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3.7. CONCEPT MAPS

Learning Through Assesment

INTRODUCTION

A *concept map* is a two-dimensional pictorial depiction of knowledge. It has been used extensively as an assessment technique of conceptual understanding, especially in science education. But its value is more than that. Students can learn during concept mapping. This chapter introduces a group of eighth grade students' reflections on their two-month experience with concept mapping. The findings indicate that the students had significant positive attitudes toward the use of concept mapping in mathematics. They generally agreed that concept mapping helped them to better understand the mathematical concepts presented in the class by clarifying the relations with other relevant concepts. Implications for teachers' adoption of concept mapping in school settings are also discussed.

For the last three decades, concept maps have been used quite extensively in educational settings as an effective technique for organizing and presenting information. Its use as an assessment tool has been explored in mathematics education as well (Afamasaga-Fuata'I, 2009; Mansfield & Happs, 1991; Williams, 1994). When concept maps are used as an assessment technique, rather than a static product, they take on a different larger role, which is of great value to mathematics educators and curriculum designers. Student-constructed concept maps achieve exactly that by shifting the focus toward the construction process and its meaning to the students. This chapter is grounded on an experimental study in which concept map construction was used as an assessment of secondary school students' conceptual understanding in mathematics. It focuses on the mapping processes rather than the mapping products.

This chapter explores observations of the activity of students' construction of concept maps and their attitudes toward concept maps as educational tools. The findings suggest that concept mapping can be a worthwhile tool in a teachers' repertoire of assessment of students' learning. Implications for the use of concept maps in classroom settings are also discussed.

LITERATURE REVIEW

A concept map is a graphical representation of knowledge within a particular domain. It is a network consisting of nodes and labelled lines. Nodes correspond

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to key terms that represent concepts. They are usually enclosed in boxes or circles. The relationships among the concepts are indicated by connecting line segments or arrows. The labels on the lines are called *linking words* or *linking phrases*, and indicate how the joined concepts are related. The linked concepts together with labels indicated along the connectors form a meaningful statement. This statement is called a *proposition*. Ruiz-Primo (2004) considered the proposition as the basic unit of meaning in a concept map, and the basic unit used to judge the validity of the conveyed relationship between any two concepts.

From its inception, in the early 1970s, concept mapping was described as an assessment technique to trace students' conceptual development (Novak, 2005). Since then, many efforts have been made toward the exploration of the concept map's use for diagnosing conceptual understanding and detecting conceptual development. Broad theories exist to support its use for capturing the attributes of an individual's knowledge structure. In cognitive psychology, it is generally agreed that human knowledge is stored in memory as information packets, or *schema* (Jonassen, Beissner, & Yacci, 1993). When learning occurs, an individual incorporates new information into his or her schema through assimilation and/or accommodation (Piaget, 1977). The balance between these two processes conveys the idea of how knowledge develops in the mind. Studies into concept formation, concept acquisition, and conceptual learning in mathematics (Sfard, 1991; Skemp, 1986) also support the pattern of relations among mathematical concepts and the equilibration processes. Thus, the concept map, with its specific features (nodes, links, linking phrases, and structure), may be viewed as an explicit representation of individuals' knowledge structure. Once the knowledge structure is represented externally, it can be assessed by others. It is generally recognized that concept maps offer an effective way to track students' learning through structural complexity and quality of propositions (Hasemann & Mansfield, 1995; Pearsall, Skipper, & Mintzes, 1997).

Different activities and applications of concept mapping can be found throughout literature. Ruiz-Primo, Shavelson, Li, and Schultz (2001) provided a systematic description of the mapping formats and characterized the tasks along a continuum, from high-directed to low-directed, according to who chooses the concepts, who links the concepts, who generates the linking phrases, and who structures the concept map. The lower the direction of the concept map-based task, the more opportunities it will have to reveal students' conceptual understanding. However, free-style mapping is too open-ended, and presents difficulties for researchers in developing a reliable scoring system since different students may provide quite different sets of concepts and relationships (Jin, 2007; Ruiz-Primo, Schultz, & Shavelson, 2001). By comparing the limitations and strengths of different mapping tasks, the experimental study reported in this chapter used a low-directed concept mapping format, with a given concept list. The concept list could guide the students to focus on a specific knowledge domain; at the same time, the students were free to make connections among the concepts and label the lines with their own words.

