SIR Model of Swine Flu in Shimla

Vinod Kumar, Deepak Kumar and Pooja

Abstract In this work, an attempt has been made to study the SIR model on swine flu, which produced by viral infection agent in Himachal Pradesh. The SIR model is analyzed on the numerical data obtained from IGMC, Shimla, H.P. This Mathematical model is based on ordinary differential equations for communicable disease. The concept of basic reproduction number R_0 is used to analyze the stages of the disease. The result is helpful for health policy makers and reducing the number of deaths spread from the Swine flu.

Keywords Epidemic disease · Stability analysis · Swine flu · Basic reproduction number · Mathematical modelling

1 Introduction

Swine Flu is highly contagious respiratory disease which "refers to any strain of the influenza of infectious virus, called the swine influenza virus that is epidemic infection in pigs". The signs and symptoms of swine flu are fever with a body temperature over 100 °F, body weakness, cold and runny nose, cough or sore throat, body aches, chills; fatigue, diarrhea and vomiting sometimes, but more commonly seen with seasonal flu. The subtypes of influenza A disease are also known as H1N1, H1N2, H2N3, H3N1 and H3N2. This work is representing that,

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how changes in human behaviour and social mixing influence the epidemic by the study of basic reproduction number, equilibrium points and stability condition [1, 2] (Figs. 1, 2 and 3).

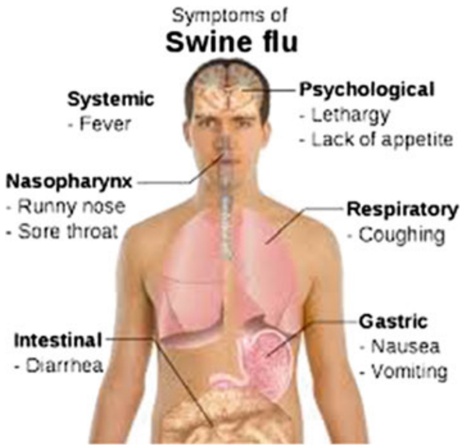

Fig. 1 Symptoms of swine flu

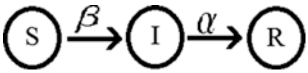

Fig. 2 Structure of SIR

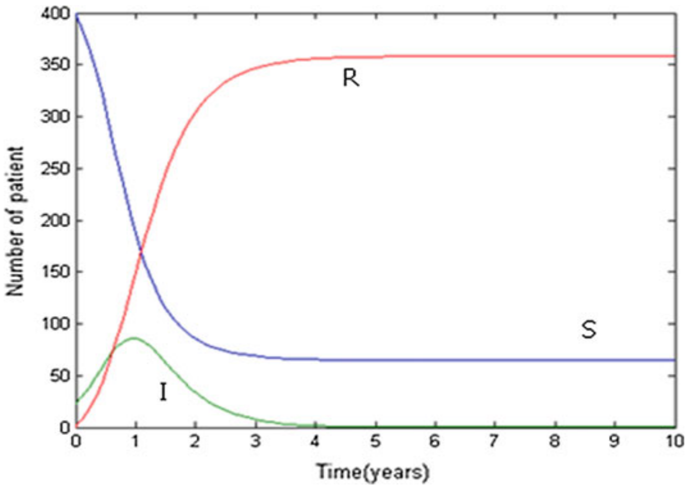

Fig. 3 In this graph, *S* represents the susceptible to the infectious stage with the *blue color*, *I* represent the total number of infected individuals with *green color* and *R* represents the recovery rate with respect to time with *red color*

In the past few months, the number of diagnosed swine flu and influenza A (H1N1) cases has raised, in area of Himachal Pradesh. In this region, the number of confirmed infections has increased, but this only covers a small proportion of the cases which have occurred in that region [3]. The world health organization defined the situation for pandemic from infectious flu and describing an optimal control policy to protect [2]. The application of mathematical modeling on chronic disease through SEIR, SIR and many other models providing the concept of secondary infection and analyzing various stages of epidemic disease with their mathematical ecology in the population [4, 5]. The mechanism of swine flu infection spreading and procedure of providing the treatment is given by a new algorithm [6]. The compartment model SIR discussed by Kermack and Mckendrick in 1927 and many mathematical models are used by many mathematicians to discussed various stages of endemic, epidemic and pandemic diseases with their serious effects on living things [7].

