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CHAPTER NINE

Developing Mathematical Proficiency and Democratic Agency through Participation – An Analysis of Teacher-Student Dialogues in a Norwegian 9th Grade Classroom

INTRODUCTION

A central line of argument within mathematics education has been that learning mathematics provides individuals with tools to make considered choices, and that developing mathematical proficiency is beneficial because it informs human individual actions. In line with this argument, it is claimed that a mathematic literate population will contribute to society's political, ideological, and cultural maintenance and development, and as such, strengthen a nation's democratic processes (Niss, 1996).

Fostering democratic citizens is an important overarching educational goal in many countries and training students in communicational processes is considered to be one of the ways of achieving such a goal (L 97).[i] Communicational processes means providing opportunities for participation in social interactions, for sharing thoughts with others and listening to others share their thoughts.

Within the social sciences, having the ability to base one's actions on deliberate choices is expressed through the concept of *human agency* (Bandura, 2001). Building on the concepts and arguments from Bandura and Niss, in this chapter we will use the term *democratic agency* to denote the capacity to make decisions and to take actions in relation to social, cultural and political issues. In the classroom, taking part in the ongoing discussions, making judgments, formulating arguments and listening to fellow students are likely to be important elements in students' development of democratic agency.

Participating in classroom discussion, being trained in communicating one's own ideas and reflecting with fellow classmates is also seen as central to developing mathematical learning with understanding (e.g., Hiebert et al., 1997), which is said to be crucial for the development of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001). Communicating and thinking together about mathematical ideas and problems thus is likely to be critical for both the development of democratic agency and mathematical proficiency.

In Nordic curricular documents (L 97; K 06; Skolverket, 2011) there are multiple statements which emphasise the importance of developing both students' democratic agency and their mathematical proficiency. These competences are also closely linked, that is "an active democracy needs citizens that can … understand and critically evaluate quantitative information, statistical analysis and economical

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 143–160.

prognosis."[ii] Our objective in this chapter is to discuss the challenges that a mathematic teacher faces when trying to comply with this two-fold demand of the curriculum. How can students be educated/taught in order to develop both their mathematical proficiency and their democratic agency? What characterises the communicational processes in the classroom? To what degree does the teacher use student utterances to stimulate and develop mathematical proficiency and democratic agency?

Our analysis is based on teacher-student dialogues in a specific lesson in one Norwegian LPS classroom (NO2-LO1). We will also refer to statements from the follow-up teacher interview and to findings from analyses conducted on all of the thirty-eight 9th grade mathematics lessons constituting the Norwegian LPS-sample. How the teacher secures broad student participation and handles students' initiatives will be central issues in our discussion, but we will also comment upon the tasks selected by the teacher and discuss if these tasks support the development of students' mathematical proficiency.

THEORETICAL PERSPECTIVES

Socio-Cultural Theory

Within socio-cultural theory, learning is viewed as becoming a participant in a certain discourse comprising the totality of communicative activities practised within a given community (Sfard, 2000, 2006). Van Oers (2006) maintains that "learning in an activity theory approach is the extension or improvement of the repertoire of actions, tools, meanings and values that increases a person's abilities to participate autonomously in a socio-cultural practice" (p. 24).

Lave and Wenger (1991) have used the expression "legitimate peripheral participation" to account for the processes of learning by which a newcomer successively moves from a peripheral to a full participation in communities of practice. In the process of participating in collective activities Renshaw (1996) states that the opportunity to use speech with others is central to conceptual development. With regard to the learning of mathematics, Sfard (2006, p. 166) claims that:

> … the idea of mathematics as a form of discourse entails that individual learning originates in communication with others and is driven by the need to adjust one's discursive ways to those of other people.

Communication and collaborative activities as important tools for the learning of mathematics, have within mathematics education also been linked to the idiosyncratic cultural and historical aspects of this particular field of theoretical and practical knowledge. Cobb (2000) sees mathematics as a complex human activity and, leaning on Dörfler (2000), states that the task facing the teacher is that of supporting and organising students' induction into the practices that have emerged during the discipline's intellectual history. Yackel and Cobb (1996) have introduced the concept *socio-mathematical norms* to denote the normative aspects of classroom action and interaction that are specific to mathematics. They claim that these norms are interactively constituted, that they regulate mathematical argumentation, and influence learning opportunities for both the students and the teacher. The teacher is seen as a representative of the mathematical community and students' mathematical communication and reasoning viewed as acts of participation in communal practices that are established through the ongoing interactions in the classroom.