Mansfield and Happs (1989a, 1989b) reported on a project involving a group of eighth grade students. In this project, concept mapping was employed to probe students' understanding before and after a teaching sequence on the topic of parallel lines using a pre-/post-test design. To get the students prepared for the concept mapping task, the researchers provided a brief training. After that, the students were required to construct a map using ten concepts, related to the subject of parallel lines, in a given concept list. Though it was acknowledged that concept mapping was a difficult task for young students and a brief introduction may not be sufficient for them to construct informative concept maps, most of the eighth grade students were able to construct a concept map. The researchers could gather meaningful information about the students' conceptual understanding and conceptual development through the analysis of the concepts and propositions in the student-constructed concept maps. No detailed information about the students' concept mapping process was mentioned in the papers. But the findings encouraged further study with young students.

Afamasaga-Fuata'I (2006, 2009a, 2009b) conducted a series of case studies using concept maps to trace perspective teachers' conceptual knowledge of certain topics, in particular,―matrices and systems, length and volume, and fractions. After a period of learning a topic, the perspective teachers were required to generate a list of concepts for the topic and construct a map showing their understanding of the interconnectedness between the concepts. After each concept map, they presented it to the class or the researcher(s). Through discussions and negotiations, the perspective teachers further revised and expanded the maps. The progressive maps were collected and then compared by the researcher. Cycles of refinements in the student teachers' concept maps were documented.

SAMPLING AND PROCEDURES

The participants of the experimental study discussed here consisted of a class of 48 eighth grade students (24 female students and 24 male students). They were selected by convenient sampling from a junior middle school in a town in the Jiangsu province, China. The students' mathematics test scores, from exams administered during their seventh and eighth school year, were collected to gauge their performance. The tests, altogether six, were all graded on a 100-point scale. Their scores were highly correlated (the correlation coefficient, $r¹$, ranged from 0.926 to 0.950, $p < 0.001$; Cronbach's $\alpha^2 = 0.989$), indicating that the tests measured a common feature about the students' mathematics achievement. For each student, the mean of the student's raw test scores was taken as an indicator of his or her school mathematics achievement (SMA).

Since concept maps have not been extensively used in mathematics classrooms in China, the experimental study first trained the participant on the techniques of constructing informative concept maps. The four mathematical topics addressed were *algebraic expressions*, *equations*, *triangles*, and *quadrilaterals*. The studentconstructed concept maps were analysed by considering both Novak's traditional methods (Novak & Gowin, 1984), including number of links and proposition score, and methods adopted from the Social Network Analysis,―density and numbers of incoming and outgoing links (Jin & Wong, 2013). No criterion map was produced for scoring the students' maps through comparison since Ruiz-Primo and Shavelson (1996) had found different criterion maps may lead to different conclusions.

Students' attitudes toward concept mapping were collected through a selfdesigned *Attitude Toward Concept Maps* (ATCM) questionnaire combined with an interview. The questionnaire was designed following Mohamed's (1993) *attitudes toward concept mapping* questionnaire in science. Some items were adopted from Kankkunen' study (2001) in which the students' opinions about concept mapping were gathered through inquiry and interviews. The questionnaire utilized a six-point Likert Scale, with the following categories: Strongly Disagree (SD), Disagree (D), Slightly Disagree (LD), Slightly Agree (LA), Agree (A), and Strongly Agree (SA). By using this six-point Likert Scale, the researcher forced the students to choose an option from either side of the agreement spectrum, not allowing a neutral response. Interviews were conducted with selected students. For the students who were not interviewed, the interview questions were assigned as an open-ended written task at the end of the study. The interview, together with the open-ended written task, made it possible to focus research attention more directly on students' process of concept mapping, and to provide additional information about the students' attitudes toward concept mapping supplementing their responses on the ATCM questionnaire.

