The Neo-Ricardian Trade Theory and the New Theory of International Values

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Abstract Ricardo's (On the principles of political economy, and taxation, 1817) theory of comparative advantage is the first rigorous theory that demonstrates that free trade benefits every country. He explained his theory using a numerical example of two countries and two commodities. However, the fact that the theory cannot be true when we expand his model to the multicountry and multicommodity case, or to the model that assumes intermediate goods, became clear. Following the study by Graham (Q J Econ 46:581–616, 1932) and McKenzie (Rev Econ Studies 21: 165–180, 1954), the neo-Ricardian theories of international trade as developed by Steedman (Fundamental issues in trade theory, Macmillan, London, 1979) reconsidered gains from trade and showed the possibility of losses from free trade. Recently, Shiozawa (Evol Inst Econ Rev 3: 141–187, 2007) indicated the differences in the number of countries and goods and analyzed cases in which prices did not depend on demand but were determined by production cost. This chapter surveys the development of trade theories and analyzes the gains from trade using the most generalized model. Furthermore, it also considers how the new theory of international values proposed by Shiozawa (Evol Inst Econ Rev 3: 141–187, 2007) provides a new horizon to the previous results.

Keywords Neo-Ricardian • Trade theory • Gains from trade • Sraffa • New theory of international values

1 Introduction

Ricardo's (1817) comparative advantage theory is considered to be one of the few theories that is accepted as a "correct theory" by almost all schools in economics. The simple and clear conclusion of this theory is as follows. First, in free trade, every country has at least one commodity that can be produced at a lower price than that of its trade partners. Second, every country achieves gains from trade by

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specializing in producing and exporting these lower-price commodities. Until now, this theory has been a basic doctrine to support the free trade policy.

However, comparative advantage theory has some difficulties in both its theoretical ground and its applicability to the real world. As a pure theory, it has two problems: (1) international prices cannot be determined inside its system and (2) the theory crucially depends on the assumptions of two countries, two commodities, and no intermediate products. The theory also faces problems in the real world: as world trade expands, the income gap between developed and developing countries widens, and developing countries seem to suffer losses from trade, which is inconsistent with the conclusion of the theory.

Numerous studies have been conducted on such problems.[1] Some problems were solved and others were verified as unsolvable. At present, problems still exist that remain unsolved. Thus, this chapter systematically explains the development of Ricardo's (1817) theory using the most generalized model. Furthermore, it summarizes what is proven and what is not.

New claims on Ricardo's (1817) theory have recently emerged. Among these, Shiozawa (2007) indicated that the number of commodities is much larger than the number of countries and that the same commodity can be produced in many countries. In this case, the international price is not influenced by the world demand for that commodity. This assertion becomes known as "the theory of new international values." In this book, the meanings and the development of this theory are discussed in various ways. This chapter examines the theoretical meaning of the theory.

The composition of this chapter is as follows. Section 2 presents the formulation of Ricardo's (1817) comparative advantage theory through a two-country and two-commodity model and verifies Ricardo's assertions.

Section 3 introduces a utility function into Ricardo's (1817) basic model and demonstrates how Mill (1852) solved the problem of determining international prices. This section also considers the plausibility that Mill's (1852) "reciprocal demand theory" became the fundamental principle of neoclassical economics, namely, that "price is determined by supply and demand."

Section 4 expands the model to a multicommodity and multicountry case and examines the difficulties that arise in that case. Ricardo's (1817) criterion of comparative advantage was proven to remain true in the two-commodity multicountry case and two-country multicommodity case. However, if we assume that both the number of commodities and number of countries are more than three, this advantage cannot hold true. Further, we confirm a counter example presented by Graham (1932) and discuss its implication.

Section 5 introduces the intermediate goods and considers their effects on Ricardo's (1817) comparative advantage theory. McKenzie (1954) showed that

[1] Chipman (1965) is a survey article on the development of pure theory after Ricardo (1817). Emmanuel (1973) is the most famous book that criticizes the applicability of the comparative advantage theory to the real world.

Ricardo's (1817) criterion does not hold true even in the case of two countries and three commodities. The plausibility of his example is confirmed using an example in which I modify the model presented by Amano (1966).

The neo-Ricardian economic theory assumes that production cannot be completed instantaneously or that the rate of profit is positive. The economic model which has this property was called "the time-phased Ricardian economy" by Samuelson (1975). In that situation, the existence of intermediate goods causes far more difficult problems. Section 6 examines the plausibility of the neo-Ricardian trade theory offered by Steedman (1979), who addressed this situation. They showed the possibility that the comparative advantage in terms of production prices may differ from it in terms of labor values. Furthermore, they showed that in such a situation, some countries may suffer losses from trade.[2] On this point, Smith (1979) presented a counterargument that the equilibrium should satisfy a condition of intertemporal efficiency. We rigorously formulate the generalized model and evaluate the meaning and the limit of the neo-Ricardian trade theory.

Section 7 presumes a model in which the number of commodities is much larger than the number of countries, namely, "the new theory of international values," and considers new findings that can be added by this theory to the traditional trade theories.

The final section summarizes the contents of this paper and provides prospects for the future development of such studies.

2 Ricardo's Comparative Advantage Theory

In this section, instead of presenting a numerical example as Ricardo (1817) did in his book, I formulate a general mathematical model and confirm the correctness of Ricardo's (1817) argument.

We assume that two countries, A and B, produce the same two commodities. The prices of the two commodities in the two countries are given by the following equation:

$$\begin{aligned} 1 &= \left(1+r^A\right) w^A l_1^A \qquad & 1 &= \left(1+r^B\right) w^B l_1^B \\ p^A &= \left(1+r^A\right) w^A l_2^A \qquad & p^B &= \left(1+r^B\right) w^B l_2^B. \end{aligned} \tag{1}$$

Here, p^h indicates the price of commodity 2 in terms of commodity 1, r^h indicates the profit rate, w^h indicates the wage rate, and l_j^h indicates the labor input coefficient of the jth commodity in country h ($h = A, B$).

[2]Important articles are collected in the study by Steedman (1979).

Let us assume that commodity 1 is relatively cheaper in country A than in country B. Thus, we have

$$p^A > p^B. \tag{2}$$

In this case, Ricardo's (1817) principle teaches us that country A specializes in producing commodity 1 and that country B specializes in producing commodity 2. If two countries specialize in such a manner, the international prices of the two commodities are given by Eq. (3).

$$\begin{aligned} 1 &= \left(1 + r^{AT}\right) w^{AT} l_1^A \\ p^T &= \left(1 + r^{BT}\right) w^{BT} l_1^B \end{aligned} \tag{3}$$

In this equation, superscript T indicates that the variable is in a free trade situation. The production in each country that is conducted in this specialization pattern indicates that the unused production processes are not profitable. Moreover, in these production processes, the production cost evaluated by the rate of profit, wage rate, and international prices under free trade exceeds its international price. Thus, we have

$$\begin{aligned} p^T &< \left(1 + r^{AT}\right) w^{AT} l_2^A \\ 1 &< \left(1 + r^{BT}\right) w^{BT} l_1^B. \end{aligned} \tag{4}$$

From Eqs. (1), (2), (3), and (4), we have the following relations:

$$p^A = \frac{l_2^A}{l_1^A} > p^T > \frac{l_2^B}{l_1^B} = p^B \tag{5}$$

Thus, the price of commodity 2 in terms of commodity 1 under free trade p^T must be determined between the prices in the two countries in an autarky. In other word, the international price should be determined in Ricardo's limbo.

