

An Algorithm for Image Denoising Based on Adaptive Total Variation

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Abstract Although the traditional TV (Total Variation) model owns excellent image denoising ability, there are staircase effect problems for TV model. In this article, two detection operators for staircase effect problem are proposed. The staircase effect problem can be solved effectively by introducing two operators into traditional TV model. On the basis, it proposes an adaptive total variation model for image denoising. When dealing with image edge, it can still use the traditional TV model. Its purpose is to maintain the advantages in edge protection for TV model. When it is in the smooth area of image, linear diffusion is used to avoid the staircase effect.

Keywords Image denoising · Staircase effect · Detection operator · Total variation

1 Introduction

In the field of image processing, image denoising technology has been the focus of the study. In recent years, the image denoising methods based on partial differential equation had made great breakthrough. In 1990, Perona and Malik proposed the image denoising method based on anisotropic diffusion, called PM model [1]. The model used Laplace operator instead of the traditional nonlinear operator. Though it had achieved good denoising effect, there were serious step effect problems. In order to remedy the defects of the PM model, You and Kaveh proposed a four order partial differential equation, named Y-K model. But the Y-K model had brought the new “dot effect” problems [2]. And the partial differential equation of higher order increases the computational complexity greatly. In 1992, Rudin, Osher and Fatemi proposed the image denoising methods of total variation, called TV model [3].

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The TV model was a functional minimization problem substantially. It can control the image to diffusion in the gradient direction orthogonal. The TV model made a qualitative leap compared with the PM model and the Y-K model. Although the TV model showed strong advantage in the image denoising, it also appeared serious staircase effect problems in the smooth area.

2 Traditional TV Model

Traditional TV(Total Variation) model was proposed by Rudin, Osher and Fatemi. It was also known as the ROF model. And it was a image denoising model based on partial differential equation [4]. The equation is as follows:

$$E_{TV} = \int_{\Omega} \left(|\nabla u| + \frac{1}{2} \lambda (u - u_0)^2 \right) dx dy \quad (1)$$

This equation can be further expanded into the form of the following:

$$\frac{\partial u}{\partial t} = \frac{1}{\sqrt{(u_x)^2 + (u_y)^2}} \frac{u_{xx}(u_y)^2 - 2u_x u_y u_{xy} + u_{yy}(u_x)^2}{(u_x)^2 + (u_y)^2} - \lambda(u - u_0) \quad (2)$$

$\frac{u_{xx}u_y^2 - 2u_x u_y u_{xy} + u_{yy}u_x^2}{u_x^2 + u_y^2}$ is the second order derivative along the tangent direction of the isophotes of image. $u(x, y)$ and $1/\sqrt{u_x^2 + u_y^2}$ is the diffusion coefficient. It can be found by the above formula that the TV model can only diffuse in the gradient orthogonal direction. It will cause the serious staircase effect in image smooth regions inevitably.

3 ATV (Adaptive Total Variation) Model

In order to overcome the staircase effect problem of the traditional TV model, two detection operator which is created from the steerable filter are proposed.

3.1 Steerable Filter

Oriented filters are useful in many early vision and image processing tasks, such as texture analysis, edge detection, image data compression, motion analysis, and image enhancement [5–8]. Here the steerable filters are designed in quartering pairs to allow adaptive control over phase as well as orientation. There are four

applications below: Orientation and phase analysis, angularly adaptive filtering, edge detection and shape from shading [9]. Edge detection is the key factor for image restoration. Here, the steerable energy $E(\theta)$ can detect edges effectively by using the n th derivative of a Gaussian and its Hilbert transform [10].

$$E_n(\theta) = [G_n^{\theta}]^2 + [H_n^{\theta}]^2 \tag{3}$$

In this article, $n = 2$, $0^\circ \leq \theta \leq 360^\circ$ and G is a Gaussian function.

Thus, a steerable quartering pair based on the frequency response of the second derivative of the Gaussian G_2 and its Hilbert H_2 is designed as the following functions:

$$\begin{aligned} G_2^\theta(x, y) &= k_1(\theta) * G_2^0(x, y) + k_2(\theta) * G_2^{\frac{\pi}{2}}(x, y) + k_3(\theta) * G_2^{\frac{2\pi}{3}}(x, y) \\ H_2^\theta(x, y) &= l_1(\theta) * H_2^0(x, y) + l_2(\theta) * H_2^{\frac{\pi}{2}}(x, y) + l_3(\theta) * H_2^{\frac{\pi}{3}}(x, y) + l_4(\theta) * H_2^{\frac{3\pi}{4}}(x, y) \end{aligned} \tag{4}$$

where $k_j(\theta)$, $l_j(\theta)$ are the interpolation functions of G_2 and H_2 , they are

$$\begin{aligned} k_j(\theta) &= \frac{1}{3} * [1 + 2 * \cos(2 * (\theta - \theta_j))] \\ l_j(\theta) &= \frac{1}{4} * [2 * \cos(2 * (\theta - \theta_j)) + 2 * \cos(3 * (\theta - \theta_j))] \end{aligned} \tag{5}$$

