Katie Makar · Shelley Dole
Jana Visnovska · Merrilyn Goos
Anne Bennison · Kym Fry
Editors

Research in Mathematics Education in Australasia 2012–2015

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Editors and Contributors

About the Editors

Katie Makar is Associate Professor of Mathematics Education at The University of Queensland. Her research aims to improve classroom practices in teaching of mathematics through inquiry. She is known internationally for her work on statistical reasoning and informal statistical inference at the primary school level. Katie served on the MERGA Executive and on editorial boards of two statistics education journals. Previously, she was a secondary mathematics teacher for 15 years in the USA and in international schools across Asia.

Shelley Dole is Professor and Head of School of Education at the University of the Sunshine Coast. She has been involved in several research projects that focus on embedding numeracy across the curriculum and the professional development of teachers. She has also been exploring in depth the importance of proportional reasoning as essential to numeracy. Her other research interests are children's mathematics thinking and learning, inquiry pedagogy, mental computation, as well as rational number curriculum in the middle and early years of schooling.

Jana Visnovska is Lecturer of Mathematics Education at The University of Queensland, Brisbane, Australia. Her research focuses on ways to effectively support mathematics teachers' professional development. To this end, she is interested in ambitious and equitable classroom instructional practices, and in the design of environments and teaching resources that support teachers' development of such practices. As part of *Inspiring mathematics and science in teacher education* project, she investigates ways to ignite interest in mathematics teaching and learning with primary and non-specialist secondary pre-service teachers.

Merrilyn Goos is Professor and Head of the School of Education at The University of Queensland. She has co-authored chapters in three previous MERGA research reviews: on digital technologies in mathematics education, the pre-service education of teachers of mathematics, and tertiary mathematics education. In addition to her current research interest in numeracy across the curriculum, she fosters collaborations between mathematicians and mathematics educators to support interdisciplinary research and development in university mathematics teaching and pre-service teacher education.

Anne Bennison is nearing completion of her doctoral studies at The University of Queensland in which she used a sociocultural approach to identify ways to support teachers to embed numeracy across the curriculum. She has published articles in major mathematics education research journals and reviewed for two of these journals. Anne has contributed to MERGA conferences as an editor of the proceedings, and as a member of organising committees and review panels. She was awarded an Australian Association for Research in Education Postgraduate Student Research Award in 2013.

Kym Fry is a Ph.D. student at The University of Queensland who is completing her study into assessing learning in inquiry mathematics classrooms. She is a teacher in the primary classroom and with 13 years of experience, has also taught at the university level for the past 7 years, working with pre-service teachers. Her involvement in a number of research projects has seen her work in the capacity of research assistant providing her with experiences seeing research projects in action in mathematics classrooms all over the south-east Queensland.

Contributors

Judy Anderson is Associate Professor of Mathematics Education and Director of the STEM Teacher Enrichment Academy at the University of Sydney. She has conducted research into middle years' students' motivation and engagement and teachers' knowledge and understanding of problem solving approaches to the teaching and learning of mathematics. As an advocate for innovative curriculum, with a particular interest in teachers as designers of integrated STEM curriculum, she is currently President of the Australian Curriculum Studies Association (ACSA).

Glenda Anthony is Professor of Mathematics Education and Co-Director of the Centre of Research in Mathematics Education at Massey University. Her research interests included effective pedagogies in the mathematics classroom, teacher education reforms, and teacher learning. Glenda is a recipient of the MERGA Career Research Medal and Fulbright-Harkness New Zealand Fellowship. She is a member of multiple editorial boards, and an Associate Editor of the MERGA *Mathematics Education Research Journal*.

Catherine Attard is Associate Professor of Primary Mathematics and Senior Researcher at the Centre for Educational Research at Western Sydney University. Her areas of research are engagement with mathematics, the effective use of mobile technologies to enhance the teaching and learning of mathematics, the effectiveness of sustained professional development on building teacher capacity and improving student learning outcomes and the use of financial literacy education as a tool to engage children with mathematics.

Bill Atweh as Adjunct Professor Curtin University, Visiting Professor Philippines Normal University and Professor Extraordinarious at University of South Africa, has successfully led six international teams in the publications of six books in mathematics and science education with contributions from many countries. He has been principal author of several literature review chapters on topics of direct relevance to equity and access to education for the Mathematics Education Research Group of Australasia. He was a Vice President (Publication) for MERGA for over 6 years.

Robin Averill works in initial teacher education at Victoria University of Wellington, Te Whare Wānanga o Te Ūpoko o Te Ika a Māui where she is Associate Dean (Teacher Education). Robin's research focuses on ways to enhance equity of access to mathematics achievement particularly in relation to indigenous Māori and Pasifika learners. She grounds her work in classroom practice and in the views of teachers and students. Robin is currently editor of *Mathematics Teacher Education and Development*.

Kim Beswick is Professor of Mathematics Education and ARC Future Fellow at the University of Tasmania. Her research interests centre on the beliefs and knowledge that underpin the practice of teachers of mathematics and how professional learning can catalyse change. She is particularly interested in how teacher beliefs and knowledge relate to their expectations of and aspirations for their students. She is a member of the International Committee of the International Group for the Psychology of Mathematics Education.

Janette Bobis is Professor of Mathematics Education at the University of Sydney and Pro Dean Research in the Faculty of Education and Social Work. Her teaching, research and publications focus on the development of primary and middle years teachers' knowledge, beliefs and practices; and on student learning, particularly the development of their computational thinking strategies, their motivation and engagement in mathematics and the impact of teaching interventions on each of these aspects.

Jill Brown is Senior Lecturer of Mathematics Education at the Australian Catholic University (Melbourne campus). She has taught mathematics at secondary school and university, and researched its learning and teaching in both of these domains and in primary mathematics education. Her current research interests include mathematical modelling as real-world problem solving, mathematical thinking and the use of digital technologies in teaching and learning mathematics.

Nigel Calder is Senior Lecturer at University of Waikato. His research interests include the use of digital technologies in mathematics education. His present research projects are: the influence of mobile technologies on the engagement of 16–18 year-old reluctant learners; the use of apps in primary school mathematics;

and how mathematical thinking emerges through student-centred inquiry learning in secondary schools. He has published various book chapters and journal articles as well as authoring *Processing Mathematics through Digital Technologies: The Primary Years*.

Rosemary Callingham is Associate Professor of Mathematics Education at the University of Tasmania. She has an extensive background in mathematics education in Australia, at school, system and tertiary levels. Her experience includes classroom teaching, mathematics curriculum development and implementation, large-scale testing, pre-service teacher education, and supervision of research students. She has numerous publications including books, book chapters and journal articles. Her research interests include statistical literacy, teachers' pedagogical content knowledge, assessment, and uses of technology in mathematics.

Ban Heng Choy is Assistant Professor of Mathematics Education at the National Institute of Education in Singapore. As part of his Ph.D. study at the University of Auckland, he developed a theory for describing and analysing what and how mathematics teachers notice when making instructional decisions that promote students' reasoning. His research interests include use of technology in teaching and learning, teacher beliefs and knowledge, assessment in mathematics, and teacher professional development.

Barbara Clarke is Adjunct Associate Professor of Mathematics Education at Monash University (Peninsula Campus). The major focus of her writing and research has been concerned with mathematics teachers, their practice, and their professional development. She has also worked on a number of research projects that focus on the teaching and learning of children with Down syndrome, particularly related to mathematics education.

Audrey Cooke is Lecturer of Mathematics Education at Curtin University. She is interested in how people interact and engage with mathematics, particularly pre-service teachers. Her research includes disposition towards and engagement with mathematics, relationships between disposition and mathematics classroom discourse, and use of ICT in education. Audrey is a reviewer for journals and conferences and contributes to mathematics education conferences nationally and internationally.

Mary Coupland is Senior Lecturer in the School of Mathematical and Physical Sciences at the University of Technology Sydney and Leader of SciMERIT: Science and Mathematics Education Research and Innovation Team. This group researches learning and teaching of science and mathematics at tertiary level. Recently she has been involved in evaluations of mastery learning for first-year mathematics subjects. Other interests include activity theory, curriculum design, and collaborations with engineering educators.

Sue Dockett is Professor, Early Childhood Education at Charles Sturt University, Albury. Sue's research interests include educational transitions, particularly the transition to school, participatory rights-based research involving children and young people, children's play, and early mathematical experiences.

Peter K. Dunn is Associate Professor of Biostatistics at the University of the Sunshine Coast, where he teaches large introductory statistics courses. He has a Ph.D. in Mathematical Statistics from the University of Queensland, and has previously worked as a statistical climatologist. His current research interests include teaching for statistical literacy.

Cris Edmonds-Wathen is Postdoctoral Researcher in the Umeå Mathematics Education Research Centre at Umeå University, Sweden. Her research interests include how mathematics is expressed in different languages. She is working on a project to improve the teaching of cultural mathematics in Papua New Guinea. Previously she has investigated spatial language in Iwaidja, a language of northern Australia, and taught both indigenous school students and indigenous pre-service teachers.

Lyn D. English is Professor at Queensland University of Technology. Her research areas include mathematics education, engineering education, and STEM education more broadly. Specifically, she undertakes research in mathematical modelling, problem solving and posing, statistics education, and early mathematical reasoning and development. Professor English was awarded the *Mathematics Education Research Group of Australasia Career Research Medal*, 2012, and the Vice Chancellor's Award for Excellence in Research in 2015. She is a Fellow of *The Academy of the Social Sciences in Australia*, and is founding editor (1997) of the international journal, *Mathematical Thinking and Learning*.

Rhonda Faragher Ph.D. is an academic teacher and researcher working in the fields of mathematics education and disability studies. Having always enjoyed learning mathematics, she has worked throughout her teaching and research career to find ways to assist others to experience success with learning mathematics as well. Her focus on the mathematics attainment of learners with intellectual disabilities became central to her work following the birth of her daughter with Down syndrome in 1996.

Helen Forgasz is Professor of Education in the Faculty of Education, Monash University. Her research expertise is in the field of equity and mathematics (and STEM) education, with a particular focus on gender issues. She has won several Australian Research Council (ARC) Discovery and Linkage grants and her work has been published widely in books and book chapters, and in scholarly and professional journals.

Peter Galbraith is Honorary Professor at The University of Queensland. He has taught mathematics at secondary school and university, and researched its learning and teaching in both of these domains. His current research interests focus around mathematical modelling as real-world problem solving, an activity which crosses some conventional educational boundaries. Peter is a past president of MERGA and of ICTMA, and is a recipient of the MERGA Career Research Medal.

Linda Galligan is Associate Professor with the Faculty of Health Engineering and Sciences at the University of Southern Queensland. After teaching high school and bridging mathematics for many years, she now teaches into first-year mathematics at university, and has strong links with schools providing mathematics programs and activities for students and teachers. Her research includes language and mathematics; and students and lecturers' perceptions of student preparation for numeracy demands of university. Her Ph.D. investigated a model of embedding academic numeracy in university programs using nursing as a case study. More recently her focus has been on the use of Tablet technology to effectively teach mathematics; and using modelling to improve pre-service teachers' deep understanding of mathematics.

Vince Geiger is Associate Professor and Research Fellow within the Learning Sciences Institute of Australia at Australian Catholic University. His research interests include the development of students' numeracy capabilities across all learning areas; teacher professional learning in relation to effective numeracy pedagogy; the teaching and learning of modelling and its applications; and the use of digital tools to enhance mathematics instruction. He is currently Associate Editor of the *Mathematics Education Research Journal*.

Wendy Goff is Lecturer at the University of Southern Queensland. She currently teaches mathematics education in the bachelor and masters courses. Throughout her career Wendy has worked in a variety of educational settings. Wendy's current research focuses on how adults come together to support the mathematical learning of young children.

Jane Greenlees completed her Ph.D. in Primary School Mathematics with a focus on assessment and test item design. After several years of continuing research in this field as well as training pre-service teachers, she has returned to the classroom and is currently enjoying the many challenges and rewards of being a primary school teacher.

Peter Grootenboer was a school teacher and leader for 12 years before moving into the tertiary sector. He received a national Jim Campbell Award for teaching excellence. He is now Associate Professor of Education at Griffith University and a member of the Griffith Institute for Educational Research. His research interests include mathematics education, educational leadership, practice/praxis theory, and action research. He recently published the book *Mathematics, Affect and Learning*.

Joanna Higgins is Associate Professor in the Faculty of Education at Victoria University of Wellington, New Zealand and specialises in mathematics education and teacher professional inquiry. Recent work from sociocultural perspectives includes facilitator knowledge and practice, the design of professional development for teachers, mindfulness-based breathing interventions in classrooms and emotional engagement in teaching and learning mathematics.

Janelle Hill is a researcher and Lecturer of Mathematics Education at Monash University. Her research focus is on gender, technology and the teaching and learning of mathematics and literacy. She has been part of a number of research projects related to gender, inclusion and indigenous students and mathematics learning.

Jodie Hunter is Senior Lecturer of Mathematics Education at Massey University, at the Albany Campus in New Zealand. She completed her Ph.D. in the University of Plymouth and this work focused on how primary teachers could learn to develop early algebraic reasoning with their students in primary classrooms. Her research interests include early algebraic reasoning, practice-based pedagogy in teacher education, the development of culturally responsive teaching for Pāsifika students and home/community and school partnerships.

Roberta Hunter is Associate Professor at Massey University in the Institute of Education. Over the past 15 years she has actively engaged in large research and professional development projects which support teachers to develop culturally responsive, ambitious mathematics pedagogy, particularly with Pāsifika and Māori students in high poverty areas. Her research focuses on mathematical practices and culture, language, and identity linked to student participation in mathematical reasoning and communication.

Chris Hurst teaches primary mathematics education at Curtin University. He is an active researcher with interests in teacher content knowledge, children's multiplicative thinking, and characteristics of effective schools. Chris regularly presents at national and international conferences and was recently involved in the national project Building an Evidence Base for Best Practice in Mathematics Education and the CEMENT project. He also enjoys providing research-based professional learning for teachers and working with teachers as part of his research work.

Naomi Ingram is an experienced mathematics teacher in the secondary sector. She is now lecturing and researching in primary and secondary mathematics education at the University of Otago, College of Education. She won the Bevan Werry Speaker Award and the MERGA Early Career Award for work related to her Ph.D. on students' relationships with mathematics. Her research interests include affect, identity, engagement, and the use of mobile technologies and challenging tasks in mathematics classrooms.

Robyn Jorgensen is Professor of Education: Equity and Pedagogy at the University of Canberra. She has spent her career working in the most disadvantaged schools seeking to identify practices that marginalise particular groups of learners, and to find ways to bring about success in mathematics for those students who are most at risk of failing. Her work focuses on teaching practices and how these can be developed for the needs of the learners while ensuring that success is achieved.

Virginia Kinnear is Lecturer of Early Childhood at Flinders University, South Australia and has taught at the University of South Australia and the University of Tasmania. She was a lawyer prior to studying and teaching Montessori education (3–6 years) in the USA and early childhood education in Australia. Virginia's doctoral research investigated young children's statistical reasoning in the early school years. Her research interests include young children's statistical and mathematical learning, mathematics curriculum and the philosophical dimensions of education.

Janeen Lamb is Associate Professor of Mathematics/Leadership in the Faculty of Education and Arts, and Director of Higher Degree Research at Australian Catholic University. Her research has focused on the leading of change in mathematics education from early years through to secondary schooling. She is currently working with school leaders on projects that are investigating their role in leading educational change in mathematical modelling, numeracy across the curriculum, and the integration of digital resources into the mathematics curriculum.

Kevin Larkin is Lecturer in mathematics education at Griffith University. He is a member of a number of research teams investigating mathematics education in primary and middle school contexts and has published widely in national and international publications in the areas of mathematics education, ICT, school leadership, and activity theory. He is the current Vice President Communications for the Mathematics Education Research Group of Australasia (MERGA).

Gilah Leder is Adjunct Professor at Monash University and Professor Emerita at La Trobe University. Her research has focused particularly on gender issues in mathematics education, on exceptionality—predominantly high achievement, and on affective factors. Gilah is a past President of MERGA and PME, a Fellow of the Academy of the Social Sciences in Australia, the recipient of the Felix Klein medal awarded for her outstanding lifetime achievements in mathematics education research and development and recipient of the 2013 MERGA Career Research Medal.

Tracy Logan is Assistant Professor at the University of Canberra. She is currently completing her Ph.D. on secondary data analysis in mathematics education. Tracy has been actively involved in a number of Australian Research Council grants in various key support roles and is currently part of a team of researchers investigating children's mathematics sense making on assessment tasks in digital and non-digital environments.

Esther Loong is Lecturer of Mathematics Education at Deakin University. She has expertise in the uses of Information and Communications Technology (ICT) and cloud technology for researching and teaching mathematics in primary, secondary and tertiary mathematics education. Her research and teaching is informed by theories of interactivity between the organism and the environment. Because of this theoretical perspective, her approach to research is often field-based. She has worked with people from diverse ethnic backgrounds and brings high-level intercultural skills to her teaching and research.

Tom Lowrie is Centenary Research Professor at the University of Canberra, and the current President of MERGA. His body of work has focused on the extent to which primary-aged students use spatial reasoning and visual imagery to solve mathematics problems and the role and nature of graphics in mathematics assessment. More recently, his research has expanded to include students' use of digital tools and dynamic imagery to solve problems.

Amy MacDonald is Senior Lecturer of Early Childhood Studies at Charles Sturt University, Albury-Wodonga and a member of CSU's Research Institute for Professional Practice, Learning and Education. Amy's research interests are around the mathematics experiences and education of infants, toddlers, preschoolers, and children in the early years of primary school; with a particular focus on transitions in mathematics education. Amy currently coordinates the early childhood mathematics education subjects in CSU's Bachelor of Education (Birth to Five Years) program.

Dr. Colleen McMurchy-Pilkington has recently moved into the field of Adult Literacy and Numeracy after spending 10 years teaching in primary schools, and 20 years as principal lecturer in Teacher Education at the University of Auckland. She has written on low decile schools, exploration of optimal learning environments for Māori learners, ethnomathematics, mathematics education for Māori and indigenous learners. Research interests include Māori education, policy development, socio-political and cultural aspects of education, and mathematics education.

Tribal Affiliation: Ngati Pikiao, Ngati Rongomai

Tamsin Meaney is Professor of Mathematics Education at Bergen University College. She has worked with indigenous education since 1990, when she worked as a teacher on a remote Aboriginal community. She has worked with indigenous educators in Australia, New Zealand and Norway. Her continuing interests include indigenous language and cultural revitalisation through mathematics education and professional development with indigenous teachers.

Jodie Miller is an Early Career Research fellow in the STEM in Education program at the Learning Sciences Institute Australia (LSIA) at Australian Catholic University. Her research areas include early algebra, semiotics, and mathematics teaching and learning in disadvantaged contexts. Jodie was awarded the 2015 MERGA early career research award.

Tracey Muir is Senior Lecturer of Mathematics Education at the University of Tasmania. Her research interests include effective teaching of numeracy, enhancing teaching practice through the use of ICT, parental involvement in mathematics education and pre-service teacher content and pedagogical content knowledge. She is a past Vice President (Development) of MERGA, a member of two journal editorial boards, and a regular journal reviewer. She has also published extensively in mathematics education and is a regular contributor at state, national and international conferences.

Kit Ee Dawn Ng is a Lecturer in the Mathematics and Mathematics Education Academic Group at the National Institute of Education, Singapore. She holds a Ph.D. in Mathematics Education from the University of Melbourne, Australia. She is involved in teacher professional development at both primary and secondary levels. Her research interests include the use of contextualised tasks in the teaching and learning of mathematics and fostering students' mathematical thinking and metacognition.

Greg Oates is Senior Lecturer of Mathematics Education within the Mathematics Education Unit, Department of Mathematics at the University of Auckland, New Zealand. Previously Greg has taught secondary mathematics, and in July 2016 will take up a new role in the Faculty of Education, University of Tasmania in Launceston. His current interests include the effective integration of technology and the curriculum in undergraduate mathematics; professional development for university teachers and pedagogical and technology knowledge for teaching (PCK & PTK) at the undergraduate level.

Bob Perry is Adjunct Research Professor, following his 'retirement' from Charles Sturt University in February, 2016. He continues to supervise doctoral candidates and conduct research projects through CSU, as well as through Peridot Education Pty Ltd. Bob's main research interests are in educational transitions, early childhood mathematics education, evaluation of educational programs and the education of indigenous children and young people. In 2015, he was awarded an honorary doctorate from Mälardalen University, Sweden and the MERGA Career Research Medal.

Penelope Serow is Associate Professor of Mathematics Education at the University of New England. She has taught in the contexts of primary, secondary, pre-service teacher education, and postgraduate studies. She is Project Leader of the Nauru Teacher Education Project and this experience has informed her research directions. These include ICT as a teaching tool in the mathematics classroom, curriculum development in developing countries, meaningful assessment strategies, and building local teacher capacity in Pacific Island Countries.

Andrew Skourdoumbis is Senior Lecturer and Early Career Researcher in Education (Pedagogy and Curriculum) at Deakin University, Australia. His recent research engages with matters of curriculum theory encompassing policy analysis, teacher practice and educational performance. He investigates global reform efforts in education that impact teacher practice and the way that exacting methods of research govern school education policy and teacher performance. Andrew has published widely in high-quality international education research journals and has a keen interest in teacher effectiveness research. As a former public high school teacher, Andrew is well versed in matters of public education and how research and policy impacts on teaching in classrooms.

Gloria Stillman is Associate Professor of Mathematics Education at the Australian Catholic University (Ballarat campus). She has taught mathematics at secondary school and mathematics, statistics and mathematics education at university. Her current research interests include a focus on mathematical modelling as real-world problem solving and metacognition. Gloria is a past vice president of MERGA in research and publications and is currently the president of the International Community of Teachers of Mathematical Modelling and Applications (ICTMA).

Hazel Tan is Lecturer in the Faculty of Education at Monash University. Her research focuses on teaching and learning secondary mathematics with technologies and related teacher education, comparative education and gender issues in mathematics education. She has expertise in using mixed-methods approaches in research. Hazel is familiar with educational practice and policy-making, having been in various roles as a mathematics teacher, a head of mathematics department, and an educational officer at Singapore's Ministry of Education.

Dr. Steve Thornton works at the Australian Academy of Science as Executive Director of Mathematics by Inquiry, an Australian government funded project to promote inquiry approaches to mathematics in schools. He has previously lectured in mathematics teacher education at the University of Oxford, the University of Canberra and Charles Darwin University. His research interests include teacher knowledge, mathematics curriculum and the intersection of mathematics education and philosophy.

David Tout has worked in schools, Technical and Further Education institutes, community providers, universities, migrant education and industry with a major focus on adult numeracy and mathematics education. He has wide experience not only in teaching and training, but also in working at a state, national and international level in research, curriculum, assessment and materials development. Dave was involved in the numeracy conceptual framework development and test development for PIAAC, and helped coordinate the mathematical literacy component of PISA 2012.

Sven Trenholm is Lecturer of Mathematics Education. He received his Ph.D. (Studentship) from Loughborough University (UK), M.Sc. (Distinction) from the State University of New York at Albany (US) and Graduate Diploma in Teaching as well as B.Sc. from McGill University (Canada). His research is situated in the intersection between the fields of mathematics education and instructional technology where he is primarily interested in the development of mathematical thinking in fully asynchronous online instructional contexts.

Tony Trinick is a former primary and secondary school teacher who for the past 20 years has worked in tertiary education, lecturing graduate and postgraduate students in Maori-medium mathematics. He has been commissioned by various NZ governments to develop and guide the development of national curricula for Maori-medium schooling. His primary research focus area is on the ideologies/politics associated with curriculum development for indigenous peoples, curriculum development as de facto language planning and the complex relationship between mathematics and language.

Tribal Affiliation: Te Whānau-a-Apanui

Colleen Vale is Associate Professor of Mathematics Education at Deakin University. Her research concerns equity and social justice in mathematics education and current and recent research projects focus on out-of-field STEM teachers, different models of teacher professional learning such as Japanese Lesson Study and demonstration lessons, mathematical reasoning, inquiry-based pedagogies, and pre-service secondary teacher education. Colleen is a former MERGA Vice President Publications and former editor of *Mathematics Teacher Education and Development*.

Margaret Walshaw is Professor in the Institute of Education at Massey University, New Zealand. She has taught mathematics at the secondary and tertiary levels. Her research interest is in developing theory to explain processes in mathematics education and to understand effective and equitable practices. The focus of that interest is identity formation, reflective practice, self/other relationships, and ethical deliberation in mathematics teaching and, more broadly, on mathematics education practices.

Jennifer Way is Associate Professor of Mathematics Education at the University of Sydney in the Faculty of Education and Social Work. Her work has focused on the development of conceptual understanding in problematic areas of mathematics learning for both teachers and students (early-years to middle-years), particularly regarding contextual factors such as digital technologies, and motivation and engagement. Current research projects are centred on the role of mathematical representation in conceptual understanding.

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Chapter 1
Introduction: Research in Mathematics Education in Australasia 2012–2015

Katie Makar, Shelley Dole, Jana Visnovska, Merrilyn Goos, Anne Bennison and Kym Fry

Abstract This chapter presents an overview of the most recent volume in a series of reviews entitled *Research in Mathematics Education in Australasia*. Each of MERGA's four-yearly reviews proudly highlights and critiques the research in mathematics education in Australasia over the previous 4 years. In this chapter, we provide an overview of the history of the four-yearly review and explain the processes of how this review was brought together. Each chapter is briefly introduced within four key themes: Issues and contexts for mathematics education; learning and teaching; teacher preparation and development; and the future.

Keywords Research in mathematics education in Australasia · RiMEA · MERGA review · Mathematics teaching and learning · STEM

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1 The Research in Mathematics Education in Australasia Series

Research in Mathematics Education in Australasia 2012–2015 is the ninth four-yearly review of research in mathematics education organised by the Mathematics Education Research Group of Australasia (MERGA). The first review was published in 1984 (Briggs 1984) to coincide with Australia's hosting of the fifth International Congress on Mathematical Education (ICME) in Adelaide. Brigg's *Summary of research in mathematics education in Australia* was intended to showcase mathematics education research being done in Australia to the broader international research community. A compilation of themed chapters reviewing and critiquing research in the region every 4 years, to be launched in time for ICME, was continued. The volume series Research in Mathematics Education in Australasia (RiMEA) is now a well-established publication by MERGA. RiMEA (also referred to as the MERGA *Research Review,* or simply the *Review*) is a resource for the MERGA community, for those outside of the region as a focused review of recent Australasian research—and key Australasian researchers—in mathematics education and is frequently the first contact for new Australasian researchers in mathematics education.

Briggs' (1984) initial summary targeted Australia. Since then, the review summary extended from Australia to Australasia, reflecting a change in MERGA's membership. The definition of Australasia has shifted since then and the version used in the previous two RiMEAs was:

> This review, entitled Research in Mathematics Education in Australasia 2008–2011, uses the same definition of Australasian mathematics education research as the previous one did: "The editors have defined 'Australasian research' as research conducted in Australasia, about the Australasian context, or by Australasians. Australasia comprises: Australia, New Zealand, Papua New Guinea, and the Pacific Islands closely allied to Australia and/or New Zealand. (Forgasz et al. 2008, 1–2)" (Perry, Lowrie, Logan, MacDonald, & Greenlees, 2012, p. 2)

However, since 2008 Singaporean researchers have had a larger presence in MERGA, hosting the annual conference in 2012. Therefore, for the current RiMEA, the following description was given to authors to describe the regional focus:

> "Australasia" primarily refers to Australia and New Zealand. However, papers published in MERGA conference proceedings and articles published in MERGA journals by researchers from countries in the South Pacific and south-east Asian regions and with particular relevance to these regions should also be considered for inclusion in the review.

The purpose of RiMEA, as expressed by the previous editors (Perry et al., 2012), is to "highlight significant findings, demonstrate links among research, identify trends and foreshadow possible future research directions" (p. 2). As with previous RiMEA volumes, only Australasian research readily accessible (e.g., books, book chapters, national or international journal articles, papers in refereed conference proceedings, presentations at major national or international conferences and research higher degree theses) with a publication date in the 2012–2015 window were included. Since it is impossible to report on all publications of Australasian mathematics education research in this period, chapter authors were asked to be selective in the research they reported.

Previous volumes of RiMEA were used as a model for the current volume where relevant in order to preserve a sense of continuity—past, present and future—in the series of RiMEA volumes. The previous publication dates of MERGA's four-yearly reviews and their editorial teams were:

- 2012: Perry, Lowrie, Logan, MacDonald, and Greenlees
- 2008: Forgasz, Barkatsas, Bishop, Clarke, Keast, Seah, and Sullivan
- 2004: Perry, Anthony, and Diezmann
- 2000: Owens and Mousley
- 1996: Atweh, Owens, and Sullivan
- 1992: Atweh and Watson
- 1988: Blane and Leder
- 1984: Briggs

2 Process of Development

The current editorial team was selected by the MERGA Executive from a pool of expressions of interest following a call for editors in 2013. The current editorial team are all MERGA members who (at the time of proposal) were in the School of Education at The University of Queensland. To continue the tradition of building capacity in the field, the team included three experienced editors and three early career researchers, two of which were PhD students.

The proposal by the current team contained 16 chapters: an introduction by the current editorial team; a reflection from the editorial team of RiMEA 2008–2012; 13 chapters across three key themes—Issues and contexts for mathematics education, Learning and teaching, and Teacher preparation and development; and a concluding reflective chapter by a distinguished member of the MERGA community. The selection of chapters was prioritised based on consideration of recurring topics in previous RiMEAs (important for tracking shifts in the field over time) and areas of pressing importance in the region. For example, a chapter on theories in mathematics education research (see Chap. 3, this volume) was proposed as a broader version of the chapter in RiMEA 2004–2007 on sociocultural perspectives. In the previous *Review,* there was not a separate chapter on political perspectives; however, recent shifts in the political climate combined with the new Australian Curriculum have had undeniable influences on the climate of Australasian research, prompting an inclusion of a chapter focused on this area (see Chap. 4, this volume). A single chapter originally proposed on equity was split into two chapters to accommodate strong and distinct proposals on social justice and inclusion (see Chaps. 6 and 7, this volume).

To recruit author teams for each chapter, a general call was made for author teams from the MERGA membership and encouragement of expressions of interest from key researchers. Proposed teams were urged to demonstrate their diversity in terms of experience, geographic location and gender. Some negotiations were required to balance expressions of interest against proposed chapters. Lead chapter authors met with the editorial team at the 2014 and 2015 MERGA conferences to promote discussion between chapters (e.g., potential gaps or overlaps), outline processes and clarify questions.

Chapters went through internal and external review, with each chapter receiving collated (non-blind) "collegial feedback" from three experts in the field plus at least one member of the editorial team. Final chapters were formatted and copyedited by Bronwyn Lacken, with final checks by the editorial team and authors made before being sent to the publisher. The first six volumes in the series were published by MERGA; 2008 was the first publication of RiMEA by an established international publisher (Sense Publishers). Springer was selected by the MERGA Executive to publish this volume following Springer's handling of the *Mathematics Education Research Journal*.

3 Overview of Chapters

Four sections comprise *Research in Mathematics Education in Australasia 2012–2015*:

- Issues and Contexts for Mathematics Education
- Learning and Teaching
- Teacher Preparation and Development
- The Future

3.1 Issues and Contexts for Mathematics Education

The seven chapters in this section highlight the common issues and contexts that affect researchers in mathematics education. A reflection by Perry, MacDonald, Greenlees, Logan, and Lowrie, the editorial team of the previous volume, Research in Mathematics Education in Australasia 2008–2011, provides a starting point for this section. The chapter *Reflections on the MERGA Research Review 2008–2011: Taking stock* reminds readers of the structure and themes of the previous 4-yearly review and identifies a set of major policy initiatives that provide a context for the chapters in the current review. The five policy areas discussed are (1) the early childhood reform agenda, (2) national curricula, (3) national and international assessment, (4) teacher accreditation, and (5) Closing the Gap. While the authors acknowledge that other initiatives could have been included, their selection affords an astute and dynamic analysis of changes in the political and educational environments of Australia and New Zealand since the last review. Their discussion also highlights the extent to which these changes have influenced mathematics education research in the past 4 years, and anticipates many of the research themes emerging in the chapters that follow.

An empirical reflection on the philosophical underpinnings of Australasian research in the period of the current review was undertaken by Thornton, Kinnear and Walshaw in the next chapter. *A philosophical gaze on Australasian mathematics education research* provokes the reader to consider ontological, epistemological, aesthetic, ethical and logical questions underpinning research that are sometimes overlooked. In their “Philosophical Gaze” of research in 2012–2015, Thornton et al. took a hermeneutic approach to seek insight into authors’ philosophical position. To do this, they conducted three investigations. The first analysed keywords of research reported in major international journals read by mathematics education researchers. Then, taking a sample of 26 papers across different topics in Australasian research, they used a framework to analyse and make inferences about these papers in terms of ontological, epistemological, aesthetic, ethical and logical perspectives these papers conveyed (often tacitly). Finally, the authors took a close examination of explicit use of the term “epistemology” in over 130 Australasian mathematics education research articles they collected to analyse how authors expressed this aspect of their philosophical stance and (if relevant) an implied ontological stance. What stood out in these authors’ analyses was how issues of aesthetics (values), ethics and logic are often left unspoken and hence assumed. The chapter closes by reminding us to regard philosophical tensions as creative forces for our work rather than divisions.

The chapter on *Researching curriculum, policy and leadership in mathematics education* by Way, Bobis, Lamb and Higgins provides a stimulating analysis of how researchers have investigated the interrelationships within and between these topics. Recent curriculum development initiatives, particularly in Australia and New Zealand, have motivated much of this research. The chapter is framed by Remillard and Heck’s (2014) model of curriculum policy, design, and enactment, which distinguishes between factors influencing the official curriculum and the operational curriculum. This model allows the chapter authors to organise their discussion of the different curriculum elements as well as to situate within this framework research on issues such as curriculum policy, assessment and testing policy, textbooks, curriculum leadership, curriculum differentiation and equity, and numeracy across the curriculum. The research reviewed in this chapter revealed many mismatches and tensions between the official and operational curriculum, which often resulted from the political motivations behind curriculum reform and national testing. Greater attention to educational leadership was urged by many researchers as a means of assisting teachers to enact the curriculum in ways that support diverse learners in different contexts. Largely as a result of recent curriculum reforms, research is now focusing on the operationalisation of the official curriculum and the demands this places on teachers. Because of the centrality of the teacher in Remillard and Heck’s model, the chapter authors draw attention to the potential role of school and classroom-based research in influencing national and system-level curriculum decisions.

Mathematics education and the affective domain is the fourth time that a chapter has dealt with affective issues in mathematics education in RiMEA. This chapter is concerned with beliefs, attitudes, identity, anxiety, and engagement and motivation. Although gender is also discussed in Chaps. 6 and 7 of this review, current concerns about the under representation of females in advanced and intermediate level mathematics subjects in schools and Science, Technology, Engineering, and Mathematics (STEM) fields led Attard, Ingram, Forgasz, Leder and Grootenboer to include a section that reviewed research concerned with the relationship between gender and affective issues. The chapter reports on increased attention to research on student engagement and attitudes towards mathematics in this review period. However, several areas were identified as having little research including affective issues about mathematics in primary and early childhood education, links between student engagement and mathematics achievement, and affective issues related to pedagogy. The influence of technology on affect was also identified as needing further research, particularly given that technology in now an integral part of teaching and learning mathematics.

In the next chapter, Vale, Atweh, Averill and Skourdoumbis critically examine theoretical stances and types of research conducted in equity, social justice and ethics in mathematics education. Socio-economic contexts, rural and remote status, and ethnic and language context of school communities, as well as gender, are rich and relevant foci of much of Australasian mathematics education research. The chapter *Equity, social justice and ethics in mathematics education* highlights that while identification and description of conditions in which mathematics education happens is indeed important if issues of social justice are to be advanced, a more proactive research agenda is also needed. Vale et al. bring attention to questions of how researchers can contribute to advancing inclusion agendas in mathematics education and discuss research designs that might be appropriate in doing so. The reviewed literature collectively illustrates that addressing this issue requires coordination of all levels at which mathematical learning is organised, including need for (a) systems to fund and support school organisational structures, resources and cultures; (b) the development of purposeful partnerships between schools and their communities; (c) including awareness of equity, social justice and ethics issues throughout the education and professional learning of teachers of mathematics and (d) creating learning environments, with the help of teachers, that would focus on mathematical learning and knowledge building.

In Chap. 7, *Inclusive practices in mathematics education*, Faragher, Hill and Clarke organised their review around the main themes of access, learning and teaching, with reference to gender, learning difficulties, giftedness, location, and cultural and linguistic diversity. In relation to access, the authors overview Australasian research into the impact upon learners of school leadership, allocation of learners to classes, socio-economic status, and school location. The second part of the chapter looks at mathematics learning experiences from the point of view of learners. The chapter authors argue that an astute teacher capitalises on the knowledge that children bring and is mindful of listening and interpreting how children view mathematical situations. The affective domain, the use of technology and issues of assessment are addressed through consideration of diverse approaches to learning. The third section of the chapter specifically discusses inclusive teaching of mathematics. Direct instruction is critiqued, with research into differentiation through good tasks presented. The authors complete the chapter with recommendations for continued and further research, particularly around longitudinal studies to provide evidence of long-term effects of programs for inclusive practices in mathematics. The authors raise the question of out-of-field teachers, modification of assessment, the role of teacher aides and parents in the mathematics education of all learners. The chapter closes with a powerful statement about the absolute necessity of continually striving for access to quality mathematics learning experiences for all students.

The final chapter in this section, *Distribution, recognition and representation: Mathematics education and Indigenous students*, reviews the teaching and learning of mathematics for Indigenous students in the Australasia region over the last 4 years. Meaney, Edmonds-Wathen, McMurchy-Pilkington and Trinick use Nancy Fraser's (2005) model for social justice—distribution, recognition and representation—to structure the chapter through economic, cultural and political perspectives, respectively. Pedagogies to enhance learning and the language of teaching and learning mathematics are analysed using this social justice framework, including in relation to the mathematical topics of number, patterns and early algebra, probability, space and geometry. The chapter promotes building capacity in the field by reviewing research on professional development for Indigenous and non-Indigenous teachers about teaching mathematics. The authors further apply Fraser's social justice elements to evaluate research in this area and recommend that professional learning improves in being more ongoing and more collaborative. Finally, the authors discuss the importance of parent-community involvement in professional development projects to engage the representation element of social justice. A comprehensive body of Australasian literature related to the topic has been sourced and the authors have taken a deliberate and thoughtful critical stance to the often difficult and sensitive issues raised.

3.2 Learning and Teaching

The second major section of the *MERGA Research Review* includes six chapters which focus on specific aspects of teaching and learning. In previous *Reviews*, this section has focused on various ages and content areas. Indeed, the Section begins by bookmarking two age groups: early childhood and tertiary mathematics. A chapter on innovative pedagogies is followed by two topics—assessment and technology—that were in the "Contexts" section of the last Review, moved here to re-emphasise their strong links to learning and teaching. The section closes with a review of literature on modelling and applications in mathematics education.

In *Mathematics education in the early years*, MacDonald, Goff, Dockett and Perry structure the chapter around key themes of curriculum, assessment, content and contexts. Evident from this chapter is the publication of two significant edited books and the extensive research undertaken in Australasia through large commissioned projects. In relation to curriculum, the chapter emphasises political contexts of early childhood education through reference to Australia's and New Zealand's curriculum frameworks aimed at young students; it signals the slippage and overlaps between curriculum documents produced by different writers, overviewing Australasian authors who have researched in this space. Research into early years assessment highlights rich, conceptual mathematical understanding that young children bring to school, and accentuates the importance of celebrating what children can do, rather than only reporting on what they cannot. In relation to early learners' content knowledge, the review shows the research activity around algebra, measurement, number, and data, but a commensurate lack of focus on geometry. The activity around the use of inquiry for solving ill-structured problems with early learners emphasises the depth of reasoning that young children can demonstrate in such activities. Research into contexts in the early years is associated with environments (technologies, rural and remote locations, and exciting learning environments), the human context (parents), and the socio-cultural context (culturally sensitive practices). Whilst this chapter summarises intense activity in early years education research, its authors identify gaps including transitions from home to school, assessments prior to school, teacher professional learning, families, and young children's understanding of geometry and problem solving.

At the other end of the spectrum, the authors of *Tertiary level mathematics learning and teaching* document the continuing interest in and growing sophistication of research in this field. Conferences such as MERGA, Delta, ICME, OZCOTS and the ICMI Study series provide important fora for researchers at the tertiary level to share their work; an increase in journal publications and books also suggests a stronger theoretical foundation is being built. Coupland, Dunn, Galligan, Oates and Trenholm review research on the teaching of specific mathematics and statistics topics, which continues to capture the interest of many tertiary academics, while also noting where others are pursuing broader questions of curriculum and policy, for example, concerning how to embed quantitative skills in Science, Technology, Engineering and Mathematics (STEM) courses and degree programs. Likewise, tertiary mathematics research topics in education that appeared in previous MERGA *Research Reviews*—for example, learning support, service teaching, technology, and statistics education—are of continuing interest. New in this *Review* is research on the development of tertiary pedagogies and lecturer development, and a more strongly theorised investigation of transitions between school and university. The topic of transition has also been expanded to encompass transition to work, vocational education, and adult numeracy—all key policy spheres both internationally and nationally. Overall, the field is maturing through increasing involvement of cross-disciplinary teams of researchers who can share knowledge and expertise between mathematics academics and mathematics education academics, between university mathematics teachers and school teachers, and between the university, vocational education, government, and employment sectors.

In *Innovative and powerful pedagogical practices in mathematics education*, chapter authors B Hunter, J Hunter, Jorgensen and Heng overview advances in research on the powerful and innovative pedagogical practices aimed to enable all students to learn meaningful mathematics. The chapter is structured to discuss (1) innovative and powerful mathematical learning environments; (2) innovative practices which promote mathematics teaching and learning as inquiry; and (3) mathematical tasks that promote deep learning. Drawing on recent studies, the chapter authors highlight how teachers both proactively act to establish classroom cultures which open up space for student voice, and position students' mathematical reasoning as central to the classroom mathematical activity. Impact of discursive practices that include reasoned argumentation, centrality of student engagement and persistence in all mathematical endeavours, and the use of thoughtfully selected, challenging mathematical tasks are themes addressed in most studies. Research findings are contrasted with policy directions prevalent in Australia and New Zealand and the consequences of this contentious space highlight concerns about basing such policies on behaviourist theories of learning. The authors forecast the implications this is bound to have for advancing the agenda of mathematical learning for all students.

The next chapter by Serow, Callingham, and Tout highlights Australasian research related to assessment during the review period, at the international, national and classroom-based levels. In *Assessment of mathematics learning: What are we doing?*, the authors raise questions regarding how information about students, gained through assessment, is reported and used. Results from international assessments are considered to inform possible directions for future research and to provide a current report card on Australian students and young adults. An in-depth exploration of research exploring the National Assessment Program - Literacy and Numeracy (NAPLAN), the most prominent external assessment in Australia, emphasises a number of issues concerning the impact NAPLAN has on schools and students. Research into innovative, classroom-based assessment practices ranges from research into high-stakes assessment systems in Singapore, to authentic assessment which provides rich contextualised data, and online tools. This chapter argues for a need for more research specifically addressing assessment in mathematics and suggests a possible research agenda for continued research.

The next chapter highlights how the incorporation of digital technologies in mathematics classrooms can transform learning and teaching in these contexts. Specifically, these transformations are outlined throughout *Transformations of teaching and learning through digital technologies* as cognitive, pedagogical and professional changes. The research reviewed in this chapter by Geiger, Calder, Tan, Loong, Miller and Larkin is based in a range of theoretical frameworks such as transactional distance theory; technological pedagogical content knowledge (TPACK); affordances; Roger's (1995) innovation framework; and the subsequent map of pedagogical opportunities. Drawing specifically on the map of pedagogical opportunities, technological innovations are considered at the task, classroom and subject levels. Australasian literature exploring technological innovations in mathematics classrooms has been further organised in relation to learners and learning and teachers and teaching. Based on the critique of literature that the authors present, future directions for research into improving the learning and teaching of mathematics education through the use of technology conclude the chapter.

The final chapter in this section, *Research into mathematical applications and modelling*, establishes the leadership in this field that Australasian region researchers have provided for over two decades. Stillman, Brown, Galbraith and Ng first delineate the different perspectives on teaching through mathematical modelling and applications. Against this background, they review exemplary literature focusing on the teaching of applications, and in particular whether and how genuine problem contexts are or can be used as applications contexts. The chapter authors make a link to numeracy research as an area within which mathematics is intended to be applied meaningfully within different content domains. The rest of the chapter leads with the modelling research, commenting on applications where appropriate. The sections review in turn theoretical developments in the field, how studies in the field addressed general goals of education (such as development of students' communication and collaboration capacity), and methodological tools used across the reviewed studies. The chapter concludes by discussing the current state of research in modelling and applications and progress since the previous *Review*, with an eye to possible future developments.

3.3 Teacher Preparation and Development

The third major section of the MERGA Research Review, *Teacher Preparation and Development*, like the corresponding section in the previous review, comprises two chapters, one dealing with pre-service teacher education and the other with practising teachers. As pointed out by the editors of the previous review, these are areas of ongoing interest to Australasian researchers in both mathematics education and teacher education.

Initial teacher education (ITE) has received considerable attention from policy makers during this review period because of the widespread belief that improving teacher quality will lead to improved student outcomes. Moreover, ITE needs to prepare beginning teachers who are responsive to changing curriculum and assessment frameworks, and the increased use of technology. In the chapter *Challenges, reforms, and learning in initial teacher education programs*, Anthony, Cooke and Muir review research on teacher preparation in three areas: accountability, effectiveness, and policies; for the knowledge society; and for diversity and equity. The limited number of studies that addressed diversity and equity was identified as an area of concern. During the review period a move from research that explores existing practices to the more proactive approach of designing and trialing innovative reforms, increased attention on social activity as a part of learning, and a greater focus on research on the work of teacher educators were noted. Anthony and her colleagues identified a need to look at ways to continue to build a sound research base on ITE, including investigating ways to scale up research projects, and to combat the potential of top-down policy mandates in the current political climate.

The importance of ongoing professional learning (PL) for practising teachers has also been the focus of increased attention over this review period as policy makers search for ways to improve student outcomes. The research reviewed in *The education and development of practising teachers* extends beyond professional knowledge and includes research on theoretical approaches to understanding professional learning for teachers and professional learning programs. Beswick, Anderson and Hurst note increased attention to theoretical aspects of PL with research contributing to understanding the nature of both teacher learning and teacher capabilities. The research on PL programs was concerned with the content foci of the PL, characteristics of effective PL, approaches to PL, and evaluation of PL. Areas that were identified as in need of further research included the potential of online delivery, cross disciplinary collaborations involving mathematicians and mathematics educators, and investigating ways to improve the effectiveness of PL. Like research on ITE, there appears to have been little attention given in the research reviewed on PL to issues of scale and sustainability. Beswick and her colleagues call for Australasian researchers to investigate ways to scale up successful PL initiatives in order to influence mathematics teaching within the Australasian context and to situate their research within broader contexts in order to have greater impact internationally.

3.4 The Future

The final section and final chapter of *Research in Mathematics Education in Australasia 2012–2015* follows the tradition of previous three *Reviews* by asking one of MERGA's most eminent scholars to write the culminating chapter. The aim of this chapter is for the author to promote a vision for the future of mathematics education in Australasia and respond to contributions of the chapters in the *Review* for elaborating this vision. The final chapter of this volume of RiMEA is written by Professor Lyn English, 2012 winner of the MERGA Career Research Medal and founding editor of the international journal, *Mathematical Thinking and Learning*. In her chapter, *Advancing mathematics education research within a STEM environment*, English focuses on the current Science, Technology, Engineering and Mathematics (STEM) environment and sets out a vision for the future of the field in negotiating our research within STEM. The chapter begins by considering possibilities for advancing mathematics education research within the current STEM environment. Noting that the current RiMEA seems to focus less on mathematical content areas than previous Reviews, English uses the chapter reviews and other research to emphasise and provide "suggestions for developing content and processes through idea-generating problems, for promoting in-depth content understanding, and for fostering general skills and processes" with particular emphasis on modelling and problem solving as a vehicle.

4 Concluding Comments

Putting together the *Research in Mathematics Education in Australasia 2012–2015* has been a tremendous pleasure for the editorial team. The Review's 65 chapter authors have done an impressive job in casting their nets widely to generate in-depth, cogent, critical and insightful discussions of the content *and relevance* of Australasian research in mathematics education over the past 4 years. Their contributions will inform not only researchers in the field—new and experienced—but also provide support for future research agendas.

As this volume goes to press, there will already be discussions in the MERGA Executive to enlist the next team of RiMEA editors, who will construct the list of chapters and recruit the author teams, who will again cast their net across 2016–2019 for research now or shortly to be conducted, written and published. Although there are elements of tradition in each volume of RiMEA, its content is adapted according to the demands of the times. This is the call of *Research in Mathematics Education in Australasia*.

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Chapter 2
Reflections on the MERGA Research Review 2008–2011: Taking Stock

Bob Perry, Amy MacDonald, Jane Greenlees, Tracy Logan and Tom Lowrie

Abstract This chapter reflects on mathematics education research in Australasia as it was represented in the review immediately preceding the current volume—*Research in Mathematics Education in Australasia 2008–2011*. It is written by the editors of the earlier review at the invitation of the editors of the current review. In recognition of government policy reforms in Australasian countries, the chapter is structured around five of these major reforms: early childhood reform; national curricula; national and international assessment; teacher accreditation; and closing the gap. The chapter looks back at the previous review and forward to prospective mathematics education research through the lenses of these reforms. It considers the implications of the reforms on mathematics education and endeavours to stimulate mathematics education researchers to work on the major challenges created by the reforms. In looking forward to the new review of mathematics education research, the chapter highlights some of the areas of mathematics education research which may prove fruitful to researchers and helpful to individuals, families, communities and societies throughout Australasia.

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Keywords Early childhood reform agenda · National curricula · National assessment · International assessment · Teacher accreditation · Closing the gap

1 Introduction

Research in Mathematics Education in Australasia 2012–2015 is the ninth such 4-yearly review undertaken by the Mathematics Education Research Group of Australasia (MERGA). These reviews have become expected and much anticipated aspects of the Australasian and international mathematics education research scene. The timing of publication is linked to the 4-yearly International Congress on Mathematical Education (ICME) at which the launch has become a permanent feature.

In this chapter, the task of the authors, who edited the previous review: *Research in Mathematics Education in Australasia 2008–2011* (Perry, Lowrie, Logan, MacDonald, & Greenlees, 2012), is to reflect on the 4 years following the publication of this review, consider the directions the review foreshadowed and provide an overview of the context for the current review. The chapters in *Research in Mathematics Education in Australasia 2008–2011* (Perry et al., 2012) were grouped into three main sections:

- Contextsfor Mathematics Education
  - Reflections on the MERGA research review 2004–2007
  - The affective domain and mathematics education
  - Equity, diversity, social justice and ethics in mathematics education
  - Indigenous students and the learning of mathematics
  - Supporting exceptional students to thrive mathematically
  - Technology in mathematics education
  - Assessment beyond all: The changing nature of assessment
- Mathematics Learning and Teaching
  - Early childhood mathematics education
  - Powerful pedagogical actions in mathematics education
  - Mathematics curriculum in the schooling years
  - Growth and new directions? Research in tertiary mathematical science education
  - Uncertainty in mathematics education: What to do with statistics?
- Mathematics Teachers
  - The professional education and development of prospective teachers of mathematics
  - Professional knowledge of practising teachers of mathematics.

A final chapter *Taking Stock: From Here to the Future* (Leder, 2012) appraised the other chapters in the review and set some possible directions for future consideration in mathematics education research in Australasia. It is from this chapter that the subtitle for the present chapter has been borrowed.

For the most part, the chapter titles in *Research in Mathematics Education in Australasia 2008–2011* (Perry et al., 2012) were not significantly different from those in the 2004–2007 MERGA review, suggesting that mathematics education research in Australasia had reached a period of some stability and consolidation. However, there was a stronger emphasis in the later review around the mathematics education of young people "on the margins" and on curriculum and assessment, perhaps suggesting the impact of national curricula that were introduced during the latter review period.

Most of the chapter authors completed their critiques of research with suggestions for possible future directions. Leder (2012) summarised these noting that some were simply for further research in the particular field while others were more specific. For example, Atweh, Vale, and Walshaw (2012) noted "a movement from the disparate agendas such as equity, diversity and inclusion to a more comprehensive and perhaps unifying construct of social justice" (pp. 57–58). In relation to assessment, Lowrie, Greenlees, and Logan (2012) echo the social justice theme in future research by considering "the appropriateness of assessment particularly with minority groups and the extent to which assessment practices consider the needs of all learners" (p. 158). Surprisingly, given the curriculum revolutions in education, particularly in Australia, there was little research undertaken in relation to new curricula and their implementation. Specifically noted as requiring continued emphasis was "mathematics learning and teaching across the prior-to-school and school transition" (MacDonald, Davies, Dockett, & Perry, 2012, p. 186). From the section *Mathematics Teachers,* Leder (2012) noted the need for future investigations on

> … cultural perspectives on teacher knowledge; the impact of politically driven pressures to influence the timing and setting of teacher education programs; the putative link between teacher knowledge of and about mathematics and student learning outcomes; and how, what and when teachers learn from their own experience without interventions from outside sources. (p. 360)

Since 2011, the final year covered by the previous MERGA review, Australia has undergone political turmoil at both the federal and state/territory levels. There have been five changes of Prime Minister and at least eight changes in State Premier or Territory Chief Minister. The Australian government has changed from a progressive and creative Labor party to a more conservative coalition determined to be fiscally responsible but constrained by the bicameral parliament. Whilst, constitutionally, the states and territories are responsible for school education, the federal government, in conjunction with the states and territories, has been successful in introducing sweeping changes in early childhood and school education and has been endeavouring to do the same in higher education, with only marginal success. On the other hand, New Zealand, with its unicameral parliament and the same Prime Minister for the period covered by this review, is more stable in its educational directions. Nonetheless, the social and contextual changes in New Zealand still require education to respond to and lead change. In the remainder of this chapter, we consider some of the major changes in government policy that have occurred in Australia and New Zealand since 2011 and use these to provide a context for the chapters that follow. In particular, we have structured the chapter around the implications of five major initiatives, knowing that this is a choice of the authors and that other areas could have been included. Two that have been suggested by chapter reviewers, but not discussed in the chapter, are the research policy and accountability environment in both New Zealand and Australia and the introduction of the national disability insurance scheme in Australia. The five areas to be discussed are:

- early childhood reform agenda;
- national curricula;
- national and international assessment;
- teacher accreditation; and
- Closing the Gap.

2 Implications of the Early Childhood Reform Agenda

In the previous MERGA review, it was noted by MacDonald et al. (2012) that during the period 2008–2011 there was unprecedented political interest in early childhood education in Australasia. This interest in the early years came as a result of curricular developments across both Australia and New Zealand. In Australia, the first ever national curriculum framework for early childhood, *Belonging, Being and Becoming: The Early Years Learning Framework for Australia* (EYLF) (Department of Education, Employment and Workplace Relations, 2009) was implemented and its impact was beginning to be seen during the 2011–2015 review period. In the schooling years, Phase 1 of the implementation of the *Australian Curriculum* (Australian Curriculum, Assessment and Reporting Authority, 2013) had begun. In New Zealand, a review of the early childhood curriculum framework, *Te Whāriki* (Ministry of Education, 1996), had been recommended.

In the years since the previous review, the early childhood education landscape—particularly in Australia—continued to change. Perhaps the most significant development in Australia was the early childhood education and care reform agenda, agreed to by the Commonwealth and the State and Territory Governments in November 2008 (Council of Australian Governments [COAG], 2012). This reform agenda—still in effect today but radically transformed by the conservative coalition in power at the federal level—means that current early childhood educators, who having already obtained their TAFE qualification and been working in the profession, were now undertaking University study to obtain a teaching qualification. The reform agenda aimed to ensure that every child has access to a quality early childhood education program that is delivered by a 4-year university-trained early childhood educator, for 15 hours a week, 40 weeks a year, in the year before formal schooling (COAG, 2012). This initiative has significantly increased the demand for 4-year qualified early childhood educators.

As an impact of the national early childhood education and care reform agenda, Australia now has large numbers of early childhood educators—many of whom have years of experience—who are undertaking Bachelor of Education programs and consequently are, for the first time, undertaking mathematics education at a University level. This is a significant advance in the education of young children, because international research provides compelling evidence of the importance of children's early mathematics learning in the years before school (Lago & DiPerna, 2010). However, there is a significant body of research which suggests that many early childhood professionals are reluctant to engage in intentional teaching of mathematics (Anthony & Walshaw, 2009; Lee & Ginsburg, 2009), and that this reluctance may be explained by concerns about overly didactic programs, privileging other parts of the curriculum such as language and literacy, and teachers' anxieties about their own mathematics knowledge (Cohrssen, Church, Ishimine, & Tayler, 2013). A further challenge is that those early childhood educators who *do* include mathematics education as part of their curriculum typically hold a very narrow view of what constitutes mathematics, stressing the ability to count and knowledge of numbers (Department for Education and Child Development [DECD], 2012, cited in Carrington & Feder, 2013; Hunting et al., 2012). As such, a key role of tertiary early childhood education programs in the current reform climate is to promote educator content knowledge in mathematics as a means of providing children with access to high-quality mathematics education programs in the years prior to starting school.

To date, it appears that no research specifically focusing on the impact of the early childhood reform agenda on early childhood mathematics education has been reported. However, we suggest that this is a significant area for future investigation and research examining the impact of the reforms is encouraged.

3 Implications of National Curricula

Australia has a national school curriculum, for the first time in its history (Stephens, 2014). For other countries (such as New Zealand and Singapore) such curricular consistency is commonplace. Although Australia's previous state and territory curricula (especially with regard to mathematics) have always had many more commonalities than differences, most debate and research tended to identify those aspects of the curriculum that were not aligned across Australia. The scoping and projection for the national curriculum was forged from a common framework of assessment—with the advent for the National Assessment Program—Literacy and Numeracy (NAPLAN) in 2008 (National Assessment Program, 2013). This consistency led to push for a common curriculum, among other things.

Sustained research has been undertaken on the national curriculum, especially in relation to development of the curriculum and the content within each discipline area. Anderson, White, and Wong (2012), in their curriculum chapter in *Research in Mathematics Education in Australasia: 2008–2011* (Perry et al., 2012), highlighted that both New Zealand and Australia have had major mathematics curriculum reform in the previous 8 years. They described the background to the reform and the processes undertaken during the development of both curricula. New Zealand was the first to undertake curricular reform in 2007, with complete implementation by 2010. The new national curriculum in New Zealand focused not only on the three content strands of Number and Algebra, Geometry and Measurement and Statistics, but also placed value on social inclusion, diversity and having high expectations. The *Australian Curriculum: Mathematics* was introduced into all states and territories across Australia between 2012 and 2015. Again, the focus of the curriculum was not solely on the three major content areas of Number and Algebra, Geometry and Measurement and Statistics and Probability, with four proficiency strands identified to link the content and processes of working mathematically: Understanding, Fluency, Problem Solving and Reasoning. The Australian mathematics curriculum also included a general capability called numeracy, which links across all new national curriculum subjects, such as English and History.

The development and implementation of these curricula has received much focus within the mathematics education community. For example, Zhang and Stephens (2013) and Gallagher, Hipkins, and Zohar (2012) have considered the curriculum reform from cross-cultural perspectives. Others have identified certain topic areas and how they have been enacted or portrayed through the curriculum (Anderson, 2014; Lowrie, Logan, & Scriven, 2012; Watson & English, 2013). There have also been publications critiquing the Australian Curriculum such as Atweh, Goos, Jorgensen, and Siemon (2012) and Luke (2010). Both the Atweh et al. and Luke works considered the extent to which new curricula would contribute to national goals and external cohesiveness. Politically, both New Zealand and Australia have undertaken reviews of the respective curricula (Hipkins, Cowie, Boyd, Keown, & McGee, 2011 and Australian Government, 2014 respectively). Not surprisingly, some reactions to these curricula have focused on key competencies and content. Ell and Grudnoff (2013), for example, considered the extent to which the new curriculum can address New Zealand's challenge to present its high standard of education to all learners, especially those from disadvantaged backgrounds, with a particular focus on the growing disparity between Māori and Pakeha students. In addition, they argue that the effect of international testing and comparison has given rise to addressing teacher quality through student outcomes. It may well be another case of not teaching to the test, but rather creating the teacher for the test. Since mathematics (and numeracy) is so prominent in the national and international comparisons of student performance, it would be interesting to consider the extent to which curriculum design is influenced by a country's perceived strengths and weaknesses.

The new "consistency" in curriculum content may lead to further studies in analysing student learning across states and territories in Australia. As Anderson (2014) pointed out, there is increased opportunity to focus on students' problem-solving skills now that four proficiencies (processes) have been described. Where the mathematics education field needs to move forward is through a better understanding of the implementation of the curriculum in the classroom. How is the content being taught? Are there disparities between state department syllabus documents? Has having a new curriculum made a difference to teachers or schools?

4 Implications of National and International Assessment

It was anticipated from the 2008–2011 MERGA review that the National Assessment Program—Literacy and Numeracy (NAPLAN) debate would only intensify after its introduction in 2008. Many studies have focused on various aspects of the recently introduced high-stakes test (Lowrie, Greenlees et al., 2012). This, in association with a heightened awareness of international assessment regimes such as the *Programme for International Student Assessment* (PISA) (Organisation for Economic Co-Operation and Development, 2015) and the *Trends in International Mathematics and Science Study* (TIMSS) (International Association for the Evaluation of Educational Achievement, 2015), would result in an increase in research focused on the nature and design of such instruments. Initially this was the case, with a substantial amount of Australasian research devoted to the field. Leder (2015) reported that 10 % of the work presented at the joint conference in 2011 of the Australian Association of Mathematics Teachers [AAMT] and Mathematics Education Research Group of Australasia [MERGA] were in reference to the NAPLAN. It was for this reason that for the first time an entire chapter was devoted to assessment in the 2008–2011 MERGA review.

However, an analysis of the current Australasian research environment indicates that assessment practices such as the NAPLAN are no longer high on the research agenda despite unanswered big questions surrounding the controversial testing regime. Many of the studies focusing on the advantages and disadvantages of large-scale testing are from an international perspective, despite some efforts to address these issues on a national scale. Subsequently, Leder (2015) concluded that although small, contextualised investigations of participation and engagement issues are important, more large-scale research is called for in regards to the efficacy of national tests. This is particularly pertinent as the NAPLAN moves towards a digital form in 2016.

A theme that has emerged from the research has been the role of the learner and the impact high stakes testing such as NAPLAN has on school students and their families, not only in terms of curriculum and learning but also in regards to students' health and well-being (Polesel, Dulfer, & Turnbull, 2012). Aspects such as IQ, family socio-economic position and parental education have been identified as predictive factors for children's numeracy performance on a standardised mathematics test (Carmichael, MacDonald, & McFarland-Piazza, 2014). Students have even been classified as commodities, with the school's role being one of adding value by processing these raw materials (Lange & Meaney, 2014). These findings all point to a need to find alternate systems of accountability that recognise the complexities of assessment purposes, modes, conditions and contexts. These include "national tests complemented by teacher assessment and moderation practice and sampling rather than census testing" (Klenowski & Wyatt-Smith, 2012, p. 76).

Research suggests that a re-evaluation of current assessment practices and reporting would also be beneficial for schools and teachers as Australian States and Territories compete for Federal "reward funding" based on NAPLAN performance. Lingard and Sellar (2013) highlighted several "perverse effects" (p. 634) of this new accountability regime including a partial dissolution of the State and Territory education systems and an added emphasis on improving or maintaining the reputation of schools to secure funding, rather than the intended objective of improving numeracy outcomes of students. Similarly Klenowski and Wyatt-Smith (2012) identified other effects including changes in teachers' pedagogical practices, principals feeling unfairly "threatened" if failing to improve test performance, unfair distribution of resources for students most likely to show improvement, parents encouraged to keep their children at home on test day and claims of teachers providing assistance to students while sitting the tests to improve their results. These findings warrant further study into teachers' practices at both macro and micro levels to help substantiate such claims and assist in necessary reforms.

A final implication of national and international assessments has been the growing body of literature on test item design, in particular the role of graphics and its impact on student performance. As high-stakes testing is now inevitable on a national and international scale, it is imperative that "assessment tasks are designed appropriately to measure the intended mathematics outcomes" (Lowrie, Diezmann, & Logan, 2012). This includes a more comprehensive understanding about the differentiation between redundant graphics that are unnecessary to students and those that are not (Greenlees & Logan, 2014) as well as the role of language and the use of contexts within an item. These issues will only gain momentum, particularly as NAPLAN moves to an online environment.

Research continues to suggest that high stakes tests such as the NAPLAN have a direct bearing on student well-being and a further impact on students' learning and experience of education by virtue of their effects on educational practices (Polesel et al., 2012). Consequently further research is necessary to ensure that, in both respects, such tests advance the interests of Australian students.

5 Implications of Teacher Accreditation

Since 2005, all teachers who hold or aim to hold a teaching position within an Australian school must become registered and accredited through the respective state authorities. However, it was only in 2012 that the Australian Professional Standards for Teachers were introduced by the Australian Institute for Teaching and School Leadership (AITSL). Teacher registration and accreditation is guided by the Australian Professional Standards for Teachers and previously the national professional standards for teachers. Implications for mathematics education have followed these regulations and national policies, with the main impact concerning initial teacher education programs and the quality of students who enter these programs, especially in terms of their levels of literacy and numeracy. As Anderson et al. (2012) maintained, the requirement for teachers to demonstrate teaching standards provided opportunities for new research into the impact such monitoring and self-assessment would have on teachers' practices and school-based policy development. However, such research has not been forthcoming, possibly due to the evaluative nature of the work. It is difficult for such personalised research not to be judgmental.

A position paper from the Teacher Education Ministerial Advisory Group (Australian Government, 2015a) was commissioned to consider teacher education and standards. A number of questions were posed that related directly to mathematics education, especially associated with the broad question of "What is the balance between understanding what is taught and how it is taught?" Contributing questions around quality, discipline knowledge, pedagogy, specialisation and expert shortages were posed. Reactions have been varied. AITSL, for example, has provided a requirement that students who wish to enter initial teacher education programs have levels of literacy and numeracy broadly equivalent to the top 30 % of the population (AITSL, 2014). They provided examples of Year 12 subjects and study scores that might reflect this standard for the various states and territories. However, AITSL have stated that these subjects and scores at this standard are not pre-requisites for admission. Despite this claim, some state teacher regulatory bodies have indicated that there will be pre-requisites for entry into initial teacher education programs. Within NSW, for example, The Board of Studies, Teaching and Educational Standards (BOSTES) (2015) have stated that, from 2016, pre-requisites for admission to a degree in primary teacher education will include achieving 80 % or higher in a minimum of three subjects including English. There is no specific mention of mathematics, even though a minimal standard in mathematics was required for entry up to 2015. The Queensland College of Teachers has stated that all incoming students will need to meet pre-requisites of sound achievement in English, Mathematics, and for primary and early childhood programs, Science also. Other State and Territory bodies mention or refer to the literacy and numeracy standards for initial teacher education from AITSL without providing explicit pre-requisites. Hence, teacher education institutions are able to make their own decisions about how students are assessed against this top 30 % standard.

In conjunction with this standard, AITSL and the Australian Council for Educational Research (ACER) have developed online literacy and numeracy tests that will assess pre-service teachers' competency levels with the first cohort sitting in 2016. Although details of how the testing program will be enacted are continuing to emerge, it is proposed that all pre-service teachers will need to "pass" these tests in order to graduate. The Australian Council of Deans of Education (ACDE) has reacted quite strongly to this initiative. Collectively, they are concerned about the impact on student enrolments in their courses—and this is especially the case for Deans in non-metropolitan universities. As these students must graduate meeting specific criteria set out by the Standards, there is a need to ensure the students entering teacher education programs have the capacity to complete the degree at these newly prescribed levels. The publication *Teaching for Excellence* (ACDE, 2014) argues that pre-service education programs should focus on the iterative relationship between content knowledge and pedagogy, raising examples from classrooms that are culturally and contextually different. One interesting suggestion is the desire to have specialist Science, Technology, Engineering and Mathematics (STEM) teachers to support the generalist teachers in primary schools. In fact, this form of targeted expertise has begun to be enacted in some pre-service courses and education jurisdictions. It seems likely that the debate surrounding the role and nature of discipline knowledge and pedagogical content knowledge will heighten in the coming years. This is especially the case for mathematics and numeracy knowledge, since the current challenges of attracting high quality mathematics students into the teaching profession will remain. The reform movement has gathered momentum in the cyclic nature of raising entrance requirements at a time of new student-driven university reforms—where universities are encouraged to enrol as many students as possible whilst raising the standards (in terms of quality) of those students (O'Meara, 2011). Such actions become increasingly complex in mathematics education, since some universities (especially those outside of the strong metropolitan institutions) are already starting from a relatively low enrolment base. As Anthony, Beswick, and Ell (2012) indicated in the 2008–2011 review, improvement in teacher education must be enacted through quality programs, and the effectiveness of the teacher educators. Little seems to be gained from research that focuses on single sites of practice or innovation and yet research is scant on analysing programs across multiple sites with common frameworks.

In New Zealand, a change to the requirements for all university entrance, not just for teacher education, was enacted in 2014. In order to attain a university entrance mark, students are required to competently complete units in both literacy and numeracy as defined by the New Zealand Qualifications Authority. This highlights the emphasis the New Zealand government has put on literacy and numeracy as pre-requisites for a university education. Since 2009, The New Zealand Teachers Council (NZTC) has endorsed the Registered Teacher Criteria, which outline the quality teaching standards that need to be demonstrated and upheld in order to be a registered teacher in New Zealand. Further to this, since 2007, NZTC have implemented the Graduate teacher standards aimed at addressing the quality of graduates into the teaching profession.

Adler, Ball, Krainer, Lin, and Novotna (2005) and Norton (2012) highlighted the fact that students who typically enter teacher education programs have limited mathematics knowledge, with many demonstrating content knowledge similar to that of a Year 9 student. Those who enter with lower levels of mathematics often leave with lower levels of content knowledge and pedagogical content knowledge compared to other graduates. Indeed, the policies put in place by AITSL and NZTC serve to address this concern. The question remains as to how individual universities will enact the various policies. Will such policies affect the mathematics subjects taught within initial teacher education programs, and is there any evidence that this has already happened? What types of bridging courses are available for students who are admitted with lower levels of mathematics? Do these make a difference? At what point in the program will students be given the literacy and numeracy tests?

The impact on the mathematics education research community with regard to such policies is likely to be varied and ongoing. Nevertheless, the delivery of pre-service teacher education programs and the ongoing professional development of mathematics teachers will remain a central focus of government education initiatives and, hopefully, mathematics education researchers, into the foreseeable future.

6 Implications of the Closing the Gap Agenda

In both New Zealand and Australia, there are government agendas designed to "close the gap" on Indigenous disadvantage in health, education and employment. The agendas have existed for some time—in New Zealand, the notion of "closing the gap" was introduced through Te Puni Kōkiri (2000), while in Australia, the National Indigenous Reform Agreement (COAG, 2009/2011) committed all Australian governments to six ambitious "closing the gap" targets relating to life expectancy, infant mortality, education and employment.

Progress has been made in both jurisdictions but it has been much slower than desired. There have been improvements in areas such as infant mortality and life expectancy, particularly in New Zealand. The growing success and reach of Kōhanga Reo (Māori immersion preschools)—there are 460 across New Zealand and others in Australia and the United Kingdom (Te Kōhanga Reo National Trust, 2015)—has resulted in advances in early childhood participation with follow-on impacts in schools. There are also some positive indications about improvements in senior school achievement but Māori and Pasifika students continue to perform less well than Asian and Pakeha on international testing and participation rates in more advanced mathematics subjects in senior high school (Buntting, Jones, McKinley, & Gan, 2013). In Australia, progress toward the goals has been slow. In the Foreword to the 2015 *Closing the Gap Report* (Australian Government, 2015b), the Prime Minister wrote:

> This is the seventh Closing the Gap Report produced since targets were set by the Council of Australian Governments (COAG) in 2008. Despite good intention and considerable investment by successive governments, the disparity in outcomes remains. Although there has been some improvement in education and health outcomes for Indigenous Australians, in many areas progress has been far too slow. It is profoundly disappointing that most Closing the Gap targets are not on track to be met. (p. 1)

For the three education goals, the data are not good.

- *Ensure access for all Indigenous 4-year-olds in remote communities to early childhood education (by 2013).* This target has not been met. In 2013, 85 % of Indigenous 4-year-olds were enrolled, compared to the target of 95 %.

- *Halve the gap in reading, writing and numeracy achievements for Indigenous students (by 2018).* Australia is not on track to meet this target. There has been no overall improvement in Indigenous reading and numeracy since 2008.
- *Halve the gap for Indigenous Australians aged 20–24 in Year 12 attainment or equivalent attainment rates (by 2020).* Australia is on track to meet this target as the gap is narrowing in Year 12 or equivalent attainment. (Derived from Australian Government, 2015b, p. 5)

There has been one more educational target added in this report:

- *Close the gap between Indigenous and non-Indigenous school attendance within 5 years (by 2018).*

Progress on each of the "closing the gap" targets and, perhaps, more fundamental issues such as considering the nature of, the reasons for, and the potential of any gap, open up many opportunities for studies by mathematics education researchers in both Australia and New Zealand. Many MERGA members are undertaking such research with quite spectacular results in individual or small numbers of contexts. The challenge of running these innovations to scale remains.

7 Conclusion

Much has happened in Australasia since 2011 and much of this impacts on mathematics education and mathematics education research. Reforms in the early childhood arena—and their potential dismantling—provide very rich sources for research studies in mathematics education. Many researchers have asked the question about continuity of learning between prior-to-school settings and school, given the two curriculum frameworks in both Australia and New Zealand. We would expect to see this work critiqued in the following pages of this book. The same could be said of the other policy changes that have been delineated above:

- national curricula;
- national and international assessment;
- teacher accreditation; and
- Closing the Gap.

Each of these has provided mathematics education researchers with opportunities to build their agendas and to make a difference to children's learning outcomes. It will be a pleasure to read of the opportunities that have been grasped and the differences that have been made. The MERGA review *Research in Mathematics Education in Australasia: 2012–2015* will provide critique, commentary, and celebration of the increasingly important research in mathematics education that is being conducted by Australasians, in Australasia, for the benefit of our children, young people and all learners. It will be a very good read, and a very useful addition to the other eight MERGA reviews.

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Chapter 3
A Philosophical Gaze on Australasian Mathematics Education Research

Steve Thornton, Virginia Kinnear and Margaret Walshaw

Abstract This chapter examines the philosophical underpinnings of Australasian mathematics education research between 2012 and 2015. It takes a hermeneutic approach, seeking to uncover often hidden assumptions about ontology, epistemology, aesthetics, ethics and logic. The first part of the chapter explains the approach taken and outlines the set of papers considered. The chapter then examines the set of keywords used in Australasian mathematics education research papers published in major international journals, seeking to identify broad themes or omissions. It then takes a more detailed look at a purposive selection of 26 papers, chosen to reflect the themes in this Review. Finally it examines papers that explicitly discuss epistemology to identify researchers' underlying assumptions about the nature of knowledge and its acquisition. The chapter points to some important tensions within the research and suggests that such tensions can be used as a creative force to enable mathematics education researchers to better identify and question the philosophical assumptions that underpin their research.

Keywords Aesthetics · Education research · Epistemology · Ethics · Hermeneutics · Logic · Mathematics · Ontology · Philosophy

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1 Introduction

> …the present community of mathematics educators lives in an academic environment with colleagues driven by a variety of frameworks, including various socio theoretical perspectives, and postmodern views of the world. Given that the resulting environment creates pressures and tensions for scholars subject to a cross fire of paradigms, what are the implications for individuals for whom both MATHEMATICS and EDUCATION are spelled in capitals? (Galbraith, 2014, p. 40)

Peter Galbraith's keynote at the 2014 Mathematics Education Research Group of Australasia (MERGA) conference, from which the above quotation is taken, highlighted the dilemmas faced by mathematics education researchers bombarded with a range of perspectives, ranging from what might be termed an extreme "post" view of the world, where everything is open to question, to an extreme "reductionist" view of the world, where absolute statements are made and research is removed from context. Of course, we can learn much from different perspectives, no matter how extreme, but how do we choose between competing views of the world? Have we, as mathematics education researchers, become hostage to a particular view of the world, and if so, how do we become more aware of the implications? Or have we become blind to the underlying assumptions about the world that drive our work?

In this chapter we attempt to address these deep questions about the assumptions that underpin mathematics education research. For we maintain, like the philosopher Alasdair MacIntyre (2011, p. 72), that "every action is the bearer and expression of more or less theory-laden beliefs and concepts; every piece of theorising and every expression of belief is a political and moral action." We are aware that trying to identify the theories and beliefs underpinning the research carried out in mathematics education in Australasia from 2012 to 2015 is a monumental task, and even were we to read and comment on every piece of literature we would undoubtedly get it wrong. Nevertheless we hope that in adopting a philosophical gaze on the mathematics education research literature we can at least heighten the awareness of the mathematics education community to those theories and beliefs, and hence hint at what might be "the implications for individuals for whom both MATHEMATICS and EDUCATION are spelled in capitals."

2 What Do We Mean by Philosophical?

In this section, we position the chapter by discussing five major dimensions of philosophical endeavour and raising questions about how these relate to mathematics education. These are:

- Ontology (metaphysics): the nature of being, becoming, existence, or reality. We ask what aspects of mathematics education are "taken as given" in the research. What do different research paradigms or theoretical frameworks used in the research assume about the nature of reality?

- Epistemology: the study of knowledge and justified belief and how we come by it. We ask are different views of knowledge evident in the mathematics education research. If so, how do these epistemological positions impact upon mathematics in the classroom and on the methods and outcomes of the research?
- Aesthetics: judgement about matters of value. We ask what values appear to underpin the research and how do they impact upon the positions adopted by teachers and researchers with respect to curriculum or pedagogy.
- Ethics: systematising, defending and recommending concepts of right and wrong conduct. We ask how does the research embed or contribute to a heightened sense of ethical awareness.
- Logic: the use and study of valid reasoning and argumentation. We ask what different approaches to reasoning are used in the research. How do these different approaches position the research with respect to its capacity to be generalised or contextualised?

We first distinguish between our use of the term philosophy and terms such as theory, paradigm, methodology and model. The boundaries between these concepts are obviously blurred, however in this chapter we focus our attention on the five philosophical dimensions outlined above. Hence, while theoretical perspectives such as sociocultural theories of learning used, for example, in research by Goos (2014), have epistemological and ontological underpinnings, we do not discuss these theories or their use per se, except as they relate to the dimensions outlined above. Nor do we specifically discuss poststructuralist paradigms used, for example, by Klein (2012) or Walshaw (2013), models of mathematics education such as mathematisation used, for example, by Stillman and Brown (2014), or methodological approaches such as design research used, for example, by Cortina, Visnovska, and Zúñiga (2013), again, except as they relate to the dimensions discussed above. Of course, each of these, and every other, theory, paradigm, model or methodology has philosophical underpinnings that make assumptions about the nature of reality, the nature of knowledge, what is valued and how choices are justified. Hence, rather than discussing this literature explicitly, our survey of the research adopts a hermeneutic approach in trying to uncover underlying philosophical positions adopted, although most commonly not explicitly identified, by researchers across a broad spectrum of mathematics education research.

We are also conscious that philosophy has a strong political dimension, as any position on the nature of reality or knowledge, or set of values, necessarily has political implications. Four studies in particular addressed political aspects of mathematics education (see Chap. 4, this volume). Thornton (2013) discussed metaphors of mathematics education, arguing that dominant metaphors of education, including the school as factory and school as clinic, have been replaced in political rhetoric by a metaphor of education as a race, evidenced by the drive for competitiveness in tables of educational rankings such as the Programme for International Student Assessment (PISA) and the Trends in International

Mathematics and Science Study (TIMSS). He proposed an alternative metaphor, termed *Slow Maths*, in which culture and context are at the forefront of educational thinking. Thornton (2014) also discussed how the drive for success in system-wide tests is evident in a state education system policy document, arguing that the dominant rhetoric is located in Heidegger's concept of the technological enframing. The technology of assessment was also discussed by Seddon et al. (2013) in relation to the impact of the Excellence in research for Australia (ERA) on educational research in Australia. They noted that knowledge-based regulatory tools such as the ERA produced "unintended consequences…that present contradictory imperatives and expectations that create moral and political dilemmas" (p. 435). Lange and Meaney (2014) surveyed press releases and news articles regarding national testing, and argued that in such articles students are "positioned as commodities with mathematics achievement being the value that can be added to them" (p. 377). They concluded that this view of students has both social justice and pedagogic implications. They argued that it disadvantages schools, predominantly those populated by students of low socioeconomic status that are perceived to perform poorly, and narrows teaching and testing approaches to those that most obviously fit the test. Such studies offer a word of caution to those who place undue emphasis on the outcomes of system-wide tests, or to those who unquestioningly adopt the rhetoric of learnification (Biesta, 2010, as discussed in Atweh, Miller, & Thornton, 2012) assumed by such testing regimes. While a more thorough discussion of political dimensions within mathematics education research would be both informative and timely, we restrict ourselves here to the observation that the critique within each of these papers suggests that dominant paradigms in systemic approaches to mathematics education are underpinned by a somewhat reductionist and positivist ontology in which results in accompanying testing regimes are taken as true indicators of the outcomes of education.

