NÚRIA PLANAS AND PAOLA VALERO

13. TRACING THE SOCIO-CULTURAL-POLITICAL AXIS IN UNDERSTANDING MATHEMATICS EDUCATION

INTRODUCTION

Research is always carried out from a standpoint, an epistemological stance that shapes the ontological assumptions about what is being researched, even though the researchers might be unaware of or unconcerned about what it is. Despite apparently being evident, this assertion needs to be revisited when reviewing the insertion of socio-cultural approaches to mathematics thinking, learning and education in the last 10 years of research in PME. The way in which theoretical tools and frameworks from other areas of study have been appropriated into mathematics education and how they have been transformed for the purpose of studying it are important issues in carrying out a research review. In this review we pay attention to the stances and assumptions that, together, articulate a logic about what it means to understand mathematics education as social and cultural. Connected to this, an initial observation, from which we derive the structure of this review, is the very same meaning that the category *socio-cultural* seems to have in the research community of PME – and more broadly in the international research in mathematics education. Our present investigation asks three leading questions:

1. What are the meanings that the authors who explicitly frame their research work as socio-cultural have given to this category?
2. Which are the identifiable directions and specific lines of concern in this body of research?
3. How have these meanings, directions and lines been enlarged and transformed in the span of 10 years?

It is our contention that there has been a growing adoption of socio-cultural frameworks in mathematics education research, and that such an expansion has led to important developments in the field, within and outside PME. This is not new; it continues along the path outlined by Gates (2006), among other authors, in the previous decades. We also argue, however, that there has been a move towards the configuration of a *socio-cultural-political axis*. While the progressive expansion of *socio-cultural* frameworks is not new, the clear featuring of the *political* framework

Á. Gutiérrez, G. C. Leder & P. Boero (Eds.), The Second Handbook of Research on the Psychology of Mathematics Education, 447–479.

is undeniable. Thus, the configuration of such an axis is particular to the last decade. From the very beginnings of PME, there has been discussion about the socio-cultural aspects of mathematics education research: what defines a socio-cultural approach, what constitutes a socio-cultural trend in the history of PME… This discussion became evident in the first *Handbook of Research on the Psychology of Mathematics Education*, in the chapters by Confrey and Kazak (2006), Gates (2006) and Lerman (2006) and their review of 'Social aspects of learning and teaching mathematics' for the period 1977–2005. The intersecting perspectives adopted in these chapters respectively addressed: constructivism, the understandings of the social, equity issues and access, and socio-cultural research. To a certain extent these perspectives could be identified separately in research; however, things have changed a great deal since then. In the decade 2006–2015, the adoption of a series of related theories allowed researchers to address not only the micro-constitution of mathematical thinking, but also its macro-configuration within larger societal fields. Consequently, it is no longer possible or at least not feasible to demarcate the boundaries between what the social, cultural and political embrace, although particular theoretical tools may emphasize one aspect more than others. This expansion to incorporate the political contributes to realising the ambitions of a research field that provides deep understandings of the complexity of mathematics education in contemporary societies.

Institutionally, the engagement of PME with an emerging socio-cultural-political axis has had several key moments. One such moment can be located at PME31, where the theme of the conference was 'School Mathematics for Humanity Education'. This took place just three years after PME28, where the theme had been 'Inclusion and Diversity'. In his plenary in PME31, Breen (2007) emphasized the fact that "individuals do not operate outside of a context – the social and political are ever-present in our teaching" (p. 76). Breen went on as follows:

> Thinking about PME, one might argue that PME conferences have always been held with the express purpose of annually celebrating the light … We each have our own template of what that light looks like and how it should be explored, and we judge each other's contributions against this template in our search for certainty. [D]evelopments indicate a welcome willingness on PME members' part to look beyond the light of mathematics education and embrace the shadow as an integral part of our field. (pp. 76–77)

The metaphor of integrating light and shadow represents well the challenges that socio-cultural-political research poses to the field. This metaphor points to the importance of how decisions and choices are made on what to research, why, with whom and for what purposes. As researchers we have the power, privilege and responsibility to illuminate the complexity of mathematics teaching and learning both towards the details of children's thinking processes in meaning mediation, and towards the broader significance of mathematics and mathematics education in contemporary societies. On tracing the paths of the socio-cultural-political axis in the last 10 years of research in PME, we will map out how these decisions and choices

have been made and how they have constituted lines of thought about mathematics thinking, teaching and learning.

We have organised the chapter in three main sections. In 'Expanding views: what does the socio-cultural-political axis mean?' we look at recent work that has contributed to an expansion of the socio-cultural views of mathematics teaching and learning, embracing the political views. From there we go to 'Mapping the socio-cultural-political axis: what is it like?', where we provide an overview of some clusters of topics and findings in PME research. In the last section, 'Moving the field forward: what is next?', we discuss strengths, challenges, gaps, in addition to future directions in PME research and open this up to mathematics education research as a whole. Before we discuss the main sections, we will explain how we proceeded methodologically to conduct the research review.

REVIEWING RESEARCH: HOW DID WE PROCEED?

Reviewing academic literature also follows a logic. In our case, the logic adopted was framed by our knowledge of existing socio-cultural research in the field and its expansion in certain directions. Our three leading questions allowed us to identify the lines of concern, connections between those lines and who represents them, as well as the resonances between the perspectives expressed in particular papers. All this work around the state-of-the-art followed the stages of *selecting* (literature), *organising* (connections) and *analysing* (novelties). These stages were planned to be inductively accomplished, with the elucidation of connections and novelties being highly iterative in nature. We assumed that for any area of study to encourage the emergence of new ideas and trends, connections and advances between what has come to be known in the most recent past are necessary.

The *selecting stage* consisted of choosing the set of papers for the literature review. In this first stage, we drew on the whole of Plenaries, Panels, Research Fora and Research Reports (RRs), and identified and counted the papers that explicitly declared a theoretical perspective identifiable as socio-cultural for 2006–2015. Plenaries, Panels and Research Fora were read almost in totality in order to decide whether they added significant new debate to socio-cultural research. In the case of Research Fora, where a number of traditions are usually represented by a collection of short papers, we searched for evidence of such debate in at least one of the papers. Some more work was needed for the study of RRs. Six sets of PME Proceedings from this period include in their first volume an index of the authors of RRs within a system of research domains. This index helped us to trace the collection of RRs with socio-cultural approaches that were presented that year. For the Proceedings without an index of this kind, a selection of candidates from among all RRs came after reading titles, abstracts, introductions and references. We thus addressed the issue of *how much*, that is to say the relative weight of the socio-cultural approaches with respect to the total number of papers year by year. Later in this chapter we will provide the more generally obtained quantitative data. It is important to note

that, although many authors refer to socio-cultural theories in some form, we only considered work with explicit statements of justification for the relevance of these theories in the investigation reported. This option reflects our idea of the research that can genuinely be seen as socio-cultural in its orientation because social and cultural principles are declared in substantial ways.

From our reading of research papers, the observation that a number of articles were conceptually close to the socio-political emerged strongly. In a second stage of the review, the nuances in the interpretation of the socio-cultural and political were identified. This allowed us to follow the traces of authors and their work presented in the entire material, together with the connections with other authors. In this strategy of mapping the networks of relationships and authors, it was possible to identify the traces of major directions and lines of concern that delineate the socio-cultural-political axis. Far from a rigid view of a structure, directions and lines of concern were explored as an interconnected system of ways in which socio-cultural-political PME researchers study and make sense of mathematics education and mathematics education research. This *organising stage* served not only to examine related insights in the sample of papers; it also guided the analysis of salient topics and issues for the purposes of delineating new paths of present and future socio-cultural-political research. In this respect, the analysis of new paths and emerging topics and issues in current research reveals our dynamic interpretation of the directions and lines of concern.

A third stage was to see, through the lines of concern, which topics and issues were addressed, and which new insights with respect to former research were provided in the papers. This *analysing stage* was planned to detect some of the newly integrated ideas in the context of PME that could be taken to the next period of follow-up socio-cultural-political research. We privileged the detection of topics or themes instead of or complementary to the detection of methodologies and methods in the narrow sense of techniques. In relation to this issue it can easily be found that a wide range of empirical papers show a qualitative analysis of qualitative data, commonly based on the development of small-scale qualitative studies. From among these, many draw on specific methodological orientations with their own technical language such as grounded theory, discourse analysis, narrative analysis, ethnography, interpretivism or phenomenology. To detect major topics and issues, we looked for relationships in the socio-cultural papers selected from one set of Proceedings as a first step to guide the search in another set of Proceedings. All in all, we encountered a number of emerging topics and issues which indicate theoretical links among several papers and authors concerning the diversity of lines of concern identified. In this stage, therefore, the approach was centred on detecting topics and issues that somehow play a role in unifying and extending the socio-cultural-political lines of concern through the introduction of pioneering conceptualisations in the context of contemporary PME work.

On using this type of logic to review literature we were looking for alignments and recurrences in the theoretical perspectives and findings in the papers, thus

permitting the depth, breadth and progress of socio-cultural-political PME research from 2006–2015 to emerge to the surface. The intention was to map the field and not to give a detailed account of each single paper. Therefore, our review cannot be exhaustive on referring to each paper, nor can it be complete on detecting all the newly integrated topics and issues in and across the socio-cultural-political lines of concern. However, we hope that, on the one hand, authors can see their work represented in our mapping of the field and, on the other, the lines of concern, topics and issues examined provide a sufficiently rich state-of-the-art.

EXPANDING VIEWS: WHAT DOES THE SOCIO-CULTURAL-POLITICAL AXIS MEAN?

