Particle Swarm Optimization and Schelkunoff Unit Circle to Minimize the Interference Near to the User

Smita Banerjee and Ved Vyas Dwivedi

Abstract This paper analyses the mathematical design of a linear antenna array in overcoming the problem of interfering signal near to the user signal. A mathematical modelling of modified version of Schelkunoff polynomial method with Particle Swarm Optimization has been presented. The radial displacement and phase on Schelkunoff unit circle are fixed for maintaining the direction of user and interferer. Reduction of sidelobe level constraint is done by searching the best location of phases. Parameters like sidelobe level and directivity have been considered in showing the usefulness of this technique. Effectiveness and limitations of placing nulls near to the main beam have been shown by relevant examples through variation of interferer positioning.

Keywords Linear antenna array · Schelkunoff unit circle · Particle swarm optimization · Sidelobe level · Directivity · First null beam width

1 Introduction

The interference can be rejected or suppressed by either putting low or minimum gain towards that direction or reducing the sidelobes. Sidelobes causes degradation of the actual signal and hence reduce the efficiency of the antennas. Placing null in the sidelobe region can reduce the effect of interference. However, this will lead to an increase of the sidelobe level (SLL). This increment of SLL will depend upon the number of interferers and their locations. If the interferers are near to the main beam, then it will directly affect the beam width: this condition needs to be further studied [1–4].

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R.K. Choudhary et al. (eds.), *Advanced Computing and Communication Technologies*,
Advances in Intelligent Systems and Computing 452,
DOI 10.1007/978-981-10-1023-1_58

Different pattern synthesis techniques such as adjusting the element position, inter-element spacing, amplitude excitation, phase excitation and complex excitation are available in the literature are useful to cancel the undesired interferers [5, 6]. Selection of the parameters alone or combination is cumbersome to achieve desirable properties. Hence, the synthesis problems are highly non-linear in nature and need to be solved using non-linear optimization algorithm [7–11].

The analytical method of synthesis using Schelkunoff Polynomial Method (SPM) was developed to put nulls in the desired direction. The main beam location and the main beam width (also known as first null beam width-FNBW) will depend upon the number of nulls and their location [12]. Several studies of SPM for the synthesis of antenna array have been reviewed [13–19]. The characteristics of the main beam have been synthesized using the largest degree of the sub-polynomial in [13]. From the specific nulls along with target points, the radiation pattern is synthesized to minimize the error between the desired and optimized radiation pattern [14]. The beamforming approach using the null points and target points is proposed in [15, 16]. Design of conformal antenna array and the effect of various parameters on the radiation pattern is discussed in [17]. By dividing the polynomial into different sets and controlling one of the sub-polynomials, the number of optimized parameters required were lesser than that of the classical complex synthesis method [18]. From a given number of antenna element and number of nulls, the radiation pattern is synthesized for reduced SLL [19].

In this study, modified SPM (MSPM) is used to put main beam and null along the specific direction by fixing the radial position and phase on the Schelkunoff unit circle (SUC). Other left over roots are taken as the optimization parameters in Particle Swarm Optimization (PSO) with cost function to reduce the SLL near to the main beam. This paper offers a detailed comparative study of the available polynomial techniques used to put the null in the desired direction. The parameters for comparison are maximum SLL, mean SLL, directivity, number and position of the interferer keeping direction of main beam and FNBW constant.

The rest of the paper is arranged as follows: Sect. 2 describes the mathematical analyses of SPM for linear antenna array, Sect. 3 describes the design procedure using PSO, Sect. 4 compares the results and Sect. 5 concludes the whole study.

2 Mathematical Analysis of Schelkunoff's Polynomial Method for Linear Antenna Array

Consider a linear array of N radiating antenna elements which are equally separated by a distance d arranged along a line. Mathematically, the AF is given by [1],

$$AF(\theta) = \sum_{n=1}^{N} w_n e^{j(n-1)\Psi} \tag{1}$$

where $w_n = a_n * \exp(jb_n)$ = complex array weights at each antenna element, a_n = amplitude weight and b_n = phase weight, $\Psi = kd\sin\theta + \beta$ = phase variation due to time delay between the elements, $k = 2\pi/\lambda$ = wave vector which specifies the variation of the phase as a function of position, $\theta =$ incidence angle w.r.t. array normal and $\beta =$ progressive phase.

