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CHAPTER THREE

Students Speaking Mathematics: Practices and Consequences for Mathematics Classrooms in Different Countries

STUDENT-STUDENT MATHEMATICAL TALK

The research reported in the companion chapter (*Spoken Mathematics as an Instructional Strategy*) revealed significant differences in the *public* mathematical discourse practised in various classrooms around the world. It is clear that the pedagogies practised in many mathematics classrooms also permit and even promote *student-to-student* mathematical speech. In fact, the pedagogies of some classrooms are dependent on the provision of opportunities for student-to-student mathematical speech. The analyses reported in this chapter suggest that at least some of the goals of those advocating student-student mathematical conversations in the classroom may be met by other instructional strategies, such as whole class public discussion. Since our data set included some classrooms where student-student mathematical conversations were encouraged and some where they were not, we were well positioned to address the question: "What differences in practice exist between classrooms where student-student mathematical talk is encouraged and those where it is not, and what appear to be the consequences for learning of those differences in practice?"

CONNECTING MATHEMATICAL TALK AND LEARNING

The role of language in learning has been widely researched and variously conceived (Alexander, 2008; Kim & Markus, 2004). Different theories attend to different aspects of language and the learning process and some of these have been discussed and relevant research cited in the companion chapter to this one. The adoption of a cognitive perspective towards learning directs the researcher's attention to the content represented by the language used. The assumption seems to be that the learner's language use can be taken to reflect their thought processes. In studies with a more socio-cultural emphasis, the focus tends to be on the discursive functions of spoken and written language (e.g., Inagaki, Hatano, & Morita, 1998). From this perspective, language is a cultural resource through which the learner is initiated into a particular community of practice (van Oers, 2001). Studies adopting a sociolinguistic perspective address the distinctive linguistic features of specialised or technical language (for example, mathematical or scientific language). In such studies, facility with language is taken to be prerequisite to any

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 33–52.

effective communication and consequently to any learning (see Walshaw & Anthony, 2008, for an overview of Western research).

Mathematics learning can be conceptualised in terms of participation in forms of social practice, where discourses form key components of that practice. Language plays a central role in mediating and constituting this participation, which is performed as classroom discourse (see Yackel & Cobb, 1996). Traditionally regarded as only auxiliary to thinking, active mathematical communication is nevertheless believed to enhance mathematical learning. It is a useful exercise, however, to conceptualise mathematics as a special form of communication. From this perspective, the expression "learning mathematics" becomes tantamount to developing mathematical discourse (Sfard, 2001, 2008).

In our analysis, we have employed student spoken use of technical mathematical terms as indicative of the students' developing confidence and skill in using the concepts and procedures signified by the technical terms in social interaction. Such growing competence in engaging in what might be called technical mathematical discourse can also be taken to indicate improvement in their capacity to participate in the community of practice constituted by the teacher and their fellow students in the mathematics classroom.

Our concern in this chapter is the construction of a connection between student learning and the way in which the practices of each classroom afforded or constrained the students' use of technical mathematical terms in public and private speech. Having established in the companion chapter the significant differences between classrooms in patterns of public discourse, we now shift attention to the spoken utterances of individual students in both public and private contexts, and, importantly, the connection between individual spoken mathematics and observable learning outcomes.

MATHEMATICAL DISCOURSE IN THE CLASSROOM FROM THE PERSPECTIVE OF THE LEARNER

Analyses were conducted of 110 lessons documented in 22 classrooms located in Australia (Melbourne), China (Hong Kong and Shanghai), Germany (Berlin), Japan (Tokyo), Korea (Seoul), Singapore, and the USA (San Diego). In this chapter, we focus our analysis on the spoken acts of the focus students (most commonly two per lesson) and on their use of mathematical language in post-lesson interview settings. The complete LPS research design is set out in the Appendix to this book. Three types of oral classroom interactions were recorded: whole class interactions, teacher-student interactions, and student-student interactions. All whole class and teacher-student interactions were documented and transcribed, but student-student interactions could only be recorded for selected focus students in each lesson. In selecting the focus students for each lesson, the researcher would typically choose two students sitting side by side (or as near as possible given the prevalent seating arrangements). Wherever possible, acting on advice from the teacher, each particular pair of students were chosen because they would normally sit near each other. In this way, any student-student conversation would be most likely to resemble the students' normal practice. A different pair of focus students was chosen for each lesson. Each focus student then participated in a post-lesson video-stimulated interview and these interviews were also transcribed. Transcription and translation were carried out by the local team responsible for data generation and were therefore undertaken by native speakers of the local language. Transcripts were then translated into English, where necessary. Technical guidelines specified the format to be used for all transcripts and the conventions for translation (particularly of colloquial expressions) (Clarke, 2006 and the appendix to this book). The analyses reported in this chapter were undertaken on the English version of each transcript (both public and private classroom dialogue and student interview).

Examining the public and private classroom utterances of 222 focus students distributed across 22 mathematics classrooms in several different cultures, we were able to study the extent to which student mathematical talk was encouraged in one classroom, in public and/or private contexts, and discouraged in another. The final stage of our analysis examined student use of mathematical terms in 191 post-lesson video-stimulated interviews.

As noted elsewhere, the study design was not intended to support any claims of national representativeness with respect to the teacher, the classroom, or the students. Instead, the research design delivered privileged access to the language used in class by approximately 10 students in each of 22 mathematics classrooms, situated in widely differing cultures and school systems. As will be seen, this language use could be connected to the development of student facility with mathematical language through the analysis of the post-lesson interviews.