Twenty years ago, Lerman (1996) pointed to a central distinction in the study of mathematics education. The conceptualisation of the relationship between the individual and the social is the core difference between Piagetian inspired studies of mathematics education and socio-cultural studies. Lerman defined the latter as research involving "frameworks which build on the notion that the individual's cognition originates in social interactions … and therefore the role of culture, motives, values, and social and discursive practices are central, not secondary" (p. 4). The focus on the classroom *context* and how it influences teaching and learning was an entry point for theories that went beyond the (social) constructivist assumptions on the effects of external factors – including other people – on individual cognition. In the second half of the 1990s, a series of conceptualisations from other fields of study (see Bartolini Bussi, 1998, for elaboration on this) had been incorporated into mathematics education in an attempt to give an account of the "individual in context". Vygotsky's cultural historical psychology was an important ground for further interpretation in the form of Cultural-Historical Activity Theory (CHAT) (Engeström & Middleton, 1996), together with the work of Lave (1988) on situated cognition, and the works of Lave and Wenger (1991) and Wenger (1998) on communities of practice. Less known but still in the same area were alternative discursive psychology approaches, in particular the work of Walkerdine (1988, 1998). Sociological and political theories of education (Bernstein, 1990; Bourdieu & Passeron, 1977) had not been so broadly adopted in PME, even though they had started to provide a frame for dealing with problems emerging from contexts of mathematics education (Appelbaum, 1995; Mellin-Olsen, 1987; Skovsmose, 1994).

In the decade between 1996 and 2005, there is varied research on mathematics thinking, teaching and learning that could be identified as socio-cultural and which was part of PME, as reported in Lerman (2006). In the period 2006–2015, attention to mathematics and school mathematics as social, cultural and political gained recognition as a principle for a large number of PME researchers (i.e. the researchers who participate and present their work at PME conferences). Gates (2006) situates the origins of such recognition in the preparation of PME29 in 2005, when the International Committee decided to broaden the domains of research through the

inclusion of the category 'Equity, diversity and inclusion' for participants to submit their work. Moreover, during the General Assembly of that conference, a proposal was approved to remove from the PME constitution the preference to consult psychology as the fundamental field of scholarship for the PME community. Our analysis indicates that this opening up has enlarged PME research by adopting various integrated interpretations of the social, cultural and political in mathematics education. This trend has persisted over the last decade, articulating what we call the socio-cultural-political axis.

For further characterisation of the socio-cultural-political axis in this section, we first discuss the identifiable, constitutive directions and lines of concern within PME in relation to the expansion movement. The description of lines of concern serves as an initial survey of some of the work and authors that have contributed to socio-cultural research in forms that did not exist or at least were quite rare ten years ago. We then elaborate on the newer cultural-historical and socio-political trends in PME, addressing particularly some of the authors whose works have been crucial in grounding the socio-cultural-political axis. We finish this section by relating the newest orientations in PME to research in the broader international field of mathematics education (i.e. the research that has not been presented at PME conferences and reported in the PME proceedings, although some PME researchers may strongly draw their PME research from it).

The Micro-Macro Constitution of the Socio-Cultural-Political in PME

In the early nineties, the interest in understanding individual mathematical thinking in context was the beginning of how some of the approaches in PME research, which originate in the work of Vygotsky, would later result in the demise of the dichotomy between the individual and the social. More than 20 years later, a refined language to engage in such an endeavour has been achieved. A basic theoretical distinction has been constructed between saying that individual mathematical thinking is influenced by interaction with others, and saying that there is no thinking – mathematical or of any other kind – outside the relationship between the self and the other. The "social" is not simply a matter of the "influence" of "the other" on the "person" – as if these were entities with a recognizable separate existence. The inseparability of the individual – the I or the self – from the other – one and many, now and in a past that is constantly present – in the production of the material and symbolic world through practice is a grounding premise to think about humans, their life and activities.

The issue of the inseparability of the individual from the other is a basic assumption of Vygotsky's cultural historical psychology rooted in Marx's historical materialism. Opposing Western European rationalism, which places the defining element of humanity in thinking understood as inner, mental activity, historical materialism breaks with the idea of the individual as a monad and proposes a configuration of three elements – *people in activity*, *artefacts and products of*

activity, and *systems of meaning* – as the inseparable unit to think about the social world. This view has a number of implications for the notion of "individual in context" as a cultural-historical phenomenon rather than socio-cognitive or social psychological. Radford (2008a, 2008b) presents a delicate elaboration of the difference between the paradigms involved – socio-cognitive/social psychological and cultural-historical, – together with a discussion of how the long-established, two-way relationship between the individual and the social has raised the issue of the inseparability of the two.

Ideas about people in activity, artefacts and products of activity, and systems of meaning are tantamount to the newer cultural-historical and socio-political trends in the field, and are displayed in the papers that, in PME, can be mapped in relation to the socio-cultural-political axis. These are ideas that, expressed in diverse forms and with different emphases, have been present in many of the theories that mathematics education researchers have drawn upon in the study of mathematics thinking, learning and teaching in context. For example, Lave (1988, pp. 178–179) refers to three levels in the analysis of human cognition in social practice: the level of the *lived experience*, where people in activity, and activity and settings, are the constitutive elements of thinking-in-doing in everyday life; the level of the *semiotic systems*, with the structures they entail in a *constitutive order* of meaning; and the level of the *dialectic relationship between the lived experience and the constitutive order* in the generation of sense, meaning and thinking.

Our main point here is that, in 10 years of research outside and inside mathematics education, and inside and outside PME, more nuanced and rich languages to study thinking and education in mathematics outside of rationalist and socio-cognitive paradigms are now available. The original issue of understanding the "activity of the individual mind in context" from a social standpoint has been progressively unpacked and given precision by means of two major directions and several interrelated lines of concern. The two directions concerning the socio-cultural-political, which will be called "micro" and "macro" throughout the chapter, are complementary in that they dialectically connect local and systemic forces in contexts of mathematics education and mathematics education research.

In the direction of the micro-details of knowledge and meaning-making in cultural configurations, two lines have become evident in PME:

- The line of the micro-genetic analysis of semiotics, to which groups of scholars contribute, together and overlapping but also with some nuanced distinctions in their approaches to classroom activity in mathematics teaching and learning. Here we can mention the work of groups such as Radford and collaborators (e.g. Radford, Miranda, & Guzmán, 2008); and Arzarello and collaborators (e.g. Arzarello & Paola, 2007).
- The line of the micro-analysis of classroom discourse, in which a variety of research methods and central concepts coexist. Here we find the later work of

Sfard and her progressive turns toward participation and commognition, which have been researched by various scholars (e.g. Heyd-Metzuyanim, 2013); the work of Wagner, Herbel-Eisenmann and collaborators (e.g. Herbel-Eisenmann, Wagner, & Cortes, 2008); and that of Morgan and collaborators (e.g. Morgan & Tang, 2012).

As developed in the context of PME since 2006, the lines concerning semiotics and classroom discourse share important similarities in the light of attention paid to some primary notions. In particular, the more traditional assumption that people communicate as individuals has been replaced in both cases with frameworks around the idea of people communicating in activity across contexts of various kinds and through a variety of artefacts that are historically and socially realised.

A second direction studies the macro-details of the connections between the different participants in mathematics education and how they relate to each other in institutional arrangements in classrooms, schools, and outside schools. In this direction, work that adopts theoretical tools to study power becomes more evident. The study by Wagner, Herbel-Eisenmann and Cortes (2008) that we referred to above, for instance, is also about power. In the "macro" direction, however, power is taken to mean a decisive feature of broader social and political structures, while in Wagner et al. (2008) the decisive feature to be researched is the classroom discourse from the perspective of micro-level actions.

Again, this second direction has been expressed through different not mutually exclusive lines of study and intellectual traditions in recent PME research:

- The line of identities and identity-construction along different combined dimensions such as language, age, socio-economic status, immigrant background, race, ethnicity, gender, etc. Here we find the work of Barwell (2013) on language and language users, and the work of Lerman (2012) on socio-economic status and working-class students.
- The line of communities of practice in contexts of research such as teacher education, professional development or out-of-school mathematics. Here we find the work of Jaworski and Goodchild (2006) on mathematics teachers' professional learning communities, and the work of Bose and Subramaniam (2011) on children knowledge-building communities.
- The line of ex/inclusion of particular groups of students from access to and full participation in school mathematics. Here we find the work on the creation of teaching and learning opportunities of Planas and Civil (2015) with bilingual immigrant children in urban contexts, and the work of Hunter and Anthony (2014) with Māori and Pāsifika students.
- The line of society and the politics of mathematics education and mathematics education research. Here we find the work of Walshaw and Anthony (2006) on the power of discourse and hegemonic discourses of power, and the work of Setati (2006) on the critical role of language ideologies in institutions of mathematics teaching and learning.

The identification of these two distinct – but related – directions in PME work allows us to organise the diversity of theoretical and empirical frameworks in use for the understanding of mathematics education as social, cultural and political. The logic of this organisation is represented in Figure 1. As researchers choose to focus on the micro-dimensions of the constitution of mathematical thinking and learning, cultural-historical approaches with an emphasis on semiotics or on classroom discourse are productive ways of researching. Complementarily, as researchers choose to direct their gaze towards the constitution of thinking, learning and education in relation to the broader systems of signification that articulate the practices of mathematics education in society, a socio-political trend would offer ways of linking mathematics education practices to broader macro-issues and dimensions. In that case the study of identities, communities of practice, processes of ex/inclusion, and the linkage between society, politics and mathematics education become productive. This logic of moves towards the micro-details, and the macro-tendencies provides a different way of thinking about the field of the socio-cultural-political as complementary analytical moves, rather than boundary crossing between discrete, not connected categories and problems.

We will now further elaborate on the micro-macro constitution of the socio-cultural-political axis by referring to some of the PME works that have made important contributions to the grounding of theoretical tools and analyses with emphasis on one of the levels – micro or macro, – but with explicit mention of the two of them. We will map the newly arrived PME cultural-historical and socio-political research. This body of research is critically confronting some of the taken-for-granted relationships between interaction and knowledge in ways that may be seen as one of the features of contemporary PME work.