In the next step, using this relation, we show that trade certainly brings gains to both countries and that free trade can create an efficient production pattern in the world. Let L^A and L^B denote labor endowment in countries A and B, respectively. Then, in an autarky, the quantities of produced commodities in each country should satisfy the following labor constraints:

$$\begin{aligned} l_1^A X_1^A + l_2^A X_2^A &\leqq L^A \\ l_1^B X_1^B + l_2^B X_2^B &\leqq L^B \end{aligned} \tag{6}$$

Here, X_j^h indicates the production of the jth commodity in country h ($h = A, B$). These labor constraints are shown in Figs. 1a and 1b. In Figs. 1a and 1b, the solid lines denote the inequalities (6), and the southwest area of the solid line represents the production possibility set in each country. We easily understand

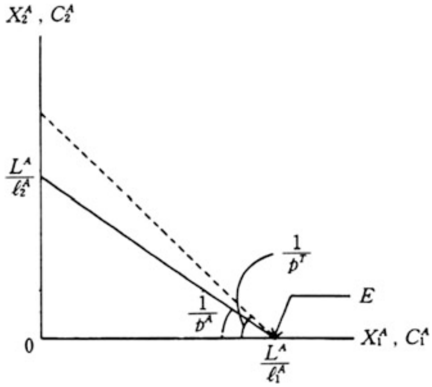


Fig. 1a Production possibility set and consumption possibility set in country *A*

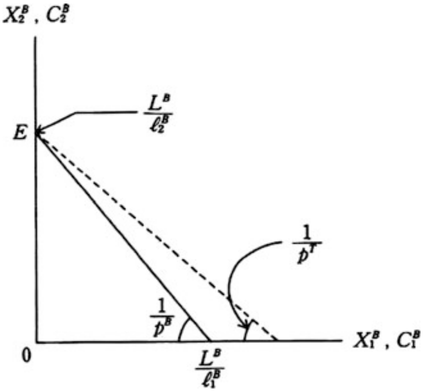
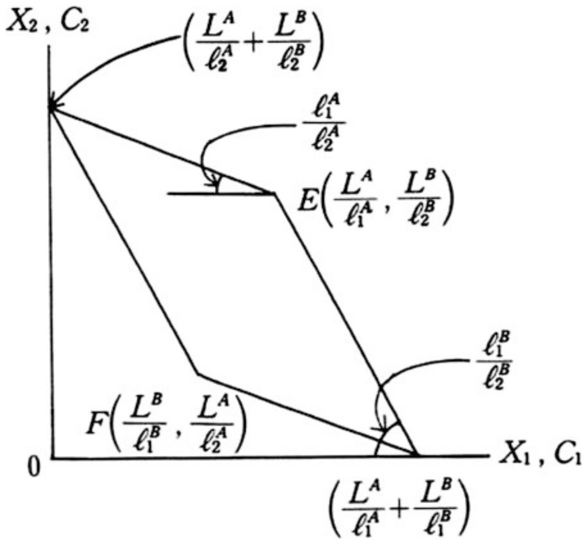


Fig. 1b Production possibility set and consumption possibility set in country *B*

that the inclination of the line equals the relative price of the two commodities in each country. From these two figures, the world production possibility set can be depicted, as shown in Fig. 2.

When both countries open trade, country *A* specializes in producing commodity 1 and country *B* specializes in producing commodity 2. The production of country *A* is depicted at point *E* in Fig. 1a, and the production of country *B* is at point *E* in Fig. 1b. The combination of the production in countries *A* and *B* is depicted at point *E* in Fig. 2. We see that point *E* is situated northeast of point *F*, where country *A* specializes in commodity 2 and country *B* in commodity 1. Thus, we confirm that production is efficiently conducted under free trade. We also see that international price p^T exists between p^A and p^B, and the consumption possibility set—the southeast area of the dotted line—is larger than the set before trade in both countries and represents the gains from trade. Thus, in Ricardo's (1817) model, when capitalists specialize in production to maximize their profits, an efficient production is realized in the world, and both countries gain from trade in the sense that they consume more commodities.

Fig. 2 Production possibility set in the world



3 Determination of the International Price by Mill

One problem in Ricardo's (1817) comparative advantage theory is that the terms of trade cannot be determined inside his model. In a free trade situation, the terms of trade should exist between the relative price in country *A* and in country *B*, when they are in an autarky. However, determining the definite level of the terms of trade is impossible.

To determine the terms of trade, we should specify the demands of both countries on the two commodities. Thus, we should introduce the demand functions to Ricardo's (1817) analysis. The economist who first made such an introduction was Mill (1852). In Section 6–8, which was added in the third edition of chapter XVIII of *Principle*, Mill (1852) assumed a demand function in a specific form and showed how international prices and consumption of both countries are determined. Using the terminology of modern economics, Mill (1852) can be said to have formulated Ricardo's (1817) model as a general equilibrium model and derived the solutions. In this section, we reformulate Mill's (1852) analysis by expanding his model to a more generalized one and consider the plausibility of his analysis.

First, let us briefly explain Mill's (1852) demand functions. Mill (1852) provided the following explanation in his book:

> As the simplest and most convenient, let us suppose that in both countries any given increase of cheapness produces an exactly proportional increase of consumption or, in other words, that the value expended in the commodity, the cost incurred for the sake of obtaining it, is always the same, whether that cost affords a greater or a smaller quantity of the commodity. Mill (1965, p. 609)

In other words, Mill assumes that the elasticity of demand with price is equal to one and the cross-elasticity of demand is equal to zero. The necessary and sufficient condition for the demand function having this property is that the utility function is a type of Eq. (7).

$$U = \varnothing \left(C_1^{\alpha} C_2^{1-\alpha} \right) \tag{7}$$

Next, let us see how the terms of trade is determined in the model that assumes this type of demand function. We assume that the supply side of the model is the same as in the previous model. Therefore, Eq. (1) is on hold in an autarky. In this case, country A specializes in producing commodity 1, and country B specializes in producing commodity 2.

Let us assume that the demand function in country A is

$$U^A = \left(C_1^A \right)^{\alpha} \left(C_1^A \right)^{1-\alpha} \tag{8}$$

Here, $1 > \alpha > 0$ is a parameter. Then, country A faces the following maximization problem.

$$\begin{aligned} \text{Max.} \quad & U^A = \left(C_1^A \right)^{\alpha} \left(C_1^A \right)^{1-\alpha} \\ \text{s.t.} \quad & C_1^A + p^T C_2^A \leqq \tfrac{L^A}{l_1^A} = X_1^A \end{aligned} \tag{9}$$

From the necessary condition of the maximization problem, we have

$$\alpha p^T C_2^A = (1 - \alpha)\, C_1^A \tag{10}$$

If we assume that the utility function of country B is given by Eq. (11),

$$U^B = \left(C_1^B \right)^{\beta} \left(C_1^B \right)^{1-\beta} \tag{11}$$

Here, $1 > \beta > 0$ is a parameter. Then, we have the following equation.

$$\beta p^T C_2^B = (1 - \beta)\, C_1^B \tag{12}$$

From Eqs. (10) and (12) and the budget constraints of country A and B, we have the following demand and supply equalities.

$$\begin{aligned} C_1^A + p^T C_2^A &= \tfrac{L^A}{l_1^A} = X_1^A \\ C_2^B + p^T C_2^B &= \tfrac{p^T L^B}{l_2^B} = p^T X_2^B \end{aligned} \tag{13}$$

Because the quantities of commodity supply are determined by the labor endowments of both countries and labor coefficients, we have

$$
\begin{aligned}
C_1^A + C_1^B &= \frac{L^A}{l_1^A} \\
C_2^A + C_2^B &= \frac{L^B}{l_2^B}.
\end{aligned}
\tag{14}
$$

Thus, the international price is

$$
p^T = \frac{(1-\alpha)\, l_2^B L^A}{\beta l_1^A L^B} \tag{15}
$$

and the production and consumption of two commodities are given by

$$
\begin{array}{ll}
C_1^A = \alpha L^A / l_1^A & C_1^B = (1-\alpha)\, L^A / l_1^A \\
C_2^A = \beta L^B / l_2^B & C_2^B = (1-\beta)\, L^B / l_2^B \\
X_1^A = L^A / l_1^A & X_1^B = 0 \\
X_2^A = 0 & X_2^B = L^B / l_2^B
\end{array}
\tag{16}
$$

Thus, we determine the terms of trade and the consumption of the two commodities in two countries in the model to which we additionally introduce the utility functions.