Positioning philosophy as a provocateur for thinking about mathematics and education thus raises challenging and unsettled questions for which there are no easy or exact answers. It contains both descriptive and normative elements in that it discusses how things are and how things ought to be. It provides a way of addressing things that are important to us, and of questioning the values and beliefs that we hold. This was very much our approach in writing this chapter. We make no claims regarding the veracity of our findings in any absolute sense. Our use of a framework such as that described above necessarily privileges a particular reading of the research literature. Rather, we used our reading and analysis as a vehicle for critical reflection on how we view the world and what we value. We invite the reader, not only in this chapter but also throughout the Review, to do likewise and engage in critical reflection about the ontological, epistemological, aesthetic, ethical and logical questions highlighted above.

3 What Do We Mean by Gaze?

In this section we discuss our approach to the literature, and the methodology used in analysing it. An initial literature search using keywords such as "mathematics AND education AND philosophy" or "mathematics AND education AND ethics" revealed no results from the Australasian mathematics education research literature. However this does not mean that the philosophical questions raised above are unimportant. Rather, we suggest that they are so much a part of researchers' identities that they are often hidden to their own gaze. We therefore adopt a hermeneutic perspective to try to see below the surface and identify the philosophical positionings that are embedded in the research.

The hermeneutic approach maintains that a text, context and reader are inextricably related. A text cannot be fully appreciated without understanding the context in which it is set, and the reader is thus obliged to try to understand the author's context, and to make sense of the text in her own context (Lerman, Xu, & Tsatsaroni, 2002; Lester & Wiliam, 2002). A key part of the context in which a text is set is the author's philosophical, that is their ontological, epistemological, ethical, aesthetic and logical, position. Hence we developed a template with which we examined a selection of Australasian mathematics education research papers. Appendix 1 shows an example of how the template was used to examine a paper by Fielding-Wells, Dole, and Makar (2014). Each of us independently read and examined this paper and made comments relating to the assumptions that appeared to underpin the research. We compared responses to ensure a degree of consistency in our interpretation and approach, and collated our responses into one document. Even in this reading of one paper it was apparent that certain assumptions are made in the research about the nature of knowledge and how it is developed, the purpose of mathematics education and what is valued, and the purpose of mathematics education research and its relation to teachers. A close reading, such as this, of every Australasian mathematics education research paper published between 2012 and 2015 would, of course, be impractical. Hence to ensure that we examined a selection of papers dealing with a variety of topics and likely to adopt a range of philosophical positions we collated the keywords from every Australasian-authored paper in the major mathematics education journals: *Mathematics Education Research Journal*, *Mathematics Teacher Education and Development*, *Educational Studies in Mathematics*, *Journal for Research in Mathematics Education*, *Journal of Mathematical Behavior*, *Mathematical Teaching and Learning*, and *ZDM—The International Journal on Mathematics Education*. We then searched for keywords likely to be closely linked to each of the subsequent 13 chapters in this Review, and selected, either at random or based on recommendations from the chapter authors, two papers that we felt were likely to be prominent in each of these 13 chapters, giving us a purposive sample of 26 research papers, for each of which one of us completed a template similar to that shown in Appendix 1. The findings from this hermeneutic examination are presented in Sect. 5.

We further used our keyword analysis as a data set in its own right. By examining the frequency of each of the keywords and looking for trends, we hoped that we would uncover philosophical dimensions of the mathematics education research that are particularly prominent or absent. We present these data in Sect. 4.

Finally we searched our database of some 130 papers in mathematics education research journals for the term *epistemology*, or variants thereof. While we recognise that ontology logically underpins epistemology, an author's epistemological stance suggests much about their ontological beliefs. Hence, in the literature epistemology is likely to be both prominent and revealing. We found more than 20 occurrences, and examined each of these for use and meaning. We present the results of this examination in Sect. 6.

4 What Do the Paper Keywords Suggest?

In an attempt to make sense of what the keywords suggested we classified them according to their focus. Our initial categorisation included 23 concepts such as research methods, levels of schooling, discourse or knowledge and cognition. We then further categorised the keywords into the six main headings shown in Table 3.1. We are aware that such broad categories mask the finer details of the initial 23 categories, but we suggest that they do say much about the focus of the studies carried out in Australasian mathematics education research. Not surprisingly mathematics content, classroom practices and how students understand or relate to mathematics teaching were the major foci of Australasian mathematics education research. Research methods, theories and the contexts in which research is conducted were also highlighted in a significant number of papers, while a large number focused on teacher knowledge or teacher education. Only 13 keywords related to goals of education or ethical considerations, which we suggest reflects the relative lack of explicit attention paid to the philosophical underpinnings of mathematics education research. Given the attraction of a specific journal to like-minded authors and readers, it is likely that authors did not consider the necessity to make such positions explicit. While we cannot claim that keywords alone indicate authors' philosophical positionings, they are suggestive of what authors consider to be of primary importance in a paper. We therefore suggest in passing that researchers give consideration to using more explicit terms in their keywords.

Table 3.1 Categorisation of keywords

Mathematics (content, application, curriculum, processes)	142	29 %
Classroom (pedagogy, assessment, tools, discourse)	128	26 %
Students (levels, geolocation, cognition, beliefs, affect)	105	21 %
Research (methods, theoretical underpinnings, contexts)	57	12 %
Teachers (education, development, knowledge)	50	10 %
Goals and ethics	13	3 %

We turn now to a closer examination of the keywords in a selection of papers. All research rests on ontological assumptions about the form and nature of the reality being studied. Few keywords made explicit the authors' assumptions relating to the issue of what reality is like, however many implied different ontological assumptions. For example, Meaney and Evans (2013) who used the keyword *ethnomathematics* argued that the Western worldview is at odds with the worldview of Indigenous communities. Counting as accumulation, it is proposed, is more appropriately replaced, in some Indigenous communities, with counting for sharing. Underpinning this argument is a position that asserts that realities are local, specific, and constructed, and hence, everyday understandings, as well as symbols such as language, are prominent in such research. In contrast, Lim and Chapman (2015) used the keyword *scale development* and described the development and validation of an academic motivation scale in Singapore. Such research measures student attributes in an objective system where different types of motivation have different worth.

Many papers included epistemologically-related keywords such as *indigenous knowledge*, *mathematics teacher knowledge*, *pedagogical content knowledge*, and *conceptual knowledge*. Others gave explicit attention to the philosophies of knowledge production associated with, for example, *Vygotsky*, *enactivism* or *inquiry-based learning*. Keywords that identify specific types of knowledge or draw on seminal work of key figures make epistemological assumptions about what constitutes knowledge of the reality being studied. Research reported in these papers addressed epistemological questions concerning who has access to valued knowledge, how such knowledge is acquired and which knowledges are valued over others.

A few papers used keywords with an obvious aesthetic dimension such as *creative mathematical problem solving*, *persistence*, *authentic investigation* and *values*. Such keywords highlight that mathematics should be more than a body of knowledge: they are concerned with highlighting mathematics as something that is meaningful and relevant to students, allowing them opportunities to solve problems in a variety of ways and to develop productive habits of mind.

Keywords such as *equity* (see Chap. 7, this volume) and *care theory* highlighted ethical dimensions, exposing structures, arrangements, beliefs and practices that are inequitable and that impose constraints to students' or teachers' constructions of knowledge. These papers highlighted the goal of emancipation: participants will be able to change their circumstances and will be able to create a more just and more democratic place for themselves within the world of mathematics education.

A large number of keywords focused on particular aspects of mathematics and reasoning. These included such terms as *functional thinking*, *spatial reasoning*, *proportional reasoning* and *informal statistical inference*. Such keywords hint at the logic employed in mathematical reasoning, in some cases suggesting that mathematical reasoning is to be valued above everyday logic and in others suggesting that informal knowledge is a key aspect of children's mathematical development. Implicit within papers emphasising informal reasoning is the understanding that people are constantly making sense of their worlds and that truth is socially and experientially based, embedded in ongoing social interactions.

5 What Does a Hermeneutic Reading of a Sample of Papers Suggest?

Using the methodology and conceptual framework described above, we examined 26 research papers from the Australasian mathematics education research literature, comprising two papers likely to be the subject of review from each of the following 13 chapters of this Review.[1] At this more sophisticated level of analysis, we looked beyond keywords. We searched the publications for evidence of how authors' ways of "reading" the world played out in their research. Our hermeneutic approach sought to uncover some unstated assumptions that lay beneath the topic chosen, the way the research was conducted, or the conclusions drawn. We now present our analysis of how each of the five philosophical dimensions discussed above may have been embedded in the mathematics education research literature.

Our analysis suggested not unexpectedly, that authors held a range of ontological perspectives, however these perspectives were on the whole inferred from the epistemological dimensions of the paper rather than explicitly stated. Most papers (e.g., Barton, Oates, Paterson, & Thomas, 2015) appeared to position mathematical truth as a socially constructed reality located in collective and agreed meaning making, with mathematical concepts thus seen as fluid and evolving, rather than fixed and stable. Further to this approach, some papers viewed culture as the determiner of mathematical purpose, with mathematical thinking and concepts differing culturally. From this perspective, mathematics is a subjective experience, with meaning residing in the individual.

Conversely, some papers (e.g., Stillman & Brown, 2014) emphasised mathematics as accessed through and existing in the physical systems of the world. As a consequence, models of the world can be constructed and represented mathematically, which in turn enables real problems to be solved. The mathematical modelling process therefore makes certain assumptions about the nature of reality and the capacity of mathematics to encode that reality. Several papers (e.g., Zhang & Stephens, 2013) implicitly assumed mathematics to be hierarchical and sequential, with an accompanying deconstruction of whole to part, a perspective on mathematics that we suggest is strongly located in and defined by formal curriculum.

Unsurprisingly, given the ontological perspectives we identified above, the template analysis indicated a strong epistemological focus on the acquisition and development of conceptual knowledge. In mathematics *education* research, acquisition of conceptual knowledge targets how we come to know and therefore

[1]A complete list of these 26 papers is given in Appendix 2.

teach mathematics. The prevalence of this theme accords with Schoenfeld's (1992) observation that ontological perspectives about mathematics drive goals, and hence form the basis for mathematics instruction. Epistemology, ontology and pedagogy are therefore inextricably intertwined in this regard, and we now discuss the dominant *pedagogical* themes found in our analysis.

Across the papers, mathematical learning was strongly positioned as dynamic (e.g., Cavanagh & Garvey, 2012), in that we come to know mathematics through active co-construction and social participation, by discussing and engaging in argumentation and questioning, and by engaging in communities of practice. As a result, mathematical knowledge can be intuitive, contested, and subjective. Many papers (e.g., Fielding-Wells, Dole, & Makar, 2014) suggest that we learn mathematics by working in disciplinary practices; through generalising, conjecturing, inquiring and proving, and by working with mathematical procedures (e.g., Roche & Clarke, 2013). We gain mathematical knowledge by interpreting, reflecting, playing, making errors and risk-taking (e.g., Gervasoni & Perry, 2015). We access the physical reality of mathematics through solving problems we encounter in the world, a process that enables connected, systems (relational) knowledge to develop and we can represent and model the real world problems we encounter and solve (e.g., Ho & Lowrie, 2014). Because real world problems are contextual, some research (e.g., Owens, 2015) highlighted that culturally dissimilar mathematical knowledge is accessed in different ways, and mathematical knowledge is therefore dependent on cultural identity.

Aesthetics was the least visible dimension in our template analysis. Our definition of aesthetics was broad, encompassing matters of value and the relationship between values and curriculum and pedagogy. As a result, we found underpinning themes about the worth of mathematics (e.g., Thomas & Klymchuk, 2012), the valuing of student-teacher and peer relationships, and well-being (e.g., Averill, 2012) which were not clearly attributable to a paper's ontological or epistemological dimensions. However, the moving, beautiful and sublime dimensions of aesthetics that many of us appreciate in mathematics were notably absent in the papers we reviewed. We wonder if it is time to re-evaluate and re-invigorate discussion of aesthetic dimensions of mathematics, and to consider and investigate how such dimensions might positively influence mathematics teaching and students' learning.

Although the ethical dimension was less visible than the ontological or epistemological dimensions, many papers explicitly discussed ethical responsibilities to different groups of learners (see Chap. 7, this volume for a more detailed discussion of diversity). Here the calls were for greater acknowledgement of, and accommodation for, diversity in culture and student needs, such as those of Indigenous, special needs and gifted and talented students (e.g., Bicknell & Riley, 2012; Clarke & Faragher, 2014), and for teachers to adopt a culturally responsive approach to meet and embrace these differing needs (e.g., Meaney & Evans, 2013). In some cases (e.g., Anthony, Hunter, & Hunter, 2015) these ethical considerations extended to asking practising or pre-service teachers to consider the importance of reflective practice when examining the moral and ethical dilemmas found in their professional experiences with diverse learners. We note however that outside the 26 papers surveyed here, there has been growing discussion of the importance of ethics in Australasian mathematics education (see also Chap. 6, this volume). Atweh (2013) highlighted the apparent exile of ethical considerations in mathematics education research, arguing that much research focuses on *good* mathematics, rather than on mathematics *for the good*, making an implicit assumption that if the general population becomes better at mathematics, society will necessarily become more just. He maintained (Atweh, 2012) that ethics should precede ontology and epistemology in considering what counts in mathematics education, "making the assertion that ethics is not an add-on to the concerns in mathematics education. It lies at the very foundation of every decision in the field" (p. 340).

Ethical dimensions specific to the social value and purpose of mathematics education and learning mathematics (e.g., Lange & Meany, 2014) were also visible. While some research (e.g., Pierce & Stacey, 2013; Wilkie, 2014) implicitly assumed that there was a right way of "doing" mathematics or applying mathematical knowledge, other research (e.g., Muir, 2014) positioned knowledge, resources or procedures as something that could be freely chosen and applied openly and flexibly when learning and problem solving. We suggest that the extent to which students are free to choose and apply particular methods for solving problems is an ethical dimension of mathematics learning. Regardless of the outcome of such decisions however, we suggest that fostering intellectual dispositions in learning mathematics should characterise all mathematical learning and be made a more explicit focus of mathematics education research. Described by Sockett (2012) as intellectual virtues, these include qualities such as engaging accuracy, truthfulness, impartiality and open-mindedness.

Our analysis of the dimension of logic suggested that most mathematics education research papers use inductive approaches to research, data analysis and questions of generalisation. We identified consistency in the logical structure in the way research was presented, in connections between methodology and method, and in the congruence between data and discussion. We also observed that researchers remained faithful to the chosen methodology; for example, a hermeneutic study (e.g., Calder, 2012) applied the interpretive logic inherent in the theory. The importance of logical reasoning in mathematical *learning* was also evident in many papers (e.g., Fielding-Wells et al., 2014; Logan, 2015), where the role of explanation and justification and of normative validity was discussed. Few if any of the papers we examined however could make claims to using forms of logic that would be expected when engaging the discipline of mathematics itself. We suggest that there may be a place for reinvigorating logical, evidenced reasoning and rigorously argued reporting in mathematics education research.

6 What Do Specific References to Epistemology and Ontology in the Mathematics Education Research Literature Suggest?

As one might anticipate, in the mathematics education research literature the term *epistemology* was by far the most commonly occurring of the five areas of philosophical inquiry framing our chapter. Of the 139 papers that we identified in major educational research journals that were at least co-authored by a researcher associated with an Australasian university, 24 included the term epistemology or a variant thereof somewhere in the paper. In this section we attempt to draw together the epistemological perspectives in these papers, and to highlight the implications of particular epistemological positionings for mathematics education.

However, as highlighted by Galbraith (2014) in the quotation at the beginning of this chapter, there is by no means a common view on either what we mean by epistemology or on how knowledge building is best promoted in the mathematics classroom. This diversity of epistemological perspectives was the subject of a paper by Adam and Chigeza (2014) who discussed binary oppositions between different pedagogical approaches and perspectives and showed how these are related to epistemically relevant binaries. They maintained that “the coordination of these different and seemingly contradictory assumptions presents a ‘wicked problem’ for mathematics educators” (p. 109) that ultimately impacts significantly on students’ attitudes towards mathematics.

Reflecting on a long involvement in mathematics teacher education, Klein (2012) similarly highlighted the “inadequacies of contemporary theoretical and philosophical orthodoxies to fully address pedagogic change” (p. 25) and used a bifocal lens of psychological and post-structuralist constructs to highlight how power relations are inextricably connected to the construction of knowledge among pre-service teachers. In writing her paper, Klein aimed to “encourage fellow educators and researchers in mathematics education to continue to search out new perspectives in relation to theories, philosophies and ontologies that inform changes in instructional practice” (p. 39). From a similar post-structuralist perspective Walshaw (2012) argued that social justice is an epistemological issue. She claimed that a post-structuralist perspective and vocabulary provides ground for taking ethical practical action in a new epistemological context. For Walshaw, this is more than a mere construction; post-structuralist perspectives “open up the possibility of intervention through a commitment to social and educational change” (p. 117). Similarly Valero and Meaney (2014) argued that “scholarly work has the ethical commitment of pushing the limits of existing research discourses in the forming of the epistemological frameworks that format conceptions of practice” (p. 984).

Such commitments have been explored by a number of the authors who specifically highlighted epistemological issues. McMurchy-Pilkington, Trinick, and Meaney (2013), for example, discussed curriculum reform in New Zealand, describing how contestation over language and epistemology enabled a mathematics register for the Maori language to be modernised, in the process revitalising language and ideally leading to a more inclusive and culturally responsive curriculum. Meaney and Evans (2013) similarly discussed the values and purposes of school mathematics for Australian Indigenous students, arguing that we must take seriously both Indigenous epistemologies evidenced in traditional mathematical ideas and ways of knowing, and the imperative to learn school mathematics. They argued that achievement in system-wide assessments should not be considered the pinnacle of success for Indigenous students. Writing in a special issue of the journal *Learning Communities* on ethnographic stories of disconcertment, MacMahon (2013) described the both-ways approach to mathematics at Yirrkala in northern Australia, highlighting the disconnect between the assumption of epistemic equality that lies at the heart of the individualistic epistemologies of western mathematics and the person-specific meanings that underpin Indigenous epistemologies in that context. Lipka, Wong, and Andrew-Ihrke (2013) also discussed how Indigenous epistemologies, in their case those of the Yup'ik Eskimos, are brought into dialogue with academic mathematics, while Hooley and Levinson (2014) compared the educational experiences of UK Roma gypsies and Indigenous Australians, asking whether it is possible for formal educational systems to be inclusive and democratic by connecting with the epistemological views of marginalised groups and acknowledging their history, culture and identity. Averill (2012) suggested that it is, and explained how through explicit attention to Maori ways of knowing and being, it is possible to develop a culture of care in the mathematics classroom that is both inclusive and responsive. While it is not our purpose to pre-empt the discussion of mathematics for Indigenous students later in this Review (see Chap. 8), we suggest that the issues highlighted by these authors are deeply epistemological in nature, and that efforts to raise the achievement of disadvantaged or marginalised groups will not be effective unless such epistemological questions are addressed.

Several of the studies discussed above are specific in identifying the epistemic impact of different worldviews held by Indigenous and non-Indigenous people. An interesting variation of this is a study by Chan and Wong (2014), who examined the connection between ontology, epistemology and religious beliefs in mathematics. They described three representative teachers of mathematics: one Buddhist, one Christian but strongly influenced by a Confucian worldview, and the third an evangelical Christian. They suggested that Catholic and Protestant religious views tend to result in beliefs about mathematics as calculable and precise, while worldviews of Chinese origin orient the believer to see mathematics as primarily involving thinking. The Buddhist teacher in their study had what they considered to be stronger constructivist views about mathematics, and saw greater unity between mathematics and their view of the world. They described the Buddhist teacher as having a "connective epistemological worldview" (p. 268). Calder (2012) critiqued a view of mathematics similar to that found by Chan and Wong in the teacher of Christian persuasion as fixed and precise, arguing that mathematical concepts evolve rather than present themselves as fixed realities. In a study of the use of digital media such as spreadsheets and *Scratch* in pre-service teacher education, he adopted a hermeneutic perspective, stating that "understanding is a process rather than a position and a 'concept' is a shared consensus rather than an irrevocable truth" (p. 272). Mathematics itself, then, is "an evolving set of perceptions, with each iteration of interaction, interpretation and explanation either extending its edges or refining it core identities" (p. 282), and learning is an ongoing condition of becoming. Nason, Chalmers, and Yeh (2012) also examined the use of ICT tools, in particular *Knowledge Forum*, in pre-service teacher education. The tool is explicitly epistemic, providing a vehicle through which students can collaboratively build knowledge through the processes of wondering, conjecturing and hypothesising.

Bautista and Roth (2012) discussed underlying ontological assumptions about mathematics, which are exemplified in children classifying three-dimensional shapes through bodily movements and physical manipulation. Their study was framed within a theory of what they term "mathematics in the flesh" in which "mathematics does not constitute a corpus of transcendental and decontextualized abstract ideas, but a phenomenon only existing with/in our lived/living body" (p. 91). The mind/body duality highlighted in this study was further discussed by Roth (2012) in a theoretical exploration of the application of cultural historical activity theory (CHAT) to mathematics education research. He argued that traditional applications of Vygotskyian social constructivist theories maintain an external/internal duality and tend to emphasise static perspectives of activity rather than highlighting its dynamic nature. An interesting contrast to Roth's external/internal duality was provided by McDonough and Sullivan (2014) who looked at the beliefs and knowledge of young children through creative interviewing procedures. While Roth maintained that knowledge and beliefs represent external and internal manifestations of an individual's mathematical persona in a social context, McDonough and Sullivan explicitly started from the premise that beliefs are an internal individual construct, while knowledge is an external social construct. Afamasaga-Fuata'i and Sooaemalelagi (2014) also highlighted different aspects of mathematical action using a modification of Gowin's epistemological vee diagram. The thinking and doing sides of the epistemological vee enabled Samoan pre-service teachers to record and reflect on their attitudes, investigations and metacognitive tools.

A number of authors who highlighted epistemological aspects of their research discussed sociocultural approaches of learning. Goos (2014) used Valsiner's zone theory to examine how sociocultural perspectives can inform research that seeks to have an impact on classroom practice in the context of professional learning on technology integration. She described how a teacher's Zone of Proximal Development, in interaction with the Zones of Free Movement and Promoted Action, has deep epistemological underpinnings; it becomes "a set of possibilities for the development of new knowledge, beliefs, goals and practices created by the teacher's interaction with the environment, the people in it, and the resources it offers" (p. 523). Anthony, Hunter, and Thompson (2014) used cultural historical activity theory (CHAT) to trace one teacher's learning journey, highlighting the dialectic tensions at the epistemological level of the classroom. The teacher's increasingly rich understanding of these tensions, examined through Activity Theory, empowered him to think in new ways and to transform his teaching through what the researchers termed "expansive learning". Roth and Gardner

(2012) used CHAT to examine how electrical apprentices in Canada cross boundaries between formal schooling and the workplace, suggesting that the gap between the formal and work discourses appears to arise from an epistemology that "tends to endorse the valuation of abstract knowledge over actual practice and, as a result, to separate learning from working, and, more significantly, learners from workers" (p. 187).

Several other authors pointed to the epistemological gap between abstract mathematical knowledge and the contextual knowledge required in the workplace. Coben and Weeks (2014) facilitated the boundary crossing highlighted by Roth and Gardener (2012) through the provision of dynamic online virtual environments that closely match the workplace environment of nurses who are required to accurately administer medication dosages. Ramful and Narod (2014) discussed the epistemological gaps between students' reasoning in mathematics and chemistry, using Vergnaud's theory of conceptual fields to examine students' use of proportional reasoning. Building on the premise that "mathematical concepts exist in relation to each other and draw their meaning from a variety of situations" (p. 30) they described the complexities involved in the simultaneous use of chemistry knowledge and mathematical knowledge and argued for collegial collaboration between chemistry and mathematics education researchers.

Epistemological gaps and obstacles were also the subject of several mathematics-specific research papers. Hong and Thomas (2014) identified epistemological gaps and changes required in students' understanding of differentiation and integration in the transition from school to university. They described how digital materials designed to provide an improved cognitive base through a flexible, proceptual understanding of key ideas of calculus help to address these gaps and develop versatile thinking. Cortina, Visnovska, and Zúñiga (2013) used Brousseau's classification of ontogenetic obstacles, epistemological obstacles and didactical obstacles, arguing that didactical obstacles should be avoided, but that ontogenetic and epistemological ones should be faced. They outlined three images of equipartitioning of fractions that present didactical obstacles, arguing for a more widespread re-examination of assumptions about teaching and learning. The gaps between children's perceptions of reality and the world of mathematics were highlighted by Ben-Zvi, Aridor, Makar, and Bakker (2012) in their investigation of grade 5 students' development of informal statistical reasoning. They used Polanyi's theory that when faced with a problem people first develop a hypothesis drawn from personal beliefs and experiences, and that when faced with contradictory evidence, such evidence is often ignored or unseen. They claimed that the act of reconciling evidence with beliefs is an epistemological act, and described how the epistemological gap in informal statistical reasoning can be bridged through the use of growing sample sizes. Makar (2014) also investigated grade 3 students' informal inferential reasoning about the typical heights for grade 3 children, suggesting that an inquiry process built on epistemic argumentation can help to bridge the real world/mathematical world gap. Fielding-Wells et al. (2014) reported on the impact of epistemic argumentation to promote proportional reasoning. The grade 4 children in their study were able to progressively develop more sophisticated concepts of proportional reasoning as they developed mathematical models to represent proportions in the human body.

As discussed above every piece of research in mathematics education has underlying epistemological assumptions. This section has highlighted those papers that explicitly discussed some aspect of these epistemological assumptions within the paper. It has not been our intent to replicate the discussion of these papers elsewhere in this volume, as no doubt most will inform the discussion in subsequent chapters. However, we have attempted to show how the explicit epistemological perspectives in those papers influence the research frameworks and priorities within the research.

7 Conclusion

Our philosophical gaze has moved from a general overview of the keywords in more than 100 Australasian mathematics education research papers published between 2012 and 2015 to a more in-depth themed analysis of a purposive sample of 26 of these papers, to a detailed analysis of those papers that explicitly discuss epistemological aspects of the research. What stands out in our reading of the literature is that, with the exception of epistemology, the five philosophical dimensions discussed at the beginning of this chapter are largely unremarked. This does not mean that they are absent; rather most research papers implicitly assume particular positions relating to the nature of reality, how knowledge is produced, and issues of aesthetics, ethics or logic.

We have also identified a number of tensions inherent in the approaches embedded in the literature. At the beginning of this chapter we asked how we, as mathematics educators, choose between competing views of the world. The competing views we have identified include, for example, mathematics as a human construct or a model of reality, knowledge or beliefs as individual or social, mathematics learning as conceptual or procedural, research as interpretive or transformative, and curriculum as given or open to critique. We suggest that none of these tensions is one or the other. Rather, we suggest that mathematics education research (and hence education itself) is always both.

Thus rather than regarding these tensions simply as "wicked problems" (Adam & Chigeza, 2014), we suggest that mathematics education researchers and teachers have to live with them and use them as a creative force. When we, as mathematics education researchers, recognise and live with such tensions we do not become hostage to a particular view of the world, nor do we become blind to the underlying assumptions about the world that drive our work. Rather we recognise that research is underpinned by particular views of the world and the nature of knowledge, and that this has significant implications for both research and practice. We hope that the philosophical gaze adopted in this chapter provides a vehicle for identifying and questioning these underlying assumptions, and that it might provide the stimulus for ongoing philosophical inquiry within the Australasian mathematics education research community.

Appendix 1: Template Used in a Hermeneutic Reading of a Mathematics Education Research Paper

Paper: Fielding-Wells, J., Dole, S., & Makar, K. (2014). Inquiry pedagogy to promote emerging proportional reasoning in primary students. *Mathematics Education Research Journal*, *26*(1), 47–77.

Dimension	Observations
Ontology (metaphysics)—the nature of being, becoming, existence, or reality. What aspects of mathematics education are "taken as given" in the research? What do different research paradigms or theoretical frameworks used in the research assume about the nature of reality?	Maths is there "proper concept" (p. 48) Real life context crucial, mathematisation (p. 59), reasonableness (p. 69), transfer Challenges image of maths as unproblematic (p. 62)
Epistemology—the study of knowledge and justified belief and how we come by it. Are different views of knowledge evident in the research? If so, how do these epistemological positions impact upon the methods and outcomes of the research?	Student learning is "foundational" (p. 47), "developmental" (p. 48), "prerequisite" (p. 48), "difficulties with proportion" (p. 70) Maths as hierarchical, structured, sequential challenged (p. 50) Also p. 62—complex problems simultaneous with conceptual development (cf traditional approach). Challenge curriculum (p. 73)
Aesthetics—judgement about matters of value. What values appear to underpin the research and how do they impact upon the positions adopted by the researchers with respect to curriculum or pedagogy?	Efficiency, applicability, elegance, decision making, justification, analysis Affective and intellectual challenge and goals (p. 71) "Value mathematical practices that cut across particular content" (p. 55) Social impact (body image, Barbie), perhaps unstated
Ethics—systematising, defending and recommending concepts of right and wrong conduct. How does the research embed or contribute to moral or intellectual virtues such as truthfulness, impartiality, open-mindedness, courage and justice?	Goals to develop problem-solving, application, thinking tool Ill-structured better than well-structured Evidence stressed by children Philosophical approach to teaching (inquiry and epistemic argumentation) (p. 54) "Intellectual rigour", "authentic practice", "investigative spirit", "ownership", "accountability" (in the group learning sense) (p. 70) "Scrutinise role of mathematics as gatekeeper" (p. 71)
Logic—the use and study of valid reasoning. What different approaches to reasoning or argumentation are used in the research? How do these different approaches position the research with respect to its capacity to be generalised or contextualised?	Enquiry, evidence (p. 72 and following)—Geneva's story, public argumentation Frameworks to summarise process Question, evidence, conclusion, purpose (p. 53) Modeling with unifix cubes Norms

Appendix 2 List of Papers Used in Hermeneutic Reading of Mathematics Education Research Papers

Chapter numbers	Keywords used to search for papers	Papers used
Chapter 4	Policy, curriculum, leadership	Sullivan, Clarke, Clarke, Farrell, and Gerrard (2013), Zhang and Stephens (2013)
Chapter 5	Affect, affective, attitudes	Attard (2013), Lim and Chapman (2015)
Chapter 6	Equity, diversity, social justice	Averill (2012), Lange and Meaney (2014)
Chapter 7	Inclusive, disabilities, special needs, gifted	Bicknell and Riley (2012), Faragher and Clarke (2013)
Chapter 8	Indigenous, second language, Maori, Torres Strait Island, Aboriginal	Meaney and Evans (2013), Owens (2015)
Chapter 9	Early years, early childhood, pre-school, prior to school, young children, babies, toddlers	Cohrssen, Church, and Tayler (2014), Gervasoni and Perry (2015)
Chapter 10	Tertiary, university, undergraduate	Barton, Oates, Paterson, and Thomas (2015), Thomas and Klymchuk (2012)
Chapter 11	Innovation, pedagogy, problem solving, transformative	Ho and Lowrie (2014), Pierce and Stacey (2013)
Chapter 12	Assessment, evaluation, testing, standards, formative, summative	Logan (2015), Roche and Clarke (2013)
Chapter 13	Digital, media, computers, devices, tablets, apps	Calder (2012), Muir (2014)
Chapter 14	Mathematical applications, mathematical modelling, real world, mathematising	English (2012), Stillman and Brown (2014)
Chapter 15	Pre-service, initial teacher education	Anthony et al. (2015), Cavanagh and Garvey (2012)
Chapter 16	Professional learning, community of practice, teacher-development, early career	Goos (2014), Wilkie (2014)

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Chapter 4
Researching Curriculum, Policy and Leadership in Mathematics Education

Jennifer Way, Janette Bobis, Janeen Lamb and Joanna Higgins

Abstract This chapter reviews research regarding the official mathematics curriculum and its enactment, the educational leadership to support this enactment, and the associated influential policy, such as national testing. It explores the interrelationships between inherent issues such as the potential influence of textbooks, curriculum equity, and the complexities of implementing numeracy across disciplines. Substantial research has led to the development of robust theoretical models to inform both future research and practical developments across a range of aspects of curriculum, policy and leadership. However, the seemingly diverse research perspectives are all drawn towards the teacher in the classroom as the critical context for further research.

Keywords Curriculum · Policy · Leadership

1 Introduction

In this chapter we focus on research regarding the official mathematics curriculum and its enactment, the educational leadership to support this enactment, and the associated influential policy, such as national testing. Our literature search for this

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review began with the broad topics of “curriculum”, “policy” and “leadership” in mathematics education. As expected, researchers had identified issues within these topics, with clusters of studies around the policy-curriculum relationship (including national testing policy), the role of educational leadership, the potential influence of textbooks, curriculum equity, and the complexities of implementing numeracy across disciplines within the curriculum. A major goal of the review became to explore the interrelationships within and between these topics and issues.

While we acknowledge close relationships between curriculum and other enactment factors such as teacher professional development, pedagogy and assessment of student learning, this research is dealt with in other chapters of this book, and therefore not considered in depth in this chapter. Included here are studies of curriculum-related *numeracy*, in particular, the involvement of mathematical skills across other learning areas such as English and Science. We also acknowledge the importance of recent curriculum developments in early childhood education, but refer readers to Chap. 9 of this book for review of such research. Similarly, Chap. 10 is dedicated to tertiary level mathematics and so research on matters pertaining to curriculum design and implementation, and leadership in this context have been excluded. Consequently, the content of this chapter is dominated by research relevant to primary and secondary school education.

The reason for the focus of Australasian research on primary and secondary education becomes obvious when the strong influences of recent political agendas and national curricular initiatives in Australia and New Zealand are realised. Therefore this chapter begins with information about recent political-educational directions in these two countries. For background information on the preceding curriculum development or implementation phases for Australia and New Zealand, we refer readers to the curriculum chapter of the previous 4-yearly review (see Anderson, White, & Wong, 2012). As pointed out in the previous review, each country took a very different approach to initial curriculum development. New Zealand began “with a vision and principles for the whole curriculum”, whereas the Australian Government began “with four subject areas including mathematics” (Anderson, White, & Wong, 2012, p. 226). The previous review also concluded that “Curriculum reform through the written or intended curriculum does not necessarily lead to reform in the enacted curriculum” (Anderson, White, & Wong, 2012, p. 238). Hence we see value in the inclusion of “leadership” in the current review. Understanding these contexts helps to reveal the complex relationships amongst policy, curriculum, educational leadership, and the teachers who are expected to bring curriculum intentions to fruition in classrooms. With the purpose of bringing some clarity to the complex system of curriculum policy, design, and enactment, this chapter is framed by a model developed by Remillard and Heck (2014)—as presented in the second section. While this model was not referred to in most of the studies reviewed here, it served as a valuable organiser for much of the chapter, and we foresee its theoretical utility for future research.

2 The Australasian Context

2.1 New Zealand

The current *New Zealand Curriculum for English-medium teaching and learning in years 1–13* (Ministry of Education) was launched in 2007 and mandated for implementation in early 2010. Underlying the curriculum are eight principles: (i) High expectations, (ii) Treaty of Waitangi, (iii) Cultural diversity, (iv) Inclusion, (v) Learning to learn, (vi) Community engagement, (vi) Coherence, and (vii) Future focus. In addition, five key competencies are identified: (i) Thinking, (ii) Relating to others, (iii) Using language, symbols and texts, (iv) Managing self, and (v) Participating and contributing. The introduction of the curriculum was soon followed by an initial evaluation of its implementation, leading to the report titled *Directions for Learning: The New Zealand Curriculum Principles, and Teaching as Inquiry* (Education Review Office [ERO], 2011). The focus of the evaluation was to "investigate how schools were using the eight principles and the teaching as inquiry process as outlined in *The New Zealand Curriculum*" (ERO, 2011, p. 1). It was found that 82 % of the schools evaluated had developed school-based curricula that reflected the principles. School leadership was found to be a significant influence, particularly in achieving the further enactment of the curriculum in classrooms. Of interest is the emphasis the review placed on the pedagogical-guidance function of the curriculum, and the importance of leadership in the realisation of curriculum aims.

The implementation of the Mathematics Curriculum component of the broader curriculum was evaluated in terms of (a) the design and review of each sample school's mathematics curriculum, (b) the use of achievement information by trustees, leaders, teachers and students, and (c) the acceleration of progress of priority learners. The published report, *Mathematics in Years 4–8: Developing a Responsive Curriculum* (Education Review Office, 2013), suggests that although some schools were highly effective in all three aspects, many schools needed increased support and leadership to achieve the curriculum implementation expectations.

2.2 Australia

Research published on school mathematics and numeracy in Australia over the past 4 years has taken place in the context of the initial implementation phase of the first *Australian Curriculum*, produced by the Australian Curriculum, Assessment and Reporting Authority (ACARA) and released in stages from 2010 to 2015. The introduction of a national curriculum has historical significance, because for the first time, the state education jurisdictions have ceded substantial curriculum responsibility to a national authority. Gerrard et al. (2013), Anderson (2014) and Stephens (2014) provide informative historical perspectives of the policy shift from state to federal responsibility for curriculum reform, leading to the release of the first national mathematics curriculum in December 2010. It is important to note that each state and territory still holds responsibility for the implementation of the curriculum, and as pointed out by Anderson (2014), deeply held beliefs and previous practices typically produce variations in the enactment of curriculum, not only at the school level, but at system or state levels.

Some academics have questioned the political motivations and social/economic drivers that have shaped the content and inherent values in the Australian Curriculum, posing questions such as "Whose knowledge is valued? Who decides? And who benefits?" (Ditchburn, 2012, p. 268). The relevance of such questions becomes apparent when contemplating the seven *general capabilities* and three *cross-curriculum priorities* featured in the new curriculum. The general capabilities are: (i) literacy, (ii) numeracy, (iii) information and communication technology capability, (iv) critical and creative thinking, (v) personal and social capability, (vi) ethical understanding, and, (vii) intercultural understanding. The cross-curriculum priorities are: (i) Aboriginal and Torres Strait Islander histories and cultures, (ii) Asia and Australia's engagement with Asia, and (iii) sustainability (ACARA, 2015a). There are perhaps some interesting contrasts in the political orientations of the New Zealand and Australian governments to be revealed by examining the *principles*, *key competencies*, *general capabilities* and *priorities* of the two curricula.

Concerns have also been raised about the competitive "curriculum pressures" of the National Assessment Plan—Literacy and Numeracy (NAPLAN) in Australia, and publication of results on the *MySchool* website (http://www.myschool.edu.au/). These pressures, amplified by debate about school funding inequalities and political attention to ranking in international testing programs (Programme for International Student Assessment [PISA] and Trends in International Mathematics and Science Study [TIMSS]), have highlighted differences of social advantage/disadvantage in student access to the broader curriculum (Yates, 2013). Some of the issues regarding differentiation of the curriculum to support equity of learning opportunity have been investigated by researchers and are dealt with later in this chapter.

As well as attending to the mathematics content in the *Australian Curriculum: Mathematics* (ACARA, 2015b), teachers are expected to consider the teaching approaches required to develop in students the four *Proficiencies* of (i) understanding, (ii) fluency, (iii) problem-solving, and (iv) reasoning. Amongst the academics closely involved in the design of the Mathematics component of the broader Australian Curriculum are those with the optimistic view that the principles underlying its structure and presentation will provide educators with decision-making opportunities that will benefit the learning of all students (Sullivan, 2012). This view implies that teachers will take note of isolated statements, such as "It encourages teachers to help students become self-motivated, confident learners through inquiry and active participation in challenging and engaging experiences" (ACARA, 2015b), and that teachers will be able to translate this intent into specific classroom pedagogy. Some researchers argue that the pedagogical intent of the mathematics curriculum may not be communicated strongly enough to inspire the desired teaching practices (Atweh, Miller, & Thornton, 2012). Indeed, Zhang and Stephens (2013), in their study of Australian and Chinese teachers, concluded that

> effective implementation of any curriculum reform depends on teachers' subtle interpretations of official curriculum documents and their professional dispositions to act on those ideas, which go well beyond general descriptions or statements of intent that are usually embodied in official curriculum advice. (p. 499)

On another level, schools are required to attend to the development of numeracy across all learning areas (subjects)—numeracy being one of the seven cross-curricular "general capabilities". This requirement places demands on teachers from all disciplines to recognise inherent mathematics concepts and skills and incorporate their development into teaching plans, raising questions about teacher preparedness to effectively enact the numeracy development requirements. Interpreting this array of curriculum intentions, designing appropriate teaching plans and effectively implementing such plans arguably requires significant school-level educational leadership.

The implementation of new school curricula in New Zealand and Australia clearly presents a rich context for research. A substantial portion of this chapter critically explores the questions being asked by researchers and their responsiveness to the issues arising from the new curriculum context, but first we establish a framework for comprehending the relationships amongst the broad topics of policy, curriculum and leadership.

3 Relationships Amongst Policy, Curriculum and Leadership

As noted by others (e.g., Remillard & Heck, 2014), the term *curriculum* takes on different meanings around the world. It was also noted during preparations for this chapter that *curriculum* is often used in educational contexts without clarification. This lack of clarification made it difficult at times to precisely determine what was being referred to, since the term is applied to a variety of aspects comprising a broad spectrum of curriculum planning and enactment processes. Consequently, our initial search for literature was driven by the desire to establish some clarity around these aspects that might also assist in the structuring of chapter content emerging from our review. Here we present our perspectives on key constructs and processes influential in the planning, enactment and assessment of curriculum. Drawing and building upon the definitions and views of prominent researchers in the broader international field of mathematics curriculum research (e.g., Remillard & Heck, 2014; Schmidt et al., 2002) we delineate key terms and present a systematic perspective on curriculum in which research about policy, curriculum and leadership, discussed in the rest of this chapter, is situated.

The mathematics curriculum is broadly defined by Remillard and Heck (2014) "as a *plan for the experiences* that learners will encounter, as well as the *actual experiences* they do encounter, that are designed to help them reach specified mathematics objectives" (p. 707). They propose a framework that conceptualises

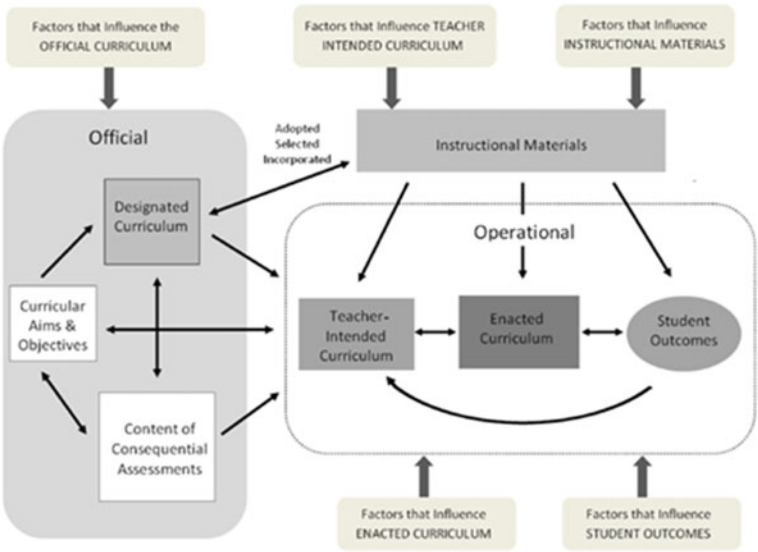

Fig. 4.1 Visual model of the curriculum policy, design, and enactment system. Remillard and Heck (2014), p. 709; Fig. 1, with permission of Springer

various curricular elements (policy, assessments, textbooks, student outcomes etc.) existing within a curriculum policy, design and enactment system (see Fig. 4.1).

The framework focuses on two components of the curriculum system. First is the *official* curriculum, specifying what should be taught. It is sometimes referred to as the "intended" curriculum (Schmidt et al., 2002). The official curriculum incorporates curriculum elements including official curriculum elaborations, curricular aims such as the achievement standards contained within the *Australian Curriculum* (ACARA, 2015a) or achievement objectives in *The New Zealand Curriculum* (Ministry of Education, 2007), and mandated assessments such as the *National Assessment Plan—Literacy and Numeracy* [NAPLAN]. It is within this domain that research relating to curriculum policy is most pertinent, as the official curriculum is heavily influenced by social, cultural and political factors—all of which are in constant flux. Herein lies a potential limitation of the Remillard and Heck (2014) model. It presents the official curriculum as absolute, rather than a more dynamic view of curriculum "in the making" that should also be informed and revised with input from experts and practitioners in mathematics and mathematics education (Kemmis et al., 2014).

The second component highlighted in Remillard and Heck's (2014) system framework is the *operational* curriculum. This component specifies what actually occurs during the enactment process—some of which exists outside the official, sanctioned curriculum. It comprises the enacted curriculum, which includes aspects of curriculum leadership, teacher development in terms of their pedagogy and knowledge, interactions between students and teachers during instruction, the tools and resources used by teachers, and the actual mathematics presented. The enacted curriculum has the greatest potential for impacting a broad range of student outcomes—their achievement, attitudes and their motivation and engagement in mathematics (Thompson & Huntley, 2014). Hence research has mostly focused within the operational domain of the curriculum system, and particularly the enacted curriculum and student outcome components.

4 The Official Mathematics Curriculum

Two major issues regarding the *official mathematics curriculum* were identified in the research reviewed for this chapter: the official curriculum as a form of policy; and the role of student assessment and national testing in an official curriculum. A key message from the papers reviewed is that the official curriculum is a political tool, perceived as a means for ensuring the social and economic well-being of citizens and a country at large, as well as for enhancing student performance (Walshaw & Openshaw, 2011). Measurement and monitoring of student performance on a national scale is therefore a consequence of a national curriculum.

4.1 Curriculum Policy

Stephens (2014) emphasised the importance of seeing the development of an official curriculum as a socio-political process nested in political cycles of government, with curriculum development initiated as successive new governments come into power. A change in official curriculum indicates an attempt at social-political-economic change. Therefore, the curriculum embodies current imperatives and is intended to be future-focused, and reform-oriented (Anderson, 2014; Goos, Dole, & Geiger, 2012a). However, the directions chosen by the government in power at the time of curriculum development may not be in harmony with a new government's political agenda, prompting curriculum reviews as recently seen in Australia (see *Improving the Australian curriculum*, ACARA, 2015c).

A common catalyst for curriculum policy is perceived declining standards and associated declining international ranking, generated through international studies such as TIMSS and PISA. Leung (2014) warns against using country-rankings as impetus for "changes in education policies without due consideration of the nature and limitations of these studies" (p. 579). Instead, attention should be given to trends in achievement scores, differences between strands of mathematics and to the attitudes of the students (Leung, 2014). Unfortunately, it is the country-rankings that make media headlines, with attention rarely given to the informative data on variables such as curriculum, resources and instruction.

In the Australasian context, the *mathematics curriculum* is viewed as a component of the broader national curriculum. The *mathematics curriculum* refers to both the selected mathematics content (often called the syllabus), and to the social/cultural values and pedagogical expectations communicated through the aims and principles underlying the curriculum. One of the factors specified by Remillard and Heck (2014) as an influence on the official curriculum is “Values and beliefs about mathematics and the goals of education as held publicly and by individuals and groups wielding power” (p. 714). Accordingly, the mathematics curriculum can be interpreted as a vision for the discipline (Atweh, Miller, & Thornton, 2012). The mathematics curriculum communicates its purpose and value in society, the mathematics that should be taught, the ways in which it should be taught and assessed, and the type of mathematical thinking that is important. Atweh, Miller, and Thornton (2012) critically examined the internal and external cohesiveness of the *Australian Curriculum: Mathematics*—in other words, the alignment of the broader curriculum goals (*General Capabilities* and *Cross-curricular Priorities*), mathematics *Proficiencies*, and “the rationale behind the content selection and organisation that may guide teachers and schools in their construction of their school curricula, pedagogical and assessment practices” (p. 16). They conclude that there are missed opportunities for providing teachers with sufficient guidance to achieve the goals of “inter-disciplinary approaches”, “deep knowledge” and “complex problem-solving” in the intended curriculum.

Other researchers have contributed to debates about curriculum structure and content by investigating key aspects of the mathematics curriculum (such as numeracy and problem solving), looking at how the content is communicated and what mathematical knowledge is valued. For example, the focus on numeracy can be seen as reflecting the importance of social and economic well-being (Goos, Dole, & Geiger, 2012a, 2012b). It has been argued that a critical orientation to numeracy is important in developing a citizenry that is equipped for the numeracy demands of the 21st century. The inclusion of *Numeracy* as one of the General Capabilities in the Australian Curriculum suggests its importance—yet the effectiveness of its representation in each subject’s curriculum (including Mathematics) has been questioned by researchers (Goos, Dole, & Geiger, 2012a, 2012b). Similarly, although Problem Solving and Reasoning are stated as two of the four *Proficiencies* permeating the new *Australian Curriculum: Mathematics*, analysis of the content descriptors reveals minimal representation of the higher order mathematical thinking depicted in the definitions of these two *Proficiencies* (Anderson, 2014; Atweh & Goos, 2011; Atweh, Miller, & Thornton, 2012). In contrast, Problem Solving is presented as the central feature and primary goal of the Singaporean curriculum (Kaur, 2014). However, in both contexts, the researchers call for more investigation of teachers’ enactment of problem solving in their classrooms.

4.2 Assessment and Testing Policy

Here we refer only to national assessment imposed by policy. Chap. 11 of this book deals more broadly with research on assessment of mathematics learning. An accountability agenda associated with curriculum policy is evident through the instigation of national assessment regimes—some testing-oriented such as *The National Assessment Program—Literacy and Numeracy* (NAPLAN) in Australia, and others standards-oriented such as *National Standards* in New Zealand. Indeed, Stephens (2014) identified national assessment and school reporting as system-level levers leading to the Australian Federal government taking greater central control of the curriculum. Although both countries have central reporting requirements for student assessment results, the approaches are very different. Australia imposes a strict testing regime for school years 3, 5, 7 and 9 and publishes school data on the public *MySchool* website. Commentary on NAPLAN, perhaps not surprisingly, dominates recent research, with very little reported about the impact of New Zealand's *National Standards* processes.

The New Zealand *National Standards* are "broad descriptions of expected achievement derived from curriculum achievement objectives" for school years 1–8 (Ministry of Education, 2011). Emphasis is placed on the value of formative assessment practices, with "the use of professional teacher judgment underpinned by assessment for learning principles rather than a narrow testing regime" (Ministry of Education, 2011). The policy is based on international research and key publications are provided for teacher professional reading, along with a range of resources, via the *Assessment Online* website. In contrast, the Australian *National Assessment Program* bypasses classroom teachers in data collection, with its purpose stated as being "the measure through which governments, education authorities, schools and the community can determine whether or not young Australians are meeting important educational outcomes" (ACARA, 2015d). National standardised tests in literacy and numeracy (NAPLAN) are administered in school years 3, 5, 7 and 9, and results are returned to schools some months later. The claimed benefits are "to help drive improvements in student outcomes and provide increased accountability for the community (ACARA, 2015d).

Concerns have been raised regarding the impact of Australian NAPLAN on teachers and students. Drawing on a survey of 8000 educators across Australia, Polesel, Rice, and Dulfer (2014) conclude that high-stakes testing has resulted in "a narrowing of curriculum, a restriction in the range of skills and competencies learnt by students and a constriction of pedagogical approaches" (p. 653). (See also the full report, Wyn, Turnbull, & Grimshaw, 2014). Contrary to the government's purported intention of supporting schools through providing data about student progress, the emphasis that has been placed on rapid gains in student performance, and the comparison of schools in terms of success or failure, has produced negative influences on the quality of learning in the majority of schools. (See Polesel, Rice, & Turnbull, 2012, for a review of literature.) However, it should be noted that some schools have more productively used NAPLAN data by analysis in conjunction with other school-based assessments to identify learning needs for both students and teachers (Polesel, Rice, & Dulfer, 2014). Hardy (2015) offers an interpretation of the difference in school responses after applying a Bourdieuian framing to the interviews of 55 participants from three Queensland primary schools. He concluded that the teachers had collectively "repurposed" the government's NAPLAN agenda and "appropriated from solely performative and political purposes—for more educational purposes" (Hardy, 2015, p. 10). However, the capacity of most educators to effectively analyse and interpret the NAPLAN data may be a substantial barrier to many other schools.

Picking up on this issue, Chick and Pierce (2013) investigated the statistical literacy needed by government personnel, principals, and particularly teachers to interpret the results of large-scale statistical reports such as NAPLAN. Through identifying the nature of statistical knowledge needed by teachers to appropriately interpret graphical representations of data, they propose a 3-level framework for professional statistical literacy. The framework emphasises the importance of professional and local contexts, and groups the skills required to draw statistically valid conclusions under *Reading Values*, *Comparing Values*, and *Analysing the Data Set*. Chick and Pierce (2013) argue that teachers (both in-service and pre-service) need targeted professional learning to develop the required statistical understanding and critical thinking.

Other researchers have viewed the influences of NAPLAN from a social justice perspective, questioning whether the national testing policy supports the social goals of the curriculum. Lange and Meaney (2014) conducted Bernsteinian analysis of NAPLAN—that is, an analysis of the structuring of knowledge and the framing of pedagogical practice. They reflect on how "raising standards" is used as a euphemism for "social justice", and is distorted to become schools' accountability for student achievement. Children are positioned as commodities "to add value to", frequently through deficit language in the public discourse around national high-stakes testing. Lange and Meaney argue that this situation is contradictory to the purpose of mathematics education for citizenship as well as limiting what is generally understood as being numerate. A useful overview of the diversity and scope of mathematics education research associated with NAPLAN is presented in a paper by Leder (2012). She points to areas needing further investigation such as gender, Indigenous students and the needs of the highly able. The importance of considering the learning needs of particular groups of students and specific school contexts is not limited to national testing programs, but appears to extend to other national assessment approaches. Following the negotiation of a culturally responsive assessment protocol, the *Māori-medium National Standards* (New Zealand) were implemented in 2011, with a reasonably optimistic forecast for averting the anticipated negative effects of national data gathering on Maori education (Özerk & Whitehead, 2012).

Overall, commentators agree that the policy lever of high-stakes testing to increase student achievement in mathematics has a negative impact through limiting the public's understanding of the mathematics curriculum, as well as limiting what is taught in schools. There is a need for researchers to counter the drive of political leaders and policy makers for such testing regimes as NAPLAN, by providing large-scale evidence of the impact on curricula, amongst other aspects of schooling. However, there is less agreement on specific aspects of the impact of high-stakes testing and how it can be managed at different levels of the system, so targeted studies are also required to better understand specific contexts. Also lacking in the research literature are comparative studies of the Australian and New Zealand approaches, including the cultural responsiveness of assessment programs to Indigenous students, and other groups such as recent migrants and refugees.

5 The Operational Curriculum

The operational curriculum in Remillard and Heck's (2014) framework relates to what actually occurs during the enactment process. During times of curriculum change the enactment process can be strongly influenced by the textbooks and resources used by teachers in selecting the actual mathematics being taught. One view is that the combination of the official curriculum and textbooks and resources provides opportunities for teacher development, with a view to improving learning through enhanced pedagogy and knowledge (Sullivan, 2012). However access to these opportunities relies on curriculum leadership that guides the transition from old practices to new and improved practices. This leadership can take the form of teacher leadership, principal leadership and/or system leadership (Gaffney, Clarke, & Faragher, 2014a).

Naturally, not every learner will experience the curriculum in exactly the same way. Teachers, influenced by their own knowledge and beliefs, perception of student needs, and local contexts, will transform the official curriculum through planning their *teacher-intended curriculum* (Remillard & Heck, 2014). Further transformations will occur during *instructional interactions* with students, including the *pedagogical moves* made in response to students (Remillard & Heck, 2014). Much of the research regarding instructional interactions is dealt with in the *Learning and Teaching* section of this book, but here we include studies that have focused on the redesign or differentiation of the curriculum, and scrutiny of the curriculum framed by issues of equity in its implementation. An aspect of the new Australian Curriculum that has received much attention is the enactment of *numeracy* across the range of disciplines taught in schools.

5.1 Textbooks

Textbooks and resources are usually designed to align with the official curriculum. In the Remillard and Heck framework they are considered to be *instructional materials* that often take a transitional role between the *official curriculum* and the *operational curriculum*. Frequently textbooks are revised or created in response to the advent of a new curriculum and, as Shield and Dole (2013) commented, can be a means of advancing mathematics reform in the classroom. Kaur (2014), in her article on the enactment of the mathematics curriculum in Singapore, commented that textbooks in the Singaporean setting adhere very closely to the official curriculum, and so are a critical component of teachers' enacted curriculum. However, other education jurisdictions have considerably less influence on the development of new textbooks, highlighting the need to scrutinise their content.

Internationally, researchers have gradually developed a better understanding of the role of textbooks in operationalizing the curriculum (Fan, Zhu, & Miao, 2013). Broadly, international studies have focused on the three main areas of (a) textbook analysis, (b) the ways in which textbooks are used, and, (c) textbook comparisons (Fan, Zhu, & Miao, 2013). The main focus of recent textbook studies in the Australasian context has been to explore the potential of particular textbooks to realize the intent of the curriculum, so relate mostly to (a) textbook analysis. Various analytical models have been developed and utilised to investigate the potential of textbooks to influence the teaching and learning of mathematics. In a pro-active approach to textbook design, Debritz and Horne (2013) describe a model for developing curriculum resources that embed a guided inquiry process, with the materials intended to support teachers' interpretations of the *Australian Curriculum* and provide a starting point for curriculum planning. In another study, Shield and Dole (2013) expressed concern about the extent to which textbooks could support the development of deep and connected knowledge. They applied a framework to analyse five textbook series for middle-school mathematics and found limited support for the development of the multiplicative structures required for proportional reasoning.

Rafiepour Gatabi, Stacey, and Gooya (2012) reported a comparative study of Iranian and Australian textbooks related to problems that promote mathematical literacy. They recommended that countries adopt a framework to identify the capacity of a textbook to promote mathematical literacy, and give attention to including diversity in problem contexts. Siemon, Bleckly, and Neal (2012) also take up this call, arguing that textbooks have too many low level problems that focus on practising skills that do not advance the intent of the *Australian Curriculum*. Instead, they suggest that by focusing on connecting the "big ideas" there is an opportunity to rationalise the over-crowded curriculum.

Collectively, these studies suggest that textbooks can play an important intermediary role between the official and the operational curriculum, but they also raise concerns about the efficacy of textbooks to support teachers in realising the intended learning outcomes of the official curriculum. The variety of analytical models developed in these studies provides other researchers with tools for further investigations. However, without centralised monitoring of textbook quality in Australia, the impact of research findings on the representation of content may be limited.

5.2 Curriculum Leadership

Considerable research has also been undertaken in Australasia on teachers leading change in mathematics teaching and more specifically numeracy (e.g., see Faragher, Gaffney, & Skoss, 2014; Gaffney, 2012; Geiger, Goos, & Forgasz, 2015; Jorgensen, 2015a). These research projects have covered primary, secondary and university level mathematics across urban and remote regions and generally report changes in teacher practice that lead the way to improved student outcomes. Moreover, the three cross-curriculum priorities in the *Australian Curriculum*, Aboriginal and Torres Strait Islander histories and cultures, Asia and Australia's engagement with Asia, and Sustainability, each provide a wealth of opportunities for linking mathematics to other subject areas and promoting numeracy (Watson & Neal, 2012). Importantly, it has been noted across many projects that teachers are actively engaged in decision making through every step of the planning and delivery of their teaching (e.g., Sullivan, Clarke, Clarke, Farrell, & Gerrard, 2013; Jorgensen, 2012, 2015b; Miller & Warren, 2014). It is during these planning moments that teacher leaders come to the fore as they guide and support their colleagues (Hudson, Spooner-Lane, & Murray, 2013). This support can come in the form of assisting others to see the need for change, as well as facilitating the transition. However, it has also been reported that once the research team completes their project, support for teacher leaders ceases, leaving a vacuum that is often difficult to fill for other leaders within the school system (Sexton & Downton, 2014).

The search for leadership to further support the operationalisation of the curriculum is most often directed to title-holder leaders. The recent book, *Leading Improvements in Student Numeracy* (Gaffney & Faragher, 2014b) outlines the case that, for improvements to be long lasting and effective, leadership must stem from a range of sources: (a) the educational system, (b) the school principal or other title holders within the school, and (c) untitled teacher leaders. This perspective is consistent with scholarly writing in the area of educational leadership where an understanding of leadership has evolved from one synonymous with "positional authority" and the province of single individuals in those positions, to one which views leadership as an "influencing relationship" in educational settings. The latter perspective on leadership is evident in those capable of influencing others, either individually or in teams, to further student learning (Jorgensen, 2015a). The definition of leadership provided by Rost (1993) is useful because it assists in understanding leadership as "an influence relationship among leaders and collaborators who intend real changes that reflect their mutual purposes" (p. 99).

Current leadership theory is beginning to describe more comprehensively what it means to be a leader across a range of contexts. This includes being authentically "in" the group you are leading; being a champion for the group; transforming the group; and aligning the group with external expectations (Branson, 2009). Changes in the context merely change the manner by which these characteristics are enacted. In this contemporary understanding, educational leadership can and should be distributed across the entire education system (Branson, 2010). This approach to leadership ensures that all those involved in enacting the curriculum are better prepared to meet their obligations. Lamb and Branson (2015) provide a visual, Zonal Theory, representation of the possible roles for key school players in school change processes (Fig. 4.2). Important in this representation is that the actions and reactions of the principal and teachers are not independent of each other but are in fact co-constructed. Research supports the notion that a strong professional relationship between the principal and each participating teacher influences the way in which the curriculum is operationalised (Batiste, Walker, & Smeed, 2015).

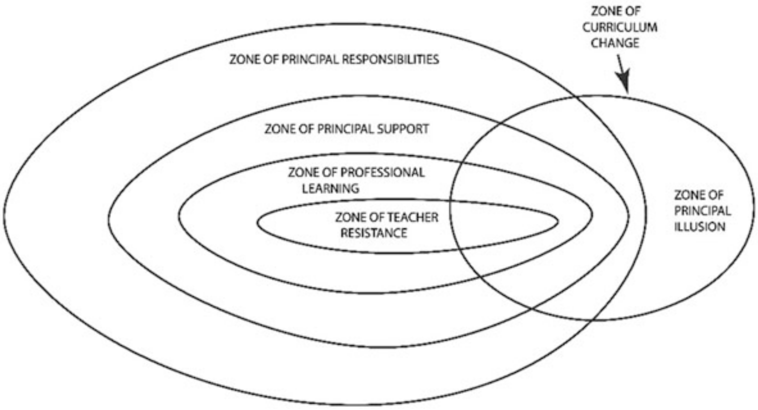

Fig. 4.2 Representation of Zonal Theory applied to introduction of curriculum change in a school. Lamb and Branson (2015), pp. 1010–1026; 21 July 2015, SAGE Publications, doi:10.1177/1478210315588840

Leadership does not stop at the school gate as system leaders have an important role to play in operationalising the curriculum (Ashhurst & Gaffney, 2014). Best outcomes for all are achieved with the alignment of thinking and practice across the education system that includes the education department, local education districts, schools and classrooms. Alignment can be strained in times of rapid curriculum change as is evident within Australia with ACARA and in New Zealand with the Education Review Office (ERO). Initial findings from a study of the introduction of the *Australian Curriculum* (English and Mathematics) reveal a lack of alignment between policy makers and schools, with "very different notions of teachers' work in relation to curriculum planning and enactment" (Gerrard et al., 2013, p. 70). There is also inconsistency in the implementation approaches across schools, with "dramatic differences in the extent to which curriculum planning is currently embedded within school processes" (Gerrard et al., 2013, p. 70). These findings resonate with the evaluation conducted by the New Zealand Education Review Office (ERO, 2011) that found much of the difference in curriculum-reform progress at the school level was due to leadership within the school. However, schools also work within a system. Mathematics education researchers are working across a range of contexts including Indigenous communities (Jorgensen, 2012; Jorgensen & Perso, 2012; Warren & Quine, 2013) with an effort to understand how systems can lead change on a large scale and how this can support the school principal to implement change within the school environment.

5.3 *Curriculum Design, Differentiation and Equity*

The role of language and how different languages enable individuals to engage—or not engage—with mathematical thinking, is an important theme in research concerned with the design and enactment of the curriculum. When issues of equity are prioritised, McMurchy-Pilkington, Trinick, and Meaney (2013) argue that curriculum development can be an enabling process. The researchers contrast two iterations of mathematics curriculum development in New Zealand to reveal how it supported the revitalisation of the Maori language. They, along with Meaney, Trinick, and Fairhall (2013), describe how the development and enactment of mathematics curricula has had positive linguistic, cultural and political consequences for Indigenous language communities and recommend that the language of instruction be a consideration in future curriculum development.

Issues of equity and fairness in the curriculum also underpin Edmonds-Wathen's (2013) investigations of the challenges faced by mathematics teachers of Australian Indigenous students in remote locations of the Northern Territory. It seems that the decision to teach mathematics at a level lower than that which is officially designated as "age-appropriate" is an informed response by teachers to the learning needs of their students. While the research on the learning of mathematics by Indigenous students is discussed elsewhere in this volume (see Chap. 8), Edmonds-Wathen's findings serve to highlight the mismatch that often exists between the content contained in official curriculum and what is actually enacted in the classroom and how teachers can unwittingly work to widen the gap in achievement between various groups in the general student population. Despite the introduction of a national curriculum as an explicit attempt to provide common experiences to *all* Australian students, Jorgensen and Perso (2012) claim that "equity in provision does not guarantee equitable learning outcomes" (p. 131). They argue that a robust national curriculum is necessary in promoting high expectations and providing equitable access to mathematics for all students, but the curriculum will not be able to achieve these goals unless it is part of a multi-faceted approach.

One such approach was explored by Rampal and Makar (2012). They discuss the ideals and practicalities of embedding authenticity and cultural relevance in the primary mathematics curriculum. Using two diverse contexts—India and Australia—they highlighted how the implementation of innovative curriculum materials incorporating authentic and culturally relevant experiences familiar to students helped students connect more easily with the mathematics content. Similar to Jorgensen and Perso (2012), Rampal and Makar argue the necessity of a multi-faceted approach with united efforts from all stakeholders, including curriculum developers and providers of professional development, to achieve true curriculum reform that is both high quality and equitable.

Differentiating the enacted curriculum for students of various achievement levels is also an equity issue. Zmood (2014) outlines different drivers of high achievement and explores the main curriculum differentiation strategies schools and teachers can use with high achieving mathematics students, including acceleration, enrichment and extension experiences. She argues that teachers need strategies and resources at their disposal to enable them to maximise the mathematical potential of the most capable students. Similarly, Sullivan (2012) discusses issues surrounding some practices used by schools in their attempts to differentiate the curriculum for students demonstrating various levels of achievement. He reiterates ACARA's claim that "all students should have access to all of the mathematics in the compulsory years" (p. 184) and is critical of "ability" streaming practices, citing past research that confirms the inequitable outcomes of such practices with only minimal or no gains for capable learners and negative attitudinal impacts for less capable students of mathematics. Sullivan continues to espouse the benefits of "extending prompts" as a teaching strategy for high achieving students (Sullivan, Mousley, & Zevenbergen, 2004). While Chap. 7 of this volume provides a closer examination of research pertaining to inclusive teaching practices in mathematics education, there was a noticeable paucity of research on the operationalization of the curriculum in classrooms that explicitly addressed the diversity of achievement and for building equitable outcomes in mathematics. For further research concerning equity in education and Indigenous education we refer readers to Chaps. 6 and 8 respectively.

5.4 Numeracy Across the Curriculum

The impetus for much of the recent research regarding numeracy has been the official *Australian Curriculum* for schools that names Numeracy as one of the *General Capabilities*. In this context, numeracy is described as

> the knowledge and skills to use mathematics confidently across all learning areas at school and in their lives more broadly. Numeracy involves students in recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully. (ACARA, 2015b)

In enacting the curriculum, teachers are expected to identify the specific numeracy demands of their discipline and design learning experiences that support application of mathematical knowledge and skills. However, there is limited information about the capacity of teachers to effectively perform these professional tasks.

Building on their previous research, Goos, Dole, and Geiger (2012b) explored the "challenges of moving from the intended outcomes framed by formal curriculum documents to the enacted practices of teaching and learning" (Geiger, Goos, & Dole, 2014, p. 489). In a series of workshops and action research cycles, ten pairs of South Australian primary and secondary teachers used the *Numeracy in the 21st Century* model (Fig. 4.3) as a basis for planning, implementing and evaluating numeracy experiences with their students.

Central to the numeracy model, depicted in Fig. 4.3, is the key element of real-life *contexts*, including work, citizenship, and personal and social contexts. Three further elements (the corners of the triangle) are the "deployment of *mathematical knowledge*, the use physical and digital *tools*, and consideration of students' *dispositions* to the use of mathematics" (Geiger, Goos, & Dole, 2014, p. 477). An important feature of the model is the embedding of the four elements in a *critical orientation* to the use of mathematical skills and concepts. Although firmly focused on numeracy development, the model captures a range of other *General Capabilities* from the Australian Curriculum, such as ICT capability, critical and creative thinking, and personal and social capability.

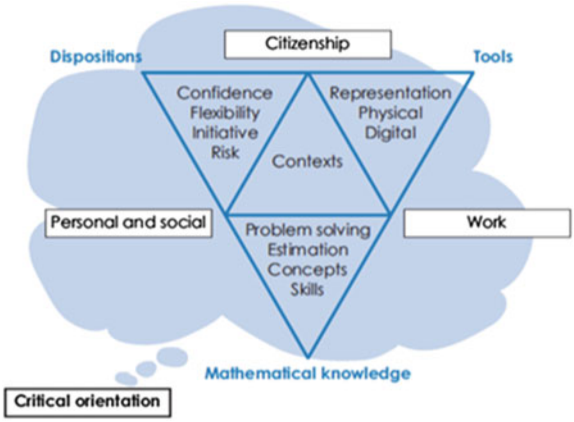

Fig. 4.3 A model for numeracy in the 21st century. Goos, Dole, and Geiger (2011); Fig. 1, with permission of Springer

The numeracy research project was strongly focused on the realities of the participating teachers' own attempts to recognise and develop numeracy within their own disciplines. Appropriately, the researchers and teachers together produced a special issue of the professional journal *Australian Mathematics Teacher* (Volume 68, Issue 1, 2012).

While the teachers increased their confidence in planning for numeracy integration, developed the use of digital tools, and influenced student dispositions, achieving the *critical orientation* in learning activities proved to be the most challenging aspect (Goos, Dole, & Geiger, 2012b). A key message is that teachers require guidance and support in planning and implementing numeracy across the curriculum—and when they receive it, rich numeracy contexts emerge that are beneficial to learners. Similarly, Callingham, Beswick, and Ferme (2015) concluded that "considerable systemic support over time is needed" (p. 559) to support teachers in decision-making about numeracy in their classrooms, particularly when they lack confidence in their own mathematical understanding. The complexity of such decision-making in the context of teacher-identity has also been highlighted through the case studies of Bennison (2015).

A potential perspective for further research is to better understand the origins of teachers' struggle with the "critical orientation" of mathematical practices within the knowledge and practices of the contextual discipline. For example, are there mismatches between ways of "knowing and doing" in Mathematics and ways of "knowing and doing" in English that create some pedagogical conflict?

6 Conclusion

The purpose of this chapter was to critically review the research pertaining to the areas of policy, curriculum and leadership in mathematics education and to explore the interrelationships between these areas. The framework for the curriculum policy, design, and enactment system developed by Remillard and Heck (2014) proved to be highly useful in the conceptual organisation of the review. The Remillard and Heck model neatly represents the interacting components of the complex system, and supported recognition of the interrelationships amongst the seemingly disparate research directions under review. In response to the provocative context of the recent introduction of a new curriculum, both New Zealand and Australian researchers focused on aspects of the official curriculum and operational curriculum—which we grouped around the issues of: reflection of political agendas in the curriculum, national testing, textbooks, equity issues, leadership and numeracy across disciplines.

Although not entirely pessimistic, a key message from the research was a warning about inconsistencies, mismatches and tensions between the official curriculum and various aspects of the operational curriculum. For example, the timeframes and political motivations of national testing and the application of the data generated from such tests contrast with the educational motivations and shorter-term goals for assessment data of teachers. In another example, while carefully designed textbooks can be supportive of the pedagogical intent of the curriculum, other textbooks fall short in their support of aspects such as deep understanding, problem solving and reasoning. Researchers were consistent in their call for increased leadership to assist teachers in appropriate enactment of the curriculum, perhaps most importantly in making decisions about mathematics curriculum differentiation that actually achieves the desired equity goals rather than maintaining achievement differences. Similarly, non-mathematics teachers require guidance to fulfil the syllabus requirement of integrating numeracy development within their discipline areas.

While it was not the intention of this review to conduct a comparative analysis between the New Zealand and Australian curricula, comparison at a basic level served to reveal some similarities and differences that may be worthy of further investigation. For example, in comparing the principles and competencies, and capabilities and priorities of the respective curriculum documents, we see that both countries highlight cultural diversity and attention to the educational needs of their Indigenous populations, but Australia extends to an awareness of regional engagement with Asia. We know little about how these cultural themes are represented in each Mathematics curriculum, and how they are enacted in classrooms. Even allowing for different ways of expressing similar ideas (e.g., “thinking” vs “critical and creative thinking”), there are some notable inclusions and absences, such as “learning to learn” (New Zealand) and “information and communication technology capability” (Australia). How are these differing emphases reflected in the instructional practices of teachers? The New Zealand curriculum explicitly states the expectation for inquiry-based pedagogies, whereas specific teaching strategies are less clearly articulated in the Australian curriculum. We see this difference reflected in the number of Australian researchers exploring the differences between the intent of the official curriculum and the enacted curriculum in schools and individual classrooms. We also note a much stronger focus on educational leadership in operationalising the curriculum in New Zealand, from both the government and researchers. The stronger presence of textbook-focused research in Australia also raises questions. Anecdotal evidence suggests there has been a decline in mathematics textbook use in New Zealand in recent years. If this is indeed the case, and considering that textbooks play a substantial intermediary role between the official curriculum and the operational curriculum, what now fills that role? Perhaps there is a connection to the emphasis on leadership in New Zealand.

A strength of recent research in the broad field of curriculum is the emergence of robust theoretical models with practical applications as well as research utility. In particular, the *Model for numeracy in the 21st century* (Goos, Dole, & Geiger, 2012b) shows great promise in its usefulness as a theoretical framework for further research, but also as a practical professional learning support for teachers exploring numeracy development in curriculum areas other than mathematics. Like the numeracy model, much of the research reported in this chapter has developed from earlier research noted in the previous review (see Anderson, White, & Wong, 2012). Not surprisingly, particularly in the Australian context, there is now greater emphasis on the *operational curriculum*—or rather, issues surrounding the operationalisation of the *official curriculum*. The consequential "spotlight" on the professional work of teachers highlights the incredible complexity of interpreting the official curriculum, and implementing it to serve the needs of all students. This remains a rich area of research.

As a closing comment we refer back to the Remillard and Heck curriculum system model (Fig. 4.1) and draw the reader's attention to the direction of arrows leading to, and crossing the boundary of, the *operational curriculum*. All but one arrow is uni-directional and all point inward to the teacher (and school). What then, according to this model, is the role of school and classroom research in influencing system-level and national curriculum decisions?

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Chapter 5
Mathematics Education and the Affective Domain

Catherine Attard, Naomi Ingram, Helen Forgasz, Gilah Leder and Peter Grootenboer

Abstract This is the fourth chapter on affective issues to appear in the Mathematics Education Research Group of Australasia (MERGA) reviews of research in mathematics education. In our review of topics and findings of studies that have been published in Australasia during the period 2012–2015, some themes and issues have been identified that appear noteworthy. Past reviews on affect and mathematics have noted a limited amount of theorising, with many studies providing description of teacher and student perceptions of affective issues. This does not appear to have changed significantly in this period, however it does appear that the foci of affective research within specific topics is beginning to broaden, with a shift in attention on student engagement and attitudes towards mathematics. It is noted that although research has increased in the area of engagement, there have been no substantive explorations of any links between engagement and academic achievement.

Keywords Mathematics education · Affect · Anxiety · Engagement · Beliefs · Identity · Gender

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1 Introduction

This is the fourth chapter on affective issues to appear in the Mathematics Education Research Group of Australasia (MERGA) reviews of research in mathematics education and, as in previous research periods, it is clear that affective issues continue to attract research attention from the mathematics education community. Issues considered to influence learning outcomes are captured evocatively by the excerpt below:

> Individuals' attitudes, beliefs and emotions play a significant role in their interest and response to mathematics in general, and their employment of mathematics in their individual lives. Students who feel more confident with mathematics, for example, are more likely than others to use mathematics in the various contexts that they encounter. Students who have positive emotions towards mathematics are in a position to learn mathematics better than students who feel anxiety towards that subject. Therefore, one goal of mathematics education is for students to develop attitudes, beliefs and emotions that make them more likely to successfully use the mathematics they know, and to learn more mathematics, for personal and social benefit. (OECD, 2013, p. 42)

It is widely recognised that the boundaries between affective factors tend to be porous and interconnected (see e.g., Leder, 2012; Mason, 2004). It is noteworthy that much of the material reviewed in the chapter is generally considered only once, that is under only one heading—even though a careful reading of the work might well reveal that an alternate categorisation would not be unreasonable. In other words, the affective term chosen by the author(s) of the original work reviewed has been, pragmatically, accepted by the writers of this chapter.

The location of papers accepted and published in refereed journals in the period 2012–2015 varied widely in terms of likely audience and favoured methodological approach (e.g., *Creative Education* vs. *Journal of Educational Psychology*). Some were aimed specifically at the mathematics education community (e.g., *Educational Studies in Mathematics; Mathematics Educational Research Journal*); others at the educational research community more widely (e.g., *Educational Psychology*). Given this diversity of terminology, instruments, and perspectives it is more realistic to consider the 2012–2015 pool of contributions collectively.

The articles reviewed in this chapter are relatively evenly distributed among the different factors highlighted in this chapter. Over two decades ago, McLeod (1992, p. 580) noted "some of the most consistent literature in the affective domain had emerged from research concerned with gender differences in mathematics learning". Perhaps therefore not surprisingly, the largest group appeared under the heading *gender and the affective domain.*

About one-third of the work reviewed in this chapter appeared in MERGA or International Group for the Psychology of Mathematics Education (PME) conference proceedings. Although the space limitations imposed on conference papers inevitably restrict the scope of the material presented, strategic use of conference presentations can serve as an introduction to a subsequent article or chapter (e.g., Attard, 2012, 2013; Attard & Curry, 2012; Darragh, 2013, 2015; Leder & Forgasz, 2011). Alternately, conference contributions over successive years can be used to unveil extra information or explain further developments and thus enable a more solid body of work to be shared with the broader mathematics education research community (e.g., Brady, 2012, 2014; Wilson, 2013; Wilson & Raven, 2014). While some papers reviewed focused primarily on affect (e.g., Chaman & Callingham, 2013; Lim & Chapman, 2013; Plenty & Heubeck, 2013), in many others concern about affective factors was a secondary consideration.

In our review of topics and findings of studies that have been published on the affective dimension of mathematics education in Australasia during the period 2012–2015, some themes and issues have been identified that appear noteworthy. Past reviews on affect and mathematics have noted a limited amount of theorising, with many studies providing description of teacher and student perceptions of affective issues. This does not appear to have changed significantly in this period, however it does appear that the foci of affective research within specific topics is beginning to broaden, with a shift in attention on student engagement and attitudes towards mathematics. It is noted that although research has increased in the area of engagement, there have been no substantive explorations of any links between engagement and academic achievement.

In this chapter the reviewed research has been organised into the topics of identity, beliefs, attitudes, engagement and motivation, and anxiety. Although issues relating to gender are addressed in Chaps. 6 and 7 of this volume, the heading of gender and the affective domain has also been included in this chapter as this topic is often related to other affective variables and is deemed relevant to current concerns relating to an under-representation of females in the Science Technology, Engineering, and Mathematics (STEM) fields and in advanced and intermediate grade 12 mathematics subjects. The research reviewed was conducted on students and teachers (practicing and pre-service). In addition, the section on gender and affect reports on a series of studies by Forgasz, Leder, and colleagues, in which data were collected on gender-stereotyped views from the general public. Research in the sections on motivation and engagement and attitudes involved only students, whereas all the other sections included research on teachers and students.

There is a "suite of constructs" (Clarke, 2015) that characterises research into affect and in this chapter, the discussion and analysis of research follows the continuum of stability and intensity as suggested by McLeod (1992), where we begin with a discussion on beliefs, followed by gender. We continue with an exploration of research on attitudes, identity, and anxiety. Research on engagement and motivation, the outcome of affective processes, follows. The chapter concludes with an analysis of methodological issues that have emerged in the review period, and which have not already been covered in the text.

2 Beliefs

There has been a continued interest in the beliefs of teachers and students related to mathematics education, although perhaps not in the same quantity as has been reported in previous reviews. Many of the studies have been reported in conference proceedings, with a few reported in journal articles. The studies focused on beliefs about the nature of mathematics and mathematical pedagogy, and there were a range of methods employed including qualitative, quantitative, and mixed-method approaches. In general, the findings were consistent with the understandings developed previously, although there were a few interesting new nuances in some of the research methods employed, the participants involved, and the theorising of the findings.

Historically, the participants in many studies about mathematical beliefs have been teachers, and this trend is still evident in the studies reviewed in this volume. Beswick has a substantial track record in exploring teachers' beliefs about mathematics and mathematics education, and she has continued this work during the review period (Beswick, 2012; Beswick & Callingham, 2014). In investigating the often pernicious problem related to the distinction between teachers' beliefs about mathematics as a discipline and mathematics as a school subject, Beswick (2012) suggested that the disparity is complexly related to prior mathematical experiences in a range of contexts. The contextual nature of these beliefs about mathematics means that they can be conflicted and disconnected. Furthermore, these beliefs have an impact on mathematics education practice, and so one implication is that professional development needs to focus more explicitly on teachers' mathematical experiences as school students, as university students, and as mathematics school teachers, so teachers can reconceptualise their views of the discipline and how it relates to their teaching. Beswick's study involved secondary teachers, but Brady (2012, 2014) noted in her qualitative study that primary school teachers' mathematical beliefs were not particularly related to their beliefs about mathematics teaching. As Brady highlights, this result seems contrary to some other studies that found a correlation between discipline-based and educational mathematical beliefs, so this warrants further investigation. Bobis, Way, Anderson, and Martin (2015) also examined the beliefs of primary school teachers, and they found that the views were mediated by a range of personal and contextual factors, and their developing beliefs were influential in their professional development in mathematics education.

