

# The Short-Term Forecast of BeiDou Satellite Clock Bias Based on Wavelet Neural Network

Qingsong Ai, Tianhe Xu, Jiajing Li and Hongwei Xiong

**Abstract** According to nonlinear and nonstationary characteristics of BeiDou satellite clock bias time series, this paper proposed a method using the wavelet neural network (WNN) based on the first-order difference of adjacent epoch to predict the satellite clock bias. Experimental data with sampling interval of 15 min rapid and ultra-rapid satellite clock bias provided by Wuhan University is used to test the validation of the method. The results show that the forecast precision of 6 h for BeiDou satellite can reach 1–2 ns, and the 24 h can reach 2–4.6 ns using the proposed method. The test results also show that the accuracy and stability of the model prediction can be improved greatly using the proposed method compared to the traditional gray model and quadratic polynomial model.

**Keywords** Satellite clock bias · First-order difference of adjacent epoch · Wavelet neural network

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## 1 Introduction

It is well known that the high precision satellite clock is the basis of the navigation system, and the forecast accuracy of satellite clock parameters directly influence the performance of the navigation system services. Research shows that spaceborne atomic clock is quite sensitive, easily disturbed by the outside environment or their own physical properties. This characteristics makes it difficult to understand the detailed changes, and hard to predict the satellite clock bias with high accuracy [1]. Based on the properties of satellite atomic clocks, many domestic and foreign scholars have put forward a lot of prediction models in order to improve the forecast accuracy of satellite clock bias. The most common forecast model is quadratic polynomial model, and gray model GM (1, 1); In addition, spectrum analysis (SA) model, autoregressive integrated moving average (ARIMA) model, Kalman filtering (KF) model, and relevant improvement models are also widely used. The quadratic polynomial model is first proposed by Allan to forecast the rapid satellite clock bias [2]. Although quadratic polynomial model can generally reflect the physical properties of the atomic clock, it is inevitable to be influenced by the periodic term and random term. On the other hand, the prediction accuracy will be greatly decreased with the increase of the step size using quadratic polynomial model. Gray model can take advantage of less information to forecast in short term, but it is hard to use longer historical data to improve the prediction accuracy further. Its essence is to use index function as predict model, and the prediction value is easily affected by the coefficients of index function. It is difficult to determine a set of reasonable coefficients to predict satellite clock bias using gray model [3].

In terms of the BeiDou Navigation Satellite System (BDS), the satellite is equipped with rubidium (Rb) atomic clock. In order to provide the user with high precision navigation, positioning and timing services, the prediction of BeiDou satellite clock bias becomes more and more important. Because of the shortage of the traditional forecast models above-mentioned, a new method of combining the wavelet neural network (WNN) and the first-order difference of adjacent epoch for satellite clock bias prediction is developed in this paper. The proposed method absorbs not only the advantages of the wavelet commendable time-frequency localization characteristics and strong nonlinear mapping ability [4], but also the advantages of the first-order difference with good numerical stability and weakening influence of system error.

## 2 First-Order Difference Algorithm for Satellite Clock Bias

Assume the satellite clock time series is expressed as  $X = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(m)\}$ , then a new sequence  $\Delta X^{(0)} = \{\Delta x^{(0)}(k), k = 1, 2, \dots, m - 1\}$  can be obtained after making a first-order difference between the adjacent epoch.

The novel algorithm just uses the new clock series that forecasts the difference in the next period and restores the undifference clock time series which used the predicted value. The formula can be expressed as

$$x^{(0)}(\tau) = x^{(0)}(m) + \sum_{m+1}^n \Delta x^{(0)}(\tau - m), \quad m + 1 \leq \tau \leq m + n$$

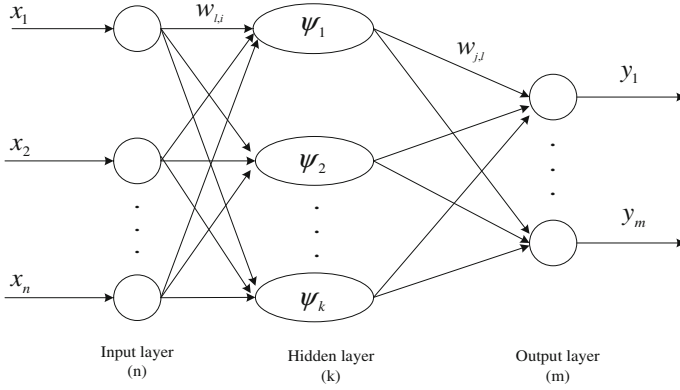
It improves the numerical stability of satellite clock bias, eliminates part of influence of the system error, and enhances the fitting and prediction accuracy.

