Teaching and Learning Mathematics in Multilingual Classrooms

Issues for Policy, Practice and Teacher Education

Anjum Halai and Philip Clarkson (Eds.)

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Issues for Policy, Practice and Teacher Education

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NOTES ON CONTRIBUTIONS

All chapters included in this volume underwent a blind review process. We acknowledge the reviewers who were kind enough to participate in this process.

This volume gives voice to a range of contributions from a variety of countries many that rarely if ever have a voice in the international research literature. We first list the home countries (and where appropriate place of residence) of authors:

- Australia
- Congo-Brazzaville
- England (Canada)
- France
- Germany (3)
- India
- Iran-England (Chile)
- Mali
- Pakistan (England)
- Pakistan (Tanzania)
- Tanzania
- South Africa (7)
- Sweden (2)

The countries in which various studies are situated are (noting there are three general chapters):

- Congo-Brazzaville
- England
- Germany (2)
- India
- Mali
- Pakistan
- Papua New Guinea
- Tanzania
- South Africa (5)
- Sweden

SECTION I

REVIEW AND CRITIQUE OF MATHEMATICS EDUCATION IN MULTILINGUAL CONTEXTS

ANJUM HALAI AND PHILIP CLARKSON

1. TEACHING AND LEARNING MATHEMATICS IN MULTILINGUAL CLASSROOMS

An Overview

INTRODUCTION AND RATIONALE

Mathematics education has increasingly in the last 30 years acknowledged the crucial role that language plays in learning. However, this has mainly been the role of language in cognition, such as students' understanding of mathematics concepts and relationships, and not necessarily its impact on social, cultural, and political issues and learning in mathematics. Furthermore, multilingualism in mathematics classrooms has been underpinned by a deficit discourse that viewed languages other than the language of instruction as a "problem". This situation is beginning to change, and there is a shift away from the traditional deficit discourse and views on the advantages that can be conveyed by additional language/s in the classroom.

Contemporary concerns in mathematics education recognize that in the increasingly technological and globalized world, with concomitant change in population demographics (e.g., immigration, urbanization) and a change in the status of languages (e.g., English as a dominant language of science and technology) multilingualism in classrooms is a norm rather than an exception. Shifts in perspective also view language not simply as an instrument for cognition with all learners equipped with this instrument in service of learning, although clearly in the classroom that remains of importance. Rather, it is now also being acknowledged, as it has been in other areas of cultural construction, that language use is inherently political. Hence the language that gets official recognition in the classroom is invariably the language of the powerful elite, or the dominant societal language, or in the case of post colonial contexts the language of the colonisers. Using this socio-political role of language in the learning frame, quite different issues arise for teaching, learning and curriculum for linguistically marginalized learners than that of only cognition (e.g., immigrants, second language learners, other).

In researching the issues noted above, this edited collection draws on recent and emerging insights, as well as understandings about the approaches to improving policy and practice in mathematics education and mathematics teacher education in multilingual settings. The main objective of the book is to present, and discuss critically, examples of work from a range of contexts. In doing so the authors use these examples to draw out key issues for research in mathematics education in

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 3–10.

language diverse settings including in teaching, learning and curriculum, and fit these with appropriate policy and equity approaches.

Another strong theme within the book is that the policy environments both nationally in most countries, and globally, are overwhelmingly concerned with improving student performance in mathematics achievement. Towards this end, policies on language in education are being considered and reconsidered with specific reference to mathematics teaching and learning. However it appears that most of the time policy changes seem to be made for only political ends, with scant attention paid to the relevant research. For example, the language in education policy swings in Malaysia over the last decade, or the current shift towards changing the language of instruction in upper primary mathematics classrooms to English in Pakistan and Zanzibar (Halai & Muzaffar this volume; Kajoro this volume) were made with no acknowledgement of either pertinent national and international educational research. Given such a global policy environment, this publication both teases this issue with some case studies, and challenges both researchers and politicians and their advisors to find better ways for making decisions.

A further significant dimension of this book is that it brings insights mainly from developing countries where relatively less research activity takes place. We have drawn together both established researchers who are able to give perspectives that reach back across years of involvement with these issues, with new colleagues who bring fresh new insights. In particular there are a number of examples drawn from different contexts in Africa, which brings a new and exciting perspective to bear on this area of mathematics education research.

BOOK STRUCTURE

The book is divided into four sections that provide a focus on some of the different dimensions of the issues of mathematics teaching and learning in multilingual settings. The first section entitled '*Review and Critique of Mathematics Education in Multilingual Contexts'* provides both an historical overview of this area of research, but goes beyond that to critique the work that has been undertaken. Strengths are acknowledged, but research gaps and inadequate approaches are also noted. The second section entitled '*Policy and Mathematics Education in Multilingual Contexts'* examines three specific contexts in different ways to show that this area of education, like any educational process, is not immune from politics. Exemplars of policy interventions in language are shown to impact on mathematics education in multilingual contexts quite directly. The third section, namely *'Learning Mathematics in Multilingual Classrooms'*, includes chapters from a range of geographical contexts. It mainly provides issues of learning mathematics in contexts where the language of instruction is not the first or the second language of the teachers and learners. Finally the fourth section, '*Mathematics Teaching and Teacher Education in Multilingual Classrooms*' looks at strategies and approaches to teaching and teacher education in the context of multilingual mathematics classrooms. We now give a brief overview of the substantive issues and discussions as presented in the various sections of the book.

This opening chapter is followed by Phakeng's, in which she provides a historical overview of research on mathematics education and language diversity through a review of research published in selected international journals. She maintains that research on language and learning started with a focus on bilingualism and the bilingual learner. The 'problem' at that stage was mainly located in the learner and was based on an underlying assumption that there is something wrong with the bilingual learner. Studies in the eighties moved from focusing on the bilingual learner to the bilingual classroom. In the nineties there was another shift to a focus on multilingualism, a global phenomenon, which until then was not taken into consideration by research in mathematics education. In recent years, Phakeng notes that research on mathematics education and language diversity has come to recognize the socio-political role of language. This shift also brought with it recognition that fluency in more than one language per se has no necessary effects (either negative or positive) on learners' mathematics achievement or the cognitive and intellectual development of children in general. Her review suggests that the contradictory results reported in the literature may be accounted for by the socio-economic and psychosocial differences between learners, and not their fluency in multiple languages per se. Phakeng highlights significant advances, findings, gaps and future research directions.

In the final chapter of this opening section, Barwell offers a critical examination of research on the learning and teaching of mathematics in contexts of language diversity, multilingualism, second language learners, among other issues. He draws on ideas from the contemporary sociolinguistics of multilingualism, including the concept of superdiversity, to explore three aspects of previous research in mathematics education. Specifically, he shows how research on the learning and teaching of mathematics in contexts of language diversity is often based on simplistic ideas about language, about language groups and speakers, and about communication. Barwell maintains that the ideas presented in his chapter reframe quite fundamentally some of the challenges faced by learners, teachers and policymakers in this area of mathematics educational research.

The next three chapters in the second section provide specific cases of language education policies that have impinged directly on mathematics learning and teaching. In chapter four, Clarkson details some of the sociocultural and linguistic context of the journey that the Papua New Guinea education system has travelled since independence in 1975 looking specifically at the teaching of mathematics. Starting by endorsing the colonial political policy of using English only in teaching, then gradually moving over some 20 years to privilege vernacular languages for teaching in the early years, to a sudden and surprising reversion of policy to that of independence in 2013. Clarkson provides a succinct overview of the issues for mathematics teaching and learning within the changing policy landscape of Papua New Guinea and clearly shows that mathematics teaching at least in this context has never been divorced from political decisions made by others outside of the education system. He ends with a plea for researchers to become involved with the political process and indeed see this engagement as a crucial part of their professional life.

In the following chapter Halai and Muzaffar examine the paradoxical effects of a policy whose fundamental aim was to achieve greater equity in distribution of cultural capital by mandating English as a language of instruction for all learners in the education system. It draws on data from a large-scale empirical study carried out in the Punjab province in Pakistan. Taking a social justice perspective on the issue they maintain that the attempt to distribute the cultural capital, including linguistic and mathematical capital, among learners is a nuanced and a political process. For any anticipated success, this process must recognize the role of learners' first language, or the proximate language, as a resource to learn mathematics. In turn, this recognition would require challenging some deep-seated assumptions about the role of learners as recipients of knowledge in the classroom dynamics of teaching and learning mathematics.

Next, Kajoro traces the history of national educational language policy in Tanzania from its colonial past to post independence, highlighting particularly the change of medium of instruction in primary schooling from English to the national language, Kiswahili. With insightful cases from the mathematics classrooms, he illustrates how Kiswahili, though officially the medium of instruction at primary school level, is taken to be a mere linguistic placeholder for the real language of education, that is English. Kajoro then identifies the political and socioeconomic forces that are working against the promotion of Kiswahili and also explores why intense pressure is currently being exerted on the government by many educational stakeholders, especially academics, to review the school language policy and look into the possibility of either reinstating English as the medium of instruction for all disciplines and at all levels of schooling, or using Kiswahili as a medium of instruction throughout the schooling years.

The third section of the book looks closely at issues of mathematics learning in multilingual classrooms. The section opens with Prediger's and Krägeloh's work with immigrant learners in Germany, focusing in particular on conceptual understanding of variables, a crucial topic for school success in algebra. They draw on a case study from a larger design research project in which multilingual low-achieving students are supported to gain access to this topic in a 'content- and language-integrated learning arrangement.' Through rigorous empirical analysis of videotaped teaching–learning processes they show the epistemic role of the language of schooling, a register to which these underprivileged students have limited natural access in either of their languages.

In the following chapter Noren and Andersson explore theoretically the construct of students' agency, and then use it for an analysis of classroom social interactions with a combination of sociocultural and critical theoretical perspectives. Using empirical evidence obtained through intensive engagement with learners from Arabic speaking homes in Swedish classrooms they illustrate how agency works and how students' agency varies in different contexts. They maintain that in learning mathematics, student's agency is much more powerful if the classroom discourse enables the use of bilingualism.

Bose and Clarkson look at how multilingual students in a multi-school institution in Mumbai India negotiate the meanings of and process problems in mathematical contexts. They show that students switch between languages and registers as well as drawing on available contextual cues as they engage in their mathematical learning. Their findings show that in this process students utilized a wide range of cultural resources and cues in their negotiation of meanings. These cues in many cases are only accessible by the students if they switch to their home languages and then back to the school and formal technical mathematical languages.

Reporting from his study with learners having another first language (Turkish) than the language of instruction at school (German), Meyer holds that every German classroom can be characterized by the presence of a certain language variety: that is, some languages spoken in the classroom are not shared by all students. He maintains that learners can make use of their first language to learn mathematics in the classrooms, even though the teacher does not share their first language. According to his results, a great advantage of use of a language, which is not the language of instruction but is the first language of the learners, is that learners can use this language in flexible and multiple ways in the course of learning mathematics.

Nkambule's work is also with immigrant learners but in multilingual classrooms in South African schools. She explores discourse practices with immigrant and local learners during the teaching of linear programming in an urban school in South Africa. Through empirical data collected from immigrant learners from the Democratic Republic of Congo, she found that the teacher supported immigrant learners by switching to two additional languages, French and English during the teaching of linear programming. She concludes that the teacher's support for the immigrant learners by resorting to their additional language paradoxically raised questions about the extent to which local learners were marginalized in the process of learning.

The fourth and the final section took account of issues for teaching and teacher education in the context of multilingual mathematics classrooms. It opens with Essien's and Adler's work with Wenger's (1998) communities of practice (CoP) theory to understand and describe pre-service mathematics teacher education practice in multilingual classrooms. Drawing on empirical data to operationalize theoretical constructs, they show how Wenger's communities of practice theory was expanded into a framework that could productively analyse the nature of pre-service mathematics teacher education classrooms in multilingual settings. They argue that this elaborated framework enables researchers to examine, in an integrated manner, the mathematics content, the interactional context and the discourses in multilingual pre-service teacher education multilingual classrooms.

Next, Webb and Webb look at teaching strategies that could promote numeracy achievement and mathematical reasoning in bilingual classes in South African township schools where both the teachers and pupils were English second-language speakers. To this end the pupils in six purposively selected grade seven mathematics classes in three township schools engaged in pre- and post-tests of numeracy and reasoning skills and their teachers were observed over a period of nine months, teaching them mathematics using strategies to promote numeracy and mathematical reasoning. Their findings concur with international research which suggests that using selective questioning, demonstrating the relevance and procedures for solving problems, and developing a social and dialogic space using exploratory talk improves mathematical reasoning and numeracy skills in language diverse mathematics classes.

Tshabalala and Clarkson provide two illustrative classroom vignettes from a classroom in an informal settlement, west of Johannesburg to explore the impact of the teacher's language practices when the teacher's home language was different from that of the learners in a grade 4 mathematics multilingual classroom. The study provides convincing evidence that the teacher's use of the learners' home language, positioned as a tool to enhance conceptual understanding, was not always effective. Confusion and misconceptions in teaching arose because the teacher was not proficient in the home language of the learners (Setswana or Zulu), or in the English mathematical language that was the focus of the teaching.

In the following chapter Farsani illustrates how the bilingual orientation of a particular complementary school in the UK developed a different pedagogy to what is perceived to be the norm in monolingual contexts. This different bilingual pedagogy provided a space for British-Iranian bilingual learners to incorporate not only their languages, but aspects of histories and experiences of how complex fractions (for example) were solved in Iran. He further maintains that the bilingual orientation of the complementary school not only offered a perspective on how complex fractions can be seen differently, but how this knowledge can be transferred in different tasks and settings.

In the final chapter of the fourth section Galisson, Malonga-Moungabio and Denys look at the case of Mali and Congo-Brazzaville where the 'Harmonization Project Mathematics (HPM)' was launched in 1992 to support reform in mathematics curricula and teaching, mainly in the post independent French-speaking region of Africa. Their study looked at the teaching of mathematics in the early years of secondary education (students 11–15 years of age) in each of these two countries. They maintain that in both countries French was the official language of instruction in secondary schools, but local languages were treated differently. What was taught remained influenced by the French curricula, but teaching methods developed differently in the two countries. They conclude that the different paths of evolution in Mali and Congo-Brazzaville since 1992 show that the HPM has led the two countries to teach mathematics in a way which takes into account, to a lesser or greater extent, the difficulties encountered in their educational systems and their socioeconomic contexts. However, both the Congolese and the Malian curricula bear witness to the persistence of an educational discipline (mathematics as taught in the first years of French secondary education) produced by a Western educational system.

CONCLUDING REFLECTIONS

This edited volume arose mainly, though not exclusively, from the deliberations in the Topic Study Group 30: ‘Mathematics Education in Multilingual and Multicultural Environments’ in ICME 2012 in Korea (Halai & Barwell, 2015). Some participants chose post conference to develop their ideas into quite new contributions. We also invited some additional authors to contribute to round out some sections. As in the case of the TSG 30, the volume brings together contributions from diverse geographical contexts including technologically advanced countries with increasingly large immigrant populations (e.g., Canada, Germany, Sweden, UK), postcolonial countries with concomitant colonial languages as the medium of instruction (e.g., Pakistan, Papua New Guinea, South Africa, Tanzania) and countries with varied indigenous and official languages (e.g., India, Papua New Guinea, South Africa). It is reaffirming the changing and increasing impact of research into the role of language in mathematics learning that the International Commission on Mathematics Instruction commissioned the Study 21 entitled *Mathematics Education and Language Diversity* (Barwell, Clarkson, Halai, Kazima, Moschkovich, Planas, Phakeng, Valero & Villavicencio, 2015).

In this volume the conversation is progressed by illustrating the extent and breadth of issues that impinge on teaching and learning mathematics in multilingual classrooms. One significant contribution that it makes among others is it identifies new questions and issues for research. For example, a crucial issue is the need for cross-disciplinary approaches and frameworks for informing issues in teaching and learning of mathematics in multilingual classrooms. Although in the past researchers looked elsewhere for ways to approach this issue (e.g., Cummins’ work), Barwell not only illustrates this well, but by using new perspectives for mathematics education research from linguistics he revitalizes and informs our area anew. Likewise, Halai and Muzaffar raise significant social justice issues when they examine a ‘policy of language of instruction’ that takes into account issues of redistribution of cultural capital, but does not necessarily take into account issues of cognition and learning of mathematics. These contributions suggest that the mathematics education research including in multilingual settings needs to shift its somewhat inward looking stance and be open to fresh ways of advancing our research, but without losing sight of the main aim.

The issues of mathematics teaching and learning in multilingual contexts as noted in this volume are strongly located in the dynamics of a highly globalized society of the 21 century, especially the issue of rapidly changing demography in the wake of rapid immigration and urbanization (Atweh, Clarkson, & Nebres, 2003). However, a significant aspect of the global society of 21st century is that of an increasingly technological world. What is the interface of technology, teaching and learning in the context of multilingual classrooms (see Borba, Clarkson, & Gadanidis, 2013 for a small beginning in this area)? These are further issues, among others, that need to be investigated.

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2. MATHEMATICS EDUCATION AND LANGUAGE DIVERSITY

Past, Present and Future

INTRODUCTION

There is a growing body of research on mathematics education and language diversity and increasingly this research is published in international mathematics education journals as well as linguistics journals focusing on language and education. The first journal paper on mathematics and language diversity to be published in an international mathematics education journal appeared in 1979. The paper, entitled "Language and mathematical education", was authored by Austin and Howson and published in Educational Studies in Mathematics (ESM). ESM is the oldest English international mathematics education journal, which was first published in 1968. An interesting question to ask is why the first journal paper on mathematics and language diversity was only published in 1979.

This Chapter provides a brief review of research on mathematics and language diversity internationally. The review focuses on research published in selected key international journals and was guided by the following questions:

- What research has been published in this area of study internationally?
- What contribution has this research made to our understanding of the complexities of teaching and learning maths in contexts of language diversity?
- What are the gaps and silences visible in research in this area?

The phrase language diversity is used to refer to contexts in which any of the participants (learners, teachers or others) are potentially able to draw on more than one language as they go about their work. The presence of these languages, however, does not necessarily mean that language diversity is recognised as an asset in that context. I deliberately use the phrase language diversity rather than bilingualism or multilingualism to highlight the significant differences between what I refer to as the politics of bilingualism and politics of multilingualism. While multilingualism is about inclusion and recognition of all languages, bilingualism is about competition between two languages to the exclusion of others. In all the contexts that are labelled as bilingual there is an existence of other languages that are wittingly or unwittingly silenced. For a detailed discussion on this matter see Phakeng (forthcoming).

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 11–23.

I begin this Chapter with a discussion of research on language and learning published before 1979. What follows is a brief background on how this discussion began in mathematics education. Here I highlight the important role that the second International Congress on Mathematical Education (ICME-2) held in the United Kingdom in September 1972 as well as the international symposium on "Interactions between linguistics and Mathematical Education" held in Kenya in 1974 played in shaping the debates. While the review presented in this Chapter does not include conference papers, I specifically focus on these two conferences because they gave the impetus for the Austin and Howson paper published in ESM in 1979. These discussions provide a theoretical context for what follows: a description of the methodology used for the review and an analysis of research done in this area of study internationally. From these bases I highlight gaps and possibilities for future research.

Setting the Scene: Research on Language and Learning before 1979

While the first paper on mathematics education and language diversity was only published in 1979, there were extensive debates among researchers and educators about the effects of bilingualism on the learner before then. Many of these debates happened in psychology journals and books (e.g., Child development) while there was silence in mathematics education journals. There are authors who argued that bilingualism has negative effects on language development, educational attainment, cognitive growth and intelligence (Reynold, 1928; Saer, 1963 both cited in Grosjean, 1982). Others argued that under certain conditions bilingual skills can have positive effects on the learning process (Ianco-Worrall, 1973; Been-Zeef, 1977; Pearl & Lambert, 1962).

A great majority of studies completed before 1979 concluded that bilingualism had negative effects on learners' linguistic, cognitive and educational development. Bilingualism was seen as unnatural and it was argued that a bilingual child hardly learns either of the two languages as perfectly as he would have done if he had limited himself to one. There was also a widespread view that the brain effort required to master two languages instead of one diminishes the child's power of learning other things, which might and ought to be learned. Leo Weisgerber (1933 in Saunders, 1988), a highly regarded German linguist, argued that bilingualism could impair the intelligence of a whole ethnic group, while Reynold (1928 in Saunders, 1988) was concerned about the fact that bilingualism leads to language mixing and language confusion which in turn results in a reduction in the ability to think and act precisely, a decrease in intelligence, an increase in lethargy and reduced self-discipline. From his study of Welsh-English bilingual children in rural areas Saer (1923) concluded that bilingual learners had lower IQ scores than monolingual children, and this inferiority became greater with each year from age seven to eleven. Saunders (1988) warned, however, that caution must be exercised when comparing monolinguals and bilinguals on tests of intelligence, particularly on the tests of verbal intelligence, and particularly if, as often happens, the bilinguals are tested in only one of their languages, perhaps the second language.

It was in 1962 when Pearl and Lambert conducted a study that indicated that bilingualism is an asset to the child. They studied the effects of bilingualism on the intellectual functioning of ten year-old children from six Montreal schools. They found that instead of suffering from 'mental confusion' bilinguals were profiting from a language asset. They concluded that:

> Intellectually (the bilingual's) experience with two language systems seems to have left him with a mental flexibility, a superiority in concept formation, and a more diversified set of mental abilities, in the sense that the patterns of abilities developed by bilinguals were more heterogeneous. It is not possible to state from the present study whether the intelligent child became bilingual or whether bilingualism aided his intellectual development, but there is no question about the fact that he is superior intellectually. In contrast, the monolingual appears to have more unitary structure of intelligence, which he must use for all types of intellectual tasks. (Pearl & Lambert, 1962, p. 20)

Although these results were criticised on the grounds that only the intellectually brighter children were chosen for the bilingual group (e.g., by Macnamara, 1966), the studies that followed also indicated that bilingualism is an asset. Ianco-Worrall's (1972) study of Afrikaans-English four to nine year-old bilingual children in South Africa showed that bilinguals reach a stage in semantic development two or three years earlier than their monolingual peers. They analyse language more intensively than do monolinguals. Been Zeef (1977) found the same results in a similar study with Hebrew-English bilinguals and monolingual English and Hebrew children. Bilinguals realise sooner the arbitrary nature of language because the link between a word and its meaning is less strong in bilinguals than in monolinguals. This result had some implications for the bilinguals' cognitive abilities. As Cummins (1981, p. 33) argued, the ability to separate the meaning of a word from its sound is necessary if a child is to use language effectively as a tool for thinking.

In 1979, Swain and Cummins compared the positive and negative studies and concluded that the positive findings are usually associated with majority language groups in immersion programs. In such cases there is a high value attached to knowing two languages. The second language is added at no cost to the first and the parents are of relatively high socio-economic status. Negative findings, on the other hand, are found with submersion students who are surrounded by negative attitudes. They are forced to learn the majority language and are not encouraged to retain their first language. They also do not live in a social environment that is conducive to learning. Swain and Cummins also argued that while there were a variety of factors impacting children's intellectual development, bilingualism was one of the significant factors that could have a positive impact. While research in this area of study at this stage did not foreground the role of the social, it is clear that there was an acceptance that it is possible that bilingualism per se might have no necessary effects (either negative or positive) on the cognitive and intellectual development of children in general. What may account for the contradictory results reported in the literature during this period are the psychosocial differences between bilinguals and monolinguals, and not bilingualism per se.

The Beginning of the Conversation in Mathematics Education Journals

During the second International Congress on Mathematical Education (ICME-2) held in the United Kingdom in September 1972, the need for fundamental research on the relationship between the learning of basic mathematical structures and the language through which they are learnt was highlighted as critical. It was as a result of this ICME-2 decision that an international symposium on “Interactions between linguistics and Mathematical Education” was held in Nairobi, Kenya from 1st to 11th September 1974. The symposium was sponsored by UNESCO in cooperation with the International Congress on Mathematical Instruction (ICMI) and the Centre for Educational Development Overseas (CEDO). Prior to 1974, it seems that there were no formally organised international conferences focusing exclusively on the relationship between mathematics and language. The Symposium highlighted the lack of research on the relationship between language and mathematics and concluded that difficulties in mathematics learning depend on the language of learning. It further affirmed that all languages include linguistic features of benefit for the acquisition of mathematical concepts and thus can be used for mathematics teaching and learning.

One of the issues that the symposium highlighted is the fact that the problems of learning mathematics in an additional or foreign language are not peculiar to learning in a world language such as English or French because there are many other countries such as Tanzania and India, where many learners have to learn mathematics in a national language (e.g., Kiswahili, Hindi) which is not their home language. This practice still continues and increasingly so in European countries that do not have any of the now world languages as the main language (e.g., Spain, Italy) and are experiencing the pressure to ensure that their learners are fluent in at least one of the world languages. In my view this is an important matter that remains a gap in research in this area of study. So far research published in the selected journals focuses on bilingual and multilingual contexts and not yet on the specificity of trilingual contexts where learners are exposed to a home language, national language and official language. The specificity of trilingual contexts in mathematics teaching and learning lies in the fact that unlike in multilingual contexts where there is a presence of multiple languages but only two languages (home language and LoLT) that are in competition, learners in trilingual contexts have to deal with three languages, each of which has its own power and influence – one as a home language, the second as a national language and the third as a world language.

The paper published by Austin and Howson in Educational Studies in Mathematics in 1979 was a follow up on the Nairobi symposium and it concludes that the challenge of language and mathematics learning and teaching is not just an issue for developing countries but for the whole world. In developing countries the challenge is that of learners learning mathematics in a language that is not their mother tongue; in developed countries such as Wales, the USA, Belgium and Canada there are communities of immigrants with well-established ‘minority’ languages and in some countries there are instances where problems arise because of the non-standard nature of the local vernacular (e.g., Jamaica, England, USA, etc.). Austin and Howson acknowledged the fact that bilingualism is a political matter and thus change in society may lead to policy change. Indeed much has changed since 1979: the world has become more multilingual and some countries have changed their language policies and practices, which makes this review timely and relevant. The section that follows focuses on the methodology used in this review – essentially, where and how relevant research published was identified.

METHODOLOGY

Research on mathematics and language diversity is published in mathematics education journals as well as linguistics journals focusing on language in education. In completing this review it was thus important to consider journals across these disciplines. Focusing specifically on published research in journals means that other research that is completed on mathematics and language diversity was excluded because it is not published in the selected journals. The decision to focus only on research published in specific journals was influenced by the need to pay attention only to work that has gone through a rigorous process of review and published in generally recognised leading journals in mathematics education international.

In identifying papers focusing on mathematics education and language diversity, there were also papers focusing broadly on different aspects of language and communication in mathematics education, for example work of Pimm, Pirie, Morgan, Rowland and others. These papers are excluded from the review because they do not focus specifically on language diversity in mathematics education, but on the nature of the mathematical language or ways of communicating mathematically. The Table 1 provides details of the journals selected for the review, the year of inception of the journal as well as the number of papers identified as relevant for the review.

The main limitation of this methodology is that it covers only international journals that only publish in English and thus excludes authors who do not write in English as well as research conducted in regions where English is not the language of research. Table 2 shows how the number of publications has increased per decade since the seventies.

Most of the research completed in this area of study is empirical and the data is analysed qualitatively. The section that follows explores the content of the research that has been published, its contribution as well as the gaps and possibilities for future research.

Table 1. Details of journals selected for the review

	Name of Journal	*Year of inception*	*Number of papers*
Mathematics Education Journals	Educational Studies in Mathematics (ESM)	1968	18
	Journal of Research in Mathematics Education (JRME)	1970	6
	For the Learning of Mathematics (FLM)	1980	8
	Mathematics Education Research Journal (MERJ)	1989	9
	International Journal of Science and Mathematics Education (IJSME)	2000	2
	Sub-Total		43
Linguistics Journals	Journal of Multilingual and Multicultural Development (JMMD)	1980	0
	Language and Education	1987	5
	International Journal of Bilingual Education and Bilingualism (IJBEB)	1998	2
	International Journal of Multilingualism (IJM)	2000	1
	Sub-Total		8
Total			51

Table 2. The number of papers published per decade

Period	*Number of articles published*
1970 – 1979	1
1980 – 1989	6
1990 – 1999	11
2000 – 2009	25
2010 – 2012	8
Total	51

REVIEW OF RESEARCH IN THIS AREA OF STUDY

Table 3 tabulates the most dominant topics or themes that the research has focused on. In order to systematise the review of the papers I developed a framework for looking at the papers. I looked at the journal in which the paper is published, the author, level (i.e., primary/secondary/tertiary), central problem, research approach and the arguments the paper is making. This enabled me to look across the papers and it also made visible the themes and trends emerging from the review. While on the surface it may seem unproblematic to decide which paper focuses on one theme rather than another, in practice the distinctions were more complex. So in deciding on the theme I focused more on the central problem that the paper is addressing rather than issues that come up in the process of the exploration. For example, while Moschkovich (1999) refers to the practice of code-switching, the central problem that the paper is exploring is how teachers can support the participation of English Language Learners in mathematical discourse.

Table 3 shows in brief what research has been undertaken in this area of study. It is not surprising that learner performance has the highest number of papers published because the concern with the performance of learners who learn mathematics in a language that is not their home language is at the core of most of the research completed in this area of study. As I argued elsewhere, at the core of this concern is the need to address the uneven distribution of mathematical knowledge and success (see Setati, 2012). Studies that focused on learner performance compared the performance of learners who learn mathematics in their home language and those

Table 3. Research topics covered in the papers published

Research topics/themes	*Number of papers*		
	Mathematics Ed Journals	*Language journals*	*Total*
Code-switching	8	3	11
Teachers supporting bilingual or multilingual learners	6	0	6
Learner performance	18	3	21
Curriculum planning & Development	4	0	4
Policy	1	1	2
Learner participation	1	1	2
Conversation between researchers from the north and the south	2	0	2
Research Methodology/theory	2	0	2
Research Review	1	0	1
Total	43	8	51

who learn in a language that is not their home language. Research concluded that poor performance is due to lack of understanding the language of the test (Adetula, 1989; De Courcy & Burston, 2000; Evans, 2007; Farrell, 2011; Llabre & Cuevas, 1983; Ni Riodan & Donoghue, 2009; Zepp, 1982). What we have learned from this research is that for the performance of learners who learn mathematics in a language that is not their own to improve it is important that the language, culture and the logic or reasoning system of the learner should match with that of the teacher, the textbook and the curriculum (Berry, 1985; Evans, 2007; Zepp, 1982). Recent research suggests that competence in both the home and the language of learning and teaching (LoLT) can be an advantage in mathematics achievement (Clarkson, 1992; Clarkson & Galbraith, 1992). While Farrell (2011) and Gerber, Engelbrecht, Harding, and Rogan (2005) caution that causal relationships should never be assumed when it comes to the relationship between language fluency and learner performance; he agrees with Clarkson that competence in the home language and the LoLT has a bearing on learner performance. These findings encourage bilingualism and in many ways are at odds with those of the sixties, which positioned bilingualism as a problem.

Research in this area of study does not only encourage bilingualism but also argues for the development of the learners' home languages as a strategy to motivate them to succeed in mathematics (e.g., Barton, Fairhall, & Trinick, 1998). While encouraging the development and the use of the home languages may be an ideal for many countries, it is due to the hegemony of what is regarded as the language of power (e.g., English) that the use of code-switching to support learners has become a common practice in many classrooms all over the world (Adler, 1998, 1999; Barwell, 2003a, 2005; Clarkson, 2007; Heng, 2006; Khisty & Chval, 2002; Lim & Presmeg, 2010; Moschkovich, 1999; Planas & Setati, 2009; Setati, 1998; Setati & Adler, 2000, Setati, 2005). This is mainly because teachers are trying to ensure that while they use the learners' home languages to support learning they do not disadvantage their learners by not ensuring that they have access to English, which is seen as a language of international communication.

The research theme/topic that has the least number of papers in Table 3 is the one on reviews. This is because there has not been a review since the 1979 paper by Austin and Howson that provides a bibliography indicating the wide variety of relevant articles and books in this area of study. The other categories that have fewer than five papers published are the category on research methodology/theory, north-south conversations, policy issues and learner participation. The first paper in the category on methodology/theory highlights the fact that research in mathematics education is mainly published in English and discusses how this may discriminate on the basis of language use both within the community of researchers and in the practice of research (Barwell, 2003b). Discrimination here refers to differential opportunities afforded for using language with resultant effects of unequal access to power and resources. Barwell (2003b) observes that most of the research in mathematics education is carried out in multilingual settings and thus the languages and the language practices in such settings influence findings of the research even if it is not exploring issues of language.

What is most interesting is the fact that the two publications that focus on issues of language policy are both based on the Malaysian experience (Heng & Tan, 2006; Lim & Presmeg, 2010). These papers are as a result of the language policy changes that happened in Malaysia, which implemented its new education policy of teaching mathematics and science in English in 2003 in a move to keep abreast with global developments and have greater access to science, technology and business knowledge. The research was mainly to understand the impact that this new policy has on classroom practice and to find out how teachers were dealing with the challenges of teaching mathematics in English. Given the recent (2011) switch again in Malaysia on language policy, it might be anticipated that further studies will be undertaken to track its impact on learning and teaching mathematics. It is interesting that while policy changes also happened in several countries in Africa during the nineties none of the papers focusing on policy were published in the linguistics and mathematics education journals selected for this review.

The papers in the north-south conversations category focus on interactions between researchers in South Africa, Britain and the USA about language diversity issues in mathematics education (Barwell & Setati, 2005; Phakeng & Moschkovich, 2013). The papers specifically compare how some mathematics teachers and learners in the different countries deal with the complexities of learning and teaching mathematics in linguistically diverse classrooms. On the one hand, Barwell and Setati (2005) foreground code-switching as a common practice in multilingual classrooms in South Africa, but it is never used in UK classrooms. On the other hand, Phakeng, and Moschkovich (2013) raise two important issues that until then had not been attended to by research in this area of study. First, is the fact that while research in this area of study refers explicitly to language and culture, it does not foreground race. There is no doubt that language plays an important role in the social construction of race, racism and racial identity in mathematics classrooms and thus interesting that research in this area of study has ignored these important links in its analyses. The second issue is the fact that research in this area of study in the USA refers to bilingualism despite the multilingual nature of the country and the classrooms. While the political agendas of bilingualism are different from those of multilingualism, it is clear that research in this area of study uses the two labels as a proxy for race and socio-economic status.

It is perhaps important at this stage to indicate that research in this area of study has tended to treat bilingualism as a form of multilingualism, which is convenient but problematic because it ignores the different political agendas of bilingualism and multilingualism. It is often true that in contexts that are regarded as bilingual are in fact multilingual but foreground two dominant traditions that are in competition. For example Canada is regarded as a bilingual country, with English and French as official languages, despite the fact that there are indigenous people who speak a variety of languages that are never counted. A bilingual language policy is often used as an apparatus of politics to appease two competing language traditions. These politics inevitably shape language choices, and language use in mathematics classrooms in these countries. It is Adler (1997, 1998, 1999) and Setati (1998) who introduced multilingual mathematics classrooms through their publications, which came out in the nineties. This move has also shaped the thinking in this area of study internationally.

What Are the Gaps and Silences Visible in Research in This Area?

While research in this area of study conducted in the USA and Europe involves immigrant learners, most of it does not focus on the specificity of this group of mathematics learners. In my view this is a weakness because as Planas and Gorgorio (2004) argue, challenges faced by immigrant mathematics learners in linguistically diverse classrooms are different from those faced by other learners. While the challenges faced by other learners may be limited to language fluency, immigrant learners also have to deal with issues of cultural, political and linguistic identity. As Kazima (2007) argues, in addition to language, learners bring different cultural practices that are relevant for their mathematics learning. Thus to focus only on how the language of their new country shapes their mathematics learning does not give a full understanding of the challenges that immigrant learners have to deal with. Furthermore, research conducted in developing countries has so far not focused much on immigrant learners and thus gives an impression that immigrant learners are only a feature of mathematics classrooms in developed countries, while in fact there are immigrant learners all over the world.

There is a dearth of research in this area of study focusing on teacher education. Only two papers were identified as focusing on teacher education, however, were not categorised as such because the focus of their analysis was not on how the teacher educators support their learners (Stacey & MacGregor, 1991; Chitera, 2009). While the participants in Chitera's research (2009) were teacher educators the paper essentially focused on code-switching as a practice in teacher education classrooms, hence it was listed under code-switching. The second paper focuses on immigrant pre-service teachers in Australia with limited English language skills (Stacey & MacGregor, 1991). The authors highlight these teachers' limitations when teaching mathematics in English and then argued that they need to be provided with opportunities to develop and improve their language skills during teacher education.

While research in this area of study continues to grow, very little of it focuses on how mathematics teachers should deal with the complexities of teaching and learning mathematics in linguistically diverse classrooms. While research focuses on the analysis of what currently is, teachers on the ground continue to hope and ask for what could or should be. Herein lies another opportunity for further research.

CONCLUSION

This paper has given an overview of research in mathematics and language diversity. It has specifically focused on the development of research on mathematics education and language diversity, highlighting significant advances, findings, gaps and future research directions. It has further highlighted not only the paucity but also the slow growth of research in this area of study – 51 papers were published in the selected international journals between 1979 and 2012. This is clearly a slow growth that also signal the small number of researchers worldwide working in this area of study. Elsewhere I have argued that this area of research is politically charged with interdisciplinary demands as well as the need for multilingual research teams. This is perhaps what accounts for the slow growth and hence the challenges are not just about growing knowledge in this area of study but also about growing capacity.

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3. MATHEMATICS EDUCATION, LANGUAGE AND SUPERDIVERSITY

INTRODUCTION

There is a growing awareness in the social sciences that globalisation has changed the nature of 'diversity'. The old certainties of distinct ethnic groups, with distinct cultures, speaking distinct languages and living in distinct communities in specific locations are increasingly untenable. This shift is as applicable in mathematics classrooms as anywhere else and is particularly relevant to research on the teaching and learning of mathematics in multilingual classrooms (Clarkson, 2009a). The literature on teaching and learning mathematics in multilingual classrooms includes many detailed accounts showing the wide range of forms of multilingualism that have been documented in mathematics classrooms, and the linguistic complexity of each one of these situations (for some examples, see Cocking & Mestre, 1988; Barwell, Setati, & Barton, 2007; Barwell, 2009a). It is important to consider whether the underlying assumptions about language and multilingualism that frame this work are still relevant in the light of these new kinds of diversity.

What, then, has changed? One consequence of globalisation is the huge increase in the number of categories needed to talk about any given slice of society (Vertovec, 2007). Labels based on nationality, language, ethnicity, race, religion and so on, no longer necessarily apply uniformly or constantly to identifiable communities or in particular locations. Individuals may identify with several nationalities or racial groups and may speak combinations or mixtures of several languages. Vertovec (2007), whose research is focused particularly on migration, referred to this apparently new situation as 'superdiversity.' While this idea has emerged from research on migration, particularly as it affects societies in developed countries (Vertovec's work is about the UK), it is applicable very widely. Indeed, from a historical perspective, there is scarcely anywhere that has not been affected by migration at some point.

Research in applied linguistics, and particularly in linguistic ethnography or contemporary linguistic anthropology has noted a similar transformation in the nature of language and its place in society (e.g., Blommaert, 2010; Blommaert & Rampton, 2011; Blackledge & Creese, 2010, though for a prescient analysis that informs much of this work, see Vološinov, 1986). This transformation has not simply influenced our understanding of how people use language or languages; it has led to changes in how the nature of language is understood. Blommaert and Rampton (2011) identify three aspects to this shift:

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 25–39.

> Over a period of several decades – and often emerging in response to issues predating superdiversity – there has been ongoing revision of fundamental ideas (a) about languages, (b) about language groups and speakers, and (c) about communication. Rather than working with homogeneity, stability and boundedness as the starting assumptions, mobility, mixing, political dynamics and historical embedding are now central concerns in the study of languages, language groups and communication. (p. 3)

The purpose of this chapter is to examine research on multilingualism in mathematics classrooms in relation to these recent changes in sociolinguistics in the context of superdiversity. I show how some of the assumptions in this research are compatible with a superdiversity perspective, while others are open to critique.

The chapter is organised as follows: in the next three sections, I summarise key points relating to Blommaert and Rampton's three sets of fundamental ideas: about language, about language groups and speakers, and about communication. In each case, I explore how these ideas are relevant to research on the teaching and learning of mathematics in multilingual classrooms, drawing on examples of such research. The final sections of the chapter consider the implications of this analysis for research methodology, educational policy and mathematics teaching practice.

ABOUT LANGUAGES

What is a language? The concept of language is often treated as a category of the natural world, akin to species of plants or animals. Linguists have organised languages into family trees that resemble the taxa of biology, showing the relationships between contemporary languages and tracing their historical evolution from earlier languages. Hence, French, Italian and Spanish are understood to be Romance languages, all related and all derived from Latin. Latin and its descendant languages are themselves part of a broad Indo-European language group that includes most languages spoken across contemporary Europe, the Middle East, Central Asia and the northern part of South Asia. This language group can in turn be traced back to a single hypothetical 'proto-Indo-European' language from some time in prehistory (see any reasonable textbook on the history of languages; for example, Jansen, 2012).

This view of language is widespread and clearly reflects our experience of language to some extent. As a theoretical position, however, it is problematic. In particular, it treats languages as autonomous entities, like natural organisms. Pure languages can be identified and described, while new languages arise from interbreeding between these pure forms. In academic linguistics this view can be traced back to the establishment of the field in nineteenth-century Europe, at a time when modern European nation-states were in the process of emerging from a more disparate collection of territories and language varieties. Languages were associated with these new nation-states; they played an important ideological role in the creation of a single people with a single language (e.g., German in Germany, Italian in Italy, French in France). This perspective also informed

European colonisation, with colonial administrators and missionaries combining to enumerate and describe the languages of their subjugated territories, the better to manage them and convert the people to Christianity (see for example, Pennycook & Makoni, 2005).

These developments in linguistics continued into the twentieth century, with Saussure's work perhaps the most well-known. Saussure (1974) proposed a strict theoretical division between language as a system, which he called 'langue' (the French for tongue or language) and language as used, which he called 'parole' (the French for spoken or written expression). Until the mid-20th century, linguistics studied langue, seeking to describe, analyse and understand the structure of different languages and the structural relationships between them. This perspective is apparent in the alphabetisation of many previously unwritten languages, the creation of dictionaries and grammar guides and other forms of language description and prescription.

In the second half of the twentieth century, linguistics has increasingly focused on language as it is spoken and written, leading ultimately to a profound critique of the perspective described above. Bakhtin (1981), for example, proposed a view of language with two poles, which he referred to as 'unitary language' and 'heteroglossia'. These two poles are similar to Sassaure's language and parole. However, for Bakhtin, *unitary language* referred not to language in terms of structure, but to the ideology that languages are ever fixed and complete. Bakhtin contrasted unitary language with the diversity of language as it is used: the languages of different places, times, activities, speakers and so on, for which Bakthin used the term *heteroglossia*. Arguably, for Bakhtin, it is this heteroglossia that is the reality of language, rather than any underlying structure. Heteroglossia is not simply diversity, however; Bakhtin develops a complex theoretical position in which different varieties of language-in-use dynamically interact and influence each other. The idea of unitary language is ever-present, however, and influences how language is used, even as heteroglossia exerts a diversifying pressure for variation.

More recently, Makoni and Pennycook (2007) have extended the idea of a unitary language ideology, arguing that languages are 'inventions.' At a basic level, the idea of invention signifies that no utterance ever precisely reflects the dictionaries and grammars that purport to describe any given language. Makoni, however, is also thinking at a socio-ideological level. In his work, linguistic description is better understood as linguistic invention: by describing a language, a particular version is frozen and is subsequently taken to be the definitive account of that language. Pennycook and Makoni (2005) describe, for example, the rather arbitrary separation and naming of languages in southern Africa by colonial linguists. These invented languages then obscure the many diverse varieties of ways of speaking and communicating that heteroglossia entails, particularly in the context of superdiversity, in which many languages are used in different and changing ways, often mixed together, by different people and for different purposes within relatively localised activities or locations.

This emerging critique of the concept of language is summarised by Blommaert and Rampton (2011):

> The traditional idea of 'a language', then, is an ideological artifact with very considerable power – it operates as a major ingredient in the apparatus of modern governmentality; it is played out in a wide variety of domains (education, immigration, education [*sic*], high and popular culture etc.), and it can serve as an object of passionate personal attachment. But as sociolinguists have long maintained, it is far more productive analytically to focus on the very variable ways in which individual linguistic features with identifiable social and cultural associations get clustered together whenever people communicate […] If we focus on the links and histories of each of the ingredients in any strip of communication, then the ideological homogenization and/or erasure achieved in national language naming becomes obvious, and a host of sub and/or transnational styles and registers come into view, most of which are themselves ideologically marked and active (Agha, 2007). Instead, a much more differentiated account of the organization of communicative practice emerges, centring on genres, activities and relationships that are enacted in ways which both official and commonsense accounts often miss. (p. 4)

A key point in Blommaert and Rampton's account is that while the idea of language is widespread and perfectly reasonable in everyday life, it has become *analytically* problematic: it is often difficult to examine interaction and clearly distinguish different languages; and such distinctions often turn out to be less significant than other forms of difference, such as those of accent, or style or genre.

How can this critique of the notion of language inform research on multilingualism in mathematics education? First, a great deal of research in this area is implicitly based on a unitary language ideology. It is, for example, more or less standard practice to describe multilingual mathematics classrooms in terms of the different named languages that the students speak. While this practice may seem natural, it obscures any 'unnamed' languages or ways of speaking on which students may also draw. Some researchers have gone further, and attempted to categorise different multilingual mathematics classroom contexts. In my own earlier work, I have used a framework borrowed from applied linguistics (Barwell, 2005a) and Clarkson (2009b) has made similar comparisons. This kind of approach becomes increasingly difficult to sustain, however, in the light of the strong diversification of language use in any give context. For example, in a current research project of my own,[1] I set out to compare students' participation in mathematics classroom interaction in a range of different multilingual settings in the Ottawa area of Canada. In Canada, both French and English are official languages and schooling is available in both languages, although through separate school systems. One mathematics classroom in the study was included as an immersion setting: students who are speakers of English are taught in French, so that they learn French. In fact, for many students in the class from immigrant backgrounds, English is also an additional language. Similarly in another class in the study, students considered to be learners of English as a Second Language (ESL) include recent immigrants, second-generation immigrants, francophone Canadians or are from Canada's First Nations—and these backgrounds are not mutually exclusive. Hence the clarity of clear, language-based distinctions between different multilingual contexts quickly becomes rather messy and blurred.

Second, a unitary language approach leads to an analytic focus on distinct languages. In particular, interest in the practice of code-switching is derived from a unitary language perspective that sees languages as discrete and distinct. Code-switching is defined as the use of two languages within a single interaction and, as such, has been noted as a highly prevalent phenomenon in some multilingual mathematics classrooms (e.g., Setati, 1998, 2005; Farrugia, 2009; Halai, 2009; Adler, 2001; Planas & Setati, 2009; Parvanehnezhad & Clarkson, 2008). Research has shown how the use of different languages may have different functions in the learning of mathematics (Setati, 2005); how mathematics teachers may not be aware that code-switching is happening (Setati, 2005; Parvanehnezhad & Clarkson, 2008); and how code-switching can lead to dilemmas for mathematics teachers (Adler, 2001). Historically, code-switching has been seen as a degenerate language practice and mathematics educators have been at pains to counteract such negative associations in the context of mathematics classrooms (e.g., Moschkovich, 2002).

The unitary language perspective means, however, that while code-switching is highly salient, other practices or situations are obscured. For example, in many multilingual mathematics classrooms, only one language may be heard, despite the wide range of language practices that students draw on in the context of their broader multilingual lives (see, for example, Setati & Barwell, 2006). A strong focus on distinct languages may therefore lead to the arbitrary separation in research and policy of multilingual mathematics classrooms in which only one language is used, from those in which two or more are used.

The unitary language ideology is also apparent in models designed to map how students may come to learn to do mathematics in the required language of instruction. Both Setati and Adler (2000) and Clarkson (2009b) have proposed such models, which see students moving from informal expression of mathematical thinking to formal mathematical language, from oral expression of mathematical thinking to written expression and from a non-official language to the language of instruction. For example, Clarkson (2009b) discusses how a student in Papua New Guinea might move from expressing basic fraction concepts in their local village language, but is encouraged (or expected) to quickly move to the national language and then to English. As with the focus on code-switching, such models make other language practices less visible (despite the use of double-headed arrows). In particular, practices that involve drawing on several languages at once do not fit neatly into the boxes in the model.

Finally, a unitary language perspective is apparent within mathematics itself. In particular, the notion of 'mathematical language' or 'the mathematical register' (in whichever language) suggests a standard, fixed way of speaking and writing about mathematics. In multilingual contexts, as Setati (2008) has pointed out, 'mathematical language' may be associated with a particular prestige language, such as English. Mathematical language is not, however, uniform, but, like language in any domain, varies according to the situation (Moschkovich, 2002). While many researchers accept this point, the discourse of *the* (i.e., a single) mathematical register is powerful and is often apparent in mathematics curricula in particular (Barwell, 2013).

ABOUT LANGUAGE GROUPS AND SPEAKERS

Just as a superdiversity perspective includes a critique of prevailing ideas about languages, a parallel and inter-related argument applies to speakers, whether individually or in groups. Under this heading, attention is turned to concepts such as 'native speaker', 'mother tongue' and 'speech community' (Blommaert & Rampton, 2011). Much as a unitary language perspective reifies languages as distinct, uniform entities, speakers have also been idealised. The concept of the native speaker, for example, is based on assumptions of growing up in a linguistically and culturally homogenous community, in which a single language is acquired, primarily through family and community interactions. For native speakers, the language they grow up speaking is considered to be their mother tongue, the link with the mother indicating the importance of the language spoken within the family, and particularly by mothers, as the most significant in a child's development. While this may be true, the emphasis on a single language in this characterisation is increasingly questionable. It is apparent that speakers can and do draw on a range of languages and varieties of language for the range of social activities in which they participate (Blackledge & Creese, 2010). Moreover, children growing up in the context of superdiversity are socialised into this complex range of practices, rather than into a single mother tongue. It is therefore more productive to understand these different sets of practices as *repertoires*, on which speakers draw according to the situation. The advantage of this approach is that it:

> dispenses with a priori assumptions about the links between origins, upbringing, proficiency and types of language, and it refers to individuals' very variable (and often rather fragmentary) grasp of a plurality of differentially shared styles, registers and genres, which are picked up (and maybe then partially forgotten) within biographical trajectories that develop in actual histories and topographies. (Blommaert & Rampton, 2011, pp. 4–5)

This position also challenges the notion of proficiency, at least when it is understood in absolute terms: that is, the idea that the goal of every language learner is to develop a 'native-like' level of proficiency.

The critique of the native speaker can also be extended to much educational language policy. For example, in the UK, many students are categorised as 'additional language leaners' and provided with additional support, even while they join mainstream classrooms, while elsewhere, terms such as English as a second language (ESL), English Language Learner (ELL) or Non-English-Speaking Background (NESB) have been used. Leung et al. (1997) highlight three assumptions on which such an approach is based:

1. that linguistic minority pupils are, by definition, bilingual, having an ethnic minority language at home while at school they are learning and using English;
2. that these pupils' language development needs can be understood and categorised broadly in the same way; that is, there is a universal L2 [i.e., second language] learner phenomenon, which, since the 1960s […] has been conceptualised as someone learning English as a social and linguistic outsider; and
3. that there is an abstracted notion of an idealised native speaker of English from which ethnic and linguistic minorities are automatically excluded. (pp. 545–546)

Leung et al.'s critique, along with the more recent work discussed above, challenges each of these assumptions, and highlights the complex interactions of linguistic repertoires, identity and social change. Hence many students: grow up speaking multiple languages rather than a single mother tongue; learn and develop language in different ways; have a range of strengths and needs with respect to schooling; identify themselves with different languages at different times; and along with all of this, may easily be 'native' to the place in which they live and go to school.

Much of this reorientation in the notion of native speakers and mother tongues raises questions about research conducted in multilingual mathematics classrooms. The characterisation of multilingual learners as second language learners and, hence, as outsiders is apparent in much of this research (see Barwell, 2003). In contexts of immigration, the application of Leung et al.'s (1997) critique is fairly clear. Research in mathematics education has tended to go along with the assumptions they highlight. Students are described largely in terms of the language that they do not speak (usually English) and are assumed to learn this language in a uniform, fairly standard way (for example, see Planas & Civil, 2013; Clarkson, 2007; Barwell, 2003, 2012). In particular, the question of language proficiency plays an important role in research that seeks to understand whether language learners (perhaps a nativist term) do better or worse in learning school mathematics. Clarkson (1992, 2007; Clarkson & Galbraith, 1992), for example, sought such a relationship, drawing on Cummins's (2000) threshold hypothesis. A similar approach was adopted by NíRíordáin and O'Donoghue (2009), who analysed the mathematics performance of students in Irish immersion programs. In both cases, the research is based on the assumptions that the students speak two distinct languages, and that it is possible to measure the proficiency in each of these languages separately. These studies undoubtedly uncovered a relationship between language proficiency and mathematics performance, supporting Cummins's hypothesis that low proficiency in either language would lead to lower performance in subjects like mathematics.

Clarkson (2007) also offers some evidence that high proficiency in two languages is related with higher than average performance in mathematics. However, this approach does not look in any depth at the context of the students' language use. In many cases, for example, testing each language separately may not reflect their daily experience of working with a repertoire of several languages and varieties. The research still uncovers a relationship, however, because schooling is also based on these same assumptions. The danger is that some students are seen as having a language deficit, when, in fact, this deficit is constructed by the particular structures and assumptions of the school system.

The critique of the concept of mother tongue has been recognised to some extent by researchers in some mathematics education contexts. For Setati, who is from South Africa, the notion of a mother tongue makes little sense, since almost everyone in South Africa grows up speaking multiple languages (Barwell & Setati, 2005). Nevertheless, her research provides evidence that the othering effect of an idealised view of native speakers and second language learners still operates. In particular, her research shows that students and teachers perceive a choice: mathematics can be taught in English or in learners' home languages (i.e., one of nine 'African' languages with official status in South Africa – although there are many other languages that do not have this status). Most students and teachers accept that teaching mathematics in the students' home languages will mean that they are likely to develop a deeper understanding of the subject. But English offers other advantages, including access to jobs, higher education and the upper middle class, even if this is at the possible expense of a good understanding of mathematics. Of course, this choice has an ideological dimension, since it implies that only one language can be used to teach and learn mathematics, when it may be possible to use more than one or to use mixtures of languages (Setati et al., 2008). Indeed, while the use of English is, in the end, preferred, in practice, multiple languages may in fact be used, even if their use is not fully acknowledged (e.g., Setati, 2005). From this perspective, students are positioned as 'learners of English', indeed, as 'second language learners' of English. This positioning 'others' the students: in effect they are 'non-native' speakers of a language that is widely used in the country in which they may always have lived. Since English is also seen as 'the language of mathematics' (Setati, 2008), students are framed in deficit terms in relation to the learning of mathematics that are difficult to escape. A similar framing can be seen in relation to bilingual students in mathematics classrooms in the USA, where students may grow up in Spanish-English-speaking communities but are labelled as 'ESL' students and, in some states at least, placed in special programs (see, for example, Planas & Civil, 2013).

ABOUT COMMUNICATION

The assumptions about languages and about language speakers discussed so far in this chapter have important implications for the nature of communication in the context of superdiversity. Blommaert and Rampton (2011) highlight several specific points, drawing on a tradition of empirical research that dates back to the development of the ethnography of communication in the 1970s (e.g., Hymes, 1974) and subsequent ethnographic research focused on literacy (e.g., Heath, 1983; Street, 1984). A key finding demonstrated by this ethnographic approach is that while the structure and semantics of language remain important aspects of meaning-making, they are certainly not pre-eminent. A speaker's choice of language, accent, style or register are also important aspects of communication. For example, when a teacher starts a class by saying something like 'Right everyone, pay attention please', the semantic content of the utterance may not be as significant as the teacher's tone of voice, and the fact that within a school context, this kind of formulation is indicative of teachers and lesson beginnings. Of course, students need to be familiar with such ways of talking in order to recognise them as such. The link between a particular utterance and broader discourses and institutionally organised forms of talk is known as indexicality. The teacher's tone of voice, for example, indexes teacher ways of speaking and acting, and, by extension, the teacher herself as a teacher.

Beyond the various features of language, meaning-making is also multimodal in nature: it draws on the physicality of human expression, as much as on its verbal/aural form. This physicality includes gestures, facial expressions, bodily movement and so on. As with the broader features of language itself, the multimodal aspects of communication depend on participants having shared interpretive frameworks. Gestures, for example, can mean different things in different places. Hence, speakers' repertoires consist not just of different languages, styles, accents and so on, but also of different multimodal practices (Blommaert & Rampton, 2011).

