On Failure Theories for Composite Materials

Ramesh Talreja

Abstract Limitations inherent in failure theories formulated on homogenized description of composite materials are discussed. Failure mechanisms in composite materials, as understood today, are reviewed. Based on this knowledge, arguments are put forth to abandon the classical approach to formulation of failure theories for composite materials, and to instead use a computation-based failure assessment methodology. Such a methodology is proposed. In conjunction with this, the idea of virtual testing to supplement experimental determination of material response characteristics is discussed.

1 Introduction

As applications of composite materials have expanded from the aerospace to non-aerospace fields, such as automotive, wind turbines, and subsea structures, design for safe performance has become critical. Current application of carbon-epoxy composites in civilian aircraft is essentially a replacement of aluminum (coined as "black aluminum") with limited use of the potential of these materials. The main roadblock to a higher level of utilization is lack of reliable failure analysis capabilities in the industry. The current approach in the aerospace applications is to conduct extensive tests to demonstrate safe performance, which has forced the industry to keep the load levels excessively low. Although the low weight of polymer composites still gives

R. Talreja (✉)
Department of Aerospace Engineering, Texas A&M University,
College Station, TX 77843, USA
e-mail: talreja@tamu.edu

R. Talreja
Department of Materials Science and Engineering, Texas A&M University,
College Station, TX 77843, USA

R. Talreja
Department of Engineering Science and Mathematics, Luleå University of Technology,
SE 971 87 Luleå, Sweden

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significant fuel savings in aircraft, other applications cannot afford such conservative designs.

A compelling case was made in a recently completed survey of composite failure theories (Soden et al. 1998; Hinton et al. 2002, 2004; Kaddour et al. 2004) that their predictive capabilities judged against test data are not reliable. This is a sad commentary on the state of affairs in view of the long history of the development of the theories. Perhaps this long history is the reason for the problems underlying the lack of success of the theories. As will be argued below, the path taken in developing the failure theories has been flawed. The nature of the flaws is such that efforts to "improve" the theories are frustrated by lack of agreement with some or the other test data. An alternative path must be taken, and it may lead to the painful realization that the current theories are best left behind.

The following sections will discuss first the fundamentals of the common theories of failure in unidirectional composites, drawing upon a previous treatment (Talreja 2014). The physical nature of the failure in these composites will be summarized next. The models that account for the known mechanisms will be reviewed and the challenges remaining in this direction will be described. Finally, a scheme to connect the models into a failure assessment methodology will be presented.

2 The Classical Failure Theories for Unidirectional Composites

The first significant theory of failure in unidirectional composites appeared in 1965 (Azzi and Tsai 1965), known as the Tsai–Hill theory. In fact, associating Hill's name with this theory is questionable since Hill (1948) did not in any way suggest that his theory, developed for yielding of anisotropic metals (metals with texture), may be applied to fiber-reinforced polymers. Hill's yield criterion for anisotropic solids was a simple mathematical generalization of the von Mises criterion for isotropic yielding when expressed in terms of the deviatoric stress components. In this generalization, the important connection with the distortional energy density was lost. This fact is significant because having energy concept to capture the physics of metal plasticity strengthens the basis of the yield criterion. In any case, the weak form of the criterion in Hill's proposal still only applies to yielding that is driven by shear at the microscopic level. The underlying mechanism of yielding for anisotropic (orthotropic) metals in Hill's formulation manifests itself in six yield constants—three normal stress thresholds and three shear stress thresholds—as a generalization of a single yield stress in the isotropic case.

Adopting the Hill criterion to another material (a unidirectional composite) can only be justified if that material yields (or in another way becomes critical) as a result of a single shear-driven mechanism. Although a polymer may satisfy this requirement, it is doubtful that it will continue to do so when reinforced with fibers. In fact even an unreinforced polymer shows pressure sensitivity in its inelastic behavior, thereby violating the shear-driven yield requirement. The presence of fibers significantly alters "failure" in the form of deviation from elasticity, as will be discussed later.

After reducing the Hill criterion to two-dimensional form and assuming isotropy in the cross-sectional plane of unidirectional composites, the criterion in (Azzi and Tsai 1965) takes the following form

$$\left(\frac{\sigma_1}{X}\right)^2 - \left(\frac{\sigma_1\sigma_2}{X}\right)^2 + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\sigma_{12}}{S}\right)^2 = 1 \tag{1}$$

where σ_1 and σ_2 are the normal stresses in the fiber and transverse directions, respectively, and σ_{12} is the in-plane shear stress. X, Y, and S are the "yield" stresses corresponding to σ_1, σ_2, and σ_{12}, respectively.

