Chapter 21
Enacting App-Based Learning Activities with Viewing and Representing Skills in Preschool Mathematics Lessons

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Abstract This chapter comprises discussion on research findings of this study on how apps can be used in the classroom to promote children's construction of mathematical knowledge by setting up specific learning contexts in ways that fundamentally transform the instructional environment. The study results identify how children enact viewing and representing skills through digital texts to acquire new strategies in their addition and subtraction learning. These skills enable children to externalise their understanding and internalise new meaning-making when interacting with peers. However, these dual reciprocal learning approaches require due consideration of the elements of the learners' learning styles, the standard of the game designs and the community settings of the classroom, all of which are crucial in determining the learners' engagement in a learning activity and active involvement in associated learning processes. With the appropriate level of autonomy and opportunity for choice, learner engagement will contribute to subsequent learning, with behavioural intensity and emotional quality at optimal levels. A detailed examination of the meaning-making processes through which viewing and representing skills mediate children's knowledge acquisition while seamlessly switching between individual and social interactions has led to the development of the framework in the preschool classroom's learner-centred mathematics learning model presented here.

21.1 Introduction

Mathematics competencies are cumulative over time (Jordan et al. 2009; National Mathematics Advisory Panel 2008). If not properly addressed and overcome, difficulties encountered at any stage of learning will lead to poor achievement in subsequent mathematics learning. For example, competencies in whole numbers are

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essential to arithmetic. Therefore, the effectiveness of the classroom teaching pedagogy during early education in developing fundamental numeric skills is crucial (Egan and Hengst 2012). Problem solving skills in addition and subtraction are dominant aspects of the fundamental competency domains in mathematics. When children practise solving problems, their underlying conceptual and procedural knowledge in addition and subtraction determines their competency (Canobi et al. 1998; Reeve et al. 2003). The causal relations between these two areas of knowledge have been found to be bidirectional; increases in conceptual knowledge will help to increase procedural knowledge and vice versa. The iterative relationship influences the development of conceptual–procedural knowledge, particularly in the competency domain of addition and subtraction (Bethany and Schneider 2015; Canobi 2009; Rittle-Johnson et al. 2001). Therefore, to develop an effective classroom teaching pedagogy, the integration of content knowledge (i.e. conceptual and procedural) into learning approaches is important (Chiu and Churchill 2015a, b). These approaches must engage children in the learning activities while leading to the process of meaning-making utilising the content knowledge (Hiebert and Wearne 1996; Star et al. 2011).

The conceptual and procedural knowledge of learners are observed explicitly via strategies applied during their routes to problem solutions. Some of the strategies applied in addition problem solving are direct modelling (represented by objects, which are all counted), counting on from first, counting on from larger and recalled with no apparent counting; those applied in subtraction problem solving are direct modelling (counting objects by separating from the total and counting those remaining), counting down from (a backward counting sequence from bigger numbers) and counting up from (a forward counting sequence from smaller numbers) (Carpenter and Moser 1984).

Problem solving is central to mathematics. One of the challenges in mathematics education is to help children to become skilled problem solvers rather than rote learners. Even after 30 years of reform, rote thinking is still common in classroom mathematics problem solving practices (Lithner 2008). Students often complete exercises in their textbook in which similar tasks are provided as exemplified in the book (Granberg and Olsson 2015). Rote learners are imitative; learners imitate a solution procedure memorised from the textbook. Conversely, creative reasoning engages students by allowing them to develop well-founded mathematically anchored arguments for their choice of methods in non-routine problem solving processes. Studies have shown that in most problem solving attempts students who engaged with creative reasoning performed significantly better than students who used imitative reasoning (Boaler 1998; Jonsson et al. 2014; Kapur 2011). In conjunction with challenging non-routine problems, collaboration is often suggested, since it can improve students' conceptual understanding (Boaler and GReeno 2000; Stahl et al. 2011). However, to accomplish collaborative creative reasoning, a suitable learning environment needs to be established. In this learning environment, students need to apply new strategies repeatedly with the objective being the advancement of their competency in addition and subtraction problem solving. Collaboration on a challenging problem cannot be automatically initiated within groups. The process of negotiation to seek the new knowledge (i.e. the correct strategies) must be made visible to the learners during their collaborative efforts. Therefore, mathematics learning that is solely print based and structured by content printed in a book is therefore inadequate (Clausen-May 2013).

Research evidence over the last 40 years regarding the impact of digital technology on learning consistently identifies positive benefits. In terms of teaching and learning technology resources, there are a number of free online mathematical problem solving digital artefacts. These tools mainly afford opportunities to learn interactively with ideas, content and modalities that were not previously possible (Yelland 2015). Research results also have indicated that the integration of digital devices in a classroom learning context facilitates cooperative participation of young learners with other classmates and teachers (Lindahl and Folkesson 2012; Wakefield and Smith 2012). That said, teaching could create and facilitate learning contexts, but not the actual learning. Learning mathematics and acquiring the competency to solve problems have been largely understood as a rational cognitive process (Chiu and Churchill 2015b; Zan et al. 2006). The actual learning takes place when learners make sense of mathematics through a meaning-making process.

The process of meaning-making is implicit and indirect (Seeger 2011). This interaction with digital text incorporates the four macro-skills of listening, speaking, reading and writing, but requires additional skills including frequent use of visuals, dynamic information and interaction (i.e. viewing and representing) (Khoo and Churchill 2013; Kress 2010; MOE Singapore 2010). Therefore, the focus of this study is not students' learning, but rather how, during the children's collaboration, the technology facilitates instant immersion in mathematical problem solving practices.

