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CHAPTER FOUR

Students at the Front: Examples from a Beijing Classroom

INTRODUCTION

As we know, the front is where the teacher's desk, blackboard and/or a projection screen are located. In this chapter we focus on classroom episodes with "Students at the Front" as an event in which "a student presents information publicly in written form, sometimes accompanied by verbal interaction between the student and the teacher or other students about the written work; other students may attend to this information or work on an assignment privately" (Jablonka, 2006, p. 108). In China's mathematics classes, "Students at the Front" events can be categorised into two types: (i) writing down the procedures for the solution on the blackboard; and (ii) giving oral descriptions of the approaches, a highly valued practice in the new-century curriculum reform. Based on the video recordings of a sample of 6 lessons in a eighth grade mathematics class in Beijing, the authors carried out an analysis of the activity "Students at the Front". The findings presented in the chapter illustrated the various forms of activity involved in the "Students at the Front" event. We explained the nature of interactions within the event by considering norms for presentations of problems, the extent of oral explanation, and the form of teacher-student exchanges.

The results reported in this chapter reflected the latest trend in China's mathematics education. In 2001, "The Standard of the Full-time Compulsory Education to the Mathematics Courses (Trial version)" was promulgated, which indicated the official beginning of the nation's new mathematics course reformation. The new mathematics curriculum policy has made significant changes in the basic values, development mechanism, development process, implementation system, and support system for curriculum in China. Students now have more opportunities to engage in classroom discussions, give comments, and ask questions (Liu, Wang, Sun, & Cao, in press).

To ensure that students take a major role in mathematics classes with real participation in class and full expression of thoughts, it is important for the teachers to organise the "Students at the Front" activity. In aligning with the mathematics curriculum reform it has become increasingly important for teachers to encourage students to share their own thinking procedures for solutions with the rest of the class. As a result, great changes have taken place and nowadays in the classes given by excellent teachers, students enjoy opportunities to speak publically in class (Cao, Liao, & Wan, 2008).

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 53–64. © 2013 Sense Publishers. All rights reserved.

There have been some previous studies concerning "Students at the Front" in mathematics classes. Jablonka (2006) conducted a comparative study of students' behaviour at the front of the classroom (on the board or in front of the teacher's desk) focusing on forms and functions in six mathematics teachers' classes from Germany, Hong Kong, and the United States. In these six classrooms, Jablonka found that students were hardly initiated into 'talking mathematics'. She argued that the classroom practices do not afford public student argumentation for different reasons (p. 120). In another study, by Begehr (2006), research on students' oral behaviour in mathematic classes in Germany investigated the scope of verbal actions. Begehr found that the German teachers "outtalked" their students, without being aware of it and the students' verbal participation was restricted to "disjointed fragments" (p. 180).

This chapter analysed 6 lessons selected from the video recordings of 12 lessons taught by a mathematics teacher (BJ1) in Beijing. Through analysing the coded teaching video sets, the research investigated the following aspects of the "Students at the Front" event: (i) information about the lesson events, including features such as the type of student activity, percentages of the events durations of the total teaching time, the frequencies of occurrence of the events, and the individual time for each student in the "student at the front" activity; (ii) analysis of the types and characteristics of the "Students at the Front" events; (iii) problems found in students' performances in the activity and effective methods for the teachers to promote higher learning efficiency, which can improve the relationship between teachers and students and create good classroom atmosphere through the activity "Students at the Front".

RESEARCH DESIGN AND METHODOLOGY

The overall LPS research design as set out in the Appendix of this book was adopted by the study. Sequences of at least ten lessons were recorded in the classroom of three teachers who were selected as representatives for the normal level of all teachers. In Beijing, efforts were made to ensure that the three classrooms were in demographically different parts of the city. Three video records were generated for each lesson (teacher camera, focus student camera, and whole class camera), and this video record was supplemented with post-lesson videostimulated interviews with two students after every lesson and with the teacher three times during the period of data generation. This combination of classroom video material plus teacher and student interviews constituted the primary data source for the analyses reported in this chapter.