In the 4 years covered in this chapter, there seems to have been an increased focus on students' beliefs with about half of the articles/papers investigating the mathematical views of school students. These studies reported on primary students' self-beliefs in mathematics (Dimarkis, Bobis, Way, & Anderson, 2014), subjective beliefs about probability (Watson & Callingham, 2015), primary students' espoused and enacted mathematical beliefs (Perger, 2013), and the impact of inquiry-based learning (McGregor, 2014) and authentic tasks (Marshman, 2015) on junior secondary students' beliefs about mathematics. A substantial contribution came from McDonough and Sullivan (2014) who continued a long-standing program of research into children's beliefs. In their article they did not so much report on their findings about children's mathematical beliefs, but rather on the development of some useful tools to access and understand primary school students' mathematical beliefs. These interview-based prompts included visual, verbal, and text-based dimensions. These proved to be useful for researchers and also teachers in providing insights into young students' beliefs about mathematics and learning mathematics.

Apart from the studies reported by McDonough and Sullivan (2014) and Watson and Callingham (2015), most of the other studies employed fairly standard and traditional methodologies and methods including large-scale surveys (Beswick & Callingham, 2014), case studies (Beswick, 2012; Perger, 2013), narrative enquiry (Brady, 2012, 2014), and mixed methods (Dimarkis et al., 2014). However, McGregor (2014) conducted a design experiment in which he employed ill-structured tasks—"open-ended, context-based questions that have a deliberate 'messiness'" (p. 453), to simultaneously challenge and investigate secondary students' mathematical beliefs. He found that the open-ended investigative tasks and pedagogies created opportunities to make students' beliefs overt and to challenge their views and positions. Like McDonough and Sullivan (2014), this approach was useful for both researchers and teachers.

3 Gender and the Affective Domain

There is a long history of the pivotal role of affective variables in understanding the under-representation of females in the more challenging mathematics subjects studied at school level, at the tertiary level, and in related careers, as well as in the relative underperformance of females compared to males in mathematics. The seminal Fennema and Sherman studies in the USA in the 1970s (e.g., Fennema & Sherman, 1977) established this relationship; in the Australian context, Leder's early research (e.g., Leder, 1982) extended on this research.

In more recent times, there has been renewed concern about the under-representation of women in the STEM fields (e.g., Office of the Chief Scientist, 2014). The close monitoring of enrolments in grade 12 mathematics subjects across Australia (e.g., Barrington & Evans, 2014) indicates that females remain under-represented in the advanced and intermediate level subjects. Thomson, De Bortoli and Buckley (2013) reported on recent Programme for International Student Assessment (PISA) results; the data revealed that in Australia the gender gap in mean scores has increased in favour of males between PISA 2003 and PISA 2012. Australia-wide National Assessment Plan—Literacy and Numeracy (NAPLAN) results at all grade levels also show that, on average, males achieve higher than females, although higher proportions of females than males reach the national benchmarks (see NAPLAN reports from 2008–2014, downloadable from http://www.nap.edu.au/results-and-reports/national-reports.html).

Recently, Thomson (2014) conducted a secondary analysis of Australian PISA 2003 and 2012 data to see whether the gender differences in students' PISA scores were reflected in gender differences in particular affective variables. She found that the strongest predictor of achievement for both males and females was mathematics self-efficacy. The next strongest predictor for females was self-concept in mathematics, whereas for males this variable was not a significant influence on mathematics achievement. For males, it was mathematics anxiety that was the next strongest predictor; surprisingly anxiety was not a significant influence on the mathematics achievement of females.

In the period under review, 2012–2015, rather than being the main focus of studies in which affective variables were explored, gender was more often included as a variable of interest. The main foci and the participants in the studies in which gender was included were quite varied. The vast majority of these studies involved quantitative data gathering, mainly through surveys/questionnaires. In general, however, the findings were little different from those reported in the past. Among students, males continued to hold more functional attitudes and beliefs than females, that is, views more likely to lead to future success in mathematics. Other people, including significant others in the social milieu of students, also appear to hold stronger and more positive expectations of boys than of girls with respect to mathematics performance and career directions.

3.1 Differential Beliefs About Males and Females

Those holding beliefs or attitudes consistent with the construct "mathematics as a male domain" (a field/discipline considered more suited to males than to females) are considered to hold traditional gender-stereotyped views. Forgasz, Leder, and colleagues have reported on a series of related studies in which they gathered the views of the general public about the gender-stereotyping of mathematics. Data were gathered in the streets of Victoria, Madrid (Spain), the UK, Seoul (Korea), and Canada, as well as online using Facebook to recruit participants. The same short survey was administered to all participants, translated into other languages as required. The findings were reported in journal articles (e.g., Forgasz, Leder, & Tan, 2014; Leder, Forgasz, & Jackson, 2014) and book chapters (e.g., Forgasz, 2012; Forgasz, Leder, Mittelberg, Tan, & Murimo, 2015; Leder & Forgasz, 2012; Leder & Forgasz, 2014); earlier versions of some of these articles were reported at MERGA (e.g., Forgasz, Leder, & Gómez-Chacón, 2012; Leder & Forgasz, 2011), PME (Forgasz, Leder, & Tan, 2013), and other international conferences. Traditional gender stereotyping was found to persist, that is, mathematics remains a "male domain" in the views of a noteworthy proportion of respondents. While many respondents in the streets and online were found be gender neutral (i.e., did not to hold gender-stereotyped views), among those whose beliefs were gender-stereotyped, more respondents considered males than females to be better at mathematics and in using calculators, more suited to being scientists and for the computer industry; they also believed that parents and teachers would hold similar views. The extent of these views varied by country; among the online respondents, a clear majority of those from China, for example, strongly reflected traditional gender-stereotyped beliefs. Spanish respondents in the street were more likely to be gender neutral than were Australian respondents.

A surprising, and somewhat disturbing finding, was that younger respondents (under 40) in Australia were more likely to hold traditional gender-stereotyped beliefs than those who were older (over 40). The research team also gathered data on the gender-stereotyping of English using a modified version of the survey used to tap views on mathematics. A sample of pedestrians in the streets of Victoria was asked about both mathematics and English. While many were gender neutral about both subject areas, those who were gender-stereotyped believed that English was the domain of females and, consistent with other samples in this extended study, mathematics was the realm of males (Leder & Forgasz, 2014).

Carmichael (2014) reported on a secondary analysis of The Longitudinal Study of Australian Children (LSAC) data and found that parental expectations influence their children's mathematics achievements, and that parents' perceptions of sons and daughters differed. The parents (mostly mothers) "tended to perceive sons as being better at mathematics than daughters yet they tended to have non-academic long-term expectations for these boys" (p. 145). Carmichael cautiously concluded that causative relationships could not be identified and suggested factors that might have shaped parents' expectations, including feedback from teachers about classroom achievement and NAPLAN test results.

3.2 Males' and Females' Beliefs About Themselves as Learners of Mathematics

Tan (2012) reported on the relationships between the beliefs about knowing and learning mathematics of senior high school students in Singapore and Victoria, and how they engage with calculators. Among both the Singaporean and Victorian students, gender differences were found in reported learning styles: males scored higher than females for "connected knowing and deep approaches to learning", whereas females scored higher than males on "separate knowing and surface approaches". The effect sizes were higher for the gender differences in the Victorian data than in the Singaporean data. No gender differences were found in the correlations between beliefs about mathematics and how students engaged with calculators.

3.3 Other Studies of Gender and Affective Variables

Significant gender differences in primary-aged children's preferences for the type of games played were reported by Lowrie, Jorgensen, and Logan (2012). While females tended to prefer games that required logic and problem solving, games containing maps were preferred by males. These gender differences were found to be more pronounced in non-metropolitan locations.

In studies focusing or including measures of anxiety, several researchers included gender as a variable of interest. Chaman and Callingham (2013) found a positive relationship between a lack of anxiety and attitudes towards mathematics, with no gender differences for this relationship. Wilson (2013) reported no gender differences in the anxiety towards mathematics of primary pre-service teachers. Aimed at examining changes in mathematics motivation for boys and girls during high school, Plenty and Huebeck (2013) found that girls reported “stronger anxiety, uncertain control, and failure avoidance than boys, despite also reporting greater mastery focus” (p. 14).

For the Australasian studies conducted during the period of review, the pattern of gender differences on affective variables associated with the learning of mathematics were generally consistent with previous research in the field.

4 Attitudes

In this four-year period there appeared to be an increased number of studies that focused on mathematical attitudes, with a significant number being undertaken in East Asian countries. Of particular interest related to this trend was a review conducted by Leung (2014) that examined the relationship between the cultural values, including attitudes towards mathematics, of East Asian students and their achievement in mathematics. In general, the studies showed that negative attitudes towards mathematics still prevail across a number of groups including primary and secondary students (Larkin & Jorgensen, 2014), and pre-service teachers (Afamasaga-Fuata’i & Sooaemalelagi, 2014; Young-Loveridge, Bicknell, & Mills, 2012). However, other studies reported some more positive attitudes from at least some of the participants, including a majority of junior primary school students (Leder & Forgasz, 2012).

There were some interesting findings that have implications for educational practice. Of note, Leder and Forgasz (2012) reported from quite a large scale study (n = 321) that there were no clear patterns of differences among Aboriginal and Torres Strait Island students in Years K-2 compared to their non-Indigenous peers. In the New Zealand context, Winheller, Hattie, and Brown (2013) found that secondary students’ self-efficacy in mathematics was primarily related to their learning outcomes, and their perceived quality of learning was connected to “confidence in” and “liking mathematics”. In a study with “reluctant mathematics learners aged 16–18 years”, Calder and Campbell (2015) found that the use of iPads was instrumental in changing the students’ attitudes towards mathematics because content was presented in a more appealing and engaging manner, although these researchers did not report if this resulted in improved conceptual development. Also, in relation to teacher education, Young-Loveridge et al. (2012) noted that there were some improvements in primary pre-service teachers' mathematical attitudes over their initial teacher education program. More specifically, Bailey (2014) found that open-ended investigations were effective in improving the mathematical attitudes of pre-service primary teachers who had initially struggled with their mathematics education courses.

In the period since the last four-yearly review, some new methods and approaches to understanding students' mathematical attitudes have appeared. Larkin and Jorgensen (2014) employed iPads and video diaries in a "Big Brother" tent to allow students to express their attitudes about mathematics in a novel and insightful manner. In the tent the students were able to record a video about their mathematical learning and experiences at school. This method seemed to allow the Year 3 and 6 students the opportunity to reflect on their mathematical experiences and their resultant attitudes. In their quantitative study, Lim and Chapman (2013) developed an updated and revised shorter version of the *Attitudes toward Mathematics Inventory*, which measures four sub-scales: enjoyment, motivation, self-confidence, and perceived value. Their revised inventory was administered to 1601 participants and it was found to be statistically internally consistent ($\alpha = 0.93$) and reliable over time (mean $r_{xx} = 0.75$), and it strongly correlated with the original scale (mean $r = 0.96$). Lim and Chapman (2013) argued that the revised instrument could be a valuable tool for researchers as it is more appropriate for contemporary mathematics learners and it is more efficient (i.e., it took less than ten minutes to complete). Finally, there were two studies (Areepattamannil & Kaur, 2013; Ng, Lay, Areepattamannil, Treagust, & Chandrasegaran, 2012) in which Trends in International Mathematics and Science Study (TIMSS) data were used to investigate students' mathematical attitudes, and these studies had very large samples but drew on a very limited number of TIMSS items. While these large samples provide some interesting insights about students' mathematical attitudes, including a positive relationship between attitudes towards mathematics and mathematical achievement (Areepattamannil & Kaur, 2013), the results need to be taken with a degree of caution given the context of the data collection and the limited number of items considered.

Finally and importantly, Jorgensen and Larkin (2015) have started developing a more robust theoretical foundation for researching and developing attitudes towards mathematics. Their emerging work attempts to draw on a range of theories including psychology, sociology, and postmodernism to develop an explanatory model for different attitudinal responses across a range of contexts. This theorising is overdue and as it develops will add to the robustness of research in this field.

5 Identity

Identity research has continued to be a focus in the period 2012–2015, evidenced by the special issue of identity in the Mathematics Education Research Journal in 2015 (Jorgensen, 2015). This reflects the interest in mathematics education on the social and cultural context of the classroom, and on theories that see meaning, thinking, and reasoning as social products (Lerman, 2000). The strength of the identity research is that the researchers define the concept well and have situated their research within a theoretical perspective. Furthermore, the research published during this period explored the identities of a range of participants from the various sectors.

5.1 Operationalising Identity

A theme of identity work in this research period is to operationalise the notion of identities to "make it amenable to investigation through empirical studies" (Bennison, 2015, p. 1). Bennison (2015) developed a conceptual framework for teacher identity as an embedder-of-numeracy, which includes five domains of influence: knowledge, affective, social, life history and context. Bennison suggested that researchers can use the framework to capture the complexity, situatedness, and overlapping nature of teachers' multiple identities while at the same time practically focusing on social and cognitive characteristics related to a teacher as an embedder-of-numeracy.

Ten years after Sfard and Prusak (2005) advocated for the notion of identities to be operationalised, their narrative approach is widely used in mathematics education research within Australasia. They equated identities with the reifying, endorsable, and significant stories that surround a person. Meaney and Lange (2012) used Sfard and Prusak's framework to explore 104 pre-service primary teachers' content testing results and views on content testing regimes. The participants' views of teachers of primary school mathematics, combined with their previous identities as school students, meant that they had a restricted view of how mathematics understandings connected to the institutional identities of being a teacher. The pre-service teachers were focused on their own mathematical performance and rarely discussed mathematics teaching as helping students to understand the subject conceptually. Meaney and Lange suggested that pre-service teachers need to reconceptualise what it means to do mathematics by personally engaging in forms of mathematical activity that differ from those that they experienced as students.

Andersson and Seah (2012) used Sfard and Prusak's framework to explore one upper secondary student's learning of mathematics within a social science programme in Sweden. The student's engagement was affected by the learning contexts and this was studied through changing identity narratives. Complex interplays between values, agency, and context were found. The data suggests that the relatively stable, sociocultural valuing of achievement affected the student's state of mathematical wellbeing and engagement.

Ingram's (2013, 2015) longitudinal research into the mathematical journeys of 30 secondary students extends Sfard and Prusak's notion of identities by placing identities within an affective framework of students' relationships with mathematics. This connects identities with four other elements: views of mathematics, affective responses, knowledge, and habits of engagement. The students produced their identity narratives through negotiating meaning between their relationship with mathematics and their interaction with classmates, families, by the class they were placed in, where they were positioned in the class, assessment results, performances, prizes, and through their doing of mathematics.

5.2 *Other Conceptions of Identity*

Darragh (2013, 2015) used the metaphor of performance to portray identity. Through investigating the mathematics identities of 22 students and their 16 teachers in the students' transition from primary school to secondary school, three themes of what is valued emerged: asking questions, perseverance (persistence), and confidence. Differences and tensions in the ways these themes were valued by the students and teachers, promoted by classroom experiences, and performed by the students were explored. Darragh (2015) also found there were a number of different "scripts" students can call upon in their recognition of a performance of "good at mathematics". Differences were highlighted between the teachers' view of what were important performances for students' successful learning and what the students thought they should be doing in order to be successful in mathematics. She suggested that there was a need for teachers to be explicit about how, when and what type of question they wish to hear from their students to ensure that the stage enables all students to feel comfortable to enact this script.

Walshaw (2013) argued that mathematical identifications are tied to the social organisations of power. She advocated for post-structuralist identity research because this allows insights to be developed into the discourses and practices of school life. In this view, identities are constructed through social interaction and daily negotiation, and within contexts laden with the understandings of others. Walshaw examined the identity constructions of one secondary school girl and one pre-service secondary teacher at the end of her course. These participants experienced constant tensions while negotiating meaning about being a student, or a teacher, because of the need to confront understandings of others and the power relationships that are part of the discourses and practices within a classroom and school.

6 Anxiety

The concept of anxiety has been explored in different contexts in research during 2012–2015. There has been a range of qualitative and quantitative methodologies used to study anxiety specifically, and anxiety within a broader affective framework. The revised mathematics anxiety scale, developed from Richardson and Suinn's (1972) version, continues to be used (e.g., Wilson, 2013). Open-ended surveys and interviews have also been widely employed to investigate the phenomena (e.g., Boyd, Foster, Smith, & Boyd, 2014; Ng, 2012).

In terms of new frameworks, Wilson (2014) suggested that a tool developed in the field of intellectual and developmental disabilities, the scientific and societal view of quality of life tool (QOL), can be applied effectively to studies of mathematics anxiety. Perception, exclusion, choices, self-image, empowerment, and personal control are principles of QOL that relate to maths anxiety. Wilson illustrated how bibliotherapy, which has the potential to reconstruct pre-service teachers' self-perception and identity as learners and teachers of mathematics, can be seen through the QOL lens.

6.1 Tertiary Students' Mathematics Anxiety

The anxiety experienced by pre-service teachers has continued to be a main area of focus, although it is pleasing to note that a range of pre-service teachers have been included as well as the oft-researched, proximal primary pre-service teachers. Boyd et al. (2014) linked anxiety with self-efficacy in 223 primary and early childhood pre-service teachers. Larkin and Jamieson-Proctor (2013) found the use of transactional distance theory in planning and implementing units in an online mathematics course reduced student anxiety in pre-service secondary teachers.

Wilson (2013) found that mathematics anxiety was related to, and experienced differently by, 219 pre-service primary teachers in Australia depending on their age, and, with Raven (Wilson & Raven, 2014) continued to report on the effectiveness of bibliotherapy techniques with primary pre-service teachers. These researchers found that the use of bibliotherapy can assist teacher educators in understanding and addressing mathematics anxiety and outlined the themes generated from their exploration into the causes of pre-service teachers' mathematics anxiety. Individual "disabling" teachers had a lasting impact on pre-service teachers' anxiety, as well as a continuing cycle of fear, failure and avoidance, the nature of mathematics, the pre-service teachers' image of themselves as learners, and the influence of parents and families.

In the wider tertiary sector, Gyuris, Everingham, and Sexton (2012) measured students' mathematics anxiety at entry to their science course and compared this to their subsequent performance. These researchers found that students who preferred the maths-physical sciences were significantly less maths anxious overall. Their results indicated that mathematics anxiety was not a useful predictor of performance, however, they noted that assessment schedules seem to have an effect on mathematics performance anxiety and argued that this relationship merits further investigation.

6.2 School Students' Mathematics Anxiety

There has only been a minor focus on anxiety research into school-aged students. Chaman and Callingham (2013) found that there was a relationship between anxiety and attitude towards mathematics among Indian students. Ng (2012) explored the cause of anxiety of students within her secondary school in Singapore. In this research a number of themes emerged in common with Wilson and Raven's (2014) research of pre-service teachers, although perhaps because of Ng's teaching role in the school, the participants did not cite their teacher as a major cause of anxiety. Interestingly, the influence of students' parents and families on students' anxiety was a common theme and this is an area of research that warrants review and further exploration across a range of cultures and affective constructs.

Whyte and Anthony's (2012) review of anxiety begins this process well. Setting their discussion of the literature in the context of New Zealand, they considered the role of mathematics anxiety and its impact on students' learning practices and outcomes. Literature relating to the potential origins of mathematics anxiety—home, society, and classroom—is reviewed. The authors comment on the recent paucity of anxiety literature and suggest their paper has "reopened a discussion that has largely lain silent in the last few years" (p. 11).

It will be of interest in the next review period to see if there is development in anxiety research in mathematics education. Anxiety has been linked in varying ways to other affective factors. It is these links that may explain why anxiety is not often explicitly researched. Rather, anxiety is often included as one of a wide range of affective responses to mathematical situations that a person can experience (e.g., Plenty & Heubeck, 2013).

7 Engagement and Motivation

The study of engagement and motivation appears to be a growing area of interest in mathematics education research. Several studies were considered for this section as having a significant focus on engagement and/or motivation. Only two of the studies were conducted in New Zealand, with the remainder conducted in Australia. One study incorporated data from Singaporean and Australian students and teachers. The studies were published in an even spread of conference proceedings and journal articles, and one published research report.

Previous review chapters on the affective domain included limited mention of motivation and engagement, however in the current period several studies have emerged, particularly in the area of engagement. Although the constructs of motivation and engagement are often mentioned interchangeably in educational contexts, they are quite distinct, so some clarification is now being provided. Attard (2012) explained the distinction by stating that motivation derives from a person's beliefs and orientations towards schoolwork and learning, while engagement manifests through a person's thoughts, behaviours, and actions. The two constructs are related in that "when an individual is engaged with mathematics, he or she has been influenced by motivation" (Attard, 2012, p. 10) and when a person is substantively engaged, motivation is increased.

A study by Plenty and Heubeck (2013) appeared to blur the constructs of motivation and engagement. While the authors cited engagement in the title of their paper, their longitudinal study was primarily focused on motivation, comparing mathematics motivation to general school motivation amongst high school students in regional New South Wales. Findings from this study explored other affective topics including student anxiety and differences between the motivation of boys and girls; hence this paper receives some brief mention in other sections of this chapter. Plenty and Heubeck reported that all students reported low mathematics self-efficacy in Year 7, but not in Year 8 or 9. Mathematics motivation appeared to be less positive compared to general academic motivation, and this was apparent in the youngest cohort of participants: "a focus on changes in motivation during high school is warranted as students' beliefs and behaviours towards maths relative to others appear to be malleable rather than firmly set" (p. 26).

There appears to be limited theorising in relation to engagement and/or motivation, with the majority of studies primarily providing descriptions of teachers' and students' experiences with mathematics and mathematics pedagogy. However, in a longitudinal study that explored the experiences of 20 children during their transition from primary to secondary schooling, Attard (2013, 2014) developed a Framework for Engagement with Mathematics (FEM) that defines the necessary components required for promoting substantive engagement with mathematics. The framework was informed through a review of current literature on student engagement with mathematics and then inductively from the results of the study.

Martin, Anderson, Bobis, Way, and Vellar (2012) conducted a large quantitative study on student disengagement involving 1601 middle school (Year 6–8) students from 200 classrooms in 44 Australian schools. In this study, engagement was viewed from the perspective of "switching on", where students were oriented towards mathematics in their future academic lives, and "switching off", where they were disengaging from mathematics. Disengagement and future intent were an important focus and unique factor in this study, which was guided by Bronfenbrenner's ecological perspective (Bronfenbrenner, 1979). The use of an ecological perspective differs from perspectives previously used in affective research in that it places the child at the centre of the environment to explore the interactions between the child and various factors such as home, classroom, school, and time. Motivation and engagement were viewed as a function of the different levels within an educational ecology. Martin et al. claim that no research has previously "tested the extent to which mathematics disengagement and future intent are part of a broader educational ecology relevant to child and youth development" (p. 14). Apart from this work, the remaining studies do not offer new theory, but explore classroom experiences from pedagogical, teaching and student perspectives.

7.1 The Influence of Relationships on Student Engagement

Classroom relationships and their influence on student engagement have featured heavily in recent studies on engagement and motivation. As mentioned earlier, Attard's study (2013, 2014) explored the influence of the teacher and the importance of developing positive pedagogical relationships as a foundation for engagement. Similarly, Skilling (2014) conducted a study that investigated the teaching practices of 31 Year 7 teachers in Australia and the way these teachers perceived that their practices influenced student engagement. In her study Skilling found the teachers identified practices that promoted or hindered engagement, and not all teachers reported using practices that were effective for promoting engagement. Practices identified as promoting engagement included those that emphasised the relevance and future value of mathematics, practices that emphasised mathematics applications and connections, the promotion of student autonomy, and sensitivity to students' interests. Practices identified that hinder engagement included teachers' low expectations of students, the perception that engagement is separate to the teaching of content, uncertainty about how to engage students, and high levels of teacher control. Skilling also acknowledged the link between motivation and engagement: "by making links between motivational factors and types of engagement the physiological processes that involve emotions and cognitions that influence mathematical learning processes are emphasised" (p. 595).

7.2 Pedagogy and Engagement

Three studies focused on pedagogical aspects of engagement. Bragg (2012) investigated the incorporation of games into mathematics lessons to engage students and improve learning outcomes. Data informing this study were drawn from classroom observations of six primary school children. In this study engagement was linked to "on-task" behavior and the outcome was that the use of games was deemed to be a useful pedagogical tool that promoted engagement. Sullivan et al. (2014) focused on the use of challenging tasks in their paper. The data reported were drawn from a larger study on persistence with mathematics tasks. Participating students were more engaged with challenging classroom tasks and preferred to persist with such tasks prior to receiving explanations or scaffolding from teachers.

A third study that focused on pedagogy was conducted by Calder (2013), who investigated the use of a democratic approach to student-centred inquiry. Calder gathered data from one Year 10 class in New Zealand. He found that the use of contextual, needs-based learning was highly engaging and motivating for those particular students. There were strong indications that the high level of engagement experienced during the study was a result of student involvement in determining the learning context and research question explored by the students.

7.3 Other Studies of Engagement

Ingram (2013), took a different approach in her study on student engagement by focusing her exploration on the skills and habits of engagement. Using data drawn from a longitudinal qualitative study of 31 Year 10 students in New Zealand, she described the quality of engagement of two specific students and concluded that individuals have differing sets of engagement skills and habits of engagement and tend to disengage, engage fully, or superficially engage as a result of differing relationships with mathematics.

Surprisingly, there was only one study reporting on the link between students' engagement with mathematics and the use of technology. Conducted by Attard and Curry (2012), the study explored how the introduction of iPads in one Australian primary classroom influenced students' engagement. The findings indicated that students were more engaged when iPads were involved. However there were multiple issues relating to the use of iPads and the way they were used, due to a lack of teacher experience and limited opportunities for professional development at the time the study was conducted. For example, the teacher's inexperience led to a narrow use of the devices for drill and practice of number computation, rather than using them to promote a broader, problem solving approach. Given the emphasis on Information and Communication Technology (ICT) in current curriculum documents, this is of some concern and there is a need for more research into the influence of ICT use on student engagement with mathematics.

8 Methodological Issues

Although in this chapter different components within the affective domain have largely been considered separately this distinction is not maintained in this section. As mentioned at the outset, the material reviewed is relatively evenly distributed among the different aspects of affect highlighted in this chapter. Throughout, when appropriate, comparisons have already been made in the text between the scope and direction of earlier work and the relevant research identified for the 2012-2015 period. Several key methodological descriptors pertinent to the research captured in this chapter are presented next.

8.1 Samples

Sample sizes involved in the research varied enormously, for example, from N = 1 (Andersson & Seah, 2012) or N = 2 (Beswick, 2012; Ingram, 2013) to N = 4599 (exploration of TIMSS data—Areepattamannil & Kaur, 2012). The composition of samples, too, varied, in terms of the number and type of participants involved in different studies (e.g., students of different ages, backgrounds, or ethnicity, pre-service and experienced teachers). These differences can affect the nature and direction of findings obtained and mitigate against a ready synthesis of findings reported in the literature. In some of the work reported researchers were meticulous about putting their results in a broader context. But in other reports there would certainly have been scope for a greater emphasis on a careful, evidence based synthesis of diverse findings. A cautious accumulation of relevant findings is needed if our understanding of the affective domain and its influence on the learning and teaching of mathematics is to move forward.

8.2 Methodological Approaches

As in previous years, self-report instruments, frequently consisting of Likert scale items, featured prominently among the instruments used to capture affect. Many relied as well on open-ended items or classroom observations for supplementary information and used both quantitative and qualitative methods to analyse the data collected. Given the wide range of publications in which the material covered in this chapter has been published it is not surprising that various methodological approaches were used in the research reviewed: quantitative, and qualitative using interviews, observations—including videotaping, and narrative inquiry. As well, an innovative use of technology was incorporated in a small number of studies. This included: Attard and Curry (2012) in their investigation of the influence of iPads on student (Year 3) engagement; Larkin and Jorgensen's (2015) use of iPad video diaries for data gathering; and Forgasz, Leder, and Tan (2014) who relied on Facebook for data gathering in their study. Common to all these approaches is the need to infer aspects of affect from observable behaviour or responses to specifically designed instruments or settings.

8.3 From Bricks to Walls

Qualitative or case studies were often highlighted in the research reports included in conference proceedings. While such work is particularly valuable for the generating of new hypotheses or the tentative explorations of new ideas, the robustness of inferences or conclusions drawn from such work is inevitably questionable. It is thus pleasing to see that some researchers have extended carefully on work first reported at a mathematics education research conference. With respect to quantitative studies, the more complex design and analysis described in a small number of studies is worth noting. The next *MERGA Research Review* may reveal whether this is indicative of a new trend—fuelled by the ready availability of new, user-friendly software.

8.4 Towards the Future?

Three decades ago Leder (1985) wrote that physiological indicators of the impact of affect on mathematics learning, and particularly of attitudes to mathematics, had been used in a number of research studies but that their use, particularly in a classroom setting was likely to remain limited. Perusal of the current literature confirms the continuing accuracy of this observation. However, while still attracting only limited research attention, the relevance of cognitive neuroscience to mathematics education, and within it the affective domain, is beginning to be recognised in research conducted and published beyond the MERGA community. More widespread usage of such new tools in the design and execution of future research may prove a useful adjunct to the instruments and approaches currently available in our explorations of mathematics education and the affective domain.

9 Concluding Comments

Over the last four years there has been an increase in the number of studies investigating affective factors relating to mathematics education. However, there are areas that appear to be lacking and some clear pathways for future research have emerged. The previous review (Lomas, Grootenboer, & Attard, 2012) noted an increase in student-focused studies and a reduction in pre-service teacher and classroom teacher studies. However this appears to have evened out, with a renewed interest in pre-service teachers, particularly in the area of anxiety. The majority of studies included in this chapter were focused on secondary school contexts with few studies relating to the primary years and no studies relating to the early childhood area. This concern has been voiced in all three past reviews on affect in mathematics education.

A need to improve the scope of research on affect to investigate how teachers' beliefs influence mathematical pedagogy, students' beliefs and other outcomes has been recognised in previous reviews on affect in mathematics education, and this has happened to some degree, in McGregor's study (2014). Very few studies across the various themes and topics have provided insight into pedagogical considerations that address the range of affective issues, and this warrants further research if improvements in attitudes, identity, beliefs, anxiety, and engagement relating to mathematics are to occur.

As mentioned earlier in this chapter, past reviews on affect and mathematics have noted a limited amount of theorising, with many studies providing description of teacher and student perceptions of affective issues. This review period has seen some development; however theorising affect continues to be a challenge within the Australasian context.

Methodological approaches have appeared to increase in variety over this review period, with the majority of the studies reported using a range of qualitative approaches. Past reviews have lamented a lack of classroom observational data and this appears to have improved, with an increase in data generated from students. Advancements in technology are seeing new approaches to data collection and it is expected that these will continue to evolve as the next review period unfolds.

The use of technology and its influence on affective components is an area that has received minimal attention during this review period. Given that technology is now an integral aspect of teaching and learning and in many jurisdictions is embedded within mathematics curricula, it is surprising that only one small study has investigated its relationship to student engagement. Consideration of the impact of technology on this digital generation of teachers and students and the ways it influences their perceptions of and interactions with mathematics education is imperative.

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Chapter 6
Equity, Social Justice and Ethics in Mathematics Education

Colleen Vale, Bill Atweh, Robin Averill and Andrew Skourdoumbis

Abstract The performativity policy mindset driving national and international testing highlights issues of equity in access and success according to socio-economic status, geographic location, ethnicity, gender and combinations of these factors. Researchers seek explanations for these inequities in terms encompassing engagement, participation and achievement to identify socially just and ethical practices at system, school and classroom level. The emergence of a theoretical perspective involving redistribution, recognition and participation (Fraser, Fortunes of feminism. From state-managed capitalism to neoliberal crisis, 2013) is evident in a range of studies concerning leadership, professional learning, pre-service teacher education, and pedagogies that focus on equity and social justice in mathematics education. The challenge of ethical and socially just practices at all levels and social groups is in providing access to deep learning in mathematics and success in "knowledge making" (Jorgensen, Proceedings of the 37th Annual Conference of the Mathematics Education Research Group of Australasia 2014).

Keywords Equity · Social justice · Redistribution · Recognition · Participation · Ethics · Socio-*economic status* · Rural · Ethnicity · Gender

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1 Introduction

Research on equity and social justice in mathematics education shows that "Australia lags behind many other OECD countries in terms of our equity outcomes" (Jorgensen, 2014, p. 311; Thomson, De Bertoli, & Buckley, 2013; Thomson, Hillman, & Wernert, 2012; Thomson, Hillman, Wernert, Schmid, Buckley, & Munene, 2012). This is also true of New Zealand "where around 75 % of the between-school variation in performance is accounted for by the socioeconomic background of students and schools" (Thomson et al., 2013, p. 278). Socio-economic status, gender, Indigeneity,[1] and ethnicity have typically featured in research concerning equity and social justice. Previous literature reviews on equity and social justice (Atweh, Vale, & Walshaw, 2012) identified a growing concern regarding the geographic context of school communities and the disadvantage of students in rural and remote locations. In this chapter we review studies concerned with socio-economic context, geographic location, language and culture, and gender. We identify theoretical frameworks and themes, and critique these studies in order to further understand these issues in differently-advantaged school communities.

A number of studies reviewed acknowledge the duplicitous nature of disadvantage within a school community, for example Jorgensen and Lowrie (2013), so that research which focused on one equity factor often involved others. Literature that specifically focused on Indigenous students is reviewed in Chap. 8 (see this volume) and the research concerning diversity within the classroom, differently-abled students, and inclusive practice in Chap. 7 (see this volume).

Given the over-riding policy culture of performativity, systemic responses to international and national achievement tests continue to focus on teacher quality (Skourdoumbis, 2012). Hence many of the studies reviewed focus on teacher quality through research of professional learning opportunities and programs; some also researched pedagogical approaches for social justice. We therefore expect that themes explored here are likely to intersect with the chapters on education policy and teacher professional learning. Implications for policy, teacher education, school communities, and teaching practice will be drawn along with issues for further research. We begin this chapter with a discussion of the theoretical perspectives informing this review.

2 Theoretical Perspectives

The previous review of the Australasian research in this field (Atweh, et al., 2012) revealed divergent theoretical foundations and whilst still dominated by a deficit discourse, emergent theoretical frameworks were evident:

[1]We acknowledge that Indigenous people in different places in Australasia prefer to use the term Aboriginal or their own cultural or tribal name to describe themselves.

> There seems to us to be a movement from the disparate agendas such as equity, diversity and inclusion to a more comprehensive and perhaps unifying construct of social justice. Likewise, a few authors are beginning to understand the agenda of social justice in terms of ethics. (Atweh et al., 2012, p. 57–58)

This aspect mirrors the chapter's alignment with the critical tradition given the concern with distributions of power, resources and knowledge, and links to schooling. With this in mind, the chapter draws on the work of Nancy Fraser (2013), in particular her notions of justice in terms of *redistribution, recognition* and *participation* reiterating her concern for an integrated conceptualization of justice in socio-economic, cultural and political terms.

The new "facts" driven sensibilities of measurement systems change what tallies and what is calculated for social justice in education. Lingard, Sellar, and Savage (2014), for instance, suggested that the proliferation of national and global testing and data substructures has re-articulated social justice as equity in current schooling policy. This represents a marked difference from previous considerations of social justice and equity in the education research literature with its emphasis on equality of opportunity, inclusion, diversity, fairness, and access. In many respects, the new technologies of governance in education and their performativity overlays determine the empirical research investigations of our time.

To be true to the critical stance we adopt here, we note that some researchers in the field have reflected upon some of the theoretical stances that different researchers on equity and social justice have adopted and, by implication on the types of research that is being conducted. Jorgensen (2014) discussed the general shift from cognitive and psychological theories that have guided early research in the field to more sociocultural perspectives that Lerman (2006) called the social turn in mathematics education. Jorgensen identified several emerging theoretical constructs guiding many of the new researchers. Her paper presented the robust view that

> These social theories have gained precedence in the field up to this point in time, but I want to disrupt this power base and question whether this position is creating a sense that the social conditions within which learning mathematics occurs is shifting focus away from the core learning of mathematics. My reason for this challenge is the continuing (and perhaps even growing) number of students from socially, culturally and linguistically diverse backgrounds who are still not performing well in mathematics. (Jorgensen, 2014, p. 313)

She went on to say that these theories "may have explanatory value but they may be causing educational research to be 'barking up the wrong tree'" (p. 313). Jorgensen concluded her call for a new paradigm that is needed in researching mathematics education and she provided examples suggesting that

> Learning environments that powerfully shape the potential for mathematical knowledge making for ALL students becomes the agenda for the future paradigm … This paradigm is one where all students are effectively scaffolded by excellent teachers who are able to create knowledge making. (Jorgensen, 2014, p. 317)

In a similar vein, Atweh and Graven (2016), using the construct of "ethical imagination", raised some issues for researchers working with excluded students. They argued that researching inclusive education demands a level of empathy and responsibility from researchers towards the “subjects” of their research. They argued that ethics in research, particularly with marginalised groups, should go beyond “ticking boxes” for informed consent and confidentiality on standard ethical clearance forms towards a commitment to enter into meaningful dialogue with participants focusing on anticipated benefit. They urged researchers to engage critically and embrace dialogue with teachers in the research relationship avoiding deficit discourses which further shut down the space for teacher learning enabling redress. The authors raised some questions for researchers to consider. Is research that identifies problems and complexities in questions of inclusion, and research “on” the excluded and their helpers sufficient? Does the researcher have an obligation to lobby for and work towards inclusion? What research designs allow for the understanding of exclusion and at the same time attempts to redress it?

3 Socio-Economic Context of School Communities

Links between educational achievement including aspiration and socio-economic context are predictably consistent (Jorgensen, 2012b; Jorgensen, Gates, & Roper, 2014; Thomson et al., 2013; Thomson, Hillman, & Wernert, 2012; Thomson, Hillman, Wernert, Schmid, et al., 2012; Vale, Weaven, Davies, Hooley, Davidson, & Loton, 2013). Contributing factors include: student mix, student family background, parental connection(s) to school, teacher quality, student language skill(s), curriculum alienation, and so on (Jorgensen, 2012b; Jorgensen et al., 2014).

Education systems are, as Bok (2010) pointed out, “a fundamental aspect of the social mechanisms that reproduce unequal access to, and outcomes from, education for students from low SES backgrounds” (p. 164). In contrast to their “elite” counterparts, students from low socio-economic communities cannot draw upon the requisite cultural and social capital needed that conditions and positions them for schooling and beyond. Schools often reflect and transmit the structured dispositions of a pedagogic order, including those framing teaching practice. The teaching of mathematics is particularly susceptible to routinised practice, the usual arrangement one of teacher-led lecture style presentation(s) with minimal student interaction(s) followed by individual student work through text based exercises. Atweh, Bose, Graven, Subramanian, and Venkat (2014) argued that such approaches also raise social justice issues since:

> Research provides consistent evidence that suggests that teachers often adjust their teaching to their perceptions of students’ achievement levels. While this may appear to be appropriate, it can restrict the opportunity to learn for low-achieving students. This is of particular concern when it involves groups of students from certain social, cultural or language backgrounds. Sztajn (2003) noted the tendency of using rote teaching for low SES students and problem solving with high SES students.… Luke (1999) warned that the “dumbing down” of the curriculum for low-achieving students excludes them from developing high order thinking and intellectual quality work. It also diminishes their opportunity to learn content needed at higher levels of schooling. (Atweh et al., p.17)

In a social and political context where the grand equalizer of public education no longer holds, teaching actions and practices are likewise re-set so that a ‘back-to-basics’ logic in teaching gains favour over experimental and holistic teaching approaches. Yet, in mathematics education, studies (see Duru, 2010) showed that innovative teaching approaches are conducive to sustained gains in achievement, particularly for the disadvantaged.

Articles reviewed for this section of the chapter generally indicated one major noteworthy point, namely that socio-economic disadvantage is still a major determiner of student achievement. The articles also illustrated effective features of programs for the early years of schooling and number learning (Gould, 2014; Perry, Gervasoni, & Dockett, 2012), the value of using technological tools in under-resourced and disadvantaged communities (Goodwin & Gould, 2014), pedagogical beliefs that enable changing practice (Atweh & Alai’, 2012) and the importance of curriculum leadership in disadvantaged schools (Jorgensen, 2012a).

3.1 Challenging Hegemonic Practice

Jorgensen’s work (2012b) on scholastic mortality rates among disadvantaged students (working class and Indigenous) highlighted the difficulties these students encounter in schooling. She used Pierre Bourdieu and his theoretical notion of “miscommunication” and “habitus” to outline how the education system and its inherent system(s) work to the detriment of the most disadvantaged. The symbolic violence of communicative codes, the language (linguistics) used in school education and in various disciplines, for example mathematics, is complicit in the learning outcomes of students. Jorgensen (2012b) suggested that student and teacher “behaviour is complicit in the stratified outcomes of learning school mathematics” none more so perhaps than when there is “no recognition of the linguistic codes that learners bring to school mathematics” (p. 37). In suggesting that students of different class backgrounds *do* school differently, that is by virtue of their habitus, she has acknowledged that the “process of miscommunication becomes a subtle form of exclusion of which the child and teacher may be totally ignorant” (Jorgensen, 2012b, pp. 37–38).

Learning school mathematics is about mastering the codes of the discipline. Most current mathematics teaching involves the transmission of knowledge and less reliance on rich or authentic engagement (see Boaler & Staples, 2008). Successful acculturation to school mathematics means mastering its sedimented disciplinary knowledge. Jorgensen reminded us that students “most likely to succeed in the discipline are those whose habitus is strongly aligned with the objective structuring practices of the field” (2012b, p. 38). The impact of habitus was further revealed in another study by Jorgensen et al. (2014). This study showed how a teacher’s practice of streaming in their classroom, which provided more or less access to hegemonic mathematics knowledge, reproduced cultural dispositions and disadvantage as the teaching practices for lower streamed students contributed to delayed progression and underachievement. Jorgensen advocated that teachers and in particular mathematics teachers be mindful of how their teaching practices or indeed their beliefs may actually reinforce disadvantage, for instance by presenting mathematics problems in class in a reified and de-contextualised way or by denying disadvantaged students access to creative mathematical thinking.

Work by Atweh and Ala'i (2012), like Jorgensen's (2012b), addressed a core code of the education system, pedagogy (teaching) and its relationship to learning. These researchers, like others (for example, Gutstein, 2006) in the field of mathematics education, noted that teaching practices of mathematics teachers contributed to the engagement and presumably achievement of learners. Importantly their work on specific teaching practices, what they term a "socially response-able approach to mathematics education" (Atweh & Ala'i, 2012, p. 98) with its notable concern about social justice, provided one approach towards developing students' responsibility through mathematics education.

A key part of their study involved working with several teachers to develop and enact, within their teaching, socially response-able mathematics activities. The Atweh and Ala'i study pointed to the reticence that many mathematics teachers display towards alternate "open ended pedagogies" (2012, p. 103). It also illustrated that when teachers use approaches other than what may be conveniently termed "direct instruction", students invariably demonstrate a "deeper understanding and engagement in the class" (Atweh & Ala'i, 2012, p. 103). The Atweh and Ala'i study reinforced how important teacher beliefs about the epistemological nature of mathematics are in shaping teacher attitudes and beliefs about the discipline of mathematics that then determine their "readiness to take risks in changing classroom practices" (Atweh & Ala'i, 2012, p. 104).

3.2 Addressing Disadvantage in the Early Years

The Gould (2014) study of the association between students' number knowledge and social disadvantage at school entry, was concerned with the number knowledge of students from different socio-economic backgrounds at point of school entry. Gould cited evidence suggesting that disadvantaged children enter school already behind their more advantaged counterparts and, importantly, that early mathematics knowledge tends to predict subsequent school achievement. His study suggested that early intervention aimed primarily at developing basic number knowledge, namely object counting, identifying numerals, and flexible use of oral counting (core knowledge that Gould has found correlates with Family Occupation and Education Index of the School) of disadvantaged young children is needed. Indeed, it is not simply that disadvantaged children need support but that planned and "designed experiences in early number are particularly important in preschool settings servicing low socio-economic communities to reduce the disparities in the background knowledge" (Gould, 2014, p. 261) of the already disadvantaged. Interestingly his study also pointed to the contested nature of the early childhood curriculum in Australia and the place of mathematics education within it (see Cohrssen, Church, Ishimine, & Tayler, 2013). Nonetheless like the researchers he drew upon, Gould advocated for a programmed approach to the teaching of mathematics in the early years. There is a simple and sound reason for this: identifying "what needs to be addressed to reduce the risk of those starting behind in mathematics learning staying behind in their mathematics learning" (Gould, 2014, p. 262).

In another study Perry et al. (2012) also showed that targeted early mathematics programs such as *Let's Count* in low socio-economic communities provided opportunities to enhance mathematics learning outcomes. Early childhood educators also reported that they too benefited as learners and teachers of mathematics from programs such as *Let's Count* as these programs "build or maintain positive dispositions and increased confidence towards mathematics" (Perry, Gervasoni, & Dockett, 2012, p. 600).

3.3 Leading Curriculum Change

Jorgensen's work (2012a) on Curriculum Leadership focused on how vital it is that local school and community context be considered before major curriculum change is enacted. In considering the specific model of devolved leadership, the most common type of model, Jorgensen was able to examine how common curriculum practices were enacted in particular types of schools (regional and remote). Issues identified included the use of commercial numeracy programs, sustainability, high expectations and curriculum leadership and community. There were particular issues linked to sustainability including the high turnover of staff and the "constant change in provision of numeracy programs within schools" (Jorgensen, 2012a, p. 374). Remote settings were at particular risk here as constant staff changes including Principal/Leadership changes often resulted in program change. Her study indicated the general acceptance of high expectations in all schools and communities analysed. Strong and accepting relationships between curriculum leadership and community were seen as vital to the success of any numeracy curriculum program.

The achievement outcomes arising from initiatives to provide curriculum leadership to networks of disadvantaged primary and secondary schools and to develop whole school and network approaches to teaching were investigated by Vale et al. (2013) using longitudinal statistical analysis of achievement data. They found that growth exceeded expectations during Terms 2 and 3 of the school year, but decreased and tended to be below expectations in school terms 4 and 1 (the Spring/Summer months), mirroring US data on the "summer slowdown" phenomenon. The significance of this issue takes on a heightened importance when one considers that in Australia there are "large gaps in achievement between students from the highest and lowest socio-economic backgrounds" (Vale et al., 2013, p. 2). Schools servicing low socio-economic communities need to work at reducing the impact of the "summer slowdown" if they are to close the achievement gap.

Alternative teacher certification pathways (Teach For Australia for instance) are marketed as replacement modes of teacher training and education designed specifically to address stagnating student achievement and reducing educational disadvantage. Despite this, disadvantaged Australian school students continue to trail their more advantaged peers (Skourdoumbis, 2012). Further research of such programs is warranted.

The studies reviewed here reveal the duplicitous nature of social disadvantage, as schools serving low socio-economic communities are often also schools in rural or remote locations, and have significant school populations of Indigenous students or students of other cultural and language backgrounds. The next section focuses on research concerning equity issues of geographic location.

4 Rural and Remote School Communities

Research regarding disadvantage and inequities in mathematics outcomes for students in rural and remote communities have addressed systemic and structural issues of staffing encompassing teacher retainment, curriculum leadership and quality of teachers and teaching in these schools. Each of the studies reviewed in this section were conducted in Australia as the literature search did not reveal studies conducted in New Zealand, a change from the previous review period. Attending to issues of teacher quality these studies report on teacher preparation, support for teachers and the outcomes of professional learning for teachers in schools in rural and remote locations which usually service Indigenous communities.

International tests show that 25 % of Year 4 students are taught by teachers who are not "very confident" in teaching mathematics and 34 % of Year 8 students are taught by teachers without qualifications in mathematics (Thomson, Hillman, & Wernert, 2012; Thomson, Hillman, Wernert, Schmid, et al., 2012) and that students of teachers with lower levels of qualification or confidence score lower than others. These studies do not identify the location of the less qualified and less confident teachers, but reports of *Staff in Australian Schools* (McKenzie, Weldon, Rowley, Murphy, & McMillan, 2014) consistently show that less qualified and beginning teachers are disproportionately located in remote and rural locations and low socio-economic metropolitan communities. The studies reviewed below either directly confront the issue of attracting, training and retaining staff to teach in rural and remote pre, primary and secondary schools or consistently identify less qualified or beginning teachers as the target of professional learning and curriculum innovation projects in remote and rural communities.

4.1 Teachers, Identity and Practice in Rural and Remote Schools

Building on previous studies Handel et al. (2013) conducted a survey involving 191 secondary school mathematics and science teachers from 27 schools in New South

Wales to find the factors that determined their intention to stay or leave the school or the profession. Their previous research had found that high proportions of teachers returned to the coast or left the profession after completing their required tenure. This study identified instructional, school organisational, and curricula issues that impacted retention. These included being the only trained teacher in the subject, being expected to teach in another discipline, that is out-of-field, few opportunities for professional development, lack of support services such as mentoring and coaching within the school or district and lack of funding for resources and materials because of small school budgets. As inexperienced teachers they were also expected to take on administrative and leadership responsibilities. The respondents indicated that inducements to take up rural and remote appointments were not sufficient to out-weigh these professional or personal factors concerning rural or remote living. These findings confirm the critical role of leadership for social justice in rural and remote schools (Jorgensen, 2012a).

Aware of the high incidence of out-of-field teaching in rural and remote schools and that poor attraction and retention factors contribute to the extent and longevity of out-of-field teaching, Hobbs (2013) used socio-cultural theories of learning, boundaries and identity to identify factors contributing to out-of-field identity. She interviewed 18 secondary teachers from three rural secondary schools in Victoria who identified as out-of-field and developed the "Boundary Between Fields" model to conceptualise three factors contributing to out-of-field identity: context, including rurality and school culture and organisation; support mechanisms such as: provision of professional learning; mentoring; coaching and resources; and personal resources, including adaptive expertise, teacher knowledge and dispositions. Concurring with Handel et al. (2013) and Jorgensen (2012a), Hobbs found that access to collegial support and professional learning and leadership practices impacted on their identity as out-of-field. Adaptive expertise enabled teachers who might otherwise identify as out-of-field to take the initiative in developing their knowledge and engage in professional learning. Hobbs concluded that "rurality…demands adaptive expertise" (p. 285).

Three case studies explored professional learning programs to respond to transience of teachers in rural and provincial schools, in-field/out-of-field identity and the absence of leadership or professional learning opportunities (Owens, 2015; Sandhu, Kidman, & Cooper, 2013; Warren, Quine, & De Vries, 2012). The pedagogical frameworks used in these projects were culturally responsive and related to Fraser's notion of participation and involved engagement with the community in different ways and to varying degrees. Sandhu et al. (2013) tracked the pedagogical shifts of an in-field mathematics teacher with six years' experience of teaching in a remote secondary school where at least 30 % of the student population were Indigenous. This teacher was a participant in a professional learning and curriculum development project conducted in nine schools in Queensland involving in-field and out-of-field teachers using the Reality-Abstraction-Mathematics-Reflection (RAMR) pedagogical framework. Warren et al. (2012) reported on the first stage of a longitudinal study of the professional learning of beginning teachers of Foundation to Year 3 students. Their pedagogical framework, RoleM (representations, oral language and engagement in mathematics) used socio-cultural theories of learning and involved teachers in dialogue with experts and collaborative planning, enactment and sharing. Owens (2015) investigated changes to pedagogical practice of schools serving communities with significant populations of Indigenous students in New South Wales. The case study school participated in three curriculum development projects, each involving culturally responsive teaching: *Stronger Smarter Learning Communities*, a project for school leaders, *Make It Count*, a project to develop approaches for teaching Indigenous students, and *8 ways*, a project to develop teachers' cultural competence in the classroom.

Sandhu et al. (2013) found that in-field teachers can identify as out-of-field when they don't know their students or how to address the learning needs of their students. The RAMR enabled the teacher to become more flexible in their teaching methods to meet the needs of underachieving students. Warren et al. (2012) reported positive changes in beginning teachers' attitudes, beliefs and practices about the teaching of mathematics, expectations of students and confidence to be innovative. Sustainability of these collegial practices in the context of the high levels of leadership and teacher transience will be tested in the next phase of their study. Key findings from Owens' (2015) study included the importance of funding to enable involvement of a critical mass of teachers and to give the Indigenous community a voice and role in decision-making. The curriculum frameworks used in these studies are also reviewed in Chap. 8 (see this volume).

4.2 Preparing to Teach in Rural and Remote Contexts: Pre-service Teacher Education

One of the strategies allegedly employed to overcome shortages of qualified teachers in rural and remote schools is the alternate teacher education pathway *Teach for Australia* (TFA) that recruits elite, high performing graduate students and places them in underperforming, hard-to-staff schools (Weldon, McKenzie, Kleinhenz, & Reid, 2012). The education program consists of a 6-week intensive teacher education program, followed by appointment as an associate teacher for 2 years in a disadvantaged school. At the beginning of the school year following completion of the TFA program only 26 of the 42 initial cohort had secured a tenured position (Weldon, et al., 2012). Skourdoumbis (2012) provided a critique of this teacher education pathway drawing on Bourdieu's (2000) critical theory. He argued that the initiative is deficit focused and employs a "teacher-hero" scenario, where high achieving, inexperienced teachers are expected to solve the problem of low achievement without addressing the reproduction of social inequalities, evidenced in the studies tracking retention and border-crossing reviewed above. Skourdoumbis (2012) argued that policy responses such as TFA contribute to, rather than subvert, the reproduction of social disadvantage.

One study of pre-service teacher education concerned with teaching rural locations specifically addressed primary mathematics teaching, while another involved pre-school teachers. Wilson (2013) was concerned that beginning teachers may pass on mathematics anxiety or use inappropriate teaching practices to students in rural and remote schools. She compared the level of mathematics anxiety of primary pre-service teachers in a rural campus and metropolitan campus of a university using the Revised Mathematics Anxiety Rating Scale (RMARS) instrument. She found that the mean level of mathematics anxiety was higher for rural pre-service teachers than for metropolitan pre-service teachers, though the difference was not statistically significant. Hunting, Mousley, and Perry (2012) conducted a study of rural pre-school teachers' perspectives of young children's mathematical thinking using structured individual interviews with 64 preschool teachers across three Australian states. The interviews focused on five themes: awareness of children's mathematical thinking, support for mathematics teaching, use of technology and computers, their attitudes and feelings about mathematics and assessment and record keeping. Missing from their study were perspectives and practices on engaging with parents on mathematical activity and cultural or context-based pedagogies for pre-school children in rural, remote or Indigenous communities.

In summary these studies reported inexperience in teaching mathematics or in the school and community context, transience of leadership and absence of support structures and opportunities for mentoring and professional learning. They revealed the complexity of rural school communities and the importance of school culture, organisation and leadership structures and provision of resources to enable schools to form partnerships with their communities and that support teachers to develop cultural and pedagogical knowledge and be adaptable and culturally-responsive. Teachers and leaders in rural and remote schools must want to stay and contribute to sustained change in pedagogical practice that makes a difference to students' mathematical learning. Socio-cultural and critical theory informed the research studies, with Fraser's (2013) meaning of social justice foregrounding some studies and Jorgensen (2014) arguing for a shift in paradigms to enable a focus on mathematics knowledge making. Our review now turns to consider another intersecting cultural factor of equity and social justice, namely the ethnic and language contexts of school communities.

5 Ethnic and Language Context of School Communities

Articles reviewed for this section of the chapter were initially sorted by whether they related to Indigenous students' learning, culturally responsive practices, or language issues. This process resulted in around half falling within two or all of the three categories, illustrating the complex and interrelated issues inherent in examining equitable approaches to teaching and learning mathematics for students in Indigenous and minority ethnicity groups, particularly when language issues are also pertinent. Such complexities demonstrate the suitability for this review of utilising Fraser's (2013) conceptualisation of social justice as an integration of socio-economic, cultural and political factors.

The overarching theme of the work reviewed is that of enhancing equity of access to mathematics learning and achievement through teachers being aware of and attending to students' cultural capital in mathematics instruction (e.g., Averill, 2012a, 2012b; Averill & Clark 2012, 2013; Edmonds-Wathen, 2014; Meaney, Trinnick, & Farihall, 2013; Owens, 2014a). Cultural capital discussed includes the ways of being, knowledge, and skills that students possess and the ways of being, knowledge, and skills inherent within their heritage cultures. Emerging themes include the increasing emphasis on recognising the suitability and importance of involving the people that are closest to students (their parents, families and school communities) in decisions about and awareness of their learning (e.g., Averill, 2012a, 2012b; Meaney, Trinnick, & Fairhall, 2013; Owens, 2014a), and recognising the essential nature of "place" within mathematical learning, such as through understanding and acknowledging customary links between environmental and cultural activity in order to utilise "ecocultural" mathematics within teaching (e.g., in space and geometry, see Owens, 2014b). Areas such as these provide opportunities for rich and valuable future inquiry.

5.1 Culturally Responsive Teacher Practices

Most of the reviewed literature focused on culturally responsive teacher practices shown to assist or advocate for learning that improves achievement. Methods used included: recognising the role of contextual artefacts and gesture in young Indigenous students' learning of growing patterns (Miller, 2014; Miller & Warren, 2012), focusing on mathematisation and contextualisation to help make mathematics meaningful, in turn enhancing mathematical resilience (Thornton, Statton, & Mountzouris, 2012), and considering learning in relation to a holistic model of health and wellbeing encompassing cognitive, social, physical and spiritual aspects of classroom learning and interactions (Averill, 2012a). In contrast, Jorgensen and Lowrie (2013) discussed ways in which schools help their Indigenous students successfully navigate their school experiences by explicitly illustrating how to "play" the "game" of school mathematics.

An increasing number of articles focus on describing the value of recognising or incorporating culturally linked knowledge and practices into instruction and learning (e.g., Grootenboer & Sullivan, 2013; Warren & Quine, 2013). These researchers seek to align classroom practices and pedagogies with the diverse experiences, identities, values and norms students bring from their out of school lives to their learning. The examples given are likely to enhance not only the learning of indigenous and/or minoritised students, but of all. For example, Averill and Clark (2013) found teacher professionalism, consistency, courteousness, flexibility, and one-to-one teacher-student interactions contribute to respectful classroom environments, developing effective teacher-student relationships. Teachers and students knowing each other including knowing individuals' learning preferences and needs, and teachers' use of specific feedback and encouragement, contributed to students' learning (Anderson, Averill, Te Maro, Taiwhati, & Higgins, 2013).

Language-based equity issues discussed in the reviewed literature included challenges associated with English, which is often the language of instruction and assessment, yet not the first language of some learners, teachers, or researchers (Edmonds-Wathen, 2013; Matang & Owens, 2014). Further issues included cultural differences in mathematical understandings between students and their teachers (Edmonds-Wathen, 2014), and classroom metaphors which can create culturally-bound concepts, particularly for Indigenous students (Edmonds-Wathen, 2012). Language-linked research also reported achievement improvements resulting from a classroom focus on representations, oral language, and engagement (Warren & Miller, 2013, 2015), and reduced language dependency questions impacted positively on overall numeracy scores (Wilson & Barkatsas, 2014).

Equity issues relevant to instruction in students' heritage languages included challenges for teachers and students in adopting mathematical terms often new to these languages and language revitalisation (Edmonds-Wathen, Sakopa, Owens, & Bino, 2014; Trinick, Meaney, & Fairhall, 2014). Evidence from the enactment of two iterations of curriculum development, demonstrated the part mathematics curriculum can play in language revitalisation. McMurchy-Pilkington, Trinick, and Meaney (2013) described how, despite and in part due to, curriculum development occurring within contested spaces in relation to Ministry of Education expectations and Maori aspirations, processes and products were used to support revitalisation of te reo Maori. Despite such affordances, substantial challenges can still exist for mathematics instruction in languages other than English. For example, Trinick et al. (2014) outlined societal, policy, in-school, mathematical, and linguistic factors that can assist and hinder the adoption of the registers of mathematics and mathematics education by teachers within Maori medium schools, themselves second language-Maori learners, found within their study of 19 teachers across two schools.

Challenges to culturally responsive mathematics learning identified across the reviewed literature included teachers' (lack of) culturally-based knowledge, teachers viewing mathematics learning and students' heritage cultures as distinct (Averill, 2012b), and the likelihood that the place of English language in instruction and research may constrain mathematics education possibilities (e.g., Meaney, 2013). Given these substantive challenges to advancing culturally responsive mathematics teaching, surprisingly little of the reviewed literature focused primarily on initial or in-service teacher education. Exceptions include Hurst and Sparrow's (2012) study into a pilot project training teaching assistants to plan for helping individuals and small groups with mathematics learning. The study found not only that teachers had an enhanced confidence and ability in their teaching, but the teaching assistants too became integral to their professional learning communities. Other promising work includes Owens's (2012; 2014a; 2014b) explorations of student teachers' project reports, which illustrated how activities linking culture and mathematics can help develop their mathematical identities, and Owens, Edmonds-Wathen, Kravia, and Sakopa's (2014a, b) use of design principles for teacher professional learning in Papua New Guinea. In addition, Anthony, Hunter, and Thompson's (2014) description of one teacher's learning journey following an inquiry-based intervention showed the importance of safe learning environments and including individual and collective learning for successful continued use of high leverage and culturally responsive intervention strategies. Given persistent achievement differences due to ethnicity (e.g., Forgasz, Leder, & Halliday, 2013; Leder & Forgasz, 2014), further work in this area is needed to build on research into effective culturally responsive teaching practices.

In summary, recent work in the areas of culture, language and ethnicity adds to the development of understandings of factors that impact on the mathematics learning and achievement of Indigenous and marginalised students, including those for whom language issues exist, whether related to English, a heritage language, or the language/s of instruction. A theme, that although present explicitly (e.g., Anderson et al., 2013; Owens, 2014a; Warren & Quine, 2013) and implicitly (e.g., Miller, 2014), appears underdeveloped is that of the importance of partnerships in advancing understanding of effective culturally responsive practice. Themes across the reviewed literature suggest that such partnerships best provide suitable ways forward in Australasian schooling contexts towards increased consistency in practice reflecting Fraser's notions of participation and recognition, and through this, increased equity of access to mathematics achievement. The three equity factors explored thus far have often been entwined in the school communities and have revealed strong overlaps in theoretical frameworks, themes and findings. The final equity factor, gender, whilst also present in the disadvantaged contexts reviewed, is also an equity issue in otherwise advantaged school communities.

6 Gender

Australasian research has continued to investigate the incidence of gender inequity and the factors contributing to gender differences in achievement, participation and attitude. International studies, the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA), continue to show gender differences in achievement favouring boys in Year 4 (Thomson, Hillman, Wernert, Schmid, et al., 2012), Year 8 (Thomson, Hillman, & Wernert, 2012) and at 14 years of age (Thomson et al., 2013). These differences are significant for secondary students in Australia and New Zealand, have increased since 2003, and are higher than the OECD average in the PISA study (Thomson et al., 2013; Thomson et al., 2012a). Socio-cultural identity theory dominates the theoretical frameworks informing the research in this field of equity and social justice. The literature search found only one study that investigated social justice pedagogy for women and girls (Tanko & Atweh, 2012).

6.1 The Widening Gender Gap

The proportion of students participating in senior secondary mathematics is continuing to decline. Mack and Wilson (2015) reported that the steepest declines and lowest participation rates in New South Wales from 2001 to 2014 are for girls. Observing that the gender gap revealed in international studies was widening, Forgasz and Hill (2013) analysed results of the highest achievers for all three Year 12 mathematics units in the Victorian Certificate of Education from 2007–2009. The factors explored included gender, socio-economic status, geographic location and learning setting. They found that “males, students from higher socioeconomic backgrounds and those attending metropolitan schools predominated amongst the highest achievers in all three VCE mathematics subjects” (Forgasz & Hill, 2013, p. 481). Moreover the gap increased with the level of difficulty of the mathematics subject. However, their study did not use inferential statistics to test for statistical significance. Their findings show that gender differences were not as large as differences for socio-economic status and geographic location.

Carmichael (2013) used data from the large Longitudinal Study of Australian Children, for children aged six years; and two years later when aged eight years, to investigate the influence of prior achievement and teachers’ assessment on gender differences in achievement. Using Rasch modelling to analyse data he found that gender differences favouring males whilst small at this age, did increase in the two-year period. Gender differences were also related to content as teachers rated girls more highly than boys for data content, but boys more highly than girls for place value and computation.

6.2 Socio-Cultural Perspectives and Identity

Concerned about declining participation of boys in post-compulsory mathematics subjects, Easey et al. (2012) surveyed Year 10 boys at an all boys’ school in Queensland. They found that boys who intended to study at least one of the two more advanced Year 12 mathematics subjects valued the relevance of mathematics for their professional career aspirations, whereas the boys intending to study the less demanding mathematics subject were more likely to base this decision on their perceived lower mathematics ability. This group also believed that mathematics was “not critical in society” (p. 248). These findings suggest a shift away from gender stereotyping of mathematics as a male domain. Other studies continued to search for any shifts in this long established phenomenon.

Carmichael (2014) explored the influence of parents’ attitudes on gender differences in mathematical outcomes. Parents were asked to predict how far their child would progress in their education and how well they were progressing in mathematics. He found that “parents of boys tended to have more positive perceptions about their son’s mathematics achievements than parents of girls”

(Carmichael, 2014, p. 124). However they were more likely to predict that their daughters would achieve a tertiary education and their sons a trade qualification. This finding is somewhat surprising given the mean level of socio-economic status reported for the sample.

Adopting a more complex view of the social context of learning, “that is the attitudes, actual and perceived, of critical ‘others’ in students’ homes, at school, and societal beliefs more generally” (p. 373), Forgasz, Leder, and Tan (2014) conducted an international study of the gendered perceptions of mathematics, technology capabilities and Science, Technology, Engineering and Mathematics (STEM) related careers. They used Facebook to recruit participants from 81 countries but focused their analysis on nine countries with larger numbers of participants. Analysis showed significant differences by country on gendered perceptions of mathematics capability, parents’ gendered perceptions of mathematics and teachers’ gendered perceptions. In most countries, but not all, these perceptions favoured males, especially in China. Non-gender stereotyped perceptions predominated in six countries including Australia. Participants in all nine countries agreed that there was no gender difference in importance of mathematics learning. Further studies involving a similar instrument are reviewed in Chap. 5 of this volume. Together these studies report changes to gendered perspectives of mathematics, at least in English-speaking countries.

One study reviewed here explored students’ gendered mathematics perceptions. Tan (2012) conducted an online survey involving students from Singapore and Australia about their beliefs about learning and knowing of mathematics with graphics and CAS calculators. Her study was informed by feminist theory of women’s ways of knowing (Belenky, Clinchy, Goldberger, & Tarule, 1986) and metaphors of interacting with technology (Goos, Galbraith, Renshaw, & Geiger, 2000). She found significant gender differences in ways of knowing and learning mathematics with males scoring higher on Connected Knowing—Deep Approach and girls scoring higher on Surface Knowing—Surface Approach. She also established association between these ways of knowing and the use of calculators as Master with Surface Knowing and the use of calculators as Collaborator with Connected Knowing. Tan argued that these findings have implications for students’ mathematical achievement. They also indicate that further research of the learning environment and learning expectations are warranted.

6.3 *Social Justice Pedagogy*

Tanko and Atweh (2012) used Gustein’s (2006) framework for teaching mathematics for social justice with a group of Arab women participating in a tertiary mathematics bridging course. This framework included goals for mathematical learning as well as goals for using mathematical knowledge for change and social justice. The mathematics program involved student selected mathematical projects on issues of significance to the women along with worksheets to enhance mathematical content and skills relevant to these projects. Interviews confirmed increased confidence among participants, and the women's project work and exercises displayed development of mathematical understanding and skills beyond the basic numeracy skills expected. The researchers noted the challenge of selecting problems that provided opportunity for engagement with challenging mathematical ideas and meeting the social justice goals of the students.

In summary these studies continue to provide evidence of gender differences in achievement and approaches to learning. They also reveal that whilst gender-stereotyped perceptions of mathematics persist for some social groups, shifts away from the perception of mathematics as a male domain are also evident, especially in the English-speaking world. However, more research on pedagogical approaches that transform deficit and gendered perceptions are needed.

7 Concluding Remarks

Every care was given in this review to identifying research studies conducted by researchers from and within Australasia. We note that other studies relevant to this chapter theme are reviewed in Chap. 7 or 8 (see this volume). Summarising the complex issues researched by the various studies reported here is difficult. Here we offer overall comments about the patterns arising from this review and consider some of the gaps we have identified, some pending research questions and some implications for policy and teacher training.

First, equity, social justice and ethics concerns remain high in Australasian mathematics education research. Research is diverse, incorporating a wider range of social groups not equitably participating in mathematics learning and achievement. We note in particular, that in addition to long standing concerns about gender and socioeconomic status, there is an increase in the number of studies around Indigenous, language and culture issues and rural and remote schools.

Second, we note a common theme and finding regarding the development of partnerships between schools and their communities. Social justice through representation and participation (Fraser, 2013) is a prominent theoretical perspective. Leadership and school culture are pivotal to forming partnerships with community to develop cultural understanding and whole school approaches to teaching mathematics. These approaches have been variously described as inclusive, culturally responsive, socially response-able, and place-based pedagogies. Jorgensen (2014) argued that these approaches must place mathematics learning and knowledge making as the learning objectives. Questions regarding the way in which recognition, representation and participation is enacted in these approaches and their impact on mathematics learning and success are addressed in Chaps. 7, 8 and 11 (see this volume).

Third, support mechanisms, including curriculum leadership, and professional learning opportunities and culture enable teachers to develop cultural and pedagogical knowledge, and to be adaptable, flexible and committed to social justice.

These findings speak to the redistribution aspect of social justice (Fraser, 2013) and the need for systems to fund and support school organisational structures, resources and cultures if social disadvantage is not to be continually reproduced.

Fourth, in terms of future directions for research in the topics addressed here, we call for an increase in research that looks to how learning environments can be created with the help of teachers, to focus on mathematical knowledge building. In other words, we call for more research designs and theoretical stances that directly target the elevation of disadvantage enabling teachers and communities to gain mathematical knowledge towards that aim.

Lastly, we consider the implication of research in this area for pre-service teacher education. A number of studies reviewed in this chapter conducted research involving pre-service teachers. The studies show that teacher education needs to prepare teachers who are adaptable and ready to implement equitable and socially just pedagogies appropriate for the students in their school community. In many teacher education courses students undertake general education classes that deal with issues of exclusion and equity, and study separately the teaching of mathematics. Atweh et al. (2014) call for greater care in the education of mathematics teachers so that their teaching reflects awareness of equity, social justice and ethics issues. Issues of this kind should be included as integral components of all pre-service teacher education mathematics courses.

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Chapter 7
Inclusive Practices in Mathematics Education

Rhonda Faragher, Janelle Hill and Barbara Clarke

Abstract Inclusive mathematics education acknowledges human diversity and involves supporting the diverse learning needs of all students in general mathematics classrooms. In this chapter we review Australasian research concerning the various categories of diversity using the three themes of our framework: Access to the curriculum through policies and leadership practices; Diverse approaches to learning mathematics; and Teaching approaches for inclusion. Our analysis of the literature explored commonalities in research approaches and issues across the field. Our framework deliberately avoids reviewing literature under categories of diversity which would only serve to further segregate. Our review focused on issues arising in the teaching and learning of mathematics and the policies and practices that enable those endeavours. We were unable to identify any research that indicated some groups of learners needed to be taught away from other students. Those strategies or techniques needed for some could be used to enhance the learning of all. Following our review under the three themes, we propose areas of needed research and encourage mathematics education researchers in our region to further develop this field.

Keywords Inclusive education · Approaches to teaching · Mathematics attainment · Numeracy · Educational leadership · Diversity

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1 Introduction

The scope of this chapter is mathematical attainment for all learners, which is a fundamental concern for all interested in understanding mathematics teaching and learning. Inclusive education is founded on the recognition of human diversity and involves "supporting the belonging and full participation of all people together" (Cologon, 2014, p. 4). Inclusive mathematics education requires welcoming, valuing, and supporting the diverse learning needs of all students in the shared general mathematics classroom (Faragher, 2015; Thousand & Villa, 2000). Therefore, inclusive education encompasses, but is not a synonym for, special needs or learning difficulties. In this chapter we review Australasian research findings concerning the various categories of diversity such as gender, learning difficulties, giftedness, location, and cultural and linguistic diversity as they relate to mathematics education. These are considered thematically in terms of access to the curriculum through policy and leadership, as well as approaches to learning and teaching mathematics. The categories and themes are not exhaustive but reflect the extant literature, particularly in the Australasian context. We have used this structure in the development of our conceptual framework described below and represented in Fig. 7.1.

A major aspect of research in inclusive education pertains to the overlap between research disciplines, particularly special education. The relationship between mathematics education literature and special and inclusive education literature is of relevance as is the extent of overlap of definitions of construct (e.g., Direct Instruction). This overlap of fields brings richness in the variety of methodologies but also challenges in the development of a shared corpus of knowledge. As part of this review, we consider methodological issues related to conducting research in the area of inclusive education and offer suggestions for greater synergies between fields. Our analysis of the literature explores commonalities in research approaches and issues across the categories of diversity.

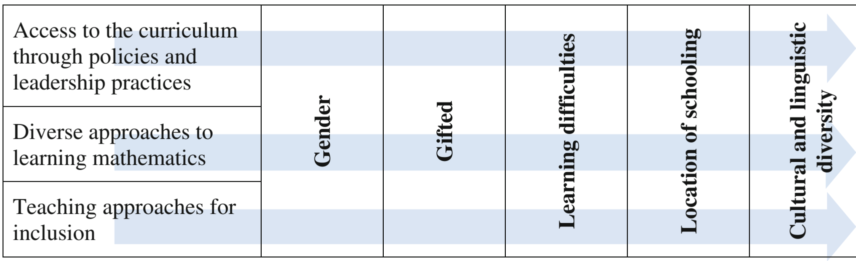

Fig. 7.1 Conceptual framework

2 Conceptual Framework

The conceptual framework that supports this chapter is shown in Fig. 7.1.

Equity has been a central focus of policy and curriculum documents in Australasia for many years, for example, The Melbourne Declaration on Educational Goals for Young Australians (Ministerial Council for Education, Employment, Training and Youth Affairs [MCEETYA], 2008); Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2009); The New Zealand Curriculum (Ministry of Education, 2007); Early Childhood Education curriculum framework, Te Whāriki (Ministry of Education, 1996). Research, however, has tended to focus on specific aspects of diversity and inclusion. This chapter provides a synthesis of relevant research across the range of categories of diversity and interprets them in the context of inclusive mathematics education for improved learning outcomes for all students. We aim to contribute to the maturing of the field by providing a more holistic framework through which diversity can be examined, with the ultimate goal of improving inclusive mathematics education practice.

In structuring our review, we have taken the framework themes (see Fig. 7.1) for our chapter. This is a deliberate approach to underline the philosophy behind inclusive practice. Every learner is diverse in some way and thinking in categories of learners merely serves to segregate. It is recognised, however, that there are common factors in groups of learners that have an impact on their learning of mathematics and this is reflected in the research literature. Therefore, we review these areas as they arise under the overarching themes, exploring the literature with a view to curriculum and learning practices in the primary and secondary schooling years. Our work here overlaps material reviewed in other chapters (see e.g., Chap. 6, this volume, that takes a socio-cultural perspective to inclusivity and diversity, and Chap. 8, this volume, on Indigenous learners). The different emphases in analyses make all three chapters on diversity and inclusivity distinct. We now turn to review literature under the three themes of our framework.

3 Access to the Curriculum

Access to a rich mathematics curriculum is at the heart of inclusive educational practice. Education is a right for all; however, disparity continues to exist in the nature of mathematics education provided. For some students, particularly in the secondary years, access to the general mathematics curriculum continues to be an aspiration (Faragher, 2014). Along with most other countries in the Asia Pacific region, Australia and New Zealand are signatories to the United Nations Declaration on Human Rights and the Convention on the Rights of Persons with

Disabilities. These two international agreements are binding, and national legislation has followed in both countries to enshrine the right to inclusive education in law. From these national laws, policies have been implemented to ensure compliance in various educational jurisdictions. The implementation of policies is the responsibility of those in educational leadership positions and it is at this point where considerable variance can occur. In this section, research that considers access to the mathematics curriculum is reviewed from the perspective of leadership approaches, policy and practice.

3.1 The Impact of School Leadership

School leadership teams have a significant impact on the mathematics attainment of students (Gaffney & Faragher, 2010). They also have an impact on inclusion of students with disabilities (Bawa Kuyini-A & Paterson, 2013). The leadership dimension in mathematics education has received some attention from researchers in recent years, and the impact on inclusive mathematics education practice has begun to emerge. However, our search of relevant literature returned very little published in the years of this review. It would seem that the 2010 special issue of the Mathematics Education Research Group of Australasia (MERGA)'s *Mathematics Teacher Education and Development* (Volume 12, Issue 2) was a major publication in the area and little has emerged since. Here we review the few exceptions and encourage future research in this field. An exception is a chapter by Gaffney, Bezzina, and Branson (2014). Reporting findings from a larger study (Gaffney & Faragher, 2014), the chapter identified key aspects of the practices of principals and other school leaders that have an impact on student achievement in learning mathematics. Similar to Bawa Kuyini-A and Paterson (2013), who identified links between school principals' expectations of teachers to implement inclusive education practices and the practices of teachers in affecting learning outcomes for students, Gaffney, Bezzina, and Branson (2014) emphasised the importance of alignment between vision, organisational structures, teaching approaches, and community engagement on student learning outcomes. Policies and approaches around community engagement, particularly engagement with parents, have been reviewed in a study by Clinton and Hattie (2013). Their analysis would suggest that parental involvement in schooling is important but the type of involvement matters. This would resonate with research by Averill (2012) and Polidano, Hanel, and Buddelmeyer (2013) that indicated the importance of high expectations of student achievement by parents, teachers and the students themselves.

The value for student learning, of a vision shared by school leadership and teaching teams, was noted in a case study report by Jorgensen (2015). Research by Mills et al. (2014) exposed problems that occur when common understandings of differentiated learning are not shared by school leaders and the teachers who are implementing the policies. Without a shared vision and clear conceptualisation of differentiation, a variety of approaches resulted with varying success.

3.2 Allocation to Classes

A very common approach for managing diversity of learners is the practice of allocating students to classes according to individuals' school achievement. Studies from the 1960s began to report the detrimental effects of such practices for the majority of learners, and particularly those in the bottom streams. Macqueen (2013) has studied a variation on this practice that is seen as more palatable to a community becoming aware of the negative impacts of streaming. She investigated the use of regrouping in primary schools, defined as the practice of allocating students to "separate achievement-based classes for individual subject areas such as English or mathematics" (p. 296). The mixed methods study involved schools in New South Wales and examined the practices of eight schools—four that used regrouping practices matched with four like schools that did not. All schools in the study were considered disadvantaged. Macqueen's research indicated that the practice of regrouping showed the same equity issues as streaming. An overview of these issues can be found in the National Numeracy Review (Council of Australian Governments Human Capital Working Group, 2008). Therefore, regrouping should not be regarded as a way to respond to diverse learners of mathematics.

Macqueen made an interesting methodological point in the analysis of the quantitative data arising from the Quality of School Life survey used in her study. Numbers in the low-achieving classes were smaller than the high-achieving groups due to school policies. Small numbers of students providing data from the low-achieving class made obtaining statistically significant differences more challenging. The use of a mixed methods approach in this study enabled the use of qualitative data to investigate trends in the quantitative data.

Another approach to accessing Australasian education is the provision of single sex schools, most frequently in non-government schools. A study by Forgasz and Hill (2013) analysed data from examination results published in metropolitan newspapers that highlighted boys' achievement in mathematics. They reported, "students in single-sex schools, particularly boys schools, were over-represented amongst the highest achievers in all three VCE [Victorian Certificate of Education] mathematics subjects" (2013, p. 493). However, they also discussed their challenge in untangling the impact of socio-economic advantage on the results. In discussion of their findings, the authors posed questions about how the public may interpret these results. Without the awareness of other factors at play, they suggested that the general reader may gain support for gendered views of mathematics achievement. They may also make assumptions about school quality without considering the impact of the socio-economic status (SES) of the school.

3.3 Socio-Economic Status

Perry and McConney (2013), explored the effect of school socio-economic status (SES) on student reading and mathematics learning outcomes. It is well-established that students' and schools' SES are strong predictors of educational attainment, although the strength of the relationship varies between countries (OECD, 2013). Perry and McConney compared results from Australia and Canada as these two countries were considered to have similar educational challenges such as remote populations. However, it was noted that Canada appears to ameliorate some of the effects of SES without compromising quality. The authors argued that the difference is attributable to policies that have led to a system in Australia that is much more marked by "school choice, privatisation and social segregation" (Perry & McConney, 2013, p. 138) than Canada. Attending a high SES school in Australia has a much more marked positive effect on achievement. This finding links to Forgasz and Hill's (2013) study, who pointed out these effects can flow into other areas and perhaps confuse other issues, such as inviting the public to the view that there is a gender difference in mathematical attainment. This serves to underline the risk of considering categories of diversity in isolation.

Conflicting findings on the impact of SES on school achievement were generated from an analysis undertaken by Polidano and colleagues (Polidano et al., 2013). Their study used statistical techniques from the field of economics to analyse data from two related sources: the 2003 OECD Programme for International Student Assessment (PISA) Australian cohort and its linked sample from LSAY, the Longitudinal Survey of Australian Youth (Department of Education Employment and Workplace Relations (DEEWR), 2011). LSAY is one of only a few datasets that links to PISA and the only one in Australasia. LSAY tracked 10,000 students who were 15 years old when they sat for PISA in 2003 until they reached the age of 24 years. The datasets provide information on school completion and school and individual characteristics through surveys completed by principals and students. The Polidano et al. study investigated factors contributing to the differences in school completion rates with respect to school SES. Their results suggest that school characteristics for students after age 15 are relatively unimportant. What mattered in predicting school completion were educational aspirations of students and their parents as well as achievement levels of students at 15.