In the context of relative socio-cultural homogeneity, the interpretive frameworks through which these different resources derive their meaning go largely unremarked, since they are widely shared. In the context of superdiversity, however, interpretive frameworks may often not be shared and furthermore, may no longer be as stable as was previously assumed. Blommaert and Rampton (2011) highlight two specific issues that arise from this situation. First, the nature of interaction in the context of superdiversity becomes much more complex:

> In situations where linguistic repertoires can be largely discrepant and nonverbal signs may do little to evoke solidarity, or alternatively in settings where there is a surfeit of technologically mediated texts and imagery, the identification of any initial common ground can itself be a substantial task. (Blommaert & Rampton, 2011, p. 7)

In such situations, concepts like 'negotiation of meaning' are more difficult to sustain, when the basis for such a negotiation may not be apparent. What participants do *not* know therefore becomes as important as what they do know. Blommaert and Rampton (2011) make the significant point that an analysis based on intercultural difference is insufficient; attention should instead focus on inequalities in interpretive resources.

Second, the intercultural mixing that occurs in the context of superdiversity leads to a diversification in communicative forms, in ways that often defy easy analysis. This diversification is often based on a good deal of creativity and innovation, with speakers drawing on others' communicative resources as much as on those that they might be assumed to use. As they point out, such creativity has always existed: students, for example, have always mimicked the way teachers talk, usually for comic effect. In the context of superdiversity, however, the variety and complexity of such practices has increased. An additional challenge for the researcher is to distinguish practices that appear to be innovative to them from those that are innovative for the participants:

> It is easy for a practice's novelty to the outside analyst to mislead him/her into thinking that it is a creative innovation for the local participants as well. (Blommaert & Rampton, 2011, p. 7)

By focusing on how new practices are produced, analysts get some insight into the shifts occurring in the underlying interpretive frameworks that go with them. These new interpretive frameworks, however, redefine the context of the interaction. This perspective therefore emphasises the reflexivity of communication, with the consequence that context cannot be assumed. Instead, context must be a focus of analysis (Blommaert & Rampton, 2011, p. 10).

Much research on multilingualism and mathematics education has examined the communicative resources that students use in mathematics classrooms (although not all necessarily use the term 'resources'). This work includes:

- Setati's (1998, 2005) research in South Africa on code-switching, in which students' multiple languages are seen as resources;
- Moschkovich's (2002, 2008) research in Spanish-English mathematics classrooms in the USA, in which she examined a range of resources, including students' 'native language', gestures, metaphors and multiple meanings;
- My own research on bilingual students' interpretation and construction of mathematical word problems in the UK (Barwell, 2005b, 2005c, 2009b);
- Halai's (2009) study of students' use of grammar in an English-medium classroom in Pakistan;
- Planas's research on the meanings, and the related classroom norms, relating to the participation of immigrant students in mathematics classrooms in Catalonia, Spain (Planas & Gorgorio, 2004; Planas & Civil, 2013).

While the research listed above highlights a range of different resources used by multilingual students to make meaning in mathematics classrooms, there has been little attempt to systematically describe students' communicative repertoires. Instead, research has tended to examine specific practices or resources. For example, it is notable that in much of this work, the practice of code-switching, where present, is highly salient for the researchers. In the light of Blommaert and Rampton's (2011) position, however, it is worth considering whether code-switching is a significant practice for students and teachers. Clearly in some contexts it will be more significant than others; the point is that researchers should not always assume that code-switching is the most significant resource in students' communicative repertoires.

In my research in the UK, in primary school mathematics classrooms in which multilingual students rarely used languages other than English, I examined some of the discursive resources such students used to make sense of mathematical word problems. These resources included their knowledge of the generic features of word problems, their knowledge of the grammar and spelling of written English, the use of narrative accounts of their experiences outside of school, and their understanding of the mathematical structure of the word problems (see Barwell, 2005b, 2009b). I showed how the use of these different resources facilitated students' understanding of word problems, but were shaped by the social nature of students' interaction. For example, narrative accounts of their personal experience were often used to interpret the scenario of a word problem, something students are known to find difficult, particularly those from minority backgrounds. At the same time, however, these narrative accounts were implicated in negotiations of students' identities and their relationships with each other (Barwell, 2005b, 2005d).

Moschkovich (2002, 2008) has perhaps gone furthest in examining different resources used by Spanish-English bilingual students in mathematics classrooms in the USA. In one study, for example, she describes how a student called Alicia attempts to explain the relationship between the lengths of the sides of a rectangle and its perimeter. Moschkovich summarises the resources that were used:

> Alicia used gestures to illustrate what she meant, and she referred to the concrete objects in front of her, the drawings of rectangles, to clarify her description. Alicia also used her native language as a resource. She interjected an invented Spanish word into her statement. In this way, a gesture, objects in the situation, and the student's first language served as resources for describing a pattern. Even though the word that she used for rectangle does not exist in either Spanish or English, it is very clear from looking at the situation that Alicia was referring to a rectangle. It is also clear from her gestures that even though she did not mention the words length or width, she was referring to the length of the side of a rectangle that was parallel to the floor […] Although Alicia was missing crucial vocabulary, she did appropriately (in the right place, at the right time, and in the right way) use a construction commonly used in mathematical communities to describe patterns, make comparisons, and describe direct variation: "The longer the _____, the more (higher) the _____." (pp. 201–203)

These examples are consistent with many of the ideas about communication discussed by Blommaert and Rampton (2011). Moschkovich focuses on resources, rather than simply semantic content and pays attention to the multimodal nature of communication. In a more extended analysis of two students' work on interpreting a graph, Moschkovich (2008) goes beyond examining students' meanings to look at the different frameworks for interpretation on which these meanings seemed to be based. Similarly, in my research, I focused on broader aspects of communication, such as genre and narrative, rather than the structure and semantics of language. My analyses make visible some of the interpretive frameworks the students drew on to make sense of word problems.

Such work could go further, however. In particular, it could look more systematically at students' repertoires, rather than simply pointing out particular practices or resources. It would also be valuable to examine the distribution of such resources among students in a class or across a wider range of settings, an approach that would more explicitly raise issues of equity in multilingual mathematics classrooms. Such work could also examine more carefully how the context, including the language context, is reflexively produced by participants, rather than simply taking it is a stable background.

CONCLUSIONS AND IMPLICATIONS

Globalisation has resulted in some significant changes in human society. People move around the world for many different reasons and in many different circumstances. The children involved in these movements will find themselves in mathematics classrooms, more or less anywhere they may go. Ultimately we are all affected by migration, whether our own or that of other people (Moore, 2012). Teachers are increasingly faced with students who draw on a variety of different languages and other language practices, many of which are unfamiliar to them. Understandably, this language diversity presents teachers with challenges. To make sense of these challenges, it is important to understand the nature of language use in the context of this superdiversity.

In this chapter, I have provided an overview of some key ideas about language that contribute to a more productive understanding of language than that offered by a nineteenth-century structuralist perspective based on discrete language, described in terms of their abstract structures, rather than their pragmatic uses. From this alternative perspective, languages are seen as fluid, changing and intermingling. There are no clear boundaries between them. Similarly speakers are seen as users of repertoires, drawing on a variety of languages, ways of talking and other resources, tailoring what they say to suit the audience and the situation. Communication is seen as creative, indexical and reflexive, constructing the speakers and the context in which it takes place.

Research in multilingual mathematics classrooms reflects some, but by no means all of this perspective. Research has highlighted how languages may be mixed in such classrooms, although such mixing is still seen as unusual, when it may, in fact, represent the norm, at least in some settings. Research has documented a range of resources that students may draw on to make mathematical meaning. A valuable next step would be to examine how these resources are distributed. Finally, research in multilingual classrooms tends to take the language context for granted, when the reflexive nature of all language use means that context is produced by the participants.

Superdiversity has implications for mathematics teachers and policymakers, who must deal with greater and more complex forms of language diversity. The ideas discussed in this chapter reframe some of the challenges that they may face. It is important, for example, to recognise that language learners are not all the same and do not all learn and develop a new language in the same way. It is also important to recognise that such learners are already sophisticated users of complex repertoires of communicational resources. Teaching mathematics in such contexts should not be viewed as replacing previous languages and versions of mathematics with the language and mathematics that we do 'hear'. The task is better understood as one of developing and extending students' existing repertoires. Of course, extending means adding new resources, including new languages, but it can also mean extending the use of resources that students already use. Such an approach demands that the resources students bring into the mathematics classroom be recognised and that they be encouraged to make use of them.

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SECTION II

POLICY AND MATHEMATICS EDUCATION IN MULTILINGUAL CONTEXTS

PHILIP CLARKSON

4. THE INTERTWINING OF POLITICS AND MATHEMATICS TEACHING IN PAPUA NEW GUINEA

EDUCATION IN PAPUA NEW GUINEA AT AND AFTER INDEPENDENCE

Michael Somare, Papua New Guinea's (PNG) first prime minister pleaded with his people to "accept old traditional values but at the same time, adapt easily to an alien electronic age of the twentieth century" (Griffen, Nelson, & Firth, 1979). The second prime minister, Julius Chan, was reported to say; "There's no 'Melanesian Way' to pilot aircraft" (Lancy, 1983). The foundation leaders of PNG saw clearly the dilemma of their peoples whose cultures have lasted for 40,000+ years, and have served them well, but now exist as a modern state in a world dominated by very different cultures. A key to this dilemma was through western education, quite different to their traditional education (McLaughlin, 2011).

Prior to 1960 there was a system of Christian mission primary schools (Guy, 2009; Meere, 1968). There was no secondary education readily available to PNG students. The 1960s saw rapid expansion of the education system, with many village-based governmental Primary Schools started. Secondary schooling was initiated and was divided into Provincial High Schools, years 7 to 10, at least one in each province, and four year 11/12, National High Schools (see Figure 1).

PNG has the highest density of languages in the world. Some 4.5 million people speak some 800+ language, with pidgin languages, Tok Pisin and Hiri Motu, and English as a third common language. Hence one of the crucial issues that had caused deep and long-term debate is what teaching language to use. Prior to independence, the schools run by missions invariably used the village vernacular, certainly in the early grades. When Australia in the 1960s drew all primary schooling into one system, a common curriculum was deemed, as was a mandated teaching language, English. This decision was justified as English was a world language that would enable PNG to eventually have a cohort of well-educated citizens who would be able to lead their country on the world stage.

There was much opposition to choosing English from the mission schools and many academics. It was known and now documented that many students sat through their early years of schooling wondering what they were suppose to do since they understood nothing of the school context, let alone the language being used (Clarkson, 1991a; Muke, 2012).

Even though the PNG government at independence stayed with the policy of English as the language of teaching, research continued to accumulate which argued

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 43–55.

that students' home languages were a key factor in students' ability to perform well in school (Downing & Downing, 1983). By the early 1980s the *Tok Ples Skul* (local language schools) began in which the local vernacular was used in the early year of schooling with a clear aim of integrating the official school curriculum with local knowledge sources (Kemelfield, 1983). Hence although a decision had been made regarding the language of schooling, the debate continued.

MATHEMATICS EDUCATION IN PNG AT AND AFTER INDEPENDENCE

By independence mathematics teaching in PNG was in large part based on the work of Dienes (1971). However little thought was given to the underlying issues of culture and the conflicts between what was embedded in this type of mathematics and traditional values and mathematical systems (Clarkson, 2011). Although Dienes noted the importance of language, most PNG teachers thought little of this as an issue that impinged on their teaching.

There had been scattered research into the mathematics embedded in the cultures of PNG (Lean, 1992; Saxe, 1982; but see much later Saxe, 2012). At this time some research studies that targeted students and teachers doing school mathematics began to appear. Through these it was recognized that language did play an important role in students' mathematics learning (Jones, 1982; Souviney, 1981, 1983; Suffolk, 1986). The author contributed to this by arguing that it was not only the language of instruction (English), but also students' home languages that may be influential in their school mathematics learning (Clarkson, 1983, 1984a). The results of working intensively with six urban community schools at this time were later published in a series of articles confirming this thesis (Clarkson, 1991b, 1992), as well as the impact on all the external year 6 assessment results of students (Clarkson & Clarkson, 1993). The expanding *Tok Ples Skul* movement coupled with the above research lead to a rethinking of the language policy for education in Papua New Guinea.

EDUCATION IN PAPUA NEW GUINEA DURING THE MID 1980S AND BEYOND

In 1986 the Matane Report was published (Matane, 1986; Department of Education, 1986). Although few immediate outcomes from this Report eventuated and it being cogently critiqued (e.g., Guthrie, 2011), it has had a far-reaching influence, setting the scene for decades of education policy in PNG. It advocated two oppositional aims: education should leap forward using the latest western approaches available to prepare PNG to take its place in the world economy, but at the same time education should be of service to the everyday person and village cultures. This report envisaged a PNG developed education system drawing on the ancient cultures of PNG blending with them the best from the west. The core changes flowing from the Report were encapsulated in the *National Education Plan* (Department of Education, 1995; Guy, 2009). The structure of the school system was changed with three-year

Elementary Schools based in villages, Primary Schools (years 3 to 8), and Secondary schools (years 9 through 12) (Department of Education, 2003a; Glover & Ikupu, 2002) (see Figure 1).

It was the Elementary School that was most revolutionary. Village communities were to have a large control over the financial support of the schools and decide on who would teach (Glover & Ikupu, 2002; McLaughlin, 2011). The village was also to determine what teaching language would be employed in their school. It was assumed that rural schools would choose the local vernacular, although urban schools might adopt a *lingua franca* or English. Muke (2012) suggests that urban Elementary Schools almost always chose English, with a number of rural schools doing likewise, even though the standard of English of the teachers was questionable (Gerry, 2011). The reasons for choosing English seemed to be that community elders believed going to school meant getting good jobs in the long run, and hence speaking English was paramount for that long term goal. Hence the earlier students started with this language the better. For them there was little weight given to the notions of preserving the traditional cultures of PNG through schooling. That remained the province of the village. Hence although the Elementary School was suppose to

Schooling structure before education reform			**Schooling structure after education reform**		
	Grade	Age	Grade	Age	
	1	8	P.1	6	
	2	9	P.2	7	**Elementary**
Community	3	10	1	8	**School**
School	4	11	2	9	
	5	12			
	6	13	Grade	Age	
			3	10	
	Grade	Age	4	11	
	7	14	5	12	**Primary**
High	8	15	6	13	**School**
School	9	16	7	14	
	10	17	8	15	
	Grade	Age	Grade	Age	
National	11	18	9	16	**Secondary**
High	12	19	10	17	**School**
School			11	18	
			12	19	

Figure 1. Comparing the structure of the PNG education system in the 1970s before education reform, and 2000s after education reform (Muke, 2012)

reflect the local community, perhaps the age old divide between what was important culture, and what was schooling leading to jobs, and the language divide that went with this, was still alive and well.

Given that students made a transition from the Elementary School to Primary School after year 2, year 3 was designated the 'bridging year.' During year 3 teaching began in the language of the feeder Elementary Schools. By the end of the year the teachers would be using in the main English for teaching, although reverting to the vernacular when it was deemed necessary. The sharing of languages in teaching would continue although by year 6 the teaching would be in English (Muke, 2012).

Clearly this policy assumed that all students came from Elementary Schools that used the same language of teaching. In the majority of schools this was not a problem. However given the huge variety of languages in PNG, sometimes in a small geographical area, there were a number of exceptions. As well if one or more Elementary Schools had used English as their teaching language, their students may have some difficulty reverting to their vernacular in year 3. On the other hand such students may well outpace their peers as the teacher used more and more English. However local schools were left to make adjustments to the policy as best they could; no guidelines for these and other dilemmas seemed to have been developed.

The notion of beginning schooling in a language that students knew well was not only going back to one of the threads of the decades long language debate for schooling in PNG, but by then (the late 1980s and early 1990s) was well attested to in the educational research literature (e.g., Tucker, 1999). But how to teach was also an issue. Whether or not there should be a particular way in which PNG children should be taught, peculiar to PNG cultures, was an issue that had been consider to some degree before (Clarkson, 1984b; Clarkson & Leder, 1984). Swain (1996) had made the key point concerning education systems in general, and teaching in particular, that systems need to be forever evolving by testing changes and thinking anew, what was best for them. Importing so called best practice from elsewhere, often from the west, was no substitute for self-development.

In 1998 the project 'PASTEP' (Primary and Secondary Teacher Education Project) began to revise the teacher preparation of primary teachers. The project was based around the new structured school system and the latest curriculum that was being formulated and was published in 2003 (Department of Education, 2003a). Sadly however, much of what was stipulated for the PASTEP project by the Department was 'best practice' from Australia rather than developing a truly PNG style of school teaching (Nongkas, 2007; Pickford, 2008), although not the language policy. That was one of the few markers of deep PNG influence on the project.

One result from the evaluation report of the PASTEP project is relevant here. It had been accepted for some time that in bilingual programs one of the crucial factors is that teachers did need to be fluent in the languages that would be used in their classrooms (Tucker, 1999). However that is not always possible given the variety of contexts that exist for bilingual teaching programs. But critically, if there was to be a formal switch in the language of teaching at some point, in PNG's case in year 3 to English, then the teachers needed to be, among a range of abilities, fluent in this target language of teaching. The PASTEP evaluation found that by and large the beginning primary teachers had poor standards of English, well below the level that would be normally expected in PNG (Clarkson, Hamadi, Kaleva, Owens, & Toomey, 2004; see also Zeegers, 2005). Given that a number of these exit students would be teaching in grade 3 in their first years of teaching, it did not argue well for the strength of the year 3 bridging program.

MATHEMATICS EDUCATION IN PAPUA NEW GUINEA DURING THE MID 1980S AND BEYOND

The school mathematics program was impacted by the changes that gradually transformed the education system from the mid 1980s. During the late 1980s the Department continued a long delayed publishing program of text-books for the primary schools. However there was no reference to the importance of language in these books. The continuing emphasis on the use of materials, which reached back to Dienes, was still evident, as was the new advice for teachers to link the mathematics learning to PNG contexts. The latter was accomplished in the texts, although not very well, by use of local pictures and the insertion of Tok Pisin words from time to time. The influence of the original 1970s curriculum derived from Australian mathematics curriculum was still very evident.

The international publishing company, Oxford University Press (OUP), had been contracted to print the 1980s texts. In 1990, OUP was contracted again, but this time to publish a new series of mathematics texts for years 1–6. In the subsequent texts there was clear evidence of the use of materials, various pictures of PNG artefacts and PNG people were featured. Some Tok Pisin words were also used. But there was no real change to the way the curriculum was interpreted to bring into focus the possibilities of ethno mathematics. One real change was a limited acknowledgement that when teaching mathematics, language was important. Both in the introduction to the teacher's resource book for year 1, and in the Department of Education Secretary's message at the front of the year 1 textbooks, it was stated; "the children should talk about what they are doing in every lesson. Since learning is most effective when it has meaning and is enjoyable, allow the children to use the language that they are most comfortable with. … pay attention to what they are saying" (Department of Education, 1991a, 1991b, 1991c). Here for the first time was an explicit statement regarding the importance of language in mathematics teaching. Moreover there was permission for teachers to 'allow' children to use their vernacular and/or a lingua franca. There was no stipulation that the then official language of teaching, English, must be used in all teaching. Teachers were regarded as professionals who could make the best decisions for their students in the classroom. However there was little in the resource book that encouraged the use of multiple languages, and no explicit teaching strategies. Of the 14 units that the year 1 material was divided into, one unit was explicitly on language, but was entitled 'Learning mathematics in English'

(Department of Education, 1991a). Admittedly in the teacher's resource book in Unit 1, there were a few suggestions in the introductions to some lessons that suggested teachers use vernacular words for some ideas. But that strategy was not pursued in later units.

Importantly the Secretary's message in these texts published in the early 1990s indicated how the Department was thinking and which later became quite explicit. Although mathematics was seen sometimes as different to much of the rest of the curriculum, the Department regarded its new language policy to apply to the teaching of mathematics. This was made clear in the subsequent mathematics syllabi published for Elementary Schools and the lower years in Primary Schools, and the accompanying teacher guide (Department of Education, 2003b, 2003c; 2004a, 2004b). In these documents the policy of using a vernacular for teaching in the Elementary School was clearly stated, and the progression to English was to begin only in year 3. However in the actual syllabus there were some deviations to this policy. In the Elementary syllabus in aspects dealing with number and operations, a partial move to English was clearly indicated, although there was nothing concerning the use of English in the other areas. In the syllabus for years 3–5, although there were statements in the opening pages about the application of the general language policy applicable to all school teaching, there was nothing in the year 3 mathematics syllabus that even suggests how teachers should use vernacular language and how to bridge from vernacular to English.

One might expect if teachers were to be using vernacular languages in their teaching, then there would be a clear emphasis on ethno mathematics utilizing the local mathematics. This was so in some places in the Preparatory year of the Elementary School syllabus, but after that this emphasis rapidly reverted to a curriculum that looks very much like any other western mathematics curriculum (Vagi & Green, 2004). Hence Matang's (2002) continued argument for an ethno mathematics approach seemed to have had little impact (see also Owens, 2015).

It has already been noted that both Elementary and lower Primary School teachers needed to have good competence in English for teaching, including for the teaching of mathematics, and that there were doubts regarding such competence. For teaching mathematics, these teachers also needed to have good competence in mathematics for the year levels they are teaching, and for some years beyond. Vagi and Green (2004) suggested that the Elementary School teachers could be expected to have a good intuitive grasp of their local cultural mathematics. However with only one week of 'training' in the teaching of western mathematics in their on-the-job delivered certificate studies, it was doubtful they would have any real depth in understanding the western mathematical ideas to which the syllabus so rapidly moved. These teachers would have no alternative but to revert to what and how they were taught mathematics during their own schooling. Given that many of these teachers also experienced their first years of schooling in a language fog (see above), it is doubtful they can remember much of what and how they were taught mathematics in their first years.

It seems that in the early years of Primary School there are also difficulties in the teaching of mathematics. In the evaluation of PASTEP (see earlier, Clarkson, et al., 2004), teacher college mathematics education lecturers were asked both in interviews and through questionnaires whether they prepared their students to teach mathematics in the bridging year 3 and beyond. Although they said they did, clearly their responses only referred to mathematics per se. There was no attempt to look at the issue of how language impacts on mathematics teaching and learning. Furthermore, they believed that the issues of bridging did not apply to mathematics teaching. They gave no indication that any aspects of ethno mathematics would have implication for teaching mathematics. Hence it came as no surprise that final year college students and beginning teachers of one or two years experience also believed that there were no language impacts on their teaching of mathematics, and bridging did not apply to their mathematics teaching. The final relevant result from the evaluation of PASTEP for here deepens the difficulties for the teaching of mathematics in the bridging year. Results showed that as for English, students fell short in mathematics performance.

Given that the above results suggest that much professional development work was needed with both young Elementary and Primary teachers, there is another very interesting result reported in the literature that suggests that experienced teachers were somewhat different to their young colleagues. Muke worked with a number of experienced year 3 teachers in the Whagi Valley in the Western Highlands (Muke, 2012; Muke & Clarkson, 2011a, 2011b). In these classrooms, teachers were competent in all three languages used; Whagi, Tok Pisin, and English. Muke's study focused on how the teachers used these languages to teach mathematics to their year 3 students. All the Elementary Schools had used Whagi as the language of teaching. The year 3 students were fluent in Whagi and Tok Pisin, with some having a small facility in English. Most lessons used all languages, but the frequency of use of each language varied considerably depending on the teacher and on the subject matter being taught. However there were three results that are worth noting here. Firstly, all teachers believed that students should be taught in English at the earliest point in time in Primary School, given that it was facility with this language that would be important for them in the future if they wished to go on with their studies and for their future careers. They did not have a problem with the students having been taught in Whagi in the Elementary School, but now in Primary School it was time for them to change. Muke felt that this was a belief born from their teaching experience. Secondly they did use both Whagi and Tok Pisin in their mathematics lessons, but they used these two languages as a strategy to teach their students the language of mathematics in English. Hence the teachers were in no doubt that language competence did impact on mathematics learning. Thirdly these teachers did from time to time use examples from village life in their mathematics teaching, but again this was a teaching strategy to move to the real, that is western, mathematics in the syllabus. Hence these well-regarded teachers were moving their students as rapidly as possible to an English teaching environment and focusing on western mathematics. They gave little credence to the notions embedded in the curriculum that encouraged teaching that valued the village cultures of PNG, at least as far as mathematics went. Not for them the notions of ethno mathematics and the related notion that it is important to learn and use the local vernacular to have a deep understanding of your own mathematical heritage. Well at least the Primary School was not the place for that type of teaching.

Clearly by the end of the 2000s, a huge change had gradually evolved within the structure of schooling in PNG, and the curriculum. This had impacted the mathematics teaching in Elementary and lower Primary Schools and was more or less aligned to what the mathematics research had recommended for some time, at least concerning language issues. Although there was some movement in implementing broader ethno mathematical ideas, the journey was beginning, even though the official mathematics curriculum was still little different to what was found in many western countries.

CODA

Under the heading of 'Language Policy in all Schools' the statement in Figure 2 was the core of a new directive issued by the Acting Secretary of Education on 28th of Jan 2013, the beginning of the school year (Department of Education, 2013). This was a complete change of policy, instantly and very surprisingly reversing the direction of education language policy of 25+ years.

> All member schools operating under the National Education System will use **English as the medium of instruction commencing on Monday the 4th of February 2013**, the first day of the 2013 academic year. English is the official language of instruction and for pupils to speak and write from Elementary Preparatory Grade to Grade 12.

Figure 2. Key statement on the new 2013 education policy: Emphasis given by being 'boxed' and using 'bold' type face in the original (Department of Education, 2013, p. 2)

In the body of the circular it was made very clear that all subject teaching would be in English, starting from the preparatory year in Elementary School, replacing the use of all other languages. It was only in exceptional circumstances that other languages would be permitted, and then only to explain difficult ideas, reverting to English as soon as possible. The new policy was promulgated to "address the concerns raised by the society including parents, members of the community, teachers, former students under the reformed curriculum, academics, and political leaders who demanded the policy change. These people blamed the poor standard of spoken and written English because of the new vernacular in schools" (pp. 1–2). The circular went on to note that in the opinion of the government "It is important to use English for teaching and learning early in our schools as this is the language mostly used for administration and business in Papua New Guinea and around the world" (p. 2).

Not surprisingly people who had been involved in the production of vernacular based materials for school use were surprised and wondered what would happen to the resources they had developed (Candee, 2013). Interestingly new research projects designed to refine the use of vernacular in teaching mathematics in early years had just been awarded by an aid agency in Australia [2013–2015 – *Improving the teaching of mathematics elementary schools by using local languages and cultural practices (Papua New Guinea),* Chief investigator: Dr Kay Owens (Charles Sturt University), funding: AusAID.]

At the time of writing (Oct, 2013), it is clearly far too early to assess how this new policy will impact on the PNG school system. However one issue seems clear. It has been argued above that even before this change of policy the national government needed to provide extensive mathematics professional development for both Elementary and lower Primary School teachers, and English language professional development for lower Primary School teachers. Now such funding requirements are hugely increased. Up to now a core assumption for Elementary Schools was the teaching would be in the vernacular. But now, having to teach in English, all Elementary teachers surely will need to be competent in that language as well, quite a new criteria for these teachers to meet. Extra funds will clearly be needed, and provided rapidly, to deal with this emergency.

CONCLUSION

Education cannot be separated from politics. There have been times when politicians have taken a 'hands off' approach and allowed their departments to develop advice and policy based on educational research. However when education is seen as a key factor in social policy of a country, with all the demands to meet the competing factions within the electorate, then the slow evolution of education policy will not fit the immediate needs of the political masters. In such an environment long-term research with its inevitable provisional statements of what is best seems underwhelming. Politicians need answers that promise programs that deliver before the next election comes around. This should be no surprise to education researchers. Changes are normal in government policies in economics, social welfare, defence, etc., and change happens more often when there is a change of government. However it regularly surprises educational researchers that firstly politicians do not take more notice of their results, and secondly that politicians change policy often with little or no consultation to the relevant research, let alone the researchers.

Interestingly Malaysia also experienced an unexpected policy change concerning the language that would be used to teach mathematics in the early 2000s only to reverse the policy some 8 years later (Azmi & Maniam, 2013; Clarkson & Indris, 2006). Both times the changes were driven by public and political opinion with little reliance on educational research. Clearly PNG and Malaysia are not the only countries to change policy in this way. But what is so intriguing with the process in PNG is that the long process that lead to the initial change was measured and evolutionary, with many in the curriculum and research branches of the Department coming to a clear understanding of what could be, and in the end what should be, the policy for PNG. This seemed to be in line with the way things are done in PNG. It also gave time for teachers to be attuned to the possibilities and then the reality of the change. It gave them time to build up resources, so when the official curriculum documents codified the change in 2003, this was as much a recognition of what was happening in many classrooms, just as much as a document that told teachers of the 'new way forward'. This was not so in 2013. In 2013 all changed, following the election of a unifying national government in late 2012 and after much political turbulence that had continued on the national political stage for a number of years. In one way the new school language policy, can be understood as a new government wanting to quickly stamp its authority on one of their largest and most public departments.

There are many reasons and many ways why and how vernacular languages may be used in school systems. These are always political decisions, but often the making of them and the aftermath is a turbulent time for those involved (Liddicoat, 2008). One important distinction in such a process is whether when vernacular languages are used as the language of teaching, is this to be understood as a vehicle for cultural understandings of the local people, or whether vernacular language is used as a strategy to introduce students to the more global language of teaching to be used in later years of schooling? Till now PNG seemed to be trying to do both, and had evolved a strategy that held potential for achieving both.

What then of the interplay between research and policy? In part such interplay must mean that researchers should have some interaction with those who develop and implement policy. The world renown science educator, Peter Fensham, has argued that education researchers were not entitled just to sit and moan that politicians and bureaucrats take little notice of research in creating and enacting education policy (Fensham, 2008). He argued that researchers have an obligation to engage both groups on a continuing and deep basis, even though that may take time away from their own research. It is the researchers' responsibility to initiate and ensure this interplay continues over the long term. This partnership must continue whether the results of their research form the basis of policy at any point in time or not. For Fensham this was simply part of the profile of a researcher. One hopes that this obligation will continue to be fulfilled by education research colleagues in PNG.

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5. LANGUAGE OF INSTRUCTION AND LEARNERS' PARTICIPATION IN MATHEMATICS

Dynamics of Distributive Justice in the Classroom

INTRODUCTION

The policy makers in education, in the post-colonial contexts, often introduce the ex-colonial language with perceived or real power and privilege as a medium of instruction, ostensibly for distributive justice for all learners. Since language of power is part of the cultural capital needed for social mobility, its use in classrooms is assumed to help distribute this capital through formal education. However, such attempts create a paradoxical effect as learners, often from low socio-economic background, face the twin challenge of learning both a language and the subject knowledge (mathematics in this case) presented in that language. The learners are systematically perceived as deficit laden and ultimately marginalized from optimal participation in the course of learning mathematics (Halai, Muzaffar, & Valero, 2015).

In this chapter, we illustrate this paradoxical consequence of language in education policies by examining the case of Pakistan's Punjab province where the state introduced in 2009, English as a medium of instruction in schools serving a largely Punjabi and Urdu speaking population. A major contention of this policy was to distribute the advantage of English language, perceived or real, to all learners in the education system. We illustrate the paradoxes that followed the implementation of this policy by deploying Nancy Fraser's framework consisting of three dimensions of social justice; i.e., redistribution, recognition, and participation in the mathematics classrooms (Fraser, 2008, 2001). Following Fraser we argue that participation in educational processes is not simply a matter of distribution of resources. Rather, it is inherently linked to the politics of recognition of the socially and culturally marginalized learners. We show that learners' cultural resources remain unrecognized in classroom interactions that privilege a language other than their first or a proximate language as the language of instruction, thus resulting in their marginalization. In this paper, first language is used similarly to the term mother tongue to refer to a language that the learners learnt first or they identify with; proximate language refers to a language that is commonly used in the learners' proximate environment and is familiar to them. Based on this analysis, we argue for a more socio-culturally

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 57–72.

embedded and inclusive use of language in the classroom instead of an abrupt move from one language to the other as the language of instruction.

LANGUAGE OF INSTRUCTION AND LEARNERS' PARTICIPATION IN MATHEMATICS: ASPIRING FOR SOCIAL JUSTICE IN PRACTICE

Learners' participation in mathematics has been approached from a variety of perspectives, including the cognitive psychological perspective that looked at learning as construction of knowledge through learner's interaction with the physical and social world (e.g., Piaget, 1959); socio-cultural perspective that looked at learners' participation in and through social interactions employing the tools of culture such as language and symbol systems in mathematics (Vygotsky & Luria, 1994); equity and social justice perspective that seeks to understand learners' participation in terms of their negotiation of social and intellectual space for participation and through teachers' creation of opportunity for all learners irrespective of their language, ethnicity, gender and socio-economic background, to participate in the process of learning mathematics (Atweh & Brady, 2009; Atweh, 2007; Valero & Pais, 2011).

Nearly all of these perspectives assume the differential nature of learners as distributed at various points on a scale of cultural and economic advantage. More often than not, the cultural and the economic are imbricated with each other. For instance, learners who do not share the dominant culture and language are also the ones that come from economically disadvantaged households. Thus the call for making all learners learn better, or providing quality education for all, can be interpreted in terms of a requirement to provide all learners with the tools that are traditionally available only to a few, i.e., to implement a certain kind of distribution of cultural capital to those who don't have it. The notion of cultural capital employed in this paper draws on Bourdieu's perspective, according to which cultural capital is familiarity with the norms of the dominant culture mainly the competence to use language of the educated and higher social class (Bourdieu, 1977). Within this perspective it is not enough to take account of cultural resources such as language or mathematics to which the learners are being introduced, rather the significance is in the norms and practices of use of these resources that collectively constitutes the cultural capital.

Arguably, redistribution of the cultural capital in the form of languages especially a global language like English is a concern for those well intentioned and social-justice oriented decision makers. However, in mathematics classrooms a fundamental concern is or should be to construe the cultural capital to be distributed in terms not of language but of mathematical knowledge and ways of knowing, and seek its distribution to all learners. For the sake of argument, let us substitute mathematical capital for the cultural capital. The mathematical capital, then, would include a combination of mathematical knowledge, skills and attributes that enable learners to succeed in examination (Bourdieu, 1977;

Zevenbergen, 1998). In an increasingly globalized and technological world, such mathematical capital would include application of mathematics knowledge, communication and interpretation of mathematics, problem solving and creativity (Hirsh, 2010). When conceptualized from this perspective, mathematical capital would be different from the traditional emphasis in mathematics classrooms on routine algorithms and procedures.

When education systems use the mother tongue or first language as the language of instruction at pre-primary and early primary level they are not just responding to the insights from scientific research but also to the political imperative to recognize and value the existing cultural capital of the learners. In the case of some countries with multiple major languages in use, one out of the several languages in use is recognized as the national language, which may also be different from the learners' first language or mother tongue but which it is necessary to learn due to its status as the national language. The later introduction of the national language as a language of instruction at elementary or upper primary level of education, and of a global language such as English as the language of instruction (e.g., the case of Tanzania, India) are instances of the ways in which policy attempts to distribute cultural capital (Brock-Utne, 2012; Halai & Karuku, 2013). The language of instruction assumes its status in an intricate web of social, cultural, political, and cognitive preferences. In a world formatted by the current tide of globalization, the language(s) of instruction in the national and sub-national setting is influenced by the patterns of global cultural dominance (Atweh, Clarkson, & Nebres, 2003). Thus, it is not unusual to find the language of instruction in mathematics to be other than the first or the proximate language of the learners. These differences raise new problems for learners' participation in mathematics, making it increasingly difficult for those not competent in the language used as the medium of instruction. Thus learners' proficiency in the language of instruction becomes a key determinant of their ability to participate [or not] in mathematics.

The difference in language of instruction and the learners' first or proximate language is regarded as both a cause as well as an effect of power differential within particular societies. If the disadvantages were solely economic, redistribution of incomes through taxation and philanthropy could make the societies more equal. However, in this case the cultural and economic disadvantages coincide. By privileging a particular culture and language, the education systems do not recognize the cultural resources associated with the learners' first languages. This situation raises issues of social justice for the linguistically marginalized learners.

Fraser's (1997) notion of three key dimensions of social justice, i.e., redistribution, recognition and participation, is a useful way of understanding issues of social justice in education. This framework is often employed at the macro level, where the dynamics of reform are focused on redistributing the benefits of education through improved access to education across the socioeconomic boundaries. However, the framework can also be employed in classrooms where social justice issues are experienced first-hand. For example, in mathematics classrooms distributive justice would imply equal access by all learners to mathematical capital in the form of knowledge, skills and ideas important for success in mathematics. Likewise, recognition within the classrooms would require that the teachers acknowledge and respect the diverse backgrounds and needs of various individuals and groups such as gender, ethnic or linguistic minorities. Participation from Fraser's perspective means challenging the hierarchical power structures and norms in the classroom so that opportunity is created for all learners to be active learners. Of course a practical implication of this framework at the classroom level would be a pedagogic process that is radically different from the traditional teacher directed pedagogy. Thus Fraser's framework for social justice in education is inherently political in nature (for a further elaboration of social justice in education also see Tikly & Barrett, 2013).

To address issues such as those noted above, Fraser (2001) elaborates that two kinds of remedies are often employed to deal with issues of redistribution and recognition, affirmative "aimed at correcting inequitable outcomes of social arrangements without disturbing the underlying framework that generates them" (p. 82), and transformative, "aimed at correcting inequitable outcomes precisely by restructuring the underlying generative framework" (p. 82). Extending this discussion on social justice with specific reference to mathematics education, Atweh (2007) maintains that none of the three dimensions in Fraser's framework are reducible to the other. Indeed parity in participation can only be achieved if a dialectic relationship is established between redistribution and recognition. In what follows we will illustrate how redistribution without adequate attention to recognition and participation led to a paradoxical situation for the teacher and the learners where the good intentions of the policy makers instead led to consequences for the learners where they learnt neither language nor mathematics.

DISTRIBUTIVE JUSTICE IN ENGLISH MEDIUM MATHEMATICS CLASSROOMS: CASE OF THE PUBLIC PRIMARY SCHOOLS IN PUNJAB

Pakistan is a linguistically diverse country with over 300 dialects and approximately 57 languages spoken throughout the country's four major provinces, and Urdu as the national language and the lingua franca. Despite being designated as the national language, Urdu is the first language of less than 10 per cent of the population (Rahman, 2005). English remains the preferred language due to its status as an abiding colonial heritage and a language that continued to be associated with power and privilege after Pakistan's independence. Schools that offer instruction in English are called English medium schools. These schools, mostly privately managed, are found in both urban and rural areas. Learners in Pakistan's English medium schools learn their subject matter content and the English language simultaneously and are expected to become proficient in both.

There are five main levels in the education system in Pakistan: Primary (Classes K[1] through five, ages 6 yrs.–10 yrs.), middle[2] (Classes six through eight); high school or matric level (Classes nine and ten) leading to a secondary school certificate, and higher secondary or intermediate level (Classes eleven and twelve) leading to a higher secondary certificate, and finally tertiary education.

Typically mathematics is a compulsory subject that learners have to study throughout the course of their primary and secondary school cycle. Performance in mathematics, however, has been an enduring concern. For example, on the basis of a comparative study of the quality of education in public and private schools in Punjab, Andrabi, Jishnu, Khwaja, Vishwanath and Zajonc (2008) claim that "By the end of class three, just over 50% of the tested children have fully mastered the Mathematics curriculum for grade I. They can add double-digit numbers and subtract single-digit numbers but not much more. They cannot subtract double-digit numbers, they cannot tell the time, and double-digit multiplication and simple long division are beyond reach for all except a small minority" (p. 19). Similar concerns abound about the quality of primary education and especially learners' achievement in mathematics in the country.

Language in education has always been seen as an issue in need of a policy resolution in Pakistan. Since the report by the Shariff Commission in 1959, an influential first document on education policy and those that followed soon after, aimed to distribute to a wider cross section of society, the cultural capital encoded in formal education delivered through the mother tongue and later the national language and ultimately English (Ministry of Education, 1959). Yet, English remained the primary medium of instruction in the elite private schools. More often the children going to these schools also had access to English at home. The policy decisions about language in education in the country, especially about the language of instruction have oscillated from privileging regional languages and Urdu (the national language) as the medium of instruction to using English as the medium of instruction. Since English remained the language in which the state conducted its business, it constituted part of the cultural capital accessible only to a very small elite in Pakistan. This gave rise to high level of inequality in the country.

More recently, the policy has attempted to remediate this situation by making English the language of instruction for all students. The National Education Policy (NEP, 2009) required the use of English as a medium of instruction for science and mathematics in class four onwards. As an example of distribution of cultural capital, the policy sought to provide opportunities for "children from low socio-economic strata to learn English language." (Ministry of Education, 2009, p. 28). In 2009, the provincial government in Punjab, the largest and arguably the most developed province, followed up on the NEP by introducing English as language of instruction in its schools at the primary and secondary levels. In compliance with this policy several textbooks, teacher guides and assessment were rendered into English for use by teachers and learners.

Implicit in this change in language of instruction policy were two main elements typical of a redistributive motivation. First, was the perception that English is the language of power and opportunity and all learners needed to become proficient in English. Second, a change in the language in education was expected to result in social justice through redistribution of the cultural capital, mainly comprising of access to English, to the disadvantaged and marginalized sections of the society.

THE LANGUAGE IN EDUCATION PROJECT

This present chapter draws from a large project carried out in six selected districts of Punjab to investigate the extent to which introducing English as language of instruction supports quality teaching and learning in public primary schools. Here it must be noted that Punjabi language, together with its several dialects, is the mother tongue of most learners in Punjab. Yet Punjabi has never been used as a medium of instruction. The schools were either English or Urdu medium until the decision of the government of Punjab to introduce English as the language of instruction in all schools. The study involved classroom observations in schools implementing the new policy. The observations were undertaken in a total of 126 primary classes in English, science and mathematics in public primary schools where English had been introduced as a language of instruction. Transcripts of lesson observations were read and coded under the following emergent categories:

a. Utterance in Urdu;
b. Utterance in English;
c. Mixed Utterance;
d. Teachers' imperative prompts for management of behavior;
e. Teachers' procedural instructions in mathematics;
f. Questions posed by the teacher.

In addition interviews were conducted with teachers, head teachers and parents (For details about the study see Rashid, Muzaffar, & Butt, 2013).

Specifically this chapter draws on the quantitative discourse analysis of teacher and student talk as reported in the project report and the transcripts from mathematics classrooms (n=41). These transcripts were analyzed on the basis of Fraser's framework to understand the extent to which learners were able to participate in mathematics in the context of classrooms where the language of instruction was not the first or the second language of the teachers and the learners. In the section that follows we present the key findings together with illustrative data.

CREATING SPACE FOR LEARNERS' PARTICIPATION

An overall pattern borne out in almost all the lessons observed was the three-phase lesson structure. Phase one was introductory where the teacher reviewed or referred to the previous lesson and introduced the topic of the new lesson. Phase two was the main body of the lesson where the teacher explained a mathematical procedure or the concept that was the topic of the day. During this phase the textbook and the chalkboard were the main resource for teaching. The third phase invariably meant that learners worked in their notebooks at tasks taken from the textbook but similar to those introduced by the teacher in the main body of the lesson.

In terms of verbal interactions there was opportunity for learners to participate in phase one and phase two of the lessons. Quantitative analysis of teacher talk showed that teachers typically used a mix of Urdu and English with Urdu as the main language of the classroom discourse; 62% of all teacher utterances were using this mixed mode. When learners did participate in the interactions they seldom uttered a full sentence in English, except when asked to read from the textbook. Full sentences in English constituted only 5% of all learners' utterances, all of which were reading from the textbook.

Observations also showed that a significant corpus of the mathematics lessons comprised of teaching procedures and routines for computation (e.g., sum or product of fractions, HCF), measurement (e.g., area, perimeter). A relatively small corpus of the mathematics lessons observed comprised of "word problems." Certain key features emerged in both these genres that raise questions about the extent to which the policy aspiration of social justice in the classroom was met. In teaching mathematics procedures, an emphasis was to ensure that learners know the names of mathematics terms in English. This emphasis was also present in lessons on 'word problems', where a significant effort to introduce names of mathematics terms in English was prevalent. In addition, it was noticeable that teachers tried to convert the 'word problems' into specific procedures and routines by recognizing key words or phrases that could provide a hint of the mathematics operation to employ. Illustrative data extracts are provided from both genres of mathematics lessons in the corpus.

Data Extract One

Provided below is an extract from a lesson in class four. The teacher (T) introduced the topic of Highest Common Factor (HCF) by Prime Factorization and worked on the chalkboard to demonstrate to the learners (L) the procedure for deriving the HCF of 50 and 75 by Prime Factorization.

1. T: Bachon kal hum nay kya parha? [Children what did we study yesterday?]
2. L: HCF (Chorus)
3. T: HCF ka matlab kya hai? [What is the meaning of HCF?]
4. L: Highest Common Factor (Chorus)
5. T: Aaj hum nay parhna hai HCF by Prime Factorization. —ki choti choti tajziyan banti hain. [Today, We have to study HCF by Prime Factorization – small small factors are made]

In line 3 above the teacher asked the learners to provide the "meaning" of HCF. But line 4 shows that learners simply gave the full name of the mathematical term HCF. Teacher's acceptance of the full name in English was symptomatic of an emphasis on learning mathematical names in English without necessarily probing the meaning that learners made of those terms. In line 5 the teacher made a pedagogic move by introducing the topic of "HCF by prime factorization". In the same line she stated that, "small small factors are made" (choti choti tajziyan banti hain). Presumably, "small small factors" referred to prime factors as 'small' because they cannot be further factorized. Of course, small (choti) can be interpreted in a number of ways and not all of them would lead to this conclusion. Additionally the word 'tajzian' has its root in tajzia that means to analyse or split apart. A use of tajzian could potentially provide the learners with a conceptual link to the notion of factors. It is noteworthy that an attempt to explain prime factorization, however limited and inaccurate, was made in Urdu.

To continue with the lesson above, interactions from line 6–25 (full transcript in Appendix A) showed that the teacher worked on the chalk board through the procedure of finding the HCF of 50 & 75 by taking their prime factors. She found the prime factors of 50 (2, 5, 5) and of 75 (3, 5, 5) and then the common factors (5, 5) and the highest common factor (25). Once completed she set the class to do similar work in their notebooks.

This is an instance of interactions that were dominated by the teacher and did not involve meaningful participation by the learners where they don't just learn mathematical procedures and their names in English, but could also have had an opportunity to learn concepts and mathematical relationships. For example, the teacher accepted the learners' response in line 5 and moved towards introducing the topic of the day 'HCF by Prime factorization.' However, it showed no evidence of learner's engagement with mathematics concepts, ideas and relationships around highest common factors and prime factors. While, some of the issues illustrated in the data are about pedagogy that emphasized procedures above concepts and relationships, they were compounded due to an additional effort required by the teacher and the learners to become familiar with mathematics terms in English.

In extract two below, we see that similar patterns of procedural discourse persist in a lesson with a focus on word problems.

Data Extract Two

In this lesson in class four the topic is "statement problems" also known as word problems. The class was working on the problem as read by one of the learners "Amna bought four point fifty (sic) (4.50) metre of cloth. Ayesha bought ten point fifty (sic) (10.50) metres of cloth. How many metre of cloth did they both buy?"

1. T: Statements kaay savalaat hain. [(These are) questions with statements]
2. L: Miss mein Parhun. [Miss may I read]
3. T: Chalain beta koi parhay-yeh statement parhain-savaal number eik ki-ji [OK child one of you read the statement of question number one-yes (points towards one learner)]
4. L: (Reading from the book). Amna bought four point fifty (sic) matter of clothes
5. T: Metre, matter nahi metre. [Metre, not matter, metre]

6. L: Metre. Ayesha bought ten point fifty (sic) metres of clothes (sic). How many metres cloth did they both buy?
7. T: Jee. [Yes]
8. L: Eik Jaisa. [Alike]
9. T: Metre, matter nahi metre. [Metre, not matter, metre]
10. L: Plus (chorus)
11. T: Bachon dono ka kya matlab hai? [Children what is meant by 'dono']
12. L: Plus (chorus)
13. T: Bachon dono ka kya matlab hai [Children what is meant by 'dono']
14. L: Ten?
15. T: Dono ko aapnay kya karna hai? Plus karna hai, minus karna hai, multiply karna hai, divide karna hai?[what do you have to do to both? Plus, minus, multiply or divide?]
16. L: Four point five, aur (and) ten point fifty (sic)

In the extract above, the teacher attempts to convert the process of problem solving into a procedure for identifying 'key words' in the statement of the problem and converting them into commands for mathematical procedures.

In line 11, 13, and 15 the teacher directs learners' attention to the word "dono (both)" and prompts them to use the word "dono (both)" to identify the operation—plus, minus, multiply or divide—that should be carried out to provide a solution to the problem. Learners have already got the idea and are shouting "plus" (line 10, 12). In line 16 they offer the two values that are to be added.

From line 20–29 (Appendix B), she takes the class through the procedure of addition of decimal numbers (4.50 and 10.50) by cautioning them to vertically align the decimal points by placing one below the other (line 27). Working through the procedure, in line 30, learners offer the correct answer fifteen point zero zero. However, they do not offer the unit of length and she prompted them to do so in line 35.

Moreover, learners utter choral brief responses, mainly consisting of mathematical terms or numbers, in response to the teachers' prompts and procedural instructions. The only instance of a complete and extended contribution in English was when one learner read the statement of the problem. The learner mispronounced the word "metre" as "matter" and the teacher corrected her pronunciation.

The above extract was illustrative of a pervasive pattern in teaching solution of word problems. Teachers prompt learners to focus on a key word or phrase in the problem statement that provided a clue to the operation to be used in solving the problem. However, in multilingual classrooms, such as the one shown here, it involved an additional process of translation. Hence we saw that the word "both" was translated as "dono" which could mean 'the two combined' or 'first and second numbers together.' Some learners interpreted "dono" as a signal to combine or plus (line 10 & 12). While others interpreted it as first and the second number, "four point five zero and ten point five zero" (line 16 & 18). The teacher accepted "Plus" as the correct answer (line 19) and moved ahead with adding the two numbers 4.50 and 10.50. However, had she accepted the second answer provided by the learners "four point five zero and ten point five zero", it is likely that learners would have had to justify explicitly the decision to add the numbers. In the extract above, it remained unclear whether or not all learners recognized the reason for taking the decision to plus.

Locating the two illustrative extracts within Fraser's framework, the focus in the classroom interactions was on ways of naming terms in English language so that the mathematical capital in terms of conceptual knowledge and mathematical relationships was not being distributed to the learners. Participation was limited in nature to 'safe talk' with little evidence of conceptual learning. Significant conclusions and recommendations can be drawn for a dynamic that would support the social justice intentions of the policy of language of instruction when implemented in the classrooms.

DISCUSSION AND CONCLUSIONS

A key conclusion is that the policy aim of redistribution of cultural capital including knowledge of mathematics and proficiency in English, did not appear to be achieved because the policy positioned English in a position of power and did not recognize the teachers' and learners' marginalized position as non-English speakers. As far as the language in use was concerned, it was evident that neither the teacher nor the learners could use English for meaningful communication. While the policy required the use of English, what came across as English in practice was merely names of mathematical concepts. If the policy aimed at distributing *English* as the cultural capital, it was clearly failing in achieving this aim.

What was of greater concern in mathematics classrooms was the lack of evidence about transfer of mathematical capital. Nature of interactions showed that the classroom talk was mainly in the realm of procedural discourse of mathematics. For example, the teachers' questions were limited to asking students to apply procedures and learners' contributions and questions were concerned with taking the procedures forward. While, emphasis on a procedural discourse was not entirely due to learners' and teachers' lack of proficiency in the language of instruction, the procedural dictations were arguably 'safe talk' which were strategies to escape from the difficulties of engaging in meaningful communication in a second or third language (Chick, 1996). More significantly, when safe talk dominated classroom interactions, little cultural capital was traded between teachers and learners.

It was reasonable to expect that in the course of teaching and learning processes, learners' participation would be reflected in the quantum of their contribution and in the quality. However, the profile of language use in this case showed that the students were largely mute. This pattern of students' lack of participation was not limited to this lesson but was noted throughout all the observed lessons. As noted in the data on overall interaction patterns, learners seldom uttered a full sentence in English, i.e., only 5% of all the utterances, and that too usually when asked to read something from the textbook. The mixed language utterances typically involved substitution of Urdu by English terms inserted in sentences in Urdu. In short, learners were not engaged in meaningful mathematical communication. The learners were neither learning English nor mathematics. However, acquiring academic knowledge and higher order thinking is not just a cognitive function, it is also dependent on the tools of thinking that were provided by culture, mainly being the language. Situated within the context of the social justice framework, the extent and nature of participation actually marginalized the linguistically marginalized twice over. Not only were they denied the opportunity of exposure to use of acceptable acadamic language of instruction, they were also marginalized from a conceptual discourse in mathematics. Essence of social justice from Fraser's perspective was in parity of participation, according to her "overcoming injustice means dismantling instutionalized obstacles that prevent some people from particpating on a par with others as full partners in social interaction "(Fraser, 2008, p. 16). At the level of the classroom, instutionalized obstacles were those cultural norms and patterns of engagement that denied access to the learners to resources essential for interaction with their peers. Significanlty these resources included the language(s) that formed the collective cultural captial in the classroom and forms of mathematical knowledge essential for their success in examination and beyond. In the context of the case study being considered, the policy of English as a language of instruction had however inadvertantly further entrenced those obstacles by not recognising the cultural and linguistic diversity of the learners.

Several recommendations could be made to enable the social justice aspriations implicit in the current policy of language of instruction so that learners benefit from a meaningful participation in learning and transfer of mathematical capital. First, for parity of participation in the classroom interactions the structural hierarchies in the relationship of learner and teacher would need to be questioned. As it stands the classroom dynamics were tightly controlled through a structured pedgagic practice with implicit norms that did not necessarily empower the learners. For learners marginalized due to language and culture or other forms of exclusion (e.g., gender, social class, disability) teaching and learning strategies would need to be adapted to enable a wider participation through creating space in the classroom dynamics for learners' voice to be heard (Tikly & Barret, 2013).

Second, an assumption underpinning the language of instruction policy was that all the education processes would be conducted in the target language once the policy was mandated. This assumption was reflected in the prescribed textbooks that were written in English and the end of year examination that learners were expected to write in English. In the classroom all 'official processes' were conducted in English, these include the work on the chalkboard, setting of assignments for learners to do in their notebook, and in-school examination. However, classroom interactions showed that learners and teachers employed Urdu to negotiate mathematics problems encoded in English. Street (2003) proposes a view of language as a "socially situated practice and recognizes the diversity in language as a resource and an approach to democratize the educational process and contribute to greater equality and opportunity" (p.134). An implication of this theoretical positioning is to problematize the assumption that teachers and learners bring clearly defined systems of language in classrooms because language in practice is fluid, moves across boundaries and takes meaning in context.

To conclude, in mathematics classroom, developing learners' participation in mathematics is the valued ideal, and redistribution of linguistic capital could support learners' participation in mathematics if it recognised the differentiated backgrounds, experience and needs of the mathematics learners. A nuanced interpretation and implementation of the language of instruction could mean that learners' first or proximate languages are seen as a resource that the teachers could employ to facilitate the participation of learners in the process of learning. This recognition would not simply be a technical change introduced through teaching techniques but would entail a different mindset to accommodate the learner as a participant. Deep seated assumptions about appropriate language and pedgogic practices would need to be challenged for transformative social arrangements in the classroom.

ACKNOWLEDGEMENT

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NOTES

[1] In some schools the traditional 'kutchi class' is offered to prepare learners for schooling. The primary school age 6-10 yrs., is given in NEP 2009. Other sources note the age as 5-9yrs.

[2] According to the Education Policy 2009, the Primary and Middle school levels are being merged to form the Elementary Level (Classes one-eight).

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APPENDIX A

1. T: Bachonkal hum nay kya parha? [Children, What did we study yesterday?]
2. L: HCF (Chorus)
3. T: HCF ka matlab kya hai? [What is the meaning of HCF?]
4. L: Highest Common Factor (Chorus)
5. T: Aaj hum nay parhna hai HCF by Prime Factorization. —ki choti choti tajziyan banti hain. [Today, We have to study HCF by Prime Factorization—small small factors are made]
6. T: Sab say pehlay 2 aab 2 par iss ko taksim karna hai [First 2, Divide this [number] by 2].
7. T: 2 par yeh taksim nahi hua [This [number] was not divisible by 2]
8. T: Aab 3 par isko taksim karna hai [Now divide this [number] by 3].
9. T: 3 par yeh taksim hogaya [This [number] was divisible by 3]
10. T: 2—nikalay 75 teen par taksim hogaya [75 was divided by 3]
11. T: unintelligible
12. T: 3 multiply 5, 3x5=15 [Number facts in English]
13. T: Yeh 3 par taksim hogaya- aab 75 kaay, factor of 75 liktay hain [This was divided by 3. Now we write the factor of 75]
14. T: Yani iskaay 3x5x5=75 [So, its factors are…number facts written].
15. T: Aab hamnay HCF nikala tau hum aab iss kaay common factor likhain gay [Now we have to find HCF…we will write the common factors]
16. T: Common factor hain? [Common factors are? *A prompt*]
17. T: Common kaun kaun say hain? [Which ones are common?]
18. T: Idhar 2 hain idhar 2 nahi hai [2 is here, but 2 is not there]
19. T: Idhar 3 hai aur idhar 3 nahi hai [3 is here, 3 is not there]
20. T: Idhar 5 hai aur idhar bhi 5 hai [5 is here, 5 is also there]
21. T: Yeh common hain, 5x5=25 [These are common…number facts]
22. T: aab yah common hain [Now these are common]
23. T: Humain common factor mil gayaye hain [We found the common factors]
24. T: Aab humain inka HCF nikalna hai. [Now we have to find the HCF]
25. T: 5 ko 5 kay sath multiply kariain gay tau humara HCF nikal aya [multiply 5 by 5 and we have our HCF]
26. T: HCF kya hai 25 [What is HCF? 25]
27. T: Samjh aya saval? [Did you understand?] Dobara duhrana hai? [Repeat it?]
28. T: HCF kya hai? [What is HCF?]
29. L: Highest Common Factor. (Chorus)
30. T: [–]
31. T: Agla saval likhain 70, 49 [write next question 70, 49]
32. T: Bana logay? [will you be able to do it?]