However, depending on the demand volumes, the terms of trade might not be left in limbo. Before analyzing this case, we first return to Mill's (1852) *Principle* and see how Mill (1852) treated this case. Mill stated the following:

> Let it be supposed that in England 100 yards of cloth, previously to the trade, exchanged for 100 of linen, but that in Germany 100 of cloth exchanged for 200 of linen. When the trade was opened, England would supply cloth to Germany, Germany linen to England. Mill (1965, p. 609)

Thus, our model completely accords with the example in Mill's (1852) *Principle* if we change names as follows: from country A to England, country B to Germany, commodity 1 to cloth, and commodity 2 to linen. We also assume the price in an autarky $p^A = 1$ and $p^B = 1/2$.

In this case, England has a comparative advantage in cloth and specializes in producing it. Germany specializes in linen. As was previously shown, if the demand function has the property that Mill (1852) assumes, the proportion of total income spend on the consumption of each commodity in each country is constant after opening trade. Thus, the quantity of consumption of the commodity for which England and Germany specialize in production after establishing trade is the same as the consumption before trade. If demand equals supply for two commodities, the quantity of the commodity produced in England and that is not consumed in the domestic market is exchanged for the quantity of the commodity produced in Germany and that is not consumed in the country as an equivalent value. Mill stated the following:

> Let the quantity of cloth which England can make with the labor and capital withdrawn from the production of linen, be $= n$. Let the cloth previously required by Germany (at the German cost of production) be $= m$.

> Then n of cloth will always exchange for exactly $2m$ of linen.
> Mill (1965, p. 611)

If England specializes in producing cloth, the production of cloth is $2n$ and the quantity of cloth not consumed in England is n. Similarly, because the price of linen in terms of cloth in Germany is 1/2, the quantity of linen that can be produced with the labor withdrawn from the production of linen is 2 m. Thus, n of cloth always exchanges for 2 m of linen. In this case, can England and Germany gain from the benefits of trade? Mill stated the following:

> If $n = 2m$, the whole advantage will be on the side of Germany.
> If n be greater than m, but less than $2m$, the two countries will share the advantage; England getting $2m$ of linen where she before got only n; Germany getting n of cloth where she before got only m.
> Mill (1965, p. 611)

We can easily confirm the resembling result in our generalized Mill model. If we assume

$$p^B > \frac{(1-\alpha)\, l_2^B L^A}{\beta l_1^A L^B}, \tag{17}$$

then $p^T = p^B$, and country A specializes in producing commodity 1, and country B produces both commodities. The consumption and production of two commodities in two countries are given by

$$\begin{array}{ll} C_1^A = \alpha L^A / l_1^A & C_1^B = \beta L^B / l_1^B \\ C_2^A = (1-\alpha)\, l_1^B L^A / l_1^A l_2^B & C_2^B = (1-\beta)\, L^B / l_2^B \\ X_1^A = L^A / l_1^A & X_1^B = \beta L^B / l_1^B - (1-\alpha)\, L^A / l_1^A \\ X_2^A = 0 & X_2^B = (1-\alpha)\, l_1^B L^A / l_2^B l_1^A + (1-\beta)\, L^B / l_2^B \end{array} \tag{18}$$

Here, $X_1^B = C_1^A + C_1^B - X_1^A$, which is positive from (16). Thus, if the sum of the demand for commodity 1 in two countries is larger than the production of commodity 1 in country A—in other words, if country A is a relatively small country—this situation might happen, and all of the benefits of trade will be on the side of country A.

As we have seen so far, Mill's (1852) analysis is almost perfect as a general equilibrium analysis although its defect is using a specific type of demand function. We cannot criticize his analysis even from the viewpoint of contemporary economics. Thus, we can state that his analysis was ahead of his time or was too advanced. For precisely that reason, Chipman (1965) stated in his article that Mill's contribution was not correctly understood for a long time.

Mill seemed to lead economics from the classical price theory, which states that prices are determined by the production cost of a commodity to the neoclassical theory, which states that prices are determined by an equilibrium between a commodity's demand and supply. If two countries specialize in producing a commodity for which a country has a comparative advantage under free trade, then prices are certainly not proportional to labor input and depend on demand. In this case, the neoclassical approach seems more appropriate than the classical approach for explaining price determination. In that sense, we state that Mill killed classical economics, and Chipman (1965) and Negishi (1981, 1983) evaluated this point.

However, we should be careful about this subject. First, in the case of incomplete specialization, prices are in accord with production costs in a large country, and classical economics is restored. Second, is classical economics defined as the economic doctrine that assumes that prices are independent of demand or that price is proportional to its labor value correct? The dependency of prices on demand also occurs when we consider rent.[3] However, should we really believe that dependency means the end of classical economics and the rise of neoclassical economics? When we regard classical economics in a broader context and define it as economics that stresses the importance of analyzing the economy from the viewpoint of reproducibility, the most important point in Ricardo's analyses should be considered to be the existence of intermediate goods rather than the dependence of prices on demand. However, this point had not been considered after Ricardo until McKenzie (1954) and Jones (1961) analyzed it using the modern linear programming method in the 1950s.

4 Many Commodities and Many Countries

Another problem of comparative advantage theory is that this theory crucially depends on three assumptions, namely, two countries, two commodities, and the nonexistence of intermediate goods. What types of difficulties arise if we relax these assumptions and generalize the theory to the situation of a multicommodity, a multicountry, and the existence of intermediate goods?

It is confirmed that the theory is robust if we increase only the number of countries from two to many, assuming that the number of commodities is two. The theory is also robust if we increase only the number of commodities.

However, if we increase both of the number of countries and commodities, a difficulty arises. Let us consider an example. We assume that three countries, A, B, and C, produce three types of commodities before trade. The necessary labor input to produce a unit of each commodity and the labor endowment in the three countries are shown in Table 1. In Table 1, l_j indicates a labor input to produce a unit of the jth commodity and L indicates a labor endowment.

In this example, which production specialization pattern satisfies the principle of Ricardo's (1817) comparative advantage theory and which pattern is efficient? First, let us consider the pattern $\circ$, which indicates that country A specializes in producing commodity 2, country B specializes in commodity 1, and country C specializes in commodity 3. As is easily confirmed, the pattern $\circ$ satisfies the standard of

[3] See, for example, Montani (1975), Kurz (1978), and Takamasu (1983).

Table 1 Counterexample to Ricardo, three countries, three commodities case

	Country A	Country B	Country C
l_1	•100	∘100	100
l_2	∘ 50	70	• 30
l_3	40	• 30	∘ 20
L	4500	4500	3000

Ricardo's (1817) comparative advantage theory because every country specializes in producing the commodity with a comparative advantage for any pair of two countries and two commodities. For example, when we check for countries A and B and commodities 1 and 2, we have inequality (19) and country A has a comparative advantage in commodity 2.

$$\frac{50}{100} = \frac{l_2^A}{l_1^A} < \frac{l_2^B}{l_1^B} = \frac{70}{100} \tag{19}$$

This statement is also true for the country B and country C pair, and for the country A and country C pair. Thus, the pattern ∘ is consistent with Ricardo's (1817) standard in comparative advantage theory.