FINDINGS

Observations of Participants' Concept Mapping

This section includes observations made during both the student training and concept mapping stages.

During the training, the students, first, treated concept mapping as a simple drawing task instead of a test of their mathematical understanding. They provided only brief and general linking phrases in their maps. For example, one of the students initially constructed a proposition that in written form reads, "A triangle may be an isosceles triangle". When asked to explain what she meant by *may be*, the student said, "when it has two equal sides, it is an isosceles triangle; when it has no equal sides, it is not an isosceles triangle." In general, when prompted, most of the students could add more links to their concept maps. This finding suggested that the students needed further training before they can construct meaningful concept maps that can be used to represent their levels of understanding. Hence, more detailed training (Jin & Wong, 2010) was provided before the concept mapping tests.

After the training, four concept mapping tests were administered, consisting of free-style mapping using ten or eleven given concepts. The tests were given on different days. Students were given 30 minutes to complete each test. It was

noticed that, after the students worked on the mapping tasks for about 10 minutes, some of them put the test paper aside, indicating that they were done; while some others seemed to have been struggling with the possible connections. The researcher checked several students' maps, privately, during the test time, and inquired as to what their specific difficulties with the mapping test were. Most of them said they could not find more connections among the given concepts, but, at the same time, were unsure whether they had included all the expected connections. During one of such informal exchanges, the researcher provided a prompt: "How about exponents and like terms? Is there any relation between them?" The student thought for a while, then seemed suddenly enlightened, and started to add the link. As a result, for all four concept mapping tasks, after the students had worked on constructing their drafts of concept maps for about 25 minutes, the researcher provided four or five prompts to the class to assist with the task. Given the prompts, most students added information to their concept maps.

Examples of Well-Constructed and Poorly-Constructed Concept Maps

The correlation coefficients between the students' school mathematics achievements (SMA) and their proposition scores for the four topics ranged from 0.709 to 0.753, with $p \leq 0.01$, indicating that the students' ability to build informative concept maps did, to a certain extent, reflect their mathematics achievement. Focusing on the topic of triangles, [Figures 1](#page-5-0) and [2](#page-6-0) are examples of a well-constructed concept map and a poorly-constructed concept map, respectively. Carefully comparing the two, one may gain more insight into the usefulness of student concept mapping in addressing their conceptual understanding.

[Figure 1](#page-5-0) shows that the student possessed a comprehensive understanding of the concepts. All of the eleven given concepts were involved in the map. The most inclusive concept, *triangle*, was placed at the centre with special types of triangles around it. The links from triangle to the six special types of triangles were labelled with definition-based linking phrases. The relationship between *acuteangled triangle* and *equilateral triangle* was indicated. The student noted out that an acute-angled triangle whose angles are all equal to 60° is an equilateral triangle. The relations between the other triangles were not shown in the concept map, indicating that the student may find the relationship between acute-angled triangle and equilateral triangle more obvious than the others. *Angle* was placed close to *acute-angled triangle*, *right-angled triangle,* and *obtuse-angled triangle*, since these different types of triangles are categorized using their angles. Isosceles triangle and equilateral triangle both have special properties of *median* and *midline*. This might be the reason that the student placed median and midline right below the two triangles. *Symmetry axis* was placed near *isosceles triangle*. This is reasonable since, among the special types of triangles, isosceles triangle (considering an equilateral triangle as a special case of an isosceles triangle) is the only one that always has one axis of symmetry. The student constructed 25 links for the 11 given concepts. The

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number of links constructed is greater than most of the other students' concept maps. The propositions in the concept map are all correct, and some even indicated deeper insights into the relationships. Therefore, each proposition was assigned a score of 2; the entire map earned a proposition score of $2 \times 25 = 50$.

Figure 2 shows an example of a poorly-constructed concept map. Although it included all 11 given concepts, much fewer links were constructed in this map relative to the one in [Figure 1](#page-5-0).