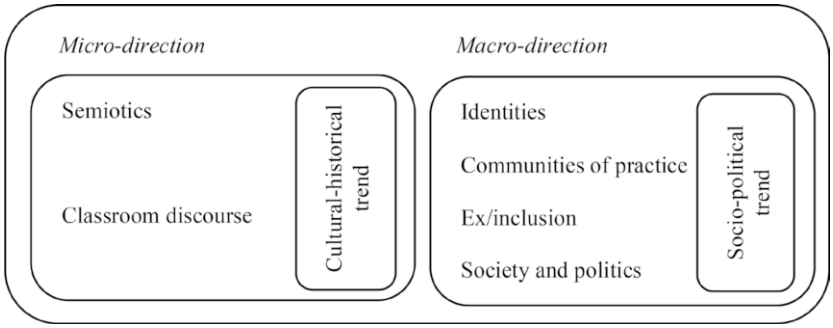

Figure 1. Representation of elements of the socio-cultural-political axis

Newer Cultural-Historical and Socio-Political Trends in PME

The micro-macro constitution of the socio-cultural-political axis has been possible through the number of works that, for the last decade of PME research, have included

neighbouring disciplines such as sociology, history, philosophy, anthropology, political science and linguistics. The study and inclusion of related disciplines have allowed an expansion of what is meant by "the individual's cognition being originated in social interactions" (Lerman, 1996, p. 4) in terms of the inseparability of the individual from the other. In line with the issue of inseparability, what is new in contemporary socio-cultural PME research is the dual emphasis on *how* mathematics learning occurs (i.e. the nature of interaction) and *what* mathematics learning is (i.e. the nature of knowledge and learning). Such an emphasis has led to changes in earlier socio-cultural interpretations of cognition as socially *occurring* and individually *being*. In the last decade, the interpretations of cognition in which the social is only considered in part of the "story" – learning is socially shaped but the products of learning shape the individual rather than the social – have been largely problematised. A move has taken place from viewing the social as only part of the "story" to conceptualising cognition as unified processes of cultural, social, historical and political induction into communities of thinking and practice. This newer conception of cognition as socially *occurring* and socially *being* appears in works of the micro and macro directions.

In order to explain more carefully the expansion of PME socio-cultural research and the micro-macro constitution of the socio-cultural-political axis, for this part of the chapter we have chosen influential work within two newly-established PME trends: the cultural-historical and the socio-political. Our analysis of the PME literature over the period 2006–2015 mostly relates the emergence of the cultural-historical trend to works on the micro level of semiotics and classroom discourse, and that of the socio-political trend to works on the macro level of identities, communities of practice, ex/inclusion processes, society and politics (see Figure 1). We indicate some of the authors who have been contributors to the advancement of either cultural-historical or socio-political ideas in recent PME research.

The rise and development of cultural-historical orientations in the context of PME can be illustrated through the research presented by Luis Radford and collaborators. Earlier in this chapter, we related Radford to the body of socio-cultural-political research in the direction of the study of knowledge and meaning-making at the micro level of specific cultural configurations, and particularly to the line of semiotic analyses of practices in the mathematical culture of the mathematics classroom. This is certainly an important part of the work by Radford inside and outside of PME. Nevertheless, his theoretical work on possibilities and boundaries of cultural-historical orientations in mathematics education research has also served to reinforce the complementary character of the two directions, micro and macro, in the constitution of the socio-cultural-political. The problem is not only to examine the configuration of local practices but also to consider broader cultural and historical forces – including the intellectual traditions in the field – intervening in the configuration, development and study of such practices.

In his plenary at PME38, Radford (2014) observed a cultural and historical concept of the individual, and reflected on the production of mathematical knowledge as a

historically-based social process. In that plenary, Radford highlighted the decisive role given to culture, history and society in the understanding and formation of knowledge and thinking: knowledge is always articulated and enunciated in social, cultural, historical and political conditions, and these are the conditions that structure how it is materially accessible in culture. Some years before, in his commentary for a PME Research Forum, Radford (2010) reflected on the cultural and historical nature of theory and knowledge in mathematics education research:

> Theories in mathematics education reflect and refract implicit specific national-cultural "world views". They are unavoidably immersed in those symbolic systems of cultural significations... of the symbolic structures of society structures from where (implicitly or explicitly) our theories draw their views of what constitutes a good student, a good teacher, and, of course, a good researcher. (pp. 169–170)

When reviewing PME research with a focus on *what* is mathematical knowledge and learning from the perspective of its historical construction, we encountered the term *objectification* very frequently over the last decade in socio-cultural research on meaning-making in the mathematics classroom. This is how Radford calls the social process involved in becoming progressively aware of the cultural logic of mathematical entities and knowledge. Radford and collaborators have taken many initiatives to illustrate how the cultural theory of knowledge objectification, as a strong socio-cultural and historical framework to think about the ontology and epistemology of mathematics teaching and learning (Radford, 2008b), works and what it adds to the understanding of mathematics teaching, learning and thinking processes. For example, Radford, Bardini and Sabena (2006) show how the objectification of generalisation in mathematical tasks requires the coordination of eye, word and gesture in a rhythm. The latter provides ways for students to become gradually aware of the moves involved in generalising. Radford, Miranda and Guzmán (2008) examine how students recreate cultural and historical mathematical meanings around the objectification of Cartesian graphs in school mathematics. Radford (2011) reports another investigation in which young children interact with different manifestations of school knowledge constructed around early algebraic thinking by means of material artefacts and gestures.

A related but not identical research agenda is the socio-cultural path taken by some Italian groups interested in a semiotic analysis of mathematics teaching and learning embedded in the idea of classroom activity. With a variety of topics of concern, the semiotic analysis undertaken in mathematics classrooms shows how learning unfolds in the interaction of individuals and groups in a space of action, production and communication – the APC space (e.g. Arzarello, Bazzini, Ferrara, Robutti, Sabena, & Villa, 2006). The APC space is built up in the mathematics classroom as a dynamic system, where the different components – body, physical world and cultural environment – are integrated into a unity that reports on the embodied and cultural nature of the mathematical concepts. Further elaborating on

Vygotsky's activity theory, the issue of mediation in mathematics learning through specific social classroom practices and cultural frameworks of rationality is taken up in Arzarello and Paola (2007).

The study of the expansion movement and of the micro-macro constitution of the socio-cultural-political axis also points to the rise of socio-political orientations in PME. Here, the dynamics of power in mathematics education and mathematics education research is examined in relation to a variety of notions such as identity, positioning, disposition and agency, often borrowed from neighbouring disciplines and placed in relation to issues of social class, gender, age, race, ethnicity or language. In her introduction to the PME38 Panel (Phakeng, Halai, Valero, Wagner, & Walshaw, 2014), Phakeng (formerly Setati) traced the rise of the socio-political trend back to the intellectual tradition of critical mathematics education:

> In his paper entitled, 'Critical mathematics education for the future', Skovsmose (2004) argues that while mathematics education can empower, it can also suppress, and while it can mean inclusion, it can also mean exclusion and discrimination. Mathematics education, Skovsmose explains, does not contain any strong 'spine', because it can collapse into forms of dictatorship and support the most problematic features of any social development, or it can contribute to the creation of a critical citizenship and support democratic ideals. (p. 56)

The rise and development of socio-political orientations in the context of PME can be illustrated through the research presented, often in collaboration, by Lerman and Jorgensen (formerly Zevenbergen). Lerman and Jorgensen have contributed to the body of socio-cultural-political research in the direction of the study of connections between participants in mathematics education at the macro level of institutional arrangements in classrooms, school and out-of-school, and in respect of the lines of identities and ex/inclusion of groups of students. However, their investigations are especially valuable because they have contributed to widening, reinforcing and connecting the various lines of concern in the configuration of the macro direction of socio-cultural-political research.

In the introduction to the Research Forum entitled 'Sociological frameworks in mathematics education research' in PME33, Lerman (2009) indicated the importance of drawing on the work of authors such as Bernstein, Bourdieu or Foucault in order to "discuss how such [sociological] frameworks can shape our research questions and methodologies and form a basis for change in mathematics education" (p. 217). These frameworks are expected to help to examine *what* is mathematical knowledge and learning from the perspective of its political construction in ways that can complement the role and use of the philosophical frameworks previously used and revisited by Skovsmose (1994). More generally, in a number of PME collaborations, Jorgensen and Lerman (e.g. 2006, 2007) drew on sociology to understand the (educational) role of mathematics and knowledge in society, and the problematic nature of (school) mathematical knowledge. In these works, the investigation of a

pure historical subject, whose ideal cannot be empirically found, is replaced with the investigation of concrete dynamic subjects engaged in attempts to reconstruct meanings and relations of power in culture, in the spaces of mathematics classrooms.

Even if classrooms have remained the privileged site for PME socio-cultural-political research over the last decade – although with efforts to include non-prototypical classrooms, – some of the socio-political research traces the political in relationships among people and institutions beyond the classroom. Walls (2006), for instance, addresses the impact of standardised tests on students' learning and their identities as mathematics learners. In a primary school community, she investigated the accounts of children, parents, teachers and managers on their perception of Year 5 Aspects of Numeracy Test in Queensland, Australia. Using a Foucauldian framework, Walls traced how testing was not just a way of detecting what children know mathematically, but rather a way of comparing and differentiating them, creating hierarchies and, thus, in/excluding some. The test was seen "as a major event in school and home life in its perceived authority to tell the truth, that is, to objectively measure and rank each child. In this perception, school management and teacher behaviour were modified, pupil identity reworked, and relationships within families adjusted" (Walls, 2006, p. 359). This type of research reveals that the different technologies of mathematics education – including testing – affect children and their identities, but also many of the participants in the network of practices of mathematics education.