Mathematical Competence and Mathematical Proficiency

Two of the most influential descriptions of the concepts *mathematical proficiency* and *mathematical competence* have been provided by Kilpatrick et al. (2001), and Niss and Jensen (2002), respectively. The work of Kilpatrick's group has been used as a basis for informing the educational authorities in the U.S. Department of Education for the improvement of quality and usability of educational research (Ball, 2003). The work conducted by Niss and Jensen has been central to the refinement of the concept of mathematical literacy in PISA (OECD, 2009). The eight competences listed in the PISA Mathematics Theoretical Framework as constituting different aspects of mathematical literacy are as follows:

– Mathematical thinking and reasoning
– Mathematical argumentation
– Modelling
– Problem posing and solving
– Representation
– Symbols and formalism
– Communication
– Aids and tools

As noted by both Niss and Jensen (2002), and in the PISA framework (OECD, 2009), there is substantial overlap between these competences, and students are likely to draw on more than one of the competencies when solving mathematical problems. Of particular interest for this chapter are the connections between "Communication," "Representation," and "Symbols and formalism." Even though communication of mathematics does not necessarily involve the use of specific mathematical "tools," (e.g., symbols), if a goal is to develop students' communicative competences in mathematics, it is important to give them opportunities to be involved in mathematical discourses where mathematical symbols and representations are being used. This is closely connected to the central tenets within socio-cultural theory, where dialogues and participation in subject informed discourses are considered to be activities of key importance for student learning. For a student to be able to improve his/her participation in meaningful mathematical discourses, it is necessary to be introduced to mathematical symbols and formulas and to have opportunities to partake in classroom discussions applying subject specific concepts and categories.

Mathematical proficiency is a concept closely related to mathematical competence, as defined by Niss and Jensen. According to Kilpatrick et al. (2001), mathematical proficiency is made up of five strands:

- Conceptual understanding – comprehension of mathematical concepts, operations, and relations;
- Procedural fluency – skill in carrying out procedures flexibly, accurately and appropriately;
- Strategic competence – ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning – capacity for logical thought, reflection, explanation, and justification;
- Productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

As with mathematical competence, Kilpatrick et al. (2001) stress that these five strands are interrelated, representing "different aspects of a complex whole" (p. 116). They go on to argue that mathematical proficiency is a multi dimensional concept and that it cannot be achieved by giving attention to just one or two of these strands. Rather, they claim that the five strands provide a framework for discussing important notions related to mathematical proficiency, like knowledge, skills, abilities, and beliefs.

Democratic Agency

Carlgren, Klette, Myrdal, Schnack, and Simola (2006) argue that a particularly important trait of the Nordic comprehensive school system, characterised by being unified, unstreamed, and open for all students, is *individualisation* which includes the idea of the "active child" as well as attention to the needs of each child. As a result of the large school reforms throughout the 20th century, students from all socio-economic groups and with different socio-cultural backgrounds were given educational opportunities. According to Carlgren et al., teachers at this time adopted individualisation as the best way to accomplish differentiation within the mixed ability classrooms. In recent years, individualisation of working methods has been particularly widespread in Swedish and Norwegian classrooms. As a result, Carlgren et al. argue that the "idea of the educated citizen seems to have been replaced by the separated individual responsible for his/her own life" (p. 303). Furthermore, Carlgren at al. claim that individualisation is expected to strengthen each student's belonging to his/her community and his/her ability to be actively involved in civic activities. In this respect the practice of individualisation can also be considered as a tool for the development of students' democratic agency. This belief in stimulating personal active involvement can be linked to the central tenets of social-cognitive theory. Bandura (2001) contends that the power to shape actions for specific purposes is the key feature of personal agency. He argues that the challenge in collaborative activities is "to melt diverse self-interests in the service of common goals and intentions collectively pursued in concert" (p. 7).

In the Norwegian core curriculum, there are several statements aimed at defining the overarching goals for the national compulsory school system. Many of these statements stress that education must contribute to the development of personal agency as a pre-requisite for participating fully in a democratic society. These statements are closely connected to central and fundamental thoughts within western political philosophy related to beliefs in democratic institutions and civil rights. Statements related to a fundamentally scientific worldview and to the training of abilities that will secure high proficiency levels in academic subjects can also be found:

> Education in this … tradition entails training in thinking – in making conjectures, examining them conceptually, drawing inferences, and reaching verdicts by reasoning, observation and experiment. Its counterpart is practice in expressing oneself concisely – in argument, disputation and demonstration.

