Multi-objective Optimization-Based Design of Robust Fractional-Order $PI^{\lambda}D^{\mu}$ Controller for Coupled Tank Systems

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Abstract In this paper, optimal control of coupled tank systems has been proposed using H_{∞} fractional-order controllers. Controller tuning has been posed as multi-objective mixed sensitivity minimization problem for tuning the fractional-order PID (FOPID) controllers and multi-objective variants of bat algorithm (MOBA) and differential evolution (MODE) has been used for optimization. Use of fractional-order controllers provides better characterization of dynamics of the process and their tuning using multi-objective optimization helps in attaining the robust trade-offs between sensitivity and complementary sensitivity. Both the FOPID controllers tuned with MOBA and MODE present robust behavior to external disturbance and the compared results show that MOBA-tuned controller presents efficient tracking of the reference.

Keywords Fractional-order control · Differential evolution · Bat algorithm · Coupled tank systems · Multi-objective optimization

1 Introduction

Most real-world control engineering problems are multi-objective in nature and it is challenging to meet the design constraints. In control systems, the objectives range from time domain specifications like rise time, percentage overshoot, ISE, ITSE, etc., to frequency domain requirements such as gain and phase margin, sensitivity, etc. Most of these objectives are usually conflictive in nature. So, a trade-off among several objectives has to be achieved for optimal performance. Thus, the controller tuning can be posed either as an aggregate objective function (AOF) or as generate-first choose-later (GFCL) multi-objective optimization problem [1].

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In this paper, a fractional-order $PI^{\lambda}D^{\mu}$ controller has been optimized considering multiple robustness objectives. PID controllers because of their simplicity are most popular and effective controllers and account for the 90 % of the total controllers used in industry today [2]. Fractional-orders systems provide a better characterization of the real dynamic processes [3]. Podlubny [4] introduced the concept and demonstrated the effectiveness of fractional-order $PI^{\lambda}D^{\mu}$ controllers (FOPID) over standard integer-order PID controllers. The extra degrees of freedom introduced by the fractional-order integrator λ and fractional-order differentiator μ increase the difficulty in finding their optimal gains. Obtaining the optimal gains for FOPID such that it satisfies the various time domain and frequency domain characteristics is of great significance both in theoretical and practical terms.

The optimized FOPID controller has been implemented on a coupled tank liquid level control system. In coupled tank systems, liquid level control in multiple connected tanks is a nonlinear control problem and is center to various diverse industrial establishments ranging from petrochemical, wastewater treatment to nuclear power generation. Several researchers globally have applied different control methodologies for the same, such as, second-order SMC [5], FOPID [6], bat algorithm optimized PID [7], multi-objective GA-tuned PID [8], and fuzzy PID controllers [9].

The work focuses on the optimization-based design of FOPID controllers using multi-objective bat algorithm (MOBA) and multi-objective differential evolution (MODE). The optimization problem has been posed as a mixed sensitivity problem, in which the H_{∞} norm of the sensitivity $S(s)$ and complementary sensitivity $T(s)$ has been used to define the multi-objective problem. Thus facilitating obtaining optimal FOPID parameters such that the system offers robust behavior to model uncertainties and external disturbances.

The paper has been organized into the following sections; Sect. 2 provides the mathematical modeling of the coupled tank liquid level control system. In Sect. 3, the prerequisites of fractional-order calculus and FOPID controllers have been discussed. Section 4 deals with defining the mixed sensitivity optimization problem. Sections 5 and 6 deal with basics of multi-objective bat algorithm and multi-objective differential evolution. In Sect. 7 several results obtained have been discussed followed by conclusions and references.

2 Mathematical Modeling of Coupled Tank System

The schematic representation of the coupled tank liquid level control system is shown in Fig. 1. Considering the flow balance equations, the nonlinear mathematical model of the system has been derived as under [7, 8].

