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CHAPTER EIGHT

Students and Their Teacher in a Didactical Situation: A Case Study

INTRODUCTION

Giving students the space to actively participate in the introduction of new knowledge through their own independent discovery is one of the demands of pedagogical theory and curricular documents. For example, Czech official pedagogical documents demand that pupils develop their problem solving competence by "making use of the acquired knowledge to discover/identify various ways of a problem solution" (Framework Education Programme for Basic Education, 2007, p. 12). Prerequisite to such approach is providing the space in which the pupil may apply informal knowledge. Informal knowledge is often subconscious, chaotically connected, and unclearly formulated. If it is to be used, the teacher must be able to listen to his/her students' voices and make it the basis for the construction of a knowledge network (Kaur, 2009). It seems that this is more difficult in mathematics than in other subjects, as mathematical knowledge has a rigorous structure. Our case study demonstrates that a competent teacher who believes in the appropriateness of this approach may use it to activate and motivate her students.

The theoretical background to our considerations is Brousseau's Theory of didactical situations (TDS); namely the concept *a*-didactical situation and the role of students in it. The organisation of an *a*-didactical situation as such (Brousseau, 1997) involves listening to students' voices. This can be observed in the whole *a*-didactical situation, in the situation of action, but much more distinctly in the situations of formulation and of validation. Students not only (for themselves) draw some conclusions from the activities they are involved in but they also share them with their classmates and the teacher. It is the organisation of the situation that makes them formulate their ideas, not explicit summons by the teacher.

A-DIDACTICAL SITUATION AND ITS PHASES

In our previous work (Novotná & Hošpesová, in press) our focus was on the development of TDS. We explored the institutionalisation phase in *a*-didactical situation and the role of the teacher in it. In this chapter we would like to

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 133–142.

investigate the role of students. We use several concepts from TDS (for more details see e.g., Složil, 2005).

Brousseau (1997) formulated the concept of a *didactical situation;* a system in which the teacher, student(s), milieu and restrictions necessary for creation of a piece of mathematical knowledge interact "to teach somebody something". The educator "organises a plan of action which illuminates his/her intention to modify some knowledge or bring about its creation in another actor, a student, for example, and which permits him/her to express himself/herself in actions" (Brousseau & Sarrazy, 2002, p. 3). In a special case, *a-didactical situation,* the educator enables the student(s) to acquire new knowledge in the learning processes without any explicit intervention from him/her. It is possible to distinguish three phases of an *a-didactical situation*:

– *Situation of action* – its result is an anticipated (implicit) model, strategy, initial tactic
– *Situation of formulation* – its result is a clear formulation of conditions under which the situation will function
– *Situation of validation* – its result is verification of functionality (or non-functionality) of the model

In our data analysis we focused on the different roles played by the teacher and the students in the different phases of an *a*-didactical situation. Our work led us to ask several questions, which we want to focus on in this text:

– How is an *a*-didactical situation initiated? Is it always planned in advance? Do sometimes students bring it about?
– What is the role of teachers and students in exploring the situation?

The data processed in this chapter were obtained by video recording of 10 consecutive lessons of mathematics in the 8th grade (students mostly aged 14). The teacher was an experienced educator with 30 years of teaching practice. The lessons were given in a middle sized school in Mnichovo Hradiste in January 2010. The data format is based on the LPS design (Clark, 2006). The lessons were video recorded using three cameras. One camera focused on the teacher, the second camera recorded the whole class and the third camera monitored a selected pair of students. This pair was different in every lesson. In the course of the 10 recorded lessons almost all pupils became members of the monitored pair. In addition to lesson recordings, post-lesson interviews (based on the video recording) with the teacher and the selected pair of students were carried out immediately after each lesson. The recorded sequence of lessons dealt with the solution of system of equations.