With reference to the last part of this statement, this is largely in line with constructivist and socio-cultural learning theories where participation is considered to be a crucial factor for students' learning (Yackel, 1995; Cobb, 2000; Van Oers, 2008). It also links the training of academic skills to key aspects of democratic agency. Students should not only get the opportunity to learn academic subjects, like for instance mathematics, but they should also be trained in actively using and communicating their knowledge socially.

Giving the students the opportunity to express their thoughts through participation in classroom discussions is strongly recommended in the Norwegian curriculum. Competency in expressing oneself verbally (oral skills) is one out of five "basic skills" in the national curriculum, (the other ones being "writing skills," "digital skills," "arithmetic skills" and "reading skills").

A prerequisite for developing "oral skills" in mathematics seems to be that the instructional formats used in the mathematics lessons are not dominated by individual work only, but also by formats supporting oral communication. Later in this chapter we will present findings related to the analysis of instructional formats across all the 38 mathematics lessons included in the Norwegian LPS-study. In the method section of this chapter, the analytical dimensions applied will be further explained, but some considerations related to the term whole class instruction will now be introduced.

Whole Class Instruction

Individual seat work and whole class instruction have been the traditional cornerstones of mathematics lessons. In TIMSS it is documented that on average about 70% of activities in mathematics classrooms in the participating nations consisted of these two activities (Mullis, Martin, Gonzalez, & Chrostowski, 2003). In TIMSS individual seat work is defined as "working on problems with or/and without teacher guidance," while whole class instruction is categorised as "listening to lecture-style presentations and/or reviewing homework."

To what degree students are given opportunities to actively participate in classroom discourse seems to be a particular interesting aspect of variations related to whole class instruction. As argued in the introduction of this chapter, communicational processes are seen as essential both for the development of democratic agency and mathematical proficiency.

Several studies have reported a high degree of teacher dominance and little student involvement during whole class instruction (e.g., Stodolsky, 1988; Hiebert & Wearne, 1993). In the analysis of public talk in the mathematics classrooms that were included in the TIMSS 1999 Video Study, the ratio between teacher and student talk – as measured in number of words spoken – was found to be quite high in all countries, varying between 16:1 in Hong Kong SAR, to 8:1 in the USA (Hiebert et al., 2003). However, lately some classroom studies have reported findings which indicate higher levels of student participation. Clarke and Xu (2008) compared patterns of utterances in mathematics classrooms in six nations. They report some interesting differences both related to frequency, to mathematical content of utterances, and to opportunities for student participation. Emanuelsson and Sahlström (2008) analysed and compared teacher-student dialogues in a U.S. and a Swedish mathematics classroom and report several deviations in patterns of student participation and in possibilities for influencing the content of the conversations during whole class instruction. In the U.S. classroom the teacher generally exerted a strict control of the discourse and the activities. In the Swedish classroom students' utterances seemed to influence the discourse and the teacher's instruction to a large extent. However, Emanuelsson and Sahlström (2008) argue that a consequence of high levels of student participation in the observed classroom was that "… the mathematics gets lost in the tangle of talk" (p. 218).

METHODS

The analysis in this chapter is based on empirical data collected by the Norwegian research group as part of the Learners Perspective Study (LPS) (Klette, 2009), and the characteristic design of the LPS study has been followed closely (see Clarke, Keitel, & Shimizu, 2006). Our present analysis is anchored in video data from one particular lesson in one particular mathematics classroom, and in data from the follow-up interview with the teacher. In addition to this we will present selected findings from a quantitative analysis performed on all the 38 lessons of the Norwegian LPS-study. This is done to provide an overview of some central traits characterising the teaching of mathematics in these classrooms, especially with regard to dimensions such as "main instructional format," "forms of communication" and "teacher-student relation." It is argued that information regarding these dimensions is relevant as it affords a background for the discussion of the selected classroom episodes with regard to the main themes of this chapter, challenges related to developing mathematical proficiency and democratic agency for all students.