SPM is the classical approach for the synthesis of antenna array based on the number and location of interferers. The advantage of this method lies in its placement of minimum gain or nullification of the interference from undesirable directions [12]. Apply Euler's relation $z = x + jy = e^{j\Psi} = e^{j(kd\sin\theta + \beta)}$ and rewrite Eq. (1) in terms of z as:

$$AF(z) = \sum_{n=1}^{N} w_n z^{(n-1)} \tag{2}$$

From the Fundamental Theorem of Algebra, for an N element array, the array factor can be viewed as polynomial of degree $(N-1)$ and can be expressed as a product of $(N-1)$ linear terms which represents $(N-1)$ roots

$$AF(z) = W_N(z-z_1)(z-z_2)(z-z_3)\ldots(z-z_{N-1}) = w_N \prod_{n=1}^{N-1}(z-z_n) \tag{3}$$

where $z_1, z_2, z_3 - - - z_{N-1}$ are the roots of the polynomial. $z_n = \exp(j\Psi_n)$; Ψ_n = $kd\sin\theta_n$ is the phase of the nth root. The complex variable z can be rewritten as: $z = |z|e^{j\Psi} = |z|\angle\psi = 1\angle\psi$.

Instead of varying the weights w_n, appropriate placement of all the roots on the SUC is carried out for certain array pattern.

In Modified form of SPM (MSPM), the direction and beam width can be achieved by fixing 2 roots z_{M1} and z_{M2} for main beam out of $(N-1)$ roots. I interferers from undesired direction can be rejected by placing null in the required pattern and fixing the roots z_{In} on the SUC. Rest of the $(N-I-3)$ roots can be used to control the other constraint of the array pattern. The value of $z_{Rn} = \exp(j\Psi_{Rn})$; Ψ_{Rn} are the phase of the nth root. The equation can be written as [18]:

$$AF(z) = w_N \prod_{n=1}^{2}(z-z_{Mn}) \prod_{n=3}^{I+2}(z-z_{In}) \prod_{n=I+3}^{N-1}(z-z_{Rn}) \tag{4}$$

3 Design Procedure

Figure 1 shows the position of the user and the interferer which are placed near to the main beam at the peak of the first and second sidelobes. Following are the steps in the design procedure:

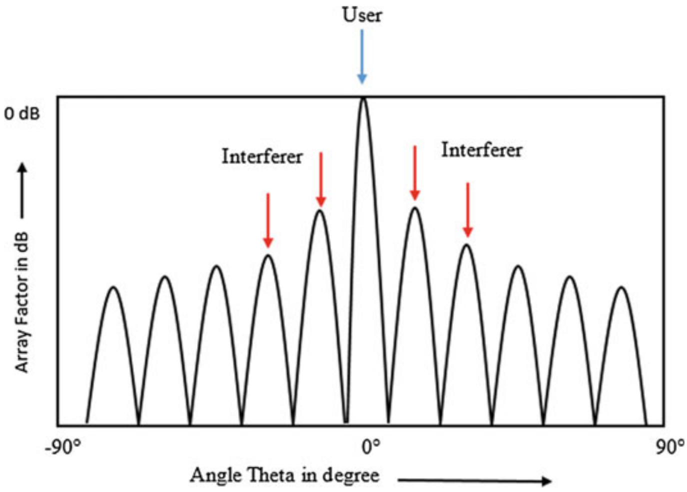

Fig. 1 User and interferer position

Step (1) Specify the size of the array as the number of interferers that can be introduced in the design will be one less than the size of the array.

Step (2) Specify the position of the main beam and calculate the phase of the 2 roots responsible to form first nulls. MSPM fixes the main beam position accurately by placing the phases Ψ_{M1} and Ψ_{M2} on the SUC.

Step (3) Specify the points $\theta_{In}(n = 3, 4, - - I)$ where I is the number of interferers. Calculate the phases of the interferers $\Psi_{In}(n = 3, 4, -I)$. MSPM fixes the interferer position accurately by placing the phases on the SUC.

