The Relation Between Value and Demand in the New Theory of International Values

Tosihiro Oka

Abstract The principal theorem of the new theory of international values for a Ricardo-Sraffa trade economy is presented and then illustrated using a two-country, two-commodity model and a two-country, three-commodity model. It is shown that the classical vision of values as independent of demand is preserved, even when international trade takes place. In other words, values are mainly determined by costs of production or, ultimately, by technology. The values are, however, not determined uniquely, and demand plays a role in selecting a set of values from among those that are admissible under present technology and mark-up rates. Three different production possibility frontiers are introduced: R-efficient locus, physical maximal frontier and capitalistically feasible frontier. It is argued that distinguishing among these three frontiers is necessary in order to comprehend the role of demand in determining international value. Lastly, the similarity of this relation of value and demand to that of rent theory is pointed out.

Keywords International values and demand • Production possibility frontier • R-efficient locus • Capitalistically feasible frontier • Growth rate and profit rate

1 Introduction

The classical theory of value is characterized as the value determined by production costs; the relative prices are determined by technology, independent of demand, if distributive variables, a set of profit rates, are given. This is true for a closed economy, which has no international trade. The new theory of international values retains the classical characteristic in the case of open economies in which commodities for final consumption and intermediate commodities are traded. In the case of international trade, however, value is not determined uniquely, even when a set of profit rates is given, and there is a room for demand to play some role in determining which value is selected from those that are feasible. The new

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theory has been developed for the 'equivalent economy', as defined by Shiozawa (2014, p. 110). That is an economy where the input coefficients include profits on advance capital. In an equivalent trade economy, the relation between prices and demand is established in such a way that the price vector is normal to the facet of the production possibility set on which the demand vector exists. An equivalent economy is, however, a hypothetical economy, defined using input coefficients that include profits. Thus, the production possibility set for the economy is different from that of a real economy. The relation between demand and value should be real, and thus the relation in an equivalent economy should be restated for a real economy. This is the objective of this chapter. We first define a Ricardo-Sraffa (RS) trade economy, for which the principal theorem of the new theory of international values is presented in a general form. Then, the theorem is illustrated using a two-country, two-commodity and a two-country, three-commodity examples for an RS trade economy. Next, we distinguish among three kinds of production possibility frontier, and, using these concepts, we identify the relation between demand and value. Here, we describe how demand affects the selection of the system of techniques and then determines international values.

2 The Principal Theorem of New Theory of International Values

The new theory of international values, developed by Shiozawa (2014), has established the existence of a combination of prices and wages that enables a set of production techniques to be adopted competitively and that gives no incentive to change to other techniques. This theorem is established for the Ricardo-Sraffa (RS) trade economy, which is defined as follows.

There are M countries and N commodities; a technique τ of producing a commodity is identified by the vectors of net output and labour input coefficients:

$$\boldsymbol{a}(\tau) = (a_1^\tau, a_2^\tau, \cdots, a_N^\tau), \quad \boldsymbol{u}(\tau) = (u_1^\tau, u_2^\tau, \cdots, u_M^\tau).$$

Here, a_j^τ represents the net output of commodity $j (j = 1, 2, \cdots, N)$, and u_k^τ denotes the labour input of country $k (k = 1, 2, \cdots, M)$ for technique τ. The net output of commodity j is the gross output minus the input of commodity j, though 'input' here includes profit on advance capital. Thus, $a_j^\tau = b_j^\tau - c_j^\tau(1 + r^\tau)$, where b_j^τ is the gross output of j, c_j^τ is the physical input of j, and r^τ is the mark-up rate, or the profit rate, for technique τ.

Simple production is assumed, so there is no multiple production. Thus, every technique produces only one commodity; suppose commodity n is produced using technique τ, $a_n^\tau > 0$ and $a_j^\tau \leqq 0$ for $j \neq n$. Every technique is assumed to belong to only one country. If technique τ belongs to country m, $u_m^\tau > 0$ and $u_k^\tau = 0\,(k \neq m)$. Since any scalar multiple of a technique is assumed to be feasible, we can assume

$u^{\tau}_m = 1$, for normalization. Therefore, only one component of $\boldsymbol{u}(\tau)$ is positive, with a value of unity, and the values of the other components are 0.

Arranging the vectors $\boldsymbol{a}(\tau)$, $\boldsymbol{u}(\tau)$ for T techniques vertically, we have the following matrices:

$$A = \begin{bmatrix} a^1_1 & a^1_2 & \cdots & a^1_N \\ a^2_1 & a^2_2 & \cdots & a^2_N \\ \vdots & \vdots & & \vdots \\ a^T_1 & a^T_2 & \cdots & a^T_N \end{bmatrix} \quad J = \begin{bmatrix} u^1_1 & u^1_2 & \cdots & u^1_M \\ u^2_1 & u^2_2 & \cdots & u^2_M \\ \vdots & \vdots & & \vdots \\ u^T_1 & u^T_2 & \cdots & u^T_M \end{bmatrix}.$$

The vector $\boldsymbol{y} = (y_1, y_2, \cdots, y_N)$, defined as

$$\boldsymbol{y} = \boldsymbol{s}A,$$

represents the net products, where $\boldsymbol{s} = (s^1, s^2, \cdots, s^T)$ is the vector whose component, s^{τ}, represents the size of the operation of technique τ. The labour inputs are represented by $\boldsymbol{s}J$. When country m has q_m of labour, the labour quantities in the world are represented by $\boldsymbol{q} = (q_1, q_2, \cdots, q_M)$.

Using this notation, the production possibility set $\mathcal{P}$ is defined as

$$\mathcal{P} = \{\boldsymbol{y} \in \mathbb{R}^N \mid \boldsymbol{y} = \boldsymbol{s}A, \boldsymbol{s}J \leqq \boldsymbol{q}, \boldsymbol{s} \geqq \boldsymbol{0}, \boldsymbol{s} \in \mathbb{R}^T\}.$$

An element of $\mathcal{P}$, $\boldsymbol{y}$, is called a maximal element, when $\boldsymbol{z}$ meeting $\boldsymbol{z} \geq \boldsymbol{y}$, $\boldsymbol{z} \in \mathcal{P}$ does not exist.[1] The set of maximal elements is called the maximal boundary or the production possibility frontier (PPF). The PPF consists of a finite number of facets. The interior of such a facet is called a regular domain.[2] Note that a maximal element of $\mathcal{P}$ represents a combination of commodities that can be consumed with leaving sufficient capital for all techniques to grow at a rate of $r^{\tau}(\tau = 1, 2, \cdots, T)$. The hypothetical economy, with the growth rates equivalent to the profit rates, is called an 'equivalent economy' (Shiozawa 2014, p. 110).[3]

Based on this concept of an RS trade economy, the following principal theorem of the new theory of international values is established:

Theorem 1 *Provided that* $\boldsymbol{y}$ *is a maximal element of the production possibility set, there exists a vector of commodity prices,* $\boldsymbol{p} = [p_1, p_2, \cdots, p_N]'$*, and a vector of wages,* $\boldsymbol{w} = [w_1, w_2, \cdots, w_M]'$*, under which no technique obtains extra profit* ($J\boldsymbol{w} \geqq A\boldsymbol{p}$) *and the total value of the net products is equal to the total sum of wages*

[1]Here, $\boldsymbol{z} \geq \boldsymbol{y}$ means $\boldsymbol{z} \geqq \boldsymbol{y}$ and $z_i > y_i$ for at least one component i.