Figure 2. An example of a poorly-constructed concept map for triangle concepts (translated from Chinese)

Triangle was placed at the centre of the concept map. Seven concepts were placed around it. The links between *triangle* and the seven concepts were all labelled with definition-based linking phrases. However, no connections among the seven concepts was shown in the map. *Equilateral triangle* was placed below *isosceles triangle* since, as recognized by the student, an equilateral triangle is an isosceles triangle with a 60° angle. *Median* and *symmetry axis* were, most likely, the last two concepts added to the map. On the one hand, there seemed no space in the student's map for the two concepts to connect directly to *triangle*. On the other hand, the relationships of *median* and *symmetry axis* with equilateral triangle seemed to be

the most obvious to the student. Thus, he simply linked the two concepts with the equilateral triangle. The student did not seem to have attempted to construct further connections among *median* and *symmetry axis* and other concepts in the map. It appears that the student was satisfied with his map since no concept was isolated. As a result, only ten links were constructed, each with a varying proposition score. This concept map earned a proposition score of $0 + 2 + 1 + 1 + 0 + 2 + 1 + 2 + 1$ $+ 2 = 12$. This is far less than the obtained mean class proposition score of 25.75. The linking phrases used indicate that the student does possess some understanding of the relationships; however, he did not express his ideas clearly. For example, he linked *triangle* to *midline* with the linking phrase "the midpoints of two sides." He may know that a midline is a segment connecting the midpoints of two sides of a triangle, but did not explicitly state that a midline is a *segment* in the linking phrase. In practice, some instructors may wish to reduce this strict requirement of *detailed* linking phrases.

ATCM Questionnaire

The questionnaire covered the following five aspects:

- Ease of constructing concept maps (*ease*);
- Confidence with concept mapping (*confidence*);
- Enjoyment of concept mapping (*enjoyment*);
- Usefulness of concept maps (*usefulness*), and, finally,
- Preference of using concept map for further study (*preference*).

The Cronbach's alphas of the five aspects, except for *ease,* were higher than 0.70, showing that these aspects have acceptable internal consistency. The *ease-*related items had low internal consistency (Cronbach's $\alpha = 0.339$), possibly because they did not belong conceptually to the same aspect. The mean scores of the other aspects ranged from 4.16 to 5.06, showing different levels of students' agreement. The results are shown in Table 1 where the different aspects are shown in a descending order according to their means.

Table 1. Cronbach's α, means and standard deviations (SDs) of the five aspects of the Attitudes Toward Concept Mapping (ATCM) Questionnaire

Aspects	Cronbach's α	Mean	S.D.	
<i>Usefulness</i>	0.724	5.06	0.20	
Preference	0.867	4.37	0.20	
Enjoyment	0.830	4.22	0.35	
Confidence	0.724	4.16	0.88	
Ease	0.339			

The students shared a significantly positive view toward the *usefulness* of concept maps. All 48 students agreed that using concept maps one can clearly describe relationships among mathematical concepts. Over 90% of them indicated that constructing concept maps is, indeed, an effective assessment technique. They generally thought that concept maps appropriately reflect their understanding of mathematical ideas, and considered the maps a fair tool for gauging their conceptual achievement. Although it was found that with prompts students could add more propositions to their concept maps, from the students' perspective, concept mapping had its value as an assessment technique. The students readily admitted that they could benefit from concept mapping. For example, more than 90% of the students concurred that concept mapping is helpful for understanding mathematics concepts, and that they could see more clearly how concepts are related after building a concept map. Eight students *strongly agreed* that they are able to come up with new ideas when engaged in concept mapping. These eight students include both high and low achieving students, suggesting that students of different academic levels can benefit from concept mapping.

The *preference* aspect measures the students' preference of using concept maps in their further study of mathematics. In general, the students' responses to this aspect were positive. Majority of the students indicated that they would like to use concept maps in their further study, and hope that their teachers can use concept maps to teach mathematical concepts. However, more than 40% of the students indicated *slight* agreement only. There are two possible reasons for this hesitation. First, even though the students seemed to have a desire to use concept maps, as was evident from their appreciation of the maps' usefulness, they were uncertain whether they would continue to use this technique by themselves. Secondly, as was discovered through the informal talks, students generally viewed problem-solving as the most important issue in learning mathematics. They did not think that concept mapping is helpful for problem-solving. Among the students who indicated disagreement to the items related to *preference*, most were female students. For example, among the six students who showed disagreement to the item "I'd like to use concept map in mathematics," five were female students. This finding suggests that male students may have higher preference for further use of concept maps. The gender difference needs to be studied further.