Equation (1) was shown in Azzi and Tsai (1965) to agree well with test data for a unidirectional composite loaded in tension at an angle inclined to the fiber direction. However, other data generated later did not show good agreement, suggesting a need for another theory.

Observing that the stress components in Eq. (1) were "interacting" in a quadratic manner, a more general quadratic polynomial than what appears here could improve the lacking agreement with test data. This idea seemed to have led to the search for a more general formulation, which was found in the tensor polynomial for strength proposed in Goldenblatt and Kopnov (1965). A simplification of the polynomial to plane stress state resulted in the quadratic expression proposed in Tsai and Wu (1971) as

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{12}\sigma_1\sigma_2 + F_{66}\sigma_{12}^2 = 1 \tag{2}$$

after using the fact that the sign of the shear stress has no effect on the shear strength. The coefficients of the terms in Eq. (2) are inverses of strength values and these can be found by testing with one imposed stress component at a time, except the constant F_{12}, which requires a biaxial stress test. Since a biaxial test can be done with an arbitrary combination of the two normal stresses, determination of this constant becomes non-unique. This flaw in the theory cannot be removed, but can only be "fixed" by adopting a biaxial test as a convention. The consequence of the lack of a physical reasoning for choosing the biaxial stress for determining F_{12} was discussed among others by Hashin (1980), who also pointed out that a single surface in the σ_1-σ_2-σ_{12}-space, represented by Eq. (2), was inadequate in describing failure in different combinations of stresses. He suggested instead to formulate the failure criteria in a piecewise smooth form with each branch of the failure surface dealing with a certain failure mode. He suggested to separate fiber failure mode from the matrix failure mode because of different governing mechanisms and to formulate quadratic interaction equations in each case.

Hashin's work (Hashin 1980) provided a direction away from the totally curve-fitting schemes toward utilizing some understanding of the failure process. In particular, his suggestion to consider a failure plane in the matrix of a composite to formulate the matrix mode failure criteria was taken up by Puck and coworkers (Puck and Schürmann 1998, 2002). These authors devised elaborate procedures for determining additional material constants needed to characterize the failure envelopes in different stress combinations.

Many other works have also proposed formulations of failure criteria using various assumptions regarding how criticality of failure occurs. As mentioned above, assessments of a large number of such failure theories reported in Soden et al. (1998), Hinton et al. (2002, 2004), and Kaddour et al. (2004) concluded that no single theory was able to agree with all test data used for comparison.

One can ask why the proposed theories are not satisfactory. Perhaps the answer lies in the failure mechanisms, which are rich in detail and varied in how they initiate, progress, and become critical. The next section will discuss the failure mechanisms.

3 Failure Mechanisms in Unidirectional Composites

3.1 Fiber Failure in Tension

Fiber failure in a unidirectional composite under imposed overall axial tension has been studied extensively. A recent work using high resolution X-ray microtomography (Aroush et al. 2006) provides clear evidence of the statistical nature of this process (Fig. 1). As reported in that work, fiber breaks appear initially at low loads as single failures at discrete locations because of failures at weak points. On increasing the applied load, more fibers fail, mostly near the previously broken fibers, and the so-called doublets form. This process continues until one or more of the broken fiber clusters grow unstably to failure of the composite.

3.2 Fiber Failure in Compression

Although the strength of a unidirectional composite under axial compression is characterized as a fiber failure mode, as by Hashin (1980) and subsequently by many, the failure mechanisms involved in this case depend significantly on the matrix behavior. As described by Jelf and Fleck (1992), the fiber failure in compression may be categorized as elastic microbuckling or plastic microbuckling, depending on whether the matrix stress–strain behavior is linear or nonlinear. For the latter case, the compressive strength is predicted well by Budiansky's kink band model (Budiansky 1983), which has the fiber misalignment and matrix shear yield stress as parameters. Kyriakides et al. (1995) further verified the dependence of the compressive strength on fiber misalignment angle and generalized it to fiber waviness as the microstructure (defect) scale. The microstructural features of the kink bands and mechanisms of their formation were clarified by these authors. Their analysis found that in the presence

