Fuzzy Soft Set Theory and Its Application in Group Decision Making

T.R. Sooraj, R.K. Mohanty and B.K. Tripathy

Abstract Soft set theory was introduced by Molodtsov to handle uncertainty. It uses a family of subsets associated with each parameter. Hybrid models have been found to be more useful than the individual components. Earlier fuzzy set and soft set were combined to form fuzzy soft sets (FSS). Soft sets were defined from a different point of view in Tripathy et al. (Int J Reasoning-Based Intell Syst 7(3/4), 224–253, 2015) where they used the notion of characteristic functions. Hence, many related concepts were also redefined. In Tripathy et al. (Proceedings of ICCIDM-2015, 2015) membership function for FSSs was defined. We propose a new algorithm by following this approach which provides an application of FSSs in group decision making. The performance of this algorithm is substantially improved than that of the earlier algorithm.

Keywords Soft sets · Fuzzy sets · Fuzzy soft sets · Group decision making

1 Introduction

The Fuzzy set introduced by Zadeh [1] in 1965 has been found to be a better model of uncertainty and has been extensively used in real life applications. In order to bring topological flavour into the models of uncertainty and associate family of subsets of a universe to parameters, soft sets were introduced by Molodtsov [2] in 1999. The study on soft sets was carried forward by Maji et al. [3, 4]. As mentioned

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in the abstract, hybrid models obtained by suitably combining individual models of uncertainty have been found to be more efficient than their components. Several such hybrid models exist in the literature. Following this trend Maji et al. [5] put forward the concept of FSSs will systematize many operations defined upon them as done in [5]. Extending this approach further, we introduced the membership functions for FSS in [6]. Maji et al. discussed an application of soft sets in decision making problems [3]. Some applications of various hybrid soft set models are discussed in [1, 7–12]. This study was further extended to the context of FSSs [6] where they identified some drawbacks in [3] and took care of these drawbacks while introducing an algorithm for decision making [6]. In this paper, we have carried this study further by using FSS in handling the problem of multi-criteria group decision making.

2 Definitions and Notions

A soft universe (U, E) is a combination of a universe U and a set of parameters E

Definition 2.1 (*Soft Set*) We denote a soft set over (U, E) by (F, E), where

$$F : E \rightarrow P(U) \tag{2.1}$$

Here, P(U) denotes the power set of U.

Definition 2.2 (*Fuzzy soft set*) We denote a FSS over (U, E) by (F, E) where

$$F : E \rightarrow I(U) \tag{2.2}$$

3 Fuzzy Soft Sets (FSS)

Here, we discuss some definitions and operations of FSSs. Let (*F, E*) be a FSS. In [13] the set of parametric membership functions was defined as $\mu_{(F,E)} = \left\{ \mu^a_{(F,E)} | a \in E \right\}$ of (*F, E*).

Definition 3.1 For any $\forall a \in E$, the membership function is defined as follows.

$$\mu^a_{(F,E)}(\mathrm{x}) = \alpha, \alpha \in [0, 1] \tag{3.1}$$

For any two FSSs (F, E) and (G, E) the following operations are defined.

Definition 3.2 $\forall a \in E$ and $\forall x \in U$, the union of (*F, E*) and (*G, E*) is the fuzzy soft set (*H, E*), is given by

$$\mu^{a}_{(H,E)}(\mathrm{x}) = \max\left\{\mu^{a}_{(F,E)}(\mathrm{x}),\ \mu^{a}_{(G,E)}(\mathrm{x})\right\} \tag{3.2}$$

Definition 3.3 $\forall a \in E$ and $\forall x \in U$, the intersection of (*F, E*) and (*G, E*) is the FSS (*H, E*), is given by

$$\mu^{a}_{(H,E)}(\mathrm{x}) = \min\left\{\mu^{a}_{(F,E)}(\mathrm{x}),\ \mu^{a}_{(G,E)}(\mathrm{x})\right\} \tag{3.3}$$