PUBLIC MATHEMATICAL DISCOURSE

In our first analytical pass reported in the companion chapter, we counted the number of utterances made by anyone participating in a whole class or teacher-student interaction (a "public utterance" from the student perspective), a construct we designated as *public oral interactivity*. Our second analytical pass considered mathematical terms rather than utterances. The specific terms, of course, reflect the topic being taught in each class. Eighteen of the twenty-two classrooms were studying algebra topics, while three were studying geometry (Tokyo 2 and Melbourne 1 and 2), and one decimals and percentage (Melbourne 3). With the possible exception of Melbourne 3, all topics could be associated with a vocabulary of sophisticated mathematical terms. Since we had recorded the public and private talk of two focus students in each lesson, and could supplement these with the transcripts of interviews with those focus students after each lesson, the prevalence of student spoken use of technical mathematical terms provided an entry point for the fine-grained study of how such terms were used, in response to what teacher prompts, and with what consequences for student learning. This provided the focus for the analysis reported in this chapter.

THE SIGNIFICANCE OF STUDENT-STUDENT INTERACTIONS

The private conversations recorded in any one lesson were only those of the two focus students and their immediate neighbours. Two different focus students were recorded in each lesson. In this section, we report the frequency of utterances (uninterrupted oral communications) and key mathematical terms (as defined in the companion chapter and below) in both public and private arenas with respect to the two focus students. All utterances made by the two focus students were differentiated according to whether the utterance was targeted at a public audience or a private audience. Public utterances were those made to the teacher (either in response to a teacher question during whole-class discussion or in one-on-one interaction) or to another student, but intended to be audible to the whole class. Private utterances included statements made to a student peer in private or to oneself.

In Figures 1 and 2, the results given for both public and private Oral Interactivity and Mathematical Orality are per focus student per lesson and have been averaged over the spoken contributions of around 10 students per classroom. This should minimise the effect of individual student timidity or extroversion, although awareness of being recorded was a common characteristic of all focus students (and of their teachers). The number of utterances and key mathematical terms was normed to a standard lesson length of 45 minutes.

Three classrooms stand out in Figure 1 because of their extremely low frequency of student-student interaction: Shanghai 1, and Seoul 1 and 3. In these three classrooms, student-student conversation can be discounted as an instructional strategy (or as a subversive practice by students). For example, in Seoul classroom 1, there were no instances of student private talk in the first four recorded lessons and only two private utterances from one of the focus students in lesson five, an average of 0.2 utterances per student per lesson. The first utterance was “That’s yours” and the second was “No.” Obviously, neither involved any technical mathematical terms.

The corresponding figures in the companion chapter show relatively high levels of whole class public mathematical orality in the Shanghai classrooms, but this is not evident in Figures 1 and 2 because the typical public contribution of an individual Shanghai student occurs within a class of fifty students (at least ten more than the average for classes in any of the other cities) and a specific individual’s contributions will consequently be less frequent than in smaller classes.

Rather than characterising aggregated whole class behaviours, Figures 1 and 2 express their findings in terms of the individual student. At least three observations are noteworthy: (i) The complete absence of a spoken mathematical term by all ten recorded students in each of the three Korean classrooms; (ii) The relatively low frequency of private (student-student) use of mathematical terms in all three Shanghai classrooms (which in public discourse were sites of relatively frequent student mathematical orality); and, (iii) The remarkable result for Tokyo 2: averaging 9.44 privately spoken mathematical terms per student per lesson across a

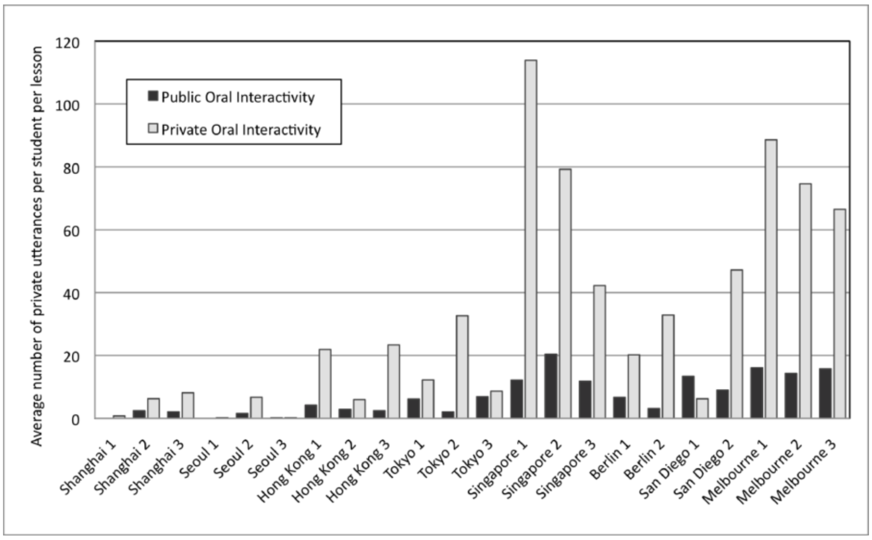

Figure 1. Public and Private Oral Interactivity: Frequency of utterance per student per lesson (each bar represents the average of two students for each of five lessons – i.e., an average over ten students per class)