For the purpose of this chapter we studied six lessons (5th-10th) of one teacher, BJ1. For the six lessons of BJ1 that we analysed, "Students at the Front" was identified according to the description: In the classroom teaching, the activity "Students at the Front" starts from the moment a student leaves his/her own seat to go to the front, where (s)he writes on the blackboard or gives an oral presentation concerning certain teaching content, knowledge points or problems in front of the whole class, and ends when the student is back to his/her own seat. It was noted

that the tasks for the "Students at the Front" may vary – some required students to answer a complete question, while others required students to figure out certain steps of a solution.

Object of Study and Relevant Basic Information

Analysis was undertaken of the video recordings of the fifth to the tenth lessons out of the twelve consecutive lessons in natural settings given by an eighth grade mathematics teacher (BJ1) in Beijing. Teacher BJ1 was an experienced teacher who had previously taught classes ranging from the seventh to the twelfth grade and participated in many professional development activities. The focus of the lesson sequence was about knowledge of quadrilaterals for the eighth grade. The topics of the lessons are listed in Table 1.

Table 1. Contents of the fifth to the tenth lessons given by Teacher BJ1

Lesson	Content of courses
L5	Theorem and Property of Median of Triangle
L6	Rectangle & Square(1): Property of Rectangle
L7	Rectangle & Square(2): Decision Theorem of Rectangle
L8	Rhombus
L9	Rhombus & Square
L10	Special Quadrilaterals: Internal Relations among Parallelogram and Rectangle, Square, Rhombus

The teaching objectives of the lessons were: (i) to enable students to know different aspects of learning geometry; (ii) to gain a general perspective on the knowledge of lines, surfaces, and cubes; and (iii) to appreciate that learning mathematics could be an approach for improving one's analytical capabilities.

RESEARCH PROCEDURE

The first step of the analysis was to use the *Studiocode* software to code events in the video and generate relevant statistics. These statistics were then combined with the text records of the classes and after-class interviews with students to carry out the qualitative and quantitative analyses. The analysis included the following steps: (i) observe the lesson videos, consult relevant documents and set the primary codes; (ii) use the primary codes and the *Studiocode* software to carry out quantitative analysis of the video material. This process includes modifying the codes and establishing final codes; (iii) use the final codes to conduct quantitative analysis of the coded material, gather relevant statistics and then combine the statistics with the classroom record and the relevant video records to carry out qualitative analyses of the classroom videos.

In relation to the "Students at the front" lesson event, it was discovered that there were two types of representative behaviour evident in the data: (i) blackboard

presentation, meaning that students do in-class exercises on the blackboard with the procedures written down, and (ii) oral presentation, in which students stand at the front and explain their own approaches, understanding or thinking procedures to the rest of the class with occasional writings on the blackboard as support.

The analysis of students' presentations was based on frameworks used in Begehr's (2006) and Jablonka's (2006) research. For the coding, we looked into the verbal communication between the teacher and the students in each "Students at the Front" event, identifying such features as selection of the presenting student, the number of problems addressed in this mode, the type of problems, any evaluation of the presentation by the teacher and/or the class, the length of time taken by each and all students participants, and any consequent actions that could be associated with the event.

Through primary analysis, it was found that most students 'at the front' were selected by the teacher, while a few volunteered to give presentations, and sometimes the teacher utilised the analogous strategy of displaying students' inclass working via the projector. Sometimes several students gave presentations on the same problem, while at other times different students talked about different problems. The evaluation of the students' presentations could arise from: teacher's comment, peers' comment, and teacher-student mutual comment. The content of any particular evaluation could vary. Table 2 shows the codes in evaluating students' presentations.

	Codes Explanation
PS1	The teacher appoints a student to give a presentation on a problem.
PS2	Students volunteer to give the presentation.
PS3	Students volunteer to supplement the current presentation.
PT1	One student talks about one problem.
PT2	Several students talk about the same problem.
PC1	Students explain approaches to a certain problem.
PC2	Students elaborate specific steps and reasons for their solution and
	describe the procedure.
PM1	Oral speech only.
PM2	Oral speech together with written procedures, drawings and marks on the blackboard.
-	The teacher comments on the presentation.
SC	Students comment on each other's presentation.
TSC	The teacher and the students comment on the presentation
	together.
	PS2 PS3 PT1 PT2 PC1 PC2 PM1 PM2 TC SC