It is increasingly commonplace today for preschoolers to make use of computers in their out-of-school activities (MDG Advertising 2012). Although most children aged 2–5 years are more competent in interacting with a tablet computer than tying their shoe laces (Lunn 2012), the place of ICT in formal education in kindergarten has been contentious (Zaranis et al. 2013).

There is an emerging gap between the capabilities of digital learning in meaning-making and how preschoolers appropriate computers in their mathematics learning. Based on this concise review, we may conclude that the skills applied when preschoolers interact with digital texts are important in revealing the meaning-making processes. These interactions may facilitate the users externalising their understanding and internalising new knowledge. In order to develop a sustainable pedagogy utilising digital technology, it is imperative to investigate how these skills combine with collaborative interactions and result in the gaining of new knowledge. Moreover, we need a more explicit framework to integrate elements in the digital-based learning context of these sustainable practices in institutionalised education.

21.2 Literature Review

21.2.1 Designing a Creative Reasoning and Collaborative Learning Environment

Creative reasoning promotes the development of conceptual understanding in mathematical knowledge (Lithner 2008). In the creative reasoning learning environment, children construct their solving strategies and in this aspect, they are required to struggle with mathematics problems that are somewhat new to them. As they encounter challenging and non-routine problems, they undergo the process of testing and developing their solving strategies, visualising and verifying their arguments as to why their idea does or does not work. Brousseau suggested a didactic design that allows students to be responsible for arriving at solutions. In this design, teachers should not interfere or guide children (Brousseau 1997). Children's autonomous engagement in a teaching and learning activity is particularly important because it functions as a behavioural pathway through which their motivational processes contribute to their subsequent learning and development (Connell and Wellborn 1991; Hyungshim et al. 2010). However, in this learning design, if the students do not receive proper supportive activities, stagnation will likely result, hindering them from moving forward in their problem solving process (Ploetzner et al. 2009).

Therefore, an appropriate level of feedback in response to the students' actions should be introduced: collaborative engagements such as discussions, mutual explanations and elaborations are often suggested as means to assist children to improve their understanding (Hoffkamp 2011). However, having students work in groups do not automatically initiate collaboration. To design a creative reasoning and collaborative learning environment, the problem solving activity must be able to facilitate sharing. That is, students must be able to visualise the meaning-making processes and their representations (Rakes et al. 2010). Moreover, the activity must also allow them to effectively distribute their collaborative efforts through verbalization of mathematical concepts, referencing, testing, visualising, etc. One of the suggested methods is the use of dynamic software (Granberg and Olsson 2015). The proposition that digital texts may support problem solving activities brings us to the question as to how these texts facilitate creative reasoning and a collaborative learning environment.

21.2.2 Viewing and Representing Skills with Digital Texts

The notion of literacy in the twenty-first century has changed with the emergence of digital texts. The advancement of technology has led to some fundamental changes in the ways we receive and produce texts on screens. Digital interfaces can support user input and system output, with multi-mode capabilities including touch, eye gaze, speech, movement and hand gesture as a means of input or synthetic speech, graphical displays, and gesture as output. Thus, these texts are multimodal in nature; the users are required to design their path of engagement actively and continuously on screens both spatially and temporally to make sense of texts and interact when called for. However, the process of meaning-making is always about the interactions between one's perception and the mediums of communicative exchange. Perception is defined as a process of collecting information from the environment based on vision, touch, hearing and muscle to construct an internal representation (Gibson 1979). Therefore, the fundamental skills of the users are crucial in this aspect.

To make meaning with digital texts, one needs to understand various digital functionalities. For example, to search, translate, utilise the affordances of different modes in effective meaning-making, navigate digitally in different ways, and make meaning by placing elements of information with different modes in appropriate spatial/temporal positions. These profound changes brought about by digital texts have led to the development of emerging skills used to interact with digital texts—i.e. viewing and representing.

First, the viewer's interests draw attention to an element that is then selected; via the same process, another element is then selected and so on. In between the selections, an attempt is made by the viewer to integrate the selected elements to form meaning (Kress 2010). In the process of selections and integrations, the meaning will be translated from one mode into another (Mills 2011). In so doing, the viewer alters the meaning of the elements along the lines of their interest (Khoo 2012).

To represent messages with digital text, the producer must have an objective regarding what to show, what message to convey and what he wants to achieve socially, culturally or for other purposes (Kress 2010). The process of composing digital texts incorporates the competence of making meaning with multimodal elements, utilising the affordances of mode, creating meaning by contextually linking elements of different modes and utilising digital functions in meaning-making and navigation. Further, the composer's designs are derived from physical structures in real world settings. The skill of composing with an objective in mind is termed 'representing' (Khoo 2012). Table 21.1 summary of viewing and representing skills includes two levels of engagement with at least five aspects of competencies (Khoo and Churchill 2013). When the children interacted with digital texts in the mathematical problem solving apps, they applied different strategies to arrive at the answers. Viewing and representing skills are necessary when interacting with digital texts on screens to externalise what appears in the user's mind or to internalise the information.