### 3 The Basic Model of Neural Networks

Numerous studies and mathematical theory have proved that artificial neural networks (ANNs) can successfully approximate any complicated nonlinear time series, which provides a great support for the study of nonlinear problems [5, 6]. Currently, the widely used method is the feed-forward neural network in which the most famous is BP (Back Propagation) networks. The ideal result of the BP network is to be used in forecasting the satellite clock bias series of nonlinear, nonstationary. In mathematics, BP algorithm is a local search optimization method, but its aim at seeking general extremum of complex function that may fall into local minimum and lead to network training failure. In view of the above shortcomings, many scholars have improved the primary arithmetic, which mostly focus on increasing the rate momentum of learning; Although these proposed methods can improve the predict accuracy to a certain extent, but still unable to resolve the problem of local extremum. In order to overcome the above shortcomings, wavelet theory has been gradually introduced [7].

#### 3.1 The Principle of Wavelet Neural Network

Wavelet analysis uses the multi-resolution processing signal or function by translation and dilation. It has great advantage in the local information extraction and analysis about signal and function. The method combines the self-learning approximation ability, adaptive and self-organizing, fault tolerance, and other characteristics of ANN. The WNN not only inherits the good time–frequency localized feature of wavelet transform, but also retains the self-learning and many other features of ANNs. The main difference between the WNN and the BP network is the difference in the neuron excitation function of the hidden layer. In WNN the wavelet function replaces the role of sigmoid function in the hidden unit, and the wavelet parameters and wavelet shape are adaptively computed to minimize an energy function for finding the optimal representation of the signal. Simultaneously,



**Fig. 1** Network structure of wavelet neural network

the WNN’s excitation function is introduced in the dilation and translation coefficients. The network is trained with BP algorithm in batch way [8, 9]. In the following, a three-layer WNN model with both “sigmoid neuron nonlinearity” and “wavlon nonlinearity” is proposed and described for the purpose of satellite clock bias series prediction [6] (Fig. 1).

$n, k, m$  is the sum of input, hidden and output layer nodes

$x_i (i = 1, 2, \dots, n)$  is input vector,

$y_j (j = 1, 2, \dots, m)$  is output vector

$\psi_l (l = 1, 2, \dots, k)$  is wavelet basic function,  $\eta$  is learning rate

$a_l, b_l$  is dilation and translation coefficient of wavelet hidden layer, respectively

$w_{l,i}$  is the connection weight between the hidden unit,  $l$  and input unit  $i$

$w_{j,l}$  is the connection weight between the output unit,  $j$  and the hidden unit  $l$

The output vector could be expressed as  $y_j = (\sum_{l=1}^k w_{j,l} \psi_{(a,b)}(\text{net}_l))$

$\text{net}_l = \sum_{i=1}^n w_{l,i} x_i$  the error function can be written as  $E = \frac{1}{2} \sum_{j=1}^m (y_j - d_j)^2$

$d_j$  is the desired target output

Under the idea of gradient descent, the derivation procedure of the corresponding connection weights, dilation, and translation coefficient could be referred to the literature [10, 11]. The network is trained by the gradient algorithm based on the forward and backward propagation, and is adjusted until the error function value fall into a limited range (Fig. 2).

### 3.2 *Determine the Network Structure of Wavelet Neural Network*

A very important step of WNN application is to determine the suitable network architecture. The hidden layer nodes of the network have a great impact on the forecast accuracy of WNN. If the nodes are few, the network learning cannot be very well which needs to increase the number of training, and the accuracy is also affected. If the nodes is too many, the training time increases a lot and the network is likely to over-fitting. The optimal selection of hidden layer nodes can refer to [12]. Based on the range of the nodes, the iteration of learning process can be stopped if it meets the error conditions [refer to the formula( $E$ )]. If it still does not satisfy the error condition after reaching the maximum number of iterations, the number of nodes should be increased by one layer, then the implicit iterative process is repeated until the error meet the requirement. In practice, the number of hidden layer nodes can be adaptively determined using the above-mentioned method. In this paper, the WNN architecture was finally chosen as net 5-8-1(one input unit, eight hidden units and one output unit) through trial-and-error processes of practical examples of data test. The connection weights, the parameters of wavelet dilation, and translation were initialized as [10].

