and international testing programs can become powerful levers for curriculum change; not necessarily the most desirable change. A recent Research Committee's report featured in the *Journal for Research in Mathematics Education* (Herbel-Eisenmann et al., 2016) further indicated how the viewpoints and decisions of policymakers are often shaped by outside organisations including the media, various companies, and other academic areas. Some of these organisations promote media storylines that conflict with the research undertaken by mathematics educators, resulting in a lack of consensus on how mathematics and mathematics education should be portrayed to the broader community.

These socio-political factors can impact on how the official curriculum is implemented, which could result in an operational or enacted curriculum that often exhibits features extending beyond what is mandated. Residing within this operational curriculum are factors that impact on students' learning including the nature and extent of teachers' disciplinary and pedagogical knowledge, how students and teachers interact, the resources used, and the mathematical content and learning experiences provided. It is thus not surprising that the operational curriculum has the greatest potential for influencing students' achievements, and is where the bulk of the research is conducted.

A core message from Way et al.'s review is a warning about the "inconsistencies, mismatches and tensions between the official curriculum and various aspects of the operational curriculum" (p. 16). Such a warning is timely, given the changing political perspectives on what is needed in advancing education across the board, especially with respect to STEM education, coupled with the desire to improve national and international assessment outcomes. For example, the Australian Government's *Review of the Australian Curriculum* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2015b) cited "uncrowding" and "rebalancing" of the primary curriculum as two key objectives in improving the curriculum (ACARA, 2015b, p. 1). As a consequence, the endorsed and improved *Australian Curriculum* was launched in September, 2015 (ACARA, 2015a). At the same time, competing forces emerged from national and international calls for advancing STEM competencies to promote innovation, productivity and overall economic growth. Potential tensions can thus arise as mathematics educators attempt to implement the improved *Australian Curriculum* in the face of increased demands from political, business and industry leaders to increase STEM achievements.

3 The STEM Spotlight

Australia's current focus on increasing STEM outcomes reflects the escalating international concerns for advancing the field. In the United States for example, the 2013 report from the Committee on STEM Education stressed that "The jobs of the future are STEM jobs", with STEM competencies increasingly required not only within, but also outside of, specific STEM occupations (National Science and Technology Council, 2013, p. vi). Developing skills in the STEM disciplines is thus regarded as an urgent course of action in many education systems, fuelled in part by perceived or actual shortages in the current and future STEM workforce (e.g., Caprile, Palmen, Sanz, & Dente, 2015; Charette, 2013; Hopkins, Forgasz, Corrigan, and Panizzon, 2014; The Royal Society Science Policy Centre, 2014). Outcomes from international comparative assessments (e.g., OECD, 2013a, 2013b) have further sparked the STEM urgency seen across many nations.

Within Australia, the long anticipated *National STEM School Education Strategy, 2016–2026*, was unveiled in December 2015 as a comprehensive plan for the nation's STEM education. Designed to build upon and better coordinate the existing range of reforms, the document emphasises the significance of the *Strategy*:

> A renewed national focus on STEM in school education is critical to ensuring that all young Australians are equipped with the necessary STEM skills and knowledge that they will need to succeed. (Education Council, 2015, p. 3)

This renewed focus is intended to lift foundational skills across the STEM disciplines, develop mathematical, scientific and technological literacy, and promote the growth of 21st century problem-solving skills including creative thinking and critical analysis. The need to commence with the early school years and continue throughout the levels of education, as emphasised in the *Strategy*, was highlighted in a subsequent press release by the Federal Minister for Education and Training, the Hon. Simon Birmingham (2015, December 14):

> Developing an early interest in subjects like science, maths and IT will help school students prepare for life and work beyond school. We need to do more and we need to do it differently to encourage more young students to engage with science, technology, engineering and maths subjects.

Earlier in 2015, the Australian Industry Group expressed similar recommendations in its report, *Progressing STEM Skills in Australia* (AIG, 2015). Included in the report's key points were the importance of STEM skills for the workforce and the competitiveness of the national economy, the urgency to tackle our students' under-performance in STEM compared to nations that perform well, and the need to "develop more engaging school curriculum and pedagogy to attract students to STEM" (AIG, 2015, p. 6). Calls for increasing the pool of qualified STEM teachers were also included in the report. Given these industry recommendations, coupled with those of the *National STEM School Education Strategy, 2016–2026*, opportunities for advancing mathematics education could be both enriched and diminished. Obviously we aim for the former and thus need to ensure that the profile of mathematics is neither weakened nor overshadowed by the other STEM disciplines, especially science.