Definition 3.4 Given (*F, E*) is said to be fuzzy soft subset of (*G, E*), $(F,E)\subseteq(G,E)$ and $\forall a \in E$ and $\forall x \in U$,

$$\mu^{a}_{(F,E)}(x) \le \mu^{a}_{(G,E)}(x) \tag{3.4}$$

Definition 3.5 (*F, E*) is said to be equal to (*G, E*) written as (*F, E*) = (*G, E*) if $\forall x \in U$,

$$\mu^{a}_{(F,E)}(x) = \mu^{a}_{(G,E)}(x) \tag{3.5}$$

Definition 3.6 The complement (*H, E*) of (*G, E*) in (*F, E*) is defined $\forall a \in E$ and $\forall x \in U$.

$$\mu^{a}_{(H,E)}(x) = \max\left\{0, \mu^{a}_{(F,E)}(x) - \mu^{a}_{(G,E)}(x)\right\} \tag{3.6}$$

4 Application of FSS in Group Decision Making

Several applications of soft sets theory are given in [2]. In [5] Maji et al. provided an application of FSSs in a decision making system. But the algorithm given in that paper has some issues and those issues are discussed in [6]. Tripathy et al. rectified the issues and provided suitable solution for the problems addressed in [3] and also introduced the concept of negative and positive parameters in [6].

Most of the real-life problems cannot be effectively resolved by a single decision maker. Depends on the uncertainty and the amount of knowledge available, it is not easy to take a suitable decision for a single decision maker. So, it is needed to gather multiple decision makers with different knowledge structures and experience to conduct a group decision making (GDM). Here we discuss an application of group decision making in FSSs.

Algorithm

1. Input the priority given by the panel (J_1, J_2, J_3, …J_n) for each parameter, where ‘n’ is the number of judges.
2. For each judge J_i (i = 1, 2, 3, …, n) repeat the following steps.

a. Input the fuzzy soft set (*F, E*) provided by Judge J_i and arranges it in tabular form.
b. Construct the priority table (PT). This table can be obtained by multiplying priority values with the corresponding parameter values. Also, calculate the row-sum of each row in the PT.
c. Construct comparison tables (CT). This can be achieved by finding the entries as differences of each row sum in PTs with those of all other rows.
d. Find the row sum for each row in the CT to obtain the score.
e. Construct the decision table by taking the row sums in the CT. Assign rankings to each candidate based upon the row sum obtained.

3. 3. Create a rank table based on the results obtained from the above step which contains rankings provided by all the judges.
4. 4. Calculate the row-sum of each candidate in the rank table to find the rank-sum of each candidate. The candidate with lesser row sum value is the best choice. If more than one candidate is having the same rank-sum, then the candidate having higher value in highest absolute priority column will be selected. This process is continued till final ranking list is obtained.

Assume that 'n' candidates are applying for a job in an organization. From these n candidates, the organization filters out many candidates based on some criteria (For e.g.: Those who got more than 60 % marks are eligible to attend the interview). The candidates, who passed the elimination criteria, will be eligible to attend the interview. The interview performance of each selected candidate is analyzed by a panel of different judges. Here, the panel assigns some parameters to evaluate the performance of each candidate. Some parameters are communication skills, personality, reactivity etc.

Let *U* be a set of candidates $\{c_1, c_2, c_3, c_4, c_5, c_6\}$. The parameter set E be {knowledge, communication, reaction, presentation, extracurricular activities}. Consider a FSS (*U, E*) describing the 'performance of candidates'. Consider J_1, J_2 and J_3 are the judges who analyze the performance of the candidates and each judge is assigning a rank to each candidate according to his/her performance.

The panel of judges assigns priority values to the parameters and based upon the impact of the parameters, they assign rankings to each parameter. This is shown in the following Table 1. The parameters knowledge, communication, behaviour, presentation, extracurricular activities are represented by e_1, e_2, e_3, e_4 and e_5.

The parameter values assigned by each judge to the candidate depend upon the performance of the candidate in the interview. The FSS for the candidates from the judge J_1 is shown in Table 2.

The priority for parameters e_1, e_2, e_3, e_4 and e_5 is given by the judge panel as 0.4, 0.3, −0.15, 0.05 and 0.1. Here, the parameter 'e_3' is a negative parameter.