THE TEACHER AS THE INITIATOR OF THE *A-DIDACTICAL SITUATION*

In our set of data the effort to create an *a-didactical situation* was evident in all lessons. The incentive was almost in all cases on the teacher's side. Her statements in the lessons and in the post lesson interviews clearly show that she had prepared the situation deliberately. For example, she stated at the beginning of the second lesson [CZ 3-L02, 00:03.27][i]: "Today we will continue … solving the task from the end of the last lesson. And let's see what will happen; what we'll discover; if we will manage to figure it out or solve something so that we won't have to guess the solution any more, as we did yesterday."

In the CZ3 lessons the *a-didactical situation* was started by students' independent activities as they worked individually, in pairs, or in groups on teacher assigned problems. The students were able to solve the problems, but without any previously learnt and practiced algorithms. The solution of the problems was based on the students' real life experience or on application of previously acquired knowledge or experience. Let us now look at several examples.

The sequence of the lessons was designed around one unifying concept (systems of two linear equations with two unknowns) to which the teacher kept referring. She decided to start from the solution of word problems using the trial and error strategy. She posed several word problems which led to a linear equation with two unknowns (in lessons 1 and 2). The knowledge of the context allowed the students to solve the problem without actually knowing the mathematical procedure. In the next step the teacher used this non-mathematical context to introduce systems of equations and different solving methods:

– [CZ3-L01]: Divide 3 l of water into cups sized 0.5 l and 0.2 l so that the cups are full to the mark. You must use all the water and cups of both sizes. Once you have a solution, you can use the cups and water over there to check correctness of your solution.
– [CZ3-L02]: A task from your skiing course. When you were on the skiing course in Janov, Veronika and Lucka went to the shop to buy some goods for themselves and for others. When counting and distributing chocolate bars and packets of nuts they found out that the shop assistant only gave them the total cost of two bars of chocolate and three packets of nuts, which was 49 CZK. Find out the price of a bar of chocolate and a packet of nuts.
– [CZ3-L07]: You will remember that in one of the previous lessons we bought nuts and chocolates. Let's now try different purchases. For example: 6 bars of chocolate and 9 packets of nuts cost 147 crowns. 6 bars of chocolates and 4 packets of nuts cost 92 crowns. Can we now say what the price of a packet of peanuts and a bar of chocolate is?

The students were asked to solve the problems on their own. Then they showed the different solutions on the blackboard. In most cases the teacher supported the discussion by questions asking for reasons, justification, and opinions. Her original idea was that the students would use their everyday life experience for solving this problem. However, it turned out that the teacher's and the students' perception of the situation differed. The teacher explained in the post-lesson interview that her intention of introducing pouring out water related to: "Hyperactive children … When they can do something manually, it is very useful for them. What was crucially important was how they selected the unknowns. Correction of wrong mathematisation – that's the point of discovery for some of the children." [CZ3-L01, post-lesson interview with the teacher, 00:12:40]: The students who commented on the same lesson said:

[CZ3-L01 post-lesson interview with the student 1, 00:00:34]:

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Student1    The pouring out of water—all of us know that, but it was
            good to see it. It was not difficult today.
Exp         Was it useful that you had the chance to try it out?
Student1    It wasn't boring.
Student1    I found it simple to say which cup is big or small. If it
            is x or y. I did not enjoy it all the time but sometimes
            I'm more tired.
Exp         If there were greater numbers, would you enjoy it more?
Student1    If it's too easy, I don't want to think about it. I
            understood all of it, how it should be. I discovered the
            formula later.
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[CZ3-L01 post-lesson interview with the student 2, 00:00:34]:

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Student2    It started with the trial. It was good that we could see
            it practically. But I didn't enjoy it, because we only
            did one thing.
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However, the progress does not necessarily have to be smooth. Sometimes a student's voice brought in an inappropriate answer, sometimes a student did not answer at all despite the teacher's expectations. At that point the teacher needs much self-control to give students the chance to be heard as illustrated in the following extract from lesson 4.