However, this production pattern cannot be compatible with a competitive equilibrium. Let us show the incompatibility. As was seen in the section that explains Ricardo's (1817) comparative advantage theory, for unused production processes, the cost of producing a unit of the commodity measured using current prices, the wage rate, and the profit rate exceeds the price. In contrast, the cost equals the price for the actually operating production process. Thus, we have the following inequalities and equalities for three commodities.

$$\begin{array}{lll} p_1^T < \left(1 + r^{AT}\right) 100 w^{AT} & p_1^T = \left(1 + r^{BT}\right) 100 w^{BT} & p_1^T < \left(1 + r^{CT}\right) 100 w^{CT} \\ p_2^T = \left(1 + r^{AT}\right) 50 w^{AT} & p_2^T < \left(1 + r^{BT}\right) 70 w^{BT} & p_1^T < \left(1 + r^{CT}\right) 30 w^{CT} \\ p_1^T < \left(1 + r^{AT}\right) 40 w^{AT} & p_1^T < \left(1 + r^{BT}\right) 30 w^{BT} & p_1^T = \left(1 + r^{CT}\right) 20 w^{CT} \end{array} \tag{20}$$

By eliminating the profit rate and the wage rate in Eq. (20), we have Eq. (21).

$$\begin{array}{lll} p_1^T < \frac{100}{50} p_2^T & p_2^T < \frac{70}{100} p_1^T & p_1^T < \frac{100}{20} p_3^T \\ p_3^T < \frac{40}{50} p_2^T & p_3^T < \frac{30}{100} p_1^T & p_2^T < \frac{30}{20} p_3^T \end{array} \tag{21}$$

When we start from the upper left of Eq. (21) and use the lower right and lower middle, we have

$$p_1^T < 2 p_2^T < 3 p_3^T < \frac{90}{100} p_1^T \tag{22}$$

Obviously, no positive price exists, and the profit rate and the wage rate satisfy (22).

Table 2 A three-country and three-commodity case in which Ricardo's comparative advantage theory does not hold

	Pattern ∘	Pattern •
Commodity 1	45	45
Commodity 2	90	100
Commodity 3	150	150

The production pattern ∘ can also be verified as not being efficient in the sense that no Pareto-dominant production pattern exists for that pattern. Table 2 shows the outputs of each commodity for the production patterns ∘ and •. The outputs of commodities 1 and 3 in the world are the same, and the output of commodity 2 is larger in the pattern •. Thus, we see that the pattern ∘ is not efficient.

Is the pattern • truly efficient and a competitive equilibrium? McKenzie (1954) and Jones (1961) clarified this point. McKenzie (1954) showed that a competitive equilibrium in free trade is an internationally efficient production pattern. We provide proof of this equivalency in a generalized model in Sect. 6. Before proceeding to the proof, we consider the meaning of intermediate goods in an open economy in Sect. 5. For efficient production patterns, Jones (1961) showed that an efficient production pattern is one that minimizes the product of labor inputs of the produced commodities, such as $l_1^A l_2^B l_3^C$ if the number of countries equals the number of commodities and each country specializes in producing only one commodity.

5 Intermediate Goods

The case in which Ricardo's (1817) comparative advantage theory does not hold also exists in the situation in which we assume intermediate goods. McKenzie (1954) showed this phenomenon in the case of three countries and three commodities. Amano (1966) also showed this phenomenon in the case of two countries and three commodities.

Following Amano (1966), we make an example of two countries and three commodities for which ordering the comparative advantage in an autarky and in free trade does not accord. The method for providing an explanation is slightly different from that of Amano (1966).

Let us assume that two countries, country A and B, exist, and both countries produce three commodities. The production technique of both countries is assumed to be as follows.

Country A

$$
\begin{array}{llll}
a_{11}^A = 0 & a_{21}^A = 0 & a_{31}^A = 0 & l_1^A = 100 \\
a_{12}^A = 0 & a_{22}^A = 0 & a_{32}^A = 0.8 & l_2^A = 50 \\
a_{13}^A = 0 & a_{23}^A = 0 & a_{33}^A = 0 & l_3^A = 200
\end{array}
$$

Table 3 Labor directly or indirectly required to produce a commodity, namely, labor value in countries A and B

	Country A	Country B
v_1	100	100
v_2	210	140
v_3	200	100

Country B

$$\begin{array}{llll} a_{11}^B = 0 & a_{21}^B = 0 & a_{31}^B = 0 & l_1^B = 100 \\ a_{12}^B = 0 & a_{22}^B = 0 & a_{32}^B = 0.4 & l_2^B = 100 \\ a_{13}^B = 0 & a_{23}^B = 0 & a_{33}^B = 0 & l_3^B = 100 \end{array}$$

Here, $a_{ij}^h (h = A, B)$ is the quantity of the ith commodity required to produce one unit of the jth commodity, and l_j^h is the labor input required to produce one unit of the jth commodity in country h. When we assume that the profit rate in countries A and B equals zero, prices in countries A and B are calculated as in Eq. (23).

$$\begin{array}{ll} p_1^A = 100w^A & p_1^B = 100w^B \\ p_2^A = 0.8p_3^A + 50w^A & p_2^B = 0.4p_3^B + 100w^B \\ p_3^A = 200w^A & p_3^B = 100w^B \end{array} \tag{23}$$

The quantity of labor directly and indirectly required to produce a unit of a commodity can be calculated, as shown in Table 3. In Table 3, v_j indicates the labor input required to produce one unit of each commodity or labor value.

Because the commodity price is proportional to the labor value if the profit rate is zero, we have the following relationships.

$$\frac{p_1^B}{p_1^A} > \frac{p_2^B}{p_2^A} > \frac{p_3^B}{p_3^A}$$

Hence, country A should have a comparative advantage against country B in the order of commodity 1, commodity 2, and commodity 3. Thus, in free trade, country A must specialize in producing commodity 1.

However, we easily show that the production specialization pattern for which country A specializes in commodity 1 and country B in commodities 2 and 3 cannot be compatible with a competitive equilibrium. As was previously shown, in a competitive equilibrium, we have the following equalities and inequalities.

$$\begin{array}{ll} p_1^T = 100w^{AT} & p_1^T < 100w^{BT} \\ p_2^T < 0.8p_3^T + 50w^{AT} & p_2^T = 0.4p_3^T + 100w^{BT} \\ p_3^T < 200w^{AT} & p_3^T = 100w^{BT} \end{array} \tag{24}$$

Substituting the middle left of Eq. (24) for the upper left and lower right, we have

$$p_2^T < 80w^{BT} + 0.5p_1^T$$

Considering the upper right of Eq. (24), we have

$$p_2^T < 130w^{BT} \tag{25}$$

However, from the middle right and lower right of Eq. (24), we have

$$p_2^T = 140w^{BT}$$

which is inconsistent with Eq. (25). Thus, no nonnegative prices enable this production specialization pattern.

In addition, we show that the same phenomena occur even in the case of two countries and two commodities if the rate of profit is positive. Thus, if we assume the intermediate goods, the order of a comparative advantage in an autarky does not coincide in general with the order in free trade.

6 Intermediate Goods and a Positive Profit Rate: Critique by the Neo-Ricardian

In the analysis of Sect. 4, the assumption is that no intermediate goods are required to produce a commodity and the production period is the same for every commodity. Thus, the labor hours required directly or indirectly to produce one unit of a commodity, namely, labor value, equal the production price for every commodity. In Sect. 5, we introduce intermediate goods. However, because we assume that the rate of profit is zero, the labor value or labor required directly or indirectly to produce a unit of commodity still equals its price.

When we assume that the production periods differ from each other, or assume that intermediate goods are required to produce commodities and the rate of profit is positive, the labor value is not proportional to its price. In that case, ordering the comparative advantage in terms of the production price could not be in accord with ordering in terms of the labor value.

In that case, can every country still gain benefits from trade? This situation was analyzed by the neo-Ricardian trade theory. Instead of assuming intermediate goods, Steedman and Metcalfe (1973) presented an example in which the production periods differ from each other and the ordering of the comparative advantage in terms of price and labor values is different. In contrast, Takamasu (1991, pp. 44–49) assumes intermediate production goods and a positive profit rate and presents a similar example. Following Takamasu (1991), we provide an example that has the same property and consider the type of results that will ensue.

Let us assume that country A has the following input coefficients.