The students indicated moderate *enjoyment* toward concept mapping. They showed the strongest disagreement on the item "I find concept mapping boring". Only 7 out of the 48 students slightly agreed with this item. More than 80% of the students agreed that concept mapping is interesting. About three quarters agreed they liked spending time on concept mapping. The students' responses to these items consistently suggest that they did enjoy concept mapping.

With respect to the *confidence* level, students seemed relatively insecure in their ability to construct and utilize concept maps. However, they admitted that given more practice they would be able to assemble better concept maps.

The *ease* aspect actually dealt with levels of easiness, anxiety, and timeconsumption issues. Most of the students admitted that concept mapping is challenging, but more than half of them denied that they felt anxious when they were asked to construct a concept map. This is likely due to the students' awareness that their concept map scores would not be used to judge their performance in school. The students held different views on whether concept mapping is an unusually timeconsuming activity. It was noted that among the eight students who chose *strongly disagree* to this item, six were male students; while, among the seven students who chose *agree* or *strongly agree* to this item, only two were male students. It appears that the female students in this study required more time to construct concept maps compared to male students. This study did not aim to study gender differences, but such differences might prove to be a fruitful area for future research; for example, to further investigate Edwards' (1993) finding that female students produced significantly more complex concept maps than male students.

In summary, the above findings about students' attitudes toward concept mapping encourage further exploration of the utility of concept maps as an assessment technique of conceptual understanding, as well as a learning strategy in mathematics.

Interview and Open-Ended Written Task

Twelve students were selected for interviews, based on their school mathematics achievement level. Six students were high-achieving and the other six were lowachieving students. The interviews were conducted one-on-one. Each interview took about 15 to 30 minutes and was audio-taped. The interview questions were provided to the remaining 36 students as an open-ended written task at the end of the study. They were given 30 minutes to write down their answers. The interviews and the open-ended written task together provided rich information about the students' thoughts and opinions about the use of concept maps as an assessment technique, a learning strategy or, even, a teaching method. Below are two of the most revealing questions posed to the students:

Question 1: *In which aspect(s) do you think concept mapping is helpful?*

- (*A*) *help with review*,
- (*B*) *help with memorization*,
- (*C*) *help with problem-solving*,
- (*D*) *support understanding of concepts*,
- (*E*) *others (please specify), or*
- (*F*) *no use at all*

Please indicate your answer by selecting one or more of the choices above, and provide your explanations below.

Question 2: *Can you summarize your major achievements during this concept mapping period?*

[Table 2](#page-11-0) below summarizes the results of students' responses to Question 1. The percentages are shown in parentheses below each choice, and typical corresponding explanations (translated from Chinese) are also provided.

As is demonstrated in [Table 2](#page-11-0), students agreed that concept mapping benefited their learning of mathematics in different ways. The students' responses to Question 2 were mostly positive. The positive statements reflected (A) , (B) , (C) , and (D) choices of Question 1. Students indicated that concept mapping would be beneficial for review, memorization, problem-solving and understanding. Questions 1 may have influenced students' answers to Question 2; this is an unintended limitation of the design of the questions. Below are some examples of students' responses to Question 2 (translated from Chinese):