Illustration

[CZ3-L04, 00:32:32]:
The teacher's intention was to support students to construct and solve of equations with one unknown (the two equations express the same unknown) and to the comparison of the "right sides". She wrote on the blackboard: $x = 3 + 2y$, $x = 9 - 3y$. The explanation went on as follows:

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Teacher     Can you construct a valid equation for one unknown? ...
            Let's think about it together. Can anybody see it? We
            have two equations: x equals 3 plus 2 ypsilon, and the
            second: x equals something diferent, 9 minus 3 ypsilon.
            What must hold for equalities? If the left sides equal,
            what does it mean for the right sides of the equations?
            Any ideas? Peter?
Peter       3 plus 2 y equals 9 minus 3 upsilon.
Teacher     What do you say, Thomas? Could we write it like this? ...
            Yes? No? ...
Thomas      I don't think so.
Teacher     Why?
Thomas      If I substitute 2, so in one (equation) I get 7 and in
            the
            other 3.
Teacher     Hm. When we substitute 2 for y, are both equalities
            right? If we substitute 2 for y, do we get here the same
            x as here? [She points at the original equation on the
            blackboard.]
Students    Yes.
Teacher     So this is not what satisfies both equations. See? So 2
            was not well chosen. Veronika?
Veronika    If it should have the same solution it must be equal.
Teacher     Exactly. If both equations must have the same solution,
            the same number for x in the first and the second
            equation, so they must be equal and the second x must
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therefore be equal to its counterpart. Solve one equation for the unknown *y*.

DIFFERENT ROLES OF THE TEACHER AND STUDENTS IN SITUATIONS OF FORMULATION OF CONCLUSIONS OF STUDENT INDIVIDUAL WORK

This section focuses on the situation of formulation when the relevant information is transmitted from one student who knows it to other students in a group. The analyses concern its forms and quality, as well as other students' reactions in situations when conclusions are transmitted by students. It is compared to similar situations when the information is transmitted by the teacher.

In this section, the following terminology is used: The person who formulates the conclusions and explains them to the others is called the *transmitter*, and those who get the information are called *receivers*. Students have both roles, that of a transmitter and a receiver.

Illustration

This extract comes from the 7th lesson. In the final part of the 6th lesson, students were divided into groups of four. Each group was given 4 problems A, B, C, and D with each member of the group being responsible for one of those problems. Then students left their "home groups" and met in four "expert groups" – in each group one of the four problems was solved collectively. The "expert groups" were given two tasks: to solve the assigned problem correctly and to learn how to explain the correct solution to all members of their "home group". The activity of explaining in "home groups" was scheduled for the beginning of the 7th lesson.

The following extract is a recording of the work in one "home group". The students are labelled S1, S2, S3 and S4. The problem discussed is B (transmitter S1). This problem involved the same system of linear equations as in problem A

$$3x - y = -3$$
$$2x + y = -2$$

but this time it was to be solved by substitution (solving one equation for one of the unknowns and substituting its value into the other equation). This episode follows the presentation of the solution to Problem A (transmitter S2, system of equations solved by comparison, i.e., by eliminating the same unknown from both equations, setting the two expressions equal to each other and then solving this equation). In the beginning S2's explanation was understood by the group. However, when they got to the equation $0 = -5y$ they remembered that there was a problem with division by 0 and did not know what to do with it. They failed to solve the problem until the teacher gave them a hint.

The group continued with S1's solution to problem B.

