The New Theory of International Values in the Context of the Ricardo-Sraffian Theory of Value and Distribution

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Abstract In this article we introduce readers to the new theory of international values by placing it in the context of the Ricardo-Sraffian theory of value and distribution. Ricardo's theory is described as that in which exchangeable values of commodities are regulated by the quantities of labour bestowed in their production, on which he established his theory on the distribution of the produce of the earth. Contemporary classical theory, founded by Sraffa, is described as preserving Ricardo's perspective of the value independent of distribution and of demand by replacing the labour theory with the production-cost theory. After noting that Ricardo left the question of determination of values of the commodities traded internationally, it is shown that J. S. Mill argued that the law of demand and supply determines them, which conflicts with the classical perspective. We then demonstrate how the new theory of international values solves the question in line with the classical vision. Lastly, the similarity between this theory and Sraffa's treatment of multiple products is indicated.

Keywords J. S. Mill • Sraffa on multiple products • Classical theory of value and distribution

1 Introduction

In this article we will introduce readers to the new theory of international values. The classical theory of value is characterised as determination of relative prices by production costs, which is contrasted with the neoclassical theory in which relative prices are determined as equilibria between demand and supply. In the case of a closed economy with simple production, the minimum price theorem (often referred to as the non-substitution theorem) guarantees that prices will be determined by production costs. International trade and multiple production (including existence

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of other production factors than labour) are the two causes that undermine the dominance of the cost side in price determination. We will show how the new theory retains the classical nature of value theory when international trade exists.

First, I will describe Ricardo's theory of value and distribution and his vision on the dynamics of distribution. Second, I will show Sraffa's way to retain Ricardo's vision of distribution that is independent of value, without the labour theory of value, on which Ricardo depended. It is explained that the classical theory of value is reformulated as the production-cost theory of value. Third, it is shown that the production-cost theory of value seems to become invalid when foreign trade is introduced, because Ricardo left the determination of international values unsolved and J. S. Mill presented a solution using a law of supply and demand, which is in opposition to the production-cost theory of value. By using Mill's numerical example, I will show that Mill's solution is not valid and international values can be determined by cost-side factors according to the new theory. Lastly, it is indicated that international trade is, in a sense, regarded as a case of the same nature as that in which multiple products exist.

2 Ricardo's Theory of Value and Distribution

David Ricardo, quoting Adam Smith, argued that utility is necessary for a commodity to have exchangeable value, but that utility is not the measure of exchangeable value. He then identified two sources of commodities' exchangeable value; one is scarcity and the other is quantity of labour required to obtain them (Ricardo 1951a, pp. 12–13). He insisted, however, that commodities that derive their exchangeable value from scarcity are limited to those whose supply cannot be increased by labour and that the greatest part of goods that are the object of desire are produced by labour, and their exchangeable value is determined solely by the quantity of labour bestowed on their production.

Thus, the theory of value of the classical school is characterised as the labour theory of value; relative prices are in proportion to the quantity of labour bestowed on their production. The quantity of labour includes the labour used directly in production and the labour embodied in the goods used in production, such as raw materials, fuels, chemicals, machines and tools. This theory can be expressed well by the equation

$$v_j = v_1 a_{1j} + v_2 a_{2j} + \cdots v_n a_{nj} + l_j, \tag{1}$$

where v_j represents the value of a good j, l_j the quantity of labour directly used in the production of one unit of j, a_{ij} the quantity of an input $i (i = 1, 2, \cdots, n)$ required to produce one unit of j and v_i the value of the good $i (i = 1, 2, \cdots, n)$.

On the basis of this theory of value, Ricardo established a proposition on the law of the distribution of products: profits will fall only when wages rise. Wage is the price of labour, and it is regulated by the same law that regulates all prices of commodities: relative prices are in proportion to the quantities of labour bestowed on their production; thus, wage is in proportion to the 'value of labour', in other words, the amount of labour required to produce the foods, necessaries and conveniences to enable the labourers 'to subsist and to perpetuate their race without either increase or diminution' (Ricardo 1951a, p. 93). If we let ω represent the value of one unit of labour, then the equation (1) becomes

$$v_j = v_1 a_{1j} + v_2 a_{2j} + \cdots v_n a_{nj} + \omega l_j + \pi_j, \tag{2}$$

where π_j represents the profit from the production of one unit of commodity j. Comparing (1) and (2), we have

$$\pi_j = (1 - \omega) l_j. \tag{3}$$

This equation expresses straightforwardly the relation between wage and profit. It also shows that a change in ω does not have any effect on the value of the products; it affects only the distribution of the products between capitalists and labourers.

Ricardo dismissed scarcity as a source of exchangeable value for the greatest part of goods, the supply of which can be increased by labour. It is only the scarcity of land that plays a role in Ricardo's theory of value and distribution for such goods. In the progress of population, demand for foods increases, which requires land of inferior quality to be called into cultivation. Cultivation of such land will require a larger quantity of labour to produce the same amount of product, which will raise the value of the product. The increase in the value gives rise to a surplus in the cultivation of land of superior quality. The surplus will be obtained by the landowner as a rent (Ricardo 1951a, p. 70).

If $a_{in}^1 (i = 1, 2, \cdots, n)$ and l_n^1 denote the quantity of input i and labour, respectively, to produce one unit of the good n, say, corn, on the land of superior quality, and $a_{in}^2 (i = 1, 2, \cdots, n)$ and l_n^2 the quantity of input i and labour, respectively, to produce one unit of corn on the land of inferior quality, then

$$\begin{cases} v_n = v_1 a_{1n}^1 + v_2 a_{2n}^1 + \cdots v_n a_{nn}^1 + l_n^1 + \tau \\ v_n = v_1 a_{1n}^2 + v_2 a_{2n}^2 + \cdots v_n a_{nn}^2 + l_n^2, \end{cases}$$

where τ represents the rent that accrues from the land of superior quality. When lands of further inferior quality come into cultivation, the values would have to meet the equations

$$\begin{cases} v_n = v_1 a_{1n}^1 + v_2 a_{2n}^1 + \cdots v_n a_{nn}^1 + l_n^1 + \tau^1 \\ v_n = v_1 a_{1n}^2 + v_2 a_{2n}^2 + \cdots v_n a_{nn}^2 + l_n^2 + \tau^2 \\ \quad \vdots \\ v_n = v_1 a_{1n}^{\nu-1} + v_2 a_{2n}^{\nu-1} + \cdots v_n a_{nn}^{\nu-1} + l_n^{\nu-1} + \tau^{\nu-1} \\ v_n = v_1 a_{1n}^{\nu} + v_2 a_{2n}^{\nu} + \cdots v_n a_{nn}^{\nu} + l_n^{\nu}, \end{cases} \tag{4}$$

where a_{in}^k, l_n^k $(k = 1, 2, \cdots, \nu)$ represent the quantity of input i and labour, respectively, to produce one unit of corn on the kth superior land, and τ^k $(k = 1, 2, \cdots, \nu - 1)$ is the rent to that land. The ν-th land is one of the poorest qualities, or the marginal land, and generates no rent.