[2]See Definition 3.3 in Chap. 1 of this volume.

[3]When the actual rate of growth for the technique τ is different from its rate of profit, the actual production possibility set will be different from $\mathcal{P}$. This difference matters when we deal with the role of demand in determining the international value. We will return to this question later.

$(\langle \boldsymbol{y}, \boldsymbol{p} \rangle = \langle \boldsymbol{q}, \boldsymbol{w} \rangle)$,[4] *under which every country has at least one competitive technique and labour is fully employed. Conversely, if there is a set of* $\boldsymbol{p}$ *and* $\boldsymbol{w}$ *that satisfies* $\langle \boldsymbol{y}, \boldsymbol{p} \rangle = \langle \boldsymbol{q}, \boldsymbol{w} \rangle$ *and* $\boldsymbol{J}\boldsymbol{w} \geqq \boldsymbol{A}\boldsymbol{p}$, *then* $\boldsymbol{y}$ *is a maximal element.*[5]

A vector $(\boldsymbol{p}, \boldsymbol{w})'$ is called an 'international value'. An international value that satisfies $\boldsymbol{J}\boldsymbol{w} \geqq \boldsymbol{A}\boldsymbol{p}$ and $\langle \boldsymbol{y}, \boldsymbol{p} \rangle = \langle \boldsymbol{q}, \boldsymbol{w} \rangle$ is called an 'admissible value', and an admissible international value corresponding to a net product vector on a regular domain is called 'regular value'.[6] When technique τ is competitive under an admissible international value $(\boldsymbol{p}, \boldsymbol{w})'$, the τth component of the inequality $\boldsymbol{J}\boldsymbol{w} \geqq \boldsymbol{A}\boldsymbol{p}$ is satisfied with equality (i.e. $\langle \boldsymbol{u}(\tau), \boldsymbol{w} \rangle = \langle \boldsymbol{a}(\tau), \boldsymbol{p} \rangle$), and for uncompetitive techniques, a strict inequality holds. The set of competitive techniques is called a 'system of techniques'.

3 The Two-Country, Two-Commodity Model for an Equivalent Economy

A diagram illustration is a useful way to explain this theorem. Since David Ricardo (1951, p. 135), two-country, two-commodity models have been used repeatedly. The model has a risk of opening the way to the supply and demand theory of international value, but it can express the principal characteristics of the new theory.

Table 1 gives an example of the labour and commodity input coefficients of a two-country, two-commodity RS trade economy. Here, the numbers represent the quantity of inputs per unit of gross output. Assuming that the rate of profit is unity for all the techniques, the net output coefficients per unit of labour for the equivalent economy are as shown in Table 2. Let us suppose that the quantities of labour in countries A and B are unity and five, respectively. When all existing labour in both

Table 1 An example of a two-country, two-commodity RS trade economy: input coefficients

		Input coefficient		
		Labour	Commodity 1	Commodity 2
Country A	Production of commodity 1	1/10	0	1/4
	Production of commodity 2	1/50	1/20	9/20
Country B	Production of commodity 1	1/10	9/20	1/4
	Production of commodity 2	1/10	1/20	0

[4] $\langle \boldsymbol{y}, \boldsymbol{p} \rangle$ and $\langle \boldsymbol{q}, \boldsymbol{w} \rangle$ represent the scalar products of $\boldsymbol{y}$ and $\boldsymbol{p}$ and $\boldsymbol{q}$ and $\boldsymbol{w}$, respectively.

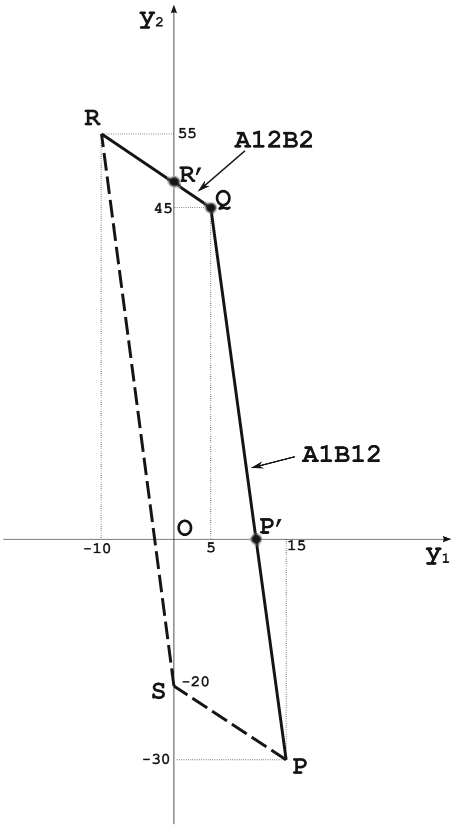
[5] This theorem is equivalent to Theorem 3.4 in Chap. 1, the proof of which is presented in Appendix to Section 3 of Chap. 1.

[6] See Definition 3.7 in Chap. 1. Shiozawa (2014) gave different definitions; an international value that satisfies $\boldsymbol{J}\boldsymbol{w} \geqq \boldsymbol{A}\boldsymbol{p}$ is called 'admissible', and an admissible international value that satisfies $\langle \boldsymbol{y}, \boldsymbol{p} \rangle = \langle \boldsymbol{q}, \boldsymbol{w} \rangle$ is called 'regular' (Shiozawa 2014, p. 351).

Table 2 Net output coefficients for the two-country, two-commodity example per unit of labour

		Net output coefficient	
		Commodity 1	Commodity 2
Country A	Production of commodity 1	10	−5
	Production of commodity 2	−5	5
Country B	Production of commodity 1	1	−5
	Production of commodity 2	−1	10

Fig. 1 Production possibility set of a two-country, two-commodity model



countries is directed to the production of commodity 1, the combination of the net products will be represented by the vector

$$1 \times (10, -5) + 5 \times (1, -5) = (15, -30),$$

which is also represented by point P in Fig. 1, where y_1 and y_2 represent the net products of commodities 1 and 2, respectively.

When all labour in country A is directed to the production of commodity 1, and all labour in country B to commodity 2, the net products are

$$1 \times (10 - 5) + 5 \times (-1, 10) = (5, 45),$$

represented by point Q in Fig. 1. When all labour in both countries is used to produce commodity 2, the net products will be (−10, 55), represented by point R. Point S represents the case where all labour in country A is directed to commodity 2 and all labour in country B is directed to commodity 1.