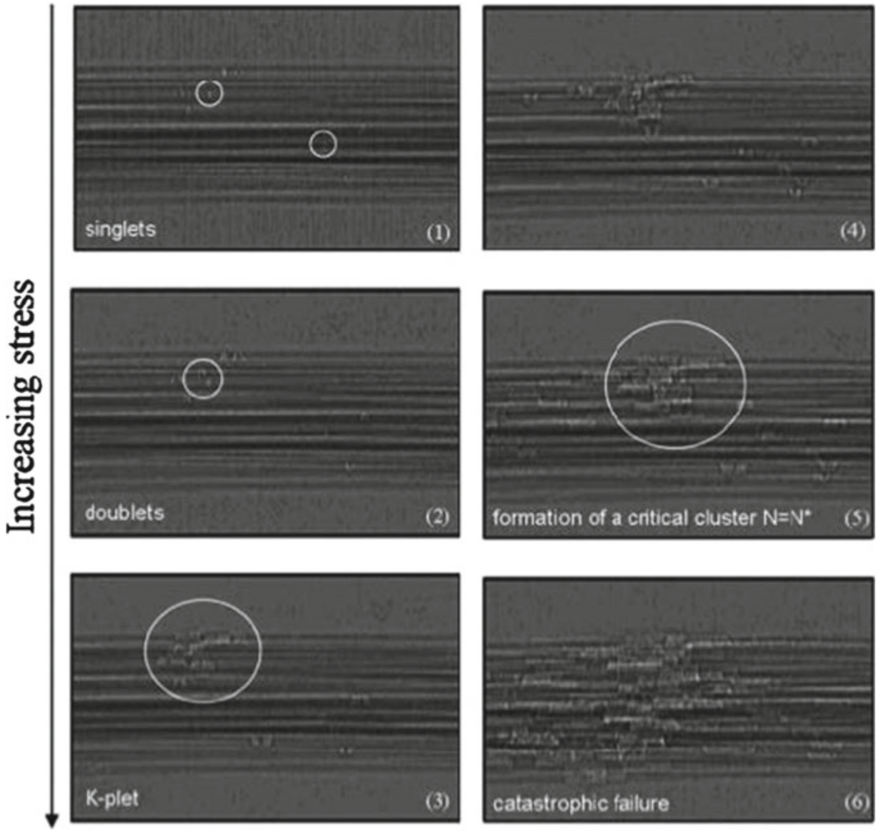

Fig. 1 Sequence of fiber failures in a unidirectional composite under axial tension (from Aroush et al. 2006)

of fiber waviness, localization of shear deformation occurs in the matrix. This forms bands and the flow of the matrix in the bands results in bending of fibers and eventual breakage, which results in the formation of the observed kink bands. Figure 2 illustrates schematically the early stage of microbuckling leading to formation of a kink band (Berbinau et al. 1999).

3.3 Matrix Failure in Transverse Tension or Compression

On loading a unidirectional composite under tension normal to fibers, failure occurs suddenly and at low stress levels. The mechanisms leading to the catastrophic failure are conveniently studied by observing the initiation and progression of cracks within the plies of a laminate. The appearance of these cracks is typified by the images shown in Fig. 3 (Gamstedt and Sjögren 1999).

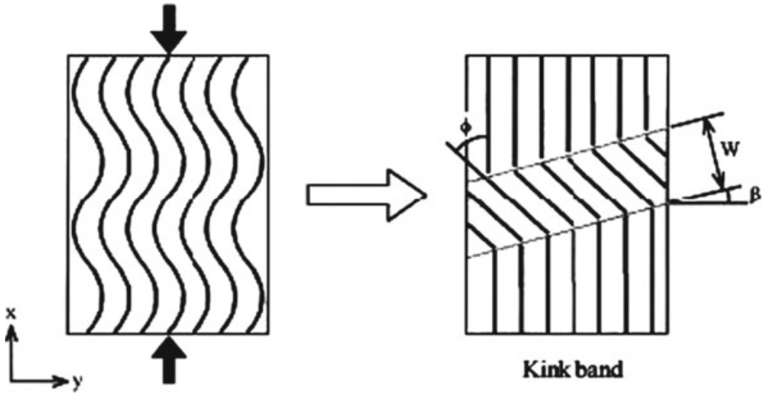

Fig. 2 Schematic illustration of fiber microbuckling and kink band formation in a unidirectional composite subjected to axial compression (Berbinau et al. 1999)

When compression is applied normal to the fibers, the failure is found to occur along a plane that is inclined to the loading direction, as reported in González and LLorca (2007) (Fig. 4). On closer examination, it is found that microscopic cracks formed due to shear and the coalescence of these cracks led to failure along the inclined plane.