Collecting the equation (1) for n commodities, we have

$$\begin{cases} v_1 = v_1 a_{11} + v_2 a_{21} + \cdots v_n a_{n1} + l_1 \\ v_2 = v_1 a_{12} + v_2 a_{22} + \cdots v_n a_{n2} + l_2 \\ \quad \vdots \\ v_n = v_1 a_{1n} + v_2 a_{2n} + \cdots v_n a_{nn} + l_n. \end{cases} \tag{5}$$

This defines the system of values. When rents exist for scarce land, the last equation in (5) is replaced by (4).

The principal problem for Ricardo was to determine the laws that regulate the distribution of the produce of the earth, that is, how it is divided among the three classes of the community (i.e. how it is divided among rents, profits and wages) (Ricardo 1951a, p. 5). The most important part of the law is given by equation (3), which shows that profits decrease only when wages increase. As the population grows, lands of inferior quality are called into cultivation with the emergence of rents, accompanied by a rise in the value of corn, which brings about a rise in wages and a fall in profits. This is an outline of the dynamics of distribution viewed by Ricardo.

The rate of profit is the profit divided by the value of capital; using the notation in (2), the rate of profit, ρ_j, is expressed as

$$\rho_j = \frac{\pi_j}{v_1 a_{1j} + v_2 a_{2j} + \cdots v_n a_{nj}} = \frac{(1 - \omega) l_j}{v_1 a_{1j} + v_2 a_{2j} + \cdots v_n a_{nj}},$$

when wages are paid after production and as

$$\rho_j' = \frac{\pi_j}{v_1 a_{1j} + v_2 a_{2j} + \cdots v_n a_{nj} + \omega l_j} = \frac{(1 - \omega) l_j}{v_1 a_{1j} + v_2 a_{2j} + \cdots v_n a_{nj} + \omega l_j},$$

when wages are paid before production.

The rates of profit for different commodities are, in general, not equal to each other (i.e. $\rho_j \neq \rho_k$, $\rho_j' \neq \rho_k'$), unless the capital-labour ratios of two industries are equal:

$$\frac{l_j}{v_1 a_{1j} + v_2 a_{2j} + \cdots v_n a_{nj}} = \frac{l_k}{v_1 a_{1k} + v_2 a_{2k} + \cdots v_n a_{nk}}.$$

The higher the capital-labour ratio, the lower the rate of profit. Inequality in the rate of profit would cause shifts of capital between industries, which would lower the relative price of the products of the industry that has a higher profit rate. Such shifts of capital will continue until the profit rates in all the industries become equal.

Ricardo, thus, acknowledged that the law that relative prices of commodities are in proportion to the quantity of labour bestowed on their production must be modified when taking the difference in the capital-labour ratio into account. In his expression the cause of the modification was the difference in the degree of durability of capital or in the time which must elapse before one set of commodities can be brought to market (Ricardo 1951a, p. 34), but that is the same as the difference in the capital-labour ratio.

Ricardo's proposition on the distribution must also be influenced by this modification in the theory of value: a change in the value of labour does not affect the value of the commodity, but only affects the amount of profit. Ricardo, however, did not think this modification had a serious impact on the whole body of his theory of value and distribution, because, he thought, the effects of a change in distribution on relative prices were slight in comparison to the effects of a change in the quantity of labour required for production (Ricardo 1951a, p. 45).

3 Contemporary Classical Theory

The above-mentioned problem, the incompatibility between the labour theory of value and a uniform rate of profits, was solved in the direction of abandoning the labour theory while preserving the fundamental views Ricardo intended to advocate: first, the distribution that is independent of the values and second, the values that are independent of the demands. This solution was proposed by Sraffa, who had been engaged in editing *The Works and Correspondence of David Ricardo*. He discovered a rudimentary version of the first fundamental view in Ricardo's *An Essay on the Influence of a Low Price of Corn on the Profits of Stock* (Ricardo 1951b) and found that it was held through all the editions of the *Principles* (Sraffa 1951, pp. xxx–xxxv).

According to Sraffa, the system of prices is formulated as

$$\begin{cases} p_1 = (1+r)(p_1 a_{11} + p_2 a_{21} + \cdots + p_n a_{n1}) + wl_1 \\ p_2 = (1+r)(p_1 a_{12} + p_2 a_{22} + \cdots + p_n a_{n2}) + wl_2 \\ \vdots \\ p_n = (1+r)(p_1 a_{1n} + p_2 a_{2n} + \cdots + p_n a_{nn}) + wl_n, \end{cases} \tag{6}$$

where $p_j (j = 1, 2, \cdots, n)$ is the price of commodity j and w is the wage rate.[1] Sraffa devised a composite commodity which, when used as a measure of value,

[1]This expression is different from Sraffa's in that the equations are written in terms of one unit of output.

held constant the ratio of outputs to inputs, or capital, and expressed the relation between the wage rate and the profit rate without being influenced by the change in relative prices. The composite commodity is called the 'standard commodity' and is defined as the set of commodities, $(x_1, x_2, \cdots, x_n)$, that satisfies

$$\begin{cases} x_1 = (1+R)(a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n) \\ x_2 = (1+R)(a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n) \\ \qquad \vdots \\ x_n = (1+R)(a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n). \end{cases} \tag{7}$$

Assuming the total quantity of labour employed to produce $(x_1, x_2, \cdots, x_n)$ unity (i.e. $\sum_j l_j x_j = 1$), adopting this commodity as the standard of value and defining the value of the net product of this commodity produced by one unit of labour as unity (i.e. $\frac{R}{1+R}\sum_i p_i x_i = 1$), we have the equation

$$r = R(1-w) \tag{8}$$

by comparing the sum of each equation in (6) multiplied by x_j:

$$\sum_j p_j x_j = (1+r)\sum_i \sum_j p_i a_{ij} x_j + w \sum_j l_j x_j$$

with the sum of each equation in (7) multiplied by p_i:

$$\sum_i p_i x_i = (1+R)\sum_i \sum_j p_i a_{ij} x_j.$$

Equation (8) shows that wage rate and profit rate are in a relation expressed as a straight line.