Another feature of the newer PME socio-political trend is the view of systems of meanings and activity as changeable by the people involved in them. When focusing on the social and political conditions that constitute school mathematics, and on how such conditions are partially made accessible to marginalised groups in the dominant culture, a number of studies have emphasized the transformative dimension of power by conceptualising ideas of change, resistance and agency. In Jorgensen (2015), mathematics teaching and learning are seen in terms of progressive access to dominant worldviews and induction into mainstream cultural systems in ways that leave room for alternative realities. Jorgensen explores two rural schools and shows teachers who move to the very limits of the mainstream culture to modify interpretations of mathematics and professional learning. While the stories of successes and failures provided by the teachers' activity in these schools remind us of the lights and shadows in Breen (2007), the appreciation of power as "positive" refreshes the principles of the critical theory pioneer project by Skovsmose (1994) and outlines the consideration of Foucault (1980).

So far we have characterised the socio-cultural-political axis in the context of PME by (1) pointing to two main identifiable directions – with respect to the emphasis on either micro or macro levels of analysis; (2) describing a number of researched lines of concern in relation to these directions; and (3) illustrating two influential theoretical orientations which cross over lines of concern and, by doing so, contribute to the conceptualisation of the socio-cultural in newer forms, namely the cultural-historical and the socio-political (see Figure 1). The ways in which we

have characterised the socio-cultural-political axis offer an idea of its richness and complexity. This is a complexity that goes beyond describing different dimensions, combinations between them and zones of intersection, as the geometrical image of an axis might suggest. We are documenting constructions in development, which depend on the dynamics of theories and groups of researchers continually contributing new analyses and interpretations to the field.

Some Evidence of the Socio-Cultural-Political Axis Outside PME

We have discussed the emergence and vitality of the newer cultural-historical and socio-political trends in PME research, as well as their transversal role in the configuration of the directions and lines of concern of the socio-cultural-political axis. However, it is fair to say that socio-cultural-political research is not a PME creation, at least not solely. The traditional perspectives for which mathematics education research is primarily linked to the search for didactical responses to technical problems, faced by students in their learning and by teachers in their teaching, currently coexist close to the newest perspectives that address problems in the field which are of a political or ideological nature.

When looking at the whole field of mathematics education research, cultural-historical and socio-political trends are identifiable along with renewed interpretations of the relationship between the individual and the social in ways that demand a variety of theories which originated in other fields. Several of the international initiatives on how to move the field forward through the adoption and recontextualisation of theories from other fields have been driven by researchers who are active in the PME community. In their report of the ICME Survey Team on 'The notions and roles of theory in mathematics education research', Assude, Boero, Herbst, Lerman and Radford (2008) indicate the potential and some of the benefits of using theories of an "external type" rooted in sociology or anthropology. Another initiative in pushing the integration of disciplines for the development of theories and perspectives in mathematics education research can be found in the section 'Social, political and cultural dimensions in mathematics education' of the *Third International Handbook of Mathematics Education*, in the chapter by Jablonka, Walshaw and Wagner (2013). These authors provide a critical overview of how diverse social, political and cultural theories are allowing us to widen our contemporary perspectives of mathematics education and mathematics education research. This chapter actually addresses the extent to which notions from literary theory, discourse analysis, social linguistics, sociology, positioning theory and postmodern approaches were also present in the PME proceedings from 2007 to 2010.

Additional evidence of the constitution of the socio-cultural-political axis outside of PME comes from initiatives on how to move the field forward through theorizing work. In this respect, the theorizing work in a collaboration between Roth and Radford (e.g. 2011) has had an enormous impact in that it has contributed to frame CHAT perspectives (Engeström & Middleton, 1996) in mathematics education research.

Through the conceptualisation of mathematics thinking and learning as dynamic systems of meanings and activity, these authors have challenged the psychologically-based distinctions between the individual and the social. Their work is simultaneous with the theoretical revision of the affordances and hindrances of social and cultural psychology in mathematics teacher education research. In this respect, the third and the fourth volumes of *The International Handbook of Mathematics Teacher Education*, edited respectively by Krainer and Wood (2008) and Jaworski and Wood (2008), illustrate the international opening up of a significant part of mathematics teacher education researchers to cultural-historical and socio-political perspectives, and more generally the theoretical interrogation of dominant ideologies at work in teacher education research designs. In her introductory chapter of the fourth volume, Jaworski (2008) refers to this opening up to theorizing within newer terrains from outside psychology:

> [T]here has been a shift. One obvious difference is that constructivism has moved from a largely cognitive, psychological focus to take into account social contextual and institutional factors... In parallel, socio-cultural theories, rooted in the work of Vygotsky and followers have become better known and understood in mathematics education, with a challenge, implicit or explicit, to constructivism… and social, cultural, political and policy issues have become more evident in the mathematics education literature... [P]erspectives of teacher educators have moved into more social frames... with recognition also of the wider influences of system and society. (p. 4)

Particularly in relation to socio-political perspectives, Valero and Zevenbergen (2004a) challenged the socio-cultural-psychological approaches by offering sociologically-oriented alternative ways to theorize mathematics education and mathematics education research. At the time of writing, the collaborative project of that book – which was linked to their participation in PME activities – remained unique in many senses. It meant bringing together authors from different parts of the world, socio-cultural traditions and emerging critical political perspectives, sharing the challenge of rethinking mathematics education research in relation to the political, economic and social conditions of schooling and other institutions with a role in the regulation of practices on the micro level. The book chapters provided a number of early responses to why mathematics educators and mathematics education researchers should care about power, equity, social justice and critical pedagogy. In their introduction, Valero and Zevenbergen (2004b) anticipated the need for the revision of the socio-cultural-psychological dominant orientations in socio-cultural research toward the inclusion of the political:

> [I]t is possible to identify another trend strongly rooted in sociology, critical theory and the politics of education. This trend stands on the assumption that mathematics education is, in essence, a *social* and *political practice*. This practice is social because it is historically constituted in complex systems of

> action and meaning... This practice is political because the exercise of power, both in it and through it, is one of its paramount features. (p. 2)

Gutiérrez (2013) outlined the enormous potential for the field, the socio-political turn having been initiated in the ways of thinking about mathematics teaching, learning and education, but also in the critical ways of thinking about research and research methodologies. As in Valero and Zevenbergen (2004b), Gutiérrez distinguishes between socio-cultural approaches to mathematics education research, with an emphasis on understanding psychological processes from a social base (in which the social is only a part in the "story"), and socio-political approaches, with an emphasis on understanding social and political processes on their own. She argues that, although socio-cultural-political issues still remain under-researched in the field, the dominance of socio-cultural-psychological orientations in socio-cultural research is being reduced. There are now more authors who are bringing power and mathematics (education) together, and placing issues of equity to the fore in their investigations. These investigations pose crucial challenges to the whole field, which is however still operating under poorly articulated approaches and agendas surrounding issues of equity, and threatened by tacit deficit-based beliefs that not *all* students and groups can learn.

In a similar way to the refinement of theoretical language to study the cultural constitution of mathematics education practices, the theoretical language to grasp the political in mathematics education has also become more nuanced. Nowadays, the adoption and recontextualisation of a variety of tools to study power have been brought to the field of mathematics education. Besides the interest in thinking about pedagogies that help improve the achievement of students from socially minoritised groups who have not succeeded in mathematics, the field is producing more solid analyses of why and how mathematics education practices, in a broad network inside and outside the school, operate inclusion/exclusion and differentiation of groups and learners. Examples of this type of work can be found in recent special issues of international journals, such as *Educational Studies in Mathematics*, on social theory in mathematics education (Morgan, 2014a), or *ZDM* on socio-economic influences on mathematical achievement: what is visible and what is neglected (Valero & Meaney, 2014).

MAPPING THE SOCIO-CULTURAL-POLITICAL AXIS: WHAT IS IT LIKE?

Overall, the absolute and relative frequencies of the socio-cultural RRs indicate a stable high representation of this domain for the last decade of PME. While in Lerman (2006), the result was that "the number of Research Reports classified as socio-cultural has grown substantially from 1990 onwards" (p. 353), in 2016 it can be said that the domain has become firm and consolidated, and it represents between a quarter and a third of the total RRs, by year and across years. To gain a more complete picture, it can be added that we also identified about a total of thirty

Plenaries, Panels and Research Fora as contributing to the socio-cultural-political axis in any of the identified directions, lines of concern and/or newer trends (see Figure 1) for the period 2006–2015.

The reality created by the numbers above points to the *extent*, in intensity and volume, of socio-cultural-political research. But these numbers are poor data in terms of understanding *how* the socio-cultural-political axis has been framed in PME over 10 years. In the previous section, we presented the existence of two major related directions, a diversity of lines of concern and two newer trends with a role in the rise and development of the socio-cultural-political axis inside and outside PME. We argued that these directions, lines of concern and newer trends cross over different topics of study in the field, groups of researchers and intellectual traditions around the world. Below, we map some of the topics newly addressed within the socio-cultural-political axis. These topics cross over more than one line of concern (i.e. semiotics, classroom discourse, identities, communities of practice, ex/inclusion and society and politics), and provide a window to PME contemporary socio-cultural-political research.

Knowledge Creation and Knowledge Use

The question of what mathematics is in/for teaching/learning is addressed through a series of topics of knowledge creation, knowledge use, frameworks and field development, which are present in practically all lines of concern through a number of papers. Although all papers are theory-building in some form, we refer here to those that search for evidence and arguments mostly in theory.

An example of this type, in the micro-direction of meaning mediation in classroom settings, is Hershkowitz, Tabach, Rasmussen and Dreyfus (2014). These authors expand the idea of knowledge *agent* to knowledge *agency* by considering an empirical bottom-up approach. The study combines two approaches – Abstraction in Content and the Documenting Collective Activity – to place knowledge and its mechanisms at the core of research on classroom discourse and the teaching and learning processes involved in it. Some more papers of this type can be found in the Research Forum in PME37 by Tabach, Nachlieli, Heyd-Metzuyanim, Morgan, Tang and Sfard (2013) on the development of "strong" discursive research. Different issues around the theory of commognition (Sfard, 2008) are addressed to make the argument that mathematics is a discursive activity and that mathematical objects result from the ways of communicating about them. In this framework, mathematics knowledge is conceptualised as the development of mathematical ways of using discourse, and mathematics learning as participation in a certain discourse. In particular, this means that mathematics knowledge is a kind of discourse, and consequently discourse is a topic of research in its own right, not a window to something else. This standpoint raises questions concerning how participation in one discourse is subject to participation in other discourses. Drawing on a socio-cultural-political agenda, we see the relative status of different discourses and the

access to them by different participants situated as strong lines of reasoning for commognitivist research at present and in the years to come.