APPENDIX B

1. T: Statements kaay savalaat hain. [*(These are) questions with statements*]
2. L: Miss mein Parhun. [*Miss may I read*]
3. T: Chalain beta koi parhay-yeh statement parhain-savaal number eik ki-ji [*ok child one of you read the statement of question number one-yes (points towards one learner)*]
4. L: (Reading from the book). Amna bought four point fifty matter of clothes
5: T: Metre, matter nahi metre. [*Metre not matter metre*]
6. L: Metre. Ayesha bought ten point fifty meters of clothes. How many meters cloth did they both buy?
7. T: Jee. [*Yes*]
8. L: Eik Jaisa. [*Alike*]
9. T: Metre, matter nahi metre. [*Metre not matter metre*]
10. L: Plus (chorus)
11. T: Bachon donoka kya matlab hai? [*children what is meant by 'dono'*]
12. L: Plus (chorus)
13. T: Bachon donoka kya matlabhai? [*children what is meant by 'dono'*]
14. L: Ten?
15. T: Dono ko aapnay kya karna hai? Plus karna hai, minus karna hai, multiply karna hai, divide karna hai? [*what do you have to do to both? Plus, minus, multiply or divide?*]
16. L: Four point five, aur ten point fifty
17. T: Ji plus karna hai-donoka kya matlab hai? [*yes you have to plus- what is meant by 'dono?*]
18. L: Four point fifty aur ten point fifty
19. T: Plus dono-theek hai-vo likh raha hai Amna bought four point fifty meters of clothes. Amna ney kitna kapra khareeda? 4.50 metre, theek hai? Ayesha bought? Kitna karpra khareeda? [*Plus both, all right? It is written that Amna bought four point fifty meters of clothes. How much cloth did Amna buy? 4.50 metre, all right? Ayesha bought? How much cloth (did she) buy?*]
20. L: Ten
21. T: Ten point fifty meters
22. L: zero zero zero
23. T: How many clothes both buy? Dono nay kitna kapra mil kar khareeda? [*how much cloth did both buy altogether?*]
24. L: Ten
25. T: Aap nay kis kis ko plus karna hai? [*which ones do you have to plus*]
26. L: five
27. T: Point kaay neechay point hoga-yeh 50 vaisay hi aagaya aur yehan yeh kya aayay ga-10 [*place point under the point- 50 will come as it is. What will come here, 10*]

28. L: .1 Aisay hi one [*.1 as it is. 1*]
29. T: Theek hai aab aap inko pura karlain [*all right now you complete it*]
30. L: Fifteen point zero zero
31. T: Zero zero zero
33. L: Fifteen hundred
34. T: Five five?
35. L: Fifteen metre
36. T: Zero 1 carry ka hai-point kaay neechay point aa gaya. Point four or one [*Zero, 1 is for carry over, place the point under the point. Point four or one*]
37. L: Miss mei

PETER MAYENGO KAJORO

6. TRANSITION OF THE MEDIUM OF INSTRUCTION FROM ENGLISH TO KISWAHILI IN TANZANIAN PRIMARY SCHOOLS

Challenges from the Mathematics Classroom

INTRODUCTION

Tanzania, along with Kenya, Uganda, Burundi and Rwanda, belongs to the East African community, a regional intergovernmental organization that aspires to lead the five countries into a political federation (East African community website). Although Kiswahili is widely spoken in all these East African member states, it is only in Tanzania that the language has been given the status of both the national language and that of the medium of instruction in all basic education institutions. This means that almost all official business is normally conducted in Kiswahili and all teaching in public pre-primary and primary schools is undertaken in Kiswahili. English, the language of the last colonial power in Tanzania, continues to be used in some official matters to a very small extent. It is however still the official language of instruction for all post-primary education.

Tanzania came into being in 1964 after the union of Tanganyika (hereinafter referred to as Tanzania mainland) and Zanzibar (the two main islands off the Indian Ocean coast of Tanzania mainland, Unguja and Pemba). Although this chapter will concentrate on Tanzania mainland, there has been, until very recently, so many similarities in language policies between the two parts of the union that an extrapolation of what was asserted about one part of the union in terms of language policy and language of instruction could, to a large extent, be made to the other with very slight variation. The current Zanzibar education policy however, while it still maintains Kiswahili as the medium of instruction at primary school level, makes English the medium of instruction from year five of primary schooling for both mathematics and science (Government of Zanzibar, Ministry of education and vocational training, 2006). It should also be noted here that although Kiswahili is widely spoken in both parts of the union, it is the mother tongue of almost all Zanzibaris, which is not the case for the mainland where the majority of the rural population speak their respective ethnic tribal languages as their mother tongue and learn Kiswahili at school.

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 73–85.

This chapter looks at why, despite the fact that Kiswahili is Tanzania's national language and that the language has been used as the medium of instruction at primary school level for more than four decades, it has not been phased in as an official medium of instruction at post-primary level of schooling. It also looks at some challenges in the teaching and learning of mathematics that came about as a result of the change in the medium of instruction from English to Kiswahili at primary school level. It furthermore delineates more challenges regarding students' transitioning from Kiswahili medium primary schools to English medium secondary schooling. It concludes by calling for a review of the national language policy to gradually phase in Kiswahili as the sole language of instruction at all levels of education by initially providing for both English and Kiswahili as languages of instruction.

HISTORY OF THE NATIONAL LANGUAGE POLICY

Tanzania mainland received her political independence from Britain in 1961. During the British colonial rule, which lasted for about 40 years, the British government had instituted an educational system in which initial instruction in the first year of primary education was given in a mixture of a people's mother tongue and Kiswahili, a language which was mostly spoken along the Indian Ocean coastal strip and the Zanzibar islands, but which had partially gained the status of a lingua franca over the entire territory, thanks to the language policy instituted by the Tanzania mainland's first European colonizers, the Germans. The German colonizers had made Kiswahili the language of administration and education (Swilla, 2009). It is important to note that in the context of rural Tanzania mainland, teaching in a mixture of the indigenous languages and Kiswahili in the first and second years of schooling, was the most natural option given the fact that there were more than a hundred indigenous ethnic groups in the country (Sa, n.d.), each with its own language and only using Kiswahili for the purpose of communicating with outsiders. This meant that children from these different ethnic groups, especially from rural areas, who were enrolled in the eight-year primary schools, would hardly speak Kiswahili in their initial years of schooling. It therefore made a lot of sense in teaching to use a mixture of the children's mother tongue and Kiswahili in a bid to transition them into Kiswahili for later years of schooling. English as a language was introduced in year three but the official medium of instruction continued to be Kiswahili for the first four years. The language of instruction started moving towards English in year five and completely changed from Kiswahili to English in year six (Swilla, 2009), with English as the sole language of teaching in years seven and eight.

Post-independence primary education was characterized by various educational policy changes, many of which were principally prompted by the wish to distance the new nation and her people from their colonial past. Furthermore, the new nation wanted to mark out its own identity in terms other than just the name and flag. Legère (2006) argued that Tanzanian language policy after independence in 1961 put emphasis on Kiswahili as an *authentic symbol of the Tanzanian nation* (my italics for emphasis). Consequently, one of the most conspicuous policy changes in education that were instituted after independence were those which saw the primary school medium of instruction become Kiswahili throughout the primary education cycle and the primary education cycle being reduced from eight years to seven years.

Although the medium of instruction in primary schooling was completely changed from English to Kiswahili, English was maintained as the medium of instruction at secondary school level. A number of reasons for maintaining English as the language of instruction at that level have been suggested. Swilla (2009) observed that high costs in financial terms that would have been required in the preparation of the teaching force and teaching materials for Swahili-medium post-primary education might have influenced the decision to retain English. He further observed that in order for Tanzania mainland to continue being part of regional and international communities, knowledge of English as an international language was of importance and therefore it was necessary for secondary school students to continue getting exposure to the language, not only in English language lessons, but also in all the other subjects (except Kiswahili) taught at that level so as to increase their exposure to the English language and that way guarantee their mastery of the English language.

Maintaining English as the language of instruction was meant to be a temporary measure as the ground was being prepared to make Kiswahili the medium of instruction at secondary school level and beyond. Legère (2006) posited that a compilation of terminology wordlists, focusing mainly on the subjects taught at secondary school level, had been embarked on by the Tanzania Kiswahili Council following a 1969 decision to gradually phase in Kiswahili as the medium of instruction beyond the primary schooling cycle. Although the National Kiswahili Council did its best in compiling this terminology wordlists, the phasing in of Kiswahili was met with many political and socio-economic forces that prevented it in becoming the medium of instruction at secondary school level. These political and socio-economic forces will be discussed in detail later in the chapter.

CHALLENGES FROM THE MATHEMATICS CLASSROOM

Challenges Related to the Mathematics Register

This envisaged major education policy change was not without challenges for mathematics teaching. Two of these were; firstly, not all pupils from the different peoples that constituted the new nation, Tanzania mainland, had the requisite competency in the language of instruction to be able to effectively interact with the primary school curriculum, and secondly lower primary school (standards one to four) teaching and learning materials had been in Kiswahili even before the medium of instruction policy change, however, upper primary school (standards five to seven) materials were all still in English and needed to be translated into Kiswahili.

More importantly however were the challenges brought about by the technical vocabulary for subjects like science and mathematics. Hence it was planned that the technical mathematics register was to be developed, sometimes from the scratch. A lot of input was required in these two subjects because of the existence of many discipline-specific terminologies; to this end a National Kiswahili Council (in Kiswahili: Baraza la Kiswahili la Taifa abbreviated to BAKITA) was established to spearhead the development of technical terms for, among many others, school curriculum teaching documents (Legère, 2006). Furthermore, teachers for the upper school, who were used to teaching in English, were now supposed to familiarize themselves with the new Kiswahili technical vocabulary so as to be able to teach their respective disciplines in Kiswahili. It should be remembered that although the teachers were fluent speakers of Kiswahili, the National Kiswahili Council had developed some completely new terms that were introduced into the Kiswahili language to form subject-specific register. Teachers had to learn these as well as other Kiswahili terms, which though familiar, had been assigned an additional technical sense.

Hence completely new terms were introduced into the Kiswahili vocabulary, which were hitherto not part of the ordinary day-to-day Kiswahili vocabulary. For instance, for circumference (and the deemed Kiswahili equivalent term Kivimbe), diameter (Kipenyo), radius (nusu-kipenyo), lowest common multiple (kigawe kidogo cha shirika, KDS), the highest common factor (kigawo kikubwa cha shirika, KKS), significant figures (tarakimu aushi), and BODMAS (MAGAZIJUTO)[1] are a small sample of the newly developed terms that were then introduced into the Kiswahili vocabulary as mathematical terms. These terms had to be learnt by the teachers of the mathematics syllabus, before they could effectively use them in the classroom.

On the other hand, some words were coined from words that were already in use in the day-to-day Kiswahili vocabulary, and they were then assigned new mathematical meanings. The Kiswahili equivalents (in brackets) for transversal line (mkingamo), integers (namba kamili), brackets (mabano), to name but a few, became part of the mathematics vocabulary. These terms too had to be learnt by teachers before using them effectively in the mathematics classroom.

Considering the fact that language is a critical component of teachers' pedagogical content knowledge (Meaney, Trinick, & Fairhall, 2014), teachers' unfamiliarity with the new mathematics register must have compromised the teachers' efficacy in managing the mathematics teaching and learning process. Perhaps what did not pose a challenge to mathematics teachers were mathematical words that were directly adopted from English, in phonological terms, but were orthographically adapted to Kiswahili. Some examples were: graph (grafu), algebra (aljebra), set (seti), kilometre (kilometa), centimeter (sentimeta), kilogramme (kilogramu). There is every reason to praise this move for it is a strategy used in many languages. English itself borrowed a number of words from both Greek and Latin and anglicized them.

Challenges Regarding Textbooks

Although borrowing words from English and adapting them in accordance with Kiswahili orthography has been lauded in the previous section, there were however a number of oversights on the part of mathematics textbook developers, which, in my opinion and experience, served to undermine the status of Kiswahili in the minds of the students and implicitly worked against the adoption of Kiswahili as the medium of instruction. Right at the outset, the retention of the letters c, q, and x in algebra, despite the fact that these three letters are absent from the Kiswahili alphabet but of course are part of the English alphabet, seemed to be saying very saliently, but perhaps implicitly that Kiswahili was not "fully developed" as a resource for the expression of mathematical ideas and English had to be called in to rescue the situation. Mbena, Haule, and Masota (1976) used the letter x to name angles (p. 65) and the letter Q to name points in a plane figure (pp. 66 & 68). Furthermore, the learner is asked to simplify algebraic expressions 7x+3x+x and 4b+8q+2v as an exercise (p. 143), both of which contain letters that are not part of the Kiswahili alphabet. There were many options, within the Kiswahili alphabet, which could have been used to stand for the unknown values or for variables.

Perhaps the phenomenon above would have been tolerable if it had been corrected in subsequent years. But unfortunately it has continued, for many years to date. Sichizya and Kwalazi (2010) in a mathematics book currently in use in standard six used the letter Q to name angles (p. 88) and the letter x many times in chapter eight on algebra and on many occasions elsewhere in the book. Although this would be taken as comparable to the move described above where English words were directly adopted but adapted orthographically, the presence of many other options in the Kiswahili alphabet that would have furnished letters to stand for variables and unknown values mitigates the strength of any reasons that could be advanced in favour of retaining x, q, and c in mathematics textbooks written in Kiswahili.

Moreover, teams of textbook writers seemed to have adopted some formats of writing mathematical symbols for units that were inconsistent with the way symbols for units of physical quantities are generally written in Kiswahili. There is a marked difference between the Kiswahili syntax and the English language syntax in using units of measurements. English places the unit of measurement after the quantitative numeral representing the amount or size of the physical quantity being measured. For instance, in measuring distance between two points, one would announce the result as 'the distance between the two points is say 15km or 50cm etc.' In Kiswahili however, this would have to be rendered as km15 and cm50 respectively (written in Kiswahili as km 15 and sm 50, respectively), since the Kiswahili syntax requires that the unit be placed before the quantitative number that represents the amount/size of the physical quantity being measured. This rule applies to all cases of measurements irrespective of the physical quantities being measured.

Unfortunately, the rule above was completely ignored when it came to measurement of angles and temperature. In English, the size of an angle would be presented as 15° or 175° or the degree of hotness/coldness would be given as 60°C. This, in Kiswahili and according to the rule above, ought to have been rendered as °15 and °175 respectively while the temperature would have been given as °C60.[2] But writers of Kiswahili mathematics textbooks (e.g., Mbena, Masota, & Haule, 1976; Sichizya & Kwalazi, 2010) have declined to observe this rule and have simply adopted the English way. The same observation would be made for 56% in English, which ought to have been written as % 56 in Kiswahili, but textbook writers have continued to use the English way of writing percentages. It is argued here that such scenarios as these are exposing the mathematics learners to a number of issues all of which implicitly point towards reinforcing the belief that English is the 'proper' language of mathematics teaching and learning and not Kiswahili.

An additional pointer to this conclusion is found in the mathematics textbooks currently on the market. For example in one common textbook which was written just after the policy to change the medium of instruction in primary schools from English to Kiswahili, the circumference of a circle was given as $V = \pi p = 2\pi k$ (Mbena, Haule, & Masota, 1976, p. 74), where V is the circumference (kiVimbe in Kiswahili), p is the diameter (kiPenyo in Kiswahili) and k is the radius (nusu kipenyo in Kiswahili, which literally translates into half the diameter). Books that are on the market currently give the circumference of a circle as $C = \pi d = 2\pi r$ and d is defined in Kiswahili as 'kipenyo' while r is defined in Kiswahili as 'nusu kipenyo' (Sichizya & Kwalazi, 2010, pp. 109 & 111). Here, it clearly shows that the English letters d and r are being given preference to the Kiswahili terms and symbols.

Mathematical formulae act as mnemonics to the learners. That is why a first language English speaking child learning mathematics in his/her mother tongue would very easily remember the formula $V = ⅓ \pi r^3$ if V stands for *V*olume, r stands for *r*adius. How difficult would it have been to remember the formula if it had been given as $V = ⅓ \pi k^3$? Substantively, if P stood for volume, and k stood for radius, the formula would be perfectly correct, but pedagogically speaking, it would have very limited value in helping the child remember the formula. We have a similar situation in $C = \pi d = 2\pi r$ with C given as 'mduara' d as 'kipenyo' and r as 'nusu kipenyo.' The letters do not have visual markers/indicators that would assist the learner remember the formula and thus they do not act as mnemonics. In summary, these examples from textbooks give the distinct impression that Kiswahili simply acts as a placeholder as learners wait to embark on serious mathematics studies at a later stage when they do the learning of mathematics in English.

Post-Primary Education Challenges

There were additional challenges when it came to primary-to-secondary school transition: mathematics and science technical terms had to be learnt in English, which posed an additional task to the learning of mathematics or science. Moreover, as years went by, it became increasingly difficult for pupils joining secondary schools to understand lessons delivered entirely in English (Brock-Utne, Desai, Qorro, & Pitman, 2010), given the fact that English had continued to be the medium of instruction in secondary schools. One reason was that secondary school teachers were finding it increasingly difficult to conduct meaningful lessons exclusively in English. But since both primary and secondary school teachers had to be drawn from graduates of this same education system, a point was reached when both students and teachers in secondary schools were unable to effectively conduct lessons exclusively in English. Qorro (2006) aptly summarized this scenario when she argued, "in Tanzania secondary school classrooms and higher education the language of instruction is not well understood by the majority of teachers and most students" (p. 4).

A direct result of the foregoing was that code switching increasingly became the order of the day (Brock-Utne, Desai, Qorro, & Pitman, 2010). Although this phenomenon is not unique to mathematics lessons, it has far reaching implications in the mathematics classroom. In this context code switching, often noted as a help to learning when teachers and learners are competent in both languages (Clarkson, 2007), here it impedes learning since both teachers and learners are not competent in the English language. Thus switching between languages to try and understand the mathematical concepts, and given the fact that mathematics is a heavily concept-laden discipline with concepts strongly linked to language, no cognitive advantage accrues to the learners. The latter was confirmed by Lukari (2010) who conducted research in mathematics classrooms in a public secondary school in Dar-Es-salaam. Although the researcher originally extolled code switching in that it facilitated the learning of mathematics by enabling the learners to interact with one another during the teaching and learning process, and hence allowing for thorough understanding of the mathematical concepts, she nevertheless found in this context it had detrimental effects as a direct result of both teachers and learners not being competent in English. Hence, during translations of words and reformulation of tasks into the other language, such code switching led to confusion and misconceptions. Perhaps this, I think, gives a clue that it is a good thing to use the two languages in the classroom, but strategies should be put in place to ensure that competent English language teachers handle English language teaching, and that all teachers, including mathematics teachers, be given English language courses to polish their English language skills.

Moreover, the argument that code switching can assist learners to understand mathematical concepts in the Tanzanian context where both teachers and students are incompetent in English would still be of very limited use when it came to sitting for national examinations in which they would have to read and comprehend instructions and questions in English; formulate their responses and write them in English. If code switching has become the order of day, perhaps examination policy ought to be flexible enough to reflect this classroom reality by moving towards bilingual examinations.

Rote learning and blind recitation are inevitably a direct result of the learners' limited English language proficiency (Brock-Utne, Desai, Qorro, & Pitman, 2010). Having well understood the mathematical concepts via the medium of a well-understood language, Kiswahili, the learners are now obligated to recast this understanding in a language they are not conversant with. The best option then becomes rote learning and memorization. Since mathematics is a concept-laden and concept-driven discipline, any attempt to learn it using the memorization strategy unescapably leads to a cul-de-sac of their mathematics-learning trajectory.

POLITICAL AND SOCIO-ECONOMIC FACTORS AGAINST KISWAHILI

The challenges highlighted above all serve to undermine the effectiveness of mathematics teaching and learning. Mathematics is a vital subject whose importance in transforming the society's economy has been acknowledged in Tanzania'a development vision 2025. The language issue in education is one of the major factors that have continued to undermine government efforts to remedy mathematics teaching and learning.

As noted above, the use of English in secondary school setting was initially taken as a temporary state of affairs. Although there were initial plans to extend the use of Kiswahili as a medium of instruction to secondary schools, which would have got rid of the problems associated with primary school to secondary school linguistic transition, the plan has not been realized to date. There were several political and socio-economic forces at work against the promotion of Kiswahili as a medium of instruction, so much so that today, more than fifty years after independence, Kiswahili has not been made the medium of instruction at secondary school level and beyond.

Quite a number of people in the country acknowledge the fact that English is not well mastered by secondary school students. Its continued use as the sole official language of instruction at that level would appear to be sheer lack of serious intent on the part of educational policy makers. However, in order to understand the immensity of the problem of replacing English with Kiswahili as a language of instruction or even officially accepting the two languages to be used as official languages of instruction at secondary school level, it would be imperative to study the forces that have worked tremendously against that move. Rajani, Scholl, and Zombwe (2007) attributed this inertia to principally two factors: political factors and socio-economic factors. Whilst lauding a total re-orientation of the educational system in the country from the contemporary 'lots-of-schooling-but-little-learning' scenario to a learner capability approach that would provide true learning, the authors list a number of elements that would need to be redressed before such a transformation could be realized. One of these elements was language. They then go on to say;

> … the issue of language cannot be ignored, in Tanzania primary education is taught in Swahili and secondary suddenly shifts to English, even when most learners are far from proficient in its use. As a result, most students find themselves having to learn different subjects in a language that they do not understand. The scientific evidence is clear: students learn better in their mother tongue or other language in which they are very competent. However, government policy and public perception seems to equate English with progress and achievement. Quality education in Tanzania cannot be achieved without a thorough consideration of the language of instruction issue. (p. 10)

But why has the government not been able to heed the calls from a number of researchers for reconsideration of the language policy and act in a way that would appease those who, like the authors cited above, are discontented with the use of English as the sole medium of instruction? Why hasn't, for instance, Kiswahili been given the status that English has at secondary school level, so that at least both are recognized as official languages of instruction? The most resounding response to this query seems to have come from Neke (2005) who, citing a special Correspondent of The *African* newspaper of the 5th November, 1999, concurred with the correspondent in saying that the use of English in Tanzanian schools created conditions that would help to maintain inequality between the poorer people and the middle and upper socioeconomic class. The majority, who did not speak English, was likely to remain poor peasants and unskilled labourers. This is concretized by the fact that, according to Swilla (2009) "99.17% of private primary schools are English-medium and 99.59% of government schools are Swahili-medium. Therefore, private primary education is synonymous with English-medium, and government primary education, with Swahili-medium education" (p. 5). Given the fact that poor peasants and all the other common people cannot afford to enrol their children into private primary schools, it would seem like Kiswahili is good as a medium of instruction only for the children of the poor but not for the children of the well-to-do, for whom education in English is the most natural thing that makes sense. It further disadvantages the children of the poor when they move on to secondary school level where the language of instruction becomes English. Those who come in from private primary school have the upper hand since they have been learning in English during the entire primary school cycle.

Just like English was a language of power spoken by the colonial masters and a few 'privileged' Africans who worked close to their masters, so has it been replicated in post-independence Tanzania mainland. Neville Alexander (2000, p. 11) quoted by Brock-Utne and Holmarsdottir (2004) asserts that in post-colonial Africa it is knowledge of English and/or French [for example] that sets leaders apart from the vast majority of their African compatriots and which keeps them and their offspring in the privileged middle and upper classes.

Neke (2005) has also argued that decisions about language-in-education issues were socio-economic, since potentially they entailed a reversal of power relations and could lead to certain groups in the community, whose language is not selected, finding themselves on the fringes of the socio-economic and political spectrum. What this actually means in the Tanzanian context is that the "neo-colonial elite" referred to above cannot allow the replacement of English by Kiswahili, or even the use of both, because such a move would inevitably redefine power structures and distribution in the society, the end results of which may not be easily predicted. The most viable option therefore becomes that which maintains the status quo, which assuredly continues to privilege them.

Furthermore, there is another force that has continued working against the adoption of Kiswahili as a medium of instruction at secondary school level. The general public has continued to equate education with English (Rajani, Scholl, & Zombwe, 2007), a fact that has been exploited massively by policy makers in favour of continued use of English as a sole language of instruction. This perspective among the general public was a direct result of British colonial rule lasting for about forty years. During colonial rule, schooling was equivalent to engaging in a process that brought the educated person closer to the white man's ways of life, including mastering the master's language. It was on the strength of this mentality and attitude awash in the minds of the general public that the then Minister of Education, when responding to a question about the follow up of the proposal to start teaching in Kiswahili in the secondary schools in Tanzania from 2001, as reported by Brock-Utne and Holmarsdottir (2004), said:

> I hear there is some pressure to change. It mostly comes from professors. My own opinion is that I have to take into account what the community wants. Is it the community that has asked for this change? I get a large number of applications from groups that want a license to start English medium primary schools. I have not had a single application from anyone who wants to start a Kiswahili medium secondary school. The Tanzanian community is not thinking about this language issue. I hear it from professors. I don't hear it from the community. The day I hear it from the community I shall start thinking about it. (p. 4)[3]

The minister exploits the community's ignorance to uphold a national language policy that is absolutely untenable considering the classroom linguistic realities in the country. The minister failed to see or chose not to see the fact that those who apply for licenses to operate English medium schools are driven by the false relationship between education and English that the public has been forced to establish in their minds. The minister's responsibility was to straighten this crooked thinking by explaining to the public what ought to be happening on the basis of the professors' researched pronouncements, instead of ridiculing the professors. If medical doctors had advised a minister of health to ban the use of chloroquine because research had indicated that continued use of the drug would have serious side effects on the patients, would it be in order for the minister to insist on wanting to hear it from the community first before he /she implements the ban? Matters pertaining to education are just as technical as those that pertain to other fields like medicine, and pronouncements from professors of education should be taken seriously and given due consideration.

Little wonder therefore, current mathematics textbook at primary school level are written in an "anticipatory" manner. They would rather give the measure of an angle as 57^0, the way it would rightly be written in a mathematics textbook in English, even when it is in a mathematics textbook written in Kiswahili. It would be inappropriate to write it as ^{057}, the way it ought to appear in accordance with Kiswahili syntax, because it would not blend well with what they will encounter at secondary school level, where mathematics books are in English. Likewise, such textbooks use $C = \pi d = 2\pi r$ with C given as 'mduara' d as 'kipenyo' and r as 'nusu kipenyo', because that is what they will interact with at secondary school level. $D = \pi k = 2\pi(½k)$, where

D stands for m*D*uara (i.e. Circumference); k stands for *K*ipenyo (i.e., diametre) and ½k stands for nusu kipenyo (i.e., radius, but literally half-diametre) would simply waste their precious time and would ultimately be a source of confusion at secondary school level where 'real' education commences.

WHAT IS THE WAY FORWARD?

Considering the importance of language in concept formation and in learning in general, given the widespread belief that language and concept formation are very closely linked (Vygotsky, 1978; Wildsmith-Cromatry & Gordon, 2009), and given the fact that mathematics is a heavily concept-laden discipline, the language-of-instruction scenario in Tanzania becomes critical in relation to the teaching and learning of mathematics in Tanzanian classrooms. Literature abounds which discusses which path the country should follow as far as the medium of instruction is concerned. The most cogent arguments have come from Qorro (2002), quoted by Brock-Utne and Holmasdottir (2004), who has argued that to continue using English as the medium of instruction defeats the very purpose for which it is claimed to have been maintained, that of helping learners improve their proficiency in the English language and be able to use it for learning other subjects. Likewise Heugh (2011) who, on the basis of findings from many research projects, suggested that:

> Successful education requires mother-tongue-medium education throughout, […]. It can also include the teaching and learning of a second language for use as a second, complementary medium,[…]. Successful education everywhere requires mother tongue based systems. In Africa, this means African language-based systems. The end target of school cannot be the former colonial/official language only. The target must be a high level of proficiency in at least two languages – that is, academic bi- or tri-lingualism which include the mother tongue (or a language closest to this) plus an international language of wider communication (French, Portuguese, Spanish or English). (p. 154)

In the Tanzanian context Heugh's comment can be seen to be advocating for the use of Kiswahili throughout the schooling career. In his opinion, to have English only being used as the sole language of instruction at any point in the schooling career would be unacceptable. He would argue that English should only be used as a complementary medium of instruction and an international language for wider communication. This suggestion opens the doors to learning even more international or regional languages drawn from the four listed above and even elsewhere (e.g., Chinese or Rwandese).

Nonetheless, since the Tanzanian general public at large still believes that English is the most appropriate language of education (Rajani et al., 2007; Pitman, Majhanovich, & Brock-Utne, 2010), the translation of such a suggestion into reality may prove to be very problematic both socially and politically. Nevertheless a national language policy should be put in place that will see Kiswahili being gradually phased in as the medium of instruction throughout the national education system. The policy would still maintain English not only as a parallel, complementary language of instruction but also as a subject taught by very competent English language teachers for the purpose of communicating with the international community. An abrupt switch over from English to Kiswahili at secondary school level might deny learners the opportunity to make use of technological facilities that have become the prime movers of the 21st century economies with English considered to be the interface with which to interact with that technology.

NOTES

[1] The 'O' in BODMAS for 'OF' did not have an equivalent in MAGAZIJUTO. The author of this chapter would have preferred BODMAS to be translated to MAYAGAZIJUTO, in which case the YA would have stood for OF.

[2] C stands for Celsius (in honour of a scientist). Names may not be translated, the C, though not in the Kiswahili alphabet, may therefore be retained in Kiswahili text for this reason.

[3] See chapter by Clarkson in this volume for a very similar justification given by the Secretary of Education in Papua New Guinea for an abrupt change in language policy in their schools.

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SECTION III

LEARNING MATHEMATICS IN MULTILINGUAL CLASSROOMS

SUSANNE PREDIGER AND NADINE KRÄGELOH

7. "X-ARBITRARY MEANS ANY NUMBER, BUT YOU DO NOT KNOW WHICH ONE"

The Epistemic Role of Languages While Constructing Meaning for the Variable as Generalizers

INTRODUCTION

Language challenges in mathematics classrooms do not only appear for multilingual students (including bilinguals, and in Europe mostly immigrant students), but also for monolingual learners with underprivileged socio-economic background. As only 4% of students have immigrated to Germany themselves, almost all monolingual and multilingual students grew up in Germany and have developed good basic interpersonal communication skills (BICS, Cummins, 1979). In spite of these skills, large-scale studies show that many multilingual students and monolingual underprivileged students experience substantial language barriers resulting in limited school success and in particular limited achievement in mathematics (OECD, 2007). This discrepancy has been explained by the difference between BICS and cognitive academic language proficiency (CALP, Cummins, 1979) since the acquisition of CALP seems to necessitate access to learning opportunities that are not equally provided by all families. More recently, the construct of CALP has been linguistically elaborated by the constructs 'language of schooling' (Schleppegrell, 2004; Thürmann, Vollmer, & Pieper, 2010) or, in German, 'Bildungssprache' (Gogolin, 2009; Feilke, 2012; Morek & Heller, 2012). These two constructs explain why even many German native speakers experience multilingual challenges in mathematics classrooms: all students have to mediate between *three or six registers*: their everyday language, the language of schooling, and the technical language of mathematics; and possibly each of them in first and second languages (Prediger, Clarkson, & Bose, 2015).

Although this distinction of different registers is now omnipresent in the academic discourse on multilingual classrooms, and even in the European policy discourse (Thürmann et al., 2010), substantial further research is needed for investigating the mechanisms on the micro-level of *mathematics* learning processes. Topic-specific empirical insights into these questions are necessary for supporting students in overcoming these language barriers, as Schleppegrell (2010, p. 107) has claimed.

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 89–108.

This chapter contributes to these research needs with respect to an algebraic concept. The importance of algebra is clear since it counts as a gatekeeper especially for (language) minority students' middle school success (Moses & Cobb, 2001). In particular, within algebra, the meanings of variables are crucial, which is why the exemplary mathematical topic 'meaning of variables as generalizers' has been chosen for investigating the following research questions with underprivileged low-achieving multilingual eighth graders (age 13/14 years):

Q1 *Specification of topic-specific linguistic means:* Which kind of linguistic means are crucial for the algebraic topic 'meaning of variables as generalizers'?

Q2 *Impact of language on learning processes:* How does students' proficiency in the mediating language influence the individual learning pathways to constructing the meaning of variables as generalizers?

Q3 *Designs for fostering topic-specific language learning:* How can the reconstructed language-determined limits be overcome by suitable language- and content-integrated learning arrangements?

These questions combine two general aims, foundational empirical insights into complex processes and developing concrete learning arrangements. The combined aims are treated within the research program of Didactical Design Research (Gravemeijer & Cobb, 2006; Prediger & Zwetzschler, 2013).

After presenting the theoretical background on the algebraic topic and the transitions between languages in Section 1, the methodological background of Didactical Design Research will briefly be sketched in Section 2. Section 3 offers some empirical snapshots from the design experiments that show the epistemic role of languages for constructing meanings.

THEORETICAL BACKGROUND

The Algebraic Topic: Constructing Meaning for Variables as Generalizers

Variables are among the most important concepts in algebra. Their conceptual understanding comprises two essential meanings:

- The *variable as unknown* is dominant for solving equations, more generally in the conception of algebra as "a study of procedures for solving certain kinds of problems" (Usiskin, 1988, p. 12). The unknown stands for a fixed number that has to be found by using the given relations.
- The *variable as generalizer* is needed in contexts where algebraic expressions or equations are formulated or interpreted in algebraic and non-algebraic contexts, mainly in the conception of "algebra as generalized arithmetic" (ibid, p. 11)

or "algebra as the study of relationships of quantities" (ibid, p. 13). Unlike the unknown, the generalizer stands for many numbers at the same time or for successively changing numbers.

Empirical studies have repeatedly shown that these well-specified target meanings are not attained by all students. Many students conceptualize variables simply as meaningless symbols that can be transformed according to formal rules or as a "symbol for an element of an replacement set" (Usiskin, 1988, p. 9; similarly Malle, 1993) or provide other deviant interpretations of variables (e.g., Küchemann,1981; Kieran, 2007).

So far, the role of languages in this limited success has rarely been addressed in research: although some empirical written assessments show a strong statistical connection between language proficiency and algebra skills (MacGregor & Price, 1999), little is known on the role of different languages in students' pathways to the meanings of variables.

A textbook analysis gives first hints on possible obstacles: many German textbooks try to support the construction of the second meaning, the variable as generalizer, by referring to typical linguistic expressions that are used outside algebra classrooms; for example the textbook task in Figure. 1 with "x-beliebig" and "x-mal" (literally meaning "x-arbitrary"[1] and "x-times").

Textbook task (original)	**Translation to English**
1 Was ist damit gemeint? a) Claus zieht eine x-beliebige Karte. b) Diana hat x-mal versucht, Beate zu erreichen.	**1** What is meant here? a) Claus picks an **x-arbitrary** card. b) Diane has tried **x-times** to contact Beate.

Figure 1. Textbook introduction to variables by reference to typical linguistic expressions (Böttner et al., 2006, p. 136)

Assuming these expressions are known by the students, the authors' intention is to remind students of out-of-school-language resources to help their individual construction of meaning. However, the empirical section will show that these linguistic resources cannot be taken for granted for *all* students since they are part of the language of schooling, not necessarily of students' everyday register.

Hence, substantial experiences are needed for constructing the meaning of these generalizing expressions. These experiences can be gained by the well-established shapes and pattern approach (e.g., Mason et al., 1985; Stacey & MacGregor, 2001; Kaput, Blanton, & Moreno, 2008), in which the investigation of growing patterns for shapes and/or number sequences leads to variables and algebraic expressions as means to express general patterns. For example in Figure 2, the algebraic expression $2+x\cdot 6$ gives the general rule for calculating the numbers of dots in the shape at the x-th position. (Note that · is the German sign for multiplication ×).

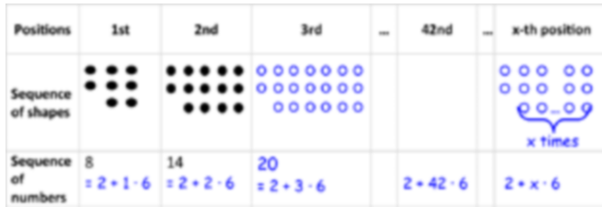


Figure 2. From patterns of shapes and numbers to the variable as generalizer

In our design research project, the shapes and patterns approach was used for a remediating course on variables and algebraic expressions for low-achieving multilingual grade 8 students. Although having some potential for learning mathematics, the students had difficulties during their first encounter with the variables (following other approaches) in their regular classrooms. Hence, the intended learning pathway for this second encounter with the variable comprised the following three stages: The students:

1. find growing patterns for different sequences of shapes and numbers,
2. express these growing patterns informally (with pictures, tables, own verbal descriptions, or with quasi-variable numbers such as 42), and
3. remember or re-discover (instead of invent) the variable as a means to express a symbolic rule for the general case.

As this approach relies heavily on informal expressions that the students invent (e.g., Akinwunmi, 2012), we hypothesized that language plays a crucial role in their constructed pathway. This would give an explanation for the strong connection of learning outcomes in algebra and language proficiency (as shown by MacGregor & Price, 1999). As little is known about the processes so far, this chapter investigates how the interplay between different languages contributes or hinders the intended learning pathways.

Communicative and Epistemic Role of Languages

As *sociolinguists* have pointed out, multilingual challenges in mathematics classrooms cannot only be linked to different minority languages, but also to different registers. In sociolinguistics, a register is defined as a "set of meanings, the configuration of semantic patterns, that are typically drawn upon under the specific conditions, along with the words and structures that are used in the realization of these meanings" (Halliday, 1978, p. 23). The social embeddedness of the communication situation is often emphasized: "A register can be defined as the configuration of semantic resources that a member of a culture typically associates with the situation type. It is the meaning potential that is accessible in a given social context" (Halliday, 1978,

p. 111). Hence registers are characterized by the types of communication situations, their field of language use, and the discourse styles and modes of discourse. In this sense, the language of schooling can be linguistically conceptualized as a register that is situated between, but overlapping with, both the everyday register and the technical register (Prediger et al., 2014; see Figure 3).

The *didactical relevance* of the three registers has been outlined by Pimm (1987): learning mathematics always involves the transition between these registers, and this means moving consequently forward and backward, not only moving in the direction of the technical register (Prediger et al., 2014). For the multilingual learners, the three registers appear in their first and second language (L1 and L2, or even more; see Figure 3).

The *socio-educational relevance* of distinguishing the three registers has been explained by Gogolin (2009). Most teachers are aware that the technical register needs to be acquired in school, and hence give explicit learning opportunities for the technical language. In contrast, the school register (to which only students of privileged socio-economic background are already acquainted) is sometimes treated as *a learning condition*, instead of *a learning goal*, such as the expression "x-arbitrary" in Figure 1. Empirical studies show that limited proficiency in the language of schooling is an important challenge for many (monolingual or multilingual) students in mathematics (Schleppegrell, 2004; Thürmann et al., 2010). This is immediately evident for language difficulties in *test situations* as shown in many American studies (Abedi, 2006), but also for the German language context where the proficiency in the language of schooling could be reconstructed as the background factor with the highest impact on mathematics achievement (Prediger et al., 2013).

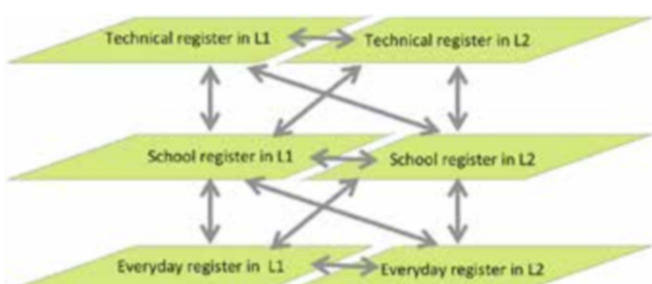


Figure 3. Three registers relevant for mathematical learning (Prediger et al., 2014)

However, the communicative role of the language of schooling is not only relevant in test situations. Every classroom interaction requires language as a *medium for the transfer of knowledge*. Many researchers have stressed that students with limited academic language proficiency are often hindered in showing their mathematical competences in classroom interactions (e.g., Schleppegrell, 2004). In contrast to these accounts of the *communicative role* of language, this contribution focuses on the *epistemic role* of the involved languages in the (individual or/and social) processes of knowledge construction and enculturation into mathematical practices.

Authors who emphasize the *epistemic role* of the language of schooling (e.g., Schleppegrell, 2010; Thürmann et al., 2010; see also Morek & Heller, 2012) especially point to its importance for higher order thinking practices such as abstracting, generalizing, or specifying causal connections (Morek & Heller, 2012, p. 75; Feilke, 2012). These sociolinguistic, didactical and socio-educational considerations show that the distinction of registers and their characteristics offer an insightful theoretical background for explaining possible language difficulties on a macro- and meso-level. However, the *methodological* potential of the distinction of registers for the empirical *micro-analysis* of concrete learning situations is limited by the situatedness of registers and their large overlap. Both features can hinder a unique assignment of utterance one of the registers.

For this reason, the methodological approach for empirical data analysis operationalizes the distinction of languages by *situated repertoires* with a higher potential on the micro-level of concrete learning situations. For analytical purposes, we do not distinguish the sociolinguistic registers but three situational activated linguistic repertoires: the technical repertoire (usually being a part of the technical register, see Figure 4); the individual linguistic repertoire (that students bring into the situation, and which can comprise linguistic means from different registers); and the mediating repertoire (by which the teacher intends to mediate between the others, usually comprising different registers). These repertoires will be operationalized in Section 2.3.

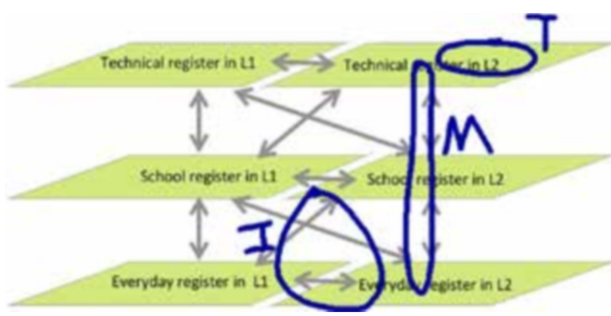


Figure 4. Individual (I), mediating (M), and technical (T) linguistic repertoires within and between the registers

METHODOLOGICAL FRAMEWORK

The specific epistemic role of different languages in student's pathway to constructing meanings of a variable was investigated within a design research framework.

Topic-Specific Didactical Design Research as Methodological Frame

Our framework of Topic-Specific Didactical Design Research (Prediger et al., 2012; Prediger & Zwetzschler, 2013) relies on the iterative interplay between designing teaching–learning arrangements, conducting design experiments, and empirically analyzing the processes. It is shown diagrammatically in Figure 5 (following Gravemeijer & Cobb, 2006).

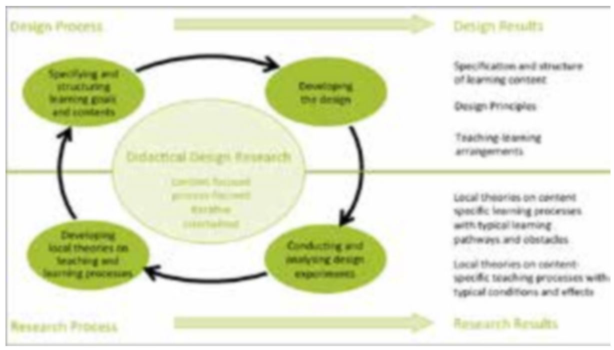


Figure 5. Four working areas for Didactical Design Research

Design Experiments as Method for Data Collection

The design research project deals with a learning arrangement (comprising 10 sessions of 45 minutes each) designed for low-achieving multilingual grade 8 students in a remediating course on variables and algebraic expressions (hence, it is their second encounter with variables). In the overarching project, we conducted six design experiment cycles with a total of 68 students. In sum, about 190 × 45 minutes of design experiments were completely video-recorded and partly transcribed (selections made as they pertained to the research questions).

The case study presented in this chapter uses data from cycle 4 in which the design experiments were conducted in a laboratory setting (see Prediger & Zwetzschler, 2013) by the second author. The four multilingual girls involved in the case study, Ayla and Gözden, Meliha and Gülnur, were 14/15 years old. Their parents immigrated from four Middle East countries before their birth or one year after.

Methods for Data Analysis

The methodological background of the analysis starts from the assumption that mental and interactional processes are linked, but should be carefully distinguished in order to reconstruct the trains of thought and the evolution of linguistic means in the interaction. For our sequential interpretative analysis of the transcripts, we reconstructed, for each speaker’s utterance;

(a) the speaker's individual construction of meaning (shortly called *individual mental model*),
(b) the speaker's linguistic means to express the mental model (shortly called *linguistic realization*), and
(c) the listener's interpretation of the linguistic means (shortly called *interpretation*).

Points (a) and (c) give hints to the mental processes of both students and researcher/teacher, and (b) refers to the interactional processes and the development of linguistic means. Point (c) is only mentioned if necessary.

For classifying the linguistic means and their mutual transition in (b) according to the used languages, the distinction of three situationally activated linguistic repertoires is used (see Figure 4). The word level, sentence level, text level, and discourse level in different semiotic representations (words, signs, graphs, gestures), together with their mutual meanings are all used in this classification process. They are operationalized as follows:

- *Technical Linguistic Repertoire:* Linguistic means and their intended meanings are assigned to the technical linguistic repertoire in a specific learning situation when they belong to the general technical register and are part of the target language. In our concrete learning situation, the variable x and its meaning as generalizer are the most prominent targeted linguistic means.
- *Mediating Linguistic Repertoire:* Linguistic means are assigned to the mediating linguistic register in a specific learning situation when it is used by the teacher or the material to mediate between the technical repertoire and learners' language for communicating mathematical contents, meanings, or tasks. This includes especially numerical and graphical representations or artifacts. Hence the criterion for assigning this repertoire draws on didactical or interactional intentions and on an *a priori* specification of the target language that is to be mediated, not *a priori* on sociolinguistic categories.
- *Individual Linguistic Repertoire:* The reconstruction of the individual linguistic repertoire needs the strongest methodological control. For each linguistic means used by the learners in a learning situation, we check whether its activation can be traced back to an external model by the teacher or material, or whether it was initially activated by the student without external model. In the second case, we assign it to the initial individual repertoire; in the first case, we reconstruct an act of *integrating a linguistic means from the technical or mediating repertoire into the student's individual repertoire*. In this way, we can reconstruct the micro-process of individual language development.

Hence, in contrast to the sociolinguistic registers, the linguistic repertoires are operationalized with respect to observable characteristics in the analysed learning situation (see the analytic table in the next section for an example). The analysis focuses on the transitions between repertoires, for example marked by [$T \to M$]

if an idea or content is expressed first in the technical and then transformed to the mediating linguistic repertoire.

The complete sequential analysis and classification with respect to the transition of linguistic repertoires was carefully discussed between at least two researchers in order to achieve a communicative validation.

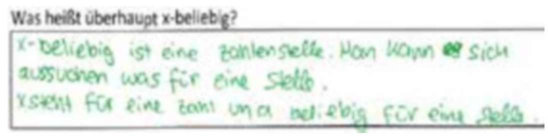
EMPIRICAL SNAPSHOTS FROM THE DESIGN EXPERIMENTS

We Do Not Know X-Arbitrary

Episode 1: Ayla, Gözden, and unknown mediating terms. For embedding the variable x (an important element of the intended technical repertoire) in the students' individual linguistic repertoires, the course for low-achievers intended to use the mediating expression “x-arbitrary.” For investigating whether students are familiar with the expression x-arbitrary, it is briefly mentioned early in Stage (1) of the course (see Section 1.1) for diagnostic purposes: after having specified the number of dots for several positions (see Figure 2), the girls Ayla and Gözden are confronted with the question on the worksheet: “What is the number for an x-arbitrary position?” The researcher/teacher (RT) of the design experiment diagnostically explores the girls' thinking in the following transcript where 2/97 stands for the second transcript, line 97):

2/ 97	RT	What does x-arbitrary mean, do you have an idea?
2/ 98	Ayla	No, our teacher also says that, but we don't know what it means.
…		
2/ 102	Ayla	She never explains it, not really.
2/ 103	Gözden	[*whispers to Ayla*] I only know that in algebraic expressions, there … there is x.
2/ 104	Ayla	[*to Gözden*] Yes, but THIS is a position of the numbers [points to the worksheet]
…		
2/ 111	Ayla	That's how it is said in German. [*laughs*]
2/ 112	RT	[*laughs*] Right, that's how it is said in German. But, ehm, what does arbitrary mean, though?
2/ 113	Ayla	any position of the numbers
2/ 114	RT	Yeah [*nods for acknowledging*]
2/ 115	Ayla	But you don't know which one.
2/ 116	RT	Yeah, exactly … And that is how you hit the concept. That is the concept x-arbitrary. That means any position, nothing more.
2/ 117	Ayla	Then, we can choose a position and then -
…		

2/ 128 Gözden [*writes down their working definition*]
"x-arbitrary is a position of number. You can chose which one. x stands for a number and arbitrary for a position."



The girls' relation to the mediating linguistic expression x-arbitrary is multifaceted: they know that it is used in German (#2/111) and that their teacher uses it (#2/98), but they say they do not know its meaning (#2/98). By formulating "how it is said in German" (#2/112) they signal that they do not feel part of this language community; this is one of the rare explicit indications for students' awareness that the mediating repertoire belongs to the language of schooling in the described sociolinguistic sense. However, the girls realize that the mediating expression is connected to the variable x, the corresponding sign of the technical repertoire (Gözden in #2/103). The researcher believes that Ayla constructs the intended meaning when she translates "x-arbitrary" into her individual linguistic repertoire by "any position" (#2/113), since "any" can refer to the intended meaning "all or arbitrary." However, the researcher does not realize that Ayla and Gözden stick to the meaning as *one* number instead of *all* numbers (#2/115 and #2/128). Although both girls activate the word "any" by which generalization usually can be expressed, they do not yet have access to the underlying practice of generalization. Due to space restriction, we show only one of the analytic tables, that of the dialogue above (see Table 1).

The diverging meanings become evident two sessions later. After having written down many arithmetic expressions (see scan in the transcript), the teacher gives them the algebraic expression 2+3·x for another sequence. However, they are still unsure what x and x-arbitrary mean. The excerpt of transcript from the fourth session starts when they discuss again what values to insert for x:

4/ 102 Ayla […] But how can you know what number you should calculate there?
4/ 103 RT Well, first, we do not need to calculate. We first want to ...
4/ 104 Ayla ... or is it for all?
4/ 105 RT Exactly. It is for all positions.
4/ 106 Ayla Ah! Okay!
...
4/ 110 Ayla [*to Gözden*] Look, that stays always, doesn't it? [*points to the arithmetic expressions in the table*] And then, [*points to the algebraic expression 2+3·x*] you can insert every number you want, and that is, then every calculation becomes the same.

Only after having written down the arithmetic expressions for many positions (hence after a hands-on experience of generalizing; see Figure 9 for the activity) can the

Table 1. Analyzing the interaction of Episode 1

Line in tran-script	*Students' mental processes*		*Interactional processes: linguistic realization [and change of register]*		*Researcher/ teacher's mental processes*
#2/97f	A,G: no meaning, but identified as part of teacher's German language	←	RT: "x-arbitrary" [*T*→*M*] A: "our teacher also says that, but we don't know what it means" [*M = alien register*]	←	Intention: give an explanation for technical symbol x as generalizer by a mediating term x-arbitrary
#2/103	G: x-arbitrary contains symbol x	→	G: "in algebraic expressions, there … is x" [*M*→*T*]		
#2/104	A: x in x-arbitrary does not correspond to symbolic variable x	→	A: "Yes, but THIS is a position of the numbers" [x-arbitrary] [*T* ≠ *M*]		
#2/113-2/115	A: constructs meaning for x-arbitrary as one unknown	→	A: "any position of the numbers", "but you don't know which one" [*M* → *I*]	→	Interpretation of Ayla's "any": constructed meaning as generalizer
			RT confirms right word "any" without addressing wrong meaning unknown vs generalizer		

students extend their individual meaning of x from the unknown ("know which number to calculate there," #4/102) to the variable as generalizer. Now the linguistic means x and x-arbitrary are successfully integrated into the individual repertoire, and Alya (#4/110) can explain its meaning to Gözden (#4/110).

The scene shows that constructing the meaning of a variable as generalizer does not only depend on isolated (technical or mediating) words, but is deeply connected to the practice of generalizing itself. That is why Lee (1996) talks about an *initiation into the culture of generalizing*.

Episode 2: Meliha and Gülnur and the repdigit. The episode of Meliha and Gülnur starts in a similar way when they are asked to search the 42nd position of three sequences. The scan of their work in Figure 6 shows that the process of finding shortcuts (and hence a generalizable pattern) starts quite slowly and only begins in the third sequence: they stop writing down all numbers until 42, but the video shows repeated additions of 6 on the calculator. So far, no multiplicative shortcut has been found.

Similar to Ayla and Gözden, they do not know what x-arbitrary means and guess many different possibilities, such as repdigits (#3/110 in the original transcript, not shown here), prime numbers (#3/116), or letters that stay the same (#3/123). Then the teacher explicitly refers to the mathematics classroom (#3/126):

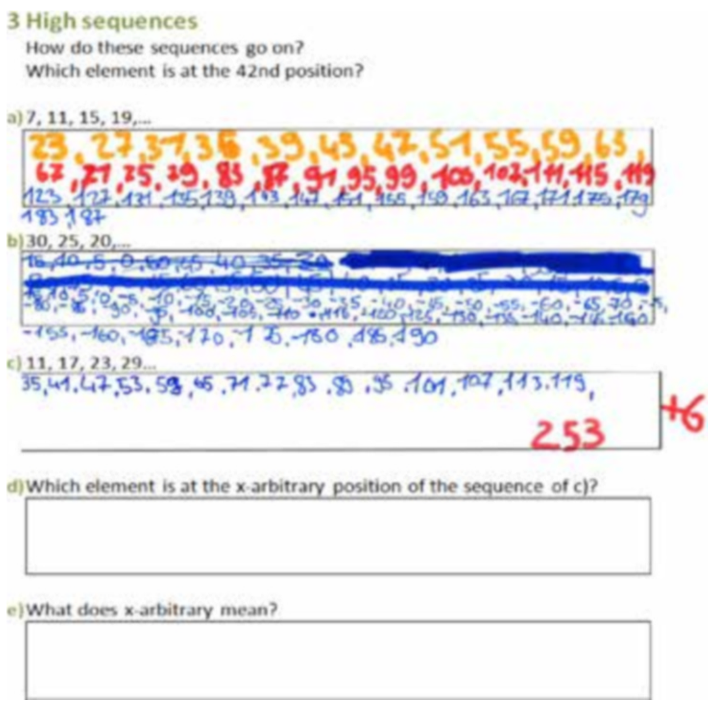


Figure 6. Meliha and Gülnur's pathway to generalizing and the variable as generalizer

3/	116	Gülnur	[the numbers] that you can only divide by itself
	…		
3/	123	Gülnur	[the letter x] that it stays the same
	…		
3/	126	RT	Perhaps you know the term x-arbitrary from math classrooms?
3/	127	Gülnur	We only know the variables. For example x. That we insert, though.
3/	128	RT	Yes exactly, there you can insert. But a variable, it can be even more. Or?
3/	129	Gülnur	Calculating formulas

The term x-arbitrary (that was meant to mediate the variables from the technical repertoire) again poses difficulties; as earlier, the shift $T \rightarrow M$ did not help to construct an adequate meaning. Instead, the shift back to the technical repertoire (suggested by the researcher/teacher's explicit reference to math classrooms in #3/126) helps the students to connect the term "x-arbitrary" to the technical concept of variable. Gülnur activates the meaning of the variable as a place holder into which one can insert values (#3/127), although with a certain vagueness of imprecise grammar.

We conclude that in both case studies, the mediating register did not work in the intended way for students who do not feel acquainted with the term x-arbitrary. This empirical finding gives a first contribution to research question Q1: students' individual meaning construction for variables as generalizers cannot be initiated simply by giving them a translation using the mediating repertoire. Instead of explaining the concept by one mediating term, the complete thinking practice of generalizing first has to be established.

