Modeling of Networked Control Systems Based on Multiple Communication Channels with Event-Triggering

Li-Ying Zhao, Qian Li and Jun Wang

Abstract The work here examines the stability problem of a linear system and controller design of the networked control system with transmission delays. An innovative model is constructed based on the discrete event-trigger communication mechanism and multiple communication channels between sensor and controller. Lyapunov stability theory and liberty matrix method are applied to derive sufficient conditions for the exponential stability and the design controller. Two simulation examples are given to show that the proposed theorems are superior to other event-trigger methods in some published literature.

Keywords Networked control systems (NCSs) · Discrete event-triggered scheme · Multiple communication channels · Exponentially stability · Linear matrix inequality

1 Introduction

Recently, Networked Control Systems (NCS) have become popular among scholars due to a range of advantages such as easy installation and high efficiency [1, 2]. The signal transmits through a common network medium rather than point-to-point wirings. Due to the insertion of a network, induced delay can decrease the stability of the systems. In an NCS, the signals of the plant are sampled periodically and then

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released to the controller through the network. If the sampling period is too small, the number of data packets will increase quickly, which can lead to an overloaded communication bandwidth. However, when there is little fluctuation of the measurement signals, fewer sampled signal are must be transmitted for control design task [3].

Therefore, the event-triggered mechanism has been shown to be more effective than the time-triggered method in terms of decreasing data transfer. As to event-triggered mechanism, the data packets are updated when required and needless signals can be avoided. Studies show that when the event-triggered condition is satisfied, the sampling is triggered in [4, 5]. The event-triggered condition is given as an inequality based on the state vector, when one side of the inequality exceeds a fixed threshold, the signals are sampled. The event-triggering mechanism has three advantages: (1) the mechanism can work to sample when the signals are necessary; (2) reduces the burden of the network bandwidth; (3) reduces the cost of the controller design in [5].

Most researchers in the NCSs field use a single communication channel for the transmission of information where time-delay and information missing are inevitable in [6, 7]. If the communication channel is improved, the conservation of the system can be reduced. The problem of stability analysis and controller designs are studied with multiple communication channels, which are used in NCS not concerned with event-trigger in [8, 9]. The work presented here focuses on designing the event-triggered controller for networked systems considering the multiple communication channels and time-delay. For convenient calculation, we have modeled the system with two channels and study its stability by a suitable Lyapunov functional and liberty matrix method. We than solve the controller matrix by LMI technique.

Notation in this paper is quite standard. ‘*’ denotes the entry of matrices implied by symmetry.

2 Problem Description

2.1 The Plant

Consider the following continuous-time linear system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0 \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state vector, and $y(t) \in R^r$ is the measurement output vector; system matrices $A \in R^{n\times n}$, $B \in R^{n\times n}$, $C \in R^{r\times n}$ are real constant matrices; and x_0 is the initial condition.

In this paper, the purpose is to design a state feedback controller $u(t) = Kx(t)$ to make the system stability, where K is a matrix to be determined later.

2.2 Event Generator

In this section, event generator consists of a register and a comparator [3]. The register stores the last released data $x(i_k h)$ ($i_k = 1, 2\ldots$), while the comparator is used to check whether the next sampled signal $x((i_k + j)h)$ ($j = 1, 2\ldots$) satisfies the following judgement algorithm:

$$\left\|\Omega^{\frac{1}{2}}[x((i_k+j)h)-x(i_kh)]\right\|_2 \leq \sigma\left\|\Omega^{\frac{1}{2}}x((i_k+j)h)\right\|_2 \tag{2}$$

where Ω is a positive definite matrix, $\sigma > 0$, and $\sigma \in [0, 1)$.