Another example of work about knowledge creation and knowledge use, with emphasis on the movement toward the socio-political in classroom discourse research, is the Plenary by Morgan (2009) in PME33. The question of *what* is mathematical knowledge and learning is here linked to the questions of *what* is language and *what* is discourse. Morgan addresses the need to connect the various perspectives in mathematics education research for the construction of powerful theoretical tools that can help gain insight into ways of integrating social structures and individual processes. She refers to the analytical challenges posed by the integrated study of the social and the individual as follows:

> My concern with social inequalities precludes adoption of a perspective that denies or ignores the influences of social relationships and structures on individual experience and achievement. My personal search for theory has thus been shaped by a need to understand how individual and social may be connected... Theories of learning and activity based in the Vygotskian tradition offer powerful ways of understanding such connections... However, because many of my questions seek to address the uneven distribution of knowledge and educational success, I intend to focus here on the contributions of sociolinguistic, discursive and sociological theory to my way of understanding. (p. 51)

In her research, Morgan draws on a variety of theories such as linguistics and social semiotics, critical discourse analysis and social theory. The articulation of these three "toolboxes" allows her to unfold an analytical strategy in which any classroom data are understood through knowledge of the immediate context of the practice and knowledge of the broader socio-cultural context shared by participants in this practice. Within this framework, classroom interaction is investigated along with more general social practices and larger structural arrangements of education that frame the particular contexts of mathematics education. Morgan's work constitutes a perspective that binds the micro direction of classroom discourse and semiotics sensitive analysis to the macro political structuring, that is, "how mathematics education functions in society for individuals and for various social groups" (Morgan, 2014b, p. 130). As in the case of commognition, where the combination of communication and cognition in the very same name of the theory grasps the unity of the individual and the social, Morgan's social semiotics assumes the socio-cultural-political principle of inseparability.

Some more papers centred on theoretical issues of knowledge creation and knowledge use, now located in the direction of the politics of mathematics education and mathematics education research, can be found in the Panel by Phakeng et al. (2014) in PME38. The hypothetical case of building a new school project in South Africa intending to provide mathematics education to improve the living

conditions of children in a poor marginal area is used as a basis for a discussion of how mathematics education research, with its many discourses, relates to concrete social contexts and makes theory work for action. One of the foci of the papers in that panel is the ways in which discourses about what counts as an adequate organisation of mathematics education in schools are constituted by the social, political, material and cultural conditions of the schools, communities and countries. A second focus is the approach to theory and theorizing as the preceding necessary stage for performance and political action toward more democratic dynamics in the case of the imagined South African school. Similarly to Jorgensen (2015), theory and theorizing are viewed as transformative tools for the development of alternative more equitable worlds.

Community Work and Participant Development

The conceptualisation of theory as action, present in many papers regarding knowledge creation and knowledge use, is linked to another prominent topic in socio-cultural-political PME research: community work and participant development. As further interpretations of Lave and Wenger (1991) and Wenger (1998), there are a number of papers that move their focus towards communities of practice, identity, ex/inclusion and the politics of mathematics education. Some of these papers engage with community work, community development and action for social change, both in school and out-of-school contexts.

Mathematics education research has expanded to include a variety of cultural, historical and political considerations that have led some researchers to become engaged in intense community work with groups of students, teachers and families. This topic of research has been addressed by Civil and Planas with data from very different political contexts but similar methods and findings. These authors have presented a number of RRs of their work together and with their own teams in Arizona and Catalonia about the mathematics education of minority groups of students in the relationship between schools, families and other groups of students (e.g. Planas & Civil, 2008). In the RR by Phakeng, Bose and Planas (2015), a case is made concerning research on the relationship between educational policies and ideologies of mathematical achievement of language minority groups in contexts of poverty, with direct participation and political action by the team of researchers. An example of this type is the work with families and after-school programmes conducted by Civil (e.g. 2008, 2012). With a focus on the role that language plays in the mathematics classroom placement of some of the children, in her Plenary in PME36 Civil (2012) reports issues of parental engagement in mathematics education for Mexican-American working-class communities of the US. She reflects on how to establish bridges between home and school for mathematics learning and teaching with attention to the diversity of social practices, institutional discourses and out-of-school identities that students meet and struggle with:

> Largely due to the restrictive language policy affecting the schools where my work was located, I became interested in the interplay between language and mathematics, particularly for students whose home language is different from the language of schooling... The tension between in-school and out-of-school mathematics often goes hand in hand with what forms of mathematics are more valued... I see three elements at play as I reflect on opportunity to learn in the context of non-dominant communities: the nature of the mathematics problem; the language(s) involved; and the valorization of knowledge. (p. 45)

An increasing number of PME papers over the last decade have addressed issues of community work and participant development, while bringing out the tensions and possibilities of "bridging" communities and discourses during the research process. In Cooper, Baturo, Duus and Moore (2008), a researcher-teacher collaboration for the teaching of mathematics in vocational education for indigenous blocklayers in Australia is analysed. Thinking about the relationship between three main actors – researchers, an experienced blocklaying teacher, and indigenous, blocklaying students – in the context of vocational education, the concept of communities of practice (Wenger, 1998) is fruitful. In this research, the notion of community does not only include the series of practices of learning in school, but also extends beyond the school as practices of blocklaying happen outside vocational education and are important for the Torres Strait Communities to which students belong. The analysis of mathematics learning in such a context demonstrates that learning extends to community service, and in this way in-school and out-school practices are connected in a sense of community. In this broad sense of community, also including the participation of the researchers, new initiatives that would allow students to make sense of mathematics in blocklaying could emerge.

In Planas, Iranzo and Setati (2009), the analysis of classroom events with bilingual students in Catalonia is part of a community project with mathematics teachers in schools with a high percentage of working-class students learning the language of instruction. In the project, classroom events are examined and selected for inclusion in mathematics teacher education programmes to be conducted by the researchers under the principle that the use of the students' languages has positive effects on the increase in mathematical participation and learning. Developmental work with teachers is also present in the shared design, implementation and evaluation of the pedagogic practices and tasks in a sequence of lessons for the exploration of mathematical participation, in line with issues of authority and power, and the ways in which these are encoded in classroom discourse. Planas and Civil (2015) report the learning of some of the teachers who were engaged in that project for several years, and who gained awareness of the extent to which issues of authority and power are pervasive in their lessons. On the basis of these teachers' accounts about the criticality of the language practices in their classrooms, developmental work is conceived as a site where practices and identities can be modified in ways that allow

us to imagine new forms of mathematical participation for all students, regardless of their dominant languages. This is therefore another example of study where theorizing is expected to promote change and identity work by students, teachers, and also researchers.

Teachers, Student Teachers and Teaching

With respect to communities and developmental work, a large number of papers address teachers and student teachers in studies that have been conducted in different parts of the world. Johnsen Høines and Lode (2006) examine the initial education of mathematics teachers in Norway and the use of post-teaching, collaborative, subject-based discussions as part of conversations in teaching practicum as a powerful learning setting. Student teachers and their tutors were invited to collaborate with the researchers. The intention of the collaboration was to explore the qualities of the conversations after a period of students' practicum to learn about the different aspects of the practice of a mathematics teacher. Inspired by Alrø and Skovsmose's (2002) proposal of learning as dialogue, they found that the institutionally dominant *evaluative discourse* in teacher education can be challenged by an ongoing *investigative dialogue*. The former is the type of discourse to which student teachers are often subjected, due to the fact that many conversations on their activity when they meet students during practicum are of an evaluative nature. The collaboration in the setting introduced by the researchers and among the researchers, tutors and student teachers, opens up for conversations that explore possibilities for what the student teachers could have done in the practicum. The emergence of a new possible conversation challenges the power of institutionally framed traditions, and offers a new rich collective learning opportunity not only for student teachers, but for tutors and researchers alike.

Adler and Ronda (2014) also bring to the fore the complexities of stimulating dialogue and participation in either a mathematics lesson or a teacher education context in South Africa. In their report, they explain a framework for describing teachers' mathematics discourse in instruction to be used later in teachers' professional development. This framework provides for a responsive and responsible description of the teacher as a professional who is in the process of challenging pedagogies and increasing her or his professional knowledge by paying attention to the opportunities for dialogue and interaction with students in the classroom, and to how these opportunities can be provoked and supported by the use of particular types of mathematics discourse in instruction. Under the influence of the literature on teaching dilemmas (Adler, 2002), Adler and Ronda elaborate on teacher development, classroom discourse and learner participation as the core elements of their teaching and teacher education framework.