Segment PQ represents the net products that can be produced by directing all labour in country A to the production of commodity 1 and labour in country B to the production of both commodities. Similarly, segment QR represents the net products that can be produced by directing labour in country A to both commodities and all labour in country B to commodity 2. Let us call the activity that produces the net products on segment PQ 'production A1B12' and the activity that produces the net products on segment QR 'production A12B2'. Let us also call the set of techniques that produces a point on segment PQ 'system of techniques A1B12' and that on segment QR 'system of techniques A12B2'. Similarly, segment PS corresponds to production A12B1 or system of techniques A12B1, and segment RS corresponds to production A2B12 or system of techniques A2B12. Using this notation, point P can be said to correspond to production A1B1, Q to production A1B2, R to production A2B2 and S to production A2B1. Parallelogram PQRS represents the production possibility set, and OP′QR′ represents its non-negative section. PQR is the maximal frontier, and P′QR′ is its non-negative section.

Let w_A and w_B represent the wage rates in country A and country B, respectively, and p_1 and p_2 represent the prices of the first and the second commodity, respectively. We can assume $w_B = 1$ without losing generality. In order for an international value $(p_1, p_2, w_A, 1)$ to be admissible, it should meet

$$\begin{cases} 10p_1 - 5p_2 \leqq w_A \\ -5p_1 + 5p_2 \leqq w_A \\ p_1 - 5p_2 \leqq 1 \\ -p_1 + 10p_2 \leqq 1. \end{cases} \tag{1}$$

When production A1B12 becomes competitive, the first, third and fourth inequalities should be met with equality:

$$\begin{cases} 10p_1 - 5p_2 = w_A \\ p_1 - 5p_2 = 1 \\ -p_1 + 10p_2 = 1, \end{cases}$$

which implies $p_1 = 3, p_2 = 2/5$, and $w_A = 28$. Under these values, the second inequality of (1) is met with a strict inequality, which means the production of commodity 2 in country A is not competitive and will never be carried out. The vector of prices $(3, 2/5)'$ is normal to segment PQ. Any point on segment PQ, $\boldsymbol{y}$, has a value equal to the y_1 coordinate of point P′, in terms of commodity 1, when valued by the price vector $(3, 2/5)'$ (i.e. $\langle \boldsymbol{y}, \boldsymbol{p} \rangle / p_1 = 11$). Thus, $\langle \boldsymbol{y}, \boldsymbol{p} \rangle = 33$, which is equal to $\langle \boldsymbol{q}, \boldsymbol{w} \rangle = 1 \times 28 + 5 \times 1$. Therefore, this international value is admissible. Since any point on the interior of segment PQ can be produced using competitive techniques under the value $(3, 2/5, 28, 1)'$, this international value is regular.

In order for production A12B2 to be competitive, the first, second and fourth inequalities of (1) should be met with equality:

$$\begin{cases} 10p_1 - 5p_2 = w_A \\ -5p_1 + 5p_2 = w_A \\ -p_1 + 10p_2 = 1, \end{cases}$$

which implies $p_1 = 1/14, p_2 = 3/28$, and $w_A = 5/28$. Under these values, the third inequality of (1) is met with strict inequality. The production of commodity 1 in country B is not competitive. The vector of prices $(1/14, 3/28)'$ is normal to segment QR. Any point on this segment, $\boldsymbol{y}$, has a value equal to the y_2 coordinate of point R′ in terms of commodity 2, when valued by the price vector $(1/14, 3/28)'$ (i.e. $\langle \boldsymbol{y}, \boldsymbol{p} \rangle / p_2 = 145/3$). Thus, $\langle \boldsymbol{y}, \boldsymbol{p} \rangle = 145/28$, which is equal to $\langle \boldsymbol{q}, \boldsymbol{w} \rangle = 1 \times 5/28 + 5 \times 1$. Therefore, this international value is admissible and regular.

Point Q corresponds to production A1B2. In order for this production to be competitive,

$$\begin{cases} 10p_1 - 5p_2 = w_A \\ -p_1 + 10p_2 = 1 \end{cases}$$

must be met, which implies

$$p_1 = \frac{2w_A + 1}{19}, \quad p_2 = \frac{w_A + 10}{95}. \tag{2}$$

The other two techniques will not become competitive when

$$\begin{cases} -5p_1 + 5p_2 < w_A \\ p_1 - 5p_2 < 1 \end{cases}$$

are met. Combining the above result, this implies

$$\frac{5}{28} < w_A < 28.$$

The change in the value of w_A from 28 to 5/28 corresponds to a change in p_2/p_1 from 2/15 to 3/2. As long as (2) is met within this range,

$$\langle \boldsymbol{y}, \boldsymbol{p} \rangle = \langle \boldsymbol{q}, \boldsymbol{w} \rangle = w_A + 5.$$

Therefore, the international value that meets (2) is admissible, but is not regular, because points on the interior of segments PQ or QR cannot be produced with competitive techniques under this value.

Production A2B12 realizes the net products on segment RS. The value that makes production A2B12 competitive must meet the equations:

$$\begin{cases} -5p_1 + 5p_2 = w_A \\ p_1 - 5p_2 = 1 \\ -p_1 + 10p_2 = 1, \end{cases}$$

which implies $p_1 = 3, p_2 = 2/5$, and $w_A = -13$. In other words, the wage rate in country A is negative, and under this wage rate, the production of commodity 1 in country A earns extra profit (i.e. $10p_1 - 5p_2 = 28 > -13$). Therefore, the value, $(p_1, p_2, w_A, w_B) = (3, 2/5, -13, 1)$, is not admissible. Similarly, the international value that would make production A12B1 competitive is not admissible; the variables cannot all be non-negative—assuming $w_B = 1$, $(p_1, p_2, w_A, w_B) = (-2/13, -3/13, -5/13, 1)$ and assuming $w_A = 1$, $(p_1, p_2, w_A, w_B) = (2/5, 3/5, 1, -13/5)$.

Consequently, the maximal frontier of the production possibility set consists of two segments, each of which has a vector that is normal to it, and that vector represents the prices that construct a regular international value. Only for production at the vertex of the segments can there be admissible international values, the price vector of which does not have a unique slope, but the slope must lie between the slopes of the two vectors that are normal to the two segments. This is the case which Graham (1948) called ‘limbo’, as described in Chap. 10 by Sato in this volume.

Like Graham (1923, 1948), the new theory stresses the importance of production on segments other than their endpoints and regards the limbo case as improbable. This point will be made clearer in the two-country, three-commodity case. The two-country, three-commodity model is the minimal model needed to represent the RS trade economy where the number of countries, M, is smaller than the number of commodities, N.

4 Two-Country, Three-Commodity Model for the Equivalent Economy

Table 3 presents an example of the two-country, three-commodity case. When the profit rate is unity for all techniques (the uniform rate of profit is not a necessary assumption, just for simplification), the net output coefficients per unit of labour will be as shown in Table 4.