Remark 1 Under the constraint (1), the sampled state $x((i_k + j)h)$ will not be transmitted if it is satisfying the inequality (2). Assuming that h is the sampled period, release times are i_0h, i_1h..., where $i_0 = 0$ is the initial time, and i_kh is the sensor sampling instant. Supposing that the delay in the network communication is τ_k and $\tau_k \in (0, \bar{\tau})$. t_k ($k = 1, 2\ldots$) is the time instants at which the data packet $x(i_kh)$ arrives at the controller. It is concluded that $t_1 < t_2 < \cdots t_k\ldots$ because packet dropouts and packet disordered do not occur in the transport process. The signal transmitted to the controller can be written as $x(t) = x(i_kh)$, $t \in [t_k, t_{k+1})$.

Let:

$$\rho_k = \min\{j|t_k + jh \geq t_{k+1}, j = 1, 2\ldots\}. \tag{3}$$

The interval $[t_k, t_{k+1})$ can thus be written as $[t_k, t_{k+1}) = \cup_{j=1}^{\rho_k} I_j$, and $I_j = [t_k + (j-1)h, t_k + jh)$, $j = 1, 2, \ldots, \rho_k - 1$, $I_{\rho k} = [t_k + (\rho_k - 1)h, t_{k+1})$.

Two functions $\tau(t)$ and $e_k(t)$ in $[t_k, t_{k+1})$ can be described as

$$\tau(t) = \begin{cases} t - i_kh & t \in I_1 \\ t - i_kh - h & t \in I_2 \\ \vdots & \vdots \\ t - i_kh - (\rho_k - 1)h & t \in I_{\rho_k} \end{cases},$$

$$e_k(t) \in \begin{cases} x(i_kh) - x(i_kh) & t \in I_1 \\ x(i_kh) - x(i_kh + h) & t \in I_2 \\ \vdots & \vdots \\ x(i_kh) - x(i_kh + (\rho_k - 1)h) & t \in I_{\rho_k} \end{cases}.$$

Then $x(i_kh) = e_k(t) + x(t - \tau(t))$, $\tau_m \leq \tau_k \leq \tau(t) \leq h + \bar{\tau} = \tau_M$.

Remark 2 From the definition of $e_k(t)$ and the triggering Algorithm (2), it can be seen that, for $t \in [t_k, t_{k+1})$,

$$e_k^T(t)\Omega e_k(t) \leq \sigma x^T(t - \tau(t))\Omega x(t - \tau(t)). \tag{4}$$

2.3 Multiple Channels

Since the system transmits the useful information through multiple channels, we consider two channels to transmit the data packet and Eq. (1) can be rewritten as

$$\dot{x}(t) = Ax(t) + BKM_\sigma(x(t - \tau(t)) + e_k(t)), \tag{5}$$

where $M_\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}$, $\sigma_i = \begin{cases} 1 & \textit{though the ith channel} \\ 0 & \textit{though the other channel} \end{cases}$ $(i = 1, 2)$, $P(\sigma_1 = 1) = p$, $P(\sigma_2 = 1) = 1 - p$, $K \in R^{1\times 2}$.

Remark 3 As shown in Fig. 1, the closed-loop NCS is composed of a plant, a sensor, an Event-trigger mechanism, a controller and an actuator. Two channels are established which change the original characteristics of single channel data transmission. A number of sampled data packets will be discarded under the discrete event-triggering mechanism, which will make the network loads reduced greatly. If we can improve the channel, a good quality of the network can be possibly ensured.

2.4 Main Lemmas

Definition 1 [10] The system (5) is said to be exponentially stable if for $\forall \varepsilon > 0$, there exist scalars $\alpha > 0$, $\beta > 0$, such that

$$E\left\{\|x(t)\|^2\right\} \leq \alpha e^{\beta t} \sup_{-2\bar{\tau} \leq s \leq 0} E\left\{\|\phi(s)\|^2\right\}$$

where $\phi(.)$ is the initial function in Eq. (5), $x(t) = \phi(t)$, $t \in [-\bar{\tau}, 0]$.