Table 2.	Codes used	for the	student	oral	presentation

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	TR1	The teacher approves of the student's presentation in simple ways
	TR2	such as nodding, applauding and saying "good". The teacher points out the incorrect or incomplete points in the presentation in simple ways such as shaking head, saying "that
	TR3	isn't correct", or "that isn't good". The teacher interrupts the presentation without letting him/her
Teacher's	TR4	finish the speech or task. The teacher supplements the presentation, completing and
Responses		improving.
(TR)	TR5	The teacher corrects the student's mistakes during presentation.
	TR6 TR7	The teacher further elaborates on the student's presentation.
	IK/	The teacher encourages and guides the student according to his/her presentation
	TR8	The teacher speaks instead of the student when (s)he encounters
	TR9	difficulty in presentation. The teacher gives no direct reaction to the student's presentation
	11()	and continues the teaching.

The following is a sample coded transcript from L10 using the student oral presentation coding (Table 2):

Teacher	[PS2]Which two angles? Who will come to the front? (A student stands up) Ok, Fan Xiaoshu, please. Which two angles?
Teacher	Here, use this and speak at the front. Draw it out on the blackboard by yourself, do it on your own. Which two angles are equal?
Student A Teacher	[PM2][PT1] These two. [TR7] Right! Why?
Student A	[PM2][PC2] These two are the same, and equal angles lead to equal sides.
Teacher	[TSC] Hmm, he said these two are the same, right? (Students: Yes.) But he didn't give the justification. Lack of justification. What did you say?
Student A Teacher	[PC1] Just use the parallel interior alternate angles. [TC][TR6] Which parallel interior alternate angles? Which? Come to the front, yes, come here. Aha, I see you are anxious to speak at the front Aha, he is so eager to speak out here. Yes, just come here and point out where he has made a mistake.
Student B	[PS3][PC1] Just now we said this equals this, so that's an interior alternate angle.
Teacher	[TC][TR7] Point it out clearly, which and which forms an interior alternate angle.
Student B	[PC2][PM2] This and this, and this and this are equal. And this is a bisector. These two angles are equal, so these two are equal, and equal angles lead to equal sides.
Teacher Students	[TSC] Is it OK? Right? Right.

STATISTICS AND ANALYSIS

The researchers used the *Studiocode* software to gather statistics for the videos of six coded lessons.

PS	L5	<i>L6</i>	<i>L</i> 7	<i>L8</i>	L9	L10	AVG
PS1	2	2	0	0	3	3	1.67
PS2	1	0	2	1	0	0	0.67
PS3	0	0	2	0	1	0	0.50
TOTAL	3	2	4	1	4	3	2.83

Table 3. Frequency of presentation source in BJI's classes

From Table 3 it is apparent that while students volunteered to do the presentations or offered supplementary presentations, more than a half of the presentations were still directed by teacher appointment (average frequency of PS1: 1.67, of PS2 & PS3: 1.17). It should be noticed that the pattern of participation was influenced by the lesson objective. For example, in Lesson 8, in order to introduce a new concept, the teacher taught for most of the time, only setting aside time for one presentation. However, in Lesson 7, the second lesson of rectangle and square, since the students were comparatively familiar with the content being taught, more time was used for students presentations.

Table 4. Length of time (minutes) of different types of presentations in BJI's classes

PT	L5	<i>L6</i>	L7	<i>L8</i>	L9	L10	AVG	% of Lesson Time
PT1	2.08	0.00	2.20	4.50	1.02	2.09	1.98	4.95%
PT2	7.45	2.56	6.02	0.00	2.41	1.31	3.29	8.23%
TOTAL	9.53	2.56	8.22	4.50	3.43	3.40	5.27	13.18%

From Table 4 we can see that the average time of students' presentations accounted for 13.18% of the total length of lesson time which is 6 hours, since the length of a typical single lesson is 40 minutes. Also shown is that the time of cooperative presentations (PT2) lasted almost twice as long as the time of solo presentations (PT1), especially in Lesson 5 and Lesson 7, where PT2 lasted 7.45 minutes and 6.02 minutes respectively, taking up more than 15% of the length of time of a single class.

Most notably, there were no student-only comments during student presentations. The main form of comment was teacher-student mutual comment, taking up 61.75% of the total comments based on students' presentations, however, this accounts for only 4.4% of the total lesson time. From Table 5 we can see that in L5, L6, L9 and L10, teacher-student comments obviously outnumbered the teacher-only comments, especially in L6 where there was only TSC. However, in L7 the teacher-only comments dominated and in L8 there were only teacher's

comments. It appeared that this was determined by the content being taught, which corresponds to the characteristics of lessons shown in Table 1.