Table 21.1 Viewing and representing skills framework (from Khoo and Churchill 2013)

Macro process	Element selection	Element integration
Skills	*Multi-mode*	*Contextual link*
	The skill to interpret or create elements of different modes to form information	The skill to interpret and create contextual links (in spatial/temporal layouts) with different elements to form information
	Affordances of mode	*Navigation*
	The skill to apply and engage with the affordances of different modes in elements to form information	The skill to move around a screen to integrate different elements to form information
	Digital functionality	
	The skill to assimilate digital functionalities in elements to form information	The skill to assimilate digital functionalities to integrate elements to form information

21.3 Research Design

The current study applies the Activity Theory (Engestrom 1987, 1999) as the theoretical framework in the current mobile technology-related contexts of learning. A close examination was made of the relations between the students (subjects), objectives and tools used in the learning activities (see Fig. 21.1). The study included observations, video recording and interviews, and employed an inductive research strategy that intuitively developed abstractions from the research (Merriam 1988).

The research design is qualitative (Merriam 1988; Yin 1994). Two research questions emerging from the literature reviews guided the data collection and analysis of the current study:

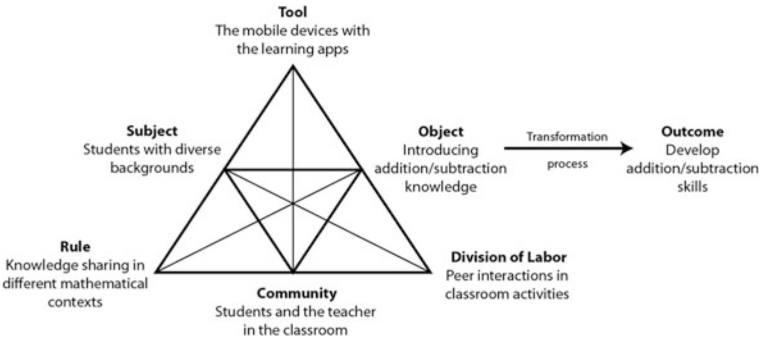
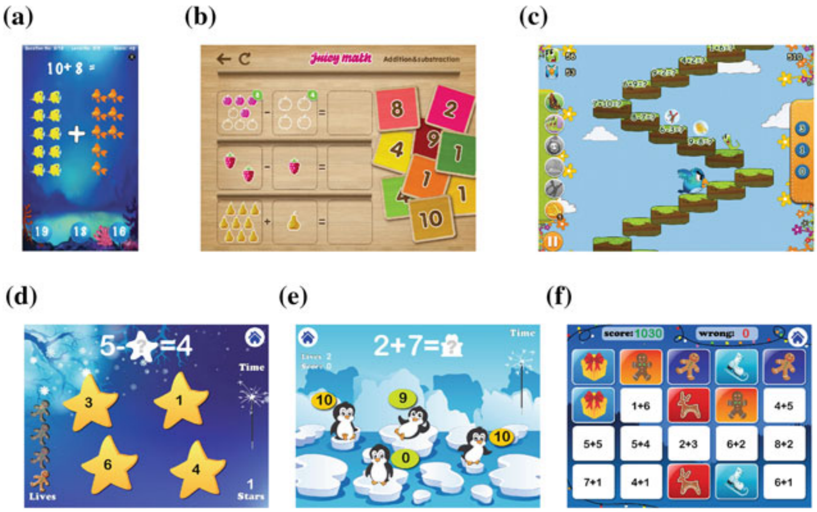


Fig. 21.1 The research framework of the study



A: Addition & Subtraction for kids
B: Juicy Math
C: Worm Jump
D: Math Adventure (Penguin Craze)
E: Math Adventure (Catch A Star)
F: Math Adventure (Winter Match)

Fig. 21.2 Screen captures of the apps understudied **a**: Addition & Subtraction for kids, **b** Juicy Math, **c** Worm Jump, **d** Math Adventure (Penguin Craze), **e** Math Adventure (Catch A Star), **f** Math Adventure (Winter Match)

- How do digital texts facilitate collaboration in mathematics problem solving activities?
- What elements of digital-based problem solving activities might contribute to or obstruct students' collaborative creative reasoning practices?

Data were collected over a period of 8 months. During the two-level study (see Table 21.2), the study was explorative and a constant comparative method of data analysis was employed.

The study commenced by selecting suitable participants (the sampling units). The selection was terminated when no new information was forthcoming from new sampled units during the research period. Four participants were identified according to their learning styles and their community settings in the classrooms: their profiles are presented in Table 21.3. Pseudonyms are used to preserve their anonymity.

During the 4 months' study at the kindergarten, the teacher used different mathematics apps in children's practices in the classroom. The apps were introduced after the formal mathematics lessons. Each of the apps was used repeatedly two to four times per week. All the children were given the opportunity to use the

Table 21.2 Research methodology procedure and aims

Level 1	Level 2		
Observation	Classroom observations	Interviews with the teachers	Interviews with the children (before and after the lessons)
To find out whether: • The kindergartens were teaching mathematics with mobile devices • The children used the devices individually, and received appropriate feedback in the process of learning, resulting in acquiring new knowledge	• To observe how the participants learn in the classroom with mobile devices • To study how the participants interacted with the artefacts, their peers, the community and the rules in the lessons • To video record the lessons	Unstructured interviews • To determine the objectives of using the mobile devices in the classroom • To identify the addition and subtraction strategies taught by the teacher to the participants	Structured interviews To determine how well the participants understood the questions in the apps • To study how the participants interacted with their peers in lessons and understood the rules, and the maths concepts in the artefacts
• Kindergarten(s) that applied mobile learning in mathematics resulting in authentic learning being observed were shortlisted	• To determine how the students used viewing and representing skills • To ascertain the social learning skills emerging from use of mobile devices	• To verify the researcher's observations on how the teachers mediated mobile technology in learning	• To investigate how the emerging skills mediate digital text in learning addition/subtraction. • To confirm the observational data during the lessons
1st–4th month	5th–8th month		