2 Mathematical Model

The SIR model for mathematical epidemiology was proposed by Kermack and Mckendrick. The governing equations on swine flu are given below:

$$\frac{dS}{dt} = -\beta SI \tag{1}$$

$$\frac{dI}{dt} = \beta SI - \alpha I \tag{2}$$

$$\frac{dR}{dt} = \alpha I \tag{3}$$

2.1 Diagram of the Model

2.1.1 Variables and Parameter Description

S = Susceptible class, I = infective class, R = Total recovered, β = rate of transmission, α = rate of recovery.

3 Analysis of the Disease Free Equilibrium

If the individual contains no symptoms of swine flu in the susceptible class. Then the numbers of infective will be zero. i.e. I = 0. This state of the condition in the absence of the infection is said to be as disease free equilibrium in the population. Therefore, we have

$$\beta SI - \alpha I = 0 \Rightarrow S_0 = \frac{\alpha}{\beta} \tag{4}$$

Thus, the disease free equilibrium point is given as (S_0, 0, 0, 0, 0).

3.1 The Basic Reproduction Number

The basic reproduction number is defined for an epidemic SIR compartmental model for the Swine flu. If $R_0 < 1$, then the disease free equilibrium is stable; otherwise, it is unstable for $R_0 > 1$. Mathematically defined as,

$$R_0 = \frac{\textit{The transmission rate}}{\textit{The rate of recovery}} \tag{5}$$

This basic reproduction ratio has important role in determining the rate of infection. Hence, the epidemic outbreak of the disease is stable or unstable and the host will recover or not recover from the disease.

3.2 Equilibrium Analysis of the Disease

At equilibrium,

$$\frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0 \tag{6}$$

$$\begin{aligned} &\text{Case 1:} \frac{dS}{dt} = \frac{dI}{dt} \\ &\Rightarrow S \propto \frac{k_1}{R_0}; \\ &\text{Where } k_1 = \frac{1}{2} \end{aligned} \tag{7}$$

Therefore, the susceptibility increases as the basic reproduction ratio decreases and vice versa.

$$\begin{aligned} \text{Case 2:} \frac{dI}{dt} &= \frac{dR}{dt} \\ \Rightarrow S &\propto \frac{k_2}{R_0}; \\ \text{Where } k_2 &= 2 \end{aligned} \tag{8}$$

Therefore, the susceptibility increases as the basic reproduction ratio decreases and vice versa.

$$\begin{aligned} \text{Case 3:} \frac{dS}{dt} &= \frac{dR}{dt} \\ \Rightarrow S &\propto \frac{k_3}{R_0}; \\ \text{Where } k_3 &= -1 \end{aligned} \tag{9}$$

Therefore, the susceptibility increases as the basic reproduction ratio decreases and vice versa.

3.3 *Stability Analysis of the SIR Model*

Jacobian matrix of the governing equation of the system is given as,

$$J = \begin{bmatrix} -\beta I & -\beta S & 0 \\ \beta I & \beta S - \alpha & 0 \\ 0 & \alpha & 0 \end{bmatrix} . \begin{bmatrix} S \\ I \\ R \end{bmatrix} \tag{10}$$

$$\text{Now, Det(J)} = \begin{vmatrix} -\beta I - \lambda & -\beta S & 0 \\ \beta I & \beta S - \alpha - \lambda & 0 \\ 0 & \alpha & 0 - \lambda \end{vmatrix} = 0 \tag{11}$$

$$\Rightarrow (-\lambda) \left[\lambda = \frac{-(\beta I - \beta S + \alpha) \pm \sqrt{(\beta I - \beta S + \alpha)^2 - 4\beta\alpha I}}{2} \right] = 0 \tag{12}$$

i.e. $\lambda_1 < 0,\ \lambda_2 < 0$ and $\lambda_3 < 0$; if $-(\beta I - \beta S + \alpha) > \sqrt{(\beta I - \beta S + \alpha)^2 - 4\beta\alpha I}$

Since, all the eigen values are negative then given model is stable, otherwise the model is unstable.