Table 3 An example of a two-country, three-commodity RS trade economy: input coefficients

		Input coefficient			
		Labour	Commodity 1	Commodity 2	Commodity 3
Country A	Production of commodity 1	1/10	0	1/4	0
	Production of commodity 2	1/50	1/20	9/20	0
	Production of commodity 3	1/100	3/200	1/20	0
Country B	Production of commodity 1	1/10	9/20	1/4	0
	Production of commodity 2	1/10	1/20	0	0
	Production of commodity 3	1/20	1/40	0	0

Table 4 Net output coefficient for the two-country, three-commodity example per unit of labour

		Net output coefficient		
		Commodity 1	Commodity 2	Commodity 3
Country A	Production of commodity 1	10	−5	0
	Production of commodity 2	−5	5	0
	Production of commodity 3	−3	−10	100
Country B	Production of commodity 1	1	−5	0
	Production of commodity 2	−1	10	0
	Production of commodity 3	−1	0	20

Assuming the quantity of labour in country A is 1 and that in country B is 5, the production possibility set is the nonahedron shown in Fig. 2. This diagram is drawn in the same way as in the two-country, two-commodity case. In this diagram, A_iB_j represents the net products by applying all labour in country A to the production of commodity i and all labour in country B to the production of commodity j, and the nonahedron is drawn by connecting all A_iB_js $(i, j = 1, 2, 3)$. The non-negative section is described by hexahedron OGHIJKL in Fig. 3. Triangles GHI and HKL and tetragon HIJK are the maximal frontier.

Assuming $w_B = 1$, an admissible value should meet the inequalities:

$$\begin{cases} 10p_1 - 5p_2 \leqq w_A \\ -5p_1 + 5p_2 \leqq w_A \\ -3p_1 - 10p_2 + 100p_3 \leqq w_A \\ p_1 - 5p_2 \leqq 1 \\ -p_1 + 10p_2 \leqq 1 \\ -p_1 + 20p_3 \leqq 1. \end{cases}$$

When production A123B2 is carried out competitively, the first, second, third and fifth inequalities must be met with equality, which implies

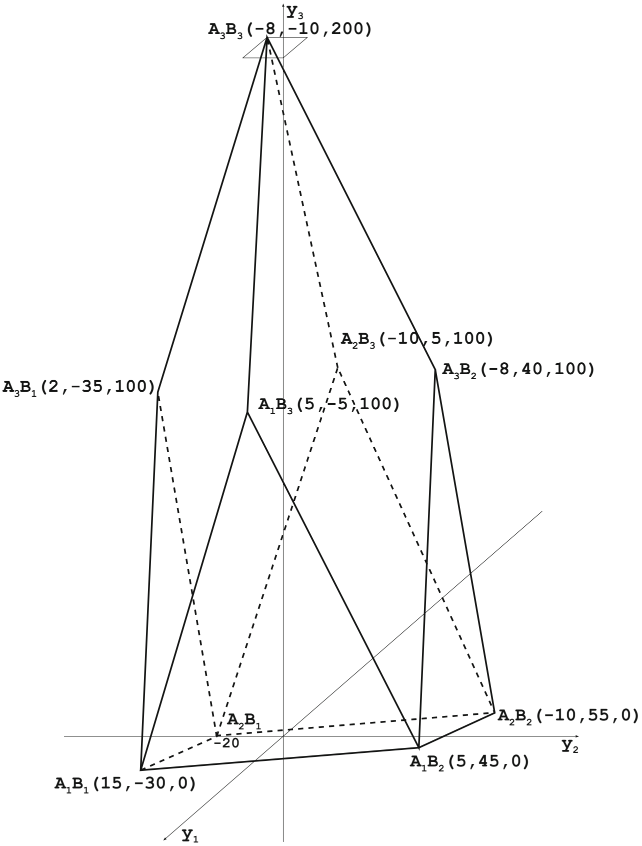


Fig. 2 Production possibility set of the two-country, three-commodity example

$$\begin{cases} p_1 = 1/14 \\ p_2 = 3/28 \\ p_3 = 41/2800 \\ w_A = 5/28. \end{cases} \tag{3}$$

Under this international value, the residual inequalities are met with strict inequality. Production A123B2 can realize any point on triangle HKL, and because the price vector $(1/14, 3/28, 41/2800)$ is normal to HKL, any point (y_1, y_2, y_3) on the plane satisfies

$$\frac{1}{14}(y_1 - 5) + \frac{3}{28}(y_2 - 45) + \frac{41}{2800}y_3 = 0,$$

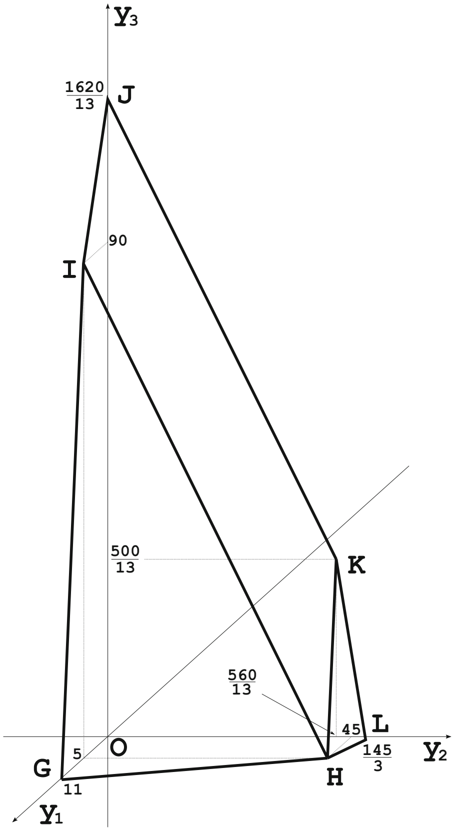


Fig. 3 Production possibility set of the two-country, three-commodity example: the non-negative section

or

$$\frac{1}{14}y_1 + \frac{3}{28}y_2 + \frac{41}{2800}y_3 = \frac{145}{28},$$

while the sum of wages is

$$1 \times \frac{5}{28} + 5 = \frac{145}{28};$$

thus, $\langle \boldsymbol{y}, \boldsymbol{p} \rangle = \langle \boldsymbol{q}, \boldsymbol{w} \rangle$. The international value (3) is, therefore, admissible. It is also regular, because any point in the interior of triangle HKL can be produced with the techniques that are competitive under this international value.

The same reasoning establishes that

$$\begin{cases} p_1 = 9/17 \\ p_2 = 13/85 \\ p_3 = 13/170 \\ w_A = 77/17 \end{cases}$$

is a regular international value which makes production A13B23 that produces a point on tetragon HIJK competitive. The international value

$$\begin{cases} p_1 = 3 \\ p_2 = 2/5 \\ p_3 = 1/5 \\ w_A = 28 \end{cases}$$

is also regular and makes production A1B123 that produces a point on triangle GHI competitive.

Other than those regular international values, there are two admissible international values that bring limbo-type productions into existence. One is the value that satisfies

$$\begin{cases} p_1 = (2w_A + 1)/19 \\ p_2 = (w_A + 10)/95 \\ p_3 = (27w_A + 23)/1900 \\ 5/28 < w_A < 77/17, \end{cases}$$

and the other is

$$\begin{cases} p_1 = (2w_A + 1)/19 \\ p_2 = (w_A + 10)/95 \\ p_3 = (w_A + 10)/190 \\ 77/17 < w_A < 28. \end{cases}$$

The former makes production A13B2 competitive, and the latter makes production A1B23 competitive. In the two-commodity case, the relative price p_1/p_2 could change freely between the upper and the lower limits for the limbo-type production to be carried out. Here, in the three-commodity case, the relative prices p_1/p_2 and p_3/p_2 are constrained as

$$p_1 = 10p_2 - 1 = \frac{200p_3 - 1}{27}$$

in the case of A13B2, and

$$p_1 = 10p_2 - 1 = 20p_3 - 1$$

in the case of A1B23. This is because these limbo cases bring about net products on the edges HK and HI, respectively, and do not bind them at any vertex of the polyhedron. There is no vertex in the positive octant, because the number of countries is less than the number of commodities. As the difference between the numbers expands, the degree of freedom in the relative prices decreases, and the degree at which demand affects prices becomes smaller.