In addition to this straightforward expression, Sraffa gave a proof for the proposition that the real wage rate must be lowered in terms of any commodity as a standard when profit rate, r, increases. Suppose the commodity i is the standard of value (i.e. $p_i = 1$); if w does not fall when r rises, $p_1a_{1i} + p_2a_{2i} + \cdots + p_na_{ni}$ must decline, and the price of some commodities, therefore, must decline. Suppose j is such a commodity; then $p_1a_{1j} + p_2a_{2j} + \cdots + p_na_{nj}$ must decline at a greater rate than p_j does, but for the commodity whose price declines at the greatest rate, that is impossible, and consequently, w must fall (Sraffa 1960, p.47).

That values are independent of demands is self-evident in equation (6), where demands never appear. The significance of this view of values being independent of demands, however, becomes great when the choice of production techniques is considered. Let us assume labour as the standard of value (i.e. $w = 1$), in (6), and assume that a new technique or a method of production appears for, say, commodity 1 with a set of coefficients, $a'_{11}, a'_{21}, \cdots, a'_{n1}, l'_1$, which, under the present prices satisfying (6), brings about a surplus to the production of commodity 1:

$$p_1 > (1+r)(p_1 a'_{11} + p_2 a'_{21} + \cdots + p_n a'_{n1}) + l'_1.$$

The surplus, or extra profit, will make this new technique prevail in industry 1, which will, in turn, eliminate the extra profit by lowering the price of commodity 1 until it equals its cost of production including normal profit. In this state, the prices must satisfy the equations:

$$\begin{cases} p'_1 = (1+r)(p'_1 a'_{11} + p'_2 a'_{21} + \cdots + p'_n a'_{n1}) + l'_1 \\ p'_2 = (1+r)(p'_1 a_{12} + p'_2 a_{22} + \cdots + p'_n a_{n2}) + l_2 \\ \qquad \vdots \\ p'_n = (1+r)(p'_1 a_{1n} + p'_2 a_{2n} + \cdots + p'_n a_{nn}) + l_n. \end{cases} \tag{9}$$

Here, p'_1 must be smaller than p_1 when $w = 1$ in (6). If commodity 1 never enters the production of other commodities (i.e. if $a_{1j} = 0$ for $j = 2, 3, \cdots, n$), their prices will not change (i.e. $p'_j = p_j$ for $j = 2, 3, \cdots, n$). In this case, we will have

$$p'_1 < (1+r)(p'_1 a_{11} + p'_2 a_{21} + \cdots + p'_n a_{n1}) + l_1,$$

because the first equation in (6) is met, $p'_i = p_i (i = 2, 3, \cdots, n)$ and $1-(1+r)a_{11} > 0$. This means the previous method for commodity 1 runs a deficit under the present prices; this method will never revive. If commodity 1 enters the production of any other commodities, say, commodity 2 (i.e. $a_{12} > 0$), then p'_2 must also be smaller than p_2. Furthermore, if commodity 2 enters the production of another commodity, its price will be lowered. The prices of the commodities, the production of which commodity 1 enters directly or indirectly, thus, should be lowered. Suppose commodity k is the one with the greatest rate of reduction in price among those commodities whose price is lowered. Because from (6)

$$p_k[1-(1+r)a_{kk}] = (1+r)(p_1 a_{1k} + \cdots + p_{k-1} a_{k-1,k} + p_{k+1} a_{k+1,k} + \cdots + p_n a_{nk}) + l_k,$$

and $p_k[1-(1+r)a_{kk}] \geqq (1+r)p_1 a_{1k}$, if the rate of reduction in p_k is larger than that in p_1, the production of commodity k will run a deficit. The rate of reduction in the price of commodity k must, therefore, not be larger than that in the price of commodity 1; as a result, the rate of reduction in p_1 is the greatest. Because from the first equation of (6)

$$p_1[1-(1+r)a_{11}] = (1+r)(p_2 a_{21} + p_3 a_{31} + \cdots + p_n a_{n1}) + l_1,$$

and p_1 will face the greatest rate of reduction, we will have

$$p_1'[1-(1+r)a_{11}] < (1+r)(p_2'a_{21} + p_3'a_{31} + \cdots + p_n'a_{n1}) + l_1.$$

This means the previous method for commodity 1 will run a deficit under the new set of prices; therefore, that method will never revive.

Consequently, with a given rate of profit, r, a method of production for a commodity that has the least cost of production under the present prices will be chosen. If the chosen method is different from the one that consists of the set of methods of the products on which the present prices are based, the new method will bring about a new set of prices, under which the newly chosen method will continue to be competitive; under this set of prices, producers have no incentive to return to the previous method.

Let us refer to a set of production methods, or techniques, each of which produces a certain net quantity of each of the products demanded by the society as a 'system of techniques'; a set of the coefficients of production of the commodity j

$$(a_{1j}, a_{2j}, \cdots, a_{nj}, l_j)$$

defines a method of production of j, and a set of such sets of coefficients for all the commodities

$$\begin{cases} (a_{11}, a_{21}, \cdots, a_{n1}, l_1) \\ (a_{12}, a_{22}, \cdots, a_{n2}, l_2) \\ \qquad\qquad \vdots \\ (a_{1n}, a_{2n}, \cdots, a_{nn}, l_n) \end{cases}$$

defines a system of techniques.

Through consecutive application of the above process, a system of techniques and a set of prices, each of which is the lowest in terms of wage, are determined independently of demands. Shiozawa (1981, pp. 104–109, 2007, p. 143, 2014, p. 71) called this proposition the 'minimum price theorem'.[2] The demand side has a relation to the quantities of the commodities supplied but does not concern their prices.

When the rate of profit changes, the system of techniques chosen will also change, but the direction of the change has no relation to the intensity of capital or labour; we cannot say that when the rate of profit rises, the methods of production with smaller intensity of capital become more competitive.

In the neoclassical theory of prices and distribution, a rise in the rate of profits, r, causes a change in production techniques in the direction of decreasing capital

[2]This theorem is generally known as the 'non-substitution theorem' (Dorfman et al. 1958). Pasinetti (1977) discussed the meaning of the theorem from the Sraffian viewpoint.

intensity or increasing labour intensity. This fact, when combined with discrepancies in capital (or labour) intensity among products and with product prices dependent on demands, brings about the effects of changes in demands on profit rates (or wage rates); thus, when demand increases for a product with high capital intensity and its price rises, the rate of profits will rise and the rate of wages will decline. Those relations are the key in the Heckscher-Ohlin-Samuelson theory of international trade.