CT	L5	L6	L7	L8	L9	L10	AVG	% of Comment Time	% of Class Hour
TC	0.17	0.00	3.28	0.97	1.97	0.17	1.09	38.25%	2.73%
TSC	4.52	1.08	1.13	0.00	2.27	1.54	1.76	61.75%	4.40%
TOTAL	4.69	1.08	4.41	0.97	4.34	1.71	2.85	100%	7.13%

Table 5. Length of time (minutes) for comment types used in student presentations

Only five types of responses were found in the data: TR1, TR4, TR5, TR6, and TR7. Within the six lessons there were no incidences found where the teacher negated the student's ideas (TR2), interrupted the student (TR3), spoke over the student (TR8), or gave no comment (TR9). Among the five types of response codes recorded, the most prevalent was TR7, as shown in Table 6, which related to encouraging and guiding students in their presentations.

Table 6. Length of time (minutes) on teacher's different responses to students' presentations

TR	L5	<i>L6</i>	L7	L8	L9	L10	AVG	% of Comment Time	% of Class Hour
TR1	0.14	0.00	0.00	0.20	0.21	0.11	0.11	3.89%	0.28%
TR4	0.67	0.00	0.00	0.00	0.43	0.51	0.27	9.54%	0.67%
TR5	0.60	0.00	1.23	0.07	0.00	0.26	0.36	12.72%	0.90%
TR6	0.00	0.00	0.35	0.41	0.45	0.20	0.24	8.48%	0.59%
TR7	2.84	1.09	2.57	0.39	2.51	0.71	1.60	56.54%	4.00%
TOTAL	4.25	1.09	4.15	1.07	4.60	1.79	2.83	100%	7.06%

However, whilst TR7 responses were the preferred form, it is noted that they comprised only 4% of the lesson time as shown in Table 6. Overall, the distribution of response types suggested that the teacher feedback was geared to provide correction, guidance and encouragement; she never interrupted or negatively criticised students' presentations. In other words, the teacher valued very much each student's presentation.

SUMMARY OF FINDINGS

Our analysis of classroom events involving student presentations "at the front" in six mathematics lessons of one teacher BJ1 is summarised as follows.

There is Rich Variety in the Types of Students' Presentations at the Front

The most common scenario was that the teacher selected certain students to give the presentations, while students also competed for the chance to do a presentation or to supplement others' presentations. The number of student presentations was close to three (2.83) times per lesson, regardless of how the presenter was selected. The overhead projector served as an important tool for the students to do the presentations.

The format of the student presentation typically began with the Teacher BJ1 asking a question with students volunteering to present their solutions or ideas at the front. When there was no volunteer, BJ1 appointed some students to do the presentation. If students failed to offer complete solutions or their answers needed to be supplemented and further explained, or when other students had different ideas, BJ1 usually encouraged other students to give supplementary presentations as seen in the following transcript:

```
Teacher ...these two triangles? Who can help him? Who has got any
idea? Great! You! Come on!(BJ1-L05)
Teacher (to the rest students) Do you have anything else to
add?(BJ1-L07)
Teacher So you got it? Well, you please! (BJ1-L08)
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One or More Students Give Presentations Concerning One Problem

On completion of a presentation, Teacher BJ1 often encouraged other students to share their different ideas. Allowing as many students as possible to express their own thoughts encouraged discussion about the best idea and enabled each student's ideas to be known. Sharing ideas of several students took 8.23% of the total lesson time (about 3.3 minutes per lesson), almost twice as long as presentations (4.95%) in which one student talked about one problem.

Students' presentations are important in that they provide an opportunity for the teacher to know the extent to which the students have grasped the knowledge of the subject matter, as well as find out their existing problems and barriers in applying their knowledge. In addition, knowing the students' cognitive thinking means that the teacher can more effectively guide and help them. To support this point, the following is a sample transcript from Lesson 7 in which three students at the front were working to find the area of a parallelogram. The teacher, instead of telling the students the answer, encouraged them to ask and listen to each other, express their opinions to their classmate's answers.