4 Mathematics Within the STEM Spotlight

Fitzallen (2015) expressed succinctly the above points in her Mathematics Education Group of Australasia (MERGA) research paper:

> The emphasis on science, technology, engineering, and mathematics (STEM) education in recent times could be perceived as business as usual or as an opportunity for innovation and change in mathematics classrooms. Either option presents challenges for mathematics educators who are expected to contribute to the foundations of a STEM literate community. (p. 237)

In highlighting the many reports that claim STEM provides contexts for fostering mathematical skills, Fitzallen pointed out that these reports do not acknowledge the reciprocal relationship between mathematics and the other STEM disciplines. That is, the ways in which "mathematics can influence and contribute to the understanding of the ideas and concepts of other STEM disciplines" (p. 241) are not being addressed. Numerous researchers have argued for increasing the spotlight on mathematics especially when science seems to dominate the STEM landscape (e.g., English, 2015; English & Kirshner, 2016; Marginson, Tytler, Freeman, & Roberts, 2013). As Marginson et al. (2013) noted, many nations refer to the role of STEM education as one that fosters "broad-based scientific literacy" with a key objective in their school programs being "science for all" with increased efforts on lifting science education in the primary, junior, and middle secondary school curricula (p. 70). Interestingly, Marginson et al. pointed out that STEM discussions rarely adopt the form of "mathematics for all" even though mathematics underpins the other disciplines. They thus argued that "the stage of mathematics for all should be shifted further up the educational scale" (p.70).

Foundational to discussions on how the profile of mathematics might be raised are the various perspectives on what STEM education entails and hence the research needed. It is not the intention of this chapter to address the various perspectives on STEM, as there are already numerous articles addressing this issue (e.g., Bryan, Moore, Johnson, & Roehrig, 2015; Charette, 2014/2015; Vasquez, Sneider, & Comer, 2013). The perspective of the STEM Task Force Report (2014) in the US, however, is worth noting, especially given its focus on mathematics as an integral component of each of the other STEM disciplines. The Report maintains that STEM education is far more than a "convenient integration" of its four disciplines, and that the disciplines "cannot and should not be taught in isolation, just as they do not exist in isolation in the real world or the workforce" (p. 9). STEM education from this perspective encompasses "real-world, problem-based learning" that integrates the disciplines "through cohesive and active teaching and learning approaches" (p. 9). In defining each of the disciplines from an integrated perspective, the Report defines "mathematically literate students" as not only knowing "how to analyse, reason, and communicate ideas effectively", but also being able to "mathematically pose, model, formulate, solve, and interpret questions and solutions in science, technology, and engineering" (p. 9).

5 Promoting Mathematics Education Research Within STEM: A Focus on Problem-Solving and Modelling

The chapters of this book offer significant insights into how we might lift the profile of mathematics across the STEM landscape, given the potential tensions and inconsistencies that can arise between the official and operational curriculums. Not only do we need to ensure that mathematics receives the attention it deserves within our STEM climate, but also that our students are provided equitable opportunities to develop the mathematical literacy for successful participation in their current and future worlds. In offering recommendations for addressing these concerns, I have chosen to consider a few ways in which we might promote more effective problem-solving and modelling, taking into consideration some of the learning and teaching issues raised in this book.

Included in several chapters are calls for increasing students' competencies in mathematical problem-solving, modelling, and reasoning processes. MacDonald et al. (Chap. 9, this volume), for example, highlight research demonstrating the capabilities of young learners in innovative problem-solving, while Stillman et al. (Chap. 14, this volume) illustrate the advances that have been made in modelling and applications ranging from innovations in pedagogy through to developments in theory and methodological tools. Hunter et al. (Chap. 11, this volume) report on a range of innovative and powerful pedagogical practices that can advance learning and problem-solving, as well as promote more equitable outcomes for students with diverse needs and backgrounds.