Image information features of each direction can be extracted based on principle steerable filter. Figure 1 is the result that the simple geometry are dealed after fixed direction filtering and steerable filtering. Among them, (a) is as the original image, (b) is for Canny operator, (c) is for 0° fixed direction filtering, (d) is for 90° fixed direction filtering, and the final picture (e) is filtered through a steerable filter. Although figure (b) after Canny operator treatment can identify image edge information greatly after, the square has appeared double edge. It makes a big deviation in the details. Obviously, the details and features will always be lost in a certain single direction, which is not beneficial to the analysis of the image, such as Figure (c) and (d). In contrast, (e) has almost no loss of detail after the steerable filter is. It shows all the edge features of the original image excellently. It can be seen from this experiment that the steerable filter has great advantages in edge detection.



Fig. 1 Comparison chart for Different algorithms. **a** Original image, **b** Canny operator, **c** 0° fixed direction filtering, **d** 90° fixed direction filtering, **e** steerable filter

3.2 Detection Operator

The spectral power $E(\theta)$ of the steerable filter is the orientation strength along a particular direction by the squared output of a quartering pair of band pass filters steered. The responses of the same pixel are different by different phases. Here the max value by the phase is needed which is the main direction. As the maximum, where

$$m_{(x,y)} = \max(E_{(x,y)}(\theta)), \quad 0^\circ \leq \theta \leq 360^\circ \quad (6)$$

Based on the new indicator m from the steerable filter, we present a natural way to improve the TV model. The STV model is presented as follows:

$$AE_{TV} = \int_{\Omega} \left(|\nabla u|^{q(\bar{m})} + \frac{1}{2} \lambda(\bar{m})(u - u_0)^2 \right) dx dy \quad (7)$$

where the functions $q(\bar{m})$ and $\lambda(\bar{m})$ are as follows [11]:

$$q(\bar{m}) = 1 + \sqrt{\bar{m}}, \quad \lambda(\bar{m}) = k * \sqrt{\bar{m}}, \quad \bar{m} \in (0, 1], \quad k > 0 \quad (8)$$

In this way, the TV model can achieve adaptive by changing their coefficient. And when $\bar{m} \in (0, 1]$, $q(\bar{m}) \in [1, 2)$, $\lambda(\bar{m}) \in (0, k]$ when $q(\bar{m}) \rightarrow 1$, $\lambda(\bar{m}) \rightarrow k$. This model is close to TV model; when $q(\bar{m}) \rightarrow 2$, $\lambda(\bar{m}) \rightarrow 0$. At this time, the model is close to the least squares method.

Thus, it can achieve good denoising ability and edge preserving by using the TV model in image edge region. And it can effectively solves the problem of the step effect by using the least squares method in the smooth region.

4 Comparative Analysis and Experimental Results

There are a large number of “false edge” in the ramp images. This kind of image will have a relatively serious staircase effect in the process of denoising [12]. Therefore, the ramp image is a typical example for staircase effect testing. Here select a typical ramp images as the original image. Figure 2 is a comparison diagram of using two algorithms for image denoising.

The figure (a) is the original image, figure (b) is Noisy image after adding Gauss white noise, figure (c) is the denoised image for TV model diagram. It is easy to see that the image has been very clear. But there are obvious step effect in the image. figure (d) is the denoised image for adaptive TV model. It not only avoid the staircase effect partly, but also has strong denoising ability. It is one of the most ideal method.

