STUDENT VOICE IN MATHEMATICS CLASSROOMS AROUND THE WORLD

Student Voice in Mathematics Classrooms around the World

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SERIES PREFACE

The Learner's Perspective Study provides a vehicle for the work of an international community of classroom researchers. The work of this community is reported in a series of books of which this is the fourth. International comparative and cross-cultural research has the capacity to inform practice, shape policy and develop theory. Such research can reflect regional, national or global priorities. Cross-cultural comparisons of social practice in settings such as classrooms can lead us to question our assumptions about what constitutes desirable learning or effective instruction. International comparative research offers us more than insight into the novel, interesting and adaptable practices employed in other school systems. It also offers us a new perspective on the strange, invisible, and unquestioned routines and rituals of our own school system and our own classrooms. In addition, a cross-cultural perspective on classrooms can help us identify common values and shared assumptions across geographically disparate social settings, which in turn can facilitate the adaptation of practices from one classroom for use in a different cultural setting. The identification of structure and recurrence within cultural diversity can help us to distinguish between fundamental commonalities and local conventions. Research into the phenomenon of student voice in different classroom settings can provide profound contrasts and unexpected similarities, supporting the constructive interrogation of entrenched practices and established theory.

David Clarke
Series Editor

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GLENDA ANTHONY, BERINDERJEET KAUR, MINORU OHTANI
AND DAVID CLARKE

CHAPTER ONE

The Learner's Perspective Study: Attending to Student Voice

INTRODUCTION

Learning environments are never identical. Research findings from the Learner's Perspective Study (LPS) affirm just how "culturally-situated are the practices of classrooms around the world and the extent to which students are collaborators with the teacher, complicit in the development and enactment of patterns of participation that reflect individual, societal and cultural priorities and associated value systems" (Clarke, Emanuelsson, Jablonka, & Mok, 2006, p. 1). In this book we attend closely to this collaboration with our focus on the voice of the student. Collectively the authors consider how the deliberate inclusion of student voice within the LPS project can be used to enhance our understandings of mathematics classrooms, of mathematics learning, and of mathematics outcomes for students in classrooms around the world.

As noted by the originators of the LPS project, the LPS design with the deliberate inclusion of the student voice, was initially conceived to address what was noted as a major limitation of international comparative studies at the turn of the century—an exclusive focus on the curriculum and the teacher. In reference to what was the major source of international comparative data at this time Thorsten (2000) notes:

> What is absent from nearly all the rhetoric and variables of TIMSS pointing to the future needs of the global economy is indeed this human side: the notion that students themselves are agents. TIMSS makes students from 41 countries into passive object of 41 bureaucratic gazes, all linked to the seduction of one global economic curriculum. (p. 71)

In contrast to the position taken by Thorsten at the beginning of this century, contemporary intercultural research, while still contestable in some forms (Wiseman, 2010) is characterised by a broadening of theoretical perspectives aligned to socio-cultural and political dimensions of mathematics education (see Shimizu & William, 2013). Contemporary educational research has increasingly drawn our attention to the importance of the social processes within the classroom. Quality mathematical experiences that enhance a range of student outcomes are premised on the understanding that knowledge is necessarily social (Bell & Pape, 2012; Wagner, 2007; Walshaw, 2011). Researchers draw on social learning theory to look at how competence is constructed and constituted within the unique activity

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system of a classroom (Gresalfi, Martin, Hand, & Greeno, 2009), and to explore the formation of learners' identities (Solomon, 2007), learners' dispositions (Hunter & Anthony, 2011), and learners' participatory and mathematical practices (Boaler, 2008). The socio-cultural influences on learning are well represented in the LPS project. In particular, the research design involving intensive video capture of micro and macro classroom events and post-lesson video-stimulated recall interviews gives primacy to the voice of the student.

Attending to student voice also serves to enhance our understanding of the ongoing relationship between the teacher and student as co-constructors of knowledge and practice within the classroom. No matter where the classroom is situated within the world stage, effective teachers—such as those selected as participants in the LPS project—are those that focus on enhancing student outcomes and achieve their purpose. That is to say, a pedagogical practice that is effective is linked to student outcomes. Achievement outcomes related to mathematical proficiency encompass conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning (Nation Research Council, 2001). Added to those outcomes is another set that underwrites a quality mathematical experience. These are the social and cultural outcomes relating to affect, behaviour, communication, and participation (Anthony & Walshaw, 2007; Sullivan, 2011). Proposed in this way, Walshaw (2011) contends that effective pedagogy results in the development of mathematical proficiency and aptitude over time and is "characterised by an enhanced, integrated relationship between teachers' intentions and actions, on the one hand, and learners' dispositions towards mathematics learning and development on the other" (p. 94).

In attending to the relational and social nature of learning we also need to acknowledge that mathematics learning is embedded within both the cultural and political dimensions of mathematics education (Jablonka, Wagener, & Walshaw, 2013). Acknowledging the socio-political setting of the classroom learning environment, the design enables us to foreground the agency of the student, the nature of learner practice, and the cultural specificity of that agency and that practice (Gutierrez, 2013). Teachers, learners, (and researchers) bring to the teaching and learning encounter a history that is entwined with their experience of the social and political work. These approaches, underpinning chapters within this text, have been used to understand learning and development in a way that takes culture as a core concern. Locating social and cultural processes as mediators of human activity and thought highlights the importance of local activity settings (Nasir & de Royston, 2013).

While it is not intended that this collection of chapters provide a comprehensive inventory or a summary of all our separate learning about the form and role of student voice in LPS classrooms, collectively the chapters serve to highlight the varied ways that students' mathematical, social, and political voices are implicated in the social interactions and range of learning outcomes within the mathematics classroom. Not surprisingly, given the diversity of the classrooms and the theoretical perspectives of the authors, student voice is given 'voice' in a multiplicity of ways. In the following sections, we introduce the varied ways student voice has been framed by the authors of the text based on their theoretical and local perspectives.

Mathematical Discourse in the Classroom

The nature of productive talk in the mathematics classroom has been the focus of considerable research. As highlighted in the two chapters by Clarke, Xu, and Wan, talk can occur in both public and private arenas and involve student and teacher, several students, or a self-conversation. Within much of the research originating in the West, the research focus has been on talk that occurs in group work and whole class discussions (Walshaw & Anthony, 2008; Wood & Kalinec, 2012). The focus of much of this research has been on participatory and communication practices associated with the development of mathematical argumentation discourse. Research has acknowledged that effective and equitable implementation of group and whole class discussion is challenging. With groups, in particular, many studies have found that students are rarely focused on mathematical content for the entire portion of their small group time (Wood & Kalinec, 2012) and some students are excluded from equitable participation (Esmonde & Langer-Osuna, 2013). In Chapter 2, Clarke, Xu, and Wan focus their attention on the adoption of the discursive practices of the academic mathematician— that is, the written and spoken language endorsed by the wider mathematics community. Specifically, they analyse the opportunities for students to hear and speak mathematical terms. Drawing on LPS classrooms from Melbourne, Hong Kong, Shanghai, Berlin, Tokyo, Singapore, Seoul and San Diego their analysis highlights considerable variance in the expectation and opportunities for students to engage in spoken articulation of mathematical terms as part of public classroom discourse. Importantly, their analysis reveals relative differences in the levels of public talk versus mathematical public talk, a feature in common with analysis of mathematical talk within small group situations. In Chapter 3, Clarke, Xu, and Wan's triangulation of the classroom video data with the student interview data leads the authors to suggest a link between classroom mathematical orality and student learning outcomes. They claim that "those classrooms that promote student spoken use of mathematical terms do develop in those students the capability to use mathematical terms to describe their mathematics classroom and their mathematics learning" (p. 50). This finding informs discourse practices associated with Western curricula that advocate expectations that secondary level students will communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary, and a variety of representations, and observing mathematical conventions (Barwell, 2012). However, given the disparity of pedagogical discourse practices—regarded as effective within their local settings—associated with the respective classrooms the question posed by the authors as to whether such fluency in spoken mathematics is associated with higher forms of mathematical understanding is timely and significant.

In Chapter 4, Cao, Guo, Ding, and Mok examine public student voice within a specific classroom episode termed “Students at the Front.” “Students at the Front” activity is advocated as part of China’s mathematics reforms. The goal of the activity is to encourage students to share their mathematical thinking about problems with peers. In light of an earlier comparative study (Jablonka, 2006) of classes drawn from Germany, Hong Kong, and the United States that highlighted the challenges involved in orchestrating and supporting productive public mathematics discourse, Cao and colleagues wanted to look closely at the form and frequency of student talk during this activity within a sample of six lessons from one class in Beijing. Lesson excerpts provide examples of students sharing their solution strategies, students providing assistance to other students, students building on other students’ thinking and of active listening. Quantitative analysis of the classroom video suggests that time spent on explanation and discussion was significantly greater than time spent on procedural explanations. Another feature was the teacher evaluation and summary after each student presentation. The authors hypothesise that the integration of student voice in teaching orchestrated by the teacher is a characteristic of effective pedagogy linked to the development of student thinking.

Moving to Singapore, Kaur, in Chapter 5, also provides an analysis of students’ engagement in public talk. In this chapter, however, the analysis occurs across a series of 10 lessons each for two teachers. Characterised according to levels of discourse complexity, with Level 0 involving students giving short answers to the teacher’s questions, and Level 3 involving students initiating clarifying questions and building on peers’ thinking and explanations, Kaur’s analysis of lessons within these two classrooms offer limited evidence of student engagement in content-learning discourse in the classrooms studied. While exhibiting individual patterns of discourse, Kaur concludes that both classes were dominated by teacher talk and student listening. Student-teacher interactions, for the most part, were related to the teacher’s assessment of students’ progress in understanding the demonstrated problem solution methods, and this was classified as calculational orientation. Kaur hypothesises that the almost total lack of student-initiated public talk was expected, given the instructional organisation of teacher demonstration, seatwork and whole class review of written work by the teacher. Without activities such as “Students at the Front” as described in Chapter 4, student-initiated public talk was not likely to occur in a classroom that privileges individual attainment via practice during seatwork. Kaur hypothesises that the effectiveness of the teachers’ pedagogies is related to the close attention to monitoring and attending to students’ progress against very specific instructional objectives.

Students’ Participatory Practices within the Classroom

Within our classrooms, students must learn to engage in classroom discourse and practices that serve both social and cognitive functions. A research focus on the social nature of learning activity must include the co-construction of classroom norms, participation structures, and collaboration (Nasir & de Royston, 2012).

These social activities, “where teacher and students improvise their interactions within the constraints and affordances of cultural, societal and institutional norms” (Clarke et al., 2006, p. 8) involve “reciprocity and a pedagogical attention that moves students towards independence” (Walshaw, 2011, p. 94). Students are implicated in the classroom practices that both constitute and are constituted by the norms and interactions of the classroom on a daily basis. A focus on the student gives voice to these practices from the inside (while still subject to the researcher’s interpretation).

Chapter 6 by Gallos Cronberg and Emanuelsson, featuring the student Martina’s voice within a sequence of ten lessons from a Swedish classroom, provides a unique insight into cultural variation concerning student independence. Martina learns mathematics in a classroom environment in which students are required to plan and work on their own on different tasks, independent of other students and to a large extent independent of the teacher. Prompted by concerns raised by Hansson (2010) that question the extent to which such an environment and associated pedagogical practices can support students to develop appropriate levels of mathematical proficiency, the authors examine how Martina negotiates her learning environment. In particular, how she exhibits agency and how she interact with others in her community are considered in relation to learning outcomes associated with mathematical practices and reasoning. Using Martina’s voice, the authors examine how Martina was able to contribute to the sociomathematical norms in the class, engaging in public mathematical discourse that involved explanation, justification, and argumentation. However, the authors note that opportunities to learn were mediated by access to mathematical tasks and by Martina’s interpretation of the didactical contract (Brousseau, 1997). In contrast to many of the learning environments featured in this text, Martina regarded the textbook as the main source of support, and as a consequence, learning outcomes were dependent somewhat on the suitability of the instructional text to assist movement to the next zone of learning development (cf. Vygotsky, 1986).

In looking at participatory practices within the lesson, Nyman and Emanuelsson in Chapter 7 have chosen to focus their analysis on the enactment of the mathematical task using the construct of task-related attention. As noted by Sullivan, Clarke and O’Shea (2010), effective learning is not solely dependent on the quality of the tasks, “but also on the ways the teacher implements the task, and whether the students are able to take advantage of the opportunities that working on the task might offer them” (p. 531). Situated within the social interactions of one Swedish mathematics lesson, Nyman and Emanuelsson give voice to students’ task-related attention through the categories of relevance, solution methods, and validation of tasks. Again the learning outcomes are a focus. In this chapter, the outcome that takes centre stage is that of student interest and its relationship to participation in mathematics learning practices. For example, they illustrate how interest constructed during a student generated discussion on task relevance acts as a segue for the student to both solve the task and see the meaning of the task. Linking the documented practice to effective pedagogies, the authors hypothesise that the teacher’s efforts to clarify the relevance of the task was instrumental in developing student interest and consequent engagement with the intended content matter. The authors contend that task-related attention can be enhanced by specific teacher-student interactions that support and acknowledge student interest.