In the contemporary classical theory, as formulated above, the quantity of capital cannot be defined other than as a value of the commodities which enter production, and it is dependent on the prices of the commodities, which, in turn, are dependent on the rate of profits. The rate of profits, therefore, cannot be 'explained' by such a concept as 'the marginal products of capital'. This is a characteristic of the classical theory that is in contrast to the neoclassical theory, and the famous 'capital controversy' concerned this point.[3]

When the demand for foods increases and Ricardian rents arise, the n-th equation in (6) is replaced with

$$\begin{cases} p_n = (1+r)\left(p_1 a_{1n}^1 + p_2 a_{2n}^1 + \cdots p_n a_{nn}^1\right) + w l_n^1 + \tau^1 \\ p_n = (1+r)\left(p_1 a_{1n}^2 + p_2 a_{2n}^2 + \cdots p_n a_{nn}^2\right) + w l_n^2 + \tau^2 \\ \qquad\qquad\qquad\qquad\vdots \\ p_n = (1+r)\left(p_1 a_{1n}^{\nu-1} + p_2 a_{2n}^{\nu-1} + \cdots p_n a_{nn}^{\mu-1}\right) + w l_2^{\nu-1} + \tau^{\nu-1} \\ p_n = (1+r)\left(p_1 a_{1n}^{\nu} + p_2 a_{2n}^{\nu} + \cdots p_n a_{nn}^{\nu}\right) + w l_n^{\nu}. \end{cases} \tag{10}$$

Sraffa (1960, pp. 75–76) pointed out there can be another type of rent—a rent which would emerge from the coexistence of two different methods applied to the land with a uniform quality. In the increase in demand for foods, a method of producing foods that produces more corn per acre with higher cost per unit of product may be required. Rent would have to emerge in order for that method of production to be equally competitive with another method with smaller cost. In this case,

$$\begin{cases} p_n = (1+r)(p_1 a_{1n} + p_2 a_{2n} + \cdots p_n a_{nn}) + w l_n + \tau t \\ p_n = (1+r)\left(p_1 a'_{1n} + p_2 a'_{2n} + \cdots p_n a'_{nn}\right) + w l'_n + \tau t', \end{cases} \tag{11}$$

[3]The controversy began with the question raised by Joan Robinson (1953–1954) on the concept of 'capital' in the aggregate production function, which became popular in growth theory (Solow 1957). The possibility of the 'reswitching' of techniques indicated by Sraffa (1960) became a focal point, because it implies that capital intensity has no relation to the rate of profits. The proposition posed by Levhari (1965) that reswitching does not occur when all the commodities are 'basics' turned out to be false (Levhari and Samuelson 1966). Solow (1967) tried to save the neoclassical capital theory by proposing an interpretation that the profit rate is regarded as the rate of the increase in consumption for the saving necessary for switching from one technique to another, but the interpretation was criticised by Pasinetti (1969) as nothing but an accounting identity.

where $(a_{1n}, a_{2n}, \cdots, a_{nn}, l_n, t)$ and $(a'_{1n}, a'_{2n}, \cdots, a'_{nn}, l'_n, t')$ are two sets of coefficients (t and t' are the coefficients for land input) representing two different methods applied to land of the same quality.

Sraffa included the chapter titled 'Land' in Part II of his book, which deals with 'multiple-product industries and fixed capital', and regarded the case where rents arise for the land with a uniform quality as of the same nature with the case of multiple products from a single industry, in the sense that 'at least one commodity is produced by *more than one* method'; that is, multiple processes producing the same products can coexist (Sraffa 1960, p. 78).

The contemporary classical theory of value is characterised as the cost of production theory, which retains Ricardo's vision of distribution that is independent of values without relying on the labour theory of value; it established the vision of values that are independent of demands taking choice of techniques into account.[4]

4 Foreign Trade and J.S. Mill's Solution for the Determination of International Values

In Ricardo's theory, profits decrease only when wages rise, and vice versa, and wages fall and rise according to the fall and rise in the quantity of labour required to produce the necessaries on which wages are expended (Ricardo 1951a, pp. 48–49, 126, 132). On the basis of this theory, Ricardo argued that 'the natural tendency of profits is to fall; for, in the progress of society and wealth, the additional quantity of food required is obtained by the sacrifice of more and more labour' (*op. cit.*, p. 120). Foreign trade is regarded as a factor that lowers wages through the imports of cheaper food (*op. cit.*, p. 132) and maintains the rate of profit against the tendency to decline.

Here, it is meant that the labour to produce a certain quantity of an export exchangeable for a quantity of food produced in a foreign country is less than the labour to produce the same quantity of food domestically. The absolute quantity of labour bestowed to produce food in the foreign country does not matter; the quantity of an export needed in exchange for a certain quantity of an import 'is not determined by the respective quantities of labour devoted to the production of each' (*op. cit.*, p. 135). The famous example given by Ricardo demonstrates that: to produce cloth requires the labour of 100 men for one year, and to make wine requires the labour of 120 men for the same time in England; to produce the wine in Portugal requires the labour of 80 men for one year, and to produce the cloth requires the labour of 90 men for the same time. In this situation, Portugal exports wine in exchange for cloth from England, and it is advantageous for both countries.

[4]Shiozawa (2016) proposes to rename Ricardo's theory of value the *cost of production theory of value* from the modern viewpoint.

The labour bestowed in Portugal is not exchanged for the same quantity of labour in England; if it is, neither cloth nor wine will ever be exported from England because they are too expensive.

In autarky, 100 units of wine is worth 120 units of cloth, or the relative price of wine is 6/5 in terms of cloth in England; 90 units of wine is worth 80 units of cloth, or the relative price of wine is 8/9 in terms of cloth in Portugal.[5] When the relative price of wine is greater than 6/5, cloth will not be produced in England; when it is smaller than 6/5, wine will not be produced in England. When the relative price of wine is greater than 8/9, cloth will not be produced in Portugal; when the relative price of wine is smaller than 8/9, wine will not be produced in Portugal. In order for cloth to be produced in England and for wine to be produced in Portugal, the relative price should be between 8/9 and 6/5. When the relative price of wine is 8/9, only cloth is produced in England, both wine and cloth can be produced in Portugal, and one unit of labour in Portugal is equal to $8/9 \times 100/80 = 10/9$ units of labour in England; when the relative price of wine is 6/5, only wine is produced in Portugal, both wine and cloth can be produced in England, and one unit of labour in Portugal is equal to $6/5 \times 100/80 = 12/8$ units of labour in England. When the relative price is greater than 8/9 and smaller than 6/5, England specialises in the production of cloth and Portugal in the production of wine.

Ricardo left the question at what point between the two poles the relative price is determined unresolved. John Stuart Mill addressed this problem and gave an answer that it is the law of supply and demand that determines the relative price of a commodity traded internationally.