A Quantitative Three Level Analysis of All Mathematics Lessons

All the video captures from the 38 lessons were analysed in the software program Videograph (Rimmele, 2002) by the use of theory-based categories developed by the Norwegian research team (Klette et al., 2005; Ödegaard, Arnesen, & Bergem, 2006). This analysis was carried out on three different levels, but only the categories relevant for the present analysis will be presented in this chapter. The coding was initially done in intervals of one second only, but later aggregated on the basis of one-minute intervals. This means that the code that dominated each minute "got" this minute. The way the coding was conducted makes it possible to present findings on different levels: one-lesson level; classroom level; or an all-comprehensive level, i.e., aggregating the results from all 38 lessons. In this chapter only a few aggregated results from the whole study will be presented.

At the first level of analysis the lessons were coded with regard to the following instructional format categories:

– Whole class instruction
– Individual seat work
– Group work

In the second level of analysis the characteristics of whole class instruction were further investigated. Several sub codes for this main category were applied, and the ones relevant for our analysis were the following:

– Dialogical Instruction: Use/mobilise students' knowledge for instructional purpose.
– Task Management: Teacher gives verbal/non-verbal instructions regarding assignments and class projects (grouping, material resources).
– General Messages: general messages and comments of classroom business.

It should be noted that *Dialogical Instruction* here implies that the students are actively involved and not only listening to the teacher's exposition.

At the third level of analysis subject specific categories were applied. Of particular interest for the issues discussed in this chapter are the findings related to the category named *Features of Dialogue*, consisting of the following three codes:

– Student initiatives: Students make comments or ask questions that initiate class discussion.
– Teacher exposition: Teacher presents or explains something monologically.
– Teacher initiatives: Teacher asks questions in order to mobilise student knowledge.

The findings from the three level quantitative analyses will be presented and discussed in relation to our main theme in a later section, and constitute the background to our main qualitative analysis of classroom interactions.

A Qualitative Analysis of Teacher Instruction and Classroom Interaction and Communication

Our qualitative analyses of teacher instruction and teacher-student interaction and communication are, as previously mentioned, based on video captures of one particular mathematics lesson in one specific 9th grade classroom. However, the lesson was taught twice as the class was split, and episodes from both lessons will be presented. This class was part of a lower secondary school situated in a suburban area characterised as being socio-economically and ethnically diverse, but mainly middle class Norwegian. The teacher described the proficiency level in this class as average, as assessed by national standard examinations. The mathematics teacher was well qualified. She was a certified teacher with several years of teaching experience, and she had recently completed a one-year study in mathematics education.

The criteria for our selection of episodes, excerpts and quotations were based on their relevance for the issues discussed in this chapter: the development of mathematical proficiency; and democratic agency. In addition to episodes from the particular lesson(s), a few quotations from the follow up teacher interview will be presented, selected on the basis of the same criteria.

FINDINGS

Findings from the Three-Level Quantitative Analysis

Based on the first level of analysis of all the 38 video taped mathematics lessons, the dominant instructional formats were found to be whole class instruction and individual seatwork. Nearly 95% of the lesson time in mathematics was used on these two activities, quite evenly distributed between the two of them. The remaining 5% of lesson time consisted of group work.

The second level of analysis revealed that almost 60% of the time allocated to Whole Class Instruction could be categorised as *Dialogical Instruction*, defined as “Use/mobilise students’ knowledge for instructional purpose” (see code definitions above). This finding indicates that, when the teacher presented new mathematical themes, the students were generally given broad opportunities to participate and to get involved in the classroom discussions. As we developed from the literature communicational processes – sharing thoughts with others and listening to others – is critical for both the development of democratic agency and mathematical proficiency. At a first glance then, our findings suggest that the Norwegian LPS teachers handle this aspect of classroom instruction with success; with students involved in the activities to a large extent. In terms of developing students’ democratic agency, affording opportunities for participation seems to be crucial. However, securing the development of mathematical proficiency presupposes that the teacher orchestrates classroom activities which involve more than one strands of this concept (Kilpatrick et al., 2001). This will be discussed in the qualitative analysis of the teaching-learning processes in the classroom (see below).

Figure 1 presents the percentage of time used on classroom discussions in mathematics lessons, under the category “Features of Dialogue.” The following three sub codes were used: student initiatives; teacher initiatives; and teacher exposition. It should be noted that it was not coded for frequency of initiatives, but for time used on discussing themes raised through student or teacher initiatives, or through teacher exposition. The latter code is here defined as the teacher speaking for a minimum of one minute. This was done to discriminate between the codes Teacher Exposure (TE) and Teacher Initiative (TI) with the intention of analysing to what degree the teacher would initiate classroom discussions through longer monologues (TE), or through quite short propositions (TI).