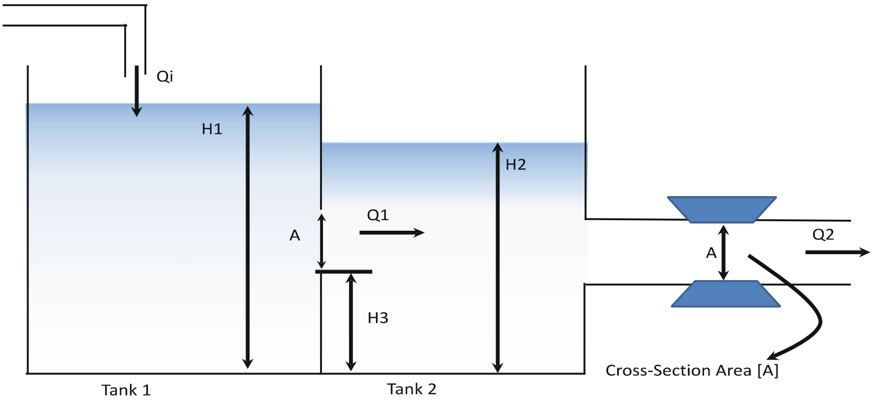

Fig. 1 Schematic representation of coupled tank liquid level system

For tanks 1 and 2:

$$Q_i - Q_1 = A\frac{dH_1}{dt} \quad \text{and} \quad Q_1 - Q_2 = A\frac{dH_2}{dt} \tag{1}$$

where, H_1, H_2 are the heights of tank 1 and 2; A is the cross-sectional area; Q_1 and Q_2 are the rates of flow of liquid.

Equation 2 shows the steady state representation of the system:

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{pmatrix} -k_1/_A & k_1/_A \\ k_1/_A & -(k_1+k_2)/_A \end{pmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1/_A \\ 0 \end{bmatrix} q_i \tag{2}$$

Transfer function of the system given by Eq. 3 has been obtained by taking the Laplace transformation of Eq. 2.

$$G(s) = \frac{1/_{k_2}}{\left(\frac{A^2}{k_1 \cdot k_2}\right) \cdot s^2 + \left(\frac{A(2 \cdot k_1 + k_2)}{k_1 \cdot k_2}\right) \cdot s + 1} = \frac{1/_{k_2}}{(T_1 \cdot s + 1)(T_2 \cdot s + 1)} \tag{3}$$

where,

$$T_1 + T_2 = A\frac{2 \cdot k_1 + k_2}{k_1 \cdot k_2} \quad , \quad T_1 \cdot T_2 = \frac{A^2}{k_1 \cdot k_2}$$
$$k_1 = \frac{\alpha}{2\sqrt{H_1 + H_2}} \quad \text{and} \quad k_1 = \frac{\alpha}{2\sqrt{H_2 + H_3}}$$

Following parameters have been considered as constants for obtaining Eq. 4; H_1 = 18 cm, H_2 = 14 cm, H_3 = 6 cm, α = 9.5 (constant for coefficient of discharge), A = 32.

$$G(s) = \frac{0.002318}{s^2 + 0.201 \cdot s + 0.00389} \tag{4}$$

3 Fractional-Order PID Controllers

Non-integer integrals and derivatives provide better characterization of the dynamic systems and the use of fractional-order calculus in control systems has an ample potential to change the way we model systems and controllers [3]. Fractional-order PID controllers (FOPID) introduced by Podlubny [4] are more flexible and offer better performance in achieving the control objectives. The transfer function of the FOPID controllers is given by Eq. 5.

$$K_{\mathrm{frac}} = k_{\mathrm{P}} + \frac{k_I}{s^{\lambda}} + k_{\mathrm{D}} \cdot s^{\mu} \tag{5}$$

Optimal design of the FOPID controllers involves the design of three parameters k_{P}, k_{I}, and k_{D} and two orders λ and μ, the values of which can be non-integer [10].