Table 21.3 Details derived from the selection criteria for the four cases

Participants	The lessons				Age	Gender
	Addition and subtraction for kids (Session 1)	Adventure math (session 2)	Juicy math (Session 3)	Worm jump (Session 4)		
Peter	Team A	Each participant was provided with one device. They practised individually	Each participant was provided with one device. They practised individually	Each participant performed in front of the group until they had completed their practice	5	Male
Mary	Team A				5	Female
Ben	Team B				5	Male
Nicole	Team B				5	Female

apps during practices. Four apps were selected for the current study: "Addition & Subtraction for Kids", "Math Adventure", "Juicy Math" and "Worm Jump". These apps were selected based on the unique characteristics of the instructions they applied in learning mathematics (see Table 21.4).

Table 21.4 The learning apps

Apps	The Nature of the apps
Addition and subtraction for kids (The first app)	The app started with one question at a time. There are 10 questions in a set. The numbers featuring in the questions were from 1 to 20 and were randomly set. Each question was either addition or subtraction. Each provided three possible multi-choice answers. There was no time limit set for the questions. The accumulated scores (10 points for each correct answer) and the number of wrong attempts appeared on the top of the screen (See Fig. 21.2a)
Math adventure (The second app)	There were five different themes (Penguin Craze, Winter Match, Catch a Star, Snowman Hunter & Math Bingo). Different themes provide different instructions in problem solving. For each of the themes selected, the player must further select the type of questions (addition or subtraction) and the number range (1–10 or 1–20). In all themes, the questions had to be completed within a time limit.”
	Penguin Craze addition allows the participants two lives (two chances to make mistakes before the game is terminated) and lasts for a maximum of three minutes. The question is an equation (i.e. 3 + 2 = ?) with four choices of answer. For every ten questions attempted, one free life is awarded (see Fig. 21.2e).
	Catch A Star subtraction has two lives and also lasts for a maximum of three minutes. The questions are in equation form and require the finding of an unknown in equations (i.e. “? – 4 = 2” or “8 − ? = 2”), with four choices of answer. For every ten questions attempted, one free life is awarded (see Fig. 21.2d).
	Winter Match addition has 20 boxes, 10 containing numbers and another 10 an incomplete addition equation (e.g. 3 + 2=). The player clicks a box and subsequently selects an answer by clicking another box. Once a question has been correctly solved, both boxes are closed. The score reduces as time advances, reaching a zero score in 110 s. The numbers of incorrect answers are displayed (see Fig. 21.2f)
Juicy math (The third app)	The game provides three choices: addition only, subtraction only or both. There is no set time limit for completion of the questions. The screen displays three questions at any one time. Each of the questions shows different objects in two boxes. The objects can be clicked. For addition, for each of the objects that the player clicks, the number counter increases by one. Once all the objects are clicked the total will appear in each box. The player will drag the answer to the answer box. It will bounce if the answer is wrong. Only when an answer is correct, will it be accepted and the two boxes on the left of the answer box change from objects to numbers. For subtraction, once an object in the second box is clicked, the object in the first box will disappear. The object remaining in the first box is the answer (see Fig. 21.2b)

(continued)

Table 21.4 (continued)

Apps	The Nature of the apps
Worm jump (The fourth app)	The bird in the game moves forward a step at a time. The worm is ahead of the bird and once a question is answered correctly, the worm will move forward a step. If the answer is answered wrongly, the worm will stay put. Once the bird reaches the worm, it will be eaten. To keep the game going, the player must answer promptly to stay ahead of the bird. Addition and subtraction questions are displayed and there are three choices per question. Some obstacles appear after the 40th–45th steps to slow down the player's moves (see Fig. 21.2c), thereby increasing tension and elevating the degree of difficulty in completing the game successfully

21.4 Observation of Participants' Enacting the App with Viewing and Representing Skills

21.4.1 Participant One: Peter

Peter was an outgoing child, asking questions spontaneously of his classmates in the classroom whenever doubts arose in his mind. His recent maths assessment result was about average. This study provides evidence of Peter's competency in enacting the viewing and representing skills (see Table 21.1). In Fig. 21.3, he enacted the viewing skills by interpreting meaning made by elements of different modes and integrating them to form contextual information. He read the question in numeric symbols "6 − 1=" (see Element 1, 3.1 in Fig. 21.3), counted the fish in the picture (see Element 2, 3.1 in Fig. 21.3) below the question and selected the numeric answer at the bottom of the screen (see Elements 3, 3.1 in Fig. 21.3). In Fig. 21.3, Peter selected and integrated the information contextually—i.e. in both spatial and temporal layouts. He chose "5 + 5" on one button and subsequently chose "10" on

3.1 3.2 3.3

Fig. 21.3 Peter enacted the viewing and representing skills another (see Element 1, 3.2 in Fig. 21.3) to complete the question, while simultaneously observing the countdown score (as in Element 2 & 3, 3.2 of Fig. 21.3). In the fourth app, he read the questions and chose the answer while simultaneously striving to solve the question fast to maintain distance from the bird (see Elements 1, 2 & 3 in Fig. 21.3). He was highly aware of the movement of the bird seeking to eat the worm as it came closer. The game used the movement of the bird as a metaphor for the time limit. In the three apps, Peter's abilities in enacting the viewing and representing skills with digital texts were observed (Khoo and Churchill 2013).