This thinking practice, as well as the concrete term, is part of the language of schooling, marked as an alien sociolinguistic register by the girls. As the linguist Feilke (2012) has emphasized, the language of schooling and the thinking practice of generalizing are directly connected, since the language of schooling provides the linguistic (lexical, grammatical, and discursive) means for higher order thinking practices. This is no surprise since, historically, every register evolved for being most suitable for specific purposes. The register of schooling is optimized for making explicit concentrating, discussing, and generalizing. Feilke defines generalizing as "presenting circumstances as independent from personal, temporal and local situational references and assuming their general validity" (Feilke, 2012, p. 9, translated). He names some typical linguistic means for realizing this practice: grammatical means (such as generalized or generic forms 'he instead of I or you,' or cutting out the agent by passive forms, generic use of articles), lexical means (as in 'all conditions,' 'always,' …), and routines such as defining.

With Feilke, we assume that Meliha and Gülnur as well as Ayla and Gözden still need to develop their linguistic means for generic forms and for many numbers addressed at the same time (simultaneous aspect; Malle, 1993). This challenge goes much deeper and is more subtle then the simple term x-arbitrary. The following Episode 3 is aimed at further exploring and perhaps strengthening this assumption.

Experiencing Generalizing and Variables as Generalizers

We continue to follow the learning pathway of Meliha and Gülnur and show how experiences with generalizing and generalizers can be established together with the necessary linguistic interchanges that support their learning.

For finding and expressing growing patterns informally, the remediating course offers (in stage (2); see Section 1.1) different ways of finding the higher positions in sequences (with pictures, tables, "Merve's" verbal description, and expressions with the quasi-variable 35, as printed in Figure 7; here with texts translated into English).

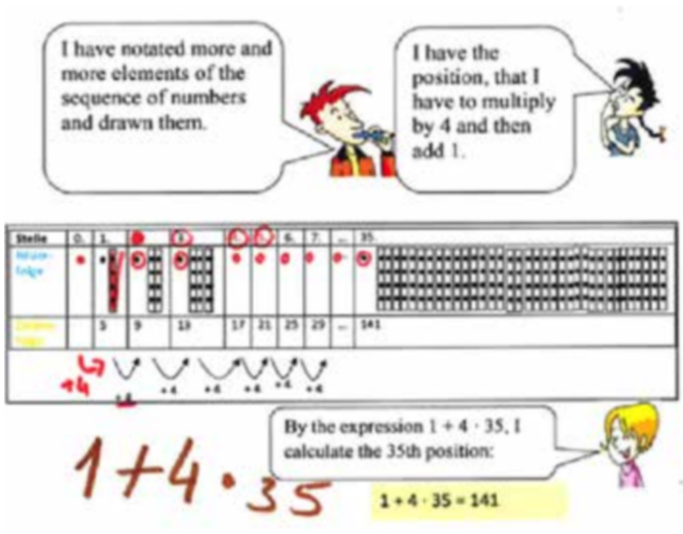


Figure 7. Three ways of finding the 45th position

Interestingly, the students' first attempt to adopt the strategies they have learnt in one context to the next sequence (in Figure 8) shows many initial mistakes and a prevalent willingness to work with given linguistic models: the girls do not only transfer the phrasing ("*I have the position, that I have to multiply by 3 and then add 1*"), but also the "+1" in the verbal and symbolic description. The first arithmetic expression 1+5·60=181 is later corrected to 2+3·60=182.

Episode 3: Again this X-arbitrary. For initiating stage (3) of the course, the teacher now comes back to the practice of generalizing and asks the girls to find the expressions for many different positions (see Figure 9).

This repeated operative experience offers the fundamental idea of coming back to the variable and x-arbitrary:

5/ 56 RT [...] ok. Now, task b) and c) ask how to calculate for an x-arbitrary position? Describe like Merve. And how does Pia write an expression for x-arbitrary positions?
5/ 57 Gülnur Ah, again x-arbitrary.
…
5/ 62 RT [...] You can consider Merve's way again and then think about what such an x-arbitrary means here now.
5/ 63 a Gülnur [*looks onto the sheet with the table from Figure 9*]
b Perhaps, it is [*points to the table with the pen*]
c the x-arbitrary position [*moves the pen up and*
d *down*]

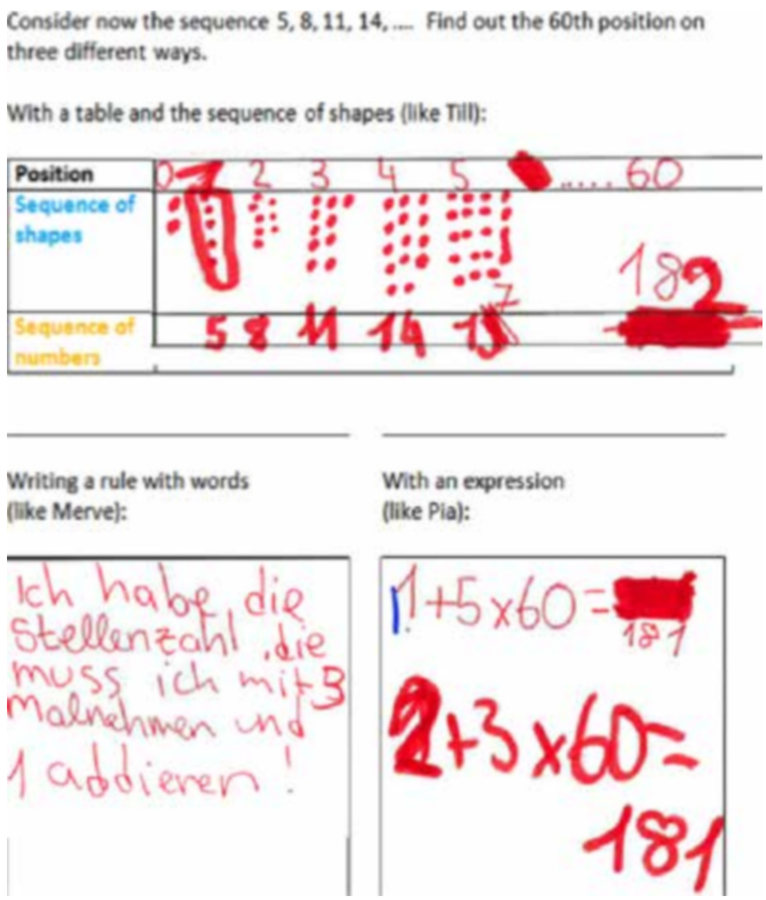


Figure 8. Meliha and Gülnur transfer to the 60th position

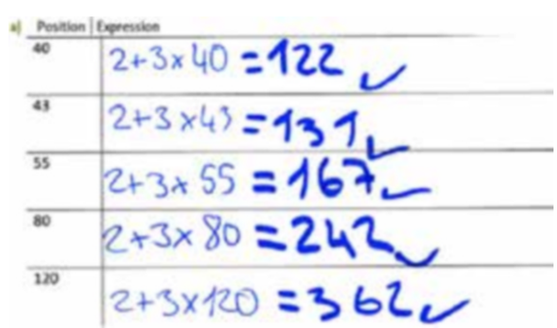


Figure 9. Meliha and Gülnur experience the generality of the initial expression 2+3·60

e	the position that always changes.
f	So, not the sequence of numbers and the
g	sequences of shapes but the position.
	Can be x-arbitrary, because it often changes. We have, we need not always make the 43rd position, for example. We sometimes need to make the 120th position.

Gülnur immediately recognizes the difficult concept x-arbitrary (#5/57) and remembers that they did not understand it in Episode 2. The researcher/teacher encourages them (in the non-printed lines #5/58–5/61) and hints at the verbal description given by the fictitious Merve from Figure 7 (#5/62). In line #5/63, Gülnur develops the idea that x-arbitrary addresses the changing positions that they have considered in the table of Figure 9. Her utterance vividly shows how linguistic means can successively evolve: before verbalizing this idea explicitly, she uses the gesture of moving the pen up and down in the table to communicate her idea (#5/63c). Based on the operative experience with many arithmetic expressions and the pointing gesture, she finds the linguistic means to address the idea within her individual linguistic repertoire: "the position that always changes" (#5/63d). The generalizing activity is translated into a *temporal* consideration for which she can find words. The generic examples by which she intends to strengthen her argument (#5/63g) are accompanied by other temporal terms such as "always," or "sometimes." Hence, the transition [*M* → *I*] now successfully takes place in several steps.

In the succeeding activities of the remediating course, the girls consolidate their constructed meanings and deepen their experiences with generalizing for sequences. The linguistic analysis shows that they gain more and more confidence in the notion of variable as generalizer and integrate the mediating term x-arbitrary into their individual linguistic register. They accomplished this even for explaining huge algebraic expressions such as x·3+x·5+x·17+2·19+468, which resulted from their mathematization of a complex word problem (with x being the varying number of students in a calculation of costs), as noted in the following:

8/	12	Meliha	Because, x-arbitrary, that is, where the students. They change. Thus we have at x, always students.

Hence, they manage the mathematizing, that is, the transition between the word problem text in the mediating repertoire and the technical repertoire of an algebraic expression [*M*→*T*]. They also manage the interpretation, that is, the transition back from the technical to the individual repertoire [*T*→*I*] that works fluently here [together *M*→*T*→*I*], showing the integration of the term x-arbitrary into the individual repertoire.

These observations and the careful analysis of the nature of linguistic means for expressing generality in the individual repertoire offer further contributions to research questions Q1 and Q2: Gülnur's use of many temporal terms in her individual linguistic repertoire shows, on the one hand, that she has found a pathway to generalizing via the operative variation of expressions in Figure 8. This operative activity makes the idea of generality accessible by offering a dynamic perspective (one position inserted after the other). On the other hand, Gülnur's use of many temporal terms confirms Feilke's emphasis that the *simultaneous* consideration of many possibilities needs the construction of new linguistic means that are not necessarily part of students' individual repertoires. In contrast, Ayla in Episode 1 succeeded in tentatively expressing simultaneous generality by saying, "you can insert every number you want, and that is, then every calculation becomes the same" (#4/110 in Episode 1).

As the *temporal* linguistic means for expressing successive changes seem to be more accessible than linguistic means for expressing *simultaneous* generality, the mental activity of dynamically changing numbers seems a very appropriate pathway to the thinking practice of generalizing. In this sense, the learning arrangement has been optimized in preceding design experiment cycles so that it now allows Gülnur to bridge the gap by intensively offering operative experiences. This provides an important contribution to research question Q3.

Episode 4: From x-arbitrary position to x-arbitrary kilometer. Once the meaning of x-arbitrary positions is consolidated for the students, a slow process of transfer starts. This can be exemplified with an episode from the tenth session with Meliha and Gülnur in which they were asked to find a general algebraic expression for the following word problem: "Every tour with the taxi costs a basic fee of 3€. For each kilometer, a price of 2€ is added." To conduct the transition [*M*→*T*], Gülnur develops the expression x·3+x·2 and explains:

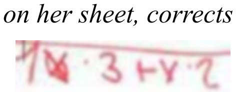
10/ 30 Gülnur And for what stands x, I have, I have no question but I have written x times 3 plus x times 2. Ah, wait [*she writes* "How much is the price for an x-arbitrary position?"]

…

10/ 34 Gülnur [*reads her question*]

10/ 35 RT Ah, ok, for an x-arbitrary position. And what does x-arbitrary position mean here?

10/ 36 Gülnur Ehm, well, we do not know how many kilometer, well [*writes on her sheet, corrects the first x into 1*]



The prompt to pose a question for interpreting the algebraic expression [*T*→*M*] urges her to think about the meaning of the variable. It is remarkable that, in her question, she does not activate her individual repertoire of speaking about changes in temporal terms but draws on the learned mediating term "x-arbitrary position," so far without connecting it to the taxi situation in the word problem (#10/30–10/34). The researcher/teacher intervention (#10/35) draws her focus to the kilometers. Instead of changing the question, she first corrects her mistake in the algebraic expression (#10/36). Hence, the initiated transition [*T*→*M*→*I*] enables her to evaluate her symbolic expression [*T*→*M*→*I*→*T*]. She then shifts from "x-arbitrary positions" to "x-arbitrary kilometers."

In the end, Gülnur succeeds in the connection of the variable to the general number of kilometers. This enables her to transfer her algebraic competencies of using the variables as generalizer to new everyday contexts. The trajectory of transitions between repertoires [*M*→*T*→*M*→*I*→*T*] shows that she could integrate the linguistic means into her individual repertoire but still needed the connection to everyday contexts: using "x-arbitrary position" does not immediately imply that the connection to kilometers is drawn, but after a prompt, it could be.

This snapshot shows that, for these students, the language- and content-integrated learning arrangement could provide a pathway to generalizing as a mathematical practice and the variable as generalizer.

CONCLUSIONS

The case study on the meaning of variables as generalizers offer empirical insights into topic-specific language challenges for underprivileged multilingual students: teachers and teaching materials often use linguistic means in the mediating linguistic repertoire that go beyond isolated terms such as x-arbitrary. Beyond the word level, the sentence, text, and even discourse level is often addressed, as in our case study the thinking practice of generalizing. These linguistic means often belong to the sociolinguistic register that has been termed *language of schooling* and plays an important epistemic role in students' pathway to conceptual understanding and higher order thinking practices. However, for socially underprivileged mono- or multilingual students, the register contains many linguistic means that are not part of their initial individual repertoires.

Successful learning arrangements for these disadvantaged students should therefore consider the mediating repertoire to be part of the *learning goals*, not the *learning resources*. If the learning arrangements provide opportunities to construct meanings for these mediating linguistic means with important epistemic roles, then the construction of meanings for mathematical concepts can be successfully supported. The thorough and deliberate shift between repertoires and registers can contribute to these processes (Prediger et al., 2014).

For the concrete exemplary topic "generalizing and variables as generalizers," the well-established shapes and pattern approach has proved to be useful when being combined with operative variations of arithmetic expressions (cf. Figure 9). These later experiences give the opportunity to get access to the "foreign" culture of generalizing (Lee, 1996), a thinking practice that is deeply connected to the language of schooling (Feilke, 2012) and should therefore be the issue of further empirical investigations.

To conclude, our design research project could contribute to four research demands (a and c being postulated by Schleppegrell, 2010, p. 107):

a. topic-specific research, here on an algebraic concept that is central for middle school mathematics, namely variables as generalizers,
b. a conceptual framework that helps to analyze the transition between languages not only on the sociolinguistic meso-level, but also on the micro-level of the concrete situation,
c. practically and empirically approved instructional designs for developing students' language of schooling together with the specific mathematical topic, and
d. empirical insights into typical learning pathways initiated by these instructional designs.

NOTE

[1] In German, "arbitrary" is often used for generalizing, as well as "x-arbitrary." The translation of the typical German term could have also been "x-any."

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8. MULTILINGUAL STUDENTS' AGENCY IN MATHEMATICS CLASSROOMS

INTRODUCTION

During the last years we have noticed an increasing attention in mathematics education research addressing agency (see for example Andersson, 2011; Björklund Boistrup, 2010; Boaler & Greeno, 2000; Grootenboer & Jorgensen, 2010; Grootenboer & Zevenbergen, 2007; Lange, 2010; Macmillan, 2004; Martin, 2000; Norén, 2010; Powell, 2004; Wagner, 2004, 2007). Why agency at this point in time? According to Ahearn (2001) one answer, that we agree with, is that there is a clear connection of interests in approaches that foreground practice on the one hand, and social movement on the other.

In mathematics education research the construct is used in different ways depending on the focus of the research and the authors' theoretical standpoint (Andersson & Norén, 2011). Agency is an elusive construct that, according to the Blackwell Encyclopaedia of Sociology (Fuchs, 2007), is hard to pin down:

> Agency is a fundamental and foundational category and puzzle in virtually all social sciences and humanities. Debates over agency have emerged together with these fields, and continue unabated into the present time, with no resolution or consensus in sight. While many agree that agency, action, and actor are basic in some sense, controversies persist over the definition, range, and explanatory status of these concepts. In addition, agency is contested because it connects to core questions in metaphysics, philosophy, and ethics, such as free will, moral responsibility, personhood, and subjective rights. Agency is tied to the legacy of liberal humanism that is part of the core of democratic citizenship. (p. 60)

In this chapter we explore the construct of agency theoretically. We then use it for analysis of social classroom interaction from a coordination of socio cultural and critical theoretical perspectives. As a starting point, we agree with Macmillan (2004) who takes the stance that students are thinking and feeling subjects acting in relationships with others. In line with van Lier (2008) we recognize that a key principle for learning depends on the student's activities and initiatives, and more so than anything a teacher or a textbook transfers to a student. Teachers and fellow students are also needed in roles of mediating functions, but the emphasis for learning has to be on a student's action, interactions and affordances (see also Skovsmose, 1994). As Andersson (2011, p. 215) wrote, "classrooms are spaces of socially

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 109–124.

organised practices that, in different ways, shape how individuals are expected to, allowed to and/or required to act". Thus we view agency as contextually enacted, and as a way of being and acting in relationships with others.

In relation to equity and accessibility we move away from theories determining multicultural and multilingual students as disadvantaged. We follow Powell (2004) who used the notions of agency and motivation to avoid deterministic theories and to resist deficiency explanations of African-American students' failure in mathematics in the USA. Powell's research study among 24 Grade 6 students gave "evidence of the mathematical achievement of students of colour as a by-product of their engagement of their agency" (p. 10). Powell found that the students initiated investigations, reasoned and progressed in building foundational understanding of certain mathematical ideas. To Powell, an understanding of agency "is particularly important since both failure and success can be located within the same set of social, economic, and school conditions that usually are described as only producing failure" (p. 6).

The purpose of this chapter is twofold. First, we theoretically elaborate on various interpretations of agency with the purpose to clarify differences between different theoretical interpretations of the theoretical construct. Second, we use empirical data from one multilingual mathematics classroom in Sweden with the aim of showing how agency works and how students' agency varies in two different contexts. In the chapter we elaborate on different theoretical standpoints of agency. In order to coordinate the constructs of agency within socio cultural and critical theories, we start with describing them with support of networking theories, as suggested by Prediger, Bikner-Ahsbahs and Arzarello (2008). Our purpose is to show that coordination of socio cultural and critical perspectives can be useful for avoiding deficiency models of multilingual students, and how individual actions impacted by societal discourses in the mathematics classroom bring resources for learning mathematics.

THEORETICAL PERSPECTIVES FOR CONCEPTUALIZING THE CONSTRUCT AGENCY

Socio-Cultural Conceptualization of Agency

According to Ahearn (2001), agency can be seen as a "socioculturally mediated capacity to act" (p. 112). Drawing on a Vygotskian tradition, agency extends 'beyond the skin' because it is frequently a property of groups and involves 'mediational means' such as language and tools (Ahearn, 2001, p. 113). The definition of agency does not have an individual character, but relates to contextually enacted ways of being in the world; that is agency is always a social event. A theoretical perspective in line with Ahearn (2001) is the ecological definition elaborated by Biesta and Tedder (2006). These authors suggest that agency should not be understood as a capacity, and particularly not an individual's capacity, but should always be understood in transactional terms; that is, as a quality of the engagement of actors with temporal-relational contexts of action (p. 18). They refer to an ecological understanding of agency, "i.e., an understanding that always encompasses actors-in-transaction-with-context, actors acting by-means-of an environment rather than simply *in* an environment" (Biesta & Tedder, 2006, p. 18). This reasoning implies that agency should be understood as achieved in relation to the particular context, and not as a possession of the individual. The context "comprises the network of relationships and available recourses in the social practises in which we act, but at the same time contexts are forming the ways and spaces where we act" (Andersson, 2011, p. 45).

A view on agency as achieved "makes it possible to understand why individuals can be agentic in one situation but not in another. It moves the explanation away, in other words from the individual and locates it firmly in the transaction" (ibid p. 19).

As we understand agency, in Biesta's and Tedder's words, it is achieved in 'relation to/transaction with' time and context, is narrow in focus, and moves the perspective from the larger purpose of education to the individual within education. We see it as one way to further elaborate the relations and intersections between the individual, society and mathematics education. This leads us into the next section where agency will be defined from a more critical perspective.

A Critical Conceptualization of Agency

Defining agency within a critical paradigm implies that the notions of discourse and power need to be scrutinised. To be able to relate to wider societal and political issues the concept of discourse will henceforth be used to examine power exercised in these particular multilingual mathematics classrooms. Power is always present in social interactions and produces discourse. At the same time the rationality of a certain discourse opens possibilities for alternative accounts and selves (Foucault, 1980, 1982/2002, 1984/2006) or in our wording, alternative agentic performances. Power can be seen as embodied in people's actions. In a classroom some actions create possibilities and others create limitations for learning. That is possible because power is exercised by and through possible conditions, and it coincides with the conditions of social relations in the classroom in general. Thus power in terms of agency is manifested in students' and teachers' actions by influence of various discourses. For example, a discourse cannot stand by itself, but it can be understood through its relationship to other discourses, ultimately an opposing discourse. Two opposing discourses in many multilingual classrooms in Sweden are the one that promotes "bilingualism" and another that promotes "only Swedish" (Norén, 2010).

Discourses impact on students and teachers in the mathematics classroom, and as a consequence – students' agency. On the other hand agency can bring about a discourse switch. For example, Norén (2011) shows how 'language-as resource' is aligned with a view on multilingual speakers in a wider, positive context of teaching and learning mathematics. The concept of discourse according to Foucault (1969/2002) is defined as ways of speaking/practice, which in some way is regulated and has coherence and supremacy in relation to broader social contexts. To speak is to take up a position and to adjust to the regulatory power within a certain discourse. Agency, that can be seen as required for discursive change or discourse switch, is enacted in discursive practices. This view of agency and power relations makes it possible to analyse classroom interactions and view students and teachers as social actors in mathematics classrooms practices, ruled and structured by discourse.

Critical theory has included contributions from structuralism, feminism, postmodernism and post colonialism (Popkewitz, 1999). These different perspectives make various assumptions regarding the definition of power and the self. One influence of post modernity comes from Foucault (1969/2002; 1971/93). Power in Foucault's terms is conceptualised as working in two directions and not as a one-way surveillance technique of power. The late Foucault (1980, 1982/2002) saw discourse as a medium through which power relations produced speaking objects – in our view this relates to the concept of agency even though Foucault did not discuss agency, but related to human beings as agents. Skovsmose (1994) also uses similar notions to agency, although he is not using the notion explicitly. In his writings it is an evasive concept that is concealed behind expressions as empowerment, intentionality, action and choice. He writes:

> Actions cannot be described in mechanical or in biological terms; and if a person's behaviour can in fact be described in such a way, then behaviour is not a part of his or her actions. It is not a personal action to breathe or to let one's hair grow. This I see as the first essential condition for performing an act: indeterminism must exist, or, the acting person must be in a situation where choice is possible. The person acting must have some idea about goals and reasons for obtaining them. (p. 176)

Critical mathematics education emphasizes social justice issues and student empowerment through mathematics education. In Skovsmose's work (1994), a basic assumption is that implicit as well as explicit functions of mathematics education are of importance for society and democracy (see also Skovsmose & Valero, 2001). When Skovsmose spoke about the formatting power of mathematics, he says it was a way to try to address the relationship between mathematical knowledge and power (2005). Skovsmose also articulates that mathematics education serves as a gatekeeper, for who will get and who will not get access to the information and communication structures in society (see Skovsmose, 1994; 1998; 2005). He concludes by saying that the learner is a member of society and mathematics can be a source for decision-making and action, which in turn makes mathematics education a critical feature in society. Skovsmose relates to Foucault and his description of technologies of the social, the connection between power and knowledge, and to discourse. To Skovsmose it seems obvious that mathematical knowledge can be expressed in ways of action, and this is what we in this article articulate as agency.

Aligning the Two Constructs of Agency

Theory encompasses methodology (Lerman, 2006). The connecting strategy between theories that we use is to coordinate (Prediger et al., 2008) the two constructs of agency. A common core within socio cultural and critical conceptualizations of agency is the construct's operationalization in social settings, such as in classroom interaction. In that sense the theories are compatible. The differences between the theories are mainly how the theories are relating to power relations, as described above. Critical theory is more clearly connected to wider societal discourses. Socio cultural theories are also related to phenomena outside the classroom, such as cultural and historical aspects of society. This justifies the aligning of the theories for this paper, however further theoretical discussion of this point can be found in Norén and Andersson, (2011). Thus in this paper we identify agency as enacted in social relations.

METHODOLOGY

To gather data we used ethnographic methods (c.f. Hammersley & Atkinson, 2007). Participant observations were regularly[1] carried out with a group of students from 8th through 9th grades. Data also consists of audiotaped interviews and informal talk with students during breaks and lunchtime.

Foucault's theoretical understanding of power is an approach that facilitates an analysis of what is going on in the mathematics classroom when people interact. His view on power is comprehensive and operationalized in terms of agency. Our analytical focus is on processes and content, with an emphasis on how things are done, what is done, and what is said in the specific context. When students and teachers are acting, it is the result of particular discourses, while the actions themselves allow us to understand how discourses take shape and change directions. Indicators for students' agency in the analysis are their forward directed engagement, initiatives, and ability to get other students and the teacher involved in their own reasoning within the classroom practises. Central questions in the first phase of analysis have been: How is agency enacted in the practices? How do multilingual students act to gain mathematical knowledge? How do students use their various resources like languages and multimodal expression? How and where do the students address their attention? To what do the students give their attention? In the second phase of analysis, the following question became central; How do multilingual students' performance of agency relate to wider societal discourses?

We now briefly sketch in some aspects of Sweden to help give a context for two glimpses into multilingual mathematics classrooms. These glimpses will exemplify, through classroom practices, our theoretical discussion above, relating to issues of power and agency in mathematics education discourses. In this way we discuss, with both a (socio cultural) relational and a critical understanding of agency, how different individual actions might actually become resources for learning mathematics.

The Swedish Context

Sweden today is a multilingual and multicultural society. One fifth of the students in compulsory school years have backgrounds from countries outside Sweden. In these years Arabic is the most common mother tongue spoken after Swedish (SOS, 2009/10).

Some suburban schools in the largest cities, Stockholm, Gothenburg and Malmoe have schools with up to 98% of students speaking first languages other than Swedish.

Low performance in mathematics among second language learners is often attributed to factors related to individual characteristics or students' cultural background (Khisty, 1995; Moschkovich, 2002; Barwell, 2009). In Sweden such deficit discourses have been used to explain low performances in mathematics among multilingual and multicultural students, in particular by pointing to factors related to students' cultural and linguistic backgrounds and/or to their families (Gruber, 2008). A strong influential discourse is 'Swedish only' demanding that only the Swedish language be used in all classrooms, and to 'learn Swedish fast' which will enable students to learn the subject content in Swedish (Runfors, 2003). Another example of a deficit discourse is the students' 'lack of Swedishness', positioning them as second language learners with shortcomings in terms of Swedish cultural capital (Runfors, 2003).

Swedish research also shows that students enact agency by affirming their identities as multilingual and multicultural learners (Haglund, 2005; Norén, 2010). In Norén (2010) the findings indicate that bilingual communication in mathematics classrooms enhances students' identity formations as engaged mathematics learners. Language- and content-based instruction in Swedish seems to contribute to the same aim. A risk is that monolingual instruction may endanger students' identities as bilinguals and normalize Swedish and Swedishness exclusively.

RESULTS

A First Classroom Example

The following example is centred on Madiha, a bilingual 15 year-old girl originally from Iraq with Arabic as her mother tongue. At the beginning of this study, Madiha had started attending her Swedish school about two years earlier. According to Madiha she was immediately placed in a preparatory class where the focus had been on learning the Swedish language at the expense of continued learning in mathematics. Madiha was self-conscious about priorities in the preparation class, saying:

> Then we worked almost nothing with math. It was just numbers (arithmetic) and plus and minus. No texts. We invested in Swedish. Nothing on the math. I have struggled and struggled. (from interview in eigth grade)

Her comment is a reflection of the strong influential discourse of "Swedish only", to learn Swedish fast, and the everyday perception that it would have enabled her to quickly start studying mathematics in her second language Swedish. When Madiha changed schools at the start of eighth grade, she accepted an offer to participate in a bilingual education project, where the languages of instruction were Arabic and Swedish. The students and the teacher used both languages. It had been noted that older students who were recent arrivals in Sweden and spoke mostly Arabic, needed this programme more than their peers who had been in Sweden for a longer time and had previously been taught mathematics in Swedish only. Further observations during several sessions made it clear that Madiha felt safe engaging in mathematical interactions with teachers and peers when she used Arabic, although she was not so sure of herself when she used Swedish. One episode about a problem with the angles and heights of the triangles illustrates this.

Three girls, Madiha, Jila and Nina, combine their two languages simultaneously as they reason about the given task, finding the heights in triangles. The girls are standing in front of a whiteboard. Jila draws two triangles, one acute and the other obtuse angled (see Figure 1).

Figure 1. An acute and an obtuse angle triangle, drawn by Jila on the white board

The bilingual teacher comes up to the white board and asks in Arabic how many heights a triangle has. Jila in responding enacts agency with words and multimodal expressions. Jila draws a right-angle triangle on the whiteboard, while saying:

Jila: I draw a right triangle, a [right-angle triangled heights][2] is the heights, there are heights? This is the height of a right triangle (Jila is now pointing to the vertical side from top to bottom).

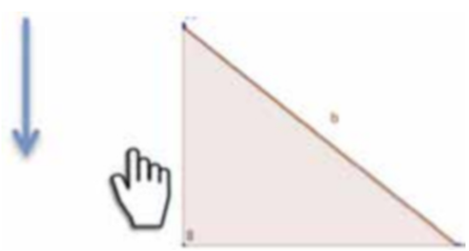

Figure 2. A right-angle triangle drawn by Jila

In social interaction with her peers and teacher, Madiha enacts agency using two languages, though mostly Swedish, and multimodal expressions to clarify the issue of, and to learn more about, the heights of triangles. Madiha shows engagement by enthusiastically pointing to the vertical height on the right-angled triangle (as above like Jila did). The extract below shows part of the interaction in which Swedish and Arabic are used:

Madiha: You can turn the triangle [up-side down], then it is easy to understand that there must be several heights. any side can be the base.

By moving both her hands to the right Madiha is pretending she is turning the triangle 90°, changing bases. Then pointing to an imaged right hand verticle side of the triangle and the angle of 90°. The teacher acknowledges Madiha's actions by nodding to her, and saying:

Teacher: It is the height of a triangle, how many heights are in a triangle? [How many base sides are there in a triangle?]
Madiha: You can do this, you can extend the …

Madiha is changing her focus and is now pointing to the obtuse-angle triangle's obtuse corner, and makes a movement with her hand to the right.

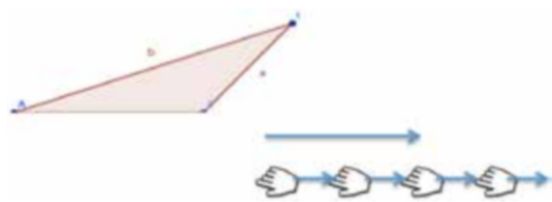

Figure 3. Madiha is making a movement with her right hand from the obtuse corner to the right

Madiha is pointing to an imagined extension (blue arrow above), and then draws a dotted extension of the horizontal leg of the obtuse-angle triangle. She ends with pointing to an imagined 90° angle. She tries to reason about how to draw the second and third of the heights and the base. She discusses with Nina:

Madiha: Angle 90, 90… Height No. 2, in a triangle there are… three heights.

It seems like Nina thinks it is easier to use the acute-angle triangle to find the third height, she starts talking about looking at the acute-angle triangle. She is using the word extend, building on Madiha's wording. She is drawing heights from the three bases, getting three right angels.

Nina: Extend there… one can do the same here. it says so in the book. You can do… extend.

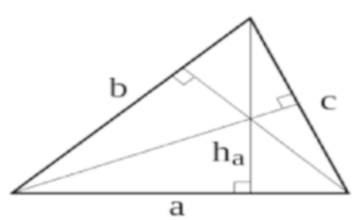

Figure 4. The three heights in an actute-angle triangle

The girls are finally getting the heights in place in all three types of triangles. In their collaboration with each other and the teacher, the white board and the pens also play an important role. Hence this is a multimodal interaction consisting of the people and the white board since the white board and the pens have an impact on the actions the girls take, as well as the discourse, promoting a use of the two languages.

The example shows that Madiha regards the task seriously. She enacts agency in that she enters into dialogue with others to learn more about triangles, angles and heights. She uses her two languanges and multimodal expressions to "come closer" to knowledge about triangles, heights in triangles, bases and angles.

It is also important to note the explicit recognition of the bilingual context which is used as a resource in the creation of learning opportunities. In an interview Madiha opined:

> I have learned more (mathematics and Swedish, authors' comment). Arabic makes it easier and possible to learn more. Language is an important issue. (from interview in ninth grade)

Learning mathematics through Arabic and Swedish opened opportunities for enacting agency, which also led to opportunities for learning interactions about heights of triangles. Throughout the observations the teacher was not the only voice. In the social interaction between students and students, and between students and teacher, the students' individual engagement and initiatives to learn more mathematics were valued. In the later interview in ninth grade Madiha stressed how much easier it is to understand mathematics when you are allowed to use your mother tongue. She also claimed that the learning process was strongly dependent on the teacher. The teacher spoke Arabic and Swedish, and used both languages regularly in her teaching. Critically these bilingual education practices used by Madiha (and other students in the classroom) were seen as related explicitly to their learning ("Arabic makes it easier").

This example highlights the advantage that is afforded with bilingual mathematics teachers. This lesson was embedded in a "language deliberately" project and there are probably not many bilingual teachers in Sweden who knowingly use both languages as a resource for learning mathematics.

Swedish is used more than Arabic in teaching mathematics, probably because of the impact of Swedish mathematics text books, and the strong influential "Swedish only" discourse. It is shown in Swedish research that multilingual students taught in their second language Swedish, often become "silente students". Second language learners are "accustomed to" not understand, and to not ask questions, even though it is clear they don't understand (León Rosales, 2010, p. 270). Enacting of agency the way Madiha and her class mates do to learn and engage with mathematics, is in contrast to León Rosales findings. In a school system where students' first languages are usually subordinate to the normative Swedish, the use of two languages in mathematics teaching and learning enhances the value of students' first languages and thus opens up avenues for agency and enhancing the status of the students themselves.

A Second Classroom Example

Another classroom example that illustrates students enacting agency and their potential for learning mathematics, concerns four girls, Marian, Norma, Payman and Rama. They are working on a group project, an oral examination, on a National Test in Grade 9 (Skolverket, 2007) (see Figure 5).

The problem they are asked to solve contains statistical data, to be used as the basis for their mathematical argumentation. The teacher is interested in whether and to what extent students use mathematical language; in particular whether students verbally analyze and interpret data in the tables and charts, as well as the extent to which they can critically examine the advantages and disadvantages of four different charts, which were distributed to each one of them. The text problem concerns students' TV viewing habits (Skolverket, 2007, p. 5).

30 Grade 8 students[3] at a school answered this question. *How many hours per week do you watch TV?* You can see the results of this survey in the table.

0	8	13.5
1.5	8.5	14
2.5	9	15
3	9	16
3.5	9	16.5
4	10	18
4.5	10.5	19
5	11.5	19.5
6	12.5	21
6	13	23

- Study the table and the diagram and explain how the students assembled the data from the table into the diagram.
- What advantages/disadvanteges does the diagram have?"

Figure 5. The times each individual had watched TV per week

The tabulated data (as in Figure 5) was distributed to the 4 students in the group. It shows the amount of time each individual student watched TV during a week. The range is from no time to 23 hours a week. For the task reported in the test it was supposed that different groups of students (group A:1 – A:12), 12 in total, drew different types of diagrams. One is shown in Figure 6.

The four girls each received one of these different types of diagrams to study individually. They read the instructions and studied the respective table and their chart type diagram – bar charts, pie charts, histograms, and a kind of bubble chart

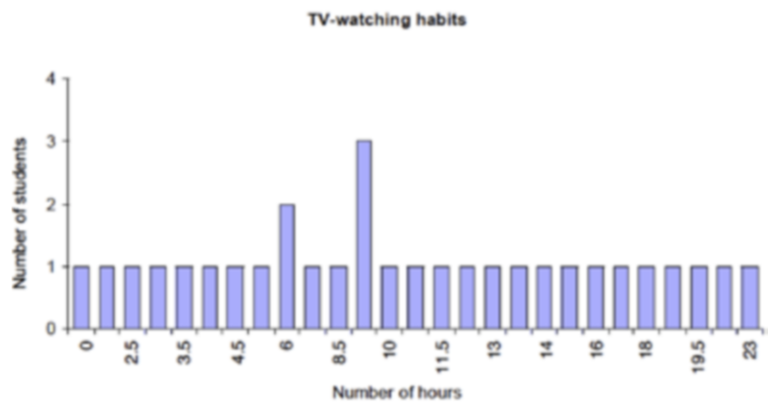

Figure 6. Group A:1's representation of hours per week

which depicted three televisions in various sizes. They first studied their charts in silence for a while and then they began to discuss with each other the types of diagram charts they had. The teacher listened to them and also spoke to them one by one, asking questions about the charts. Later she spoke to them as a group.

The teacher and students used only Swedish. A discussion between the four girls progressed. They seem to agree that the pie chart which one of them received is pretty easy to interpret, but it may not be the best representation of how many hours the 30 Grade 8 students were watching television. "A bar graph can be better" was suggested. But two of the girls are confused by the term '30 Grade 8 students' "How many are there really?" Rama points to the pie chart, and says she tried to count how many groups or classes there are.

Rama: But are there 30 classes? 38 classes? Thirty eighth graders?

Marian: 30? No! 30 classes!

Rama: 30? No! 30 eighth classes!

The girls stop to discuss and look at the text problem as it is written on paper again. They study together the three columns; they then go through the columns one by one (see Figure 5). The data represents how each of the 30 students responded to the question of how much time they watched TV every week. This inspection of the table does not help them to continue the task. Payman is reading and saying out loud: "How many hours *a week* watching *you* on TV? *30 Grade 8* students have been asked" emphasising *a week, you* and *30 Grade 8*.

Rama and Marian seem still to be confused. They look at each other and at the teacher. The teacher realizes that none of the girls understand what the term '30 Grade 8 students' signifies, that it is 30 individual students in the eighth grade.

She begins to speak Arabic. Two of the girls have understood the phrase as if the problem were posed as either '30 different classes of students in grade 8' or maybe '38 classes'. But neither of these interpretations of the problem seemed to make sense when they studied the columns. Although the students could possibly solve the problem without fully understanding the meaning of '30 Grade 8 students'. In fact it interfered so much with their understanding of the problem, they were not able to continue with the solution process. The teacher eventually translates this expression, and then finally translates the whole of the text into Arabic. She focuses particularly on the '30 grade 8 students'. When the girls go on to discuss the problem again after the teacher's intervention, they use both Arabic and Swedish for a moment, just a few sentences, and then they return to the "only Swedish" discourse. The remainder of their dialogue is only in Swedish. The girls continue to debate the pros and cons of the different types of diagrams, understanding quite clearly now what' 30 Grade 8 students' actually means in this context.

We interviewed the teacher after the test was finished. She said she did not really want to switch into Arabic because it was an important test: "And all students in Sweden do it in Swedish, and it is the language they must use when they learn in the future, when they go to secondary school level, in high school".

The test instructions to the teacher says that students should get the help needed and with the same arrangements as in ordinary mathematics lessons, when doing the test. However the teacher's interpretion is that Swedish only should be used in this test situation. This means that for both students and the teacher, Swedish acts as the most valued language. One can say that a discourse that normalizes the 'only Swedish' was privileged.

The discussion between the girls went smoothly until Rama and Marian became aware that they do not agree on the meaning of the expression '30 Grade 8 students'. Their lack of understanding of the text disrupts them from solving the mathematical task and the mathematical conversation stopped. They cannot progress, the solution process becomes slightly confused, even though they attempt to agree on the meaning of the expression. Interestingly during the regular mathematics lessons, unlike this test situation, when Arabic was used together with Swedish, confusions such as this were rare.

Hence for both the teacher and the students, they all reflected an institutional norm, a discourse where Swedish is supposed to be the language of the assessment. This is the case even if both the teacher and the students know that the use of Arabic in this situation would facilitate students' understanding and communication between them. Thus one can say that the teacher acted out of regard to a deficit discourse. When the teacher after a while realizes that the girls are confused, and that their confusion is a barrier for them to go forward and solve the problem mathematically, only then does she abandon the dominant discourse that determines that only Swedish is useful. She then chooses to use the language students use during regular mathematics lessons and in their daily lives, namely Arabic, by a discourse that recognizes bilingualism as a resource. The use of both languages clearly had an impact on the girls' learning, which is of great importance for these newly arrived students.

In the test situation, the four students were strongly influenced by the change in power relations. It was also the teacher, as in the situation to begin with showed an identity as "Swedish" math teacher. As a result, from pupils' enacted agency, she was later forced to show her identity as bilingual. The teacher then enacted agency and confirmed her students as bilingual. At this moment of recognition, she changed the current (or influencing) discourse of a dominant discourse, only Swedish, to a discourse that supports bilingualism as a resource, and thus an opportunity for students to continue to reason mathematically.

Power relations were challenged by the discourse 'only Swedish' when the teacher enacts agency and translates to and explains in Arabic. The teacher's main concern was that the students would understand what the meaning of the problem was, in order to discuss it. Students needed a translation to continue to argue mathematically. Once the students understood the meaning of '30 Grade 8 students', they could continue to discuss and argue, so that the task was solved. Besides being a test situation described above, the circumstances might also be interpreted as an opportunity for student agency and mathematical communication. There is scope for pupils' agency in student-student and teacher-student communications. In our analyses the interaction can be interpreted as follows: The students took the test situation very seriously, and in the assessment situation the teacher and the students exercised a discourse normalising "Swedish only". The students' non-comprehension of the expression '30 Grade 8 students' interfered with their mathematical understanding of the text problem, and they got stuck. This kind of situation was rare in their ordinary mathematics lessons, where both Swedish and Arabic regularly were used in exercise of a discourse promoting bilingualism. The interaction reflects an institutional value of Swedish as the language of assessment and authority. In the interaction that followed the students' enacted agency and the teacher was "forced" to switch language from Swedish to Arabic, in a discourse of solidarity recognising her students as bilingual learners. In the moment of recognition the discourse switched from a dominant discourse of 'Swedish only' to a discourse promoting bilingualism as a resource, and thus an opportunity for the students to perform mathematically.

FINAL REMARKS

In the two classroom examples Swedish serves as the language of authority, more strongly in the second classroom example than in the first. This would not have been visible without the focus on discourse and students' enactment of agency. The coordination of socio cultural and critical theories when exploring agency and using it for analysing classroom practices made the impact of the available discourses visible. Individual actions, such as Madiha's in the first example, became resources for the ongoing learning of mathematical concepts, in this example about heights in triangles, for herself but also for other students in the bilingual classroom. The second example shows that discourse switches are possible through students' agency for relational understanding to eventuate. The national test situation made the discourse promoting "Swedish only" the prominent discourse to the extent of extiguishing all other discourses, but teacher's agency changed the path.

Both the multilingual mathematics classroom glimpses are in line with earlier studies supporting the use of two languages when learning mathematics (i.e., Moschovich, 2002; Setati, 2005). The critical perspective and our focus on agency highlight different aspects of classroom institutional order principles and puts light on students' actions and intentions for learning mathematics.

In this paper the critical theoretical approach and thus the conceptualization of agency as non individual and achievable in social interaction makes it possible to put light on how wider societal discourses have impact on individual student's opportunities to engage in mathematics learning within social classroom practises, and also how students' engagement make discourse change possible.

A certain discourse that opens possibilities for alternative accounts of agency for multilingual students, or in our wording, alternative agented performances, is the official discourse that supports bilingualism. It made power relations change. Power was exercised by and through the available discourses, and it coincided with the conditions of the social relations in the classroom in general. Thus power in terms of agency was manifested.

Based on this research we make a final reflection: When bilingual students are to solve mathematical tasks, and use their mother tongue as a resource for learning, they can progress, enact agency, and take meaning to a greater extent than if only Swedish is used. Hence other discourses than "Swedish only" need to be present in the Swedish mathematics classrooms for students to be able to achieve agency for their learning. When a discourse supporting bilingualism is applied, a discriminatory enactment of "Swedish only" discourse, focusing a priori on the Swedish language and not mathematical subject content knowledge, cannot be used as a boundary between exclusion and inclusion of students' communication in the mathematics classroom.

NOTES

1 In average one mathematics lesson every week.

2 The square brackets denote what was said in Arabic but for convenience now translated to English. An Arabic mother tongue teacher did the original translation to Swedish. Norén translated Swedish to English. A description of actions is given in the normal ( ) brackets.

3 In Swedish, the wording is åttondeklassare ('30 eighth graders'). In Swedish, one can put together a few words to make new words. Eighth graders are assembling eighth and graders. *Åttonde* is the bending of *Åtta* (eight), making it the eighth. *Klassare* is a bending of *klass* (Grade), which means graders. The understanding of *åttondeklassare* is not straight forward on any other language than Swedish.

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ARINDAM BOSE AND PHILIP CLARKSON

9. STUDENTS' USE OF THEIR LANGUAGES AND REGISTERS

An Example of the Socio-Cultural Role of Language in Multilingual Classrooms

INTRODUCTION

In multilingual classrooms, use of different languages is an important, although often neglected or not appreciated, resource (Clarkson, 2007; Moschkovich, 2002). Multilingual classrooms based in multilingual and multicultural societies present potentially rich instances of language and cultural resources that can inform language sensitive teaching strategies for developing conceptual understanding among students (see, Cummins, 2000; Duval, 2006). Some of the previous studies undertaken in multilingual and multicultural societies like India, Pakistan and South Africa have highlighted how transitions between everyday registers, school registers and mathematical registers happen naturally for students by using different representations and non verbal cues (Bose & Choudhury, 2010; Halai, 2009; Setati, 2005). These studies however, did not look particularly at the cultural resources or cues that emerge during classroom lessons. It has been argued separately that in such societies, the linguistic and social nature of mathematics facilitates its social construction together with language (Barton, 2009), and such classroom phenomena can facilitate effective mathematical learning (Prediger, Clarkson, & Bose, 2015).

Researchers have also shown that the plurality of registers can be used purposefully since students' discourse and thinking are not confined to the teaching language but occurs in all languages in which the stakeholders (students and teachers) are fluent (Clarkson, 1983; Prediger & Wessel, 2011). This chapter, using empirical snapshots from some classrooms in Mumbai, India, will show that teachers are at times also prepared to use the multiplicity of languages and registers that are available in their teaching of mathematics. The chapter will first describe some of these episodes from a socio-cultural standpoint of teaching, and then use Prediger and Wessel's (2011) "Integrated Model" that underlines the representations required by the teacher.

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 125–141.

MEANING MAKING AND LANGUAGE NEGOTIATION IN MULTILINGUAL MATH CLASSROOMS

Researchers in the past have emphasized the need to look at the relationship between language and mathematics learning from a "situated-sociocultural perspective" for better "mathematics reforms" (Moschkovich, 2002, p. 189). Previous studies on multilingualism in mathematics classrooms have highlighted the challenges faced by students while tackling words problems, and when they encounter mathematical as well as technical registers and symbols. In these studies, the focus shifted from students' learning of vocabulary and comprehension skills to one of students' meaning making and knowledge construction (Clarkson, 2007; Halai, 2009; Moschkovich, 2002; Setati, 2005).

In particular, Moschkovich's (2002) notion of situated-sociocultural perspective provides an analytic tool to describe students' competences in drawing on resources derived from their experiences of out-of-school contexts as well as participation in the mathematical discourse in the classroom. This tool provides a perspective, Moschkovich has argued, that is different from but does not replace just acquiring vocabulary or constructing meaning by students. She suggests it is necessary as well to examine students' mathematical discussions and their use of resources so that more subtle meanings cued from gestures and objects can be deduced. However Moschkovich did not clearly model the various languages and registers that multi lingual students may have access to.

The research literature that looks at the interface of multilingualism and mathematics education often reflects three broad notions of language-negotiations in mathematics classrooms: transition between first (or home) and second (or school) languages, which is often referred to as code switching in relevant studies; transition between informal (everyday) and formal (technical) language; and, transition between different mathematical representations (see, Prediger, Clarkson, & Bose, 2015). There seems to be only a few research studies directly addressing the latter two of these transitions.

It is commonly seen that multilingual students often switch between informal (everyday) and formal (school and/or technical) registers favoring their "language of comfort", as Bose and Choudhury (2010) have termed it. They have argued that students' "language of comfort" could be their home (or first) language, or their commonly spoken local language, which may be different to their non-home (or second) language, or a mix of both. Their "language of comfort", in addition to other languages such as home and formal languages, affords the potential for increased switching between everyday and technical registers. This phenomenon has also been discussed by Clarkson (2009) who has noted that the three registers (everyday/ informal register, school register and technical register), may exist in more than one of the student's languages and that multilingual students often encounter them all.

He has further suggested there is often a dynamic between all three registers, across all the languages (L_1, L_2, and so on; see Figure 1).

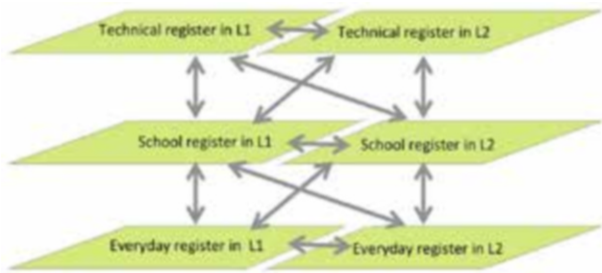

Figure 1. Three-tiered model of language and registers showing possible dynamic movement (adapted from Clarkson, 2009)

Prediger and her students have extended Clarkson's model by incorporating other teaching registers (see Figure 2). Prediger has been exploring how immigrant Turkish students utilize these registers in their learning when enrolled in German secondary schools (e.g., Prediger & Wessel, 2013). Within all of these registers she argues that various socio-cultural cues are used to good effect for the students' learning.

In this chapter we use notions from both Prediger and Wessel's model and Moschkovich's situated-sociocultural perspective to examine the language moves

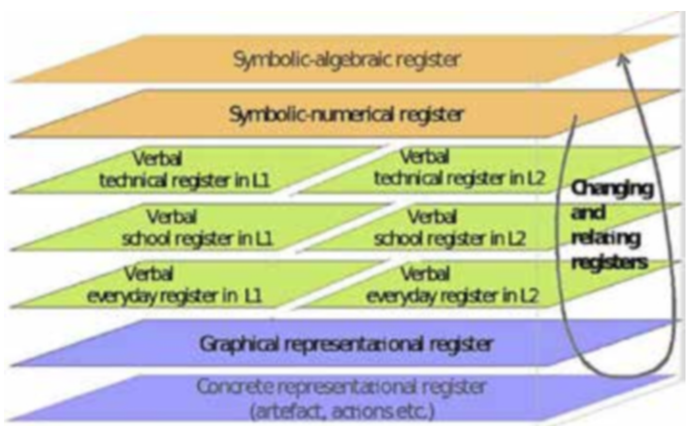

Figure 2. An integrated model that surrounds the language model (both written and verbal) with other teaching registers (adapted from Prediger & Wessel, 2011)

and utilization of cultural cues in various mathematical learning settings by multi lingual students in Mumbai, India.

CONTEXTS AND SNAPSHOTS FROM THE CLASSROOMS

We present below a number of snippets of recorded dialogue from two different types or contexts of classrooms: the first context is dialogue from two standard classrooms, while the second context is from class sessions taught during a vacation camp that was conducted in the school by the first author and his colleagues. But first we sketch in the contexts of the two types of classrooms.

The 'Ordinary' Classroom and the 'Vacation Camp' Classroom Contexts

The snippets of dialogue analyzed later come from two different Grade 6 and a Grade 7 classroom of two government-run schools in a large, densely populated low-income settlement in central Mumbai, India. The first group of students were attending an English-medium school, while the second group participated in a vacation camp run for sixth and seventh graders of the above English-medium school and another Urdu-medium school, both co-housed in the same building.

An economically active neighbourhood surrounds the school building. This neighbourhood is dotted with micro-enterprise businesses in that practically every household (single room, low height dwellings) runs a small-scale manufacturing or factory unit. Being an old and established settlement, it draws migrants from various parts of the country. These migrants come to Mumbai in search of a livelihood. This mostly unskilled workforce often gets work in different workshops. Children living in the neighbourhood participate in the income generating practices from an early age and thus have ample opportunities to gather everyday mathematical knowledge associated with various manufacturing practices.

The five-floor school building has each floor designated as a separate school. Each of these schools uses a different language as a medium of instruction; namely Telugu, Marathi, English, Tamil and Urdu, respectively from the ground floor through to the fourth floor. The local government runs all five schools. Every student, no matter which school they attend, can fluently speak at least two different languages, while many can speak three languages or more.

The English-medium school draws the largest number of students compared to the other schools since the learning of English is seen as a gateway to future welfare, and hence school education in English is in huge demand. The English school finishes at Grade 7. Unlike the Urdu school which has single sections for Grades 8, 9 and 10, students graduating from Grade 7 of the English school need to transfer to other privately run schools to complete the higher grades, or they drop out from school and stop studying altogether.

Students, whose dialogue was analyzed, came from the low-income settlement surrounding the school building and most belonged to migrant families. They all knew Hindi and Urdu while some spoke a different home language as well, for example, Marathi, Tamil, Bhojpuri, etc.

Teachers in both the English speaking and Urdu speaking schools are normally assigned a grade for the whole academic year and are expected to teach all subjects. In other words, there is no separate subject teacher in any grades in these schools. The English-medium school has a newly appointed batch of young teachers (mostly males) who joined the school in the last five years. None of the teachers live in the neighbourhood. However, through constant interaction with the students, parents and the community, and having participated in conducting the recent governmental census (an official duty assigned to the government-run school teachers), most teachers had an awareness and knowledge about the students' sociocultural and socioeconomic background, the kinds of work the families or the students were involved in, the students' everyday practices and routine, their income, wages, etc. All the teachers of the English-medium school could speak Marathi (first or home language) and Hindi fluently, while most of them could only partially speak English. Not every teacher in the Urdu-medium school was fluent in Marathi and English, but they were natively fluent in Urdu and Hindi.

Lessons in the fortnight-long vacation camp were taught by a senior colleague of the first author (a senior researcher in mathematics education). The attendance of students was voluntary. The vacation camp teacher could speak six languages fluently including Hindi (spoken Hindi is similar to spoken Urdu but the scripts are different), and the local language of Mumbai, Marathi. The camp commenced soon after the term-end examinations, and before the declaration of results. During this fortnight teachers were occupied in the grading and results preparation work. Therefore it was an ideal time for holding teaching camps during regular school hours but without disturbing any classroom teaching. The daily average attendance was around 25 with a majority of girls. The camp had daily classes for one hour and a half with a weekly holiday on Sundays.

Socio-Cultural Background and Language Context

The phenomenon of language switching is a common feature and part of the routine communication in urban India, but not a common practice during formal classroom sessions. In fact, in most English-medium schools, there is active discouragement of using multiple languages. English is expected to be the sole teaching and learning language used. However, in the English-medium school of the low-income settlement from where the following snippets originated, there is a marked departure from this convention. The researcher observed the regular teachers using multiple languages, with frequent use of English as would be expected, but also *Bambaiya Hindi* (a dialect of Hindi popularly used in Mumbai and surrounding areas) from time to time. Hence, switching between these languages by the students was expected in these classes.

In contrast, the medium of instruction in the vacation camp classes was a combination of Hindi and Urdu with occasional use of English. Hence it was expected to find the vacation camp students following code-switching and using a mixture of languages.

Classroom Situations

The classroom situations in both contexts (school and vacation camp classes) had some differences. Use of both English and Hindi/Urdu during routine teaching processes usually came during explanation of difficult terminologies with an objective to clarify the concepts in an effective way. The main motto of the teachers was to 'clear the concept' and students often opened up in their home language than in English. The English-medium school had classrooms with no desks or benches for the students and they sat on a long mat or rug put on the floor. The students attending the camp, conducted in the Urdu-medium school in the same building, had the privilege of having benches and desks. Boys and girls in both schools sat in separate groups during their regular school schedule, a sociocultural norm, which was also observed in the vacation camp classes.

While the regular classroom scenario reflected a routine teaching/learning process in that teachers commonly started a lesson with introducing the topic for the day followed by individual bookwork, in comparison the vacation camp classroom scenario was more informal and relaxed with much more of an emphasis on student discussion. School classroom teaching did not involve group-work, although students informally discussed among themselves and/or looked at each other's work while doing the given exercises. Emphasis was given on working out every problem given at the end of each lesson in the textbook. Typically, the teacher solved one problem on the blackboard with explanations and occasional discussion with students but building on students' knowledge and experience rarely occurred. Rather emphasis was given on using the formulae appropriately. In contrast, the practice of group-work was encouraged during the vacation camp classes. The vacation camp teacher encouraged students to shift from individual work towards making public (shared) comments and questions using mathematically discursive practices (Bose, 2014). During math lessons in the regular English-medium classes, textbooks were only used with no use of other aids or resources. Much emphasis was given to rote memorization of multiplication tables, facts and formulae and their application. In contrast, the vacation camp saw use of diverse artifacts and emphasis was given to understanding rather than just knowing.