```
Teacher Can you work out its height?

Student B & C Yes.

Teacher OK. Maybe you can ask them.

Student A (to B) How to work out its height?

Teacher (to B) Please. (to A) You can write down what he said.

Student B DE is known, and then MN can be obtained. So we can get

the area of this parallelogram.

Teacher Good! But I am not very sure. Which one?
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Student BThis one. Parallelogram AMND.TeacherSure? Is this parallelogram our target?Student CNo!TeacherPlease tell us your opinion.
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The Main Form of Students' Presentations at the Front is Oral Explanation of the Approaches

From the statistics of content of all the presentations from BJ1's lessons (see Tables 3-6), it can be seen in all the coded lessons that the total time spent by students *describing* procedures (PC2) was 7.57 minutes, while 24.05 minutes was required for *explaining* the approaches (PC1). This shows that teacher BJ1 placed great emphasis on developing students' thinking and preferred to ask students to present their thoughts on certain knowledge points or typical examples. By careful attention to the students' own expression of their ideas, their existing problems can be found and any difficulties in mastering the knowledge of the subject matter can also be known. Hence, the teacher can guide and help the students to overcome any barriers in their thinking process, as well as promoting their initiative in learning.

In terms of the form of presentation, Teacher BJ1 encouraged students to write down their approaches and procedures while speaking, placing emphasis on the students' abilities of thinking, speaking, and doing. An example can be seen in the previous transcript when the teacher said to one of the students, A: "You can write down what he said."

Sometimes, however, a student's presentation was entirely in oral form as captured in the following transcript:

```
Teacher This angle. Plus this angle and you can get 90 degrees, is
it this angle? Ok, tell us please.
Student Yes.
Teacher You mean the angle EGD?
Student No, it's the angle inside.
Teacher The one inside? You mean this angle?
Student Exactly, the exterior angle of the triangle AEG.
Teacher Excellent, it is exactly this exterior angle.
```

Here, it is clear that the student's presentation involved answering closed-type questions posed by the teacher.

In other cases, a student's presentation comprised both written and oral form as illustrated in the following transcript:

```
Teacher Come on share with us your ideas.
Student EF is parallel to AD. (pointing to the drawing on the blackboard) These two angles are equal, and this angle equals to the sum of the two angles, so this angle and that angle are equal. Therefore, they are parallel.
Teacher Can you understand?
Students No.
Teacher Please draw it out using the yellow chalk and tell us why the two angles are equal.
```

Student	(Drawing lines on the blackboard) this angle is the
	exterior angle of the triangle, and this big angle equals
	the sum of this angle and this angle.
Teacher	Can you understand this?
	(The rest of the students nod.)
Student	And since this is parallel to this, we can know this angle
	equals this angle.
Teacher	Can you understand now?
Students	Yes!

In L5, L6, L7, and L8 both written and oral forms were applied; in L10, only the oral form was adopted; and in L9, 2.15 minutes were spent on both forms, while 1.27 minutes was spent on the oral form only. In all the coded classes, time spent in oral form only was about 4 minutes with the rest spent in applying both forms.

The Main Evaluation Form is Teacher-Student Mutual Comment

The majority of BJ1's evaluative responses took the form of mutual comments from both teacher and students (61.75% of the total comment time) and there were no student-only comments.

Across the six-lesson sequence comments offered by BJ1 within the students presentations were mainly guidance and encouragement, (56.54% of the total comment time). Free of interruptions or negative critique there was a sense that each student's presentation was valued. As making positive comments is an art, the teacher should be good at finding students' strengths in learning activities and provide positive comments in time. In this way, the students' learning potential can be tapped and confidence boosted. Positive evaluation can increase students' self-esteem and confidence, while negative evaluation can lead to the opposite effect.

When the students were giving their presentations, Teacher BJ1 stayed at the front and maintained her interaction with the students and the rest of the class. She offered positive reaction to the correct and reasonable points in the students' presentation with "yes," "right," nodes and smiles. Meanwhile, she did not forget to interact with the rest of the class, including explaining important or difficult points to the students and asking the rest of the class whether they understood the speaker. When a student's presentation was not correct or complete, the teacher usually asked other students. When the student finished the presentation, the teacher further summarised or explained the knowledge points for the other students to better understand and grasp the knowledge. She put great emphasis on guiding and encouraging the students to think (TR7), which accounted for 56.54% of the total comment time. Generally, Teacher BJ1 guided the students to think on their own and to come up with the approaches themselves.