The conflicting findings between Perry and McConney's study and the work of Polidano et al. could have a number of possible explanations. It could be that in the study of 15 year olds, the damage had already been done and school characteristics were no longer of importance. Polidano et al.'s study recognised the lower achievement of students in low SES schools and this gap would match the work of Perry and McConney. Economic modelling is an uncommon methodology in mathematics education research and differences in methodology may account for some of the differences in findings. However, it is fair to say the studies have not addressed identical research questions. Paired studies that did so, using different methodological approaches, would provide useful comparisons between techniques.

Another finding of the work of Polidano et al. (2013), is the effect of teachers. Those contributing to a positive school culture were found to have a greater estimated effect on retaining students from low-SES backgrounds than those from higher SES backgrounds. Walshaw and Brown (2012) gave a description of the practices of one mathematics teacher who made consideration of affect an explicit part of his practice. The study used the work of the seventeenth century Dutch philosopher, Spinoza, to explore the connection between affect and thinking. For the purposes of this review, we are interested in the example that analysed teaching practices with a Year 9 low-attaining mathematics class in a low SES school. They noted the impact of policies: "Prevailing mathematics education policies and discourses at his school invoke a commitment to a wider understanding of diversity than was previously expressed through stereotypical images based on group affiliation" (p. 189) and "at the school, equitable teaching practices had become a crucially important driver to embrace diversity and to redress social injustices" (p. 189). One of the challenges the teacher faced was the erratic attendance of the students, some of whom missed significant amounts of schooling. In dealing with this issue, the teacher made explicit decisions to enable students to keep with the classroom collective, opting for "the use of rules, repetition of tasks and small procedural steps" (p. 192). Although this practice is commonly documented in low-attaining classes and even advocated in special education literature (Westwood, 2000), the authors noted that this practice encourages "ways of thinking and being in the classroom setting that may be perpetuating the marginalisation of … an already disadvantaged class" (p. 193). This idea is mentioned briefly in the paper and not pursued due to the focus on other aspects of study. It would be illuminating for further research to explore this thesis in greater depth.

3.4 Location of Schooling

We have been considering policies and practices that affect access to the curriculum. In some cases, access is dependent on opportunity to be taught. In regional, rural and remote areas, this could depend on suitably qualified teachers being available and sufficient student numbers to offer mathematics, and particularly the more specialised topics. Handal and colleagues (Handal, Watson, Petocz, & Maher, 2013) investigated factors influencing teachers to remain in non-metropolitan areas. Using a questionnaire, the authors collected quantitative and qualitative data from 191 secondary teachers in 27 rural or remote New South Wales schools. The regions were undergoing population decline and respondents described the impact of this on schools. With falling enrolments, staffing numbers fall, requiring teachers to teach across key learning areas (KLA) and often outside their area of training. One respondent noted, "… specialist areas are a luxury".

Small secondary schools also employ few staff in each teaching area and this leads to lack of mentoring of beginning teachers from more experienced subject specialists. "Despite being beginning teachers, most have to function as if they were curricular experts" (p. 23). The authors identified policy practices that were influencing the retention of the teachers in the study and suggested that lack of professional development, curricular mentoring and curricular support were problems that could be addressed using technologies to overcome distance and connect into professional communities beyond their local area. Compensation for living and working in remote areas as well as mechanisms to allow transfer to other schools were considered important policy matters to address retention of teachers in regional, rural and remote schools. Accommodating the effects on schools of declining rural populations may be a more difficult policy to develop.

Handal et al. (2013) found no significant difference between the responses of teachers of mathematics and science and those of teachers of other KLA, noting teaching in a small school necessitated teaching across a number of areas. Hobbs (2013) explicitly studied teachers in rural or regional secondary schools who were teaching "out-of-field", that is, they were teaching subjects for which they had not been trained. She identified a concerning practice of out-of-field teachers who were "content to perpetuate dominant subject pedagogies regardless of their effectiveness" (p. 293). Noting that this was not the case for all out-of-field teachers, she identified an area of needed research—the impact on the engagement and achievement of students of being assigned an out-of-field teacher. It is likely that policies and practices relating to the allocation of teachers may have a significant impact on the achievements of students. Again, it is essential that multiple factors are analysed in this required research. If there are fewer qualified teachers to be assigned, the choice of which school and which class matters. In studies on streaming, Zevenbergen (2005) noted low stream classes most often were assigned the least qualified staff. Hobbs' model suggests that some school policies may make out-of-field teachers more effective, that is, if policies encourage "communities of practice where teachers are supported and enabled to expand their professional identity" (Hobbs, 2013, p. 293).

In remote Australian communities, the student population is often largely or totally Indigenous. By contrast, with the exception of Indigenous Education Officers (IEO), the teaching staff rarely are Indigenous. Warren and Quine (2013) discussed an aspect of their research in remote schools in Queensland, describing changes in classroom structures that led to improved learning outcomes for students in mathematics. In their qualitative study using grounded theory, the IEOs were considered equal partners in the learning process and due to their stability in the community, they were deemed crucial to the success of the initiative. The paper examined changed leadership structures that explicitly involved Indigenous community leadership in partnership with non-Indigenous leadership, in seeking to address challenges arising from a context with high teaching staff turnover in remote locations. This is an example of a policy approach that has an impact on the learning outcomes of students in diverse settings.

This section of our review has considered the Australasian research studies that deal with the opportunity for diverse groups of students to access mathematics. Policies and their implementation have a substantial effect on the opportunity for students to learn mathematics. In the following section, we will discuss research relating to the learning of mathematics itself.

4 Diverse Approaches to Learning Mathematics

A key aspect of inclusive mathematics education is acknowledgement of the diversity inherent in all learners. It is recognised in the research literature that some individual learners cluster into categories that can have an impact on their achievement in mathematics. In this section, we review literature about groups of students, all from the perspective of the learner. Five general themes have emerged and in this section, and we discuss these themes in turn.

4.1 Context of the Learner

Mathematics learning theory that explored situated cognition (Lave, 1988) encouraged the teaching of mathematics in contexts that were relevant to the learner. Current research literature indicates this practice has continuing merit for inclusive classrooms.

A study undertaken by Grootenboer and Sullivan (2013) investigated the prior mathematical knowledge of 56 primary students in north-western Australia. Data were collected through a task-based, one-on-one interview that focused on mathematical concepts related to measurement, with tasks designed to connect to students' experiential world through the use of contexts and themes from their local community and their hobbies, interests and activities. With reference to inclusive practices, the researchers concluded that the students' capacity to engage with tasks and questions was influenced significantly by the context of the problem, with many students unable to answer questions with irrelevant or unknown contexts. In addition, questions that were related to familiar contexts were more likely to be answered correctly, suggesting students were able to demonstrate mathematical conceptual knowledge when they were able to personally connect to the task. Inclusive practices in mathematics need to take into account factors such as geographical location and experiences.

Challenges of context extend to understanding the language of instruction. Verzosa and Mulligan (2013) reported on an intervention phase of a study aimed to assist second grade Filipino children in solving addition word problems in English, a language they primarily encounter only in school. The researchers commented that "the fact that children who cannot understand simple statements such as 'Alvin had 3 coins' are obligated to learn mathematics in English says much about how their school experience must be too far removed from their daily lives" (p. 238). It was found that minor interventions such as providing definitions for English words commonly found in word problems were not effective as children struggled to remember what these words meant. Findings showed that children's difficulties were not confined to the lack of English language proficiency but were also related to students not possessing the mathematical knowledge necessary to handle more complex mathematical structures. Difficulties with developing mathematical understanding due to the language of instruction may serve to exacerbate feelings of exclusion in students, which may then lead to further disadvantage.

4.2 Playing to Their Strengths

Mathematics teachers have always had concern for those who struggle to learn mathematics. Likewise, researchers in the field have sought understanding of how to enhance the mathematics attainment of all learners, including those in defined groups. Research undertaken in the review years in Australasia continues to support the finding that while some approaches may make learning mathematics easier for some specific groups, these approaches are of benefit to students in general. We have not identified any approaches that are needed for some students that would not benefit others. In essence, there are no special approaches that require some groups of students to be taught mathematics away from their mainstream peers.

An indicative example was provided by Clarke and Faragher (2014) who reported data related to early number development from a larger study. It was concluded that children in this study developed alternate ways of counting and that the development of number understanding was enhanced through the use of symbols. The authors explained that there appeared to be some evidence to suggest that children with Down syndrome were more comfortable with the numerical symbol than the verbal count word. The authors argued that this can be linked to the relative weakness in verbal processing of children with Down syndrome. They raised the issue that the focus on skill development emphasising the count word first that is used with typically developing children may not be as productive as the use of models that emphasise the use of numerals. Teachers in a subsequent study (Clarke & Faragher, 2015) indicated that they found use of resources such as a number paddle and tens frames to be helpful in inclusive primary classroom settings. This study investigated the practices of effective primary school teachers in Victoria and the ACT who were including a student with Down syndrome in their regular classroom mathematics lessons. A key point here is that even though learners with Down syndrome require an emphasis on numerical symbols to enhance early number development, other learners in the classroom can make use of these connections as well. It is not necessary to teach some students separately from others.

4.3 Issues of Affect

The concept of *affect*, particularly with respect to the effect on learners and the impact of gender, also received research attention in the review years. As noted earlier, Walshaw and Brown (2012) extended the theorising of affect. They cited McLeod (1992, p. 576) to define *affect* as “a wide range of beliefs, feelings and moods that are generally regarded as going beyond the domain of cognition” and in their own synthesis of the literature, note the importance of affect for learning, stating, “affect influences thinking, just as thinking influences affect. The two interact” (Walshaw & Brown, 2012, p. 186).

The relationship between positive affect towards science and mathematics and achievement in these disciplines was explored by Ng, Lay, Areepattamannil, Treagust, and Chandrasegaran (2012) in a study of Malaysian and Singaporean Grade 8 students. It was found that positive affect towards science and mathematics indicated statistically significant predictive effects on achievement. There were also predictive effects on mathematics achievement for the students’ gender, language spoken at home and parental education. The researchers concluded that educators should consider implementing self-concept enhancement intervention programs and suggest that this may also serve to increase inclusion for students by counteracting the effect of aspects such as home and everyday influences.

In a study by Ng (2012), the origins and impact of mathematics anxiety on 294 Singaporean secondary students were examined. As early as Primary 4 (Year 4), students in Singapore are “ability-grouped” (p. 570) by four subjects, including mathematics. In Primary 6, mathematics contributes to their Primary School Leaving Examination (PSLE) score. Results on this exam are used to assign students to secondary courses. Research findings revealed a negative correlation between anxiety level and achievement. Of the top five situations that worried students, four were test-related. Even so, highly anxious students were reported to persevere and enjoy the subject.

An international, longitudinal study (Watt et al., 2012) explored gender differences in, and gendered relationships among, mathematics-related motivations towards high school mathematics participation, educational aspirations, and career plans. Participants were from Australia, Canada, and the United States in Grades 9/10 at Time 1 and Grades 11/12 at Time 2 and came from suburban middle to upper-middle socioeconomic backgrounds, primarily of Anglo-European descent. Stereotypic gender differences in educational and occupational outcomes were found only among the Australian sample. Male adolescents held higher intrinsic value for mathematics in the Australian sample. Ability/success expectancy was a key predictor in the North American samples, in contrast to intrinsic value in the Australian sample. Attainment/utility (“importance”) values were more important for female adolescents’ career choices, except in the Australian sample. The importance of gender socialisation practices and its relationship to engagement and inclusion are emphasised, with reference to differences in perceptions of mathematics-related motivations between students from Australia, Canada, and the United States.

4.4 Learning with Technology

The role of technology in inclusive mathematics practices has been the focus of a number of recent studies. Casey (2013) used action research to study curriculum design in the context of social media in secondary mathematics. As an approach to learning mathematics, Casey designed online projects around students' real-life experiences and day-to-day knowledge to help students link mathematics to their activities, inside and outside the school. It was found that students benefited from this approach in many ways, with students creating multimedia resources to help those in other classes understand particular concepts. Incorporating students' out-of-school activities assisted them to come into the mathematics classroom with a relaxed tone; their interests were more visible and using visual clues strengthened their understanding and meaning making, which also supported their literacy practices. Providing students with the means to utilise their own interests in an inclusive and supportive environment encouraged them to share their knowledge with other students.

In a similar vein, Australian researcher Daniel Shank and US colleague Sheila Cotten (2014) investigated how the use and ownership of different aspects of technology could empower urban youth through increasing their self-efficacy. It was found that compared with owning one's own computer, both not owning a computer and sharing a computer were positively related to self-efficacy in the domain of science and mathematics. Shank and Cotton speculated that not owning a computer may have driven students to collaborate more and potentially use their laptops more often. The work of Shank and Cotten (2014) and Casey (2013) suggest that collaboration and inclusion are linked to students' efficacy.

4.5 Assessing Learners

It hardly seems a revelation to note that valid assessment instruments and interpretation of results is critical to making an accurate judgement of what learners know and can do. Even so, development of assessment techniques is a continuing area of research in the field of inclusive mathematics education. These techniques are required for researchers to determine the effect of interventions as well as for classroom teachers to use in their work. The development and use of modified assessment instruments for teachers' use with learners undertaking modified programs within an inclusive classroom would appear to be an emerging area of expertise, and research into this aspect of teachers' work would be welcome.

Reviewed research made use of a variety of assessment approaches, many developed specifically for the particular studies being undertaken. Task-based assessment interviews between individual learners and their teacher or the researcher continued to be an important methodology. Faragher and Clarke (2014) discussed the use of this technique in research with learners who respond atypically, as was the case for their population of students with Down syndrome. Interviewers required expertise in mathematics pedagogy as well as understanding of learner behaviours. Task based interviews were also used by Grootenboer and Sullivan (2013) who developed their own instrument based on the lived contexts of the Indigenous students in the study. By testing students on their knowledge of mathematics in familiar and unfamiliar contexts, they noted: “The findings of this study suggest that, at least in part, the under-achievement of these students in these formal tests may be due to the relevance and veracity of the assessment instrument” (Grootenboer & Sullivan, 2013, p. 181). As a result, the researchers assert that “there are real concerns about national testing regimes that discriminate against some students, and the use of these flawed results to make claims about the students’ mathematical (or other subjects) knowledge and understandings” (p. 184).

In another study of Indigenous students and their mathematics achievement, Yeung, Craven, and Ali (2013) asked a sample of Indigenous and non-Indigenous students (n = 1342) from schools in New South Wales to respond to a survey measuring five domains of self-concept (i.e., school, reading, mathematics, art, and physical abilities), two learning-related factors (enjoyment and participation), and a self-assessment of their school work. Student scores in a NSW state-wide assessment of students’ literacy and numeracy were also obtained. The researchers found that Indigenous students scored lower in both reading and mathematics than their non-Indigenous peers and concluded that Indigenous students were clearly disadvantaged in terms of academic achievement, irrespective of region (urban or rural). They concluded that “the consistent pattern of Indigenous students displaying lower scores for both achievements and self-concepts leads us to conclude that Indigenous students were disadvantaged in both” (Yeung et al., 2013, p. 420) and that Indigenous students did not seem to have a good estimate of their abilities in reading and mathematics. With reference to the research of Grootenboer and Sullivan (2013), the possibility exists that results may be related to the “non-inclusive” aspects of the test items used in the state-wide assessments of the students. Indigenous students’ lower scores for achievement and self-concept may be a side-effect of the nature of the tests given to them, and not necessarily due to a lack of mathematical conceptual knowledge. In order to promote Indigenous students’ academic self-concept and academic achievement, methods of assessment may need to change.

The research of Verzosa and Mulligan (2013) can be seen as following the same vein in regards to the effect of question context and assessment and its relationship with inclusive practices in mathematics. It was conjectured by Verzosa and Mulligan (2013) that using Filipino to convey mathematical concepts would not prevent students from accessing the same concepts in English once they had acquired proficiency in the language, but if children had poor understanding of number concepts and part-whole relations, then even substantial linguistic support in the form of narration would fail to help them construct appropriate situation models. They found that there were very few instances when a problem in English was solved and children’s unfamiliarity with the language continued to impede problem solution.

Assessment of diverse learners remains a challenge for policy and practice. The Mills et al. study (2014) indicated that some teachers in their study were uncomfortable with modifying secondary assessment. In some subject areas, teachers were prevented from doing so by policy determination. In mathematics, they note, “in the non-senior years teachers suggested that the assessment tasks catered to different levels. However, this appeared to relate to such practices as extension tests for the high achieving mathematics students" (p. 342). The researchers argued that assessment as well as learning can be modified in a way that is challenging and meaningful, allowing all students to demonstrate their learning, however expectations around quality should not be different.

In this section of our chapter, we have reviewed studies from the perspective of the learner. In the following section, we move to the final part of our theoretical framework and consider Australasian research from the perspective of teaching.

5 Teaching Approaches for Inclusive Practice

Learners of mathematics are unique individuals. Inclusive mathematics education practice would be simply impossible to achieve if teachers had to plan for and teach each student separately. There is a growing corpus of research and practice that underpins inclusive mathematics education. In this section of our review, we examine this literature under three sub-themes: Location of schooling; Values, expectations and beliefs of teachers; and Direct Instruction. A body of work on mathematics education exists in the special education literature and the third theme arises from this work. However, it is rare that the two fields of special and general mathematics education coincide and there is little overlap between authors writing in both areas. This has implications for methodological practice and subsequent findings of research. Advice given to teachers wishing to develop their inclusive practice can be confusing when it arises from different theoretical backgrounds. We review research in mathematics education and special education research fields in an attempt to find common ground and identify areas where further work is required.

5.1 Issues of Context/Location of Schooling

Where teachers do their work has an impact on their practice. In particular, location of schooling arises in the literature as an important variable in inclusive practice and we have already considered this in the previous two themes. The impact on teaching is now discussed.

A challenge for education in areas of sparse population is the provision of a range of educational opportunities for learners. One solution adopted in the western region of New South Wales is a program for gifted and talented students, called the XSEL program. Furney, McDiarmid, and Bannister (2014) have provided a descriptive account of the program with some initial data on student learning outcomes. This program makes use of sophisticated technology to offer learning opportunities to students enrolled in their local school but attending classes with selected students across the region. The online lessons are supplemented by residential schools. Further, more rigorous research is needed into the effectiveness of programs of this type. Research questions abound, for example, in teaching approaches. The teacher's role in this program is fundamentally different from traditional classroom teaching as policies such as this point would indicate: "*xsel* teachers do not teach 'face to face' any *xsel* students in their own school. All *xsel* teachers teach only *xsel* students at other schools" (Furney et al., 2014, p. 43). The impact on schools where some students are not included in the local classes and the effect on learners of not being selected for these programs would also be rich areas of research. On the face of it, it would seem programs like *xsel* offer segregated education to a select group of students and would be contrary to inclusive education practices.

Lowrie and Jorgensen (2012, 2014) have studied a different aspect of rural education—distance education (DE) in the home. Several aspects of the research reported in these two papers are of relevance to this review, including: the changing population of students accessing DE to include students with disabilities, those disenfranchised by traditional schools, those with challenging behaviours, as well as those in rural areas; the use of parents (mostly mothers) as teaching assistants; and the changed role of pedagogical practice with the adoption of new learning technologies. The researchers used an ethnographic study conducted at a school site and a connected home site. Semi-structured interviews in conjunction with formal observations (lesson studies) were undertaken. The data analysis revealed insights into teaching approaches and changes that had occurred as new technologies were implemented. Constraints of technology, such as unreliable connectivity, and resources (provision of electronic materials supplemented by print materials) led to individualised teaching, often as one-on-one phone conversations. The authors note,

> some of the common social, environmental, and cognitive dimensions of classroom engagement cannot be replicated. … everyday social perspectives so influential in learning … are restricted by the influence of the dominant medium of communication – that is, a blended or digital resource base. Consequently, teachers may feel somewhat disconnected to the students they teach. (Lowrie & Jorgensen, 2012, p. 2)

A powerful aspect of Lowrie and Jorgensen's research is that it was undertaken at two time points separated by 8 years. This longitudinal focus allowed the initial anticipated benefits of technological innovations to be contrasted with the actuality.

Understanding the variables that affect teaching approaches for learners in different contexts and locations is critical for improving mathematics education across Australasia. Much is made of the promise of learning technologies to improve learning outcomes and yet Lowrie and Jorgensen's work would suggest that this promise is not necessarily achieved in practice. Research is needed to understand and remove impediments to achieving the hoped for benefits for learners and their teachers.

5.2 Values, Expectations and Beliefs of Teachers

> Why do we usually say that we do not know the needs of people with disabilities while we do not know anybody's needs, actually? As a matter of fact, there is some research showing that both groups can have the same difficulties in learning mathematical content. (Marcone & Atweh, 2015, p. 773)

The impact of values, expectations and beliefs of teachers on student learning outcomes has been recognised for some time, particularly when education is viewed through socio-cultural theoretical frameworks. Similarly, the teaching profession has a specific focus on student engagement as a necessary component of learning. In working with students from diverse populations, research indicates these teacher attributes are important variables and here we consider research arising in the review years.

Bishop and Kalegeropoulos (2015) reported on a small-scale study of student engagement in the mathematics classroom. In their chapter they refer to (Dis) engagement signifying that they are talking about "engagement, disengagement and re-engagement together" (p. 194). This is in acknowledgement that engagement is not a static state but one that is influenced by a range of factors including the teacher and the classroom context. They argued that consideration of teacher and student values help us understand this dynamic process. The choice to engage is the student's but the pedagogical practices and teacher expectations influence that engagement. They argue against practices of labelling and other excluding pedagogies and argue for inclusive pedagogies.

Seah and Andersson (2015) advocate for a process of values alignment to support effective inclusion in culturally diverse classrooms. They argue theoretically with support of two secondary school cases. They claim that it is important for teachers to be able to negotiate values difference and values conflict situations that arise. From a perspective of managing cultural diversity they suggest that the acknowledgement of difference in values can be important.

> Thus, teacher capacity to actualise values alignment between herself/himself and her/his students go [sic] a long way towards acknowledging students' cultures, knowledge, skills and dispositions, thereby contributing to diversity in mathematics learning and teaching in ways which are inclusive and empowering. (p. 180)

Owens (2015), reporting on case study data from one of the schools involved in the *Make it Count* project, found considerable change in teachers' practices through including Indigenous community and cultural considerations. The school was in a large regional setting with around 10 % Indigenous students. The teachers initiated small step changes "but only with consultation and mentoring and significant two-way sharing of cultural and intellectual knowledge by the Aboriginal community locally and nationally" (p. 75). She argued that an ecocultural critical pedagogy was developing and was characterised by initiatives including, "establishing a garden that can be used easily for mathematics lessons, recognising the value of land links, outside lessons, non-verbal teaching, and stories" (p. 76).

A rural setting was the focus of another research study (Hunting, Mousley, & Perry, 2012), this time investigating rural preschool practitioners' knowledge and practices concerning children's mathematical development prior to entering their settings. To undertake the research, 64 practitioners in rural areas in three Australian states were surveyed and interviewed. Site visits were also made. While respondents were noted to have good knowledge of the mathematical content displayed by young children, they were not as aware of mathematical processes. The researchers recommend that professional learning programs put greater emphasis on processes, making clear the link between "understanding basic concepts and words and the development of ways of thinking and other mathematical processes" (Hunting et al., 2012, p. 46). Recognising mathematical thinking displayed by children and knowing how to develop this further is an important aspect of expectations of teachers about their students. If they expect to see mathematical processes being used by young children, they are more likely to look for them and encourage this activity through planned learning experiences.

Averill (2012), in a mixed methods study in the context of Year 10 multiethnic classrooms in New Zealand, took a perspective of culturally responsive teaching with a focus on teacher care. In reporting on 100 observed lessons from three teachers, patterns within the data indicated that, "the lessons exhibiting the most caring teacher behaviours and practices were those with greatest student engagement (i.e., highest levels of on-task student behaviour) and the most student-initiated interactions (related and unrelated to mathematics)" (p. 121). The teachers articulated the challenges and their own personal limitations as they strove to teach to the needs of culturally diverse classrooms.

The literature reviewed here provides some evidence of the value of inclusive and alternate pedagogies. It also highlights the importance of teachers' expectations and values and the need to be mindful of those of their students. One of the difficulties with using this literature to inform teaching is that it often comes from a variety of theoretical perspectives beginning with the particular area of student need or disadvantage. There can then be a tendency to argue for inclusive practices that include those who are being studied but does it genuinely expand the opportunities for all? Sullivan (2015a) argues that while it is much more difficult to redress student differences than to identify them, "it is also possible that steps that education providers take to redress differences can sometimes exacerbate the exclusion of some students" (p. 3). He acknowledged the complexities of providing advice to teachers based on research but argues that, "age appropriate experiences are more likely to enhance the inclusion of marginalised students than merely activities that are matched to the levels achieved by the student on systemic or standardised assessments" (p. 12).

Faragher (2014), in reporting on two case examples involving students with Down syndrome in inclusive settings presents a similar argument. While these cases are anecdotal, they provide existence proofs of the possibilities for surprising mathematical development with students who have generally been excluded as they have not been considered capable of engaging in secondary mathematics. She argued for greater inclusion of students with mathematical learning difficulties rather than a limiting of their mathematical experiences.

5.3 Direct Instruction

In recent times, renewed discussion at the policy level has involved the teaching approach, Direct Instruction (DI). This is claimed to be of particular benefit for low attaining students such as those with learning disabilities and in more recent applications, Indigenous Australian students. Initially developed in the United States in the 1960s, it has been revised and refined over the years. The US Department of Education's *What Works Clearinghouse* defines the approach as, "a teaching technique based on extensive task analysis. Instruction is fast-paced, teacher-directed, prescribed, and explicit with all children receiving instruction on a pre-specified sequence of activities at the same time" (Institute of Education Sciences, 2007, p. 3).

A significant body of research into the effectiveness of DI has occurred over the decades with conflicting results. The What Works Clearinghouse considers "the extent of evidence for Direct Instruction to be small for oral language, small for print knowledge, small for cognition, and small for math" (p. 1). The challenge here is at the heart of the disparity between advocated approaches in special education and general mathematics education. Ewing (2011) reviewed evidence for and against DI and noted the tension between traditional and behaviourist approaches to mathematics teaching and learning and noted criticisms were at the level of theory (assumptions about human nature and society) and practice (classroom practice is typically different from ideal formulations of DI). Ewing's review was published prior to the time scope of this review, however, we include it here as it provides a valuable source of research findings to underpin the discussion of this approach which is receiving renewed attention from education authorities in Australia.

In 2012, the Australian Council for Educational Research (ACER) was contracted by the Queensland Department of Education, Training and Employment, to evaluate the Cape York Aboriginal Australian Academy (CYAAA) Initiative, which features the use of DI for teaching mathematics. In the final report (ACER, 2013), the authors note that the initiative had only been underway for a few years and limited data were available to assess the impact of the initiative. They were unable to determine if the initiative had made a positive impact on student learning outcomes, though they noted teacher comments that achievement had increased more for literacy than numeracy.

With increasing interest by Australian governments in the implementation of Direct Instruction (Attwood, 2015), it is clear that mathematics education researchers need to be involved in rigorous evaluation of the teaching approach. Longitudinal studies are needed to provide sufficient time for the innovation to be implemented. Investigations of actual classroom practices would address the criticism of research findings that have been sourced from clinical settings. Finally, research is needed that tackles differences in the theoretical stance of researchers from special and general mathematics education communities, where reconciliation between behaviourist approaches and socio-cultural perspectives of learning is made.

This section has reviewed literature from the perspective of teaching. Across the three themes, common factors emerged such as location of schooling. In conclusion to this chapter, we turn now to implications for mathematics education research.

6 Conclusion

Inclusive practices in teaching mathematics has been a field of great importance in Australasia in recent times. The implications of the enacting of this practice has been the subject of on-going research, but much more is needed. In our chapter, we have reviewed the literature in a way that emphasises an inclusive approach. We have considered three key themes: policies and approaches that affect access to a mathematics education for all learners; diverse approaches to learning mathematics; and mathematics teaching approaches in inclusive contexts. In essence, we have looked at policies, learning, and teaching.

6.1 Areas of Needed Research

Our literature searches identified considerable research activity in the area of inclusive mathematics education research between 2012 and 2015. Even so, much, much more research is needed. Throughout this chapter, we have indicated gaps in the literature. Here we propose a composite list of areas where we advocate research:

- The impact on students of being taught by out-of-field teachers
- How teachers modify assessment instruments for use in inclusive classrooms
- The impact of negative stereotyping on school-aged students
- Influence of information and communication technologies on learning mathematics
- Roles of teacher assistants and parents in supporting the mathematics learning of students in specific contexts such as intellectual disability or distance education
- Carefully designed objective research trials of teaching innovations, such as Direct Instruction, especially with a longitudinal focus.

6.2 Recommendations for the Research Community

Continuing high standard qualitative research is critical in Australasia to ensure that findings from research studies into inclusive mathematics education practice are recognised and valued. Small populations and therefore, sample sizes, are likely to minimise opportunities for robust quantitative studies. However, qualitative methodologies are well-established, as are protocols for gathering and analysing the data. Those used should be clearly documented in research reports. In addition to preparing scholarly publications, researchers can support each other in the development and use of qualitative techniques through master classes organised by research communities such as MERGA, training of research students, and sharing techniques in conference presentations.

While it is important to acknowledge that all classrooms are places of diverse student experience, background and capability, more specific research is needed into the effective practices of teachers in inclusive settings with specific categories of diverse learners. Even though we can learn much from research where a specific category of learner is the starting point, it is also valuable to research the classroom as the starting point. Sullivan (2015b) argues for a model of mathematics teaching designed to address the diversity of student preparedness in mathematics based on work focused on including and engaging all students. It assumes a common mathematical learning focus and requires teachers who are clear on the intent and the sequencing of tasks associated with that learning, have developed a communal classroom environment and are explicit about the pedagogies of mathematics teaching. Such work might also inform practices with specific groups of students.

Longitudinal studies are also to be encouraged. Innovations in educational practice take time to implement and become established before effects on student outcomes are observed and able to be measured (ACER, 2013). Unfortunately, these studies are rare and many innovations are not given sufficient time to become established before a new practice is implemented. Researchers may need to look to pseudo-longitudinal approaches where similar data are collected from different participants but in similar contexts at points separated by a number of years.

Research into enhanced inclusive mathematics education has the prospect of increasing the accomplishment and enjoyment of mathematics by all learners. This is a worthy goal, indeed.

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Chapter 8
Distribution, Recognition and Representation: Mathematics Education and Indigenous Students

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Abstract The research undertaken in the last four years on the learning and teaching of mathematics connected to Indigenous students is evaluated using Fraser's model for social justice, which consists of three elements: distribution (economic), recognition (cultural) and representation (political). Although at least one element, usually distribution, was the focus of the research papers, the occurrence of all three was rare—with representation seldom visible. Yet, evidence suggests that representation is an important element if Indigenous student achievement is to improve. As a consequence, there is a call for a moral change in how mathematics education research is promoted and undertaken with Indigenous students, with a need to include greater Indigenous community representation.

Keywords Capacity building · Community · Culture · Indigenous students · Language of instruction · Mathematics education · Māori students · Nancy Fraser · Parents · Social justice

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1 Introduction

In this chapter, we review research from the last four years that investigated various aspects of the teaching and learning of mathematics connected to Indigenous students. In doing this we use the same definition for Indigenous students as in previous reviews (Meaney, McMurchy-Pilkington, & Trinick, 2008; Meaney, McMurchy-Pilkington, & Trinick, 2012), namely, that Indigenous[1] students are indigenous to the land in which they are learning mathematics, such as those living in Australia, New Zealand, Papua New Guinea and the Pacific. Indigenous students living outside their country of origin, such as Pasifika students in New Zealand, are not part of this review but are included in the two previous chapters. Indigenous students should not be considered an homogenous group, but differ according to a range of factors such as geographical situation, language and involvement with traditional practices. In describing the different research projects in this chapter, we include contextual details for each study. For qualitative research, this information was generally provided in the original article. However, in most quantitative research, important distinctions in factors are often conflated (Leder, 2012). Furthermore, even when contextual details are available, there will be diversity within groups which is often lost even in qualitative studies.

In comparison with earlier reviews, there are fewer contributions from New Zealand, perhaps reflecting government funding drying up. However, there has been a resurgence of contributions from Papua New Guinea, with almost all of them connected to the work of Kay Owens. The paucity of research originating from the Pacific continues, with only three papers by two non-Indigenous researchers. The centrality of one or two researchers in Pacific countries makes this research fragile. Cutbacks in aid programmes by the larger economic powers (i.e., Australia and New Zealand), are likely to have a long-lasting impact on mathematics education in the Pacific.

In this chapter, we use the social justice framework of Nancy Fraser (2005), mentioned in Cazden (2012), because we consider Indigenous student outcomes to be inextricably tied to issues of social justice. Such an evaluation is important because, particularly in Australia, a large amount of resources has been committed to addressing the issue of underachievement of Indigenous students over the last decade, but with limited results. As Thomson, Hillman, and Wernert (2012) stated when comparing Indigenous students' mathematics literacy results in the Programme for International Student Assessment (PISA) tests between 1995 and 2011:

> None of the differences between years is significant, that is, the 2011 score for Indigenous students, as for non-Indigenous students, is not significantly different to the score in any of the other years of testing. The difference between Indigenous and non-Indigenous students is significant, as it has been in each year of testing, and has not decreased in size. (p. 30)

[1]We capitalise Indigenous when referring to people as a mark of respect.

Despite the best of intentions, the mathematical achievement of Indigenous students, at least in Australia and perhaps New Zealand (see Hāwera & Taylor, 2012), has failed to match that of their non-Indigenous peers. Therefore, without an evaluation of what has been done and the outcomes from it, we risk losing focus:

> The ultimate result of the huge effort being put into potentially futile initiatives is that we as educators will lose focus of the main purpose of education, to make it count in the lives of students. Instead, in the end we may just be counting education. (Guenther, 2013, p. 158)

1.1 A Theory of Post-Westphalian Democratic Justice

We consider that the lack of change in education achievement results is an issue of social justice:

> While it is normal and natural that educational outcomes vary between individuals, stable and substantial differences in educational outcomes between *groups* of individuals are a cause for concern. Such differences suggest that social and educational forces, policies, and structures are systematically privileging some groups over others. (Song, Perry, & McConney, 2014, p. 178, emphasis in original)

To better understand how privileging occurs and is identified in research on the teaching and learning of mathematics connected to Indigenous students, we use Fraser's (2005) three-element model for social justice. From her perspective, it extends beyond the borders and notions of egalitarian societies, based on income and wealth, in earlier perspectives on social justice. For example, earlier research—concerned with the social and political aspects of the learning of mathematics—idealised the provision of access to mathematics for everyone independent of skin colour, gender and class (Jorgensen & Perso, 2012). Jorgensen and Perso (2012) argued that such a view of social justice is limited because it does not take into account the diverse backgrounds of students. Fraser (2005) took this one step further in stating that "theories of justice must become three-dimensional, incorporating the political dimension of *representation*, alongside the economic dimension of distribution and the cultural dimension of recognition" (p. 5, emphasis in original). At the global level, representation includes acknowledging national and regional groups' demands for independence, and recognition of treaty and Indigenous rights. She deemed this version of social justice more valuable when issues were present in more than one country, calling it "a theory of post-Westphalian democratic justice" (p. 5).

In this section, we give examples of how we interpret each element. Distribution considers how social goods such as education are distributed to different groups. From analysing the appearance of the achievement gap in PISA results between Indigenous students and non-Indigenous students, Song et al. (2014) concluded that political decisions about how schools were funded contributed to higher level of inequitable resourcing in Australian schools compared with New Zealand schools: "This high level of segregation in Australia is associated with large differences in school resources, especially the ability to recruit and retain qualified and experienced teachers" (p. 194). Although their study was about results in reading, viewing teachers as resources would have a similar impact on mathematical literacy results. In New Zealand, Turner, Rubie-Davis, and Webber (2015) found that teachers had different expectations of students' capacities for learning mathematics based on their ethnicity, regardless of their achievement. Interviews revealed that they had the lowest expectations of Māori students, blaming students' attitudes and their home backgrounds as the main contributors to poor achievement. The teachers' expectations were likely to restrict the distribution of mathematics learning to these students, thus achieving a self-fulfilling prophecy.

In regard to the element of recognition, a social justice approach to mathematics education would positively recognise students' cultural experiences as a useful basis for their mathematics learning (Meaney & Evans, 2013). Grootenboer and Sullivan (2013), in considering remote Indigenous students in the Kimberley, stated:

> Indeed, we are convinced that the current national testing programs in Australia do not provide a fair platform for remote Aboriginal children to display the extent and complexity of their mathematical knowledge and skills, and the validity of their results in these assessments need[s] to be viewed with some scepticism. (p. 187)

Although Grootenboer and Sullivan (2013) suggested that these students could achieve on these tests if the teaching they received built on what they knew, there is a contradiction in expecting only the students to change and not recognising that the assessments also need to change. Like the lack of familiar problem contexts for Indigenous students, raised in previous Mathematics Education Research Group of Australasia (MERGA) reviews (see Meaney et al., 2008, 2012), the aggregating of Indigenous students' results as though they are one homogenous group (Leder, 2012) is an issue of recognition. Such aggregation can "hide, rather than identify, the strengths and needs of the different sub-groups" (Leder, 2012, p. 12), thus leading to all Indigenous students being considered low achievers in mathematics. If students take on these messages of low achievement, then there is a risk that it too contributes to a self-fulfilling prophecy (Leder & Forgasz, 2012; Trinick, 2015).

The final element in Fraser's (2005) model is that of representation. If mathematics education for Indigenous students is to be socially just, then Indigenous communities need to contribute to decisions about what should be taught, what should be researched and by whom—yet this element is rarely present in this set of research papers. For example, in discussing the design of a literacy and numeracy strategy for Indigenous students living in remote communities in the Northern Territory (Perso, 2013), scant attention is paid to the possibilities for Indigenous communities to participate in the decision making of what should occur in classrooms. Although Indigenous Education Workers (IEWs) were recognised as being knowledgeable about the students' cultural contexts, the IEWs were considered to need professional development to impart this knowledge, as well as needing more knowledge about Western schooling. Described in this way, the possibility that IEWs could represent their communities by working with teachers to develop culturally appropriate mathematics education could easily be ignored.

The literature critiqued in this chapter is divided into four sections: pedagogy to enhance learning, language of teaching and learning, mathematical topics, and capacity building. We contend that without all three elements of Fraser's model being present, intervention programmes and their associated research are likely to have only the limited success identified in the longitudinal statistics on Indigenous students' achievement in mathematics.

2 Pedagogy to Enhance Learning

In this section, we discuss research on different pedagogical approaches for teaching Indigenous students and evaluations of the effectiveness of interventions. Programmes which focused on pedagogical approaches were often linked to the social justice element of distribution, specifically the distribution of Western mathematics knowledge. For example, structured mathematics teaching, with clear and explicit expectations and learning goals, and a timetable, was described by Jorgensen (2013) in her presentation of school mathematics as a game of which the rules need to be taught.

Another example of a structured approach was Pegg and Graham's (2013) *QuickSmart* programme, implemented in 600 Australian schools. An intensive intervention for middle school students, it focused on developing automaticity with arithmetic facts. The features identified as effective for teaching Indigenous students included explicit instruction with highly structured lessons, including focused games. Although the programme showed achievement gains for both Indigenous and non-Indigenous students, there is a need for a more nuanced assessment of the contribution that improving basic skills makes to performance on complex cognitive tasks. Otherwise, what is distributed as valued Western mathematics, arithmetic facts, might prove to be of little use to Indigenous students.

Exemplary practices with Aboriginal and Torres Strait Islanders, celebrating the success of quality teaching at a very remote Australian site over eight years, were described by Jorgenson (2015a, b). The principles and strategies used in the school included high expectations, being explicit, a whole school approach and supportive leadership, with a strong emphasis on linking mathematical language with Standard Australian English in an environment rich with resources (see Fig. 8.1). How or if the cultural experiences of the students were incorporated into the learning environment (recognition) or if the local community was involved in leadership or the curriculum (representation) were not reported. Rather, the focus seemed to be on the distribution of valued Western mathematics.

In contrast to the structured approach but still with an emphasis on distribution, Jorgensen and Lowrie (2013) described using a guitar-hero, digital game to motivate students' school engagement while providing a context for mathematical learning, such as percentages in scores for the game. The authors did not describe whether the mathematics learning was part of the rich task (a concert performance) that was the culmination of the program.

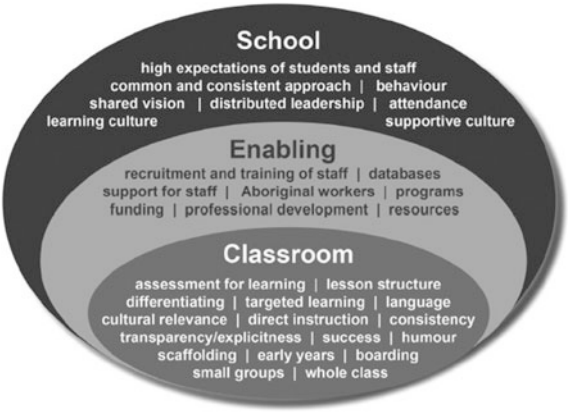

Fig. 8.1 Themes connected to levels of practice. Jorgensen (2015b, p. 666), Fig. 1 with permission from Mathematics Education and Society (MES).

The *Maths in the Kimberley* project has run over several years and sought to implement a complex programme, based on using rich mathematical tasks. It was a Australian Research Council funded linkage project in that researchers worked with the Association of Independent Schools in Western Australia (AISWA) from 2007-2011 (Jorgensen, 2015a). Related to the social justice aspect of recognition (Fraser, 2005), students' cultural practices and background knowledge were taken into consideration. For example, acknowledgement of consensus decision making processes in Indigenous communities contributed to the incorporation of group work (Sullivan, Jorgensen, Boaler, & Lerman, 2013). However, perceptions of questioning in whole class mathematics lessons and small group interactions were connected by students to the culturally-accepted response of shame, which resulted in the use of group work having to be reappraised.

Recognition of students' culture was considered valuable in linking teachers' mathematical and pedagogical content knowledge to their capacity for cultural responsiveness. Sparrow and Hurst (2012) found that teachers who began their project with low knowledge and competence in mathematics pedagogy and little specific awareness of each student's mathematical learning needs, were more able to be culturally responsive as they increased their mathematical and pedagogical content knowledge. The researchers suggested that cultural responsiveness and responsiveness to the individual are interlinked.

Yet knowing how to relate mathematics teaching to cultural activities of an Indigenous group is challenging. For example, in New Zealand the paradigm that Māori students should achieve as Māori underpins culturally responsive teaching. Investigating the views of teachers and an ethnically mixed group of Year 10 students, Averill (2012b) found that the teacher and the students identified that teachers needed to know about students as individuals and have some knowledge of their heritage cultures. However, teachers' practices were not always identified by either students or teachers as being culturally responsive. Neither teachers nor students valued teachers' knowledge of mathematical aspects of students' cultures. Students indicated that school mathematics was separate from their cultures. These findings identified that teachers needed support to integrate cultural knowledge and mathematics teaching, while students needed help to recognise that using cultural activities could contribute to their learning.

The *Make It Count* project in Australia was an initiative of the Australian Association of Mathematics Teachers from 2009 to 2012, operating in eight clusters of schools with significant populations of Indigenous students. Clusters were paired with critical friends, mostly academics with expertise in mathematics education and/or Indigenous education. This project encouraged schools to develop mathematics programmes that were responsive to their contexts, while drawing on previous research and the expertise of the critical friends. Thornton, Statton, and Mountzouris (2012) highlighted how mathematics could be embedded into everyday learning contexts. They gave the example of the engagement of a student in a school garden programme, which contributed to her developing mathematical resilience, including a more positive disposition towards mathematics and the willingness to learn from mistakes and persevere with new strategies. This is one of the few projects which illustrated all three elements of Fraser's (2005) social justice model (albeit at an individual level). The possibilities for the student to gain Western mathematics were increased with recognition of her cultural background. Having the student choose the context enabled her to connect to wider family interests, indicating that the element of representation contributed to her education.

Recognition of community-specific needs and circumstances is imperative with financial literacy education (FLE), rather than assuming that an inappropriate "one-size-fits-all" approach, delivered across a range of contexts, will suffice (Blue, Grootenboer, & Brimble, 2015). Training local people to deliver FLE that may transmit a message to their community that financial problems can be "fixed" if only one acquires budgeting skills, and without regard to culturally inappropriate delivery and contexts, proved ineffective in a case study of a Canadian Indigenous reservation community. That resources are distributed inequitably throughout society cannot be fixed by FLE training when poverty is an issue of low wages or lack of employment opportunities.

3 Language of Teaching and Learning

In this section, research on how the choice of the language of instruction affects Indigenous students' learning of mathematics is discussed, as well as Indigenous teachers' learning of the language of instruction. This issue is often tied to the distribution element of social justice through increasing Indigenous learners' opportunities to acquire Western mathematics, generally in the medium of English. Edmonds-Wathen, Sakopa, Owens, and Bino (2014) noted teaching in Indigenous languages can be contentious, even when the Indigenous population comprises the majority in a country. In Papua New Guinea and Australia, the perception of English as providing access to education and future employment makes it a valued language of instruction. While the choice of which language to use is linked to local political issues, readiness to participate in the global economy is increasingly being used as a means to suppress and further marginalise minority languages. This can be considered an example of Fraser's (2005) argument that globalisation on the politics of nation states, including language policies, can have an impact at the micro level of schooling.

One approach where students did not have the language of instruction as their first language was to reduce the emphasis on verbal language. This included considering what explicit teaching means in relation to mathematical concepts while considering students' cultural and linguistic context. This can be seen in this analysis from Halls Creek District High School:

> What I think might be a very explicit explanation or demonstration of a core idea, and can be successful in some classrooms, may well have no meaning in a class of Indigenous learners for whom standard Australian English is a second and sometimes third or fourth language. The challenge then is to make the identified core knowledge accessible to Indigenous learners without relying on traditional expository pedagogies. (Tomazos, 2012, p. 2)

Braid and Sullivan (2012) described mathematics lessons that used an "economy of words" in order to avoid the cognitive overload that can come from students being submerged in a "sea of blah" (p. 1). During these lessons, the focus was solely on the mathematics, temporarily de-emphasising a primary focus on the learning of English. Although the reduced focus on oral explanations was done to support the distribution of Western mathematics, it would have also limited possibilities for recognising Indigenous culture and language as having a place in mathematics classrooms.

Elsewhere in Australia, several studies described strategies for students to make use of their first language, Kriol. Treacy (2013) found that some Indigenous students confused everyday meanings of mathematical terms in Standard Australian English (SAE) and Kriol. Treacy (2013) suggested that "students first need to learn the concept and the associated word in Kriol, and then learn the English word that matches the concept" (p. 640). Baxter and Gilligan (2012) described a code-switching strategy where mathematical narratives were presented in both SAE and Kriol, using planned mathematical vocabulary that highlighted differences and similarities between the two languages and their mathematical registers. Jorgensen and Kanwal (2015) also described one school's planned use of SAE and Kriol as providing recognition of the students' language and culture by attributing a high status to Kriol, even if not used as the language of instruction.

There were some studies that focused on the element of representation. Wilkinson and Bradbury (2013) described the collaborative process of creating mathematical terms in the Djambarrpuyŋu language with Indigenous assistant teachers. They noted the success of this work, but also the complexity of the process. Many linguists today accept a moderate or limited Sapir-Whorf hypothesis (named after the linguists, Sapir and Whorf), that the ways in which groups of people see the world may be influenced by the language that they use (Trinick, 2015). Therefore developing terms for Western mathematical ideas in a non-Western language is likely to be challenging. Wilkinson and Bradbury (2013) highlighted the importance of schools providing Indigenous and non-Indigenous teaching teams with sufficient time to plan together.

Similarly, in a study on teacher education, Trinick, Meaney, and Fairhall (2014) focused on teachers who taught in an Indigenous language but who had completed their teacher education in English. They raised the issue of how school systems and initial teacher education programmes can support teachers in learning the registers of mathematics and mathematics education in an Indigenous language. Similarly, Edmonds-Wathen et al. (2014) found that teachers who had received their own education in English were not necessarily equipped to teach mathematics in their first languages. Thus if an Indigenous language is to be used to support both the recognition and representation elements of social justice, system support is needed.

Discussing the first two iterations of the Māori mathematics curriculum, McMurchy-Pilkington, Trinick, and Meaney (2013) highlighted curriculum development as a site of ideological contestation, i.e., international neo-liberal ideologies versus Indigenous language rights. They described how the development of Māori-medium schooling was used to produce the first Māori mathematics curriculum, enabling Māori to promote their agenda of language revitalisation. Both curricula took into account the need of Māori students to progress to tertiary education and so be competent with Western mathematics (distribution). While the first articulation of the mathematics curriculum was largely a translation of the English version (recognition), the second incorporated a stronger reflection of Māori worldviews and was more supportive of language acquisition and revitalisation goals, thus including the social justice element of representation.

4 Mathematical Topics

In the previous review (Meaney et al., 2012), the mathematical topics covered were number, probability, and space and geometry. In 2012–2015, many articles originated from two well-funded projects about the teaching of pattern and early algebra, while studies about number, space and geometry, and probability tended to be stand-alone projects. Although the main focus was generally on the distribution of Western mathematics to Indigenous students, many papers included recognition of students' cultural backgrounds. Representation was present generally when Indigenous researchers were included.

4.1 Number

Interestingly, most of the studies on number focused on Indigenous number systems and numerical thinking (recognition) rather than on the direct learning of the Western number system (distribution). The first two studies, mentioned below, included Indigenous researchers and so also included the element of representation.

In Papua New Guinea, the first three years of school (Prep to Grade 2) have been until recently taught in the vernacular language of the local community, known as *Tok Ples*. The students learn their vernacular number systems before transitioning to learning in English and the Western number system at Grade 3. Matang and Owens (2014), investigating the number understanding of students from 22 schools, found that "children learning to read and write and count in their own language Tok Ples performed better than those learning early number knowledge without Tok Ples" (p. 550). Although work discussed in previous MERGA reviews indicated that the situation is extremely complex, this result indicates that learning vernacular number systems can be used as a bridge to learning Western number systems.

Meaney and Evans (2013) drew attention to some of the erroneous historical accounts of the quantifying practices of different Australian Indigenous groups which had suggested that these groups did not quantify. They pointed out that looking only for number and reckoning systems parallel to the Western system can prevent researchers from seeing Indigenous number practices. They emphasised the need for representation by suggesting that Indigenous researchers working in their own communities should control how traditional number practices are connected to Western mathematics.

Núñez, Cooperrider, and Wassman's (2012) investigation into number concepts of unschooled Yupno people from Papua New Guinea demonstrated that number lines are a cultural construct—a widely used and useful one, but nevertheless not an innate part of mathematical thinking. This highlights the problem of mathematical artefacts being considered innate instead of the cultural products of those who design and implement mathematics education. It also illustrates that a focus only on the social justice aspect of distribution can lead to reduced rather than increased learning opportunities for Indigenous students.

Treacy, Frid, and Jacob (2015), looking at quantifying strategies of Indigenous students from the Goldfields Area of Western Australia, found that the students performed a quantity matching task without counting. Although uncertain what students did, the researchers suggested that a form of family matching or subitising might have been used. Treacy et al. (2015) concluded that there was a need to take recognition into consideration, possibly in alignment with representation: "The findings highlight a need to further examine the world views, orientations and related mathematical concepts and processes that Indigenous students bring to school" (p. 18), specifically to determine the actual strategies the students used.

Taking the approach recommended by Treacy et al. (2015), Ewing (2012, 2014) documented the mathematical practices of mothers in a Torres Strait Islander community as cultural "funds of knowledge", such as sorting through classification, repeated patterning and partitioning. She discussed similarities and differences between the community practices of sharing through partitioning in the distribution of fish that have been caught and division though equal sharing in school mathematics. Ewing (2014) argued that mathematics is located in children's lives and social relationships and that development of their mathematical understanding needs to come from these practices and contexts.

One study focused only on the learning of Western number concepts by Indigenous students, with the division strategies of 44 students (Year 7–8) from Māori-medium schools being documented (Hāwera & Taylor, 2012). Twenty-nine students used a wide variety of strategies of varying efficiency, even when they had not been explicitly taught these strategies. However, the researchers expressed concern about the students being unable to provide appropriate answers and recommended that teaching of mathematical concepts such as multiplication should include teaching relevant mathematical language.

4.2 Patterns and Early Algebra

Two externally funded projects in Australia focused on developing young children's patterning skills. The approaches for the two projects were different. The Patterns and Early Algebra Preschool (PEAP) Professional Development project (Papic, 2013b) was a 3-year early numeracy project, conducted across New South Wales and the Australian Capital Territory, that focused on developing young children's awareness of pattern and structure in order to promote the foundation for mathematical thinking (Papic, Mulligan, Highfield, McKay-Tempest, & Garrett, 2015). It thus could be considered as focusing on distribution. Children aged 4 to 5 years were assessed on their patterning understandings. Professional development was provided for the early childhood educators, who implemented an intervention which had a positive impact on the children's mathematical thinking (Papic et al., 2015). Notably, the children's Indigeneity was conflated with low socioeconomic status and a perceived lack of school readiness (Papic, 2013a, b). Making tasks culturally relevant was mentioned, with "hands-on" experiences described as critical for engaging Indigenous students (Papic, 2013b). However, the programme used the same material that had been developed for non-Indigenous students, with minimal discussion about the need to modify it. This raises questions about how recognition and representation could be included in such programmes.

The Representations, Oral Language and Engagement in Mathematics (RoleM) project from Queensland was developed specifically for Indigenous students, ESL students and students in low socioeconomic contexts; however, the researchers were careful not to conflate these groups (Warren & Miller, 2015). The researchers involved individual communities, thus recognising the importance of representation, so that they could learn with them about how best to support students' learning. "Collaboration between the school, local communities, parents, teachers, students and Indigenous education workers is seen to be crucial to success" (Warren & Miller, 2013, p. 153). A project developed alongside RoleM investigated how young Indigenous students learn to generalise with growing patterns, with findings focused on the importance of gesture as a semiotic system in generalising about such patterns (Miller, 2015; Miller & Warren, 2015).

From the *Make It Count* project, Barnes (2012) described how contextually relevant word stories were used to engage interest in algebra, and teach algebra and abstract concepts to girls at a boarding school. While the paper was not developed and structured as a research paper, Barnes emphasised that the girls were “easily engaged” with algebra, and that they managed abstraction well “once they [saw] that it [was] a powerful way of solving complex problems” (p. 6).

4.3 Probability

In one of the few papers from the Pacific, Morris (2014) discussed his initial investigation into his failure to teach probability at the university level in the Kingdom of Tonga. He used the Sapir-Whorf hypothesis to suggest that Tongan students’ interpretations of uncertainty were linked to their language not having ways to discuss uncertainty, with cultural views that suggest “future events are not uncertain but are waiting to be revealed” (p. 246). This seemed to be about the need for recognition of difference but the paper did not extend the ideas about how to resolve the issue in a way that would support students’ learning.

Pickles (2013) explored in Goroka, Papua New Guinea, how the introduced practice of gambling was connected to older traditions of competitive giving. As such, he problematised considerations of gambling as just being about chance and therefore the provenance of mathematics. This suggests that gambling is not a “natural” context for teaching probability, something also raised by Meaney and Evans (2013) in relationship to card games and number understandings.

4.4 Space and Geometry

Drawing on a wide body of research, Owens’s (2015) book on an ecocultural perspective on visuospatial reasoning argues that “education besides recognising a school, system, and global perspectives as contexts may benefit from connecting to place and culture to understand and strengthen visuospatial reasoning” (p. 12). Owens (2013, 2015) described several projects undertaken to investigate the spatial and measurement concepts of different language groups in Papua New Guinea. Acknowledging the importance of distribution and recognition, Owens was concerned that Western mathematics should neither be used to replace the existing mathematics within Papua New Guinea cultures nor be ignored so that students were not provided with opportunities to learn it.

> By ensuring mathematics is part of an ecological perspective provided by culturally competent teachers who establish educational partnerships with the communities around their activities, then the teaching of mathematics will support cultural knowledge and relationships as well as advance school mathematics. (Owens, 2013, p. 967)

In Australia, Sullivan and van Riel (2013) focused on geometrical topics because of “an often stated assumption that the prevalence of direction words in some

Indigenous languages implies that the learning of aspects of geometry may be closer to Indigenous students' experience than the learning of number" (p. 142). Sullivan and van Riel's study was on students connecting 2D and 3D representations of objects. However, Sullivan and van Riel's goal for Indigenous students to learn "conventional" mathematics precluded any questioning of "the appropriateness of an early emphasis on geometric shapes in some mathematics syllabi, when other types of spatial knowledge are more precisely defined and much more highly valued in the Aboriginal child's home culture" (Harris, 1991, p. 142). It seemed the goal for more equitable distribution of Western mathematics knowledge overrode concerns for recognition.

Edmonds-Wathen (2013) investigated spatial concepts of the Iwaidja, a northern Australian Indigenous cultural group, and suggested that spatial concepts in Indigenous languages can influence children's use of similar terms in English (Edmonds-Wathen, 2014). In New Zealand, Hāwera and Taylor (2013) described an intervention study on transformational geometry for Year 7 and 8 students in a Māori-medium class. The intervention wove mathematical language learning with cultural understandings into a range of tasks, including the use of ICT. This was one of the few studies in this review which actively sought and reported on children's own experiences of learning, thus acknowledging the need for Indigenous students' representation in research.

Trinick, Meaney, and Fairhall (2015) have begun a project to consider how traditional Māori cultural knowledge can be revived in regard to spatial orientation. Their initial results showed that students needed support to orientate themselves outside of classrooms, regardless of what system of knowledge they drew upon. Although Indigenous researchers were involved in this research, the focus was on the social justice elements of distribution and recognition.

5 Building Capacity

Research on professional development for Indigenous and non-Indigenous teachers about teaching mathematics did involve, to varying degrees, all three social justice elements. When Indigenous staff and community provide expert knowledge about the teaching of mathematics to Indigenous students, then the representation element of social justice is present (Fraser, 2005).

5.1 Professional Learning Needs to Be Ongoing and Collaborative for Sustainability

It is acknowledged that "deliver and run" professional development is unlikely to lead to sustainable transformation of teacher practices (Owens, 2014a). Instead, changes come from job-embedded professional development, with an emphasis on personal learning, reflection and pedagogical change (Warren, Quine, & DeVries, 2012), taking place over extended periods with expert mentoring and informal support within and beyond the school (Owens, 2014a). Sustainability of these changes comes when teachers engage in their own professional growth in a collaborative context with support from experts in the field over time (Hāwera & Taylor, 2014; Jacob & McConney, 2013; Owens, 2014a; Warren & Miller, 2013; Warren et al., 2012). This is particularly important in rural or remote areas in Australia where teachers often are at the beginning of their careers and/or change positions regularly (Jorgensen & Kamal, 2015).

An ethnomathematical project in Papua New Guinea, with an emphasis on the representation element of social justice, encouraged Indigenous teachers to link their cultural mathematics with school mathematics (Owens, 2014b). A strong sense of their cultural identity encouraged teachers to recognise the value and relevance of their cultural heritage to mathematics education. In turn, their mathematical identities were strengthened by linking to their community contexts. Part of the project's aim was to create a sustainable community of learners (Owens, Edmonds-Wathen, & Bino, Owens et al. 2015). Teachers living in remote areas participated in a week of face-to-face professional development. The workshop used an electronic resource package, accessible offline. This included videos of cultural activities from different parts of PNG, and videos exemplifying children's learning, the latter featuring Australian learners (Bino & Edmonds-Wathen, 2014). These videos will be replaced by others that reflect learners from PNG and their environment. In evaluating the workshop, participants reported that they had learnt about making links between cultural mathematics and school mathematics, teaching mathematics, providing group activities, and asking questions to enhance thinking (Owens et al., 2015). The facilitators recognised ongoing challenges to deliver the professional learning on a larger scale.

Teachers need good knowledge of mathematics to make links to their Indigenous learners' out-of-school contexts so the learners can "see themselves as mathematicians doing maths in their everyday lives" (Jacob & McConney, 2013, p. 98). After a year of professional development, teachers reported a growing confidence in planning for and monitoring student learning (Jacob & McConney, 2013). Nevertheless, many teachers still were not confident in areas like diagnosing learning and making mathematics explicit to learners. The classroom teachers worked together with a mathematics specialist to overcome conceptual hurdles facing Indigenous learners. As well the role of Aboriginal and Islander Education Officers (AIEO) was essential. This acknowledges the need for representation, as well as distribution and recognition, in order for socially just mathematics education programmes to be provided.

In a project to improve numeracy outcomes for early years Indigenous learners, numeracy specialists worked with teachers, Aboriginal Education Assistants (AE) and AIEOs to enhance their pedagogical content knowledge, confidence in teaching mathematics and their development as participants in professional learning communities (Hurst & Sparrow, 2012). Results indicated that the AE and AIEOs improved their confidence and ability to take on greater responsibility for teaching and began to see themselves as integral members of a professional learning community. They reported on genuine team work with the teachers and they felt valued as equal members of staff. The AEIOs believed that their confidence to teach and support children's learning came from knowing more content.

In the *Maths in Kimberley* project, Jorgensen, Grootenboer, and Niesche (2013) developed a pedagogical model to assist teachers in six remote schools to promote effective pedagogical practices in mathematics. A comparison was made between teachers' answers to a questionnaire and video recordings of their lessons. The results showed that professional development could support teachers to make significant changes to their pedagogy. However, facilitators noted the difficulties of supporting teachers, given the large distances between the researchers, development teams and schools, and suggested alternative working processes. The researchers also emphasised that communities were unique and pedagogical activity was not always transferable.

Similarly, Owens (2014a) cautioned against assuming that all Indigenous learners are strongly steeped in their cultural practices. In an urban Australian school, teacher pedagogy and the school environment changed for Indigenous students when funding supported professional development which involved the community, the social justice element of representation. The school revised their teaching approaches and curriculum to better include family and Aboriginal cultural heritage. Shared ownership developed with the community feeling welcome in the school and taking on leadership roles. Teacher perceptions, skills and pedagogy changed and a place-based mathematics curriculum resulted. Learners acknowledged their Indigenous connections and learnt the Indigenous language. Changes eventuated for students because there were not only high expectations about test results (the distribution element), but also an expectation students would be comfortable identifying as Aboriginal and being proud of their heritage (the recognition element).

Indigenous students live in a range of circumstances. For remote communities, one of the most challenging issues is high staff turnover which is considered counter-productive for initiatives designed to increase Indigenous learners' achievement (Jorgensen, 2012; Owens, 2014a; Warren & Quine, 2013). There is substantial evidence that shared leadership and power, inclusive of Indigenous culture, knowledge and values, along with high expectations of learners, can lead to improved Indigenous student learning outcomes (Ewing, Sarra, Cooper, Matthews, & Fairfoot, 2014; Warren & Quine, 2013). Distributed leadership proved valuable in schools where principals and teachers remain for short periods (Jacob & McConney, 2013; Jorgensen, 2012). As one principal stated, "Teachers come and go but the community stays" (Owens, 2014a, p. 76). Distributed leadership may be incongruent with established local practices, based on a Western perspective that uses vertical structures (Warren & Quine, 2013). However, sharing power and authority with Indigenous staff and communities helps to build capacity (Warren & Quine, 2013) and is clearly connected to the social justice element of representation.

One approach to improving representation is to increase the number of Indigenous mathematics education researchers through partnering and mentoring Indigenous academics (see Owens, 2014b; Owens et al., 2015). Dawson (2013, 2015) reported on a capacity building project for Indigenous mathematics educators from across Micronesia. The educators, who were co-researchers in the *MACIMISE* (Mathematics and Culture in Micronesia: Integrating Societal Expectations) project, involved local experts, familiar with community culture and practices, to develop 17 culturally based mathematics units (Dawson, 2015). The educators completed advanced degrees at the University of Hawaii-Manoa that honoured the mathematical practices of their respective Micronesian communities.

The mathematics education research community in Australasia has not yet developed similar projects to MACIMISE, as a way of supporting the inclusion of the representation element of Fraser's (2005) social justice model. Hāwera and Taylor (2014) discussed fluidity of engagement and power sharing between researchers and participant teachers in a Māori-medium setting and argued for the importance of working relationships, modelled on whānau lines (family), with Indigenous experts from outside the school.

5.2 Parent-Community Involvement

Parent and community involvement was visible in some professional development projects reported in previous sections, but were also the focus of other projects. Such involvement supports the inclusion of the representation element of social justice.

Preschoolers in an Aboriginal community in New South Wales transitioning to school learnt best when there was a partnership between their parents and teachers (Sarra & Ewing, 2014). This partnership promoted a sense of continuity between home and school, which enabled numeracy language and understanding to develop in contexts similar to those in the preschoolers' homes. With elders and community members sharing their knowledge, the school was able to develop a culturally rich curriculum. Culturally appropriate resources and the learning environment enabled these learners to reduce cultural, linguistic and contextual barriers and be more able to engage in mathematics learning.

Similarly, Averill (2012a) in New Zealand argued for teachers to develop culturally responsive mathematics teaching by considering the families of learners from the different cultural groups in their classrooms as cultural models and advisors. Forming relationships with these families could provide continuity between teachers and the community.

Ewing et al. (2014) explored how the learning and teaching of Vocational Education and Training (VET) courses could contribute to successful outcomes for Indigenous learners, including increasing their future employment opportunities. Effective community relationships were considered to lead to young people enrolling in Certificate courses and to act as a conduit for gaining feedback and support from community elders.

6 Conclusion

Much of the research focusing on the relationship between Indigenous students and mathematics can be broadly grouped around two major themes; research which seeks to explain why Indigenous students underachieve and solutions for redressing the underachievement. Explanations for underachievement of Indigenous students include low teacher efficacy and low teacher and student expectation (Jorgensen et al. 2013; Owens, 2014b; Turner et al., 2015); inadequate teacher subject, pedagogic and cultural knowledge (Edmonds-Wathen, 2012, 2015); and conflict between the culture of home and school (Meaney & Evans, 2013). As well, when mathematics and mathematics education are considered culturally free (Owens, 2013) and not taking place in culturally and socially loaded contexts (Averill, 2012b), Indigenous students can become alienated from the learning environment and mathematics education.

A number of researchers suggest Indigenous students' underachievement is related to the power relationships in schools and classrooms (Cazden, 2012; Jorgensen & Perso, 2012). Indigenous people, their culture and language, are frequently in subordinate positions in schools and curricula, with national priorities frequently determined by the needs and aspirations of the majority, which is most often European (Trinick & May, 2013). Trinick et al. (2015), in relation to Māori in New Zealand, provided the example of national curricula advantaging Western spatial perception to the detriment of traditional Māori cultural knowledge. The ongoing tendency of government agencies to frame initiatives in terms of mainstream education (the distribution element), in the first instance, assumes that these will naturally "translate" to the Indigenous education context.

Some programmes, such as *Make it Count*, did embrace an approach which enabled individual schools to frame their interventions to match the needs of their students, and communities (recognition and representation). However, gaining systematic data on whether these resulted in improved outcomes for students has proven difficult (Forgasz, Leder, & Halliday, 2013).

From our perspective, some of the issues raised in the last two reviews do not seem to have been resolved. Only a few studies actively included Indigenous representation, either in the planning of interventions or in undertaking research. This suggests that changes need to be made by the mathematics education research community to influence policy, if the aspirations of Indigenous communities in regard to mathematics education for their children are to be achieved. In 2015, it seems incongruous that the majority of research done in Indigenous mathematics is still carried out by non-Indigenous researchers.

One point that we have noted in doing this four-yearly review is the influence of funding bodies on the type of research which is done. Although many researchers had only the research time granted to them by their universities, the most prolific research outputs generally came from externally funded projects (see Grootenboer & Sullivan, 2013; Owens et al., 2015; Warren & Miller, 2013). In order to gain the increasingly rarer grants, applications need to be written in an acceptable way with a particular focus. Increased Indigenous representation within research environments could be achieved by including this requirement within application guidelines. Reduced aid funding from Australia, in particular, has curbed the possibilities for Indigenous researchers from the Pacific and Papua New Guinea to complete graduate degrees. Consequently, there is a need to consider other ways to support capacity building of Indigenous researchers. There is a moral responsibility for research communities, such as MERGA, to advocate for more funding for Pacific nations. Many of these nations, threatened by rising sea levels as a result of global warming, need mathematically literate advocates to support their nations' future in international gatherings, such as the one held in Paris in December 2015.

We support Fraser's contention that without all three social justice elements being considered in research—namely, redistribution (economic), recognition (cultural) and representation (political)—intervention programmes and their associated research are likely to have only limited success, as identified in the longitudinal statistics on Indigenous students' achievement in mathematics.

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Chapter 9
Mathematics Education in the Early Years

Amy MacDonald, Wendy Goff, Sue Dockett and Bob Perry

Abstract This chapter presents a synthesis of the Australasian early childhood mathematics education research which has been conducted during the review period 2012–2015. "Early childhood education" is taken to be the education of, and for, children aged between birth and 8 years old. The research canvassed in this chapter encompasses a range of early childhood contexts, including home, school, and early childhood education services. Similarly, the research presented in this chapter has been undertaken with a range of stakeholders in early childhood mathematics education, including early childhood and school educators, families, and the children themselves. Consistent with previous reviews, this chapter is structured according to four key themes which have emerged in canvassing the current research: curriculum in early childhood mathematics education; assessment in early childhood mathematics education; content of early childhood mathematics education; and contexts for early childhood mathematics education. This synthesis of research is then used to provide recommendations for future research in this field.

Keywords Mathematics content · Learning contexts · Early childhood curricula · Mathematics curricula · Assessment · Early childhood educators · Transitions to school

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1 Introduction

In this chapter, we present a synthesis of the Australasian early childhood mathematics education research undertaken during 2012–2015. Consistent with previous review, in this chapter "early childhood education" is taken to be the education of, and for, children aged between birth and eight years old. The research presented in this chapter takes into consideration a range of early childhood contexts, including home, school, and early childhood education services. Similarly, the chapter considers research which has been undertaken with a range of stakeholders in early childhood mathematics education, including early childhood and school educators, families, and the children themselves. Indeed, the views of children and families in early mathematics education are well-represented in the Australasian research, and this is to be celebrated.

The review period has seen the publication of two significant books focused on mathematics in the early years. The first of these, *Reconceptualizing Early Mathematics Learning* (English & Mulligan, 2013) examines the mathematical capabilities of young children and demonstrates that young children are developing complex mathematical knowledge and abstract reasoning a good deal earlier than previously thought. The second, *Mathematics and Transition to School: International Perspectives* (Perry, MacDonald, & Gervasoni, 2015), is the first international edited book to consider mathematics and transition to school in tandem—both of which are important aspects of any young child's life. Both of these texts are collections of international research edited by Mathematics Education Research Group of Australasia (MERGA) members, and contain many chapters written by Australasian early childhood mathematics education researchers, demonstrating the leadership of Australasian researchers in the field of early childhood mathematics education. It is anticipated that both texts will prove to be significant points of reference for research in the next MERGA review period.

This chapter is structured according to four key themes which have emerged in canvassing the current research: curriculum in early childhood mathematics education; assessment in early childhood mathematics education; content of early childhood mathematics education; and contexts for early childhood mathematics education. This synthesis of research is then used to provide recommendations for future research in this field.

2 Curriculum in Early Childhood Mathematics Education

The *Research in Mathematics Education in Australasia* (RiMEA) review period of 2012–2015 has been a tumultuous time in Australasian early childhood education, especially in Australia, where two new curriculum documents—*Belonging, Being and Becoming—The Early Years Learning Framework for Australia* (EYLF) (DEEWR, 2009) and *The Australian Curriculum—Mathematics* (Australian

Curriculum, Assessment and Reporting Authority [ACARA], 2013) have both reached their implementation phases. Similarly, in New Zealand, there are two separate curriculum documents: *Te Whãriki* (Ministry of Education [MoE], 1996) for children from birth to school entry and *The New Zealand Curriculum* (MoE, 2007) for the school sector which have mandated contextually appropriate approaches in each context (Lee & Lomas, 2015). As a consequence, there has been a lot of action in early childhood mathematics education research in the overlapping space of these two curriculum documents—the time when most children in Australia and New Zealand start school. Several key ideas pertaining to curriculum have been considered and these are now discussed separately.

2.1 Links Between the Early Years Learning Framework and the Australian Curriculum—Mathematics

The *Early Years Learning Framework* and the *Australian Curriculum—Mathematics* were developed at different times, by different groups of people and with little consultation across the two endeavours. While, as Connor (2011, p. 15), suggests, "The Australian Curriculum recognises that the EYLF establishes the foundations for effective learning in school and throughout life and aims to build on those foundations as learners move through schooling", the different structures, content and philosophies of the two documents makes it difficult to see links between them. Connor (2011) uses the general capabilities in the *Australian Curriculum* across all subject areas to make links with the learning outcomes in the EYLF. Perry, Dockett, and Harley (2012) have made these links explicit in the area of mathematics and this has been extended through the introduction of *reflective continua* to assist educators in prior-to-school settings and schools "in their quest for excellence in the development of young children's powerful mathematical ideas" (Perry & Dockett, 2013, p. 159).

2.2 Continuity of Pedagogies Across the Transition to School

"Transition from home to school, or from a prior-to-school setting to school, is often characterised by discontinuity across the areas of relationships, pedagogy, curriculum, resources and support" (Dockett & Perry, 2014, p. 8). The move from so-called "play-based and child-centred" pedagogies in prior-to-school settings to more formal, "subject-based" pedagogies in schools would seem to challenge the often heard call for continuity across the transition to school. Perhaps, however, the "ideal" of continuity of pedagogy may be an unattainable and even undesirable myth. "There does seem to be agreement that promoting continuity does not mean that contexts should become the same… Indeed, there is strong evidence that young children want school and prior-to-school to be quite different; they do not want more of the same as they start school" (Perry, Dockett, & Harley, 2012, p. 157). Approaches to developing continuity of pedagogy across early childhood transitions have been considered in both New Zealand (Lee & Lomas, 2015) and Australia (Dockett & Perry, 2014; Perry, Dockett, & Harley, 2012).

2.3 *Children's Mathematical Knowledge as They Start School and Its Relation to the Foundation Stage of the Australian Curriculum—Mathematics*

One of the consequences of the development of the *Australian Curriculum—Mathematics* and the *New Zealand Curriculum in Mathematics* has been the setting of achievement standards for children at the end of their first year of school. Recently, particularly in Australia, a number of researchers have questioned the level at which the standards are set.

Reporting on some aspects of number knowledge gathered through a standardised interview schedule from 65,000 children entering the first year of school in NSW in 2011, Gould (2012, p. 108) demonstrated that "[a]pproximately 16% of the Kindergarten [first year of school] children showed the facility with number expected of students commencing year 1". He also notes a wide range of abilities in the data set and suggests that this "sets significant challenges to establishing reasonable expectations of what all students should know and be able to do at the end of the first year" (p. 109).

In their work with the *Let's Count* project, Gervasoni and Perry (Gervasoni & Perry, 2013, 2014, 2015; Perry, Gervasoni, & Dockett, 2012) have noted similar findings to Gould, concluding that:

> Overall, it appears that the new *Australian Curriculum – Mathematics* Foundation standard is neither sufficiently challenging for children nor adequate for signaling to teachers the type of experiences and instruction that are important. Whilst acknowledging that the *Australian Curriculum – Mathematics* encourages teachers to adjust curriculum and instruction to match children's knowledge, it must also adequately reflect the mathematical capabilities of children when they begin school. The data presented in this chapter suggest that Australian education authorities need to undertake more fine-tuning to set the Foundation standard at a level that sufficiently engages and challenges children at the time of transition. (Gervasoni & Perry, 2015, p. 61)

While the two national curriculum documents relevant to mathematics in the early childhood years have provided a great deal of impetus for positive change in the ways in which mathematics is learned and taught in these years, there would still appear to be need for further work.

3 Assessment in Early Childhood Mathematics Education

Assessment in education, particularly high-stakes, normative, national or international testing, has gradually become more and more prevalent over the last decade. Mathematics—or "numeracy"—forms a key component of such testing. The work of Lange and Meaney (2014) exemplifies the potential impacts of such a testing regime and speaks importantly about the commodification of children and the reduction in what is seen to be important in mathematical content and pedagogy. While most of the "action" in such testing occurs after the early childhood years, there are very noticeable impacts on children and their educators in these years. "There is an increasing focus on assessment within curriculum disciplines in early childhood with the implementation of more specific articulation of curriculum requirements" (Clarke, 2015). Some form of school entry assessment has been developed in most states and territories of Australia, and in New Zealand. While the stated purpose of these assessments is for the teachers in the first year of school "to establish a starting point for monitoring each child's numeracy development and to plan appropriate further teaching-learning interactions" (Gould, 2012, p. 105), there is also the possibility—and the likelihood—that these school entry assessments will be linked to later national testing results in order to reinforce already held beliefs about which children can benefit most from schooling in mathematics (Claessens & Garrett, 2014). As well, there does need to be some consideration of whether or not the first year of school teacher uses information which is available—through families—from prior-to-school educators. A number of states of Australia mandate that these "transition statements" be written by prior-to-school educators with the purpose of handing them on to school educators. How these statements are used—if at all—continues to be an area of needed research.

Appropriate methods of assessment in the early childhood years have been considered by a number of Australasian researchers. Clarke (2015) provides a strong argument for the use of individual task-based interviews but promotes flexibility to allow children to show what they know.

> Children are rich mathematical thinkers. They are entitled to experience assessments that provide opportunities to show what they know and can do to researchers and educators. In advocating a place for more flexible approaches to interviewing I would argue that it provides greater richness and validity in terms of results for individual children. However, listening is vital. (Clarke, 2015, p. 43)

The work that Gervasoni and her teams have done in their *Extending Mathematical Understanding* program (Gervasoni et al., 2012) and *Let's Count* (Gervasoni & Perry, 2013, 2014, 2015; Perry, Gervasoni, & Dockett, 2012) also uses individual interviews to assess children's numeracy knowledge and skills and progress over time. Much of the flexibility suggested by Clarke is incorporated into these assessments.

It has been known for many years that language is a critical factor in children's mathematics learning and their ability to demonstrate this learning (Ellerton & Clements, 1991). In a study using conversation analysis, Mushin, Gardner, and Munro (2013) have considered video recorded class lessons in the first three years of schooling in a Queensland Indigenous community. They concluded that "language (non-)understanding may interfere with a student's capacity to demonstrate understanding of a mathematical concept" (p. 429); "student's language comprehension may pose problems for demonstrating mathematical understanding" (p. 430); and that "care… must be taken by teachers and test designers in maths assessment to ensure that there is clarity between conceptual knowledge of mathematics and the language that cloaks it" (p. 431). While the Mushin et al. (2013) study was conducted with Indigenous children, the message is clear: language is important in the development and assessment of young children's mathematical knowledge.

Not only are the words used in conversations about mathematics important. In a study embedded in the influential *E4Kids* project (Tayler, Ishmine, Cleveland, Cloney, & Thorpe, 2013), Cohrssen, Church, and Tayler (2014) consider the importance of pauses in educator-child conversations in prior-to-school settings for the development of children's mathematical concepts. Such pauses not only allow thinking time for both the educator and the child but also "markedly slow the pace of interactions, providing opportunities for teachers to provide contingent, attuned responses as they support or extend children's learning" (Cohrssen et al., 2014, p. 176). Assessment of learning and planning for the intentionality of teaching are enhanced through the careful use of such pauses. Children are able to demonstrate their understanding and educators are able to scaffold the child's learning and provide high quality feedback. The result is an example of assessment via purposeful, intentional conversational interaction.

An alternative method to interview and other more formal assessment methods is the use of learning stories. This approach to assessment and documentation originated in New Zealand (Carr, 2001) and has been used extensively across the world (Carr & Lee, 2012; Perry, Dockett, & Harley, 2012). As part of a larger study, Lim, Anthony, and McLachlan (2014) focused on "what mathematics was visible and regarded by the teacher as important" (p. 407) in a sample of 66 learning stories from three kindergartens in New Zealand. They concluded that teachers were more likely to document children's play with mathematics rather than mathematics that might emerge from children's more general play activities. The conclusion drawn is that there may be some danger to children's learning arising from such restrictions in the documentation. The restrictions in what is observed, and when, might lead to "restricted opportunities for mathematics learning" (p. 411). It is suggested that teachers need "more guidance concerning the assessment and documentation of mathematics learning" (p. 412).

4 Content of Early Childhood Mathematics Education

Australasian research in early childhood mathematics education has continued to explore specific mathematical content areas, consistent with previous MERGA review periods. Indeed, the majority of Australasian studies have considered children's understandings of a *specific* content area (such as measurement), while fewer have considered a *range* of content areas within the same study. In the previous edition of RiMEA, MacDonald, Davies, Dockett, and Perry (2012) identified three dominant content areas among the Australasian research: number, algebra, and measurement. They noted that data was an emerging area in which important work had been undertaken, and that geometry was an area which, at that point, was under-addressed. Since the previous review period, algebra (and patterns) has increased its presence amongst the content-focused research, with measurement and number continuing to attract research; though, to a lesser extent to that of the previous review period. The most significant development has been a marked increase in the amount of research focusing on data. A small amount of work concerning geometry has also emerged, as has research focusing on children's mathematical reasoning. These six content areas—patterns and algebra, measurement, number, data, geometry, and problem solving and mathematical reasoning—will be canvassed in this section.