We have 15 systems of techniques. Table 5 shows the international values that corresponds to the 15 systems of techniques. The net products that system A*ijk*B*l* can produce with the full employment of labour in both countries are shown as triangle $(A_iB_l)(A_jB_l)(A_kB_l)$ in Fig. 2, the net products of system A*i*B*jkl* as triangle $(A_iB_j)(A_iB_k)(A_iB_l)$, and the net products of system A*ij*B*kl* as tetragon $(A_iB_k)(A_iB_l)(A_jB_l)(A_jB_k)$.

The first three systems in Table 5 have regular international values, as described above. The other systems do not have admissible values. The fourth to seventh systems have positive international values, because the plane including the polygon each system can produce with full employment of labour has a positive normal vector and has a non-negative section. Their international values, however, are not admissible, because they give extra profit to some technique not belonging to the system. The 8th to 11th systems of techniques have non-negative price vectors, but the wage rate of either country becomes negative, because the plane including the

Table 5 International values in the systems of techniques

System of techniques	International value					Remarks	
	p_1	p_2	p_3	w_A	w_B		
A123B2	1/14	3/28	41/2800	5/28	1	Regular value	
A13B23	9/17	13/85	13/170	77/17	1		
A1B123	3	2/5	1/5	28	1		
A12B23	1/14	3/28	3/56	5/28	1	Positive value	Extra profit to A3
A13B12	3	2/5	41/100	28	1		Extra profit to B3
A123B3	10/31	15/31	41/620	25/31	1		Extra profit to B2
A3B123	3	2/5	1/5	7	1		Extra profit to A1
A123B1	2/13	3/13	41/1300	5/13	−1	Non-negative price vector Negative wage for either country	
A23B13	2	3/5	1/20	−7	−1		
A2B123	3	2/5	1/5	−13	1		
A23B12	3	2/5	0	−13	1		
A12B13	−2/13	−3/13	11/260	−5/13	1	Price vector with negative components	
A23B23	−3/10	7/100	7/200	37/10	1		
A13B13	2/3	−1/15	1/12	7	1		
A12B12	–					No value makes A12B12 competitive	

polygon each system can produce does not have a non-negative section. The 12th to 14th systems do not have positive normal price vectors, and the last system does not have an international value that makes all the techniques constructing the system competitive, except for the edges.

5 Discrepancy Between the Growth Rate and the Profit Rate

The matrix A denotes net outputs. 'Net output', here, means gross output minus input multiplied by one plus the profit rate; $a_j^\tau = b_j^\tau - c_j^\tau(1 + r^\tau)$. Net products $\boldsymbol{y}$ is defined using this concept, and thus, the production possibility set should also be understood in terms of the input coefficients that include profits on advance capital. Shiozawa argued, 'in this case, the production possibility set should be interpreted as the set of net surplus product in the growing economy with growth rate $1 + r$' (Shiozawa 2007, p. 146).

As is evident from the fact that the profit rate is expressed with superscript τ above, the rate can vary among techniques; thus, it can also vary among industries and among countries. Therefore, the production possibility set should be interpreted as the set of net surplus product in the growing economy with growth rates that are equal to the profit rates, which can vary among industries and among countries.

The growth rate of an industry is, however, not necessarily equal to its profit rate. With regard to the world economy as a whole, its growth rate is, in general, different from the rate of profit on capital. Let us investigate what occurs to the relation between net products and international value when growth rates are different from profit rates.

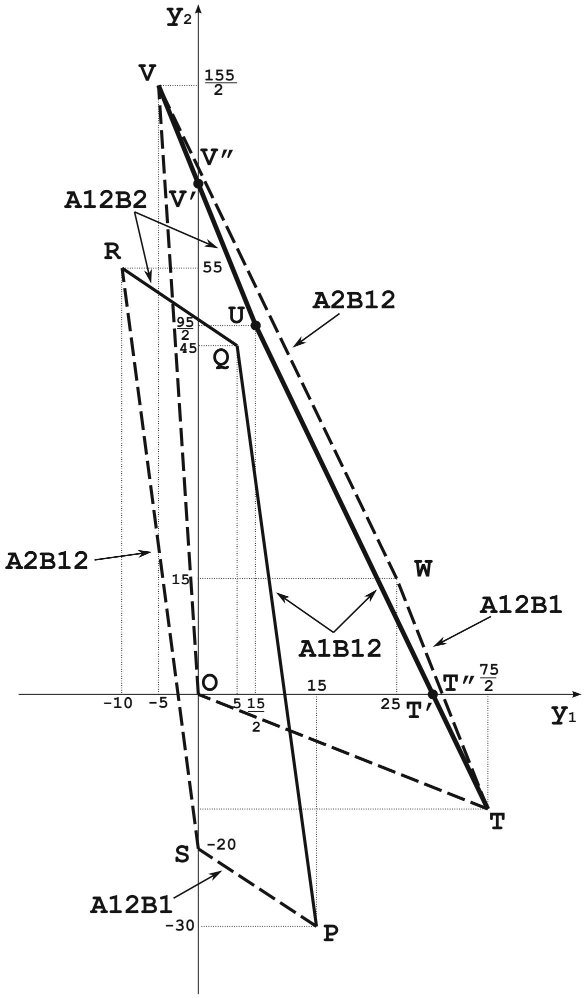
Let us consider the case where there is a unique profit rate r for all industries in the world, and the growth rate of all industries is zero. This is a special case, but the analysis can be extended to more general cases.

From the two-country, two-commodity example given in Table 1, we have the net output coefficients per unit of labour under the zero rate of growth, as shown in Table 6. The net products from full employment of labour can exist on segments TU, UV, VW and WT in Fig. 4. Allowing for underemployment, the inside of parallelogram TUVW and the area of tetragon OTUV can also be produced. Thus,

Table 6 Net output coefficient for the two-country, two-commodity example per unit of labour under a zero rate of growth

		Net output coefficient	
		Commodity 1	Commodity 2
Country A	Production of commodity 1	10	−5/2
	Production of commodity 2	−5/2	55/2
Country B	Production of commodity 1	11/2	−5/2
	Production of commodity 2	−1/2	10

Fig. 4 Production possibility set under a real growth rate for the two-country, two-commodity model



the production possibility set is represented by tetragon OTWV, and TWV is the production possibility frontier.

This production possibility frontier, however, will not be produced under the actual profit rate, which is unity in this case. This is because in order for the points on TW to be produced, production A12B1 must be carried out, but system of techniques A12B1 does not have an admissible international value, as observed above. Similarly, the points on VW cannot be produced under the existing profit rate, because there is no admissible international value that makes A2B12 competitive. Under a profit rate of unity, only systems of techniques A1B12 and A12B2 have admissible international values.