Step (4) Remaining (N − 3 − I) roots are optimized to search the location of Ψ_{Rn} roots on the SUC. In this work PSO is used to achieve the best array pattern to minimize the effect of interferer near to the main beam.

The following steps shows how PSO is used to search the best Ψ_{Rn} roots [11]

Step (i) Initially a population $(npop)$ of 100 particles are taken at random and the number of iterations $(i\max)$, tuning parameters $(\phi 1 \; and \; \phi 2)$ and weights (w) are set. The $(N - 1 - I)$ roots other than the main lobe and interferer are chosen as the variable $\Psi_{Rn}(i)$ in the optimization problem. Initially the lower $\Psi_{Rn}(i, \min)$ and upper $\Psi_{Rn}(i, \max)$ limit of phase are chosen for the design variable.

Step (ii) Initialize the position for the kth variable in the population by $\Psi_{Rn}(i, k) = \Psi_{Rn}(i, \min) + (\Psi_{Rn}(i, \max) - \Psi_{Rn}(i, \min))u(i)$ where $k = 1, 2, \ldots .npop$ and $u(i)$ is the random number generated between 0 and 1. Initialize the velocities of the kth variable as v(i, k) = 0.

Step (iii) The appropriate fitness function for suppressing the interferer is the reduction of sidelobe level. For each set of possible phase angles, the

SLL is evaluated. And objective function is to minimize the SLL. $FF(\Psi_{Rn}(i,k)) = [SLL\,|\,AF(z,\ \Psi_{Rn}(i,k))|]$.

Step (iv) Compute best fitness of the particle $pbest(i,k) = Fitness$ $function\,(\Psi_{Rn}(i,k))$ and global best $gbest(i,k) =$ min $(pbest(i,\ k))$. The location of $pbest(k)$ and $gbest$ are given by $p(\Psi_{Rn}ik)$ and $g(i\Psi_{Rn})$.

Step (v) Update the velocity $v(i+1,k) = w * v(i,k) + \phi 1(p(\Psi_{Rn}ik) - \Psi_{Rn}(i,k))$ $u(i) + \phi 2(g(i\Psi_{Rn}) - \Psi_{Rn}(i,k))u(i)$ and position $\Psi_{Rn}(i+1,k) = \Psi_{Rn}(i,k) + v(i+1,k)$ for each particle.

Step (vi) From the new position and velocity, update the fitness $FF(\Psi_{Rn}(i+1,k)) = [SLL\,|\,AF(z,\ \Psi_{Rn}(i+1,k))|]$.

Step (vii) If FF $(\Psi_{Rn}(i+1,k)) < pbest(i,k)$ then, $pbest(i+1,k) = FF$ $(\Psi_{Rn}(i+1,k))$. Update $gbest\,(i+1,k) =$ min $(pbest(i+1,\ k))$.

Step (viii) The selection continues until maximum number of iterations is reached. If $i < i\max$, then increment i and go to step (5) or else the solution $gbest\,(i+1,k)$ is the location of the phase angle where minimum SLL is obtained.

4 Numerical Simulation Results

In order to show the effectiveness of this method, a 16 element linear antenna array with $\lambda/2$ interelement spacing is taken. Synthesis using SPM is applied: this is considered as the reference. Therefore the number of nulls that can be placed using SPM is 15. Two cases are studied for different interferer position. The simulation is done using MATLAB.