Country A

$$\begin{array}{lll} a_{11}^A = 0.4 & a_{21}^A = 0 & l_1^A = 60 \\ a_{12}^A = 0.2 & a_{22}^A = 0 & l_2^A = 100 \end{array}$$

In this case, the direct or indirect labor to produce a unit of commodities 1 and 2 in country A can be calculated using the following equations.

$$\begin{aligned} 0.4v_1^A + 60 &= v_1^A \\ 0.2v_1^A + 100 &= v_2^A \end{aligned} \tag{26}$$

Solving this Eq. (26), we have $v_1^A = 100$ and $v_2^A = 120$. These values are the same as in Ricardo's example.

Next, let us calculate the prices of commodities 1 and 2 in country A. When we assume that wages are paid after production, commodity prices can be given by the following equations.

$$\begin{aligned} 0.4\left(1 + r^A\right) + 60w^A &= 1 \\ 0.2\left(1 + r^A\right) + 100w^A &= p^A \end{aligned} \tag{27}$$

As is evident by comparing Eq. (27) with Eq. (26), the price of a commodity is not proportional to the labor value if the rate of profit is positive. When we give $r^A = 1$, we have $p^A = 11/15$.

To make a comparison with the argument in Sect. 2, let us suppose that the labor endowment of country A is 4800 units. Then, we derive the consumption possibility set of country A. Provided that X_1^A and X_2^A denote the gross outputs of commodity 1 and 2 in country A, respectively, then we have the following labor constraint inequality.

$$60X_1^A + 100X_2^A \leqq 4800 \tag{28}$$

Because the net outputs of commodities 1 and 2, Y_1^A and Y_2^A, are the gross outputs minus the inputs for the production in the next period, we have

$$\begin{aligned} Y_1^A &= X_1^A - \left(0.4X_1^A + 0.2X_2^A\right) \\ Y_2^A &= X_2^A \end{aligned} \tag{29}$$

Solving (29) with respect to X_1^A and X_2^A, and by assigning (28), we have

$$100Y_1^A + 120Y_2^A \leqq 4800 \tag{30}$$

Thus, the consumption possibility set is the same as that of Ricardo's (1817) original example.

Then, let us derive the labor values, the commodity prices, and the consumption possibility frontier of country B. Let us assume the production technique of country B as follows.

Country B

$$\begin{array}{lll} a_{11}^B = 0.3 & a_{21}^B = 0 & l_1^B = 63 \\ a_{12}^B = 0.3 & a_{22}^B = 0 & l_2^B = 53 \end{array}$$

Then, v_1^B and v_2^B are calculated from (31).

$$\begin{aligned} 0.3v_1^B + 63 &= v_1^B \\ 0.3v_1^B + 53 &= v_2^B \end{aligned} \tag{31}$$

Solving (31), we have $v_1^B = 90$ and $v_2^B = 80$. These values are also the same as in Ricardo's example. The production prices can be calculated from (32).

$$\begin{aligned} 0.3\left(1 + r^B\right) + 63w^B &= 1 \\ 0.3\left(1 + r^B\right) + 53w^B &= p^B \end{aligned} \tag{32}$$

When we give $r^B = 1$ in (32), we have $p^B = 59/63$ and $w^B = 2/315$. The consumption possibility set in country B is given by

$$90Y_1^B + 80Y_2^B \leqq 3600 \tag{33}$$

Thus, excluding the commodity prices, everything is the same as in Ricardo's (1817) numerical example.

Comparing the relative price of commodity 1 in terms of commodity 2 in country A with that in country B, the relative price is smaller in country A than in country B.

$$p^A = \frac{11}{15} < \frac{59}{63} = p^B \tag{34}$$

We note that the direction of the inequality is opposite to the direction of Ricardo's example, in which prices are assumed to be proportional to the labor values.

When the capitalists in country A maximize profits, they specialize in producing commodity 2 and importing commodity 1.

Suppose that the international price p^T is 5/6 ($p^A = \frac{11}{15} < \frac{5}{6} < \frac{59}{63} = p^B$). Then, the sets of consumable commodities when country A specializes in producing commodity 2 and country B specializes in producing commodity 1 can be calculated as follows. Let us calculate for country A first. When country A uses all of its 4800 units of labor to produce commodity 2, the country produces 48 units of commodity 2. To produce 48 units of commodity 2, 48/5 units of commodity 1 are required. Consequently,

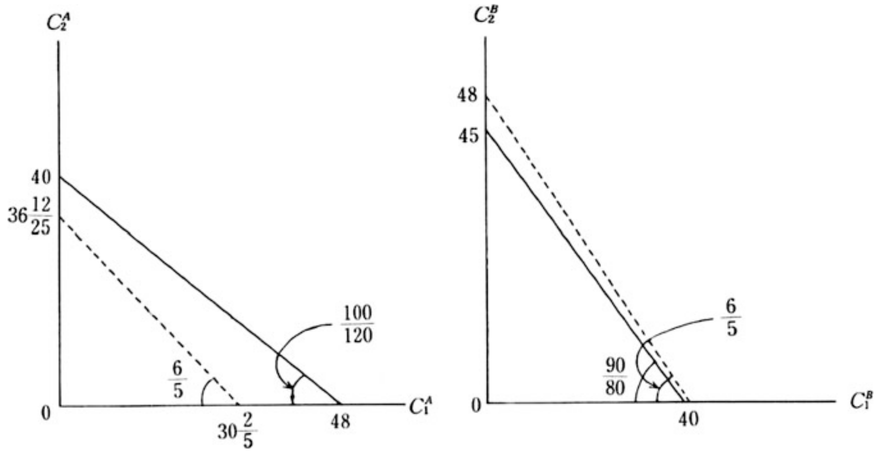


Fig. 3a Consumption possibility set when price is not proportional to labor value ($p^T = 5/6$)

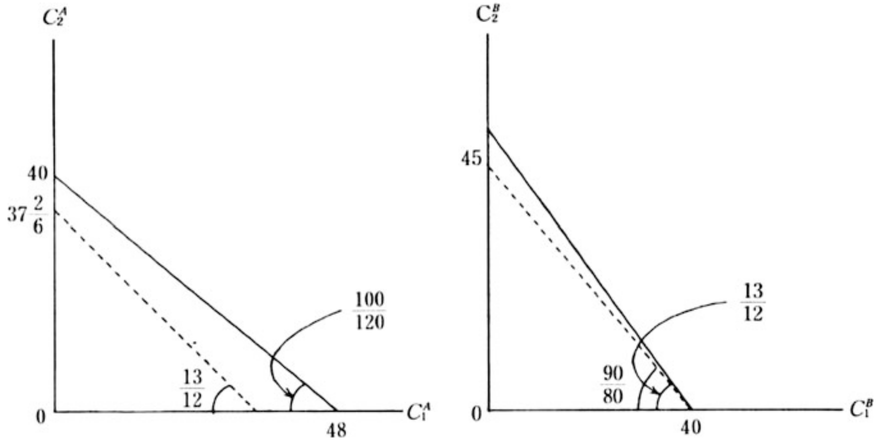


Fig. 3b Consumption possibility set when price is not proportional to labor value ($p^T = 13/122$)

$$C_1^A + \frac{5}{6}C_2^A \leqq 48 \times \frac{5}{6} - \frac{48}{5} = 30\frac{2}{5} \tag{35}$$

is the set of consumable commodities in country A. For country B, the set of consumable commodities in country B can be calculated using (36).

$$C_1^B + \frac{5}{6}C_2^B \leqq 40 \tag{36}$$

As is evident from Fig. 3a, the set of consumable commodities in country A is made smaller by opening trade. If we assume that the international price p^T is 12/13, as shown in Fig. 3b, both consumable sets in countries A and B shrink by opening trade.

Therefore, when price is not proportional to labor value, one or both of two countries is shown as possibly suffering from trade in the sense that some countries shrink their consumption possibility set. Thus, the neo-Ricardian trade theory created fundamental doubt over the benefits of free trade as proven by Ricardo's (1817) comparative advantage theory.