S1-1 Look how clear my solution is. Copy it and it will all be solved.
S3-1 Could you explain this? [S3 points at the equation where *x* is substituted by $(-3 + y)/3$]

S1-2	Oh, I forgot how I did it. Wait I'll rememeber. Substitution method, it means that ... Yes, clear. Look, this x is this [S1 circles the expression $x = (-3 + y)/3$] and you put this x here, then in fact you have it three times. S3 does not understand.
S1-3	If x was for example 2, then you ... I am explaining it to you.
S4-1	Don't explain, don't explain.
S1-4	You won't understand it. No, I will explain it to you when you don't understand. This here is x. This here is x. So in fact 3 times this x here. We only substitute in this equation.
S2-1	And what is this?
S1-5	As you have this, you know, you will only write down this. Do you understand?
S2-4	No.
S1-6	You calculate ...
S4-2	And why do you have it three times?
S1-7	Well, because here is the 3. Look. If you had 2, then you would have $3 \times 2 - y = -3$. Only x is not 2 but all this. Therefore you write there all this. Could you tell me why you don't understand it? To begin with you simply calculate how much x is. [Towards S2 who presented the solution of Problem A.] As you did it here [she points out the method of comparison].
S2-2	You said that there could as well be 2.
S1-8	No, I didn't say that. Look, you know how to find what x is from this equation, what x equals. But this x equals $(-3 + y)/3$. So our x equals this and I substitute this in that equation. Therefore the 3 is in fact this and I put there this x. So this is three times this. I substitute it in the equation, calculate it, and here is the result. [All the time when talking, she is pointing in the right places in her notation.]
S2,3,4	It is clear now.

The episode illustrates the following properties of the situation of formulation.

1. Active role of the transmitter and the receivers. The student who is in the position of the transmitter is very active in the whole episode. Although she has a clear idea what the correct procedure is and understands why it is correct, the transfer to his/her classmates is far from smooth. The receivers are active in their role. Their refusal to passively accept what the transmitter presents means that the discussion is very fierce with all participants heavily involved.

If we compare this to the situation when the teacher is the transmitter, the difference is mainly on the receivers' side. In case of transmission from the teacher, the students are much less active in trying to express their doubts than when the transmitter is one of the students. In the above transcribed episode, the transmitter had to answer questions 7 times. In a similar episode when the correct solution was presented by the teacher, only two questions were posed by students.

2. Formulations and reformulations; eliminating obstacles. When the first description of the procedure was not grasped by the other students, the transmitter tried to proceed in a way that is used by the teacher in similar situations – she tried to find reformulation of what was presented. Similarly to the teacher she tried to show an analogy to the situation with a concrete number. Although this procedure works when used by the teacher-transmitter,[ii] here it looked to be less productive, sometimes even counter-productive (see e.g., S2-2).

We offer two reasons for this outcome. One is the lower level of the language used by the student-transmitter. Her explanation was mostly based on what had been written in the model solution in the "expert group", she did not rewrite the calculation step-by-step, accompanying this rewriting by an accurate description of what she was doing in each step. As a consequence, the transmitter's discourse appears unclear to the receivers. When compared with the teacher's behaviour, the student-receivers grasped the teacher's accurate explanation much faster and more smoothly.

The other reason is linked to part of didactical contract evident within the classroom. As part of their expectation that the teacher provides students with clear and reliable information, the students trust that the teacher's explanation is correct, a trust which may not necessarily hold for a student-transmitter.

3. Originality of student-transmitter's techniques of explanation. In the analysed episodes, student-transmitters tried to apply the techniques that the teacher was using in mathematics lessons. This can be explained by the quality of the teacher's interventions during mathematics lessons. The students are well aware of the utility and good results of the teacher's techniques and therefore try to use them whenever they face the need of intervention.

4. Motivational potential of discussions in groups without the teacher's direct intervention. In the experiment, the use of students as transmitters was assessed by students as very useful. This is illustrated by the following extracts from post-lesson interviews with two students after the 7th lesson.

Interview with S3 (I denotes the interviewer)

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I           What was interesting on group work?
S3          Well, everybody can express his/her ideas. Everybody
            calculates in a different way, so.
I           But you can do it also in the whole class discussion,
            can't you?
S3          Yes, that's true. But when it's in groups, it's more. I
            don't know, I think we're discussing it more.
...