In Chapter 8, Novotná and Hospešová consider how students can develop their problem solving competence within the social milieu of the classroom. They utilise student and teacher voice within classroom discourse episodes to understand how the teacher provides space for students to access and build on informal and acquired knowledge. Drawing on the theory of Brousseau (1997), they analyse a-didactical situations looking closely at teacher and student perceptions of situations where the teacher intentionally provided activities linked to students' real life experiences or prior knowledge. The authors conclude that to be effective the teacher needs to attend to students' voice in all its possible forms. They provide evidence of the complexity of occasioning student learning, arguing that the effective teacher needs not only to be able to "work with students' suggestions on how to solve a given problem," but also the teacher needs to be able to react "without hesitation to the unforeseen situations arising in consequence to other influences than mathematics" (p. 141). Meeting this challenge, Novotná and Hospešová argue, requires that the teacher has a deep knowledge of the students as individuals.

Bergem and Pepin, in Chapter 9, examine the development of democratic agency within the mathematics lesson. Like Walshaw (2011), they argue that effective pedagogy that involves students' participation in classroom discussions has both a cognitive and social dimension. Using data from Norwegian classes, they provide exemplars of teacher student interactions that either afford or constrain opportunities for students to challenge and question the teacher's and other students' thinking. Student voice, as expressed in challenges to the teacher, was used to explore engagement levels with tasks and to examine issues of democratic participation. A feature of the Norwegian classes in the LPS project was that opportunities were provided for everyone to contribute to and participate in ongoing discussion. However, unlike the Shanghai classrooms described by Huang and Barlow in Chapter 10, this opportunity extended to a choice to agree or refuse to come to the board when asked by the teacher. Effective pedagogy, these authors conclude, occasions opportunities for students to participate and engage themselves as members of a group. They suggest that the diversity of ideas expressed within the group has the potential to promote deeper mathematical understanding.

Much of the current research work on understanding how students participate in the social and mathematical practices of the classroom is driven by the need to address systemic levels of underachievement and disengagement among disadvantaged groups of learners in our classrooms (Gutierrez, 2013). An important role of intercultural studies is that they enable us to question taken-for-granted practices within one's own culture and society that may serve to perpetuate inequities. For example, Anthony in Chapter 12, while focused on the notion of students' perception of the 'good' teacher, contrasts the learning opportunities afforded students in a top and a low set class in terms of co-constructed norms of participation. In each of these classes, students accessed significantly different activities and associated mathematical practices that variously afforded or constrained student opportunities to develop mathematical proficiency.

Students' Perceptions of the Classroom and Teacher

One of the arguments for inclusion of student voice relates not to the role of student voice within the lesson itself, but rather to student voice about the learning experience after the fact. Until relatively recently, most efforts to improve education have been based on adults' notions of how education should be conceptualised and practised and the views and opinions of young people have been traditionally discounted as having less legitimacy than the views of adults. Research seeking students' perceptions began with the premise that for teachers and researchers to be able to understand and improve learning and teaching, we need to canvas students' needs and viewpoints. Brown (2002) argued that student views of learning reflect their experiences with the activities that teachers provide and the values teachers convey as being important. That is, students construe learning in ways that they have been socialised to do, through their perceptions of what their teachers' value. These student voices can be particularly useful for informing local contexts. For example, examining the messages within the narratives of young Maori students in New Zealand, Bishop (2003) identified conditions necessary for supporting the engagement of Maori youth in school-based learning. Central to the findings was that young Maori students valued teachers who would enable them to bring their cultural experiences to the learning conversation.

Motivated by a strong belief that the characterisation of the practices of mathematics classrooms must attend to the learners' practice with at least the same priority as that accorded to the teacher's practice, several chapters in this text collate both student and teacher perspective data generated by the LPS research design. Within the video-stimulated recall interview situation both students and teachers were asked comment on aspects of the lesson that were significant to them, and invited to make more general comments about the overall experience of the teaching/teacher and the learning environment. Taking the view suggested by McGregor (2005) of treating students as 'experts' in schooling, the design assumes that students will have knowledge of the class which adults might not have. In this sense, students could hold different views regarding what are important moments within their lesson from those of the teacher. Prompted by a scarcity of research about how Chinese students perceive their classroom learning, Huang and Barlow, in Chapter 10, explore the relationship between student and teacher perceptions of important events across a set of 15 consecutive lessons in a Shanghai classroom. They describe the students' perspective as aligned to 'learner-trained learning' where students are well aware of the expected procedures and react promptly to teacher cues (Cortazzi & Jin, 2001). Thus, not surprisingly the authors noted a strong match between teachers' and students' perceptions of important events within each lesson. Again, we see how student voice can inform our understanding of effective pedagogy and learning outcomes. This match, Huang and Barlow argue, affirms the effectiveness of the particular pedagogical approach in promoting valued learning outcomes, claiming that "the students are more likely to engage in the mathematical tasks at the cognitive level the teacher intends" (p. 184).

Drawing on student interview data from one Singapore class (SG1) and one Hong Kong class (HK1), Mok, Kaur, Zhu, and Yan (see Chapter 11) compare Singapore and Hong Kong student perspectives of their respective lessons. Following on from earlier independent analysis (see Kaur (2008, 2009) from Singapore and Mok (2009) from Hong Kong) noting that both cohorts of students were very positive about their learning, the authors examine those pedagogical routines and practices that were important to the students within the exposition, seatwork and review phases that characterise lessons within these two countries. Based on student reports of significant moments in lessons, students in Singapore and Hong Kong provided closely matched responses. Both cohorts valued clear teacher explanations and demonstration of procedures, followed by individual seatwork for practice, with Singapore students also reporting appreciation of opportunities for group work as an additional source of practice. Whole class report back sessions were also valued by both cohorts, largely as a way of checking the answers and (re)learning via corrective feedback. The authors characterised the students' expectations as 'seeking a virtuoso to follow.' While the characterisation of pedagogy is different to that provided by Huang and Barlow, the harmonious match between teachers' instructional practices and students' expectations re their learning needs is offered as a potential reason for students' high performance levels.

Chapter 12 by Anthony also uses students' voice from interviews to explore the notion of a 'good' teacher, but in this chapter students' perceptions of their teacher are linked to their perception of themselves as a learner. Using the conceptual tools proposed by Cobb, Gresalfi, and Hodge (2009), Anthony looks at the interplay between social practices and the processes of self-form that are at work within two contrasting New Zealand mathematics classrooms—grouped distinctly as high achieving and low achieving students. Analysis of the alignment between what students valued in their teacher and expectations of how students should behave within the mathematics classroom, expressed as normative identity, were consistent within each class, but notably different between classes. In any mathematics classroom, mathematics knowledge is "created in the spaces and activities that the classroom community shares within a web of economic, social and cultural difference" (Walshaw, 2011, p. 95). In this chapter, exploration of these spaces by means of students' voices illustrates how the development of mathematical proficiency cannot be separated from the axes of social and material advantage or deprivation that operate to define students both within the school system and community. While both classes reported positive feelings about their mathematical learning experience, Anthony questions whether these positive feelings equated to equitable learning opportunities.

CONCLUSION

In these chapters we see how effective pedagogy—in all its various forms—takes into account the ways of knowing and thinking, language, and discursive registers made available within the physical, social, cultural, historical, and economic community of practices in which the teaching and learning is embedded (Anthony & Walshaw, 2007). The authors have used student voice to demonstrate how aspects of engagement in mathematics lessons, such as approaching unfamiliar problems, persisting in the face of challenge, and interacting with others are crucial determinants of what students come to know and do (Boaler, 1997). These behaviours all constitute aspects of disposition—an important strand of mathematical proficiency (Gresalfi, 2009).

In some chapters, the role of social, emotive, and motivational factors are shown not simply to act as influences on learning but are seen as central drivers to the learning process. Issues of identity and power bring suggestions of socio-political framing paying heed to the power and affective dimensions of the classroom in their exploration of the "relationality of the teaching/learning encounter" (Appelbaum & Allen, 2008, p. 52). For example, Anthony (see Chapter 12), Bergem and Pepin (see Chapter 9), and Gallos Cronberg and Emanuelsson (see Chapter 6) consider how individuals can act to resist classroom social and socio-mathematical norms.

Collectively, these chapters serve to affirm the underpinning assumption of the Learner's Perspective Study that the characterisation of the systems of social practice within the mathematics classrooms must attend to the learners' practice with at least the same priority as that accorded to the teachers' practice. In focusing on student voice within this partnership, as enacted in many different guises across different cultures and socio-political learning environments, we hope that we will be better informed to understand the relationship between pedagogy and learning mathematics, and between pedagogy and the empowerment of diverse learners.

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CHAPTER TWO

Spoken Mathematics as an Instructional Strategy: The Public Discourse of Mathematics Classrooms in Different Countries

SPOKEN MATHEMATICS IN THE CLASSROOM

This chapter examines the use of spoken mathematics in the public discourse of eighth-grade mathematics classrooms internationally. By "spoken mathematics" we mean the recognizably mathematical terms used in spoken interaction in the classroom. Our principal focus was the relatively sophisticated terms by which each lesson's central concepts or procedures were named. In our analysis we addressed the question(s): "What is the occurrence of publicly spoken mathematics in the different classrooms studied and what efforts do the teachers appear to make to promote students' use of technical mathematical terms in their public classroom talk?" A companion chapter examines the question of students' private spoken mathematics in the classroom and the possible learning that might result.

LANGUAGE IN MATHEMATICS TEACHING/LEARNING

Various theories of learning attach priority and even primacy to language (notably Vygotsky, 1962). Without revisiting in detail the distinction between "mathematics as a language" and the role of language in the learning of mathematics, it is important to note that "language and mathematics" is a conjunction that has been explored by a variety of scholars, particularly in the context of the mathematics classroom (for example, Pimm, 1987). In the context of science, Lemke (1990, p. 1) asserted unequivocally "learning science means learning to talk science." This identification of a discipline with a particular discourse has been taken one step further by Sfard (2008), who defines mathematics *as* a discourse. Our position in this chapter is that students participating in mathematics classrooms are initiated into a local discourse that might be called "the discourse of the mathematics classroom." The discourse of one mathematics classroom may differ significantly from the discourse of another, both in terms of the mathematical sophistication of the terminology employed and in the relative prioritisation and authority accorded to the voices of the teacher and the students.

Any theory of mathematics teaching/learning must address the role of language. In this chapter, we take the orchestrated use of mathematical language by the participants in a mathematics classroom to be a strategic instructional activity by the teacher. Our particular focus is the role of spoken mathematics in both

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instruction and learning. The instructional value of the spoken rehearsal of those mathematical terms and phrases central to a lesson's content can be justified by reference to several theoretical perspectives. Interpretation of this spoken rehearsal as incremental initiation into mathematics as a discursive practice could be justified by reference to Walkerdine (1988), Lave and Wenger (1991), Bauersfeld (1994), and Sfard (2008). The instructional techniques employed by the teacher in facilitating this progression could be seen as "scaffolding" (Bruner, 1983) and/or as "acculturation via guided participation" (Cobb, 1994). Interest in "speaking mathematically" as an important aspect of mathematics classroom interaction has an extensive history. Since language is universally accorded a central role in the learning of mathematics, the instructional use of spoken mathematics by students and teachers in classrooms warrants investigation in settings differentiated by language, by school system and by culture. Such variation in classroom setting provides the optimal conditions for the interrogation of both theory and practice regarding the role of spoken mathematics in classrooms internationally.

Research and theorising regarding the role of language in mathematics classrooms has been culturally-situated to a remarkable extent. The review by Walshaw and Anthony (2008) includes the statement: "What these researchers have demonstrated is that effective instructional practices *demand* students' mathematical talk" (p. 523, emphasis added). The review purposefully omitted consideration of classrooms situated in Asian countries, and this was acknowledged by the authors. Given contemporary interest in the success of school systems in countries such as Japan, Korea, and Singapore in international tests of mathematics achievement, this omission could be seen as unfortunate. The review does, however, provide extremely useful insights, provided this cultural specificity is taken into account.

There is an internal coherence and consistency of message in the literature about classroom discourse arising from what might be called the Western canon[i] in educational research. A key element in this message has been summarised succinctly by Silverman and Thompson (2008).

