Chapter 26
Research on Description Method of Quasi-3D Appearance Data with Mathematical Surface Function

Xinggen Qian, Yang Jin and Hongguo Wang

Abstract Appearance data in quasi-three-dimensional (Quasi-3D) space are the height data, which are distributed over the grid points of a 2D plane. The quasi-3D data can be measured from a high-resolution 3D scanning device. With the help of mathematical surface function, the discrete space appearance data can be generalized; thus, some processing (e.g., interpolation, deformation) can be implemented. Based on segmentation of a 2D plane into blocks and the application of cubic surface functions (Coons, Bezier, and B-Spline), the appearance data in quasi-3D space are generalized processed, the parameters of the surface pieces are obtained, and some technical key points are determined. The quasi-3D appearance data can be used in scientific research, industry, high-fidelity 3D animation/film, image reproduction, and so on.

Keywords Quasi-3D appearance · Surface function · Generalization

26.1 Introduction

In general, a real three-dimensional object can be constructed from some key points, each of which has its coordinates (x, y, z). If the coordinates x and y are located in the same flat plane, and the coordinate z indicates "height value," the 3D coordinates can be simplified and called as "Quasi-3D" or "2.5D" coordinates.

The coordinate data z in "Quasi-3D" space are known as "appearance data" and are often obtained by high-resolution scanner, which can capture not only the color data, but also mainly the "height value" of object surface over a plane, and the pickup positions are distributed over a regular grid. Figure 26.1 shows the surface of leather (left) and its Quasi-3D appearance (scanned by Dr. Wirth GravurSysteme). Conceivably, the surface of an oil painting has the similar attribute.

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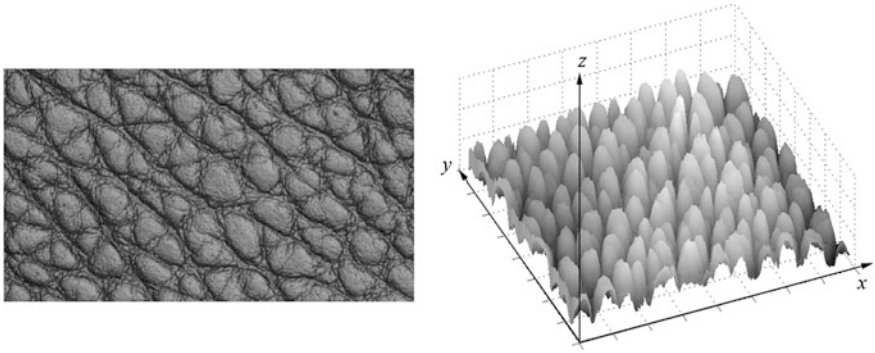

Fig. 26.1 Surface of a leather piece and its quasi-3D appearance

In order to describe and to generalize the appearance data, mathematical functions can be used [1, 2]. Since the appearance of a surface is usually quite complicated, the selection of mathematical function and the method for the generalization should be attentively investigated.

26.2 Surface Model for the Description of Quasi-3D Appearance

The natural objects possess complicate form and/or texture on their surface. In order to describe the appearance data, mathematical functions, e.g., Bezier, B-spline, and Coons, can be selected to apply. Although a higher degree of function leads to higher descriptive power, the order of complexity is also increased correspondingly. Compromised, cubic surface functions are selected. As example, Formula 26.1 is cubic surface function in the form of Bezier [3]:

$$Q_{\text{Bezier}}(x,y) = \text{UM}_{\text{Bz}}\text{PM}_{\text{Bz}}^{T}W \tag{26.1}$$

wherein

$$U = \begin{bmatrix} x^3 & x^2 & x & 1 \end{bmatrix}, M_{\text{Bz}} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$M_{\text{Bz}}^{T} = M_{\text{Bz}} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix}, W = \begin{bmatrix} y^3 \\ y^2 \\ y \\ 1 \end{bmatrix},$$

$(x,y) \in [0,1]$.