Snapshots from the Classrooms

Snapshot 1. We present below three transcripts (Transcripts 1a, 1b and 2) as snippets from regular math lessons in the English-medium school on the topic "profit and loss" from Section A and Section B classes of Grade 6. The first author observed both classes. In all the excerpts, the left-hand column indicates the segment number of the original transcripts while "T" and "S" indicate the teacher and student respectively. The next column reflects the actual utterance, while the right-most column presents the exact translation into English of the utterance. The grey shaded words represent words spoken in English in the original utterance while underlined and shaded words (only in Transcript 3) represent the words spoken in Urdu. The acronym 'BB' in the transcripts stands for the 'blackboard'.

Transcript 1a. Grade 6, Section A (English-medium school); Lesson: Profit & Loss; Teacher: T_1

1	T_1	We all know about profit and loss/ what it is? Kaun batayega mujhe?	We all know about profit and loss/ what it is? Who's going to tell me?
2	S	fayda and nuksan/	profit and loss/
3	T_1	kab hota hai profit and loss?	when do profit and loss occur?
4	S	jab.	when…
5	S	jab hum kam price mein kharidkar jyada price mein bechte hain/	when we buy for lesser price and sell for more price/
6	S	agar hum bag three fifty ka lenge aur char sau mein bechenge to profit hoga/	if we buy a bag for three fifty and sell for four hundred then there'll be profit/
7	T_1	kitna profit hoga?	how much profit?
8	S	fifty/	fifty/
9	T_1	bagseller ne kitne mein kharida? Three fifty (writes 350 on BB)/ usne Wasim ko four hundred mein bech diya/ (writes 400 on BB)/	how much did the bagseller buy for? Three fifty (writes 350 on BB)/ he sold it to Wasim for four hundred/ (writes 400 on BB)/
10	T_1	to usko kitna rupees jyada mila?	then how much rupees more did he get?
11	S	humlog jitne mein bag kharidte hain, agar kam daam mein bechenge to loss hoga/	whatever price we buy a bag for, if we sell it for less then loss occurs/

Transcript 1b. Grade 6, Section A (English-medium school); Lesson: Profit & Loss; Teacher: T_1

34	T_1	pehle bag three fifty mein kharida aur four hundred mein becha to kya hua?	first bag was for three fifty and sold for four hundred then what happened?
35	S	profit/	profit/
36	T_1	kitna?	how much?
37	S	fifty rupees/ (chorus)/	fifty rupees/ (chorus)/
38	T_1	kaise mila fifty rupees?	how did you get fifty rupees?
39	S	minus kiya/	did minus/

(*Continued*)

40	T_1	very good/ kisko kisme se minus kiya?	very good/ what did you minus from what?
41	S	four hundred mein se three fifty/	three fifty from four hundred/
42	T_1	four hundred kya tha aur three fifty kya tha?	what was four hundred and what was three fifty?
43	S	four hundred SP aur three fifty CP/	four hundred SP and three fifty CP/

Transcript 2. Grade 6, Section B (English-medium school); Lesson: Profit & Loss; Teacher: T_2

2	T_2	write the heading / (writes "Profit & Loss")	write the heading / (writes "Profit & Loss")
3	T_2	First we discuss meaning of profit and loss / jaise hum dukan mein jate hain toh mostly hum yeh use karte hain / example, dukan mein jate ho, pen buy kara, uska daam hai ten rupees / yeh uska cost price / kharidne ki kimat ko CP bolte hain / maine same pen bech diya fifteen rupees mein / to mera kya hua?	First we discuss meaning of profit and loss/ So when we go to shops mostly we use this/ example, you visit a shop, buy a pen, its price is ten rupees/ this is its cost price/ buying price is called CP/ I sold the same pen for fifteen rupees/ then what do I get?
4	S	profit/	Profit/
5	T_2	yaani CP se jyada jo mila toh that is called as a profit / CP se upar jo value mila toh profit hua/	Meaning whatever is obtained more than CP that is called as a profit/ The value that is obtained over CP becomes profit/
6	T_2	ab loss kya hai?	Now what is loss?
7	S	agar uss pen ko paanch rupaye mein beche to loss hua/	If that pen is sold for five rupees then it's a loss/
8	T_2	yeh CP hai (writes CP on BB) usse agar mujhe kam kuchh milta hai to woh loss hota hai/	This is CP (writes "CP" on BB) if I get anything less than this then it is a loss/

Both Grade 6 teachers introduced the technical terms *profit, loss, cost price (CP), selling price (SP)* and the formulae for finding profit or loss. Emphasis was given on solving the textbook problems and correctly arriving at the answers. The transcripts are parts of the discussions that happened while the teachers were explaining the concepts. The students, with their exposure and engagement in the work contexts, and knowledge about the income generating work around them, possessed an understanding of "profit and loss" and used their everyday language to encode the formal meanings of these terms.

The classroom conversations drew on such experiences. For example in Transcript 1a, line 5, 6 and in Transcript 2, line 7, students' use of the instances of profit and loss came from their routine economic transactions and shopping. Students from the settlement have a rich exposure to handling and exchanging currency notes and coins and in doing the calculations mentally (Bose & Subramaniam, 2011). Such exposure created natural everyday settings for the students to co-relate such mathematics content as was being discussed in the classroom with their out-of-school contexts. This prompted them to use their "everyday" registers; for instance in line 11, Transcript 1a, *agar kam daam mein bechenge to loss hoga (if we sell it for less then loss occurs*). Interestingly here the sentence construction used by the student also reflects an informal and everyday usage of their colloquial registers, which are different from the sentence construction when used in a formal usage.

Snapshot 2. During the vacation camp classes the teacher adopted a teaching design experiment approach to try and explicitly build connections between middle graders' everyday mathematical knowledge, work-context knowledge and identities to inform their school mathematics learning. In contrast, during these students' regular classroom lesson in the Urdu-medium school, use of their everyday mathematical knowledge was spontaneous even though their teachers only used the textbook. In the vacation camp however, the teacher planned the lessons to build on students' funds of knowledge from everyday practices with a focus on measurement knowledge and use of fractions. Transcript 3 depicts the classroom scenario when the teacher gives a task of taking different possible measures of various kinds of garments viz., shirts, t-shirts, and *kurta* (long full sleeve shirts) first using a standard measuring tape (popularly known as *inch tape*) followed by a non-standard paper-strip of fixed length. In Transcript 3, the highlighted and underlined words are the Urdu words used during conversation.

Transcript 3. Vacation camp, Grades 6 & 7 (mixed), Lesson: Length measurement

765	T_3	achha, to yeh jo size, kisime likha hai thirty eight, kisime likha hai thirty nine, kisime likha forty, kisime likha thirty eight, aur kuchh likha hai / yeh number kahan se aaya?	ok, so these sizes, some have written on them thirty eight, some have thirty nine, some have forty, some have thirty eight, and some more is written / where have these numbers come from?
766	C	kahan se aaya? Sir, yeh number collar ka hai/	where have they come from? Sir, these numbers are collar's/
767	T_3	collar se?	from collar?
768	C	yes sir/	yes sir/
769	T_3	collar ka to seventeen hai, yeh to thirty eight hai/	collar's is seventeen, this is thirty eight/
770	T_3	haan?	yes?

(*Continued*)

771		(not clear)	(not clear)
772	C	sir, uski size hai/	sir, it's the size/
773	T_3	uski size hai? Size kahan se aata hai? Size kahin na kahin to rahna chahiye / Size ka bhi koi naap hona chahiye na? Kahan se aaya?	It's its size? Where does size come from? Size must be somewhere or the other / there must be some measure for size, no? Where has it come from?
774	T_3	yeh jo thirty eight likha hai, yeh kiska naap hai? Kahan se aaya hai?	this one thirty eight written here, whose measure is this? From where has it come?
775	C	Sir, sir bolu main/	Sir, sir may I say?
776	T_3	haan/	yes/
777	C	sir, lambai aur chaurai ko zarab karke jo aaya woh hai/	sir, it is one which comes by multiplying (zarab) length and breadth/
778	T_3	lambai aur chaurai ko zarab karke dekhna kya aata hai, woh bahut bada sankhya aayega na bahut bada/ lambai yeh hai, chaurai yeh hai, isko zarab karenge to kya aayega? (indicates length and breadth on the blackboard)	see what come by multiplying (zarab) length and breadth, that'll be a large number, no, very large/ this is length, this is breadth, by multiplying (zarab) them what will come? (indicates length and breadth on the blackboard)
779	C	one hundred … (not clear)	one hundred … (not clear)
780	T_3	haan? To yeh kahan se aaya?	yes? So where has it come from?
781	C	tailor likh deta hai/	tailor puts it/
782	T_3	tailor aise hi likha rahega?	has tailor put it just like that?
783	T_3	soch ke batao na/	think and tell/
784	C	sir tailor ko pata rahta hai/	sir tailor knows it/

TRANSFER BETWEEN REGISTERS

The above transcripts show that students did move between their languages of comfort and evoked their 'out-of-school' knowledge, often encoded in their everyday language, during lessons. We observed that the context of the lessons impacted on such phenomena. Moving between languages and the 'knowledges' they encoded is now examined.

Use of Code-Switching

Transcripts 1a, 1b and 2 showed that teacher-students talk involved frequent transitions between the home language and the second language, the language of teaching (English here). There are instances when the code-switch occurred in the form of translation (Transcript 1a; lines 1 and 2), and explanation (Transcripts 1b; lines 38–43, where the action of subtraction is termed as "minus" and used as a verb reflecting an apparent use of a code-switch from their home language). Code-switches also occurred at the instances of exclamations or while complimenting a student's work (e.g., Transcript 1b; line 40). Code switching was further noted while calling out numbers like *three fifty*, *four hundred*, etc. Incidentally *three fifty* (Transcript 1a)was also used to indicate three *fifty* (or 'three fifties' as an acronym for three times fifty, referring to one hundred and fifty) in many cultures in their everyday register, but when spoken in English it assumes a different connotation that students (also some non-English speaking communities) are aware of. In the excerpts above, *three fifty* stood for *three hundred and fifty* and not for *three times fifty*. Use of language in examples such as these derives from socio-cultural roles of the languages. That is, language-use carries with it varying nuances and connotations and a shared understanding, which was not clearly visible at times in these contexts to an observer.

Transition from Everyday to Technical Register

Two key issues in the teaching/learning of mathematics are presenting explanations by the teacher and encouraging student to do the same, related scaffolding, and the need for reducing the cognitive load. These two issues are addressed below.

Explanations/scaffolding. Language switching during the vacation camp classes often occurred between everyday (informal) and technical (formal) registers (see Figure 2). Code switches also occurred laterally between home and other languages while transiting to technical registers (see Figure 2). Such instances occurred naturally while giving explanations and also reflected the use of technical registers embedded in the everyday parlance. In Transcript 3, the vacation camp teacher asked students to explore the meaning of the numbers printed on the collars of the garments they were measuring. These numbers are generally referred to as the "size" of the garment. Students deliberated upon questions like, which measure did those numbers signify, and how were the numbers arrived at? They had already taken measures of different attributes of the garments, such as length (L), breath or width (W), *kamar* (waist) (K),*gala* (neck) (G), *asteen* (sleeve-width) (A), and shoulder (S). Transcript 3 showed students' familiarity with the tailoring work, which is one of the popular livelihoods in the settlement. Drawing on their available knowledge resource, or the 'concrete representational register' of Figure 2 (or as some cultural anthropologists call it 'funds of knowledge', (see Gonzalez, Moll, & Amanti, 2005), students were making sense of the different measures. For example, it was easy for the students to figure out that for measuring the waist one needed to double the width's measure at that location. Though the measurements were taken initially with the standard measuring tapes and then with the help of non-standard "paper strip" of fixed length, use of "palm-length" (called *bitta* locally) was also common. Students' descriptions first were articulated in their everyday register, but they then made a transition to the technical register. Language negotiation seemed to offer scaffolding to meaning making while engaged in doing the activity.

Exposure of children to various work practices in the economically active settlement in which they lived came in the form of language as well as their immersion and active participation in these work practices. That the language played an important role seemed to emerge from the fact that transition from a novice to an apprentice and further to an expert happened not only by learning to do the tasks efficiently but also by learning the technical registers appropriately. For example, in the tailoring work, a novice is first trained to learn different kinds of sewing and knowing their names. Sohrab (pseudonym), a 14 year-old boy from Grade 6 of the Urdu-medium school, explained the different stages of learning that a novice goes through to become an apprentice, and eventually an expert. Transcripts 4a and 4b present a glimpse of how Sohrab used the technical registers. To further explain these terminologies to the interviewer (the first author) in colloquial Hindi, Sohrab switched to the everyday register [Transcript 4a; line 31 and Transcript 4b; lines 70, 74–76].

Transcript 4a. Students' knowledge about their work-contexts; Sohrab, Grade 6, tailoring work

31	S	haan, mere ko lace waigrah lagana padhta tha / aur jo peechhe bandhte hain, naari hota hai na naari, jo gol dhaga bandhte hain peeche, woh banata tha / aur suit hua sada suit, churidaar, pyjama, woh sab banata tha sir/	So, I had to put lace and so on / and which is tied at the back, it's *naari*, which is a round thread tied at the back (of a pyjama), I made that / and suit, simple suit, *churidaar* (fitting pyjama), *pyjama*, made all of these.

Transcript 4b. Students' knowledge about their work-contexts; Sohrab, Grade 6, tailoring work

70	S	pehle straight line sikhna padta hai, dhaga jaise seete hain apun woh line seedha hona mangta hai/	Firstly [stitching on] straight line is learnt. When we sew using a thread that line ought to be straight/
74	S	… pehle to sir turuprei karna sikhayega/	… first hemming [turupei is a kind of hemming] is taught/
75	T	kya?	Kya?
76	S	turuprei/	*Turuprei* (hemming)/
79	S	yaani ki dhaaga katna, aur ghadi karna, ghadi kaise hota hai, istiri maarna/	Means cutting thr thread, and pressing it, how do you press, ironing/
80	T	ghadi matlab?	*Ghadi* means?

81	S	seedha humlog ghadi karte hain na shirt pant (gestures cloth-folding)	We iron shirt pant by making them straight (gestures cloth-folding)
82	T	achha, fold karte hain?	Okay, folding?
83	S	haan, haan/	Yes, yes/

Similarly every task has its own set of vocabulary and the requirement of learning it. Work-contexts present such different socio-cultural aspects of language. For example, Rizwi, a 12 year old boy from Grade 6 of the Urdu-medium school, while explaining his textile printing work (referred to as dyeing work) frequently referred to various sizes as *char-by-paanch* (four-by-five) for design size, or *satrah-paanch* (seventeen-five) for *stoppers'* size ("stoppers" are wooden blocks used for imprinting designs in block printing). He knew that these were measures in inches and referred to certain dimensions (see Transcript 5).

Transcript 5. Student' knowledge about their work-contexts; Rizwi, Grade 6, Textile printing

178	S	haan, char-by-paanch ka tees rupaya lega woh/	Yes, he takes thirty rupees for four-by-five/
179	T	Char-by-paanch kya?	Four-by-five what?
180	S	design/	Design/
181	T	Char-by-paanch kya matlab naap hai? Char kya hai?	Four-by-five means what measurement? What is four?
182	S	char inch aur lambai paanch inch/	Four inch and length five inch/

Many other work practices entailed similar elements of the mathematical register that over time had become elements of the everyday register or work-context register for the students. Different uses of the mathematical registers emerged while interacting with the students about their varied work-contexts. Another example was of recycling work in which Arshad (12 year old boy from Grade 6, English-medium school) used natural numbers as ordinal numbers for grading waste plastic sheets. Use of ordinal numbers and alphabetical symbols for garment sizes were other occasions when their mathematical registers had become a part of the everyday parlance, even though the underlying conceptual construction might have remained unclear and fuzzy for the students.

Reduction in cognitive load. Mathematics lessons typically invite students to engage in meaning making and subsequently arrive at the solution of a problem task, a practice that amounts to an increase in students' cognitive load. Transition between different mathematical representations as well as between technical and everyday registers (Figure 2) can be seen as a way to reduce such cognitive load in mathematics classrooms. For these students, often during the classroom discussion and routine conversation the use of acronyms in English resulted in a transition to the technical registers from everyday registers. Such a practice is not unusual and has become ubiquitous in everyday parlance in urban India and reflects the contemporary socio-cultural scenario. In Transcript 6, still in the context of the size of garments, students could make a guess of what "M" signified based on their understanding of what "S", "L" and "XL" stood for. Use of such acronym-laden technical registers helped, after the initial learning of their meaning, in reducing the cognitive load. It appeared that such synchronous use of acronyms were easier to handle for the students, and had become part of their registers in their various languages.

Transcript 6. Vacation camp, Grades 6 & 7 (mixed), Lesson: Length measurement

724	T	XL ka matlab kya hai?	what does XL mean?
725	C	zyada/	more/
726	T	extra large/	extra large/
731	T	thik hai / aur S ka matlab? S ka matlab kya hai?	alright / and S means what? What does S mean?
732	C	size / size / size/	size/ size/ size/
733	T	aur yeh kya hai?	and what's this?
734	C	M/	M/
735	T	M ka matlab? M ka matlab kya hai? XL ka matlab extra large, M ka matlab?	M means? What's the meaning of M? XL means extra large, M means?
736	C	metre, metre/	metre, metre/
737	T	kya ho sakta hai?	what could it be?
738	T	S ka matlab small, S likha rahta hai na size mein, uska matlab small / L ka matlab large/	S means small, S is written no as size, it means small/ L means large/
739	C	M matlab medium/	M means medium/
740	T	M ka matlab medium/	M means medium/
741		(boy who answered is elated)	(boy who answered is elated)

Use of Non-Verbal Cues Facilitating Transition to Mathematical Representations

Gestures and representations are useful in communication and significant cues are used not only in work practices but they are also helpful during classroom discussions (see Figure 2). During the shirt measurement activity, for example, the students used a variety of gestures for communicating their explanations as was evident from their answers. For example body language and role-playing emerged when some students behaved like tailors in the way clothing was measured and they put the meter-tape across their neck as tailors do. The prototype of a shirt (representation) that the teacher drew on the blackboard further worked as a visual cue (see Figure 3). This representational cue seemed to facilitate transitions between different mathematical representations, namely, different measures (L, B, W, A, S, K, G), and also in deriving the relation between the "size" of the shirt and these measures. Transition between such representations helped in making sense of the measures, and eventually to complete the task.

Use of Everyday and Mathematical (Technical) Registers

An analysis of the students' verbal interchanges showed there were frequent use of the English mathematical register, although the use of the Urdu mathematical register was also prominent. Switches between these and the students' everyday register were also noted (see Figure 2). One example was the use of different binary fractions, which have become part of the students' daily parlance; e.g., *aadha* (half), *paav* (quarter), *pauna* (three-quarter), *sawa* (five-quarter) and *aadha-paav* (half-quarter). Students often transitioned between *pauna*, *teen paav* (three *quarter*) and even seventy-five percent, even though knowledge of percent may not have been well grounded. However, moving between such different mathematical representations seemingly helped students in providing explanations and justifying their claims. Use of such binary units and their further divisions came from the interface between sociocultural setting and language.

Figure 3. Teacher's drawing (representation) of a shirt

SUMMARY

The switching between languages is nothing new in one sense. It has always been a normal way of communicating for multi lingual speakers, a practice in which mono lingual speakers clearly cannot participate. However in many Indian school classroom contexts, where colonial influences still have a huge influence with the belief that knowing English is essential in today's world, the notions of learning in one language (English) and only using that language for communicating and thinking prevails. Nevertheless, as emerged from the above data, students from both the English-medium and vacation camp classes switched between languages and used various registers (see Figure 2), as they engaged in their learning, even though there was very little emphasis on this strategy in the English-medium school.

While early studies on bilingual student's mathematical learning focused on vocabulary acquisition and comprehension of word problems, more recent studies underscore students' knowledge construction, meaning making and participation in mathematical communication (Moschkovich, 2002). Shift towards public expressions in the classroom from individual, private and silent activities leads to greater use of verbal and social cues (Cobb et al., 1993). It is no surprise then that "communicating mathematically" and "participating in mathematical practices" are increasingly being emphasized in the contemporary mathematics classrooms (Moschkovich, 2002, p. 190). Using *situated-sociocultural* perspective, Moschkovich argued that communicating mathematically involves participation in communities of practice, engaging in negotiations through conversation and making use of multiple and diverse resources such as words and objects from language of comfort, gestures, everyday experiences, code-switching and mathematical representations. In line with these notions, we noted that the teacher of the urban vacation class also believed that students utilized all cognitive avenues available to them when engaged in mathematical learning, including moving between their languages, when it appeared that such moves would help their understanding. These notions were not clearly evident in the regular classrooms, and hence there seemed to be a clear distinction in our data between the different teaching contexts, 'regular' and 'vacation camp' classrooms. Further, this distinction seemed to be most visible when word-problems with appropriate illustrations in the students' 'language of comfort' were introduced in the vacation camp classroom.

The above examples also show that students indeed switch between their languages but also utilized the peculiar cultural cues that are drawn from their lives outside of the classroom. In developing such contexts the vacation camp teacher allowed the students to explore their languages and their lived environment to see mathematics around them and hence potentially helped embed the mathematics into their everyday thinking without compromising the formal understanding of this mathematics.

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10. PRODUCTIVITY AND FLEXIBILITY OF (FIRST) LANGUAGE USE

Qualitative and Quantitative Results of an Interview Series on Chances and Needs of Speaking Turkish for Learning Mathematics in Germany

INTRODUCTION

In many countries all over the world, there are learners learning in a language that is different from their first language. In most of these settings, using the first language is advocated, seen as a resource, and used in order to enable the students' access to mathematics (e.g., Adendorff, 1993; Adler, 2001; Baker, 1996; Clarkson, 1991, 1992; Dawe, 1983; Moschkovich, 1999; Setati, 1998; Setati & Adler, 2001; see also an overview in Barwell, 2009). Various case studies have shown how the first language can provide wider options to participate in classroom interactions; for example by giving the students the opportunity to switch between the languages they feel most comfortable with (Moschkovich, 2007; Setati & Duma, 2009). In other studies, the cognitive benefit of first language use has been analysed: Clarkson (2007) analysed mathematically successful bilinguals and emphasized the (meta)cognitive benefit of using the first language while making sense of mathematical texts. Kern (1994) reported on the cognitive benefit in facilitating semantic processing while using the first language. Ellerton and Clarkson (1996) underlined the relevance of individual languages for making sense of mathematical expressions and for developing conceptual understanding.

This short overview shows that the benefit of first language use in mathematics education concerns the individual as well as the social functions for learning mathematics. It is problematical though, that the studies mainly originate from countries where the use of different languages, which one may call multilingualism, is part of the national identity (e.g., Moschkovich, 2007, for California, USA; Setati & Duma, 2009, for South Africa).

Similar to other European countries, the situation in Germany is determined by emigration and immigration movements which bring a great variety of languages spoken in schools. Vertovec (2007) spoke of "Super-Diversity" in order to describe the actual situation of the majority of European countries – consisting of diverse languages and cultures (for a detailed illustration see Meyer, César, Norén, & Prediger, 2015). The language of instruction in mathematics classes in Germany is almost exclusively German, although about one-fifth of the students speak another first

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 143–156.

language (Chlosta & Ostermann, 2008). These students are mostly from a Turkish migration background (second or third generation). German is usually the exclusive language of instruction in mathematics classes – aside from some bilingual classes where another language might be the language of instruction. First language use is forbidden in many German schools (normally by local boards). The quoted number of students with another first language underlines the often-expressed demand for specific support of German language skills in every subject (e.g., in the national integration plan, Bundesregierung, 2010, pp. 47–60). Some researchers advocate that classes should be open to a consistent use of first language by those students who are able to speak two or even more languages (Gogolin & Lange, 2010).

The above-mentioned results – especially the results concerning the cognitive and metacognitive value of the use of first language – are in line with several didactic studies and other approaches on language usage which support the everyday language's significance for construction and development of meaning of mathematical concepts (independent from the aspects of multilingualism). For instance, Wagenschein (1968, p. 102) has emphasized that *technical language* as *the language of the understanding* comes at the end of the learning process – as one learning target.

In the context of this chapter's intention, everyday language is mainly realized in the learner's first language. Following Lawler's concept of micro worlds, Bauersfeld (1983) developed the concept of *Subjektiver Erfahrungsbereich* (subjective field of experience). With the help of this concept, Bauersfeld described such phenomena as, for example, children being able to calculate the price of two pieces of chewing gum but could not solve the equivalent symbolic calculation. Mathematics is (at first) inseparable from everyday life and thus inseparable from the language that is spoken in everyday conversation. Carraher, Carraher, and Schliemann (1985) describe similar phenomena in the context of children determining the price of coconuts. The authors make a distinction between street mathematics and mathematics at school. When the learners' mathematical skills are bound to the prevailing context, the context is normally realized in their first language.

Thus, there are a lot of reasons to think that the first language can be a useful learning medium even if the language of instruction is another language. The study presented in this chapter can be seen as an indication of how far the results can be put into practice concerning the learning of mathematics. A series of interview-studies is described, focusing on the situation of students with a Turkish migration background using their first language for learning mathematics in Germany.

INTERVIEWS

As Turkish is the biggest language minority in Germany, this study focuses on Turkish students. Even if the students' former education has not been in the first language, the following research questions are investigated in order to gain information about how the first language can be used for elaborating mathematical concepts and what circumstances and consequences can be observed:

1. How do students react to the opportunity to use their first language in language reception and production?
2. For what purposes do the students use their first language?
3. What phenomena accompany the use of the first language?

For investigating the students' processes of thinking, communicating and understanding in depth, the method of clinical interviews for data collection has been used. The students had a Turkish migration background and could talk and read in Turkish (ability to read was one criterion for participating in the study, which reduced the number of candidates drastically). All students had grown up in Germany and had good basic interpersonal communication skills (BICS) in Turkish and German as well as varying cognitive academic language proficiency in German (CALP) (see definitions in Cummins, 1979).

For studying first language use under laboratory conditions of a clinical interview, the operationalization of the language use in an interview design is crucial. In two previous studies, we investigated the functioning of different bilingual settings with varying options for language production and reception (see Meyer & Prediger, 2011). The different options of using the first language in all interview series are presented in Figure 1; their realization in each series is described in detail below.

Turkish in language reception…	Turkish in language production…
	… encouraged among each other (option 2.1) both languages allowed in the working process, explanations only in German
… facultative[1] offer (option 1.1) simultaneous presentation of a German and a Turkish text with option to choose	… facultative offer (option 2.2) the students could choose between a German- and a Turkish-speaking interviewer
… offer pushed with delay (option 1.2) the Turkish text is given after working with the German one	… offer pushed with delay (option 2.3) at the end of the interview, they should work together in Turkish

Figure 1. Options for using the first language Turkish in the interviews

First Interview Series

Methodology in the first interview series. Concerning *language reception*, two options of using Turkish were distinguished. For a *facultative offer* (option 1.1) to use Turkish, the task (an open modelling problem) was simultaneously presented in Turkish, "Kanak"[2] and German. The students could decide to use all texts or only one text. For an *offer pushed with delay* (option 1.2), the students were confronted with the German text first. After having finished the task they could read the Turkish text. They were then encouraged to consider the text by questions like: "I also have another text that seems to be pretty similar. Unfortunately, I can't read Turkish. Can you also work on this problem here?"

Concerning *language production*, three options were distinguished. At first (option 2.1), the students were encouraged to speak Turkish to each other while solving the tasks. In option 2.2, the presence of a Turkish-speaking person enabled students to speak Turkish (*facultative offer)* with an interviewer the whole time. This option was not used in the third series because the students did not use it very often. In the third option (option 2.3), the Turkish language use *was pushed with delay* by "tricks" after the first solving processes were completed: in the first two interview series a second person entered the room and presented himself as an only-Turkish-speaking "caretaker". By sending the interviewer to the school's headmaster, he stayed alone with the students and asked them to tell him what they were working on and what they had developed so far. In the third series, the students were asked to present their solutions to an only-Turkish-speaking student behind the camera. Certainly, the caretaker's and the fictitious student's role represented another context, linguistically and culturally. This can be crucial for an interview setting, as the use of language and the accompanied construction of mathematical meaning are always context-dependent (cf. Meyer, 2010).

All interviews (in each of the three series) were videotaped and selected parts were transcribed. The qualitative analysis of the transcripts was conducted following the interpretative research paradigm (cf. Voigt, 1998).

Selected findings of the first interview series. If a text in Turkish was *offered facultative*, nearly all students used the German text. Some of the students compared both texts and found the German one easier. The Turkish text was almost exclusively used to find the meaning of some words, for being able to talk in Turkish (e.g., to the "caretaker"), but this only occurred a few times during the on-going working process. The text written in Kanak was not used at all. This phenomenon was also affected by moments of peer pressure; for example, one girl started her work with the Turkish text but had to change to the German version for collaborating with her partner. When language reception was *pushed with delay*,most students compared the texts and commented: "This is the same." Just one boy started to solve the task based on a misunderstanding of the Turkish text.

If *language production* in Turkish was *encouraged between the students*, the students rarely made use of their first language, neither in the working nor in the presenting phases. In interviews with two interviewers (option 2.2), almost all sequences in Turkish had been initiated by the second (Turkish-speaking) interviewer. The option of *pushing language production with delay* was more successful in the sense that it could initiate the students' explanation of ideas in Turkish, although with hesitations and little confidence. All students were able to present their solutions in Turkish though the explanations were, as expected, full of German technical terms and many moments of code-switching (single words as well as longer expressions), allowing the students to express themselves in richer registers.

Second Interview Series

Methodology in the second interview series. The students' utterances in the first interview series could be interpreted as the students would not change their habitualized context-specific language, which they had learned and used for years of school socialization, just for a single interview with an unfamiliar person. As German is the exclusive language of instruction in mathematics classes, the German language is the students' usual language of communicating mathematics – even if their thought processes might be in their native language (as discussed by Clarkson, 2007, p. 194). Additionally, some of the students' expressions indicated limited self-confidence in their first language capacities in the mathematical context, although their communication appeared to be successful. For these three reasons (artificiality of the research setting, habitualized context-specific language use, and missing self-confidence) led the researchers to intensify the observed moments of encouraged language production: the interviewer started by asking the children to teach her/him to count in Turkish, or to translate sentences into Turkish. This was to encourage students to value the Turkish language as a working language for the mathematics. After a period of working in two languages, the caretaker came again to *push with delay* for Turkish explanations. As the students addressed the Turkish-speaking interviewer only a few times in the first series, this strategy was used in only a limited number of interviews.

In the first series, the students did not use the text written in Kanak at all. This, and the apparent opinion of students to value the languages German and Turkish for themselves, led to the decision to reduce the number of different languages to two. The options of the bilingual settings for language reception were reduced to *pushed reception with delay*, as this caused the only remarkable phenomenon.

Altogether 31 interviews (2–3 students in each interview) were conducted with children from grade 4 to grade 6 in the second interview series. The language biographies of the children were comparable to those in the preliminary study.

The main task was a short text full of non mathematical technical terms where not all relations were clearly expressed, and synonyms were often used (original formulation: "According to a UN report, 1/4 of all adults in this world are analphabets, that means, they cannot read. Due to this, they cannot learn many professions. 2/3 of all analphabets are women" cf. Prediger, Barzel, Hußmann, & Leuders, 2012). First, the students were asked to rewrite this text in their own words in order to make it more understandable for other students. Second, they were asked to make a picture that expressed the text's content. Finally, the students were confronted with a drawing which had to be explained using the information in the text's. Thus, the students had to elaborate an understanding of 'parts of parts', which had not been mathematical content they had studied.

Selected findings of the second interview series. By analysing the second series of interviews, particular attention was given to the various functions of the learners' first language use. In attempting the task on ‘parts of parts’, the learners reached their linguistic limits and also the limits of their mathematical competences: they were forced to acquire a new mathematical conceptual term. In several interviews, this process happened in the learners’ first language, shown in the particular case of Dilara and Elina in Meyer and Prediger (2011): by a neologism (they invented the double comparative form “fewerer” in Turkish), the two students expressed the complex comparison of rates. The creation “fewerer” only seems to be an intermediate state, as the expression was later substituted by “the most”, which is a more usual way to describe rates.

Examples in which the learners had been able to gain cognitive benefits through the use of their first language were observed in several interviews. Evidence of how the first language strengthened some students’ participation in the group interaction could also be reconstructed. Previously, the students’ participation in the group was markedly less when only German was spoken. One simple reason for their engagement in the Turkish-speaking moments of the group discussion might be connected to the (in most interviews used) exclusion of the controlling interviewer. In some other interviews the children seemed to have elaborated communications rules; e.g., “Whenever the interviewer says something like ‘You can reconsider that together’, we have to switch to Turkish.”

Interestingly, it seemed that the smallest methodological change which had been made for the second interview series was the most effective: to put a spotlight on the Turkish language, or rather to establish it as a working language, by asking the students at the beginning of the interview to teach the interviewer some things in Turkish (e.g., to count to 10). This changed the perceived distribution of linguistic competence drastically: we observed that students made much more use of Turkish while solving the task. This strategy in particular – the permission and appreciation of the first language – can be used advantageously for the everyday mathematics classes.

Third Interview Series

Methodological information. In the third series of interviews, fewer methodological changes were carried out compared with the second series. Now, the options with regard to the different tasks were realized in a different way. The first task was to classify the visible sides of a tower that consisted of several dice that were put in layers one on top of each other. Based on the sum of numbers of viewable spots of each die (2*7), the number of dice(x) and the knowledge of the number of spots (y) which the upper die showed at its top, the overall number can be calculated as $14x + y$. The task can be regarded as difficult as it is normally used for gifted children of this age (Kaepnick, Nolte, & Walther, 2005). The task was chosen in order to challenge the students not only linguistically but also mathematically. Thus it was thought that the processes of explicit negotiating might be more extensive and therefore the role of language as a learning-medium could be focused on. By contrast with the previous interviews, it was not the task to form a concept but to recognize mathematical relations. For carrying out the task, the text was formulated in both languages (option 1.2, Figure 1). Instead of installing a "caretaker", the presentation of the results in the first language was guaranteed since the students had to present their ideas to fictitious student who only spoke Turkish via a video so that the fictitious student would mathematically understand the trick while watching the clip. This realization of option 2.3 (Figure 1) did not ensure an elaboration of the solution in Turkish, but at least the linguistic articulation of what they were doing was guaranteed since they had a target of explaining their processes to the fictitious student. Hence they had to express precisely the mathematical connections and relationships they thought were important in doing this task.

While working on the second task, the students were sitting back-to-back. One student was given the geometrical shape, for example in the form of a package (Figure 2). This shape was to be explained to the second student and, in so doing because of their sitting position, the first student could only use verbal and linguistic devices to convey a description.

Figure 2. A frustum as an example of a geometrical shape, which was to be described

As gestures and facial expressions were excluded, the students were limited to making use of their language competencies in order to communicate about the geometrical shape. Hence precise language expressions were necessary for giving a clear explanation. Thus, these tasks do not only demand language as a learning-medium but also (technical) language as a learning-prerequisite.

If the first geometrical shape was described in German, the students were asked to describe the second shape in Turkish. This was the task-specific realization of option 2.3 (Figure 1). In contrast to the cube-task, this task was given orally, which meant that the options of language reception were not used.

Altogether more than 80 group interviews were conducted in this series, with 2–3 students in each group. For various reasons, only 73 interviews (all with two students) could be analysed. The students were aged between 8 and 12 years (elementary school and early secondary school).

The analysis of the third series is on-going and hence only preliminary qualitative and quantitative results are presented here. The qualitative procedure stayed the same and to now the quantitative analyses have been limited to frequency and the cross-classified tables. The cross-classified tables have been created for the items "normative productive" and "interactive productive" separately for both tasks and for the item "gender", on the one hand, and all other 90 items, on the other hand. The other items consist of aspects of the biography of language-learning (cf. Grießhaber, 2003) and draw on selected items of HAVAS 5, a test to evaluate the actual state of language (cf. Reich & Roth, 2004); implying for example the number of linguistic "jokers" used (that is, general terms such as "thing" being used when the student did not know another expression, often when they did not know a technical term) or the fluency of language. Also some items concerning the use of different languages (e.g., the number of turns in the L1 or the number of code-switches) were captured.

The (normative and interactive) productivity has been defined as follows: "Normative productive" means that a student expresses mathematically stable concepts or conjectures that have not been expressed before. This also includes new task-specific suitable explanations or comparisons. Guessing, for example, has not been coded, as this has not been regarded as mathematically stable. For the coding of this item the reaction of the other participant of interaction is not relevant. "Interactive productive" means that the mathematically stable concepts or conjectures have been picked up and used by the partner. Thus, a scene can only be regarded as being productive in the social interaction if it has been productive in a normative (mathematical) sense beforehand.

Two scientists did the coding of the interviews. Based on the coding-manual an independent assistant agreed with their classifications after reviewing 15% of the interviews giving a high reliability of the results.

Selected Quantitative Findings of the Third Interview Series

The first results of the quantitative analyses show a complex picture. As had already been observed in the first two interview series, language-reception (the cube-task written in Turkish) was not used very often (only by 10% of the students, some of which switched to the text written in German after some time). In order to understand singular words or expressions (e.g., in order to present their solutions for the fictitious Turkish student) more than two-thirds of the students switched between the different texts. Thus, we can conclude that the first language appeared to be helpful for the students, but it did not turn out to be the main working text-language. Regarding language-production, 84% of the students used their first language Turkish while working on the cube-task, and 50% of them on the shape-task.

Most qualitative attributes concerning language use have been comparable across the languages, such as speaking fluently (measured by the length and the number of breaks while speaking). Interestingly, the students requested a fewer number of words (max. six times) from the speaking partner while speaking Turkish; in contrast during nearly half of the interviews, the students asked for the meaning of German words while interacting. Nevertheless, the translators (two native-speaking students from Turkey) attributed to most of the students a lower quality of speaking in Turkish compared with German. This can be explained as that the competencies in their first language were well developed and proven in their former everyday interactions, but had not been used for communicating about mathematics. The more extensive use of German technical language supports this explanation and it is also not surprising given the background of the monolingual German mathematics education of the students. Correspondingly, the students asked for fewer technical terms while speaking Turkish and, simultaneously, the number of technical terms which were used was lower compared with the number used in the German-speaking turns (both aspects are regarded in a relative way). In the phases of Turkish interaction, the students did not announce lack of knowledge as often as in the German parts (e.g., by utterances such as “I cannot express that”). This indicates the time-tested quality of the first language Turkish in moments of interaction.

From a pure mathematical point of view, in only about 15% of the working processes we could find moments of productive use of the Turkish language. This “pure mathematical point of view” is defined by disregarding non-mathematical contents, which also might have a positive influence on the working process (some of these are presented below). Apart from three exceptions, the mathematical-normative productive statements were picked up and used by the partners of interaction (interactive productivity). Astonishingly, there was no interference: if first language use of the students turned out to be productive (normative or interactive) concerning one task, it was not necessarily productive concerning the other task. On the basis of the phenomena observed in the cross-classified tables, arithmetic means were composed. This indicates that if the use of the first language (L1) turned out to be productive (instead of unproductive):

- the number of technical terms expressed in L1 was rather low,
- the number of linguistic “jokers” (in L1 and L2) was rather high,
- the number of code-switching of a single word (from L2 in L1) was rather high,
- the number of code-switching of more than one word (from L2 in L1) was rather high,
- the number of turns in L1 was relatively high, and
- using the first language was comprehensible and continuous.

This suggests that the high number of first language turns with productive first language use may explain the average higher number of linguistic jokers (e.g., “sey” – English: “thing”), or rather the shorter or longer borrowings from the second language German, the average low number of technical terms used in the first language appears to be unusual. The results (also with regard to the comprehensibility and continuity of language use) allow us to propose the following thesis: the learners were well able to communicate in their first language but they put the productivity of the mathematical-content communication before the eloquence of first language use. In other words, the flexibility of language use seems to be a characteristic of productive language use.

The productive working students seemed to have control over their first language not only in their every day life but also while talking about mathematics. They are able to replace their technical language knowledge gaps by requesting or by replacing them by code-switching or circumscriptions. Concerning the cube-task, the group of productive working students showed first language competencies also in language reception as they mostly worked with the text written in Turkish.

Selected Qualitative Findings of the Third Interview Series

The productivity of language use cannot be measured by looking at mathematical content only. The first language could also have other functions, which might be useful for learning but cannot be measured objectively. Some of the functions of first language use observed in the third interview series are (cf. Kraegeloh & Meyer, 2012) as follows.

Using the first language increases the chance to gain new explanations. Celia and Aynur were working on the shape-task. Celia was asked to describe a truncated pyramid (Figure 2) and Aynur was supposed to rebuild it by following Celia's description. The students did not know the technical (geometrical) term for the figure and could choose in which language they talked to each other. Celia starts by speaking German (translations from Turkish are written in grey):

Celia: Ehm, and above, yes, and above there are also four corners. The same on the sides, too.

Celia's orders are not sufficient to rebuild the figure. Thus, Aynur is confronted with a linguistic challenge. The interviewing person indicates that Aynur may ask questions:

Aynur: Has it a cone end?
Celia: What's up?

Explicitly, the expression in Turkish represents a question concerning the content. Implicitly, Aynur might be expressing the wish to communicate in her first language. In the following sequences, the assumed wish is repeated explicitly as Celia does not react in Turkish: "Explain it in Turkish!" There may be several reasons for this. With regard to the third interview series, many of the descriptions of figures were more precise in Turkish even though they were more limited in the use of technical terms.

Thus, some episodes in Turkish resulted in a more adequate rebuilding of the figure compared with the situation in German.

Using the first language increases the chances to take part in the interaction. In one interview, a formerly quiet student could be observed. After short episodes of talking Turkish in an intimate situation with his interview partner, the student talked more and longer in the official communication (in German) with the interviewer. This might be interpreted as the use of the first language giving students more options to express their thoughts in other, possibly preferred, ways. Consequently, the use of Turkish could be reasonable in different interviews for a changing speaking situation. Furthermore, some students could be observed talking a lot in the Turkish parts but not in the German parts of the interview, and also the other way around.

The productiveness of using the first language in language reception is limited. Concerning language reception, remarkable results could be observed. Getting a mathematical task in Turkish surprised a lot of students. Generally, one could assume that the offer of the same task written in both languages might be useful to overcome problems of understanding the task. Many of the students (especially in the third interview series) looked at the Turkish text, but switched to the text written in German and ignored the Turkish in the subsequent working process. They gave reasons such as "I understand German better" or "I do not understand some words in the Turkish text."

In order to make a more productive use of Turkish in language reception, it seems to be useful to change the conditions. This could be realized by learning mathematical technical terms in the first language for a longer period.

Using the first language in order to organize the working process. Using the first language created a situation in which the students could communicate without the interviewer's participation. In this situation, it was often observed that the learners talked about the work organization: Who presents the results? Who takes on which part of the calculation?

Using the first language in order to enable an intimate communication. It would not be honest to describe only chances of using first language if the teacher/interviewer does not share this language. Certainly, some students also used their first language to create an intimate situation making use of the interviewer not listening. Some negotiated on who was going to present the solutions or they talked about the interviewer or the setting:

Elina: Eh, I understand it. But you may present it, maybe I learn something of it. The thing on the other side [she means the camera, M.M.] is not my case, it makes me nervous (*laughing*). Come on, go ahead.

CONCLUSIONS

Although many students used their first language hesitantly in almost every interview at first, they showed quite profound first language competencies. Moments of interactional, cognitive and/or metacognitive benefits of first language use (in language production) for conceptual understanding were reconstructed. The short insights in different interview series showed that using the first language can help to elaborate mathematical concepts (e.g., parts of parts). Apart from the cognitive benefits, different functional purposes of first language use could also be observed – including social (e.g., an increasing participation) as well as metacognitive (e.g., organizing working processes) benefits.

As the students used the options of bilingual language reception only a few times in a productive way, our future research will focus on the options of language production – as already investigated in the development of the interview series. This also implies that it is not necessary to translate every mathematical task into every language in the mathematics classes, though this assertion needs long-term studies with changing conditions. Moreover, it can be expected that students will not immediately change patterns of language use that they have established over years in a monolingual culture of mathematics grade classes for a small and short-term interview setting.

With regard to first language use in language production, the quantitative results of the third interview series indicate that the learners put productivity before linguistic elegance. Therefore, they rather seem to use their language flexibly. The flexible use of a language, distinguished by many linguistic deficiencies (such as a lack of technical terms, the use of a linguistic "joker" and longer code-switching), declares the language to be an instrument for working processes. In terms of Bernstein (1971), we can say that the conscious use of the "restricted code" has positive effects on the process of negotiating (mathematical) knowledge.

Gutstein (2003) argued that learners who learn mathematics in a language that is not their first language can be successful in school as long as we take their background (e.g., first language) into account, in effect recognising who they are. Setati (2005) showed how a teacher who is sharing the first language of the learners and the language of instruction could realize this in the mathematics classroom. The results of this study indicate that learners can make use of their first language even though the teacher (the interviewer in this study) does not share their first language. In this study, the students' background has been taken into account by giving them the role of language-teachers and to allow them to work in their first language.

On the basis of the results presented in this chapter the use of a first language seems to be an appropriate and useful resource for learning mathematics. This might not only be with regard to interview situations, but also small group discussions in regular mathematics classrooms, where the teacher does not share the first language of the students. Surely in doing so appearances of segregation should be avoided. However, one might be afraid of the teacher losing control of the class if he/she does not understand the students' small-group working language. However trusting in the students' individual responsibility (cf. Clarkson, 2003) can go a long way. The working process can also be observed by checking the results of the students' work. Necessarily, the language of the official class communication has to be a shared language.

NOTES

1 'Facultative' translates into English as more or less 'optional'. Hence here this can be understood to mean that 'students were given a choice which was clearly optional'.

2 "Kanak" is a usual term for describing the language of Turkish adolescents speaking in their everyday life in Germany: the normal communication in Turkish is interrupted by words in German (cf. Zaimoglu, 1995).

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THULISILE NKAMBULE

11. SUPPORTING THE PARTICIPATION OF IMMIGRANT LEARNERS IN SOUTH AFRICA

Switching to Two Additional Languages

INTRODUCTION

What forms of discourse are valued in classrooms with immigrants in South Africa? How does a teacher determine which form of discourse is valued by immigrant learners? Who gets to participate in the valued forms of discourse? These questions focus attention on how immigrant learners were supported in classroom interaction, supporting that involves the formation of immigrant learners' mathematical identities that has implications for their achievement and access to resources within the classrooms and in their immediate environment.

In this chapter I examine the discourse practices that supported the participation of immigrant learners in multilingual classrooms in South Africa. I rely on data drawn from a wider study that investigated the teaching and learning of linear programming in three different settings in South Africa (Nkambule, 2013). In particular I explore discourses that the teacher used and further show that immigrant learners were supported during the teaching and learning, a context that has not been researched in mathematics education in South Africa. Yet South Africa is a country with a rapidly increasing immigrant population. The gap that exists in knowledge currently is addressed in this chapter hence the importance in its contribution. However, issues related to multilingualism and immigrant learners are not specific to South Africa only, it is an international work.

The larger study alluded to in the previous paragraph examined three settings; located in an urban, rural and township environments. The focus in this chapter is one of these settings, the urban environment. The analyses conducted in the wider study, the results of which are discussed in this chapter, focused on translation of some key concepts in linear programming. I observed that instead of supporting the majority learners who learn in a second language, English, the teacher switched between English and French, which are additional languages of some of the immigrant learners. The use of the two languages as a support was possible because the teacher owned resources of language. I argue that the teacher's discourse advantaged immigrant learners who understood French because the majority who learned in a second language in this mathematics classroom did not get the opportunity of a second explanation in a language they were comfortable with. This then points to

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 157–170.

the complexity of teaching mathematics in the context of language diversity thus creating unequal access to the mathematics learners are learning.

This chapter is organised as follows. Firstly, I start by briefly but adequately describing the historical, political and social context in order to establish the significance of the issues related to immigrants learning in South Africa; a country that has witnessed a rapidly increasing immigrant population. Issues related to immigrants in South Africa might or might not relate to their participation in the classrooms, and therefore could position them in the context of social practice. Secondly, I then focus on teaching linear programming in multilingual classrooms with immigrant learners with a focus on how a teacher supported immigrant learners.

Tension and Violence Directed at Immigrants in South Africa

Migration by black citizens from African countries to post-apartheid South Africa with an intention to stay, some permanently, is a phenomenon that is mirrored in and affects other countries around the globe. These migrants travel with school aged children who are found in mathematics classrooms. Some of these learners are from countries where the language of learning and teaching is French or Portuguese, but are now expected to learn through the medium of English in South Africa. However, studies document that immigrants are not well received by some South African citizens (Crush, 2008). Consequently, the past few years have seen tension and violence directed at immigrants. According to Crush and Williams (2003), this tension and violence is a demonstration of frustration by poor and unemployed citizens who accuse foreigners of stealing their jobs, as well as an increase in competition for scarce resources such as housing. The accusations are mainly because as black South Africans claim space within their country's urban areas, which were previously forbidden them,[1] they confront non-nationals also seeking safety, employment and economic opportunities in the same urban areas.

This chapter explores how a teacher supported immigrant learners in learning linear programming through the medium of English, their second additional language. The focus of the analysis will be on one important aspect of learning linear programming; understanding the symbolic meaning of key words when extracting inequalities from a given text. I analyse an excerpt from a partial transcript of a grade 11 linear programming lesson in an urban environment in Johannesburg, South Africa. The aim of the analysis is to illustrate how the teacher supported immigrant learners from a country where the language of teaching was French. During the lesson, the teacher focused on explaining the meaning of key words such as 'availability', 'maximum' and 'at least' to French as he interpreted the content.

RELATED LITERATURE AND THEORETICAL FRAMEWORK

Discursive research that looks closely at what is said and done in the classroom discourse, and how talk and action come together to offer opportunities for mathematics learning, show how language is a live issue for participants in the mathematics classrooms (Moshckovich, 2007; Kazima, 2006; Clarkson, 2007). These researchers have carried similar investigations concerning the role and use of language in the teaching and learning of mathematics. Clarkson (2007) found that Vietnamese speaking students from immigrant background in Australia would use Vietnamese to interpret mathematics tasks and carry out some calculations although their teacher was not aware of their discourse. While Kazima (2006) argues that second language learners may express themselves and understand mathematics better when learning in a language that they are fluent in. Furthermore, teachers and researchers in South Africa and elsewhere are creating opportunities for the use of learners' home languages when teaching mathematics (Setati, Molefe, & Langa, 2008; Webb & Webb, 2008; Nkambule, Setati, & Duma, 2010). These researchers argue that the use of home languages, or a language that learners are comfortable with, may enable them to access the mathematics while learning English. In addition, Levitt and Schiller (2004) states that individuals who have migrated from one country to another may continue to incorporate daily routines, activities and institutional affiliations that connect them to their country of origin even as they actively engage in their everyday lives in their destination country. It can be supposed then, following Levitt and Schiller, that immigrant learners from the Democratic Republic of Congo (DRC), a focus in this paper, can probably learn mathematics through the medium of English if they are supported by the use of French, the language of learning and teaching (LOLT) in the DRC.

Civil and Planas (2004), whose research focuses on immigrant learners who face learning in a different language to their own in Spain and USA, argue that mathematics classrooms with immigrant learners are key sites of inclusion or exclusion; privilege and disadvantage settings where learners learn to be particular kinds of people. However, there are immigrant learners who migrate to another country where language is not an issue, for example UK immigrants to Canada or Australia or Spanish immigrants to parts of South America although there maybe cultural issues. In the mathematics classrooms discussed by Civil and Planas (2004), everyday classroom interaction may support a community through particular understandings of individual learners, their unique experiences and educational uniqueness. Though, it may be difficult at times due to the number of languages and cultural backgrounds that may be represented in the mathematics classrooms with immigrant learners. There is evidence from research that in some of the classrooms with immigrant learners, immigrants are usually not provided with the opportunities to incorporate their rich cultural identity and life experiences into their formal schooling (Gorgoriò & Planas, 2001; Civil & Planas, 2004; Civil, 2008; Planas & Gorgoriò, 2004). Immigrant learners take upon themselves negotiations as they adapt to their new schools, learn a new language of learning and teaching (LOLT) as well as the official languages in the destination countries. Hence immigrant learners have less control over the cultural and social resources in the mathematics classroom context.

In this chapter the needs of the immigrant learners to develop linear programming discourses that would enable them to succeed is the specific focus. Learning linear programming is viewed in terms of a relationship between the immigrant learner's activities, and their environment in which they have to think, feel, act and interact (Gee, 2005). Any environment in which an immigrant learner finds himself or herself is filled with possible actions determined by features in the environment, like the languages used. In this chapter, a discourse perspective (Gee, 2005), stating that language is linked to the context in which it is used and of language forms that take on meaning in particular contexts, is used. Gee contends that in any situation people 'pull off' or try to use certain identities. To do this they use language and activities. He defines identities as the different ways of being involved in social groups like mathematics classrooms. In these social groups, people speak, write, act and dress in ways that portray certain identities. The speaking, the writing and the acting are all established in Discourse with a "capital D."

Gee (1996, p. 131), contends that an uppercase "D" Discourse involves much more than sequential speech or writing:

> A Discourse is a socially accepted association among ways of using languages, other symbolic expressions, and 'artefacts', of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or social network', or to signal (that one is playing) a socially meaningful 'role'.

According to the definition offered by Gee, Discourse is more than just language because it involves using languages and symbolic expressions in interactions with people belonging to specific communities. Hence Discourse also involves points of view, values, communities and artefacts. In linear programming, (in Gee's sense) Discourse then would include more than ways of talking or writing; they would also include beliefs, points of view and social practice. Discourse certainly involve using language, but also involves people and communities. In this regard, Discourse is situated both materially and socially. This means that Discourses are collective and not individual.

Talk is embedded in practices tied to communities like mathematics classrooms. In these classrooms, Discourse is connected to mathematics ideas presented during teaching and learning of content like linear programming. So Discourses involve thinking, signs, tools and meanings. Words, utterances or texts have different meanings, functions and goals depending on the practice in which they are embedded. In this regard, learning linear programming occurs in the context of practices tied to communities. These practices are constituted by actions, meaning for utterances. Discourses give meaning to every human activity and thus well-established 'within Discourses' are practices and knowledge from which people within a particular community draw when engaging in the various roles that they play. Gee emphasises that the key to Discourse is recognition. This suggests that whatever you have done must be similar enough to other performances to be recognizable; if it is not recognizable, then, you are not in the Discourse (Gee, 2005).

Therefore, participation in linear programming Discourses may be understood as explanations of content and acting in the ways that mathematically competent people explain and act when explaining linear programming. The explanations involve more than language. For example linear programming Discourse includes constructing inequalities when given statements such as "x is at least 20", representing the inequalities geometrically and finding a feasible region. Therefore, immigrant learners in mathematics classroom will use language, behaviours, actions, and tools to recognise themselves and others as belonging to a set of linear programming practices or Discourse. At the same time immigrant learners give meaning to that Discourse by reproducing or transforming it. So a Discourse is a way of doing things which projects a certain identity.

CONTEXTUAL BACKGROUND AND METHOD

This chapter specifically focuses on data collected in one school (School A in the wider study referred to above). It is located in central Johannesburg. The founder of the school was an immigrant from the Democratic Republic of Congo (DRC). The school catered for learners from all over Africa and had a student population of about 800 learners: 85% were South Africans and 15% were from elsewhere in Africa. The LOLT in the school was English. French was the only foreign language that was offered as a second language subject, although in addition there were the other official languages of South Africa. Data analyzed in this chapter were collected in a Grade 11 mathematics classroom with 26 learners: 21 were South African, three were from the DRC, one was from Malawi and one was from Zimbabwe. The only immigrants targeted in this research were French speaking, originally from the DRC. There were eleven home languages represented in this mathematics classroom, eight2 of which were official languages in South Africa while the other three (Lingala, Chichewa, and Shona) were not indigenous to South Africa, but home languages of the immigrant learners. Only eight learners shared a home language; IsiZulu.

There were 23 teachers in this school at the time of data collection, and in addition a secretary and a security officer. Six teachers out of the 23 were originally from the DRC and could speak English and French. Three teachers were from Ghana and could also speak English and French, four were from Malawi, five were from Zimbabwe and five were South African citizens. During the process of data collection, some teachers and a few learners were interacting in French within the school premises. Data analyzed in this chapter were collected through observation of linear programming lessons, and a post-observation interview with one of the immigrant learners, John (not his real name).

John arrived in South Africa towards the end of the year 2010 while he was doing Grade 10. The mathematics teacher observed was also from the DRC and was fluent in French and English. In addition to French, the teacher shared a home language (Lingala) with John, and incidentally the two other immigrant learners in this mathematics classroom. English was a language the teacher shared with the majority of the learners in his class. He had been teaching mathematics in South Africa since 2000.

In framing the chapter through the use of Gee's constructs of Discourses, I plan to show how the teacher in an urban environment supported immigrant learners in order to facilitate the growth of linear programming knowledge in English and French. This knowledge did not require a significant change of the mathematics knowledge that the immigrant learners brought from their previous school environment. For example, an immigrant learner may know how to express inequalities ideas in French, such may convey a particular status in his country of origin and that could be exchanged with the teacher in South Africa. However, Setati (2005) has shown that mathematics teachers' decisions about which language to use, and how and when to do so, do not only reflect curriculum and pedagogic decisions, but also the political context of their practices together with the identities and activities they are enacting.

