Chapter 16
The Research on Gravure Spot-Color-Matching Model

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Abstract The demand for reproduction of printing color vastly expands, which significantly promotes the application of the spot-color ink. To improve the accuracy and practicability of color matching, the Kubelka–Munk color-matching theory and the improved spectrophotometric uniform color space weight factor color-matching method are applied to construct two matching models. The four-color (C, M, Y, W) model is established on the basis of three-color model (C, M, Y), and the derivations of the two models will be shown. In addition, a new experimental program is designed for ink toner to obtain the basic data of color matching. Prediction recipes are obtained by using MATLAB software for matrix arithmetic. And the accuracy and practicability of color-matching models are verified through the experiment. The results show that two models are feasible for better utility of the spot-color matching.

Keywords Computer color matching · Spot-color ink · Kubelka–Munk theory · Spectrophotometric color matching

16.1 Introduction

Spot-color ink attained the favors of consumer and enterprise because of its constant hue, wide color gamut, good visual effects, and unique security features. However, traditional color matching mostly relies on experienced personnel, who may be disturbed by environmental and psychological conditions [1]. This renders the matching process unreliable and the matching result unstable. To improve the

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accuracy, ameliorate the efficiency, and reduce the cost, a computer-based color-matching model seems indispensable [2]. Scholars have proposed a series of color-matching theory, such as Kubelka–Munk theory, Neugebauer equations, and the Masking equation and the corresponded approaches such as tristimulus matching, spectrophotometric color matching, and the neural network color matching.

Single-constant Kubelka–Munk theory and the improved spectrophotometric uniform color space weight factor method are applied to the establishment of models after comprehensive evaluations of the accuracy and the practicability of various color-matching methods. After extensive efforts, two color-matching models that take the experimental conditions into account are established.

16.2 Theoretical Foundations

16.2.1 Kubelka–Munk Theory

Kubelka–Munk theory is a two luminous theory applied to the semitransparent medium and opaque medium. The simplified form is as follows [3]:

$$\frac{K}{S} = \frac{(1 - R_\infty)^2}{2R_\infty} \tag{16.1}$$

where K is absorption coefficient, S is scattering coefficient, and R_∞ is the reflectance at infinite thickness. Equation 16.1 approximately indicates the relationship between reflectance and the absorption and scattering coefficients of ink.

According to single-constant Kubelka–Munk theory, scattering coefficient can still be unchanged in some cases, although the colorant concentration changes. In other words, the single-constant theory assumes that the scattering ability of mixture does not change. Because the absorption and scattering coefficients are additive, when there are n kinds of toners in the ink layer and takes the substrate into consideration, calculation of K/S value can be simplified [3]:

$$\frac{K}{S} = \left(\frac{k}{s}\right)^t + c_1\left(\frac{k}{s}\right)^1 + c_2\left(\frac{k}{s}\right)^2 + \cdots + c_n\left(\frac{k}{s}\right)^n \tag{16.2}$$

where k_t and s_t are the absorption and scattering coefficients of substrate, respectively, c_1, c_2, …, c_n are the concentrations of various colorants, k_1, k_2, …, $\mathrm{k_n}$ are the absorption coefficients of various colorants, and s_1, s_2, …, s_n are the scattering coefficients of various colorants.

Equation 16.2 is the expression of the single-constant K-M theory.

16.2.2 Uniform Color Space Weight Factor Method About Spectrophotometric Color Matching

Spectrophotometric color matching minimizes the color difference between the standard and the match by fitting spectral curves directly.

Spectrophotometric color matching has a characteristic that equal changes of reflectance at different wavelengths may lead to varying degrees of color perception difference. Therefore, the wavelengths that have strong sensitivity should be emphasized in the matching process with larger weight factors. On the contrary, the wavelengths that are inert should be matched with smaller weight factors [4]. So weight factors are established in the uniform color space combining the colorimetric theory. The basic expression is as follows [4]:

$$\sum_{j} \left[\Delta E(\lambda_j)\right]^2 \rightarrow \min \tag{16.3}$$

where $\Delta E(\lambda_j)$ is the color difference caused by reflectance change at the wavelength of λ_j.

Next, specific analysis is conducted about Eq. 16.3.

The color tristimulus values can be determined by Eq. 16.4 [5]:

$$\begin{cases} X = K \sum\limits_{400}^{700} S(\lambda) R(\lambda) \bar{x}(\lambda) \Delta\lambda \\ Y = K \sum\limits_{400}^{700} S(\lambda) R(\lambda) \bar{y}(\lambda) \Delta\lambda \\ Z = K \sum\limits_{400}^{700} S(\lambda) R(\lambda) \bar{z}(\lambda) \Delta\lambda \end{cases} \tag{16.4}$$

where $S(\lambda)$ is relative spectral power distribution, $R(\lambda)$ is spectral reflectance, $\bar{x}, \bar{y}, \bar{z}$ are CIE standard colorimetric observer spectral tristimulus values, and K is a constant.