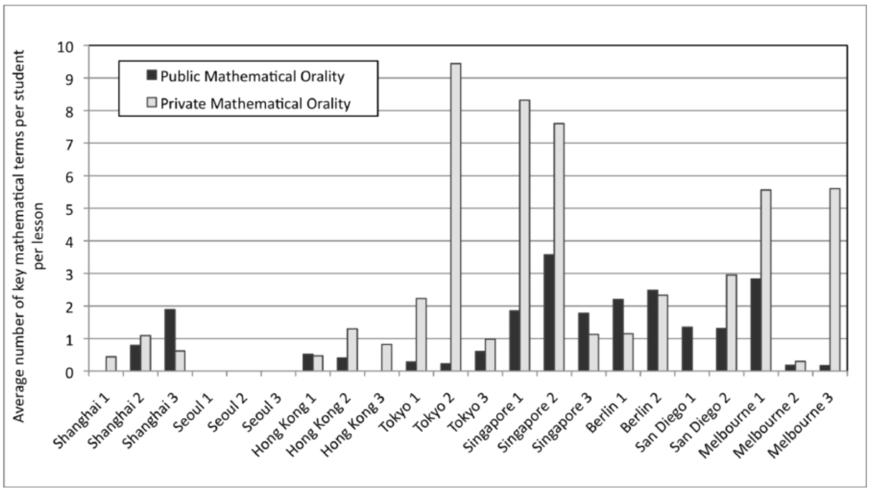

Figure 2. Public and Private Mathematical Orality: Frequency of use of key mathematical terms (each bar represents the average per student of two students for each of five lessons – i.e., an average over ten students per class)

sample of ten focus students over the five lessons studied. It is also noteworthy that the other classrooms in which student-student use of mathematical terms was most prevalent were Singapore 1 (8.32) and Singapore 2 (7.60). Of the "Western" classrooms, where student-student interaction might be expected to be much more common, only Melbourne 1 (5.56), Melbourne 3 (5.60) and San Diego 2 (2.95) were at all comparable in the private use of mathematical terms.

It is important to consider the nature of the student-student interactions and the manner in which spoken mathematical terms were employed. In our analysis of both public and private spoken mathematics, we focused on those "key mathematical terms" that constituted the content-focus of the lesson. Table 1 sets out about 3 minutes of student-student interaction recorded in lesson 2 in the second Tokyo classroom. This classroom was noteworthy for its high level of student-student (private) interaction (see Figure 2). In the episode displayed in Table 1, the students had been asked to draw a triangle with point P somewhere along the segment AB, and then draw a line running from P that divides the area of the triangle into two (see Figures 3a and 3b). The key terms have been highlighted in the transcript. Some terms, such as "line," fall into the category we have called "related terms" (see the later discussion of student interviews). These related terms did not constitute the lesson's substantive content but were relevant terms connected to that content. Figures 3a and 3b show the diagrams constructed by the two students: Wada and Kawa. There is a vitality evident in the interactive exchange between these two students that illustrates the sort of cognitive engagement valued by the advocates of spoken mathematics (see Walshaw & Anthony, 2008) and analysed in detail by Helme & Clarke (2001).

Table 1. Sample student-student "private" interaction - Classroom transcript (Tokyo School 2 – lesson 2, 29:46:12 – 33:15:19)

Kawa	[To Wada] I managed to draw that line!
Wada	Like this?
Wada	[To Kawa] If you draw that line over the middle point [mid-point], isn't that the answer, Kawa?
Kawa	Oh, I don't think so!
Wada	I think you don't have to do such a thing. I think you just have to draw a line from P.
Kawa	I don't really understand what you mean.
Wada	Um, you drew a middle point [mid-point] here, right? So if you just draw a line from here, wouldn't that do?
Kawa	Can you draw a line from P?
Kawa	You're kidding. What did you say? Are you saying that you can draw a line from here?
Wada	Yes. If you draw a line from there, if goes over the **middle point** [mid-point] so there is no problem there.
Kawa	Really? Let's try then.
Kawa	What was the name of the **theorem** again?
Wada	**Middle point** [Mid-point] **connection theorem**.
Kawa	That's it! But it isn't **parallel** there. Are you going to try drawing it there?
Wada	[To Tsutahara] Doesn't this work when you draw a **parallel line** by free hand and then draw a line that goes along P?
Tsutahara	I don't understand what you're talking about.

Wada	Never mind then.
Kawa	I'll understand it with Wada then.
Wada	Draw a **parallel line.**
Kawa	Did so.
Wada	Well, it's not going over P if you notice.
Kawa	And which one's the same here? Tell me.
Wada	These two are **parallel.**
Kawa	Yeah, I knew that.
Wada	Doesn't it look like it's the right answer?
Kawa	This one's a lot easier to see. It's nice and big, this one. Wait! Don't you have to say something about the **bottom line** [base]?
Wada	What?
Kawa	Something we discuss about every time we do this.
Wada	Never mind about that.
Kawa	Yeah, but we always prove that these two **triangles** are the same or whatever.
Wada	Well, that's my answer.
Kawa	Nothing to do with **triangles** this time? Are you sure about that?
Wada	Um, um, this one.
Kawa	Which two?
Wada	This one and this one.
Kawa	What happens when they're the same?
Wada	It's the same.
Kawa	Which two?
Wada	These two.
Kawa	How come?
Wada	Because they're **congruent.**
Kawa	Where's the **bottom line** [base] then?
Wada	This is the **bottom line** [base], I bet. God, I don't know which one is the **bottom line** [base] now.
Kawa	This one has to be the **bottom line** [base].
Wada	This has to be the (height), this one. This is the **height.** I got it now!
Kawa	Is this the **height**? Is it all right if it's now **parallel**?
Wada	Well, it doesn't have to be **parallel.** No need for that.
Kawa	But then which two become equally in half?
Wada	What the hell are you saying?
Kawa	Aren't we doing the one that we have to divide in half or something like that?
Wada	Yes, that's the one we're talking about.
Kawa	I'm starting to get mixed up now.
Wada	Well, I'm starting to get a headache.