Another example of a paper, this time on the knowledge and preparation of the mathematics teacher educators, is Jaworski and Goodchild (2006). These authors

present findings from a developmental research project, again in Norway, that seeks to create knowledge and improve practice in mathematics learning and teaching by developing inquiry communities between teachers in schools and mathematics teacher educators in a university setting. The powerful idea of inquiry communities in the collaborative work between teachers and teacher educators has the broader potential to inform about how learning opportunities can be created in the mathematics classroom. In her PME36 Plenary, Goos (2012) similarly reflects on the idea of community of inquiry as a resourceful way of understanding the creation of learning opportunities in a variety of settings in mathematics education. In particular, Goos traces out certain questions for possible future research trajectories in mathematics teacher education that consider connections between different communities of inquiry and their cultures:

> Calls for improvements to mathematics education are implicitly based on the assumption that well prepared mathematics teacher educators are available who can foster change in teachers' practices... The ethical, social, political and intellectual challenges inherent in bringing about this type of change are well known. However, much less is known about the professional preparation of the mathematics educators who undertake these tasks, or about how they continue to learn throughout their careers... Creating opportunities to learn across interdisciplinary boundaries may lead to new understanding of how to integrate the mathematical and pedagogical expertise of community members to enrich mathematics education. (p. 80)

Digital Technologies and Pedagogies

The ways in which teachers and students use technologies (commonly called ICT) as tools for mathematics teaching and learning have been examined from a variety of perspectives and theories in the last two decades of PME research. There is, however, an emerging topic in the examined PME papers over the last decade regarding the beginning of socio-cultural-political research concerning the study of the role, use and effect of digital technologies in mathematics teaching and learning within schools across a range of sites and socio-cultural backgrounds. Some of these papers provide micro-level findings of the complex communication and meaning-mediation processes involved in the production and interpretation of signs (e.g. gestures, drawings, natural talk, mathematical register) when working with technology. While ICT tools are interpreted as crystallizers of historical forms of thinking available in contemporary societies, the analyses are focused rather on how these tools mediate students' mathematical thinking and learning, as well as particular pedagogies of mathematics teaching. Dynamic geometry packages are, for instance, analysed as elements of culturally produced forms of thinking, doing and teaching mathematics in educational settings.

An example of this type of work on the micro level is the investigation by Geiger, Dole and Goos (2011) in Australia on the integration of digital technologies, such as electronic calculators, into classroom practice for numeracy teaching. This research adopts a critical framework for the understanding of numeracy in the connection between mathematical knowledge, dispositions and cultural tools, for the purposes of their use and relevance in dimensions of life such as the personal/social, work and citizenship. ICT as part of numeracy allows the well-documented gap to be bridged between, on the one hand, school knowledge and learning and, on the other, out-of-school knowledge and action. In this respect, ICTs are studied and used as mediation tools not only for learning but also for social action in relation to differential access and outcomes. The influence of a Bourdieuian approach is clear in how Geiger and colleagues relate differential access and outcomes to the various structuring practices that serve to recognise and validate particular dispositions and skills within schools and classrooms.

Also under a socio-cultural approach but now completed with a Bernsteinian analysis for an investigation on a macro level, Lerman and Zevenbergen (2006, 2007) examine how the digital divide affects students, families, educational institutions and classroom pedagogies. In their 2007 RR, these authors present a study of the ways in which teachers use interactive whiteboards in their classrooms in the curricular context of the Australian New Basics. Despite the potential of using the tool to enhance student learning, the analysis of a number of lessons showed a restricted approach in its use, for quick lesson introduction preceding whole class teaching. Interviews with teachers indicated that the approach observed was based on assumptions about students, mathematics learning and technologies. These teachers failed to recognise serious questions in terms of equity concerning the experiences and access to computers and ICT programs in the home. In the 2006 RR, the Bernsteinian framework for the same three-year research project is introduced to reflect on the potential role of technologies to support numeracy learning for all students with a focus on disadvantaged learners in particular. The notions of visible and invisible pedagogies, together with those of recognition and realisation rules, are considered in relation to "the digital divide" between children from middle-class and working-class homes. It is concluded that newer forms of pedagogy based on ICT innovation and the related approved pedagogic interactions need to be made visible in the schooling contexts of mathematics education:

> Research shows that working within a progressive paradigm, that is, where the pedagogy is invisible, but mitigating the weak framing through strengthening some of the features of the pedagogy can make a substantial difference to the success of disadvantaged students... We conjecture that, without explicit awareness by teachers of the implications of different forms of pedagogy on different social groups the aims of the New Basics in terms of more equitable outcomes are not likely to be met. (p. 55)

Out-of-School and Workplace Mathematics

From our presentation of some clusters of topics in contemporary socio-cultural-political PME research, it would certainly be inaccurate to infer that these clusters were not present to some extent in earlier times. Even though we may deal with areas of study that have persisted since earlier time spans, our purpose in this chapter is to show what is new from the perspective of what is being added to the micro and macro directions identified in PME socio-cultural-political research. In this respect, a further topic in different papers over the last decade is the investigation of mathematics out of school, and particularly of mathematics cultures other than school mathematics in the workplace. When looking at what persists, we find a first type of studies that primarily give continuity to the socio-cultural studies of children's out-of-school mathematics and adults' mathematics in the workplace under situated cognition approaches that were already common in PME in the nineties. When looking at what is new, we find a second type of studies that expand long-established approaches by including cultural, social and political considerations, along with issues about valorisation/status of knowledge in the sense indicated by Civil (2012). We have only identified a few studies of the latter type in our literature review, but we see it as the beginning of an opening of this topic to the socio-cultural-political axis.

In two related papers, Bose and Subramaniam (2011) and Subramaniam (2012) report a study on children's everyday mathematical knowledge associated with participation in work activities. In India, it is common for children in low socio-economic positions to undertake different kinds of work. Conversations with school children age 10–12 living in a slum in Mumbai showed how arithmetic strategies based on the values of the currency that they operate with allow children to complete certain complex calculations. Their knowledge of measurement units used in packages and products that they manage are also present in their calculation strategies. The research confirms what has been found in previous studies of the transition between school and out-of-school mathematical practices: while people show a quite sophisticated contextualised capacity for dealing with qualities and measurements, these do not necessarily transfer into the realm of formalised school performance. The issue remains of how teachers and educators can bridge this gap to open learning opportunities up to these groups of students. Together with the presentation of a variety of arithmetic strategies, an additional point is made: different strategies may be valued differently in mathematics classrooms depending on whose knowledge is being represented by them and whose participation may be favoured. In this way, the topic of out-of-school mathematics versus in-school mathematics in previous decades of PME work moves in a direction that studies power and issues of valorisation of knowledge more explicitly. As posed by Subramaniam (2012), the focus moves toward critical issues surrounding the relationship of school learning to knowledge accessed outside the school:

> Optimism about knowledge acquired by children outside school, especially mathematical knowledge, being a potential springboard for learning school mathematics is evident even in the early writings on 'out-of-school' mathematical knowledge... However, despite many studies exploring the contours of such knowledge and its settings, its integration with the school mathematics curriculum remains limited. (p. 107)

Regarding the study of adult workplace mathematics, in the last decade some papers have addressed mathematical activity in the workplace from a micro perspective where mathematics is embedded in the work context and is mediated through tools. While, in some of the studies examined on children's out-of-school mathematics, the socio-political trend is visible, in the majority of studies on workplace mathematics the cultural-historical trend through a variety of interpretations of CHAT perspectives is common. CHAT perspectives are useful frameworks to analyse mathematics learning to become a professional within specific institutional settings that call for new forms of practice, knowledge and resources in the development of professional agency. Triantafillou and Potari (2006), for instance, report an ethnographic study in Greece with groups of technicians, some of them with vocational qualifications and others with an academic background. The detection of mathematical strategies in the activities of all the groups (e.g. locating a fault in an underground wire-pair; installing and programming a telecommunication network; working in an Earth satellite station) is discussed in terms of what these results tell mathematics education research about differences in the valorisation of knowledge at school and in the workplace.

Social Views, Discourses and Values

Issues concerning valorisation of knowledge and knowledge users are strongly connected with the type of study that addresses mathematics in society and the framing of social views, discourses and values about mathematics and mathematics education. All these issues are highly related to the research domain on affect. Within the socio-cultural-political axis, however, this cluster of topics appears in papers mostly informed by discursive and sociological perspectives. Together with the papers that critically unpack official pedagogic discourses (e.g. Lerman & Zevenbergen, 2006, 2007), we find a few papers that address the role and use of "unofficial" media discourses in the wider cultural field. An example is the paper by Evans, Tsatsaroni and Czarnecka (2009). Here, the increasing use of mathematical images in distinct advertisements of nine English newspapers and the reproduction of certain public images of mathematics in the suggested messages are examined. These authors argue that such images in the media intertwine with pedagogical discourses of mathematics since both, as interconnected cultural productions, regulate people's construction of identity and subjectivity. Their results raise questions as to how

different mathematical images are used in the media addressing audiences with different social classes:

> We note the emergence of a trend – supported by our evidence – whereby mathematics equations or formulae are recruited as global communication technologies of subjectivity, shaping desire especially for those strata of the middle classes that are the most promising clients in the global consumers' market. This emerging strategy might undercut the use of maths as a critical discourse for citizens. (pp. 23–24)

This type of investigations opens up clearly for the discussion of mathematics and broader (un)official political and economic discussion in society. It also raises questions concerning the extent to which mathematics education can and should currently provide a space for critique in society, particularly regarding how certain cultural productions (e.g. advertisements) work to cancel out initiatives designed to improve the level of mathematical knowledge in the general population.

In relation to social views and discourses entering the field of mathematics education, there is another group of papers focused on how social views, power relationships and discourses enter the culture of the mathematics classroom in the form of values and actions of valuing. Seah (2013) refers to the category of mathematical values as those linked to the convictions that have been emphasized in the tradition of Western mathematics. Rationalism, control and progress are some of the values emerging from the development of a large-scale study with teachers from a variety of cultural settings and backgrounds, who were asked to respond to what they find important in mathematics education. This study brought together research teams from eleven regions across the world, such as China, Hong Kong, Japan, Singapore and Sweden. Rather than interpreting the values identified as individual qualities of the teachers, Seah proposes an analysis based on values as the internalisation of dominant cultural ways of viewing the world (of mathematics education), with an effect on the enactment of specific dispositions to teaching and learning (mathematics). Similarly to what is claimed by Evans et al. (2009), Seah alerts us about the risks of accepting without critique the ways in which mathematics and practices and participants in mathematics education are valued in contemporary societies.

MOVING THE FIELD FORWARD: WHAT IS NEXT?