Table 7 Net output coefficient for the two-country, three-commodity example per unit of labour under a zero rate of growth

		Net output coefficient		
		Commodity 1	Commodity 2	Commodity 3
Country A	Production of commodity 1	10	−5/2	0
	Production of commodity 2	−5/2	55/2	0
	Production of commodity 3	−3/2	−5	100
Country B	Production of commodity 1	11/2	−5/2	0
	Production of commodity 2	−1/2	10	0
	Production of commodity 3	−1/2	0	20

Therefore, under a profit rate of unity, only productions A1B12 and A12B2 can be carried out, which will produce the net products on segments TU and UV, the non-negative section of which is T′U and UV′. Hence, the actual production possibility frontier should be TUV, and the production possibility set should be OTUV. Segments TW and WV are physically possible, but will not appear under capitalistical competition. The real production possibility set OTUV is not convex, or the production possibility frontier TUV is not concave to the origin.

Takamasu (1986, 1991, pp. 63–67) showed that the production possibility frontier in a two-commodity, two-factor (labour and land) Sraffa-Leontief economy with positive profit rates is not necessarily concave to the origin. The non-convexity of the production possibility set shown here for the RS trade economy has the same characteristics as Takamasu's case.

This phenomenon is observed also for the two-country, three-commodity model. The net output coefficients per unit of labour derived from Table 3 under a zero growth rate are shown in Table 7. Under these net output coefficients, net products produced by the full employment of labour will be on the nonahedron in Fig. 5. Allowing for underemployment, all points on the segments connecting the points on this nonahedron and the origin can be produced; these points are included in the production possibility set. The production possibility frontier consists of triangle $(A_3B_1)(A_3B_2)(A_3B_3)$, triangle $(A_1B_1)(A_2B_1)(A_3B_1)$ and parallelogram $(A_2B_1)(A_2B_2)(A_3B_2)(A_3B_1)$, the non-negative section of which consists of triangle MNP, QRS and pentagon MPQST in Fig. 6.

Net products on this production possibility frontier could be produced by systems of techniques A123B1, A23B12 or A3B123. These systems of techniques, however, will never appear under the present rate of profit. As shown in Table 5, under the profit rate of unity, a negative wage rate is necessary to make A123B1 and A23B12 feasible, and the value making A3B123 feasible gives extra profit to A1. This corresponds to the fact that plane $(A_1B_1)(A_2B_1)(A_3B_1)$ and plane $(A_2B_1)(A_3B_1)(A_3B_2)(A_2B_2)$ do not have non-negative section and that triangle $(A_3B_1)(A_3B_2)(A_3B_3)$ is behind the maximal frontier in Fig. 2.

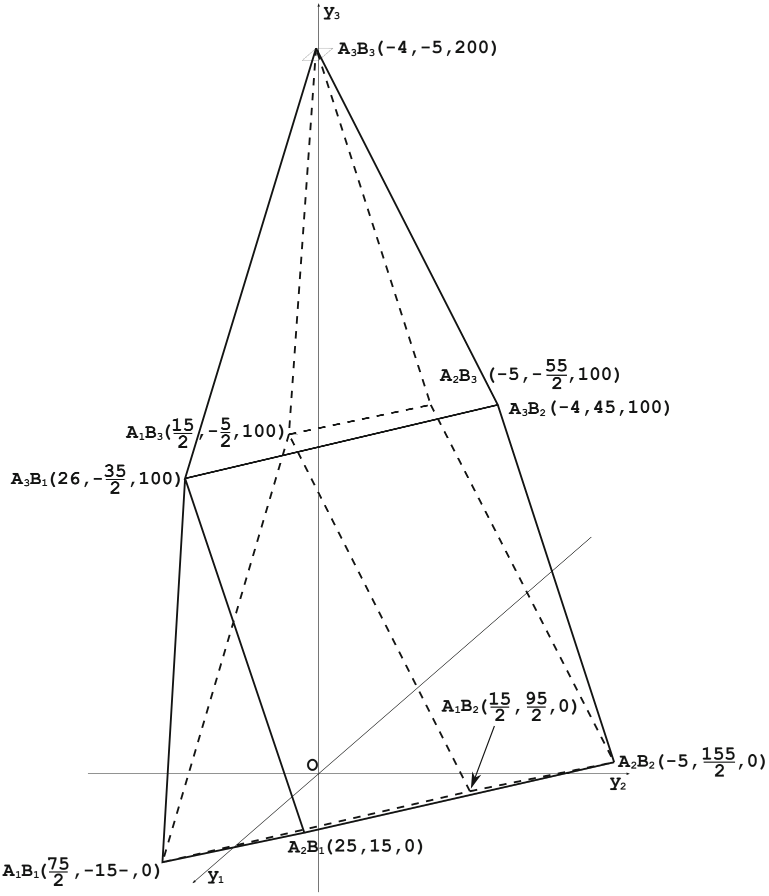


Fig. 5 Production possibility set of the two-country, three-commodity example under a zero growth rate

Only the three systems of techniques, A123B2, A13B23 and A1B123, are feasible under the profit rate of unity; thus, the maximal frontier under a zero growth rate consists of triangles $(A_1B_2)(A_2B_2)(A_3B_2)$ and $(A_1B_1)(A_1B_2)(A_1B_3)$ and parallelogram $(A_1B_2)(A_1B_3)(A_3B_3)(A_3B_2)$ in Fig. 5, the non-negative section of which is shown as UVW, UYZ and UWXY in Fig. 7. This frontier is below MNP, QRS and MPQST, and not concave to the origin, as in the two-commodity case in Fig. 4.