The transcript above also illustrates one of the difficulties associated with translation. Where a technical term is used in the original language, a literal translation may not correspond to the equivalent English form of the technical term (for example: middle point or mid-point). We have chosen to translate the Japanese wording of the technical term literally, while indicating in parentheses the corresponding English version of the technical term. In this way, the connotations and entailments of the original phrasing and the institutionalized status of the technical term are available for analysis and interpretation. Figures 3a and 3b show the written work that was the focus of the students' conversation.

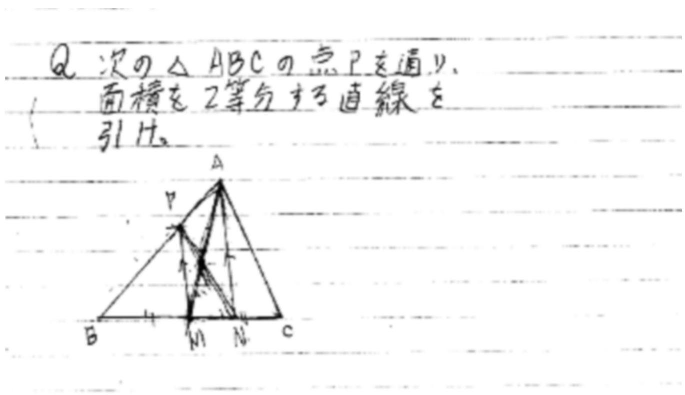

Figure 3a. Wada's work

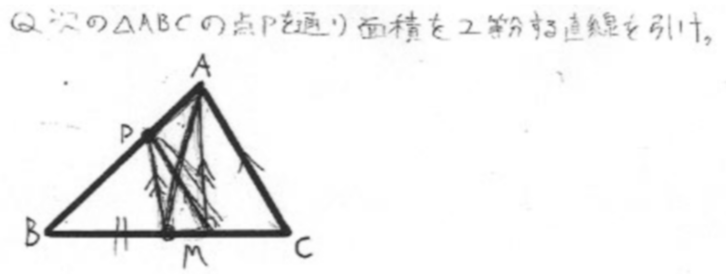

Figure 3b. Kawa's work

As has already been noted, while the frequency of utterance (oral interactivity) for the focus students in Tokyo 2 was comparable with the Western classrooms analysed, the frequency of use of key mathematical terms per student per lesson was higher than for any other classroom. Since all teachers studied were considered 'competent' by their local community, we must consider the occurrence of private student-student speech to be a deliberate affordance by the teacher within the socio-mathematical norms of the classroom. In the case of Tokyo 2, we have evidence of a pedagogical practice (occurrence of student-student talk) that appears to be much more prevalent in the Western classrooms studied than in many of the Asian classrooms. Singapore 1 and 2 also offer evidence of a significant level of student-student talk, combined with a high level of private use of key mathematical terms. In fact, what might be called the "lexical density"[i] of student-student talk in Singapore 1 and 2 is very similar to that of Melbourne 1 and 3.

Such individual cases represent an important demonstration of the viability of practices in classrooms where their use might be assumed to be precluded by cultural convention: a form of "existence proof." As displayed in Figure 2 and illustrated in Table 1, not only do we find a relatively high frequency of private oral interactivity in Tokyo 2, but student private spoken use of key mathematical terms is extremely frequent (that is, the lexical density of student private interactions is relatively high). Whatever benefits might accrue from the classroom rehearsal of spoken mathematics, we would expect these to be particularly evident for the students of Tokyo 2.

In characterising the use of key mathematical terms in student-student classroom speech, we must not forget that the Shanghai classrooms were characterised by high levels of lexical density in the public classroom discourse. The Shanghai classrooms represent a very interesting case. Shanghai Teacher 1 has been shown to value and promote student spoken use of mathematical terminology (see Clarke, Xu, & Wan, companion chapter in this book). However, constrained by the apparent conventions of Chinese classroom practice, Shanghai Teacher 1 enacts this prioritisation in the public domain only. Because of the large class size in Shanghai, this means that any particular student will have proportionately less opportunity to actually "talk mathematics" in comparison with students in smaller classes, even though the teacher's clear intention is to provide the opportunity for this to occur. The role of choral response becomes very important here. Even if it is not possible for each student in a Shanghai classroom to make spoken use of many mathematical terms in a given lesson, the teacher's classroom practice explicitly values students' spoken fluency with mathematical terms and this valuing is communicated very clearly to the class through the teacher's orchestration of public discussion. Further, the students have the opportunity to hear their classmates' oral use of mathematical terms in the public classroom discourse. This provides a sharp contrast to the pedagogies employed in other classrooms, particularly Tokyo 2 or Melbourne 1, where student-student spoken mathematics was prioritised. Consider this interview statement from the second interview with Tokyo Teacher 1.

Tokyo 1	Um, it went totally different from what I had planned ... But it was not important to do as planned. Students discuss with each other and have their own opinions is what is most important. And I think it is what was good about this lesson.

What were the consequences for the students' learning of these pedagogies, in each of which spoken mathematics was promoted, but by very different instructional means?

Spoken Mathematics in the Classroom: Key Points Summary

The prevalence of spoken mathematics in the 22 classrooms studied differed in the following respects:

– the frequency of public utterance
– the relative prominence of the teacher or the students' voices in public discourse
– the frequency of public use of spoken technical terms, most particularly by students
– the differences in the extent to which student use of spoken mathematics was strategically facilitated by teachers
– the extreme differences between classrooms in the occurrence of student-student (private) use of spoken mathematics.

In some classrooms, student-student spoken mathematics was an essential component of the dominant pedagogy. In other classrooms, it was entirely absent. These extreme differences allow us to ask the question: "With what consequences?"