In a review chapter of this kind, we cannot consider that the review has been finished just because we have come to the end. The review of literature could have gone on and on, allowing more evidence of important discussions in contemporary PME socio-cultural-political research to be added. We are aware that important clusters of topics such as multilingual mathematics teaching and learning have not been directly addressed. At this point, however, we have already made our main

arguments: PME socio-cultural-political research over the last decade has matured to distinguish a variety of related approaches to conceive of mathematics education and mathematics education research as social, cultural and political. While a majority of prior PME socio-cultural research supported the idea that cognition is a socially-originated individual process, and therefore focused on socio-cultural-psychological orientations, the domain has taken newer socio-cultural-political directions in support of the inseparability of the individual and the social. The study of works in these newer directions points to the constitution of a socio-cultural-political axis in the field inside and outside PME.

As an area of study develops, examining what kind of research and how it has been recently conducted is fundamental in order to be able to think about future work, and also to point to some of the gaps and priorities to be further researched. Our examination of papers has allowed us to identify some lines of concern, topics and issues in socio-cultural-political PME research for which there is an increasing amount of evidence, either theoretical or empirical. One of the findings from the analysis of contemporary socio-cultural-political PME work is that most of the studies reported are empirically oriented, and among these a majority are classroom-based. These studies privilege analyses of data on students and teachers in their classroom environments in order to explore their processes of interaction and engagement with school mathematics. Here, the meanings of classroom interaction and discourse have gone through multiple rounds of refinement and interpretation in the last decade of PME for the development of knowledge about mathematical identities and many other related topics. What we want to outline is the fact that a majority of the classroom-based studies reported were conducted at the school level up to the students' age of 16 years, and practically none at other levels or sites of mathematics education practice, such as pre-school education, higher education or adult education. It could be argued that research at these other levels and sites is the focus of other regular meetings of the field – such as the study group on Adults Learning Mathematics (ALM), or the Congress of Ethnomathematics. In any case, the lack of studies at these levels and sites constitutes a current gap in PME socio-cultural-political research that needs to be filled. The institutional circumstances intervening on the different levels of mathematics education may lead to differences in the type of processes involved in classroom interaction, discourse, institutional framings, processes of in/exclusion and also the forming of mathematics in these contexts.

Excluding the papers which draw on data from research in classroom contexts, we have also seen that papers which report studies in out-of-school and/or vocational contexts are rare. This was not the case in the nineties, when several PME papers were regularly presented on this topic (see, e.g., the plenary by Schliemann, 1995, at PME19). Together with the exploration of the causes involved in the progressive misrepresentation of this topic, a planning needs to be undertaken to fill this research gap. Actions towards the construction of a more extended scientific community

with its researchers in connection with other scientific communities and researchers may be essential. The development of PME socio-cultural-political work depends greatly on the expansion and enrichment of research conducted in out-of-school contexts, for which strategic collaboration with other mathematics education researchers from communities entirely focused on this type of work would be very beneficial. There is a significant amount of out-of-school mathematics education research outside the context of PME, making important, foundational contributions to the field. In particular, international groups such as Mathematics Education and Society (MES) conduct regular conferences that have become a forum for research on the social, cultural, ethical and political dimensions of mathematics education. Out-of-school and non-classroom-based research on situations of poverty and inequity has an important presence in these conferences.

Yet another finding that deserves consideration concerns issues of geographic representation in relation to the location of the authors and participants in the studies examined. The production of socio-cultural-political PME papers has been concentrated in about a dozen countries over the last decade. Taking into account that what lies at the core of socio-cultural-political mathematics education research is the need to address the uneven distribution of knowledge and success, it is significant to note the uneven distribution of geographical representation and the silence coming from the low "ranked" countries. A majority of the studies reported refer to participants in countries like Australia, Canada, South Africa, United Kingdom and the US, while fewer papers report studies including participants in regions such as East and Middle East Asia, Eastern and Southern Europe, and Central and South America. When paying attention to some of these regions, it is not always the case that socio-cultural-political research is misrepresented there. In Southern Europe, for instance, a number of researchers from Greece, Portugal and Spain are developing influential work in the domain, but they present their studies in conferences other than PME. All in all, the development of the socio-cultural-political axis requires an increased representation of regions for a better understanding of the many social and political challenges faced by participants in mathematics education across different contexts worldwide. The inclusion of more diverse settings would certainly result in stronger conceptualisations of culturally-grounded notions and theories.

Based on our review we find that, although several of the socio-cultural-political PME papers examined address important methodological questions, only some of them primarily consider these questions as a major topic of discussion and overtly elaborate on the need for, development and evaluation of particular analytical approaches. Thus, there remain a larger number of unexplored questions with respect to the possibilities, limitations and suitability of the variety of research methodologies that different authors use in their studies. One aspect of the reflexivity of research is researchers' awareness of their own participation in the reproduction of particular cultural and political relations concerning mathematics education. This topic, raised many years ago in PME (Valero & Vithal, 1998), is

taken up by Baturo et al. (2008) when reflecting on research collaboration and methodologies that strive for de-colonial knowledge relations when studying mathematics education in/with indigenous communities. Alongside the strength of rigorous small-scale qualitative research, another feature of PME socio-cultural-political research is that most qualitative research on mathematics classroom discourse typically focuses on a few episodes, and rarely provides quantitatively larger evidence (e.g. Herbel-Eisenmann, Wagner, & Cortes, 2008). Moreover, the adoption of long-term methodologies and comparative cross-cultural studies with a socio-cultural-political orientation has remained minimal. All these approaches and kinds of evidence are necessary to further advance the domain in the field.

In the previous section, we discussed the interrelated topics of knowledge creation and knowledge use to support their presence in practically all lines of concern in socio-cultural-political PME research through a number of papers. However, even in relation to these topics, our review revealed that conceptual theoretically-based papers are less frequent than empirical papers. Few theoretical discussions on the emerging trends in research in PME form part of the proceedings. One of the exceptions is Brown (2009), who discusses Radford's concepts of culture and subjectivity in his theory of knowledge objectification, from the point of view of what psychoanalytical frameworks to theorize learning may offer the field. This type of paper is quantitatively rare in comparison to papers centred on the analysis of empirical data, rather than on the discussion of the theoretical construction that precedes the identification of a particular construct. This finding has several implications for development of the field as a whole, and for the ways in which we are building the socio-cultural-political terrain. Due to the empirical tradition in mathematics education research, and particularly in PME, it is not surprising that the model of theory building for the development of the socio-cultural-political axis draws mostly on the accumulation of data and data analysis as an argument for the discussion of theory. There is, however, a substantive assumption in this way of building theory: it presupposes that socio-cultural-political phenomena can be directly observed or linked to something that can be directly observable. Cultural-historical and socio-political approaches problematise those meanings of observable based on the search for external measures of constructs. Further elaboration on what can be designed to be observable, along with what kinds of observation matter and why, is still required.

ACKNOWLEDGEMENT

Núria Planas' participation in this review has been funded by the Catalan Institute of Research and Advanced Studies (ICREA), the Spanish Grant EDU2012-31464, and the Catalan Grant SGR2014-972. For Paola Valero, this paper makes part of the research initiated under NordForsk's Nordic Center of Excellence Justice through Education (JustEd).

REFERENCES

Adler, J. (2002). *Teaching mathematics in multilingual classrooms*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Adler, J., & Ronda, E. (2014). An analytic framework for describing teachers' mathematics discourse in instruction. *Proceedings of PME38 and PME-NA36, 2*, 9–16.

Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. Dordrecht, The Netherlands: Kluwer.

Assude, T., Boero, P., Herbst, P., Lerman, S., & Radford, L. (2008). The notions and roles of theory in mathematics education research (Report of Survey Team 7). In *Proceedings of ICME-11* (pp. 338–356). Monterrey, Mexico: ICMI.

Appelbaum, P. M. (1995). *Popular culture, educational discourse, and mathematics*. New York, NY: State University of New York.

Arzarello, F., Bazzini, L., Ferrara, F., Robutti, O., Sabena, C., & Villa, B. (2006). Will Penelope choose another bridegroom? Looking for an answer through signs. *Proceedings of PME30, 2*, 73–80.

Arzarello, F., & Paola, D. (2007). Semiotic games: The role of the teacher. *Proceedings of PME31, 2*, 17–24.

Bartolini Bussi, M. G. (1998). Verbal interaction in the mathematics classroom: A Vygotskian analysis. In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 65–84). Reston, VA: NCTM.

Barwell, R. (2013). Formal and informal language in mathematics classroom interaction: A dialogic perspective. *Proceedings of PME37, 2*, 73–80.

Baturo, A. R., Matthews, C., Underwood, P., Cooper, T. J., & Warren, E. (2008). Research empowering the researched: reflections on supporting indigenous teacher aides to tutor mathematics. *Proceedings of PME32 and PME-NA30, 2*, 137–144.

Bernstein, B. B. (1990). *The structuring of pedagogic discourse*. London, UK: Routledge.

Bourdieu, P., & Passeron, J. C. (1977). *Reproduction in education, society and culture*. London, UK: Sage Publications.

Bose, A., & Subramaniam, K. (2011). Exploring school children's out of school mathematics. *Proceedings of PME35, 2*, 177–184.

Breen, C. (2007). On humanistic mathematics education: a personal coming of age? *Proceedings of PME31, 1*, 3–16.

Brown, T. (2009). Delineations of culture in mathematics education research. *Proceedings of PME33, 1*, 209–216.

Civil, M. (2008). Language and mathematics: Immigrant parents' participation in school. *Proceedings of the PME32 and PME-NA30, 2*, 329–336.

Civil, M. (2012). Opportunities to learn in mathematics education: insights from research with "non-dominant" communities. *Proceedings of PME36, 1*, 43–59.

Confrey, J., & Kazak, S. (2006). A thirty-year reflection on constructivism in mathematics education in PME. In Á. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 305–345). Rotterdam, The Netherlands: Sense Publishers.

Cooper, T. J., Baturo, A. R., Duus, E., & Moore, K. (2008). Indigenous vocational students, culturally effective communities of practice and mathematics understanding. *Proceedings of PME32 and PME-NA30, 2*, 377–384.