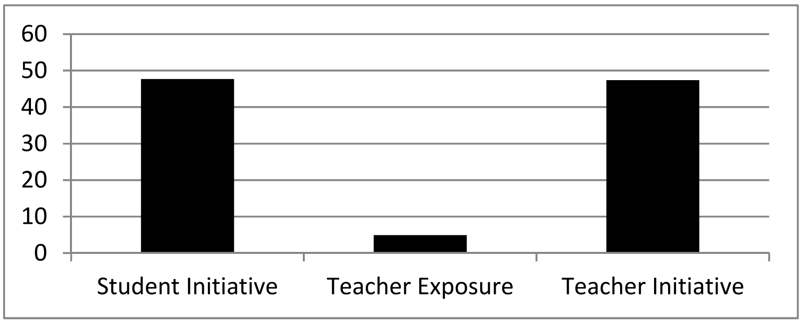

Figure 1. Features of dialogue; the percentage of time used on classroom discussions on the basis of student initiatives and teacher initiatives and the time used on teacher exposition

As revealed in Figure 1, a substantial amount of time in the Norwegian LPS classrooms was used on discussing issues raised through student initiatives. This finding indicates that the students' active involvement was given high priority, that students' opinions were valued and that students were being trained in communicating and participating in the mathematics lessons. All these are important pre-requisites for the development of democratic agency and mathematical proficiency. However, with regard to the development of mathematical proficiency, other strands of this concept also have to be activated. In the next section we will analyse particular episodes from one specific lesson from this perspective; investigating how the teacher responds to student initiatives and if other strands of mathematical proficiency come in to play.

Developing Democratic Agency and Mathematical Proficiency

As previously discussed, an important overarching educational goal expressed in the Norwegian curriculum relates to the development of democratic agency as evidenced by "critical abilities to attack prevailing attitudes, contend with conventional wisdom and challenge existing arrangements." Many student initiatives in our material can be related to this goal and a few illustrative examples are presented.

Example 1. The context of the first example is from an introductory lesson on equations where the teacher decided to use an apparently "very easy" text problem as her starting point. She wrote the problem on the blackboard and the teacher-student dialogue went like this:

Excerpt 1:

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T       Per and Kari have five apples jointly. Per has two apples. How
        many has Kari?
Frank   Are we supposed to answer that?
Hanne   Are you kidding?
T       No kidding. We'll use very simple examples at this stage so
        that we understand how one can apply equations.
Hanne   This is childish.
T       This is childish, says Hanne.
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In this dialogue the students challenge the teacher's use of the example to illustrate the theme of the lesson, an introduction to equations. It appears that they find the presented problem too easy and somewhat trivial – why use equations to solve this? Hence, they experience the teacher's choice of problem inappropriate. By referring to the question as "childish," Hanne indicates that she finds it inappropriate to be asked this kind of question and that it is not "problematic" for her, that is, she can solve this in her head and without using equations. In relation to the statements about democratic agency found in the curriculum, the students are clearly ready to express their opinions, even if this includes criticising the teacher's choice of illustrative examples and activities.

As the lesson continues another student, Alf, questions why one had to learn about equations at all:

Excerpt 2:

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Alf     What is the point of doing equations? It's just; it's easier
        to write three plus two instead of dealing with this X.
T       Yes, but the reason we are doing this now is to learn, we will
        get more difficult exercises as we go on.
Alf     But what could we use it for?
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Alf's comment initiated an extended classroom discussion (about 4 minutes) about why one should learn about equations. Many students participated eagerly in this discussion, expressing strong opinions about the value of learning about equations. Somewhat frustrated the teacher ended the discussion with the dictum that now they had to continue working on today's theme, "equations."

In this example we see how the teacher attempted to balance the students' rights to express their opinions, a central notion in the development of democratic agency, with the demands of the curriculum (to learn about equations). There are two main points here:

– The teacher's choice of example led to students doubting the purpose of equations.
– The authenticity of the example: it was not a "real life" example, but a contrived example (which did not work in this case).