He made a 2-country, 2-commodity example: 10 yards of broadcloth cost as much labour as 15 yards of linen in England and as much as 20 yards of linen in Germany. In this situation, 'it would be the interest of England to import linen from Germany, and of Germany to import cloth from England' (Mill 1909, p. 585). The relative price of linen in terms of cloth should be between 10/20 and 10/15. The demand for linen in England and for cloth in Germany depends on the relative price. Suppose it is 10/17. Under this price, if the demand for cloth in Germany is 10,000 yards and the demand for linen in England is 17,000 yards, Germany can pay for 10,000 yards of cloth by 17,000 yards of linen, and England can pay for 17,000 yards of linen with 10,000 yards of cloth; the demand and the supply of both goods coincide with each other.

If, however, the demand for linen in England is only 13,600 yards under the price of 10/17, Germany cannot pay for 10,000 yards of cloth with 13,600 yards of

[5]Here, Ricardo's four numbers are interpreted as representing the quantities of labour needed to produce a unit of wine and of cloth in both countries. Tabuchi (2006, 2017) and Faccarello (2015) argue that Ricardo's numbers should not be interpreted as such. They insist that those numbers are not technical coefficients, and Ricardo assumed from the outset some amount of wine produced by using 80 men a year in Portugal is exchangeable for some amount of cloth produced by using 100 men a year in England. See also Chap. 9 in this volume.

linen, which is worth only 8000 yards of cloth; the demand for linen is below its supply, and vice versa for cloth. The price of linen must be lowered in order for the demand to increase. Suppose when the price becomes 10/18, the demand for linen in England increases to 16,200 yards, and the demand for cloth in Germany decreases to 9000 yards. Now England can pay for the 16,200 yards with 9000 yards of cloth; the demand coincides with the supply for both commodities. When, on the contrary, the demand for linen in England under the price 10/17 is too large, say 20,400, the price should rise for the demand to meet its supply.

Mill says, 'it may be considered, therefore, as established, that when two countries trade together in two commodities, the exchange value of these commodities relatively to each other will adjust itself to the inclinations and circumstances of the consumers on both sides, in such manner that the quantities required by each country, of the articles which it imports from its neighbour, shall be exactly sufficient to pay for one another' (Mill 1909, p. 587). Mill, here, seems to regard the constraint of the equality between payments and receipts as being able to adjust the exchange value of the traded commodities. He, in fact, emphasises that 'the exports of each country must exactly pay for the imports; meaning now the aggregate exports and imports, not those of particular commodities taken singly'(*op. cit.*, p. 590) and named this the 'equation of international demand' or the 'law of international values' (*op. cit.*, p. 592). Mill considered this law to be 'but an extension of the more general law of Value, which we called the Equation of Supply and Demand' (*ibid.*), but the equation of supply and demand evidently refers to the equality between the quantities of particular commodities (*op. cit.*, pp. 446–448), and the law, if it is about the adjustment of the international values, should concern the equality of demand and supply of particular commodities.

Mill linked the determination of the relative value with the determination of the share of the advantage of trade; taking the above example of cloth and linen, when 10 yards of cloth is equal to 15 yards of linen, England will not get any share of the advantage of trade, while Germany will obtain the entire advantage, and when 10 yards of cloth is equivalent to 20 yards of linen, England will obtain the entire advantage, with Germany receiving no share. The distribution of the advantage is, thus, determined according to the law of supply and demand; the greater the intensity of demand of the exported commodity from the foreign country, the more share of advantage the exporting country will obtain.

Mill considered the law to be valid also when applied to cases of more than two commodities and presented the above cited numerical example with the addition of a third commodity, iron. In the example above, where 10 yards of cloth was of equal value with 15 yards of linen in England and of equal value with 20 yards of linen in Germany, Mill assumed the terms of interchange to be 10 yards of cloth for only 16 yards of linen, because the demand of England for linen is much greater than that of Germany for cloth. Then it was assumed that the quantity (called hundredweight) of iron which is of equal value with 10 yards of cloth in England will, if it is produced in Germany, cost as much labour as 18 yards of linen (*op. cit.*, p. 590). Iron, which is a product England can export, will improve the terms of interchange for England in comparison with the previous circumstances in which England exported only cloth. As a result, Mill supposed, the rate of interchange will be 10 yards of cloth for 17 yards of linen.

Suggesting that the same argument can be applied when the 4th, 5th and 6th commodities are included, Mill presented the following conclusions:

> If, therefore, it be asked what country draws to itself the greatest share of the advantage of any trade it carries on, the answer is, the country for whose productions there is in other countries the greatest demand, and a demand the most susceptible of increase from additional cheapness.... It gets its imports cheaper, the greater the intensity of the demand in foreign countries for its exports. It also gets its imports cheaper, the less the extent and intensity of its own demand for them. (*op. cit.*, p. 591)

It is this view that the new theory of international values denies. In the next section, I demonstrate how the view is denied by the new theory of international values by using Mill's numerical example.

5 The New Theory of International Values

Mill drew his conclusion by focusing on the case where the value of cloth is 10/17 in terms of linen for the cloth-linen-iron, England-Germany trade economy. This is an example of a 2-country, 3-commodity trade economy. The new theory follows the method of the contemporary classical theory; prices are proportional to their costs of production including the profits to the capital, and a commodity appears both as a product and as an input to other products. To describe Mill's example, however, it is appropriate to ignore the inputs of commodities, which enables one to ignore profits, but it is indispensable to express the quantity of labour and the wage rates.

A characteristic of the new theory, in contrast to the contemporary classical theory for the domestic economy, is that the labour in one country is a different factor of production from the labour in another country, and the wage rates are also different between those countries.