S3          In one case none of us knew how to calculate it, we found
            it strange. But later we grasped it.
I           And do you think that it helped you that you could
            discuss it together?
S3          Yes, here definitely yes.
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Interview with S1 (I labels the interviewer)

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I           What do you personally find good on group work?
S1          Well, that the lesson is somehow livelier and we aren't
            just sitting and looking, but we can at least discuss
            with the others.
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I           O.K., livelier, I understand, but is it also important
            from the perspective that you for example discover
            something when you're discussing?
S1          Yes, we have more ideas about it.
I           And it helps to find the solution to the problem.
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5. Facing failures. In group discussion, students listen to other students' voices. They also learn that sometimes it happens that their effort to solve a problem may not be successful, that they may fail in the activity. This is a situation they will be facing repeatedly in their life and they must treat this situation not as an endpoint but as a stimulus to look for other solution strategies, using the lesson they have learned from the unsuccessful attempt. Of course this can also happen when the transmitter is the teacher. But natural school hierarchy influences how students see their failures face-to-face with the teacher. Although the didactical contract may have some effect on this hierarchy, it is still true that students feel more at ease if they fail within peer groups rather than when the teacher is involved. The advantage of the activity based on discussion among students is that after a failure they usually do not cease trying to find another way leading to the correct solution.

DISCUSSION AND SOME CONCLUSIONS

Illustrations of situations which were used in this text clearly show that it is impossible to study students' and the teacher's voices separately. The situation may be compared to the situation of an orchestra with a conductor and musicians. The roles both of the conductor and the individual musicians are clearly indispensable. The role of the teacher strongly resembles the role of the conductor. And even when the situation in the class looks like a concert without a conductor, it is never really so.

To follow in the line of the previous metaphor: in some cases the student can play the role of the conductor to her/his classmates (this role is referred to in the previous text as the transmitter). However, when this happens we see that the course of the concert can change. The "musicians" are much more open when expressing their doubts and ambiguity and if they do not understand the situation they ask for further explanations. They are not influenced by the unerring authority that the teacher represents for them. The student transmitters are more likely to try several versions of explanations using language that is more comprehensible to peers in which may in fact promote deeper understanding. However, overall the transmitter's role is influenced by the didactical situation in the classroom. S/he does not create a new didactical situation.

Within the group activities and report back, the teacher's role is crucial even if it is not always explicit. Even when it is the student's activity which is in the central position, the student must not be let down. As part of preparing that substantial and stimulating learning environment for the students the teacher must make the decisions on how the problem will be presented to the students, what forms of representation will be used, how much space the students will be offered for discussion of the problem, and which student strategies will be supported.

The teacher in this case study was exceptionally sensitive to students' voices in all their possible forms. Not only did she work with students' suggestions on how to solve a given problem, but also she reacted without hesitation to the unforeseen situations arising in consequence to other influences than mathematics. Her reactions do not merely reflect experience of a teacher of mathematics; they are also motivated by her deep knowledge of her students and behaviour of the class. The teacher reacted to her students' voices not only verbally but if necessary also by changes in the intended lesson plan. This was transparent in all the observed lessons and the post lesson interviews.

To conclude we may say that facilitating students' individual discoveries (a-didactic situations in school practice) makes strenuous demands on a teacher's competences, especially in the area of psychology, pedagogy, content knowledge, but also in the area of class management.

NOTES

i The transcripts from the classroom are labelled as follows: CZ 3 (3rd Czech collection of data based on LPS design), L02 (2nd lesson), time of the start of the episode.

ii In the 3rd lesson, students were asked to express radius r from the formula for circumference $l = 2\pi r$. Students suggested several formulas for r. Following a short discussion three were singled out as possible. The conversation proceeded as follows:

T If I put there concrete numbers would it help? What do you think?
S Yes.
T Let's try it. Well, let's say what we know. We know the circumference, let us choose 15 cm for l. Find the radius r of such a circle.

After having solved this problem with concrete numbers, students were able to decide which of the formulas on the blackboard was correct.

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