4.1 Patterns and Algebra

Research concerning young children's pattern and algebraic thinking has advanced considerably since the 2008 edition of RiMEA, in which Perry, Young-Loveridge, Dockett, and Doig identified work on pattern, structure and early algebra as a significant new field of research in Australasia. The previous review period (2008–2011) saw a substantial body of research related to the *Pattern and Structure Mathematical Awareness Program* (PASMAP) (Mulligan & Mitchelmore, 2009), which incorporates the *Pattern and Structure Assessment* (PASA) and the *Early Mathematical Patterning Assessment* (EMPA) (Papic, Mulligan, & Mitchelmore, 2011). The PASMAP program of research again features in the current review period. Current publications emanating from this long-running project focus on the ways in which the PASMAP Kindergarten tasks develop and link the themes of grid patterns, number patterns, multiplication and base ten numeration (Mulligan & Mitchelmore, 2012). Given the prominence of the PASMAP program of research over a substantial period, it would be interesting to see future research which utilises the PASMAP data for new purposes via secondary data analysis, thus adding additional value to this significant data set.

Another prominent project during the review period has been the *Early Years Generalising Project* (EYGP) (Warren, Cooper, & Miller, 2012), which has explored children's responses to a Piagetian-style patterning interview. Within the larger project, Warren et al. (2012) have explored children's ability to determine elements in different positions when two units of a repeating pattern were shown. Another specific focus within the larger project has been on young Indigenous students' understandings of growing patterns. Miller and Warren (2012) and Miller (2014) explored Indigenous students' initial understandings of growing patterns prior to formal teaching of this concept at school, and found that these students were already capable of working with growing patterns, and that contextual artefacts and gesture played important roles in students' understandings of these patterns. In light of these findings, Miller (2014) has proposed a cultural learning semiotic model that takes account of the various stakeholders in young Indigenous children's patterning development. It is anticipated that research which implements and further examines this model will be presented in the next MERGA review.

4.2 Measurement

In the previous review, research concerning young children's measurement understandings was predominantly related to mass and length (MacDonald et al., 2012). In the current review period, the emphasis on length has diminished while mass has continued as an area of focus. In particular, Cheeseman, McDonough and colleagues have maintained their focus on young children's understandings of mass measurement, particularly in the first three years of primary school. In their study utilising a teaching experiment which explored mass concepts with children aged six to eight years (Years 1 and 2), McDonough, Cheeseman, and Ferguson (2013) found that these young children engaged with mass measurement in complex and diverse ways. Children drew on prior experiences as well as visual cues to explore ideas including the comparison of masses, choosing and using informal and formal units, and conserving masses. McDonough et al. concluded that the rich learning experiences contained within their teaching experiment provided opportunities for children to engage with challenging mathematical ideas in play-based, problem-solving contexts.

In the same study, Cheeseman, McDonough, and Ferguson (2014) utilised the task-based interview developed as part of the seminal *Early Years Numeracy Project* (ENRP) (see, for example, Clarke et al., 2002) to conduct assessments of the children's knowledge before and after the teaching experiment. Results from the pre- and post-interview data for 119 children indicate that the children made substantial gains in their learning as a result of the teaching experiment. The Year 1 children moved from an awareness of the attribute of mass and some comparison of masses to quantifying masses and using standard units, while the Year 2 children showed similar progress with the majority of children being able to quantify mass and use standard units by the end of the teaching experiment.

Young children's understandings of measurement have also been considered more holistically, rather than in relation to specific concepts (i.e., mass). MacDonald (2013) has explored what young children themselves consider "measurement" to be. In a study utilising the drawings of children aged 4–6 years, MacDonald (2013) found that the majority of children related the concept of "measurement" to *length* measurement, specifically. Children also considered "measurement" to be a process of "finding out" about an object, such as finding out how long or tall an object is. Importantly, children were able to draw and talk about *processes* of measuring, utilising both informal and formal tools and units of measurement.

Similar understandings about measurement were revealed in MacDonald's (2012) study utilising children's photography to ascertain their ideas about measurement in home contexts. The photographs, taken by children aged four to six years, revealed children's understandings of measurement concepts such as identifying measurable attributes, comparing attributes such as height or area, and using units to describe their measurements.

The various studies by both MacDonald and Cheeseman et al. have shown that when given a range of opportunities to demonstrate their knowledge and experiences, young children can show that they have sophisticated understandings about measurement which have been constructed in, and can be applied to, a range of contexts.

4.3 Number

In the previous review, it was highlighted that Australasian research focusing on number development is challenging assumptions about children's understanding of multi-digit numbers, mathematisation and subitising (MacDonald et al., 2012). In particular, the research of Gervasoni and colleagues in the *Bridging the Numeracy Gap* project (encompassing the *Extending Mathematical Understanding* [EMU] Specialist Teacher Course and the EMU Intervention Program) has shown how carefully designed tasks can "assist teachers to identify students who need further experience with multi-digit numbers to construct full conceptual understanding" (Gervasoni et al., 2011, pp. 321–322). Gervasoni and colleagues continued to publish from this research during the current review period, and their recent findings have demonstrated how children who had previously been seen as "struggling" with school mathematics can, with targeted support, progress their number learning beyond what might have been expected (Gervasoni et al., 2012). The longitudinal progress of 42 Grade 1 students who participated in a 10–20 week EMU intervention program has been examined, and it was found that overall the students made accelerated progress during Grade 1, which was maintained when they were assessed again 12 months later (Gervasoni et al., 2013). Findings suggest that "the EMU Program was successful in enabling most Grade 1 students to progress their whole number learning beyond that anticipated in a typical year (Gervasoni et al., 2013).

Another mathematics intervention program which has proven to have positive effects on children's learning of number (as well as other) concepts has been the *Let's Count* early numeracy program (Gervasoni & Perry, 2015; MacDonald, 2015;

Perry & MacDonald, 2015; The Smith Family, 2015). As Gervasoni and Perry (2015) note, "it is commonly assumed… that children living in economically and socially disadvantaged communities are over-represented in the group of children with the least formal mathematics knowledge when they begin school, and that it is important that prior-to-school experiences help to overcome this disadvantage" (p. 47). *Let's Count* is an early numeracy intervention program designed by The Smith Family (an Australian charitable organisation) and Gervasoni and Perry, which assists families in communities that might be described as "disadvantaged", to help their children aged 3–5 years play with, learn and investigate powerful mathematical ideas as part of everyday activities (Gervasoni & Perry, 2015). A longitudinal evaluation of *Let's Count* has been conducted wherein the participant children have been assessed using the *Mathematics Assessment Interview* originally developed as part of the ENRP (Clarke et al., 2002). It was found that the *Let's Count* and ENRP cohorts performed similarly with regards to tasks with small sets, part-part-whole tasks and one to one correspondence tasks, with both cohorts showing that about 75% of the children were able to demonstrate the *Australian Curriculum—Mathematics* Foundation Standard "Students make connections between number names, numerals and quantities up to ten" (ACARA, 2013) before they begin school (Gervasoni & Perry, 2015). Data also revealed that the majority of the *Let's Count* cohort could rote count to 10 and at least one-quarter could complete the rote forward count to 20, a result reinforced by Gould (2012) who found that 16% of students in New South Wales could rote forward count to at least 30 (Gervasoni & Perry, 2015). Results from the *Let's Count* Longitudinal Evaluation suggest that many children have constructed powerful mathematical ideas prior to starting school, and it is essential for teachers to critically examine curriculum documents to ensure that all children have the opportunity to thrive mathematically during the transition to school (Gervasoni & Perry, 2015).

Gould (2014) has also examined the number knowledge of children from backgrounds which might be labelled "disadvantaged" as they start school. In New South Wales, teachers in Government primary schools interview every child within 5 weeks of them starting school, using the *Best Start Kindergarten Numeracy Assessment* (NSW Department of Education and Training, 2007). Utilising the *Best Start* data of 69,545 children, Gould investigated the strength of relationships between social disadvantage as measured by the Family Occupation and Education Index (FOEI) (Centre for Education Statistics and Evaluation, 2013) and aspects of number knowledge. Unlike the research of Gervasoni and Perry in the strengths-based *Let's Count* program, Gould's research focused on what children *could not* do upon school entry; for example, "less than 12% of Kindergarten students start school not being able to produce a correct oral count to ten" (p. 258) and "almost 57% of Kindergarten students started school not being able to state the next number word without recreating the counting sequence" (p. 259). Of course, Gould's work has been framed in such a way to point out the challenges which may be met by children from low socio-economic communities and Gould draws a similar conclusion to Gervasoni and Perry in stating that "carefully designed experiences in early number are particularly important in preschool settings servicing low socio-economic communities to reduce… disparities" (p. 261). However, it would be interesting to see future research which examines the *Best Start* data from a strengths perspective and celebrates the mathematical capacity of children as they start school.

It should be noted that few studies specifically focusing on mathematical content emanating from outside of Australia have been identified during the review period. A New Zealand study focusing on number is that of Young-Loveridge and Bicknell (2014), who explored the impact of using multiplication and division contexts for developing number understandings with five- and six-year-old children. They found that although the children developed a broader range of number facts, they did not necessarily use that knowledge in solving problems. Young-Loveridge and Bicknell encourage teachers to not only give children the opportunity to work with multiplication and division problems and larger numbers, but to also encourage children to recognise the value of derived facts for finding solutions.

4.4 Data

As highlighted in the introduction to this section, data is an area of research which has continued to develop since the previous RiMEA edition. Leading this development has been English (e.g., 2010, 2011) whose work was most prominent among the small number of studies canvassed for the previous review. English has further developed her program of research during the period 2012–2015 in collaboration with Mulligan, drawing on Mulligan's expertise in children's understandings of pattern and structure. One study has been the three-year *Transforming Children's Mathematical and Scientific Development* study which uses data modelling to integrate mathematical patterns and structural relationships with learning in science (English, 2012a, 2012b; Mulligan, Hodge, Mitchelmore, & English, 2013; Mulligan & English, 2014). The project tracked a cohort of 21 highly-able children as they moved from Kindergarten through to Grade 2. The children participated in learning sessions which developed their ability to collect and represent categorical and continuous data. Results from this study demonstrate the metarepresentational competence of young children, with the participants able to develop understandings of the structural features of data representations. In particular, Mulligan et al. highlight the children's ability to notice and integrate the elements needed to create different types of graphs. The researchers suggest that allowing young children to create their own pictographs initially, without scale, is a basis for developing concepts of attribute, frequency and variation, to which they can later add scale. They argue that the creation of data representations allows children to rehearse structural features and use these patterns to gain a deeper understanding of structure and extract meaning from their data (Mulligan et al., 2013).

A study by Kinnear and Clark (2014) has explored data modelling with a group of 5-year-old children. Using a children's picture book as a context, the children participated in a data modelling activity focusing on a prediction problem. The children constructed data models which provided evidence of their ability to use existing data, knowledge of the data context, and probabilistic reasoning to make predictions. The study concluded that children had intuitive knowledge of data representation conventions and intuitive appreciation for variation and probabilistic intuitions (Kinnear & Clark, 2014).

4.5 Geometry

There continues to be a dearth of Australasian research in the area of geometry (and space). In her project examining the range of mathematical abilities possessed by toddlers in New Zealand early childhood services, Lee (2012) highlighted that spatial understanding and exploration was the most common area of mathematics evident in toddlers' activities. Roth and Gardener (2012) have presented work which is largely an examination of the issues surrounding the objective and subjective sides of mathematics, but which utilises a second-grade unit on three-dimensional geometry as a vehicle for this exploration. Their data reveals some interesting insights into young children's conceptualisation of geometry, primarily in relation to children's rationales as to "what makes a cube a cube". For example, the children noted such things as "all the sides that it has they are all the same; they are all the same on each side" (p. 330), and the fact that a cube can be rotated to expose a square each time. These results provide evidence for children's capacity for geometrical thinking, and it would seem that further exploration of children's conceptions of geometry is warranted.

4.6 Problem Solving and Mathematical Reasoning

English and Mulligan's (2013) edited book provided strong arguments that the potential for young children to engage with powerful mathematical processes such as problem solving and mathematical reasoning, is undervalued. Indeed, this is reinforced by the limited amount of Australasian research specifically focused on young children's development of these skills. Lesh and colleagues (2013a; 2013b) have explored the ways in which modelling activities can engage children with powerful mathematical ideas from an early age. Fielding-Wells and Makar (2015) have applied the process of argumentation to challenge young children's expectations of equally likely outcomes, while Makar (2014; McPhee & Makar, 2014) has also examined children's inferential reasoning in the context of statistics education, finding that children are able to use processes of reasoning to develop an understanding of the concept of average. Makar (2014) advocates for the use of ill-structured problems to encourage children to develop their inferential reasoning skills.

5 Contexts for Early Childhood Mathematics Education

Context continues to be a strong theme of much of the Australasian research in early childhood mathematics education. We continue this review by elucidating the notion of context, including how it has been depicted and defined in Australasian early childhood mathematics education literature over the review period 2012–2015. We follow this elucidation by examining what has been learned about context during this period and highlight some avenues for future research. We advocate that mathematical contexts are more than physical settings. They involve people, resources and knowledge but also values, power, perceptions and beliefs of, about and around mathematics and mathematics education. For the purpose of this review we examine the various elements of context, which we propose as, the environmental, the human and the social-cultural contexts.

5.1 *Environmental Context*

The environmental context of early childhood mathematics education in Australasia continues to be a prominent feature of the research literature. Environments both facilitate and limit early childhood mathematics education including the mathematical experiences that are afforded to young children. Over the review period 2012–2015, works focusing on the environmental context of early childhood mathematics have highlighted both affordances and limitations. These works have also not been restricted to geographical location. For example, in their work examining the use of robotics in the early years classroom McDonald and Howell (2012) highlighted the rich mathematical opportunities that are afforded to children "through the provision of an engaging context and specific mastery and performance goals" (p. 649). Similarly, in a project that examined the use of iPads to engage young children with mathematics, Attard and Curry (2012) found that "the introduction and integration of iPads into mathematics teaching and learning appears to have had a positive impact on the teaching and learning of mathematics for the participants involved" (p. 81). Both projects highlight the benefits of incorporating technologies into the physical context of the mathematics classroom. What is also highlighted in this research is the importance of the provision of professional development around the technology if mathematics teaching and learning is to be enhanced (Attard & Curry, 2012; McDonald & Howell, 2012).

The integration of a variety of new technologies into early childhood mathematics education over the review period 2012–2015 has resulted in much work in this area. Such work has offered new insight into both challenges and benefits of such technology in the teaching and learning of mathematics (Attard & Curry, 2012; Goodwin & Highfield, 2013). For example, in Highfield and Goodwin's (2013) content analysis of "educational apps" available in the iTunes App Store they found that the majority of educational Apps for mathematics learning were closely aligned with behaviourist "drill-and-practice" techniques rather than problem solving approaches. Such findings have implications for educators using such technologies in early childhood settings in that other pedagogical strategies might need to be developed to facilitate rich mathematical learning.

Hunting, Mousley, and Perry (2012) communicate a similar message in their work with practitioners in rural prior-to-school settings. In this study, 64 practitioners were surveyed and interviewed around five themes, "children's mathematics learning, support for mathematics teaching, technology and computers, attitudes and feelings, and assessment and record keeping" (Hunting et al., 2012, p. 39). A major finding from the project was the lack of professional development, support and opportunities for networking around and about mathematics in rural areas. If practitioner opportunities to learn about early childhood mathematics are restricted in rural areas, it would be reasonable to suggest that so too are the mathematical opportunities afforded to children. This study shares a synergy with the work of Attard and Curry, and also that of McDonald and Howell in that the environmental context of early childhood mathematics education must be supported by the provision of professional development to enhance mathematics teaching and learning.

It has been well documented that the environmental context of early childhood mathematics education extends beyond the physical classroom and into the sites in which children live and learn (Knaus, 2013; Perry, Gervasoni, & Dockett, 2012). This is explicitly highlighted in the work of Jorgensen (2013) where early-years swimming lessons were found to be rich contexts for mathematics learning and for building mathematical capital prior to formal education. Similarly, Gervasoni and Perry (2013) stressed the "broad range of formal mathematics knowledge that many children construct" in home and preschool contexts (p. 344). Such contexts provide holistic views of the mathematical learning of young children, and are important inclusions in early childhood mathematics education. Further research into how these contexts might be incorporated into formal educational settings is needed, particularly if mathematics is to be supported across and between the contexts in which children live and learn.

5.2 *Human Context*

Human contexts are a pertinent feature of the Australasian research literature in early childhood mathematics education. During the review period 2012–2015, adults and their images of young children have emerged as a significant consideration, particularly with regard to the importance of noticing young children as capable and proficient users of mathematics. Muir (2012) highlighted this importance in a project that investigated parents' perceptions of and about mathematics education. Survey data from 34 parents were collected and analysed. Results indicated that parents did not perceive that they were well informed about mathematics education, and in turn felt that they were limited in their ability to notice and support mathematics learning at home. A recommendation in this study was for educators to work more closely with parents in ways that informed them of and about early childhood mathematics education.

Parents were not the only focus in the literature over the review period 2012–2015. As part of a wider project investigating how mathematics learning might be supported as children start school, Goff, Dockett, and Perry (2013) presented data arising from initial discussions with school principals. Findings presented in this work suggest “that numeracy might not be a key priority for schools as children make the transition from preschool to primary school” (Goff et al., 2013, p. 367), with most principals suggesting that social and emotional development are the priority during this time. Such findings have implications for children, particularly if their existing mathematical understandings and strengths are to be supported and expanded in the primary school setting. Principals are a key driver of practice and therefore this is an area that warrants further examination.

Educators have also been a key feature of the Australasian research literature in early childhood mathematics over the review period 2012–2015, with much of this research conducted *with* educators rather than *on* educators. Perry’s (2013) long-term work with preschool and primary school teachers’ mathematics education professional development is an example of such research. Throughout this project researchers worked in collaboration with educators to “develop an instrument—*the Reflective Continua*—designed to assist such educators to notice children’s mathematics and to plan for further mathematical experiences” (Perry, 2013, p. 783). Findings from this project demonstrate the importance of working with educators in their local contexts to guide and change professional practice.

Perry and MacDonald (2015) and MacDonald (2015) have also examined the professional development of early childhood educators as part of the *Let’s Count* early numeracy program. Evaluation of the program revealed demonstrable shifts in educators’ beliefs and attitudes about, expectations of, and aspirations for the mathematical learning of young children, and these shifts have resulted in changes in the mathematical learning opportunities for children in pre-school settings (Perry & MacDonald, 2015).

Dockett and Goff (2013) call for a shift in how the adults in the lives of young children come to recognise and respond to the mathematical understandings of children. They suggest that this involves not only recognising “[t]hat mathematics exists across many diverse contexts” (Dockett & Goff, 2013, p. 772) but also recognising the expectations that adults bring to these contexts. A key message embedded within their work is that “we usually find what we expect” (Dockett & Goff, 2013, p. 773). A similar message is found in Clarke’s (2013) work with children with Down Syndrome, where she advocates the need for researchers to think carefully about the methodological approaches they adopt. Examining what educators and researchers expect to find when working with young children in mathematics education might be a noteworthy consideration in determining what has and can be found, built upon and supported. Working together might be a way to achieve this goal.

5.3 Socio-Cultural Context

Social-cultural contexts continue to be a key issue in Australasian early childhood mathematics education research, and over the review period 2012–2015 much work has been done in this area, particularly in relation to young Indigenous children. For example, Warren and Miller (2013) reported the outcomes of a program (*Representations, oral language and engagement in Mathematics: RoleM*) that focused on an oral language approach to teaching mathematics. Data derived from a sample of 230 Indigenous Australian students demonstrated that young Indigenous Australian students are capable and proficient users of mathematics, and that a language focus in mathematics education programs can both support and scaffold this learning (Warren & Miller, 2013). An important message in this study was the significance of developing "a mathematical repertoire in a culturally sensitive manner" (Warren & Miller, 2013, p. 167).

Papic (2013) discussed similar notions in her work on improving the numeracy outcomes for young Australian Indigenous children. This work presented outcomes from a series of studies that "focused on developing children's early algebraic and mathematical reasoning skills and teachers' pedagogical and mathematical content knowledge" (Papic, 2013, p. 253). A key message from this synthesis of research was the emphasis on not only developing culturally sensitive practices, but also the need to incorporate hands-on experiences in mathematics programs. Such recommendations provide valuable windows for future researcher-educator collaborations.

6 Conclusion

The synthesis presented in this chapter has demonstrated that there is much to celebrate about Australasian early years mathematics education research. Australasian researchers demonstrate strength in research which identifies the mathematical capacities of young children, and which considers the range of contexts—physical, social and cultural—in which children's mathematical understandings develop. However, as always, there are areas for further development.

A key area which should be considered in future research is the socio-political climate and its impact on mathematics education for young children. In particular, the impact of, and connections between, Australia's two national curricula for young children is still in need of scrutiny. Similar work continues to be needed in New Zealand as well.

As with the previous edition of RiMEA, Australasian research related to mathematical content areas is dominated by projects focusing on patterns and algebra, measurement and number. While the current review period has seen growth in the focus on data and modeling, geometry continues to be under researched. It would seem that there is great potential for new knowledge generation in relation to young children's engagement with geometry concepts, and we suggest that this could be an important focus of research as we enter the next review period. Furthermore, research focusing on children's development of problem solving and reasoning skills would provide further insight into the ways in which children engage with mathematical concepts.

An interesting shift in the literature in early childhood mathematics education over the review period 2012–2015 is the move toward an examination of the environmental, relational and social-cultural practices around mathematics. This shift is important as not only does it build on what is known about early childhood mathematics education but it also signifies a move toward marrying this knowledge with practice. In our opinions, such movement in the field is not only exciting but a cause for celebration.

The equivalent chapter in the previous review concluded with a list of areas for consideration in future research. While the research presented in this chapter has certainly contributed new knowledge to the field of early childhood mathematics education, it is our conclusion that the call for further research made in the previous review remains current. In particular, we encourage researchers to continue to contribute to the following important areas for research:

- Continuity and change across the prior-to-school to school transition;
- Under-researched mathematical content areas (namely, data, geometry, and problem solving and mathematical reasoning);
- Ramifications of new and/or revised curricula and standards in Australia and New Zealand;
- Impact of school entry assessments;
- Professional learning programs for early childhood mathematics educators; and
- Roles of families and communities in young children's mathematical learning.

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Chapter 10
Tertiary Mathematics Education

Mary Coupland, Peter K. Dunn, Linda Galligan, Greg Oates and Sven Trenholm

Abstract Mathematical and statistical education research relevant to students in tertiary settings is reviewed. This is an expanding field and is evolving as researchers shift their attention from the reporting of innovations in lecturing practice and course design to include a deeper consideration of the experiences of educators and learners in this space. The purposeful inclusion of group work and discussion, focus on concepts, authentic problem solving, interactions in lectures with student response systems and online learning are all changing the way mathematics and statistics are taught at this level. The authors note that traditional measures of achievement in the form of exam marks are still relied upon, and call for theory-based and theory-building research including investigations of depth of understanding, and of transfer of knowledge and skills to new situations. An emphasis on the learner's experience and the employment of cross-disciplinary teams of researchers are further suggestions.

Keywords Undergraduate mathematics and statistics education · Digital technology in learning mathematics and statistics

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1 Introduction

This chapter surveys the growing field of research into mathematics learning and teaching at the tertiary level. Significantly, the Australian Mathematical Society established in 2015 a Special Interest Group in Mathematics Education (SIGME) which has among its goals the promotion of inquiry and discussion about tertiary mathematics education. This interest is worldwide: 2015 sees the first issue of a new journal, the *International Journal of Research in Undergraduate Mathematics Education*, with six Australasians on the Editorial Board. In addition, 2016 will see the launch of a new international conference series with INDRUM 2016, the first conference of the International Network for Didactic Research in University Mathematics. The four-yearly International Congress on Mathematical Education (ICME) series continues to include a Topic Study Group on mathematics education at tertiary level, and in 2012 at ICME there was also a report of a Survey Team on *Key Mathematical Concepts in the Transition from Secondary School to University*.

Against this background, we located relevant reports by searching journals and refereed conference proceedings. The field is well served by conferences, including the annual Mathematics Education Research Group of Australasia (MERGA) conferences and the biennial Delta conferences. Increasingly, selected papers from Delta proceedings appear in special issues of the *International Journal of Mathematical Education in Science and Technology* (*IJMEST*). The teaching of statistics to tertiary students is celebrated at the Australian Conferences on Teaching Statistics (OZCOTS), and MacGillivray, Martin, and Phillips (2014) compiled topics from the three OZCOTS held in 2008, 2010 and 2012.

Space limitations necessitated favouring journal articles, book chapters and books over refereed conference presentations, and sometimes only one of a set of related papers by the same author or group was included. Research in this field will grow as collaborations between mathematicians, statisticians and mathematics educators increase, and we hope to see further research at the boundaries of technical and social fields of practice. There is a continuing place for description and evaluation of innovations in teaching practice, but we also hope to see an increasing number of reports of research grounded in theories of learning and utilising a variety of empirical research methods.

The chapter is presented in sections headed “Teaching”, “Learning”, “Transitions”, “Digital Technologies”, “Tertiary Statistics Education”, and “Suggestions for further research.” Naturally there is some overlap.

2 Teaching

This section considers the many studies whose focus was primarily on the design and delivery of courses and subjects.

2.1 Mathematical Topics

The 2008–2011 review (Perry et al., 2012) noted over 20 papers addressing a particular mathematical topic. Although the smaller number of papers explicitly discussed here may suggest a reduction in such studies, research featured elsewhere in this chapter indicates that investigating ways to teach specific content remains important. Hong and Thomas (2015) investigated students' perspectives of differentiation and integration in graphical construction. Klymchuk used paradoxes and counterexamples in statistics (Klymchuk & Kachapova, 2012) and calculus (Klymchuk, 2014). The use of technology in specific mathematical contexts was investigated by Getenet and Beswick (2014), examining graphs of logarithmic functions, and by Blyth (2013) using Maple to explore maximum problems without calculus.

Mathematical topics also remained a focus of conference presentations, including the value of the mathematics of juggling (Bier, 2013) and an argument for removing the teaching of limits from first-year calculus courses (Rice, 2013). At the 2012 ICME in Seoul, Oates (2012) discussed the implications of eleven calculus papers presented at the 2011 Delta conference.

In computational and discrete mathematics, Skerritt (2014) designed a second-year mathematics course with accompanying textbooks to introduce students to computer algebra systems in computational mathematics; Sugden and Stocks (2013) explored the potential of the television quiz program Letters and Numbers to engage student interest in a range of fundamental computer science and mathematics concepts. They concluded that treating the Numbers Game as a simple syntactical structure problem, rather than a complex algebraic problem, is a valuable metaphor for the application of computation to Computer Science and Discrete Mathematics subjects.

In algebra, Sugden (2012) continued the game-theme, describing his use of the Number Crunch game to teach elementary algebra to under-prepared students in first-year undergraduate mathematics courses. Various aspects of lecturer practice, including visual and linguistic tools, in the teaching of linear algebra were examined by Hannah, Stewart, and Thomas (2013).

Attracting attention in mathematics education is the notion of threshold concepts: concepts essential to knowledge and understanding within particular disciplines as they act like conceptual portals that, once crossed, enable students to comprehend a topic not previously understood. Hoadley, Kyng, Tickle, and Wood (2015) provided quantitative evidence for the identification of threshold concepts in the finance curriculum, incorporating finance, modelling and statistics. Loch and McLoughlin (2012) considered the teaching of threshold concepts in engineering mathematics using MathsCasts, concluding that they provided a productive basis for pedagogy worthy of further investigation. In contrast to the operation of threshold concepts, the learning of new material may be inhibited by *cognitive obstacles* when a student's thinking is constrained by features of previously learnt concepts. Mallet (2013) illustrates this difficulty in the case of scalar line integrals, where the interpretation of integration as the area between a curve and an axis inhibits the learning of more advanced integration.

2.2 Policy, Curriculum, and Course Design

Studies here considered the overarching principles for tertiary mathematics course delivery. Paterson and Barton (2013) asked questions about the outcomes expected from courses, including whether establishing a common view about learning outcomes for undergraduate students is possible. In their interviews with mathematicians internationally, they found a strong understanding of the needs and aspirations of all undergraduate students, mathematics majors or not, and described how they had gained a sense of shared purpose reaching beyond the specific matters of content. Trenholm, Alcock, and Robinson (2012) reviewed research in the transformation of tertiary lecturing practice as a result of technology advances, and made observations concerning the instructional context, inherent complexities of technology use and recommendations for effective implementation of technology. Lim, Thiel, and Searles (2012) described their success in improving pass rates in a second-year course for students enrolled in engineering and mathematics programmes through introducing quizzes, providing a mark incentive.

Policy and practice come together when researchers join the debate at both national and institutional level concerning quantitative skills in Science, Technology, Engineering and Mathematics (STEM) courses. A consistent theme in most studies and reports is how to establish collaboration between disciplines to build a unified approach to quantitative skills across subjects and institutions. Matthews et al. (2012) conducted an extensive study as part of the *Quantitative Skills (QS) in Science* project. The report exposed the detrimental effects of disciplinary isolationism on developing science graduates with the necessary quantitative skills. Rylands, Simbag, Matthews, Coady, and Belward (2013) interviewed academics about QS in science, and concluded that a significant factor in determining the success of cross-disciplinary teaching was the belief-set of the academics involved. Without a commitment to examining beliefs about the role of service teaching, other strategies to encourage collaboration (committees, encouragement from Deans) were unlikely to be successful or sustained. Flegg, Mallet, and Lupton (2012) also recommend that mathematicians collaborate with academics in other departments—in this case with engineering departments—in order to design curricula based on relevant problem-solving tasks so that students see the relevance of mathematics to their future studies and careers. They also found, through student reports in surveys and interviews, that problem-solving tasks helped students to learn and retain mathematical skills.

2.3 Numeracy and Learning Support

Rylands and Shearman (2015), and Shearman, Rylands, and Coady (2012) considered learning support for students in large first-year courses without assumed mathematics knowledge, finding that many students enrolling in mathematical subjects designed for non-STEM majors have minimal mathematical skills and poor motivation with subsequent negative effects on performance. The authors implemented alternative pedagogies in an attempt to improve student performance. Rylands (2013) examined mathematicians teaching large first-year service courses. She claimed that this is a very difficult task, and concluded it was up to mathematicians to educate those whom they serve about the challenges faced and about what is realistic for their students.

Jackson and Johnson (2013) highlighted the critical problem of students entering university lacking basic mathematical skills. They described the development of the Maths Skills programme at La Trobe University focusing on basic skills relevant to each science discipline. The programme was designed through close collaboration between science subject coordinators and the project leader, a mathematician, and included contextualised resources: the confidence of the participating students improved. Students often need support during a second subject if they narrowly passed a preliminary subject, and the report by Hillock, Jennings, Roberts, and Scharaschkin (2013) demonstrated the success (measured by pass rates and student surveys) of a programme of additional tutorials and targeted examination revision for an at-risk group of engineering students. Galligan (2013a) described a systematic approach which successfully developed academic numeracy at the university, program, course, and individual student and teacher level.

2.4 Professional Learning, Beliefs and Practices, Theoretical Perspectives

New projects and developments in the field continue to emerge, for example, in the 2012 project *First Year in Maths* which aims to improve the first-year experience for undergraduates and reduce attrition rates through building the leadership capacity of first-year coordinators in mathematics, engineering, science and commerce (e.g., see King & Cattlin, 2015).

Beliefs and practices were also the focus of several studies. Balatti and Belward (2012) described action research by a team of lecturers to improve the learning experience of first-year students. They used the three categories of saying/thinking, doing, and relating, to explore practices. Beliefs and practices that had remained unexamined were abandoned, modified, or at least, questioned. Galligan et al. (2013) analysed first-year lecturers' perceptions of topics and skills needed in the respective courses that they teach and their perceptions of students' preparedness for these topics. Surith (2015) reported on the nature of mathematical knowledge and its effect on the dynamic and static nature of lecturing. (For research on the education of pre-service teachers, see Chap. 15).

The last group of research reports reviewed in this section are examples of studies that are either seeking to establish new theoretical perspectives in undergraduate mathematics teaching, or are derived from existing perspectives in other domains. Barton, Oates, Paterson, and Thomas (2014) summarised the work in a successful professional development project based on Schoenfeld's resources, orientations, and goals framework. Two papers from this study examined individual lecturers' practices. Paterson and Evans (2013) described significant changes in Evans' practice based on the opportunities both to observe others teach, and to examine and discuss her own practice within this forum; while Kensington-Miller, Sneddon, and Stewart (2014) described the shifts in academic identity of two lecturers, a mathematician and a mathematics educator, as they both made changes to their teaching practice by implementing new questioning techniques. Phan (2013) conducted a structural equation analysis to examine expectancy-value and cognitive process outcomes in mathematics learning. He concluded that academic achievement in mathematics will be influenced positively by both non-cognitive process outcomes, in this case, self-efficacy expectations and task value; and cognitive process outcomes, including a deep-learning approach and reflective thinking. Summit and Rickards (2013) explored a constructivist approach to mathematics laboratory classes to discuss a framework for mathematics education that included higher order levels of learning.

3 Learning

3.1 The Student Experience

While many research studies focused on first-year students in service courses such as Science or Engineering, Worsley (2014) followed a cohort of mathematics students over the 3 years of their degree and investigated how their study approaches and views of mathematics changed in that time. Among her findings were that students with a "major" in mathematics reported an increase in enjoyment but a decrease in perceived relevance of their studies as they progressed to third year. Independent learning behaviours were found to be related to achievement. One is led to ask how such independence in learning is to be encouraged. Mendiolea (2013) investigated student preferences for written worked examples with five levels of increasing information and guidance, and a screencast, concluding that offering choice of level of information invokes student agency in relation to their own learning.

3.2 Group Work and Language

Group work is increasingly part of the experience of undergraduate students of mathematics. Dalitz (2014) investigated the kinds of talk that took place in a single study group of three first-year students chosen as a case study. Episodes of mathematical talk were coded for type of interaction and for cognitive level. The absence of high-level cognitive activity could be attributed to the choices made by the students not to attempt abstract questions. Dalitz questioned how interventions might encourage students to recognise the need to improve their conceptual knowledge, and support them to do so. Clark, James, and Montelle (2014) also collected data while groups of students worked collaboratively, but this time on open-ended problem-solving tasks deliberately chosen by the instructors to provide a high level of challenge in an advanced calculus subject. Tutors provided guidance, not answers. Field notes were transcribed after each session and then coded using established problem-solving dimensions based on observations of expert problem-solvers. It was found that two further dimensions were needed, Questioning and Group Synergy. The latter could be regarded as similar to Dalitz's use of co-construction as a type of interaction, but offers a richer category as it includes encouragement of group activity and evidence of leadership. Sheryn and Ell (2014) investigated students' prior opinions of group work and compared that with their thoughts and opinions after a semester of Team Based Learning. There were tentative suggestions that students benefited from the group work in a way similar to the study by Clark et al. (2014): some students reported that "as groups they reached a level that they would not have reached alone" (Sheryn & Ell, 2014, p. 875). This study also raised questions about the nature of learning in undergraduate mathematics in terms of the tension between gaining skills and knowledge on the one hand, and learning to become mathematicians on the other.

3.3 Assessment

The wide variety in approaches to research in this field is evident in the articles identified as having assessment as their major theme. While not presenting data, Seaton (2013) reflected on the success of the "La Trobe Method" of partially-marked, fortnightly assignment work, with explicit feedback to students beyond achieving the correct answers. Khan (2015) concluded that carefully designed closed-book examinations were preferable to open-book exams because more students achieve higher results, based on numerical data for one subject over three semesters. Groen et al. (2015) advocated for mastery learning in first-year cohorts, writing about three subjects over four semesters and using both quantitative and qualitative methods.

From a different perspective, Trenholm, Alcock, and Robinson (2015) examined feedback practices of instructors in fully asynchronous online undergraduate courses. Survey methods were used and relationships between assessment practices and the approaches to teaching of 66 participants were investigated. Varvasky, King, Coady, and Hogeboom (2014) provided examples of marking rubrics, a resource booklet and a website, demonstrating ways to move towards improving student learning by being clear about valuing problem solving and communication.

A ubiquitous part of the experience of teaching in higher education is the collection by universities of students' opinions of their learning experiences. King, Loch, and Rylands (2013) investigated what their own students regarded as feedback in mathematics, in comparison with the feedback to students that was provided in their subjects. An important conclusion was that "…teaching staff see a much broader range of student-staff and student-resource interactions as providing feedback than students" (King et al., 2013, p. 10). Students saw feedback as "advice about ways to improve" and many said that they did not receive this. Staff, however, saw feedback as including marked assignments, even if machine marked, as well as opportunities to talk with tutors and lecturers.

4 Transitions

In the 2004–2007 review (Forgasz et al., 2008), for the first time a chapter on adults returning to study mathematics appeared. This area has now been incorporated into the Transitions section of the present chapter and includes research on general adult numeracy, mathematics in vocational education, and transition to work.

4.1 Transitions: Tertiary to Work

As with previous reviews, few papers concentrated on the university-to-work transition. Wood, Mather et al. (2012) expanded their research to include comparisons with five other countries by investigating undergraduate students undertaking a mathematics course and their views on mathematics in future studies and careers. A report from the Australian Council for Educational Research (Edwards, Pearce, Perkins, & Brown, 2014) used focus groups to explore the experiences and attitudes of women, mature-age workers and new graduates towards the engineering labour market, and also explored the views of school students about the engineering workforce. Typical comments were related to the mismatch between the importance of mathematics on the one hand, and the lack of clarity around the prerequisites and assumed knowledge of mathematics for engineering on the other. The study suggested many employers are concerned about the decline in participation in STEM subjects at school level, and the resultant impact on the pipeline of students to relevant tertiary and employment. Edwards, Perkins, Pearce, and Hong (2015) centred their report around the investigation of 37 universities' work integrated learning (WIL) programmes in STEM, using a combination of interviews and surveys of staff to investigate the effectiveness of WIL to equip graduates with capabilities to meet the expectations of employers. One conclusion was the need for more employer engagement.

While there was a small focus on nursing numeracy in this period, a major compilation of research was published in an eight-paper series, "Safety in Numbers", in the journal *Nurse Education in Practice* and summarized in the introductory article by Weeks, Sabin, Pontin, and Woolley (2012). A complementary article (Coben & Weeks, 2014) highlighted the characterization and authentic assessment of competence in nursing and suggested the use of such a model in other vocational areas such as aviation and finance. Also suggested was the use of a boundary-crossing research approach between school-taught and work-related knowledge as an appropriate theoretical framework, as used by others in mathematics and mathematics education (e.g., Goos, 2015). Galligan (2013b) utilised a conceptual framework based on Valsiner's Human Development Theory to trace nursing students' development of essential academic numeracy skills.

4.2 Transitions: School to University

In 2012 the ICME Survey Team 4 examined the transition from secondary school to university mathematics (Thomas et al., 2012), suggesting an increased international interest. The Survey Team concluded that "coordination and dialogue across education levels" is largely absent. The research is maturing, with some theoretical frameworks emerging along with growing interest in cognitive, curricular and pedagogical issues. The theoretical frames, however, are from Europe and the US and in the main have not been transferred to Australia. The review in this chapter suggests that cross-sector dialogue is improving and greater cooperation and interest in each other's expertise is emerging.

Many papers appeared on the level of preparedness for university and the consequences at university (e.g., Galligan & Hobohm, 2015; Loughlin, Watters, Brown, & Johnston, 2015) and diagnostic testing, interventions and reduction in higher failure rates (King & Cattlin, 2015) with a review by Maltas and Prescott (2014).

As in previous MERGA reviews, there were papers on bridging mathematics programs. A study in the context of health science students (McNaught, 2013) analysed a support program designed for students before they enter programs. A series of papers on bridging mathematics (e.g., Gordon & Nicholas, 2013; Poladian & Nicholas, 2013) addressed the issue of insufficient mathematics preparation, while other papers suggested a mismatch between students' and lecturers' perceptions of preparedness (Abdulla et al., 2013; Dalby et al., 2013; Galligan et al., 2013; Wandel et al., 2015).

New Zealand appears to be leading in approaching transitions from a "know how" viewpoint. Thomas and Klymchuk (2012) reported on differences in teaching style, assessment and curricula, and called for mechanisms such as cross-sector visits to assist in establishing meaningful dialogue between those who teach mathematics in schools and universities.

4.3 Vocational and Adult Education

The 2004–07 review included a review of research in vocational education. Some of this research was supported by the National Council for Vocational Education Research (NCVER). In recent years NCVER commissioned reports that are summarised by Beddie (2015). Current research includes studies on production workers and apprentices (Black, Yasukawa, & Brown, 2013; Karmel, Roberts, & Lim, 2014). Circelli, Gillis, Dulhunty, Wu, and Calvitto (2012) mapped the Adult Literacy and Life Skills (ALLS) survey and the Australian Core Skills Framework (ACSF). A report by Berghella and Molenaar (2013) investigated the capacity of the VET (Vocational Education and Training) workforce to address workplace numeracy needs and found none of the 44 participants had a specialist adult numeracy training qualification; perhaps in response, another report described the development of materials for the Foundation Skills Training Package (Walker, 2013).

With the Australian Qualifications Framework and its Pathways Policy in support (Australian Qualifications Framework, 2013), it is not surprising that research has emerged in this field. While evidence exists that VET is an effective pathway to higher education, particularly for students from low socio-economic backgrounds (Langworthy & Johns, 2012), particular issues within the quantitative disciplines may exist. Kilpatrick (2015) and Penesis et al. (2015) found that the mathematics gap between VET and university is large with many VET qualifications in education, engineering, business and health science having little to no mathematics content.

Two NCVER reports focused on transitions between VET, higher education and work. Wheelahan et al. (2012) used both Australian Bureau of Statistics data and interviews with stakeholders and found “specific problems with students’ level of preparation in mathematics, particularly around competency-based training” (p. 7). Callan and Bowman (2015) wrote about VET providers delivering degrees and again highlighted readiness for mathematics as an issue. An earlier Australian Workforce Productivity Agency report (2013) and a report by The Australian Industry Group (2013) gathered details from employers about literacy and numeracy issues in the workplace and their impact on business. They found that 75 % of employers reported that their business was affected by low levels of literacy and numeracy with an effect on productivity. Interestingly, Black, Yasukawa, and Brown (2013, 2014), in an ethnographic investigation of three manufacturing companies, suggested that some employers were not concerned with employees’ level of literacy.

On adults learning mathematics, Roth (2014) used cultural-historical activity theory as a framework when documenting how electrician apprentices in formal classes learn a trigonometrical approach to conduit bending that is not used at all on the job. In describing how individuals reacted to this situation, he found that boundary crossing was not as useful as the theory of personality.

The ICMI working group on educational interfaces between mathematics and industry included three chapters from Australia and one from New Zealand, with each feeding the work contexts back into the school (FitzSimons & Mitsui, 2013; Geiger, 2013; Stillman & Ng, 2013). There was an emerging framework for ethnography of adult mathematical and numeracy practices (Smedley, 2013). Whitten (2013) discussed the importance of beliefs about mathematics for adults.

The Programme for International Student Assessment (PISA), Programme for the International Assessment of Adult Competencies (PIAAC), and Adult Literacy and Life Skills Survey (ALLS) are tests that inform government policy. Gal and Tout (2014) examined commonalities and differences in how adult numeracy and mathematical literacy were assessed in PIAAC and PISA, and highlighted challenges associated with interpretation of results. It is important that these tests are well scrutinised as both the New Zealand and Australian governments have strategies to improve literacy and numeracy skills to Level 3 or above based on the PIAAC assessments (Department of Education and Training, 2015).

5 Digital Technologies

Our review found 35 studies involving the use of digital technologies in tertiary mathematics education, the bulk of which were published in peer-reviewed journals and proceedings. Consistent with a wider interest in the first-year experience, most of these studies focus on the student experience of using digital technologies in introductory-level tertiary mathematics instruction, particularly in calculus and statistics.

Table 10.1 provides a summary. Many studies used a mixed methods approach with most using samples smaller than 100. As shown, most of these studies still reflect the use of technology as a “simple pedagogical adjuvant” (Artigue, 2010, p. 467) or assistive tool to traditional pencil-and paper-based instruction, rather than tools that can transform current pedagogical approaches.

In this regard, it is helpful that some primarily reflective work is adding to the academic discourse on pedagogical transformation through the use of digital technologies. McMullen, Oates, and Thomas (2015), for example, discussed how digital technologies are being “integrated” into a large calculus course while Maclaren (2014) discussed the potential for Tablet PCs to transform traditional pedagogy. Finally, Bardini, Oldenberg, Stacey, and Pierce (2013) discussed how calculus students’ understanding of the equality symbol is transformed when using technology.

This review is consistent with the 2008–2011 review (Perry et al., 2012) as well as wider analyses (e.g., Laborde & Sträßer, 2010) which suggest a gulf remains between the promise and actual benefit derived from using digital technologies. For

Table 10.1 Summary of tertiary mathematics education research focused on the use of digital technology, 2012–2015

Digital technology use		Incidence	Total
Traditional (face-to-face) pedagogy	Overall Use (e.g. “integration of ICT”, use/non-use of “hyper learning”)	4	4
Devices	Tablet PCs (e.g. “pen-enabled”, iPad)	6	8
	Electronic Voting Systems (e.g., “VotApedia”)	2	
Systems	Computer Algebra Systems (CAS; e.g., “Maple”, “TI-Nspire CAS calculator”)	6	14
	Computer-aided Assessment (CAA)	1	
	Recorded Lecture Video or Screencasts (e.g., “MathCasts”)	7	
Computer-mediated Communication	Word Equation Editor	1	1
Partially web-mediated (e.g., blended)	Course development and redesign	2	2
Fully web-mediated or online pedagogy	Tool/Device use (e.g., Tablet)	2	6
	Pedagogy (i.e., assessment, course development, adaptive CAA and synchronous tutorials)	4	
		Total	35

example, although student perceptions of recorded lecture videos continues to be positive (e.g., Loch, Jordan, Lowe, & Mestel, 2014), little is reported about the nature of mathematical thinking associated with their use. Moreover, of potential concern, some studies report opposing effects: from a relatively strong methodological basis, Varsavsky (2012) found Australian students who did not use CAS in secondary mathematics did better in their first-year mathematical methods courses than those who did use CAS. However, with some exceptions, Bardini, Pierce, and Vincent (2013) found no relation between the use of CAS in high school and first-year university students’ understanding of functions.

Equally concerning, Trenholm, Alcock, and Robinson (2012) found additional evidence that fully online (FO, where the instruction is entirely web-mediated) mathematics courses were becoming more commodified: less reliant on human interactions, compared to earlier FO courses, and more reliant on computer-human interactions. This raises the question about how digital technologies are being integrated into differing instructional contexts: if they can be transformative, then how? Indeed, as reported in earlier reviews, the ability to answer these questions is being outpaced by the advancement of digital technologies. These challenges appear even more pointed in FO teaching and learning where rapid growth is being projected (Ross, 2014). Numerous challenges in mathematics instruction are recognized (Trenholm et al., 2012), yet very few studies investigate pedagogical transformation in relation to FO instruction. Apart from the previously mentioned study only two others directly address pedagogy: McDonald (2013) investigated the contribution of online synchronous tutorials in supporting the teaching and learning of undergraduate introductory statistics while Quinn, Albrecht, Webby, and White (2015) examined FO course development processes occurring over 3 years and ten iterations.

Insights gained from working in the FO environment may benefit the transformation of traditional face-to-face course pedagogy. For example, recorded lecture videos or screencasts are being considered as a means of providing feedback (Robinson, Loch, & Croft, 2015) or for peer instruction (Croft, Duah, & Loch, 2013), although the actual benefit to learning remains unclear. Perhaps the best known example of FO pedagogy transforming traditional approaches is the so-called "flipped classroom" (O'Flaherty & Phillips, 2015). Loch and Borland (2014) provide a brief overview and suggest research questions. Seaton, King, and Sandison (2014) write about "flipping the maths tutorial" as a pedagogical innovation in tertiary settings but their work does not implicate digital technologies or lectures. Internationally, in contrast, several journal articles were found (e.g., Maxson & Szaniszlo, 2015).

5.1 Future Directions for Digital Technology Research

Associated research extends from the use of digital technology tools in traditional face-to-face courses to more immersive forms of mediation such as fully asynchronous online courses. Across this continuum, much remains to be understood about how digital technologies may (or may not) benefit tertiary mathematics education. There is a need to investigate how digital technologies may transform and not merely assist current pedagogical approaches. Importantly, we need to move beyond studies of how students and staff *perceive* or *experience* the simple (or novel) use of digital devices or systems (cf. Maxson & Szaniszlo, 2015). Though valuable to some extent, such studies are not sufficient for understanding the resultant nature of student learning, including transfer, retention, as well as depth of understanding, the latter not necessarily reflected in academic achievement measures such as examinations or course grades.

In the area of online learning in particular, work is required to produce a more fine-grained understanding that takes into account the nature of associated mathematics (e.g., introductory vs. more abstract higher level mathematics, applied vs. pure) and student demographics (e.g., students in engineering vs. pure mathematics). This includes the production of useful tasks. We need to investigate the nature of "learning" in this space, and what are the best ways to "teach", and whether these are different for more mature learners.

6 Tertiary Statistics Education

Research in statistics education continued to focus on first-year classes, given their typically large enrolments and unique challenges as service courses. Recent Australasian research has centred on course delivery, the teaching of concepts over computation, and the use of software.

6.1 Course Delivery

David and Brown (2012) redesigned their introductory statistics course, reducing the emphasis on traditional lectures and tutorials in favour of online tutorials, using Excel, computer-based skills testing, web-based learning materials and smaller group activities. A student survey yielded favourable responses, with online tutorials identified as the most useful resource, and textbooks the least.

Khan's (2013) redesign occurred in stages. He compared three different methods of teaching delivery in first-year statistics, gradually changing components over 2 years. Changes included increasing the availability of recorded lectures, adding the use of Student Response Systems (SRS), and amendments to the assessment and the face-to-face delivery arrangements. Despite this, no significant difference was detected in the final course marks, though examination marks slightly increased with each enhancement.

Others have studied specific components of delivery. Mobile phone-based SRS were studied in large statistics classes at three universities, from the point of view of the students (Dunn, Richardson, Oprescu, & McDonald, 2013) and instructors (Dunn, Richardson, McDonald, & Oprescu, 2012). In general, the use of SRS was favourably received, and students were not distracted by the use of mobile phones in class. To complement lectures, Dunn, McDonald, and Loch (2015) found that the use of short screencasts (called StatsCasts) helped students engage with important topics.

Lenard, McCarthy, and Terence (2014) argued that a first-year statistics course may include the study of ethics (e.g., in sample size calculations). To encourage this, they provided example exercises encouraging students to engage with the National Statement on Ethical Conduct in Human Research.

From a different aspect of course delivery, Bilgin, Bulger, and Robertson (2013) studied 227 students and found an association between the amount of active and collaborative learning and students' satisfaction in tutorials. To encourage collaboration and student participation, Zhang and Govindaraju (2012) used "What if?" questions, arguing that students would not ask such questions themselves for fear of appearing silly in front of peers. Petocz et al. (2014) studied the use of "peer learning" outside formal class times, finding that first-year psychology students who participated found such sessions more useful than lectures (which can be overwhelming in content) and tutorials (because others students may be less committed to their learning). Hood, Creed, and Neumann (2012) studied the relationship between achievement and attitudes to statistics of second-year psychology students, finding that performance in a first-year statistics course (22 %), effort (8 %) and expectancies (2 %) significant contributed directly to achievement in their cohort.

6.2 Conceptual Understanding

Students' conceptual understanding has been studied within specific topics such as probability (Klymchuk & Kachapova, 2012), random variables (Kachapova & Kachapov, 2012), conditional probability (Reaburn, 2013), *P*-values (Reaburn, 2014), mutually exclusive and independent events (Ollerton, 2015), and the role of sample size in parameter estimation (Noll & Sharma, 2014). More broadly, Richardson, Dunn, and Hutchins (2013) studied students' knowledge of nine words used in statistics and research. One finding was that "significant" was often defined by students using a general English definition at the start of a course, but many attempted a statistical definition after the course (though most were incorrect). The authors concluded that students may progress through phases in understanding technical words: first identifying that a general English word has a statistical meaning, and subsequently learning that meaning.

Pfannkuch et al. argue for facilitating conceptual development of statistical topics using computationally-intensive methods (e.g., Budgett et al., 2013; Pfannkuch, Wild, & Parsonage, 2012; Pfannkuch, Wild, & Regan, 2014) such as bootstrap or randomisation methods. This development is supported by "conceptual pathways" (Pfannkuch & Wild, 2012) and software (iZNight, www.stat.auckland.ac.nz/~wild/iNZight and Visual Inference Tools, www.stat.auckland.ac.nz/~wild/vit/).

Another way in which technology is used to teach concepts is through a virtual population, which eliminates many of the practical impediments to authentically teaching epidemiological and statistical concepts involving data collection from humans. Baglin, Bedford, and Bulmer (2013) examined the use of the virtual population in *The Island* for postgraduate online biostatistics students, while Donnison et al. (2014) studied on-campus undergraduate students in applied health. Both studies reported very favourable responses by students to the virtual population.

6.3 Use of Statistical Software for Analysis

While the use of statistical analysis software is becoming a key outcome of statistics courses (Bilgin, 2013), the use of technology described above only assists students to understand statistical concepts. However, technology can support performing statistical analyses also. While both ways of using software contribute to learning statistics, research has focused on the use of technology for conceptual understanding. However, Baglin (2013) proposed a theoretical framework for teaching analytical software in statistics classes. Baglin and Da Costa (2012, 2013) then compared two methods for teaching SPSS (Guided Training; Error-management Training), firstly using an experimental (2012) and then a quasi-experimental design (2013) addressing some weaknesses of the original experiment. No measurable difference in mean training difficulty, satisfaction, statistical package self-efficacy or training anxiety was detected. One difficulty in the studies was distinguishing the learning of software from the statistical concepts.

First-year statistics classes often use real data, but the impact that using real data has on students' engagement and achievement has been poorly understood. Neumann, Hood, and Neumann (2013) studied this using a small (n = 38) qualitative study. They found that using real data provided a relevant perspective in students' learning, maintained student interest, enabled students to remember more clearly, motivated and engaged students, and helped students understand. Neumann et al. (2013) also suggested features of real data that prove useful in the classroom. To this end, Brooks (2012) and Dunn (2012, 2013) provided examples of using interesting data in class to advance students' knowledge and engagement. Forbes (2014) attributes the success of New Zealanders in the international statistics education community, in part, to encouraging students to engage with real data.

7 Suggestions for Further Research

To further research in this area we suggest:

1. The involvement of cross-disciplinary teams of researchers. This would allow for sharing of knowledge and research expertise and foster the grounding of research efforts in theories of mathematics education as well as in psychological and sociological theories if relevant. Equally, research reported by educational psychologists is much more useful to mathematics educators if the pedagogical context is described.
2. Especially in the area of digital technologies, we need to know more about how mathematics academics, mathematics education academics, learning support personnel and learning technologists may productively cooperate to transform the learning experiences of our students. This research has the potential to lead to theory integration and theory building that we hope will provide a clearer understanding of not only the impact of digital technologies on mathematics instruction, but on the nature of the field as a whole.
3. Transitions into university study and then into work are research areas that will continue to attract government funding. In these fields the need to balance the "know what" content questions with the "know how" and "know why" questions of metacognitive knowledge in mathematical sciences will be increasingly important. Collaborations across sectors and with employers will be essential.

As mentioned in the opening paragraphs, tertiary mathematics and statistics education is an expanding area for research in Australasia. The breadth and depth of the research being done is impressive. A focus on students and learning will provide the direction and motivation for achieving the goals suggested here. A suitable guide in this endeavour is the book by Wood, Reid, and Petocz (2012) who reflect on years of researching the student voice and the student experience and call for a broader, more holistic curriculum in mathematical sciences. This curriculum "looks beyond the discipline of mathematics itself, beyond its techniques and components, […] to focus on the role that mathematics plays in the personal and professional lives of the students who are learning mathematics, helping to describe, explain and even change the world they live in" (p. 144).

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Chapter 11
Innovative and Powerful Pedagogical Practices in Mathematics Education

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Abstract The use of powerful and innovative pedagogical practices by teachers potentially provides opportunities for all students to learn mathematics successfully. In this chapter we offer a review of a range of Australasian studies about how teachers proficiently organise mathematical classrooms. The review shows that there are three key themes in the research literature. These include innovative and powerful learning environments, innovative practices which promote mathematics teaching and learning as inquiry, and mathematical tasks that promote deep learning. In this chapter we illustrate through the different and diverse studies we draw on that there has been shift in focus from an agenda of deficiency in mathematics to one of proficiency within student centred inquiry with problems set in authentic contexts to increase student engagement, motivation and ownership of the learning.

Keywords Pedagogy · Equity · Inquiry · Learning environments · Tasks

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1 Introduction

This chapter is about the powerful and innovative pedagogical practices teachers use to enable all students to learn mathematics successfully. In this chapter the focus is on mathematical pedagogy; that is the relationship between mathematical concepts and how these are conveyed or communicated through instructional activity (Anthony & Walshaw, 2007; Ball & Bass, 2000). We draw on Australasian studies that provide evidence over the previous 4 years (2012–2015) of powerful pedagogical practices which have influenced or hold the potential to influence positive outcomes for all learners in mathematics classrooms.

Our exploration of powerful and innovative pedagogy is timely given the challenge by Jorgensen (2014) in a recent Mathematics Education Research Group of Australasia (MERGA) presentation for the need for the mathematics community to construct a new paradigm in mathematics education. She argued that despite the many social theories of learning proposed over the last decade by mathematics educators and researchers, for some groups of learners nothing has changed and we continue to have a "significant problem with the outcomes in indigenous education in particular and equity target groups in general" (p. 311). She asked fundamental questions about whether the inequitable outcomes many students encounter might be a structural part of the system rather than an unplanned and random outcome. She suggested that perhaps we need to consider whether schools and education are structured in ways that re/produce inequality. For this reason identifying and explaining the powerful pedagogical practices that currently meet the needs of all learners is urgent; as is considering those practices promoted in theory but not yet operationalised. We also remain cognisant that for the changes in teaching and learning practices that are promoted in this chapter to become widespread there needs to be support from a wide range of stake-holders including politicians, mathematicians, and scientists (Stillman, 2013).

Our review encompasses a range of studies inclusive of small scale projects as well as those larger in size undertaken by Australasian researchers. It is organised by three important themes: (1) Innovative and powerful mathematical learning environments; (2) Innovative practices which promote mathematics teaching and learning as inquiry; and (3) Mathematical tasks that promote deep learning. The three themes are positioned within the integrated sociocultural perspective taken by Goos (2014). Goos challenged us to simultaneously be mindful of the students' and teachers' learning while also extending the lens beyond teacher and student learners to consider how opportunities to learn are presented to mathematics teacher educator researchers. In discussing the key findings we consider how the different studies we drew on fit within each theme and which might contribute to our knowledge about how teachers work effectively in classrooms to provide opportunities for all students to develop both mathematical competencies and positive mathematical dispositions. We also make links to topics which are covered in more depth in other chapters of this volume. These include equity (see Chap. 6), diversity (see Chap. 7), indigenous education (see Chap. 8) and professional development (see Chap. 16).

2 Innovative and Powerful Mathematical Learning Environments

In this first section we look at the ways in which classroom environments are structured to support productive mathematical activity. We examine the connections between teachers establishing classroom cultures, promoting productive discourse, and promoting and maintaining student engagement. These aspects fit within the framework Cobb and McClain (2006) proposed as critical to effective, inquiry-based learning environments.

2.1 Establishing Classroom Cultures

Quality learning environments exhibit a number of interrelated factors which combined support all students to be able to engage in a range of mathematical reasoning practices that advance their mathematical proficiency. However, establishing classroom cultures in which teachers position *all* students as active constructors of reasoning—"doers and thinkers of mathematics" (Leach, Hunter, & Hunter, 2014, p. 381) is challenging. Leach et al. provided evidence that explicit teacher actions were required to establish student engagement in mathematical reasoning. Using classroom vignettes, they illustrated the need for teachers to focus on the social aspects as much as on the sociomathematical aspects of learning mathematics. For example, to scaffold a range of communicative processes, the teachers needed to make sure that the students actively listened, took responsibility for their own reasoning, and asked questions to clarify their thinking. At the same time, students were supported to engage in a range of mathematical practices including providing acceptable mathematical explanations, justifications, and responses to challenges.

Other studies (Hunter & Anthony, 2014) provided evidence that when teachers used explicit scaffolding actions, including those which were culturally appropriate, equity issues were addressed. Bills and Hunter (2015), and Hunter (2013) described how a group of Pāsifika students were empowered mathematically when teachers drew on Pāsifika values within a range of pedagogical actions to structure the discourse and activity in the classroom. As a result, the students engaged in mathematical argumentation in culturally respectful ways within communities of mathematical inquiry. Both these studies were embedded within culturally responsive teaching of diverse students with a key focus on equitable outcomes (for further discussion related to equity see Chap.6, and diversity see Chap.7 of this volume).

2.2 Promoting Productive Discourse

The importance of ways in which teachers promote productive mathematical discourse within inquiry learning communities has been an on-going research theme in the past two MERGA four yearly reviews. Leach et al. (2014) positioned the development of productive mathematical discourse as integral to the engagement of diverse learners in doing mathematics and talking about doing mathematics and their construction of positive mathematical identity or disposition. Hunter (2012) provided persuasive evidence of the importance of explicit teacher scaffolding of reasoned discourse within a range of mathematical practices including mathematical argumentation to support equitable access.

Argumentation can be identified as a way to move students beyond tacit understanding to explanation of ideas and justification (Fielding-Wells & Makar, 2012). Perry and Dockett (2013) in a study set in the early years of schooling (prior to school and the first 2 years of schooling) illustrated the power of young children learning mathematics through discussion and argumentation. Likewise, Hunter (2014b) in a study of older students illustrated the importance of teachers closely attending to the discourse and applying a press on student thinking which ensured that they engaged at higher cognitive levels. She showed that through the construction of a classroom culture which supported inquiry and argumentation not only were the students responsible and accountable for their own reasoning they were also able to link numerical concepts to algebraic reasoning.

A key element of argumentation is the role of evidence. Fielding-Wells (2013) investigated how young students developed awareness of need for defensible mathematical evidence when developing an argument. In this study the teacher took a key role in firstly supporting students to define a mathematically researchable question, then prompting them to recognise the need for appropriate evidence. She achieved this through explicit discussion of claim-evidence links and specific scaffolds such as asking students to draft an imaginary conclusion and reflect on the evidence that would be needed. The teacher actions prompted significant changes in students' practice of providing evidence and by the conclusion of the study students had begun to carefully consider the effectiveness of the representations and mathematical content.

Closely aligned to facilitation of productive discourse is the shaping of student identity and disposition. What shapes student identity includes ways in which the classroom culture and task selection provides individual students with opportunities to grow as learners, talkers and users of mathematics. In exploring the use of digital games to provide an engaging context for Indigenous learners, Jorgensen and Lowrie (2013) found that these games provided the catalyst for learning mathematics in ways that were non-threatening to students while providing a context in which students could talk and identify with both the context and the mathematics.

Proactive and explicit actions have been shown as essential for establishing productive classroom mathematical discourse (Sullivan, Clarke, Clarke, Farrell, & Gerrard, 2013). For example, Walker (2014) noted the difficulties many teachers encountered in maintaining student reasoning as a focus particularly in the summary phase of a lesson. Her data showed that productive discourse occurred during the summary phase when teachers presented students with opportunities to summarise in sections of the lesson. Similarly, Ferguson (2013) demonstrated the challenges for lower-attaining students in accessing and understanding reasoning during whole class discussions. Suggestions to improve the participation of lower-attaining students and to gain greater benefits to their learning included recording the main points of discussions, the use of revoicing during discussion of student solution strategies, providing think-time, and explicit discussion with students of the purpose of whole class discussions.

2.3 Promoting and Maintaining Student Engagement

Bishop and Kalogeropoulos (2015) cautioned the self-fulfilling prophecy of labelling students as dis/engaged learners without placing attention on the classroom culture. In their comprehensive account of student engagement, Martin et al. (2012) maintained that teachers need to provide quality learning environments—a key component of which includes students engaged in risk taking and mathematical challenge through use of open ended and differentiated tasks and activities—in order to address disengagement. They showed how such activity propelled the students into both exploring networks of interconnected mathematical ideas and being adaptive with their knowledge in unfamiliar situations. Additional studies (e.g., Brough & Calder, 2012; Sullivan, Clarke, & Clarke, 2012) showed that through similar experiences students learnt the need for risk taking and persistence in mathematics.

In high quality learning environments the powerful effect of placing student reasoning central to mathematical activity rather than teaching to a set of rules and procedures is realised. Different forms of knowledge and participation are privileged in different classroom settings. Hunter (2012) provided evidence that when a teacher taught lessons procedurally student disengagement increased. In contrast, when the same teacher used pedagogical practices within an inquiry environment to engage the students in a range of mathematical practices this resulted in them using high levels of cognitive reasoning as they justified and generalised their solutions.

Williams (2013) suggested that a transmissive teaching approach is associated with an absence of inclination to explore mathematical situations and ideas. She noted that some students developed confidence and saw themselves positively in relation to their ability to reproduce teacher provided strategies and rules. She labelled this as “disabling” confidence because these students portrayed a lack of persistence. In contrast, other students had “enabling” confidence; confidence developed through their success in overcoming mathematical challenge, puzzling out ideas for themselves and engaging in mathematical argumentation to construct new knowledge.

Marshman and Brown (2014) used the notion of collective argumentation to provide opportunities for students to discuss, think, justify, refine, and validate their ideas during problem solving tasks. They showed that when students and teachers worked collaboratively in problem solving, students were empowered to participate as mathematicians and engage in interpreting and communicating mathematical ideas. They showed the re-engagement of students who had been disaffected by traditional approaches to teaching when structured scaffolding of sense-making was used. Similarly, Leong et al. (2013) used a practical worksheet to scaffold and develop problem solving dispositions in low-achieving students, and found that students were engaged to proceed beyond the initial sense-making processes of problem solving with little or no teacher intervention.

Student engagement changes within different learning contexts. At present research in this area in Australasia is not wide and so further research around this topic is needed.

2.4 Shifting Student Beliefs

Student beliefs about what makes a good learning environment are a powerful predictor of what they gain from a mathematical classroom. Anthony (2013) investigated student perceptions of what constituted good teaching and learning in two New Zealand secondary classrooms with contrasting pedagogical framings. In the more traditional classroom, the students valued the clear teacher explanations and linked these to developing their own understanding. Similarly, students in the reform or inquiry classroom valued clear explanations from their teacher but also noted need for effort on their own part to understand these explanations. What the study illustrated was that in the traditional classroom, students had limited opportunities to exercise conceptual agency which meant some students did not access tools to justify or refute claims or assess their own thinking. In the reform classroom, students were expected to exercise conceptual agency which included struggling with mathematical ideas, obligations to explain and justify, work collaboratively, and ask clarifying questions. In this classroom, student learning was focused on developing insight and understanding.

Several studies in this review sought to illustrate how shifts in the classroom resulted in changes in the roles and beliefs of both teachers and students. For students who were inducted into classroom communities using innovative pedagogies, studies provided evidence of shifts in their role as learners. For example, Kidman, Grant, and Cooper (2013) examined teacher articulations of their experience with a new pedagogy which moved from a traditional worksheet approach to drawing on structural and indigenous approaches. The teacher participants described how they felt that inquiry approaches supported students to expand their problem-solving abilities and make connections to contexts outside of school. They outlined how in teaching in this way, students were given more challenging learning opportunities and were expected to take a more participatory role. Of importance was the teachers' acknowledgement that student learning using inquiry methods often exceeded their expectations. Similarly, Calder (2015) in a study with senior students illustrated that they took increased responsibility for their learning when engaged in self-initiated mathematical inquiries. Brough and Calder (2012) suggested that student initiated inquiries are emancipatory but require flexible teachers to draw on "democratic and empowering pedagogical understandings" (p. 158).

Making explicit to students what it means to do and learn mathematics has become a feature of many classrooms and is closely connected to the beliefs they construct about themselves as doers and users of mathematics. Sullivan et al. (2012) illustrated that effective teaching involved teachers making explicit reference to the expectations they held towards specific behaviour—such as listening or how to respond. Similarly, in her comprehensive work in remote indigenous settings, Jorgensen (2015) noted that many teachers made reference to their need for making pedagogy explicit to learners so that they engaged with the mathematics rather than trying to second-guess the purpose of lessons. Likewise, Sawyer (2013) within an early years context showed that when teachers made their teaching explicit and transparent, students were better able to engage with the substantive mathematical concepts and processes. Sullivan (2015) has extended this argument to working with teachers and systems and argues that researchers need to make explicit "the ways of offering advice to schools, the specific strategies the researchers might be suggesting, and acknowledging the new challenges that these strategies might themselves create" (p. 3).

The findings in this section reiterate an argument made in the previous MERGA four yearly review that careful consideration needs to be given to the construction of learning environments that equitably meet the needs of a diverse group of learners. From these findings our attention is drawn to the ways the change in pedagogy causes changes in teacher and student beliefs. Overall the findings present a view of innovative and powerful mathematical learning environments which include a focus on (a) promoting and maintaining classroom cultures; (b) promoting productive discourse; (c) promoting and maintaining student engagement; and (d) shifting student beliefs. Although the studies are all set within social theories of learning they do open up a need for other theoretical frames to be developed particularly when addressing equity issues. Jorgensen's (2014) challenge to the MERGA community for a new theoretical frame presents the MERGA community with further research opportunities.

3 Innovative Practices Which Promote Mathematics Teaching and Learning as Inquiry

In this second section we look at the innovative practices which have emerged within what is often termed inquiry practices or ambitious instructional practices (Franke, Kazemi, & Battey, 2007) and how these provide opportunities to learn for all participants. In this section we examine the connections between teacher noticing and responding to student reasoning, and the influence of teacher beliefs on their enactment of inquiry environments.

The notion of teachers facilitating student engagement in mathematical inquiry in learning communities is a consistent theme which threads through many papers. Jaworski (2014) described the process of inquiry as one in which the teacher and students work together around rich mathematical tasks. She explained how within this process the students are learning from each other and the teacher; and the teacher "is also learning as a teacher" (p. 6). Calder and Brough (2013) described the critical role of the teacher where students collaboratively developed their own situations to investigate. They illustrated how the teacher was required to have high levels of pedagogical content knowledge to enable them to work flexibly with student ideas, reflect on student interests and evaluate both the individual and collective mathematical understanding. Hunter (2014a) extended this thinking and argued a need for teachers to take a stance of inquiry and develop a conjecturing atmosphere in classrooms if students are to develop a set of interconnected big mathematical ideas.

3.1 Teacher Noticing and Responding to Student Reasoning

Teacher noticing, which is a powerful pedagogical action that is closely associated with the notion of *professional vision* (Goodwin, 1994), as a way of seeing and interpreting observations or events, is unique to the job of teaching. Mathematics teacher noticing has received increased international attention in recent years (e.g., Sherin, Jacobs, & Phillips, 2011), but interest in this specific field of research has only recently begun to emerge within the Australasian setting. A number of studies looked at enhancing teacher noticing in different contexts. For example, Bragg and Vale (2014) examined how teachers' noticing of student reasoning was enhanced through demonstration lessons, while Seto and Loh (2015) investigated how teacher noticing was promoted through mentoring conversations between a mathematics teacher mentor and a beginning teacher. Beswick and Muir (2013) reported on the use of videos in a pre-service setting and noted that although the pre-service teachers struggled to notice aspects of teaching beyond those readily available, the use of video and a focus on noticing had possibilities. These studies highlight the potential of teacher noticing in improving teaching and learning of mathematics.

In general, if we consider noticing to be productive when a teacher responds to the observations in a way that promotes student engagement and discourse, then although people notice all the time, it can be argued that not all noticing is productive. As part of a larger doctoral study, Choy (2013) distinguished productive from less productive noticing and characterised productive noticing by drawing on Yang and Rick's (2012) *Three-Point* Framework. Choy persuasively demonstrated how lesson study groups and the notion of productive noticing helped to shape collaborative teacher dialogue related to student reasoning while at the same time provided a tool for them to further explore their own teaching and responses to student thinking in the moment. Choy (2014b) extends previous research on teacher noticing, which is focused on noticing in-the-moment of teaching and during post-lesson reflection, by arguing that productive noticing begins when teachers plan their lessons. By connecting noticing during lesson planning (Choy, 2014b) to in-the-moment noticing during classroom teaching and retrospective noticing during post-lesson discussion (Choy, 2014a), Choy presented teacher noticing as a high leverage practice that can potentially improve mathematics teaching and learning in significant ways. It remains, however, to be seen how teacher noticing can be enhanced in other contexts. Given the impact of teacher noticing on rich student learning within inquiry communities, further discussion and research is needed within the Australasian context.

3.2 Teacher Beliefs

Enacting the pedagogical actions which support students to develop rich connected mathematical understandings in what is termed an inquiry, reform or ambitious setting requires considerable skill and beliefs about the efficacy of this mode of learning on the part of the teacher. For example, Murphy (2015) illustrated that for teachers to change to a more discursive pedagogy required them to transform not only their practice, but also their beliefs about social interactions within the classroom, their role and purpose and classroom dynamics. One aspect of shifting the classroom practices towards inquiry which needs to be carefully considered is teacher self-efficacy. Engaging in teaching within an inquiry setting is a risk for many teachers and closely tied to their own identity and disposition to take risks. In Kidman, Grant, and Cooper's (2013) study described in an earlier section, the teacher responses showed how the teachers undertook an inquiry process themselves as they learnt to teach using inquiry. They identified the required changes as major and requiring significant effort. Interestingly, two key barriers to enacting innovative pedagogies such as inquiry were identified. The first, classed as technical barriers included the need for teachers to have a deep understanding of mathematics, the required pedagogy, and commitment to planning. The second group of barriers classed as cultural included the culture of dependence on the teacher or text-books, or a fear of mathematics. Of note was the importance of teacher collaboration and the need to develop a culture of reflection on values and beliefs.

Having access to a supportive and collaborative network is identified as key in a number of studies that explored how to sustain growth towards classrooms which are student focused. Skilling (2014) interviewed thirty one teachers from ten secondary schools to interrogate the individual classroom, pedagogical, and school-based features they believed influenced student engagement and achievement. Critical motivating factors which emerged included relevance and connectedness of the tasks, teacher student relationships, and practices which enhanced student autonomy and empowerment. However, factors which they identified as disengaging closely aligned with their own beliefs and practices. These included their own uncertainty on how to engage and relate to students and their need to remain in control of the learning. Lewis (2014) drew our attention to the need for consideration of the stresses placed on novice secondary teachers when asked to enact inquiry pedagogy. Lewis noted that their struggles were diverse but centred on the uncertainty of their teaching role in classrooms in which the discourse was not teacher led. Of importance in these studies were the supportive organised networks which aided their pedagogical growth and supported shifts in beliefs about inquiry classrooms.

Teacher uptake and use of powerful pedagogical practices is closely aligned to the subject knowledge and pedagogical content knowledge they hold. A number of studies (Cheeseman, McDonough, & Ferguson, 2014; Lamb & Visnovska, 2013) drew on the design research approach for examining best practice in developing different aspects of content knowledge in classroom studies as well as using it to illuminate teacher learning within professional development settings. (For further discussion about professional development see Chap. 16, this volume). In a design study Visnovska, Cobb, and Dean (2012) described the important outcomes of 5 years of collaboration with a group of teachers. Of interest for this chapter is the way in which the sustained proactive facilitation of teacher engagement with the new instructional sequences meant that the teachers did not merely assimilate them into their current practices but enacted them in ambitious ways.

The findings in this section illustrate that for powerful and innovative classroom environments to be constructed both the teachers and the students need to be repositioned as “inquirers”. Closely aligned to enacting innovative pedagogical practices are the beliefs the teachers hold about mathematics teaching and learning. Teacher knowledge remains a recurring theme in this chapter of the volume and the chapter in the previous MERGA four yearly review. Teacher noticing and responding has emerged as one possibility for teachers to enhance their pedagogical and mathematical knowledge. A focus in this area presents many research opportunities for the MERGA community.

4 Mathematical Tools that Promote Deep Learning

In this third section we explore the role of mathematical tasks which have been used to engage students in deep reasoning. We make connections between the selection of tasks, the role of challenging tasks, the importance of linking to the students’ cultural contexts, experiences and interests, and the role of digital tools in supporting and enhancing learning. As Cobb and McClain (2006) maintain the classroom culture provides the background for an effective inquiry based environment but the task is critical in positioning all participants in the intellectual rigour of authentic mathematical practices.

4.1 The Selection of Tasks

The selection of mathematical tasks is a critical aspect of innovative and powerful pedagogy. This is because these shape the teaching and learning context. In their study of five textbook series, Dole and Shields (2013) reported that textbook examples were unlikely to promote deep learning in mathematics and that other models needed to be sought. A number of studies (e.g., Cheeseman, Clarke, Roche, & Wilson, 2013; Sullivan et al., 2014; Sullivan, Clarke, & Clarke, 2013; Williams, 2013) noted the many factors which influenced student willingness to engage and persist with challenging tasks. These studies detailed key pedagogical actions which were influential in ensuring the construction of classroom cultures in which student reasoning emerged and was central to mathematical activity. These included the importance of how teachers posed the tasks, the interactive supports provided to students as they engaged in the tasks, and the summary collaborative sharing and review of task solutions.

A key criterion for selection of challenging and complex tasks used within inquiry or ambitious mathematics settings includes the need for tasks to have multiple entry and exit points and cause "sustained thinking" (Sullivan & Davidson 2014, p. 606) and argumentation. The choice of tasks is also dependent on the teachers' decision making around the richness of the tasks; the use of technology to support learning and the use of other resources (Aubusson et al., 2014). The interaction of these factors can influence better student learning. The selection of good tasks provides students with opportunities to determine their own pathways, and explain and identify patterns and justify their reasoning. Fielding-Wells, Dole, and Makar (2014) used a mathematical inquiry approach and design structure which engaged students in an investigation around a challenging problem. Important to the design was the public culture where argumentation led to students relinquishing their own beliefs and ideas to develop strong collective reasoning related to proportionality. Key to the rich learning which evolved was a task which caused the students to engage authentically in mathematising. Hunter (2014b) also used a design structure to illustrate the importance of specifically designed tasks, tools, and representations. Incorporated in her study was recognition of how the teacher's consistent press for justifying and generalising led to the students constructing rich algebraic reasoning.

Teacher subject knowledge and pedagogical content knowledge are key elements which need to be considered when teachers select and use challenging and complex tasks. Like the previous 4-yearly review, a number of studies (e.g., Brough and Calder, 2012; Burgess, 2012) reinforced the need for high levels of teacher knowledge to be able to respond in ways which extend student reasoning. Sullivan and Mornane (2014) noted that within a cohort of junior secondary teachers who engaged in research using challenging tasks, they had the subject matter knowledge required to complete the task but not the pedagogical content knowledge. In this study, although there were positive learning gains and the teachers affirmed their use of tasks when they limited their introduction so that challenge was maintained, they also encountered problems which involved a range of pedagogical decisions. These included maintaining norms for students' listening and learning from each other and how they sourced tasks at the right level of challenge. One useful way to support teachers in structuring lessons involving challenging tasks that Sullivan et al. (Sullivan, Askew, Cheeseman, Clarke, Mornane, Roche, & Walker, 2014) showed was to provide different discourse prompts to facilitate discussion. Calder and Brough (2013) argued the need for tasks with rich contexts that require mathematical organisation. They showed that the use of democratic, power-sharing teaching approaches heightened the relevance for students and their ownership of tasks whilst also raising levels of student achievement. They illustrated that such approaches have benefits both in relation to student motivation and learning. As Sullivan and Mornane (2014) found students in their study engaged with mathematical concepts beyond suggested curriculum levels and were highly motivated.

4.2 The Role of Challenging and Ill-Structured Tasks

Increasingly, importance has been placed on the role of challenging or ill-structured tasks. Both challenging and ill-structured tasks embody elements of open-endedness, context based relevance, space for differentiation and what McGregor (2014) terms "messiness" (p. 453). As McGregor explains the messiness varies across tasks but often includes vagueness with key factors missing, poorly collected or sorted data and unspecified steps leading to a solution. In his study he provided evidence of problems a messy task caused for senior students aged 16–17 and the effects on the students' self-efficacy. He argued the importance of inquiry based learning and ill-structured tasks as a way to expose and challenge both the beliefs and attitudes students hold towards mathematics.

Some studies (e.g., Fielding-Wells, 2013; Makar, 2012) promote the need for challenging or ill-structured problems and tasks as a way of combating the often decontextualized and abstract nature of well-structured problems. These require some degree of risk and challenge to both teachers and students. Through use of such tasks, Sullivan and Mornane (2014) showed how students gained positive results when teachers encouraged them to persist. Cheeseman et al. (2013) found that teachers perceived tasks as challenging for students when there were demands for mathematical reasoning, interpretation of complex mathematics, and an expectation of the development of student generated solution strategies. In their study, teachers were asked to observe a lesson which involved a challenging task and to identify the key strategies used to encourage student persistence. The teachers identified important elements such as the openness of the task, a collaborative environment that allowed for discussion and clarification, acknowledgement to students of the challenge and multiple solution strategies, the importance of observation before any teacher intervention, and an expectation that thinking would be recorded.

Powerful practices promote the need for a focus on the use of differentiated tasks which challenge and extend students in different ways. However, this does not suggest a lack of possible predicted solution pathways students might take. Some studies (e.g., Briand-Newman, Wong, & Evans, 2012; Cheeseman, McDonough, & Ferguson, 2014; Cortina, Visnovska, & Zuniga, 2012) argue for the need for teachers to have what is variously termed trajectories of learning or teaching, conjectured pathways, instructional sequences, a framework of “growth points” or teacher horizon content knowledge as key elements to support their students to progressively learn to mathematise. Sullivan et al. (2013) reported that while teachers may have a clear idea of the overall topic, they are quite diverse in how they might achieve those goals, and need to be engaged in active decision making throughout all stages of the planning process. Ellemor-Collins, Wright, and McEvoy (2013), and Cortina, Visnovska, and Zuniga (2014) illustrated the importance of teachers using carefully constructed instructional sequences. Ellemor-Collins et al. proposed that such sequences needed to be situated at the “cutting edge of the student’s knowledge” (p. 1) to maintain interest and growth. In another study, Pfannkuch, Arnold, and Wild (2014) carefully designed a sequence of learning experiences based on a learning trajectory to develop students’ understanding of sampling variability. Even though they had chosen only one of the many possible trajectories, their students had begun to develop concepts related to statistical inference.