Let us take the quantity of labour required to produce one yard of linen as unity in England and in Germany. Since the labour in England and that in Germany are different things, this assumption merely expresses a choice of the unit of those kinds of labour. In addition, let us take the labour in England as the standard of value (i.e. the wage rate in England is assumed to be unity). Suppose p_c^e, p_l^e and p_i^e represent the prices of cloth, linen and iron, respectively, in England and p_c^g, p_l^g and p_i^g those in Germany; domestic prices in England in autarky should meet

$$p_c^e = \frac{3}{2}, \quad p_l^e = 1, \quad p_i^e = 15.$$

If w is the wage rate in Germany, the prices in Germany in autarky should meet

$$p_c^g = 2w, \quad p_l^g = w, \quad p_i^g = 18w.$$

When trade begins between the two countries, which commodities produced in each country become competitive depends on w:

1. When w is greater than 1, no commodity from Germany becomes competitive; if trade takes place, production is carried out only in England, and the international prices, p_c, p_l, p_i become identical to those in England:

$$p_c = \frac{3}{2}, \quad p_l = 1, \quad p_i = 15.$$

2. When $w = 1$, the domestic prices in Germany become

$$p_c^g = 2, \quad p_l^g = 1, \quad p_i^g = 18;$$

   linen from Germany becomes as competitive as that from England, but the other two commodities are not competitive. All the commodities from England are competitive; so, the international prices, p_c, p_l, p_i, become identical to those in England:

$$p_c = \frac{3}{2}, \quad p_l = 1, \quad p_i = 15.$$

   Germany specialises in the production of linen.
3. When $5/6 < w < 1$, the domestic prices in Germany become

$$p_c^g > \frac{5}{3}\left(> \frac{3}{2}\right), \quad p_l^g < 1, \quad p_i^g > 15;$$

   so, linen from Germany becomes more competitive than that from England, but the other commodities from Germany are still not competitive. Linen in England loses competitiveness. Therefore, Germany specialises in linen, and England specialises in cloth and iron. This is Mill's case. The international prices will become as follows:

$$p_c = \frac{3}{2}, \quad \frac{5}{6} < p_l < 1, \quad p_i = 15.$$

   Mill assumed $p_l = (10/17)p_c = 15/17$, which satisfies $5/6 < p_l < 1$.
4. When $w = 5/6$, the domestic prices in Germany become

$$p_c^g = \frac{5}{3}\left(> \frac{3}{2}\right), \quad p_l^g = \frac{5}{6}(< 1), \quad p_i^g = 15;$$

   linen from Germany is more competitive than that from England, and iron becomes as competitive as that from England, but cloth is still not competitive. The international prices would be

$$p_c = \frac{3}{2}, \quad p_l = \frac{5}{6}, \quad p_i = 15.$$

Cloth is produced only in England, and linen only in Germany, but iron is produced in both countries.

5. When $3/4 < w < 5/6$, the domestic prices in Germany become

$$p_c^g > \frac{3}{2}, \quad p_l^g < \frac{5}{6}(< 1), \quad p_i^g < 15;$$

linen and iron from Germany become more competitive than those from England, but cloth from England is more competitive than that from Germany. International prices should meet

$$p_c = \frac{3}{2}, \quad \frac{3}{4} < p_l < \frac{5}{6}, \quad \frac{27}{2} < p_i < 15.$$

England specialises in cloth, and Germany in linen and iron.

6. When $w = 3/4$, the domestic prices in Germany would meet

$$p_c^g = \frac{3}{2}, \quad p_l^g = \frac{3}{4}(< 1), \quad p_i^g = \frac{27}{2}(< 15);$$

all the commodities from Germany become competitive; cloth is as competitive as that from England, and linen and iron are more competitive than those from England. The international prices should meet

$$p_c = \frac{3}{2}, \quad p_l = \frac{3}{4}, \quad p_i = \frac{27}{2}.$$

Under those prices, England is competitive only in cloth and specialises in it, while Germany will produce the full set of commodities.

7. When $w < 3/4$, every commodity will lose competitiveness, if produced in England, and production will be carried out only in Germany; the international prices will be identical to those in Germany:

$$p_c = 2w, \quad p_l = w, \quad p_i = 18w.$$

Table 1 shows how the competitive industries change according to the change in w. When the wage rate in Germany equals unity, only linen is competitive in Germany. As the wage rate declines, iron and then cloth is called into production in Germany, while in England linen and then iron are withdrawn.

At the wage rates of 1, 5/6 and 3/4, the prices are uniquely determined only by the technological factors, that is, by the costs of production; the law of demand and supply has no role here. When the wage rate is between those values, prices are not determined just by the technological factors, but they cannot be said to be

Table 1 Competitive industries and the wage rate

	England			Germany			International price		
w	Cloth	Linen	Iron	Cloth	Linen	Iron	Cloth	Linen	Iron
$1 < w$	$+$	$+$	$+$	$-$	$-$	$-$	3/2	1	15
$w = 1$	$+$	$+$	$+$	$-$	$+$	$-$	3/2	1	15
$5/6 < w < 1$	$+$	$-$	$+$	$-$	$+$	$-$	3/2	w	15
$w = 5/6$	$+$	$-$	$+$	$-$	$+$	$+$	3/2	5/6	15
$3/4 < w < 5/6$	$+$	$-$	$-$	$-$	$+$	$+$	3/2	w	$18w$
$w = 3/4$	$+$	$-$	$-$	$+$	$+$	$+$	3/2	3/4	27/2
$w < 3/4$	$-$	$-$	$-$	$+$	$+$	$+$	$2w$	w	$18w$

'$+$' means 'competitive', and '$-$' means 'uncompetitive'

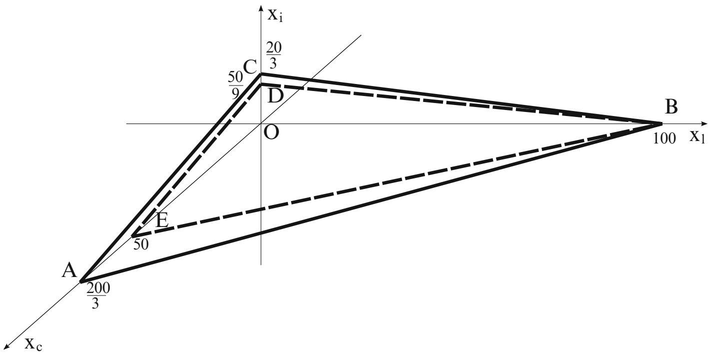


Fig. 1 Production possibility set in autarky

determined by the law of demand and supply; they are still under the constraint of technological factors, not just in the sense Mill noticed, that is, in the sense that prices have upper and lower bounds, but also in the sense that not all the prices can change independently; for example, $p_i/p_c = 10$ when $5/6 < w < 1$, and $p_i/p_l = 18$ when $3/4 < w < 5/6$.[6]

When the quantity of labour is limited, a set of coefficients for labour input gives a production possibility set. The triangle ABC in Fig. 1 represents the production possibility frontier in autarky for England when the quantity of labour is 100. The triangle EBD is that for Germany. When trade begins between them, the production possibility set is enlarged; the faces ABC, CFDA and AED in Fig. 2 are the production possibility frontier. The triangle ABC corresponds to the case where cloth, linen and iron are competitively produced in England and just linen is produced in Germany; the vector (3/2, 1, 15), which consists of the prices in this

[6]The cases with the wage rates of 1, 5/6 and 3/4 are examples of Graham's (1948) 'linkage case', whereas the cases with the wage rates between those values are examples of the 'limbo case', as explained by Sato in Chap. 10 of this volume.