Given that problem-solving and modelling are contentious and complex domains, recommendations for their advancement both within mathematics and across STEM fields remain challenging. With the diverse range of learning contexts, one set of recommendations will not necessarily apply to all education systems. Elsewhere (e.g., English & Gainsburg, 2016) I have considered some implications for fostering problem-solving and modelling drawing on studies of the competencies required by 21st-century work and life. These studies revealed that such problem-solving requires:

- A substantial and flexible grasp of foundational mathematical ideas and processes;
- General skills that are of a cognitively high level;

- An understanding of conceptual models that underlie processes or systems, which in turn requires the ability to interpret complex representations within given contexts;
- The ability to interpret quantitative data in different complex forms in unfamiliar, multiple domains;
- The ability to solve a range of novel problems.

In light of these broad competencies, I offer some suggestions for advancing problem-solving including targeting content as well as process through idea-generating problems; promoting in-depth content understanding; developing general skills and processes; and fostering interdisciplinary modelling. In considering these aspects, I also touch upon research addressing equity, motivation and engagement, and social justice.

5.1 Developing Content and Process Through Idea-Generating Problems

Many decades of debates have taken place regarding whether we should teach problem-solving per se or teach mathematics *through* problem-solving; not surprisingly, results have been inconclusive. My perspective is that both aspects should be addressed, although this is not implying that all mathematical content should be taught through problem-solving. Designing problems that are sufficiently cognitively demanding to foster both significant mathematical content and effective problem-solving capabilities would appear a powerful way of tackling this issue. Lesh and Zawojewski (2007) argue that such problems should encourage students to “develop a more productive mathematical way of thinking about the given situation” (p. 782). The focus of problem-solving then becomes one of learning or idea generation, rather than simply the application of problem-solving processes or strategies. Situating students at the centre of their learning where they are encouraged to engage with meaningful yet challenging problematic situations can lead to the application of higher levels of cognitive reasoning, as Hunter et al. indicated in Chap. 11.

Idea-generating problems that are cognitively challenging not only encourage high-level thinking and reasoning, but also offer multiple entry points, and enable students to use varied solution approaches. Furthermore, as Silver, Mesa, Morris, Star, and Benken (2009) indicated, problems with high cognitive demand require students to explain, describe, and justify; make decisions, choices, and plans; formulate questions; apply existing knowledge and create new ideas; and represent their understanding in multiple formats. Likewise, the research of Sullivan et al. (e.g., Sullivan, Clarke, Cheeseman, Mornane, Roche, Swatzki, & Walker, 2014; Sullivan & Davidson, 2014), cited in the chapters by Attard et al. and Hunter et al. (Chaps. 5 and 11, this volume), document the importance of cognitively demanding tasks where “sustained thinking” and argumentation are fostered (Sullivan & Davidson, 2014, p. 606). Exposing students to such cognitively rich problems can empower a wider range of students to “participate as mathematicians and engage in interpreting and communicating mathematical ideas” (Chap. 11, this volume, p. 4). As Sullivan et al. (2014) found as part of a large study, students appeared more engaged with challenging classroom tasks, preferring to persist with such tasks prior to intervention by the teacher.

Approaches to improving content and processes through idea-generating problems also need to take into account important social justice issues. For example, Vale et al. (Chap. 6, this volume) report on a study by Atweh and Ala’i (2012) where efforts to implement “Socially Response-able Mathematics” activities were hampered by teachers’ reluctance to use “open ended pedagogies” (p. 103). Their study revealed that when teachers use such approaches, in contrast to direct teaching, students invariably demonstrate a “deeper understanding and engagement in the class” (Atweh & Ala’i, 2012, p. 103). Alleviating reticence to implement more challenging, idea-generating activities would seem a core plank in our efforts to promote all students’ learning across the STEM disciplines.

5.2 Promoting In-Depth Content Understanding

In targeting both content and process in idea-generating problems, efforts to develop deep conceptual understanding can be hampered by an overriding focus on national and international test achievements. As Serow et al. (Chap. 12) point out in their chapter on assessment of mathematics learning, there appears to be a mismatch between ACARA’s stated objectives and national testing that assesses “some fairly conventional mathematical knowledge in straightforward ways” (p. 5). Unfortunately, although our national assessment items are rigorously trialled and validated, they are not adequate on their own for providing a sound basis for the mathematical understandings and skills required for the 21st century.