FINDINGS

Episode 1: Acknowledgement of Social Context

In this episode the teacher uses a mnemonic strategy commonly used in the two countries, the DRC and South Africa. Furthermore, he shares his experience as a teacher who has taught in the two countries. In this regard the teacher introduced two well-known methods of finding an equation of a straight line; the gradient intercept method (y = mx + c) and the dual intercept method $\left(\frac{x}{a}+\frac{y}{b}=1\right)$ in terms of the two different approaches taken in each country as shown in the transcript below.

26. Teacher	I will teach you two techniques, one technique from South Africa and one technique from Congo… (*learners laughing*) Ja! This one we do not do it here, but I will give you (referring to the dual intercept method). Then you will choose which one is easier for you. You know that the standard form of an equation of a line is y equal to mx plus c (y = mx+c), which is the general form. Then you change it to the standard form which is ax plus by equal to c (ax + by = c). The first thing is to determine the gradient and the y intercept. What is the y intercept?
27. Learner	Eight
28. Teacher	The line cuts the y-axis at …eight! Ja! Therefore your c is eight! Okay! You understand?
29. Learners	Yes
30. Teacher	Ja! Then you find the gradient, look! [*name of learner*] what is gradient? Your mother asks what's the gradient? Tell your

	mother what is gradient? If your mother ask what are you going to tell her?
31. Learner	Change in y over change in x… [*learner interrupted by teacher*]
32. Teacher	Your mother! Ah! [emphasis] your mother don't know change in y over change in x, what are you going to tell her?
33. Learner	…rise over run…
34. Teacher	Now remember I told you gradient is rise over run, do you still remember that? The rise is on the y-axis and the run is on the x-axis. Now, will the gradient be positive or negative?… which position gives the negative gradient which position gives the positive gradient? Remember I told you when you rise like that (*teacher pointing upwards direction*) the rise will give positive gradient and when you descend like that (*teacher pointing downward*) the gradient is negative. Now if you look at the line, will the gradient be positive or negative?
43. Teacher	Congolese method! Congolese method Ja! Look at here Ja! Another method Ja! Okay another method
44. Learner	Yes Congolese method
45. Teacher	Now look at here you use the formula which says x over a plus y over b is equal to one, $\left(\frac{x}{a}+\frac{y}{b}=1\right)$ where a is, a is where a is the x-intercept, b is the y intercept okay? Easy formula Okay now you look at here [*teacher pointing at positions of intercepts on the graph*] What is the x-intercept? Yes
46. Learners	The x-intercept is four and the y intercept is 8.
47. Teacher	Which one is representing a?
48. Learners	Four
49. Teacher	From there the equation becomes x over four plus y over eight equal to one. From there you find the LCM of four and eight. The LCM of four and eight is what? Yes
50. Learners	It is eight
51. Teacher	From there you multiply each term by eight. x over four times eight plus y over eight times eight equal to one times eight. You get two x plus y equal to eight which is what you have here (*teacher pointing at equation of the line*)
52 Learners	Aha!
53. Teacher	Ja! Now which one is easier for you?

The use of the verb 'give' in utterance 26 suggests a 'gift' which puts the teacher in the position of one able to bestow gifts. At the same time the students are positioned as people who should then be grateful for having been given such a gift. It also presents knowledge as something which can be 'handed over', that is transmitted, as opposed to knowledge that is co-constructed. The teacher's way of acknowledging the social context resulted in him connecting well known methods of finding equations of straight lines to two nationalities instead of referring to dual intercept method and gradient intercept method.

Instead of using these terms, his utterance shows an interesting tension between teaching the learners the discipline specific language of linear programming whilst also maintaining social relations with them. My interpretation of the social relations is that the teacher does this more to express solidarity with the immigrant learners' bicultural situation and also to present the two methods. The approach makes the social context significant by relating mathematical concepts to personal biography. The teacher's Discourse in this episode is related to content learnt as well as the social context and encouraged immigrant learners from the DRC to draw the mathematical content learnt in the DRC. Hence this Discourse supported immigrant learners from the DRC.

Episode 2. Reading Relevant Material in the Discourse

The teacher also quite deliberately used reading relevant material in the Discourse. Reading to the learners provided the opportunity to introduce immigrant learners from the DRC to new genres. Immigrant learners, as listeners, are engaged while developing background knowledge and increasing their comprehension skills. While reading, the teacher emphasized that he did not know how to pronounce some English words like *aquarium*, so he incorporated French as shown in the stanza below.

59. Teacher A school wants to take learners on an outing, a school wants to take learners on an outing to an aquar…, aquarium, eh! how do you pronounce it I don't know in English aquarium or aquarium? Because in French we pronounce it as aquarium [*teacher writes aquarium on the board and pronounce aquarium in French] i*n an outing to an aquarium full stop. At least fifty five learners must be transported…There are two types of minibuses available…Type A…can carry fourteen passengers … and type B can carry ten passengers …There are at most three type A buses …at most three type B buses … A maximum of five drivers are available, let x be the number of type A buses full stop, let x be the number of type A buses … y be the number of type B buses …

A question to consider is why is the teacher bothered with the word aquarium in the first place while reading? It is not relevant to the solving of the problem. There are a number of issues that can be raised from this episode. One of the issues might be that English is the most highly valued language in this multilingual classroom. This is not surprising because English is the language of teaching and learning. However, the spelling of aquarium is the same in both English and French; second language learners need support on how they can spot cognates across languages. Therefore, the teacher valued proper pronunciation of English words to enable learning of the language and improvement in their comprehension skills, which in the process would aid the immigrant learners' engagement with linear programming. In addition to that, the teacher might have been signalling that not everyone is equally competent in all languages and so was lending support to the immigrant learners. Furthermore, the teacher demonstrated solidarity as an additional language learner and also opened up space for immigrant learners to express their difficulties in crossing languages. Therefore reading relevant material in the Discourse supported immigrant learners from a country where French is the language of learning and teaching.

The analysis of the two episodes shows that the immigrant learners with a French background were supported during the teaching and learning. This shows that there was an awareness of language differences. In view of that, to engage in the differences between the immigrant learners' languages, the teacher incorporated this reality by introducing French pronunciation since the spelling of the word *aquarium* is the same in French and English. Choosing French pronunciation in an environment where the language of learning and teaching is English created an environment in which French was intended for those learners who had been learning through the medium of French. Then it seemed appropriate that the teacher decided to include French, particularly so that immigrant learners who have been learning mathematics through the medium of French are directly involved. Certainly, if there were no learners who had French as their language of learning and teaching before migrating to South Africa, he would not have switched to French. The switch related to the need for immigrant learners to fit in the new environment and fully integrate into their socio-cultural milieu.

Episode 3. English the Starting Point for Crossing Languages in the Teacher's Discourse

Learners have been solving linear programming tasks involving phrases like 'at least', 'at most', 'maximum' and 'minimum'. This lesson was selected because the teacher introduced French when explaining the meaning of some key words. The lesson is based on this task:

> A manufacturer of kitchen units makes two types of units, Ralto and Quatro, in a workshop which is available for only twenty days each month. Suppose he makes x units of Ralto and y units of Quatro each month. It takes two days to put one unit of Ralto together and three and a third days to put together one unit of Quatro. The paint shop can handle a maximum of eight units per month. At least two units of Ralto must be produced each month. Furthermore the number of units of Ralto must be at least a third of the number of quarto. Write down the constrain inequalities.

The analysis focuses on Discourses as the key medium for language practices and actions by teachers and learners pointing to the conditions immigrant learners as something that matters in the teaching and learning of linear programming. It will show empirically that immigrants are distinct to multilingual learners and the teacher supported immigrant learners in order to succeed when constructing inequalities.

The text below shows that the switch appears to be mostly from English to French and not often from French to English. Therefore it appears that English seems to be the procedural starting point for 'crossing languages'. In this excerpt, the teacher supported immigrant learners' activity by shaping the development of the mathematical meaning of 'available', 'at least' and 'maximum' in relation to the task by building on their linguistic skill while modelling the use of English.

79. Teacher	There are words that will indicate that this is a constraint…like availability, okay? Less than or equal to … Here they talk about availability, what does availability mean?
81. Teacher	Availability means I cannot go beyond this number, okay? (teacher pointing at 20 days)
83. Teacher	As I cannot go beyond this …it means less or equal to and here they say availability. Jah! availability …that is…this is a constrain when they say availability…Availability implies less or equal to [*et cela signifie moins de*], therefore will have the constrain here… The paint shop can handle the maximum of eight units per month … [*qui va donner le maximum*] Maximum, it's a constrain … It means I can't go beyond that one… (teacher pointing at eight) it means as I am producing x-units of Ralto and y-unit of Quatro, if I take $x + y$ those are the total unit that I will produce. It means when I will take them into the paint, the paint say I cannot take more than eight per month to produce, do you see that?
85. Teacher	And they say at least two unit of Ralto must be produce each month, [*ya au moins deux*] at least two units of Ralto. (*Alors… sens… hein plus grand que deux Ralto*). This is another constrain …they say at least two units, at least two units means what?
86. Learners	Greater than (*chorus*)
87. Teacher	Greater than or equal, at least two units of Ralto. … It means x must be at least two (teacher wrote $x \geq 2$ on the board), I cannot produce less than two units for Ralto (*Je ne peux produre moins pour que deux Ralto)*. …

The given tasks required the immigrant learner to deal with the real-world context of the manufacturer appropriately, that is, the immigrant learner must choose aspects of the task which are applicable in linear programming. This suggested that the immigrant learner was positioned as a learner who has an approach to the linear programming discourse. Therefore, such a learner should have an understanding of the situated meaning of 'available', 'at least', 'maximum', in the text. This would result in knowing the pattern associated with the constraints of time, number of units the paint shop can handle and being able to recognize the inequality to assign.

During the lesson, it gives the impression that language switching by the teacher becomes a resource to better understand the task. Therefore, the teacher introduced French to provide support to immigrant learners to develop their understanding in order to obtain correct inequalities for the given task. Obviously, the introduction of French is to support immigrant learners who have been learning through the medium of French. In the above excerpt, the teacher translated 'less than or equal to' for the symbolic meaning of availability in the context of the given task. In utterance 85 he also translated the phrase 'at least' to French. In the course of teaching and learning, it appears that the teacher established an environment that mirrors their own, which would enable a smoother transition from French to English. He provided extra-linguistic clues in a language familiar to immigrant learners and negotiated the common meaning of these words with them. The teacher's approach created space for the use of French in an environment where English is the language of learning and teaching and it served to make immigrant learners use their past experience during the linear programming lesson.

The teacher further supported immigrant learners' understanding by giving explanations like "I cannot go beyond this number" (utterance 83) in a certain way by using words they are familiar with in the discourse. This suggested that the teacher did not only focus on French translations but used several expressions in English as well. The teacher is positioned as someone who possesses resources of languages which he can use to support immigrant learners from a French speaking country in order to enable them understand the content. Along these lines, it can be argued that the teacher considered Discourses of immigrant learners as acceptable knowledge within the linear programming lesson at that time. The approach by the teacher may result in ways of thinking and interacting by immigrant learners that are re-valued and the Discourses (knowledge and language skills) were undergoing change with time.

Episode 4: French the Starting Point for Crossing Languages in the Learners Discourse

In episode 3, English was the starting point for switching when the issue tended to be about the teacher communicating (teaching content to learners), but that English failed to be the starting point for switching when the issue is more about the learner communicating his or her own understanding as shown in this episode. In other words, explaining understanding appears to be better enabled when learners use a language they are more familiar with, as shown in the following text:

85. Thuli Now I want to know which language do you use in mathematics?

86. John	[Je fais mon travail en Français pour ne pas faire des erreurs… J'utilise toujours le Français… en Anglais je ne sais pas certains mots. donc le Français je connais… j'ai fait les mathématiques en Français…] I do my work in French so that I do not make mistake… I always use French … English I do not know some of the words… so French I know…have done mathematics in French…
87. Thuli	When do you use English?
88. John	[Je n'utilise pas beaucoup l'Anglais en mathématiques… parfois j'échange… mais pour les mathématiques, j'utilise le Français…] I do not use much English in mathematics… sometimes I swap … but for mathematics I use French…
89. Thuli	Do you sometimes find yourself thinking in English and sometimes in French?
90. John	[Non… parce que… En mathématiques je pense en Français. sauf lors de la lecture du manuel … car ils sont en Anglais?] No… because … in mathematics I think in French… except when reading from the textbook… since they are English?]

John constructed himself as a learner who could communicate linear programming concepts in French as opposed to English. The episode showed that the experience of migrating from the DRC to South Africa led to an opening of opportunities for John to learn and use English at school. He was responding to the new circumstances which involved, drawing on and refashioning discourses associated with his cultural values while constructing his new identity within the private and public domain of his life. The language choices that John was making in this new environment included efforts to take up new languages at the heart of his response to these circumstances. However, John was still relying heavily on his French to facilitate and negotiate new relationships and construct new identities. These identities were contextualized, that is, were located in the support and opportunities that arose during the teacher's discourses when teaching linear programming.

DISCUSSION AND CONCLUSION

Discourses are social and historical ways of thinking, acting, interacting and talking like a mathematics teacher in the classroom with immigrant learners. The interactions in the data discussed in this chapter reflect a number of historical, interactions and talk that supported immigrant learners in their endeavour of learning linear programming. In each of the episodes there is evidence that the Discourse supports immigrant learners in the multilingual mathematics classroom in South Africa. Analysis of the social context, reading relevant material in the Discourse, switching languages enabled exploration of how immigrant learners were supported during teaching and learning of linear programming. The chapter has shown that the

Discourse of teaching and learning is complex and evolve with meaning emerging and shifting in ways that respond to the social context, at the same time construing the content and interaction.

The teaching and learning of linear programming occurred in the context of practices which are tied to communities and constituted by actions, meaning for utterances. One of the teacher's actions in the analysis is acknowledging the social context. The teacher did that by incorporating two countries, the DRC and South Africa, when teaching two methods of finding an equation of a straight line. The methods being the gradient intercept method (referred to as South African method) and the dual intercept method (referred to as Congolese method). So the teacher's practice is tied to school mathematics Discourse community and the goal was to enable success in the teaching and learning of linear programming. Furthermore, the teacher switched between two additional languages, French and English, when reading and explaining concepts during his teaching Discourse. In this regard, the switching by the teacher when explaining meaning of key concepts to immigrant learners from the DRC who understood French showed that he understood that they learnt better in an environment that promoted thinking in a language that they were comfortable with. Therefore, the teacher valued the languages immigrant learners brought into the linear programming Discourse and his goal was to create an environment that mirrored their own in order to promote success. The use of the language resources immigrant learners brought to the lesson enabled a smoother transition from a French environment to an English one and therefore they were more likely to succeed. This kind of support shows that words and utterances are tied to the social context and as a result immigrant learners were not excluded or seen as having language problems in the classroom Discourse. The teacher's resources turn out to be tools with which he provided support to immigrant learners from a French speaking country and hence connected content in both languages. The experiences of immigrant learners in learning linear programming in two additional languages created opportunities that hopefully will enable them to have greater opportunities to acquire the skill, knowledge and dispositions central to success. A relevant question to ask is how can the other immigrant and local learners who learn in a second language benefit in such a context?

NOTES

[1] During Apartheid-era South African migration policy promoted permanent white immigration and temporary black migration. The post-apartheid period is characterized by a mix of circular, permanent and transit migration (Landau & Kwabe-Segatti,2009).

[2] English, Afrikaans, IsiZulu, Setswana, Sepedi, IsiXhosa, Xitsonga and Tshivenda.

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SECTION IV

MATHEMATICS TEACHING AND TEACHER EDUCATION IN MULTILINGUAL CLASSROOMS

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12. OPERATIONALISING WENGER'S COMMUNITIES OF PRACTICE THEORY FOR USE IN MULTILINGUAL MATHEMATICS TEACHER EDUCATION CONTEXTS

INTRODUCTION

In this chapter, we draw substantially on Wenger's (1998) Communities of Practice (CoP) theory to develop and then propose a methodological approach for analysing pre-service mathematics teacher education multilingual classrooms. The approach emerged in Essien's (2013) study that investigated how pre-service mathematics teachers were being prepared to teach mathematics in multilingual contexts. Like many others in mathematics education, the theoretical frame for the study drew from a disciplinary domain in the social sciences to investigate the teaching and learning of mathematics. But why Wenger, and his theory of learning through participation in a community of practice, particularly given that Wenger's CoP theory was developed from studying informal learning settings?

The theoretical journey that led us to Wenger began in a situated frame to enable us to bring to the fore the multilingual context in which pre-service mathematics teacher education in South Africa occurs, and in which prospective teachers will teach (e.g., Brill, 2001). We soon realised, however, that cognition was central to this work. Our concerns, however, were more with teaching and learning practices in teacher education, and not teacher educator thinking. Given our interest in foregrounding multilingualism, and our orientation to this as a resource and not a problem (Adler, 2001), we went on to explore the potential of sociolinguistic theory (Eggins, 2004) for this study. This more discursive approach brought with it a detailed focus on classroom discourse, backgrounding the classroom community as we came to view it. It was through this process of engagement with a range of theoretical resources with potential to illuminate language practices in mathematics teacher education in a context of multilingualism, (coupled with pilot empirical work in teacher education institutions), that we came to appreciate multilingual mathematics teacher education classrooms as complex communities. Such classrooms have diverse participants, roles and motives, and so we returned to our initial orientation to learning and teaching as situated. Hence we drew instead on Wenger and his more explicit and stronger social situative/practice theory, together with others who have argued its salience for studying teacher learning.

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 173–193.

Clarke (2008, p. 30), for example, argues that since Communities of Practice (Wenger, 1998) theory is at once a theory of learning, of identity, of meaning, of community and a theory of practice, a CoP theory "offers considerable potential for thinking about a community of students whose common enterprise is to learn the practices of teaching". It became productive to start with this view of learning teaching as a social practice, as the major structuring frame for our study of mathematics teacher education in multilingual settings, and then to seek additional resources to develop our methodology in full.

As a start, we needed to embrace Graven and Lerman's (2003) argument that in order to use Wenger's theory of learning in formal education settings, much work needs to be done to translate his theory from workplace/informal settings to learning in more formal education contexts (such as pre-service teacher education classrooms) where teachers play a central role in promoting successful learning. This "work", and the integration of additional theoretical resources with CoP theory forms the substance of this chapter. Through it, we propose a methodological approach for analysing the nature of pre-service mathematics teacher education (TE) classrooms in multilingual settings broadly based on Wenger's (1998) CoP theory, and elaborated by a set of additional and pertinent theoretical resources.

As noted above, the methodology and framework we offer in this chapter emerged in Essien's (2013) study of pre-service teacher education classrooms. The study involved four pre-service classrooms at two universities in one of South Africa's nine provinces. Two of the teachers were from University A and the other two were from University B.[1] University A is frequented by pre-service teachers (PSTs) and teacher educators (TEs) for who English, the Language of Learning and Teaching (LoLT), is an additional language. University B is frequented by PSTs of different linguistic backgrounds taught by a good number of teacher educators whose first language is the language of teaching and learning. The study focused on the nature of CoP of these different pre-service teacher education classrooms. The findings from this study indicated that within the multiple layers of teacher education, there was an overarching emphasis given to the acquisition of mathematical content. The findings also revealed that the communicative approaches and patterns of discourse used by the teacher educators opened up different possibilities as far as preparing pre-service teachers for teaching in multilingual classrooms is concerned.

As noted, our focus in this Chapter is not the study and its results, but the enabling methodology that evolved. We use selected data excerpts from the study as we describe the various aspects of the methodology.

WHY WENGER'S (1998) COP THEORY?

In developing a methodological approach for understanding the nature of the pre-service teacher education multilingual classrooms, we started with Wenger's (1998) notion of community of practice. We conceptualised the pre-service multilingual classrooms as a non-homogeneous community where different members play different roles, have varying levels of knowledge, confidence and commitment. Fundamentally, it was where every member is in a learning position as far as the dynamics of the community is concerned. We avoided explaining communities of practice using the apprenticeship model of learning in the workplace, which deals with interaction between the newcomers and the more knowledgeable other (the experts), and how newcomers create a professional identity. Wenger (1998) rather describes a community of practice as an entity bounded by three interrelated dimensions – mutual engagement, joint enterprise and a shared repertoire. For Wenger, communities of practice are, as Aguilar and Krasny (2011, p. 219) note, "a place of learning where practice is developed and pursued, meaning and enterprise are negotiated among members, and membership roles are developed through various forms of engagement and participation." For Wenger (1998), therefore, a community of practice has three interdependent components/dimensions: Joint enterprise (what it is about), mutual engagement (how it functions) and shared repertoire (what capability is produced).

Wenger (1998) argues that in a community of practice, mutual engagement, a carefully understood enterprise, and a well-honed repertoire are all investments that make sense with respect to each other. This means that the three dimensions of learning are "interdependent and interlocked into a tight system" (p. 96) (see Figure 1). For Wenger, it is essential that the three dimensions of a community of practice are present to a substantial and meaningful degree.[2]

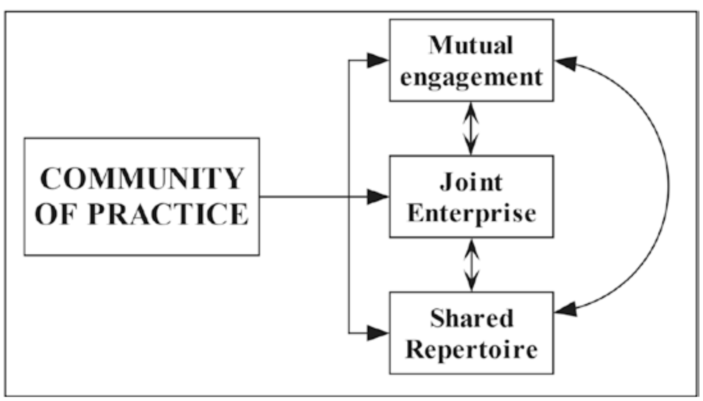


Figure 1. Dimensions of communities of practice

Practice, according to Wenger, does not exist in the abstract but resides in a community of people and the relations of mutual engagement by which they can do whatever they do. Hence, membership in a community of practice is a matter of mutual engagement (Wenger, 1998, p. 73). Mutual engagement can, thus, be defined as does Clarke (2008, p. 30) as "participation in an endeavour or practice whose meanings are negotiated among participants." A joint enterprise is the result of mutual engagement, and "refers to the focus of activity that links members of a community of practice" (Clarke, 2008, p. 31). Wenger explains that an enterprise is joint, not in the sense that everyone believes in the same thing or agrees with everything, but "in that it is communally negotiated." Wenger (1998, p. 83) defines a 'repertoire' as "a community's set of shared resources", thereby emphasising both the 'rehearsed character' and the 'availability for further engagement in practice' of a community's repertoire. Put differently, shared repertoire "refers to the common resources for creating meaning that result from engagement in joint enterprise" (Clarke, 2008, p. 31).

APPLYING AND EXTENDING WENGER'S COP THEORY TO PRE-SERVICE MULTILINGUAL TE MATHEMATICS CLASSROOMS

The process of recontextualising a 'non'-mathematics framework for analysing data in mathematics settings is not always a straightforward endeavour. So it was for us in using Wenger's (1998) Communities of Practice (CoP) theory. There were several challenges in developing a methodological approach for use in pre-service teacher education multilingual classrooms based on Wenger's theory. Firstly, Wenger is not a mathematician or a mathematics educationist and was not theorising specifically for the mathematics classroom. Wenger's theory, thus, has limitations in terms of providing tools for analysing the (nature of) mathematics pre-service teacher education communities of practice. Secondly, despite the importance accorded to shared repertoire and mutual engagement as dimensions of communities of practice, Wenger's CoP model lacks a coherent theory of language-in-use. Despite the emphasis on a jointly negotiated enterprise and on the negotiation of meaning, little insight is given into how meanings are made and interpreted (Creese, 2005). In the proposed methodological framework, this gap was addressed by using Mortimer and Scott's (2003) theoretical constructs of meaning making as a dialogic process (DP). For Mortimer and Scott (2003, emphasis in original), "meaning making can be seen to be a fundamental *dialogic* process, where different ideas are brought together and worked upon." They argue that the dialogic process makes a "shift in focus away from studies of students' alternative conceptions, and towards the ways meanings are developed through language in the … classroom" (p. 4). We contend that Mortimer and Scott's dialogic process is compatible with CoP theory by Wenger for two reasons: Firstly, just like CoP theory, DP acknowledges the centrality of purposeful discourse[3] between the teacher and the students in the classroom or learning environment as Mortimer and Scott (2003, p. 3, emphasis in original) argue that "talk is central to *meaning making* process and thus central to *learning*". Secondly (and related to the first), both theories are rooted in the premise that learning takes place in social situations where there is social exchange among members of a particular social configuration.

In general then, the challenge for us as researchers using Wenger's notion of CoP was to draw on CoP theory as a theoretical framework, and then using the teacher education community of practice classrooms that provided the empirical field for our study, to develop a methodological approach that would be relevant to (and provide tools for the analysis of) pre-service mathematics teacher education classroom contexts. In doing this, to deal with mathematical aspects of practices in the shared repertoire dimension of CoP, the works by several authors (McClain & Cobb, 2001; Sullivan, Zevenbergen, & Mousley, 2005; Tatsis & Koleza, 2008; Voigt, 1995; Yackel, 2000; Yackel & Cobb, 1996) were drawn upon. In drawing on these theoretical sources, we adapted and modified ideas to suit our purposes based on the data collected in pre-service teacher education classroom communities of practice. Limitations to Wenger's CoP were dealt with by introducing the work of Mortimer and Scott (Mortimer & Scott, 2003) into the mutual engagement process of CoP because of the ability of Mortimer and Scott's (2003) Dialogic Processes framework in charactering different kinds of discursive classroom interactions. In all of these, the three dimensions of communities of practice (as shown in Figure 1) and their associated processes as proposed by Wenger provided the backbone for the development of our methodological approach. Each dimension of CoP was subdivided into categories. The categories were then subdivided into sub-categories/ guiding questions with descriptors. While the dimensions and categories were developed *a priori* by using Wenger's CoP theory and other literature, much of the sub-categories and their descriptors were developed *a posteriori* from working with data obtained from the multilingual teacher education classrooms involved in Essien's (2013) study of pre-service teacher education classrooms.[4] In what follows, we elaborate on the characterisation of each of the dimensions of CoP.

CHARACTERISING THE SHARED REPERTOIRE

In characterising the shared repertoire of the different communities of practice in the study in which the present framework was developed, particular concepts/constructs within Wenger's notion of shared repertoire alongside categories emerging from data from pre-service teacher education classrooms were used. In so doing, three categories of analysis and their associated questions in each of the categories were identified: mathematical practices, norms of practice, and pool of shared language and shared representations that reflect and shape a joint understanding of the community's joint enterprise (see Figure 2). We also drew on the work that has been done in these three areas to characterise the shared repertoire of the different communities of practice. It is our contention that these three categories are representative of the common or shared resources (of a community such as the ones in our study) for the negotiation of meaning.

MATHEMATICAL PRACTICES

Our use of the term "mathematical practice" resonates with the way it is used by Godino, Batanero and Font (2007, p. 3) to refer to "any action or manifestation (linguistic or otherwise) carried out by somebody to solve mathematical problems, to communicate the solution to other people, so as to validate and generalize that solution to other contexts and problems." Given this definition of mathematical practices, practice for us is defined as taken-as-shared ways of doing and communicating mathematics which can be idiosyncratic of a person or shared within an institution (persons in the same problem situation). This definition of (mathematical) practices is consistent with Wenger's conception of practice and shared repertoire in that it acknowledges the fact that practices are shared (jointly owned by a community) and are common resources for the negotiation of meaning within communities.

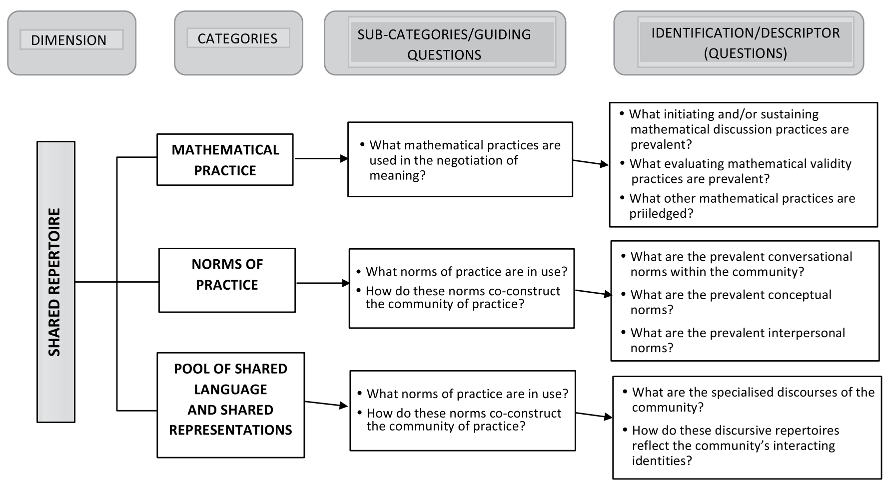
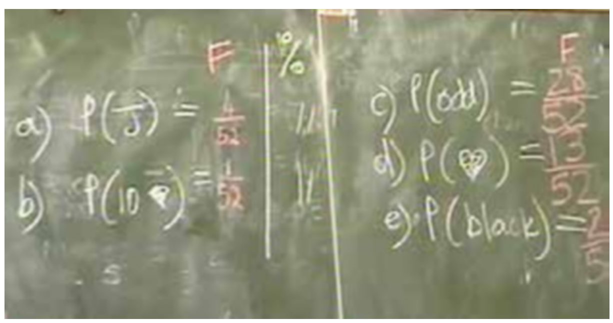


Figure 2. Categories for analysis of share repertoire and associated questions

In line with the above definition of mathematical practices, within the shared repertoire of the communities and under the category of 'mathematical practice', the analytical task as far as this category is concerned is to expound on the different practices that are in use in the negotiation of meaning in each community; and how these practices are made visible (or not) in the mathematics multilingual communities of pre-service teacher education classrooms. We use the excerpt below as a key record of classroom observation in which to illustrate some of the empirical features of the framework. In the excerpt below, the teacher educator called on a pre-service teacher to explain the reasoning in the solutions, (which were proffered by other pre-service teachers (PSTs)), after these PSTs had solved the questions on the board. The class was working on finding the probability of picking a jack, a diamond, and a club in a pack of 52 cards. The shared conversation developed as follows:

1 TE Right. Right. Ready. [looks at her watch] I'm sorry I'm pushing you. Shh. There is one more little thing I want to do, …um…, but Simon has offered to just volunteer. Now what's going to happen is he's going to go through the thinking – how these people were thinking, see if he agrees with the way they were thinking about the desired outcomes and about the, all the possibilities, OK. And then he's going to look at the fraction, he's going to look, imagine he's a teacher now that's marking this work. So what he wants to do is look at what's going on in the thinking behind these answers, OK. If … let Simon, let him go through all of these 5 first. If there's anything you disagree with we will go back to it. OK? Because one thing you must be clear on, I don't care what phase you are, if ever you are teaching Maths or you're doing a little private lesson at home, or you're helping your little sister, it makes no difference, you've always got to think how they're thinking before you can say 'You are wrong', 'You are right'. And even if they're wrong you want to see what they're thinking about. OK. But let him go through, um, starting with number 1. And I'm going to step aside for a minute and I want you to imagine that you are now looking at their thinking and… carry on. [The solution provided on the board were:]



2 PST1 Ja, so for the first one here the thinking is…

3 TE Well first of all go to the bracket, see what we want.

4 PST1 OK, [points to (a)] so in the bracket we have a Jack, so since we know that we have 52 cards all in all, so the Jacks that we have, we have 4 Jacks. So here this fraction tells us that we have 4 Js (Jacks) out of 52 cards. Right?

5 PSTs [Some students] Mmm

6 PST1 OK, let's go to the second one [points to (b)] Since… Since each card is having a dice, a heart, a spade and a…

15 PST1 Clubs. So how many 10s? The 10s which… OK, how many 10s? We have which…[laughs] Class: [laughs]

16 TE Simon, you're not teaching us. Just look at what's written there and how the person is thinking. Look at the answers.

17	PST1	OK. The person here was thinking that we have one Diamond 10, which is right because you have 4 10s – 1 is this, this, this and this. [points to ♠ ♣ ♦ ♥ which were drawn on one side of the board] So here the fraction is 1 over 52, and that is right.
18	PSTs	Yes
19	PST1	Let me go to the 3rd one after the 4th and the 5th.

A number of practices emerged. In the excerpt above, it can be argued that explanatory practices in the classroom community were intricately linked with providing justification and critiquing solution practices. Critiquing the solution was undertaken by both the teacher educator and the pre-service teacher. One of the ways in which the teacher educator encouraged the PSTs to critique solutions was to ask them to explain the thinking behind the solutions to classroom activities that have been produced by their fellow pre-service teachers as evident in turns 1 and 16.

In categorising the different practices that emerged, three major headings based on the nature of the practices and the purposes of the practices were used: 1) initiating and/or sustaining mathematical discussion practices; 2) evaluating mathematical validity practices; 3) General classroom practices. The first heading groups practices that enable what some authors have referred to as productive mathematical discussions in the class (e.g., Stein, Engle, Smith, & Hughes, 2008), and others as productive disciplinary engagement (e.g., Engle & Conant, 2002). The second heading clusters authorising practices, which deal with judgments about what is mathematically legitimate or not. Finally practices that neither belonged to the initiating mathematical discussion practices nor the evaluating mathematical validity practices were put into the third group. In coding the transcripts, where there were questions followed by an answer, the coding referred to both the question and the answer(s), provided that the answer(s) was/were direct response(s) to the question asked. For example, the question: "what do you mean by…" was coded as a call for an explanation (MP-EM). The response provided to this question formed part of the original MP-EM code. So, the question and the answer constituted one code rather than two codes of MP-EM each. Also, where a particular utterance which has already been coded (as writing mathematically (MP-WM) for example) was repeated[5] on the same task or sub-task, the utterance was not recoded as writing mathematically but as reiterating. But where there was a different emphasis on the same issue (for example, to a particular member of the community/group), then it was given the same code (in this case, MP-WM). In Appendix A, we present a selection of the practices that emerged from our study, the coding scheme and the code identification rule(s) (descriptors). The mathematical practices and descriptors presented in Appendix A are by no means exhaustive. They are intended to give indications as to how anyone who intends to use this methodological approach can categorise the emerging mathematical practices in his/her research study (see Essien, 2013 for full details). We now turn to the norms of practice category.

Norms of Practice

While mathematics practices deal with what discursive/pedagogic practices are made available in the community of practice and how this impacts on the community, the norms of practice are concerned with the rules of engagement that contribute to the stability of the mathematics discourse and the community of practice. Put differently, mathematical practices, it can be argued, are concerned with the dynamics of the learning process while the norms of interaction are concerned with the dynamics of the interaction process. Norms are regularities that guide social interactions. They are expectations/obligations (implicit or explicit) that community members have of one another (Yackel, Cobb, & Wood, 1991). Yackel et al. (1991) went on to argue that it is through the interlocking obligations in the mutual construction of classroom norms that make it possible for participants to act appropriately in specific situations giving rise to observable interaction patterns. Drawing from different works on norms in mathematics classrooms, two constructs pertaining to norms of practice in mathematical classrooms became pertinent for the present methodological framework: social norms, and sociomathematical norms (McClain & Cobb, 2001; Voigt, 1995; Yackel & Cobb, 1996). Each of these two norms were further sub-categorised into three norms:

- Conversational norms: Norms that guide interaction in the class and do not relate directly to the content of the mathematics at stake. Example: taking turns to speak norm; speak-out norm;
- Conceptual norms: Relates directly to the mathematical object under discussion: Example: Justification norm; mathematics justification norm; consensus norm; non-ambiguity norm;
- Interpersonal norms: This is related to conversational norms, but in this particular case, these are norms that guide the interpersonal relations in the class. Example: the avoidance of threat norm; one is expected not to ridicule the answer of another community member.

Appendix B provides a list of norms and their descriptors of what emerged. What was important in developing conjectures about the emergent norms of practice in the mathematics community was to look for instances, regularities and patterns in the way the pre-service teacher education classroom communities acted and interacted as they engaged with classroom mathematical activities. For example, prompts for rephrasing/reiteration would indicate the non-ambiguity norm, and words such as 'why' expressed through questions or the use of 'because' would indicate a justification norm.

For a norm to be considered to have occurred there needed to be some recurrence. Only one instance of, for example prompts for rephrasing, was not sufficient.

"Regularities" used in the definition of norms implies that there is some form of consistent reoccurrence of a particular 'instance of a norm'.

It is not the aim of this framework to delve into how norms are communally constituted. The main aim in delineating the norms of practice in this methodological approach is to make sense of how certain characteristics of the teacher education classroom CoPs and regularities in classroom activities are influenced by the social context of the community and how, in turn, they influence the dynamics of teaching and learning in multilingual pre-service teacher education classrooms.

Pool of Shared Language

The third category in shared repertoire is the pool of shared language and shared representation. A community's shared repertoire sometimes derives from the common knowledge base which is reminiscent of the common purpose of the existence of such a community and which are more often than not, unfamiliar to those outside of the community. The specialised discourse used in a community may indicate some form of reification or different mathematical practices. In analysing the pool of shared language and shared representations, the main questions that we bore in mind were: What are the common discursive repertoires or specialised discourses used in the community of practice? How do these common discursive repertoires co-construct the community and reflect the different mathematical practices of the community?

ANALYSING THE MUTUAL ENGAGEMENT OF COP

In analysing the mutual engagement dimension of the CoP, two categories were developed for use: pattern of discourse, and building of identities (see Figure 3).

Pattern of Discourse

As indicated earlier, the work of Mortimer and Scott (Mortimer & Scott, 2003; Scott, Mortimer, & Aguiar, 2006) was instrumental in developing the framework for analysing engagement in the community in general and of the pattern of discourse category in particular. Esmonde (2009) argues that in analysing mathematics classroom interactions, it is essential to focus not only on the content of mathematical talk, but also on the interactional context in which talk occurs. To this two, we would add that the nature of talk itself (that is, whether it is procedural, dialogic, authoritarian, etc.) is also crucial. To this end, while Wenger's theory provided the backbone for developing the mutual engagement dimension, the three aspects of classroom mathematics interaction provided the guiding principle. Hence, in the framework, while the content of talk is dealt with by asking the question *who makes substantive contribution*, the interactional context in which talk occurs is taken care of by analysing how participation is organised. Finally, the nature of talk was analysed through the communicative approach and patterns of discourse aspect of the framework.

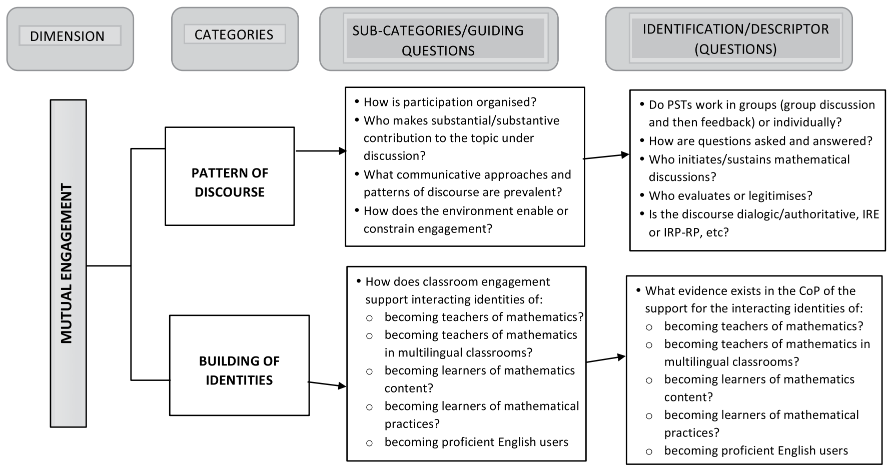


Figure 3. Categories of analysis for mutual engagement and associated questions

The term "pattern" in pattern of discourse is used in the broader sense that comprises how participation is organised, who makes substantive contributions, where authority stems from and what communicative approach is prevalent. By substantive contribution, we refer to subject-matter content talk/discourse that contributes to mathematical advancement in terms of knowledge and understanding of the mathematical content at hand, or in the teaching and learning of such content.

Building of Identities

Wenger (1998) notes that identity is in part a trajectory of where members of a community (as a collective and as individuals) have been, where they currently are, and where they are going. Examining this three-tiered trajectory of identity would entail following pre-service teachers as students, as student teachers and then as novice teachers. The methodological approach proposed in this chapter does not focus directly on this three-tiered trajectory since empirical data that informed its development was only collected during the time interval in which mathematics topics/concepts were addressed in class. The framework only focuses on the second part of Wenger's identity trajectory – *where members are currently*, while bearing in mind where they are going. As Hodges and Cady (2012) note, for Wenger, identity is in part how individuals come "to participate within a community in conjunction with how … individual[s] talk[ ] about and make[ ] sense of that participation." This means that access to where member are currently is possible through the observation of classroom practices in communities of practice. To this effect, under the mutual engagement dimension of CoP, the methodological approach made provision for the analysis of evidence present in the different CoPs in support of the interacting identities of: becoming a teacher of mathematics, becoming teachers of mathematics in multilingual classrooms, becoming learners of mathematics, becoming learners of mathematical practices and becoming proficient English users for the purpose of teaching/learning mathematics (see Essien, 2014 for full descriptors).

Examining the Joint Enterprise

The development of the joint enterprise dimension of CoP was informed by those dimensions of the community of practice that support the appropriation of mathematical knowledge and the associated processes of understanding and tuning the enterprise (Wenger, 1998). There is an overarching broad joint enterprise that brought members together in the first place. The way in which the pre-service teachers and the teacher educator (in the individual communities of practice) negotiated different aspects of the joint enterprise of teaching and learning to teach mathematics, and, therefore, how they tune this initial enterprise was analysed through: 1) the external conditions that constrain and/or enable a particular joint enterprise and how the community adapts or responds to these conditions; 2) how practices in use reflect what is valued by the community and can be perceived as the joint enterprise; 3) how responsibility is defined in the communities of practice. Figure 4 shows the categories and descriptors used in the analysis of the joint enterprise.

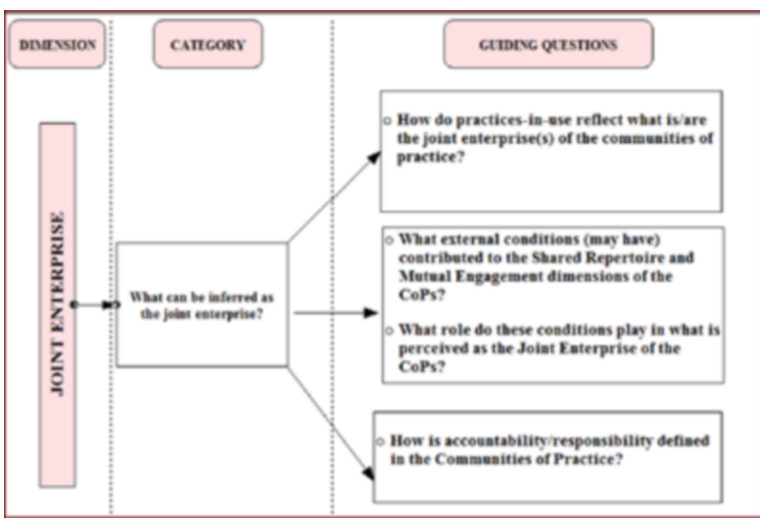


Figure 4. Categories of analysis for joint enterprise and associated questions

For Wenger (1998), mutual engagement is fundamentally defining of CoPs and such mutual engagement is directed towards a negotiated joint enterprise. In addition to this, the shared repertoire of a community as described by Wenger, "can be seen as the tangible expression of mutual engagement and the key means of carrying forth a joint enterprise" (Levinson & Brantmeier, 2006, p. 331). Hence, both mutual engagement and the shared repertoire dimensions serve as a window through which one gains entry into the communities' joint enterprise(s). Since the joint enterprise is anchored in mutual engagement and shared repertoire, what can be captured as a community's negotiated response to their specific conditions is captured through an encompassing gaze on the guiding questions related to mutual engagement and shared repertoire, and how together, all the categories and their descriptors provide a window with which to unlock the joint enterprise in mathematics pre-service teacher education multilingual classrooms. In analysing the joint enterprise of each of the pre-service teacher education community in our study, thus, the joint enterprise was taken as an outcome of the analysis of mutual engagement in the community's set of shared resources (shared repertoire) used in the negotiation of meaning.

In relation to the excerpt above, a number of features of this classroom are visible: first, in terms of the patterns of discourse, the class interaction was structured such that both pre-service teachers and teacher educators are able to explain. In turn 1, the pre-service teacher (PST1) was expected to gain an entry into how other PSTs reasoned when they solved the probability problem on the board. Hence substantive contributions were made by both the teacher educator and the pre-service teachers in this classroom. This was possible in this classroom community because of the interactive/authoritative communicative approach of the teacher educator. That said, it can be argued that the teacher educator positioned the PSTs as both becoming learners of mathematics content and as becoming teachers of mathematics. This latter positioning comes out forcefully in turns 1 and 16 in excerpt 1 above where the TE exhorts the pre-service teachers and PST1 in particular to act like a teacher. The excerpt, thus, gives an indication that for this classroom community, not only was the acquisition of mathematical knowledge an important enterprise, but also, the development of the identity of the pre-service teachers as future teachers of mathematics was a valued enterprise.

POSSIBILITIES AND LIMITS IN THE ELABORATED FRAMEWORK

In using the methodological approach described above to analyse our data, an issue that arose was the fact that the shared repertoire dimension of CoP and the mutual engagement dimension were difficult to analyse separately. For example, in working with the methodological approach, we came to realise that we could not analyse the data beyond mere description of the practices (and norms) present in the class if we analysed the shared repertoire dimension as an independent entity. For a deeper analysis, we needed to combine the analysis of the different categories within shared repertoire and mutual engagement at the micro level, and between shared repertoire and mutual engagement at a macro level. For example, it was not in the naming of the different practices present in the CoPs that we saw differences between the TE classroom communities, but in examining how these practices shape and are shaped by the norms of practice and the mutual engagement dimension of CoP. In one community, for example, explaining mathematically as a practice dealt more with explaining a procedure while in another community, it was more on clarifying a concept. In both cases, the discourse around the concept shaped the nature of the content and provided an indication as to what the pre-service teachers were enculturated into and how their identities were shaped. Thus, shared repertoire and mutual engagement dimensions analysed together provided a richer description of the classroom communities involved in our study, and *ipso facto*, enabled us to make inferences as to what the joint enterprise(s) of these communities is/are. Of particular significance, our framework foregrounded the heavy reliance of the negotiation of the joint enterprise on the dialogic processes (communicative approach and patterns of discourse used by the teacher educator) that are privileged in the community, thus confirming the importance of strong analytic tools for discourse patterns.

Our analysis of the shared repertoire and the mutual engagement dimensions of CoP enabled us to gain entry into/deduce what is/are the joint enterprise(s) in particular teacher education classroom communities that has/have been jointly negotiated (or which can be considered as their negotiated response to their specific conditions), and by so doing, the implications thereof for pre-service mathematics teacher education especially in multilingual settings. We share this methodological framework in the hope that other researchers are able to use the framework in similarly productive ways.

But even though the methodological approach is useful in thinking about teacher education communities of practice in terms of mutual engagement, shared repertoire and joint enterprise, the approach however, presents a number of limitations. First, it does not capture the effect of boundary practices (Wenger, 1998) of other communities of practice that the pre-service teachers and the teacher educators belong to and how they (boundary practices) impact on the classroom CoPs. Clarke (2008, p. 94) argues rightly that "in conceptualizing the student teachers' community of practice within the wider set of communities of practice that comprise the enterprise of education, the issue of boundaries [in which the students learn to teach through participation in the university and the school communities] must inevitably arise". With regards to this point, one general limitation of this study is that the researchers did not follow the pre-service teachers (PSTs) to their practical teaching and so, cannot analyse PSTs' boundary-crossing practices. Moreover, the methodological approach was not developed to capture and explore the extent of PSTs' enculturation into the practices that are privileged in the CoP or the extent to which the PSTs have formed each of the interacting identities.

Suffice it to say in conclusion that research conducted in mathematics multilingual classrooms has always been accused of: 1) being skewed towards analysis of language use and language practices, and 2) of being devoid of the content itself which engenders the talk. Through analysis of mathematical practices in use and substantive contributions that are made in the class, attention is paid to the mathematical object of the classroom discourse; through the joint enterprise which provides for engagement with external condition that influence the interactional context (and in making provision for engaging with classroom discourse), the framework attends to the issue of discourse in the multilingual contexts. It is thus our contention that the proposed framework provides an approach that examines the mathematics content, the interactional context and the discourses in multilingual pre-service teacher education multilingual classrooms in an integrated manner.

NOTES

1 For ethical reasons, we do not expound on the empirical context of these two universities beyond their linguistic demographics.

2 Elsewhere (see Essien, forthcoming), Essien has engaged with the issue as to whether or not the appellation of Communities of Practice can be used to describe pre-service teacher education classroom social configuration.

3 Taken in our study as language and other forms of communication that are in use within a community and define members of such a community (Monaghan, 2009).

4 Due to space limitations, only the abridged version of the framework is presented in this Chapter. A complete argument of the theory and more details of data collection and analyses can be found in Essien (2013).

5 For example if the teacher educator repeatedly shows the PSTs the correct way to write/represent a mathematical concept.

6 There is obviously a blurred boundary between conceptual norm and mathematical practices because they are both mathematical in a sense. But if the consensus norm, non-ambiguity norm, justification norm, etc are more normative (that is, taken as regularities that guided the classroom discourse), they can be talked about as norms.

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APPENDIX A

Mathematical practices in use and descriptors

Category: Mathematical practices	*Mathematical practices-in-use (sub-category)*	*Code*	*Code identification rule(s)*
Initiating and/ or sustaining mathematical discussion practices	Explaining mathematically	MP-EM	When 'what' is used in a question by a community member. Or when the intonation used by the TE or any community member indicates a call for further explanation. Also, the use of the phrase/sentence: • 'anything else', e.g., anything else you want to add to that? • 'no? why not?' • 'this is what I mean…' • 'what does it mean?' • 'Do you understand what you have to do?' COMMENTS: MP-EM could also be a call for someone to shed more light on what has been said. Eg, 'what do you mean by …' MP-EM need not necessarily start in the form of a question. It could also be the explanation of a particular concept or an explanation of another PST's reasoning or solution to a mathematics problem.
	Defining Mathematically	MP-DM	When there is a formal or informal definition of a mathematical concept by either the teacher or the PSTs

(*Continued*)

Category: Mathematical practices	*Mathematical practices-in-use (sub-category)*	*Code*	*Code identification rule(s)*
	Exemplifying (Providing examples)	MP-PE	When the PST/TE provides an example to demonstrate a mathematics method (e.g., example of an application of a mathematics procedure) and in concept development to indicate a mathematics relations (e.g., examples of a concept like triangle, etc) (Bills et al., 2006). It could also be when a community member demonstrates how something is done in mathematics, e.g., how to draw a frequency table Close to MP-EM. An explanation can be made through the provision of an example. Use of words like: "like…", "example". It can also be a call by a community member for someone to give examples.
Evaluating mathematical validity practices	Providing Justification	MP-PJ	Close to MP-EM and MP-PE. The "how" question indicates MP-EM while the "why" question would indicate MP-PJ. Instances where a PST/TE is asked to explain the procedures or steps leading to the solution of a maths problem would indicate MP-EM while a call to justify the procedure would be MP-PJ. For example: "who can tell me why the positive sign becomes negative when taken to the other side of the equation?" would be providing justification. The sentence: 'what is your evidence', could indicate either MP-EM or MP-PE or MP-PJ depending on the context of use.

Category: Mathematical practices	*Mathematical practices-in-use (sub-category)*	*Code*	*Code identification rule(s)*
	Critiquing solution	MP-CS	Involves critiquing the solution of a problem proffered by a community member. Different from MP-PJ and MP-CC. Here, a community member critiques his/her or other peoples' solution to a mathematical problem. In MP-CC, postulates are critiqued while MP-PJ involves justification for a conjecture or for the solution to any of the processes involved in the solution of a question. It can also be a call by any community member for other members to critically consider his/her solution to a mathematical problem or the processes involved in finding such solution. E.g., "what did you do wrong", "think carefully why you would make that decision"
Other mathematical practices	Proceduralising	MP-Pc	When the TE or the PST deals with the procedure/steps for solving a particular problem. For instance, if the TE or PST talks about taking a variable to the other side of the equal sign and changing the sign, that would be categorised as MP-Pc. But if a member of the community states why this procedure works, then it was categorised MP-PJ. Could also be a call for a particular procedure or aspects of the procedure to be used in solving a mathematical task: example: "Where do we start?" (which calls for the first thing that needs to be done by way of procedures) "What do we do next?"

(*Continued*)

APPENDIX B

Norms of practice and descriptors

Category: Norms of practices	*NP in use (sub-category)*	*Code*	*Code identification rule(s)*
Conversational Norms	Participation by all norm	[NP-PA]	The expectation that all member of the community participate in the classroom activity. This is evident, when for example • the teacher calls to find out if some less active students are following the lesson • the TE calls out specifically for members who have not given input in the discussion
	Speak-Out norm	[NP-SO]	The expectation that members of the community should speak loud enough for everyone to hear. Phrases like 'louder', 'speak up', etc. would indicate the speak-out norm.
Conceptual Norms[6]	Mathematically Sensible norm	[NP-MS]	The expectation that a community members solution or solution strategy makes sense to others or that a community member's explanation of a maths concept makes sense to others. Words like, 'does that make sense to you', anyone wants to challenge that' and 'do you agree' may depict such expectation
	Consensus norm	[NP-CS]	Group members are expected to reach an agreement on the solution to a maths question or explanation of a maths concept.
	Non-ambiguity norm	[NP-NA]	Expectation that mathematical expressions are clear and unambiguous, expressed through prompts for rephrasing. Example: T: What is the formula we use to calculate the distance between 2 points? S: we use the same formula [laughter] T: what is that the same formula? What is that the same formula? Yes sir.

Category: Norms of practices	*NP in use (sub-category)*	*Code*	*Code identification rule(s)*
	Justification Norm	[NP-JN]	The expectation that a community member has to justify her/his opinion(s). Expressed through words such as "because", "that is why", "would you explain why…?"
Interpersonal Norms	No Ridicule norm	[NP-NR]	The expectation that no member of the community may be derided if he/she makes a mathematically or grammatically incorrect statement.
	Collaboration norm	[NP-CB]	Relates to group work. The expectation that all members of the group must work together to solve a mathematical problem

LYN WEBB AND PAUL WEBB

13. DEVELOPING MATHEMATICAL REASONING IN ENGLISH SECOND-LANGUAGE CLASSROOMS BASED ON DIALOGIC PRACTICES

A Case Study

INTRODUCTION

International mobility has often resulted in minority immigrant pupils in many countries being immersed in a foreign language (Moschkovich, 2007). In such cases the immigrants' main language is used only in their homes or constrained social environments, and additional transition from their main language to the official language of the country is facilitated by hearing and seeing the language around them – and the very real necessity of understanding and being understood. In South Africa the situation is more complicated as it is the majority of pupils who are immersed in a foreign language in classrooms where English is chosen as the official Language of Learning and Teaching (LoLT). The reason for this situation is that, as in many ex-colonial Anglophone countries, political and historical imperatives have elevated English to a language of esteem and value (Setati, 2008). Although official policy allows schools to choose their own LoLT, the majority of schools choose the 'English only' route – despite English being the second (and sometimes the third or fourth) language of both the pupils and the teachers (Heugh, 2006).

In rural South Africa, and many township situations, English is almost only heard in schools and pupils have little opportunity to develop and practice communication skills in this language. In such situations teachers are faced with the dual responsibility of teaching mathematics as a gateway subject to tertiary education and select occupations while, at the same time, teaching the English, which is seen as imperative for upward mobility (Setati, 2008). In order to promote mathematical understanding, teachers often revert to explaining mathematical concepts in the pupils' main language or by code switching in a teacher-centred manner (Webb & Webb, 2008). However, as all assessment and mathematics textbooks are published in English, this approach places pupils in the potentially difficult position of having to bridge the gap between grappling for mathematical understanding in their main language and rigorous mathematical language in English in the texts and tests.

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 195–209.

The complex situation described above motivated this study, which attempts to identify principles that:

- Provide an effective starting point for second language pupils to journey from informal talk in their main language towards formal mathematical talk in English, and;
- Help identify and exemplify teaching strategies based on these principles that have been shown to improve mathematical reasoning

While the data generated in this study are localised in a South African context, the issue of the hegemony of colonial languages in second-language teaching and learning settings is a universal phenomenon (Setati, 2005). Our findings should therefore contribute not only to the scholarly debate in English dominated bilingual contexts, but to the problem of second language teaching and learning in bilingual and multilingual contexts in general.