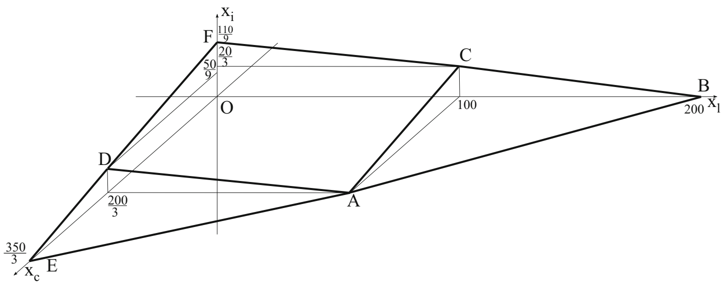


Fig. 2 Production possibility set in the trade between England and Germany

case, is normal to the triangle. The parallelogram CFDA corresponds to the case where cloth and iron are competitively produced in England and linen and iron are competitively produced in Germany; the price vector (3/2, 5/6, 15) is normal to the face. The triangle AED corresponds to the case where just cloth is produced in England and all three commodities are produced in Germany; the price vector (3/2, 3/4, 27/2) is normal to the face.

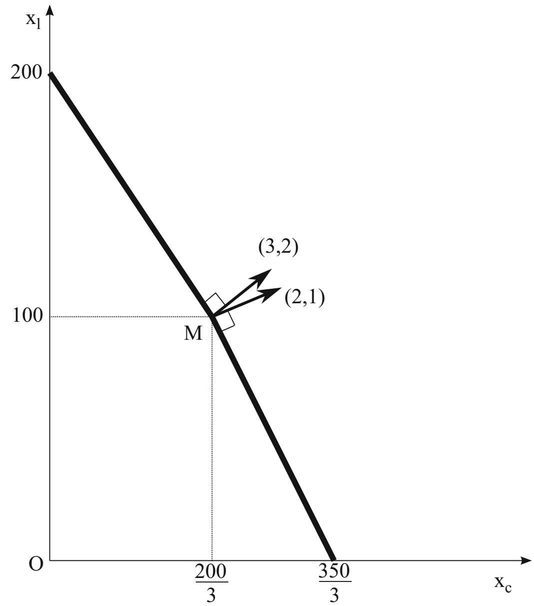
A point on one of these faces excluding its edges can be produced only by using the system of techniques consisting of the methods that become competitive under the set of prices, the vector of which is normal to the face. For example, a point on the face CFDA can be produced only by using the cloth and iron industries in England and the linen and iron industries in Germany; those industries become competitive under the price set (3/2, 5/6, 15).

Mill's case refers to a point on the ridge AC, where the price of linen has the freedom to change from 5/6 to 1; the law of demand and supply is able to influence the terms of trade.[7] The new theory reveals that those are very special points on the entire production possibility frontier. On the overwhelmingly large area, the set of prices is determined solely by technological considerations and has no freedom to change depending on supply and demand.

There is no point on the frontier at which prices can change freely to any direction and at which the law of supply and demand can be said to determine the prices, other than the points A, B, C, F, D and E, none of which exists in the positive space. This is in contrast with Ricardo's and Mill's examples of a 2-country, 2-commodity case. Figure 3 shows the production possibility frontier of Mill's cloth-linen, England-Germany example when the quantity of labour in both countries is 100. Point M is a point in the positive area at which the relative price p_c/p_l can change freely within the range [3/2,2]. The number of commodities must not be larger than the number

[7]The ridges of the production possibility frontier represent the limbo cases as explained in Chap. 10 of this volume; Mill, therefore, dealt with only a case of limbo.

Fig. 3 Production possibility frontier of 2-country, 2-commodity trade economy



of countries in order for such a point to exist in the positive space. When the number of commodities is larger than the number of countries, such points do not exist in the positive space.

6 Introduction of Intermediate Goods and General Expression of the New Theory of International Values

When wage is paid before production, profits can arise even if there are no inputs other than labour. In this case, the above analysis is made valid by replacing the sum of wage with the sum of wage multiplied by 1 plus the rate of profit, or $(1 + r)wl$, where r is the profit rate, w is the wage rate and l is the coefficient of labour input. Here, the rate of profit in one country can be different from that in the other country; it can be varied even among industries.

When there are inputs other than labour, in the case of 2-country, 3-commodity, the prices, p_1, p_2, p_3, and the wage rate of country 2, w (assuming the wage rate of country 1 is unity), should meet

$$\begin{cases} p_1 \leqq \left(1+r^1\right)\left(p_1 a_{11}^1 + p_2 a_{21}^1 + p_3 a_{31}^1\right) + l_1^1 \\ p_2 \leqq \left(1+r^1\right)\left(p_1 a_{12}^1 + p_2 a_{22}^1 + p_3 a_{32}^1\right) + l_2^1 \\ p_3 \leqq \left(1+r^1\right)\left(p_1 a_{13}^1 + p_2 a_{23}^1 + p_3 a_{33}^1\right) + l_3^1 \\ p_1 \leqq \left(1+r^2\right)\left(p_1 a_{11}^2 + p_2 a_{21}^2 + p_3 a_{31}^2\right) + w l_1^2 \\ p_2 \leqq \left(1+r^2\right)\left(p_1 a_{12}^2 + p_2 a_{22}^2 + p_3 a_{32}^2\right) + w l_2^2 \\ p_3 \leqq \left(1+r^2\right)\left(p_1 a_{13}^2 + p_2 a_{23}^2 + p_3 a_{33}^2\right) + w l_3^2, \end{cases} \tag{12}$$

where r^k represents the rate of profit in country k and a_{ij}^k and l_j^k the quantity of commodity i and labour in the production of commodity j in country k. Four of the inequalities will be met as equations, and the rest will be met as strict inequalities when a point on one of the faces of the production possibility frontier excluding its edges should be produced. For example, when industry 1 and 3 in country 1 and industry 2 and 3 in country 2 are competitive,

$$\begin{cases} p_1 = \left(1+r^1\right)\left(p_1 a_{11}^1 + p_2 a_{21}^1 + p_3 a_{31}^1\right) + l_1^1 \\ p_3 = \left(1+r^1\right)\left(p_1 a_{13}^1 + p_2 a_{23}^1 + p_3 a_{33}^1\right) + l_3^1 \\ p_2 = \left(1+r^2\right)\left(p_1 a_{12}^2 + p_2 a_{22}^2 + p_3 a_{32}^2\right) + w l_2^2 \\ p_3 = \left(1+r^2\right)\left(p_1 a_{13}^2 + p_2 a_{23}^2 + p_3 a_{33}^2\right) + w l_3^2. \end{cases}$$