In case 1, main beam is at angle 0° and 2 interferers are assumed at the peaks of first sidelobe near to the main beam. The two phase angle for the main beam roots z_{M1} and z_{M2} are chosen as Ψ_{M1} = −21.93° and Ψ_{M2} = 21.93° to form a main beam along 0° with first null beam width (FNBW) of 14°. The phase angle for the 2 interferers are Ψ_{I3} = −34.34° and Ψ_{I4} = 31.25° to suppress the gain at the peak of the first sidelobes $\theta_{I3} = -11k\,and\,\theta_{I4} = 10°$. It has been observed that the maximum SLL deteriorates from −11.40 to −4.20 dB. Hence rest of the 11 roots have been considered for optimization to reduce the maximum SLL In case 2, main beam is at angle 0° and 4 interferers are assumed at the peaks of first and second sidelobe. The main direction and main beam width (MBW) are considered same as that of case 1. The phase angle for the 4 interferers required to suppress the interference are Ψ_{I3} = −55.62°, Ψ_{I4} = −34.34°, $\Psi_{I5} = 31.25°$ and $\Psi_{I6} = 55.62°$. The maximum SLL increased from −10.36 to −1.93 dB. Only 9 roots are put in the optimization to reduce the maximum SLL. Figures 2 and 3 shows the synthesized radiation pattern after SPM, MSPM and PSO. Table 1 shows the location of the roots on the SUC and Table 2 shows the computed element complex excitation for the optimized radiation pattern of Figs. 2 and 3.

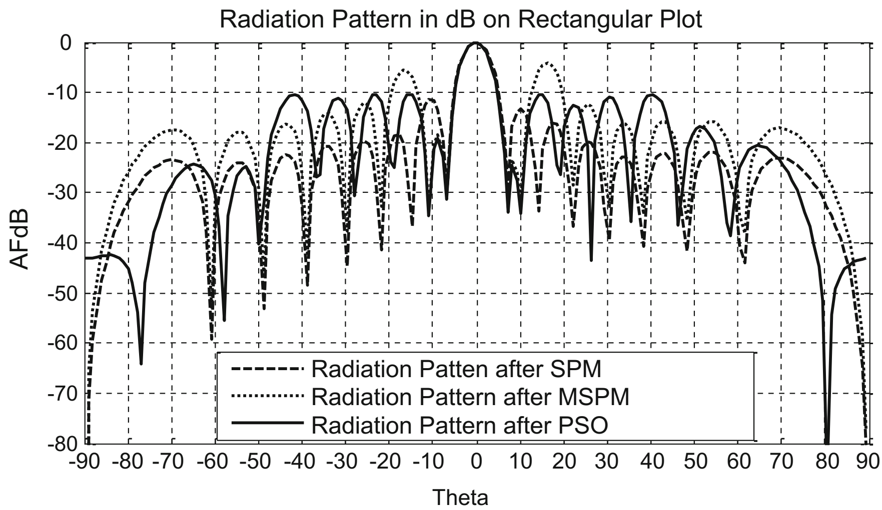

Fig. 2 Best radiation pattern found by PSO for 16 element antenna array with interferer at the peak of first sidelobe

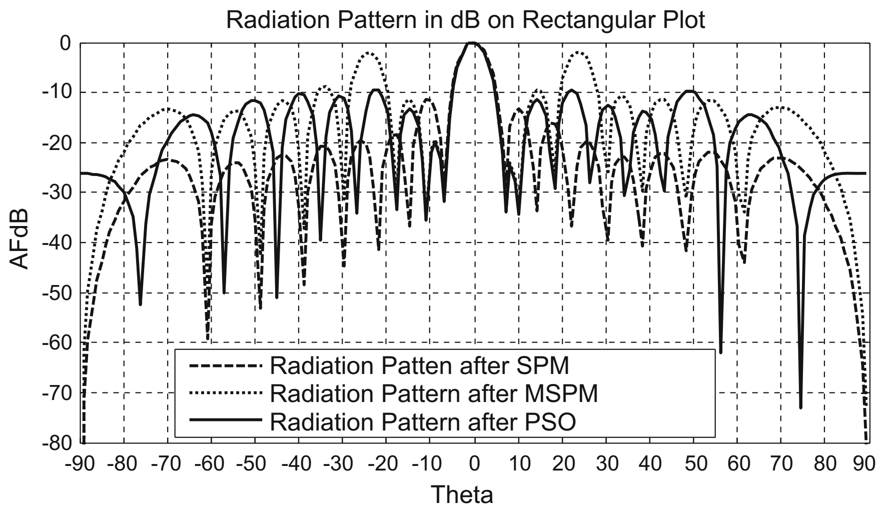

Fig. 3 Best radiation pattern found by PSO for 16 element antenna array with interferer at the peak of first and second sidelobe