Lemma 1 [5] *For positive matrices* $R > 0, X > 0, \rho$ *is any chosen constant, we have*

$$-XR^{-1}X \leq \rho^2 R - 2\rho X \tag{6}$$

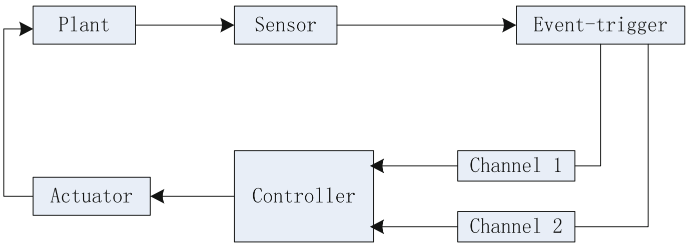

Fig. 1 The diagram for event-triggered NCS structure

Lemma 2 [11] *For any vectors* $x, y \in R^n$, *and positive definite matrix* $Q > 0$, *then the inequality holds*:

$$-2x^T y \leq x^T Q x + y^T Q^{-1} y \tag{7}$$

3 Stability Analysis and Controller Design

Theorem 1 *For given* p, σ *and matrix* $K \in R^{1 \times 2}$, *the nominal system is exponentially stable if there exist positive matrices* $P_1 > 0$, $P_2 > 0$, $\Omega > 0$, $R > 0$.

$$\begin{bmatrix} W & \Phi_{12} & \Phi_{13} & \sqrt{\tau_M} N & \sqrt{\tau_M} M \\ * & -p^{-1} R^{-1} & 0 & 0 & 0 \\ * & * & -(1-p)^{-1} R^{-1} & 0 & 0 \\ * & * & * & -R & 0 \\ * & * & * & * & -R \end{bmatrix} < 0 \tag{8}$$

where $W = \Phi_{11} + \Gamma + \Gamma^T$, $\Gamma = [N\ M\text{-}N\ {-M}\ 0]$,

$$\Phi_{11} = \begin{bmatrix} pP_1A + (1-p)P_2A + (pP_1A + (1-p)P_2A)^T & pP_1BKE_1 + (1-p)P_2BKE_2 & 0 & pP_1BKE_1 + (1-p)P_2BKE_2 \\ * & \sigma\Omega & 0 & 0 \\ ** & -Q & 0 & \\ ** & * & -\Omega & \end{bmatrix},$$
$$\Phi_{1i} = \begin{bmatrix} \sqrt{\tau_M} A & \sqrt{\tau_M} BKE_i & 0 & \sqrt{\tau_M} BKE_i \end{bmatrix}^T, \quad i = 1, 2, \ N = \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T \end{bmatrix}^T,$$
$$M = \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T \end{bmatrix}^T \quad E_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T \ E_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}^T.$$

Proof Consider the following functional:

$V(t) = V_1(t) + V_2(t) + V_3(t)$, $V_1(t) = x^T(t) P_\sigma x(t)$, $V_2(t) = \int_{t-\tau_M}^{t} x^T(s) Q x(s) ds$, $V_3(t) = \int_{t-\tau_M}^{t} \int_{s}^{t} \dot{x}^T(v) R \dot{x}(v) dv ds$. Using free weighting matrix method [9], the following equation holds:

$$2\xi^T(t) M \left[x(t - \tau(t)) - x(t - \tau_M) - \int_{t-\tau_M}^{t-\tau(t)} \dot{x}(s) ds \right] = 0,$$

$$2\xi^T(t) N \left[x(t) - x(t - \tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s) ds \right] = 0,$$

where N, M are appropriate dimensions matrices, and by Lemma 2,

$$-2\xi^T(t)M\int_{t-\tau_M}^{t-\tau(t)}\dot{x}(s)ds \leq (\tau_M-\tau(t))\xi^T(t)MR^{-1}M^T\xi(t)+\int_{t-\tau_M}^{t-\tau(t)}\dot{x}^T(s)R\dot{x}(s)ds,$$

$$-2\xi^T(t)N\int_{t-\tau(t)}^{t}\dot{x}(s)ds \leq \tau(t)\xi^T(t)NR^{-1}N^T\xi(t)+\int_{t-\tau(t)}^{t}\dot{x}^T(s)R\dot{x}(s)ds.$$