Table 1 Numerical values of the parameters

Parameters	B	α	S(0)	I(0)	R_0
Values	0.012531 per year	2.46269 per year	399 per year	23 per year	0.00125 per year

$$\lambda_1 < 0, \lambda_2 < 0 \text{ and } \lambda_3 > 0; \text{ if } -(\beta I - \beta S + \alpha) < \sqrt{(\beta I - \beta S + \alpha)^2 - 4\beta\alpha I}.$$

It is observed that the stability condition of disease free equilibrium exists i.e. the value of $R_0 < 1$ i.e. basic reproduction number is less than 1, which shows that the total number of susceptible decreases.

4 Numerical and Graphical Analysis of the Model

Data for the influenza cases from January 2013 to December 2013, is collected from IGMC Shimla, HP. Numerical analysis for the disease on the population is given below with the help of the basic reproduction number.

$$\text{From the Eq. 7, we have } R_0 = \frac{k_1}{S(t)}; \text{ for all } \mathrm{k}_1 > 0. \text{ Consider } \mathrm{k}_1 = 1/2, \text{ then } R_0 = 0.00125 > 0 \tag{13}$$

Therefore, the disease free equilibrium is unstable and the infected class will not recover completely from the infection (Table 1).

The graph shows that susceptible class is decreasing with respect to time and removal class is increasing with respect to time due to treatment on time. The infected class is increasing initially but it is approaching to zero after getting treatment.

5 Conclusion

In this study, we conclude that the recovery rate from the epidemic disease is high due to the strong immunity in the population. The SIR model helps to explain and understand the various stages of the disease, which are given graphically through MATLAB analysis. The SIR model has used the basic reproduction number to the study of stability of the disease. We found the basic reproduction number $R_0 < 1$, this value shows that epidemic will not spread. This model approaches that if the social awareness about the diseases among the susceptible, it may help to decrease the infection rate of epidemic disease.

References

1. Das, P., Gazi, N.H., Das, K., Mukherjee, D.: Stability analysis of swine flu transmission—a mathematical approach. Comput. Math. Biol. **3**(1), (2014)
2. Sebastian, M.R., Lodha, R., Kabra, S.K.: Swine origin influenza (Swine Flu). Indian J Pediatr **76** (2009)
3. Numerical Data Collected from Indira Gandhi Medical College, Shimla, January, 2013 to December, 2013
4. Herbert, W.: Hethcote: the mathematics of infectious diseases. SlAM Rev **42**(4), 599–653 (2000)
5. Henneman, K., Peursem, D.V., Huber, V.C.: Mathematical modeling of influenza and a secondary bacterial infection. WSEAS Trans. Biol. Biomed. 10(1) (2013)
6. Aldila, D., Nuraini, N., Soewono, E.: Optimal control problem in preventing of swine flu disease transmission. AMS **8**(71), 3501–3512 (2014)
7. Singh, M., Sharma, S.: An epidemiological study of recent outbreak of influenza A H1N1 (Swine Flu) in Western Rajasthan region of India. JMAS **3**(2), 48–52 (2013)
8. kumar, V., Kumar, D.: SITR dynamical model for influenza. IJETSR **2** (2015)
9. Lin, F., Muthu Raman, K., Lawley, M.: An optimal control theory approach to non-pharmaceutical interventions. BMC Infect Dis **10**, 32 (2010)