Table 3 shows a comparison of SPM, MSPM and PSO. It is observed that as the number of interferers increased from two to four, the maximum SLL deteriorates more after MSPM. An improvement of 6–7 dB in maximum SLL is achieved from the optimized array pattern as compared to the MSPM. This however comes at a

Table 1 Location of roots on the SUC

Root	Radial position	Figure 2		Figure 3	
		Phase θ	Phase Ψ_n	Phase θ	Phase Ψ_n
1	1	−7.00	−21.93	−7.00	−21.93
2	1	7.00	21.93	7.00	21.93
3	1	−11.00	−34.34	−18.00	−55.62
4	1	10.00	31.25	−11.00	−34.34
5	1	−77.17	−175.51	10.00	31.25
6	1	−58.01	−152.67	18.00	55.62
7	1	−49.58	−137.05	−75.11	−173.96
8	1	−36.36	−106.73	−57.78	−152.29
9	1	−27.56	−83.28	−44.68	−126.57
10	1	−19.27	−59.42	−35.23	−103.85
11	1	18.83	58.12	−26.43	−80.13
12	1	26.24	79.59	26.53	80.42
13	1	35.16	103.66	34.48	101.91
14	1	46.62	130.83	43.86	124.72
15	1	57.95	152.57	57.78	152.29

Table 2 Relative complex excitation of each antenna element

Element	Figure 2		Figure 3	
	Amplitude	Phase (°)	Amplitude	Phase (°)
1	1.0000	0.0000	1.0000	0.0000
2	0.8122	0.7724	0.3684	2.5817
3	0.1381	−115.1845	0.0852	−175.9010
4	0.3330	−2.5476	0.4123	5.4397
5	0.5207	−2.5370	0.2662	−3.9045
6	0.4134	−11.4519	0.5865	−10.6377
7	0.6620	−6.5656	0.5541	−3.3381
8	0.8710	−12.2772	0.4985	−3.8959
9	0.6387	−7.7301	0.5874	−3.3358
10	0.8710	−3.1829	0.4985	−2.7757
11	0.6620	−8.8946	0.5541	−3.3335
12	0.4134	−4.0082	0.5865	3.9661
13	0.5207	−12.9231	0.2662	−2.7671
14	0.3330	−12.9125	0.4123	−12.1113
15	0.1381	99.7244	0.0852	169.2294
16	0.8122	−16.2325	0.3684	−9.2533

cost of mean SLL. Compared to SPM, though optimization could not improve the mean SLL, it has been shown to improve the directivity by 3 dB as compared to the initial SPM pattern.

Table 3 Maximum SLL, Mean SLL, FNBW and directivity

Parameters	Value				
	After SPM	Figure 2		Figure 3	
		After MSPM	After PSO	After MSPM	After PSO
FNBW (°)	14.04	14.04	14.04	14.04	14.04
Max. SLL (dB)	−11.40	−4.20	−10.36	−1.93	−9.37
Mean SLL (dB)	−18.61	−13.79	−16.21	−10.82	−13.34
Directivity (dB)	6.97	8.52	9.93	9.08	10.01

5 Conclusion

This paper describes the mathematical design of a linear antenna array using a modified version of Schelkunoff polynomial method with Particle Swarm Optimization. The roots of the main beam are fixed to maintain the position and beamwidth of the user. The roots of the interferer are also kept constant to provide lower value of gain in the undesired direction. The remaining roots are varied to reduce the SLL. It has been observed that as some of the roots are fixed, the complexity of the optimization algorithm reduces as lesser number of variables are used in the optimization. This study shows the performance of null placement and its dependence on maximum and mean SLL, FNBW, directivity, number and position of nulls. The numerical results shows a good performance in terms of directivity and SLL. Simulated result shows successful placement of −30 dB gain towards the interferers at the peak of first and second sidelobes as well as considerable reduction in SLL keeping the beam width constant. Although it is implemented for linear antenna array, but it can be further studied for planar and conformal antenna array. The proposed approach can also be helpful in designing and developing microstrip patch antenna array to change the radiation pattern by changing the amplitude and phase of each of the array element.

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