> Thompson, Philip, Thompson, and Boyd (1994) and Cobb, Boufi, McClain, and Whitenack (1997) argue convincingly that students' participation in conversations about their mathematical activity (including reasoning, interpreting, and meaning-making) *is essential* for their developing rich, connected mathematical understandings (Silverman & Thompson, 2008, p. 507, *emphasis added*).

The review by Walshaw and Anthony (2008) is similarly assertive regarding the role of student conversation in mathematics classrooms. They make strong statements regarding the importance of student spoken participation in mathematics classrooms and support them with a long list of references. As acknowledged by the authors, these references are drawn entirely from Western sources, and the results are presented legitimately as unequivocal advocacy of classroom dialogue.

> There is now a large body of empirical and theoretical evidence that demonstrates the beneficial effects of participating in mathematical dialogue in the classroom. (Walshaw & Anthony, 2008, p. 523)

By way of contrast, Li (2004) in discussing "A Chinese Cultural Model of Learning" made the following observation.

> Asian students not only do not believe that speaking promotes thinking as do Western students; they believe that speaking interferes with thinking. (Li, 2004, p. 132)

Such differences in the role accorded to language in learning situations must have consequences for the form taken by the discourse of the mathematics classroom in different cultures. The distinctive character of classrooms situated in different cultural traditions has been the subject of several substantial publications (Clarke, Keitel, & Shimizu, 2006; Fan, Wong, Cai, & Li, 2004; Leung, Graf, & Lopez-Real, 2004). The analyses reported in the remainder of this chapter suggest that the instructional practices of the teachers in the various classrooms were predicated on pedagogies that assign spoken mathematics a very different function in the learning process.

The concern has been raised elsewhere (Clarke, 2006a) that the cultural-situatedness of our theorizing about classroom practice makes it very difficult to give recognition in our research to practices intended to generate behaviours or outcomes inconsistent with the cultural history of the researcher.

> The explicit promotion of student speaking in Western reform classrooms and the dominance of student listening in Asian classrooms gives the appearance of a dichotomisation of student classroom practice into an emphasis on either speaking or listening. Similarly, analyses of teacher practice reveal significant differences in teacher time devoted to speaking or listening. It is essential that the debate shift from the separate optimisation of speaking or listening to recognition of their essential interconnectedness and the role of both in any theory of teaching/learning. (Clarke, 2006a, p. 384)

Our theorizing about the connection between classroom practice and learning and, in particular, our research into the practices of mathematics classrooms must draw on (and contribute to) theories that accommodate culture as one essential aspect of the situated nature of learning, rather than ignoring the pervasive role of culture in framing our attempts at theorizing. This cultural situatedness is particularly evident in writings on the role of language in the mathematics classroom.

International classroom research projects such as the Learner's Perspective Study (Clarke, Keitel, & Shimizu, 2006) or the TIMSS-R Video Study (Hiebert et al., 2003) provide the opportunity to interrogate the capacity of our theories to accommodate classroom practice in cultural settings other than those in which the theories themselves were developed. The possible primacy of language in knowledge construction can then be examined without the distorting prejudice of a context in which particular types of oral performance (for example) are already privileged. There are at least two dangers in the cultural specificity of recent theorizing: (i) it may be that learning and instruction in non-Western contexts are simply not accommodated in simplistic (Western) frameworks, such as the popular teacher-centred/student-centred dichotomy (Mok & Ko, 2000), and (ii) typification of 'the other' may mean that learning and instruction in non-Western contexts are accorded a homogeneity or uniformity of practice and character that disregards the rich diversity in instructional practice denied by the simplistic singularity of 'the Asian classroom.'

The advocacy of "mathematical dialogue in the classroom" is based on research in 'Western' classrooms, which certainly have their own diversities of practice (see O'Keefe, Xu, & Clarke, 2006, for example). Whether that advocacy can be extended legitimately to encompass practice in classrooms situated in Asian countries remains a matter for empirical investigation. The research reported in this chapter reveals significant differences in the role of spoken mathematics in classrooms around the world. It is suggested that it may be necessary to acknowledge the cultural-specificity of our theories and the need to adapt them for use in settings culturally dissimilar from those in which they were developed.

STUDYING SPOKEN MATHEMATICS IN THE CLASSROOM

The complete LPS research design is set out in the Appendix to this book. For the analysis reported here, the essential details relate to the standardisation of transcription and translation procedures. Three video records were generated for each lesson (teacher camera, focus student camera, and whole class camera), and it was possible to transcribe three different types of oral interactions: (i) whole class interactions, involving utterances for which the audience was all or most of the class, including the teacher; (ii) teacher-student interactions, involving utterances exchanged between the teacher and any student or student group, not intended to be audible to the whole class; and (iii) student-student interactions, involving utterances between students, not intended to be audible to the whole class or to the teacher. All three types of oral interactions were transcribed, although type (iii) interactions could only be documented for two selected focus students in each lesson. Where necessary, all transcripts were then translated into English. Transcription and translation were carried out by the local team responsible for data generation and were therefore undertaken by native speakers of the local language. Technical guidelines specified the format to be used for all transcripts and the conventions for translation (particularly of colloquial expressions) (Clarke, 2006b). The analyses reported in this chapter were undertaken on the English version of each transcript of public classroom dialogue.

Analyses were conducted of 110 lessons documented in 22 classrooms located in Australia (Melbourne), China (Hong Kong and Shanghai), Germany (Berlin), Japan (Tokyo), Korea (Seoul), Singapore, and the USA (San Diego). In this chapter we report the first two stages of a stratified analysis focusing on the situated use of spoken mathematical language in these classrooms. The first and second analytical stages focused on public oral interactivity (frequency of public utterance) and public mathematical orality (spoken use of key mathematical terms) (Clarke & Xu, 2008). The third, fourth and fifth analytical stages are reported in a companion chapter. Throughout the entire analysis, we distinguish *private* student-student interactions from whole class or teacher-student interactions, both of which we consider to be *public* from the point of view of the student.

Our major concern in the first two stages of the analysis was to document the opportunity provided to students in the mathematics classroom for the public oral articulation of the relatively sophisticated mathematical terms that formed the conceptual content of the lesson and to distinguish one classroom from another according to how such student mathematical orality was afforded or constrained in the public classroom context. The specific mathematical terms employed will, of course, reflect the mathematical content of the lessons. Algebra comprises a significant component of the eighth grade mathematics curriculum in most countries. Of the 22 classrooms studied in this chapter, 18 were concerned with either systems of linear equations or the simplification of algebraic expressions. Of the four other classrooms, three addressed geometry topics (Tokyo 2 and Melbourne 1 and 2) and one "rounding decimals and percentage" (Melbourne 3). With the possible exception of Melbourne 3, the mathematical topic addressed in each classroom was associated with an identifiable vocabulary of sophisticated mathematical terms.

An essential point needs to be made here: In reporting the results of our analyses we have been careful to make explicit reference to "*the* Shanghai lessons" (or students, teachers or classrooms), meaning *only* those Shanghai lessons (or students, teachers or classrooms) for which we have data. In English usage, reference to "Shanghai lessons" or "Shanghai teachers" (without the specific use of "the") would imply generalization to all Shanghai lessons or teachers, and we have made every attempt to avoid this implication. If regularities among particular groups of classrooms or teachers appeared to indicate commonalities of practice across different settings, then the possibility of regional, cultural or national norms of practice has been suggested explicitly. On the other hand, evident disparity of practice among classrooms that might otherwise have been seen as similar can be used to contest simplistic generalised categories, such as 'Asian.'

PUBLIC MATHEMATICAL ORALITY: WHO GETS TO SPEAK PUBLICLY AND DO THEY TALK MATHEMATICS?

In our first analytical pass, we counted the number of utterances made by anyone participating in a whole class or teacher-student interaction (a "public utterance" from the student perspective). An utterance is taken to be a continuous spoken turn, which may be both long and complex. In identifying distinct utterances, we treated either a change of speaker or an extended silence (greater than 3 seconds) as the demarcation indicator separating utterances. Used in this way, the frequency of distinct public utterances constitutes a construct we have designated as *public oral interactivity*. Our premise here is that the higher the frequency of utterances in a given time period the higher is the level of interactivity. This approach does not make use of either the length of time occupied by an utterance or the number of words used in an utterance, both of which would be problematic units of analysis in a multi-lingual study like this one.

Having identified who was talking and how frequently, the question arises, "But are they talking mathematics?" We have chosen to examine the use of technical mathematical terms in our consideration of "talking mathematics." Our focus on technical mathematical terms is only partly motivated by an interest in the teacher's purposeful promotion of oral mathematical fluency. We are also concerned with student agency. In an earlier analysis, the spoken use (and re-use) of technical mathematical terms was employed as a surrogate variable for the distribution of responsibility for knowledge generation in mathematics classrooms (Clarke & Seah, 2005). In conducting these prior analyses, it became clear that the classroom use of technical mathematical language was of significance as an indicator of the dominant pedagogy in each classroom, as a marker of teacher and student agency, and as an entry point for the interrogation of theories related to the role of language in learning.

An utterance may contain more than one distinct mathematical term, and our second analytical pass recorded the occurrence of mathematical terms rather than utterances. However, the same mathematical term may appear more than one time in one utterance. For example, consider the utterance "Oh, … it's a **solution** of the equation three x plus four y equals two. A **solution**, right?" The same mathematical term "solution" appears twice in this utterance. When such a situation occurred, the mathematical term was only counted once as we regard it to be one single conceptual contribution to the classroom discussion. Of course, the term "equation" would be counted separately, even though it occurred in the same utterance as "solution." For the purpose of this chapter, we restricted our second-pass analysis to those mathematical terms and phrases that were central to the content of a lesson, which we will refer to as 'key mathematical terms' or 'key terms' hereafter. These are the terms that constituted the formal content of the lesson. This emphasis is appropriate, given that our concern in this chapter is with the teacher's instructional intentions and the classroom practices that resulted. In considering the use of mathematical language from the perspective of the learner, a later analysis distinguished different types of mathematical terms and these are discussed in the companion chapter to this one. In this chapter, we are primarily concerned with whether or not the teacher of each classroom intended to promote student use of these technical mathematical terms.

It is also important to note here that our focus was on what might be called "technical mathematical vocabulary" rather than general vocabulary put to mathematical use; that is, on terms such as "equation" but not on the mathematical use of logical connectives such as "because" or "if." We acknowledge that the use of language in mathematics classrooms includes much more than technical mathematical terms, but for the purposes of this analysis our focus was on evidence of the purposeful development of student oral fluency with the technical mathematical vocabulary prioritized in the teacher's lesson plan and in the syllabus.

A total of 110 videotaped lessons were analysed. The 'Asian' data set analysed included sequences of five lessons from three mathematics classrooms in Shanghai, three similar sequences from Hong Kong, three sequences from Tokyo, three sequences from Seoul, and three sequences from Singapore. 'Western' classroom practice was represented in this analysis by three sequences of five lessons from Melbourne, two sequences from Berlin and two sequences from San Diego. Five lessons were analysed for San Diego 1, but for San Diego 2, we analysed six lessons rather than five. The six lessons were taught as three double-period lessons with a two-part structure, where the substantive content for the lesson was usually taught in the first period and cooperative learning activities were conducted in the second period. We felt that inclusion of all six lessons was required to reflect language practice in that classroom in a balanced fashion. Since all results were reported as either per lesson or per student, this difference in the number of lessons did not affect our capacity to make comparison between classrooms. The data from San Diego 3 and from Berlin 3 were excluded because of difficulties in distinguishing "public" and "private" statements in those classrooms.

The video-coding software *Studiocode* combines basic descriptive coding statistics with a capacity to reveal temporal patterns in a highly visual form (see Figure 1). *Studiocode* connects a time-coded transcript to the video record of a lesson and supports the coding of either events in the video record or the occurrence of specific terms in the transcript. Using *Studiocode*, a timeline display could be generated of the occurrence of selected mathematical terms throughout a given lesson.

Figure 1 shows the occurrence of specific mathematical terms and phrases: linear equations in two unknowns; equation; unknown; solution; integral solution; and solution set in the public discussion occurring in one lesson in the classroom of Shanghai Teacher 1. We are employing 'public' in the same sense as previously: that is, spoken participation in whole class or teacher-student interaction. The occurrence of each distinct term or phrase is indicated here by a particular shade of grey. Within a shaded band, each line represents the use of a particular term, such as "equation," by an individual in the classroom discussion. The width of a shaded band is an indication of the number of individuals who made use of the term in public discussion. Not surprisingly, the teacher (signified by "T") made the most frequent use of each term. All other timelines refer to student use of each term.