## 4 Experimental Result

Now BDS has provided continuous navigation positioning and timing services to China and its surrounding areas. At present there are three types of BeiDou satellite in orbit, GEO satellite represented by C1–C5, IGSO satellite represented by C6–C10 and MEO satellite represented by C11–C14, respectively [13, 14]. The satellite clock bias series presents the overall approximate one-way increasing or decreasing trend, which is shown in Figs. 3, 4, and 5. With the purpose of analyzing the applicability of WNN in the satellite clock bias prediction, the C03, C09, and C11 were selected for the experiment. Test data is the rapid and actual measured ultra-rapid satellite clock bias provided by Wuhan University in the second and third day of BeiDou weeks of 490. In order to evaluate the accuracy of the prediction, we adopt final clock bias of corresponding time provided by Wuhan University. The following two schemes is performed for the experiment.

The first scheme is: Using the clock bias sequence of the second day modeling and forecasting the next 6 h.

The second scheme: Using the clock bias sequence of the second day modeling and forecasting the next 24 h.

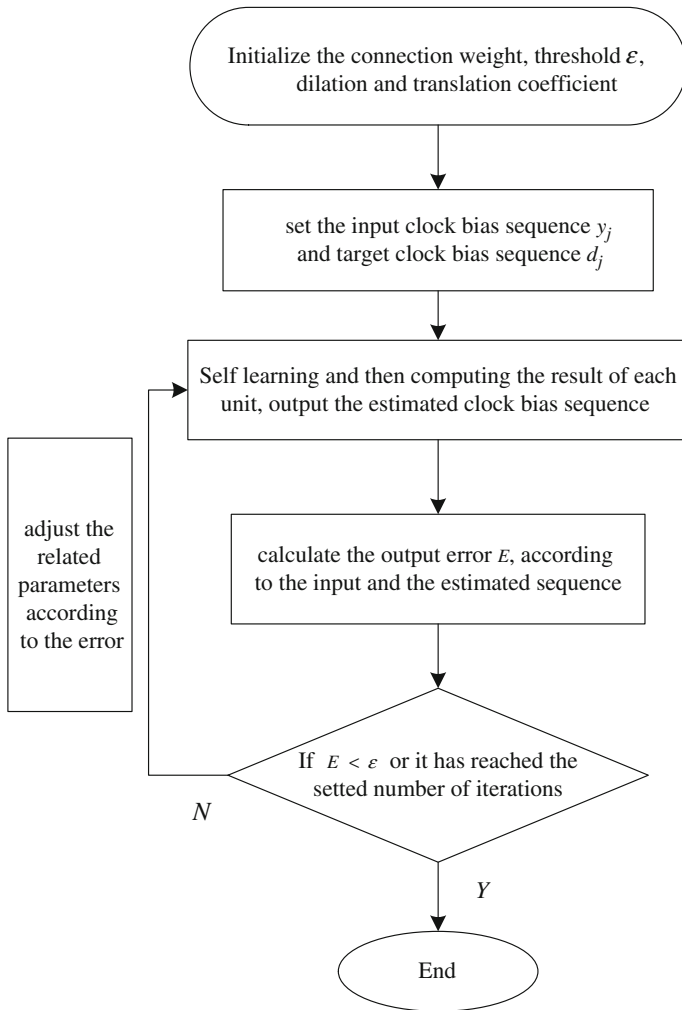
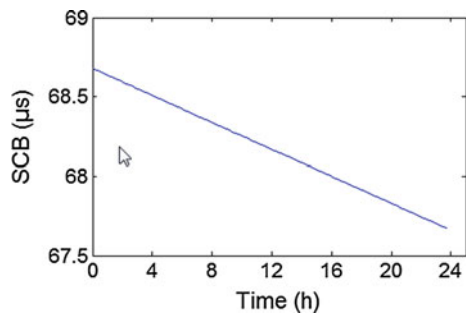
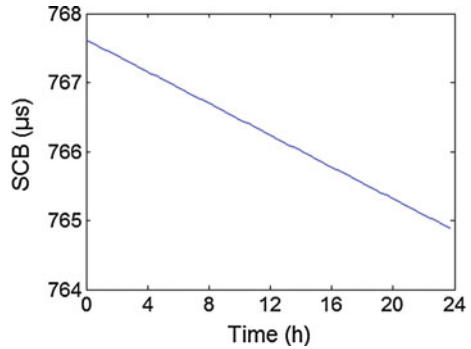