4.3 The Importance of Linking to Students’ Cultural Context, Experiences and Interests

Making links to the students’ cultural and social experiences within problem contexts and activity in mathematics has the potential to support powerful engagement. Cheeseman, McDonough, and Ferguson (2014) outlined how a group of teachers taught a sequence of five lessons which involved young students’ construction of deep understanding of mass measurement. This was achieved through a combination of rich hands on mathematics experiences including real world applications, problem solving and challenging conversation. Hunter (2013) showed the value of using culturally appropriate tasks initially to ensure that students perceived the tasks as worthy of their engagement. Cortina, Visnovska, and Zuniga (2014) illustrated the value of developing problem narratives around an ancient Mayan people with marginalised Mexican students. Similarly, Seah and Wong (2012) explored what high achieving Asian students valued in the teaching contexts of mathematics. Bills and Hunter (2015) also illustrated that posing challenging problems to students in Pāsifika contexts created powerful connections to mathematics and at the same time enhanced their cultural identity and mathematical disposition.

Other ways of making links with students' cultural context includes attending to their language. In the Australian setting in many Indigenous communities, local people are employed in the mathematics classroom to support students as they come to learn the dominant language (Standard Australian English) and mathematics. Wilkinson and Bradbury (2013), in working with the Yolngu people of northern Arnhem Land, emphasised the need for scaffolding young children in the use of appropriate number language. Working with the paraprofessionals at the school, strategies were developed collectively that supported instruction and content for the students. Within a similar context Warren and Miller (2013) showed how the use of hands-on activities for Indigenous learners, assisted in their learning of mathematics, and supported the learning of the language of mathematics. The common thread that strings through these studies to support the development of mathematical thinking of students from indigenous settings is the two-way collective sharing, mentoring, and consulting to transform schools and teaching (Owens, 2014). As Owens (2014) points out, supporting indigenous education in a culturally-responsive way goes beyond sharing strategies and resources, but more importantly, it requires a concerted effort to provide peer support, leadership training, funding, and mentoring in classroom processes.

4.4 The Role of Digital Tools in Supporting and Enhancing Learning

Digital tools have also featured in innovations in mathematics education. More recently, the use of hand-held digital media—such as phones, iPads, and digital games—have moved the discussion of innovation from computers and interactive whiteboards to the potential of hand-held resources. Day (2013) discussed the use of digital media in the classroom while others, such as Muir (2014) have raised the potential of on-line resources to provide engaging learning contexts for mathematics learning. In her work, Muir specifically looked at on-line tools such as Google, Mathletics and the Khan Academy, and noted that the role of the teacher can be significantly changed in these new learning environments where the on-line tool offers much to learners. A further area of research has been the use of hand-held digital technologies, including the range of cheap, free, and accessible apps. Larkin (2013) critically appraised the use of apps in supporting and enhancing mathematics learning and sought to evaluate apps for quality learning in mathematics. The use of digital games has also emerged as an area of innovation in mathematics. Lowrie and Jorgensen (2015) produced an international collection of research on the use of digital games to support mathematics learning as well as providing a critique and limitations of this work to support mathematics learning.

Collectively, this is an important area of work that is covered in more detail in the technology chapter in this collection (see Chap. 13, this volume) but does need reference in this chapter to acknowledge the value of digital tools for supporting innovation and learning in mathematics.

The findings in Sect. 3 illustrate the importance of teachers selecting mathematical tasks which hold the potential to challenge student reasoning and extend it into mathematical argumentation. A number of papers promote the need for ill-structured tasks with multiple exit points and which add another level of challenge. Other findings draw our attention to the importance of linking to students' cultural context, experiences, and interests. The role of digital tools as a way to support and enhance deeper understandings is briefly explored in this chapter which contrasts with the extensive explorations of this topic in the previous 4-yearly review in the chapter titled "Powerful Pedagogical Actions".

In the final section we return to the introduction and consider Stillman's (2013) contention that for significant changes in teaching and learning practices support must be available from a wide range of stake-holders including politicians. We consider policy driven initiatives which have emerged in recent times and which rather than moving us forward to develop new theoretical frames to better support equitable outcomes for all, hold a possibility of returning us to the past.

5 Policy Driven Innovations: Important Considerations

Within the Australasian context, there is now a growing pressure on education systems to adopt "research-informed" practice into school practices. Schools across the nations have been particularly influenced by Hattie (2012) and his work on visible learning. Part of the rationale for this is a perception that standards are falling in mathematics and that new, better ways of teaching are needed (Ministry of Education, 2015). In his work, Hattie draws on meta-analysis of large international studies to provide guidelines as to what makes for effective learning. His work has been applied across many countries and systems (e.g., Winheller, Hattie, & Brown, 2013) across Australasia. Part of the Hattie model has been to advise that instructions need to be explicit so that students are aware of the demands expected of them and that feedback is provided so that students can see their successes (and errors).

In implementing Hattie's (2012) recommendations, there has been a strong push for models around direct and/or explicit instruction. Instruction is seen to be the touchstone for organising school practices. Many of these reforms smack of the teaching approaches of the 1960s based on behaviourist models of learning and so rather than new theories Jorgensen (2014) argues for, these return us to out-dated theories of learning. In Australia, the lessons are scripted for teachers which parallels with what happened in the latter stages of the New Zealand Numeracy Development Project (from 2006 to the current date). The books became a formula which scripted what teachers were to say and do and teachers were expected to record clear learning intentions and, with students, establish success criteria. This provided the students with a clear message about who was driving the learning as evidenced by Hunter (2012). In this study set in a high poverty school, Hunter described how a teacher used a scripted Numeracy lesson which left the students without a voice or autonomy.

In Australia, in very impoverished schools where numeracy/mathematics achievement is quite low in comparison to other contexts, there has been considerable reform (and funding) to support the direct instruction model founded on Hattie's (2012) recommendations. The recent roll-out of the "Good to Great Schools Australia" (GGSA) has seen $22 million invested in "effective instruction" aimed at the lowest performing schools (usually seen as low socio-economic schools, and schools serving remote Indigenous communities). Initially starting with The Cape York Institute in 2014, considerable funding was allocated to implement Direct Instruction across its campuses in North Queensland for literacy and numeracy learning. The model is now being rolled out into other remote Indigenous communities and low SES communities. But as Luke (2014) cautions there is little evidence to suggest that the program has traction in terms of learning mathematics (or literacy).

In a chapter that has focused on many aspects of innovative teaching, there are powerful messages that can inform and challenge practices based within the GGSA reform rollout. Why is it that despite considerable research—much of it cited in this chapter (and others throughout this book), governments fail to listen to that research and adopt a wide-scale reform that is counter to the abundance of research in quality learning of mathematics. (For further discussion of the political implications please see Chap. 4, this volume).

6 Conclusion

This chapter has considered a range of effective and innovative pedagogical practices which teachers use in order to provide opportunities for all students to access and learn mathematics with success. While the different studies focused on specific aspects of effective pedagogical practices, collectively they allow us to envisage what powerful mathematics teaching and learning might look like.

The research has contributed to our knowledge of the ways in which effective teachers establish high quality learning cultures which open up space for student voice and position their mathematical reasoning as central to the mathematical activity. Notions of students engaged in different forms of mathematical inquiry and the discourse of reasoned argumentation are evident in most studies described in this chapter. Many studies focused on the centrality of student engagement and persistence in all mathematical endeavours and placed the use of challenging mathematical tasks as key within a learning trajectory which provided opportunities for students to learn rich mathematics. The different and diverse studies signal a change in focus from an agenda of deficiency in mathematics to one of proficiency within student centred inquiry with problems set in authentic contexts to increase student engagement, motivation and ownership of the learning. They recognise the importance of students who are motivated to engage with inquiry mathematics; stimulated to think by skilled teachers; and challenged by intriguing tasks. These studies open a pathway for continued research within an agenda of proficiency in mathematics learning and teaching.

The importance of sound teacher knowledge and beliefs underwrites many of the reviewed studies. These factors are prerequisite for the successful establishment of a community of teachers and students as reflective "inquirers". A diverse range of studies combine to offer us an emerging view of what might be ways in which teachers grow their content and pedagogical knowledge while in the "messy" act of teaching. Further research about both the impact of professional development programmes which scaffold the growth of teachers as reflective and inquiring practitioners, and the growth of teacher educator researchers is warranted within the MERGA community.

The studies we reviewed established the key role teachers play in establishing and maintaining innovative and powerful pedagogical practices in mathematics education. Looking back at the number of studies undertaken in the previous 4 years in Australasia reported by authors that were focused on mathematics pedagogies it is apparent that the MERGA community have been systematically gathering substantial evidence about effective approaches to mathematics education. It would be reasonable to suggest that they have a lot to offer in this field to the national and international mathematics community.

Recommendations

- The MERGA research community needs to continue to maintain and extend the focus on the culture of mathematics classrooms as critical sites of learning.
- The wider cultural context(s) needs consideration because it is an important element of who gets to participate and engage equitably in discourse intensive inquiry classrooms.
- Teacher and student beliefs are powerful determinants of how the different participants in mathematics inquiry classrooms participate in mathematical reasoning. Change takes time and teacher voice and student voice provide a window to understand the gradual transformation of the participants into "inquiring" learners. More research is needed to track change over time.
- Task selection, including the possibility of "messiness" and careful and thoughtful enactment is a key aspect of innovative pedagogy. Teachers need support to scaffold tasks so that they retain a high level of cognitive challenge.
- The MERGA research community needs to actively engage with policy makers and other stake holders to advocate for best practices in mathematics education.

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Chapter 12
Assessment of Mathematics Learning: What Are We Doing?

Penelope Serow, Rosemary Callingham and David Tout

Abstract During this review period it is evident that Australasia is in need of a research agenda that critically analyses the impact and use of national external and international assessments, as well as issues related to classroom-based assessment. Although these are investigated at many levels, the question remains: Are we assessing what we really need to assess? Issues remain in relation to what national and international testing reports to the community, as well as validity issues around how it is used, and accountability. In addition, school-based assessment includes assessment "of", "as", and "for" learning with pockets of research applying a fine-grained review. The role that technology plays in classrooms and large-scale assessments is worthy of continued exploration in the light of available and changing tools. Assessment in the mathematics classroom has the potential to alter the experiences of children in our schools at every level. This chapter provides a balanced review and critique of research related to assessment completed within the timeframe of this review. The authors position the discussion in the context of current agendas and consider the impact the mathematics education community has had to date, and the future potential of the community to inform assessment practices and wider discussion.

Keywords Classroom-based assessment · National assessment · International assessment · Mode of delivery of assessments

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1 Introduction

The review of Australasian research targeting assessment finds that although assessment is a driving force in education systems, classrooms, and in developing policies there has been little critique of assessment in recent years. Schools, Governments, and caregivers use national and international assessment data to make important educational decisions that impact upon the school experience of individual students in all contexts. Of particular interest is the decline in targeted research that provides empirical evidence indicating the benefits or deficits associated with different forms of assessment. To make informed decisions concerning the possible directions that such research could take, it is essential for the mathematics education community to have an understanding of the different assessment tools, particularly the national and international assessment tools. This chapter addresses some of these questions and considers the different assessments used in the Australasian context and what the results represent. The next phase of assessment will certainly involve the greater use of ICT as both the assessment and the analysis tool, hence the use of technology in assessment will continue to be an important area of research requiring in-depth investigation. Studies that target technologies' roles in assessment will be pertinent to external assessment at a national level, international assessment tools, and classroom-based assessment.

The following section examines perspectives regarding the mathematical skills expected of both adults and school children, comparing ideas developed as part of three major multinational comparative assessments of mathematics skills. The theoretical frameworks, the results and data and the research that sits behind such international assessments can provide rich information for mathematics educators and researchers about teaching and learning, although it appears that most of the resulting Australian focused mathematics research published is from the national body that implements two of the studies—the Australian Council for Educational Research (ACER).

2 Deepening Understanding from International Assessments

In this section, the conceptual and assessment frameworks developed for three international assessment programs are considered to shed light on the commonalities and differences between the constructs of mathematics, numeracy and mathematical literacy. This consideration can be used to inform current debate about directions for developing and enhancing mathematical skills in the twenty-first Century. There are three major multinational comparative assessments of mathematics skills pertinent to the Australasian context:

- Trends in International Mathematics and Science Study (TIMSS)
- Programme for International Student Assessment (PISA)
- Programme for International Assessment of Adult Competencies (PIAAC, also known as the OECD Survey of Adult Skills).

Each of these assessments will be considered separately to provide the background necessary to interpret their results and consider future research directions.

2.1 Trends in International Mathematics and Science Study (TIMSS)

The Trends in International Mathematics and Science Study (TIMSS) study is directed by the IEA (International Association for the Evaluation of Educational Achievement), an independent international cooperative of national research institutions and government agencies that has been conducting studies of international achievement in a wide range of subjects since 1959. In Australia, TIMSS is implemented by the Australian Council for Educational Research (ACER), which is Australia's representative to the IEA. TIMSS is part of the National Assessment Program, and has been conducted at Year 4 and Year 8 on a 4-year cycle since 1995, with the last cycle in 2011. Australia has participated in TIMSS since its inception, providing rich data about trends in mathematics and science achievement over 16 years.

2.2 Programme for International Student Assessment (PISA)

The Program for International Student Assessment (PISA) is an international assessment that measures 15-year-old school students' reading, mathematical, and science literacy skills every 3 years. First conducted in 2000, the major domain of study rotates between mathematics, science, and reading in each cycle. PISA also includes measures of general or cross-curricular competencies, such as collaborative problem solving. PISA is coordinated by the Organisation for Economic Co-operation and Development (OECD) in member and non-member nations. PISA is also implemented by the Australian Council for Educational Research (ACER). Australia has participated in PISA since its inception, providing data about trends in performance for 12 years. In each cycle of PISA, the focus is on one of the three domains, called the major domain, and so the bulk of the assessment is in this area, while the remainder of the assessment is distributed across the other two minor domains for that period. In 2012, the majority of the focus of the PISA assessment was on Mathematical literacy.

2.3 Programme for International Assessment of Adult Competencies (PIAAC)

The Program for the International Assessment of Adult Competencies (PIAAC) is a large-scale study developed under the auspices of the OECD. PIAAC builds on previous international adult literacy related assessments—the International Adult Literacy Survey (IALS) and the Adult Literacy and Lifeskills Survey (ALL). The direct-assessment component of the survey evaluates the skills of adults in three fundamental domains. These are considered to constitute “key” information processing skills in the sense that they provide a foundation for the development of other, higher-order cognitive skills and are prerequisites for gaining access to and understanding of specific domains of knowledge. In addition, these skills are necessary in a broad range of contexts, from education through work to everyday life. The competencies assessed are:

- Literacy
- Reading components
- Numeracy
- Problem solving in technology-rich environments.

In Australia, this household study was conducted in 2011–2012 by the Australian Bureau of Statistics (ABS) with a nationally representative sample of adults between the ages of 15 and 74. Similar samples of adults were surveyed in each of the 24 other participating countries. The goal of PIAAC is to assess and compare the basic skills and the broad range of competencies of adults around the world. The most recent results from all three international assessments, no matter how you read them, show reasons for Australia to be concerned about the performance in mathematics or numeracy of its young people and adults.

Australia’s PISA 2012 results showed that 20% of 15-year-old students were found to be performing at the lowest two levels (out of six), which covered only basic mathematical skills. Overall 42% of 15 year olds were working at Level 3 or below which is the baseline for PISA in Australia set by the Australian Curriculum, Assessment and Reporting Authority (ACARA) (Thomson, De Bortoli, & Buckley, 2013a). Australia’s average score in the PISA 2012 mathematical literacy assessment was significantly higher than the OECD average, and sixteen countries scored significantly higher in mathematical literacy than Australia. Shanghai–China achieved the highest score—the difference between Shanghai–China’s and Australia’s mean scores represents just over 3 years of schooling. In reading literacy, however, the PISA 2012 reading literacy assessment was also significantly higher than the OECD average. Only nine countries scored significantly higher in reading literacy than Australia. Shanghai–China achieved the highest score—the difference between Shanghai–China’s and Australia’s mean scores represents more than one-and-a-half years of schooling (Thomson, De Bortoli, & Buckley, 2013b, pp. 7–9). The relative performance in mathematics is significantly weaker compared to reading. In PISA in both reading and mathematics, Australia's performance has declined significantly since 2003.

For the youth and adult populations, the results from PIAAC demonstrate that a significant number of people (54% of those who took the test) aged from 15 to 74 years old in Australia did not have access to sufficient numeracy and maths skills to be able to cope equitably with life in the twenty-first century. Considered also by age group, young people between 15 and 19 years had lower levels of numeracy than did those aged 25 to 43 years, although among the younger age group women performed slightly better than men (Australian Bureau of Statistics, 2014). It should be noted that levels are not equivalent between PISA and PIAAC (Gal & Tout, 2014) but the same trend occurs—that is Australian people perform relatively worse on international assessments than their Pacific rim neighbours.

The findings from international assessments should be informing a research program to identify why the performance of Australian students has declined significantly. Despite considerable effort and money being allocated to mathematics education during the period of this review, there does not appear to be any systematic review of reasons for the decline, especially when results from national testing suggest that performance has remained the same over a number of years (Goss & Hunter, 2015).

3 Issues Relating to External National Assessment of Mathematics

The value of mathematics to society is reflected in the extent to which it is externally assessed—that is assessed by agents external to the classroom and outside of the teacher's control. The tests are usually standardised, given under the same conditions to all students, and are intended to measure performance. The most prominent of these external assessments in Australia is the National Assessment Program—Literacy and Numeracy (NAPLAN) (http://www.nap.edu.au/naplan/naplan.html) undertaken by all students in Years 3, 5, 7 and 9 of schooling. Gable and Lingard (2013) summarised the policy context of NAPLAN and concluded

> NAPLAN has a real presence and impact in Australian schooling stretching from the political through the media to state systems, schools, teachers and students in classrooms. These technologies have strengthened the national presence in schooling and are helping to constitute a more national system framed by an emergent global policy field. They have also become central to the governing of performance at system, school, principal and teacher levels. (p. 18)

National large-scale tests in numeracy commenced in 2008 with the introduction of NAPLAN. The research literature on national mathematics assessment is notable by its absence in recent years. What material has been published during the period of this review has often been conducted by researchers outside the mathematics education community, and focuses less on mathematics or numeracy than on equity issues. In this section, issues arising from NAPLAN testing will be considered in the broader context of mathematics assessment generally.

3.1 What Does NAPLAN Measure?

The construct of numeracy in the Australian Curriculum is of a cross-curriculum capability, one of seven that provide the basis for "The skills, behaviours and attributes that students need to succeed in life and work in the twenty-first century" (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2013a, p. 8). There is an explicit acknowledgement of a disposition towards using mathematics to solve every-day problems in that people need to have "the confidence, willingness and ability to apply mathematics to their lives in ways that are constructive and meaningful" (ACARA, 2013b, p. 8). In respect to NAPLAN tests, however, many of the test items appear to be classic "word problems" in which the mathematical ideas are embedded in a story or situation, and others are straightforward mathematics problems. Some typical examples are shown in Fig. 12.1.

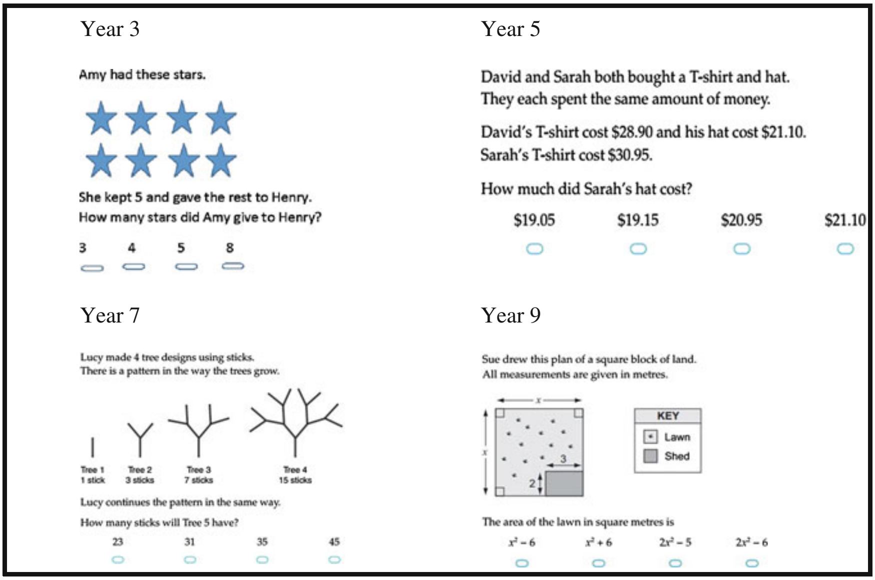

Fig. 12.1 NAPLAN items from Years 3, 5, 7 and 9 exemplar numeracy tests. *Note* The Year 3 NAPLAN item has exactly the same wording as a released item that could not be used for copyright reasons. ©Australian Curriculum, Assessment and Reporting Authority (ACARA) 2011 to present, unless otherwise indicated. This material was downloaded from the National Assessment Program website (www.nap.edu.au). *Source* Australian Curriculum, Assessment and Reporting Authority, 2012 (http://www.nap.edu.au/naplan/the-tests/the-tests.html)

These items appear to show a mismatch between ACARA's stated objectives and the assessment of some fairly conventional mathematical knowledge in straightforward ways. Although the kinds of skills addressed in the NAPLAN assessments underpin the use of mathematics in a variety of settings, they are not sufficient on their own to provide a sound basis for the twenty-first century. NAPLAN assessments clearly provide an indicator of potentially useful mathematical skills. As test items, they are rigorously trialled and validated, and are technically robust. The issue is whether they provide a basis for making judgements about students' capacity to deal with the mathematical problems of twenty-first century daily life. Despite the apparent mismatch between the nature of the items and the intended construct, there does not appear to have been any considered critique of NAPLAN numeracy tests in recent years. Perhaps the Australasian mathematics education research community should undertake work that acknowledges the strengths of NAPLAN but also proposes new approaches to mathematics and numeracy assessment that has the potential to capture some aspects of mathematical proficiency, such as reasoning, that are critical foundations for working mathematically in productive ways.

NAPLAN has been criticised on a number of grounds by researchers outside of mathematics education. Many of these studies are applicable to all subjects, not just mathematics, highlighting the need for a mathematics-specific program of research. Some of the recent discussions are described in the next section.

3.2 Uses of NAPLAN Data

NAPLAN data are used mainly for summative or evaluative purposes, driven by accountability. They are reported at national, state, school and individual student level in a variety of formats, including some interactive online resources (ACARA, 2013a). Wu and Hornsby (2012) have criticised uses of NAPLAN data on a number of grounds. They state, for example, that inferences based on a relatively short (about 40 items) test are not valid. The sample of the curriculum represented by the test is small, and does not cover all the possible content adequately. In this situation, Wu and Hornsby claim that it is not possible to discuss achievement at either school or student level because it is based on a narrow range of content and skills. Further, they say that the nature of NAPLAN is insufficient to use the tests for diagnostic purposes. There is not enough information about any one domain of mathematics or numeracy to be able to make sound decisions about areas of weakness. The fact that a group of students in a school has a low performance on a particular item may be due to contextual influences rather than on any deep misconceptions. NAPLAN also has a fairly large error margin and this can create a perception that a particular student is performing better or worse than he or she is in reality. The measures for a particular student are estimates only and should be read in conjunction with other assessment information. Tests such as NAPLAN can identify high and low performance at a point in time, but they are not sufficiently sensitive to provide fine-grained information.

3.3 Student Stress

Students themselves have indicated that they found NAPLAN tests to be very stressful (Wyn, Turnbull, & Grimshaw, 2014). Using focus groups and interviews, 70 students were asked about their experiences of NAPLAN. Almost all the students reported some level of anxiety, ranging from not being able to sleep the night before through to significant symptoms of severe stress, such as hyperventilating and headaches. Sources of stress included the time factor, and the language of the tests, especially in mathematics. For example, students indicated that they were familiar with talking about multiplication as “times” and did not know the word “product” unless they had been specifically taught its meaning. Some schools chose to focus on mathematical language in the lead up to the test in order not to disadvantage their students. Rather than adopting the mathematical lexicon as part of their practice, the emphasis appeared to be on a short-term quick fix. The next sub-section considers issues around language use in NAPLAN.

3.4 Language Issues in NAPLAN

In addition to being a source of stress, several researchers have pointed out that the nature of the language use in NAPLAN mathematics tests impacts more on some students than others, creating an equity issue. In a review of NAPLAN mathematics items for Years 7 and 9 undertaken across several years, Quinnell and Carter (2011) indicated the range of terms used for the same mathematical idea, such as subtract, minus and take-away being used interchangeably. They also list many words that have different uses outside the mathematics classroom, such as mean and product. These language issues are not new to mathematics educators but may not have been considered by teachers outside mathematics. It is interesting to note that Quinnell and Carter’s article was published in a literacy journal despite its focus on the mathematical language of NAPLAN. Similarly, Vista (2013) examined the role of reading comprehension as a mediator of mathematics achievement. Based on nearly 6000 government school students in Victoria, the study found that language background had no apparent influence on mathematics achievement but that reading comprehension had a mediating effect on problem-solving ability that, in turn, predicted growth in mathematics achievement. Over time, however, the influence of reading comprehension weakened. Although NAPLAN numeracy tests are often criticised on the basis of language use, it may be that language is less of an issue than might be thought.

3.5 Mode of Delivery of Assessments

A highly important area of research concerning the mode of delivery of external assessments in our rapidly changing technological world has been targeted by a growing number of researchers in Australasia. With the impending introduction of computer-based national testing, the representation of test items is critical to providing the optimum environment for examining students' cognitive abilities. Whatever the chosen mode, the representation of assessment tasks and the impact of modifications are essential factors to be considered by test designers. Lowrie, Diezmann, and Logan (2012, p. 169) devised an iterative study with the aim to "better understand the composition of graphical tasks commonly used to assess students' mathematical understandings". The researchers found that modifications to the graphical elements of a task did result in changes in terms of student success. Test designers are urged to closely consider the graphics embedded in tasks as they "greatly influence student understanding". Lowrie, Ramful, Logan, and Ho (2014) found that when comparing two test modes, pencil-and-paper and iPad, that the modes appeared to be influential in students' mathematics performance on graphic tasks with spatial demands and that student strategy selection changed when items were presented in different modes. The researchers recommend that "visuospatial reasoning skills will need to be taught more explicitly—and from an Australian perspective—in a new national curriculum that does not draw explicit attention to any such skills or abilities" (p. 434). In addition, Lowrie and Logan (2015), found that the participants preferred to use non-analytic strategies for most of their problem-solving processing and such processing was much more effective in the pencil-and-paper environment. "Perhaps when presented with the tasks in CBT mode, the participants found it more challenging to draw diagrams and encode information—such that using the iPads resulted in higher cognitive demands" (p. 653). Ho & Lowrie (2014) reported on one aspect from a larger cross-cultural study that investigated "students' interpretations of assessment tasks in numeracy and mathematics learning" (p. 91). The researchers designed a tool known as the Mathematics Processing Instrument (MPI) which comprised 24 questions sourced from NAPLAN and targeted Year 6 students from Singapore's PSLE (Primary School Leaving Examination). Responses to two questions were analysed closely in relation to the students' use of the model method as a problem solving tool. The findings recommended that teachers provide opportunities for students to explore the suitability of the model method to different tasks and to value other forms of visual representations that can be applied to problem-solving situations.

3.6 Predictors of Outcomes

The issues of inequity in educational outcomes are important, particularly in mathematics. There is evidence that performance in mathematics has an immense impact on future life opportunities (Lim, Gemici, & Karmel, 2013). In this sub-section, some of the factors that impact on performance on large-scale tests are examined.

Socio-economic status. Socio-economic status (SES) is a strong predictor of students' outcomes and this situation is shown in almost any large-scale testing program, including NAPLAN (Buckingham, 2014). Recent work by Marks (2015), however, suggests that school level SES is less strong an influence than prior achievement. Year 5 NAPLAN numeracy scores were best predicted by Year 3 scores, and similar results were shown for Year 7 scores based on Year 5. Carmichael, MacDonald, and McFarland (2014), using data from the Longitudinal Study of Australian Children (LSAC) (http://www.growingupinaustralia.gov.au/) and Year 3 NAPLAN results, indicated that a complex interplay among child, parental and school variables influenced numeracy outcomes measured by NAPLAN. SES was one factor that was statistically significant. In particular they pointed to home-school interactions as being a strong predictor of students' numeracy performance, but warned that more research is needed into the nature of these interactions. For example, parent help with homework had a negative influence on numeracy performance, which may be a function of parents feeling less confident about helping with numeracy. Children who liked mathematics tended to perform better, but again this relationship was intermixed with other factors.

Gender. Carmichael et al. (2014) found that gender was a significant predictor of NAPLAN performance with boys generally doing better than girls. They posited two potential explanations for this situation: possible stereotyping by parents (mathematics is a male subject), and a suggestion that many NAPLAN items rely on a spatial sense, on which boys have tended to score better. Leder (2012) considered trends in NAPLAN data over time. She found that between 2008 and 2011, for each Year level, boys consistently had higher mean scores than girls. The standard deviations, however, indicated that there was a greater spread in the boys' results. There was no constant pattern in the proportions scoring at or above the national minimum standard in Year 9, but at Year 3 girls did outperform boys on this measure. Leder summarised a number of papers that attempt to provide an explanation for the patterns of gender difference throughout the world, and called for a "nuanced rather than uni-dimensional reading" (p. 9) of the NAPLAN data. The gender issue extended beyond NAPLAN, and was shown in other large-scale external assessments. For example, using data from the Victorian Certificate of Education (VCE) Year 12 results, Forgasz and Hill (2013) identified different trends in enrolment for males and females, with slightly more boys than girls taking the two specialist mathematics subjects, Specialist mathematics and Mathematics Methods, with the reverse being true for the generalist pre-tertiary subject, Further Mathematics. When achievement was considered, males clearly outperformed females in all subjects. Additionally, SES—represented by school sector—single-sex schools, and location (metropolitan versus non-metropolitan) were factors that appeared to play a part.

Indigenous Status. Non-indigenous students have performed better than Indigenous students at all levels in NAPLAN from the time it started in 2008

(Leder, 2012). Many explanations have been offered, including SES, remoteness and language factors (e.g., Jorgensen & Perso, 2012). Schwab (2012) criticised standardised testing per se, arguing that tests such as NAPLAN can be detrimental to minority and Indigenous students. Carmichael et al. (2014) also found that Indigenous status was a significant predictor of lower performance, but tended to be subsumed into SES factors in more complex models. NAPLAN has been criticised as being an inappropriate approach to determining Indigenous students' mathematical abilities (Meaney, McMurchy-Pilkington, & Trinick, 2012).

The factors that impact on NAPLAN outcomes can obviously be interpreted in different ways. Leder's (2012) call for a more considered and subtle approach seems warranted in light of the often conflicting reports. NAPLAN does provide a gross measure of performance in mathematics. It is difficult, however, to gain permission to use NAPLAN data in ways that could potentially provide the nuanced approach called for by Leder. If NAPLAN data were made available on the same basis as the large PISA and TIMSS data sets, a potentially fruitful field of research could be opened up. Drake, Wake, and Hoyes (2012), using data from large-scale assessments in England, have developed a protocol for assessing assessment items. It could be productive to apply such an approach to NAPLAN, especially since the approach includes an indicator of authenticity, which would seem to be in keeping with the stated focus of NAPLAN.

3.7 Other Large-Scale Assessment Research

MacDonald and Carmichael (2014) considered the mathematical competency of 4 and 5-years-olds based on data from the Longitudinal Study of Australian Children (LSAC). Children's teachers completed a questionnaire that included a question about the competency of each child on six aspects of number, including interest in number. Whereas most children could sort, classify and count objects, fewer could recognise numbers or undertake simple addition. Of interest was the finding that these results appeared to be related to the nature of the program undertaken by the children prior to entering school, with school-based programs having a more positive effect than child-care centre programs. In addition age was a significant factor.

Despite the lack of recent work specifically addressing assessment in mathematics, Australasian researchers are undertaking assessment research that could potentially apply to mathematics. For example, one approach to considering twenty-first century skills and knowledge was that undertaken by the Assessment and Teaching of Twenty-First Century Skills Project (ATC21S) (Griffin, Care, & McGaw, 2012). In this study two broad categories of skill were identified: Learning Through Digital Networks, and Collaborative Problem Solving. Each of these two overarching groupings were further subdivided into several strands, such as Knowledge Building, and Intellectual Capital—defined as collective intelligence in networks. Assessment tasks were developed using technology as an intermediary.

What this project lacked was any consideration of the nature of the knowledge base. The importance of cognitive aspects was explicitly acknowledged but the assessment focus was on the processes of learning through digital networks and collaborative problem solving rather than the actual knowledge developed or used in this process. It is inconceivable, for example, to think of problem solving completely devoid of any mathematical thinking. The nature of the mathematics, as well as the contextual background on which students draw, is likely to affect the process of problem solving as has been reported in several studies (e.g., Langrall, Nisbet, Mooney, & Jansem, 2011). Again, however, there is no recent research that has a focus on the interaction of mathematical knowledge and processes in problem solving. Research into the uses of technology has also been more focused on the use of the technology itself, and how its use plays out in the classroom (e.g., Callingham, 2011; Geiger, Goos, & Dole, 2014).

4 Classroom-Based Assessment

Although curriculum documents and policies (ACARA, 2013b) make reference to the importance of classroom-based assessment, in relation to "for", "as" and "of" learning, research over the last 4 years has mainly focused on assessment "of" learning. This may be due to the social and political focus on external assessments. Despite the attention to NAPLAN and other national and international tests politically, within schools, and within homes, there has been some interesting and valuable research targeting student-centred classroom-based assessment that uses innovative assessment tools.

This section, which considers research targeting classroom-based assessments begins with an important study carried out by Hogan et al. (2013) in Singapore. The Singapore education system has received international recognition due to its "extraordinary record of achievement over the past two or three decades" (p. 57). The study found that in Secondary 3 English and Mathematics classrooms in Singapore "the national high-stakes assessment system, by virtue of its considerable institutional authority, both shaped the pattern of instructional practice at the classroom level and constrained opportunities for instructional improvement" (p. 57). The researchers offered four recommendations to the Ministry of Education in Singapore in regards to assessment practices that are of interest to other education sectors who are moving down the track of high-stakes national testing:

1. Improve the quality of high-stake national assessment tasks so that they prioritise "extended, elaborated, authentic, multidimensional twenty-first century knowledge building tasks (including tasks that are both collaborative and ICT-mediated) that will drive instructional improvement";
2. Introduce "a school-based, professionally moderated, standards-driven component … into the national high stakes assessment system";

3. Enhance “professional capability, authority and pedagogical autonomy of teachers”; and
4. Undertake a “modest deregulation of the assessment market”. (pp. 100–101)

The findings of this study highlighted that

> the current assessment regime incentivises and rewards teachers to teach (and students to learn) in ways that maximise assessment performance rather than the kinds of teaching and learning called for in national policy documents and generally associated with teaching for understanding frameworks (p. 99).

The researchers found “prior achievement as consistently the single strongest predictor of student achievement” (p. 99), similar to findings in Australia. This indicates a need to address sources of inequality such as social class effects at the individual level. It is interesting to note that one of the key issues raised was the inconsistency between internal structures and the overarching policies. Education systems such as that in Australia have the same inconsistencies when considering assessment for learning principles driven by curriculum documents compared with summative assessment and internal assessment schedules used at the school level. There appears to be a disconnection between all three levels; curriculum and policy, school structure, and classroom practice. The relationships and boundaries which impact upon mathematics learning and assessment between these levels is an interesting research direction flagged for future exploration.

In relation to classroom-based assessment, there have been some useful Australasian studies over the past 4 years. Although these are authentic and provide rich contextualised data, their significance can be lost as the wider community focuses on external assessment. Cheeseman and McDonough (2013) explored the use of photographs and diagrams within pen-and-paper assessment items to view students’ understanding of mass concepts. The researchers observed that the students were able to engage in the unfamiliar questions within a familiar setting and that the “open-ended response formats” elicited “explanation, deductive reasoning and justification of thinking of young children” (p. 152). Although only one sub-strand of measurement provided the context of the study, the research prompts the mathematics education community to continue investigations of authentic assessment practices in other domains. Complementing this field, Clark, Page, and Thornton (2013) studied the design of learning tasks that promote the transfer of mathematical knowledge to new contexts. These studies align with the work of Rogers (2013) who investigated the affordances of online assessments. Despite the practical advantages such as speed, Rogers found that many factors can obstruct the validity and reliability of the online tool, particularly the item design, similar to the work of Lowrie and his team discussed earlier. These studies raise many issues related to online assessment and contextualised open-ended response formats in classrooms as assessment “for” learning.

In addition, Stacey, Price, and Steinle (2012) considered the item design and usefulness of online tests as forms of formative assessment, and have highlighted the need to enable the feedback of student achievement to be available to teachers "at the point of teaching" (p. 400), using the computer generated feedback to inform teachers about what students know, and assist in determining future student tasks. In a subsequent study, Stacey (2013) explored the notion of developing teachers' pedagogical content knowledge using students' responses to online assessments to examine students' understandings, thus considering the responses as a means for reflective practice. A key finding of the research involves the careful consideration of the role of the "machine" (technology) and the role of the "human" (teacher). The importance of professional conversations between teachers were emphasized by White and Anderson (2012), who have added to this debate through their investigation of a professional learning model which considered using data from different sources, such as school-based and national tests to identify the learning needs of students. Secondary Mathematics teachers were encouraged to consider errors and misconceptions, and students' confidence in their attempts to answer context-based questions, and the development of key ideas in the mathematics classroom.

Fry and Makar (2012) utilised five effective strategies for formative assessment using an inquiry approach in a Year 3 mathematics classroom. This research justified "the richness of assessment opportunities in this pedagogy" and the statement was made by the researchers that "as pedagogy and assessment in primary mathematics classrooms move away from a focus on isolated facts, skills and procedures, pedagogy and assessment practices need to align with the new ways of thinking and understanding" (p. 1). Despite the evidence provided by this research and others' studies to validate the effectiveness of seamless assessment and learning tasks, there is a tendency for schools to take safety in focusing their attention on "teaching to tests" in the face of school "league tables". Studies have continued in pockets of the mathematics education community of Australasia relating to the assessment of particular mathematics concepts, such as place value (Rogers, 2012), and the concept of average investigated by Watson, Chick, and Callingham (2014). Cheng (2013) investigated the technique of asking students to problem-pose to gain an insight into students' understanding of fraction concepts. The importance of language and context was emphasised as opposed to a focus on whole numbers. Dockett and Goff (2013) also studied the notion of maintaining relevance for children's interests in mathematics assessment tasks and of documenting learning in ways that promote student engagement and interest in mathematics tasks.

Papic, Mulligan, Highfield, McKay-Tempest, and Garrett (2015) investigated the impact of a patterns and early algebra program on children in transition to school in Australian Indigenous communities. The researchers highlighted that although internationally there is increasing interest in enhancing early mathematics curricula and assessment, there is nevertheless still a gap in research on numeracy assessment tools to assess the learning outcomes of Indigenous students prior to commencing formal schooling. The findings highlight the need to engage students in tasks and to provide teachers with the necessary support to plan effective rich assessment tasks. The study also reiterated the work of Jorgansen and Perso (2012) which described the importance of scaffolding learning and that to be "effective in scaffolding, and thus to extend students' zone of proximal development, teachers must have knowledge of students' current understanding" (p. 136). Language differences remain an underlying issue in many parts of Australasia. This is of particular concern in the early years of schooling for Indigenous Australian students whose challenges when developing new mathematical ideas are compounded by the standard Australian English they are expected to engage in which is very different to the language the students speak in their own community (Mushin, Gardner, & Munro, 2013).

The Early Mathematics Patterning Assessment (EMPA) (Papic, 2015) provides educators with an interview-based assessment tool that provides a window for viewing students' understanding of patterns and early algebra. It is interesting to note the relationship between the development of engaging assessment tools, professional development opportunities using student responses to tasks as stimulus for discussions, the importance of teachers listening to their children, and making more use of teaching moments.

Assessment tools and frameworks relating to specific strands of mathematics have been investigated by various researchers in later years of schooling as well. These studies include the work by Hilton, Hilton, Dole, and Goos (2013) who describe a two-tier diagnostic tool which considered different types of student reasoning and highlighted the inability of students in their sample to apply proportional reasoning in different situations. The sample was very large, being 2000 middle-school students (Years 5–9). The team's work highlights the benefits of research into specific instruments that allow educators to view students' reasoning when providing correct and incorrect responses. Other researchers have responded to identified assessment issues through in-depth studies which map the developmental journeys students take when developing conceptual understanding of particular concepts. Examples include the work of Major (2012) who studied student responses to place value questions and identified a three-factor structure supporting the literature on the underlying concept of place value and providing critical ideas to consider when designing teaching/learning tasks.

Many schools purchase and use prepared tests to monitor student achievement in mathematics. Such tests frequently consist entirely of multiple choice items. One such test, the ACER Progressive Achievement Test (PAT) was examined by Rogers and Zoumboulis (2015). They reported that

> although these items are placed on a scale providing a score for students, there is value in taking the time to examine the carefully constructed questions in terms of the key and the distracter reasoning in order to identify misconceptions and adjust teaching and learning where necessary to challenge these misconceptions. (p. 119)

This comment highlights the formative and summative capability of some multiple choice items.

Logan, Lowrie, and Diezmann (2014) interestingly considered "the role and nature of co-thought gestures when students process map-based mathematics tasks" (p. 87). The findings highlighted the prevalence of silent gesturing when students were engaged in unfamiliar contexts or those that were spatially challenging. This research points to the value in teachers developing an awareness of co-gesturing as an indicator of cognitive challenges for students, emphasising the important role of teacher observation whilst students complete assessment tasks. Tate-McCutcheon and Drake (2015) iteratively developed an Individual Basic Assessment Tool (IBFA) to provide teachers with formative information about students' mastery of basic facts. The test employs four-second viewing of PowerPoint slides and a combination of teacher questioning and visual prompts to enable access to a range of abilities. This innovation is another example of teachers being involved in classroom assessment processes.

5 Conclusion

It is evident that in many parts of Australasia the mathematics education community needs to maintain an active role in providing the evidence-based research so that informed decisions are made concerning the direction of assessment practices. As a community, it is imperative that we provide information that enables informed decisions about the direction of national, international, and classroombased assessment.

Although there is tendency to focus on the limitations of external assessments, a way forward to positively contribute to national testing, is to focus on the strengths of tests such as NAPLAN and investigate additional approaches to large-scale assessment that will capture reasoning and working mathematically in productive ways. This would require a mathematics specific research agenda. Issues such as students' stress, language, and inequity are factors that have been shown to impact on student NAPLAN results. It is time to take the lead from researchers discussed in this chapter who have investigated alternative approaches to NAPLAN which carefully consider the authenticity and applicability of assessment items.

There are a number of potential areas that researchers, teacher educators, curriculum developers and teachers could study and learn from in the three international assessments. First, there is an opportunity for research and cross-fertilisation between the ideas and constructs underlying TIMSS, PISA and PIAAC. How does each construct relate to each other, and how do they relate to what is described, taught and assessed in schools and in lifelong learning?

Second, there is the ability to compare and analyse the data and information available, including from the available background variables, and the results themselves. The results from all three assessments show that proficiency levels in mathematics, numeracy or mathematical literacy varies widely in most countries. The combined findings from these assessments, which have many common elements as argued earlier, cannot be dismissed as being based on a small number of items that are divorced from the content of regular mathematics curricula on the one hand, or from the nature of real world mathematical tasks on the other. The results themselves should raise several red flags for mathematics researchers and educators.

Third, the PISA and PIAAC frameworks are potentially a rich starting point for considering what range of factors makes solving a mathematical literacy or mathematical problem solving task easy or difficult. The PISA scheme builds on the mathematical modelling or mathematisation cycle, while PIAAC builds on the evolution of the numeracy construct from a Quantitative Literacy origin, and hence more explicitly distinguishes between the influences of factors related to textual and mathematical aspects of numeracy tasks. Whilst initially they seem to be quite different, the two schemes are not as different as may at first appear.

Also, to date, studies on adult mathematics and numeracy and school-based mathematical literacy have developed in some isolation. Having the survey of adult performance, PIAAC, sitting alongside two international school based assessments would enable further research to occur about the differential performances within and across countries and to see how these differences can be accounted for. Investigating and reflecting on the conceptual and assessment frameworks developed for these three international assessment programs and their results can inform current debate about directions for developing and enhancing mathematical skills in the twenty-first Century.

Continued research into classroom-based assessment, is required at all levels for assessment to change and adapt as our classroom toolkit of technological tools evolves. Our children deserve the right to have mathematics assessment tasks that allow them to demonstrate “what they do know”, engage them, and consider their interests in an environment that does not cause stress. The evidence-based nature of assessment that considers individual student responses alongside commonalities and differences, prompts balanced feedback, and professional dialogue which informs future strategies remains an important research direction for Australasia. As a mathematics community we have identified inconsistency between internal assessment structures and external assessment programs. The level at which external assessments are driving classroom practice as opposed to the curriculum and student-centred practices is alarming. Targeted research that considers the relationship between assessment practices and mathematics learning experiences are invaluable and will provide an avenue for moving forward in our attempt to find balanced mathematics assessment approaches.

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Chapter 13
Transformations of Teaching and Learning Through Digital Technologies

Vince Geiger, Nigel Calder, Hazel Tan, Esther Loong, Jodie Miller and Kevin Larkin

Abstract This chapter is a critical synthesis of research related to the transformations that take place when digital technologies are incorporated into teaching and learning practices. In developing this synthesis, research from all levels of education was reviewed with a focus on the opportunities digital technologies offer for cognitive, pedagogical, affective and professional change. The chapter is structured in alignment with Pierce and Stacey's (Pierce and Stacey, Int J Comput Math Learn 15(1):1–20 2010) map of pedagogical opportunities in which three dimensions for educational transformation were identified: tasks, classroom, and subject. A discussion of future directions for research into technology enhanced mathematics education concludes the review.

Keywords Mathematics · Technology · Digital technologies · Digital tools · Computers · Digital pedagogies · Virtual objects · ICT

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1 Introduction

This chapter is a critical review of Australasian research, over the period 2012–2015, related to the transformations that take place when digital technologies are incorporated into teaching and learning practice. The review considers research conducted at all levels of education and focuses on opportunities digital technologies offer for cognitive, pedagogical, affective and professional change to the and teaching and learning of mathematics.

In the previous Research in Mathematics Education in Australasia (RiMEA) chapter on digital technologies (Geiger, Forgasz, Tan, Calder, & Hill, 2012), the following issues were identified as foci for future research:

- How digital technologies influence learning trajectories
- The influence of digital technologies on developing mathematical dialogue and argumentation
- The role of digital technologies in promoting and enhancing social interaction within learning communities
- How digital technologies can be integrated into student-centred inquiry learning approaches
- The equity issues which emerge with the wide-spread deployment of digital technologies and how these can be addressed.

Many of these issues receive attention within the current review, along with new and emerging research themes. It might be expected that these themes were influenced by the process of curriculum reform associated with the advent of the Australian Curriculum, although some commentators, for example Goos (2012), raise concerns that this may have been a lost opportunity for the integration of digital technologies into mathematics classrooms.

Fundamental to the progress of the field is the development of theoretical frameworks as a foundation for research. A number of studies reviewed in this chapter have contributed to new knowledge by extending frameworks previously utilised to frame research into the use of digital tools within mathematics education including transactional distance theory (Larkin & Jamieson-Proctor, 2013, 2015), TPACK (Handal, Campbell, Cavanagh, Petocz, & Kelly, 2013; Larkin & Jamieson-Proctor, 2013; Larkin, Jamieson-Proctor, & Finger, 2012) and affordances (Brown & Stillman, 2014). Others have used theoretical frames drawn from outside of mathematics education in new ways, such as Pierce and Stacey's (2013) use of Roger's (1995) framework for the diffusion of innovation. In some cases, more than one framework has been used to generate new theoretical insight, for example, Goos (2013) employed both the *Master-Servant-Partner-Extension of self* (MSPE) framework (Goos et al., 2000) and Valsiner's (1997) zone theory of child development to provide complementary perspectives on the evolution of teachers' technology influenced pedagogical identities. A number of chapters related to digital technologies in the RiMEA series have been organised around the types of technologies available at that time, such as, computers, multimedia, and the internet

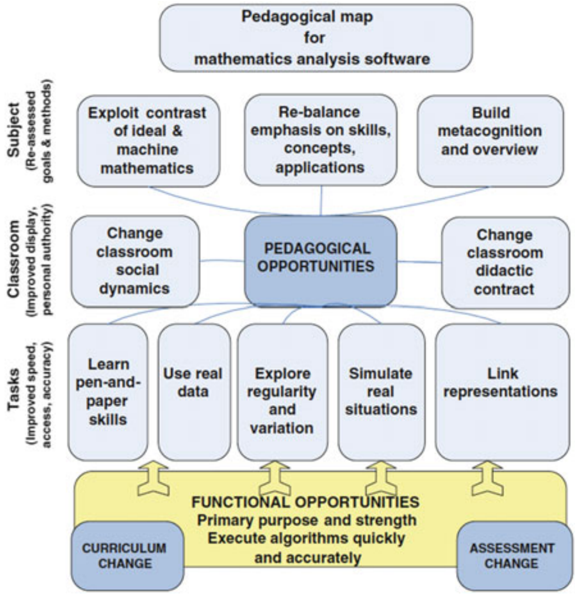

Fig. 13.1 Pedagogical map for mathematics analysis software. Pierce and Stacey (2010, p. 6), Fig. 2 with permission from Springer

(Goos & Cretchley, 2004), while others have taken a more thematic approach, for example, learning contexts and curricular design; learners and learning; teachers and teaching; and gender and affect (Geiger et al., 2012). This chapter is structured via the dimensions identified in the *map of pedagogical opportunities* (Fig. 13.1), developed by Pierce and Stacey (2010) (henceforth referred to as *the pedagogical map*), which positions technology as an agent that provides opportunities for three types of educational transformation: curriculum, assessment, and pedagogy. These transformations are made possible by the functionalities of digital technologies. While this map was initially created with a focus on the pedagogical opportunities afforded by mathematics analysis software, we see the framework as more broadly relevant to any digital technology that offers teachers and/or learners the technological functional capacities that allow for the dimensions of:

- *Tasks* they will set for their students
- Their *classroom* interaction
- The *subject* being taught

Within each of these dimensions, transformations are seen to take place within specific classroom activity. In the case of *Tasks*, for example, digital tools provide opportunity to change the: learning of pen-and-paper skills; investigation of real world data; exploration of regularity and variation; simulation of real situations; and the linking of representations. Transformations that take place within these types of activities, and others identified in research related to emerging technologies, will be outlined and discussed.

In the sections that follow, we review the role of digital technologies in promoting or mediating these types of transformations from the perspectives of *learners and learning* and *teachers and teaching*.

2 Tasks

In this section, we discuss research on the opportunities provided by digital technologies to transform tasks used for learning and teaching mathematics. Tasks related to activities presented in the *pedagogical map*, such as *learning pen-and-paper skills*, *investigating real world data*, *exploring regularity and variation*, *simulating of real situations*, and the *linking of representation*s, are considered along with innovative students activities made possible by the availability of digital technologies.

2.1 Learners and Learning

Research related to tasks from the perspective of learners and learning coalesce around five different aspects: *linking representations; using real data for exploring regularity and variation; simulating real or fantasy situations through game;* and *designing virtual objects*.

2.1.1 Linking Representations

The effectiveness of Apps used on different types of technologies (e.g., computers, tablets, some calculators) for transforming the learning of students within early childhood and primary education has been one focus of research into the use of digital technologies to link mathematical representations. Goodwin and Gould (2014), for example, found that the App *Race in the Outback* was effective in developing young children's numeral identification and ability to match numerals to quantity skills. This was effected through the linking of multiple representations of pictures of Australian animals and symbolic representations of number in the App. The potential of digital tools to mediate the linking of representations was also investigated by Yeh (2013), who found that the use of an online environment for integrating learning about 3D objects with direction, location and movement helped Year 4 students develop the capacity to mentally and visually construct virtual 3D objects.

In her research within secondary contexts, Brown (2015a) explored the potential of graphing calculators to link representations. Using data drawn from two Year 11 mathematical methods classes, and one Year 9 mathematics class, she identified 16 affordances that a Technology-Rich Teaching and Learning Environment (TRTLE) could offer student learning. These affordances were supported through technology by the linking of representations included in activities such as translating a situation (e.g., a task presented by teacher) into a strategy; perceiving relevant affordances of technology; choosing affordance bearers (e.g., the GRAPH key on the Graphics Calculator); applying mathematical and technical knowledge; interpreting mathematical output; and responding to the output. In contrast to the potential of digital tools to afford learning opportunities, Brown (2015b) also found that "the transformational power of technology was not realised by many students" (p. 440) despite having the technical and mathematical knowledge, due to the lack of perception of key affordances of the graphics calculator.

2.1.2 Using Real Data for Exploring Regularity and Variation

In a study of 27 students (12 years old), Allmond and Makar (2014) described the use of Tinkerplots (Konold & Miller, 2005) software that represented life-related data using box plots in an inquiry based teaching unit. Two investigations were described; an inquiry on estimating the time taken for a paper helicopter to drop, and a second inquiry comparing jumps of origami animals. Students collected data and created hand-drawn dot plots as part of both activities to visually identify point and interval estimates to account for variability. Data were then imported into Tinkerplots for further analyses and exploration where students attempted to standardise interval estimates and box plots. Findings included that the software: alleviated computational boundaries allowing for a focus on analysis; supported students in operating accurately and efficiently; and provided a flexible platform for the exploration of data.

In a study using generic digital tools, Geiger (2013) described a task where Year 8 students, in order to monitor their levels of physical activity as part of their Physical Education (PE) program, each wore a pedometer for one week. The number of steps students recorded in a day was entered into a spreadsheet that was on public display in the classroom. Students used the chart facilities of the software to explore the variability that lay within the data for different variables, for example, days of the week and gender. Additionally, students made use of arithmetic calculators to convert the number of steps travelled by individuals over a week into kilometres. Geiger reported on the high level of engagement among students and how the combination of task and technology allowed the teacher to move to a more investigative teaching approach.

While also reporting on high levels of motivation and engagement among students working with Web-based real-life scenarios and real data into the mathematics lessons, Loong and Herbert (2012) found that these experiences did not translate into an appreciation for the use of mathematics in everyday life. They argued that there must be greater emphasis on students working with "messy" data in order that they develop the contextual and strategic know-how needed to apply mathematics in the real world.

2.1.3 Simulating Real or Fantasy Situation Through Games

Some research reported on the effect of using software that simulates real and fantasy situations as a way to transform learning. Lowrie, Ramful, Logan and Ho (2013), for example, investigated the use of a game (The Legend of Zelda: Phantom Hourglass) that contained challenging spatial features that needed to be interpreted and decoded with respect to the game storyline. The research suggested there were gender differences in the type of games students enjoyed and in the approaches adopted to play the game. Male players generally utilised a trial and error approach and routinely accessed the spatial features of the games (including the dual screens) to monitor thinking and to record the pathways explored. Female players tended to read more text, use more deliberate movements, and utilised less experimental methods to determine where they wanted to go. The authors suggested that games such as this could be used as a stimulus for operating in out-of-game environments where similar skills were needed.

Games also featured in a study by Attard and Curry (2012) of Year 3 students' iPad supported mathematics learning. They claimed App-based games provided the interactivity, feedback, and information needed for higher levels of participation and so enhanced the learning of mathematics. Consistent with these findings, Calder and Campbell (2015) found similar outcomes with reluctant 16–18 year old learners when using app games within numeracy and literacy programs. Further evidence of the positive effect games-based mathematics instruction can have on student engagement is found in a study by O'Rourke, Main and Ellis (2013), where researchers made use of *Handheld Game Console Nintendo DS* technology to explore the perceptions of Year 4/5 students, from nine schools in Western Australia, in relation to the development of mental computation skills. While there was anecdotal evidence of improved mastery of mathematical skills, the study provided stronger evidence for increased levels of engagement in mathematics. Student participants reported that they were challenged, had developed strategies for success, and were able to make independent choices—factors that the authors claimed impacted on students' self-improvement approaches, self-determination, and self-esteem. While these findings are promising, Calder (2015) cautions that when evaluating the positive impact of games on student learning, the importance of appropriate pedagogy when using apps should not be overlooked.

2.1.4 Designing Virtual Objects

Past studies have also investigated the potential of tasks that require students to create virtual objects using digital media for learning mathematics. In a recently published study, Yeh and Chandra (2015) analysed how students designed a 3D virtual world using LOGO programs in a 10 week long teaching experiment. Groups of three Year 5 students worked in a team and chose a design project based on the creation of a virtual world. In the design and construction of the virtual world, students: explored relationships between geometrical objects (e.g., creating a simple 3D tree using a cone and a cylinder); made use of maps and plans; and utilised mathematical skills associated with number, measurement and scale, patterns and generalisation, and chance. The researchers reported that through employing trial and error strategies, students were able to see patterns and make generalisations. Further, they claimed that students' mathematical abstraction and logical thinking and reasoning were enhanced through the process of designing and creating their own virtual 3D world.

In a different approach to building students' mathematical capabilities by involving them in the process of design, Calder (2012) reported the use of *Scratch* programming software by 10 year olds to develop mathematics activities for their buddy class of 5-year olds. Scratch utilised a building block command structure with LOGO elements for students to create environments with graphic, audio and video elements. The study revealed that the immediate, visual and onscreen feedback, in addition to support from their teachers, served to modify and transform students' thinking about mathematics as they designed mathematics activities.

2.1.5 Summary for Learners and Learning—Tasks

The studies discussed in the preceding section show the variety of approaches and technologies that can be brought to bear within tasks in creating pedagogical opportunities that enhance mathematics learning. Aspects of the *pedagogical map* related to *tasks* such as *investigating real world data*, *exploring regularity and variation*, *simulating of real situations*, and the *linking of representation*s are all evident in current research; however, there did not appear to be any research relevant to the *learning pen-and-paper skills aspect.* The research reviewed here also investigated aspects of tasks not covered by the task aspect of the *pedagogical map*, such as technologies that allow students to operate within virtual worlds; either created for them or designed by them. These new themes are worthy of further investigation.

2.2 Teachers and Teaching

In this section we review literature related to technology enhanced mathematics tasks from the perspective of teachers and teaching. The themes that emerge from the research considered here represent a departure from the types of activity outlined in the *pedagogical model* under tasks, as they are concerned with the *challenges faced by teachers in designing tasks in ICT-rich environments* and how digital technologies are vital for *transforming tasks in pre-service teacher education*—while at the same time being mindful of the *task* aspects of the *pedagogical map*.

2.2.1 Challenges Faced by Teachers in Setting Tasks in ICT-Rich Environments

In order to design tasks that provide pedagogical opportunities, teachers need to be able to see the affordances of technologies for mathematics learning (Brown, 2015a) and have the motivation and the means (e.g., knowledge and skills; availability of hardware, software and support; professional development) to design such tasks.

Early career teachers (ECTs) represent a particular case. Attard and Orlando (2014) conducted four case studies of ECTs' use of technology within mathematics classes. They found that there was an unrealistic expectation, from established teachers, that ECTs were highly skilled with integrating technology into their teaching. Further, while ECTs seemed more familiar with the use of IWB, they found the use of iPads more challenging. Despite reporting that ECTs were aware of the potential advantages offered by the iPads available within their schools, the researchers reported that Apps were mainly used for drill and practice purposes. Attard and Orlando saw this behaviour as a reliance on familiar resources and a reluctance to develop their own tasks, or adapt other resources, to use with iPads—a situation they termed a *pedagogical dilemma*.

Established teachers can also face such a dilemma. Zuber and Anderson (2013) used a mixed-methods approach to investigate the experiences and beliefs of 28 mathematics teacher at five secondary schools during the second year of the New South Wales digital education revolution laptop program. Teachers were grouped into categories the "Non Adopters", "Cautious Adopters" and "Early Adopters" according to their self-reported classroom use of laptops. Overall, it was found that most teachers did not allow frequent student use of laptops in the classroom and so they rarely developed technology active tasks. This appeared to be due to a belief that "students authentically learn mathematics only using pen and paper" (p. 279). Additionally, "Cautious Adopters" and "Non Adopters" argued that laptops exacerbate classroom management problems, especially for lower-achieving students.

In a study where new technologies were more readily embraced, Pierce and Stacey (2013) observed four "early majority" teachers (those who accept the necessity for change but wish to move forward carefully and thoughtfully) who did not have a special interest in technology. They found that these teachers gradually integrated new uses for mathematics analysis software (e.g., spreadsheets, function graphers, symbolic algebra manipulation and dynamic geometry) into their existing repertoire of teaching approaches, typically by adapting existing tasks. Teachers viewed these new technologies as incremental improvements to earlier calculators and so it was important to introduce them into the classroom in order to keep students up to date with technology. Pierce and Stacey comment that this positive intention comes with the challenge of accommodating rapid changes in software and hardware design. The constant need to learn and teach new technical skills at the expense of looking for more opportunities to approach mathematics concepts in new ways is a dilemma that must be addressed when introducing new digitally enhanced approaches to mathematics teaching and learning.

2.2.2 Transforming Tasks in Pre-service Teacher Education

There is now great potential for technology embedded learning and teaching activities to transform teaching within pre-service education courses. The range of technologies utilised is broad and includes video-based digital tools, online Apps, electronic whiteboards, and online quizzes. Digital technologies are used to enhance the learning experience of pre-service teachers and to prepare them for utilising technological tools when they become fully active within the profession.

Martin (2012) reported on a study of the effectiveness of on-line quizzes as a formative and summative assessment instrument on mathematics subject content within a first-year primary teacher education program. The quizzes were included as a "hurdle" task within the course, that is, a pass grade or higher was necessary for a student to complete the subject. The quizzes were administered to student teachers at the end of the previous semester by way of preparation for the upcoming unit. During the semester, pre-service teachers were allowed to take the quizzes at any time and were encouraged to collaborate with peers, or others, when completing items online. When completing the online quizzes, immediate feedback and directions for further help on the content were provided. Pre-service teachers reported that the online quizzes were generally a useful tool for self-assessment and for improving their mathematical understanding.

Providing models of "good practice" that are readily accessible to pre-service teachers has been a long-term challenge in teacher preparation programs. In attempting to address this problem, Beswick and Muir (2013) used video excerpts of an experienced teacher engaging with four Grade 8 students in a course consisting of 176 pre-service teachers. The videos were used as stimulus for discussions about the connection between theory and practice within mathematics education. Although many participants struggled to see beyond obvious aspects of teaching, approximately three-quarters of the 176 pre-service teachers either agreed or strongly agreed that the videos: provided good examples of mathematics/numeracy practice; increased their understanding of effective mathematics/numeracy practice; provided a link between theory and practice; provided them with useful strategies to use in the classroom; and helped them to reflect on their own teaching beliefs and practices. The researchers also highlighted the importance of the quality of prompts following the video excerpts in directing the pre-service teachers' focus and discussion.

Researchers have also reported efforts to enhance the capabilities of primary pre-service teachers related to integrating digital technologies into their developing teaching practice. Bate et al. (2013) reported on the outcomes of a thirteen-week university mathematics education unit that was redesigned to model the use of technologies in tasks within mathematics classrooms as part of the $8 million Teaching Teachers for the Future initiative. The learning sequence involved attention to: concrete (physically building or creating objects); representational (using ICT to extrapolate concepts through accessing large data sets or using virtual manipulatives), and abstract (using ICT to manipulate large data sets or generalising from pattern generation). The researchers reported that throughout the course, pre-service teachers' conceptions of digital technologies changed from viewing technologies as motivational and/or drill and practice tools to envisioning possibilities of incorporating technologies into tasks meant for collaborative problem solving and student-centred discovery. The authors also claimed that pre-service teachers exhibited increased discernment toward digital technology integration by developing the ability to critique available software.

In a separate study, McDonald (2012) conducted research with third year students enrolled in a preparatory mathematics education course that embedded digital technologies such as Fun With Construction (2011), Smartboard notebook, various interactive websites, and productivity tools such as PowerPoint and Excel. Pre-service primary teachers were required to work in pairs to design an original interactive Smartboard application to support student learning for a specific mathematics concept. The results of pre- and post-program surveys, in addition to post-presentation reflection questions, indicated that confidence with using IWBs and digital technologies to teach mathematics had increased and attitudes towards the use of technology as a means of promoting student engagement and enjoyment were more positive. At the same time, pre-service teachers anticipated that there would be challenges when attempting to use digital tools in schools including access to software and hardware, networking issues, the principal's opinion of using technology in the classroom, and time needed for students to prepare for the use of digital technologies.

2.2.3 Summary of Teachers and Teaching—Tasks

In the research studies examined above, innovative approaches were reported in relation to tasks utilised to enhance pre-service teacher mathematics learning or as preparation for integrating digital technologies into their future classroom practice. It would seem that because of the nature of this research the task aspects of the pedagogical map were not as relevant as for learners and learning. This provides opportunity, however, to identify those aspects that are unique to teachers and teaching. While most studies reported positive outcomes, they also remind us of the potential challenges pre-service, early career, and experienced teachers face when they attempt to introduce technology active approaches to teaching and learning into schools. This challenge is strongly related to the beliefs and attitudes of current and future teachers about the nature of mathematics learning and the role of technology in this enterprise. Another challenge, identified in the research of Larkin (2015), is how teachers can discriminate between high and low quality digital resources, given the plethora of free Apps and commercial software now available. While there is encouragement from research that it is possible to develop teachers' capacities to make such judgements, this remains an important area for ongoing research.

3 Classroom

This section addresses research related to the classroom and consequently the dimensions from the *pedagogical map* of *changes in classroom social dynamics* and the *classroom didactic contract*. Thus the focus is on how learning mathematics plays out in individual and collective relationships between students, teachers and digital tools. In exploring this focus, we embrace a broad view of what constitutes a classroom by examining research related to both traditional proximate learning environments (i.e., face to face contexts) and non-proximate forms such as online courses, forums and communities. Past RiMEA chapters devoted to the role of digital technologies in mathematics education have considered teachers, learners, and digital tools as separate entities in the acts of teaching and learning (Geiger et al., 2012). In this chapter, we take a more integrated approach by looking at technology active classrooms from the perspective of *learners and learning* and *teachers and teaching*.

3.1 Learners and Learning

Two themes emerged when considering research related to technology active classrooms from the perspective of Learners and Learning. These themes related to the *classroom* aspects of the *pedagogical map*: *changing interactions and digital tools* and *peer interactions in the online space*. Research findings are drawn from primary, secondary, and tertiary environments.

3.1.1 Changing Interactions and Digital Tools

Tan (2012) explored the relationship between students' beliefs about mathematics and their interactions with graphing calculators through the lens of the *Master-Servant-Partner-Extension of self* (MSPE) framework (Geiger 2009; Goos, Galbraith, Renshaw, & Geiger, 2003). The study involved 964 Singaporean and 176 Victorian secondary students who were surveyed using an online questionnaire. Findings included that students' ways of interacting with advanced calculators were associated with their beliefs about mathematics and their experience of mathematics teaching. The use of calculators as *Master* (being subservient to technology) was associated with what Tan terms a high Separate Knowing-Surface Approach, and the use of calculators as *Collaborator* (high level of sophistication) was associated with a high Connected Knowing-Deep Approach.

Technology can be used to enhance and transform student learning when engaging in mathematical modelling. Redmond, Brown, Sheehy, and Kanasa (2012) conducted a study with both primary (Year 4 and 5) and middle school (Year 8 and 9) students using TI-Nspire handheld technology and digital probes when engaging in a mathematical modelling challenge. Students' survey data indicated that collaboration was necessary in order to collect data using the available technology. It was noted by the authors, however, that such comments were more prevalent among the older cohort of students.

By contrast, a study conducted by Whyburn and Way (2012) on the use of interactive white boards (IWB) with primary school students found that interactions between the teacher and student were limited, with the teacher asking closed questions with students seated at their desks for the duration of instruction with the IWB. Using *Bubble Dialogue* templates, group interviews and lesson observations to gather data on student views, a theme-based description of student perceptions was developed. The majority of students' comments related to engagement factors rather than deep learning and understanding, and the lesson observations revealed teacher-centred approaches, with limited strategies to promote student interaction and higher-order thinking. The findings highlight the need for supporting the professional learning of teachers even though they may have been using an IWB for several years.

3.1.2 Peer Interactions in the Online Space

In a study that considered how middle school students communicated with each other, and with the teacher when online in mathematics class, Mojica-Casey, Dekkers, and Thrupp (2014) found that students reported both positive and negative experiences. Students commented that while the use of ICT during mathematics lessons provided greater autonomy when communicating with peers, there was potential for all attention to be directed toward the computer screen, lowering the degree of interaction with others.

Promoting and managing productive interactions between peers has also been noted as a challenge for tertiary instruction in mathematics education. Larkin and Jamieson-Proctor (2013, 2015) considered how transactional distance theory (TDT) can be used in the design of an online mathematics education methods course for first year pre-service teachers that aimed to promote interaction between learners. The researchers found this framework to be helpful in understanding the balance between dialogue and structure for learners. Further, they claim there was a positive increase in student attitudes towards mathematics and a reduction in anxiety related to this subject. In a study that supports these findings, Galligan and Hobohm (2013) investigated the use of screencasts as an assessment tool for undergraduate education students. They found that the screencasts provide a catalyst for student/student interaction, thus reducing transactional distance.

Taking a socio-cultural approach, Goos and Geiger (2012) examined the peer-to-peer interactions between students enrolled in the third year of a pre-service mathematics education course. The research considered how software can promote student learning of mathematics and collaboration (e.g., Internet based networks for working with mathematics objects) and assist collaboration but not necessarily encourage mathematics learning (e.g., databases or virtual communities in online education). The researchers argued that technology-mediated collaboration can lead to different forms of knowledge and relationships between students. Thus, digital technology can transform the way students interact with each other and technology while constructing personal knowledge within a community of mathematics learners.

3.1.3 Summary of Learners and Learning—Classroom

Each of the studies reported above relates to *changes in classroom social dynamics* and the *classroom didactic contract* in some form. Research into the peer interactions in online space, however, represents a huge shift in what can be considered a classroom and, as a consequence, very different types of social dynamics need to be negotiated between peers as well as new and unique didactic contracts established between teachers and learners.

3.2 Teachers and Teaching

The themes that emerged from literature focused on teachers and teaching were: affordances that support interactions; transforming learning communities with digital pedagogy; and using digital technology to enhance interactions in research contexts.

3.2.1 Affordances of Digital Technologies for Supporting Teacher–Student Interactions

In a study of affordances in technology-rich teaching and learning environments (TRTLE), Brown (2013) provided a theoretical frame for understanding the role of teachers. Results of the study indicated that secondary mathematics teachers engaged in seven roles when interacting with students: evaluator; interpreter; hermeneutic facilitator; enabler; promoter; scaffolder; and follower. These roles were strongly linked to the management of students' enactment of affordances. The roles identified from this study provide opportunity for teachers to promote an engaging and interactive environment for the teaching and learning mathematics.

In a case study of one senior secondary teacher's initial attempts to adopt a Flipped Classroom approach, Muir and Chick (2014) reported that teaching practice shifted from a teacher-directed approach to one where classroom time was more focused on one-on-one tutoring—dramatically changing the way the teacher had previously interacted with his students. The authors suggest that this initial finding provides incentive for further research to be conducted into the new teacher-student interactions that emerge from this innovative approach.

3.2.2 Transforming Learning Communities with Digital Pedagogy

There were a number of studies concerned with reducing the barriers between lecturers and students, thus providing students enrolled in online course with a "similar" experience to those students who attend face-to-face courses.

Higher education is transforming the delivery of learning in mathematics with the development of new technologies, with blended learning environments becoming increasingly common as a mode of delivery. Building on Engelbrecht and Harding's (2005) desirable attributes for teaching online, Prieto and Holmes (2014) conducted a study with postgraduate students (practicing teachers $n = 60$) in which they investigated the factors that contributed to a quality online environment. The researchers reported that they moved from an asynchronous delivery to a blended approach, where the interactions with students were transformed to virtual face-to-face experiences. This was achieved through delivering content to enable: (a) students' self-regulation and engagement with digital resources (books, short videos, interactive blogs, discussion forums); and (b) student directed learning (assessment tasks with some providing feedback). Students indicated that engaging in discussion forums was the preferred mode of interaction within this community. Overall the participants who agreed to partake in the study accessed more of the online materials and had more interactions in the discussion forums, than those students enrolled in the course but not participants in the study.

Galligan, Hobohm, and Loch (2012) report on a study that considered the impact of digital technology in reducing the "distance" between first year university students studying within an online environment. Students reported that interactions with other online participants provided opportunity to demonstrate personal mathematical understandings, and for others to provide feedback and additional explanations. Additionally, the researchers claimed that these interactions created a "humanness" within the environment. It would appear, however, that the level of engagement for distance students was lower than those studying face to face, possibly due to the asynchronous adoption and delays in communication and support for students.

In a different take on tertiary learning within online environments, Muir (2012) conducted mathematics lessons with pre-service teachers using "Second Life" to provide pre-service teachers with the opportunity to focus on the teaching of mathematics in a virtual environment without the concern of impact upon the learning of real students. It allowed the instructor to both model the teacher's role and also to act as a facilitator to debrief, reflect and discuss the pedagogy involved in the learning experience. The author claimed that Second Life provided an alternative to more traditional approaches in that this virtual environment can be "controlled" by the instructor, providing the opportunity to focus on particular topics, misconceptions, learning difficulties, and pedagogical approaches. Feedback from participants indicated that despite some limitations, it had potential for enhancing their practical experience as pre-service teachers.

3.2.3 Using Digital Technologies to Support Interactions in Research

Larkin and Jorgensen's (2014) research on primary school students' attitudes and emotions towards mathematics extends the role of digital tools in transforming teacher-student interactions to include researchers. Within this research, students (Year 3 and Year 6) used iPads to record their personal aspects of learning mathematics. Results indicate that while 64 % of students (n = 105) recorded a video diary, most students only recorded one entry over the ten weeks. Despite this low rate, the researches claim that this method was more successful than traditional methods, such as surveys or interviews, for collecting this type of data. This study adds to methodological approaches to research involving young students and provides an alternative to the way in which interactions can occur between teachers, researchers and students.

3.2.4 Summary of Teachers and Teaching—Classroom

In this section, studies have each dealt with *changes in classroom social dynamics* and the redefinition of the *classroom didactic contract*. Online teaching environments appear to bring with them the challenges of isolation and disconnection of teachers and learners to their communities—hence the importance of attempting to create a more "human" experience (Galligan et al., 2012) within such environments. The notion that digital tools can catalyse a renegotiation of a different type of classroom contract between research, teachers and students (Larkin & Jorgensen, 2014) is also a new and novel direction for further research.

4 Subject

This section is concerned with research that aligns digital pedagogical media with particular mathematical content areas. In previous reviews (e.g., Geiger et al., 2012; Goos & Cretchley, 2004), there has been considerable reference to digital media that were developed to enhance learning in particular content areas. For instance, research that involved Computer Algebra Systems (CAS) in algebra or *Fathom* in exploratory data analysis. Pierce and Stacey's *pedagogical map* indicates that the potential for change in the *subject* being taught lies in three directions: *exploit contrast of ideal and machine mathematics*; *rebalance emphasis on skill, concepts, and applications*; and *build metacognition and overview*. Australasian research that related directly to specific mathematical content areas is considered with these directions in mind. As in previous sections, research literature is viewed through the twin lenses of *learners and learning* and *teachers and teaching*. Within these two areas, the review is organised under topic headings: numeracy and modelling; number and algebra; statistics and probability; and geometry and measurement.

4.1 Learners and Learning

4.1.1 Numeracy and Modelling

Several studies focused on the development of the ability to apply mathematics to the real world through attention to numeracy or mathematical modelling approaches. Redmond et al. (2012) sought to compare the reflective writings of two cohorts of students (Year 4/5 and Year 8/9) who participated in a series of mathematical modelling challenges. Whilst the reflections of the younger cohort were results oriented, the older cohort's reflections spoke more to the affective domain, group processes, the use of technology, and the acquisition of mathematical knowledge. This study supports the idea that with scaffolding, middle years students can engage in reflective practice to develop mathematical modelling skills.

Taking a different perspective, Geiger, Goos, and Dole (2015) investigated the role of digital technologies in promoting effective numeracy teaching and learning. Drawing on data from a case study of a single teacher and her class of Year 8 students, possibilities for technology integration into classroom numeracy practice were illustrated and discussed. This research provided evidence of the positive influence of digital tools on students' development of skills, mathematical knowledge, dispositions, and orientation towards using mathematics critically.

4.1.2 Number and Algebra

While there has been a focus on numeracy among Australasian educational jurisdictions, there has been continuing research into basic skills, such as number sense, that are an important foundation for the study of more abstract topics such as algebra.

In a study that investigated underlying skills in number sense such as counting and spatial skills among four year old children, Edens and Potter (2013) examined participants' patterns for engaging in spontaneous mathematical activities (primarily block construction and computer games) involving numerosity in free-play activity. Findings indicated that children who spontaneously focused on numerosity were advanced in their counting skills. Teacher rating of student motivation and interest was also correlated with improved counting and spatial skills, while child self-reports of persistence in mathematics correlated with improved spatial skills.

Research was conducted by Spencer (2013) into how iPads can support learning in number within the early years and made use of the App "Know Number Free" to assist young children to recognise and count numbers. The author claims the advantages of using iPads in promoting student learning in number include the personalisation of learning, providing opportunity for kinaesthetic and play-based learning, and motivation.

A study that linked understanding in number and algebra was conducted by Cooper (2012) and investigated the variety of ways pre-service primary teachers used an applet to demonstrate zero pairs as a mechanism for understanding algebra. In addition, Cooper also examined how the applet could be used as a tool for creating representational models, transforming the engagement, and bridging the gap between concrete and symbolic representations.

4.1.3 Statistics and Probability

When considering the judgements that students make about differences in data sets, Stack and Watson (2013) discussed some of the difficulties students have in conceptualising probabilistic and statistical concepts. In this research, they observed that students developed visual and intuitive understandings of the issues of sampling, randomness and populations. Further, the researchers argue that to create meaningful learning experiences for students requires considerable teacher knowledge and skill, especially in regards to drawing out key themes from a range of diverse activities. In a complementary report that investigated opportunities for the early development of informal inference with *Tinkerplots*, Fitzallen and Watson (2014) presented two examples that challenged the placement of inference in the school curriculum. The first study investigated Grade 5 and 6 students' understanding of covariation; the second was the study of Grade 10 students' understanding of resampling. Common to both studies were the opportunities for the development of informal inference. In both cases, many students based their conclusions on visual aspects such as the shape of a simulated distribution rather than relying upon statistical measures for these conclusions.

In another study that used *Tinkerplots* as a digital tool to support research, Prodromou (2012) examined a pedagogical theory of introducing the classicist and the frequentist approach to probability by investigating important elements in Year 9 students' learning process while working with a combinatorial problem. Results indicated that after students had seen the systematic construction of the event space via combinatorial analysis, they viewed the sample space as an essential property that regulated the results of the distribution of each sum's theoretical frequency.

4.1.4 Geometry and Measurement

Research within geometry and measurement was also conducted by a number of researchers. Yeh (2013) documented how an online learning environment, for investigating geometry and spatial abilities, transformed the ways that young children can mentally and visually construct virtual 3D objects using movements in both egocentric and fixed frames of reference (FOR). The research also included a focus on how the digital environment afforded the opportunity to integrate learning about 3D shapes with direction, location and movement. Findings suggested that Year 4 (aged 9) children can develop the capacity to construct a cube using egocentric and fixed FOR separately or in combination.

While findings that have implications for student learning are valuable, Lowrie et al. (2014) conducted research with the aim of providing insight into the challenges students face when attempting to respond to graphic-rich mathematics tasks—often used to make judgements about students' mathematical knowledge. This research compared Singaporean Grade 6 students' performance and strategy preference on two graphic-rich mathematics tasks, which were presented via pencil-and-paper and iPad modes. These researchers found statistically significant differences between students' performances on the two tasks, one in favour of the paper mode and the other in favour of the iPad. However, the students who possessed higher spatial ability were more likely to solve the tasks correctly in either mode. The implications of the study are timely given the fact that high-stakes tests are likely to be presented in a digital form in coming years.

4.1.5 Summary of Learners and Learning—Subject

While this section does not appear to cover research that addresses the *exploit contrast of ideal and machine mathematics* aspect of the pedagogical map, there are certainly instances where digital technologies have allowed for the *rebalance emphasis on skill, concepts, and applications* with students. Research on modelling and numeracy (e.g., Geiger, Goos, & Dole, 2015) offers two examples where applications of mathematics are facilitated by the use of technology and the work within statistics and probability (e.g., Stack & Watson, 2013) demonstrates how digital tools can be used to promote deeper understanding of concepts such as resampling. Perhaps because of the focus on *learners and learning*, there have been no instances of research related to *build metacognition and overview* and this remains an important avenue for future research as this is a key teacher responsibility.

4.2 Teachers and Teaching

4.2.1 Number and Algebra

Goodwin and Gould (2014), investigating the use of technology to support number development in preschool students, identified features of instruction related to the use of Apps that supported or inhibited early number knowledge development. Features such as giving instantaneous feedback were effective for particular groups of students, while they also found that teachers needed to expand and consolidate students' on-screen learning.

The use of Computer Algebra Systems (CAS) continues to be a focus in ongoing research. Heid, Thomas, and Zbeik (2013), examined the influence of CAS on the role of algebra in the school curriculum, and indicated that changes occur not only in the tasks, but also effect instruction and ways of working within mathematics classrooms. They contend that school classrooms that incorporated CAS encouraged new explorations of mathematical invariants, active linking of dynamic representations, engagement with real data, and simulations of real and mathematical relationships. Further, CAS created opportunities to both extend some algebraic procedures while also introducing and assisting the exploration of new structures, offering a range of opportunities for the enrichment of multiple views of algebra.