In Peter's engagement with the first app (see Table 21.3), two students were assigned one mobile device and took turns to answer the questions. The teacher briefed all the children in the class before they started with the strategies of "counting on from first" and "counting down from" for solving both addition and subtraction questions. Peter started the game before Mary, with the task being to complete ten consecutive questions. He counted all the objects and chose the answer for two of the ten questions. The addition strategy of modelling (Goldin 1998) was observed. Peter's teammate, Mary, explained the strategies as briefed by the teacher. Subsequently, he answered the remaining questions with the new strategies. He turned to Mary to confirm his answers each time he had made a tentative choice of answers. Mary nodded to confirm her agreement with his choices. The situation demonstrated the process of peer learning (Hwang and Hu 2013; Liu and Carless 2006) where it was mediated in the context of using a mobile device as a learning tool.

For the second app, Peter was assigned a personalised device. He selected the Winter Match in his attempt to solve the set problems. There were two criteria for monitoring the participant's learning outcomes: the speed of solving the questions and the maximum number of wrongs allowed. A countdown timer limited to 110 s the time Peter was given to finish the questions. After the 110 s had elapsed, the game could still be continued but the score was always zero (see Table 21.4). The teacher set a rule that each of the children had to score 200 to finish the activity. "Engaging learners in thinking about achieving outcomes to certain agreed criteria is a learning process" (Liu and Carless 2006, p. 280). Peter started to count using the strategy of modelling. Slowly, he switched to "counting on from the first". He sought feedback from the teacher each time he was in doubt. The teacher guided Peter from time to time. Peter demonstrated his artfulness in engaging with the learning context, while his imprompt interactions with the teacher and the teacher's feedback regulated his learning. His ability to seek feedback was observed.

In the third app, Peter was also given a personalised mobile device, and the instructions of the app provided neither scores nor time limit. Peter attempted the questions using the strategy of "counting all" on the objects in the two boxes. He applied the same strategy to the rest of the questions. It was also observed that Peter could not obtain the answers for some of the questions in his initial attempts.

In the fourth app, Peter played the game in front of the teacher and the group of classmates. It was a 40-min lesson and the lesson was repeated on 3 consecutive days. The teacher set the condition that each child should answer at least ten questions before the termination of the game. Otherwise, he/she had to repeat. Peter started his first attempt with the “counting on from” strategy: e.g. he would count, “4 [pause], 5, 6, 7, 8”, then clicked 8. The game was terminated after two questions. Peter observed how other children completed their attempts. The group was noisy. Some would speak out the numbers and answers without any apparent attempt at counting, e.g. 3, 5 [pause]… 8. Peter succeeded on his third attempt with a score of 120 (twelve correct answers). He acquired the number fact strategy by retrieving the recalled number fact from his memory (Carpenter and Moser 1984). His abilities to learn a new strategy along a learning hierarchy from acquisition to fluency were observed. His observational skill in learning was evidenced (Browder et al. 1986).

The interviews were conducted with Peter immediately before and after his lessons. The purpose of the interviews was to confirm his understanding of the strategies observed in the research. The researcher found that he applied the strategy of modelling before the lesson with the first app; he could correctly answer addition/subtraction questions involving small numbers (i.e. $2 + 3 =$, $4 - 3 =$ etc.). After the lessons, the results of the interview indicated that he could answer addition questions involving regrouping two one-digit addends (i.e. $5 + 8 = 13$); he could also answer one and two-digit subtraction questions (i.e. $18 - 4 =$). The interview revealed that he applied new counting strategies of “counting on from” (for addition) and “counting down from given” (for subtraction). In the second app, his ability to adopt the new strategy from “counting all” to “counting on from the first” was also observed.

The third app did not evidence his ability to gain any new strategies, but in the interview his ability to learn a new strategy from the “counting on from” to the “number fact” was evidenced in the fourth app.

21.4.2 Participant Two: Mary

Mary had an easy-going manner, and demonstrated good social assertive skills in the class. She scored good results in her schoolwork in mathematics and was very helpful to her classmates when asked for assistance. As with Peter, Mary demonstrated competencies in engaging with digital texts on screens in meaning-making by applying appropriate viewing and representing skills.

In her engagement with the first app, she had a vicarious experience from teaming with Peter and guiding him to complete the questions. Thus, when it was her turn, she completed the same app exercise quickly and with a full score. In her second app attempt, she was provided with an individualised mobile device to operate on her own. She clicked two consecutive buttons with the same values to close the two buttons (i.e. she clicked 4 & 4; 5 & 5, etc., instead of $1 + 3$ then 4; $2 + 2$ then 4; $1 + 4$ then 5; $2 + 3$ then 5). The remaining buttons panicked her because the problem solving questions were unfamiliar to her (i.e. $4 + 2$, $1 + 5$, $2 + 2$, $1 + 3$ etc.). She paused and pondered, then sought assistance from her teacher, who popped in and clued her up. The teacher introduced a new strategy, the “recall number fact” with no apparent counting (Carpenter and Moser 1984), and repeated the same strategy in the two subsequent questions. Thus, she coached Mary to the point where she was able to handle the problem solving on her own. Mary then independently solved the rest of the questions. The scaffolding process was evidenced (Beishuizen et al. 2010), which Mary embraced, learning the new strategy. In the fourth app, with the assistance of answer hints from the teacher and her peers, she demonstrated her ability to answer the two-digit and one-digit questions.