This episode can also be related to Kilpatrick's concept of mathematical proficiency: the fifth strand, productive disposition, is referred to as the "habitual inclination to see mathematics as sensible, useful, and worthwhile." By questioning the use-value of equations, Alf challenged the teacher and his fellow students to come up with possible arguments "in favour" of learning equations. During the classroom discussion, and it should be noted that the teacher invited students to comment on Alf's challenge, mixed opinions were expressed. While some students agreed with Alf in the futility of learning about equations, other students argued that increasing one's mathematical knowledge, including knowledge about equations, could be useful in relation to future studies. The teacher added to this by emphasising that equations were an important theme in mathematics, and that the students were likely to value it once they had learned it. In the follow up interview the teacher commented on this episode:

> At the same time I want those students that would like to continue with maths and science in upper secondary and in their working life, that they … They are dependent upon equations.

So the teacher tries to argue both to the students in the classroom and in the interview the importance of viewing mathematics as "sensible, useful, and worthwhile." However, her choice of example to introduce equations does not seem to be in line with this argumentation, as it does not connect to the students' real life experiences and has little meaning for them, in particular in terms of learning about equations. In fact, it provokes student questions about the purpose of learning about equations.

This also links to Niss and Jensen's (2002) concept of mathematical competence. As pointed out by Blomhøj and Jensen (2007), it is the activity aspect of mathematical competence that is foregrounded by Niss and Jensen, meaning that a mathematical competent person should be ready to act with insight upon problems faced. The problem posed by the teacher in this lesson is not challenging for the students, and they are not invited to participate in a meaningful discussion involving mathematically relevant symbols and representations. Consequently, their communicative competences in mathematics were not stimulated and further developed.

However, this episode illustrates that the threshold for asking questions and challenging existing arrangements in this Norwegian LPS-classrooms was quite low. Students did not hesitate to critically comment on the teacher's statements or decisions and several incidents like the one presented were observed in other lessons in the Norwegian LPS material. Again, creating an open classroom climate where critique is tolerated and welcomed seems to be essential for the development of students' democratic agency. But these elements do not necessarily secure the development of mathematical competence/proficiency in the ways these skills are defined by Niss and Jensen (2002) and Kilpatrick et al. (2001) respectively.

Example 2. The interwining of two strands of mathematical proficiency – Productive disposition and Conceptual understanding is illustrated in the teacher's perception of the nature of mathematics, as formulated in the post lesson interview:

> … mathematics is a lot of numbers, and if you put it into words, or write or explain it to others, it might make things clearer. Then they get to see how to use equations for other things than just mathematics. But that is a method of problem solving. No matter what, if you have a problem you can set up an equation!

Later in this interview she explains how she intends to stimulate and widen the students' conceptual understanding of the concept "equations." She argues that in order to have the students learn more about equations they should apply it when working on various kinds of mathematical problems:

> We can't demand of the students to understand "equations" after just one week. They have to work on it. It has to be processed. We have to make use of equations in other topics, after we have learned about equations. When we start on volume for instance; then they are calculating volumes and stuff like that. Then they can apply equations on the formulas, and then they will understand a bit more of how to make use of equations.

Based on lesson observations it can be argued that the teacher provides opportunities for the development of students' conceptual understanding by using the following:

- Different representations (e.g., the scale drawing; apples; words; algebraic notation; numbers);
- Different explanations (students' and teacher explanations);
- Different solutions (e.g., pupils presenting their solutions on the board).

However, it is not clear that the teacher used these opportunities purposefully. Looking at her use of examples in the introductory lesson, these did not seem to trigger student curiosity of the concept of equation, nor did these examples seem to stimulate or deepen a conceptual understanding by clarifying important aspects of the concept. Indeed, when faced with a word problem, the students were guided by the teacher in following particular procedures. By remaining at a purely procedural level, the students were not given the opportunity to develop the conceptual strand of mathematical proficiency ("Conceptual understanding"). One could argue, however, that the teacher provided opportunities for developing procedural fluency, defined by Kilpatrick et al. (2001) as "skill in carrying out procedures flexibly, accurately and appropriately" (p. 5).

In summary, the way the teacher argued in the interview seem to indicate that she wanted to give the students a "handle" on equations by teaching them how to go about an easy word problem. However, given that they could solve the problem in their heads it did not make sense for pupils to utilise equations in the solution process. In addition, her teaching of the "equations" stayed at a rather procedural level, where students were told what to do and how.