4 Mixed Sensitivity Multi-objective Problem Formulation

Mixed sensitivity optimization allows achieving simultaneous trade-offs between performance and robustness. Reduction of the sensitivity $S(j\omega)$ ensures disturbance rejection and complementary sensitivity $T(j\omega)$ reduction handles the robustness issues and the minimization of control effort [11]. The mixed sensitivity optimization problem is given by Eq. 6.

$$\underset{K \quad \text{stabalizing}}{\min} \left\| \begin{matrix} w_1 \cdot S(j\omega) \\ w_2 \cdot T(j\omega) \end{matrix} \right\|_{\infty} \tag{6}$$

In the work presented in this paper, mixed sensitivity reduction has been posed as aggregate objective function (AOF) given by Eq. 7:

$$\|w_1 \cdot S(j\omega)\|_{\infty} + \|w_2 \cdot T(j\omega)\|_{\infty} \le 1 \tag{7}$$

5 Multi-objective Optimization Using Bat Algorithm

Bat algorithm (BA) was introduced by X.S. Yang in 2010 and extended it for solving multiple objectives in 2012 [12]. Bat algorithm mimics the echolocation behavior of bats, which they use to locate their prey and differentiate between different insects even in complete darkness. In the initial population, each bat updates it position using echolocation in which echoes are created by loud ultrasound waves which are received back with delay and specific preys are characterized by specific sound levels. Following equations characterize the bat motion, i.e., x_i is the position, velocity v_i and the new updated position x_i^t, and velocities v_i^t at time t.

$$f_i = f_{\min} + (f_{\max} - f_{\min}) \cdot \beta$$

$$v_i^{t+1} = v_i^t + \left(x_i^t - x_*\right) \cdot f_i$$

$$x_i^{t+1} = x_i^t + v_i^t$$

where, $\beta \in [0, 1]$ is a uniformly distributed random vector, $x*$ is the current global best location of all n bats in population. Initially, uniformly derived frequency from $[f_{\min}, f_{\max}]$ is randomly assigned to each bat.

When a bat finds its prey, the loudness A_i usually decreases and the rate of pulse emission r_i increases. Initial loudness A_0 can be set to any value; here A_0 is taken as 1 and considering that bat has bound its prey $A_{\min}$ is taken as 0.

$$A_i^{t+1} = \alpha \cdot A_i^t, \quad r_i^t = r_i^0\left(1 - e^{-\lambda \cdot t}\right)$$

α and γ are constants and for $0 < \alpha < 1$ and $\gamma > 0$,

$$A_i^t \to 0, \quad r_i^t \to r_i^0, \quad \text{as} \quad t \to \infty$$

For defining the objective function as mixed sensitivity, the problem has been expressed as aggregate objective function (AOF), i.e., expressing all objectives J_k as weighted sum given by Eq. 8. As mixed sensitivity problem has been considered, so K is taken as 2.

$$J = \sum_{k=1}^{K} w_k \cdot J_k, \quad \sum_{k=1}^{K} w_k = 1 \tag{8}$$

Since, to introduce diversity in population of bats, loudness and pulse emission rates have been generated randomly and while searching for the solution, these values are updated, only if the new solution shows scope of improvement while converging toward the global minima.

6 Multi-objective Optimization Using Differential Evolution

Differential evolution (DE) is a population-based stochastic direct search optimization algorithm and is based on the use of similar operators like those of genetic algorithm; crossover, mutation, and selection. DE is advantageous in several aspects such as; it can handle nonlinear and multimodal cost functions, less computationally intense, has few control variables so it is easy to use and has very good convergence properties [13].

DE pivots on mutation operator for generating better solutions, whereas GA is dependent on crossover. In DE for searching and selection of global solution in the prospective search space, mutation operation is used. In pursuit of better solutions, scattered crossover among the parents efficiently intermix the knowledge about successful combinations [14].

The optimization index for mixed sensitivity optimization problem has also been formulated as weighted sum of objective functions given by Eq. 9.

$$J = \min_{K \ \text{stabalizing}} \left\{ \|S(j\omega)\|_\infty + \|T(j\omega)\|_\infty \right\} \tag{9}$$

In this work, a variant of DE with jitter [15] has been used for optimization.