The Use of Dialogue

Vygotsky (1978) maintains that learning is facilitated by dialogue and that the teacher plays a vital role in creating and maintaining this dialogue. Game and Metcalfe (2009, p. 45) note that dialogue is often simplistically defined as an interaction between two parties holding designated positions and that it "arises from interaction, competition, opposition and the reconciliation of positions". They posit however, that the prefix ('dia-') of dialogue means 'through' in the sense that dialogue passes through the participants and vice versa, and allows participants "to have thoughts that they could not have had on their own, yet to recognise these thoughts as developments of their own thinking".

Gorsky, Caspi, and Trumper (2006), who posit a theory of instruction that is based on dialogue, highlight two models, namely intrapersonal and interpersonal dialogue. Intrapersonal dialogue mediates individual learning and refers to the interaction between the 'I' and the subject matter that is being learned, in our case mathematics. More pertinently to this study, research has revealed that, in the school context, interpersonal dialogue between peers and teachers in the class produce notable results in pupil achievement, particularly when such dialogue is framed as 'exploratory talk' (Mercer & Littleton, 2007).

> Exploratory talk is talk in which partners engage critically but constructively with each other's ideas. Statements and suggestions are offered for joint consideration. These may be challenged and counter challenged, but challenges are justified and alternative hypotheses are offered. Partners all actively participate and opinions are sought and considered before decisions are jointly made. In exploratory talk, knowledge is made publicly accountable and reasoning is visible in the talk. (Mercer & Littleton, 2007, p. 59)

These authors structured guidelines for ground rules that teachers should negotiate in their classes for the development of dialogue in groups. For example, pupils should share relevant ideas and help each other to understand the problems set; they should listen to each other's contributions and respect their ideas, even if they disagree; pupils can challenge and counter-challenge arguments, but they should give reasons and substantiate their challenges with sentences such as, 'I think… because…' and, if possible, the groups should work towards an equitable consensus.

While much of the research on exploratory talk was carried out in English first-language settings in the United Kingdom, Rojas-Drummond and Mercer (2003) studied interactions in Mexican classrooms and found that teachers whose pupils achieved the best results in their study demonstrated the following characteristics:

- They used question-and-answer sequences not just to test knowledge but also to guide the development of understanding and to discover the pupils' initial levels of understanding. They used 'why' questions to get pupils to reason and reflect about what they were doing;
- They taught not just 'subject content', but also procedures for solving problems and making sense of experience. This included demonstrating the use of problem-solving strategies for children, explaining the meaning and purpose of classroom activities and using their interactions with children as opportunities for encouraging them to make their own thought processes explicit;
- They treated learning as a social, communicative process and used questions to encourage pupils to give reasons for their views, organizing interchanges of ideas and mutual support and generally encouraging pupils to take a more active, vocal role in classroom events.

Rojas-Drummond and Mercer (2003) incorporated these characteristics into a classroom observation checklist that they devised to document teachers' practices during their study.

Webb and Treagust (2006) tested the effects of exploratory talk in second language teaching and learning situations in a science context in South Africa where they encouraged the participating science teachers to allow their pupils to conduct their discussions in the language in which they felt most comfortable (isiXhosa or English). Their study revealed statistically significant improvements in the groups who received the treatment (exploratory talk in the language of preference) in terms of improved reasoning skills (based on Raven's Standard Progressive Matrices – as was the measure used in the UK and Mexican studies described above) over the comparison group, and that code-switching was the language use preference during exploratory talk sessions. Studies by Webb (2010), and Webb and Webb (2013), using an English/isiXhosa mathematics context concurred with these results.

Questions therefore, can be used to model useful ways of using language that children can appropriate for themselves in peer group discussions and provide opportunities for them to make more extensive contributions in which they express their current state of understanding, articulate ideas and reveal problems they are encountering (Mercer & Littleton, 2007, p. 36). One specific example is Socratic questioning. This approach focuses on a central issue and each question is answered with a question in order to tease out the reasoning behind it (Paul & Elder, 2008). This process continues with answers probed with questions until students can arrive at their own conclusions via discussion (Frick, Albertyn & Rutgers, 2010). It was this type of questioning which formed a critical aspect of the study outlined below. But there are other important aspects of language use that are also relevant before detailing the study.

From Informal Talk to Mathematical Talk and Reasoning

Setati (2005, p. 78) describes communicating mathematically as (i) managing the interaction between ordinary English (OE) and mathematical English (ME), (ii) facilitating formal and informal mathematics language and, (iii) distinguishing between procedural and conceptual discourses. Monaghan (1999), who believes that teachers are responsible for the development of both OE and ME, notes that there are three types of vocabulary in mathematics – general (e.g., chair, water); technical (e.g., trigonometry, rhombus) and specialist (e.g., point, similar). He warns of the pitfalls inherent in thinking that pupils understand the meanings of specialist words in ME and points out that pupils often bring their OE meanings into the mathematics context. For example, if a pupil is asked, "What is the difference between 12 and 7?" in an ME context the answer is 5; however, in an OE context the pupil could legitimately answer that one number has two digits and the other one digit; or that one is an even number and the other odd (Monaghan, 1999, p. 8).

Pimm (1991) highlights a difficulty that confronts all teachers, which is how to encourage movement in their pupils from the predominantly informal spoken language in which they are fluent, to the formal written language that is frequently perceived to be the landmark of mathematical activity. Three routes are suggested:

- from informal spoken language to formal written language;
- from informal spoken language through more formal spoken language to formal written langauage;
- from informal spoken language through informal written language language to formal written language (Pimm, 1991, p. 21).

The diagram in Figure 1, adapted from Pimm (1991) and Setati and Adler (2000), illustrates a possible progression in spoken mathematics by second language learners.

All of the issues mentioned above form the framework of this investigation which attempts to identify underpinning principles for an effective starting point for second language pupils to journey from informal talk in their main language towards formal

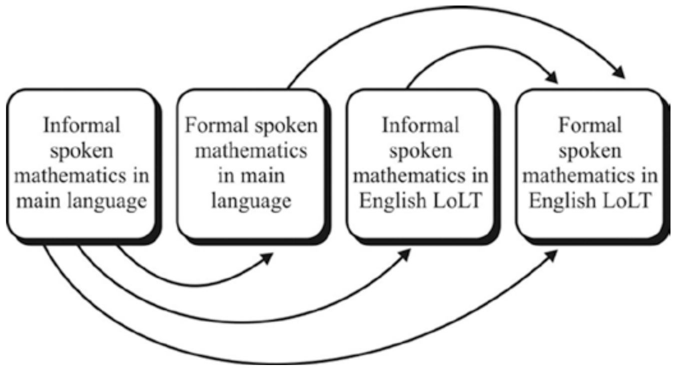

Figure 1. Progression from informal to formal spoken mathematics (Webb, 2010)

mathematical talk in English, and to help identify and exemplify teaching strategies that improve mathematical reasoning.

RESEARCH DESIGN AND METHODOLOGY

The study's design was framed in the general notion of dialogue, which includes issues of exploratory talk, questioning, disciplinary and Socratic dialogue, bilingualism and code-switching. The research was undertaken in three parts. Firstly, a purposive convenience sample of six teachers in three schools was chosen. Three of these teachers (one in each school) were introduced to the notion of introducing exploratory talk in their classrooms and the legitimate use of code-switching in their classes (where the official language of teaching and learning is English) and formed the experimental group. The other three teachers were not included in the intervention and formed a comparison group. All of the pupils (experimental and comparison groups) were tested to ascertain their baseline numeracy and reasoning skills. Secondly, the experimental group of teachers were expected to introduce the strategy in their classrooms and were observed in order to determine their level of ability to foster this type of talk amongst their pupils. Thirdly, the pupils' numeracy and reasoning skills were retested after the intervention.

Numeracy and reasoning skills were pre-identified as the constructs that we wished to track as they were considered appropriate for testing skills that were both directly related to (i) the content (numeracy) that the teachers were expected to teach and (ii) the generic reasoning skills not directly related to the activities as recorded in previous studies on exploratory talk (Rojas-Drummond & Mercer, 2003; Webb & Treagust, 2006; Webb, 2010). The teachers' abilities to introduce exploratory talk and effect changes in their pupils' numeracy and reasoning were observed and compared. One teacher was identified as a successful exemplar. This teacher's practice was analysed in order to highlight teaching strategies based on dialogic practices that appear to contribute to improved numeracy and reasoning acquisition in second-language classrooms.

Sample and Setting

The three schools (called North Primary School, South Primary School and West Primary School for the sake of their anonymity) that participated in this study were situated in a predominantly isiXhosa-speaking Black community in an urban area in South Africa. They were selected as a purposive sample which would suitably reflect, to an appropriate degree, the reality in many South African schools in terms of school and class size, resources, socio-economic standing and, most importantly, the use of English as a second language of teaching and learning. While the schools were purposively sampled, there was also a degree of convenience sampling involved as they were relatively easily accessible and had had good previous relationships with the researchers and welcomed them into the schools as observers.

As noted earlier, six grade-seven mathematics teachers in three schools (two teachers from each) volunteered to be part of the study. Three teachers who expressed interest formed the experimental group (one teacher in each school), while the other three (again, one teacher per school) agreed to act as a comparison group. Only the experimental group teachers were introduced to and participated in exploratory talk activities. The comparison group continued to teach in their normal manner. All of the teachers (both experimental and comparison groups) were isiXhosa first language speakers, as were their pupils.

Intervention

Initial workshops with the experimental group of teachers focused on discussion of the difficulties that the teachers experienced when teaching mathematics to pupils whose main language was not English. Classroom discussion and dialogue, particularly exploratory talk, were introduced and activities that could be introduced in class to ameliorate the difficulties the teachers had noted were explored collectively. Throughout the workshops we continually revisited the questions: Is it important for the pupils to use dialogue in their main languages? What form should classroom dialogue take? How can dialogue be sustained in a bilingual mathematical environment?

Thereafter the experimental group of teachers were trained in the use of reasoning cartoons and supplied with cartoons appropriate to the topics being taught during the duration of the intervention. These cartoons were informed by the concept cartoons, which were first conceptualized by Keogh and Naylor (2000) in a science context in order to develop skills of argumentation and discussion in pupils. The cartoons represent visual situations in familiar contexts and make explicit a variety of viewpoints on a topic, one correct and others presenting typical misconceptions and alternative answers.

The cartoons used in this study were specifically designed for the project and focused on topics that were pre-determined in the grade-seven syllabus (see Figure 2). The teachers were required to firstly use an example of a cartoon to introduce the pupils to, and provide opportunities to practice, the ground rules of exploratory talk. Thereafter, they were provided with a curriculum specific set of cartoons to stimulate use of exploratory talk in their classes. In conjunction with the use of cartoons to promote exploratory talk, the teachers were expected to implement the strategies to which they had been introduced in terms of questioning, disciplinary language and Socratic dialogue.

Figure 2. Example of a mathematical reasoning cartoon used in this study

Data Generation

The teachers' lessons were videotaped on average once every three to four weeks during the nine-month study. Their lessons were transcribed and coded deductively using an observation schedule based on the expectations of exemplary teaching that takes into account the issues relating to exploratory talk, questioning, disciplinary and Socratic dialogue. Qualitative observations were also recorded on the observation schedule. After each observed lesson a researcher and the teacher, who had been allocated free time from the school principal for this purpose, reflected on the lesson in terms of the expectations raised by the intervention. The strategy for the next lesson was planned jointly and the teacher was reminded that it was legitimate to encourage their pupils to engage in exploratory talk in the language of their choice.

As noted earlier, the children's numeracy skills and reasoning skills were tested pre- and post-intervention using the same tests. The well validated Raven's Standard Progressive Matrices (RSPM) test was used to test for reasoning skills as it is readily available and has been used in similar previous studies (Mercer, Wegerif, & Dawes, 1999; Webb & Treagust, 2006; Webb, 2010). The RSPM test is purported to be language- and culture-free (Raven, Raven, & Court, 2004). The numeracy test used, the Admissions and Placement Assessment Programme (APAP test), is a standardized, criterion-referenced test with proven reliability and validity (Foxcroft, Watson, Seymour, Davies, & McSorley, 2002; Watson, 2004).

Data Analysis

The quantitative data generated via the APAP and RSPM tests were treated statistically using analysis of variance (ANOVA) techniques. Testing for statistically significant differences in changes in test scores between pre- and post-tests was applied not only to the experimental and comparison groups within each school, but also between the experimental and comparison groups across schools. Cohen's d was applied to determine whether the statistically significant data were also practically significant (effect size).

Qualitative data were gathered through classroom observations focusing on language use, exploratory talk, questioning, and disciplinary dialogue. A ten-criterion, four-point Likert scale checklist was used to provide quantitative data on teacher practice and qualitative observations were recorded on this observation instrument under heading prompts which invited comments on the use of language, exploratory talk, questioning techniques, etc. The data generated for each teacher could then be compared to provide an ordinal and qualitative comparison of how well they applied what was expected of them as presented during the intervention workshops.

Trustworthiness and Reliability

The target (experimental) teachers were visited regularly (see data generation) by the researchers over a period of nine months and their lessons were observed and videotaped. An attempt was made to maximise the trustworthiness of the qualitative data by both researchers transcribing the classroom observations separately, after which the transcriptions were viewed and validated separately, and discussed to achieve consensus when differences in opinion emerged.

Similarly, two fellow experienced researchers ensured that the pre- and post-intervention quantitative tests were administered in exactly the same way in each class. The numeracy test is a standardized, criterion-referenced test that measures numeracy skills with proven reliability and validity, as is the RSPM test. In both cases, identical post-tests were administered to the same pupils nine months later in the same calendar year. Cronbach α scores were used to determine the reliability of the test scores.

RESULTS

Statistical analysis of variance (ANOVA) revealed that both experimental (n=113) and comparison groups (n=111) mean scores (Δx) had improved statistically significantly ($p \leq 0.05$) in both the numeracy and RSPM tests. These findings may be attributed to maturation over the nine month period, the fact that the pre- and post-tests were identical and, in the case of the numeracy test, there had been teaching of the topic to both groups during the period under review. However, when comparing the mean score changes and the practical significance of these data it became evident that the gains were notably higher for the experimental group. The overall practical significance (effect size) for the experimental group's RSPM change in score was high (d=1.24) as opposed to the moderate practical significance in the comparison group (d=0.63) at a reliability level of $\alpha = 0.93$. In the numeracy skills tests there was a moderate practical statistical difference in both the target group (d=0.73) and the comparison group (d=0.58) at a reliability level of $\alpha = 0.62$.

The changes in mean scores (Δx) and the effect sizes (d score) of the experimental schools in the numeracy (APAP) and the reasoning (RSPM) tests are shown in Table 1.

Table 1. Comparison of the change in the experimental groups' pre-post-test mean scores (Δx) and effect size (d score) for numeracy and reasoning

School	*Numeracy*		*Reasoning*	
	Δx	*Effect size (d)*	*Δx*	*Effect size (d)*
North	19.92	0.82 (large)	9.95	1.8 (large)
West	5.53	0.74 (moderate)	6.52	1.3 (large)
South	0.69	0.4 (small)	2.73	0.4 (small)

Cohens d effect size of practical significance indicates a small practical significance when $0.2 < d < 0.5$; a moderate practical significance when $0.5 < d < 0.8$; and a large practical significance when $d > 0.8$. Effect sizes are only calculated when statistically significant differences are found.

The statistical analyses reveal discrepancies between the schools in terms of success, not only between the experimental and comparison groups, but also among the three experimental schools, with the North School providing the most notable improvements. Analysis of the quantitative and qualitative data generated by the teacher observations revealed that Mr X of North School had numerically scored the highest in terms of his pupils' results on the tests and had provided a number of exemplary activities in his classroom. As such, it appeared that we had identified a teacher who had not only effectively implemented the desired strategy, but who had achieved the most in terms of improved pupil performance in terms of numeracy and reasoning.

There are many dangers associated with attributing causality to correlation, but the apparent relationship between Mr X's improved ability to facilitate dialogic practice and his pupils' improvements in mathematical reasoning appear to hint at such a possibility. While we recognise that such a claim would be much more robust had we made detailed comparisons of the practices of the teachers who performed well and those who did not, we still believe that the evidence points to enough of a link between Mr X abilities and his pupils' achievement to warrant closer scrutiny of his activities as exemplar practices.

Exemplars of Practice

Analysis revealed firstly that the strategies Mr X had developed resonated with Rojas-Drummond and Mercer's (2003) findings that the best results are obtained when teachers used question and answer sequences to assess knowledge and understanding, they encouraged their pupils to give reasons for their views, and learning was treated as a social, communicative process. The overarching impression at the end of our study was that Mr X's lessons took the form of a dialogue between himself and his pupils. He modelled exploratory talk throughout the lesson by emphasising that, "It doesn't matter if you are wrong. What matters is that you are able to say what you think – and why you think that way". The pupils thus responded by interacting with the teacher and each other in a supportive, encouraging manner.

Mr X used questioning to encourage the development of dialogue so that the pupils could reveal their thought processes and reasoning, "It is their opinion. What do other people think? Do you agree with that or don't you agree with that? Why? What do you think about what she has just said?" As the pupils shared their ideas about the cartoon statements he continued to draw them out and encouraged them to find reasons for their answers:

> What have they actually done to the fraction?
> Do we want to do this?
> So what do you think of that?
> I can see the idea. Why?

Even when the pupils expressed incorrect logic Mr X was encouraging and gently led the class to the right answer:

> Let us get another answer. It is not about being wrong but about asking what they mean by their ideas'.

English was used extensively to give instructions to the whole class and the groups and to provide explanations, but the pupils were free to use isiXhosa to their peers in their groups. The pupils were attentive and interested and readily shared their ideas. The teacher not only scaffolded the terminology and the language that could be useful to the pupils, but also scaffolded their critical thinking and problem solving skills through his questioning techniques. Mr X repeatedly modelled the language and processes that pupils should engage in during problem solving. He clarified the strategy, language skills, mathematical knowledge and critical thinking skills that he required them to use. Throughout the lesson Mr X modelled the language he expected his pupils to use. He repeated what pupils said and re-voiced their statements:

> Mr X: Divide into groups? A very important word? Those groups should be? Yes, *buthi*?
> Pupil: Equal.
> Mr X: All those groups should be equal.

Mr X pointed to more than one way of working out the problems by putting the onus on the pupils to discover alternative solutions:

> Is there another way of working it out?
> Do you see the difference between?
> Can you do it in the quickest and easiest way?

While discussing the properties of the shapes in their groups the pupils employed code-switching. However, their gestures were more eloquent than their halting words. Mr X sensed this and had given them manipulatives to use which encouraged them to make gestures. Such physical movement gave the pupils a means to express their thoughts and to solve the problem set, even if they did not yet have the mathematical vocabulary to do so verbally in either language.

Mr X mediated the language and vocabulary he wanted them to use in their groups and when they reported back in a plenary. For example:

> I am going to give you some vocabulary which you must use. (He holds up words written large on paper)
>
> Because I can hear you say *'la macala athe nca.'* I would like you to use now the correct vocabulary. So I will be distributing the vocabulary in your groups. But continue with the discussion.

He gave each group its own list of relevant disciplinary vocabulary so they could take ownership of their new knowledge and have the necessary mathematical

English words to refer to and use. Mr X introduced mathematics terminology by using everyday examples before using the same terminology in a mathematical context. When he introduced new vocabulary he wrote it on the board and asked the pupils to read it aloud. He thus combined aural and visual recognition. The children were encouraged to be independent and were able to read and follow instructions, with him mediating only when they requested assistance. He moved from group to group checking understanding and clarifying where necessary. He engaged with the pupils in their groups and listened intently to their questions and suggestions and gave them feedback by means of Socratic questioning, or by reinforcing their ideas with positive comments.

The pupils worked in groups and their body language mirrored their involvement. They reached over the desks to point at each other's books and looked into the interlocutor's eyes while she or he was talking. They read the problem together in chorus, seeing the English mathematical terms in written form as well hearing them aurally. At the same time they practiced the pronunciation of mathematical vocabulary, but broke into isiXhosa at times when engaging with the problem. When the talk appeared to be disputational Mr X intervened and encouraged his pupils to use exploratory talk, give reasons for their views, and express their ideas confidently. He was sufficiently confident in his own knowledge and experience to guide the class using the strategies that created opportunities for the pupils to engage in exploratory talk.

Mr X also encouraged his pupils by responding positively to their suggestions with comments such as: "We are doing very well. I think that was a bright idea." He used 'we' and 'us' to express solidarity and to build their confidence. The pupils were allowed to move around the class and often one pupil would spontaneously move to another group to find out how they were solving the problem. This freedom of movement epitomized the relaxed but focused atmosphere in the class, which helped create a classroom climate conducive to the practice of exploratory talk by encouraging the pupils to make explicit their thoughts, reasons and knowledge, and to share them. Mr X had a good grasp of the demands of the curriculum and the content knowledge and language strategies that facilitate learning. His lessons were not presented in isolation, but formed part of a planned continuum aimed at teaching a concept and ensuring understanding through practice. The activities he developed drew on previous mathematics knowledge and language acquired, enabling them to engage in directed, meaningful exploratory talk.

> What I want you to do is to read the problem first, discuss what it is about. OK? And think of ways that you can use to solve the problem. OK? And then solve the problem together in a group. Then the second problem – you are going to first discuss the problem. Understand the problem and then solve the problem. I will be coming around the groups to listen to you, not to judge in any way. OK? If you need to talk in isiXhosa, talk in isiXhosa.

The pupils were keen to answer questions; they did not seem afraid of making errors; and they did not wait passively for Mr X to give the answer. It was quite clear, from the way they smiled and leant towards each other and engaged with the problems, that they enjoyed the activities. When some groups were quick to complete an activity, Mr X praised them by clapping his hands and saying “Well done, well done!” The pupils were visibly pleased with themselves and smiled and used positive gestures and body language

DISCUSSION AND CONCLUSION

From the observations and transcripts certain aspects of teaching practice emerged that might account for the marked difference in the statistical evidence of pupils’ progress in Mr X’s class as compared to the other classes. These included judicious questioning and the informal development of interpersonal dialogue in the pupils’ main language in group discussions, which helped them move towards mathematical English in whole class discussion. These findings concur with Mercer and Littleton’s (2007) view that pupils achieve when the teacher uses question and answer techniques that guide the development of understanding; when teachers “guide pupils in problem solving techniques and sense making during a social communicative process” (Mercer & Littleton, 2007, p. 40).

Mr X also demonstrated that both intrapersonal and interpersonal dialogue could be developed to enable the pupils to rely on their own reasoning before resorting to asking the teacher who they knew would most probably reflect the question back to them (Gorsky et al., 2006). He used open questions where there was often no definitive correct answer and encouraged his pupils to engage in critical reasoning (Frick et al., 2010). He also continually connected ordinary English (OE) and mathematical English (ME) by introducing new mathematical terms through sight, sound and familiar contexts before using the terms in problems (Setati, 2005).

He encouraged his pupils to speak in their main language in groups; he re-voiced their ideas in English and demonstrated the use of problem solving techniques. He taught language skills when he gave the pupils the vocabulary necessary for the mathematics they were doing, both orally and in writing on the board or on textual hand-outs. He also reinforced sentence structure and terminology in an unobtrusive way. He was, thus, giving them the tools to communicate in mathematical English, not just speaking mathematics to them in English. Perhaps the most appealing aspect of visiting Mr X’s classes was the warm buzz of conversation that pervaded the classroom atmosphere which reflected the pupils’ belief that mistakes were opportunities to develop mathematical reasoning.

The statistically significant improvements in achievement by Mr X’s pupils suggest that this study helped to identify and exemplify teaching strategies that could improve mathematical reasoning. The approaches discussed could be an effective starting point for second language learners on their journey from informal talk in their own language towards formal mathematical talk in English. The approaches may also be pertinent to current debates in teachers and curriculum development circles in terms of using question-and-answer sequences to guide pupils to understanding; teaching problem solving strategies that expose the meaning and purpose of the problems; and using dialogic strategies that transform teaching and learning mathematics in second language contexts into a social, communicative process.

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14. MATHEMATICS TEACHER'S LANGUAGE PRACTICES IN A GRADE 4 MULTILINGUAL CLASS

INTRODUCTION

Multilingualism is rapidly becoming a serious challenge for many schools in South Africa, most noticeably in the Gauteng province. Not only do many schools have learners with a variety of South African indigenous languages as their home language, but numerous schools also have learners from other African countries. As a result of this language complexity many schools have opted for English as the language of teaching and learning despite that fact that many teachers and learners are not fluent in English (Gauteng Department of Education, 2011). As such, the teaching context in such schools is highly complex, particularly with regards multilingualism.

When learners learn mathematics in a language in which they are not fluent, and are also taught by a teacher who is a second language English speaker, promotion of conceptual understanding becomes a challenge. During the course of teaching and learning mathematics numerous language issues emerge including the language of the teacher as well as the language of the learners. These raise the following critical question: What impact does the teacher's home language have on the promotion of learners' conceptual understanding in the mathematics classroom when teaching mathematics in the English language?

This chapter draws on a study that examined whether the teacher's language practices enabled or hampered the conceptual understanding of mathematics of a class of English second language learners. It explored various language practices and investigated how the teacher tried to helped her students cope when participating in mathematics that was being taught in a language in which they were not fluent.

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

In the mathematics classroom there are always interactions between the participants: learner-initiated questions, learner-learner interactions, as well as the normal teacher-learner interactions. Such interactions often require the use of at least some mathematical terminology. Yushau and Bokhari (2003) state that mathematical terminology can be very difficult for learners since the words used can have meanings

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 211–225.

that can be completely different from their normal usage. As well there is some mathematical terminology that simply does not exist in common language usage.

Others have suggested that although mathematical terminology can cause difficulties, the issue is much deeper than that. Pimm (1981) argues that many children's difficulty with mathematics may be due to the complexity of language rather than the mathematical task itself. Hence language and mathematics become problematic, since learners cannot meet the desired objectives of their mathematics lessons due to lack of communication skills. This in itself raises a dilemma for teachers as to how to correctly assess the sources of the learners' difficulties. Teachers may end up not knowing whether the problem is with the mathematics or the language (Secada & Cruz, 2000) because the mathematics is embedded in the language, and the language is embedded in the mathematics: neither can be separated into distinct, separated entities, rather they are in a synergistic, dynamic relationship. This suggests that English language can be a barrier in the understanding of mathematics in a multilingual classroom where English is a second language not only for the learners but for the teacher as well. The problem is greater for multilingual learners who are acquiring a language of instruction especially in lower grades. They have to cope not only with the language of instruction, but also with the difficulty of learning the special terminology and syntax of mathematics (see Brodie, 1989; Durkin & Shire, 1991).

In a more general sense, as well as the cross language issues and those linked directly to mathematics, there are differences within a language that learners need to comprehend as well. Cummins (1979, 1981a) draws the distinction between basic interpersonal communicative skills (BICS) and cognitive academic language proficiency (CALP). The distinction was introduced in order to draw educators' attention to the timelines and challenges that second language learners' encounter as they attempt to catch up to their peers in academic aspects of the school language. Cummins uses BICS to refer to conversational fluency in a language, while CALP refers to learners' ability to understand and express, in both oral and written methods, concepts and ideas that are relevant to success in school. He suggests that it takes learners, on average, approximately two years to achieve a functional, social use of a second language (BICS), but that it may take five to seven years or longer, for many bilingual learners to achieve a level of academic language proficiency (CALP), which allows them to learn mathematics in a deep and nuanced manner when taught in English.

For the learners to participate actively in a lesson, they need cognitive academic language proficiency (CALP). In other words they are expected to have attained a level of language learning, which is essential for learners to succeed in school (Cummins, 2000). When the language of teaching is neither the teacher's or the students' home or main language, such attainment is clearly a challenge for all. Cummins (2000) states that cognitive academic language is always abstract and has literacy demands that are high. He therefore argues that it is imperative that teachers start with contextualized tasks and practical activities that are of low cognitive demand and for which BICS can be used as the introductory language in order to accommodate learners who are still struggling with cognitive academic language proficiency.

At this point, a specific example is useful to more fully explore these issues. While learning mathematics, learners are expected to solve word problems. When the language of teaching is English, clearly these word problems are in English, not in the students' home language. In this context, learners have to overcome at least three challenges to accomplish the solving process successfully. Firstly, before they can tackle the problem, they have to learn to read the English language. However this is no guarantee that they can understand the mathematical meaning of the sentences and the mathematics terminology used in the text (Newman, 1983). Secondly, the challenge for learners is to understand and comprehend the language in which the word problem is presented. The third challenge facing the learners is that they need to understand the mathematics which will allow them to solve the problem (Fillmore, 2007). This suggests that while English second language learners manifest the need for help with mathematics, they may well need help with reading and understanding the English in which the written mathematical problems are presented (Clarkson, 1983). However the situation is even more complicated since within the English used for the mathematical word problems, technical mathematical language is also likely to be used (CALP), compared to the ordinary or everyday English (BICS) with which students may well be more conversant. Pimm (1981, p. 2) noted that "most mathematical classes actually take place in a mixture of ordinary English and mathematical English in which ordinary words are used with a specialised meaning." The question that arises then is, how does the use of language in a multilingual class enable or constrain the use of this specialised language of mathematics?

The Role of the Teacher

While language is demanding and this academic demand of language learning is placed upon learners as they are expected to cope in learning mathematics, teachers are always expected to enable conceptual understanding of the mathematics. Cummins' suggests that teachers should not assume that learners who are fluent in everyday spoken English (BICS) are also proficient in cognitive academic language (CALP) (Cummins, 2000). Hence it is through language that teachers need to enable students to develop their thinking skills, justification of answers, dealing with word problems, following instructions, which all include understanding and using mathematical vocabulary correctly. This suggests that without proper development of CALP in both teachers and learners, the deep conceptual learning of mathematics will become difficult.

A number of authors have emphasized that the mathematics teacher's role is to support that active process through exploration and dialogue (e.g., Duffy & Cunningham, 1996; Windschitl, 2002). Such a position is also emphasised by Jaworski (1996) who argues that the mathematics teacher's role is to support the students' active process of learning mathematics through exploration and dialogue (Jaworski, 1996). She argues that teachers do not serve as pipelines that seek to transfer their thoughts and meanings to the passive learners. She further argues that for effective teaching and learning to take place, the teacher has to ensure that there is constructive dialogue within the classroom, which suggests effective communication. For such dialogue and communication, there is a need for familiarity with distinctive grammatical structures and rhetorical devices used in mathematical discourse (Fillmore, 2007). In effect a command of mathematical CALP.

Thus Jaworski's (1996) understanding of the role of the teacher in the mathematics classroom suggests that learners are also active participants in the mathematics classroom where there is interaction between the teacher and the learners. During the teaching and learning of mathematics in this kind of a classroom the teacher has to be able to manage the interaction that takes place such that at the end of the lesson all learners would understand what was being taught. In terms of multilingual mathematics classrooms the question is, in which language do the interactions occur? Clearly there is the official language, but there are also possibilities that in learners' interactions, which may or may not be private from the teacher, learners may use other languages. But in the end they may be expected to display what knowledge they have in the official language of teaching; in this study, English. Hence such interactions may create a problem for the English second language learners, but as well perhaps their English second language teachers, as they are faced with a challenge of mastering mathematics content, mathematics language and English language.

This issue of students displaying what knowledge they process is particularly relevant when it come to conceptual understanding. Here the meaning attached to conceptual understanding is that it is evident when there is comprehension of mathematical concepts, operations, and relations. In a similar manner Kilpatrick, Swafford and Findell (2001) argue that learners become able to demonstrate conceptual understanding in mathematics when they can provide evidence that they can recognize, label, and generate examples of the concept. Kilpatrick et al. suggest that the evidence of the presence of conceptual understanding is when the learners are able to use and interrelate models, diagrams, manipulatives, and varied representations of concepts; when students are able to identify and apply principles; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts. In a nutshell Kilpatrick et al. (2001) suggest that conceptual understanding reflects a learner's ability to reason in settings involving the careful application of concept definitions, relations, or representations of concepts.

If teachers are teaching for conceptual understanding, this is clearly dependent on language use in the explanation of mathematical concepts. During such explanations it clearly helps if learners are proficient in the language of teaching. But when they are not, Krashen (2003) has argued that it is imperative that language learning also progresses so that the learners become comfortable in the language environment.

Setati (2005a) also makes it clear that in order for students to develop their mathematical thinking, they have to be able to communicate mathematically, and this obviously depends to a high degree on the teacher's role to ensure that this communication is effective and efficient. Hence part of the teacher's role is to encourage language learning during mathematics learning so that the appropriate mathematics CALP level of language development can be attained. However Krashen (2003) cautions that it is important not to force the production of such new language. This language has to emerge naturally, through stages and teachers must be aware of the stages and allow learners to produce the language when they are ready, not when the teacher thinks the students should. This implies that the mathematics teacher who teaches English second language learners should ensure that the environment for learning is pleasant, non-threatening and welcoming.

Within this framework, the study described below investigated how a teacher tried to use cognitive academic language support as well as mathematical support to encourage conceptual understanding in her learners who were not fluent in the language of teaching, in this case English. It explored how the teacher assisted learners who were still using a form of English that is for day to day living (BICS) to move to using CALP, the language necessary to understand and discuss mathematical content in the classroom.

METHODOLOGY

The study described here was a qualitative case study focussing on one multilingual Grade 4 classroom in an informal settlement in Gauteng West District in Gauteng Province, South Africa. The purpose of the study was to find out what impact the teacher's home language had on the promotion of her learners' conceptual understanding in the mathematics classroom. In this study the teacher taught mathematics in a language, English, that was not the learners' home language.

In order to understand the language issues that emerged during the promotion of conceptual understanding there was a need to interrogate meanings, intentions, positionings (in relations to language spoken, used, manner of use, etc.), as well as the teacher's actions in relation to the learners. The qualitative approach used helped in providing in-depth descriptions of the spoken language used by the teacher as she taught and interacted with the learners.

Sampling

The study was conducted in one grade four primary classroom, in a school in the Gauteng Province, in an informal settlement. A purposive sampling approach was used. The purpose of choosing this particular school was that both the teacher and the learners were English second language speakers. Most of the learners who lived in the area around this school never communicated in English at home. Most of their parents belong to a 'working class' while others are completely unemployed.

'Informal settlement' refers to a residential area where disadvantaged people from rural areas come and reside in order to get employment and a better life in urban areas. There is usually no infra-structure in such areas, including this one. What typically happens is that a group of people/families erect a shack on unoccupied land, without the permission of the municipality. There is usually no electricity and no proper ablution in such areas; although over time what was initially a squatter camp may have some infra-structure developed with (Reconstruction and Development Programme) RDP houses.

For this study what was needed was a grade four mathematics teacher who taught in a multilingual class where learners learn mathematics in a language that is not their home, first or main language. It was important that this teacher was appropriately qualified in teaching mathematics which was some assurance that she was able to promote conceptual understanding in her teaching. The evidence of her capability was in the curriculum monitoring report by both the school and the mathematics facilitator at the district. The teacher has a three-year diploma majoring in Mathematics and an honours degree in mathematics education. She had been teaching Mathematics for the past nine years.

The language of learning and teaching in the school was English. Learners and the teachers spoke Setswana, Zulu and Xhosa. The learners study their home languages as subjects. In addition to these languages, the learners also study English as a subject. The class was heterogeneous in terms of gender and ability and had a range of learners from those who achieve high marks in mathematics to those who are struggling to pass especially in mathematics.

Data Collection

Data in this study were collected through teacher and student interviews and lesson observations. The three grade four lessons observed were; a lesson on shape and space, a lesson on fractions, and a lesson on expanded notation. These lessons were given in the normal course of teaching in this classroom and were not specifically taught for this project. The vignettes given below were with learners whose home language is Setswana.

Interviews. Two teacher interviews were conducted by the first author (later referred to as the 'researcher'). The first one was before the lesson observations (pre-observation interview) took place, and second interview was conducted at the end of the sequence of lessons observed (reflective interview). The two interviews were aimed at finding out how the teacher saw the importance of her role in promoting conceptual understanding, and how the language impacted on the promotion of conceptual understanding. There were also short interviews held with some students after each of the observed lessons to clarify particular episodes or incidents that occurred during the lessons.

Lesson observations. The lessons observed were video recorded to capture as much as possible of what the teacher and the learners did in class during the lesson. Using a video camera enabled the capturing of interactions during the lesson that could not have been easily undertaken if only an observation sheet had been used. Teacher-learners, learner-teacher and learner-learner interactions were recorded. The use of only one video camera did prevent more detailed data being recorded. However it did mean that neither the teacher nor learners were overwhelmed by the presence of lots of cameras, where perhaps they would end up focusing on the cameras instead of the lesson.

In reviewing the video recorded episodes, the researcher focused on how the teacher helped the learners unpack their solutions in tasks and opened up opportunities for explanation and justification of their answers. In particular, opportunities of how the learners used language to deal with these tasks focusing on conceptual understanding were examined.

RESULTS

The results from the study are discussed using three vignettes from the lessons observed.

Vignette 1

This vignette is taken from the first lesson on shape and space. This and the one that follows came from a topic centred on 3D and 2D shapes. Prior to this lesson students had seen and drawn examples of rectangles, so the start of the lesson rehearsed some ideas with which they should have been familiar.

The lesson began with the learners presented with the figure below on the chalkboard:

Teacher: Okay, let's see, what is the name of this shape?
Mpho: It is a rectangle ma'm
Teacher: Why do you say it's a rectangle?
[Learners are quiet and seem to be thinking]
Teacher: Okay, what do you see here? Tell me, how can you explain to someone else that this is a rectangle?

Thabo: It has four parts
Teacher: Four parts?
Thabo: Yes mam, the two parts here are short and the other two parts are long
Teacher: Come and show us
Thabo: One, two, three, four mam *[pointing at the sides of the rectangle]*
Teacher: Oh! Thabo, we don't call that parts, we call them sides. We talk about parts in fractions but now we talk about sides because these are shapes. Do you understand?
Learners:[silent and looking at the diagram]
Teacher: Why are you quiet; don't you understand? Niyabona [*can you see this*], shebang mo [*look here*], side and part are not the same dinthotse ha di tshwane [*these things are not the same*]. Mh. [*Scratching her head*], Okay remember fractions have parts and shapes have sides. Ka Setswana e yi, okay niyasazi isiZulu nonke ne [*okay, you all know Zulu*]?
Learners: Yes
Teacher: Lama part e shape [*these parts of a shape*] we call them sides and then [*drawing a diagram with four equal parts*] lama piece ale diagram wona we call them parts [*these pieces of the diagram are called 'parts'*]; [*the teachers keeps quiet for some time as if she is in deep thoughts*]

When the teacher presented the learners with a rectangle she wanted them to define what a rectangle is. In other words the teacher wanted the learners to be able to explain why the diagram is a called a rectangle. The role of the teacher in such a situation should be to encourage conceptual discourse by allowing learners to speak informally about mathematics; that is exploring, explaining, and arguing for their interpretations and ideas (Sfard, 2007), in this case, of a rectangle. When the communication with the learners started to be challenging the teacher seemed to lack the appropriate mathematical language that was required. Though the teacher was expected to be the one who helps learners with the appropriate mathematical language, in this instance she was the one who was facing a dilemma. The challenge here was for the teacher to know when and how to lead learners from their informal talk to formal spoken mathematics. While moving from informal to formal spoken mathematics the teacher should have been alert, ensuring that the learners use correct mathematical language.

In this classroom the teacher seemed to be failing to ensure that the informal language used in the classroom did not interfere with the conceptual understanding especially while trying to code-switch. The teacher's language dilemma led to misconceptions for the students. In essence while the teacher was supposed to be helping English second-language learners to overcome barriers and thus facilitate communication in the teaching and learning of mathematics, she however, in this instance, became the barrier.

This issue was further explored in the post teacher interview:

Researcher: Why were you quiet?

Teacher: I realised that mh… I also did not know how to explain the difference. I did not know how to explain in Setswana, Zulu or English. I realised that in our everyday language in the township we use the word 'part' for almost many things i.e. role, side, pieces of a whole, located place. When I wanted to use Setswana I could not explain further.

This suggests that the teacher realised that she was deficient in the home language of the learners as well as the mathematical language. While studies have shown that English second language learners, even at university level, are confusing the meanings of some of these mathematical terms (Setati, 2003), however in this instance it was the teacher who was confusing the meanings.

Researcher: Why did you use Zulu instead of Setswana, how sure were you that they understood because I heard you say 'niyasazi nonke isiZulu mos (I suppose you all know Zulu)

Teacher: Well, I wanted to feel comfortable in the language I was using so that I could have enough vocabulary to explain the difference between part and side. Now that you are asking me, it makes me feel like I manipulated the minds of the learners because of my authority as a teacher.

The teacher's response suggested a line of questioning during the interview with the learners; to enquire as to why they could not explain the difference between the 'side' and 'part'. The learners' and the teacher's linguistic background affected the conceptual understanding of the learners. It hampered the enablement of conceptual understanding. Hakuta and McLaughlin (1996) argue that the person's use of language depends on what is understood to be appropriate in a given social setting. This implies that linguistic knowledge is situated in a group's collective linguistic norms. In this case the teachers' daily language practices affected the teaching and learning of mathematics as it brought confusion in the mathematical setting.

It made the researcher realize that while teachers try to use language to explain mathematics, promotion or enablement of conceptual understanding may be at times negatively affected by using the language which students are more familiar with. The vignette above shows that sometimes when the teachers teach language in a mathematics class, mathematics lesson may be negatively affected. The interview with the teacher suggests that at this point she did not have a solution for this dilemma. In this case oversimplification of language use may have complicated the mathematics language in this mathematics class.

This notion was followed up in an interview with a learner:

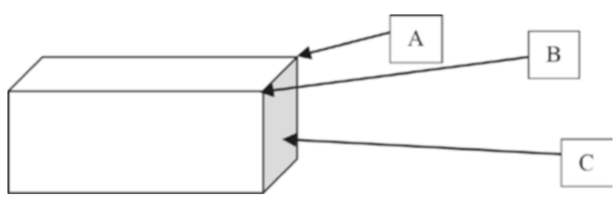
Researcher: Why didn't you answer the teachers' questions? You can explain in any language.
Learner: (*looks at the researcher and smiles, lifting her shoulder, indicating that she doesn't know*)
Researcher: Okay talk to me, why were you quiet?
Learners: nnake bona go tswana (*I see them as the same*). I don't know.

Setati (2005a) argues that part of learning mathematics requires fluency in mathematical language which includes words, phrases, symbols, abbreviations and fluency in a range of discourses that are specific to mathematics. Clearly in this instance the learners, like the teacher, did not have command of the appropriate mathematical language in either their own language or in English.

Vignette 2

The following vignette comes from the lesson that followed the one referred to above and focussed on 3D shapes.

Teacher: Okay, let's look at the diagram, can you see that it is different from the ones we worked with i.e., square and rectangle. By the way what do we call this one?
Learner: Prism
Teacher: What class
Class: Prism
Teacher: Let's see the parts of this prism. Remember I told you the names of all these parts of this prism.



(*Learners gave the following answers:* A – corner; B – corner; C – Face)

Teacher: This is incorrect; you should not call these parts corners in this diagram.
Learner: But it has a corner mam, it also looks like the corner of L, like in a rectangle
Teacher: No, we don't say that here, you see these corners are not the same as the ones you see in the rectangle.

Later after the lesson in the second teacher interview, the researcher asked:

Researcher: Can you explain what this 'letter L that has a corner' the learner is talking about?
Teacher: No, that's what we did in 2 D shapes. This is different.

Again the teacher did not use the correct mathematical language. Learners could not differentiate between the vertex, edge and the angle. They associated all the parts with the corner because of the teacher's explanation of 'letter L' shape they saw in a rectangle. The researcher continued to explore this issue:

Researcher: Why are the learners using the word 'corner' and also why are they talking about the letter 'L' shape?
Teacher: I thought if I use the word they are familiar with, it will make easy for them to understand. Now I can see that it is bringing confusion as they see other 3 D objects.

It has been argued that it is very beneficial to transition from a learner's everyday language to mathematical language and not start with mathematical language, which often has little or no connection to learners' everyday lives. However in this instance the use of an example from the student's everyday life had a negative effect on the understanding of the mathematical concepts in this class. While everyday meanings and experiences can be useful resources, they need to be used carefully and with insight by the teacher. In this instance the everyday meaning ended up creating obstacles for mathematical communication in this classroom (Moschkovich, 2002).

Vignette 3

During one point of the second teacher's interview the issue of language dilemma emerged when the teacher raised the challenge she face while teaching expanded notation. The teacher indicated that learners were finding it difficult to understand expanded notation. The following exchange took place:

Researcher: How do you see language impact in your mathematics class?
Teacher: Well, I have a serious challenge. When I was teaching them expanded notation, I ended up experiencing problems that led to misconceptions.
Researcher: What happened?
Teacher: I gave them a test after teaching. I wanted them to write 127 in an expanded notation, and the learner wrote '100 + 18 + 9 = 127. I marked it incorrect. She came and told me she was correct. I told her that was not expanded notation. What the student said to me was "Mam you said we must make it long because expand means long, I am right mam, if add all of them it's 127 which is short."

It seems that the teacher at this point abandoned the mathematical language and focused on teaching English as a language, and as a result the mathematics got lost in the process. Here the mathematical confusion arose for the student when the teacher was trying to find a simple way of describing what happens when you use expanded notation, but the student interpreted the English words used in quite a different manner.

At school the language becomes more cognitively demanding (CALP). New ideas, concepts and language are presented to the learners at the same time. Setati (2008) argues that in a mathematics class, teachers find themselves having to explain concepts in English while they assist learners to get to the mathematics language (Setati, 2005). Clearly in this instance the teacher did not achieve success.

DISCUSSION AND RECOMMENDATIONS

Looking at what happened in this classroom it was evident that the teaching and learning of mathematics could not be isolated from language. It is also evident that the interactions and the growing understanding were not easy in this class because of the teacher's and the learners' tenuous command of the language of teaching, English. When the teacher gave mathematical tasks to learners, she did not just have to deal with the mathematics, she also had to deal with the language of mathematics, and as well, the English language. Schiffer (2001) argued that before the teacher gives a task to the learners s/he should consider how s/he can attend to the mathematics in what the learners will be saying and doing. This was evident during these lessons. Attending to the mathematics involved language. It was therefore imperative for this teacher to know how to use the correct mathematical language to ensure that mathematics did not get lost in the process of grappling with the language itself. It was also important how the teacher transitioned from the everyday language that the learners started with (BICS), to the mathematical language (CALP). Of course that transition process may take place over a number of lessons. It's not something that can be accomplished in few seconds.

At school, language becomes more cognitively demanding. New ideas, concepts and language are presented to the learners at the same time. Setati (2005) argues that in a mathematics class, teachers find that they have to explain concepts in English first before they can get to the mathematical language. It follows then that teachers should ensure that they use correct mathematical language to assess the mathematical validity of learners' ideas. This does not mean that teachers should reject the everyday language that the learners are familiar with and in which they will begin to discuss the ideas. However teachers should be able to encourage learners to begin to use correct mathematical language, and move beyond oversimplification, which may arise if they only use everyday English language. Clearly the teacher in this study had difficulty in doing this at times.

The fact that most teaching in this multilingual classroom took place in English, which is not the main language of either the teacher or the learners, made participation more difficult for the learners and the development of ideas (Brodie, 2005). Yushau and Bokhari (2003) comment on the type of context that this study encountered. They suggest that for learners who are acquiring a language of instruction, as well as being taught mathematics in the new language, the language of mathematics can be another source of difficulty and confusion. This problem however does not just affect the learners only; it can also affect the teacher, if they are also not fluent in English even though they have to use it as a main language of teaching in their mathematics classrooms. This was clearly the case in this study.

Teachers need to be aware that, while the process of grappling with a language can interfere with learning, at the same time it can also nurture the learning process (Carpenter, Franke, & Levi, 2003). According to Moschkovich (2010) the teacher should know how to build on to the language resources the learners bring into the mathematics classroom. In doing this, teachers need to be tactful while using learners' home language as an enabler in the understanding of mathematics. But they should also ensure that mathematical language is correctly used at all times which will include that learners are able to distinguish between everyday usage of words and their use in a specialized mathematical context. Overall teachers need to develop a mediated teaching approach that will reduce the language barrier of these new learners, by providing language support that cushions the process of the learners' language transition as well as improving their understanding in mathematics and, at the same time, abide by the language policy regarding English as a medium of instruction. And that clearly is not an easy task, as shown by the examples drawn from this study.

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15. COMPLEMENTARY FUNCTIONS OF LEARNING MATHEMATICS IN COMPLEMENTARY SCHOOLS

INTRODUCTION

Mathematics is believed by some to be a universal language that all human beings share (Singh, 1997; Guedj, 2000). In this school of thought, mathematics has its own particular syntax, genre and ways of argumentation. For example it is commonly believed doing arithmetic is the same regardless of whether one is performing arithmetic in Chinese, Farsi or in English. However although the result is the same, the linguistic support that is behind the arithmetic process is not necessarily identical. Fuson and Kwon (1991) have observed that a language like Chinese offers a linguistic support in basic tasks such as addition or subtraction. They have observed that number words make the recomposition processes for addition and subtraction easier. For example, in adding 8 + 7, one would recompose 8 + (2 + 5) to make it 'ten five'. In English, one interpretation of the phrase 'ten five' is translated to 'ten remainder five' which makes fifteen, but in Chinese, due to the fact that number words explicitly name the tens and the units, 'ten five' is directly translated to fifteen. Therefore the conceptual tools provided by the language are not the same, even for something as basic as addition or subtraction. Therefore the language in which mathematical operations are embedded is far from being universal (Gorgorió & Planas, 2001).

Languages are socially and culturally constructed. To be specific, a language (just like any other cultural artefact) is a meaning making process that is culturally given and socially shaped (Kress, 2010). A natural language (both as a medium and message) that is used to convey mathematical meanings can at times facilitate but at other times hinder how certain ideas are developed in doing mathematics. Therefore it is not surprising that each language may enable different perspectives in how mathematical ideas can be understood.

In this chapter I will present an in-depth ethnographic example drawn from video recording and field-notes to show how a particular mathematical task (complex fraction) can be understood differently in different languages (namely Farsi and English). Moreover, I will discuss and evaluate the complementary functions of learning mathematics in the two languages in a multilingual setting (complementary school), versus learning in a monolingual setting (mainstream school).

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 227–247.

BACKGROUND AND MY RESEARCH INTEREST

Coming from a Persian background and being familiar not only to the British-Iranian community (by knowing Farsi and English and being familiar with both cultures), but also having the knowledge of the content of the study (knowing the subject knowledge and materials used in mathematics), has placed my role in this research study as an 'insider'. Insiders in research not only can easily engage with research participants through the same language and culture, but are believed to make insightful observations denied to researchers who are 'outsiders' (Martin, Stuart-Smith, & Dhesi, 1998). I became interested to look at the experiences that British-Iranian bilingual learners have in learning mathematics in two different institutional settings in the UK: in their local complementary school (weekend school which I shall describe later) where they can draw on more than one language when engaged in mathematical tasks, compared to their experiences in learning mathematics in their mainstream school (Monday-Friday school) where only English is used. My focus is firmly on the bilingual abilities of the students; Farsi/English speaking students of Persian origin.

Bilingual Education

Multilingualism and multiculturalism is now seen as one of the main social factors of 21st century. As a result of migration,[1] marriage, education, temporary residence or being the offspring of couples who are themselves members of a bilingual community, education in many countries of the world takes place in multilingual contexts since approximately "between half and two-thirds of the world population is bilingual" (Baker, 2001, p. 8). It is not uncommon now to observe multiculturalism and multilingualism in classrooms, playgrounds and in most public places (Hoffmann, 1991). Although bilingual education is becoming the norm, there are still negative connotations with the notion of bi/multilingualism. For example, in the UK bilingualism is often seen as a threat to national unity, social cohesion and the British identity (Blackledge, 2005).

Education in many countries of the world takes place in multilingual contexts. Mathematics among many other subject disciplines is also taught and learnt in two or more different national languages in many parts of the world; in United States (Khisty & Chval, 2002; Moschkovich, 1999, 2002), in South Africa (Adler, 2001; Setati, 1998), in Wales (Jones & Martin-Jones, 2004), in Malta (Farrugia, 2007), in Australia (Clarkson, 1992; Ellerton & Clements, 1991), in Papua New Guinea (Clarkson & Galbraith, 1992), in Iran (Fardinpour, 2011), in Italy (Gajo & Serra, 2002) and in the UK (Farsani, 2011). Research in bilingual education sites not only has raised awareness of linguistically diverse mathematics classrooms (Gorgorió & Planas, 2001; Barwell & Setati, 2005; Barton, 2008; Parvenehnezhad & Clarkson, 2008) but also brings to light the interplay of certain cultural artefacts such as idioms in teaching and learning. A bilingual education not only offers a greater linguistic dynamism, but in addition provides a space to integrate learners' experiences in home, community and school (Pérez & Torres-Guzmán, 1992).

Unfortunately bilingual education in many parts of the world mostly operates in monolingual education systems based on monolingual lenses and ideologies (García, 2009). Such a focus fails to acknowledge language fluidity and movement as a resource, and can raise many problems, for example when it comes to assessment. A bilingual learner's linguistic competence might be "measured and compared with a native monolingual speaker of that language which is unfair because bilingual learners will typically use their two languages in different situations and with different people for different purposes" (Baker, 2001, p. 8). More generally, teaching and learning mathematics bilingually "will continue to seem 'odd' only if it is compared to a monolingual norm, to some imagined set of 'pure' or 'normal' language practices" (Moschkovick, 2007, p. 140). This does not have to be the case.

Complementary Schools

Complementary schools are community educational institutions in which both learners and teachers have a greater access to a range of linguistic resources which "seem to offer a window onto a multilingual England [which is] often hidden from the view of policy makers in mainstream education" (Blackledge & Creese, 2010, p. 11). Complementary schools provide a space for performance of alternative languages, heritages and histories (Creese, Bhatt, Bhojani, & Martin, 2006). Li Wei (2006, p. 78) claims that complementary schools in the UK "were set up in response to the failure of the mainstream education system to meet the needs of ethnic minority children and their communities". From a socio-linguistic landscape it appears that languages with a longer history of migration and bigger populations such as Gujarati, Bengali, Turkish and Chinese have a considerable literature and research on their heritage language in the UK (see Creese, Bhatt, & Martin, 2007a; Creese, Blackledge, & Hamid, 2007b; Creese, Lytra, Barac, & Yagcioglu-Ali, 2007c; Creese, Wu, & Li Wei, 2007d).

Complementary schools usually run outside the hours of mainstream schools and are often based in a variety of settings including private home or a community centre. Unfortunately because of the unofficial nature of complementary schools, the teaching and learning that takes place is often invisible and unrecognised by the mainstream sector. Complementary schools are "non-statutory schools, run by their local communities, which students attend in order to learn the language normally associated with their ethnic heritage" (Blackledge & Creese, 2009, p. 459). Depending on the nature of complementary school, some learners have the opportunity to learn subjects such as sciences and mathematics through the mediums of English *and* their heritage language.

In the United Kingdom, the term 'complementary schools' is used for these voluntary educational settings that teach community language and draw upon cultural/religious practices that are acceptable and valued to that specific community. These community educational institutions are also referred to as ‘supplementary schools’ in the UK, ‘heritage language schools’ in Unites States and Canada (Hornberger, 2005), and ‘community language schools’ or ‘ethnic schools’ (cf. Blackledge & Creese, 2010) in Australia. It is only in recent years that the term ‘complementary schools’ has been employed to emphasise “the positive complementary function of these teaching and learning environments in relation to mainstream schools” (Martin et al., 2006; Blackledge & Creese, 2010, p. 47). I will settle on the same term because this chapter reflects the complementary functions of complementary schools in relation to the teaching and learning of mathematics. The multilingual orientation of a mathematics classroom in a complementary school often provides an opportunity to develop different pedagogic possibilities for British-Iranian learners (Farsani, 2012b). It provides a different pedagogic possibility that encounters aspects of history and culture. Hence at times insights on how mathematics can be done differently arise that are not necessarily included in the mainstream curriculum. I will later scrutinise and describe how students and teachers draw on their various languages and the additional values and resources that bilingualism brings to mathematics performance, which differs from doing the task monolingually.

What seems to be at the heart of what complementary schools are about is creating multilingual spaces (Creese et al., 2006), using language flexibly (Creese et al., 2011), and encouraging the use of the full range of young learners’ linguistic repertoires (Creese & Blackledge, 2010). Both the classroom teacher and learners code-switch constantly to co-construct meanings, mediate understanding and to increase participation in the classrooms (Creese et al., 2007a; b; c; d).

Code-Switching and Bilingualism

To interchange between two or more languages within a single communicative event in the field of socio-linguistics is often referred to as code-switching (Gumperz & Hymes, 1972; Gumperz, 1982; Heller, 1999). In the literature, ‘code-switching’ or ‘language alternation’ is also referred to as ‘language mixing’ (Redlinger & Park, 1980), ‘code-alternation’ (Auer, 1984) and ‘crossing’ (Rampton, 1995). Recently García (2007, 2009) and Creese and Blackledge (2010) have employed the term ‘translanguaging’ to depict language fluidity and movement rather than linearity. Translanguaging includes code-switching but pays more attention to the choices that are made through the agents (speaker’s perspective) and not from language perspective as code-switching has often been studied. Often bilingual discursive practices are discussed from a monolingual perspective or a monoglossic lens that views each language as a separate autonomous system. Translanguaging looks at the discursive practices and language choices from a bilingual’s perspective not a monoglossic perspective.