Fig. 2 The flow chart of wavelet neural network forecasting satellite clock bias

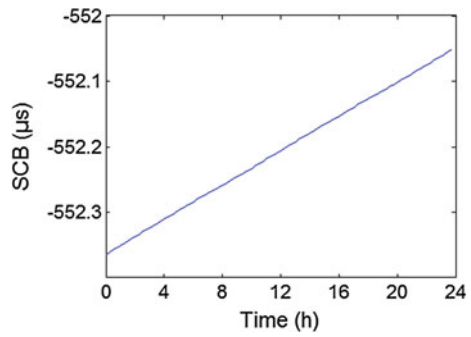
Fig. 3 C03 satellite clock bias



**Fig. 4** C09 satellite clock bias



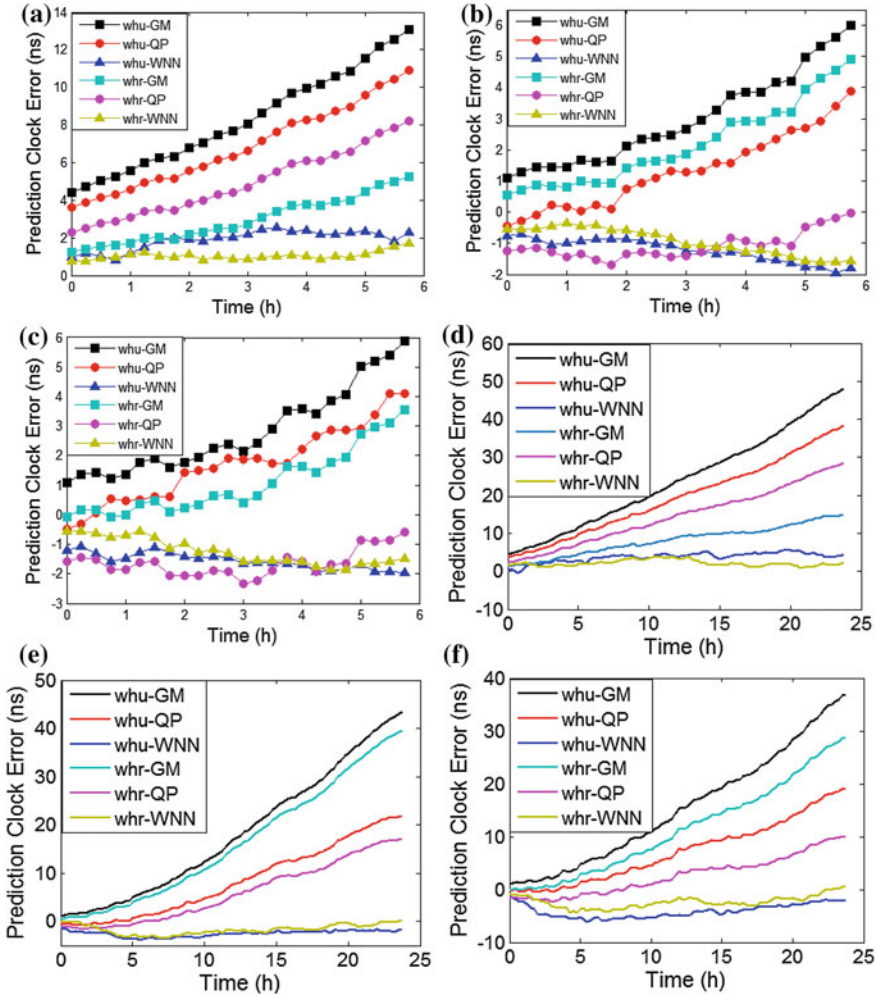
**Fig. 5** C11 satellite clock bias



### 4.1 6 h Prediction Scheme

This scheme uses the traditional quadratic polynomial, the gray model (GM), and WNN based on the first-order difference to forecast the clock bias in the next 6 h. Due to reference clock of the ultra-rapid, rapid, and the final product is different, so there exists a systematic deviation among the three types of products. In order to eliminate the impact of different reference clock, C01 is taken as the reference satellite of clock bias to carry out the precision evaluation [15]. The following Fig. 6a–c show the prediction errors and Table 1 shows the prediction precision of 6 h.