SPOKEN MATHEMATICAL FLUENCY AS A VALUED LEARNING OUTCOME

It is clearly the case that some mathematics teachers value the development of a spoken mathematical vocabulary and some do not. If the goal of classroom mathematical activity was competence in the use of written mathematics, then the teacher may give little priority to students developing any fluency in spoken mathematics. On the other hand, if the teacher subscribes to the view that student understanding resides in the capacity to justify and explain the use of mathematical procedures, in addition to technical proficiency in carrying out those procedures in solving mathematics problems, then the nurturing of student proficiency in the spoken language of mathematics is likely to be prioritised, both for its own sake as a valued skill and also because of the key role that language plays in the process whereby knowledge is constructed.

In the final stage of our analysis, the transcripts of 191 student post-lesson interviews were examined for the occurrence of the key terms that constituted the instructional focus of the lesson, together with those mathematical terms closely related to the key terms (*related terms*). In addition, we also coded *other terms*, not used in the lesson but employed by the student in interview to describe or explain some aspect of their classroom activity. We analysed transcripts of the post-lesson interviews with the same focus students whose private classroom conversations were recorded and analysed above and for the same lessons. The three categories of mathematical term are defined below.

The *key terms* were the mathematical terms or phrases explicitly identified in the teacher's lesson plans, or in explicit teacher statements, as constituting the goal(s) of the lesson. For example, in Hong Kong 2, some key terms would be "simultaneous equations" and "method of elimination." These key terms were coded for both public and private conversations during lessons.

The *related terms* were the mathematical terms or phrases, closely connected to the key terms. These terms were used by the teacher or students during the lesson and repeated by the students in interview. For example, in San Diego 1, the mnemonic "Please excuse my dear Aunt Sally," introduced by the teacher to help students remember the order of operations to be 'parenthesis, exponents, multiplication, division, addition and subtraction,' and similarly, the coined term "sub," employed by students to mean 'substitute' were considered to be related terms. More conventionally, related terms were frequently simply mathematical terms that were used in class to help to explain the key terms that were the actual content focus of the lesson.

The *other terms* were other mathematical terms not used in the lesson being described in interview. These could include mathematical terms or phrases that were categorised as either key or related terms in the other lessons analysed for that class or any other mathematical terms employed by the student. Student use of such other terms could be interpreted as indicative of connections made by the student between the content of that lesson and other content studied or known.

For each classroom, the transcripts of student post-lesson interviews examined were those that corresponded to the five (or six in the case of San Diego 2) consecutive lessons analysed for public and private orality. In the post-lesson interviews, the number of utterances was not the main area of interest. Only instances of the student articulation of mathematical terms or phrases were counted. The categorisation of mathematical terms (key, related, and other) employed in analysing the student interviews was consistent with the usage of mathematical terms or phrases in public and private conversations during the lessons. It is important to reiterate at this point that there are other aspects of student speech that might be of mathematical significance: for example, the use of logical connectives, but these were not the focus of this analysis.

The analyses already reported indicate that the classroom practices of some teachers deliberately facilitated the development of a spoken mathematical vocabulary by students, while other teachers did not do this. Since the classroom use of spoken mathematics by students has been strongly advocated in various sections of the mathematics education literature (for example, Walshaw & Anthony, 2008; Silverman & Thompson, 2008), it is important that research examine differences in the occurrence, form, and promotion of spoken mathematics in classrooms that are differently situated with respect to school system and culture. Further, research should address the question, "To what purpose and with what consequences are students encouraged to engage in spoken mathematics?" These are the issues that we have attempted to address in our research.

The post-lesson interviews undertaken in the LPS provide a unique indication of student facility with a spoken mathematical vocabulary. It is important to note that this may not be either a valued or intended consequence of mathematics instruction in some of the classrooms studied. However, the development of this facility appears to underlie instructional advocacy within the Western canon and for that reason warrants investigation.

In conducting the post-lesson interviews, students were asked to comment on what they had learned or felt was important from that day's lesson. Following which, the video for the lesson was played and the student could pause, fast forward, or rewind to any parts of the lesson that they felt were important or that they wanted to comment upon. After viewing the video, the students were asked if they had any other comments about the lesson before ending the interview session. The legitimacy of our comparison of student use of mathematical terms in these post-lesson interviews is dependent on the consistency with which the interview protocol was followed. Careful examination of all interview transcripts confirmed that the student language use analysed was in response to the same interview stimuli.

It is important to note that the interview text analysed in this study was the English translation (where required) of the original interview transcript and that both transcription and translation were carried out by the local research team in the particular country generating the data. As a result, there appeared to be slight changes to the wording of the interview prompts. For example, in the student interviews in Shanghai School 1, typical interview prompts included: "What do you think [this lesson] was about?" and "What do you think you have learned in this class?" The equivalent prompts for Seoul School 2 were translated as, "Tell me about today's class" and "What did you learn today?" The important point for our analysis is that in neither situation did the prompts suggest particular mathematical terms to the students. That is, any mathematical terms employed by the students in the post-lesson interviews were chosen by the students, rather than being suggested by the interviewers.

During the interviews, it was not unusual for students to pause for more than five seconds when pondering how they should reply to the interviewer or what they wanted to comment upon. Hence a continuous turn, uninterrupted by the interviewer, was considered as one utterance. In each turn, more than one mathematical term might occur. However, the occurrence of a particular mathematical term or phrase has been counted only once as a single conceptual contribution, even if it was mentioned more than once in a particular turn. For example, in the turn "I thought, using the - like *powers*. Like to the first and second *power* and *cubed* and stuff," two mathematical terms would be counted, namely 'power' and 'cubed'."