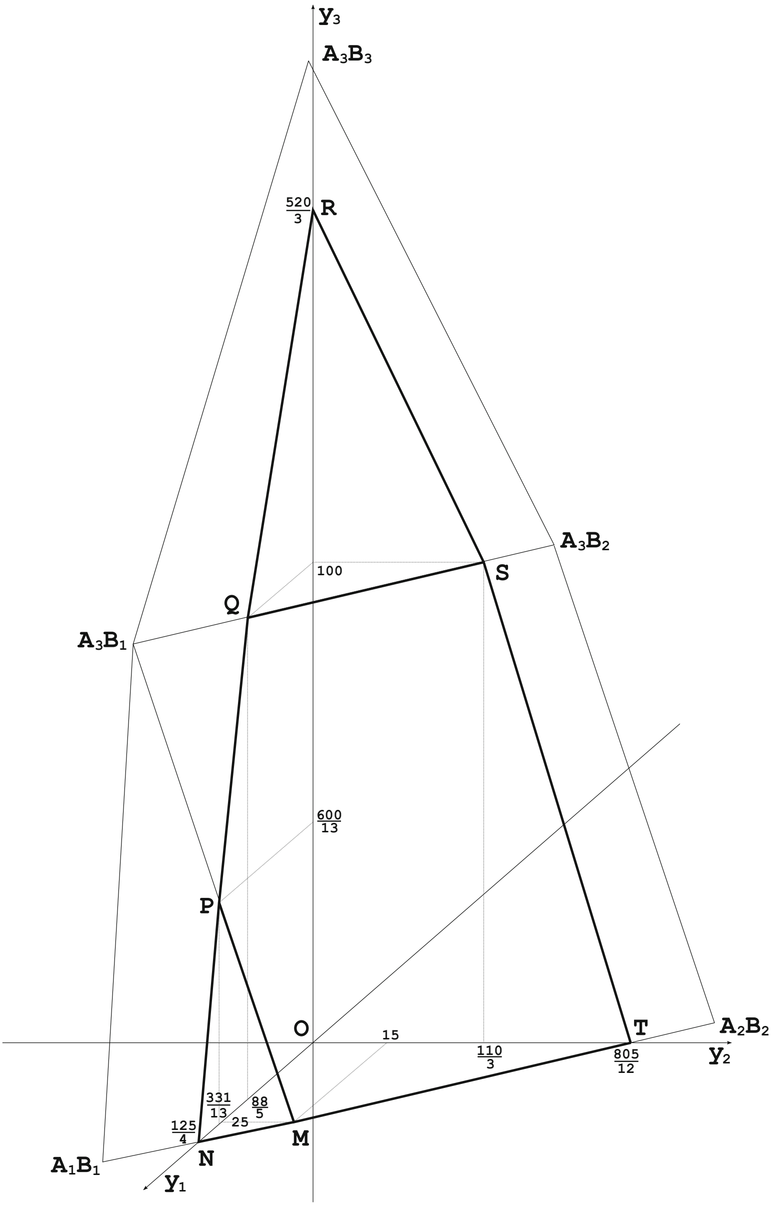


Fig. 6 Production possibility frontier of the two-country, three-commodity example under a zero growth rate: the non-negative section

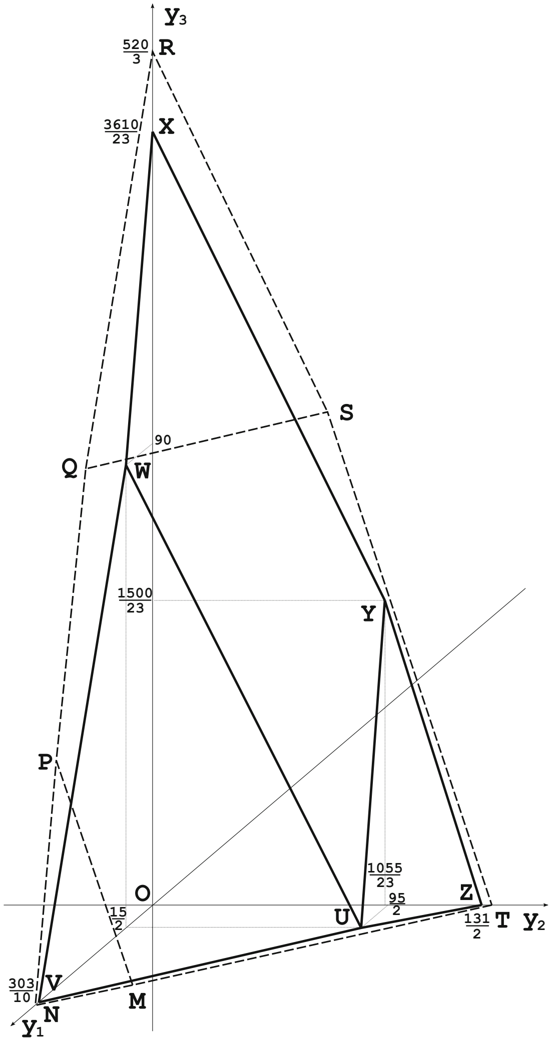


Fig. 7 Actual production possibility frontier of the trade economy with zero growth rate and unity profit rate

6 Relation Between Final Demand and Value

The non-negative sections of the three production possibility frontiers are assembled in Fig. 8. GHIJKL is the production possibility frontier under the hypothetical growth rate equal to the profit rate (here unity). This is the same one referred to as an 'R-efficient locus' (Takamasu 1991, p. 65; Mirlees 1969). We denote this REL. MNPQRST is the physical maximal frontier (PMF) under the actual growth rate of zero. PMF is above REL. Both are concave to the origin, but they will never appear; REL is a hypothetical one, and PMF can be realized only by the systems of

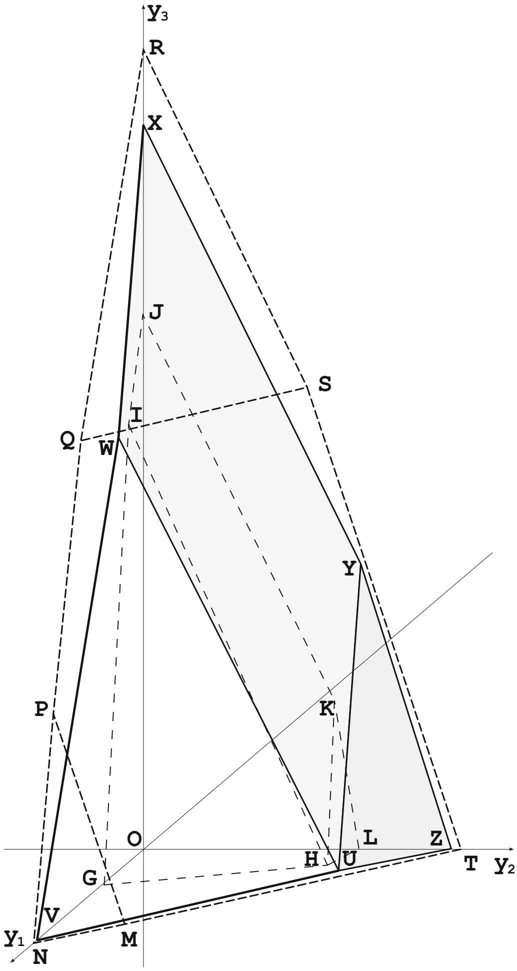


Fig. 8 Three production possibility frontiers of trade economy

techniques that never become feasible under the profit rate of unity or only under international values that are not admissible or that are not non-negative under a profit rate of unity.

UVWXYZ, which is not concave to the origin, is the production possibility frontier realized under a profit rate of unity. Let us call it the ‘capitalistically feasible frontier (CFF)’. If labour is fully employed, actual products must be on the CFF. That the CFF is below the PMF indicates inefficiency in capitalistic production.[7]

When demand requires net products on, for example, the tetragon UWXY, production A13B23 has to be carried out. In order for that to happen, the international

[7]In Fig. 4, PQR is the RFF, TWV is the PMF, and TUV is the CFF.

value must meet $(p_1, p_2, p_3, w_A, w_B) = (9/17, 13/85, 13/170, 77/17, 1)$. The price vector (9/17, 13/85, 13/170) is normal to the plane HIJK, not to UWXY, on which the net products meeting the demand will exist. When world demand moves to a point on, say, triangle UVW, the system of techniques must become A1B123, and the international value must be $(p_1, p_2, p_3, w_A, w_B) = (3, 2/5, 1/5, 28, 1)$, which is normal to GHI.