Interestingly, the studies reviewed by English and Gainsburg (2016) indicated that many of the problems arising in work and life only require basic mathematics, but importantly, this knowledge needs to be used and applied far more fluently than it is today. There appears to be the need to enrich students’ understanding of topics such as algebra, geometry, statistics, and data analysis, and to develop their skills in applying this understanding to a variety of mathematical and other STEM-based authentic problems.

Research by Hoyles and her team (e.g., Hoyles, Noss, Kent, & Bakker, 2010) on the problem-solving needed by mid-level workers in technology intensive settings found, among others, that a facility with graphs, charts, spreadsheets, and computer simulations was paramount. Their findings demonstrated the importance of understanding the conceptual models underlying real-world processes and the ability to generalise, to some extent, deep conceptual knowledge. These aspects appeared more efficacious in promoting problem-solving ability, at least within a given domain, than shallower, situation-specific procedural knowledge. Hence a key recommendation for increasing the application of mathematics across the STEM domain would appear to be the development of in-depth understanding of underlying principles and concepts, whatever the content and context. Statistical reasoning and a facility with a range of data representations emerged as key areas in need of greater attention.

Another interesting facet from workplace studies, as evident in the sentiments of employers and observations by workplace ethnographers, is the impact of employees' learning while on the job. For example, successful engineers, scientists, and technology personnel use mathematics to better understand the systems that are at the core of their work, while at the same time refine their mathematical or quantitative "tools" for future problem-solving. It is thus recommended that students be made aware of this important learning cycle observed in the work of STEM personnel. General skills and processes form a significant component of "learning while on the job."

5.3 Fostering General Skills and Processes

The importance of generic skills and processes including metacognition is underscored by several authors in this book, including Stillman et al. and Geiger et al. (Chaps. 13 and 14). Implications from their reviews align with recommendations from various employer groups on the broad skills and processes required for effective problem-solving. Although perspectives on what is required do vary considerably, they do share common features. Some of the frequently cited employer-desired skills and processes that have been identified by the *Partnership for 21st-Century Skills* (2015) appear particularly pertinent to STEM education. These processes include effective reasoning, using systems thinking, making judgements and decisions, and solving different kinds of novel problems in both conventional and innovative ways.

General skills and processes with respect to mathematical problem-solving have received substantial attention over the decades with numerous debates on the effectiveness of teaching strategies and heuristics (e.g., Lesh & Zawojewski, 2007; Lester & Kehle, 2003). It is beyond the scope of this chapter to address these various debates. However, it is worth acknowledging the important role of metacognition, with research indicating that more sophisticated levels of self-awareness and explicitness about strategies are associated with greater success in solving problems (Kapa, 2001; Schneider & Artelt, 2010). Over the years, numerous instructional interventions have been developed and implemented to enhance metacognition as one means of improving problem-solving competence (e.g., Goos, Galbraith, & Renshaw, 2002; Stillman & Galbraith, 1998). Metacognition is thus being increasingly recognised as playing a critical role in successful problem-solving and modelling, both within and beyond the curriculum including in workplace settings (e.g., Chap. 14, this volume;

Lester, 2013; Pellegrino & Hilton, 2012). Interesting advances on earlier studies on metacognition are discussed in Stillman et al.'s chapter (Chap. 14), together with Geiger et al.'s (Chap. 13) reporting further on these developments.

One such advance is the notion of "anticipatory metacognition," as addressed in Stillman et al.'s chapter. Adding a new direction to the existing work on metacognition, this notion holds considerable promise for advancing mathematics across the STEM landscape. Anticipatory metacognition includes Galbraith's (2015) concept of "noticing" when one is engaged in modelling as part of real-world problem-solving. Rather than just "looking back" on actions that have been taken in solving a problem, the problem-solver looks forward to potential cognitive actions that might be feasible, desired, or even essential. Such anticipatory metacognition encompasses the "mathematical, cognitive and physical resources necessary to mathematise real-world situations into mathematical models" (Chap. 14, p. X). As such, fostering anticipatory metacognition could potentially enhance students' competencies in modelling across STEM contexts.