Taking into account the possible occurrence of mathematical terms or phrases not categorised as key mathematical terms, the other two categories (related terms and other terms) were constructed for the purpose of reflecting the student's capacity to use mathematical terms other than those central to the substantive content of the lesson. Student use of these three categories of mathematical terms is illustrated below (Table 3).

Table 3. Interview data related to San Diego 2 - Lesson 3

00:00:07:02	I	I know it's been a few days since Friday[ii] ... since the last lesson, but can you think back and tell me what you thought the lesson was about on Friday?
00:00:16:16	Nahoku	It was just telling us - there was one ***equation*** with - there was four different ways you can show it.
00:00:24:12	Nahoku	There's the ... the ***verbal***. That one [Nahoku points at notepad], //the ***equation***, the ***graph***, and the ***T chart***.
00:00:28:06	I	//Okay.
00:00:31:26	I	Okay. That makes sense. Anything else you want to add about those four expressions?
00:00:37:16	Nahoku	They all mean the same thing.

00:00:40:02	I	Okay. What do you mean by that? What do you //mean by "the same thing?"
00:00:41:27	Nahoku	//Like, um, X Y is ***equal*** to two.
00:00:46:23	I	Uh-huh.
00:00:47:13	Nahoku	X ***multiplied*** by Y is ***equal*** to two. And then the, um, the ***T chart*** tells the same thing as all of 'em.
00:00:55:17	I	Okay. And then what does a graph tell you?
00:00:58:16	Nahoku	It's just plotting out the ***points***. Like this [points at notepad], ***negative*** two and ***negative*** one, is ***negative*** two, ***negative*** one.
00:01:07:03	I	Oh, I see. Okay. Does the graph tell you anything else about the ... representations?
00:01:13:20	Nahoku	It tells you like, if it's a ***linear line***, or a ... um ... a ***non-linear line***.
00:01:21:02	I	And what does that mean, "Linear line"?
00:01:22:23	Nahoku	***Linear*** means a ***straight line***.
00:01:24:19	I	Oh, okay.
00:01:25:13	Nahoku	[points at her paper] This is a ***non-linear***.
00:01:26:27	I	Okay. Do you know what that's called when it's non-linear?
00:01:30:19	Nahoku	I think it's this one [looks through her notes]. ***Parabola***.
00:01:35:03	I	Oh, okay.
00:01:36:03	Nahoku	Or a ***curve***.
00:01:37:25	I	Okay. Great. Okay.
00:01:41:14	I	Tell me what you think, um, you understood during the lesson on Friday. What do you think that you got worked out? An th- and then, what are some of the things that you think maybe you don't have worked out?
00:01:52:22	Nahoku	I have, um, how you can tell the ***graph*** is gonna be ***linear*** or ***non-linear*** by the ... um, the ***coordinates***.
00:02:02:24	I	Oh, okay [nodding].
00:02:06:17	I	And anything else?
00:02:11:06	Nahoku	No.
00:02:11:29	I	No? What a- what about things that you still are a little bit confused about? Anything?
00:02:16:28	Nahoku	[points at notepad] How these can be- I don't know how to tell 'em ... if they're ***curved*** or not. All I know how to tell is if they're ***linear***.

Key terms: equation, verbal, graph, coordinates
Related terms: T chart, linear, linear line, straight line
Other terms: multiplied, equal, points, negative, non-linear, non-linear line, parabola, curve(d)

The relative frequency of occurrence of each of these categories of mathematical terms expressed as the average number of mathematical terms used per student is displayed in Figure 4. Student descriptions of lesson content and learning provide a different type of mathematical performance from that displayed in student performance on mathematics tests. The classrooms studied in this project appear to differ in the value accorded to such performances.

Data from interviews with Berlin focus students were not included in Figure 4. Unlike the individual interviews conducted with students elsewhere, the Berlin post-lesson student interviews involved two and sometimes three students simultaneously. This situation arose because of the reported unwillingness of the German students to be interviewed individually. As a result, while it was possible to calculate the number of terms employed by students as a total per student (Berlin 1: 6.31 key terms, 3.23 related terms, and 7.00 other terms, totalling 16.54 terms; Berlin 2: 3.33, 3.33, 9.75 and 16.42, respectively), the nature of a group interview meant that students were less likely to mention a mathematical term that had already been introduced in the same interview by another student. As a result, the figures just cited are likely to underestimate and therefore misrepresent the facility with mathematical terms of the German students interviewed.