In another study that focused on the use of CAS by students transitioning to tertiary mathematics, Varsavsky (2012) reported findings of research which included two versions of Year 12 intermediate-level mathematics in the state of Victoria: one where students learn and are examined with CAS and another where students can only use scientific calculators. Their study compared the performance of 1 240 students as they transitioned to traditional university-level mathematics according to whether they learned intermediate mathematics with or without the assistance of a CAS. Varsavsky (2012) concluded that students without CAS demonstrated improved performance; however, the most important factor affecting student first-year university performance was whether or not they had completed advanced-level mathematics studies in secondary school rather than whether or not they had used CAS.

Other work has explored the benefits of instructional approaches that link algebraic, graphic, and numeric representations through the use of software such as *GeoGebra*. For example, Aventi et al. (2014) investigated how a class of Year 9 students reacted to *GeoGebra* when using this software for the first time. The authors claim that as a result, there was a shift in students' use of more formal language and, while students were not immediately comfortable with the dynamic environment, demonstrated an enhanced understanding of linear relationships.

4.2.2 Statistics and Probability

Digital tools were central to investigations into teachers' approaches to statistical inference and to assessment at secondary and tertiary levels.

Tinkerplots was utilised in an action research project involving two Year 10 teachers in the investigation of the implications of resampling (Watson & Chance, 2012). The authors claim that the use of this software was effective in assisting students to build intuitions about the effect of sample size and within-group variability and notions of inference based on resampling. Watson (2013) also reported that students were able to gain intuition about what is required to accept or reject a claim made for a difference of two populations, without formal testing procedures. Further, the concrete-visual approach of *Tinkerplots* facilitated such intuition (Watson, 2014).

In attempting to meet the significant challenge of assessing students understanding of statistical concepts at tertiary level, Neumann et al. (2012) report on the use of interactive computer-based simulations in informal and formal assessment. The authors reported that students believed the simulations improved their understanding of statistics and its practical application, gave them a way to practice statistics, motivated them to complete course tasks, and was interesting and engaging. Students attributed their perceived improvement to the immediate feedback provided by the simulation software.

4.2.3 Summary of Teachers and Teaching—Subject

Again, there was no research reported on the aspect *exploit contrast of ideal and machine mathematics*, perhaps an indication that this element is somewhat opportunistic in nature, rather than something teachers and/or researchers plan for. Most research in this section was related to *rebalance emphasis on skill, concepts, and applications* with the work of Heid et al. (2013) indicating clear advantages for teachers to create lessons in which students explore mathematics at a conceptual level, and engage with real data through the use of CAS. This potential should be considered with some caution given Varavsky's finding that the most important factor affecting student first-year university performance was whether or not students had completed advanced-level mathematics studies in secondary school rather than their use of CAS. Research in statistics education appears to address the aspect of *build metacognition and overview* as, for example, the work of Watson and Chance (2012) required students to begin with life related data sets in developing a conceptual understanding of resampling—approaching this topic from a different point than may be the case in many classrooms.

5 Conclusions and Future Directions for Research

This review has shown that there is a broad research agenda around technology and mathematics education. This agenda includes investigations into the use of a wide variety of digital tools for the purpose of improving the teaching and learning of mathematics. Pierce and Stacey's (2010) map of pedagogical opportunities has been used to guide the review of literature in this field by considering *tasks*, *classroom* and *subject* and the finer grained aspects of these dimensions. While not a specific focus of this review, it seems apparent that the various aspects of the *pedagogical map* are more applicable to either *learners and learning* or *teachers and teaching*, suggesting that the development of a pedagogical map for each of these perspectives might be a worthwhile research exercise. There were also areas where the map might be extended, especially in relation to learning and the opportunities provided by virtual learning environments, somewhat unsurprisingly, since the *pedagogical map* was developed to identify and describe opportunities made available by proximate technologies situated within conventional learning spaces. The extension of the pedagogical map to accommodate a broader range of technologies is an important future research enterprise. These suggestions are part of a larger issue of developing theoretical frameworks that can be used to guide research in the area of digital technologies and mathematics education.

The use of mobile digital technologies is gaining increasing prominence in both the broader community and also in mathematics classrooms. In addition, the plethora of Apps available for these technologies, including games that potentially embed aspects of mathematics that can be exploited by teachers to promote student learning, require further investigation.

A somewhat overlooked aspect of the role of digital tools in mathematics education is the use of technology in teacher pre-service and in-service learning. This review provides insights into the challenges facing teachers when introducing or expanding technology integrated mathematics classrooms. While some studies indicate there is great potential for technology to mediate teacher professional learning, it has been observed that teachers can adopt technologies in a manner that reinforces their current practice rather than transforms it (Pierce & Bardini, 2015).

Teaching and learning with online environments has clearly taken a foothold and will likely expand—in secondary and tertiary contexts at least. How new and emerging technologies mediate productive student-student and student-teacher interaction within these environments is an ongoing theme within research. These environments demand new types of interaction and collaboration between students and students and students and teachers, especially as digital tools become "smarter".

How to improve teaching and learning within specific subject disciplines is an ongoing topic of research, especially as product development in mathematics enabled technologies continues. While research into the use of existing technologies such as CAS, *Tinkerplots*, and *GeoGebra* are of current interest; new or as yet unknown technologies, offering different affordances and constraints, will continue to emerge into the future.

In considering how and in which direction the field might progress, attention must also be paid to the equity issues that emerge when teaching and learning with digital tools (Bardini, Oldenburg, Stacey, & Pierce, 2013), particularly in rural and remote communities (e.g., Goodwin & Gould, 2014; Lowrie & Jorgensen, 2012). Issues of access remain a point of division between the advantaged and the disadvantaged in all aspects of education and thus work must be done to minimise inequity exacerbated by differing levels of access to digital technologies.

This review has provided insight into future directions for research into technology enhanced mathematics education including the need to:

- develop principles that help teachers discern the quality of Apps and other software
- provide insight into how to participate in and manage interactions and relationships in online environments
- explore the potential of virtual worlds and technologies that allow students to design or influence technology formed environments

- investigate new modes of instruction, for example, blended learning and flipped classroom approaches
- document the role of digital technologies in pre-service and in-service mathematics teacher education
- explore the notion of *futures* in relation to new digital tools
- investigate new ways of structuring teaching approaches that leverage off the affordances of digital tools.

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Chapter 14
Research into Mathematical Applications and Modelling

Gloria Stillman, Jill Brown, Peter Galbraith and Kit Ee Dawn Ng

Abstract Using mathematics to solve or make sense of real-world problem situations and examining how mathematics has been used in context is of growing interest and importance in research within Australasia. Simultaneously curricula writers are increasingly recognising the importance of educating mathematically literate citizens of the world who can solve real-world problems. This chapter considers the different perspectives on teaching through mathematical modelling and applications and briefly reviews exemplary literature focusing on the teaching of mathematical modelling and applications. Not surprisingly, clear differences emerge in classroom practices engaged in applications, using known mathematics, versus modelling—where the mathematics that might be useful needs to be determined by the student. Theoretical developments in the field are reviewed. Recently, the research field has expanded from being mainly school focused to include pre-service and in-service teacher education. Methodological tools used in modelling and applications research that are being adopted or adapted in the region are then reviewed. Finally, the current positive state of Australasian modelling and applications research and its progress since last being reviewed in 2008 is discussed with an eye to possible future developments.

Keywords Applications · Mathematical modelling · Real-world · Metacognition · Mathematise

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1 Introduction

Mathematical applications and modelling have been part of the research and development agenda of the international mathematics education community for more than 30 years (English & Gainsburg, 2016). Although inclusion of applications and mathematical modelling in school mathematics curricula in the Australasian region (Ang, 2013; Australian Curriculum, Assessment and Reporting Authority [ACARA], 2014) has had a chequered history (Stillman & Ng, 2013), research into their teaching and learning has been ongoing, fuelled by "its potential to add another dimension to the mathematical experience and skill of learners" (Stillman, Brown, & Galbraith, 2008, p. 141). Shimizu and Williams (2013), amongst others, draw attention to the usefulness of modelling as an approach to deep learning.

Members of the Australasian research community have provided leadership in this research field internationally for over two decades. Galbraith and Stillman have done so at the secondary level, and English at the primary level, most recently through data modelling studies (e.g., English, 2012). The biennial International Community for the Teaching of Mathematical Modelling and Applications (ICTMA) conferences have been a major impetus for researchers as well. ICTMA 15 was held in Melbourne and the subsequent edited book (Stillman, Kaiser, Blum, & Brown, 2013) includes several chapters authored by Mathematics Education Research Group of Australasia (MERGA) members. Another ICTMA volume edited by Stillman, Blum, and Biembengut was published in 2015. At ICME 12 in Seoul in 2012, the importance of the field was recognised with a keynote plenary lecture on modelling and applications and invited regular lectures on related topics by Stacey (2015) and Stillman (2015). Research on applications and modelling from Australasia (e.g., Mousoulides & English, 2012; Visnovska & Lamb, 2012) was showcased in several forums. Against such a continually widening field of interest, this chapter reviews research and development in this region.

We structure the chapter as follows. Firstly, the different perspectives on teaching through mathematical modelling and applications are considered. Next, exemplary literature focusing on the teaching of applications is briefly reviewed. This is followed by a review of theoretical developments in applications and modelling in the Australasian region and how studies in the field have addressed general educational goals. Methodological tools used in modelling and applications research that are being adopted or adapted in the region are then reviewed. Finally, the current state of Australasian modelling and applications research and its progress since last being reviewed in 2008 is discussed with an eye to possible future developments.

2 Perspectives on Teaching Through Modelling and Applications

Modelling and applications have traditionally been coupled together; however, for the field of research in particular this does not imply that each has received similar attention. When the volume of Australasian research is considered applications comprise only about 10–15 %, especially among work advancing the field. One reason is that modelling is both more demanding and more complex than applications but contains more transformative potential that "should promote the kind of learning valued in twenty-first century work and life" (English & Gainsburg, 2016, p. 329). The chapter development will reflect this relativity.

To distinguish between modelling and applications we refer to Niss, Blum, and Galbraith (2007) who describe modelling as focusing more directly on the *processes* involved in going from reality to mathematics, that is, "standing outside mathematics looking in…trying to find some mathematics to help with the problem" (p. 10). In contrast, applications focus on going from mathematics to reality, emphasising the *objects* involved—particularly those parts of the real world for which mathematical models already exist. When considering applications, "we are standing inside mathematics looking out" (p. 10) trying to find somewhere the mathematics already chosen can be used.

Researchers in the teaching and learning of mathematical modelling and applications share a domain of interest; however diversity in interest shapes research and teaching agendas. Some applications approaches set out to link mathematics with real situations and claim links to mathematical modelling in some fashion (e.g., Nolan & Herbert, 2015). Approaches vary from small changes in emphasis to different genres (Galbraith, 2013). Examples are: using contextualised examples to motivate studying mathematics (e.g., Wells, 2014); curve fitting (e.g., Brown, 2015a); and word problems that use practical settings (e.g., Downton, 2013). For various reasons, if taken alone, these approaches lack one or more elements necessary for making a significant contribution to mathematics education if the full power of modelling as an authentic source of real-world problem-solving expertise and individual empowerment is to be realised for students—as emphasised in curriculum documents (ACARA, 2014; Ministry of Education, 2006). It is acknowledged, however, that curve fitting often appears as an important component skill.

Modelling-as-vehicle for teaching other mathematical material (e.g., Lamb & Visnovska, 2015) and *modelling-as-content* in its own right (e.g., Chan, Ng, Widjaja, & Seto, 2012) are terms coined by Julie (e.g., Julie & Mudaly, 2007) to delineate between two fundamentally different purposes when teaching mathematical modelling. Modelling-as-vehicle is used generically to include approaches that target the pursuit of curricular priorities for which elements of modelling are seen as a means to an end. It encompasses contextualised examples, curve fitting and word problems as particular instantiations but if problems of available teaching time emerge then further modelling aspects, such as model refinement, typically give way to moving on to teach new content (Galbraith, 2012).

Modelling-as-content, also described as “modelling as real world problem solving” (Galbraith, 2013), aims to equip students with skills that enable them to use mathematics in their living environment, whether workplace related, in their personal lives, or as responsible citizens. It contains two complementary purposes —to solve particular real-life problems, and to grow this expertise in students over time (Chan et al., 2012). These are the qualities emphasised in the curriculum documents above in relation to student goals, and this is the approach to modelling within which the authors of this chapter primarily work.

Given the reviewing context, we remind readers unfamiliar with the field that authors do not always make clear the orientation they take when using the term “mathematical modelling” to describe their activity. In essence the number of genres can be reduced to two (Galbraith, 2012), distinguished by the ultimate authority determining how the modelling is permitted to unfold, that is as within one or other of the vehicle or content descriptors. Whilst the content and vehicle approaches differ in some key emphases and purposes, their goals are not necessarily mutually exclusive as is captured in the work of Ang and Tan (2014). In seeking to solve genuine problems the need for new mathematical content may emerge and be developed, whilst real-world contexts can provide legitimate vehicles for the structured introduction of desired mathematics as applications.

3 Applications

Curriculum writers have been aware of the relevance of mathematical applications for many years. However they are less forthcoming in providing illustrations to enliven classroom mathematics, as the following statements from the Australian National Curriculum (ACARA, 2014, p. 3) show for a senior mathematics subject:

- use exponential functions and their derivatives to solve practical problems (ACMMM101)
- use trigonometric functions and their derivatives to solve practical problems (ACMMM103).

Such general statements provide little guidance as to the context of suitable examples, or the performance level expected. Without such guidance there is little prospect of improvement over past efforts. Furthermore, there is no shortage of information from the past (e.g., Stillman, 2002) indicating the unrealistic nature of some alleged applications, and the disdain with which such examples are treated by students.

It is a constant challenge to find new applications based in genuine real-world contexts. Sometimes these can be produced as structured questions following modelling problems in which mathematical insights have been generated along the way (see Jennings & Adams, 2013). With respect to materials currently available, Stillman, Brown, Faragher, Geiger, and Galbraith (2013) selectively analysed Australian senior secondary textbooks to examine the role textbook tasks might play in developing a socio-critical perspective on mathematical modelling. Many examples were found of standard applications utilising a variety of functions in simple plausible contexts. In some texts these were evidently influenced by high stakes examination task formats. Some contexts were authentic to real life but many were not, and instead of enhancing students' understanding of the world were more likely to reinforce the view of the limited applicability of school mathematics. Whilst the development of a critical disposition emphasising awareness of the social/political context of modelling and applications was not the overt intention of the textbook authors, some examples provided opportunities for such development.

Word problems are a genre with a long history and a large literature, and some research and development has been directed towards attempts to give them a more realistic base as applications of mathematics and even claims as modelling problems (e.g., Verschaffel, van Doreen, Greer, & Mukhopadhyay, 2010). However, both simplicity and artificiality continue to present challenges to their capacity to act as serious applications of mathematics.

Gatabi, Stacey, and Gooya (2012) compared a sample of year 9 Iranian and Australian textbooks for content relevant to mathematical literacy, with the analysis preceded by a review of the modelling process using an OECD framework. They identified a substantial number of examples classified as extra-mathematical world problems, noting that "the very large number of problems in the Australian textbooks enables a very wide variety of contexts to be encountered by students" (p. 417). They further noted that all the extra-mathematical world problems in the Iranian textbook could be regarded as standard applications (rather than mathematical modelling). However the nature of the Australian content was similar (as the article authors agree), being essentially of the familiar word problem genre.

Has anything really changed? Australasian research around mathematical applications, outside the word problem genre, was less overtly evident within the review period than research on modelling. However there are life and classroom contexts in which skills of application in real contexts are needed, other than as a contrast to modelling in the Niss et al. (2007) sense. One of these is numeracy. Geiger, Goos, and Forgasz (2015) provide a comprehensive survey of numeracy literature and the pedagogical practices associated with it. One practice uses real-world vignettes from the media or in the lived environment (see examples in Geiger, Forgasz, & Goos, 2015; Goos, Geiger, & Dole, 2013) to provide contexts for students to apply mathematical knowledge to understand, critique, or suggest quantitatively informed responses.

Geiger, Goos, and Dole (2013) illustrate use by a primary teacher of a five component numeracy model where the dimensions of contexts, mathematical knowledge, tools and disposition are embedded in a critical orientation to using mathematics. The approach involved the recognition and deployment of opportunities to apply mathematics in a situation affecting the immediate environment of students, namely, a building development within the school. A lesson sequence was designed where students considered how school activities needed adapting to accommodate changes necessitated by the construction. Teacher structuring enabled mathematics to be seamlessly integrated into the adaptations, providing a range of opportunities for students to apply their mathematical knowledge to relevant and clearly defined real-world situations. Goos et al. (2013) provide further examples of use of the numeracy model by a secondary teacher. Both papers exemplify the use of students' living environment to enrich classroom numeracy tasks.

According to Bennison (2015a), the richness of student numeracy experiences that particular teachers provide is related to their identities as embedders-of-numeracy in their everyday classroom practice. Bennison developed a conceptual framework for *identity as an embedder-of-numeracy* identifying and characterising five domains of influence—knowledge, affective, social, life history and context. A limitation of the framework is that it provides temporal snapshots of teachers' identity which are static. Identity is dynamic. To overcome this limitation, Bennison (2015b) used Valsiner's zone theory (1997) to capture the fluid nature of a teacher's identity in the context of promoting numeracy. Bennison's on-going work during the review period has implications for future professional learning programmes to assist teachers to promote numeracy through use of applications in real contexts.

Another area of progress is problem posing, which offers students valuable opportunities to create, adapt and then solve their own problems. Downton (2013), for example, advances the case for the applicability of problem posing in the primary years. She provides documented evidence that students as young as 6 years are capable of generating questions that can be investigated using mathematical modelling. She argues that using problem posing and real-world artefacts in the early years could provide the foundation for later modelling and applications experiences. English and Watson (2015) have also integrated problem posing into Year 4 classes in statistical literacy aspects of the curriculum demonstrating that statistical literacy is a rich context for the development of problem posing skills.

Applications of particular mathematical content need not always be framed as stand-alone examples. For example, once particular mathematics has been identified as relevant within a modelling problem the solution phase requires accurate application in the identified context, and appropriate interpretation of outcomes. Tan and Ang (2012) focused on issues surrounding the development of Pedagogical Content Knowledge (PCK) in mathematical modelling instruction for Singaporean teachers new to modelling. Three levels of learning experiences (Ang, 2015) were defined to guide teachers in designing or choosing tasks at a level where students have the skills and competencies to complete them successfully. The levels are: (1) basic (mathematical and technological) skill acquisition, (2) modelling competency development through application of known models, and (3) undertaking modelling. While the overall context is modelling, tasks at the second level involve applying existing mathematics to practical situations. In this sense application skills of solution and interpretation were built into the development of a systematic modelling structure as part of a teacher professional development tool. Thus, applications are accorded (rightfully in the judgment of the reviewers) a valued place in the transformation of teacher practice and the student learning environment.

Considering another dimension of application, in order to apply mathematical skills proficiently, students need to understand different mathematical representations of a concept, and be able to move smoothly between them. Nolan and Herbert

(2015) investigated student understanding of linear functions following an introduction using scatterplots and least-squares lines employing real-life data related to students' previous experience. To ensure different representations were worked with, the class using this alternative approach made extensive use of technology. Results of an end-of-topic test for the experimental class were compared with those of a traditionally taught class. There were no differences statistically between the two classes' results except with respect to sketching graphs from symbolic forms and use of direct proportion and linear models in worded real-world applications. For sketching of vertical or horizontal lines, the traditional class was superior but for sketching linear functions using the gradient-intercept method and responding to the worded real-world applications tasks the experimental class was superior.

In summary, a final comment is appropriate with implications for classroom practice that emerge from the inherent differences between applications and modelling as referred to in Sect. 2. Applications invite the perenniel relevance question from students long lamented by teachers (see, e.g., Brown, Redmond, Sheehy, & Lang, 2015). Particularly if the treatment of applications is unsystematic, this issue will arise frequently—and it is a question no-one can hope to answer consistently for individuals. On the other hand modelling provides two significant potential insights. Firstly, it becomes evident that it is not known in advance what individual pieces of mathematical knowledge will prove relevant ahead of engaging with a particular real-world problem. From this, it follows that the more mathematics an individual knows and is competent with, the better they are equipped to apply their mathematics to problems in their world.

4 Theoretical Developments

Theoretical development in the field of applications and modelling, as in other fields, is characterised by different forms of contribution. In addition to refinements of existing constructs, frameworks, and approaches that advance a field in previously defined directions, are new directions that are developed to meet challenges in the field or create new initiatives. In either case, but particularly the second, the discussion of developments may need to draw from a wider set of literature than the geographical or temporal window of a particular review. This section reviews contributions from both these genres.

4.1 The Dual Modelling Cycle Framework (DMCF)

The Dual Modelling Cycle Framework (DMCF), introduced by Saeki and Matsuzaki (2013) in an attempt to support students with different capabilities in modelling, is a new theoretical framework being used by researchers in the Australasian region (Lamb, Kawakami, Saeki, & Matsuzaki, 2014). Sometimes, when modellers are unable to anticipate a model or solve a task, they imagine models for a similar task from their prior experience to help progress the solution of the first task. Saeki and Matsuzaki used this idea to design two similar tasks that could be used in teaching to scaffold such a process with struggling modellers. By solving the analogous second task using a second modelling cycle, the modellers are able to apply the results to the location on the modelling cycle for the first task where they were struggling, thus forming linked dual modelling cycles.

Lamb et al. (2014) investigated the use of the DMCF with Australian primary school students. Several students were able to use their understanding of one task to support interpretation of a second task in keeping with the DMCF. They worked quickly to identify the way each model could be used to validate their solutions. Other students did not easily understand the first task but when they came to the second task could easily see the similarity in solutions. This finding supports the work of Saeki and Matsuzaki (2013), in particular their finding that the DMCF enables some students to engage more fully with modelling and so deepen their mathematical knowledge. This work supports the DMCF being potentially fruitful in the development of modelling competencies by providing a structured approach within situations that afford the identification of well known accessible models in one task that are relevant in solving another modelling task.

4.2 Modelling and Applications and the Epistemic Fallacy

National curriculum statements continue to espouse the ability to solve problems arising in everyday life, society, and the workplace as a major goal of mathematics education. However, support for such high sounding rhetoric is typically weakened (or absent) when the specifics of curricula are elaborated. In this context central concerns are what constitutes the "real world" in terms of the experiences of students' everyday life activity and the nature of problems located in this environment (Stacey, 2015). Statements like: "There is no objective reality; the physical world exists but is not accessible to human endeavour" (Sarantakos, 1998, p. 37) permit the imposing of "realities" that are social constructs alone. In the real-world modelling context it is necessary to explicitly take into account the structure of the natural world where modelling is enacted and solutions are evaluated—the world of human endeavour. This should also be true when choosing contexts for applications.

To illustrate implications for applications or mathematical modelling, Sfard (2008) argued that the minute an *out-of-school problem* is treated *in school* it is no longer an out-of-school problem, so the search for *authentic problems* to be modelled or used as applications is necessarily in vain—as they lose their authenticity. This argument seeks to define the *location* of the activity as the determinant of its authenticity.

In real-world problem solving authenticity has several dimensions (Galbraith, 2013), and a central one is that the solution be testable in the context in which the problem is situated, which is external to the classroom. Problems involving the physical and biological worlds, for example, rest on ontologies defined by the nature of scientific realities that must be respected. These testable attributes (e.g., of physics and biology) cannot be replaced by mental constructs, which effectively do away with ontology by placing all authority in the hands or minds or caprices of individuals, for in that case there remains no external authority from which to build epistemological choices as to what constitutes valid knowledge. Valid epistemological choices regarding the type of knowledge to pursue (properties of the modelling conducted) must be consistent with the implications of underlying ontologies. The conflation of ontology and epistemology led to the development of Critical Realism (Bhaskar, 1975), in particular "The Epistemic Fallacy". This term is used to describe the confusion of the nature of an underlying reality with knowledge of it. The increased significance of the Epistemic Fallacy (Galbraith, 2015a) has emerged because of the emphasis being placed upon the goal for students to apply their mathematical knowledge in the world outside the classroom.

The position illustrated by Sfard (2008) assumes an epistemology which privileges certain notions about what teaching is like and what classrooms are allowed to be. The assigning of a classroom as the defining "ontological" reality, rather than the environmental realities within which the problem is located, leads to problems set in the real world outside the classroom being deemed inauthentic. This is precisely back-to-front. It is the requirements of the real-world problem which determine the validity of the solution process and outcome, and this has implications for what a classroom must encompass—not vice versa.

As a corollary we note by implication that the meaning given to *authenticity* itself lies at the heart of mathematical modelling as real-world problem solving. In arguing that the term has been used too uncritically, Galbraith (2013) described *authenticity* in terms of four dimensions, which are important whether the setting involves individual activity on a private project, a workplace example, or a classroom modelling project. These dimensions are: *Content authenticity:* The problem involves genuine real world connections and its solution is within students' mathematical knowledge. *Process authenticity:* A valid modelling process underpins the approach. *Situation authenticity:* The requirements of the modelling task drive the problem solving activity not vice versa. *Product authenticity:* Given time constraints, the solution is mathematically defensible and it suitably addresses the real world problem.

The purpose of making the meaning of authenticity more explicit is to enable engagement with the substance of papers that use the term in non-explicit or idiosyncratic ways. The adaption of the Epistemic Fallacy construct to apply within mathematical modelling by Galbraith (2015a) provides a lens through which individual modelling and the general approach to modelling for students can be examined.

4.3 Anticipatory Metacognition

Metacognition has long been central in the research and practice of mathematical modelling (Ng et al., 2015; Stillman, 2015) and applications (Stillman & Galbraith, 1998). Both experts and learners in the field know the importance of reflection on actions undertaken in addressing a real-world problem, whether checking mathematical accuracy, evaluating a solution against contextual implications, or examining interim decisions. Such abilities remain important, but as a research theme metacognition is moving in new directions as seen in literature during the review period. In Galbraith (2015b), for example, the anticipatory nature of metacognition associated with "noticing" in the practice of modelling as real-world problem solving is highlighted. In terms of modelling activity, *anticipatory metacognition* shifts emphasis towards reflection pointing *forward* to cognitive actions not yet undertaken, but identifiable as possible, desirable, or necessary. *Anticipatory metacognition* is modellers' metacognitive processes as they attempt to anticipate the mathematical, cognitive and physical resources necessary to mathematise real-world situations into mathematical models. Work to date suggests there are three distinct dimensions: (1) meta-metacognition (Stillman, 2015), (2) implemented anticipation (Stillman & Brown, 2014), and (3) modelling oriented noticing (Galbraith, 2015b).

(1) *Meta-metacognition*

Meta-metacognition as explored in Stillman (2015) and Galbraith (2015a) can be understood in the following ways with respect to how teachers function in their mentoring role. In considering whether metacognitive activity by students is appropriate or being effectively conducted, a teacher is reflecting on metacognitive activity itself, both in its role in the overall modelling process and specific to the situation. That teacher may be thought of as undertaking mental activity that is *meta-metacognitive* in nature. The nature of *meta-metacognition* that teachers undertake in relation to student activities and subsequent teaching actions impact on how mathematical modelling is conducted and promoted in their classrooms. Students' capabilities to develop skills in transitioning between phases in the modelling cycle (Stillman, Brown, & Geiger, 2015) and releasing blockages in the solution process (Stillman, 2015) depend upon how they are facilitated and supported in learning and applying the modelling process, and metacognitive strategies central to it (Stillman, 2015). The ultimate goal is nurturing an effective modelling problem solver rather than helping with the solution of a particular modelling problem as an end in itself.

(2) *Implemented anticipation*

In recent work Stillman and Brown (2014) have provided empirical evidence for the construct "implemented anticipation" (Niss, 2010, p. 55). Niss (2010) used the notion of *anticipating* to produce a theoretical model of ideal mathematisation. Anticipation "implemented" as successful mathematisation involves anticipating what will be useful mathematically in subsequent steps and using that anticipation in decision making and completion of actions fulfilling those next steps. Implemented anticipation involves foreshadowing and feedback loops which come into play in decision making during mathematisation. Figure 14.1 is an

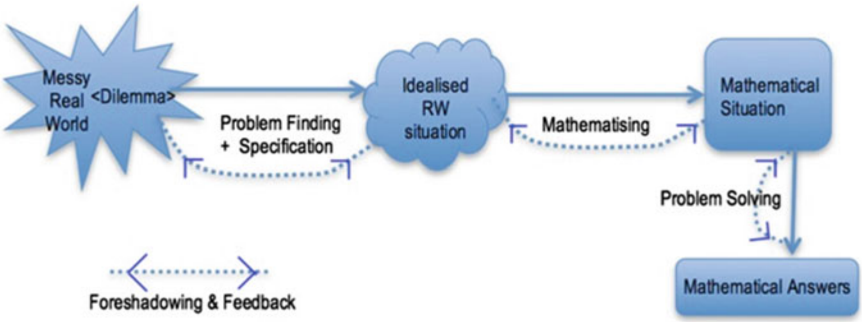

Fig. 14.1 Interpretation of Niss's model of ideal mathematisation. Stillman, Brown, and Geiger (2015, p. 95), Fig. 7.2 with permission from Springer

interpretation by Stillman et al. (2015) of the three-step mathematisation process proposed by Niss (2010). Stillman and colleagues (e.g., Galbraith, 2015b) contend that such activity employs anticipatory metacognition as defined above.

Niss (2010) proposed four enablers for modellers to successfully use implemented anticipation in mathematising: (1) possess relevant mathematical knowledge, (2) be capable of using this when modelling, (3) believe modelling real phenomena is a valid use of mathematics, and (4) have persistence and confidence in their mathematical capabilities. Stillman and Brown (2014) found all four enablers present in modelling contexts where the situation to be modelled was chosen by the modellers and contexts where the situation was chosen by the teacher. However, as they point out, further research is needed regarding the necessity for all enablers in the model.

Stillman et al. (2015) propose that implemented anticipation be extended to technology use when modelling in Technology-Rich Teaching and Learning Environments (TRTLEs) (Brown, 2015a). They present a scaffolding framework that novice modellers in school could use during the transitions (shown as solid arrows in Fig. 14.1). The framework includes a set of scaffolding questions that modellers might pose to themselves or team members so as to "facilitate productive discussion that moves novices forward from a problem context focus to actualising model construction" (p. 102).

The two works reviewed establish connections with earlier and more general notions of anticipation (e.g., Dewey, 1917), adding to the accumulated knowledge and understanding of the demanding processes involved in successfully performing a full modelling cycle. This work serves to consolidate and provide empirical evidence for a theoretical construct put forward by other researchers in the field.

(3) *Modelling oriented "noticing"*

A common approach to the skill of "noticing" in education (e.g., Choy, 2013; Jacobs, Lamb, & Phillip, 2010) is how teachers identify, interpret, and act upon classroom events aiming to enhance learning. Indeed “noticing” as a skill has been identified more widely as an essential competence to be developed within any profession (Jacobs et al., 2010, p. 170). The concept of “noticing” was extended by Galbraith (2015b) to cover this wider professional sense as “modelling oriented noticing”. This involves “noticing” how mathematicians as well as educators act when modelling, from both mathematical and teaching viewpoints as a means for studying elements central to modelling that both precede student engagement and occur during student activity. These elements include (1) the identification of situations containing modelling or application potential, (2) the formulation of appropriate mathematical questions to pursue, and (3) subsequent task development. In all three elements the emphasis is on how a current activity stimulates thinking that provides focused and incisive direction for future modelling or application action, or the mentoring of its achievement. This theoretical extension to “modelling oriented noticing” creates a research agenda involving the identification of characteristics yet to be pursued systematically.

In this section, we have reviewed several theoretical developments (the DMCF, the epistemic fallacy and authenticity, and anticipatory metacognition) mainly with respect to modelling research although aspects of authenticity and anticipatory metacognition apply to application research as well. All four are in their infancy in application in Australasian work and some, such as anticipatory metacognition, await empirical verification and development.

5 Application and Modelling Studies Addressing General Goals of Education

The demands of twenty-first century life and work have reshaped the attributes that education systems set as educational goals for young citizens to be reflected in curricula designed to achieve these goals. From the *Melbourne Declaration on Educational Goals for Young Australians* (MCEETYA, 2008) seven general capabilties were specified as underpinning the Australian National Curriculum (ACARA, 2014). The general capability of numeracy is relevant to applications and modelling and is addressed in other sections. Here we address the personal-and-social capabilities which are interpersonal capabilities concerning communication and collaboration in teams, amongst other things. Similar capabilities of Communication and Collaboration are framed by the Ministry of Education in Singapore as “emerging twenty-first century Competencies necessary for the globalised world we live in” (2014) to be possessed by an educated Singaporean citizen who is an active contributor. In this section we review exemplary studies from the period that address these social capabilities/competencies in the context of mathematical modelling or applications.

English (2013) and English, Hudson, and Dawes (2012) argue that interdisciplinary problem solving and models and modelling are a means of addressing future competencies within an increasingly complex world. Illustrative case studies involving complex, interdisciplinary modelling activities in Years 1 and 7 were presented. In all case studies, students' sharing of ideas, both in group and peer reporting, encouraged awareness, respect, and tolerance of others' thinking and feelings which is similar to findings from inquiry-based learning with Pacifika in a respectful community of practice reported by Bills and Hunter (2015). However, a more overt focus on these under-represented aspects of modelling in the classroom would likely have broadened and enriched the students' learning. According to English (2013), modelling activities should include these critical elements, from the early primary years, to fully develop the future competencies highlighted.

How teachers might facilitate classroom learning opportunities that foster student communication or collaboration has been the subject of research into teacher professional development (see Chap. 16 for further examples). Lamb and Visnovska (2013) report on one episode from a research and development project where teachers were learning to orchestrate classroom discussions with a view to providing opportunities for their students to apply mathematical understanding and skills in context. Comparing and contrasting different mathematical models of realistic situations is one way in which the relative strengths and weaknesses of the models, and the mathematics underpinning them, can become the focus of discussion in a mathematics classroom. Whilst the teachers discussed a range of models they also experienced difficulty in reconciling the conflicting ideas represented in the models. As Makar (2011) points out, it is essential for teachers when developing lasting pedagogical changes to their practice to take on the perspective of the learner and experience difficulties for themselves as in Lamb and Visnovska's study, in this instance those associated with developing appropriate communication skills.

Teaching statistical numeracy in middle school classrooms requires high quality instruction that promotes opportunities to use mathematics in modelling problem situations. Lamb and Visnovska (2015) report on a professional development session involving secondary teachers from rural and remote schools. Results indicated that some teacher groups focused on the mathematics they would teach, limiting numeracy opportunities, whilst others focused on making sense of the problem by modelling, thereby promoting statistical numeracy. The researchers suggest that ongoing learning opportunities, where differences in focus become the point of professional discussions, support teachers' understanding and appreciation of the role of modelling in promoting statistical numeracy. By harnessing the potential of similar differences in focus between student groups for fostering productive exploratory discourse and collaborative interaction, teachers can develop pedagogical practices that cultivate personal-social capabilities in the context of modelling and application tasks at the same time as extending numeracy capabilities.

Chan's study (2014) of the modelling attempts of Year 6 (aged 12) Singaporean students investigated students' mathematical thinking and modelling processes using model eliciting activities (MEAs) (Lesh & Doerr, 2003) in small-groups in a problem-based learning approach. From his investigation of the dynamics of group collaborative efforts during modelling, Chan found groups where members predominantly function as "strategists" and "analysers" led to more effective learning.

The mathematical discourse was richer. Although there were “seekers” (i.e., question askers) and “followers” (i.e., doers who carry out instructions) in every group, the more productive groups had strategists and analysers raising questions rather than the seekers. Groups comprising mainly seekers and followers were unable to go deeper in the discussion and learning was curtailed. An implication from Chan’s findings is that consideration of group composition in facilitating student modelling attempts is important to develop productive communication and collaborative competencies; having a mix of initiators, strategists, analysers, seekers and followers may result in better productivity and develop appropriate competencies than a group that comprises predominantly seekers and followers.

In this section we have briefly reviewed studies that researched personal and social competencies that are articulated as general educational goals. Modelling tasks are often tackled in small groups, requiring effective discourse and interpersonal skills in collaboration. Effective facilitation of groups so that this occurs is thus paramount as well as in-service professional development targeting such aspects together with the mathematics involved. The studies reviewed contribute to this aspect of our knowledge base of effective teaching and organisational strategies and are relevant to other approaches such as inquiry and problem-based learning.

6 Methodological Tools in Applications and Modelling Research

Many methodological tools in modelling research have been adopted or developed to date. Most exemplified here are used in qualitative case studies often involving intensive data collection and analysis. In the review period document analysis, intrinsic qualitative case studies, grounded theory, teaching experiments, design research and mixed methods have featured in studies. Most of these have been adopted from other areas but at times their use has been extended by adapting tools to change the researcher-researched dynamic or to bring a different lens to bear on on-going teaching and learning issues.

6.1 Document Analysis on Theoretical Approaches

Geiger and Frejd (2015) explore the nature of theoretical approaches used in research literature focusing on mathematical applications and modelling; in particular, the orientation and diversity of theoretical approaches used within the field of mathematical modelling and how this has changed over recent times. The study is based on the document analysis of a sample of book chapters from significant scholarly volumes dedicated to applications and modelling published between 2002 and 2011. The data predate the review period but form a foundation for comparison with current Australasian activity. Their analysis reveals that: research is oriented towards learners and teachers rather than on contexts for learning; the number of publications that tied theory to practice increased while chapters that were focused on purely professional issues decreased. The diversity of theoretical approaches utilised within the analysed sample increased over time; and, the use of local theories specific to mathematical modelling strongly coalesced around two approaches—the *modelling cycle* and *modelling competencies.* These local theoretical approaches were more frequently used than general theoretical approaches within the selected sample.

6.2 Empirical Study on the Extent of Student Engagement with Real-World Tasks

Classroom research using intrinsic case studies involving exploration of classroom phenomena are common in applications and modelling research. Brown's empirical study (2013) of three Australian primary classes investigated whether applications and mathematical modelling were perceived as being outside of mathematics rather than an integral part, requiring students and teachers to operate in a "culture of mathematising as a practice" (Bauersfeld, 1993 cited in Yackel & Cobb, 1996, p. 459). She found that students were inexperienced in interpreting mathematical problem situations, believing this was not a normal part of school mathematics leading to a belief that they had no personal experience and knowledge to bring to the task solution. However, in the third class, following an introduction to modelling, students saw their role as making sense of a new situation and mathematising it themselves. Brown concluded that tasks where the context could not be ignored, that required student reflection on the mathematics and exposing thinking to their peer group and potentially publicly in a class poster, contributed to students perceiving themselves as active interpreters of real-world problem situations and appliers of mathematics to the real world.

6.3 Grounded Theory to Study Students' Modelling in Digital Environment

Several studies have used grounded theory to explore modelling and/or applications in the last few years; for example, Brown (2015a, b) explored the extent to which Year 11 students realised the affordances of TRTLE's to help them visualise, clarify, and refine their mental models in solving a real-world function task. Brown (2015a) illustrates diagramatically the complexities of the process of enacting affordances beginning from a function situation through five intermediate stages to situation resolution. Also contributing to successful modelling in a TRTLE is the range of mathematical strategies that are possible to implement by "teachers-and-students-with-digital technologies" (p. 122) when modelling. Brown (2015b) showed that the students often did not take up the opportunities, for example, displaying data plots on a technological device to inform their choice of function model, even though they had the technological and mathematical knowledge to do so. This suggests that teachers and students must realise the cognitive role played by visualisation in modelling, in particular how this supports mathematisation, which in turn should be supported by an anticipation of the role of digital technology in a modelling context (Stillman et al., 2015). After all, affordances of TRTLEs must first be perceived in order to be enacted. Unless this happens the transformational power of technologies in modelling will not be unlocked.

6.4 Use of Videos in Teaching Experiments and Design Research for Teacher Implementations of Modelling Activities

According to Ng et al. (2015), using video recording in Multi-tiered Teaching Experiments can be pivotal to the success of a non-prescriptive professional development approach to prepare teachers of mathematical modelling. Non-prescriptive approaches give voice to teacher knowledge in teacher-researcher collaboration. Ng and her colleagues integrated a Multi-tiered Teaching Experiment with Design Research, thus allowing the co-construction of knowledge through the negotiation of meaning between the students (Tier 1), their teachers (Tier 2) and the researchers (Tier 3). Participating teachers and their primary classes were involved in two 4-phase design cycles. Cycle 1 focused on familiarising teachers with modelling task features, scaffolding strategies for mathematisation, and predicting possible student outcomes through the use of a researcher-designed MEA. Cycle 2 scaffolded teacher design of a second modelling task and planning its implementation. Video-recording of sessions enabled teacher and researcher discussion of the *same grounded image* of each lesson, so the teachers became both observers and evaluators of their own actions.

The study found critical teacher competencies in MEA implementation were: (a) striking a balance between questioning and listening and (b) fostering student metacognition. Video use was instrumental in their identification as video created grounded images allowing researchers to mediate their understanding of the teacher's conception of mathematical modelling and the classroom dynamics during implementation. Furthermore, the Design Research stages embedded in the Teaching Experiment enabled teacher development of an in-depth understanding of the cyclical modelling process as they undertook the dual roles of modeller and facilitator. The researchers also noted that archived videos provide a means of scalability for time intensive professional development programs to be used with other teachers and have potential for enhancing teachers' meta-metacognition (see Sect. 4).

6.5 Mixed Methods Researching Teacher Education in Mathematical Modelling

Tan and Ang (2013b) reported on a study which used an open-ended questionnaire to investigate Singaporean pre-service teachers' knowledge of mathematical modelling following a teacher education course, during which participants attempted two modelling tasks in small groups. Field notes of in-class discussions, artefacts of class work, and post-class reflections through a course blog were used to triangulate findings. The pre-service teachers were able to interpret real-world situations for the modelling tasks, identify important quantities involved, and make realistic assumptions during model development. However, some participants had difficulty realising that there was a need to uncover relationships between variables and look at how a mathematical model changes over time. Pre-service teachers failed to interpret the mathematical solution in the real-world situations and did not validate their mathematical models. These findings are similar to those of Widjaja (2013) who investigated pre-service Indonesian teachers' awareness of mathematical modelling through a case study of pre-service teachers solving one extended modelling task.

In another questionnaire-based study, Chan (2013) investigated Singaporean teachers' conceptions of mathematical modelling and the value they attached to mathematical modelling as part of their students' total mathematical learning. Positive features of mathematical modelling as seen by the teachers were identified along with negative perceptions. Chan concluded that future professional development needed to ease teachers into mathematical modelling by understanding concerns and needs as raised in his study.

The adaptation of an analytical tool (Schoenfeld, 2015) to parse a lesson from its video-record was used by Tan in his doctoral study to bring theoretically grounded initial results (Ang & Tan, 2014) from the evaluation of "a teacher-centric, school-based professional development (SBPD) programme" (p. 34). The programme targeted teachers' competence in teaching mathematical modelling through enhancement of teacher decision-making using Ang's framework (2015) for planning/designing mathematical modelling learning experiences. If rigorously applied, the framework appears to the reviewers to be intended to address all dimensions of authenticity identified by Galbraith (see Sect. 4). The programme supports the development of mathematical content knowledge (MCK) and pedagogical content knowledge (PCK) needed to teach modelling in Singaporean secondary classrooms.

A pilot study (Tan & Ang, 2013a) showed the framework alone was insufficient to engender rigorous application by teachers who lacked modelling experience as they did not have unpacked knowledge of the process on which to base selection of teaching strategies or anticipation of possible obstacles and learning opportunities. Modelling experiences for teachers were therefore incorporated into the training phase.

The programme was school-based to facilitate transfer of teacher learning into the local school context. The programme consisted of two transformative learning cycles of training, applying and reflection. Planning and designing meetings and classroom enactment were video-recorded. Adaptation of Schoenfeld's goal-based decision-making tool to record and analyse data facilitated detection of shifts in patterns of teaching decisions and teachers' corresponding orientations, resources and goals in teaching mathematical modelling. Lesson segments corresponding to each stage of the modelling process were parsed into lesson episodes. Episodes were subsequently parsed into lesson issues (teacher positive/negative teaching and learning moves and learning opportunities lost).

First results indicate that teachers changed focus from learning mathematical skills to student development of mathematical modelling competencies after the programme. Typically teachers adopted a modelling-as-vehicle orientation before the programme but a modelling-as-content orientation after it (see Sect. 2). Other outcomes included improved MCK (of the modelling process) and PCK (related to task design, facilitation skills, anticipation of learning opportunities). Ang and Tan (2014) concluded that there was "a definite paradigm shift in the teachers' orientation and goals in teaching mathematical modelling" (p. 33). Tan's adaptation of Schoenfeld's (2015) tool is appropriate and adds to the tools available that are localised within the study of modelling. A shortened version with episodes based only on the stages of the modelling process common with applications could be used for similar purposes in studies with teachers embracing teaching with real-world applications or contextualised numeracy tasks.

In this section we have briefly reviewed studies from the review period that have adopted or adapted a variety of methodological tools to address issues related to the learning and teaching of applications and modelling as well as the preparation of teachers to teach modelling and applications effectively. The approaches taken in these studies provide evidence of how different methodologies give insight into the issues that challenge teachers and learners in complex teaching and learning environments.

7 Conclusion: Current State of Modelling and Applications Research and Development Since 2008 Review

Since the last review chapter (Stillman et al., 2008) was published there have been clear shifts in the Australasian research field with respect to teaching of applications and modelling which was characterised then as an emerging field. Some changes have been stimulated by changes in curriculum documents in the region. The vast majority of the research has been conducted in schools but in contrast to the 2008 review there is research in pre-service and in-service teacher education as well.

There is also representative work from researchers in Singapore researching the many problems that arise with the introduction of new foci in syllabuses and the inevitable changes in classroom practice and forms of teacher support. Australasian research activity has been concentrated on teaching and learning practices in primary and secondary classrooms, modelling competencies of both teachers and students and support for teachers in implementations of applications and modelling in practice. However, teacher professional development, and the professional knowledge and competencies pre-service teachers need to teach and assess both applications and modelling effectively are areas where further research is necessary.

The vast majority of this research is qualitative mainly from case studies. Quantitative studies have not been common in Australasian work during the review period. The Geiger and Frejd (2015) meta-analysis of the use of theory in research and scholarly book publications from 2002 to 2011 showed an increased use of theory to underpin reporting of results or scholarly chapters but local theories were more evident than general theories. The field of research into mathematical applications and modelling education is an exemplary case of a field in mathematics education that is developing home grown theories related to the purposes of that research and therefore it is true that there is an emphasis on local theories (e.g., the modelling cycle, modelling competencies) that serve these purposes. This is not to say that more general theories are ignored, on the contrary these are being mined (e.g., Geiger, 2013) to add a diversity of ideas to the field. The work reviewed in this chapter also shows an emphasis on local theories as clearly these are adequate for the researchers' purposes.

New theoretical developments provide an overview of conceptual tools developed within this field to support understanding of crucial elements of teaching modelling and applications. Several of these conceptual tools have been developed specifically with modelling in mind such as the DMCF (Lamb et al., 2014) or the foreshadowing and feedback framework to engage beginning modellers in implemented anticipation (Stillman et al., 2015). Some, such as the notions of anticipatory metacognition or meta-metacognition, however, could be of more general application within learning contexts other than the modelling and applications contexts within which they arose.

In this research field a broad base of research finds its home in ICMI and ICTMA publications. Although this might have contributed in the past to restricting access to research, the availability of eBook chapters for the last four ICTMA books has increased the visibility of the work of the field as is shown in Bookmetrix data (19/1/2016) particularly for Kaiser et al. (2011) and Stillman et al. (2013) with 57444 and 47446 chapter downloads respectively since publication. An accompanying trend evident since the last review is an increasing publication in journals to augment this biennial reporting of the state of the art in the field.

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Chapter 15
Challenges, Reforms, and Learning in Initial Teacher Education

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Abstract This critical review of Australasian research concerns initial teacher education published in the period 2012–2015. The contribution to the field is organised into four broad areas: (a) research on teacher preparation: accountability, effectiveness, and policies; (b) research on teacher preparation for the knowledge society, which forms the bulk of the reviewed research; (c) research on teacher preparation for diversity; and (d) research focused on the work of teacher educators. Situated within educational settings that are undergoing continuous change and politicised attention, we note, in particular, research efforts to critically explore, design, and trial pedagogies, tasks, and partnerships associated with occasioning productive learning opportunities for prospective teachers to learn both the knowledge and the core practices of ambitious teaching.

Keywords Initial teacher mathematics education · Pre-service · Teacher preparation · Teacher knowledge · Teacher educators

1 Introduction

This critical review of Australasian research concerns initial teacher education (ITE) published in the period 2012–2015. During this review period directions in ITE have been influenced by the widespread belief that improving the quality of school systems and student outcomes can be achieved by enhancing the capabilities

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of teachers (e.g., Teacher Education Ministerial Advisory Group [TEMAG], 2015). Regaled by policy makers as both the cause of and a solution for education problems, teacher education has been criticised for not producing teachers of sufficient quality while simultaneously being viewed as "an ideal site for increasing teacher quality, providing it is subject to reform" (Ell & Grudnoff, 2013, p. 79). In an era of unprecedented attention to teacher quality and accountability, Cochran-Smith and Villegas (2015) noted that "changing conceptions of how people learn and what they need to know to thrive in a knowledge society" coupled with "growing social and school inequality" (p. 9) exert a major influencing role on current research priorities and directions for teacher education. This chapter considers how research in Australasia has responded to and informed calls to ensure the preparation of quality teachers.

We draw on Cochran-Smith and Villegas' (2015) recent international review of the "sprawling and uneven field" (p. 8) of research on teacher preparation and certification to structure our chapter. Following their lead, we organise our discussion of the Australasian contribution to the field into the three broad areas: (a) research on teacher preparation: accountability, effectiveness, and policies; (b) research on teacher preparation for the knowledge society, which forms the bulk of the reviewed research; and (c) research on teacher preparation for diversity and equity. We note also that calls for more accountability within ITE inevitably impact on teacher educators and add an additional section focused on the work of teacher educators, inclusive of research that investigates teacher educator identity, professionalism, and learning.

We conclude our chapter by reflecting upon the overall contribution of the studies to furthering the field as measured against the recommendations posed by Anthony, Beswick, and Ell (2012) in the previous review period. Based on our analysis of the strengths and weaknesses in the existing field, we also frame a set of recommendations regarding the potential future contribution of Australasian research to the research agenda of initial mathematics teacher education both locally and internationally.

2 Teacher Preparation: Accountability, Effectiveness, and Policies

Across the review period the challenge of improving the quality of schooling outcomes and the closely linked influence of the quality of teachers and their teaching has increased in scale and urgency (Hattie, 2012). Associated with calls for accountability, mathematics teacher education research has both informed and been shaped by issues concerning program design, entry and graduating standards of pre-service teachers. For example, mirroring an earlier trend in Australia, New Zealand funding exigencies have seen several universities reconsider their commitment to teacher education, with two major universities moving to postgraduate options only. While we do not see the shift to largely school-based teacher education programs as mandated by the Teacher Training Agency in England, there is a significant move towards ITE programs that partner with schools. Indeed, from 2014 the New Zealand Ministry of Education has prioritised funding to Masters-level ITE programs that involve close collaboration between partner schools and universities. In addition there was a requirement that these programs demonstrate a commitment to a teaching as inquiry stance (Aitken, Sinnema, & Meyer, 2013) and a focus on quality pedagogy for diverse learners. Meanwhile in Australia, the trend towards postgraduate qualifications has stabilised, with a mix of undergraduate and postgraduate options common within the larger educator providers.

While ITE innovations have focused on quality “teaching”—with teacher educator exponents viewing this as the key to school improvement (Australian Council of Deans of Education [ACDE], 2012), a focus on accountability and quality “teachers” continues to pervade policy. In Australia, the Australian Institute for Teaching and School Leadership (AITSL, 2013) has established an expectation that ITE programs develop processes that ensure prospective teachers have sufficient numeracy and literacy skills to engage with their course, with the added specification that pre-service teachers are in the top 30 % of the population prior to graduation. This expectation has been operationalised with national testing; the first phase of national Numeracy (and Literacy) testing commenced in late 2015, with full national implementation from mid-2016. With regard to graduate exit testing, Leder, Forgasz, Kalkhoven, and Geiger (2015) noted the need to ensure that pre-service teachers have the capacity to meet numeracy demands associated with teaching beyond their teaching domain. Their exploratory study indicated that pre-service teachers currently underestimate the numeracy demands within teaching, for example in relation to interpretation of assessment data.

These policy and associated program initiatives are driven, and in some cases informed, by a range of studies concerning prospective teachers’ mathematical knowledge. Notably, as in the previous review, studies (e.g., Linsell & Anakin, 2012; Norton, 2012) highlight weaknesses in prospective primary teachers’ entry mathematical content knowledge. Forgasz, Leder, Geiger, and Kalkhoven’s (2015) exploratory study of 151 pre-service teachers (both primary and secondary) provided an alternative perspective—claiming that most of the respondents were able to solve numeracy items considered suitable for 15 year-olds. However, these researchers also noted that only just over half of the respondents “believed that they had studied enough mathematics to be a competent teacher” (p. 319). In response to concerns about access to ITE for school leavers from regional, rural, and remote locations, where school achievement is generally described as lower than in metropolitan areas, de Silva Joyce, Feez, Chan, and Tobias (2014) examined the nature of numeracy knowledge and skills demanded of prospective teachers as they engaged in a regional university 4-year Bachelor of Education (Primary) program. In challenging a deficit view of prospective teachers, their findings point to lack of coherence and relevance of assessment activities and inadequacies in academic support.

In addition to mathematics content knowledge, Young-Loveridge, Bicknell, and Mills (2012) took the stance that attitudes to mathematics were also important entry attributes. From tests with 319 prospective primary teachers, using a Mathematics Thinking and Reasoning tool combined with attitudinal Likert measures, Young-Loveridge et al. reported that mathematical knowledge was relatively weak with fewer than half of the prospective teachers liking mathematics. They noted that those prospective teachers with positive attitudes tended to perform well on mathematics tasks.

An Australian study (Beswick & Goos, 2012), involving 294 prospective primary teachers, drew on the conceptual framework used in the Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto et al., 2008) to investigate different aspects of prospective primary teachers' knowledge for teaching mathematics. In comparing these outcome measures for groups of participants based on a range of future teacher characteristics and teacher education performance characteristics, it was found that measures of mathematics content knowledge (MCK) and overall teacher knowledge were linked to teaching efficacy. In another attempt to link entry and graduate competencies, Norton (2012) investigated the capacity of prospective teachers' prior study of mathematics to predict 122 primary graduating teachers' success on tests of mathematics and pedagogical content knowledge (PCK). Norton found that graduating teachers who were proficient at mathematics were also effective at explaining how to teach it.

What these studies have in common are concerns about the sufficiency and nature of teacher mathematics knowledge. Collectively these studies provide empirical support for the relationship between mathematics content and teaching knowledge and contribute to the establishment of an evidence-base for ITE.

3 Teacher Preparation for the Knowledge Society

This review period was marked by national goals for all learners to have sufficient mathematics capability for meaningful employment and engagement in our knowledge-based technological society. As English (2015) noted, developing competencies in the Science, Technology, Engineering and Mathematics (STEM) disciplines is "regarded as an urgent goal of many education systems, fuelled in part by perceived or actual shortages in the current and future STEM workforce and also by outcomes from international comparative assessments" (p. 3). However, preparing students for future knowledge work requires new ways of teaching that are consistent with changing conceptions of how people learn and what they need to know (Cochran-Smith et al., 2015; Jorgenson, 2014; Timperley, 2013). Reforms in mathematics education—variously described as "ambitious" (Lampert et al., 2013), "dialogic," "responsive" (Stylianides & Stylianides, 2014), "inquiry-based" (Stillman, 2013), and "responsible" (Ball & Forzani, 2011)—require different approaches to teacher education that involve a shift in the kind of learning experiences provided for prospective teachers (Anthony & Hunter, 2012; Sullivan, 2015). In reviewing the research about "what and how" prospective teachers are supported to learn we have organised our discussion around the broad, and sometimes intersecting themes of (i) curriculum; (ii) opportunities to learn within coursework; (iii) opportunities to learn within field experience/practicum; and (iv) learning to teach over time. Within these themes, we prioritise those studies that explore curricula and pedagogical innovations and directions that align prospective teachers' beliefs and understandings with ambitious teaching practices and the development of professionals who are focused on better learning for themselves and their students—adaptive experts.

3.1 Curriculum

Both in Australasia and internationally, the review period was noted for an increased focus on (re)defining the curriculum of teacher education in terms of what teachers should learn and be able to do. Using Hammerness, Darling-Hammond, and Bransford's (2005) curriculum framework—a vision of practice, knowledge of students and content, dispositions for using this knowledge, and a repertoire of practices and tools—it is clear that for our community, attention to knowledge of students and content (be it mathematical or pedagogical) is central to many researchers' agendas. This is possibly because many researchers are themselves involved in teaching with prospective teacher knowledge at the fore of their attention. Thus we begin this section with a review of studies that relate directly to teacher knowledges for mathematics teaching.

Following on from reviews of teacher quality and associated concerns about sufficiency of entry and graduating knowledge, researchers have advocated for increased focus within ITE on areas of numeracy (e.g., Cooke, 2015)—inclusive of mathematical skills and disposition towards mathematics, and statistical literacy (Chick & Pierce, 2013). Prospective teachers' mathematics knowledge linked to the school curriculum strands (e.g., number sense, multiplication, place value, decimals, proportional reasoning, fractions, geometric properties, circles and ellipses, perimeter, area, measures of centre, and probability) is evidenced in a number of MERGA and PME conference papers and journal articles (e.g., Chinnappan & Forrester, 2014; Livy, Muir, & Maher, 2012; Muir & Livy, 2012; Murphy, 2013, Wright, 2013). Collectively, authors of these publications expressed concerns that the strong emphasis on procedural knowledge displayed by prospective teachers would negatively impact on their capacity to develop appropriate conceptual and pedagogical content knowledge (PCK). Beswick (2015), in exploring pre-service teachers' PCK provided an interesting analysis of a pre-service teacher's interview around a PCK scenario test item to illustrate the interrelationship between beliefs and knowledge. In other interview studies (e.g., Chinnappan & White, 2015; Maher & Muir, 2013) pre-service teachers' capacity to identify errors in student work samples and to determine appropriate approaches to address those errors was clearly linked to levels of mathematics content knowledge (MCK).

The assessment of prospective teachers' knowledge base combined with reported school-based experiences linked to repetition and recitation of procedures raises concerns about how to deepen prospective teachers' mathematical understandings and challenge their beliefs that procedural understandings are sufficient for teaching. As Klein (2012) noted, it is not enough for "novice teachers to know and understand mathematical ideas, they must also be able and willing to implement new interactional patterns that allow their students to sense proficiency in energetic participation with and in mathematics" (p. 35).

Several studies explored learning opportunities to advance prospective teachers' MCK in ways that align with the notion of mathematical proficiency. Exploring the prevalence of mathematics anxiety among pre-service teachers, Hurst and Cooke (2014) found evidence that these teachers were often unaware of what they needed to know. These researchers concluded that the need to develop a conscious awareness of incompetence preceded the development of conscious competence. Perkins (2015) found that the novel approach of utilising a classroom teacher, independent of the practicum mentor, as mentor proved successful in developing pre-service primary teachers' confidence, and alleviated mathematics anxiety.

Chinnappan and Forrester (2014) found that the use of a model-based teaching approach enhanced prospective teachers' conceptual knowledge of fractions: they attributed this improvement to using visual forms for the representation and interpretation of the operations of fractions. Daniel and Balatti (2013) provided an example of a collaborative (prospective teacher and researcher/teacher educator) analysis of videoed episodes of prospective teachers teaching a task involving area concepts. The simulated review activity revealed both prospective teachers' areas of strength and omission in terms of utilisation of MCK. While the collaborators viewed the activity as a positive learning experience, the issue of time to effectively provide follow-up learning activities was raised. Also looking at the affordances of knowledge building communities, Nason, Chalmers, and Yeh (2012) studied the growth of prospective teachers' understanding about teaching geometry as they engaged in a study that investigated teachers' lesson planning within a computer-supported collaborative learning environment. Using a teaching experiment design Nason et al. documented how teams of prospective teachers "engaged in extensive knowledge building activity and made considerable advances to their repertoires of PCK about teaching primary school mathematics at both theoretical and practical layers" (p. 238). Changes to participants' repertoires of PCK (made through comparisons of each teams' initial and final lessons plans) focused on changes to teacher interventions within lesson plans and structuring of lesson content. Factors that influenced the growth of PCK included the nature of the lesson planning task, the cognitive scaffolds, the meta-language scaffolds, and the provision of both private and public discourse spaces.

Beswick and Goos (2012), like others, contend that MCK remains important for the development of PCK. As noted in their study involving 122 graduating primary teachers (see previous section), they found a lack of any relationship between pedagogical content knowledge (PCK) and level of prior education or highest mathematics or statistics studied. Beswick and Goos cited this finding as evidence of the central place PCK development must take within the teacher education curriculum. However, taking a different and somewhat novel approach, Ell, Hill, and Grudnoff (2012) analysed entry pre-service primary teachers' ability to recognise the key features, as an expert would, of a range of student responses to mathematical tasks. They found that approximately half of the teacher candidates "recognised the key features outlined by the experts" (p. 59). Although based on a limited sample from one institution, Ell et al. argued that these findings offer a challenge to the widely espoused view that prospective teachers bring largely conservative views of teaching and learning mathematics based on their own experience. Additionally, their study provides an exemplar of how teacher educators can themselves develop an inquiry-based approach as a way of increasing responsiveness to prospective teachers' prior knowledge.

Collectively, these studies endorse Goos' (2013a) claim that "content knowledge apparently learned during secondary schooling is not necessarily secure, and it needs to be revisited during teacher preparation" (p. 982) in conjunction with new learning associated with knowledge for mathematics teaching.

3.2 Opportunities to Learn Within Coursework

In designing learning opportunities, mathematics teacher educators are well aware of the importance of prospective teachers' attitudes and beliefs concerning mathematics, mathematics learners/learning, and mathematics teachers/teaching (see also Chap. 5, this volume). Dayal's (2013) and Wilson's (2015) exploration of prospective teachers' recounting of "good and bad" teachers reaffirms the importance of acknowledging prior learning experiences. However, the trend evident in the previous review (Anthony et al., 2012) to look closely at teacher beliefs has generally been replaced by a more holistic approach to changing and challenging beliefs. This approach is informed by theoretical arguments around social theories of learning (Goos, 2013b) and represented by the growth in attention to coursework design that affords opportunities for prospective teachers to develop attributes of professionalism associated with inquiry, collective responsibility, and knowledge co-construction (Afamasaga-Fuata'i & Sooamalelagi, 2014; Lane, McMaster, Adnum, & Cavanagh, 2014).

Explorations of opportunities to learn within online environments focused on prospective teachers' emergent identity and professionalism. Goos and Geiger (2012) investigated the social and multimodal affordances associated with online group assessment tasks that required the creation of a video presentation and set of questions that would engage primary school students in mathematically rich learning. Conceptualising digital mathematics performance "as a way of transforming mathematical identities" (p. 709), Goos and Geiger found the use of technologies supported the creation of an original performance that illustrated the use of mathematics in a real life context. Moreover, the technologies also afforded opportunities to participate in some of the "practices of a professional community in that they [pre-service teachers] created and evaluated teaching resources and engaged in professional discussions with peers" (p. 713). Larkin and Jamieson-Proctor (2015) used the theoretical framework of transactional distance theory to analyse dialogical interactions within an online course. In seeking to understand the relationship between pre-service teachers' pedagogical practices and the development of teacher knowledge and attitudes, Larkin and Jamieson-Proctor argued that the use of virtual classrooms and forums provided opportunities for high levels of dialogue comparable to traditional courses. Pre-service teacher feedback indicated the presence of the online community of practice supported students' development of positive attitudes towards mathematics.

Addressing professionalism, Klein (2012) argued that teacher change involved the constitution of a vision of teaching that embraces "new interactional patterns that centre the mathematics and learner in dynamic, productive interaction" (p. 25). Applying a bifocal lens containing psychological and poststructuralist constructs to examine prospective teachers' identity and potential to teach in innovative ways, Klein offered her experiences of teaching a mathematical inquiry course as a way to challenge the prevailing humanist inspired discursive practices that suggest nurturing and having fun are paramount. In a similar vein to Klein's efforts to provide prospective teachers with experiences that support authorship over a learning process of knowledge construction, Hunter and Anthony (2012) and Bailey and Taylor (2015) explored the provision of opportunities for prospective teachers to work on mathematical problems in small groups followed by teaching groups of children in schools. Collectively, these studies concluded that experiencing collaborative problem-solving approaches led to significant shifts in prospective teachers' thinking about what it means to do mathematics and support the learning of mathematics.

In looking at new spaces for "learning the *work* of teaching" (Lampert, 2010, p. 21, emphasis in original), other researchers have aligned with the international trend towards practice-based teacher education and associated pedagogies (Zeichner, 2012). With the goal to support prospective teachers to learn not just about teaching but also how to use knowledge of teaching in action, pedagogies of practice comprises three elements: (i) representations of practice (e.g., video records of lessons or records of student work); (ii) decompositions of practice (e.g., core or high-leverage practices such as professional noticing, building on learners' thinking, leading a discussion of solutions to a mathematics problem, and positioning students as competent); and (iii) approximations of practice (e.g., simulations of certain aspects of practice through activities such as role play and rehearsal) (Grossmann, Hammerness, & McDonald, 2009).

Several emergent studies focused on the use of representation and decomposition of specific core practices. For example, using video excerpts of mathematics teaching as representations of practice, Beswick and Muir (2013) explored teacher learning in terms of inclinations and abilities to identify and focus on the development of students' mathematical understanding. Analysis of the tutor-led discussions of each video indicated that the "pre-service teachers struggled to see beyond readily evident aspects of teaching, such as the use of concrete materials"

(p. 27). Disappointed with the limited challenge to the pre-service teachers' beliefs, the researchers conjectured that more foundational work around "what constitutes evidence of understanding and how teachers can elicit such evidence from students" (p. 49) would be beneficial as a precursor to this type of activity. Another video study, by Muir, Allen, Rayner, and Cleland (2013), investigated the use of *Second Life* as a simulacrum of a teaching environment. The virtual environment, acting as an approximation of practice, provided the opportunity to focus on particular topics, misconceptions, learning difficulties, and pedagogical approaches. Although the technology was cumbersome to master, the prospective teachers appreciated the alternative way to learn the work of teaching and receive feedback from their peer community.

Cavanagh, Bower, Moloney, and Sweller (2014) also studied the use of videos of teaching, but this time using secondary pre-service teachers' own simulated practice as examples. Cavanagh et al. concluded that the iterative process of viewing and reflection improved the teachers' communication processes across cognitive, behavioural, and affective domains, with the peer critique and support via blog postings considered a valuable part of the learning process. Likewise, Prieto et al. (2015) explored the effectiveness of video analysis of prospective teachers' own teaching episodes. A teaching rounds process based on the Quality Teaching model (see NSW Department of Education and Training, 2003) generated focused reflections on micro-teaching conducted in an after-school homework context. As with other practice-based studies, participants expressed unanimous approval for the targeted opportunities to practise teaching in a safe and approximated setting. Analysis of the 40 pre-service secondary teachers' abilities to plan lessons and reflect on their practice-based experiences led the researchers to conjecture that those teachers with poor MCK were limited in their capacity to develop PCK—a finding also noted in Beswick and Goos' (2012) study of pre-service primary teachers.

In a larger practice-based study, researchers across two universities (Anthony, Hunter, Anderson, et al., 2015) collaborated in a 3-year design-based study. Key to pedagogical and curriculum change was the use of purposefully designed "instructional activities" (IAs)—containers of core practices, pedagogical tools, and principles of high-quality teaching (Kazemi, Franke, & Lampert, 2009). Within their mathematics method courses, pre-service teachers regularly planned and taught IAs to a group of peers in the format of public rehearsals with teacher educator coaching. Analyses of rehearsals within a range of mathematics methods courses have explored pre-service teachers' perceptions of the learning activity (Averill, Drake, & Harvey, 2013), opportunities to practise and learn about how to listen and respond to students' mathematical thinking (Anthony, J. Hunter, & R. Hunter, 2015a), and the development of cultural competencies (Averill, Anderson, & Drake, 2015).

These reviewed studies reflect changes in pedagogies and associated opportunities to learn within ITE programs. With many of the studies focused on discrete learning activities, research concerning the integration of opportunities for learning the work of teaching into a more comprehensive set of experiences, both within the ITE setting and across the school setting, should be a next step.

3.3 Designing Opportunities to Learn with School Settings

It is well known that much of what pre-service teachers learn about teaching and learning occurs in a school based context, and interest in how this learning can be supported has typically focused on mentor partnerships with the pre-service teacher. For example, Livy (2015), drawing on two contrasting cases, reported how differential support and expectations by school-based mentors around teacher planning impacted on prospective teachers' development of MCK. In particular, opportunities to co-plan with a mentor teacher and to experience teaching across a wide range of class levels were significant factors that supported the development of MCK.

In line with the growing trend for new partnerships between teacher education institutions and schools, this review period has seen the emergence of studies that focus on partnerships that are inclusive of a wider community of support. In the practice-based study discussed above (Anthony et al., 2015) prospective teachers taught the Instructional Activities with small groups of students in schools. Following each teaching session video records of that work were discussed in the university setting. Cavanagh and Garvey (2012) researched a university course that included extended opportunities for mentor teachers, mathematics teacher educators, and pre-service teachers to engage in co-teaching experiences within secondary mathematics classrooms. Participant responses suggested that "everyone developed a greater appreciation of the importance of mathematical problem solving as a practical way of implementing the reform agenda for secondary mathematics" (p. 71).

Possibly because of the trend to integrate university and school learning environments, this review period was notable for the relative absence of studies that focus on the practicum experience per se. Instead, researchers involved in studies that involved partnership arrangement have argued that the co-teaching activities were valued as a way of promoting quality evidence-based discussions around effective pedagogical practices and reciprocity among the learning community members. Further research is needed to look at ways these partnerships can support engagement in theory building and development of shared conceptual framework aligned to adaptive teaching expertise (Anthony, Hunter, & Hunter, 2015b)—the hallmark of the professional teacher (Aitken et al., 2013).

3.4 The Continuum of Teacher Learning

Mathematics educators have for some time had their eye on the goal of preparation for "teaching that is more socially and intellectually ambitious than the current norm" (Lampert et al., 2013, p. 241). To dampen the effects of enculturation into existing modes of teaching, beginning teachers need to be able to take up an agentic position towards improving practice. However, Nolan and Walshaw (2012) question how realistic it is to expect beginning teachers to make significant shifts from a traditional practice to a teaching practice centred on inquiry approaches. Applying Bourdieu's social field theory they illustrate how Toni, a pre-service teacher, developed a set of hybrid pedagogies. "Steeped in the as-yet-still-developing habitus as inquiry teacher, she tapped into both fields [inquiry and traditional practices] that, operating to some extent below her conscious awareness, vied for position to constitute her as a teacher of mathematics" (p. 360). Nolan and Walshaw argue that "the challenge lies in persuading pre-service teachers to take risks and consider trying an uncomfortable habitus on for size" (p. 360).

Enabling prospective teachers to take risks was the focus of a Classroom Inquiry course in which pre-service teachers taught groups of students rich mathematical tasks. Anthony et al.'s (2015b) analysis of a series of journaling activities and pre and post course interviews were used to illustrate pre-service teachers' shift in focus from self to student and towards more complex understandings of teaching and learning—representative of shifts from routine to adaptive expertise. They argued that the teaching as inquiry model (adapted from Timperley, Wilson, Barrar, & Fung, 2007), focused on linking teacher actions to student learning outcomes, combined with the use of research frameworks aligned to the orchestration of productive classroom discourse (e.g., Hunter, 2008; Smith & Stein, 2011) supported prospective teachers in developing expertise.

Building on earlier studies of teacher learning across the boundaries of ITE and the classroom, Goos (2013b) frames taking risks as "productive tensions between teachers' thinking, actions, and professional environments" (p. 521)—tensions that can become opportunities for change. In reflecting on the nature of the researcher-teacher relationship within studies involving transitions from university to the school-based workplace, Goos (2014) argued that while both researcher and pre-service teacher are learners, for the pre-service teacher the relationship facilitates entry to a professional community of mathematics teaching. Building on an established research relationship, Anthony, Hunter, Hunter, and Duncan (2015) explored the impact of a beginning teacher's emergent adaptive expertise (Hatano & Oura, 2003). Their analysis highlighted how the beginning teacher's responsive, reciprocal power-sharing relationships with her learners supported the continuous co-construction of knowledge for herself and her students.

These studies acknowledge the tensions and challenges—both for prospective teachers and teacher educators—in working across boundaries. They serve to remind us that in striving for a more ambitious vision for learning, we must continue to work towards designing learning environments that attend to the ways in which learners are entitled, expected, and obligated to act. In particular, in looking to develop teacher graduates as agents of change, it is important that we seek to position them as sense-makers in dialogue about the challenges of ambitious mathematics teaching.

3.5 Summary

The reviewed studies in this section contribute to understanding how we might better prepare teachers to learn the work of teaching. In doing so, these studies focus on how mathematics teacher educators might support both the learning of knowledge and repertoires of practices alongside the development of dispositions for using this knowledge in a way that contributes to professionalism. In looking for a third space (Zeichner, 2012), the approximation of practice afforded by instructional innovations such as pedagogies of rehearsal and simulacrum learning environments are worthy of further investigation.

4 Teacher Preparation for Social Justice

This section considers research that focuses on how teacher preparation has responded to the challenges associated with an increasingly diverse population, the differences in lived experience between many prospective teachers and their students, and the disparities in educational opportunities and outcomes for socially disadvantaged groups. The deep commitment by Australasian mathematics teacher educators/researchers to social justice is evident in large scale professional learning and development projects with indigenous and socially disadvantaged communities (see Chaps. 6 and 7, this volume). In this section we review research that adds to that agenda—looking at innovations in ITE programs that focus on supporting prospective teachers to develop cultural awareness and responsiveness.

Nicol, Bragg, and Nejad (2013) explored pre-service teachers' ability to make mathematical problems more accessible and challenging for diverse learners. Results from their study indicated that while participants were able to draw upon a range of strategies to vary the mathematical content, the context, and the questions asked, participants were less likely to notice or attend to how their adaptations changed the mathematical structure of the problem. These insights into the lack of thought about the mathematical implication of changes suggest that task adaption may indeed be a core practice that needs explicit attention for learning the work of inclusive teaching practices.

Owens (2014) reported on a study involving pre-service teachers from Papua New Guinea. Curriculum exemplars drawn from a range of local contexts prompted teachers to "reflect upon their cultural heritage and recognise that it was valuable and relevant to school mathematics" (p. 202). In turn, teachers reported that their increased awareness of ecocultural pedagogy positively impacted on the learning and development their students' sense of worth. Afamasaga-Fuata'i and Sooamalelagi (2014) also turned to culture to investigate ways to make mathematics learning more meaningful for pre-service teachers. Aligned with curriculum reforms in Samoa, the course they reported on provided opportunities for pre-service teachers to engage in mathematical investigations of authentic contexts.

That some, but not all, students reported positive attitudinal changes provides evidence of the difficult task involved in shifting understandings of and attitudes toward mathematics.

Averill, Anderson, and Drake (2015) looked at how the teacher educator modelling and coaching roles associated with public rehearsals of teaching might support pre-service teachers to develop cultural competencies (see *Tātaiako—Cultural competencies for teachers of Māori learners,* Ministry of Education, 2011) associated with enhancing equity of access to achievement in the mathematics classroom. Retrospective analysis across rehearsal activities from two mathematics methods courses illustrated how the processes inherent in modelling and decomposition of core practices (Grossman et al., 2009) provided opportunities to professionally notice culturally responsive teaching in action. For example, wānanga—participating with learners in dialogue for the benefit of learners' achievement—was evident through modelling how the teacher could talk with learners about their learning and through listening to their ideas. Ako—a concept that suggests that each member of the learning setting brings knowledge with them from which all are able to learn—was exemplified through the reciprocity of teaching and learning and the modelling of high expectations, risk taking, and collaboration.

These studies document innovative ways to support prospective teachers in their development of equitable and culturally responsive practices, with explicit links to lesson planning and rehearsal activities. However, we cannot be complacent with our efforts. Jorgensen (2014), in advancing a new approach to equity reforms, argued that we need research that can shape "new agendas to support teachers' work in bringing about deep mathematical knowledge making … for ALL students" (p. 317). Reform efforts require that we understand how to include pedagogies that support prospective teachers' learning of culturally responsive and responsible mathematics teaching, and we actively research what effects, if any, such practices have on pre-service teachers and ultimately their diverse students.

5 Research on and with Teacher Educators

Internationally there is increasing recognition that research in ITE needs to be inclusive of research concerning teacher educators (Knight et al., 2014). With intensified calls for accountability and performance indicators in ITE, the need to know more about the learning, practices, and preparation involved in "teacher educator education" is all the more pressing. Understanding and articulating the knowledge and skills that mathematics teacher educators need to prepare teachers for the challenges of ambitious mathematics teaching and determining how teacher educators can acquire this expertise has the potential to make a significant difference to the field of mathematics education in general.

A small number of studies which focus on teacher educator knowledge, identity, and learning are highlighted in this review, which occurs at a time when special issues are underway on this topic (e.g., *Journal of Mathematics Teacher Education,* *Journal of Teacher Education*). In relation to the knowledge types identified by Shulman (1987) and others (e.g., Ball, Thames & Phelps, 2008), the knowledge required by mathematics teacher educators goes beyond MCK and that required of a classroom mathematics teacher. According to Beswick and Chapman (2015), mathematics teacher educators require a kind of meta-knowledge—described as knowledge for teaching knowledge for teaching mathematics. Building on a PCK framework conceptualised by Chick, Baker, Pham, and Cheng (2006), Chick and Beswick (2013) adapted the framework, with pre-service teachers taking the place of students, and school mathematics teachers PCK (SMTPCK) taking the place of mathematics as the teaching domain. Their framework provides a useful set of filters for examining the PCK of a teacher educator who teaches mathematical PCK. Indeed, using this framework, Marshman and Porter (2013), linked pre-service teachers' difficulty with identification of student misconceptions in fractions with the need for teacher educators to be more specific in terms of identifying the cognitive demands of tasks.

In addition to teaching prospective teachers PCK, for many mathematics teacher educators, a large part of their role involves teaching mathematics, particularly when working in early childhood and primary ITE. There have been some attempts to measure both teacher educators' mathematical content knowledge (MCK) and PCK. Callingham et al. (2012), for example, used an online survey to measure teacher educators' MCK and PCK. Their findings indicated that teacher educators found the items addressing PCK more difficult than mathematics content questions but that the question of what mathematics teacher educators need to know in order to develop quality mathematics teachers is still unanswered. Another survey study (Getenet, Beswick, & Callingham, 2015) designed to measure mathematics teacher educators' knowledge highlighted the importance of investigating how knowledge of technology was integrated with other knowledge needed for mathematics education.

In addition to knowledge, the beliefs of mathematics teacher educators have also been investigated. Beswick and Callingham (2014), for example, reported on the beliefs of mathematics teacher educators compared to experienced and prospective mathematics teachers. Mathematics teacher educators were found to be less likely than all other groups to agree that students learn by practicing procedures and methods for performing mathematical tasks and tended to hold views aligned with a problem solving view of the discipline. In conclusion, the authors called for more exploration into these issues, recommending also that the extent to which mathematics teacher educators are perceived as being out of touch with classroom realities be investigated.

Given the specialised nature of mathematics teacher education, it is not surprising that the work of teacher educators is generally researched and reported by teacher educators; indeed, the papers considered in this review period concerned several self-study projects. Chick and Stacey (2013), for example, explored the idea that mathematics educators teaching mathematics act as applied mathematicians in applying mathematical knowledge to the resolution of teaching problems. In addition, Chick and Beswick (2013) used their newly developed mathematics teacher educator PCK framework to study the PCK used by Helen in working with pre-service teachers in an online environment. The analysis focused on teaching scenarios in which the character of Boris, a puppet who represented a hypothetical student, provided stimuli to encourage prospective teachers' questioning of students. The data were examined to identify when Helen, as the mathematics teacher educator giving voice and action to Boris, appeared to use some form of knowledge for decision-making that was intended to develop prospective teachers' PCK. Different SMTPCK types that were identified included teaching strategies, prospective teachers' thinking (e.g., misconceptions), cognitive demands of tasks, and prospective teacher goals for learning. Bragg (2015) also used self-study to explore how to improve an on-line assessment task. Bragg noted that the close examination of her practice, although challenging, was productive in "developing a stronger professional eye" (p. 294) for both herself and her students.

A number of studies have examined the role of teacher educators' practices, and the influence these practices have on prospective teachers' learning. For example, Anakin and Linsell (2014) challenged the "tacit habit of teacher educators seeing learner knowledge as lacking" (p. 723). Taking pains to emphasise that mathematics teacher educators are in a privileged position to nurture the teaching and learning of mathematics, these researchers called for a rethinking of the role from a transmitter of knowledge to a learning resource. Similarly Klein (2012), in an examination of her own teaching, found that if the "teacher educator is positioned as the one who knows" (p. 29) then this positioning is unlikely to provoke change in teaching practices.

Looking to provoke change through a greater understanding of the theory-practice nexus, Cavanagh and Garvey (2012) found that the establishment of a community of practice involving pre-service teachers, supervising teachers, and mathematics teacher educators working together supported a more productive role for the teacher educator than as a knowledge expert. Learning new roles and associated pedagogies was a feature of Anthony, Hunter, Anderson, et al.'s (2015) design study that trialed the use of public rehearsals of instructional activities within methods courses. To enhance their learning, some of the mathematics teacher educators deliberately sought out classroom experiences to develop expertise in teaching instructional activities while others engaged with colleagues in reflective critiques of videoed teacher educator coaching moves.