The interviews with Mary before and after the lesson with the first app activity revealed that she had learnt the new strategy from "counting all" to "counting on from the first". Observational learning was evidenced (Orelove 1982). The pre- and post-lesson interviews revealed that in the fourth app activity she applied the "recall number fact" strategy to addition and subtraction questions with no apparent counting for numbers in the equations less than 10. However, she applied the "counting-on from larger" strategy (Carpenter and Moser 1984) for additions involving "two digits with one digit" (e.g. $11 + 4$). After the lessons, she managed to acquire a new strategy, the decomposition strategy (e.g. $13 + 5 = 10 + 3 + 5 = 18$) (Canobi et al. 1998), learning through reflection from the teacher and peer hints.

21.4.3 Participant Three: Ben

Ben was a brilliant child. He had achieved full scores in most of his schoolwork. He was observant, quiet and able to learn quickly with less practice and repetition than typical of his peers. When dealing with the apps on the screens, he demonstrated competency in engaging with digital texts for meaning-making using viewing and representing skills.

In the second app, Ben chose "Catch a Star" with subtraction $1 - 10$. The questions were in the form of $x - 5 = 2$ and $5 - x = 2$, the task being to determine the value of x. Ben was not able to tackle the form of $x - 5 = 2$, and after two wrong attempts sought help from his teacher. The teacher showed him a trick to sum the numbers of 5 and 2. He managed to acquire knowledge of the new strategy and practised for the next few similar questions. When the question "$5 - x = 2$" appeared, the teacher showed him the strategy of subtracting 2 from 5.

The studies of Ben's classroom activities with the first, third and fourth apps were also examined, and the researcher found that observational learning and peer feedback had occurred.

The pre- and post-lesson interviews on the use of the second app disclosed that Ben had learnt the strategy of "addition to solve subtraction problems" (Peters et al. 2013). When he encountered uncertainty, he inquired in an attempt to find a solution. He also applied his prior knowledge in addition and subtraction to generate new understandings. His ability to learn reflectively was observed (Boud et al. 1985; Koong et al. 2014).

21.4.4 Participant Four: Nicole

Nicole was a shy but obedient student. Although her self-expression was awkward, she was able to learn mathematics, although her mastery of skills was relatively slow. Utilising the data collection methods as per Table 21.1, the researcher observed that Nicole had acquired viewing and representing skills in engaging with information on the screens. In her first app practice, she teamed up with Ben, observing how he completed his ten questions (i.e. counting the items and saying the numbers softly as he was solving the questions: e.g. for "9-1 = ?" he would say "nine minus one", and would then count the objects aloud, "1, 2, 3, …"; for the additional question, he pointed at the first number "9" and spoke it aloud, then counted the second pictorial quantity aloud, "10,11,12, …15"). Nicole observed and imitated what Ben had done. The imitation led to Nicole's learning to apply the new strategy of "the counting on from". She imitated the steps to start from the first number. She then counted the objects on the right and since each time the questions gave a different set of numbers, she practiced and learnt the new strategy. The imitation skill emerged from the current research with Nicole.

The interviews with Nicole were conducted before and after her lessons on the use of the first app. Before the lesson, she applied the strategy of modelling. She demonstrated that she had learnt the new strategy of "counting on from" after using the app in the lesson.

21.5 Discussions

In investigating the first research question, the current study has identified six collaborative skills. Five of these skills were identified in the literature before the data collection (Beishuizen et al. 2010; Goldin 1998; Hwang and Hu 2013; Liu and Carless 2006; Mezirow 1990; Orelove 1982; Pardo 2004): observational learning, embracing the process of scaffolding, reflection, peer learning and seeking feedback. The collaborative skill of imitation emerged in the current study. These collaborative skills take place at the social level and were enacted through interaction with digital texts. They are shown in Table 21.5 below.

In the study, activity theory was applied in categorising into four dimensions how the participants came to know new addition and subtraction learning strategies. The first such dimension is subject–tool–object. During Peter's practice with the first app, he enacted the app with the viewing and representing skills and his mathematical knowledge in addition, i.e. the strategy of modelling. Mary perceived the strategy used by Peter (the externalisation of Peter's understanding was observed) and she applied verbal and non-verbal responses with the alternative approach of pointing at the first number then continuing with the objects on the right (the strategy of counting on from), subsequently obtaining the answer. Her action led to Peter's new meaning-making and he used the new strategy and

Table 21.5 The collaborative skills enacted through interaction with digital texts

	Collaborative skills	
Skills	*Peer learning*	*Reflection*
	Verbal and non-verbal responses of peers to a person's actions or behaviour that lead to new meaning-making for that person	Intellectual activities in which one's own experiences are explored in order to generate new meaning-making
	Embracing scaffolding	*Observational learning*
	The ability to elicit information and reactions during the scaffolding processes that leads to new meaning-making	The learning skill that one develops along a learning hierarchy from acquisition and fluency to generalisation of initiative behaviour
	Seeking solutions	*Emulating*
	The ability to seek information from various sources or channels through social interaction in a learning context	The act of imitating someone else's successful steps in completing a task, while also applying one's own prior knowledge