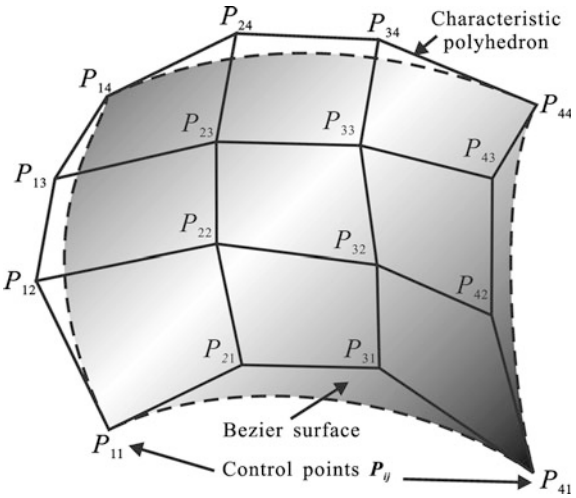

Fig. 26.2 Bezier surface, control points, and characteristic polyhedron

In the equation, the elements $P_{i,j}$ $(i, j \in 1, 2, 3, 4)$ of matrix P are control points, which can be applied to build up "characteristic polyhedron" for the construction of surface. To construct a cubic surface function, matrix P with 4×4 control points $P_{i,j}$ must be obtained. Figure 26.2 shows the control points, characteristic polyhedron, and the Bezier surface.

It can be conceivable that in order to describe the quasi-3D appearance with mathematical function, the plane X–Y must be separated in blocks, each of which includes 4×4 height (z) data. With the 4×4 height (z) data, one piece of surface can be constructed, and the whole appearance surface consists of many pieces of subsurface. Two neighboring pieces of surface are shown in Fig. 26.3. It can be seen that the four appearance data at the common border of the subpieces are applied for the two pieces of subsurface.

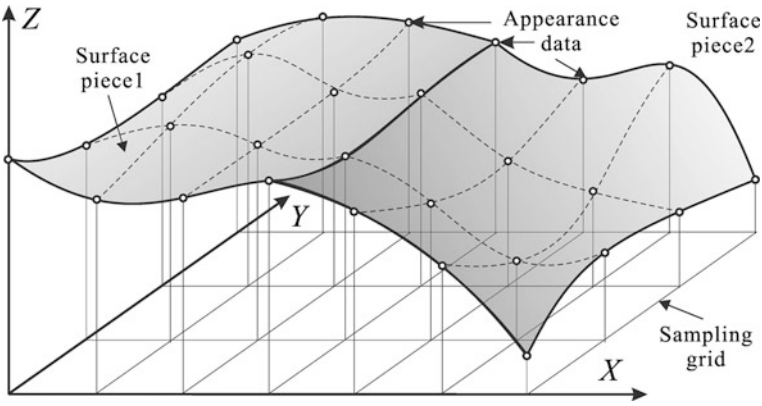

Fig. 26.3 Subsurfaces with appearance data

26.3 Basic Processing Flow and Algorithm

Essentially, the description of Quasi-3D appearance is to obtain the control parameters of the surface and/or surface pieces, which fit to the appearance data [4]. Under the condition of cubic surface function, the element data of matrix P in Formula 26.1 must be computed out. Deriving from Formula 26.1, the matrix P can be described as follows:

$$P = \left(\mathrm{UM}_{\mathrm{Bz}} M_{\mathrm{Bz}}^{T} W\right)^{-1} Q(x, y) = T^{-1} Q(x, y) \tag{26.2}$$

where $Q(x, y)$ is the appearance data at position (x, y), and M_{Bz} and M_{Bz}^{-1} are shown above. In every surface piece, the coordinates x and y are normalized to the range [0, 1]. If the sampling grid is uniformly distributed (it can also be un-uniformed arranged), the values of x and y are 0, 1/3, and 2/3, 1, respectively. Thus, the matrices U and W are also determined. The processing flow is as follows (Fig. 26.4).

It should be careful that the appearance data at the common border of surface pieces must be brought into both neighboring blocks, so that the continuity between the surface pieces is kept.

As example, a gray-level image with 511 × 430 pixels is processed. The gray level of the pixels is denoted as "appearance data," and 24310 pieces of subsurfaces are processed. Figure 26.5 shows the restored image from the surface function.

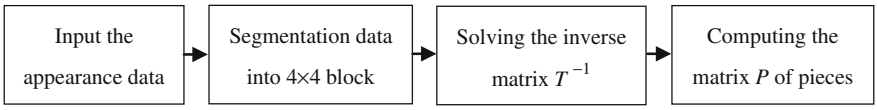

Fig. 26.4 The processing flow of computing matrix P

Fig. 26.5 Described as cubic Bezier function and from the function restored image

26.4 Discussion and Conclusions

The description and generalization of Quasi-3D data with mathematical function are implementable. Not only the "appearance data" can be from a real object body, but it can also be the data from different sources (image pixel data, frequency data, etc.). Different math functions can be applied. In the application of image processing, the surface pieces at the sides of the border can possess higher or lower degree of continuity and smoothness. The method can be used in area of object modeling, image processing, data analyzing, and fitting.

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