The highly visual nature of the timeline display can reveal temporal patterns in the occurrence of the coded terms. In the case of Shanghai Teacher 1, the solicited articulation of a key mathematical term (e.g., "equation" or "solution") from a sequence of students seems to be a distinctive characteristic of that teacher's practice. Once identified, such distinctive patterns can be examined in more detail. Below is the transcript of a one-minute interaction (min: sec) focusing on the term "solution."

This level of frequency of student spoken articulation of key mathematical terms was evident in the five lessons analysed from this Shanghai classroom. The pattern of elicited rehearsal of a key term, so visible in Figure 1 and Table 1, was also clearly evident in the practice of Shanghai Teacher 2 and Shanghai Teacher 3.

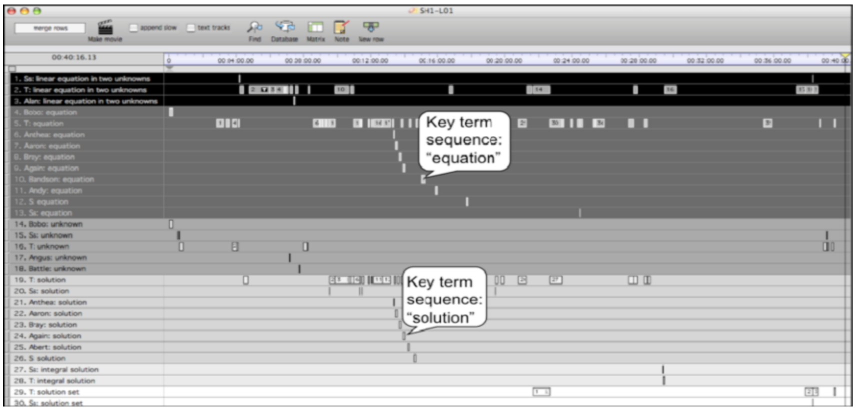

Figure 1. The occurrence of mathematical terms and phrases in SH1-L01

Table 1. Elicited public rehearsal of "solution" - Classroom Transcript (SH1-L01)

12:42(m:s) T	So let's read ... ah, let's read question one, question one. It says... in the following pairs of number value, each of them can be matched with a pair of x and y. So, let's read this. It is asking, which of them are the **solutions** of the equation two x plus y equals three? Which are the **solutions** of the equation three x plus four y equals two? Come on, have a try.
13:10 T	So, let's take a look. How about the first one? Oh, ok, you.
13:14 Anthea	x is equal to zero, y is equal to three. It is.
13:17 T	It's an equation. That means, x is equal to zero, y is equal to three. It is... ?
13:21 Anthea	It is a **solution** of the equation two x plus y equals three..
13:24 T	A **solution**. Okay, sit down please. How about you, Aaron?
13:28 Aaron	x equals zero and y equals one over two is a **solution** of the equation three x plus four y equals two..
13:35 T	Ah, a **solution** of this. Sit down please. Let's continue. Question three, question three. Come on, (...) [APOLLO and AMANDA raising their hands]
13:41 Bray	If x equals negative two, y equals two, it is the **solution** of the equation three x plus two y equals two.
13:48 T	Oh,...... it's a **solution** of the equation three x plus four y equals two. A **solution**, right? Ok, sit down please. Let's continue. Come on.
13:55 Again	When x equals one over two, y equals two, it is the **solution** of the equation two x plus y equals three.
14:00 T	Okay, it is a **solution** of two x plus y equals three. Okay, sit down please. So now, x equals one, y equals one over two, come on, (...) Tell me.
14:12 Albert	When x equals one, y equals negative one over two, it is a **solution** of three x plus four y equals two.

[Students whose names are given in full were subsequently interviewed; T = teacher, throughout]

Figure 2 shows the average number of utterances per lesson occurring in whole class and teacher-student interactions in each of the classrooms studied in Shanghai, Seoul, Hong Kong, Tokyo, Singapore, Berlin, San Diego, and Melbourne. An utterance is a single, continuous oral communication of any length by an individual or a group (choral).

The average number of public utterances per lesson provides an indication of the public oral interactivity of a particular classroom. Figure 2 distinguishes utterances by the teacher (white), individual students (black) and choral responses by the class (e.g. in Seoul) or a group of students (e.g. in San Diego) (grey). Any teacher-elicited, public utterance spoken simultaneously by a group of students (most commonly by a majority of the class) was designated a "choral response." Lesson length varied between 40 and 45 minutes and the number of utterances has been standardized to a lesson length of 45 minutes.

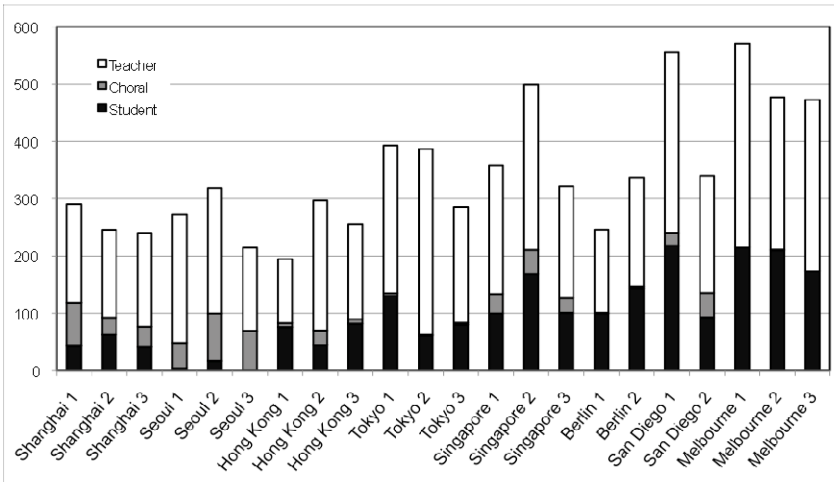

Figure 2. Average number of public utterances per lesson in whole class and teacher-student interactions (Public Oral Interactivity)

The term 'oral interactivity' is intended to signify oral communicative interchange. If one voice is dominant to a great extent, then the interactivity is likely to be low. For this reason, the relative weighting of teacher and student utterances has some significance in characterising the discourse of the classroom. If choral utterances were excluded, the relative weighting of teacher utterances over student utterances would be much higher in the three Shanghai and Seoul classrooms, and Hong Kong 2. The nature of the students' public utterances is also of interest. Shanghai 1 and the three Seoul classrooms were characterised by highly frequent choral utterances. By contrast, the classrooms in Tokyo, Berlin, and Melbourne do not appear to attach significant value to this type of utterance. The level of individual student contribution to the public classroom interactions also varied considerably. The students in the three Melbourne classrooms, Singapore 2, and San Diego 1 appeared to be much more publicly oral than the students in the three Seoul classrooms. It is worth re-emphasizing that our analysis did not compare the temporal length or complexity of utterance. It was the frequency of utterance and interchange of speaker, namely the oral interactivity that was compared.

The classrooms studied can be also distinguished by the relative level of public mathematical orality of the classroom (that is, the frequency of spoken mathematical terms or phrases by either teacher or students in whole class discussion or teacher-student interactions) and by the use made of the choral recitation of mathematical terms or phrases by the class. This recitation included both choral response to a teacher question and the reading aloud of text presented on the board or in the textbook.

Figure 3 shows how the frequency of occurrence of key mathematical terms varied among the classrooms studied. In classifying the occurrence of spoken mathematical terms, we focused on those terms that were central to the lesson content (e.g., terms such as "equation" or "co-ordinate," referred to as 'key terms'). This meant that our analysis did not include utterances that consisted of no more than agreement with a teacher's mathematical statement or utterances that only contained numbers or basic operations that were not the main focus of the lesson. In the case of the Korean lessons, in particular in Seoul 1, the choral responses by students frequently took the form of agreement with a mathematical proposition stated by the teacher. For example, the teacher would use expressions such as, "When we draw the two equations, they meet at just one point, right? Yes or no?" And the class would give the choral response, "Yes." Such student statements did not contain a mathematical term or phrase and were not included in the coding displayed in Figure 3.

Similarly, a student utterance that consisted of no more than a number was not coded as use of a key mathematical term. It can be argued that responding "Three" to a question such as "Can anyone tell me the coefficient of x?" represented a significant mathematical utterance, but, as has already been stated, our concern in this analysis was to document the opportunity provided to students for the oral articulation of the relatively sophisticated mathematical terms that formed the conceptual content of the lesson. Frequencies were again adjusted for the slight variation in lesson length.

From the results displayed in Figures 2 and 3, we suggest that the instructional practices of the teachers in the various classrooms assigned the spoken use of technical mathematics a very different function in public classroom discourse. The Melbourne 2 and 3 classrooms were highly oral and yet made relatively infrequent use of the mathematical terms that constituted the focus of the lesson's content. In general, the oral style of Melbourne Teacher 2 was highly colloquial and it is likely that the reduced prominence of technical terms reflected the teacher's rhetorical style. In interview, Melbourne Teacher 2 referred to the possibly confusing and alienating effect of technical mathematical terms, suggesting that in his practice conceptual understanding was prioritised over (and distinguished from) fluency in technical mathematical discourse. In the case of Melbourne Teacher 3, the particular mathematics topic (decimal place value and percentage) contained only limited technical vocabulary.

These two Melbourne cases illustrate the possible influence of factors such as teacher's rhetorical style and particular mathematical content on the evident promotion of spoken technical mathematics. The influence of such factors serves as a reminder that the analyses reported in this chapter are best seen as a set of case studies and that any claim to national or cultural representativeness must be explicitly argued rather than assumed.

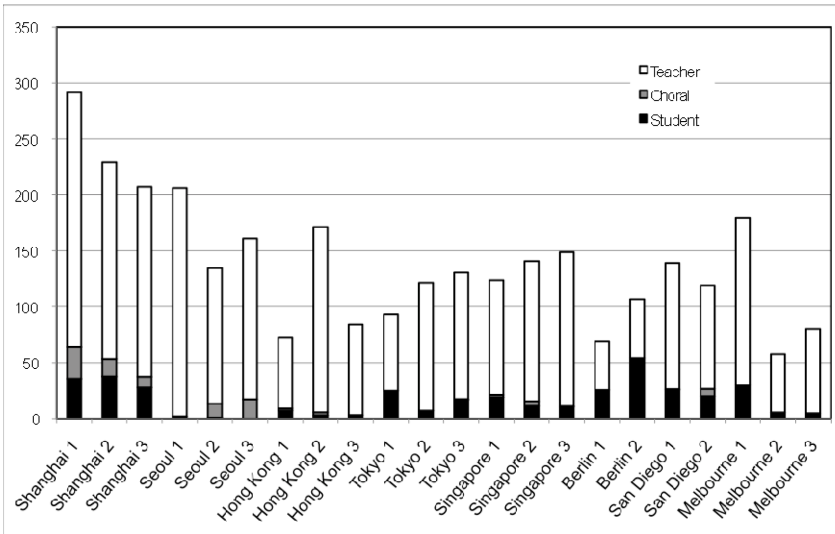

Figure 3. Frequency of occurrence of key mathematical terms in public utterances (Mathematical Orality)

By contrast, the less oral classrooms studied in Shanghai made much more frequent use of key mathematical terms and phrases. Although we did not attempt to measure either length or complexity of utterance in detail in this analysis, it should be noted that in the Shanghai lessons both teacher and student utterances appeared to be longer and more complex than elsewhere.

The orchestrated public rehearsal of spoken mathematical terms that is so evident in Figure 1 and Table 1, was also explicitly valued in the interviews with Shanghai Teacher 1. These teacher interviews made several references to the language used by the students, indicating the value attached by the teacher to both the use of accurate and standard mathematical language as well as to the completeness of student answers. For example, in the second interview, Shanghai Teacher 1 said:

Shanghai 1 I asked one student to answer me. He could tell me what was the first step, what was the second step. The answer

> was quite complete. Especially, he said the first step is to transform an equation to an algebraic expression with unknown to represent another unknown. What he said is very good. He said the second step was … put this algebraic expression into another equation to substitute the unknown in that equation. That is to make the system of linear equations in two unknowns into an equation in one unknown. Then … after that … what to do after finding out this unknown. Find another unknown by substituting the value of the other unknown. This language, that is, this mathematics language is good.

This explicit valuing of student spoken mathematical language appeared to be a characteristic of all three Shanghai teachers. Comparison between those classrooms that might be described as "Asian" is interesting. Key mathematical terms were spoken less frequently in the Seoul classrooms than was the case in the Shanghai classrooms. Even allowing for the relatively low public oral interactivity of the Korean lessons, the Korean students were given proportionally fewer opportunities to make oral use of key mathematical terms in whole class or teacher-student dialogue. In contrast to the teachers in Shanghai, Tokyo and Singapore, the teachers in the Hong Kong and Seoul classrooms did not appear to attach significant value to student spoken rehearsal of mathematical terms and phrases, whether in individual or choral mode.