Taking $\Delta\lambda = 10$ nm, the color tristimulus value differences ΔX, ΔY, and ΔZ are as follows:

$$\begin{cases} \Delta X = K \cdot S(\lambda) \bar{x}(\lambda) \Delta R(\lambda) \\ \Delta Y = K \cdot S(\lambda) \bar{y}(\lambda) \Delta R(\lambda) \\ \Delta Z = K \cdot S(\lambda) \bar{z}(\lambda) \Delta R(\lambda) \end{cases} \tag{16.5}$$

Color difference ΔE is an ideal function to evaluate the difference between two colors in the $L^*a^*b^*$ color space. The value of L^*, a^*, and b^* can be calculated by the tristimulus values, so that ΔL^*, Δa^*, and Δb^* can be obtained by Eq. 16.6 [4]:

$$\begin{aligned}\Delta L^* &= L'(Y)\Delta Y \\ \Delta a^* &= A'_X(X,Y)\Delta X + A'_Y(X,Y)\Delta Y \\ \Delta b^* &= B'_Y(Y,Z)\Delta Y + B'_Z(Y,Z)\Delta Z\end{aligned} \tag{16.6}$$

Substituting Eq. 16.5 into Eq. 16.6 [4]:

$$\begin{aligned}\Delta L^*(\lambda) &= K \cdot S(\lambda)\Delta R(\lambda)\bar{y}(\lambda)L'(Y) \\ \Delta a^*(\lambda) &= K \cdot S(\lambda)\Delta R(\lambda)\left[\bar{y}(\lambda)A'_X(X,Y) + \bar{y}(\lambda)A'_Y(X,Y)\right] \\ \Delta b^*(\lambda) &= K \cdot S(\lambda)\Delta R(\lambda)\left[\bar{y}(\lambda)B'_Y(Y,Z) + \bar{z}(\lambda)B'_Z(Y,Z)\right]\end{aligned} \tag{16.7}$$

This color-matching method has been improved in this paper according to the actual situation and the difficulty level of MATLAB programming implementation.

Because of color difference formula

$$\Delta E = \sqrt{(\Delta L*)^2 + (\Delta a*)^2 + (\Delta b*)^2}$$

We found that if we want to make

$$\sum_j [\Delta E(\lambda_j)]^2 \rightarrow \min$$

It is equal to make $[\Delta L^*(\lambda_j)]^2$, $[\Delta a^*(\lambda_j)]^2$ and $[\Delta b^*(\lambda_j)]^2$ approach minimum, respectively:

$$\begin{aligned}&\sum_j [\Delta L^*(\lambda_j)]^2 \rightarrow \min \\ &\sum_j [\Delta a^*(\lambda_j)]^2 \rightarrow \min \\ &\sum_j [\Delta b^*(\lambda_j)]^2 \rightarrow \min\end{aligned} \tag{16.8}$$

Consequently, new weight factors are established. These weight factors can minimize color difference caused by reflectance change at a certain wavelength.

16.2.3 Deduction of Color-Matching Models

16.2.3.1 Model Based on Three Colors (C, M, Y)

Three primary inks C, M, and Y are used as basic inks to establish a color-matching model. The wavelength range of reflectance will be measured from 400 to 700 nm at 10 nm increments.

It is known that the smaller the color difference between standard and match, the higher the precision of the model. An ideal state is assumed $\Delta E = 0$.

According to the analysis above and combining Eq. 16.7:

$$\begin{aligned}
\Delta L^*(\lambda) &= K \cdot S(\lambda)\Delta R(\lambda)\bar{y}(\lambda)L'(Y) = 0 \\
\Delta a^*(\lambda) &= K \cdot S(\lambda)\Delta R(\lambda)\left[\bar{x}(\lambda)A'_X(X,Y) + \bar{y}(\lambda)A'_Y(X,Y)\right] = 0 \\
\Delta b^*(\lambda) &= K \cdot S(\lambda)\Delta R(\lambda)\left[\bar{y}(\lambda)B'_Y(Y,Z) + \bar{z}(\lambda)B'_Z(Y,Z)\right] = 0
\end{aligned} \tag{16.9}$$