In summary, this section has looked at the research in ITE which has focused on the teacher educator as practitioner. While Lerman (2014) suggested that the identity of mathematics teacher educators is regulated by structural factors such as the values of education embedded in government policies, there is value in pursuing the agenda set by Beswick and Chapman (2015) and others. For example, "How does the knowledge needed by mathematics teacher educators differ from that required by mathematics teachers?" "How is knowledge for mathematics teacher education acquired?" and "Why is it important to articulate knowledge for mathematics teacher educators?" It appears that this is an area that is ripe for further research, with the potential to contribute to making a difference in the field of teacher education. In addition to studies that investigated mathematics teacher educator knowledge, the field is represented by small-scale self-studies in which mathematics teacher educators aim to advance the quality of course-based ITE learning experiences.

6 Conclusions: Looking Back and Looking Forward

Graduating teachers will begin their teaching careers in educational settings that are undergoing continuous change, at local and national levels. As beginning teachers they will need to respond to broad developments in national curricula and assessment frameworks, and to the increased use of technology in all aspects of education and social/work settings. While concerns around recruitment and graduation of quality teachers continue to drive policy initiatives within ITE, there is a real sense that the research agenda is being driven by teacher educator/researcher awareness of the need to address three related aspects of program design and implementation: "organizational structures and policies, content and curriculum, and teacher education pedagogy" (McDonald et al., 2014, p. 501).

Research reviewed in this period represents the consolidation of the groundswell of studies from the previous review (Anthony et al., 2012). However, in contrast to the previously tentative explorations of "*practice*—and the problem of doing it effectively" (Ball & Even, 2009, p. 255, emphasis in original), we now see deliberate moves to design and trial innovative practice-based teacher education reforms. Previous attention to the commonly touted divide between practice and theory has to some extent been displaced by the turn to practice-based education (Zeichner, 2012). Research (e.g., Anthony, R. Hunter, Anderson, et al., 2015; Muir et al., 2013) offered a range of innovative instructional activities designed to support prospective teachers learn the work of complex relational teaching practices associated with communities of mathematics inquiry.

While many of the studies that created new learning opportunities for prospective teachers sought to address the challenge of preparing teachers to teach in new and ambitious ways, the limited number of studies that specifically addressed issues of diversity and equity was of concern. To make inroads on meeting the "grand challenge" of providing equitable learning opportunities in the mathematics classroom (National Council of Teachers of Mathematics [NCTM] Committee, 2015) this is one area that needs heightened attention.

Critical efforts to interrogate the ITE curriculum include a focus on teacher knowledge and identification of core or high-leverage practices for mathematics teaching. However, this review has identified that there is still much to learn about the relationship between knowledges for teaching (e.g., Beswick, 2015) and about core practices such as professional noticing of students' mathematical reasoning that are crucial to equitable and culturally responsive teaching practices (Averill et al., 2015). A way forward ought to involve studies that develop a common language and a framework for aggregating explorations on teacher knowledge and pedagogical tools and opportunities for learning the work of ambitious teaching.

In line with current focus on social learning theories (Morgan, 2014), it was notable that those studies that offered innovation, for the most part, featured collaborative and reflective activities, be it with peers (e.g., in planning rehearsal activities, in experiencing problem solving activities, in critiquing online assessment and presentation) or with a wider community involving teachers, teacher educators, and prospective teachers. Likewise, with the normalisation of online learning environments and access to technological tools and representations, there was an increased awareness of the role of the performance and social activity within the learning process (Goos, 2013a, b).

This review period has seen a considerable strengthening of research involving teacher educators. Notably, practice-based teacher education innovations offer an alternative way for developing pedagogies of enactment that support prospective teachers to learn the work of teaching while simultaneously placing the teacher educator in contexts where they can learn from and with their prospective teachers. However, studies such as Goos and Geiger's (2012) and Anthony, R Hunter, Anderson, et al.'s (2015) remind us that change carries significant implications for the teacher educator. In many of the reviewed studies involving innovation, teacher educators noted the risks involved in making changes to curriculum, assessment, and pedagogies—citing pressures of space/time and challenges to identity associated with their new learning.

The politicised attention to teacher preparation and the press to institute reforms will not abate in the near future. A sound research base will be crucial if we are to avoid being at the whim of top-down policy mandates, and potentially mitigate the scenario endured in England where the research community experienced limited input into debates about reforming or improving mathematics teacher preparation (Brown, Rowley, & Smith, 2014). With evidence-based teacher education a current political priority, we must continue to build on the existing large-scale studies concerning mathematics teacher entry and graduating knowledge/testing to address concerns around accountability, equity, and access for teacher candidates.

In addition, we must look to scale-up research projects from single site studies to large-scale, national and international projects. The current *Inspiring Mathematics and Science in Australian Teacher Education* (see Goos, 2015) that provides an example of collaboration between academics from different communities of practice bodes well for the opportunity for mathematics teacher educators to open up their practice, to share their practices, and learn in, from and for practice. Only then will mathematics teacher educators be able to experience the benefits of a learning community of practice that we so readily advocate for teacher and student learning.

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Chapter 16
The Education and Development of Practising Teachers

Kim Beswick, Judy Anderson and Chris Hurst

Abstract This chapter reviews Australasian research on the education and development of practising teachers of mathematics. We consider developments in theoretical understandings of professional learning (PL) including conceptualisations of teacher learning and capabilities. Reports of PL programs that have been sites of research are reviewed according to their content foci and the approaches to PL that were adopted. The latter are considered in light of current characterisations of PL. Consideration of ways in which PL programs have been evaluated highlights the difficulties inherent in going beyond teacher self-reports and in linking specific PL programs to outcomes in relation to students' learning as well as system and policy impacts. In conclusion we present avenues for further research. These include addressing issues of scale and sustainability, assessing the affordances of online delivery of PL, enhancing collaboration between mathematics educators and mathematicians, and better understanding the mechanisms and conditions that contribute to effective PL.

Keywords Professional learning · Professional development · Practising teachers · Teacher knowledge · Teacher capabilities · Pedagogical content knowledge

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1 Introduction

The importance of the ongoing development of practising teachers has been increasingly recognised over the period 2012–2015. In New Zealand an advisory group was established in 2013 to advise on professional learning and development, then in 2015 a plan was announced in which mathematics was included as one of four areas to be prioritised from 2017 (Ministry of Education, 2015). In the same year the Business Council of Australia called for the Commonwealth to guarantee teachers ongoing professional development related to either their subject area content or pedagogy, including masters level qualifications where appropriate. In addition the Teacher Education Ministerial Advisory Group (TEMAG) (2014) drew attention to the importance of ongoing professional learning (PL) for teachers and highlighted the lack of a research base related to approaches to both initial teacher education and ongoing PL that are effective at the level of improving student learning. Although many calls for attention to and resourcing of teacher PL have been made in relation to teachers generally, these calls have occurred in the context of intensified focus on science and mathematics education (e.g., Australian Council of Learned Academies, 2013).

This chapter focuses not only on formal PL programs for practising teachers but on the range of ways in which practising teachers develop their expertise. Rather than using professional development to refer to programs that may have teacher learning as an outcome, we use the words learning and development in ways more closely aligned with their broader definitions. That is, development refers to broad sequential processes that involve the unfolding of inherent potentials, whereas learning is the processes by which teachers acquire new knowledge, skills, affects or behaviours. Learning is thus a means by which teachers develop. In interpreting the scope of this chapter broadly we have taken a different approach from the most closely aligned chapter in the 2008–2011 review (Bobis, Higgins, Cavanagh, & Roche, 2012) that was devoted entirely to the professional knowledge of practising teachers of mathematics, and return to a scope aligned with that of the 2004–2007 review (Anderson, Bobis, & Way, 2008). This is not because there has been a decreased emphasis on the knowledge needed for teaching mathematics; indeed many Australasian researchers have contributed to important advances in conceptualisations and understanding in this area over the review period. This chapter includes a significant focus on this aspect but acknowledges that this work is necessarily related to questions concerning the ways in which such knowledge is acquired and what else, apart from knowledge, is needed for the effective teaching of mathematics. This review is structured around two major sections. The first deals with theoretical approaches to understanding PL for teachers of mathematics, while the second focuses on PL programs.

There are, of course, overlaps with issues related to initial teacher education but we concern ourselves in this chapter only with research related to practising teachers. Initial teacher education is addressed in Chap. 15 of this volume.

2 Theoretical Approaches to Understanding Professional Learning

Australasian researchers have contributed to the development of theoretical understandings of PL for teachers of mathematics in two main areas–understanding the nature of teacher learning, and understanding the nature of teacher capabilities, including knowledge that teachers need in order to teach mathematics effectively. These aspects are dealt with in turn in the following sections.

2.1 Understanding the Nature of Teacher Learning

2.1.1 Sociocultural Understandings of Teacher Learning

Researchers have used and refined existing models of teacher change, and applied theoretical perspectives developed in other contexts to teacher change. Consistent with the recognition that teacher change is indeed learning, theories developed in relation to students' learning have been extended to teachers. For example, Goos (2013) used Valsiner's (1997) zone theory of student learning to describe teacher learning. Her use of Valsiner's work, as well as Visnovska and Cobb's (2015) conceptualisation of teacher learning as participation in communities of practice, exemplify what Lerman (2000) described as the social turn in mathematics education research. Goos (2013) made the point that her use of Valsiner's (1997) zone theory demonstrates the utility of sociocultural theories not only to explain teacher development but also to design interventions likely to facilitate teacher learning. In addition, this theory allowed learning to be understood as a social phenomenon without losing sight of the agency that individuals are able to exercise within their environments.

Recognition of the fact that professional development providers cannot with any certainty cause or direct teacher learning is reflected in Goos' work and in that of Anthony, Hunter, and Thompson (2014). Anthony et al. used Activity Theory (Engström, 1987) to understand the learning of one teacher involved in a PL initiative aimed at promoting the use of inquiry oriented practices in mathematics teaching. For Anthony et al. (2014), Engström's (2001) notion of multi-voicedness allowed recognition of the unique experiences and hence learning of individual PL participants and to acknowledge that teacher educators can occasion but not cause teacher learning. Anthony et al. (2014) drew on Engström's (2001) construct of expansive learning in which the focus of activity is not on acquiring particular knowledge or skills (Engström, 2001). The notion of expansive learning accommodates the fact that rather than being pre-defined, that which is learned in PL programs is often previously unknown to any participants, including the teacher educator(s). In the process of learning, the object is reinterpreted and expanded as the learner moves through successive zones of proximal development. Anthony et al. (2014) described the learning of the teacher in their study as, in addition to a change in practice, change in his thinking, and empowerment to transform his teaching beyond the focus and duration of the PL.

Goos (2013) and Anthony et al. (2014) used their respective theoretical lenses to identify tensions that prompted teachers' learning. Specifically, activity theory allowed Anthony et al. (2014) to identify tools that appeared to support teacher learning by helping teachers to recognise and engage with discrepancies between their current understanding and practices and the learning object. On the other hand, Goos (2013) described productive tensions that arise when a teacher's zone of proximal development do not align with his or her zone of free movement and/or zone of promoted action. The notion of tensions as drivers of change has also been used in work that focused on teacher learning in terms of the development of teacher identity. Identity is central to sociocultural theories of learning including zone theory (Goos, 2013). Bennison (2015) reported on the development of a conceptual framework for analysing identity as an embedder-of-numeracy based on a framework for mathematics teacher identity comprising knowledge, affect and social domains. Bennison (2015) added two further domains; life history, based on Sfard and Prusak's (2005) acknowledgement of the role of past experience in shaping identity, and professional context that included such things as available resources and the structure of the school day.

2.1.2 Psychological Understandings of Teacher Learning

In contrast to the sociocultural developments discussed in the previous section, Chick and Stacey (2013) elaborated on the idea posited by Bass (2005, p. 418) that mathematics teaching is "a kind of applied mathematics". The focus of these researchers was on the individual teacher in the context of teaching mathematics and (less explicitly) on the process by which a teacher develops mathematics teaching expertise. Chick and Stacey (2013) illustrated how problems that arise in mathematics teaching require teachers to draw upon their knowledge of mathematics, of general pedagogy, and of students within a specific teaching context. Chick and Stacey argued that the results of solving such mathematics teaching problems contribute to principles for teaching mathematics that are analogous to the methods that arise from applied mathematics research. It is the accumulation of these principles, derived from a growing number of solved mathematics teaching problems, that constitutes the expanding expertise of a mathematics teacher. By implication, mathematics teacher development is a consequence of engaging in and reflecting upon the results of the applied mathematical problem solving work that is teaching mathematics.

2.2 Understanding the Nature of Teacher Capabilities

PL opportunities are provided based upon evidence of, or assumptions about, the existing capabilities of teachers of mathematics as well as capabilities that are deemed necessary or desirable. The knowledge needed for effective teaching of mathematics has been and remains a major focus of research in this area with Ball and colleagues' seminal work on Mathematical Knowledge for Teaching (MKT) (e.g., Ball, Thames, & Phelps, 2008) remaining the major reference point. More fundamental reconceptualisations of teacher knowledge have been rare but include that of Chick and Stacey (2013) who argued that mathematics teaching is a form of applied mathematical problem solving, and Beswick, Callingham, and Watson (2012) who described a rich construct of teacher knowledge that includes beliefs and confidence in addition to Shulman's (1987) knowledge types. The latter appears to be the only example of theoretical considerations of affective aspects of teacher capability within the scope of this review. Beswick et al. (2012), Chick and Stacey (2013), and Thomas and Yoon (2014) all referred to MKT (Ball et al., 2008) even though these researchers did not specifically draw upon this work. This further illustrates the dominance of Ball and colleagues' work in relation to teacher knowledge. This section comprises two subsections describing modifications of MKT, and alternative conceptualisations of teacher knowledge.

2.2.1 Modifications of Mathematical Knowledge for Teaching (MKT)

Australasian researchers have been interested in Ball and colleagues' development of Shulman's (1987) notion of pedagogical content knowledge (PCK) (e.g., Ball et al. 2008). Along with PCK, the mathematics content knowledge (MCK) of teachers of mathematics has received particular attention. PCK has been conceptualised in a variety of ways that have included attempts to articulate the roles played in this type of knowledge by MCK and pedagogical knowledge (PK) (e.g., Ball et al., 2008; Chick, Baker, Pham, & Cheng, 2006). There is agreement that PCK draws upon, but is more than, the sum of these knowledge types (Shulman, 1987). There are, however, variations in how these types of knowledge are defined as well as in views about the ways in which they interact. Australasian researchers have proposed minor revisions to the MKT model; considerations of aspects of this model in particular mathematical domains, and shifts in emphases within particular knowledge types. Hurrell (2013) proposed refinements of MKT (Ball et al., 2008) that addressed shortcomings that he identified primarily with the model's visual representation. He did not question, beyond re-labelling PCK as PK, Ball et al.'s (2008) conceptualisations of subject matter knowledge (SMK) and PCK in terms of knowledge types that comprise them, and proposed the existence of connections between each possible pair of knowledge types. Hurrell's work is preliminary and it will be interesting see whether evidence about the nature and extent of the various interactions among knowledge types that he suggests is eventually available.

In contrast to Hurrell's proposition that all of the knowledge types that comprise Ball et al.'s MKT are mutually interconnected, Wilkie's (2014) findings are consistent with specialised content knowledge (SCK) and knowledge of content and students (KCS) being distinct constructs. This was evident from teachers having high levels of SCK but being unable to understand how children think and learn about mathematics. Similarly, teachers appeared to demonstrate knowledge of the curriculum but struggled to suggest appropriate activities for teaching that content consistent with the independence of common content knowledge (CCK) and knowledge of content and teaching (KCT). Wilkie's (2014) work also exemplifies the domain specific nature of teacher knowledge, and the work that remains to be done in finding ways to measure relevant aspects of what teachers know. She developed a rubric for assessing one aspect of MKT, SCK, for the development of functional thinking with growing patterns.

Zhang and Stephens (2013) described the use of the construct, teacher capacity, in studying curriculum reform in mathematics education. They observed that the relatively little attention that has been paid to knowledge of the curriculum by, for example, Shulman (1987) and Ball et al. (2008), could be a result of their US context in which the curriculum differs from that in other countries including Australia and China. Furthermore, Zhang and Stephens pointed to literature that demonstrates the complex and contentious nature of content knowledge—a construct that appears to have been unquestioned and undefined in other Australasian studies over the period of this review. Teacher capacity has been used elsewhere in education but, Zhang and Stephens (2013) argued, has particular characteristics in mathematics education. They described it as comprising four elements—knowledge of mathematics, interpretation of the curriculum, understanding of students' mathematical thinking, and design for teaching. All four elements correspond to knowledge types in Ball et al.'s (2008) MKT. The key differences are the greater emphasis on knowledge of the curriculum and a shift in emphasis away from knowledge as an essentially static construct towards the ways in which teachers use their knowledge in teaching (Zhang & Stephens, 2013).

2.2.2 Alternative Conceptualisations of Teacher Capability

The alternative conceptions of teacher capability evident within the scope of this review have in common an effort to provide a more holistic view of the work of teaching than the analytic MKT framework. Some, like Beswick, Callingham, and Watson (2012) and Thomas and Yoon (2014), have developed or adopted models that have included affective aspects of teachers' capabilities, whereas Chick and colleagues (e.g., Chick, Baker, Pham & Cheng, 2006) have worked with the impossibility of clearly separating knowledge types in practice. These researchers have presented fundamental rethinks of the nature of teacher knowledge beginning with work that pre-dates the period of this review (e.g., Chick et al., 2006).

Although cited by Goos (2013) their model of PCK as a continuum from PCK in a content context to PCK in a pedagogical context appears not to have been widely adopted.

More recently Chick and Stacey (2013) compared teaching mathematics to mathematical problem solving. They emphasised a dynamic view of teacher knowledge as evident in practice and, to this end, employed Rowland et al.'s notion of contingency. They used Polya's (1962) definition of knowledge as comprising information and know-how and drew attention to the fact that for teachers to be successful they need not only knowledge but also the ability to apply that knowledge in the course of teaching. In addition Chick and Stacey claimed that the greater the teacher's fluency with the relevant mathematics the more likely it is that a solution will occur to them. The implication is that content knowledge is a necessary, though not necessarily sufficient, condition for effective teaching.

Beswick et al. (2012) conceptualised knowledge for teaching mathematics as a single rich construct that incorporated beliefs and confidence as well as each of Shulman's knowledge types. They suggested that "a holistic consideration of teacher knowledge reveals important insights that may not be evident from detailed analytic dissections of the concept" (p. 133). Based on 62 teacher responses to a comprehensive profiling instrument Beswick et al. described the characteristics of four hierarchical levels of teacher knowledge that were labelled; (1) Personal Numeracy, (2) Pedagogical Awareness, (3) Pedagogical Content Knowledge Awareness, and (4) Pedagogical Content Knowledge Consolidation. They noted that confidence in one's own mathematical ability and in one's ability to help students to understand the subject does not imply adequate levels of general pedagogical knowledge or PCK. In general beliefs aligned with student-centred approaches to mathematics teaching characterised the higher levels but, contrary to this, it appeared that the most knowledgeable teachers were inclined to equate mathematics with computation and to value the use of textbooks and exposition. Beswick et al. (2012) argued that these discrepancies reflected the fact that broader beliefs about such things as the nature of the discipline, the teacher's role, and the capabilities of students were more reliable determinants of the extent to which a teacher" approach was student-centred than his or her views about the use of specific pedagogies or tools.

Thomas and Yoon (2014) used Schoenfeld's (2011) goal-oriented decision making framework to analyse one secondary school mathematics lesson. According to Schoenfeld (2011), teachers' orientations (e.g., their beliefs, values, tastes, and attitudes) inform their goals for their teaching both at the planning stage and throughout the course of a lesson. In acting to achieve their goals teachers draw upon resources (e.g., knowledge of various kinds, problem solving heuristics, and physical objects including pens, books, and digital technologies). Thomas and Yoon (2014) reported positive impacts of the PL and suggested that detailed case studies such as the one they presented could be used as catalysts for teacher reflection upon and discussion of their own resources, orientations, and goals; and hence provided a powerful tool for professional learning.

3 Professional Learning Programs

The studies reviewed in this section have reported on the impacts of particular PL initiatives aimed at improving the teaching of mathematics in various ways. Other studies have focused primarily on either the content or processes of PL. The following sections of this review cover research on PL programs according to their content foci, the nature of effective PL, approaches to PL, and the evaluation of PL programs. Overlaps among these categories are acknowledged with many studies dealing with more than one aspect to a greater or lesser extent.

3.1 Content Foci of Professional Learning Programs

The content foci of PL programs have been diverse, reflecting contemporary developments in technologies for mathematics teaching and efforts to increase the use in mathematics teaching of student-centred pedagogies (including inquiry based approaches and culturally responsive pedagogies). Specifically mathematical foci include the development of students' mathematical thinking, the design and use of challenging tasks, and planning for mathematics teaching.

3.1.1 Using Technology in Mathematics Teaching

Providing PL to assist teachers to incorporate the use of new technologies into their teaching of mathematics has been an important activity and context for research over the period of this review. Chapter 13 is devoted to the ways in which these tools can transform mathematics teaching and so the relevant studies are treated only briefly here.

PL support for integrating technologies in teaching includes learning to use new tools as well as learning new pedagogical practices and becoming sufficiently fluent in their use to make the most of opportunities afforded. Some researchers have reported on projects in which substantial support was provided for teachers (e.g., Pierce & Stacey, 2013) while others reportedly provided limited PL support and examined the influence on teachers' practices as the technologies were introduced (e.g., Attard, 2013; Attard & Curry, 2012; Zuber & Anderson, 2013). All of these studies have been small scale, exploring teachers' adoption of various technologies. The challenge is finding ways to synthesise the findings of such studies in order to identify the key factors that lead to positive impacts of new technologies and to find scalable and sustainable models of PL provision.

At the more intensive end of the spectrum in terms of PL support, Pierce and Stacey (2013) explored four teachers' assimilation of new practices into current repertoires as they began using mathematics analysis software, specifically TI-Nspire (CAS) in their Year 10 classrooms. Pierce and Stacey used a lesson study approach which was successful in supporting teachers' functional and pedagogical uses of the new technology. Although the teachers became very competent in using the technology, they were not keen to explore what else the technology afforded.

Studies in which limited PL was offered include that of Attard and Curry (2012) who examined change in Year 3 students' engagement and change in one teacher's practice during a 6 month iPad trial. From the same study, Attard (2013) examined two different teachers' integration of iPads with analyses of observed lessons further highlighting the need for careful selection of tasks and apps as well as support for students. Zuber and Anderson (2013) investigated secondary mathematics teachers' initial response to the introduction of one-to-one laptops. The mathematics teachers lacked confidence and held beliefs that were powerful barriers to regular use of laptops. These teachers believed students needed to record mathematics using pen-and-paper and that laptops were appropriately used with classes of more able students.

Investing in new technologies must be accompanied by substantial investment in PL that addresses not only technical knowledge but also the pedagogical affordances of the technology and teachers' beliefs about the potential value of changing their current practice if change is to be realised. However, the Pierce and Stacey (2013) study reveals that even when substantial PL support is available, teachers do not easily embed new technologies into their practice.

3.1.2 Using Student-Centred Pedagogies

Several studies in Australasian contexts have investigated student-centred pedagogies including inquiry-based pedagogy and culturally responsive pedagogy particularly in schools with Indigenous students. For many teachers, embracing these pedagogies may not match with their own experiences as learners of mathematics and can require substantial reflection and reinvestment in new ways of thinking about teaching and learning in potentially challenging contexts. Thomas and Yoon (2014) cited evidence that student-centred pedagogies such as inquiry-based teaching are more likely to be used by teachers with positive attitudes to mathematics and its teaching. Therefore, PL needs to challenge teachers' beliefs about their role as "teacher" and their relationships with other members of the community within which they work.

Inquiry-based pedagogy. Despite considerable interest in inquiry-based teaching (e.g., Fielding-Wells, 2015); there has been little focus on teachers' learning as they implement inquiry or on PL aimed at assisting teachers to implement such approaches. The exception is Anthony, Hunter, and Thompson (2014) who used an activity theory framework to investigate one teacher's (Zain) journey as he introduced inquiry practices into his primary classroom in a 2-year project. The changes to his practice were facilitated by researchers as co-learners who used a Communication and Participation Framework (CPF) tool for analysis as well as researcher observations and coaching, along with self and peer critique of videos of classroom episodes. Zain's classroom environment changed so that the introduction of “mixed-ability grouping, collaborative tasks and student agency challenged his existing practice” (p. 287). The authors noted that after significant transformation, Zain engaged in “boundary crossing” since he wanted to move on to new contexts to share his experiences and to have an impact in other schools.

Culturally responsive pedagogy. Several studies over the period from 2012 to 2015 in the Australian context have explored culturally responsive pedagogies in the context of efforts to improve Indigenous students’ learning outcomes. The relevant studies are dealt with in Chap. 8 of this review and hence are treated only briefly here. Culturally responsive teaching includes “… developing a knowledge base about cultural diversity, including ethnic and cultural diversity content in the curriculum, demonstrating caring and building learning communities, communicating with ethnically diverse students …” (Gay, 2002, p. 36).

It is common to find recently graduated teachers working in remote or regional locations in Australia. These teachers must learn to work in unfamiliar contexts, frequently with students with English as a second language, whose cultural backgrounds are different from their own, with few experienced teachers in the school to act as role models, and with little access to PL. Government funding has supported several studies in these contexts (e.g., Sullivan, Jorgensen, Boaler, & Lerman, 2013; Warren & Miller, 2013) but because of the nature of and the locations of the schools, these studies are necessarily small scale. An ongoing issue remains whether such projects can be sustained or scaled up to have a greater impact on students’ learning outcomes which continue to be significantly lower than other Australian students (Thomson, Hillman, & Wernert, 2012).

All of the studies reviewed in this section aimed to enhance teachers’ abilities to enact culturally responsive teaching so focused primarily on specific aspects of mathematics teaching such as developing and/or implementing tasks (e.g., Jacob & McConney, 2013; Sullivan, Jorgensen, Boaler, & Lerman, 2013) and the use of multiple representations (e.g., Warren & Miller, 2013; Warren & Quine, 2013). These programs included significant support over several years and while improvements in teachers’ ability to implement rich tasks were reported, developing tasks proved more difficult (Jacob & McConney, 2013; Sullivan et al., 2013).

Several studies had aspects of language, and particularly the use of students’ home language, as a major focus (e.g., Jacob & McConney, 2013; Sullivan et al., 2013). There appears to be agreement about the value of home language use but also recognition of the difficulties that its management can present for teachers who do not have that language. Sullivan et al. (2013) also discussed the sometimes fraught nature of questioning where, for example, shame could result when a younger sibling answers a question, and teacher questioning might be interpreted as the teacher not knowing.

Raising teachers’ expectation of students has been another common emphasis in these programs (e.g., Owens, 2015; Sandhu, Kidman, & Cooper, 2013; Warren and Miller, 2013; Warren & Quine, 2013) with success reported in all cases. Higher expectations of students was the last of five stages that Warren and Miller (2013) described teachers moving through as they participated in the Representations, Oral Language, Engagement in Mathematics (RoleM) project. The four preceding stages were gaining teachers' interest, heightening their engagement, changes in PCK, and changes in content knowledge. Owens (2015) reported change in teachers' views of Indigenous students, at least in part as a result of partnerships developed with local community members, from blaming families for poor attendance and attainment to a focus on student learning and higher expectations.

Warren and colleagues (e.g., Warren & Miller, 2013; Warren & Quine, 2013), Sullivan et al. (2013) and Sandhu et al. (2013) each used a different framework to underpin their projects. Space does not allow detailed analysis but each of complex instruction (Sullivan et al., 2013), RoleM (Warren & Miller, 2013; Warren & Quine, 2013) and Reality-Abstraction-Mathematics-Reflection (RAMR) (Sandhu et al., 2013) have much in common with each other and with widely recommended mathematics pedagogy, suggesting that in terms of mathematics, teaching Indigenous students well might be the same as teaching well in any context. It is issues of language and such things as cultural understandings of questioning that highlight where culturally responsive pedagogy appears to be a most useful construct.

3.1.3 Developing Students' Mathematical Thinking

White, Wilson, and Mitchelmore (2012) worked with a small team of teachers to develop and teach a lesson based on the Teaching for Abstraction model—a model similar to the one used in the Sandhu et al. study (2013). The model has four phases —familiarity, similarity, reification, and application—but previous attempts to engage teachers with ready prepared materials indicated teachers had difficulty implementing the model. Consequently the authors worked with teachers to develop a unit together, thereby enhancing ownership of the material. Students were able to engage with new content because they could connect with familiar contexts but they were not able to develop deep learning because teachers were less familiar with two aspects of the model—similarity and reification.

Based on six key principles of effective mathematics teaching that include engaging students with a variety of rich and challenging tasks, Sullivan, D. M. Clarke, and B. Clarke (2013), report on their work with teachers of Years 5–8 to design and implement several task types. Over the 3 years of the project, *Task Types in Mathematics Learning*, the researchers worked closely with a small number of teachers as they designed tasks and lessons, implemented and evaluated the tasks. Students reported they preferred to be challenged, to have to think, make decisions, and communicate with each other. Some of the teachers had difficulty seeing the mathematics in tasks, turning tasks into worthwhile lessons, and maintaining challenge as students struggled to find strategies and solutions to particular tasks. The researchers concluded that teachers need to have PL opportunities that focus on specialised mathematics for teaching, be exposed to a range of task types, and be aware of the limitations of their knowledge.

3.1.4 Using Challenging Tasks

The studies reviewed in this section have arisen from the *Encouraging Persistence Maintaining Challenge* (EPMC) project—during the review period, there have been additional publications from the EPMC project that are not included here because of space limitations. Sullivan and Mornane (2014) describe the outcome of working with five Year 8 teachers from one school who were teaching a class of mixed ability students. Using a modelled lesson incorporating challenging tasks, teachers were exposed to the pedagogies required to implement such tasks, which encourage risk-taking, persistence, and collaboration. Participating teachers possessed the necessary subject matter knowledge and PCK to implement the lessons so much of the PL conversations focused on the purpose of the suggestions and associated pedagogies. The teachers were advised to "allow students to engage with the challenging tasks with limited instruction for some time so that students can consider possibilities and plan their approach" (p. 197). Discussions after the modelled lesson revealed teachers focused on the posing of the task, the strategy of encouraging individual work before small group discussion, and issues arising from students reporting their strategies. Teachers then taught the collaboratively developed unit of work with all reporting the need for a "bridge" to the challenging task, using tasks with different levels of sophistication, and constraints involved in conducting whole class discussion.

Clarke, Roche, Cheeseman, and van der Schans (2014/2015) compared two forms of PL to support teachers' implementation of challenging tasks. The first involved two full days of professional development followed by several months to implement a range of tasks. The second involved a demonstration lesson on one day followed by focus group discussions with the teachers. Both forms of PL were successful in enabling teachers to identify key features of the approach and both groups of teachers noted an increased emphasis on discussion, questioning, sharing, the development of classroom culture, and holding back from telling the students what to do. A major difference in the features identified by the two groups of teachers was that the first group emphasised "time" whereas the second group placed greater emphasis on the careful monitoring of students during the lesson. This is not surprising given the first group actually implemented the tasks and realised the amount of time required to both plan and prepare for the teaching as well as the time required for students to successfully engage with each of the tasks.

3.1.5 Planning for Teaching

In the EPMC project teachers were supported in planning effective lessons with challenging tasks and several papers from this project have focused on this aspect. Sullivan et al. (2014) worked with 34 primary teachers and 15 secondary teachers for a 2-day induction that introduced a launch-explore-summarise lesson structure for engaging students with the challenging tasks. A novel component of the PL was the requirement for each group of teachers to "work through" a set of 10 tasks and to anticipate students' responses to the task in preparation to implementing them with their own students. Teachers were encouraged to provide little indication to students of how to complete the tasks although enabling prompts were provided for students having difficulty. Teachers believed the lesson structure and tasks were helpful in implementing such lessons although they would have liked even easier enabling prompts—a suggestion which would have potentially removed the challenge of such tasks and ameliorated the whole approach.

Walker (2014) observed that even after teachers had been provided with modelled experiences of summarising at the end of a lesson after students have engaged with challenging tasks, the results tended to be a "smorgasbord offering of ideas and strategies" (p. 631). After one teacher in the project suggested she planned for the summary phase "on the run", Walker (2014) asked project teachers to trial a new approach of stopping the class once or twice during the lesson to have some students share their thinking. This strategy was found to be highly effective in prompting other students to reconsider their strategies and aided them in analysing, justifying, and explaining their thinking.

A different large-scale project, *Peopling Education Policy*, undertaken by several of the same researchers, investigated teachers' planning and use of curriculum or syllabus documents and coincided with the introduction of the new Australian Curriculum (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2015). Sullivan, Clarke, Clarke, Farrell, and Gerrard (2013) investigated the processes and priorities in lesson planning using the Australian Curriculum while Sullivan, Clarke, Clarke, and Roche (2013) described teachers' decisions about mathematics tasks when they plan mathematics lessons. Finally, Roche, Clarke, Clarke, and Sullivan (2014) presented data about primary school teachers' written unit plans in mathematics and their perceptions of the essential elements of these. Although the focus of these papers was on teachers' use of curriculum documents (addressed in Chap. 4 of this volume), it is noteworthy that the authors frequently mentioned the unique PL opportunities that the implementation of new curricula provides, and which, when embraced, enables development of mathematical subject-matter knowledge as well as review and development of PCK for the teachers involved.

3.2 Characteristics of Effective Professional Learning

The broad characteristics of effective PL for teachers have been well established and were summarised by Watson, Beswick, and Brown (2012). These characteristics include: shared purpose; underpinned by evidence and recognition of best practice; ownership by participants; connected to the school context; sustained over time; explicit development of theoretical understandings and connections to practices that challenge and extend teacher knowledge through modelling; balances individual learning needs within the development of a community of practice; and employs evaluation linked to enhanced student learning that is broadly-based and ongoing.

Based on these characteristics, the 3-year Providing the **M**athematical Foundation for an Innovative **A**ustralia within **R**eform-**B**ased **L**earning **E**nvironments (MARBLE) project involved two clusters of Tasmanian schools and provided PL through: workshops (whole day, half-day, and after school) for all participating teachers, teachers in one or other cluster, or within a single school; and case studies in which a single researcher worked intensively with an individual teacher over a period of weeks.

Other PL reported by Australasian researchers over the period of the review used a variety of approaches with characteristics consistent with all of those described by Watson et al. (2012). In particular, there appears to have been an emphasis on participants owning the PL, connection with the school context, and modelling of practice. Approaches to modeling desirable practice have included the use of demonstration lessons, both live and using video (e.g., Visnovska & Cobb, 2013), and including adaptations of lesson study (Ebaeguin & Stephens, 2014). Action research (e.g., Scott, Clarkson, & McDonough, 2012), and more traditional PL workshops (Clarke, Roche, Cheeseman, & Van Der Shans, 2014) were also sites of research on PL. In all cases, prompting teachers in some way to reflect upon their experience of the PL was considered crucial as well as a key means of data collection for studying the impacts of PL. Collectively the work highlights some aspects of effective PL that are less commonly recognised explicitly and points to several aspects of mathematics teachers PL in which there remains much to be understood.

Characterisations of effective PL, including Watson et al.'s summary, have tended to overlook the role of culture. Ebaeguin and Stephens (2014) identified several aspects of Japanese culture that align with lesson study and may explain both its widespread use and effectiveness in Japan and possible difficulties in countries that differ markedly on the relevant dimensions of culture. Specifically, they concluded that high levels of collegiality among teachers, attention to detail in planning, and a long term view of improving practice all facilitate lesson study in Japan. Finally, these researchers cautioned about the extent that lesson study might be embraced elsewhere; the cultural issues that they mention could serve to highlight for PL providers in other countries aspects of culture within the participating teacher group that may require particular or greater attention.

3.3 Approaches to Professional Learning

In this section we consider the range of approaches to PL evident in Australasian research 2012–2105. These approaches are organised in three categories—demonstration lessons, video-based approaches, and action research. They have in common an effort to facilitate teachers' engagement with the reality of classroom teaching either by observing another teacher (demonstration lessons, videos) or themselves (video), and assessing the impacts of teaching on student learning (demonstration lessons, video, action research).

3.3.1 Demonstration Lessons

Demonstration lessons share many features with lesson study that arise from both approaches being conducted in the context of a specific classroom and lesson. In lesson study the process is long term and cyclical whereas demonstration lessons are more likely to be a shorter term component of a broader PL program with affordances in terms of beginning the process of breaking down the traditional isolation of teachers in their individual classrooms which is the norm in many countries. Clarke, Roche, Cheeseman, and van der Schans (2014/2015) described the use of demonstration lessons with only a briefing session before the lesson and a focus group discussion afterwards with all conducted in a single day. Although these researchers acknowledged the limitations of the approach, including that it took no account of other PL activities with which the teachers were involved, they reported evidence that the experience was worthwhile for most participants. The demonstration lessons described by Clarke et al. (2013) were part of PL provided by the Contemporary Teaching and Learning of Mathematics (CTLM) project involving 82 Victorian primary schools between 2008 and 2012. The demonstration lessons contributed to the participating teachers' ownership of the PL by allowing them to choose the focus of their observations. Clarke et al. (2013) noted that teachers seemed to have greater difficulty identifying foci related to student thinking than teachers' actions, and noted that anecdotally teachers involved in their program in its second year appeared to have become more comfortable with focusing on observing students. This is also consistent with Visnovska and Cobb's (2013) finding from their analysis of teachers' responses to the same videos of mathematics classrooms observed 2 years apart.

3.3.2 Video-Based Approaches

Visnovska and Cobb (2013) regarded the use of video as a means by which PL providers can learn about teachers' shared concerns which are necessarily constrained by their particular school contexts. The use of video can allow PL providers to account for the resources that teacher groups bring with them to their experience of a PL program. Visnovska and Cobb (2013) emphasised that, although resources used with teachers may be chosen with particular intentions in mind for the teachers' learning, it is not possible to know that teachers will use the resources in the ways envisaged, even when PL providers carefully support their use. In later work Visnovska and Cobb (2015) also used video-analysis as the main means of teacher support, as well as the notion of scaffolding from one-to-one student support to whole class teaching that is adaptive in the sense of being responsive to students' emerging understandings. These researchers noted that teachers initially regarded the videos as a means of determining the extent to which the students had achieved the intended learning. Visnovska and Cobb (2015) reported difficulty in helping teachers to focus instead on what the videos revealed about students' reasoning as an important input into planning for subsequent teaching.

Implicit in Visnovska and Cobb's (2013) work is a tension between the choice of resources, such as videos, with clear intended learning outcomes in mind for the PL participants, and the need to recognise that teacher learning cannot be pre-determined. Their work provides insights into how PL providers might both design a PL curriculum and yet recognise the communal as well as individual nature of teachers' learning in ways that respect the understandings and practices that teachers bring to the collective activity. Questions about how appropriate learning goals and curricula might or should be determined, and by whom, remain unanswered.

3.3.3 Action Research

Action research is another way in which PL can be focused closely on the context and individual needs of teachers and was the mechanism used by Scott, Clarkson, and McDonough (2012) and, implicitly, by Driscoll (2015). An innovative aspect of Scott et al.'s study (also part of the CTLM project described by Clarke et al., 2013) was the use by teachers of digital portfolios to document their project in a way that facilitated reflection on their own learning and made their experience available for discussion with other participants and the PL providers. Factors that these teachers identified as supporting them in changing their practice were the opportunity to discuss their teaching with colleagues (teachers, mathematics leaders, and researchers), structured tools and well-organised meetings, and the ready availability of teaching resources. Driscoll asked her primary school students to reflect on their learning of mathematics, including how this could be supported by teaching. She found that her own learning was stimulated by the results of her efforts to assist her students with their reflections.

Just as the teachers in Clarke et al.'s (2013) study chose the focus of their observations, the teachers in Scott et al.'s (2012) study selected an aspect of their teaching that would be the focus of their action research project. Although this could be considered at odds with the first category of Watson et al.'s (2012) summary of the broad characteristics of PL, in both cases teacher choice was provided within the context of a broader shared purpose related to improving mathematics teaching. Of Watson et al.'s categories, the second "Underpinned by evidence of and recognition of best practice", may have been least apparent to the participating teachers. Although the researchers cited in this review who described PL programs all articulated the theoretical frameworks that informed the design and analysis of the PL, and made clear the ways in which their work builds upon and has been informed by earlier research, the extent to which this was made explicit for teachers is not clear. Nor is it clear whether the decision to share or not share relevant theory and research with teachers was consciously made. Indeed in Watson et al.'s (2012) summary it is only Sowder (2007) who suggested that PL should engage teachers with research and neither the extent to which this might be desirable nor the means by which it might be achieved is clear. Bringing teachers inside PL providers' thinking and empowering them to make their own critiques of research is certainly consistent with rhetoric (e.g., Beswick, Callingham, & Muir, 2012) and with respect to the PL literature (e.g., Visnovska & Cobb, 2013) and hence is a worthwhile field for future research.

3.4 Evaluation of Professional Learning

In this section we use Guskey's (2002, p. 47) five levels of evaluation to discuss the approaches to PL evaluation employed in Australasian studies conducted in 2012–2015. In the case of participants' learning, including of new knowledge or skills, data were sometimes self-reported and at other times attempts to measure learning were made. Self-reported data included that gathered by survey or interview. The apparent gaps in the research in relation to several of Guskey's stages are unsurprising. For example, participants' reactions to a PL activity are unlikely to provide important insights that advance the field and hence are unlikely to be reported. Other aspects such as participants' use of the knowledge and skills they acquire, and broader changes at a school or system level, are both more difficult to investigate and more difficult to link to a specific PL initiative. Given the sizeable investment of governments and school systems in PL and ongoing calls for this to be increased (e.g., Office of the Chief Scientist, 2014), it behoves researchers to pursue a focus on ways of better establishing the efficacy of PL. The Australian Senate Standing Committee of Education and Workplace Relations (2013, p. 92) specified the need for evaluation at the most demanding of Guskey's levels: "the success of professional learning should be measured not just in the benefit to the teacher, but in improved outcomes for the students."

Self-reports of changes to teachers' attitudes, beliefs, practices and confidence are commonly used as indicators of a PL program's effectiveness in spite of the acknowledged shortcomings of self-report data. For example, Jacob and McConney (2013), Warren, Quine, and DeVries (2012), and Scott et al., (2012) all presented changes in PL participants' self-reported practices towards pedagogies aligned with the relevant PL programs. Changed attitudes and beliefs, again in line with program aims, were reported by Warren et al. (2012) as outcomes of the Representations, oral language and engagement in Mathematics (RoleM) project. Changed beliefs included increased expectations of student learning.

Teacher confidence is considered an important factor impacting the effectiveness of mathematics teachers because of its connection with teachers' knowledge for teaching mathematics Nevertheless, the connection between confidence and competence is not clear cut and hence caution is warranted in interpreting changes in teacher confidence. For example, Warren et al. (2012) reported increased teacher confidence in relation to implementing the curriculum in innovative and creative ways, whereas teachers in the PL program described by Jacob and McConney (2013) reported variable results in terms of changes to teachers' confidence including reduced confidence in their ability to provide students with engaging activities. Jacob and McConney (2013) speculated that participation in the PL program may have increased teachers' awareness of shortcomings in their teaching, similarly Beswick, Callingham, and Watson (2012) warned against assuming that confidence is indicative of competence.

Many of the studies reviewed included assessments of teacher learning (Guskey's second level) based on interviews (e.g., Owens, 2015). Although valuable, these data are self-reports. Beswick et al. (2012) used a comprehensive pen and paper teacher profiling instrument to measure knowledge (including relevant beliefs and confidence) of 62 teachers at the start of their involvement in the 3-year program. Although not used as evidence of teachers' learning in this study the teacher profile has been used in this way in earlier work (Watson, Beswick, Caney, & Skalicky, 2006).

No studies reporting system change were found within the last four years in the Australasian context. This is unsurprising given that educational policy is rarely attributable to a single study. Nevertheless, it is apparent that several of the researchers whose work in the PL field has been prominent over the review period (e.g., Sullivan and colleagues) have been well supported by the educational jurisdictions in which they have worked for considerable periods of time and on successive projects.

An example of Guskey's fourth level of evaluation—participants' use of new practices—is provided by Scott et al. (2012) who used digital portfolios to collect a range of evidence of change in teachers' practice. The portfolio included a video of teaching of at least 60 s with written commentary, student work samples, and a completed chart showing changes in their practice over the life of the project. In addition, Scott et al. (2012) observed one or two lessons taught by each of the four participating teachers.

There have been attempts to measure the effectiveness of PL at Guskey's most demanding level, that of improved learning outcomes for students even though it has been recognised that connecting such change to specific PL is a difficult undertaking. For example, Warren and Miller (2013) had students in the classes of participating teachers sit Progressive Achievement Test (Mathematics) early and late in each year of their project. For each year level Warren and Miller (2013) reported statistically significant changes with effect sizes ranging from medium to large. Sullivan and Mornane (2014) collected data on student learning, as well as their affective responses to challenging tasks, using interviews, online pre- and post-tests, and a written assessment. These researchers reported that most students successfully engaged with and were able to complete challenging tasks under test conditions. Although these results are encouraging it is impossible to say what part of the gains are attributable to the PL program in which these students' teachers participated.

4 Conclusion

Throughout 2012–2015, PL has remained both an important site for and focus of research for Australasian researchers. There is evidence of increasing attention to the theoretical bases of PL and examples of frameworks being used to effectively analyse the outcomes of PL programs and to situate particular PL programs within broader international agendas (e.g., Beswick et al., 2012; Chick & Stacey, 2013; Thomas & Yoon, 2014). Australasian researchers have also contributed to theorisation of constructs relevant to PL, most notably teacher knowledge, although the dominance of existing models, particularly MKT (Ball et al., 2008) has contributed to alternative conceptions of teacher knowledge achieving limited traction to date. As noted in the next section, there is, however, plenty of scope for research related to PL and to which Australasian researchers may make important contributions into the future.

4.1 Future Directions for Research

4.1.1 Scale and Sustainability

To date there has been relatively little attention to issues of scale and sustainability, and the impacts of PL at organisational and systemic levels. Despite the methodological difficulties presented it is important that mathematics education researchers continue to investigate ways to measure these impacts since it is change at a systemic level that will make broad changes in mathematics teaching.

A further obstacle to scaling up successful PL initiatives lies in the jurisdictional and systemic arrangements that result in subtly differing language and theoretical frameworks from among countries and even between Australian states. While it is understandable that mathematics educators designing PL for teachers in a specific context need to draw upon models and language familiar to those teachers, the resulting small scale specific research findings add to the impression that this aspect of mathematics education research might be a cottage industry (Lowrie, 2015) and hence of limited impact internationally. There is a need for Australasian researchers increasingly to situate their findings within an international discourse on PL.

4.1.2 Online Delivery

Amongst recommendations related to PL included in the report of the Senate Standing Committee on Education, Employment, and Workplace Relations (2013) was that the potential use of online tools for delivery of PL for teachers be investigated. To date the use of digital and online technologies has played only a minor role in the provision of PL with the Scott et al.'s (2012) use of digital portfolios a rare exception. With issues of affordability and scale becoming increasingly pertinent, the availability of relevant technologies more widespread, and online learning becoming normalised as a means of initial and further formal teacher education, it seems inevitable that more PL for teachers, including teachers of mathematics, will make use of digital resources and be delivered in modes that include online learning or are entirely online. It will be important that developments in this space are rigorously evaluated and research conducted to assist us to understand the affordances and limitations of various delivery modes.

4.1.3 Cross-Disciplinary Collaboration

Lowrie (2015) observed that interest in teachers PCK and MCK represent part of the limited common ground between mathematicians and mathematics educators in Australasia. Attention to collaboration between mathematics education researchers and mathematicians with the aim of enhancing PL for teachers of mathematics is timely especially as mathematics educators increasingly do not have postgraduate qualifications in mathematics (Lowrie, 2015) and the acknowledged importance of content knowledge to mathematics teaching expertise.

4.1.4 Improving the Effectiveness of Professional Learning

We know relatively little about why and how apparently successful PL initiatives are successful or conversely why and how others are less so. Existing models of effective PL tend to be generic and do not take account of (nor exploit) the many factors that seem likely to influence the impact of any PL initiative (e.g., the experience of the teacher and school context). This review has identified aspects of context such as culture (Ebaeguin & Stephens, 2014), that impact PL but that are not commonly included in lists of characteristics of effective PL including Watson et al.'s (2012) summary. In addition, Clarke et al.'s (2014/2015) study, although small, suggests that PL that "breaks the rules" in terms of complying with such lists can, nevertheless, be effective. What is needed is a more nuanced approach to conceptualising PL quality including in terms of its effectiveness that takes account of the aims and contexts of particular initiatives. Such an approach is likely to be more helpful than listing characteristics that may be present irrespective of the effectiveness of the PL in terms of student learning and that may not be appropriate for PL with quite contained and tightly focused aims.

Many of the studies considered in this review have provided detailed accounts of the processes by which small numbers of teachers appear to have learned. Some researchers have attempted to distil broader lessons from the experiences of participants in their programs. For example, Warren and Miller (2013) identified five stages of teachers' learning. Just as learning trajectories have proven useful in understanding students' mathematics learning (e.g., Clements & Sarama, 2004) similar approaches to mathematics teachers' learning with fine-grained foci (e.g., Kazemi & Franke, 2004) could be fruitful.

Other researchers have used existing frameworks to analyse teachers' learning. These have included zone theory (Goos, 2013), activity theory (Anthony et al., 2014), and identity formation (Bennison, 2015). Chick and Stacey (2013) saw teacher learning as an outcome of mathematics teacher problem solving. Rather than defining a trajectory these frameworks point to contributions to teachers' growth. As such they suggest ingredients that might be useful components of PL. Combined with work on learning trajectories it might be possible to identify points in teachers' development at which particular interventions might have greatest impact. Such developments would raise questions about what a PL curriculum for teachers of mathematics might look like.

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Chapter 17
Advancing Mathematics Education Research Within a STEM Environment

Lyn D. English

Abstract In presenting the final chapter for this *Research into Mathematics Education in Australasia (RiMEA)* book, I first give consideration to the official curriculum and the operational curriculum as a basis for exploring how we might advance mathematics education research within our Science, Technology, Engineering and Mathematics (STEM) environment. Next, I present an overview of some of the core features of the current national and international spotlight on STEM education. From this basis, I argue that the roles and positioning of mathematics are in danger of being overlooked or diminished within the increased STEM framework. As one approach to lifting the profile of mathematics, I explore problem-solving and modelling across STEM contexts. In utilising findings from the chapter reviews together with my own research, I offer suggestions for (a) developing content and processes through idea-generating problems, (b) promoting in-depth content understanding, and (c) fostering general skills and processes. Next, I address the advancement of modelling across STEM contexts and illustrate this with a problem set within an environmental engineering context. I conclude by offering a few avenues for further research.

Keywords STEM education · Official curriculum · Operational curriculum · Problem solving · General skills and processes · Modelling · 21st century skills · Workplace learning

1 Introduction

Each of the chapters comprising this *Research into Mathematics Education in Australasia (RiMEA)* monograph presents an in-depth and insightful review of the Australasian mathematics education research undertaken over the previous 4 years. Major longstanding issues are addressed together with new concerns emerging from

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the changing political and educational agendas both nationally and internationally. Challenges in the learning and teaching of mathematics from early childhood through to tertiary and professional education are examined, with a number of common threads appearing across the chapters. These include broad concerns about the National Curriculum and how it is enacted, the impact of national and international assessments, how we might close gaps in students' mathematics learning, and ways to advance teacher professional development and leadership. A focus on core content topics appears not as prevalent across the chapters as the foregoing issues.

It is not the intention of this final chapter to review each of the chapters in turn; collectively they present a wealth of research for our current and future mathematics education communities. Rather, I draw upon some of the key findings in the reviews that appeared pertinent in framing my suggestions for advancing Australasian research in mathematics education. In developing this chapter, I was drawn to Way, Bobis, Lamb, and Higgins' application of Remillard and Heck's (2014) model of curriculum policy, design, and enactment (see Chap. 4, this volume). Way et al. consider various components of the model's "official" curriculum and the "operational" curriculum, from which stem many of the issues facing the mathematics education community today. One such issue lies in the escalating focus on advancing Science, Technology, Engineering and Mathematics (STEM) education both nationally and internationally, with numerous reports, policy documents, and media coverage dominating the landscape (e.g., Honey, Pearson, & Schweingruber, 2014; Marginson, Tytler, Freeman, & Roberts, 2013; *National Innovation and Science Agenda*, 2015; National Science and Technology Council, 2013; Office of the Chief Scientist, 2014). Perspectives on what STEM education entails and how it should be implemented vary widely, creating new tensions between the official curriculum and the operational curriculum with respect to each of the disciplines. For example, what is recommended by industry and political leaders might not necessarily align with existing curriculum documents, nor might each of the disciplines be given equitable attention in STEM debates. Furthermore, schools wishing to develop innovative STEM programs might face obstacles from educational authorities bound to the official curriculum. Revisiting Way et al.'s discussion on the official and operational curriculums provides a backdrop for addressing some of the challenges (and indeed opportunities) mathematics education faces in the current STEM climate.

2 The Official Curriculum and Operational Curriculum

The official or mandated curriculum is not as absolute as implied in Remillard and Heck's (2014) model. Rather, the curriculum is in a constant state of flux due to the impact of various political, social, and cultural factors. Way et al. indicate how the official curriculum is often a “political tool”, viewed as a means of ensuring the social and economic growth of society as well as for improving student performance (Walshaw & Openshaw, 2011). As noted previously, because the development of the official curriculum is subjected to forces stemming from political cycles of government, recommended changes in curriculum might not necessarily align with what mathematics educators deem important in advancing students' mathematics learning. At the same time, given that curriculum policy is often strongly influenced by perceived declining student performance, national and international testing programs can become powerful levers for curriculum change; not necessarily the most desirable change. A recent Research Committee's report featured in the *Journal for Research in Mathematics Education* (Herbel-Eisenmann et al., 2016) further indicated how the viewpoints and decisions of policymakers are often shaped by outside organisations including the media, various companies, and other academic areas. Some of these organisations promote media storylines that conflict with the research undertaken by mathematics educators, resulting in a lack of consensus on how mathematics and mathematics education should be portrayed to the broader community.

These socio-political factors can impact on how the official curriculum is implemented, which could result in an operational or enacted curriculum that often exhibits features extending beyond what is mandated. Residing within this operational curriculum are factors that impact on students' learning including the nature and extent of teachers' disciplinary and pedagogical knowledge, how students and teachers interact, the resources used, and the mathematical content and learning experiences provided. It is thus not surprising that the operational curriculum has the greatest potential for influencing students' achievements, and is where the bulk of the research is conducted.

A core message from Way et al.'s review is a warning about the “inconsistencies, mismatches and tensions between the official curriculum and various aspects of the operational curriculum” (p. 16). Such a warning is timely, given the changing political perspectives on what is needed in advancing education across the board, especially with respect to STEM education, coupled with the desire to improve national and international assessment outcomes. For example, the Australian Government's *Review of the Australian Curriculum* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2015b) cited “uncrowding” and “rebalancing” of the primary curriculum as two key objectives in improving the curriculum (ACARA, 2015b, p. 1). As a consequence, the endorsed and improved *Australian Curriculum* was launched in September, 2015 (ACARA, 2015a). At the same time, competing forces emerged from national and international calls for advancing STEM competencies to promote innovation, productivity and overall economic growth. Potential tensions can thus arise as mathematics educators attempt to implement the improved *Australian Curriculum* in the face of increased demands from political, business and industry leaders to increase STEM achievements.

3 The STEM Spotlight

Australia's current focus on increasing STEM outcomes reflects the escalating international concerns for advancing the field. In the United States for example, the 2013 report from the Committee on STEM Education stressed that "The jobs of the future are STEM jobs", with STEM competencies increasingly required not only within, but also outside of, specific STEM occupations (National Science and Technology Council, 2013, p. vi). Developing skills in the STEM disciplines is thus regarded as an urgent course of action in many education systems, fuelled in part by perceived or actual shortages in the current and future STEM workforce (e.g., Caprile, Palmen, Sanz, & Dente, 2015; Charette, 2013; Hopkins, Forgasz, Corrigan, and Panizzon, 2014; The Royal Society Science Policy Centre, 2014). Outcomes from international comparative assessments (e.g., OECD, 2013a, 2013b) have further sparked the STEM urgency seen across many nations.

Within Australia, the long anticipated *National STEM School Education Strategy, 2016–2026*, was unveiled in December 2015 as a comprehensive plan for the nation's STEM education. Designed to build upon and better coordinate the existing range of reforms, the document emphasises the significance of the *Strategy*:

> A renewed national focus on STEM in school education is critical to ensuring that all young Australians are equipped with the necessary STEM skills and knowledge that they will need to succeed. (Education Council, 2015, p. 3)

This renewed focus is intended to lift foundational skills across the STEM disciplines, develop mathematical, scientific and technological literacy, and promote the growth of 21st century problem-solving skills including creative thinking and critical analysis. The need to commence with the early school years and continue throughout the levels of education, as emphasised in the *Strategy*, was highlighted in a subsequent press release by the Federal Minister for Education and Training, the Hon. Simon Birmingham (2015, December 14):

> Developing an early interest in subjects like science, maths and IT will help school students prepare for life and work beyond school. We need to do more and we need to do it differently to encourage more young students to engage with science, technology, engineering and maths subjects.

Earlier in 2015, the Australian Industry Group expressed similar recommendations in its report, *Progressing STEM Skills in Australia* (AIG, 2015). Included in the report's key points were the importance of STEM skills for the workforce and the competitiveness of the national economy, the urgency to tackle our students' under-performance in STEM compared to nations that perform well, and the need to "develop more engaging school curriculum and pedagogy to attract students to STEM" (AIG, 2015, p. 6). Calls for increasing the pool of qualified STEM teachers were also included in the report. Given these industry recommendations, coupled with those of the *National STEM School Education Strategy, 2016–2026*, opportunities for advancing mathematics education could be both enriched and diminished. Obviously we aim for the former and thus need to ensure that the profile of mathematics is neither weakened nor overshadowed by the other STEM disciplines, especially science.

4 Mathematics Within the STEM Spotlight

Fitzallen (2015) expressed succinctly the above points in her Mathematics Education Group of Australasia (MERGA) research paper:

> The emphasis on science, technology, engineering, and mathematics (STEM) education in recent times could be perceived as business as usual or as an opportunity for innovation and change in mathematics classrooms. Either option presents challenges for mathematics educators who are expected to contribute to the foundations of a STEM literate community. (p. 237)

In highlighting the many reports that claim STEM provides contexts for fostering mathematical skills, Fitzallen pointed out that these reports do not acknowledge the reciprocal relationship between mathematics and the other STEM disciplines. That is, the ways in which "mathematics can influence and contribute to the understanding of the ideas and concepts of other STEM disciplines" (p. 241) are not being addressed. Numerous researchers have argued for increasing the spotlight on mathematics especially when science seems to dominate the STEM landscape (e.g., English, 2015; English & Kirshner, 2016; Marginson, Tytler, Freeman, & Roberts, 2013). As Marginson et al. (2013) noted, many nations refer to the role of STEM education as one that fosters "broad-based scientific literacy" with a key objective in their school programs being "science for all" with increased efforts on lifting science education in the primary, junior, and middle secondary school curricula (p. 70). Interestingly, Marginson et al. pointed out that STEM discussions rarely adopt the form of "mathematics for all" even though mathematics underpins the other disciplines. They thus argued that "the stage of mathematics for all should be shifted further up the educational scale" (p.70).

Foundational to discussions on how the profile of mathematics might be raised are the various perspectives on what STEM education entails and hence the research needed. It is not the intention of this chapter to address the various perspectives on STEM, as there are already numerous articles addressing this issue (e.g., Bryan, Moore, Johnson, & Roehrig, 2015; Charette, 2014/2015; Vasquez, Sneider, & Comer, 2013). The perspective of the STEM Task Force Report (2014) in the US, however, is worth noting, especially given its focus on mathematics as an integral component of each of the other STEM disciplines. The Report maintains that STEM education is far more than a "convenient integration" of its four disciplines, and that the disciplines "cannot and should not be taught in isolation, just as they do not exist in isolation in the real world or the workforce" (p. 9). STEM education from this perspective encompasses "real-world, problem-based learning" that integrates the disciplines "through cohesive and active teaching and learning approaches" (p. 9). In defining each of the disciplines from an integrated perspective, the Report defines "mathematically literate students" as not only knowing "how to analyse, reason, and communicate ideas effectively", but also being able to "mathematically pose, model, formulate, solve, and interpret questions and solutions in science, technology, and engineering" (p. 9).

5 Promoting Mathematics Education Research Within STEM: A Focus on Problem-Solving and Modelling

The chapters of this book offer significant insights into how we might lift the profile of mathematics across the STEM landscape, given the potential tensions and inconsistencies that can arise between the official and operational curriculums. Not only do we need to ensure that mathematics receives the attention it deserves within our STEM climate, but also that our students are provided equitable opportunities to develop the mathematical literacy for successful participation in their current and future worlds. In offering recommendations for addressing these concerns, I have chosen to consider a few ways in which we might promote more effective problem-solving and modelling, taking into consideration some of the learning and teaching issues raised in this book.

Included in several chapters are calls for increasing students' competencies in mathematical problem-solving, modelling, and reasoning processes. MacDonald et al. (Chap. 9, this volume), for example, highlight research demonstrating the capabilities of young learners in innovative problem-solving, while Stillman et al. (Chap. 14, this volume) illustrate the advances that have been made in modelling and applications ranging from innovations in pedagogy through to developments in theory and methodological tools. Hunter et al. (Chap. 11, this volume) report on a range of innovative and powerful pedagogical practices that can advance learning and problem-solving, as well as promote more equitable outcomes for students with diverse needs and backgrounds.

Given that problem-solving and modelling are contentious and complex domains, recommendations for their advancement both within mathematics and across STEM fields remain challenging. With the diverse range of learning contexts, one set of recommendations will not necessarily apply to all education systems. Elsewhere (e.g., English & Gainsburg, 2016) I have considered some implications for fostering problem-solving and modelling drawing on studies of the competencies required by 21st-century work and life. These studies revealed that such problem-solving requires:

- A substantial and flexible grasp of foundational mathematical ideas and processes;
- General skills that are of a cognitively high level;

- An understanding of conceptual models that underlie processes or systems, which in turn requires the ability to interpret complex representations within given contexts;
- The ability to interpret quantitative data in different complex forms in unfamiliar, multiple domains;
- The ability to solve a range of novel problems.

In light of these broad competencies, I offer some suggestions for advancing problem-solving including targeting content as well as process through idea-generating problems; promoting in-depth content understanding; developing general skills and processes; and fostering interdisciplinary modelling. In considering these aspects, I also touch upon research addressing equity, motivation and engagement, and social justice.

5.1 Developing Content and Process Through Idea-Generating Problems

Many decades of debates have taken place regarding whether we should teach problem-solving per se or teach mathematics *through* problem-solving; not surprisingly, results have been inconclusive. My perspective is that both aspects should be addressed, although this is not implying that all mathematical content should be taught through problem-solving. Designing problems that are sufficiently cognitively demanding to foster both significant mathematical content and effective problem-solving capabilities would appear a powerful way of tackling this issue. Lesh and Zawojewski (2007) argue that such problems should encourage students to "develop a more productive mathematical way of thinking about the given situation" (p. 782). The focus of problem-solving then becomes one of learning or idea generation, rather than simply the application of problem-solving processes or strategies. Situating students at the centre of their learning where they are encouraged to engage with meaningful yet challenging problematic situations can lead to the application of higher levels of cognitive reasoning, as Hunter et al. indicated in Chap. 11.

Idea-generating problems that are cognitively challenging not only encourage high-level thinking and reasoning, but also offer multiple entry points, and enable students to use varied solution approaches. Furthermore, as Silver, Mesa, Morris, Star, and Benken (2009) indicated, problems with high cognitive demand require students to explain, describe, and justify; make decisions, choices, and plans; formulate questions; apply existing knowledge and create new ideas; and represent their understanding in multiple formats. Likewise, the research of Sullivan et al. (e.g., Sullivan, Clarke, Cheeseman, Mornane, Roche, Swatzki, & Walker, 2014; Sullivan & Davidson, 2014), cited in the chapters by Attard et al. and Hunter et al. (Chaps. 5 and 11, this volume), document the importance of cognitively demanding tasks where "sustained thinking" and argumentation are fostered (Sullivan & Davidson, 2014, p. 606). Exposing students to such cognitively rich problems can empower a wider range of students to "participate as mathematicians and engage in interpreting and communicating mathematical ideas" (Chap. 11, this volume, p. 4). As Sullivan et al. (2014) found as part of a large study, students appeared more engaged with challenging classroom tasks, preferring to persist with such tasks prior to intervention by the teacher.

Approaches to improving content and processes through idea-generating problems also need to take into account important social justice issues. For example, Vale et al. (Chap. 6, this volume) report on a study by Atweh and Ala'i (2012) where efforts to implement "Socially Response-able Mathematics" activities were hampered by teachers' reluctance to use "open ended pedagogies" (p. 103). Their study revealed that when teachers use such approaches, in contrast to direct teaching, students invariably demonstrate a "deeper understanding and engagement in the class" (Atweh & Ala'i, 2012, p. 103). Alleviating reticence to implement more challenging, idea-generating activities would seem a core plank in our efforts to promote all students' learning across the STEM disciplines.

5.2 Promoting In-Depth Content Understanding

In targeting both content and process in idea-generating problems, efforts to develop deep conceptual understanding can be hampered by an overriding focus on national and international test achievements. As Serow et al. (Chap. 12) point out in their chapter on assessment of mathematics learning, there appears to be a mismatch between ACARA's stated objectives and national testing that assesses "some fairly conventional mathematical knowledge in straightforward ways" (p. 5). Unfortunately, although our national assessment items are rigorously trialled and validated, they are not adequate on their own for providing a sound basis for the mathematical understandings and skills required for the 21st century.

Interestingly, the studies reviewed by English and Gainsburg (2016) indicated that many of the problems arising in work and life only require basic mathematics, but importantly, this knowledge needs to be used and applied far more fluently than it is today. There appears to be the need to enrich students' understanding of topics such as algebra, geometry, statistics, and data analysis, and to develop their skills in applying this understanding to a variety of mathematical and other STEM-based authentic problems.

Research by Hoyles and her team (e.g., Hoyles, Noss, Kent, & Bakker, 2010) on the problem-solving needed by mid-level workers in technology intensive settings found, among others, that a facility with graphs, charts, spreadsheets, and computer simulations was paramount. Their findings demonstrated the importance of understanding the conceptual models underlying real-world processes and the ability to generalise, to some extent, deep conceptual knowledge. These aspects appeared more efficacious in promoting problem-solving ability, at least within a given domain, than shallower, situation-specific procedural knowledge. Hence a key recommendation for increasing the application of mathematics across the STEM domain would appear to be the development of in-depth understanding of underlying principles and concepts, whatever the content and context. Statistical reasoning and a facility with a range of data representations emerged as key areas in need of greater attention.

Another interesting facet from workplace studies, as evident in the sentiments of employers and observations by workplace ethnographers, is the impact of employees' learning while on the job. For example, successful engineers, scientists, and technology personnel use mathematics to better understand the systems that are at the core of their work, while at the same time refine their mathematical or quantitative "tools" for future problem-solving. It is thus recommended that students be made aware of this important learning cycle observed in the work of STEM personnel. General skills and processes form a significant component of "learning while on the job."

5.3 Fostering General Skills and Processes

The importance of generic skills and processes including metacognition is underscored by several authors in this book, including Stillman et al. and Geiger et al. (Chaps. 13 and 14). Implications from their reviews align with recommendations from various employer groups on the broad skills and processes required for effective problem-solving. Although perspectives on what is required do vary considerably, they do share common features. Some of the frequently cited employer-desired skills and processes that have been identified by the *Partnership for 21st-Century Skills* (2015) appear particularly pertinent to STEM education. These processes include effective reasoning, using systems thinking, making judgements and decisions, and solving different kinds of novel problems in both conventional and innovative ways.

General skills and processes with respect to mathematical problem-solving have received substantial attention over the decades with numerous debates on the effectiveness of teaching strategies and heuristics (e.g., Lesh & Zawojewski, 2007; Lester & Kehle, 2003). It is beyond the scope of this chapter to address these various debates. However, it is worth acknowledging the important role of metacognition, with research indicating that more sophisticated levels of self-awareness and explicitness about strategies are associated with greater success in solving problems (Kapa, 2001; Schneider & Artelt, 2010). Over the years, numerous instructional interventions have been developed and implemented to enhance metacognition as one means of improving problem-solving competence (e.g., Goos, Galbraith, & Renshaw, 2002; Stillman & Galbraith, 1998). Metacognition is thus being increasingly recognised as playing a critical role in successful problem-solving and modelling, both within and beyond the curriculum including in workplace settings (e.g., Chap. 14, this volume;

Lester, 2013; Pellegrino & Hilton, 2012). Interesting advances on earlier studies on metacognition are discussed in Stillman et al.'s chapter (Chap. 14), together with Geiger et al.'s (Chap. 13) reporting further on these developments.

One such advance is the notion of "anticipatory metacognition," as addressed in Stillman et al.'s chapter. Adding a new direction to the existing work on metacognition, this notion holds considerable promise for advancing mathematics across the STEM landscape. Anticipatory metacognition includes Galbraith's (2015) concept of "noticing" when one is engaged in modelling as part of real-world problem-solving. Rather than just "looking back" on actions that have been taken in solving a problem, the problem-solver looks forward to potential cognitive actions that might be feasible, desired, or even essential. Such anticipatory metacognition encompasses the "mathematical, cognitive and physical resources necessary to mathematise real-world situations into mathematical models" (Chap. 14, p. X). As such, fostering anticipatory metacognition could potentially enhance students' competencies in modelling across STEM contexts.

5.4 Advancing Modelling Across STEM Contexts

The importance of understanding the underlying models that are represented mathematically and technologically is crucial in many fields, including engineering, finance, manufacturing, and agriculture. Political debates on how national and state economies might be restructured to address budget deficits, for example, draw upon modelling to support certain points of view. The foundations of this modelling, however, including key assumptions, context, and methodology, are also open to debate. As Gittins (2016), economics editor of the *Sydney Morning Herald*, warned: "The lesson for the economic profession is that the modelling they value so highly is too often being used by other economists to mislead rather than enlighten. The reputation of models and modellers is being trashed, and with it the profession's credibility" (p. 26). One of the goals of promoting modelling across STEM contexts should be developing students' appreciation of models and modelling processes and how these can both inform and misinform society.

The notions of models and modelling have been interpreted variously in the literature, as Stillman et al. explore in detail in Chap. 14. While not elaborating further on these various interpretations, I maintain that modelling is a powerful vehicle for bringing features of 21st-century problems into the mathematics classroom. In adopting this stance, I align with Stillman et al.'s framing of their research reviews, namely, a "modelling-as-content" perspective, or as Galbraith (2013) described, "modelling as real world problem-solving." This approach aims to develop students' skills in using mathematics in a range of contexts, whether it be their current or future workplaces, their personal lives, or within the broader community.

In fostering our students' understanding of, and competence in, modelling "real-world" problems, we need to consider how contexts might be selected to approximate "authentic" problems. Galbraith's (2013) four dimensions of authenticity, as noted in Chap. 14 (this volume) are worth revisiting given their relevance to modelling problems across STEM contexts.

Content authenticity The problem comprises genuine real-world links and is within reach of students' mathematical knowledge.
Process authenticity The problem engages students in valid modelling processes.
Situation authenticity The task requirements drive the problem-solving activity not vice versa.
Product authenticity The solution can be justified mathematically and appropriately addresses the real-world problem.

Of the numerous interpretations of modelling, one form I have implemented across the primary and middle school years is that of model-eliciting activities (MEAs), drawing upon the extensive research of Lesh et al. (e.g., English, 2010; Hamilton, Lesh, Lester, & Brilleslyper, 2008; Lesh & Doerr, 2003; Lesh, Zawojewski, & Carmona, 2003). Definitions of models and modelling have varied over the years, however. I have typically considered a model to be a "system of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behaviour of some other familiar system" (Doerr & English, 2003, p.112). Lesh and Fennewald (2010) offered a more succinct definition, namely, "A model is a system for describing (or explaining, or designing) another system(s) for some clearly specified purpose" (p. 7). Both definitions are especially germane to fields beyond mathematics education, including engineering and other mature science domains. In addition to meeting Galbraith's (2013) authenticity dimensions, MEAs foster the types of general skills that employers demand in the workplace and that citizens need for maximum societal participation. As previously noted, such skills include critical and innovative thinking, complex reasoning, metacognitive actions, and collaboration and communication within and across disciplines.

MEAs focus on the processes of interpretation and re-interpretation of problematic information, and on the iterative development of mathematical ideas as models are formed, tested, and refined in response to certain specifications. This design encourages students to engage in anticipatory metacognition (Galbraith, 2013) and "implemented anticipation" (Niss, 2010) as explored in Stillman and Brown (2014). For example, as students consider the problem constraints (usually in the form of a client's requirements in an MEA; e.g., Lesh & Zawojewski, 2007) and engage in iterative processes towards a solution, they anticipate mathematical ideas and actions that might be useful in progressing towards model completion.

These modelling problems provide rich opportunities for addressing the reciprocal relationship between mathematics and the other STEM disciplines, as Fitzallen (2015) highlighted. Students are encouraged to create, apply, and adapt mathematical and scientific concepts in interpreting, explaining, and predicting the behaviour of real-world based problems such as those that occur in engineering (e.g., Gainsburg, 2006). The wide range of STEM contexts addressed by MEAs and other forms of modelling facilitate the application of mathematical ideas and processes to the other disciplines. For example, the environmental engineering activity, the *Water Storage Problem* (English & Mousoulides, 2011), which was implemented in classes of 11-year-olds in Cyprus, requires students to interpret and analyse different forms of data. Students might choose to sort, organise, select, prioritise, quantify, weight, and transfer data sets.

The *Water Storage Problem* commences with students being "sent" a letter from a client, the Ministry of Transportation, who requests a model for selecting a country that can supply Cyprus with water during the next summer period. The letter asks students to develop a suitable model using the given data, as well as search for additional information using available tools such as Google Earth, maps, and other web-based resources. The quantitative and qualitative data provided for each country includes water supply per week, water price, tanker capacity, and the facilities of the ports. Students can obtain further data on distance between countries, major ports in each country, and tanker oil consumption. Students conclude the problem by writing a letter to the client detailing how their model selects the most suitable country for supplying water. As an extension of this problem, students are given a second letter from the client including data for two more countries and are asked to test their model on the expanded data and, if required, improve their model.

The environmental engineering context of the *Water Shortage Problem* is an authentic one for the students in Cyprus, where water has been rapidly drying up since the 1970s. The lack of drinkable water in Cyprus is a major problem, with water supply to homes limited. The water issue features prominently in the Cyprus media and is thus an authentic problem for all members of the community, including students, as the solution can be hindered by conflicting political agendas.

The important role of mathematics in this problem was evident not only in students' model development but also in their consideration of environmental and socio-political issues when deciding on a final model. For example, one student group was not satisfied with the model they had created because they were concerned about sea pollution, which they discussed extensively. Based on a newspaper article they had studied during the first session of the modelling activity, one student raised the question of whether it would be wiser to buy water from Greece. He mentioned that the distance from Pireus to Limassol was more than three times greater than the distance from Lebanon and Syria, and proposed to buy water from Egypt or Syria, the second and third country in distance ranking. The group also documented in their reports that all countries in the Mediterranean Sea should be fully aware of sea pollution and therefore try to minimise ship oil consumption. Another student member suggested buying water from Syria, since water price was not that expensive (compared to the price of buying from Greece and Egypt). The students finally ranked countries as Syria, Egypt, Lebanon, and Greece, and decided to propose that the local authorities buy water from Syria.

Another student group was worried about the port facilities factor, a component that some student groups chose to ignore in the models they generated. The group decided to quantify the port facilities factor and integrate their calculations within the port facilities data. A subsequent discussion focused on the finance needed for improving the ports' facilities and how this amount of money would change the water price per ton. To assist them here, the students asked for more information about the costs for improving port facilities in Syria, Lebanon, and Egypt. They were surprised when they learned that improving the ports' facilities would cost from five to ten million euro. This feedback prompted concerns regarding socio-economic considerations.

As the group progressed in their model development, they debated issues regarding tanker capacity and oil cost, and how these factors might relate to their solution, looking beyond the terms of the mathematical relations. The students were aware of energy consumption issues, and discussed how oil consumption should be kept as minimal as possible. When their teacher prompted them to decide which factor was more important, water price or oil consumption, the students replied that it would be better to spend a little more money and to reduce oil consumption. The group also made explicit that it was not only oil consumption but also other environmental issues, like the pollution of the Mediterranean Sea, which needed to be considered. The group's final model proposed Syria as the most suitable place from which to buy water, since its costs were quite reasonable and it is the least distance from Cyprus.

The *Water Shortage Problem* is just one of many modelling problems that can serve to increase the profile of mathematics across STEM contexts. Furthermore, an important feature of these modelling problems is that students of varying school mathematics achievement levels and personal backgrounds can engage with, and succeed in, solving the problems, albeit at different levels of sophistication (English, 2016). The insights gained into students' mathematical thinking and their abilities to generate STEM concepts beyond their grade level would not be achieved through national and international assessments. In addition, the interesting STEM contexts within which the problems can be couched appeal to a wide range of students who might otherwise be disengaged when dealing with traditional mathematics problems.

6 Concluding Points

In completing this final chapter, I attempted to draw upon as many of the interesting findings from the research reviews as I could within the framework I adopted. There are numerous other issues raised in each of the chapters that I would have liked to have addressed. This omission in no way dismisses their significance in advancing mathematics education research within a STEM environment. Collectively, the authors have presented comprehensive reviews of Australasian research in mathematics education during 2012–2015, and have provided key implications and recommendations for our future research endeavours. In closing I raise only a few of the many areas I consider worthy of further attention in mathematics education research.

6.1 Rebalancing the Focus on National and International Assessment

One of the challenges our community faces is dealing with national and international assessments. How we might strike a more acceptable balance between a focus on our students' mathematics performance on these tests and their development of broader mathematical competencies that incorporate 21st Century Skills? In particular, we need to investigate ways in which we might effectively reduce the tendency for national and international assessments to become the primary levers for learning in the operational curriculum, and enable them to play a more supportive role. For example, how might we capitalise on and extend national assessment items, such as those involving statistical representations, and incorporate them within modelling and problem-solving experiences?

Of particular concern, though, in rebalancing the focus on testing are issues pertaining to the inclusive practices in mathematics education as examined in Faragher et al.'s chapter (Chap. 7, this volume). In citing Grootenboer and Sullivan's (2013) study, in which instruments were developed for assessing Indigenous students within their own contexts, it was revealed that the apparent under-achievement of these students in formal tests "may be due to the relevance and veracity of the assessment instrument" (Grootenboer & Sullivan, 2013, p. 181). Grootenboer and Sullivan's warnings are especially worth noting, namely, "there are real concerns about national testing regimes that discriminate against some students, and the use of these flawed results to make claims about the students' mathematical (or other subjects) knowledge and understandings" (p. 184).

6.2 Lifting the Profile of Mathematics Across the STEM Landscape

As discussed in this chapter, the increased focus on STEM education has generated concerns regarding the presence and role of mathematics. I have argued for the need to lift the profile of mathematics across the STEM landscape and have explored problem-solving and modelling as one means of achieving this. Statistical reasoning has featured prominently in the modelling experiences I have implemented in schools. Dealing effectively with statistics is essential across all the STEM disciplines, where a facility in handling uncertainty and data is central to making evidence-based decisions involving ethical, economic, and environmental dimensions (Office of the Chief

Scientist, 2013). The increasing need to handle contradictory and potentially unreliable online data is also critical (Lumley & Mendelovits, 2012). Given that many nations are striving to achieve social, cultural and economic prosperity in dealing with a rapidly changing and insecure world, greater recognition needs to be given to the foundational role of mathematics, in particular working with data, in building the required knowledge base.

6.3 Developing 21st Century Skills

Of the four broad areas of employer-desired skills identified in *The Partnership for 21st-Century Skills* (2015) document, learning and innovation are especially relevant to promoting mathematics education within a STEM climate. These skills are further subdivided into three categories: creativity and innovation, critical thinking and problem-solving, and communication and collaboration, all of which I consider worthy of further attention from our mathematics education community. An increased focus on critical thinking, various forms of reasoning, systems thinking, and the making of informed and evidence-based judgements and decisions would seem especially required. Likewise, with the *Programme for International Student Assessment (PISA) 2015 Draft Collaborative Problem-solving Framework* (OECD, 2013b, p. 13), there emerges further areas for attention although many of the skills and problem-solving competencies and contexts identified in the PISA document have already been explored in the present chapters. The collaborative perspective on problem-solving, however, raises further interesting research agendas.

6.4 Targeting Computational Thinking and Coding

The international push for developing students' computational thinking and coding from the earliest grades calls upon our discipline to play a greater role in this development. There appear clear links between early coding, for example, and mathematics learning in the preschool and beginning school years. Developing young children's coding skills incorporates among others, sequencing, pattern recognition, deductive reasoning, numerical reasoning, data structures and representations, and functions (Liukas, 2015). Establishing such foundational links between early coding and mathematics learning appears not to be receiving the required attention and is clearly an area demanding substantial research.

Many avenues for research await our mathematics education community not only in this domain of computational thinking and coding but also in many others. It will be interesting to see the themes addressed in the next *RiMEA* book, and ways in which the research landscape might have changed during this review period. I will not attempt to anticipate what these changes might be, except to wish that research will have facilitated ways to increase access to mathematics education for a wider range of students. It is also hoped that there is a greater community awareness and appreciation of mathematics in its foundational roles across the STEM domain.

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