Although all three of the Seoul teachers made extensive public use of spoken technical mathematical terms in their teaching of each lesson, their style of questioning did not encourage frequent student public use of mathematical terms. Most commonly, students were required to agree with a mathematical statement made by the teacher. Agreement was signified by a choral "Yes." Teacher prompts that did trigger student use of mathematical terms included, Seoul Teacher 1: "What is this called?" and Seoul Teacher 2: "What kind of number is this?" and Seoul Teacher 3: "What is the second condition of similarity?" These questions could be answered with a single word or a memorised phrase. The emphasis on memory and on very succinct student responses is evident in the brief classroom exchange shown as Table 2.

Table 2. Elicited public rehearsal of "number of cases" - Classroom Transcript (KR2-L01)

T (KR2-L01)	What do we call of the possible cases?
S1	Number of cases.
T	What do we call it?
S2	Number of cases.
T	What kind of number do we call it?
S3	Number of cases.
T	We call it number of cases. Repeat, after me, number of cases.
Ss	Number of cases.

The teacher in Hong Kong 2[ii] appears similar to the three Shanghai teachers in the sense that he conducted his teaching most frequently in the form of whole class discussion. But his lessons showed no signs of the pattern, evident in all three Shanghai classrooms, where the students were systematically 'enculturated' into the language of school mathematics. In particular, despite similarities between the public oral interactivity of Hong Kong 2 and Shanghai 1 (for example), the frequency of student use of mathematical terms in Hong Kong 2 was much lower. The Japanese classrooms resembled those in Shanghai in the consistently higher frequency of student contribution, but with little use being made of choral response.

Among the 'Western' classrooms, the teachers' questions in Melbourne 2 and 3 seemed more concerned with establishing correct mathematical procedures than with developing student ability to provide mathematically lucid explanations or justifications. By contrast, in lesson GR2-L02, Berlin Teacher 2 asked open-ended questions, such as "Are there any alternatives?" and gave advice such as "The individual steps need to be explained again." This teacher seemed committed to promoting more than the correct application of a taught procedure. Teacher-student interactions had the effect of prompting student explanations and thereby provided the opportunity for students to rehearse their use of mathematical terms, while participating in a teacher-led discussion. In addition to public discussion, the Melbourne 1 and San Diego 2 classrooms were characterised by their high level of student-student ('private') spoken interaction. The inclusion of private student speech as an acceptable medium of social interaction in the classroom dramatically changes the operative pedagogy and this will be addressed in the next chapter. In this chapter, we have chosen to focus only on public classroom discourse. As can be seen from Figures 2 and 3, there is sufficient variation in public discourse alone across the classrooms studied, to justify the separate consideration of student public classroom talk.

The oral articulation of mathematical terms and phrases by students could be accorded value in itself, even where this consisted of no more than the choral repetition of a term initially spoken by the teacher (as in the last two lines of Table 2). Teachers and students in some of the classrooms we studied clearly attached value to this type of recitation (e.g., the three Shanghai classrooms). The specific terms, of course, reflect the topic being taught in each class. Eighteen of the twenty-two classrooms were studying algebra topics, while three were studying geometry (Tokyo 2 and Melbourne 1 and 2), and one decimals and percentage (Melbourne 3). With the possible exception of Melbourne 3, all topics could be associated with a vocabulary of sophisticated mathematical terms.

The value attached to student spoken mathematics in some classrooms could indicate adherence by the teacher to a theory of learning that emphasises the significance of the spoken word in facilitating the internalisation of knowledge. The use of choral response, while consistent with such a belief, could be no more than a classroom management strategy. In other classrooms, the emphasis was on the students' capacity to produce a mathematically correct term or phrase in response to a very specific request (question/task) from the teacher. In such classrooms, both of these activities (choral response and directed student response) accorded very limited agency to the learner and the responsibility for the public generation of mathematical knowledge seemed to reside with the teacher. By contrast, in other classrooms, the instructional approach provided opportunities for students to "brainstorm" in public or to generate their own verbal (written or spoken) mathematics, with very little (if any) explicit cueing from the teacher (e.g. the classrooms in Tokyo).

SPOKEN MATHEMATICAL FLUENCY AS A VALUED LEARNING OUTCOME

It is clearly the case that some mathematics teachers value the development of a spoken mathematical vocabulary and some do not. If the goal of classroom mathematical activity was fluency and accuracy in the use of written mathematics, then the teacher may give little priority to students developing any fluency in spoken mathematics. On the other hand, if the teacher subscribes to the view that student understanding resides in the capacity to justify and explain the use of mathematical procedures, in addition to technical proficiency in carrying out those procedures in solving mathematics problems, then the nurturing of student proficiency in the spoken language of mathematics will be prioritised, both for its own sake as a valued skill (e.g., Shanghai Teacher 1) and also because of the key role that language plays in the process whereby knowledge is constructed.

It is really only through international comparative studies such as this one that we can make such comparisons between classrooms so fundamentally different in their practices. It must be remembered that the teachers in the LPS project were recruited on the grounds that the local mathematics education community endorsed their practice as competent. Given this selection criterion, it is reasonable to assume we have documented competent mathematics teaching as this was conceived in each city at the time (and possibly in each country - in all cases except Hong Kong and Shanghai, which appear to draw on fundamentally different traditions of practice). Despite within-city variations, the mathematics classrooms from some cities do seem to share sufficient common features with each other to suggest that they draw on a common tradition of practice.

The consistency of language use across the three Seoul classrooms suggests a well-established tradition of practice. It has to be considered as feasible, therefore, that the Korean national success on international tests of mathematical performance (for example in the TIMSS study, reported in Beaton and Robitaille (1999)) has been achieved through classroom practices like those documented here. Of course, even well-established practices may change and recent curricular initiatives in Korea prioritise student oral participation to a much greater extent than was evident at the time these data sets were generated. Such attempts to change established practice can generate conflicting conceptions of both accomplished teaching and valued learning outcomes. Seoul Teacher 1 expressed this tension very clearly in the second teacher interview.

Seoul 1 These days there are many open classes in which students actively discuss in the class. I think the way of teaching is changing. But I think the teacher should teach. I think it is better. In the beginning, I teach, and in the last part of the class I make students discuss what they learned. It is a good way to teach math. I don't oppose the open class. But I think teacher's explanation is more important in teaching math.

Since our concern is not national typification, the issue is not whether any of the classrooms represent current (or past) national norms of practice for any of the participating countries. Instead, each classroom offers a culturally-situated variant on local practice. We are extremely fortunate that the variation between classrooms afforded such informative comparisons.

CONCLUSIONS

The results that are reported in this chapter certainly suggest that the teachers in this study differed widely in the opportunities they provided for student spoken articulation of mathematical terms as part of public classroom discourse and in the extent to which they devolved agency for public knowledge generation to the students. These differences were apparent among classrooms that might be identified as 'Western' and among those that might be described as 'Asian.' In particular, the demonstration of such differences in the practices of classrooms situated in school systems and countries that would all be described as "Asian" suggests that any treatment of educational practice that makes reference to the "Asian classroom" confuses several quite distinct pedagogies. This observation is not to deny cultural similarity in the way in which education is privileged and encountered in communities that might be described as "Confucian-heritage." But, the identification of a correspondence between membership of a Confucian-heritage culture and a *single* pedagogy leading to high student achievement is clearly mistaken. Hatano and Inagaki (1998) also questioned the grouping of Chinese and Japanese classrooms as 'Asian' and Wong (2004) problematises the use of 'Confucian Heritage Culture' as either a generic characterisation of traditional Chinese culture or as an inclusive grouping of countries such as China, Japan, Korea and Vietnam. The recent success of countries such as Korea, Singapore and Japan in tests of international student achievement has encouraged the use of such misleading generalised categories (see Clarke, 2003, for a more complete discussion). We would like to suggest that the differences between the practices of classrooms in these countries are profound and reflect fundamental differences in implicit beliefs about effective instruction and the nature of learning. Our understanding of these differences is not advanced by simplistic grouping of classrooms into either Asian or Western categories.

The resolution of the tension between perceptions of cultural similarity and the empirical demonstration of significant pedagogical difference may lie in the distinction between macroculture and microculture. Macroculture refers to a set of ideas, communications or behaviours embraced by the majority of people in a particular society (e.g., Chinese culture), whereas microculture defines regularities and patterns of interactions specific to a social group (such as a mathematics classroom). It has been demonstrated in other research that classrooms within the one macroculture (e.g., the USA) can display significantly different socio-mathematical norms (Yackel & Cobb, 1996). Certainly, the results reported in this chapter suggest significant differences in classroom discourse patterns among the mathematics classrooms in Berlin, Melbourne and San Diego, despite their cultural identification as 'Western.' Given this, perhaps it is not so surprising that different classrooms situated within the Confucian-heritage culture should display different norms of classroom behaviour associated with different pedagogies that show high levels of local consistency. Certainly, we suggest that cultural similarity does not prescribe instructional practice.

It appears to us that the key constructs Public Oral Interactivity and Public Mathematical Orality distinguished one classroom from another very effectively, particularly when the two constructs were juxtaposed (by comparing Figures 2 and 3). The trends shown by the juxtaposition of Figures 2 and 3 usefully direct our attention to classrooms and comparisons likely to reward more fine-grained analysis. The contemporary reform agenda in the USA and Australia has placed a priority on student spoken participation in the classroom and this is reflected in the relatively high public oral interactivity of the San Diego and Melbourne classrooms (Figure 2). By contrast, classrooms such as those in Shanghai, were much less orally interactive. However, the seemingly lower level of public oral interactivity conceals differences in the frequency of the spoken occurrence of key mathematical terms (Figure 3), from which perspective the Shanghai classrooms can be seen as the most mathematically oral in the public domain. In comparison, despite other possible cultural similarities, students in the Tokyo classrooms used spoken mathematics extensively in both public and private situations. However, the relative occurrence of spoken mathematical terms is only one level of analysis. A more fine-grained analysis is required to distinguish between repetitive oral mimicry and the public (and private) negotiation of meaning (Cobb & Bauersfeld, 1994; Clarke, 2001). The analyses reported in this chapter draw to our attention significant differences in the classroom discourse of mathematics classrooms situated in different cultural settings. These differences suggest further areas for research.

Despite the frequently assumed similarities of practice in classrooms characterised as Asian, differences in the nature of students' publicly spoken mathematics in classrooms in Seoul, Hong Kong, Shanghai, Singapore, and Tokyo are non-trivial and suggest different instructional theories underlying classroom practice. The question of learning outcomes is addressed explicitly in the next chapter. The implicit assumption guiding instruction in the classrooms studied in Hong Kong and Seoul seems to be that the employment of spoken mathematics by students is not to the benefit of the students' learning of mathematics. Classrooms studied in Melbourne, Berlin, Tokyo, San Diego, Singapore, and Shanghai, despite differences in implementation, seem to make the opposite assumption. Any 'universal' theory of mathematics learning would have to accommodate, distinguish and explain the learning outcomes of each of these classrooms.

What has been demonstrated is the wide variety of practice among the classrooms of competent mathematics teachers with respect to the promotion of student fluency in spoken mathematical discourse. Local consistencies of practice, such as those found across the three classrooms studied in Seoul and in Shanghai, suggest that competent teaching is very differently conceived and performed in different cultures. The promotion of student-student interaction is an instructional strategy employed extensively in some classrooms and not at all in others. The focus in this chapter has been on public discourse. Our analysis, as reported in this chapter, did not address the role and significance of student-student interaction. Nor did we attempt to connect student participation in spoken mathematical discourse with learning outcomes. Both these considerations are addressed in the next chapter.

NOTES

[i] Our use of the term 'Western canon' draws on the literary connotations of the term and is intended to invoke associations both of claimed authority and of contested legitimacy.

[ii] It should be noted that Hong Kong 3 used English as the instructional language, while Hong Kong 1 and 2 used Cantonese, so any common features of the Hong Kong classrooms are likely to reflect dominant pedagogical practices, rather than be a specific result of the use of the Chinese or English language.