García (2009, p. 7) argues for a need to move away from ‘monoglossic’ ideologies of bilingualism that “view[s] the two languages as bounded autonomous systems to heteroglossic ones." Heteroglossia is the translation of the Russian word *raznorechie* coined by Mikhail Bakhtin which describes the coexistence of distinct variety of codes within a single communicative event. Following Bakhtin (1981, 1986) Bailey (2007, 2012, p. 499) refers to the different kinds of forms or signs of heteroglossia which includes "intra-language social variation" such as regional dialects and registers. As a result translanguaging and heteroglossia place their emphasis on the fluidity and dynamicity of languages that are used simultaneously through the user's choices. In other words, to stress and emphasise the voice/speaker rather than the code or language is at the heart of this argument and as Blommaert (2010, p. 196) has observed, "it is a matter of voice, not of language".

There is an extensive use of the traditional term 'code-switching' both within the field of socio-linguistic and mathematics education literature. Therefore in order to comply and to avoid confusion, I will employ the traditional term 'code-switching' to refer to the choice of switching between two or more languages.

It is argued that code-switching requires cultural knowledge and linguistic competence in all the languages that are involved, and often, not having the cultural experiences that goes with the linguistic expressions causes errors and breakdowns in communication (Farsani, 2012). For example many Muslim students often struggle to find a solution to the question "Calculate the volume of a wine glass". This is not because there is a problem with mathematics, but there is a cultural problem, 'What is a wine glass?' Code-switching might be argued "is not simply a combination and mixture of two languages but creative strategies by the language user" (Li Wei, 2011, p. 374). Likewise Hoffmann (1991, p. 109) speaks of code-switching to be "potentially the most creative aspect of bilingual speech." Code-switching is perceived to be the "most salient difference between bilinguals and monolinguals"[2] (Moschkovich, 2007, p. 138) and one of the social functions of code-switching is the negotiation of meaning through speakers' language choices, which is also unique to bilingual learners (García, 2009). I would also like to pay attention on the term 'bilingual' as it is a generic term that describes very complex phenomena. Bilingualism is defined as "the practice of alternately using two languages will be called bilingualism, and the person involved, bilingual" (Weinreich, 1968, p. 1). Bilinguals usually have the benefits of accessing a greater range of choices in their repertoires than monolinguals (Naldic, 2009). Bilingual learners often draw across their linguistic resources (that they have at their disposal) across different settings and tasks (Moschkovich, 2010) to exclude or include others (García, 2009) and to engage with a wider audience (Blackledge & Creese, 2010). "In England the term [bilingual] is used to refer to pupils who live in two languages, who have access to, or need to use, two or more languages at home and at school. It does not mean they have fluency in both languages or that they are competent and literate in both languages" (Hall, 1995, p. 10). I will use the term bilingual because terms such as 'English as a Second Language' (ESL) or 'English as an Additional Language' (EAL) learners are problematic from a heteroglossic perspective. The problems that are encountered as a result of employing terms such as ESL or EAL learners is twofold. Firstly there is the 'matter of definition' (e.g., is a ESL learner someone who speaks with an accent?), and secondly with regard to the 'time direction' (at what stage does one stop being a second-language speaker?). I use the term bilingual from a heteroglossic and bilingual perspective to enshrine "a full range of possibilities, and taking away the negative connotations associated with being second, and not first" (García, 2009, p. 60).

Rather than concentrating on the monoglossic type of bilingual education, I will now turn to the heteroglossic ones where simultaneous variations of different signs or languages are seen as resources to negotiate meaning and to connect with a wider audience. Hence I will now shift attention to different kinds of resources, namely nonverbal, that are used more or less in every mathematics classroom to convey mathematical meanings.

Resources for Meaning Making in Mathematics Classroom

I am interested in looking at how language is used to make mathematical meaning. My definition of language follows from Mehrabian's (1971, 1972) theory of three V's; Verbal (what is said), Vocal[3] (how something is said) and Visual (gestural representations that are created in space). For example I am fascinated to examine how a classroom teacher produces gestures to convey the instructional information that has already been expressed in words. Gestures and body-based resources appear to function as amplifiers and complement visually what the interlocutors are already expressing in words conveying aspects of mathematics register such as 'parallel', 'perpendicular' (Castellón, 2007), 'takeaway', 'addition' and 'power' (Farsani, in press).

Researchers have shown different semiotic resources or modes[4] that are used in mathematics classrooms (Lemke, 2003; O'Halloran, 2005, 2009; Radford et al., 2009). These modes can be verbal or nonverbal, written or spoken, and can be displayed by different instruments and materials (such as pen and pencil or other digital devices such as graphical calculators). Modes can also be nonvisual such as speech, soundtrack (Kress, 2009), voice and music (van Leeuwen, 1999). Researchers have focused on other social semiotic modes of colour (Kress & van Leeuwen, 2002), gesture and movement (Kress et al., 2001, 2005; Martinec, 2000), gaze and proxemics[5] (Hall, 1966; Collier, 1983; Collier, 1995; Lawson, 2001a; b) and their importance in meaning making in social interaction, cultural context and classroom practice have been discussed.

By encountering verbal, vocal and visual elements of language, I am interested in how these resources are being juxtaposed in a bilingual classroom and the ways in which they provide pedagogic possibilities by creating a coherent package of meaning and action. In other words, it is not only by taking account of what

A says to B that sends a message, but also *how* A conveys the message (Farsani, 2012).

DATA COLLECTION AND DATA ANALYSIS

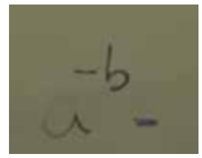
The data that I present in this section is obtained through a video recording in a complementary school and ethnographic field notes from a mainstream school.

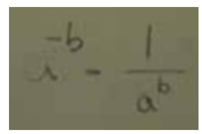
Complementary School Data: Indices

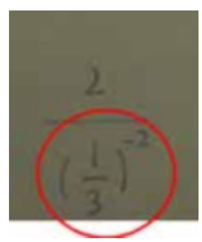
During one mathematics lesson in the complementary school, the classroom teacher (T) and learners were engaged in solving arithmetic questions from a textbook projected on the whiteboard. The transcript focuses on a particular task; students were asked to simplify $\frac{2}{\left(\frac{1}{3}\right)^{-2}}$. This task exhibited a challenge to many young bilingual learners in the classroom. The teacher recommended that the students focus just on the denominator $\left(\frac{1}{3}\right)^{-2}$ and forget about the numerator (which was 2) for the time being. In the transcription we will see how the teacher demonstrates that any number 'A' to the power of a minus integer '-B' can be rewritten as 'one over A' to the power of a positive integer 'B'. The teacher offered this method of procedure as he wanted to avoid negative powers.

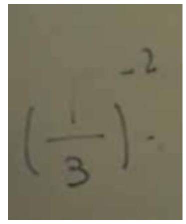
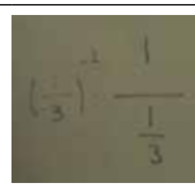
The multi-modal transcript convention I have used is as follows:

T	Teacher
B	Boy
G	Girl
[ ]	Non-verbal communication
{ }	My translation
Italics	Farsi transliterated into English
Normal font	English language
Dots	Each dot represents one second of silence
Change in font size	Change in volume of an utterance: the bigger the font is, the louder the pronunciation. The smaller the font is, the quieter the pronunciation of the term.

<table>
<tr><td>T:</td><td>chi migoftam, migoftam agar darim
{What was I saying, I said if we have}
A be power of minus B, it’s equal to what? {A to the power of minus B, it’s equal to what?}</td><td>
[The teacher writes a^{-b} on the whiteboard as he is completing his utterance]</td></tr>
<tr><td>B1:</td><td>A over B</td><td></td></tr>
<tr><td>T:</td><td>No, . . it’s one over ‘a’ be power of . . ‘b’ . . {. . No, . . it’s one over ‘a’ to the power of ‘b’. .}

chera, chon really. man be tore mamooly doost nadaram ke che kar bekonam? {why, because really,… normally I don’t like to have what?}</td><td>
[The teacher writes $\frac{1}{a^b}$ on the whiteboard]</td></tr>
<tr><td>Bs:</td><td>Negative</td><td>[A few boys said negative at the same time]</td></tr>
<tr><td>T:</td><td>Negative as a power dashte basham. {I don’t like to have negative as a power}

pas {so} I just take this one,that bit, I just take it out.</td><td>[Intonation, emphasis on the term ‘power’ as well as a hand gesture to indicate the position of power]

[The teacher refers back to the original question and indexes the denominator of the complex fraction by producing a pointing gesture to it]</td></tr>
</table>

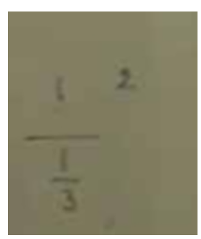
	Man daram chi, one over three *be* power of minus two. {So I have one over three to the power of minus two}	 [The teacher writes $\left(\frac{1}{3}\right)^{-2}$ on the whiteboard as he makes his utterance]
B3:	oh, *ye* one *balash mizari*{oh, put a one at the top}	 [B3 points with his index finger illustrating one at the top. He is holding his index finger up with the rest of his fingers closely curled, which could possibly indicate one. The location of his index finger is not horizontal as most pointing gestures are, but semi-vertical, more vertical than ordinary pointing gestures tend to be]
T:	*pas migam chi?* {So} XXX Inaudible I am gonna use a different colour so you know which one is which, one big fraction line, one in the top, one third in the bottom.	 [The teacher draws a red horizontal line which appears to coordinate with his utterance timing]

Khob hala mikham ino reverse *bekonam*{Ok, now I want to reverse it}

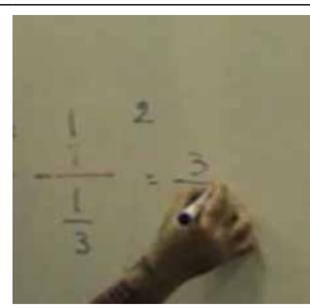
[The teacher has extended his index and mid fingers facing outwards with the rest of the fingers closed. He then turns his wrist inwards and the palm faces inside]

Khob, ino bekham reverse *bekonam chetor bayad bekonam*? {Ok, how can I reverse it?}

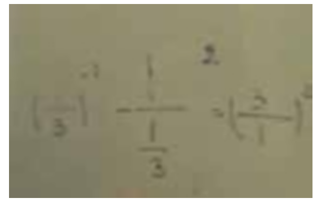
Ok, *in alan injoorie*{this is how it looks like at the moment} the whole thing to the power of two now.



[The teacher has raised $\frac{1}{\frac{1}{3}}$ to the power of two now.]

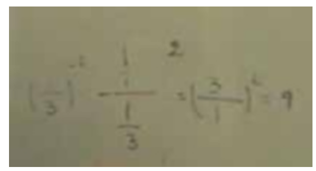
	but that is not good, we want to change it.	[The teacher created a new denominator in the top fraction by dividing one over one $\frac{\frac{1}{1}}{\frac{1}{3}}$]
B1:	*nazdik be nazdik, door be door* {near by near, far by far}	
T:	*door dar door,* {far by far,}	 $\frac{\frac{1}{1}}{\frac{1}{3}}$ [The teacher indexes the two numbers furthest apart]
	nazdik dar nazdik. {near by near}	 $\frac{\frac{1}{1}}{\frac{1}{3}}$ [The teacher indexes the two numbers closest to the main division line in the centre]
	So, *door dar door mire koja*? {So where does far by far go to?}	
B2:	*dar door* {It goes far!}	
T:	soorat, door dar door soorat {Numerator, far by far goes to numerator} or numerator. *Nazdik dar nazdik*? {Near by near goes to?}	 [Synchronically to his verbal message, the teacher writes three and a division line underneath]
Bs:	posht, jolo {Behind, in front}	

T: *makhraj*. {Denominator.}
And, three to the power of two is?

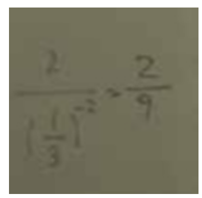


[The teacher writes one as the denominator. He also raises the fraction to the power of two]

B1: Nine



T: So that equals two over nine.

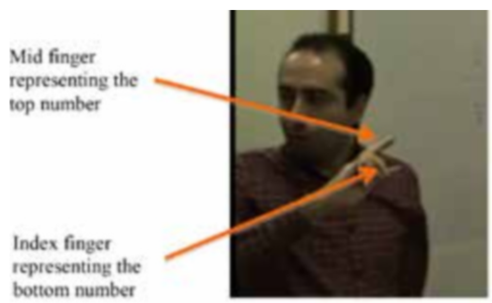


[At this time, the teacher goes back to the original question and writes two over nine as the answer to the initial question which asked to simplify $\frac{2}{\left(\frac{1}{3}\right)^{-2}}$]

B2: I get it now.

This transcript reflects the flexible movement across (and between) verbal and nonverbal language illustrating the nature of both the bilingual and dynamic discursive practices in this complementary school mathematics classroom. In the analysis of this chapter, I would first like to show the fluidity and dynamicity of flexible bilingual practices in this particular community of practice. Then I would like to extend this notion and show how the bilingual orientation of this complementary school has developed bilingual pedagogic possibilities for bilingual learners.

In the analysis of this particular transcription, I would like to first pay special attention to lines 20–22 'Negative as a power *dashte basham*' {I don't like to have negative as a power}. It is worth noting that the linguistic components of the phrase "Negative as a power *dashte basham*" belong to different languages (English and Farsi) and it is in the bilingualism of the lesson that the message is conveyed. In practice the teacher employs both languages simultaneously as a resource to negotiate meaning and to connect with a wider audience with different levels of linguistic proficiency. This particular excerpt is an example that illustrates how the teacher moves frequently between his languages at his disposal and how Farsi and English were used flexibly to convey meaning. The simultaneous use of two languages in the teacher's instructional talk illustrates the dynamic and fluid boundaries between the languages and signs at his disposal. In the teacher's instructional talk there is also variation of pitch movement [tone] used for grammatical purposes; as an emphasis not to have negative 'powers'. In other words heteroglossia is evident in the simultaneous use of different kinds of forms or signs that have been used in a flexible manner to accomplish the lesson content. From a *bilingual perspective* not only the teacher's language choices but paralinguistic styles within the language contributed to the negotiation of meaning which helps to keep the task moving forward.



What I also find of particular interest is mode-switching or the movement between meaning making materials such as a shift from verbal to visual. In lines 39–41, the teacher uttered "*Khob hala mikham ino* reverse *bekonam*" {Ok, now I want to reverse it} in words and gestures simultaneously by extending his index and mid fingers with the palm facing outwards and the rest of the fingers closed. The teacher then turns his wrist inwards and the palm faces inside. Although what was carried out in speech and gesture are materially different, they both have the same temporal and semantic presentation. Not only what was said in speech and enacted in gesture were synchronously coordinated, also the overall meaning that emerges from these two modes (verbal and visual) goes hand in hand. For example it is in the accompanying of the verbal message that the rotation of index and mid finger illustrates the concept of 'reverse.' Moreover, the rotation of index and mid fingers emphasises the verbal content visually. It is interesting to note that the rotation of index and mid finger accompanying the term 'reverse' would convey not just the concept of 'reverse' but also additional information about how the process is made. The teacher's gesture's manner of motion (how the gesture was carried out) and the trajectory (the path it has taken) illustrate how that the top number in the fraction goes to the bottom and the bottom number goes to the top.

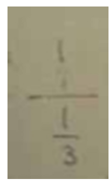
In other words, the teacher's gesture for 'reverse' can be perceived to be a grounding procedure for how the complex fractions should be solved using 'far by far, near by near' in lines 55–57 (which I shall explain later). A combination of gesture and speech not only allow the teacher to have access to a flexible and a wide range of possible resources to convey their meanings (see Alibali & Nathan, 2012) but also to present ideas that are not fully developed in speech, and expressing developing ideas (Goldin-Meadow, 1999).

From the example that I have provided, teacher's gesticulation and talk were aligned and his gesture was semantically affiliated with the on-going flow of his utterance. Teacher's employment of such gesture along with the procedural talk emerged as a powerful mechanism to visually support the relevant mathematical concept. Therefore the classroom teacher reinforced the concept and the content flexibly through the use of verbal and visual resources. Just as there are different variations of forms or signs in spoken discourse (through the user's choices), heteroglossia is evident in gestures as often a hand gesture can move in different directions and speeds. In addition the temporally coordinated and synchronous combination of speech and gesture that jointly presents the same idea unit in different mediums (verbal and visual) emphasises the fluidity and heteroglossic nature of communication in the mathematics classroom.

Now that fluid boundaries between languages (Verbal, Vocal and Visual) have been illustrated, I will reflect on how these discursive bilingual practices can be seen as a resource in teaching and learning. I would like to draw attention to how such bilingual discursive practices present a different perspective on how aspects of mathematics can be done differently, namely complex fractions.

Mainstream School Data: Variations in Solving Complex Fractions

Generally, in mainstream schools in the UK, the process of working out a complex fraction is to treat it as two distinct fractions in the form of: $\frac{1}{1} \div \frac{1}{3}$. The next stage would be to keep the first fraction and invert the second fraction and change the sign between them to multiplication, $\frac{1}{1} \times \frac{3}{1}$. The next stage is to multiply numerator by numerator, and denominator by denominator, $\frac{1 \times 3}{1 \times 1} = \frac{3}{1} = 3$. However, teacher goes on to solve this particular complex fraction differently first by creating a new denominator in the top fraction, by replacing 1 by, $\frac{3}{1}$



Then in lines 55–57 T2 offers 'far by[6] far, near by near' as the way to proceed solving the given complex fraction. 'Far by far, near by near' in Farsi is an idiom which refers to the process of simplifying a complex fraction. It simply means the product of the two numbers furthest apart over the product of the two numbers closest to the main division line in the centre, which becomes 3.

The teacher transformed a complex arithmetic process into an ornamental use of impressive words in poetry to illustrate the process of solving complex fractions. Far by far, near by near is an idiom that is extensively used for solving complex fractions in the Iranian complementary school, which contains poetic imagery and musical weight. Due to its prosodic form of verbal expression, the phrase 'far by far, near by near' in Farsi helps one to be engaged with the pedagogic possibilities in solving complex fractions. That is, it helps one not only to be able to see the corresponding arithmetical process, but also how to simplify and solve complex fractions. 'Far by far, near by near' encapsulates the process of arithmetic and helps to memorise and recollect the overall teaching of the lesson. The use of proverbs, idioms and expression in classrooms by teachers can provide a bridging pedagogy to connect abstract mathematical topics and everyday practices. Bahmanyar (1981) stresses the 'condensed expression', 'attractive meaning' and 'striking comparison' in each proverb. Moreover it has been observes that "the condensed structure of a proverb makes the remembrance and recollection of the maxim easier and at the same time increases the emphasis of its message" (Hadissi, 2010, p. 601).

The bilingual orientation of this complementary school mathematics classroom provides a space for the teacher and the learners to draw upon their linguistic resources to talk about complex fractions differently as they would in English. The classroom culture of this particular lesson allowed the interlocutors to incorporate their rich cultural identity and life experiences into their formal schooling by means of code-switching. That is, a different kind of resource which is available in a bilingual classroom and used between learners and teacher.

On a separate occasion, I carried out an informed observation into one of my key participant' (Sarah)[7] mainstream mathematics classroom. This mainstream school is a monolingual community of practice, where only English was used as the medium of instruction. The aim of this specific lesson was to identify the equations of straight lines and to be able to sketch linear graphs from their corresponding equations. The classroom teacher gave an example on the whiteboard to the class to clarify what students were expected to do. The question was to find the x-intercept of the line $y = 3x–2$. The classroom teacher solved this equation as follows:

[step 1] $0 = 3x–2$ [step 2] $3x = 2$ [step 3] $x = \frac{2}{3}$

Students were then given several equations of straight lines and they were asked to sketch and work out where each crossed the x-axis. Everyone started the task at the same time. After a short time, Sarah raised her hand and announced that she had finished the task when all the other students in the lesson were more or less half way through the given task. I was curious to know whether she was doing the same task or her results were correct.

What I noticed was she employed a different method in finding a numerical value for 'x', compared with what others in the classroom were using. Sarah employed a method which was commonly used in her complementary school by both teacher and students, which was much shorter. She used 'far by far, near by near'. I will first demonstrate the method that Sarah used and then I will show a method, different to Sarah which was used by Sarah's friend who was sitting next to her in that classroom.

The task was to 'work out where $y = \frac{1}{2}x - 5$ crosses the x-axis'. Sarah substituted '0' for 'y' as she had been shown earlier on in that lesson. She wrote: [step 1] $0 = \frac{1}{2}x - 5$ followed by [step 2] $\frac{1}{2}x = 5$ and next she wanted to reduce the coefficient of 'x' to '1' (so that to remain with an 'x' only). Subsequently she wrote [step 3] $x = \frac{5}{\frac{1}{2}}$ and finally she changed 5 into $\frac{5}{1}$ and used 'far by far, near by near' to obtain a numerical value for 'x' [step 4] $x = \frac{\frac{5}{1}}{\frac{1}{2}} = 10$. Sarah must have not only acknowledged her mainstream school teacher's method of instruction but also understood it. This is because the first two steps were exactly the same as what Sarah's mainstream teacher did e.g., substituting 0 for 'y' and taking the coefficient of '*x*' to one side. Sarah then transferred what she had already learned at her complementary school (dealing with complex fraction) and applied it into solving the equations of straight lines in her mainstream school.

Interestingly enough Sarah's peer who was sitting next to her obtained her result using a method that was employed by everyone in that classroom but Sarah. Sarah's peer could only speak English or according to some 'an uncontaminated monolingual'. Her method was as follows:

[step 1] $0 = \frac{1}{2}x - 5$ [step 2] $\frac{1}{2}x = 5$ [step 3] $x = \frac{5}{\frac{1}{2}}$

[step 4] $x = \frac{5}{\frac{1}{2}}$ [step 5] $x = \frac{5}{1} : \frac{1}{2}$ [step 6] $x = \frac{5}{1} \times \frac{1}{2} = 10$

I am interested as to whether Sarah's peer could go straight from step two to step four. If so, the question as to what extent Sarah's method of 'far by far, near by near' has actually influenced her peer's step three is not clear. But what is evident here is how Sarah drew upon her resources and incorporated what she already learnt at her complementary school in order to complement her work in her mainstream mathematics lesson. Sarah incorporates a different pedagogy that not only bridges aspects of previous histories and experiences into formal schooling but depicts how complex fractions can be seen and solved differently. She solved an equation and found a numerical values for '*x*' in less than 15 seconds in four steps as opposed to her peer who took nearly twice as much time completing the task in six steps.

CONCLUSION

Bilingual learners have different levels of language proficiencies. Within an educational context their bilingualism can impact on at least two issues concerning their learning. Firstly by how much do bilingual learners benefit from the instructional information or the procedural talk given by the teacher that takes place in a teaching session (Saxe, 1988)? Secondly, how and by how much does the interplay of different languages assist in promoting learners' comprehension? The results from this study suggest that within the bilingual context of complementary schools, bilingual learners may come to understand very quickly the positive message that complementary schools convey as they draw on their linguistic repertoire interchangeably to convey meaning. In this study learners' and the teacher's code-switching served as a powerful mechanism to construct meaning and mediate understanding. Furthermore teachers in complementary schools constantly code-switched to include a wider range of audience, and hence the instructional information that was given would have been recognised by a larger number of students (see Farsani, in press). This is in contrast to mainstream 'English-only' schools that clearly are restricted in this area (Cummins, 2005).

The bilingual orientation of this complementary school mathematics classroom developed a different pedagogy; a bilingual pedagogy whereby it provided a space for British-Iranian bilingual learners to incorporate not only their languages, but aspects of histories and experiences of how complex fractions are solved in Iran. Furthermore I have shown one example where the bilingual orientation of complementary school not only offered a perspective on how complex fractions can be seen differently, but how this knowledge can be transferred in different tasks and settings.

Other important questions still remain such as 'whether mathematics in different languages means different mathematics?' (Barton, 2008), or having access to more than one linguistic resource may afford different perspectives on how mathematical ideas can be seen? But they must wait for further research.

NOTES

1 E.g., due to wars, political factors, economical issues and religious practices.
2 Although monolinguals often 'style-switch' (García, 2009) as they "have access to a range of ways of using language and languages, which depend on who we are speaking with or writing for, what we are writing or talking about and the purposes of our oral or written interactions" (Naldic, 2009, p. 6).
3 This includes prosodic features (like stress and intonation) and other paralinguistic features like change in volume of speech (speaking louder or softer) or change of pitch (speaking higher or lower).
4 Modes are meaning materials that are culturally and socially constructed.
5 Collier (1995:235) defines proxemics by "how people regulate themselves in space and how they move through space".
6 In this case, 'by' is used in translation as a preposition of multiplication and not division. For discussion on the pragmatics of preposition in mathematics see Zagorianakos and Farsani (2012).
7 Psudonyms have been used throughout this chapter. Sarah is 14 years of age.

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16. THE EVOLUTION OF MATHEMATICS TEACHING IN MALI AND CONGO-BRAZZAVILLE AND THE ISSUE OF THE USE OF FRENCH OR LOCAL LANGUAGES

INTRODUCTION

In many African countries, the post-colonial period has been characterized by the urge to implement a new educational policy. Since the 1980's, many have begun reformulating their own curricula. Yet, the influence of the French and Belgian curricula in French-speaking Africa has survived long after the independence of these countries, not only in terms of content but also in terms of the official teaching language.

Here we analyse a few aspects of the changes from 1992 on. They concern the curricula of the early years of secondary education in two French-speaking countries, Mali and Congo-Brazzaville. Our research is based on the following questions:

- What are the curricula?
- What textbooks are used?
- What do the curricula and the textbooks teach us about the changes in the teaching of Mathematics?
- Mali and Congo-Brazzaville both use French as the official teaching language in secondary schools. Are local languages taken into account by these two countries?

The answers to these questions give us information about the official goals of educational policies. These deal with the requisite knowledge for a society in a specific socio-cultural and political context. They also promote the process of acculturation suited to a multi-linguistic environment. The answers also cast a light on what is at stake through these educational policies.

THE HARMONIZATION OF MATHEMATICS CURRICULA (HPM)[1] AND THE CIAM[2] COLLECTION OF TEXTBOOKS

In the early 1960's, most French-speaking countries in Sub-Saharan Africa became independent. In almost all of these countries, the period following their decolonization was characterized by the urge to take control of their own future. In this respect,

A. Halai & P. Clarkson (Eds.), Teaching and Learning Mathematics in Multilingual Classrooms, 249–266.

education played a major role, and the design and the implementation of educational curricula were at the heart of their development.

However, the educational curricula, based on those used in France and directly transposed from them, left very little opportunity for the primarily rural (Francophone) African socio-cultural context to be taken into account. According to Touré, the situation was as follows:

After many years of independence, it became clear that these methods had major drawbacks, because these curricula were not suited to the socio-cultural situations in these countries. Among these drawbacks were:

- the excessively abstract nature of the axiomatic approach for young Africans accustomed to audio visual messages that are connected to their rural world.
- the difficulties caused by the teaching of mathematics in a second language.
- the pedagogical problems arising from the socio-cultural diversity of these countries. The consequences of such diversity for the understanding of mathematical concepts.
- the teachers' lack of appreciation of the mathematics background of the societies in which they are teaching.
- the mixed ability classes, the shortage of teachers, their insufficient training to tackle the problems they are confronted with. (Touré, 2002, p. 175)[3]

This assessment partially coincided with the development of research in ethno-mathematics which regards mathematics as a form of knowledge socially and culturally constructed in a specific environment. The notions of ethno-mathematics emerged with the difficulties encountered in the teaching of mathematics in non-Western countries.

Two main instigators of ethno-mathematics were Bishop and D'Ambrosio. Bishop's (1990) aim was to demythologize the universal character of Western mathematics and reveal its role in supporting cultural imperialism. In the same way, D'Ambrosio (2001) postulated the existence of "different ethno-mathematics, each one responding to a different cultural, natural, social environment." He recalled that:

> One of such environments – the Mediterranean basin – gave origin to the ethno-mathematics, which we now call simply mathematics. Through the process of conquest and colonization this mathematics was imposed on the entire world. It was accepted because of its success in dealing with the ways conquerors and colonizers managed property, production, labour, consumption, and values. (D'Ambrosio, 2001, p. 67)

Toure (2002) listed the role of French-speaking African mathematicians as a drawback in the development of the new curricula suited to the African socio-cultural context. Inspired by the aims of the "Counter-Reformation" of modern mathematics in France in the 1980's (Bulletin officiel de la République française, 1977, 1981, 1985), which rejected the overly abstract nature of the axiomatic approach, the designers of the African educational programs borrowed a number of elements from the French educational programs. However, this notion took some time to mature and be implemented in African countries.

French-speaking African mathematicians, showed their will to take in hand the teaching of mathematics in the African environment early in the 1980s. This movement was based on the idea of harmonizing the curricula in the French-speaking countries of Sub-Saharan Africa and the Indian Ocean region. A common nucleus of programs for the first and second cycles was gradually developed. This was made possible, thanks to the political will manifested by the countries participating in this process, by means of the seminars which were held from 1983 onwards and thanks to the financial and human support lent by the French and Belgian overseas development agencies.

The Abidjan conference in 1992 marked the true beginning of the Mathematics Curricula Harmonization project (HPM). Common curricula were developed for all classes and sections although each country retained a degree of freedom of action. At first seventeen, and then twenty[4] countries including Mali and Congo-Brazzaville, participated in the HPM project.

International frameworks were gradually set up to ensure the further advancement of the project. Among them were the following:

- the organization of an international seminar every year from 1992 to 2002 (the last year that the French Ministry of Foreign Affairs supported a project which included HPM);
- the creation of the CIAM collection of textbooks, coordinated with the harmonized HPM curricula (discussed in various sections below);
- the creation of the PRAPs, (*Pôles de Réflexion et d'Animation Pédagogique*), bringing together inspectors, teachers from *Chantiers pédagogiques*, and sometimes teacher-trainers. (The PRAPs were the only organizations with operating budgets.)

Some of the centres, located in the countries participating in the project, had the task of developing, centralizing or diffusing information concerning specific problems. They studied ways to "renew and contextualize the curricula … enhance the scientific options for the baccalauréat,[5] … (and) identify and promote the training locations and mechanisms in this sub-region" (Malonga, Mopondi, & Denys, 2006). Thus Mali hosted the PRAP for Interdisciplinary Studies, and Congo-Brazzaville hosted the PRAP for Transition from the Primary to the Secondary Cycle.

Both Mali and Congo-Brazzaville participated in the final HPM seminar in Bamako in 2003. Its main theme, "New Technologies of Information and Communication and the Teaching of Mathematics" launched by Touré, showed that these two countries continued to be part of the international realm of mathematics education. The project overall highlighted a strong commitment to collaborate, and standardise the programs. But, crucially, the question of multilingualism was avoided.

SOME FEATURES OF THE REFORMED HPM CURRICULA

The organisation of these reformed curricula contrasts starkly with that of the earlier curricula, which represented the transposition of the French curricula of the 1980's. In fields such as algebra and geometry, taught in the fourth year of the first cycle, the curriculum consisted of lists of basic concepts. For instance, what was required in studying algebra at this level were simply listed as: square roots, […], first degree equations and inequations with two unknowns; and in the case of geometry, the Thales theorem, the multiplication of a vector by a real number, and orthogonal projections.

The HPM curricula for the first cycle (first to fourth years of secondary school) were organized in two main areas:

- geometric activities (configurations of space, configurations of planes, applications of planes, vector tools and analytic geometry);
- numerical activities (arithmetic, organisation of calculations, number calculations, literal calculations, data organisation).

In contrast to the earlier derived French curriculum, the HPM curricula were organized in three columns, with comments: this was an innovation. An example from the fourth form (third year) is given in Table 1.

Table 1. Excerpts from programme HPM (1992, p. 28)

Equations, inequations Equations reduced to the form *ax+b=0*, in Q […]	Solve in Q a linear equation in the form *ax+b=0* […]	The solving of equations and inequations will be applied to the solving of problems related to daily life

The introduction of a unit on "data organization" using proportionality and its applications (scales, percentages) in statistics was new and innovative both as an area of study, and in terms of the subjects that were taught (statistics, most notably). Based on these curricula, the textbooks in the CIAM series corresponding to the HPM curricula illustrated the intent of the editors, who came from all of the countries participating in the HPM project. They took into account the African socio-cultural milieu (Touré, 1994). Thus since 1993, Touré who wrote the preface for every work, for all classes and sections, emphasized one of the major objectives, from the point of view of the African mathematicians. For example:

> To harmonize the pedagogy of mathematics and to make available to African students and teachers high-quality textbooks that take into account the African socio-cultural environment as both the medium and the preferred vehicle for mathematical concepts. (Touré, 1994, p. 3)

The other objectives consisted of giving students access to a good grounding in mathematics and the modest price of these textbooks was to enable each student to get a book. The distribution of these works and their successive new editions testify to the effects of this large-scale movement initiated by the French-speaking African mathematicians. However the influence of the French programs and the absence of consideration for national and/or local languages were noteworthy, maybe out of a concern for unity. We now move to discussing the latter of these two issues, the influence of local languages.

LANGUAGES OF INSTRUCTION AND LOCAL LANGUAGES

Mali recognizes thirteen national languages, of which Bambara is the most widespread. In Congo-Brazzaville, the languages used are Bantu languages, which include two national languages, Kituba (or Kikongo) and Lingala. The 1992 Abidjan conference defined the objectives and goals of the teaching of mathematics without taking into account the difficulties arising from the use of a foreign language. For pragmatic reasons at the time, the conference kept French as the language of education, but did express the desire to take into account the African socio-cultural environment. Although to some this may seem a paradoxical reasoning, one explanation was that French was adopted to reduce the cost of subsequent publications.

However there were other reasons for this decision too: for the French-speaking African mathematicians who initiated the project, the universal nature of the mathematics to be taught did not require the use of mother tongues (e.g., Touré, 1994). Even more simply, in the given environment, considering the political, economic and social conditions, the question of transposing the pedagogy and language of a defined corpus of mathematics could not arise. In the 1990's, therefore, the consideration of socio-cultural context was a major question but not the use of national languages other than French for teaching.

TWO ATTEMPTS TO RENEW MATHEMATICS EDUCATION: MALI (WEST AFRICA) AND CONGO-BRAZZAVILLE (CENTRAL AFRICA)

By adopting the HPM curricula in their official texts, both Mali and Congo-Brazzaville emphasized educational situations, which took into account the culture and the socio-cultural milieu of the students. This was assimilation and not acculturation (Bishop, 1993). The student extracted the mathematical model from a familiar situation and conceptualized it (e.g., game of N'Gola and Euclidian division, cf. Mopondi & Bantaba, 2012). The student was called on to integrate his or her learning at school with his or her daily life experience.

Apprehending and capitalizing on the potential of a cultural split between mathematics education and native culture in terms of assimilation were challenges faced by the HPM curricula and by those developing the educational policies of the two countries. Our goal is therefore to identify their effects on mathematics education in the two different contexts of Mali and Congo-Brazzaville.

Mathematics as Taught in Secondary Schools in Mali Since 1990

In the 1990's, Mali stood out for two reasons:

- the educational system offered a second cycle of *Enseignement fondamental* (ages 13–16) followed by the *Lycée* (ages 16–19), which did not correspond to the HPM divisions (*Collège* for those aged 11 through 15 and *Lycée* for those aged 15 through 18).
- the official curricula for this first stage of the second cycle were published in June 1990 by the Institut Pédagogique National under the sponsorship of the Ministry of Basic Education and manifested educational, social, economic and utilitarian goals. In the case of mathematics, these curricula were distinct from the HPM curricula. Paradoxically, they seemed to transpose the French curricula of the 1980's, which was divided into two fields, algebra and geometry. These two fields emphasized the structures of number sets and the evolution from plane geometry to analytic geometry.

The formal character of this mathematical teaching was reinforced by the absence of connections that might have been made with the 'Practical Activities of Ruralisation' curriculum.[6] The mathematics curriculum lacked the utilitarian aspect of mathematics, which could have made it possible to view how it functioned in the social and economic development of the society. The textbooks published under the aegis of the Ministry (which were different from the CIAM textbooks) reflected this lack of connection to the general educational goals.

One example worth considering follows: "The calculation of proportions" in the 1990 curricula (Programmes officiels, juin 1990, p. 56) and related textbooks (Mathématiques 8[e], 9[e]). In the curriculum of the 8th form (second year of basic education) "the calculation of proportions" (Programmes officiels, juin 1990, p. 56) was integrated into the unit on rational numbers in terms of the application of the property of equal quotients. In the 9th form (the third and final year of basic education), the unit on real numbers (p. 59) took up this theme again in the unit on real numbers, at a different level of conceptualizationin terms of series of proportional numbers, and then again under the unifying title of linear applications. In the 8th form textbook, which was supposed to "lead to pedagogy based on the student's activity" (p. 3), the pedagogical schema was based on a model of "activities introducing techniques allowing the execution of tasks, "how to test the equality of two quotients", "if three numbers in a proportion are known, how to calculate the fourth", and exercises (pp. 132–135). The activities could involve certain aspects of real life (for example, the price of meat per kilogram), but at no time was the student asked to model a situation or take initiative. The knowledge applied was that of the French programs. No link was made with everyday life however easy it would have been.

The shortcomings of this educational policy were highlighted in the 2000s (Vellard, 2009, pp. 124–138). More than two thirds of the pupils of 8 and 11-year-old tested had an insufficient level in French and in mathematics. At the same time, the increase in the number of pupils led to oversized classes, and the content of the teaching did not meet the needs of potential employers. The weaknesses of the educational system prompted the Malian authorities to initiate a reform of the curriculum starting in the 2000's. This reform promoted an active model of learning which lasted for almost twenty years in elementary education (6 to 14 years of age), that of "Pédagogie convergente"[7] (Traoré, 2001). The goal of this active method of learning languages was to develop functional bilingualism, thereby justifying the use of national languages as the means of teaching in some classes. This had an impact on the way mathematics was also taught.

The APC (Approche Par Compétences)[8] (cf. Appendix 1) was essential in this process. Throughout the continuum of nine years of schooling (from 7 to 15 years of age) mathematical skills consisted of a combination of two aspects:

- reading, writing and communicating using the mathematical language and symbols.
- solving situation-problems using acquired mathematical knowledge, capacities, and skills.

These pedagogical methods under 'PedagogieConvergente' hardly seem to have been compatible with the mathematics curricula dating from the 1990's. The pedagogical methods, which seemed essential, required the reorganization of the mathematics curricula.

Gradually, the curricula – which were still in French – evolved, and the division between algebra and geometry disappeared. The gradual rewriting of the curricula using specified competencies, learning objectives and learning contents also gave an opportunity to gradually integrate the algebra and geometry ideas. For example consider Table 2 that presents excerpts from the program for the seventh form, the first year of fundamental education (students 13 years of age).

The concern evidenced by the designers to emphasize the students' mathematical activities seemed innovative. To be sure, the content remained dependent on the earlier curriculum, but the stress on mathematical language, and in particular the approach of solving word problems seemed to be a step forward. The conception of modelling used in the curriculum allowed notions from the students' real world to overlap with their mathematical world.

The example from Mali thus illustrates a process in progress (the rewriting of the curricula and the implicit use of bilingualism) in which pedagogy can lead to the reform of the mathematics curricula in terms of learning methods, if not totally in terms of the reform of content. In this process, pedagogy, and notably the use of functional bilingualism as a vehicle of learning, can only lead to the reform of the mathematics curricula and to a retreat from formalism. However this way of using bilingualism has not called into question the use of French in the teaching of older students. That is a question for the future.

Table 2. Excerpts from "Programme formation de l'enseignement fondamental niveau 4 1ère année", pp. 4–5

Competencies	*Learning objectives*	*Learning contents*
UA 6 Read, write and communicate using mathematical language and symbolism	Read and write decimal numbers	Decimal numbers No constructive introduction from examples Writing decimal numbers as floating point numbers Absolute value
Solve problem situations using mathematical knowledge, skills, abilities	Solve problem situations related to addition, subtraction, multiplication and division	Operations: addition, subtraction, multiplication, division Properties of these operations Quotient nearest 0.1; 0.01; 0.001 per excess or per default of division of two integers, or two decimal numbers. Writing decimal numbers as decimal fractions
	Construct geometrical configurations	Circle: construction and definition Disc: construction and definition
	Perform calculations on measures	Circle perimeter Disc area
	Apply problem solving approach related problem of learning unit	Problem solving approach Decoding Problem-situation modeling (comparison between situation and a solved similar situation) Different strategies enforcement Information sharing related to situation; problem solving approach enforcement

Mathematics as Taught in the Collège of Congo-Brazzaville

The general secondary school educational system consists of *Collège* (for students 11–15 years of age) and *Lycée* (for students 15–18 years of age) in *Congo-Brazzaville*.

The curricula implemented in the Congo-Brazzaville beginning in the 1990's were part of the HPM movement and employed pedagogy by objectives (defined as pedagogy aiming above all to define objectives that could be easily quantifiable and observable). The new curricula, published in 2002 by INRAP (Institut National de la Recherche et de l'Action Pédagogique), conserved the basic content of earlier curricula but moved away from the transmissive approach without usingthe mother tongue as a crutch.

These curricula result from the desire to move away from "the traditional approach" (theory – exercises – application) in order to focus "not only on the acquisition of specific knowledge, but also on their integration into the real life of the student" (Programmes des enseignements mathématiques, 2002, p. 8). They emphasise how mathematics can teach: beyond the notional content, what matters is "the power of networks of capacities to develop. What results is this paradox: the student learns to solve problems, which he has not learned to solve. He does so by means of approaches and successive steps, and thereby constructs mathematics for himself […]. This is the so-called "mathematics of action" which allows students to "build meanings in contexts and not to confine themselves to processes of deduction" (pp. 8–9).

These new programs are designed principally to meet problems related to teaching methods that are to transmissive in nature, rather than deal with the difficulties related to oversized classes or to the mastery of a foreign language of instruction.

The objectives of the new curricula are divided into *General Objectives* (OG; the final performance expected of the student in a particular domain of learning) and *Specific Objectives* (OS; the level at which a general objective is achieved). Table 3 provides a synoptic view of these objectives.

For example, the general objective "Become familiar with numbers", which is shared by all four levels, is divided into three specific objectives in the 4th form: identify a number, recognize the notation of a decimal, and identify a proportion. These specific objectives lead to conceptual content: the whole power of a relative whole number; logarithms of natural whole numbers: definition and properties; rational numbers […]; real numbers […]; the definition and properties of proportions. Although this way of organizing a curriculum is hoped to give support to progressivity coherence and a progressive stance to education, nevertheless the conceptual content is not accompanied by any comments concerning situations or potential contexts of learning, and that may be a definite drawback.

Another drawback with the new 2002 curriculum was that it no longer corresponded to the textbooks in the CIAM series, and hence the main resource in

Table 3. Excerpts from Programme des enseignements mathématiques (INRAP, p. 10)

No	*General objectives*	*Classes*			
		*6^e^*	*5^e^*	*4^e^*	*3^e^*
1	Become familiar with numbers	2 OS	3 OS	3 OS	2 OS
2	Perform numerical activities	7 OS			
	Perform numerical calculations		4 OS	4 OS	3 OS
3	Become familiar with geometrical configurations and transformations	4 OS	2 OS	5 OS	5 OS
4	Perform geometrical activities	6 OS	5OS	4 OS	4 OS
5	Perform activities in a plane defined by orthonormal coordinate				5 OS
	Perform marking activities	3 OS	3 OS		
	Perform activities in a plane related rectangular coordinate			4 OS	
6	Become familiar with data tables	2 OS			
	Make use of data tables		2 OS	4 OS	5 OS
	Organise data			5 OS	3 OS
7	Perform activities with vectors			5 OS	3 OS
8	Perform functions studies	6 OG	6 OG	8 OG	8 OG

schools used by teachers to support a curriculum was brought into question. Further the desire of the designers of the HPM curricula to place the mathematics being taught in its African socio-cultural context was not made explicit in these new 2002 curricula. The following example shows how there has been a definite disconnect between the curriculum and the supporting textbooks used by teachers.

The introduction of a new concept at the Collège level is the number logarithm to base ten. The need for this was seen to be a need from the chemistry curricula (the level of acidity or alkalinity of aqueous solutions). In the chemistry curriculum of the 3rd form, pH makes possible the measurement of the concentration of hydrogen ions in an aqueous solution. The notation is $[H^+] = 10^{-pH}$. One of the problems to be solved by the students consists of finding the value of pH when the concentration of H+ ions is known (expressed in moles per litre).

The mathematics curricula, both in 2002 and 2009 of the 4th form of Collège (students 14 years of age), introduce logarithms to the base ten for the first time: they were treated as a *new number* to identify. The teacher's guide for these curricula states (p. 36): "The student should recognize that […] logarithms are numbers defined by powers: if $a > 0$ and $a = 10^n$, then $\log a = n$". This was one year after the use of this notion in the chemistry curriculum, which was the stated need for this concept in the mathematics curriculum: clearly there is a disjunction of ideas at this point.

For each Collège level, the curriculum used one textbook (e.g., Collection Mathématiques, 4e, Nathan). In the third form textbook, logarithms to the base ten were introduced by means of a reading a passage and the use of a table of 20 values of x and 10^x without specifying that the values of 10^x are approximate values. There was no real connection with the chemistry tasks to be tackled. It was therefore the teacher's job to make any references to the chemistry exercise. There certainly was no reference either to any context of the students' own lived lives.

The mathematics curriculum developments in the Congo have lead to a process by which the transformation of teaching methods is happening, although the notions of inter-disciplinary is certainly still fragile. The example given does not talk about learning methods. If the goals of the curricula emphasized the necessity of integrating the students' scientific knowledge with their real lives, on the path to learning, which must be the rediscovery of ideas as tools for making the world intelligible, the organization of the curricula and the textbooks imply that the conception of mathematics education was not connected to the cultural context, education was de-contextualized. The stance taken by Touré at the last HPM conference in Bamako, concerning the use of new information and communication technologies in teaching, may confirm this hypothesis (Malonga, Mopondi & Denys, 2006). The developments in the mathematics curriculum in Congo have not reached the point, unlike in Mali, where the questions of using local languages and the implications that flow from this, in mathematics have been posed.

DISCUSSION

In this section of the chapter we try to answer the following questions:

- What do the curricula and the textbooks teach us about the changes in the teaching of Mathematics?
- How can we interpret this evolution by referring to ethnomathematics works?
- What are the choices of each these two countries? What perspective for a mathematics education in local languages in light of led initiatives?

An Inventory, the Current Situation with Respect to Curricula and Textbooks

Since 1990, the learning conditions of mathematics education have evolved both in Congo-Brazzaville and in Mali. The rewriting of the curricula (and in Mali, the resort to bilingualism) shows a new political awareness: education depends on its societal environment. Education is a priority, which cannot be considered without taking into account socio-economic and political constraints.

The official goals of the HPM curricula and resources (i.e. essentially the CIAM textbooks which have enabled the development of mathematics education in the French-speaking African countries) are primarily political: they are to meet the need for knowledge in non-western societies. But, paradoxically, the content of the mathematics curricula, their requirements bear witness to the staying power of an instructional text whose structure is that of the Western model. The use of French as the teaching language of mathematics is a fact in the curricula (though in Mali its introduction is gradual).

The innovative aspect of these curricula and textbooks lies in the way they have taken into account the socio-cultural environment of the African students. And yet, in spite of the fact that they rely on situations based on reality and the student's everyday life, the rooting of this mathematics in the very culture of these African countries has not been emphasized, and is still scarcely noticeable. We may suggest that the movement inspired by French-speaking African mathematicians – and which continues to this day – refers to what in the eyes of these mathematicians, is a de-contextualized corpus of knowledge (universal mathematics), which would need to be re-contextualized in ad hoc social practices.

Link between This Evolution and Ethno-Mathematical Research

Ideas put forward by Gerdès (1997) and D'Ambrosio (2001) have influenced the views of the designers of curricula and textbooks. These ideas include that the learning of mathematics should be based on social practices, which implies the daily experiences of the students should be linked to their mathematical learning, something rarely seen in Africa. Sadly however, there is no explicit mention by them, which highlights the contributions made by Africans to the development of mathematics, and hence it is no surprise they do not give examples from Africa as to how this linkage can be advanced. Some advances have been made however. For example the work of the ethno-mathematicians to rehabilitate the African heritage by deriving from a social practice the knowledge of geometric properties that may be reformulated into formal theorems (for example the construction of rectangular huts), may be an avenue that is worthwhile exploring (Gerdes, 1993). However to date such possibilities have not been linked to the curriculum and textbooks at our disposal in Mali and Congo-Brazzaville. The reconstruction of a curriculum, based on the de-contextualization of cultural practices from which mathematical models have been extracted and on their re-contextualization into teaching situations, cannot be identified in the HPM curricula, the CIAM textbooks or the curricula and textbooks now being used in Congo-Brazzaville and Mali.

As for the content of what is taught, our reading highlights the persistence of a core of universal knowledge, which is not based on the organization of knowledge advocated by Bishop (1993). The universality of the questions underlying common cultural activities meaningful for all students, i.e., "counting, locating, measuring, designing, playing, and explaining" (Bishop, 1988, pp. 182–183) should make it possible to design curricula suited to the development of universal mathematical notions.

These choices, which have important implications for thinking about and subsequently designing curricula, teacher education, temporal and financial measures, do not seem to be part of plans for educational policies in our regions of Africa, and probably for much of Africa. At least, we put this forward as a hypothesis. The socio-cultural contexts are obviously taken into account, but the interpretation of the educational strategies that they imply is not the same in Congo-Brazzaville as in Mali. The challenges are quite different.

In Congo-Brazzaville, the first cycle of general secondary education is designed "to give rise to the theoretical and practical knowledge necessary for the further pursuit of studies" (article 16 of the 2009 curricula). It is *propaedeutics* (preparatory study) for the Lycée. In Mali, the basic education system, created by the 1962 reform promotes mass education, top quality education as opposed to education limited to the staff members of the administration (inherited from the colonial period).

If we refer to Bishop's classification of educational circumstances (1993), the HPM curricula constituted a first break away from the traditional vision of de-contextualized mathematics, which did not take into consideration the split between the student's culture and that of the school. The process of acculturation that was supposed to result was to be based, on the existence of a gap between the two cultures, the transition from one to the other being made without major difficulty.

Taking into account the socio-cultural context in the HPM curricula and the CIAM textbooks makes it possible to consider an educational model, which would borrow some of its characteristics from the assimilation model (Bishop, 1993, p. 8). In this model, the student's culture can be used by way of example with the curriculum referring to cultural contexts where possible. In this formulation, the student is seen as an individual, the official language is still in use with remediation in the mother tongue if necessary.

In Congo-Brazzaville, the evolution, which we have briefly described, seems to indicate a return to a more traditional vision. On the other hand in Mali the educational model seems to be evolving towards what Bishop (1993, p. 8) refers to by the use of the term accommodation. In Mali the culture of the students has some influence on their education, and hence the curriculum has been restructured to some degree according to the culture of the students. As well the mother tongue is used up to a given level of studies. But we cannot, however, adopt the point of view that in Mali – at least officially – mathematics education vis. the product of bicultural acculturation, the meeting of two cultures, the synthesis of enculturation (initiating the student into one part of his own culture) and acculturation (initiating the student into a culture that is foreign, in a certain sense) (Bishop, 1988, p. 9). The persistence of a corpus of mathematics, which is strictly Western, is noticeable in the tests, which assess the mathematics taught in the schools.

Issues in Perspective

The issues faced by mathematics education at the middle school level thus seem different in Congo-Brazzaville and Mali during this long period of development. The study of the links between curricula in Congo and in France reveals that the influence of use of French language in mathematics education in Congo makes Congolese education less sensitive to the development and the potential of research in ethnomathematics than to the development of a mathematical rationality having no connection with cultural contexts. In viewing these curricula and their official aims, we cannot perceive a political will to take into account the difficulties particular to the Congolese environment (multilingualism, socio-cultural context).

In Mali, the principle of a new endogenous education, designed to respond to the difficulties faced by the educational system (Vellard, 2009), reveals the political awareness of the educational issues and the major role given to pedagogical standards. The reform of the curriculum was based on the implementation of convergent pedagogy, which, by eliminating the obstacles connected with the use of French (decreased learning, mental blocks), was supposed to facilitate the learning of instrumental disciplines while increasing the value of national languages. The process that was implemented (about which we only have information up to 2012, because of the current unrest in the country) is evidence both of the deep involvement of politics in the universal access to education and of the questions posed by the execution of this reform. Thus Maurer (2007, pp. 426–430) noted the necessity of developing convergent pedagogical practices in order to ensure the coherence of a curriculum based on an approach by competencies. The question of the appropriation of mathematics into an Africa language is still open.

Segla's work (2001) examined the obstacles that cannot be overcome in the mother tongue (the Yoruba language in Benin): these obstacles are connected with the creation of concepts and vocabulary. Kanouté's studies (2000) analysed the difficulties inherent in the simple translation of a French statement into Bambara (in Mali). There is no transfer of knowledge from Bambara to French, but rather a change of terminology from the French; a term that is translated is not necessarily a term that is understood; a problem that is translated is not necessarily a problem that can be solved. We presume that these studies might lead to new directions for future curricula.

In conclusion, the different paths of evolution in Mali and Congo-Brazzaville since 1992 show that the mathematics curricula harmonization project (HPM) has led the two countries to teach mathematics in a way which takes into account, to a lesser or greater extent, the difficulties encountered in their educational systems and their socioeconomic contexts. However, both the Congolese and the Malian curricula bear witness to the persistence of an educational discipline (mathematics as taught in the first years of French secondary education) produced by a Western educational system.

How far can African mathematics evolve? The teaching of mathematics in French speaking Africa is going to keep evolving. African researchers in mathematical education, African mathematicians, African politicians involved in education need to speak to one another (Mopondi, Malonga, & Denys, 2010). Nevertheless the political situations and the educational challenges in Western and Central Africa have their own specificities, as we have tried to show in this paper. These are significant factors in helping mathematical education to adjust to the cultural context of their countries. It is difficult, at this moment in time, to go into more details as to which direction this evolution will take.

One core issue remains, that of endogenous mathematics education in local languages? Is it possible that a mathematics teaching in national languages – at the middle school level – might be seen as a way of overcoming the difficulties involved in developing mathematics in French-speaking Sub-Saharan Africa? But then another question arises: how to make links between the nature of mathematics itself, the way it relates to language, and the extent to which it relies on language? But these are questions for future research.

NOTES

[1] HPM: "Harmonisation des Programmes de Mathématiques en Afrique" in French.
[2] CIAM: "Collection Inter-Africaine de Mathématiques" in French.
[3] Translation by authors.
[4] Benin, Burkina Faso, Burundi, Cameroon, Central African Republic ("RCA" in French), Union of the Comoros, Congo-Brazzaville, Ivory Coast, Djibouti, Gabon, Guinea, Madagascar, Mali, Mauritania, Niger, Democratic Republic of the Congo ("RDC" in French), Rwanda, Senegal, Tchad, Togo.
[5] High school examination qualifying for entry to university.
[6] 'Practical Activities of Ruralisation' deal with farming activities such as: calculation of the dimensions of a field; keep records of income and expenditure; practice crop rotation, the amendment, the fallow; sell plants; harvest and sell the products of the orchard; keep a holding register (under reforestation); calculate and comply with food rations; sell livestock products; prepare a monthly report, quarterly and annually on livestock activities.
[7] Convergent pedagogy in English: a method whose goal was to facilitate the passage from the mother tongue to French, based on the use of techniques of expression and communication, and especially those concerning the use of the written word in the mother tongue first.
[8] Skills-Based Approach in English.

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APPENDIX 1

The Malian Concept of the Skills-Based Approach (excerpts from official passages)

In their general principals the curriculum designers repeat their goals: to create a patriotic citizen who builds a democratic society, a participant in development who is profoundly anchored in his culture and open to universal culture, a modern man who possesses the skills associated with scientific and technological progress. The exit profile of the student expresses a set of skills (Figure 1) acquired in the course of his education. It is determined by five skills connected to educational disciplines, developed during the nine years of the curriculum.

Disciplines	*Wording of the skills*
Languages and communication (LC)	To communicate orally and in writing taking into account the circumstances of the communication
Sciences, Mathematics and Technology (SMT)	To solve problems in daily life
Social Sciences (SH)	To understand the world and participate fully in the development of one's country
Arts	To express oneself through artistic production
Personal Development (DP)	To harmoniously integrate one's surroundings

Figure 1. Skills included in the exit profile

This approach requires the decompartmentalization of the disciplines and the creation of a learning situation allowing acquisition in keeping with the requirements of living and with formative assessments.

The curriculum adopts the functional bilingualism of convergent pedagogy and uses the elements of pedagogical units as the medium of the process. These pedagogical units (for learning) represent a combination of disciplinary, transverse and life skills, learning objectives, learning content, learning activities, and evaluation activities by field. The activities cover the five fields and last one month each. There are three types of skills: skills needed for the discipline, transverse skills (intellectual, methodological, personal, social and communicational skills), and life skills (attitudes for adapting to life and connecting the learning in school with daily life).