Table 1 Priority rank table

	e_1	e_2	e_3	e_4	e_5
Priority	0.4	0.3	−0.15	0.05	0.1
Parameter rank	1	2	3	5	4

Table 2 FSS (F, E) by judge J_1

U	e_1	e_2	e_3	e_4	e_5
c_1	0.2	0.3	0.8	0.8	0.6
c_2	0.4	0.6	0.2	1	0.5
c_3	0.8	0.9	0.7	0.9	0.7
c_4	0.8	0.9	0.8	0.9	0.7
c_5	0.4	0.9	0.6	0.1	0.8
c_6	0.9	1	0.3	0.2	0.3

Table 3 Priority table for J_1

U	e_1	e_2	e_3	e_4	e_5	Row-sum
c_1	0.08	0.09	−0.12	0.04	0.06	0.15
c_2	0.16	0.18	−0.3	0.05	0.05	0.14
c_3	0.32	0.27	−0.105	0.045	0.07	0.6
c_4	0.32	0.27	−0.12	0.045	0.07	0.585
c_5	0.16	0.27	−0.09	0.005	0.08	0.425
c_6	0.36	0.3	−0.045	0.01	0.03	0.655

Parameters are classified into positive parameter and negative parameter. We use the notion of negative parameter as in [6]. The priority table is as follows (Table 3).

The comparison table obtained for the candidates by the judge J_1 is obtained is shown in the Table 4.

Comparison table (CT) shows the ranking of each candidate by the judge J_2. Here, the candidate c_6 is the best choice. Since the selection of the best candidate is governed by a panel of 3 members, we cannot take this as the best choice. So, we have to find the comparison table of the judges J_2 and J_3 to decide the optimum choice. Representation of FSS of candidates by judge J_2 is shown below (Table 5).

After applying the algorithm in the above FSS, we will get the comparison table as shown in the Table 6.

The FSS of candidates by Judge J_3 is given as follows (Table 7).

After applying the algorithm in the above FSS, we will get the comparison table for the judge J_3 as follows (Table 8).

Table 4 Comparison table

U	c_1	c_2	c_3	c_4	c_5	c_6	Score	Rank
c_1	0	0.01	−0.45	−0.435	−0.275	−0.505	−1.655	5
c_2	−0.01	0	−0.46	−0.445	−0.285	−0.515	−1.715	6
c_3	0.45	0.46	0	0.015	0.175	−0.055	1.045	2
c_4	0.435	0.445	−0.015	0	0.16	−0.07	0.955	3
c_5	0.275	0.285	−0.175	−0.16	0	−0.23	−0.005	4
c_6	0.505	0.515	0.055	0.07	0.23	0	1.375	1

Table 5 FSS (F, E) by judge J_2

U	e_1	e_2	e_3	e_4	e_5
c_1	0.3	0.2	0.6	0.7	0.7
c_2	0.5	0.5	0	0.9	0.6
c_3	0.9	0.8	0.5	0.8	0.8
c_4	0.9	0.8	0.6	0.8	0.8
c_5	0.5	0.8	0.4	0	0.9
c_6	1	0.9	0.1	0.1	0.4

Table 6 Comparison table of judge J_2

c_i	c_j							
	c_1	c_2	c_3	c_4	c_5	c_6	Score	Rank
c_1	0	−0.26	−0.45	−0.435	−0.275	−0.505	−1.925	6
c_2	0.26	0	−0.19	−0.175	−0.015	−0.245	−0.365	5
c_3	0.45	0.19	0	0.015	0.175	−0.055	0.775	2
c_4	0.435	0.175	−0.015	0	0.16	−0.07	0.685	3
c_5	0.275	0.015	−0.175	−0.16	0	−0.23	−0.275	4
c_6	0.505	0.245	0.055	0.07	0.23	0	1.105	1

Table 7 FSS (F, E) by judge J_3

U	e_1	e_2	e_3	e_4	e_5
c_1	0.5	0.4	0.8	0.9	0.9
c_2	0.7	0.7	0.2	1	0.8
c_3	1	1	1	1	1
c_4	1	1	0.8	1	1
c_5	0.7	1	0.6	0.2	1
c_6	1	1	0.3	0.3	0.6