Engeström, Y., & Middleton, D. (1996). *Cognition and communication at work*. Cambridge, UK: Cambridge University Press.

Evans, J., Tsatsaroni, A., & Czarnecka, B. (2009). Equations in a consumer culture: Mathematical images in advertising. *Proceedings of PME33, 3*, 17–24.

Foucault, M. (1980). *Power/knowledge: Selected interviews and other writings, 1972–1977*. Brighton, UK: Harvester Press.

Gates, P. (2006). The place of equity and social justice in the history of PME. In Á. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 367–402). Rotterdam, The Netherlands: Sense Publishers.

Goos, M. (2012). Creating opportunities to learn in mathematics education. A sociocultural journey. *Proceedings of PME36, 1*, 67–82.

Gutiérrez, R. (2013). Editorial: The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education, 41*, 1–32.

Herbel-Eisenmann, B., Wagner, D., & Cortes, V. (2008). Encoding authority: Pervasive lexical bundles in mathematics classrooms. *Proceedings of PME32 and PME-NA30, 3*, 153–160.

Hershkowitz, R., Tabach, M., Rasmussen, C., & Dreyfus, T. (2014). From knowledge agents to knowledge agency. *Proceedings of PME38 and PME-NA36, 3*, 281–288.

Heyd-Metzuyanim, E. (2013). From identity to identifying: Tools for discourse analysis of identity construction in the mathematics classroom. *Proceedings of PME37, 3*, 57–64.

Hunter, R., & Anthony, G. (2014). Small group interactions: Opportunities for mathematical learning. *Proceedings of PME38 and PME-NA36, 3*, 361–368.

Johnsen Høines, M. J., & Loade, B. (2006). Positioning of a subject based and investigative dialogue in practice teaching. *Proceedings of PME30, 2*, 369–376.

Krainer, K., & Wood, T. (Eds.). (2008). *The international handbook of mathematics teacher education Volume 3: Participants in mathematics teacher education*. Rotterdam, The Netherlands: Sense Publishers.

Jablonka, E., Walshaw, M., & Wagner, D. (2013). Theories for studying social, political and cultural dimensions of mathematics education. In M. A. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Third international handbook of mathematics education* (pp. 41–67). New York, NY: Springer.

Jaworski, B. (2008). Mathematics teacher educator learning and development: An introduction. In B. Jaworski & T. Wood (Eds.), *The international handbook of mathematics teacher education Volume 4. The mathematics teacher educator as a developing professional: Individuals, teams, communities and networks* (pp. 1–13). Rotterdam, The Netherlands: Sense Publishers.

Jaworski, B., & Wood, T. (Eds.). (2008). *The international handbook of mathematics teacher education Volume 4. The mathematics teacher educator as a developing professional: Individuals, teams, communities and networks*. Rotterdam, The Netherlands: Sense Publishers.

Jaworski, B., & Goodchild, S. (2006). Inquiry community in an activity theory frame. *Proceedings of PME30, 3*, 353–360.

Jorgensen (Zevenbergen), R. (2015). Leadership: Building strong learning cultures in remote Indigenous education. *Proceedings of PME39, 3*, 137–144.

Lave, J. (1988). *Cognition in practice: mind, mathematics, and culture in everyday life*. Cambridge, UK: Cambridge University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.

Lerman, S. (1996). Socio-cultural approaches to mathematics teaching and learning. *Educational Studies in Mathematics, 31*, 1–9.

Lerman, S. (2006). Socio-cultural research in PME. In Á. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 347–366). Rotterdam, The Netherlands: Sense Publishers.

Lerman, S. (Coord.). (2009). Sociological frameworks in mathematics education research. *Proceedings of PME33, 1*, 217–246.

Lerman, S. (2012). Agency and identity: Mathematics teachers' stories of overcoming disadvantage. *Proceedings of PME36, 3*, 99–106.

Lerman, S., & Zevenbergen, R. (2006). Maths, ICT and pedagogy: an examination of equitable practice in diverse contexts. *Proceedings of PME30, 4*, 49–56.

Lerman, S., & Zevenbergen, R. (2007). Interacting whiteboards as mediating tools for teaching mathematics: Rethoric or reality? *Proceedings of PME31, 3*, 169–176.

Mellin-Olsen, S. (1987). *The politics of mathematics education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Morgan, C. (2009). Understanding practices in mathematics education: Structure and text. *Proceedings of PME33, 1,* 49–64.

Morgan, C. (2014a). Social theory in mathematics education: Guest editorial. *Educational Studies in Mathematics, 87*(2), 123–128.
Morgan, C. (2014b). Understanding practices in mathematics education: Structure and text. *Educational Studies in Mathematics, 87*(2), 129–143.
Morgan, C., & Tang, S. (2012). Studying changes in school mathematics over time through the lens of examinations: The case of student positioning. *Proceedings of PME36, 3*, 241–248.
Phakeng, M. (2014). Introduction to 'The calculus of social change – Mathematics at the cutting edge'. *Proceedings of PME38 and PME-NA36, 1*, 55–60.
Phakeng, M., Halai, A., Valero, P., Wagner, D., & Walshaw, M. (2014). The calculus of social change: mathematics at the cutting edge. *Proceedings of PME38 and PME-NA36, 1*, 55–83.
Planas, N., & Civil, M. (2008). Voices of non-immigrant students in the multiethnic mathematics classroom. *Proceedings of PME32 and PME-NA30, 4*, 121–127.
Planas, N., & Civil, M. (2015). Bilingual mathematics teachers and learners: the challenge of alternative worlds. *Proceedings of PME39, 4*, 41–48.
Planas, N., Iranzo, N., & Setati, M. (2009). Language switching with a group of bilingual students in the mathematics classroom. *Proceedings of PME33, 4*, 393–400.
Radford, L. (2008a). Culture and cognition: Towards and anthropology of mathematical thinking. In L. D. English & M. G. Bartolini Bussi (Eds.), *Handbook of international research in mathematics education* (2nd ed., pp. 439–464). New York, NY: Routledge.
Radford, L. (2008b). The ethics of being and knowing: Towards a cultural theory of learning. In L. Radford, G. Schubring, & F. Seeger (Eds.), *Semiotics in mathematics education: Epistemology, history, classroom, and culture* (pp. 215–234). Rotterdam, The Netherlands: Sense Publishers.
Radford, L. (2010). Commentary to 'Networking of theories in mathematics education'. *Proceedings of PME34, 1*, 168–172.
Radford, L. (2011). Embodiment, perception and symbols in the development of early algebraic thinking. *Proceedings of PME35, 4*, 17–24.
Radford, L. (2014). On teachers and students: An ethical cultural-historical perspective. *Proceedings of PME36, 1,* 1–20.
Radford, L., Bardini, C., & Sabena, C. (2006). Rhythm and the grasping of the general. *Proceedings of PME30, 4*, 393–400.
Radford, L., Miranda, I., & Guzmán, J. (2008). Relative motion, graphs, and the heteroglossic transformation of meanings: A semiotic analysis. *Proceedings of PME32 and PME-NA30, 4*, 162–168.
Roth, W.-M., & Radford, L. (2011). *A cultural historical perspective on teaching and learning*. Rotterdam, The Netherlands: Sense Publishers.
Schliemann, A. D. (1995). Some concerns about bringing everyday mathematics to mathematics education. *Proceedings of PME19, 1*, 45–60.
Seah, W. T. (2013). Assessing values in mathematics education. *Proceedings of PME35, 4,* 193–200.
Setati (Phakeng), M. (2006). Access to mathematics versus access to the language of power. *Proceedings of PME30, 5*, 97–104.
Sfard, A. (2008). *Thinking as communicating. Human development, the growth of discourses and mathematizing*. New York, NY: Cambridge University Press.
Skovsmose, O. (1994). *Toward a critical philosophy of mathematics education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.
Skovsmose, O. (2004). Critical mathematics education for the future. In M. Niss (Ed.), *Proceedings of the 10th international congress on mathematics education* (pp. 1–19). Copenhaguen, Denmark: IMFUFA, Roskilde University.
Subramaniam, K. (2012). Does participation in household based work create opportunities for learning mathematics? *Proceedings of PME36, 1*, 107–112.
Tabach, M., Nachlieli, T., Heyd-Metzuyanim, E., Morgan, C., Tang, S., & Sfard, A. (2013). Development of mathematical discourse: Insights from "strong" discursive research. *Proceedings of PME37, 1*, 155–180.
Triantafillou, C., & Potari, D. (2006). Mathematical activity in a technological workplace: Results from an ethnographic study. *Proceedings of PME30, 5*, 297–304.

Valero, P., & Zevenbergen, R. (Eds.). (2004a). *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology*. New York, NY: Springer.
Valero, P., & Zevenbergen, R. (2004b). Introduction: Setting the scene of this book. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education. Issues of power in theory and methodology* (pp. 1–4). New York, NY: Springer.
Valero, P., & Meaney, T. (2014). *Trends in researching the socioeconomic influences on mathematical achievement*. *ZDM, 46*(7), 1–10.
Valero, P., & Vithal, R. (1998). Research methods of the north revisited from the south. *Proceedings of PME22, 4,* 153–160.
Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. London, UK: Routledge.
Walkerdine, V. (1998). *Counting girls out: Girls and mathematics*. London, UK: Falmer Press.
Walls, F. (2006). "The big test": A school community experiences standardized mathematics assessment. *Proceedings of PME30, 5*, 353–360.
Walshaw, M., & Anthony, G. (2006). Numeracy reform in New Zealand: Factors that influence classroom enactment. *Proceedings of PME30, 5*, 361–368.
Wenger, E. (1998). *Communities of practices. Learning, meaning, and identity*. Cambridge, UK: Cambridge University Press.

Núria Planas
School of Education
University Autonomous of Barcelona
Bellaterra, Spain
and
Department of Mathematics Education
University of South Africa
Pretoria, Republic of South Africa

Paola Valero
Department of Mathematics and Science Education
Stockholm University
Stockholm, Sweden