Example 3. Another goal in the Norwegian core curriculum is formulated as:

> Skill in scientific thinking and working method demands the training of … the ability to wonder and to pose new questions;

The way this goal is expressed illustrates the close link between the training of scientific thinking and the development of democratic agency. Both these skills demand an open minded and a critical attitude. There are many examples in our video material of students asking questions that demonstrate the ability to wonder. As previously mentioned, the class that we have been focusing on was split in half during all mathematics lessons, and the episode we now present is from the lessen where the teacher taught ‘equations’ to the second group. The teacher followed more or less the same script in both lessons and after some initial talk about the use of algebra and equations, she again presented the easy word problem (“Per and Kari have five apples jointly. Per has two apples. How many has Kari?”). Here is the dialogue that followed:

Excerpt 3:

T	And then I write X instead and that equals five. Do you all understand that we can write it down in this way? X is the unknown, the answer we are going to find and it represents how many apples Kari has.
Peter	But what is the known?
T	What is the known? Yes, what is that? What is known in this? Nina?
Nina	How many apples Per has and how many they have jointly?
T	How many apples Per has, because that is hers (points at the board), and how many they have jointly, right Peter? That is five, and that is what we know and this is the unknown (points at the board again).
Peter	Don’t we have to have a letter for that [the known] (pointing to the blackboard)?
T	No, we don’t, because that is not something that is unknown.

Peter’s first question seems very relevant in relation to the teacher’s explanation. It is directly related to the teacher’s use of the concept of “the unknown,” a concept that is an integral part of a mathematical discourse about equations at beginners’ level. Peter’s spontaneous challenge to the teacher’s choice of expression might seem naïve within a purely mathematical discourse, but forces the teacher to elaborate on the mathematical meaning of the concept of the “unknown” and relate it more clearly to the actual problem being addressed. The teacher first passed Peter’s question on to the rest of the class, and got an adequate answer from Nina. Following up on Nina’s response, the teacher revoiced (Franke, Kazemy, & Battey, 2007) to strengthen the explanation. However, Peter still wondered if the “known” did not qualify for having its own letter, when the “unknown” was granted one – a legitimate question in terms of mathematical thinking. Illustrating the student’s ability to “wonder and pose new questions,” it is an example of Peter’s growing scientific attitude and skills related to democratic agency. However, in terms of developing mathematical proficiency, the opportunity to explain and elaborate on concepts such as “variable” (in connection to unknown) and “constant” was not taken up by the teacher. This potentially rich situation was not taken advantage of, nor exploited in terms of stimulating and developing students’ conceptual understanding.

In terms of social norms in this mathematics classroom, it appears that these include opportunities for everybody to ask questions and contribute in class discussions. These are also important elements in developing students' mathematical proficiency, if they are linked to mathematical concepts and activities. Moreover, to develop a culture of "learning mathematics with understanding" (Hiebert et al., 1997), where student curiosity is triggered, we need to also develop socio-mathematical norms (Yackel & Cobb, 1996). That is, opportunities for learning must be grounded in discussions about mathematically significant concepts, and cognitively demanding questions and activities. However, this is not easily achieved. It requires appropriate teacher knowledge and careful lesson planning. The tasks and problems that are used as a basis for classroom discussions must be carefully selected. Boaler and Humphreys (2005) claim that teachers often seem to have a different approach; frequently introducing tasks that students can solve with a minimum of cognitive effort. As a consequence, the classroom discussions that follow will often be "unchallenging" and are not likely to contribute to the development of student mathematical proficiency.

Example 4. Another important principal in the Norwegian core curriculum is expressed in the following paragraph:

> Education shall contribute to the building of character that gives individuals the strength to take command of their own lives.

To give the students the opportunity to take command of their own lives presupposes a non-authoritarian relationship between teacher and students. Even if teacher and students often take on different roles there are many episodes in our video material that demonstrate an open classroom climate where everybody is respected. As an example of this, many students from N01 asked the teacher if they could come up to the front of the class and explain their work on the blackboard. In the spirit of egalitarianism the students could also decline to present their work on the blackboard, when asked by the teacher.

Excerpt 4:

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T       Now we'll check the answer on one of the tasks. Who would like
        to come up here and check the answer? On this task: five plus
        two X equals 25? Is there anyone who would like to do it?
        Julie, would you like to have a go?
Julia   No
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If the students refused to comply with the teacher's requests, the teacher would ask another student. These participation norms illustrate the egalitarian relationship between teacher and students, an important prerequisite for establishing an open classroom climate and for the development of democratic agency.