Table 8 Comparison table for judge J_3

c_i	c_j							
	c_1	c_2	c_3	c_4	c_5	c_6	Score	Rank
c_1	0	−0.255	−0.365	−0.395	−0.265	−0.395	−1.675	6
c_2	0.255	0	−0.11	−0.14	−0.01	−0.14	−0.145	5
c_3	0.365	0.11	0	−0.03	0.1	−0.03	0.515	3
c_4	0.395	0.14	0.03	0	0.13	0	0.695	2
c_5	0.265	0.01	−0.1	−0.13	0	−0.13	−0.085	4
c_6	0.395	0.14	0.03	0	0.13	0	0.695	1

Table 9 Rank table

	J_1	J_2	J_3	Rank-sum	Final-rank
c_1	5	6	6	17	6
c_2	6	5	5	16	5
c_3	2	2	3	7	2
c_4	3	3	2	8	3
c_5	4	4	4	12	4
c_6	1	1	1	3	1

Rank of all candidates given by judges J_1, J_2 and J_3 are shown in the rank table (Table 9). From this rank, we can find the final rank of the candidates.

From the above table, we can see that the panel has selected the candidate c_6 as the best choice.

5 Conclusions

The definition of soft set using the characteristic function approach was provided in [12], which besides being able to take care of several definitions of operations on soft sets could make the proofs of properties very elegant. Earlier FSSs were used for decision making in [5]. Some flaws in the approach were pointed out in [6] and rectifications were made. Due to the lack of information and uncertainty in real life scenarios, a single decision maker cannot able to take proper decision. So, a new algorithm is introduced in this work with respect to decision making by a group of decision makers.

References

1. Zadeh, L.A.: Fuzzy sets. Inf. Control **8**, 338–353 (1965)
2. Molodtsov, D.: Soft set theory—first results. Comput. Math Appl. **37**, 19–31 (1999)
3. Maji, P.K., Biswas, R., Roy, A.R.: An application of soft sets in a decision making problem. Comput. Math Appl. **44**, 1007–1083 (2002)
4. Maji, P.K., Biswas, R., Roy, A.R.: Soft set theory. Comput. Math Appl. **45**, 555–562 (2003)
5. Maji, P.K., Biswas, R., Roy, A.R.: Fuzzy Soft Sets. J. Fuzzy Math. **9**(3), 589–602 (2001)
6. Tripathy, B.K., Sooraj, T.R, Mohanty, R.K.: A new approach to fuzzy soft set and its application in decision making. In: Proceedings of ICCIDM 2015, Dec 5–6, Bhubaneswar
7. Tripathy, B.K., Sooraj, T.R, Mohanty, R.K.: A new approach to interval-valued fuzzy soft sets and its application in decision making. Accepted in ICCI-2015, Ranchi
8. Tripathy, B.K., Sooraj, T.R., Mohanty, R.K.: A new approach to interval-valued fuzzy soft sets and its application in group decision making. Accepted in CDCS-2015, Kochi
9. Tripathy, B.K., Mohanty, R.K., Sooraj, T.R.: On intuitionistic fuzzy soft sets and their application in decision making. Accepted in ICSNCS-2016, New Delhi
10. Tripathy, B.K., Mohanty, R.K., Sooraj, T.R.: On intuitionistic fuzzy soft set and its application in group decision making. Accepted for presentation ICETETS-2016, Thanjavur

11. Tripathy, B.K., Mohanty, R.K., Sooraj, T.R., Tripathy, A.: A new approach to intuitionistic fuzzy soft set theory and its application in group decision making. Presented at ICTIS-2015, Ahmedabad, Springer publications, (2015)
12. Tripathy, B.K., Arun, K.R.: A new approach to soft sets, soft multisets and their properties. Int. J. Reasoning-Based Intell. Syst. **7**(3/4), 244–253 (2015)
13. Tripathy, B.K., Mohanty, R.K., Sooraj, T.R., Arun, K.R.: A new approach to intuitionistic fuzzy soft sets and its application in decision making. In: proceedings of ICICT-2015, Oct 9–10, Udaipur, (2015)