From Eq. (8), we can determine that $E(\dot{V})<0$, which means that there exists a constant $\lambda > 0$, such that

$$E(\dot{V}) \leq -\lambda E\{\xi^T(s)\xi(s)\}$$

Using a method similar to that of [12], we have

$$E\left(x^T(t)x(t)\right) \leq \alpha e^{-\varepsilon t}\sup_{-\bar{\tau}\leq s\leq 0} E\left\{\|\phi(s)\|^2\right\} \tag{9}$$

Theorem 2 *For given scalars σ, ρ, if there exist positive matrices* $X > 0$, $\tilde{R} > 0$, $\Omega > 0$, N_i, M_i $(i = 1, 2\ldots8)$, $\widetilde{N}_i, \widetilde{M}_i$ $(i=1,3)$ *with appropriate dimension*

$$\begin{bmatrix}
\Xi_{11} & \Xi_{12} & -\tilde{M}_1+\tilde{N}_3^T & \Xi_{14} & \sqrt{\tau_M}XA^T & \sqrt{\tau_M}XA^T & \sqrt{\tau_M}\tilde{N}_1 & \sqrt{\tau_M}\tilde{M}_1 \\
* & \Xi_{22} & -(M_6+M_7-N_7)^T & (M_4-N_4)^T & \sqrt{\tau_M}E_1^TK^TB^T & \sqrt{\tau_M}E_2^TK^TB^T & \sqrt{\tau_M}N_6^T & \sqrt{\tau_M}M_6^T \\
** & -\tilde{M}_3-\tilde{M}_3^T-\tilde{Q} & -M_8 & 0 & 0 & \sqrt{\tau_M}\tilde{N}_3 & \sqrt{\tau_M}\tilde{M}_3 & \\
** & * & -\Omega & \sqrt{\tau_M}E_1^TK^TB^T & \sqrt{\tau_M}E_2^TK^TB^T & \sqrt{\tau_M}N_8^T & \sqrt{\tau_M}M_8^T & \\
** & ** & p^{-1}(\rho^2\tilde{R}-2\rho X) & 0 & 0 & 0 & & \\
** & ** & * & (1-p)^{-1}(\rho^2\tilde{R}-2\rho X) & 0 & 0 & & \\
** & ** & ** & -\tilde{R} & 0 & & & \\
** & ** & ** & * & -\tilde{R} & & &
\end{bmatrix}<0 \tag{10}$$

where

$$\Xi_{11}=(1-p)\varepsilon AX+(1-p)\varepsilon XA^T+pAX+pXA^T+\tilde{Q}+\tilde{N}_1+\tilde{N}_1^T,$$
$$\Xi_{12}=(1-p)\varepsilon BKE_2+pBKE_1+M_5-N_5+N_6,$$
$$\Xi_{14}=(1-p)\varepsilon BKE_2+pBKE_1+N_8$$
$$\Xi_{22}=\sigma\Omega+M_2-N_2+(M_2-N_2)^T.$$

To solve the linear matrix inequality in Eq. (10), we can get the feedback matrix K.

Proof In Eq. (8), making $P_2 = \varepsilon P_1$, defining matrix $X=P_1^{-1}$, and multiplying (8) left and right by $diag\{X, I, X, 1, 1, 1, X, X\}$. New matrix variables can be defined as:

$$\tilde{Q} = XQX, \tilde{R} = XRX, M_5 = XM_1, N_5 = XN_1, M_6 = XM_2, N_6 = XN_2,$$
$$M_7 = XM_3, N_7 = XN_3, M_8 = XM_4, N_8 = XN_4, \tilde{M}_i = XM_iX\ (i = 1, 3), \tilde{N}_i = XN_iX\ (i = 1, 3).$$

Using Lemma 1, Eq. (10) will be obtained.

4 Two Applications to Different Dimension Systems

Example 1

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

When $\sigma = 0.3$, though Matlab program, we can get K = [−0.4696 −0.6150].