5.4 Advancing Modelling Across STEM Contexts

The importance of understanding the underlying models that are represented mathematically and technologically is crucial in many fields, including engineering, finance, manufacturing, and agriculture. Political debates on how national and state economies might be restructured to address budget deficits, for example, draw upon modelling to support certain points of view. The foundations of this modelling, however, including key assumptions, context, and methodology, are also open to debate. As Gittins (2016), economics editor of the *Sydney Morning Herald*, warned: "The lesson for the economic profession is that the modelling they value so highly is too often being used by other economists to mislead rather than enlighten. The reputation of models and modellers is being trashed, and with it the profession's credibility" (p. 26). One of the goals of promoting modelling across STEM contexts should be developing students' appreciation of models and modelling processes and how these can both inform and misinform society.

The notions of models and modelling have been interpreted variously in the literature, as Stillman et al. explore in detail in Chap. 14. While not elaborating further on these various interpretations, I maintain that modelling is a powerful vehicle for bringing features of 21st-century problems into the mathematics classroom. In adopting this stance, I align with Stillman et al.'s framing of their research reviews, namely, a "modelling-as-content" perspective, or as Galbraith (2013) described, "modelling as real world problem-solving." This approach aims to develop students' skills in using mathematics in a range of contexts, whether it be their current or future workplaces, their personal lives, or within the broader community.

In fostering our students' understanding of, and competence in, modelling "real-world" problems, we need to consider how contexts might be selected to approximate "authentic" problems. Galbraith's (2013) four dimensions of authenticity, as noted in Chap. 14 (this volume) are worth revisiting given their relevance to modelling problems across STEM contexts.

Content authenticity The problem comprises genuine real-world links and is within reach of students' mathematical knowledge.
Process authenticity The problem engages students in valid modelling processes.
Situation authenticity The task requirements drive the problem-solving activity not vice versa.
Product authenticity The solution can be justified mathematically and appropriately addresses the real-world problem.

Of the numerous interpretations of modelling, one form I have implemented across the primary and middle school years is that of model-eliciting activities (MEAs), drawing upon the extensive research of Lesh et al. (e.g., English, 2010; Hamilton, Lesh, Lester, & Brilleslyper, 2008; Lesh & Doerr, 2003; Lesh, Zawojewski, & Carmona, 2003). Definitions of models and modelling have varied over the years, however. I have typically considered a model to be a "system of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behaviour of some other familiar system" (Doerr & English, 2003, p.112). Lesh and Fennewald (2010) offered a more succinct definition, namely, "A model is a system for describing (or explaining, or designing) another system(s) for some clearly specified purpose" (p. 7). Both definitions are especially germane to fields beyond mathematics education, including engineering and other mature science domains. In addition to meeting Galbraith's (2013) authenticity dimensions, MEAs foster the types of general skills that employers demand in the workplace and that citizens need for maximum societal participation. As previously noted, such skills include critical and innovative thinking, complex reasoning, metacognitive actions, and collaboration and communication within and across disciplines.

MEAs focus on the processes of interpretation and re-interpretation of problematic information, and on the iterative development of mathematical ideas as models are formed, tested, and refined in response to certain specifications. This design encourages students to engage in anticipatory metacognition (Galbraith, 2013) and "implemented anticipation" (Niss, 2010) as explored in Stillman and Brown (2014). For example, as students consider the problem constraints (usually in the form of a client's requirements in an MEA; e.g., Lesh & Zawojewski, 2007) and engage in iterative processes towards a solution, they anticipate mathematical ideas and actions that might be useful in progressing towards model completion.

These modelling problems provide rich opportunities for addressing the reciprocal relationship between mathematics and the other STEM disciplines, as Fitzallen (2015) highlighted. Students are encouraged to create, apply, and adapt mathematical and scientific concepts in interpreting, explaining, and predicting the behaviour of real-world based problems such as those that occur in engineering (e.g., Gainsburg, 2006). The wide range of STEM contexts addressed by MEAs and other forms of modelling facilitate the application of mathematical ideas and processes to the other disciplines. For example, the environmental engineering activity, the *Water Storage Problem* (English & Mousoulides, 2011), which was implemented in classes of 11-year-olds in Cyprus, requires students to interpret and analyse different forms of data. Students might choose to sort, organise, select, prioritise, quantify, weight, and transfer data sets.