DISCUSSION OF FINDINGS AND CONCLUSIONS

In our study the teacher's views on how to develop student learning of "equation" guided her use of specific instructional strategies. These included the following: "simple" word examples as introductory activity; particular representations to "picturise" equations; and her ways of setting an "equation" question out on the board. We contend that these selected strategies are likely to develop mathematical competence or proficiency, as outlined by the literature (Kilpatrick et al., 2002; Niss & Jensen, 2002).

Moreover, it can be argued that in this classroom the kinds of task (for introducing equations) set the foundation for the teacher's instruction, and it is likely that a different kind of task would have led to a different kind of instruction and a different kind of classroom discourse. At the same time the learners, and amongst them individual pupils, also voiced their preferred ways of tackling the mathematical question posed, and it appeared that the task (used to introduce the theme "equation") was not challenging for students, and hence, did not stimulate meaningful mathematical discussions. In fact, the selected tasks did not seem to offer pupils sufficient opportunities to reflect and communicate; these tasks were too easy (in terms of finding the solutions), and they were not genuine authentic problems for these learners. It is known from the literature (e.g., Hiebert et al., 1996) that appropriate mathematical tasks are those that make the mathematics "problematic" for pupils; problematic in the sense that pupils regard the task as an interesting problem, for them, something worth finding out, "something to make sense" to them (Hiebert et al., 1997; Boaler & Humphreys, 2005). An important finding from the IPN Video Study (Seidel, Rimmele, & Prenzel, 2005) was that in classrooms with high-quality classroom discourse, students are more motivated and intrigued to find things out. In the present classroom many students reacted to the non-challenging task with raised voices and arguments indicating that they did not find learning about equations to be "sensible, useful and worthwhile". However, developing an inclination to see mathematics as sensible, useful and worthwhile is an important part of mathematical proficiency, as described by Kilpatrick et al. (2001).

The differences in views between the teacher and many students regarding the value of learning about equations provide evidence of relatively well developed skills in democratic agency amongst the students. They did not hesitate to speak up, to argue or to "take actions" in relation to the emerging issues. As such, even though this analysis is based on episodes from one lesson only, it illustrates that the students had previously been given opportunities for articulating and expressing their opinions. The social norms developed in this class clearly included opportunities for everybody to contribute to and participate in ongoing discussions.

This is also documented in other episodes: students could agree or refuse to come to the board when asked by the teacher; they could probe for deeper understandings ("what is the known"); or simply answer the teachers' questions. However, creating learning "communities of practice" within the mathematics classroom, based on democratic participation and mathematical proficiency, requires that other strands of mathematical proficiency come into action and that communicational processes are extended to include other patterns of dialogues between participants (e.g., pupils develop questioning for their peers, feedback to each other, etc.). Establishing an appropriate classroom culture, where problematic mathematical tasks are tackled, depends on learners participating and engaging themselves as members of the group, and learning opportunities for deeper mathematical understanding arise as different ideas and views are expressed. Thus, developing democratic agency should not be an add-on, or optional to developing a rich and fully functioning learning community in mathematics, but rather an integral part of a classroom that attempts to foster mathematics learning with understanding and mathematically proficient students.

Boaler and Humphrey (2005) argue that the effectiveness of teaching and learning situations depend upon multiple factors: the actual students involved; the curriculum materials and tasks; and also on the myriad of decisions taken by the teacher during the mathematics lesson. They claim that teachers traditionally have been offered general educational principles, abstracted from subject specific issues and that this leaves the teacher to translate these principles into actual practice. As our analysis of selected episodes have illustrated, this can be very challenging and demanding for the teacher, for example to carefully plan the classroom activities, in particular the tasks and examples used to introduce a new mathematical topic area. Ultimately, this indicates that an important challenge for teacher education and professional development is to provide opportunities for pre- and in-service teachers to discuss subject-related and didactical challenges, and at a detailed level. Video-based studies, like LPS, can offer opportunities for analysing classroom practices that are particularly apt for teacher learning.

NOTES

[i] L97 is the core curriculum for primary, secondary and adult education in Norway. While subject curriculums have been revised since 1997, the core curriculum has been kept unchanged.

[ii] On the website http://www.udir.no/K106/MATI-03/Hele/Formal/ the objective for mathematics (1-13) is formulated (2006 Norwegian mathematics curriculum).

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