Final demand is linked to international value in this manner. This relationship of demand to value is parallel with that in Ricardo and Sraffa's theory of rent (Ricardo 1951, p. 70; Sraffa 1960, pp. 75–76). In that theory, demands impose constraints on production techniques—growth in demand for foods requires techniques that produce more corn per acre with higher cost per unit of product—and prices are adjusted to make such techniques competitive. At the same time, rents emerge. In the new theory of international values also, demands impose constraints on production techniques—including the determination of which countries produce which commodities— and prices are adjusted to realize such patterns on division of production, accompanied by adjustments in wages.

Let us express the relation between demand and value in a general formula. We have mentioned above that net output coefficients are related to gross output coefficients and input coefficients, as

$$a_j^\tau = b_j^\tau - c_j^\tau(1 + r^\tau).$$

Since simple production is assumed, any technique produces only one commodity. Let us denote the commodity produced by technique τ as $\gamma(\tau)$; thus, $b_{\gamma(\tau)}^\tau > 0$, and $b_i^\tau = 0$ for $i \neq \gamma(\tau)$. Let us define $h_j^\tau(j = 1, 2, \cdots, N), l_k^\tau(k = 1, 2, \cdots, M)$, and $\delta_j^\tau(j = 1, 2, \cdots, N)$ as

$$h_j^\tau = \frac{c_j^\tau}{b_{\gamma(\tau)}^\tau}, \quad l_k^\tau = \frac{u_k^\tau}{b_{\gamma(\tau)}^\tau}, \quad \begin{cases} \delta_{\gamma(\tau)}^\tau = 1 \\ \delta_j^\tau = 0 \quad (j \neq \gamma(\tau)), \end{cases}$$

and H, L, and I as

$$H = \begin{bmatrix} h_1^1 & h_2^1 & \cdots & h_N^1 \\ h_1^2 & h_2^2 & \cdots & h_N^2 \\ \vdots & \vdots & & \vdots \\ h_1^T & h_2^T & \cdots & h_N^T \end{bmatrix}, \quad L = \begin{bmatrix} l_1^1 & l_2^1 & \cdots & l_M^1 \\ l_1^2 & l_2^2 & \cdots & l_M^2 \\ \vdots & \vdots & & \vdots \\ l_1^T & l_2^T & \cdots & l_M^T \end{bmatrix}, \quad I = \begin{bmatrix} \delta_1^1 & \delta_2^1 & \cdots & \delta_N^1 \\ \delta_1^2 & \delta_2^2 & \cdots & \delta_N^2 \\ \vdots & \vdots & & \vdots \\ \delta_1^T & \delta_2^T & \cdots & \delta_N^T \end{bmatrix}.$$

Note that any low of L has only one component that has a positive value; if technique τ belongs to country $\mu(\tau)$, then $l_{\mu(\tau)}^\tau > 0,\ l_k^\tau = 0\,(k \neq \mu(\tau))$. Assuming the rate of profit for technique τ is r^τ and defining matrix $\tilde{H}$ as

$$\tilde{H} = \begin{bmatrix} h_1^1(1+r^1) & h_2^1(1+r^1) & \cdots & h_N^1(1+r^1) \\ h_1^2(1+r^2) & h_2^2(1+r^2) & \cdots & h_N^2(1+r^2) \\ \vdots & \vdots & & \vdots \\ h_1^T(1+r^T) & h_2^T(1+r^T) & \cdots & h_N^T(1+r^T) \end{bmatrix},$$

an admissible international value $(\boldsymbol{p}, \boldsymbol{w})'(> 0)$ must satisfy

$$(I - \tilde{H})\boldsymbol{p} \leqq L\boldsymbol{w}. \tag{4}$$

When $(\boldsymbol{p}, \boldsymbol{w})'$ is admissible, there exists $\tilde{\boldsymbol{y}}$ satisfying

$$\langle \tilde{\boldsymbol{y}}, \boldsymbol{p} \rangle = \boldsymbol{s}(I - \tilde{H})\boldsymbol{p} = \boldsymbol{s}L\boldsymbol{w} = \langle \boldsymbol{q}, \boldsymbol{w} \rangle, \tag{5}$$

which implies that, if $s^\tau > 0$, the τ-th component of (4) is satisfied with equality:

$$p_{\gamma(\tau)} - \langle \tilde{\boldsymbol{h}}(\tau), \boldsymbol{p} \rangle = l^\tau_{\mu(\tau)} w_{\mu(\tau)},$$

where $\tilde{\boldsymbol{h}}(\tau) = (h_1^\tau(1 + r^\tau), h_2^\tau(1 + r^\tau), \cdots, h_N^\tau(1 + r^\tau))$.

Suppose that the production using technique τ grows at rate g^τ and that matrix $\hat{H}$ is defined as

$$\hat{H} = \begin{bmatrix} h_1^1(1+g^1) & h_2^1(1+g^1) & \cdots & h_N^1(1+g^1) \\ h_1^2(1+g^2) & h_2^2(1+g^2) & \cdots & h_N^2(1+g^2) \\ \vdots & \vdots & & \vdots \\ h_1^T(1+g^T) & h_2^T(1+g^T) & \cdots & h_N^T(1+g^T) \end{bmatrix}.$$

Then, $\boldsymbol{y}$ that satisfies $\boldsymbol{y} = \boldsymbol{s}(I - \hat{H})$ is on a facet that is part of the CFF.

Conversely, if a final demand vector $\boldsymbol{y}$ is given on the CFF, $\boldsymbol{s}$ is determined to satisfy

$$\begin{cases} \boldsymbol{y} = \boldsymbol{s}(I - \hat{H}) \\ \boldsymbol{s}L = \boldsymbol{q}, \end{cases}$$

and such international value $(\boldsymbol{p}, \boldsymbol{w})'$ is selected as satisfies (4) and (5) for this $\boldsymbol{s}$. This is how demand affects value in the new theory.

Multiplying $\boldsymbol{y} = \boldsymbol{s}(I - \hat{H})$ by $\boldsymbol{p}$ to the right and comparing it with (5), we have

$$\langle \boldsymbol{y}, \boldsymbol{p} \rangle = \boldsymbol{s}L\boldsymbol{w} + \boldsymbol{s}(\tilde{H} - \hat{H})\boldsymbol{p}.$$

Here, the left-hand side represents the value of net products, or of final demand, and the first and second terms of the right-hand side represent labourers' consumption and capitalists' consumption, respectively.

7 Conclusion

I have illustrated the meanings of the new theory of international values using a two-country, two-commodity model and a two-country, three-commodity model in an RS trade economy and have examined the relation between values and demand in the new theory. It is found that three production possibility frontiers should be distinguished: an R-efficient locus, a physical maximal frontier and a capitalistically feasible frontier. When profit rates are given, an R-efficient locus is determined, each facet of which has a combination of prices the vector of which is normal to the facet. The combination of prices has a combination of wages that makes the techniques that can produce the points on the facet under the hypothetical growth rate equal to the profit rate competitive. When the REL is given, a capitalistically feasible frontier is determined, a facet of which will be chosen according to the final demand. The chosen facet on the CFF determines an international value, the price vector of which is normal to the corresponding facet of the REL and not normal to the facet on the CFF. Any point on the CFF is feasible by the full employment of labour but is below the physical maximal frontier under the actual growth rate.

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