The *Water Storage Problem* commences with students being “sent” a letter from a client, the Ministry of Transportation, who requests a model for selecting a country that can supply Cyprus with water during the next summer period. The letter asks students to develop a suitable model using the given data, as well as search for additional information using available tools such as Google Earth, maps, and other web-based resources. The quantitative and qualitative data provided for each country includes water supply per week, water price, tanker capacity, and the facilities of the ports. Students can obtain further data on distance between countries, major ports in each country, and tanker oil consumption. Students conclude the problem by writing a letter to the client detailing how their model selects the most suitable country for supplying water. As an extension of this problem, students are given a second letter from the client including data for two more countries and are asked to test their model on the expanded data and, if required, improve their model.

The environmental engineering context of the *Water Shortage Problem* is an authentic one for the students in Cyprus, where water has been rapidly drying up since the 1970s. The lack of drinkable water in Cyprus is a major problem, with water supply to homes limited. The water issue features prominently in the Cyprus media and is thus an authentic problem for all members of the community, including students, as the solution can be hindered by conflicting political agendas.

The important role of mathematics in this problem was evident not only in students’ model development but also in their consideration of environmental and socio-political issues when deciding on a final model. For example, one student group was not satisfied with the model they had created because they were concerned about sea pollution, which they discussed extensively. Based on a newspaper article they had studied during the first session of the modelling activity, one student raised the question of whether it would be wiser to buy water from Greece. He mentioned that the distance from Pireus to Limassol was more than three times greater than the distance from Lebanon and Syria, and proposed to buy water from Egypt or Syria, the second and third country in distance ranking. The group also documented in their reports that all countries in the Mediterranean Sea should be fully aware of sea pollution and therefore try to minimise ship oil consumption. Another student member suggested buying water from Syria, since water price was not that expensive (compared to the price of buying from Greece and Egypt). The students finally ranked countries as Syria, Egypt, Lebanon, and Greece, and decided to propose that the local authorities buy water from Syria.

Another student group was worried about the port facilities factor, a component that some student groups chose to ignore in the models they generated. The group decided to quantify the port facilities factor and integrate their calculations within the port facilities data. A subsequent discussion focused on the finance needed for improving the ports' facilities and how this amount of money would change the water price per ton. To assist them here, the students asked for more information about the costs for improving port facilities in Syria, Lebanon, and Egypt. They were surprised when they learned that improving the ports' facilities would cost from five to ten million euro. This feedback prompted concerns regarding socio-economic considerations.

As the group progressed in their model development, they debated issues regarding tanker capacity and oil cost, and how these factors might relate to their solution, looking beyond the terms of the mathematical relations. The students were aware of energy consumption issues, and discussed how oil consumption should be kept as minimal as possible. When their teacher prompted them to decide which factor was more important, water price or oil consumption, the students replied that it would be better to spend a little more money and to reduce oil consumption. The group also made explicit that it was not only oil consumption but also other environmental issues, like the pollution of the Mediterranean Sea, which needed to be considered. The group's final model proposed Syria as the most suitable place from which to buy water, since its costs were quite reasonable and it is the least distance from Cyprus.

The *Water Shortage Problem* is just one of many modelling problems that can serve to increase the profile of mathematics across STEM contexts. Furthermore, an important feature of these modelling problems is that students of varying school mathematics achievement levels and personal backgrounds can engage with, and succeed in, solving the problems, albeit at different levels of sophistication (English, 2016). The insights gained into students' mathematical thinking and their abilities to generate STEM concepts beyond their grade level would not be achieved through national and international assessments. In addition, the interesting STEM contexts within which the problems can be couched appeal to a wide range of students who might otherwise be disengaged when dealing with traditional mathematics problems.

6 Concluding Points

In completing this final chapter, I attempted to draw upon as many of the interesting findings from the research reviews as I could within the framework I adopted. There are numerous other issues raised in each of the chapters that I would have liked to have addressed. This omission in no way dismisses their significance in advancing mathematics education research within a STEM environment. Collectively, the authors have presented comprehensive reviews of Australasian research in mathematics education during 2012–2015, and have provided key implications and recommendations for our future research endeavours. In closing I raise only a few of the many areas I consider worthy of further attention in mathematics education research.