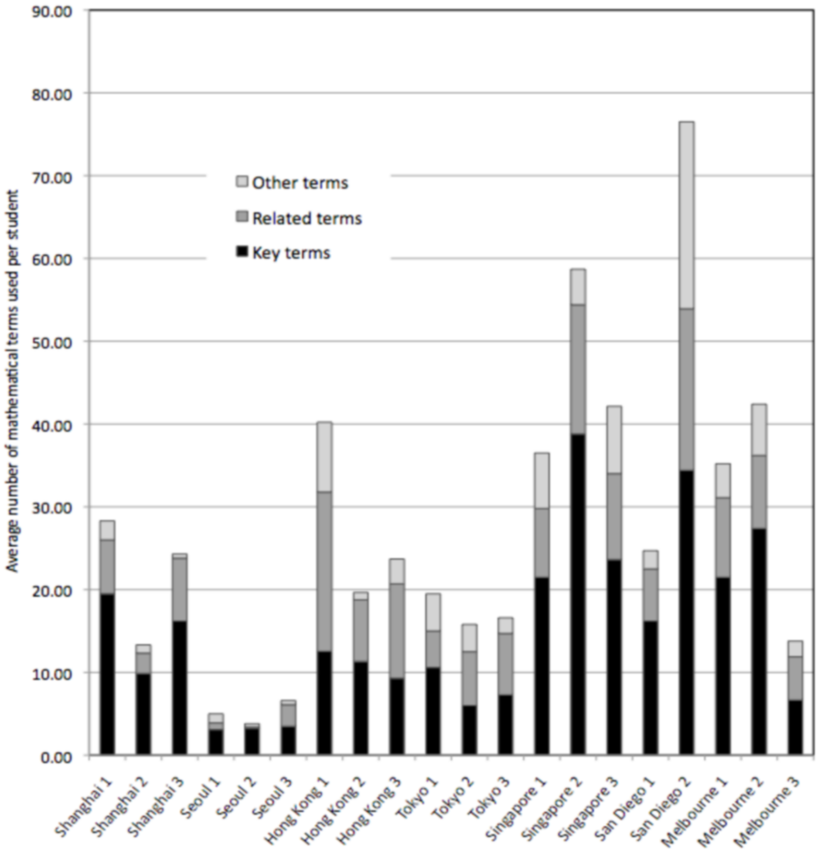

Figure 4. Frequency of use of technical terms in post-lesson interviews (each bar represents the average per student over ten student interviews for each class)

The inclusion of the Berlin results in Figure 4 would encourage misleading comparisons between the technical vocabularly of the students from the two Berlin classrooms and that of students from other classrooms. The best that can be said in relation to students from the Berlin classrooms is that the frequency of student use of mathematical terms in the post-lesson interviews was at least comparable in overall total to Tokyo 2 and 3, with a higher relative occurrence of "other" terms. Total term usage in the Berlin post-lesson interviews was lower than that for students from the other Western classrooms, except Melbourne 2. But, as noted, this is likely to be a significant underestimate, and it is possible that the term usage for individual Berlin students could lie between San Diego 1 and Melbourne 1.

Consideration of the two pairs of figures dealing with oral interactivity and mathematical orality in this and the companion chapter raised several questions regarding the learning consequences of classroom spoken mathematics. For example, all three of the Seoul classrooms provided students with little opportunity to speak mathematics, either in public or in private. When asked to describe their experience of a particular lesson, using the same interview protocol as the students from other schools, would the students from the three Seoul classrooms display comparable fluency in the use of the mathematical terms central to the content of the lesson being described? Figure 4 suggests that despite the use of the same interview protocol in all countries, the students from the three Seoul classrooms used significantly fewer actual mathematical terms to describe their experience of the mathematics classroom.

Consideration of Figure 4 suggests several interpretive hypotheses:

- If student facility with technical mathematical vocabulary is a valued outcome, then the analysis of the post-lesson interviews suggests that the public scaffolding (and explicit valuing) of student technical fluency (e.g., in Shanghai 1) can be as effective as the encouragement of student-student spoken mathematics (e.g., in Melbourne 1) in developing this facility.
- Where the classroom provided students with no opportunity to engage in spoken mathematics (Seoul), there appears to be little inclination (and possibly capacity) to do so, even in interview situations where the invitation to use spoken mathematics was explicit ("What did you learn today?").
- Student inclination to employ other mathematical terms ('other terms') in addition to those specific to the lesson could indicate a form of interconnected knowing. Detailed analysis of interview transcripts is required to determine the significance of the use of 'other terms' as indicative of sophisticated understanding. This will be addressed in more detailed case study of San Diego 2 to be reported in another volume in the LPS research series.
- Facility with mathematical speech seems to respond to personal practice (e.g., San Diego 2 and Singapore 2) but can, as noted above, also be achieved through the public promotion of student mathematical speech (e.g., Shanghai 1).

We suggest that student use of mathematical terms in interview can be used as the indicator of one type of learning outcome. Such outcomes are attributable to features of particular mathematics lessons and, may possibly be used as indicators of the success of the instructional practices of the particular mathematics classroom. Such causal claims address one of the most significant challenges of classroom research and require careful empirical justification.

GENERAL DISCUSSION

As a result of this research, we are in a position to compare types of mathematical language employed in 22 mathematics classrooms in eight cities in seven countries. The 22 classrooms offer a remarkable sample of different combinations of forms of classroom language use. Consideration of high or low frequency of utterance, together with high or low use of technical terms, each considered in both public and private contexts, suggest groups of classrooms sharing common patterns of language use:

- Mathematics classrooms of very low public interactive orality and extremely low private interactive orality – where, apart from a small number of choral responses, only the teacher makes use of any mathematical terms: Seoul 1, 2, and 3.
- Mathematics classrooms of low public interactive orality, but relatively high private interactive orality – where the student classroom use of mathematical terms is relatively low: Hong Kong 1, 2, and 3.
- Mathematics classrooms of relatively low public and low private interactive orality – where the teacher and students both make significant use of mathematical terms (that is, high lexical density): Shanghai 1, 2, and 3.
- Mathematics classrooms of high public and private interactive orality – where teacher and students make relatively infrequent use of mathematical terms (low lexical density): Berlin 1, Melbourne 2 and 3, and San Diego 1.
- Mathematics classrooms of relatively high public and private interactive orality – where the teacher and the students make relatively frequent use of mathematical terms: Melbourne 1, San Diego 2, Singapore 1, 2 and 3, and Tokyo 2.
- Mathematics classrooms of moderate public and private interactive orality – with moderate teacher and student use of mathematical terms: Tokyo 1 and 3, and Berlin 2.