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CHAPTER THREE

Students Speaking Mathematics: Practices and Consequences for Mathematics Classrooms in Different Countries

STUDENT-STUDENT MATHEMATICAL TALK

The research reported in the companion chapter (*Spoken Mathematics as an Instructional Strategy*) revealed significant differences in the *public* mathematical discourse practised in various classrooms around the world. It is clear that the pedagogies practised in many mathematics classrooms also permit and even promote *student-to-student* mathematical speech. In fact, the pedagogies of some classrooms are dependent on the provision of opportunities for student-to-student mathematical speech. The analyses reported in this chapter suggest that at least some of the goals of those advocating student-student mathematical conversations in the classroom may be met by other instructional strategies, such as whole class public discussion. Since our data set included some classrooms where student-student mathematical conversations were encouraged and some where they were not, we were well positioned to address the question: "What differences in practice exist between classrooms where student-student mathematical talk is encouraged and those where it is not, and what appear to be the consequences for learning of those differences in practice?"

CONNECTING MATHEMATICAL TALK AND LEARNING

The role of language in learning has been widely researched and variously conceived (Alexander, 2008; Kim & Markus, 2004). Different theories attend to different aspects of language and the learning process and some of these have been discussed and relevant research cited in the companion chapter to this one. The adoption of a cognitive perspective towards learning directs the researcher's attention to the content represented by the language used. The assumption seems to be that the learner's language use can be taken to reflect their thought processes. In studies with a more socio-cultural emphasis, the focus tends to be on the discursive functions of spoken and written language (e.g., Inagaki, Hatano, & Morita, 1998). From this perspective, language is a cultural resource through which the learner is initiated into a particular community of practice (van Oers, 2001). Studies adopting a sociolinguistic perspective address the distinctive linguistic features of specialised or technical language (for example, mathematical or scientific language). In such studies, facility with language is taken to be prerequisite to any

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 33–52.

effective communication and consequently to any learning (see Walshaw & Anthony, 2008, for an overview of Western research).

Mathematics learning can be conceptualised in terms of participation in forms of social practice, where discourses form key components of that practice. Language plays a central role in mediating and constituting this participation, which is performed as classroom discourse (see Yackel & Cobb, 1996). Traditionally regarded as only auxiliary to thinking, active mathematical communication is nevertheless believed to enhance mathematical learning. It is a useful exercise, however, to conceptualise mathematics as a special form of communication. From this perspective, the expression "learning mathematics" becomes tantamount to developing mathematical discourse (Sfard, 2001, 2008).

In our analysis, we have employed student spoken use of technical mathematical terms as indicative of the students' developing confidence and skill in using the concepts and procedures signified by the technical terms in social interaction. Such growing competence in engaging in what might be called technical mathematical discourse can also be taken to indicate improvement in their capacity to participate in the community of practice constituted by the teacher and their fellow students in the mathematics classroom.

Our concern in this chapter is the construction of a connection between student learning and the way in which the practices of each classroom afforded or constrained the students' use of technical mathematical terms in public and private speech. Having established in the companion chapter the significant differences between classrooms in patterns of public discourse, we now shift attention to the spoken utterances of individual students in both public and private contexts, and, importantly, the connection between individual spoken mathematics and observable learning outcomes.

MATHEMATICAL DISCOURSE IN THE CLASSROOM FROM THE PERSPECTIVE OF THE LEARNER

Analyses were conducted of 110 lessons documented in 22 classrooms located in Australia (Melbourne), China (Hong Kong and Shanghai), Germany (Berlin), Japan (Tokyo), Korea (Seoul), Singapore, and the USA (San Diego). In this chapter, we focus our analysis on the spoken acts of the focus students (most commonly two per lesson) and on their use of mathematical language in post-lesson interview settings. The complete LPS research design is set out in the Appendix to this book. Three types of oral classroom interactions were recorded: whole class interactions, teacher-student interactions, and student-student interactions. All whole class and teacher-student interactions were documented and transcribed, but student-student interactions could only be recorded for selected focus students in each lesson. In selecting the focus students for each lesson, the researcher would typically choose two students sitting side by side (or as near as possible given the prevalent seating arrangements). Wherever possible, acting on advice from the teacher, each particular pair of students were chosen because they would normally sit near each other. In this way, any student-student conversation would be most likely to resemble the students' normal practice. A different pair of focus students was chosen for each lesson. Each focus student then participated in a post-lesson video-stimulated interview and these interviews were also transcribed. Transcription and translation were carried out by the local team responsible for data generation and were therefore undertaken by native speakers of the local language. Transcripts were then translated into English, where necessary. Technical guidelines specified the format to be used for all transcripts and the conventions for translation (particularly of colloquial expressions) (Clarke, 2006 and the appendix to this book). The analyses reported in this chapter were undertaken on the English version of each transcript (both public and private classroom dialogue and student interview).

Examining the public and private classroom utterances of 222 focus students distributed across 22 mathematics classrooms in several different cultures, we were able to study the extent to which student mathematical talk was encouraged in one classroom, in public and/or private contexts, and discouraged in another. The final stage of our analysis examined student use of mathematical terms in 191 post-lesson video-stimulated interviews.

As noted elsewhere, the study design was not intended to support any claims of national representativeness with respect to the teacher, the classroom, or the students. Instead, the research design delivered privileged access to the language used in class by approximately 10 students in each of 22 mathematics classrooms, situated in widely differing cultures and school systems. As will be seen, this language use could be connected to the development of student facility with mathematical language through the analysis of the post-lesson interviews.

PUBLIC MATHEMATICAL DISCOURSE

In our first analytical pass reported in the companion chapter, we counted the number of utterances made by anyone participating in a whole class or teacher-student interaction (a "public utterance" from the student perspective), a construct we designated as *public oral interactivity*. Our second analytical pass considered mathematical terms rather than utterances. The specific terms, of course, reflect the topic being taught in each class. Eighteen of the twenty-two classrooms were studying algebra topics, while three were studying geometry (Tokyo 2 and Melbourne 1 and 2), and one decimals and percentage (Melbourne 3). With the possible exception of Melbourne 3, all topics could be associated with a vocabulary of sophisticated mathematical terms. Since we had recorded the public and private talk of two focus students in each lesson, and could supplement these with the transcripts of interviews with those focus students after each lesson, the prevalence of student spoken use of technical mathematical terms provided an entry point for the fine-grained study of how such terms were used, in response to what teacher prompts, and with what consequences for student learning. This provided the focus for the analysis reported in this chapter.

THE SIGNIFICANCE OF STUDENT-STUDENT INTERACTIONS

The private conversations recorded in any one lesson were only those of the two focus students and their immediate neighbours. Two different focus students were recorded in each lesson. In this section, we report the frequency of utterances (uninterrupted oral communications) and key mathematical terms (as defined in the companion chapter and below) in both public and private arenas with respect to the two focus students. All utterances made by the two focus students were differentiated according to whether the utterance was targeted at a public audience or a private audience. Public utterances were those made to the teacher (either in response to a teacher question during whole-class discussion or in one-on-one interaction) or to another student, but intended to be audible to the whole class. Private utterances included statements made to a student peer in private or to oneself.

In Figures 1 and 2, the results given for both public and private Oral Interactivity and Mathematical Orality are per focus student per lesson and have been averaged over the spoken contributions of around 10 students per classroom. This should minimise the effect of individual student timidity or extroversion, although awareness of being recorded was a common characteristic of all focus students (and of their teachers). The number of utterances and key mathematical terms was normed to a standard lesson length of 45 minutes.

Three classrooms stand out in Figure 1 because of their extremely low frequency of student-student interaction: Shanghai 1, and Seoul 1 and 3. In these three classrooms, student-student conversation can be discounted as an instructional strategy (or as a subversive practice by students). For example, in Seoul classroom 1, there were no instances of student private talk in the first four recorded lessons and only two private utterances from one of the focus students in lesson five, an average of 0.2 utterances per student per lesson. The first utterance was "That's yours" and the second was "No." Obviously, neither involved any technical mathematical terms.

The corresponding figures in the companion chapter show relatively high levels of whole class public mathematical orality in the Shanghai classrooms, but this is not evident in Figures 1 and 2 because the typical public contribution of an individual Shanghai student occurs within a class of fifty students (at least ten more than the average for classes in any of the other cities) and a specific individual's contributions will consequently be less frequent than in smaller classes.

Rather than characterising aggregated whole class behaviours, Figures 1 and 2 express their findings in terms of the individual student. At least three observations are noteworthy: (i) The complete absence of a spoken mathematical term by all ten recorded students in each of the three Korean classrooms; (ii) The relatively low frequency of private (student-student) use of mathematical terms in all three Shanghai classrooms (which in public discourse were sites of relatively frequent student mathematical orality); and, (iii) The remarkable result for Tokyo 2: averaging 9.44 privately spoken mathematical terms per student per lesson across a

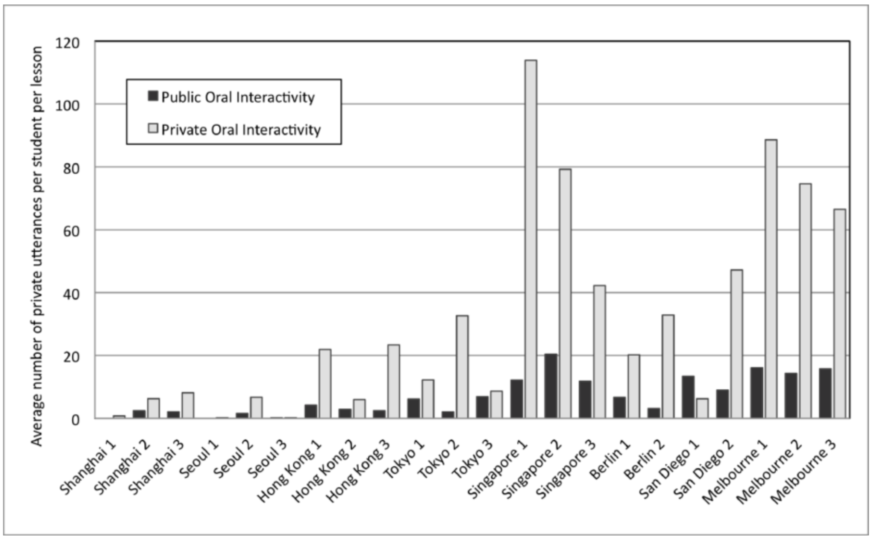

Figure 1. Public and Private Oral Interactivity: Frequency of utterance per student per lesson (each bar represents the average of two students for each of five lessons – i.e., an average over ten students per class)

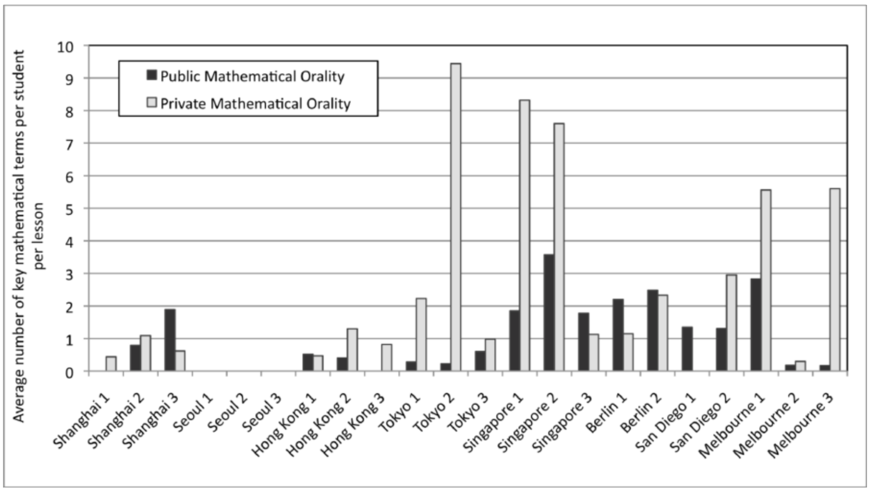

Figure 2. Public and Private Mathematical Orality: Frequency of use of key mathematical terms (each bar represents the average per student of two students for each of five lessons – i.e., an average over ten students per class)

sample of ten focus students over the five lessons studied. It is also noteworthy that the other classrooms in which student-student use of mathematical terms was most prevalent were Singapore 1 (8.32) and Singapore 2 (7.60). Of the "Western" classrooms, where student-student interaction might be expected to be much more common, only Melbourne 1 (5.56), Melbourne 3 (5.60) and San Diego 2 (2.95) were at all comparable in the private use of mathematical terms.

It is important to consider the nature of the student-student interactions and the manner in which spoken mathematical terms were employed. In our analysis of both public and private spoken mathematics, we focused on those "key mathematical terms" that constituted the content-focus of the lesson. Table 1 sets out about 3 minutes of student-student interaction recorded in lesson 2 in the second Tokyo classroom. This classroom was noteworthy for its high level of student-student (private) interaction (see Figure 2). In the episode displayed in Table 1, the students had been asked to draw a triangle with point P somewhere along the segment AB, and then draw a line running from P that divides the area of the triangle into two (see Figures 3a and 3b). The key terms have been highlighted in the transcript. Some terms, such as "line," fall into the category we have called "related terms" (see the later discussion of student interviews). These related terms did not constitute the lesson's substantive content but were relevant terms connected to that content. Figures 3a and 3b show the diagrams constructed by the two students: Wada and Kawa. There is a vitality evident in the interactive exchange between these two students that illustrates the sort of cognitive engagement valued by the advocates of spoken mathematics (see Walshaw & Anthony, 2008) and analysed in detail by Helme & Clarke (2001).