6.1 Mutation

A mutation vector produced for each target vector $x_{i,G}$ is given as in below equation.

$$v_{i,G+1} = x_{i,G} + K \cdot \left(x_{r1,G} - x_{i,G}\right) + F \cdot \left(x_{r2,G} - x_{r3,G}\right)$$

where $r_1,\ r_2,\ r_3 \in \{1,\ 2\ldots\ \mathrm{NP}\}$ are different from each other and are randomly generated, F is the scaling factor and K is combination factor.

6.2 Crossover

Crossover generates the trial vector $u_{ji,G+1}$ by mixing the parent with mutated vector and is given by equation as

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{if} \quad \left(rnd_j \le \mathrm{CR}\right) \quad \text{or} \quad j = rn_i \\ q_{ji,G} & \text{if} \quad \left(rnd_j > \mathrm{CR}\right) \quad \text{or} \quad j \ne rn_i \end{cases}$$

where, $j = 1, 2, \ldots, D$; random vector $r_j \in [1, 0]$; crossover constant CR $\in [1, 0]$; and randomly chosen index $rn_i \in (1, 2, \ldots, D)$.

6.3 Selection

From the search space, parents can be selected form all the individuals in the population irrespective of their fitness value. After mutation and crossover, the fitness of the child is evaluated and compared with that of parent and the one with better fitness value is selected.

7 Results and Discussion

The optimization and simulation of the closed loop coupled tank system has been carried out in MATLAB and the implementation of the fractional-order PID controller is done using FOMCON toolbox [10]. Table 1 shows the multi-objective bat algorithm parameters and Table 2 shows the multi-objective differential evolution parameters considered for the optimization carried out in the work.

In Fig. 2, closed loop response of the system is shown with varying inputs and it is clear from the figure that the MOBA-tuned FOPID controller tracks the reference efficiently and too with negligible overshoot. Figure 3 shows the control effort employed by the FOPID controllers to ensure the steady and smooth tracking of the reference signal. From Figs. 2 and 3, it can be surmised that in MODE-tuned FOPID controllers, tracking is not proper and a steady state error exists throughout the response.

Table 1 Multi-objective bat algorithm parameters

Parameter	Value
Population size	40
Loudness	0.5
Pulse rate	0.5
Minimum frequency	0
Maximum frequency	2

Table 2 Multi-objective differential evolution parameters

Parameter	Value
DE statergy	DE with Jitter [15]
Population size	20
Step size	0.85
Crossover probability	1

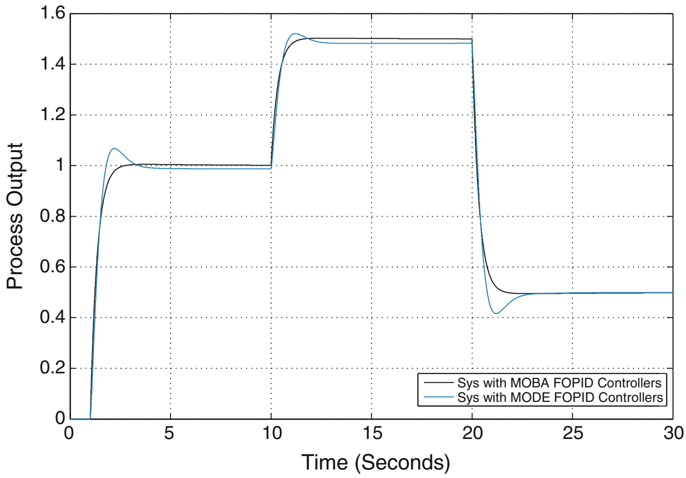

Fig. 2 Response of the closed loop system with MOBA- and MODE-tuned FOPID controllers