- With concept mapping, I know I have not fully understand the concepts; concept *mapping helps me to better understand the concepts learned; it is also helpful for the learning of new concepts.*
- With concept mapping, it becomes easier to remember the concepts. It is better *than learning the concepts individually.*
- *Now I know mathematical concepts are related and I know different connections among concepts; concept map helps me to organize the concepts I have learned.*
- *At first, I find concept mapping complex and it has nothing to do with problem solving. After I get familiar with it and did concept mapping for several times, I find problem solving is actually quite easy. When I encounter a complex problem which I don't know where to start, I construct a concept map in mind, go from one concept to another, then to other concepts, the problem becomes easier. I will continue to use concept map for my further study.*
- *Concept mapping is a kind of training on divergent thinking. When I see a concept, I am able to link it to other concepts. With such experience, I know to look at a problem from different perspectives.*
- *Concept mapping makes mathematics learning interesting; the concepts are not abstract and boring terms anymore.*
- *With concept mapping, I can easily find the connections between concepts. It helps me to review and consolidate the knowledge I've learned. It helps me to understand concepts and digest them. I think concept map can not only used for mathematics but other subjects as well; for example, some difficult and abstract concepts in physics. By doing so, I can easily understand the concepts. I want to construct a huge concept map in the future. It can include concepts in primary school, secondary school, and high school; while for secondary school, it covers secondary 1, 2, and 3, the same for primary school and high school. For each grade, I will put all the concepts into a concept map and put it in my space. So*

others can use it for study, and they can then learn mathematics, chemistry, and physics better.

Three students provided neutral to negative responses:

- *Concept mapping is just so-so. Though it did not help me too much in learning mathematics, it did help me a little bit.*
- *I do not think concept mapping is helpful for solving problems. When I try to solve a problem, the relevant concepts will come to my mind naturally. I will then solve the problem step by step. I will not think about other concepts.*
- *I don't think I will draw concept maps in most cases because it is too troublesome and time-consuming.*

The student who provided the second negative response to Question 2 above was one of the top students in the class. He had his own learning method that had proved to be effective for him, since he earned high scores on mathematics tests. Thus, he was reluctant to adopt this new technique. The other two students who gave negative responses were two academically weak students. Their mathematics teacher, who was also the teacher in charge of the class, commented that these two students were not motivated to study in general. Perhaps, this lack of motivation carried over to their opinion of the concept mapping activity, and explains their reluctance to make the effort to incorporate concept mapping into their studies.

DISCUSSION

Although, in this study, concept maps were used primarily as an assessment technique, the findings, especially the information about the students' attitudes toward concept mapping and their perceptions of the usefulness of concept maps, suggest important applications of concept maps as educational tools.

Training for concept mapping. The observations during the training and the concept mapping stages showed that, when the students were new to concept mapping, they generally had difficulty understanding the purpose of concept mapping and seemed unable to include as much information as they knew about the concepts; they needed time to get familiar with this technique. Hence, training on concept mapping solely for assessment purpose is not economical and practical for school education. Existing studies have shown that concept maps can be an effective teaching strategy. For example, in 1993, Horton and her colleagues published a meta-analysis of research concerning the effectiveness of concept mapping as an instructional tool for science teaching (Horton, McConney, Gallo, Woods, Senn, & Hamelin, 1993). The results of this meta-analysis showed that concept mapping had positive effects on both students' achievement and attitudes. Therefore, it is suggested that teachers start by introducing concept maps in classroom teaching, for example, for lesson display and information integration. During this introductory time students can gradually familiarize themselves with the attributes of concept

maps and the general concept mapping process. After that, teachers can expand the function of concept mapping as an assessment tool or a learning strategy.

Students' experiences with concept mapping. After working with concept maps for two months, students generally appreciated its use in mathematics. Aside from its use as an assessment technique, over three quarters of the students indicated that concept mapping helped them with conceptual learning. For example, one of the concept mapping tests required the students to construct a map with concepts related to *equations* and *functions*. *Equations* and *functions* were taught in separate chapters in secondary schools in China but their connections were not made explicit in the students' textbooks. Most students did not rigorously reflect on the relations between them. On the concept mapping test, they were forced to think hard about the connections between the given concepts. The students reported that they better understood the concepts when they were placed within a bigger picture. During the interviews and on the written task, students also pointed out that concept maps helped them remember different mathematical concepts by clarifying their relationships to each other.

Concept mapping provides a valuable opportunity for learning through assessment, with its many unique virtues. It is a promising instructional tool for teachers who wish to elevate their students' conceptual learning and appreciation of mathematics.

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