3.4 *Matrix Failure in In-Plane Shear*

Under an in-plane shear stress, a unidirectional composite displays nonlinear behavior. A part of this is due to a shear-induced flow of the matrix and the other part is caused by cracks formed in the matrix. Such cracks are illustrated in the image shown in Fig. 5 (Redon 2000). These cracks, as shown in the figure, form multiple

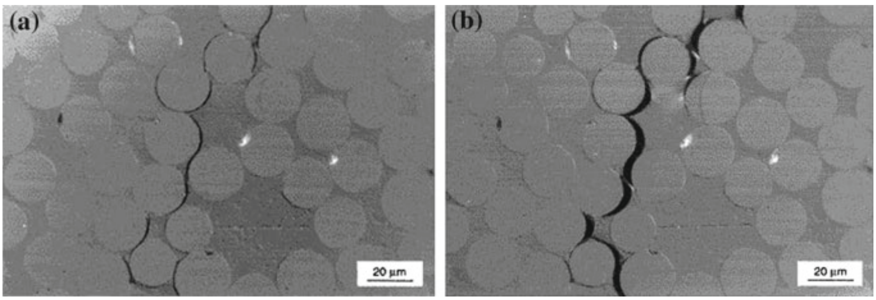

Fig. 3 Images of cracks formed under a tensile load normal to fibers. Image (**a**) is at a low load, while image (**b**) is taken at the same location at a higher load. From Gamstedt and Sjögren (1999)

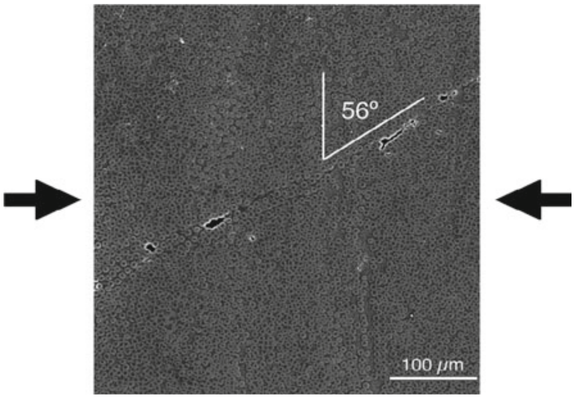

Fig. 4 Matrix failure on an inclined plane under compression normal to fibers (González and LLorca 2007)

cracks with their planes inclined to the fiber axis. On increased loading, the cracks turn along the fibers and merge together, forming a failure plane. Such a failure plane is also formed under compression normal to fibers (Fig. 4), indicating the role of the shear stress on the plane.

4 Formulation of Failure Criteria for Unidirectional Composites

In view of the brief overview of the failure mechanisms in unidirectional composites described above in Sect. 3, one can scrutinize the failure theories discussed in Sect. 2. Following points can be made.

(a) There is no basis for a single failure criterion represented by a single smooth surface in the stress space of the three stress components in the principal coordinates of the composite. Thus, the two criteria given by Eqs. (1) and (2) are each not physically based. There would be a physical basis, e.g., for an orthotropic solid where only one failure mechanism operates, e.g., yielding. Thus, the Hill criterion for yielding of a metal with texture (produced, for example, by rolling in one direction), from which Eq. (1) was derived, is a physically based criterion, while Eq. (1) for unidirectional composite failure is not.