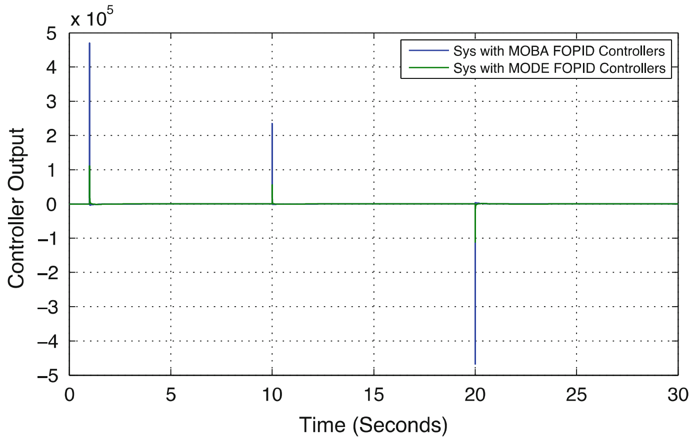

Fig. 3 Controller output with MOBA- and MODE-tuned FOPID controllers

In order to check the robustness of the system to external disturbances, a pulse has been introduced. The magnitude of the pulse is 10 % of the reference signal having a duration of 0.15 s and has been introduced at 4 s. Figures 4 and 5 show the step response of the closed loop system and the controller output, respectively. The Figs. 4 and 5 implies that the closed loop system with MOBA- and MODE-tuned FOPID controllers both efficiently tackle the noise and restore the system to original tracking of reference immediately.

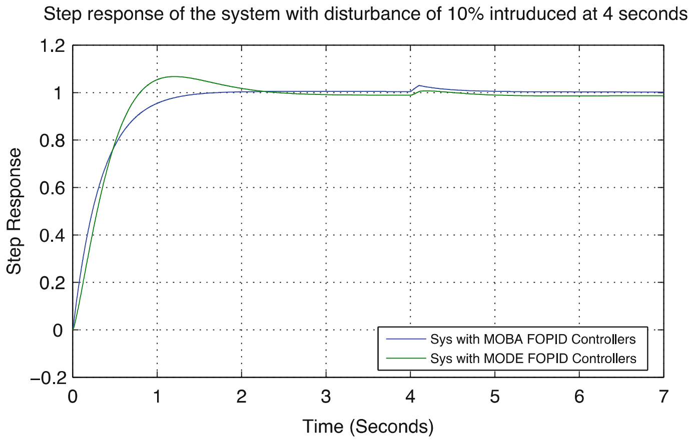

Fig. 4 Step response of the closed loop system with MOBA- and MODE-tuned FOPID controllers with 10 % disturbance introduced at 4 s

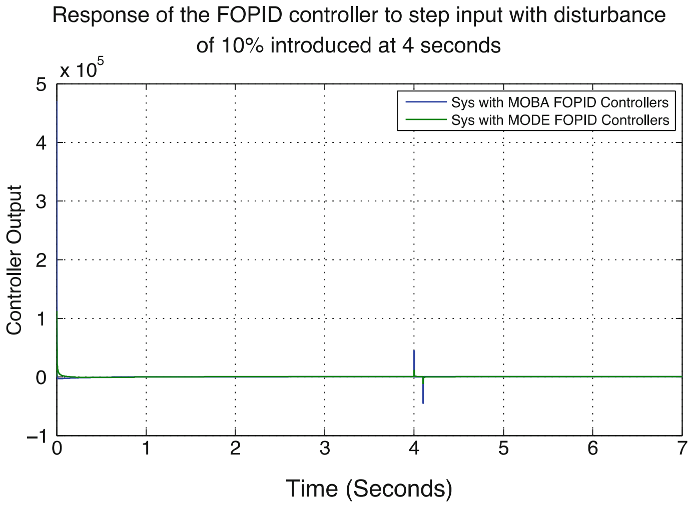

Fig. 5 Controller response of the closed loop system with MOBA- and MODE-tuned FOPID controllers with 10 % disturbance introduced at 4 s

Table 3 shows the obtained optimal FOPID parameters obtained using MOBA and MODE. Table 4 shows the H_∞ norm of the sensitivity function and complementary sensitivity function is shown and are <1 and even their sum is ≤1 which shows better robustness properties and satisfies the performance index defied in Eq. 7. In Table 5, the time domain performance indexes has been compared of both the systems and it can be seen the MOBA-tuned FOPID controllers provides better indexes like reduced overshoot percentage and reduced settling times.