Equation 16.9 is applied to single-constant K-M theory, and the concentration recipe can be obtained after a series of derivation [6]:

$$C = (TED\Phi)^{-1} \cdot TED(f^a - f^t) \tag{16.10}$$

Several matrices are defined and depicted to represent the parameters in Eq. 16.10:

$$C = \begin{bmatrix} c^1 \\ c^2 \\ c^3 \end{bmatrix} \quad T = \begin{bmatrix} L_T \\ A_T \\ B_T \end{bmatrix} \quad \begin{aligned} L_T &= L'(Y)\bar{y} \\ A_T &= A'_X(X,Y)\bar{x} + A'_Y(X,Y)\bar{y} \\ B_T &= B'_Y(Y,Z)\bar{y} + B'_Z(Y,Z)\bar{z} \end{aligned}$$

$$E = \begin{bmatrix} S_{400} & 0 & \cdots & 0 \\ 0 & S_{410} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_{700} \end{bmatrix} \quad D = \begin{bmatrix} d_{400} & 0 & \cdots & 0 \\ 0 & d_{410} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_{700} \end{bmatrix} \quad d_j = \frac{R_j^2 - 1}{2R_j^2}$$

$$\Phi = \begin{bmatrix} \left(\frac{k}{s}\right)^1_{400} & \left(\frac{k}{s}\right)^2_{400} & \left(\frac{k}{s}\right)^3_{400} \\ \left(\frac{k}{s}\right)^1_{410} & \left(\frac{k}{s}\right)^2_{410} & \left(\frac{k}{s}\right)^3_{410} \\ \vdots & \vdots & \vdots \\ \left(\frac{k}{s}\right)^1_{700} & \left(\frac{k}{s}\right)^2_{700} & \left(\frac{k}{s}\right)^3_{700} \end{bmatrix} \quad f^a = \begin{bmatrix} \left(\frac{K}{S}\right)^a_{400} \\ \left(\frac{K}{S}\right)^a_{410} \\ \vdots \\ \left(\frac{K}{S}\right)^a_{700} \end{bmatrix} \quad f^t = \begin{bmatrix} \left(\frac{K}{S}\right)^t_{400} \\ \left(\frac{K}{S}\right)^t_{410} \\ \vdots \\ \left(\frac{K}{S}\right)^t_{700} \end{bmatrix}$$

where C is the concentration matrix, E is relative spectral power distribution matrix, $\bar{x}, \bar{y}, \bar{z}$ are CIE standard colorimetric observer spectral tristimulus values matrices, superscript 1, 2, 3 are C, M, Y three-color inks, respectively, and superscript a and t are the reflectance matrices of standard and substrate.

16.2.3.2 Model Based on Four Colors (C, M, Y, W)

Only three primary inks to participate color matching do not meet the requirements of customer in the actual production. It is the first choice of many enterprises who use heat transfer printing on the fabric and cotton since the white ink has a strong ability to cover [1], thereby the white ink is added as the fourth basic ink, and the four-color-matching model will be derived based on the three-color-matching model.

Some parameters of Eq. 16.10 have changed after adding the white ink:

$$\Phi = \begin{bmatrix} \left(\frac{k}{s}\right)^1_{400} & \left(\frac{k}{s}\right)^2_{400} & \left(\frac{k}{s}\right)^3_{400} & \left(\frac{k}{s}\right)^4_{400} \\ \left(\frac{k}{s}\right)^1_{410} & \left(\frac{k}{s}\right)^2_{410} & \left(\frac{k}{s}\right)^3_{410} & \left(\frac{k}{s}\right)^4_{410} \\ \vdots & \vdots & \vdots & \vdots \\ \left(\frac{k}{s}\right)^1_{700} & \left(\frac{k}{s}\right)^2_{700} & \left(\frac{k}{s}\right)^3_{700} & \left(\frac{k}{s}\right)^4_{700} \end{bmatrix} \quad C = \begin{bmatrix} c^1 \\ c^2 \\ c^3 \\ c^4 \end{bmatrix}$$

Equation 16.10 can be used to solve three unknowns using three linear equations; however, there are four unknowns in the concentration matrix C now, and three equations are unable to solve four unknowns according to the linear algebra theory. In Eq. 16.10, the general solution of undetermined system of equations can be obtained as the following form:

$$\begin{bmatrix} c^1 \\ c^2 \\ c^3 \\ c^4 \end{bmatrix} = \begin{bmatrix} m^1 \\ m^2 \\ m^3 \\ 0 \end{bmatrix} + x \begin{bmatrix} n^1 \\ n^2 \\ n^3 \\ n^4 \end{bmatrix}$$

There is only one unknown x, and one way of solving x is to add a supplementary condition. According to Eq. 16.2:

$$\left(\frac{K}{S}\right)^m = \left(\frac{K}{S}\right)^t + c_1\left(\frac{k}{s}\right)^1 + c_2\left(\frac{k}{s}\right)^2 + c_3\left(\frac{k}{s}\right)^3 + c_4\left(\frac{k}{s}\right)^4 \tag{16.11}$$

There are 31 sets of data between 400 and 700 nm totally. The general solution is substituted into Eq. 16.11, and the least squares techniques is applied to compute the optimal solution x, so that the concentration matrix can be solved as well.