However, Smith (1979) indicated that the transition periods between autarky and trade are not considered in this argument and claimed that the intertemporal optimality can be proven, even in these examples.

We consider this point in our model. To determine the consumption of two commodities, we assume a utility function for a country, as we conducted in Sect. 3. Let us assume that every consumer has the same preference for two commodities, which is characterized by the utility function (37).

$$U = \left(C_1^A\right)^{6/17}\left(C_2^A\right)^{11/17} \tag{37}$$

When each consumer maximizes his or her utility under the budget constraint, he or she purchases commodities such that the marginal rate of substitution of commodity 1 for commodity 2 equals the relative price. Thus, in an autarky,

$$\frac{dC_2^A}{dC_1^A} = \frac{\partial U/\partial C_1^A}{\partial U/\partial C_2^A} = \frac{6C_2^A}{11C_1^A} \tag{38}$$

is equal to $1/p^A = 15/11$, and we have

$$C_2^A = \frac{5}{2}C_1^A \tag{39}$$

The intersection of Eq. (39) and the consumption possibility frontier (40)

$$100C_1^A + 120C_2^A = 4800 \tag{40}$$

is $\left(C_1^A,\ C_2^A\right) = (12, 30)$. Thus, in this combination, consumers maximize their utility and full employment is realized.

Next, let us consider the transition period of country A from autarky to free trade. Capitalists in country A specialize in producing commodity 2. To produce 48 units of commodity 2, 48/5 units of commodity 1 should be prepared. Because the gross outputs of commodities 1 and 2 in an autarky are (30, 30), the quantities of commodities that can be consumed are (102/5, 30). Thus, in a transition period, consumers maximize their utility under constraint (41).

$$C_1^A + p^T C_2^A \leqq \frac{102}{5} + 30p^T \tag{41}$$

If we assume that the international price of commodity 1 in terms of commodity 2, p^T is 5/6, (41) can be rewritten as

$$C_2^A \leqq \frac{6}{5} C_1^A + 54\frac{12}{25} \tag{42}$$

In contrast, because of the marginal rate of substitution of commodity 2 for commodity 1, $-dC_2^A/dC_1^A$ is given by

$$-\frac{dC_2^A}{dC_1^A} = \frac{6C_2^A}{11C_1^A} \tag{43}$$

which equals the international price $1/p^T = 6/5$, and we have

$$C_2^A = \frac{11}{5} C_1^A \tag{44}$$

Solving the equalized form of inequality (42) and equality (44), we have $\left(C_1^A, C_2^A\right) = (16.02, 35.25)$, which is the consumption vector in a transition period.

For the periods after two countries completely transfer to free trade, we have the consumption possibility set

$$C_2^A \leqq -\frac{6}{5} C_1^A + 36\frac{12}{25} \tag{45}$$

From (45) and (44), the consumption vector in free trade is $\left(C_1^A, C_2^A\right) = (10.72, 23.60)$.

The streams of the consumption of two commodities in an autarky and in free trade are shown in Table 4.

Although the consumption of the two commodities in an open economy is smaller than that in an autarky after opening trade, the consumption of both commodities in the transition period is certainly larger than in an autarky. To compare these two consumption streams, let us evaluate the values in terms of the international price $p^T = 5/6$ and use the rate of profit as the discount rate ($r = 1$). Then, the present discounted value of the consumption stream

Table 4 Consumption stream in time-phased Ricardian economy

	0	1	2	3	……
	Before transition	Transition	After transition		
Autarky					
$\boldsymbol{C_1^A}$	12	12	12	12	……
$\boldsymbol{C_2^A}$	30	30	30	30	……
Open economy					
$\boldsymbol{C_1^A}$	12	16.02	10.72	10.72	……
$\boldsymbol{C_2^A}$	30	35.25	23.60	23.60	……

$$C = \sum\nolimits_{t=1}^{\infty} \left(C_{1t}^{A} + p^{T} C_{2t}^{A} \right) / (1 + r)^{t} \tag{46}$$

is 37.9 in an open economy and is larger than 37 in an autarky. This intertemporal efficiency of free trade is claimed by neoclassical economists.

7 The New Theory of International Values

In this section, we introduce a basic model of the new theory of international values developed by Shiozawa (2007) and others and show theorems derived from this model. I change some economic notations from Shiozawa's (2007) original ones to the notations used in ordinary Sraffian economics or neo-Ricardian trade theory.

First, production prices in an autarky are given by the next equation, which is a standard Sraffian model.

$$p^{h} = \left(1 + r^{h}\right) p^{h} A^{h} + w^{h} l^{h} \quad h = A, B, C, \ldots, N \tag{47}$$

Here, p^h denotes a price vector in country h, r^h denotes the rate of profit in country h, A^h denotes a commodity input coefficient matrix in country h, w^h denotes the wage rate in country h, and l^h denotes a labor input coefficient vector.

Each country must satisfy the following labor constraint in an autarky and in free trade.

$$l^{h} x^{h} \leqq L^{h} \quad h = A, B, C, \ldots, N \tag{48}$$

Here, x^h denotes the column vector of output in country h, and L^h denotes the labor endowment of country h. When countries open their trade, international prices and the wage rate of each country must satisfy (49).

$$\begin{gathered} q^{T} = \left(p^{T}, w^{T}\right) = \left(p_{1}^{T}, p_{2}^{T}, \cdots, p_{n}^{T}, w^{TA}, w^{TB}, \cdots, w^{TN}\right) \\ q^{T} \begin{bmatrix} I - \left(1 + r^{T}\right) A^{A} & I - \left(1 + r^{T}\right) A^{B} & \cdots & I - \left(1 + r^{T}\right) A^{N} \\ -l^{A} & 0 & \cdots & 0 \\ 0 & -l^{B} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -l^{N} \end{bmatrix} \leqq 0 \end{gathered} \tag{49}$$

Here, p^T denotes the row vector of the international price, r^T denotes the rate of profit, and w^{Th} denotes the wage rate of country h after trade.

The gross output vector $x = \left(x^A, x^B, \ldots x^N\right)' = \left(x_1^A, \ldots, x_n^A, \ldots, x_1^N, \ldots, x_n^N\right)'$ should satisfy Eq. (50), which means that the price is equal to the cost for the production process actually used, and the cost is higher than the price for the unused production process.

$$\begin{array}{l} p_j^T = \left(1 + r^T\right) \sum_i a_{ij}^h p_i^T + w_j^{Th} l_j^h \rightarrow x_j^h \geqq 0 \\ p_j^T < \left(1 + r^T\right) \sum_i a_{ij}^h p_i^T + w_j^{Th} l_j^h \rightarrow x_j^h = 0 \\ l^h x^h \leqq L^h \quad h = A, \ldots, N \end{array} \tag{50}$$

When q^T and x satisfy (49) and (50), the situation is called a competitive equilibrium in an open economy.

We can prove some theorems for this equilibrium. First, let us check the efficiency of the equilibrium. In an economy in which the rate of profit is positive, we should consider the R-efficient locus as proposed by Mirrlees (1969) instead of the production possibilities frontier. A production vector $\widehat{x}$ is called R-efficient when no x exists that satisfies the labor constraint and (51).

$$\begin{array}{l} \left[I - \left(1 + r^T\right) A^A \;\; I - \left(1 + r^T\right) A^B \;\; \ldots \;\; I - \left(1 + r^T\right) A^N\right] x \\ \quad \geq \left[\mathrm{I} - \left(1 + r^T\right) A^A \; \mathrm{I} - \left(1 + r^T\right) A^B \ldots \mathrm{I} - \left(1 + r^T\right) A^N\right] \widehat{x} \end{array} \tag{51}$$

Here, for the convenience of a subsequent argument, let us define the column vector y by the following equation, which is the time-phased economy version of the net output vector in the world.

$$y = \left[I - \left(1 + r^T\right) A^A \; I - \left(1 + r^T\right) A^B \cdots I - \left(1 + r^T\right) A^N\right] x$$

When we introduce the notion of R-efficient production, we prove the following Theorem 1.