Remark 4 From Fig. 2, two curves $x_1(t)$ and $x_2(t)$ eventually can merge together, so the system will reach the stability if the controller insert to the open loop system. Besides, when $p = 0.3$, the value of $\tau_M = 6.7996$ is larger than the value of $\tau_M = 5.9976$ when $p = 0.5$.

Example 2

$$A = \begin{bmatrix} -1 & -5 & 0 \\ 0 & -0.5 & 1 \\ -3 & -1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 0.8 & 0.5 \\ 1.4 & -1.5 & 1 \\ 0.9 & 0.8 & -1.7 \end{bmatrix}.$$

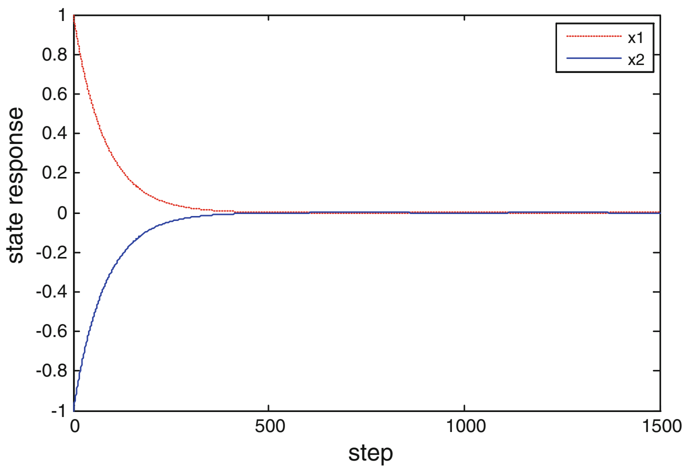

Fig. 2 State response of the system

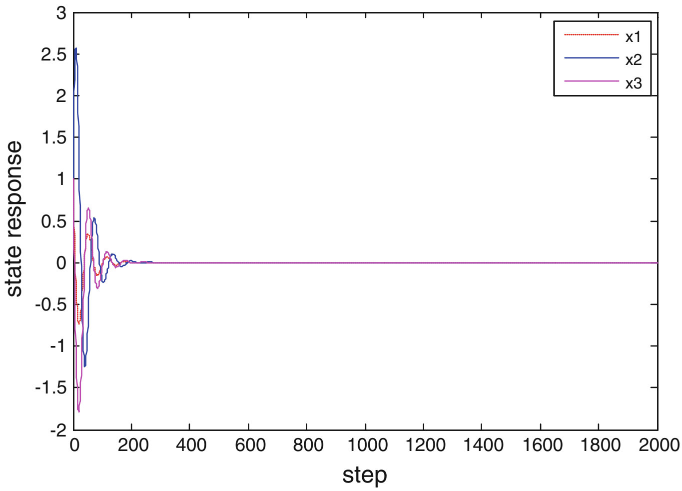

Fig. 3 State response of the system

Using Theorem 2 with $\sigma = 0.3$, $p = 0.3$, the upper bound in this system is computed as 0.4225 and corresponding controller gain and the event-trigger matrix are

$$K = [-26.4253 \quad -13.0952], \quad \text{and}$$
$$\Omega = 1.0e^{+03}\begin{bmatrix} 1.4471 & -0.1002 & 0.0124 \\ -0.1002 & 1.0006 & -0.0011 \\ 0.0124 & -0.0011 & 0.9836 \end{bmatrix}.$$

Remark 5 From Fig. 3, three curves eventually can merge together, which demonstrates the usefulness of the controller design for NCS with multiple communication channels. Besides, theorems in this paper are also applied for higher dimension.

5 Conclusion

In this study, an event-trigger sampling model has been constructed which can determine the transfer of the sampled signals. The new communication channel constructed in this scheme can avoid both unnecessary sampling signal transmission and bandwidth limitations. Criteria for stability and control design have been obtained based on Lyapunov stability method. Finally, the simulation examples show that the theorems in the work can deal with the higher dimension control problem and make the system stability, which are effectiveness and usefulness of the proposed scheme.

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