Table 21.6 The new strategies acquired by participants compared with their social level

Community	Articulated interplay level	The new learning skills adapted			
		Peter	Mary	Ben	Nicole
Team of two (1st app)	Medium	✓	0	0	✓
Individual (2nd app)	Low	✓	✓	✓	✓
Individual (3rd app)	Low	0	0	0	0
Performed one by one in the group (4th app)	High	✓	✓	✓	✓

Note "✓": the frequency of a new strategy learnt

produced the correct answers for the next questions. Peter's internalisation of the new strategy was observed. Mary had also nodded in agreement with Peter's attempts in using the new strategy. The same dual conscious mental processes—i.e. cognitive processes—were also observed in the second and fourth apps for Peter, Mary and Ben and also first, second and fourth for Nicole (Table 21.6 refers).

The participants designed their path of engagement actively and continuously on screens, both spatially and temporally, to make sense of texts. In Peter's case, with the first app as an example, he read the question "6-1", counted the fish in the picture below the question and selected the answer at the bottom of the screen. First, Peter's interests drew attention to an element that was then selected; via the same process, another element was then selected and so on. In between the selections, an attempt was made by Peter to interact with the selected elements to form meaning (see Table 21.1). The processes were visualised by peers as shared context for their joint problem space. In Peter's case, Mary noted the process (numbers, images, spatial and temporal information) by which he answered his questions. They constructed a shared conception of the given problem by applying different

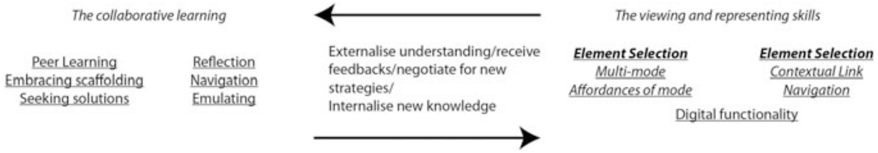


Fig. 21.4 The viewing skills facilitate collaboration amongst peers

collaborative skills as Mary explained the strategies as briefed by the teacher, emphasising parts that Peter might have missed (see Table 21.5). The crossed case study has revealed how digital texts, through viewing and representing skills, merged with the collaboration of the participants iteratively to enable the meaning-making process. These interactions facilitated the participants externalising their understanding and internalising new knowledge. The idea is demonstrated in Fig. 21.4. Research question one is answered. However, as we have observed in Table 21.6, some of the interactions might not result in the gaining of new knowledge. Results from the current research indicate that others elements must be further investigate.

In the second dimension, subject–community–object, the community setting provides the context in which the individual's learning interaction (articulated interplay) takes place through the activity (see Table 21.6). Although the findings of previous studies are that children's feelings of isolation and a sense of presence were positively correlated with the effectiveness of their learning (Cereijo et al. 2001; Rovai and Wighting 2005), the current research results show that the community setting does not have an absolute effect in new meaning-making. In the second app, all the participants gained new knowledge, but in the third app, none of the four participants gained any new knowledge.

In the third dimension, subject–division of labour–object, the individual's roles within the community differ in terms of the degree of engagement in the activities. The participants exhibited different learning styles: Peter and Mary preferred to deal with people, although Mary was smarter than Peter. Ben was excellent in inductive reasoning and Nicole was likely to solve problems in an intuitive trial-and-error manner (Carpenter et al. 1978; Kolb 1981). However, both Ben and Nicole were shy and neither was assertive when they encountered problems in answering their questions (see Table 21.7). Nevertheless, there were no significant results indicating that the different learning styles prevented them from acquiring new knowledge (see Table 21.7).

In the fourth dimension, subject–rule–object, three participants were observed acquiring new strategies in the course of attempting to adhere to the instructional requirement of the scores and speed limit when completing tasks (i.e. Peter and Mary with the second app, and Ben with the fourth). Intellectual engagement with outcomes and standards are focuses of participant involvement in solving the questions and led to clear standards in achieving high quality learning outcomes. The rules were different in different apps (see Table 21.8). The results demonstrated that the rules of the games were important in helping the participants to learn the

Table 21.7 The combination of different styles versus the new knowledge learnt

Learning style/Performance	Result is above average	Result is average
Assertive	Mary ✓✓✓	Peter ✓✓✓
Shy and not outspoken	Ben ✓✓	Nicole ✓

Note "✓": the frequency of a new strategy learnt

Table 21.8 The standard of different apps

The apps	The standard	The new strategy adopted by the four participants
Addition and subtraction for kids	Result score	✓✓
Math adventure	Time limit and result score	✓✓✓✓
Juicy math	–	–
Worm jump	Time limit	✓✓✓✓

Note "✓": the frequency of a new strategy learnt

new strategies. The third app was not designed with any rules that might help the participants to learn, although the app might have the potential to guide participants to new knowledge.

The analysis of the four dimensions also resulted in the following summaries.

- The externalisation and internalisation of the understandings might not result in new meaning-making, as observed in the first app with Mary and Ben (Table 21.6 refers).
- The differences in learning style of the participants did not significantly hinder them from learning new strategies (Table 21.7 refers).
- Community settings have no absolute effect on the meaning-making, whereas the type of rules embedded in the game designs might play important roles in this regard (see Tables 21.6 and 21.8).