6.1 Rebalancing the Focus on National and International Assessment

One of the challenges our community faces is dealing with national and international assessments. How we might strike a more acceptable balance between a focus on our students' mathematics performance on these tests and their development of broader mathematical competencies that incorporate 21st Century Skills? In particular, we need to investigate ways in which we might effectively reduce the tendency for national and international assessments to become the primary levers for learning in the operational curriculum, and enable them to play a more supportive role. For example, how might we capitalise on and extend national assessment items, such as those involving statistical representations, and incorporate them within modelling and problem-solving experiences?

Of particular concern, though, in rebalancing the focus on testing are issues pertaining to the inclusive practices in mathematics education as examined in Faragher et al.'s chapter (Chap. 7, this volume). In citing Grootenboer and Sullivan's (2013) study, in which instruments were developed for assessing Indigenous students within their own contexts, it was revealed that the apparent under-achievement of these students in formal tests "may be due to the relevance and veracity of the assessment instrument" (Grootenboer & Sullivan, 2013, p. 181). Grootenboer and Sullivan's warnings are especially worth noting, namely, "there are real concerns about national testing regimes that discriminate against some students, and the use of these flawed results to make claims about the students' mathematical (or other subjects) knowledge and understandings" (p. 184).

6.2 Lifting the Profile of Mathematics Across the STEM Landscape

As discussed in this chapter, the increased focus on STEM education has generated concerns regarding the presence and role of mathematics. I have argued for the need to lift the profile of mathematics across the STEM landscape and have explored problem-solving and modelling as one means of achieving this. Statistical reasoning has featured prominently in the modelling experiences I have implemented in schools. Dealing effectively with statistics is essential across all the STEM disciplines, where a facility in handling uncertainty and data is central to making evidence-based decisions involving ethical, economic, and environmental dimensions (Office of the Chief

Scientist, 2013). The increasing need to handle contradictory and potentially unreliable online data is also critical (Lumley & Mendelovits, 2012). Given that many nations are striving to achieve social, cultural and economic prosperity in dealing with a rapidly changing and insecure world, greater recognition needs to be given to the foundational role of mathematics, in particular working with data, in building the required knowledge base.

6.3 Developing 21st Century Skills

Of the four broad areas of employer-desired skills identified in *The Partnership for 21st-Century Skills* (2015) document, learning and innovation are especially relevant to promoting mathematics education within a STEM climate. These skills are further subdivided into three categories: creativity and innovation, critical thinking and problem-solving, and communication and collaboration, all of which I consider worthy of further attention from our mathematics education community. An increased focus on critical thinking, various forms of reasoning, systems thinking, and the making of informed and evidence-based judgements and decisions would seem especially required. Likewise, with the *Programme for International Student Assessment (PISA) 2015 Draft Collaborative Problem-solving Framework* (OECD, 2013b, p. 13), there emerges further areas for attention although many of the skills and problem-solving competencies and contexts identified in the PISA document have already been explored in the present chapters. The collaborative perspective on problem-solving, however, raises further interesting research agendas.

6.4 Targeting Computational Thinking and Coding

The international push for developing students' computational thinking and coding from the earliest grades calls upon our discipline to play a greater role in this development. There appear clear links between early coding, for example, and mathematics learning in the preschool and beginning school years. Developing young children's coding skills incorporates among others, sequencing, pattern recognition, deductive reasoning, numerical reasoning, data structures and representations, and functions (Liukas, 2015). Establishing such foundational links between early coding and mathematics learning appears not to be receiving the required attention and is clearly an area demanding substantial research.

Many avenues for research await our mathematics education community not only in this domain of computational thinking and coding but also in many others. It will be interesting to see the themes addressed in the next *RiMEA* book, and ways in which the research landscape might have changed during this review period. I will not attempt to anticipate what these changes might be, except to wish that research will have facilitated ways to increase access to mathematics education for a wider range of students. It is also hoped that there is a greater community awareness and appreciation of mathematics in its foundational roles across the STEM domain.

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