From the statistical results in Table 1 and the prediction errors from Fig. 6a–c, it can be seen that both the prediction accuracy and stability of the proposed method are obviously superior to those of GM and QP for the satellite C03. As for the C09 and C11 satellite, there is still a certain improvement, although the effect is not so significant as C03. It is because that GM and quadratic polynomial have a relatively considerable fitting ability in a short time, which lead the error accumulation is not obvious. The prediction accuracy and stability of rapid products are significantly better than those of the ultra-rapid product. It is because that the actual measurement of rapid products is only 3 h delay, while the rapid products’ delay time is 17 h. In this situation, the discrepancy of the amount of data and the number of stations will cause the difference of prediction accuracy.



**Fig. 6** a 6 h prediction error of C03. b 6 h prediction error of C09. c 6 h prediction error of C11. d 24 h prediction error of C03. e 24 h prediction error of C09. f 24 h prediction error of C11

**Table 1** The prediction precision in 6 h

Type/method		GM		QP		DWNN	
		RMS	Max	RMS	Max	RMS	Max
whu	C03	8.685	13.067	7.189	10.897	1.972	2.543
	C09	3.325	6.001	1.779	3.891	1.233	1.951
	C11	3.154	5.880	2.113	4.101	1.594	1.970
whr	C03	3.185	5.270	5.219	8.236	1.078	1.711
	C09	2.541	4.919	1.162	1.679	1.064	1.584
	C11	1.537	3.537	1.696	2.321	1.349	1.845



**Table 2** The prediction precision in 24 h

Type/method		GM		QP		DWNN	
		RMS	Max	RMS	Max	RMS	Max
whu	C03	27.068	48.234	21.769	38.390	3.881	5.931
	C09	22.623	43.416	11.179	21.910	2.567	3.872
	C11	19.005	37.058	9.310	18.939	4.558	6.105
whr	C03	8.937	14.891	16.240	28.566	2.272	3.863
	C09	20.496	39.622	8.552	17.018	1.988	3.961
	C11	14.335	28.723	4.475	10.010	2.634	4.448

### 4.2 24 h Prediction Scheme

In order to analyze the forecasting accuracy of WNN in 24 h, the paper chose all the 96 epoch to forecast next 24 h' clock basis. Figure 6d–f show the prediction errors and Table 2 shows the prediction precision of 24 h.

From Table 1, it can be seen that the cumulative error will increase with the extension of the forecast time and the stability will decrease obviously for quadratic polynomial and gray model. Nevertheless, the forecast precision and the stability of WNN are changed relatively small with the extension of the forecast time. This is related to the stability of the WNN and the good approximation ability for the nonlinear sequence. Making a difference between the adjacent epoch could improve the numerical stability of the clock bias sequence and weaken the influence of system error. The prediction accuracy and stability of rapid products is significantly better than the ultra-rapid product.

## 5 Conclusions

The characteristics of associative memory and wavelet analysis of the neural makes the WNN to have great approximation capability for nonlinear clock bias sequence. In this view, this paper proposed a WNN algorithm based on the first-order difference to modeling and prediction of BDS satellite clock bias. The ultra-rapid and rapid products are used in the prediction test for the proposed method and its accuracy is verified by the final products. The results show that the forecast precision of 6 h can reach 1–2 ns, and 24 h can reach 2–4.6 ns. The prediction accuracy of rapid products is better than the ultra-fast product, which conforms to the actual quality and precision level of the BeiDou satellite clock products. It achieves a higher accuracy than that of the gray model and quadratic polynomial model.

It should be noted that, the connection weight and the correlation parameter can be randomly initialized in the course of network learning. Through several training

adjustments and corrections, the different initial values will have an influence on the prediction results, but they are all in the acceptable accuracy. How to optimally determine the initial value is the point of the further study.

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