Table 1. Sample student-student "private" interaction - Classroom transcript (Tokyo School 2 – lesson 2, 29:46:12 – 33:15:19)

Kawa	[To Wada] I managed to draw that line!
Wada	Like this?
Wada	[To Kawa] If you draw that line over the middle point [mid-point], isn't that the answer, Kawa?
Kawa	Oh, I don't think so!
Wada	I think you don't have to do such a thing. I think you just have to draw a line from P.
Kawa	I don't really understand what you mean.
Wada	Um, you drew a middle point [mid-point] here, right? So if you just draw a line from here, wouldn't that do?
Kawa	Can you draw a line from P?
Kawa	You're kidding. What did you say? Are you saying that you can draw a line from here?
Wada	Yes. If you draw a line from there, if goes over the **middle point** [mid-point] so there is no problem there.
Kawa	Really? Let's try then.
Kawa	What was the name of the **theorem** again?
Wada	**Middle point** [Mid-point] **connection theorem**.
Kawa	That's it! But it isn't **parallel** there. Are you going to try drawing it there?
Wada	[To Tsutahara] Doesn't this work when you draw a **parallel line** by free hand and then draw a line that goes along P?
Tsutahara	I don't understand what you're talking about.

Wada	Never mind then.
Kawa	I'll understand it with Wada then.
Wada	Draw a **parallel line**.
Kawa	Did so.
Wada	Well, it's not going over P if you notice.
Kawa	And which one's the same here? Tell me.
Wada	These two are **parallel**.
Kawa	Yeah, I knew that.
Wada	Doesn't it look like it's the right answer?
Kawa	This one's a lot easier to see. It's nice and big, this one. Wait! Don't you have to say something about the **bottom line** [base]?
Wada	What?
Kawa	Something we discuss about every time we do this.
Wada	Never mind about that.
Kawa	Yeah, but we always prove that these two **triangles** are the same or whatever.
Wada	Well, that's my answer.
Kawa	Nothing to do with **triangles** this time? Are you sure about that?
Wada	Um, um, this one.
Kawa	Which two?
Wada	This one and this one.
Kawa	What happens when they're the same?
Wada	It's the same.
Kawa	Which two?
Wada	These two.
Kawa	How come?
Wada	Because they're **congruent**.
Kawa	Where's the **bottom line** [base] then?
Wada	This is the **bottom line** [base], I bet. God, I don't know which one is the **bottom line** [base] now.
Kawa	This one has to be the **bottom line** [base].
Wada	This has to be the (height), this one. This is the **height**. I got it now!
Kawa	Is this the **height**? Is it all right if it's now **parallel**?
Wada	Well, it doesn't have to be **parallel**. No need for that.
Kawa	But then which two become equally in half?
Wada	What the hell are you saying?
Kawa	Aren't we doing the one that we have to divide in half or something like that?
Wada	Yes, that's the one we're talking about.
Kawa	I'm starting to get mixed up now.
Wada	Well, I'm starting to get a headache.

The transcript above also illustrates one of the difficulties associated with translation. Where a technical term is used in the original language, a literal translation may not correspond to the equivalent English form of the technical term (for example: middle point or mid-point). We have chosen to translate the Japanese wording of the technical term literally, while indicating in parentheses the corresponding English version of the technical term. In this way, the connotations and entailments of the original phrasing and the institutionalized status of the technical term are available for analysis and interpretation. Figures 3a and 3b show the written work that was the focus of the students' conversation.

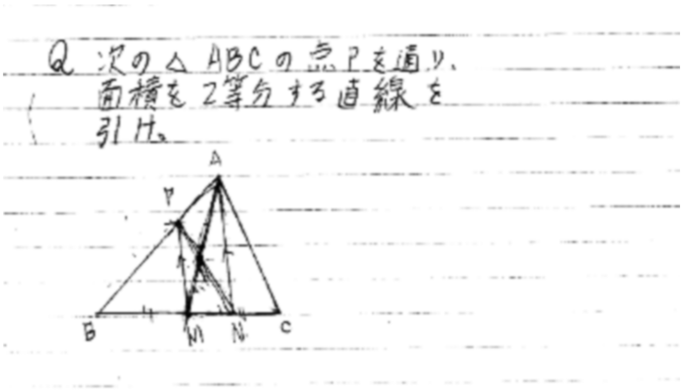

Figure 3a. Wada's work

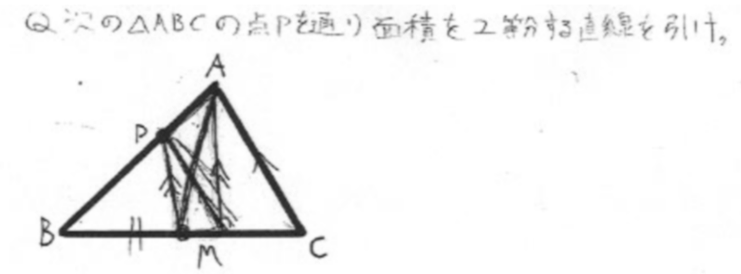

Figure 3b. Kawa's work

As has already been noted, while the frequency of utterance (oral interactivity) for the focus students in Tokyo 2 was comparable with the Western classrooms analysed, the frequency of use of key mathematical terms per student per lesson was higher than for any other classroom. Since all teachers studied were considered 'competent' by their local community, we must consider the occurrence of private student-student speech to be a deliberate affordance by the teacher within the socio-mathematical norms of the classroom. In the case of Tokyo 2, we have evidence of a pedagogical practice (occurrence of student-student talk) that appears to be much more prevalent in the Western classrooms studied than in many of the Asian classrooms. Singapore 1 and 2 also offer evidence of a significant level of student-student talk, combined with a high level of private use of key mathematical terms. In fact, what might be called the "lexical density"[i] of student-student talk in Singapore 1 and 2 is very similar to that of Melbourne 1 and 3.

Such individual cases represent an important demonstration of the viability of practices in classrooms where their use might be assumed to be precluded by cultural convention: a form of "existence proof." As displayed in Figure 2 and illustrated in Table 1, not only do we find a relatively high frequency of private oral interactivity in Tokyo 2, but student private spoken use of key mathematical terms is extremely frequent (that is, the lexical density of student private interactions is relatively high). Whatever benefits might accrue from the classroom rehearsal of spoken mathematics, we would expect these to be particularly evident for the students of Tokyo 2.

In characterising the use of key mathematical terms in student-student classroom speech, we must not forget that the Shanghai classrooms were characterised by high levels of lexical density in the public classroom discourse. The Shanghai classrooms represent a very interesting case. Shanghai Teacher 1 has been shown to value and promote student spoken use of mathematical terminology (see Clarke, Xu, & Wan, companion chapter in this book). However, constrained by the apparent conventions of Chinese classroom practice, Shanghai Teacher 1 enacts this prioritisation in the public domain only. Because of the large class size in Shanghai, this means that any particular student will have proportionately less opportunity to actually "talk mathematics" in comparison with students in smaller classes, even though the teacher's clear intention is to provide the opportunity for this to occur. The role of choral response becomes very important here. Even if it is not possible for each student in a Shanghai classroom to make spoken use of many mathematical terms in a given lesson, the teacher's classroom practice explicitly values students' spoken fluency with mathematical terms and this valuing is communicated very clearly to the class through the teacher's orchestration of public discussion. Further, the students have the opportunity to hear their classmates' oral use of mathematical terms in the public classroom discourse. This provides a sharp contrast to the pedagogies employed in other classrooms, particularly Tokyo 2 or Melbourne 1, where student-student spoken mathematics was prioritised. Consider this interview statement from the second interview with Tokyo Teacher 1.

Tokyo 1 Um, it went totally different from what I had planned ... But it was not important to do as planned. Students discuss with each other and have their own opinions is what is most important. And I think it is what was good about this lesson.

What were the consequences for the students' learning of these pedagogies, in each of which spoken mathematics was promoted, but by very different instructional means?

Spoken Mathematics in the Classroom: Key Points Summary

The prevalence of spoken mathematics in the 22 classrooms studied differed in the following respects:

– the frequency of public utterance
– the relative prominence of the teacher or the students' voices in public discourse
– the frequency of public use of spoken technical terms, most particularly by students
– the differences in the extent to which student use of spoken mathematics was strategically facilitated by teachers
– the extreme differences between classrooms in the occurrence of student-student (private) use of spoken mathematics.

In some classrooms, student-student spoken mathematics was an essential component of the dominant pedagogy. In other classrooms, it was entirely absent. These extreme differences allow us to ask the question: "With what consequences?"

SPOKEN MATHEMATICAL FLUENCY AS A VALUED LEARNING OUTCOME

It is clearly the case that some mathematics teachers value the development of a spoken mathematical vocabulary and some do not. If the goal of classroom mathematical activity was competence in the use of written mathematics, then the teacher may give little priority to students developing any fluency in spoken mathematics. On the other hand, if the teacher subscribes to the view that student understanding resides in the capacity to justify and explain the use of mathematical procedures, in addition to technical proficiency in carrying out those procedures in solving mathematics problems, then the nurturing of student proficiency in the spoken language of mathematics is likely to be prioritised, both for its own sake as a valued skill and also because of the key role that language plays in the process whereby knowledge is constructed.

In the final stage of our analysis, the transcripts of 191 student post-lesson interviews were examined for the occurrence of the key terms that constituted the instructional focus of the lesson, together with those mathematical terms closely related to the key terms (*related terms*). In addition, we also coded *other terms*, not used in the lesson but employed by the student in interview to describe or explain some aspect of their classroom activity. We analysed transcripts of the post-lesson interviews with the same focus students whose private classroom conversations were recorded and analysed above and for the same lessons. The three categories of mathematical term are defined below.

The *key terms* were the mathematical terms or phrases explicitly identified in the teacher's lesson plans, or in explicit teacher statements, as constituting the goal(s) of the lesson. For example, in Hong Kong 2, some key terms would be "simultaneous equations" and "method of elimination." These key terms were coded for both public and private conversations during lessons.

The *related terms* were the mathematical terms or phrases, closely connected to the key terms. These terms were used by the teacher or students during the lesson and repeated by the students in interview. For example, in San Diego 1, the mnemonic "Please excuse my dear Aunt Sally," introduced by the teacher to help students remember the order of operations to be 'parenthesis, exponents, multiplication, division, addition and subtraction,' and similarly, the coined term "sub," employed by students to mean 'substitute' were considered to be related terms. More conventionally, related terms were frequently simply mathematical terms that were used in class to help to explain the key terms that were the actual content focus of the lesson.

The *other terms* were other mathematical terms not used in the lesson being described in interview. These could include mathematical terms or phrases that were categorised as either key or related terms in the other lessons analysed for that class or any other mathematical terms employed by the student. Student use of such other terms could be interpreted as indicative of connections made by the student between the content of that lesson and other content studied or known.

For each classroom, the transcripts of student post-lesson interviews examined were those that corresponded to the five (or six in the case of San Diego 2) consecutive lessons analysed for public and private orality. In the post-lesson interviews, the number of utterances was not the main area of interest. Only instances of the student articulation of mathematical terms or phrases were counted. The categorisation of mathematical terms (key, related, and other) employed in analysing the student interviews was consistent with the usage of mathematical terms or phrases in public and private conversations during the lessons. It is important to reiterate at this point that there are other aspects of student speech that might be of mathematical significance: for example, the use of logical connectives, but these were not the focus of this analysis.

The analyses already reported indicate that the classroom practices of some teachers deliberately facilitated the development of a spoken mathematical vocabulary by students, while other teachers did not do this. Since the classroom use of spoken mathematics by students has been strongly advocated in various sections of the mathematics education literature (for example, Walshaw & Anthony, 2008; Silverman & Thompson, 2008), it is important that research examine differences in the occurrence, form, and promotion of spoken mathematics in classrooms that are differently situated with respect to school system and culture. Further, research should address the question, "To what purpose and with what consequences are students encouraged to engage in spoken mathematics?" These are the issues that we have attempted to address in our research.

The post-lesson interviews undertaken in the LPS provide a unique indication of student facility with a spoken mathematical vocabulary. It is important to note that this may not be either a valued or intended consequence of mathematics instruction in some of the classrooms studied. However, the development of this facility appears to underlie instructional advocacy within the Western canon and for that reason warrants investigation.