The current study shows that digital technology enables students to enact creative reasoning with digital texts. They receive positive and negative feedback relative to their actions, which allows them to modify their subsequent actions, mostly without guidance from the teachers. They collaboratively constructed and shared conceptions through visualisation of their created and enacted solving strategies with digital texts. However, as discovered in I & III, in addition to the previous studies (Granberg and Olsson 2015; Lithner 2008), the application of dynamic software to engage students in a collaborative and creative reasoning learning environment might need to consider the interplay among elements that took place at the social level of the participants—i.e. the personal learning style, the community setting and the standard of the software design. The standard of the software design has an absolute effect in determining the new meaning-making that resulted in gaining new knowledge (see Tables 21.6 and 21.8). Learning style plays a significant role in determining the intensity of gaining new knowledge. However, the community setting does not appear to play a significant role in this aspect. The findings of research question 2 were, indeed, analogous to a jigsaw puzzle. Through the activity theory, the four dimensions of analysis served as fundamental pieces of the puzzle. Once these pieces are fitted together, the answer to research question 2 was answered in totality.

The summarised framework is presented in Fig. 21.5. The interactions between the participants and the digital artefacts transcend their knowledge levels through a positive meaning-making path that might not have been possible if the contextual setting was not aligned along the path (see Fig. 21.5). This conceptual and pedagogical framework serves as a reference to classroom instructors seeking to incorporate digital apps into classroom mathematical lessons.

A pedagogical implication arises from the current research. Over the past 50 years, research findings on addition and subtraction problem solving strategies have been very well defined and consistent (Carpenter and Moser 1984). Movements are currently are underway to reform the practices of mathematics problem solving by focusing on the flexible use of appropriate strategies, rather than standard school-taught approaches (De Corte et al. 2007; Peters et al. 2013). The finding implies that the conventional theories regarding young children's learning of addition and subtraction need to be reconceptualised, and flexible use of strategies in classroom activities with digital technology promoted and adopted to facilitate collaborative creative reasoning within the proposed framework (see Fig. 21.5).

The current research has several limitations. Although the study focuses on the implementation of classroom mathematics learning through activities, the investigation was confined to the topics of addition and subtraction. Thus, findings may

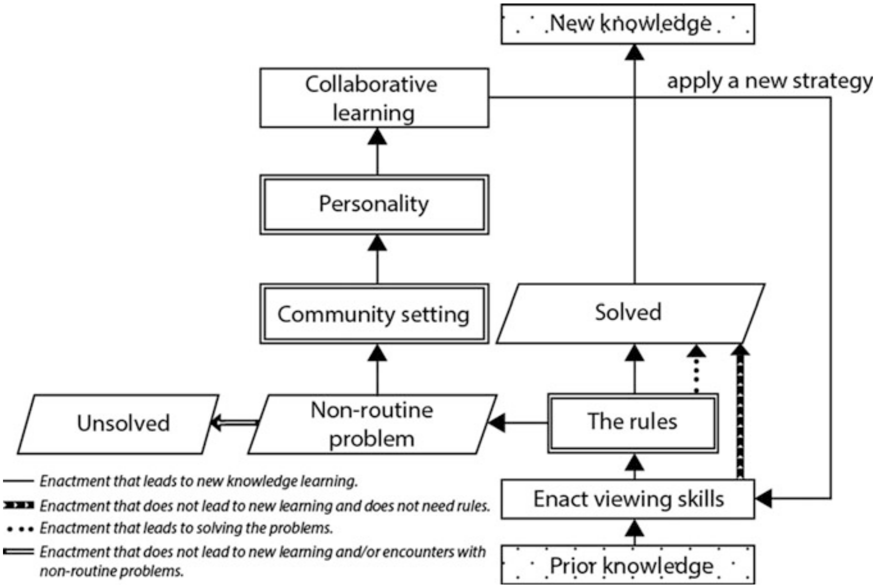


Fig. 21.5 The framework for enacting viewing/representing skills to acquire new knowledge

not be generalizable to all mathematics learning in the classroom. Future research could focus on more topics with a view to developing the current framework.

Further, the kindergarten encountered difficulties in selecting suitable apps for learning purposes. The apps used in the classroom were limited by the participants' age-related cognitive abilities (e.g. the level of written texts used and the aspects of virtual reality of the app content). Doubt must be acknowledged as to the validity and availability of apps suitable for children's classroom learning.

21.6 Conclusions

This paper synthesises relevant literature on collaborative learning and draws on viewing and representing skills to make a case for integrating digital devices in classroom mathematics learning. The study illuminates existing practices of learning mathematics from a new perspective of learning. A central concern of this new perspective is the ways in which children artfully engage with their peers and surroundings to create impromptu sites of learning. The results showed in this study demonstrated that learning apps were useful to facilitate children's classroom learning. The activities under study are learner, new knowledge, assessment and community-centred. Through digital technologies, these activities allow participants to enact autonomous tasks to construct their own meanings. The need for learners to externalise understanding is central to the activities. All parties focus on a common external representation of a subject that allows them to identify and discuss the topics (Laurillard 2002; Pask 1976).

Learning takes place effectively when learners are in control of the learning activity, able to assess and experiment with his/her ideas in the course of pursuing results, and to enquire by working with people in seeking new knowledge, then plan for new actions (Ravenscroft 2000). The current multiple case studies provide a framework for the integration of digital texts into activity-based classroom mathematics learning, along with specific recommendations that emerged from the research findings and implications.

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