DISCUSSION

In another study by Cao (2011), involving lessons taught by the same teacher from Beijing (BJ1) and another teacher from Shanghai (SH2), it was found that while

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Teacher SH2 attended mostly to the procedure and result of students' problem solving, Teacher BJ1 placed more emphasis on students' thoughts about the solution. Our current analysis of the six lessons taught by Teacher BJ1 showed that the time spent by the teacher in presenting the procedure of problem solving was 7.57 minutes in total for the six lessons, while the time spent in presenting thoughts and approaches was 24.05 minutes over the six lessons. In a comparable analysis of Teacher SH2 we found that he mostly asked the students to write down the complete procedure and steps on the blackboard without asking them to share their thoughts and ideas. SH2 preferred to give lots of comments after the students' blackboard presentation thereby placing greater emphasis on the teacher's comments and summary. His evaluation, which emphasises the procedures and steps of the solution and helps students develop more established ability in automatically solving the problems took up 68.72% of the total assessment time (about 6.7 minutes per lesson, average lesson time 40 minutes).

As the education reform in China moves forward, the idea of the studentcentered classroom will gain more significance. The *Mathematics Course Standards for Compulsory Education* points out that "the teaching activity is a process in which the teacher and the students actively participate, communicate, interact and mutually develop mathematical knowledge. An effective teaching activity requires the integration of students' learning and teacher's teaching, with the students as the main body of math learning and the teacher as their organiser, leader and partner" (Ministry of Education of the People's Republic of China, 2011). The case of Teacher BJ1 offers an encouraging model for our future classroom teaching activities.

One basic goal of mathematics education is to develop correct self-expression and communication skills. Learning mathematics requires not only solving mathematics problems but also being able to discuss, communicate, and express one's own ideas. Learning to share one's ideas with others is an important skill for everyone today. Our analysis suggests that Teacher BJ1's practice of providing open questions for the students and her push for students to discuss and share their thoughts at the front, affords an important way for students to express their own thoughts and ideas.

Students' initiative in learning is stressed to help them establish their own knowledge concepts. The task of the teacher is to help the students construct their own knowledge rather than implanting them with knowledge, for only when the students construct their own understanding can they develop interest in learning. Moreover, the students' confidence can also be boosted when they get the chance to express their own thoughts at the front, leading to higher efficiency in learning.

The teacher needs to provide some proper guidance after the student's presentation, because the teacher's encouragement and guidance can help the students think more thoroughly about the subject matter knowledge being developed. The teacher's praise and rewards after the presentation can further increase the student's confidence and interest in learning. Therefore, the teacher should seize every opportunity to guide and encourage the students, who can thus

actively explore and build their own knowledge system leading to an improved cognitive structure of mathematics learning.

REFERENCES

- Begehr, A. (2006). Students' verbal actions in German mathematics classes. In D. Clarke, C. Keitel, &Y. Shimizu (Eds.), *Mathematics classrooms in twelve countries: The insider's perspective* (pp. 167-182). Rotterdam, the Netherlands: Sense Publishers.
- Cao, Y. M. (2011). Microscopic analysis of the Chinese middle school mathematics teaching in an international perspective. Beijing: Beijing Normal University Publishing Group.
- Cao, Y. M., Liao, S., & Wan, Y. L. (2008). Opportunity to talk and mathematics teaching in Chinese mainland. *Journal of Mathematics Education*, 1(1), 59-66.
- Jablonka, E. (2006). Student(s) at the front: Forms and functions in six classrooms from Germany, Hong Kong and the United States. In D. Clarke, J. Emanuelsson, E. Jablonka, & I. A. C. Mok (Eds.) Making connections: Comparing mathematics classrooms around the world (pp. 107-126). Rotterdam, the Netherlands: Sense publishers.

Liu, J., Wang, L. D., Sun, Y., & Cao, Y. M. (in press). Chinese mathematics curriculum reform in the 21 century: 2000-2010.

Ministry of Education of the People's Republic of China (2011). *The mathematics course standards for compulsory education* (2011 version). Beijing: Beijing Normal University Publishing Group.

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