The new theory established that a set of values p_1, p_2, p_3, w exists that satisfies (12) and

$$p_1 y_1 + p_2 y_2 + p_3 y_3 = q^1 + w q^2,$$

where $y_i (1 = 1, 2, 3)$ is the net output of commodity i and $q^k (k = 1, 2)$ is the existing quantity of labour in country k that satisfy

$$\begin{cases} s_1^1 + s_1^2 = \left(1+r^1\right)\left(a_{11}^1 s_1^1 + a_{12}^1 s_2^1 + a_{13}^1 s_3^1\right) + \left(1+r^2\right)\left(a_{11}^2 s_1^2 + a_{12}^2 s_2^2 + a_{13}^2 s_3^2\right) + y_1 \\ s_2^1 + s_2^2 = \left(1+r^1\right)\left(a_{21}^1 s_1^1 + a_{22}^1 s_2^1 + a_{23}^1 s_3^1\right) + \left(1+r^2\right)\left(a_{21}^2 s_1^2 + a_{22}^2 s_2^2 + a_{23}^2 s_3^2\right) + y_2 \\ s_3^1 + s_3^2 = \left(1+r^1\right)\left(a_{31}^1 s_1^1 + a_{32}^1 s_2^1 + a_{33}^1 s_3^1\right) + \left(1+r^2\right)\left(a_{31}^2 s_1^2 + a_{32}^2 s_2^2 + a_{33}^2 s_3^2\right) + y_3 \\ q^1 = l_1^1 s_1^1 + l_2^1 s_2^1 + l_3^1 s_3^1 \\ q^2 = l_1^2 s_1^2 + l_2^2 s_2^2 + l_3^2 s_3^2, \end{cases} \tag{13}$$

where s_i^k is the level of activity for industry i in country k. This is what Theorem 4.3 in Chap. 1 means for the 2-country, 3-commodity case. Such a price vector (p_1, p_2, p_3) is normal to the face on which the point (y_1, y_2, y_3) satisfying (13) exists. This proposition has been established for more general cases with m countries and n commodities, allowing the possibility of having more than one method of producing the same commodity in a country.

The prices in autarky for country 1 should meet the same equations as (6):

$$\begin{cases} p_1 = \left(1 + r^1\right)\left(p_1 a^1_{11} + p_2 a^1_{21} + \cdots + p_n a^1_{n1}\right) + l^1_1 \\ p_2 = \left(1 + r^1\right)\left(p_1 a^1_{12} + p_2 a^1_{22} + \cdots + p_n a^1_{n2}\right) + l^1_2 \\ \qquad\vdots \\ p_n = \left(1 + r^1\right)\left(p_1 a^1_{1n} + p_2 a^1_{2n} + \cdots + p_n a^1_{nn}\right) + l^1_n, \end{cases} \tag{14}$$

under the assumption that the wage rate is in unity. An equation for the n-th commodity from country 2

$$p_n = (1 + r^2)(p_1 a^2_{1n} + p_2 a^2_{2n} + \cdots + p_n a^2_{nn}) + l^2_n$$

cannot coexist in general with the system of equations (14). If the method for the n-th commodity from country 2 is superior to that in country 1, the n-th equation in (14) will be excluded. If, however, the wage rate in country 2, w, is included, the equation

$$p_n = (1 + r^2)(p_1 a^2_{1n} + p_2 a^2_{2n} + \cdots + p_n a^2_{nn}) + w l^2_n$$

becomes compatible with the equations in (14); we can have a system of international values:

$$\begin{cases} p_1 = \left(1 + r^1\right)\left(p_1 a^1_{11} + p_2 a^1_{21} + \cdots + p_n a^1_{n1}\right) + l^1_1 \\ p_2 = \left(1 + r^1\right)\left(p_1 a^1_{12} + p_2 a^1_{22} + \cdots + p_n a^1_{n2}\right) + l^1_2 \\ \qquad\vdots \\ p_n = \left(1 + r^1\right)\left(p_1 a^1_{1n} + p_2 a^1_{2n} + \cdots + p_n a^1_{nn}\right) + l^1_n \\ p_n = \left(1 + r^2\right)\left(p_1 a^2_{1n} + p_2 a^2_{2n} + \cdots + p_n a^2_{nn}\right) + w l^2_n. \end{cases} \tag{15}$$

If the method for commodity $n-1$ in country 2 is to join, either the production of commodity n or commodity $n-1$ in country 1 must retire. Suppose the production of the n-th commodity in country 1 retires; we have a system of international values:

$$\begin{cases} p_1 = \left(1 + r^1\right)\left(p_1 a^1_{11} + p_2 a^1_{21} + \cdots + p_n a^1_{n1}\right) + l^1_1 \\ p_2 = \left(1 + r^1\right)\left(p_1 a^1_{12} + p_2 a^1_{22} + \cdots + p_n a^1_{n2}\right) + l^1_2 \\ \qquad\vdots \\ p_{n-1} = \left(1 + r^1\right)\left(p_1 a^1_{1,n-1} + p_2 a^1_{2,n-1} + \cdots + p_n a^1_{n,n-1}\right) + l^1_{n-1} \\ p_{n-1} = \left(1 + r^2\right)\left(p_1 a^2_{1,n-1} + p_2 a^2_{2,n-1} + \cdots + p_n a^2_{n,n-1}\right) + w l^2_{n-1} \\ p_n = \left(1 + r^2\right)\left(p_1 a^2_{1n} + p_2 a^2_{2n} + \cdots + p_n a^2_{nn}\right) + w l^2_n. \end{cases} \tag{16}$$

In autarky, there can be n independent equations with n price variables. In the 2-country trade economy, $n + 1$ independent equations become able to exist by including a new variable w. When a third country joins the trade economy, an additional variable w' will appear, and for another commodity, say the $(n - 2)$-th one, two equations will become able to coexist. According to Sraffa's logic about the similarity of the case with rent to the multiple-product case cited above, foreign trade can also be regarded as another case at which 'at least one commodity is produced by *more than one* method', and it is thus of the same nature as the multiple-product case.

7 Conclusions

The new theory of international values established the existence of international values that can be interpreted as being independent of demand and being mainly determined by technology. The fact that wage rates are different among countries enables the coexistence of processes in different countries of producing the same commodity and enables a system of equations sufficient to determine prices and wages just on the basis of technological information, except for a distributional variable—the rates of profit. Difference in wage rates among countries is the principal indicator of inequality in international income distribution. According to the new theory, the difference in wage rate should be understood as a result of the difference in technology among countries.

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