PSNR (Peak Signal to Noise Ratio, PSNR) is the most popular objective evaluation method. It is widely used for measuring image quality. The bigger of the

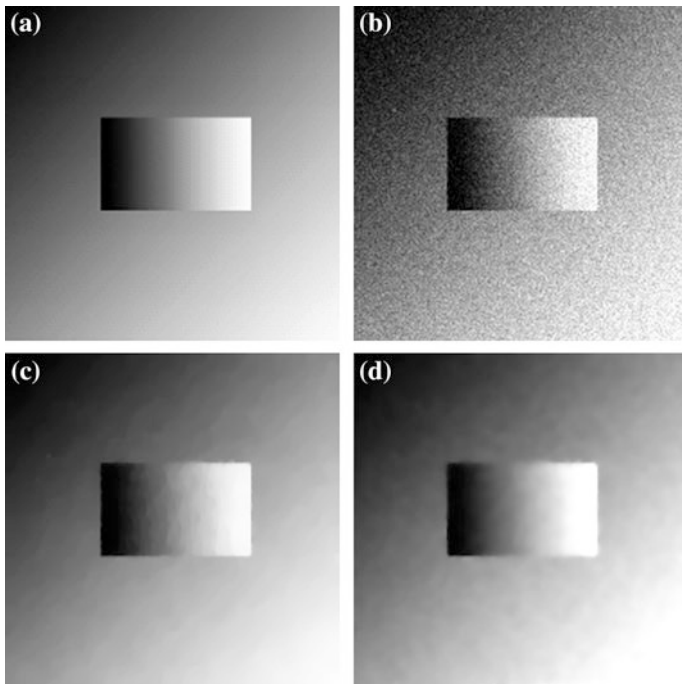


Fig. 2 The comparison diagram of using two algorithms for image denoising. **a** Original image, **b** Noisy image ($\sigma = 20$), **c** TV model, **d** ATV model

PSNR value, the better of the image quality, it mean less distortion. MSSIM (mean structure similarity, MSSIM) is also an evaluation method of image quality. It can evaluate the content similarity degree of two images. Table 1 gives the MSSIM and PSNR value for Fig. 1. It is not difficult to find that the MSSIM and PSNR for ATV model are higher than the traditional TV model.

The noise has little effect to image when the Gauss white noise $\sigma = 20$. Table 2 gives a comparison of MSSIM and PSNR for TV model and ATV when the image

Table 1 MSSIM and PSNR for TV model and ATV model

Model	MSSIM	PSNR	Iterations
TV	0.901	29.001	55
ATV	0.915	29.430	80

Table 2 MSSIM and PSNR of different noise value for TV model and ATV model

Noise	Model	MSSIM	PSNR	Iterations
$\sigma = 20$	TV	0.901	29.001	55
	ATV	0.915	29.430	80
$\sigma = 30$	TV	0.893	29.123	85
	ATV	0.900	29.334	100
$\sigma = 40$	TV	0.898	28.968	90
	ATV	0.899	28.812	100

is in different noise environment. Experiments show that, although in strong noise environment, the adaptive total variation model not only has great image denoising ability, but also deal step effect excellently.

5 Conclusion

The results show that the proposed new method achieves a great balance in denoising ability and avoiding the staircase effect after a number of experiments. However, it has some limitations in selecting parameters for some partial differential equations in this paper. Therefore, it is very necessary to conduct more experiments and select more accurate parameter values in the follow-up study.

References

1. Perona P, Malik J (1990) Scale-space and edge detection using anisotropic diffusion. *Pattern Anal Mach Intell* 12(7):629–639
2. You Y, Kaveh M (2000) Fourth order partial differential equations for noise removal. *Image Process* 9(10):1723–1730
3. Rudin L, Osher S, Fatemi E (1992) Nonlinear total variation based noiseremoval algorithms. *Physica D* 60:259–268
4. Xu J (2006) Iterative regularization and nonlinear inverse scale space methods in image restoration. University of California, Los Angeles
5. Kass M, Witkin A (1987) Analyzing oriented patterns. *Comp Vision Graph Image Process* 37:362–385
6. Knutsson H, Granlund GH (1983) Texture analysis using two-dimensional quadrature filters. In: IEEE computer society workshop on computer architecture for pattern analysis and image database management, pp 206–213
7. Knutsson H, Wilson R, Granlund GH (1983) Anisotropic nonstationary image estimation and its applications: Part 1—restoration of noisy images. *IEEE Trans Commun* 31(3):388–397
8. Zucker SW (1985) Early orientation selection: tangent fields and the dimensionality of their support. *Comp Vision Graph Images Process* 32:74–103
9. Freeman WT, Adelson EH (1991) The design and use of steerable filters. *IEEE Trans Pattern Anal Mach Intell* 13(9):891–906
10. Yokono JJ, Poggio T (2004) Oriented filters for object recognition: an empirical study. In: IEEE international conference on automatic face and gesture recognition processing, pp 755–760
11. Guo X, Wu R (2014) Application research on adaptive total variation image denoising. *J Hebei North Univ (Nat Sci Ed)* 30(5):21–24
12. Chan T, Esedpglu S, Park F et al (2005) Recent developments in total variation image restoration. Technique report, UCLA