Table 3 Optimal FOPID parameters obtained after optimization

Optimization method	K_p	K_i	K_d	λ	μ
Multi-objective bat algorithm	282.9	6.9	1276.5	0.9	1
Multi-objective DE	100	6.65	1500	0.6905	0.744

Table 4 Compared H_∞ norm of the $S(j\omega)$ and $T(j\omega)$

Optimization method	$\|S(j\omega)\|_\infty$	$\|T(j\omega)\|_\infty$
Multi-objective bat algorithm	0.0036	0.9964
Multi-objective DE	0.0068	0.9932

Table 5 Compared time domain performances

Optimization method	Overshoot (%)	Rise time	Settling time
Multi-objective bat algorithm	0.36	0.7983	1.36
Multi-objective DE	6.09	0.5385	1.98

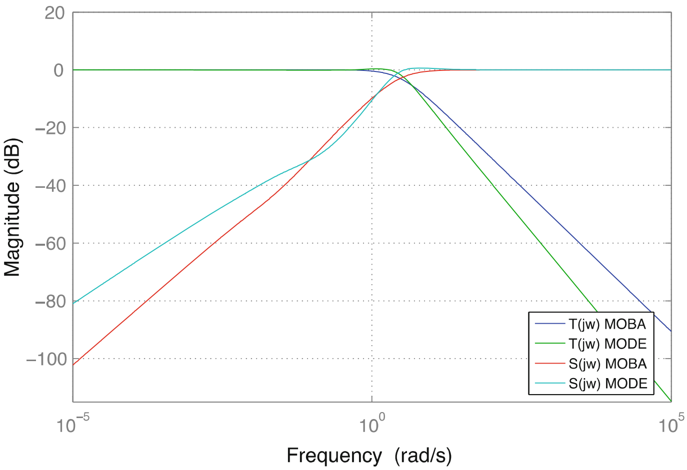

Fig. 6 Frequency domain analysis of sensitivity and complementary sensitivity with MOBA- and MODE-tuned FOPID controllers

Figure 6 shows the frequency domain representation of the sensitivity and complementary sensitivity functions using both the controllers. Gain curve of the Bode plot of sensitivity and complementary sensitivity (Fig. 6) shows how feedback influences the disturbances. Figure 6 shows that maximum sensitivity $S(j\omega)$ for MODE-tuned FOPID controllers is 0.66 dB at 6.11 rad/s, whereas for MOBA tuned FOPID controllers maximum sensitivity is ≈0 dB at that too till 10^{15} rad/s. Thus following inferences can be obtained that feedback will reduce the disturbances less than the gain crossover frequency (ω_{gc}) and for higher frequencies (beyond ω_{gc}) the disturbances will be amplified and the largest amplification factor of 0.66 (MODE) and ≈0 (MOBA). From Fig. 6 for the complementary sensitivity function $T(j\omega)$, maximum peak gains of 0.0184 dB at 0.192 rad/s for MOBA-tuned controllers and 0.351 dB at 1.27 rad/s for MODE-tuned controllers has been obtained. This provides insights about the stability margins and allowable process variations.

8 Conclusions

In real-world engineering applications, most design requirements are multi-objective in nature and for optimal performance several constraints have to be satisfied. Robust behavior of controllers in presence of uncertainties is one of the most complicated control design objectives and use of fractional calculus in controller/system designing offers flexibility and extra degree-of-freedom in achieving such goals. In this paper, mixed sensitivity minimization has been posed as a multi-objective problem. MOBA and MODE have been used for tuning the FOPID controllers implemented for the liquid level control in coupled tank systems. Results obtained show that efficient disturbance rejection and satisfaction of the performance indexes. Such optimal robust behavior ensures the process safety and the quality of products.

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