Theorem 1 An equilibrium is an R-efficient production.

Proof Let us assume that $\widehat{x}$ is not an R-efficient production. Then, a gross output vector x exists that satisfies (51). Multiplying p^T to (51) from the left-hand side and deducing $w^T L$, we have

$$\begin{array}{l} p^T \left[I - \left(1 + r^T\right) A^A \; I - \left(1 + r^T\right) A^B \ldots I - \left(1 + r^T\right) A^N\right] x - w^T L \\ \quad > p^T \left[I - \left(1 + r^T\right) A^A \; I - \left(1 + r^T\right) A^B \ldots I - \left(1 + r^T\right) A^N\right] \widehat{x} - w^T L = 0 \end{array} \tag{52}$$

Here, $L = (L^A, \cdots, L^N)$, which is a contradiction of $(p^T,\ w^T)$, and $\widehat{x}$ is an equilibrium.

Next, let us show that a price vector exists that is part of an equilibrium for an R-efficient production.

Theorem 2 Prices and a wage vector q^T exist that are part of an equilibrium and are compatible with the R-efficient production $\widehat{y}$.

For this proof, we need Lemma 1.

Lemma 1[4] For a matrix C, if

$$Cz \geq 0 \qquad z \geqq 0 \tag{53}$$

have no solution, then the following inequalities

$$qC \leqq 0 \qquad q > 0 \tag{54}$$

have a solution.

Proof We first show that the matrix on the left-hand side of (55) has the property of the matrix C in Lemma 1.

$$\begin{bmatrix} -\widehat{y} & I-\left(1+r^T\right)A^A & I-\left(1+r^T\right)A^B & \cdots & I-\left(1+r^T\right)A^N \\ L^A & -l^A & 0 & \cdots & 0 \\ L^B & 0 & -l^B & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ L^N & 0 & \cdots & 0 & -l^N \end{bmatrix} \begin{bmatrix} \lambda \\ x^A \\ x^B \\ \vdots \\ x^N \end{bmatrix} = \begin{bmatrix} v^1 \\ v^2 \end{bmatrix} \geq 0 \tag{55}$$

Let us assume that this matrix has a nonnegative solution $(\lambda, x^A, \cdots, x^N)' \geq 0$. We can rewrite this equation as the equivalent form.

$$\begin{aligned} -\lambda\widehat{y}+\left[I-\left(1+r^T\right)A^A\right]x^A+\left[I-\left(1+r^T\right)A^B\right]x^B+\cdots+\left[I-\left(1+r^T\right)A^N\right]x^N &= v^1 \\ \lambda L^h - l^h x^h = v^{2h} \qquad h = A, B, \ldots, N \end{aligned} \tag{56}$$

Let $\left(\widehat{x}^A, \cdots, \widehat{x}^N\right)'$ be the gross output vector corresponding to $\widehat{y}$. Then, consider $\bar{x} = (x+\widehat{x}) \,/\, (1+\lambda)$. As shown in (57), $\bar{x}$ satisfies the labor constraint.

$$L^h - l^h \frac{x^h + \widehat{x}^h}{1+\lambda} = L^h - \frac{l^h x^h}{1+\lambda} - \frac{L^h}{1+\lambda} = \frac{\lambda L^h - l^h x^h}{1+\lambda} = \frac{v^{2h}}{1+\lambda} \geqq 0 \tag{57}$$

[4] Nikaido (1961, pp. 157–158).

The net output vector $\bar{y}$, which corresponds to $\bar{x}$, is larger than $\widehat{y}$ if $v^1 \geq 0$, as shown in (58).

$$\frac{y}{1+\lambda} + \frac{\widehat{y}}{1+\lambda} = \frac{v^1}{1+\lambda} + \widehat{y} \geq \widehat{y} \tag{58}$$

Because this equation is contrary to the definition of efficiency, the nonexistence of a nonnegative solution is verified. If $v^1 = 0$, we increase an element of vector x^h, of which the element of $v^2 > 0$.

Thus, from Lemma 1, it is shown that (59)

$$\left(p^T, w^{TA}, w^{TB}, \ldots, w^{TN}\right) \begin{bmatrix} -\widehat{y} & I-\left(1+r^T\right)A^A & I-\left(1+r^T\right)A^B & \cdots & I-\left(1+r^T\right)A^N \\ L^A & -l^A & 0 & \cdots & 0 \\ L^B & 0 & -l^B & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ L^N & 0 & \cdots & 0 & -l^N \end{bmatrix} \leqq 0 \tag{59}$$

has a positive price and wage rate vector.

Thus, if there exists an R-efficient output vector, then the existence of the competitive equilibrium in the model of the new theory of international values is certified.

Here, we note that we do not consider the efficiency of the net output but, instead, the R-efficiency proposed by Mirrlees (1969). Although the R-efficient frontier is concave to the origin, the production possibility frontier calculated from each point on the R-efficient locus is, in general, not concave to the origin. This point is argued in detail in Takamasu (1986).

We can prove that the equilibrium, which is a point on the R-efficient locus, is intertemporal efficient.

Theorem 3 A competitive equilibrium is an intertemporal efficient production.

Proof For the convenience of proving Theorem 3, let us define the commodity input coefficient matrix in the world and the labor coefficient vector in the world as follows.

$$A \equiv \left[A^A \, A^B \ldots A^N\right]$$
$$l \equiv \left(l^A, l^B, \ldots, l^N\right)$$

Using these notations, prices and commodity production in an autarky are given by

$$p_t = (1 + r_t)\, p_{t-1}A + w_t l$$
$$x_t = Ax_{t+1} + y_t$$
$$lx_t \leqq L_t$$

International prices and commodity production in free trade must satisfy (60).

$$\begin{aligned} & p_t^T x_t^T = \left(1 + r_t^T\right) p_{t-1}^T \mathrm{A} x_t^T + w_t^T l x_t^T \\ & p_t^T x_t^T = p_t^T A x_{t+1}^T + p_t^T y_t^T \\ & l x_t^T \leqq L_t \end{aligned} \tag{60}$$

When we assume that the world economy is in an autarky at time 0 and transits to free trade at time 1, i.e., $x_0 = x_0^T$, the value of net outputs in an autarky evaluated by international prices is given by

$$\begin{aligned} Y_t = p_t^T y_t &= p_t^T x_t - p_t^T A x_{t+1} \\ &\leqq \left(1 + r_t^T\right) p_{t-1}^T A x_t + w_t^T l x_t - p_t^T A x_{t+1} \end{aligned} \tag{61}$$

In contrast, the value of net outputs in an open economy is given by

$$\begin{aligned} Y_t^T = p_t^T y_t^T &= p_t^T x_t^T - p_t^T A x_{t+1}^T \\ &= \left(1 + r_t^T\right) p_{t-1}^T A x_t^T + w_t^T l x_t^T - p_t^T A x_{t+1}^T \end{aligned} \tag{62}$$

Inequality (63) holds for the difference between the values of consumption in an autarky and in free trade.

$$Y_t^T - Y_t \geqq \left(1 + r_t^T\right) p_{t-1}^T A \left(x_t^T - x_t\right) - p_t^T A \left(x_{t+1}^T - x_{t+1}\right) \tag{63}$$