Since the characterisation of each classroom is based on detailed analysis of at least five lessons per classroom, and the private language use of about ten students in each classroom, the patterns of language use outlined above should be quite robust as characterisations of the practices of each classroom. As acknowledged earlier, the nature of the mathematical language employed will reflect the topic taught in each classroom. However, each topic (with the possible exception of Melbourne 3) required a variety of technical mathematical terms, sufficient to provide evidence of a classroom emphasis on spoken mathematics or not.

To repeat the point made in the companion chapter: It is really only through international comparative studies such as this one that we can make such comparisons between classrooms so fundamentally different in their practices. The teachers in the LPS project were recruited on the grounds that the local mathematics education community endorsed their practice as competent. Given this selection criterion, it is reasonable to assume that we have documented competent mathematics teaching as this was conceived at the time of data generation in each city. Despite within-city variations, the mathematics classrooms from some cities do seem to share sufficient common features to suggest that they draw on a common tradition of practice.

Since it is the use of mathematical language that is the focus of this analysis, student facility in the use of mathematical language to describe the activities and content of particular mathematics lessons seems an appropriate outcome to examine. Given the popular (Western) advocacy of student participation in mathematical dialogue in the classroom, the classrooms studied in Seoul provided an interesting testing ground for this advocacy, since they represent the antithesis of this practice. The consistency of language use across the three Seoul classrooms suggests a well-established tradition of practice, even if contemporary curricular reforms require that this tradition be supplanted by a more discursive pedagogy. It has to be considered as feasible, therefore, that the Korean national success on international tests of mathematical performance (for example in the TIMSS study, reported in Beaton & Robitaille, 1999) was achieved through classroom practices like those documented here.

CONCLUSIONS

The Asian classrooms in this study varied in their practice from no spoken mathematics by students (Seoul), through almost entirely public spoken mathematics by students (Shanghai), to spoken mathematics by students in both public and private classroom settings (Tokyo and Singapore). Differences in outcome in terms of facility with spoken mathematics (as displayed in interviews) may reflect differences in aspiration (rather than simply differences in success) – different cultures valuing different types of mathematical performance. What is essential is that our theories of learning should not unwittingly incorporate culturally-specific assumptions about the nature of classroom practice and about valued outcomes. Instead, our theories should anticipate application in culturally-differentiated settings and be sensitive to the constraints and affordances that culture places on practice.

To summarise: Students in the mathematics classrooms in Seoul had few opportunities to speak in class (either privately or publicly) and seldom employed spoken mathematics. Students in the Hong Kong classrooms were publicly and privately vocal, but made very little use of spoken mathematical terms in either context. Students in the mathematics classrooms in Shanghai were guided through the public orchestrated rehearsal of mathematical terms by their teachers, but seldom spoke to each other in private during class time except when explicitly asked by the teacher to conduct group or peer discussions. Students in the mathematics classrooms in Tokyo and Singapore participated orally in both public and private discussion and employed mathematical terms to a significant extent in both. By comparison, the students in Melbourne classroom 1 were highly vocal in both public and private contexts, and made more frequent public use of mathematical terms than any of the three Japanese classrooms, but less frequent use of mathematical terms in their private conversations. These different combinations of oral interactivity and mathematical orality suggest distinct pedagogies.

The essential question is, of course, whether or not students are advantaged in terms of their mathematical achievement and understanding by classroom practices that afford the opportunity to develop facility with spoken mathematics. The post-lesson interviews provide some evidence of a connection between classroom mathematical orality and student learning outcomes. This evidence suggests that those classrooms that promote student spoken use of mathematical terms do develop in those students the capability to use mathematical terms to describe their mathematics classrooms and their mathematics learning. If we use the term "mathematical orality" to signify this fluency in spoken mathematics, then our analysis suggests that, if mathematical orality is promoted in the classroom, whether in the public or the private domain, then students can develop this facility. The question of whether such mathematical orality can be associated with some higher form of mathematical understanding requires further consideration, both empirically and theoretically. It is our hope that the analyses reported in this and the preceding chapter will provide the basis for further work on this important issue.

This research also has significance for the development of theory. The contemporary advocacy of student spoken mathematics in classroom settings is prompted by research conducted in Western classrooms. The analyses reported in this and the preceding chapter can be interpreted as problematising such unqualified advocacy. Since the research cited to justify such advocacy is entirely Western, it is possible that the prescribed instructional practices might only be practicable in "Western" classrooms. As proposed in the preceding chapter, interpretation and application of the Western advocacy of spoken mathematics should be subject to three considerations: (i) The advocated practices may be non-viable in a culture dissimilar to that in which the research studies were conducted; (ii) The advocated practices may target outcomes that are not valued in school systems different from those studied; and (iii) The theories of teaching/learning by which such advocacies are rationalised may themselves be culturally-specific. Contrast such advocacy with evidence of belief in the capacity of active listening (rather than oral participation) to promote student learning (Li, 2004; Remedios, Clarke, & Hawthorne, 2008).

The results of our analyses of classrooms in Singapore and Tokyo suggest such practices are at least feasible in some non-Western settings. Research is currently being undertaken into the cultural-specificity of the constructs (particularly pedagogical terms) from which our theories of teaching/learning are constructed and through which they are expressed. It is our hope that research in the classrooms of competent mathematics teachers around the world might lead to an expansion in the instructional repertoire of all teachers and to a more inclusive reconstruction of the theories by which accomplished mathematics teaching and learning are conceived.

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NOTES

i Lexical density here refers to the relative concentration within sampled utterances of technical terms drawn from the mathematics lexicon.

ii As this example shows, it was not always possible to interview the student immediately after the lesson. The majority of interviews occurred on the same day as the relevant lesson, but sometimes it was necessary to delay an interview over a weekend.

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