Consequently, a mathematical model has been established to solve the concentration of the four-color recipe. This model is based on the three-color model and solves the concentration of four-color model by adding a supplementary condition.

The spectral reflectance of basic inks is stored into the computer previously, and the recipes can be obtained according to the color-matching models after inputting the reflectance and colorimetric values of the standard color.

16.3 Experimental Programs

Toner is putted into the basic recipe of ink to make four-color ink in this paper. After stirring evenly, the ink is daubed on the white paper by scratching a small amount with ink knife. Then, ink is wiped evenly using the spreader on white paper which is put on a heated glass pane. Each kind of ink requires multiple groups of

coating samples, and one of the best coating quality is selected as the sample which will be used in the experiment. In addition, the weight of toner, the date of sample making, and the other information should be recorded carefully. Last but not the least, samples should be measured after 24 h because of the dry fading phenomenon of ink.

16.3.1 Determining the Reflectance of Basic Ink

Firstly, the toner is put into the basic recipe of ink at certain weight increments, and the color difference will be measured between the sample and the former sample. When the color difference is small enough, it can be considered that the color will not change significantly even with more toner. So we add this weight into basic recipe of ink as the initial basic ink.

Secondly, regarding the initial basic ink as standard, toner is put into the basic recipe of ink according to different concentrations (5, 10, 20, 40, 80, 100 %). The reflectance data of the corresponding concentration can be obtained after measurement.

Finally, after analysis and comparison between the reflectance of each concentration of four-color inks and various types of four-color ink products, the concentrations that have better agreement are chosen as the basic inks, and then corresponding data are stored in the matching database.

16.3.2 Determining the Reflectance of Substrate

The basic recipe of ink is stirred uniformly without any toner and coated over white paper. It is regarded as the substrate. And then, its reflectance is measured and corresponding data are substituted into color-matching calculation.

16.4 Experimental Verification and Analysis

MATLAB R2009a software is used for programming [7]. Then, the relative spectral power distribution values of CIE standard light source (D65), CIE standard colorimetric observer spectral tristimulus values, and the reflectance of the basic ink are written into the program. After the commissioning test run correctly, the relevant data of standard are substituted into the program and the concentrations of basic inks can be computed. The concentrations that obtained by calculation are converted into toner weight, and experimental verification is conducted. The experimental results are shown in Table 16.1.

Table 16.1 The measurement values of standard and the computed values of color matching

No.	Toner weight (g)				Chromatic values				ΔE
	Actual values		Computed values		Standard colors		Matching colors		
1	C	0.080	C	0.075	L^*	48.658	L^*	46.755	2.84
	M	2.000	M	1.823	a^*	−2.443	a^*	−4.523	
	Y	0.600	Y	0.712	b^*	33.374	b^*	33.374	
2	C	0.080	C	0.077	L^*	50.357	L^*	51.562	3.08
	M	1.000	M	0.990	a^*	−4.643	a^*	−7.283	
	Y	0.400	Y	0.526	b^*	29.162	b^*	30.205	
3	Y	0.200	Y	0.239	L^*	68.887	L^*	67.980	5.80
	M	1.000	M	0.869	a^*	27.695	a^*	22.500	
	W	10.000	W	8.870	b^*	43.666	b^*	41.237	
4	C	1.000	C	1.048	L^*	33.643	L^*	37.794	6.37
	M	4.000	M	4.772	a^*	−13.433	a^*	−16.778	
	Y	2.000	Y	1.698	b^*	7.026	b^*	3.538	
	W	18.000	K	20.037					

According to Table 16.1, white ink is not added in Sample 1 and 2; and the matching model shows high precision, and the color difference is less than 5 which meets industry standards. Sample 3 consists of yellow, magenta, and white inks. The same three-color-matching model shows a larger color difference when the white ink is added. Sample 4 is four-color matching and shows a large color difference though still obtain a match color that is similar to the standard.

16.5 Conclusions

Experimental results show that the color-matching models established in this paper have considerable precision and practical significance to solve the spot-color-matching problem. The efficiency of the color matching has been improved to a great extent, although there is still some color difference in the four-color-matching application. There are many factors affecting the accuracy of color matching, including the limitations of Kubelka–Munk theory itself, inevitable errors during the experiment process, and precision of the experimental apparatus [8, 9]. The accuracy of color-matching models will be further enhanced if the experiment conditions and the data precision of basic inks can be improved.

Because of the limitation of experimental conditions and time, this subject remains to be further studied.

First, the precision of four-color-matching model can be further improved, and the characteristics of white ink should be studied thoroughly. The improvement of computer algorithm can also help improve the accuracy of models.

Second, there is a wide range of substrates in the actual printing, while in this paper, we only use white paper as the substrate to discuss. In future study, the color-matching models should be modified according to different substrates, so that the models can be applied more widely.

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