The differences from period 0 to period n are shown as

$$\begin{aligned} & Y_0^T - Y_0 \geqq \left(1 + r_0^T\right) p_{-1}^T A \left(x_0^T - x_0\right) - p_0^T A \left(x_1^T - x_1\right) \\ & \mathrm{Y}_1^\mathrm{T} - \mathrm{Y}_1 \geqq \left(1 + \mathrm{r}_1^\mathrm{T}\right) \mathrm{p}_0^\mathrm{T} \mathrm{A} \left(\mathrm{x}_1^\mathrm{T} - \mathrm{x}_1\right) - \mathrm{p}_\mathrm{t}^\mathrm{T} \mathrm{A} \left(\mathrm{x}_2^\mathrm{T} - \mathrm{x}_2\right) \\ & \vdots \\ & \mathrm{Y}_\mathrm{n}^\mathrm{T} - \mathrm{Y}_\mathrm{n} \geqq \left(1 + \mathrm{r}_\mathrm{n}^\mathrm{T}\right) \mathrm{p}_{\mathrm{n}-1}^\mathrm{T} \mathrm{A} \left(\mathrm{x}_\mathrm{n}^\mathrm{T} - \mathrm{x}_\mathrm{n}\right) - \mathrm{p}_\mathrm{n}^\mathrm{T} \mathrm{A} \left(\mathrm{x}_{\mathrm{n}+1}^\mathrm{T} - \mathrm{x}_{\mathrm{n}+1}\right) \end{aligned} \tag{64}$$

When we divide each inequality of (64) by $\left(1 + r_0^T\right)$, $\left(1 + r_0^T\right)\left(1 + r_1^T\right)$, $\ldots$, and $\left(1 + r_0^T\right) \cdots \left(1 + r_n^T\right)$ and summate them, we have

$$\begin{aligned} & \left(\mathrm{Y}_0^\mathrm{T} - \mathrm{Y}_0\right) / \left(1 + \mathrm{r}_0^\mathrm{T}\right) + \left(\mathrm{Y}_1^\mathrm{T} - \mathrm{Y}_1\right) / \left(1 + \mathrm{r}_0^\mathrm{T}\right)\left(1 + \mathrm{r}_1^\mathrm{T}\right) \ldots \\ & \geq p_t^T A \left(x_{n+1}^T - x_{n+1}\right) / \left(1 + r_0^T\right) \ldots \left(1 + r_n^T\right) \end{aligned} \tag{65}$$

If we increase n to infinity, the left-hand side of (65) converges to 0, and the value of net outputs in an open economy evaluated by the international prices and the rate of profit in the open economy is evaluated as being larger than that in an autarky.

What new findings or new theorems can we derive from this theory when we consider that the number of commodities is much larger than the number of countries?

The simultaneous equations contain $n + N$–1 unknowns that determine the international equilibrium. For n prices, N wage rates, and one rate of profit, if we take one commodity as a numeraire and assume the rate of profit given, the number of unknowns is $n + N$–1. In contrast, because the number of price equations is n, prices can be determined without depending on demand if more than N–1 commodities are produced in the same countries.

Let us confirm this concept using an example of two countries and three commodities. We assume two countries A and B, with commodities 1 and 2 produced in country A and commodities 2 and 3 produced in country B. In this situation, the following equations hold true.

$$\begin{aligned} p_1^T &= \left(1 + r^T\right)\left(p_1^T a_{11}^A + p_2^T a_{21}^A + p_3^T a_{31}^A\right) + w^{TA} l_1^A \\ p_2^T &= \left(1 + r^T\right)\left(p_1^T a_{12}^A + p_2^T a_{22}^A + p_3^T a_{32}^A\right) + w^{TA} l_2^A \\ p_2^T &= \left(1 + r^T\right)\left(p_1^T a_{12}^B + p_2^T a_{22}^B + p_3^T a_{32}^B\right) + w^{TB} l_2^B \\ p_3^T &= \left(1 + r^T\right)\left(p_1^T a_{13}^B + p_2^T a_{23}^B + p_3^T a_{33}^B\right) + w^{TB} l_3^B \end{aligned} \tag{66}$$

From (66), if we assume the price of commodity 1 as a numeraire and that the rate of profit is given, all prices and the wage rate of the two countries can be determined. Note that if production does not change, a tradeoff exists among the profit rate, the wage rate of country A, and the wage rate of country B.

The condition under which at least one commodity exists that is produced in more than one country is, approximately, that the world demand of that commodity is larger than the quantity of production that can be produced in one country. However, we have not analyzed the details of this condition. This task is left for future research.

8 Concluding Remarks

In this paper, we examined the development of Ricardo's (1817) comparative advantage theory subsequent to his work using the most generalized model. Through our analyses, we clarified that Ricardo's theory crucially depends on the assumptions of two countries, two commodities, and the nonexistence of intermediate goods. We also showed that Mill's argument on the determination of international prices depends on the assumption of perfect specialization.

Thus, we should extend Ricardo's analysis to a model that assumes multicountries, multicommodities, and the existence of intermediate goods. The positive rate of profit or the existence of a production period is also important for analyzing international trade. In this situation, prices do not depend on demand and may be determined by the production cost. When we consider intertemporal efficiency, we cannot say that trade may damage some countries. However, we should be more careful about the benefits of trade.

These assumptions are more similar to reality; because they may change the results of traditional trade theories, we should accept them and attempt to develop the analyses using them. Such analyses will be conducted by numerous researchers in the future. I am pleased if this chapter provides some assistance to these researchers.

References

Amano, A. (1966). Intermediate goods and the theory of comparative advantage: A two-country, three-commodity case. *Weltwirtshaftliches Archiev, 96*, 342–345.

Chipman, J. S. (1965). A survey of the theory of international trade: Part 1, the classical theory. *Econometrica, 33*, 132–147.

Emmanuel, A. (1973). *Unequal Exchange*. New Left Books.

Graham, F. D. (1932). The theory of international value. *Quarterly Journal of Economics, 46*, 581–616.

Jones, R. W. (1961). Comparative advantage and the theory of tariffs: A multi country model. *Review of Economic Studies, 28*, 161–175.

Kurz, H. D. (1978). Rent theory in a multisectoral model. *Oxford Economic Papers, 30*, 16–37.

McKenzie, R. W. (1954). Specialization and efficiency in world production. *Review of Economic Studies, 21*, 165–180.

Mill, J. S. (1852), *Principles of Political Economy with Some Applications to Social Philosophy*, Third Edition, London. The Collected Works of John Stuart Mill, Volume II – The Principles of Political Economy with Some of Their Applications to Social Philosophy (Books I-II), ed. John M. Robson, introduction by V. W. Bladen (Toronto: University of Toronto Press, London: Routledge and Kegan Paul, 1965).

Mirrlees, J. A. (1969). The dynamic non-substitution theorem. *Review of Economic Studies, 36*, 67–76.

Montani, G. (1975). Scarce natural resources and income distribution. *Metroeconomica, 27*, 68–101.

Negishi, T. (1981). *Classical economics and modern economics*. Iwanami Publishing. (in Japanese).

Negishi, T. (1983). *History of economics*. Toyokeizaishinposha. (in Japanese)

Nikaido, H. (1961). *Linear mathematics for economics*. Baifukan. (in Japanese)

Ricardo, D. (1817), On the Principles of Political Economy, and Taxation.

Samuelson, P. A. (1975). Trade pattern reversals in time-phased Ricardian systems and international efficiency. *Journal of International Economics, 5*, 309–363.

Shiozawa, Y. (2007). A new construction of Ricardian trade theory a many-country, many-commodity case with intermediate goods and choice of production techniques. *Evolutionary and Institutional Economics Review, 3*, 141–187.

Smith, M. A. M. (1979). Intertemporal gains from trade. *Journal of International Economics, 9*, 239–248.

Steedman, I. (1979). *Fundamental issues in trade theory*. London: Macmillan.

Steedman, I. & J.S. Metcalfe (1973), On foreign trade, *Economia Internazionale*, pp. 516–528, reprinted in Steedman (1979).

Takamasu, A. (1983). Neo-Ricardian rent theory. *Keizai Ronso, 132*, 119–140. (in Japanese)

Takamasu, A. (1986). Production possibility frontier in a Sraffa-Leontief economy. *The Manchester School, 59*, 202–207.

Takamasu, A. (1991). *Neo-Ricardian trade theory*. Sobunsha. (in Japanese)