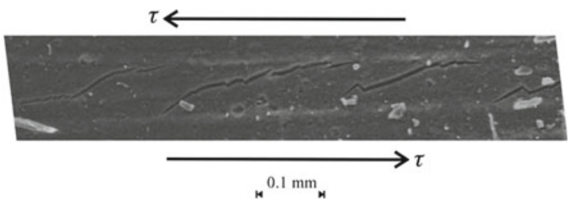

Fig. 5 Matrix cracks inclined to the fiber direction (*horizontal*) formed under in-plane shear in a unidirectional composite (Redon 2000)

(b) Under combined loading, i.e., when two or three stress components in the principal coordinates of a unidirectional composite are simultaneously applied, the "interaction" cannot a priori be assumed as quadratic, as in Eq. (2). Hashin (1980) had argued that not knowing the nature of this interaction, assuming it to be linear would be too restrictive. Therefore, quadratic interaction should be assumed, also for the reason that it would allow more flexibility in curve-fitting with experimental data. To be sure, Hashin did caution against taking the quadratic form of the criterion to be physically based, pointing out that justification for this existed only for the metal yielding.

(c) Separating the failure of a unidirectional composite into fiber failure mode and matrix failure mode, as suggested by Hashin (1980), is a good first approximation, but the mechanisms underlying the two failure modes are not amenable to such separation. Failure criteria should result from analysis of the failure mechanisms, and the critical condition for each mechanism should define failure.

The observations (a)–(c) above suggest that the current (classical) approach of formulating failure criteria based on a homogenized description of unidirectional composites is simply not capable of accounting for the physical nature of the failure mechanisms. A common feature of all failure mechanisms is that they are governed by "local" conditions, i.e., by the stress states at the microscopic level. For instance, in the tensile fiber failure, the formation of the multiple fiber failures and their critical cluster size, illustrated in Fig. 1, depend on the local (triaxial) stress state, which is responsible for the fiber failure progression (Zhuang et al. 2016). In the compression fiber failure mode, the formation of the kink band, illustrated in Fig. 2, also depends on the local (triaxial) stress state developed due to microbuckling of fibers. In this mechanism, the role of fiber imperfections has been found to be significant in triggering the formation of a kink band (Kyriakides et al. 1995).

For the matrix failure mode, Hashin (1980) suggested failure to occur on a failure plane, which is inclined to the fibers but does not intersect the fibers. Such a plane is shown in Fig. 4. The failure of this plane was proposed by Hashin to take place under the combined action of the traction components acting on the plane. A closer examination of the failure mechanisms, described above, however suggest that this may be inaccurate. The effect of shear on a plane depends on the local (triaxial) stress state resulting in inclined cracks shown in Fig. 5. Thus, although macroscopically the failure seems to occur on a plane, the initiation, progression, and criticality of the failure is governed by microscopic scale conditions.

5 Discussion and Concluding Remarks

Composite material failure is fundamentally a failure process at the scale of the fibers and matrix. Homogenizing the two constituents removes the possibility of analyzing the failure and thereby determining the conditions of criticality. Efforts since the first proposed failure theory (Azzi and Tsai 1965) in 1965 have led to increasing complexity of formulations resulting from assumptions to capture the features of the essentially microscopic scale process at the macroscopic (homogenized) scale. Not only have such efforts not succeeded (Soden et al. 1998; Hinton et al. 2002, 2004; Kaddour et al. 2004), they have introduced a great deal of uncertainty for the designers of composite structures. Each new theory coming out in the literature promises to fit the data well until another theory finds discrepancy and proposes to improve the "predictions" by yet other assumptions.

The way forward must be a multi-scale approach that is guided by the observations of the failure process at the microscopic scale. In such an approach, one must avoid making assumptions like the cohesive zones that are not evidenced by the observations. Instead, the local stress states should be calculated by the finite element type of models and failure criteria for fiber and matrix failure should be applied based on known mechanisms of failure. The techniques for observing failure at small scales are quite sharp today, e.g., based on micro-focus computed X-ray radiography (Scott et al. 2014). Constructing a scheme to analyze failure should be guided by the details revealed by such high resolution observations.

Reliance on testing for validation of the failure theories is part of the classical approach. This principle has guided the development of the failure theories in the past. The new developments in computational simulation provide another way to validate (and develop) failure theories. This is known as "virtual" testing (LLorca et al. 2011). Essentially, the idea is to conduct tests by simulation on the computer. This allows examining failure under combined loading that would be difficult, if not impossible, to do in physical tests. The risk, of course, is to do "virtual" tests that would not correspond to reality. This can happen if either the stress analysis is incorrect or if the failure criteria at the microscopic scale are not physically justified, or both. A good approach would be to validate the virtual testing approach by simpler physical tests before expanding the approach to more general cases. Still, this cannot guarantee the validity of the virtual testing.

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