In conducting the post-lesson interviews, students were asked to comment on what they had learned or felt was important from that day's lesson. Following which, the video for the lesson was played and the student could pause, fast forward, or rewind to any parts of the lesson that they felt were important or that they wanted to comment upon. After viewing the video, the students were asked if they had any other comments about the lesson before ending the interview session. The legitimacy of our comparison of student use of mathematical terms in these post-lesson interviews is dependent on the consistency with which the interview protocol was followed. Careful examination of all interview transcripts confirmed that the student language use analysed was in response to the same interview stimuli.

It is important to note that the interview text analysed in this study was the English translation (where required) of the original interview transcript and that both transcription and translation were carried out by the local research team in the particular country generating the data. As a result, there appeared to be slight changes to the wording of the interview prompts. For example, in the student interviews in Shanghai School 1, typical interview prompts included: “What do you think [this lesson] was about?” and “What do you think you have learned in this class?” The equivalent prompts for Seoul School 2 were translated as, “Tell me about today’s class” and “What did you learn today?” The important point for our analysis is that in neither situation did the prompts suggest particular mathematical terms to the students. That is, any mathematical terms employed by the students in the post-lesson interviews were chosen by the students, rather than being suggested by the interviewers.

During the interviews, it was not unusual for students to pause for more than five seconds when pondering how they should reply to the interviewer or what they wanted to comment upon. Hence a continuous turn, uninterrupted by the interviewer, was considered as one utterance. In each turn, more than one mathematical term might occur. However, the occurrence of a particular mathematical term or phrase has been counted only once as a single conceptual contribution, even if it was mentioned more than once in a particular turn. For example, in the turn “I thought, using the - like *powers*. Like to the first and second *power* and *cubed* and stuff," two mathematical terms would be counted, namely ‘power’ and ‘cubed’.”

Taking into account the possible occurrence of mathematical terms or phrases not categorised as key mathematical terms, the other two categories (related terms and other terms) were constructed for the purpose of reflecting the student's capacity to use mathematical terms other than those central to the substantive content of the lesson. Student use of these three categories of mathematical terms is illustrated below (Table 3).

Table 3. Interview data related to San Diego 2 - Lesson 3

00:00:07:02	I	I know it's been a few days since Friday[ii] ... since the last lesson, but can you think back and tell me what you thought the lesson was about on Friday?
00:00:16:16	Nahoku	It was just telling us - there was one ***equation*** with - there was four different ways you can show it.
00:00:24:12	Nahoku	There's the ... the ***verbal***. That one [Nahoku points at notepad], //the ***equation***, the ***graph***, and the ***T chart***.
00:00:28:06	I	//Okay.
00:00:31:26	I	Okay. That makes sense. Anything else you want to add about those four expressions?
00:00:37:16	Nahoku	They all mean the same thing.

00:00:40:02	I	Okay. What do you mean by that? What do you //mean by "the same thing?"
00:00:41:27	Nahoku	//Like, um, X Y is ***equal*** to two.
00:00:46:23	I	Uh-huh.
00:00:47:13	Nahoku	X ***multiplied*** by Y is ***equal*** to two. And then the, um, the ***T chart*** tells the same thing as all of 'em.
00:00:55:17	I	Okay. And then what does a graph tell you?
00:00:58:16	Nahoku	It's just plotting out the ***points***. Like this [points at notepad], ***negative*** two and ***negative*** one, is ***negative*** two, ***negative*** one.
00:01:07:03	I	Oh, I see. Okay. Does the graph tell you anything else about the ... representations?
00:01:13:20	Nahoku	It tells you like, if it's a ***linear line***, or a ... um ... a ***non-linear line***.
00:01:21:02	I	And what does that mean, "Linear line"?
00:01:22:23	Nahoku	***Linear*** means a ***straight line***.
00:01:24:19	I	Oh, okay.
00:01:25:13	Nahoku	[points at her paper] This is a ***non-linear***.
00:01:26:27	I	Okay. Do you know what that's called when it's non-linear?
00:01:30:19	Nahoku	I think it's this one [looks through her notes]. ***Parabola***.
00:01:35:03	I	Oh, okay.
00:01:36:03	Nahoku	Or a ***curve***.
00:01:37:25	I	Okay. Great. Okay.
00:01:41:14	I	Tell me what you think, um, you understood during the lesson on Friday. What do you think that you got worked out? An th- and then, what are some of the things that you think maybe you don't have worked out?
00:01:52:22	Nahoku	I have, um, how you can tell the ***graph*** is gonna be ***linear*** or ***non-linear*** by the ... um, the ***coordinates***.
00:02:02:24	I	Oh, okay [nodding].
00:02:06:17	I	And anything else?
00:02:11:06	Nahoku	No.
00:02:11:29	I	No? What a- what about things that you still are a little bit confused about? Anything?
00:02:16:28	Nahoku	[points at notepad] How these can be- I don't know how to tell 'em ... if they're ***curved*** or not. All I know how to tell is if they're ***linear***.

Key terms: equation, verbal, graph, coordinates
Related terms: T chart, linear, linear line, straight line
Other terms: multiplied, equal, points, negative, non-linear, non-linear line, parabola, curve(d)

The relative frequency of occurrence of each of these categories of mathematical terms expressed as the average number of mathematical terms used per student is displayed in Figure 4. Student descriptions of lesson content and learning provide a different type of mathematical performance from that displayed in student performance on mathematics tests. The classrooms studied in this project appear to differ in the value accorded to such performances.

Data from interviews with Berlin focus students were not included in Figure 4. Unlike the individual interviews conducted with students elsewhere, the Berlin post-lesson student interviews involved two and sometimes three students simultaneously. This situation arose because of the reported unwillingness of the German students to be interviewed individually. As a result, while it was possible to calculate the number of terms employed by students as a total per student (Berlin 1: 6.31 key terms, 3.23 related terms, and 7.00 other terms, totalling 16.54 terms; Berlin 2: 3.33, 3.33, 9.75 and 16.42, respectively), the nature of a group interview meant that students were less likely to mention a mathematical term that had already been introduced in the same interview by another student. As a result, the figures just cited are likely to underestimate and therefore misrepresent the facility with mathematical terms of the German students interviewed.

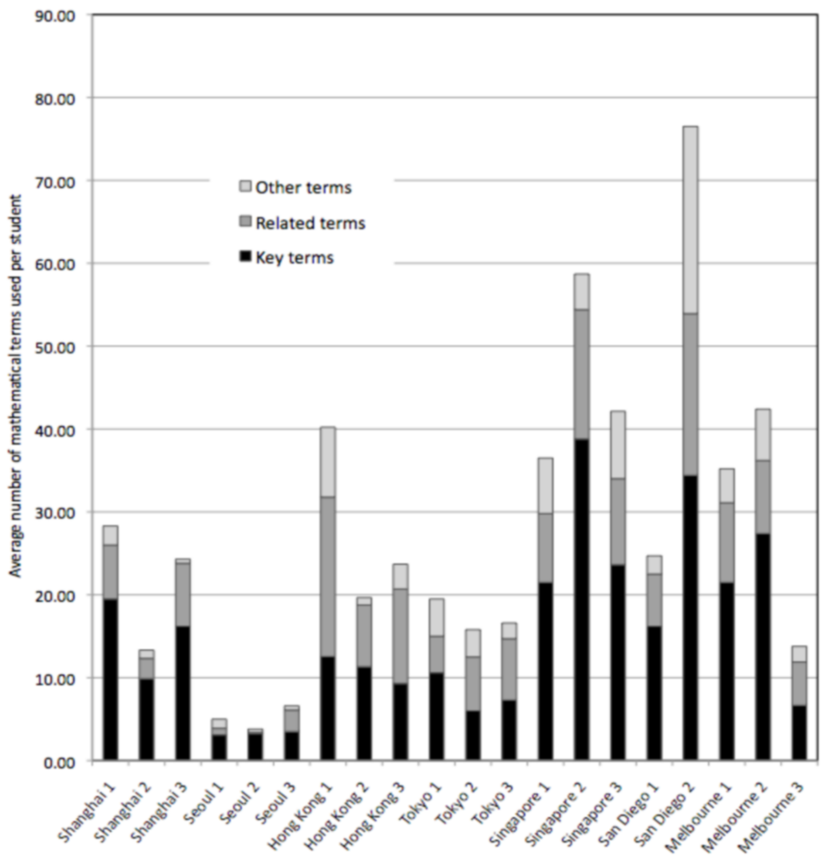

Figure 4. Frequency of use of technical terms in post-lesson interviews (each bar represents the average per student over ten student interviews for each class)

The inclusion of the Berlin results in Figure 4 would encourage misleading comparisons between the technical vocabularly of the students from the two Berlin classrooms and that of students from other classrooms. The best that can be said in relation to students from the Berlin classrooms is that the frequency of student use of mathematical terms in the post-lesson interviews was at least comparable in overall total to Tokyo 2 and 3, with a higher relative occurrence of “other” terms. Total term usage in the Berlin post-lesson interviews was lower than that for students from the other Western classrooms, except Melbourne 2. But, as noted, this is likely to be a significant underestimate, and it is possible that the term usage for individual Berlin students could lie between San Diego 1 and Melbourne 1.

Consideration of the two pairs of figures dealing with oral interactivity and mathematical orality in this and the companion chapter raised several questions regarding the learning consequences of classroom spoken mathematics. For example, all three of the Seoul classrooms provided students with little opportunity to speak mathematics, either in public or in private. When asked to describe their experience of a particular lesson, using the same interview protocol as the students from other schools, would the students from the three Seoul classrooms display comparable fluency in the use of the mathematical terms central to the content of the lesson being described? Figure 4 suggests that despite the use of the same interview protocol in all countries, the students from the three Seoul classrooms used significantly fewer actual mathematical terms to describe their experience of the mathematics classroom.

Consideration of Figure 4 suggests several interpretive hypotheses:

– If student facility with technical mathematical vocabulary is a valued outcome, then the analysis of the post-lesson interviews suggests that the public scaffolding (and explicit valuing) of student technical fluency (e.g., in Shanghai 1) can be as effective as the encouragement of student-student spoken mathematics (e.g., in Melbourne 1) in developing this facility.
– Where the classroom provided students with no opportunity to engage in spoken mathematics (Seoul), there appears to be little inclination (and possibly capacity) to do so, even in interview situations where the invitation to use spoken mathematics was explicit (“What did you learn today?”).
– Student inclination to employ other mathematical terms (‘other terms’) in addition to those specific to the lesson could indicate a form of interconnected knowing. Detailed analysis of interview transcripts is required to determine the significance of the use of ‘other terms’ as indicative of sophisticated understanding. This will be addressed in more detailed case study of San Diego 2 to be reported in another volume in the LPS research series.
– Facility with mathematical speech seems to respond to personal practice (e.g., San Diego 2 and Singapore 2) but can, as noted above, also be achieved through the public promotion of student mathematical speech (e.g., Shanghai 1).

We suggest that student use of mathematical terms in interview can be used as the indicator of one type of learning outcome. Such outcomes are attributable to features of particular mathematics lessons and, may possibly be used as indicators of the success of the instructional practices of the particular mathematics classroom. Such causal claims address one of the most significant challenges of classroom research and require careful empirical justification.

GENERAL DISCUSSION

As a result of this research, we are in a position to compare types of mathematical language employed in 22 mathematics classrooms in eight cities in seven countries. The 22 classrooms offer a remarkable sample of different combinations of forms of classroom language use. Consideration of high or low frequency of utterance, together with high or low use of technical terms, each considered in both public and private contexts, suggest groups of classrooms sharing common patterns of language use:

- Mathematics classrooms of very low public interactive orality and extremely low private interactive orality – where, apart from a small number of choral responses, only the teacher makes use of any mathematical terms: Seoul 1, 2, and 3.
- Mathematics classrooms of low public interactive orality, but relatively high private interactive orality – where the student classroom use of mathematical terms is relatively low: Hong Kong 1, 2, and 3.
- Mathematics classrooms of relatively low public and low private interactive orality – where the teacher and students both make significant use of mathematical terms (that is, high lexical density): Shanghai 1, 2, and 3.
- Mathematics classrooms of high public and private interactive orality – where teacher and students make relatively infrequent use of mathematical terms (low lexical density): Berlin 1, Melbourne 2 and 3, and San Diego 1.
- Mathematics classrooms of relatively high public and private interactive orality – where the teacher and the students make relatively frequent use of mathematical terms: Melbourne 1, San Diego 2, Singapore 1, 2 and 3, and Tokyo 2.
- Mathematics classrooms of moderate public and private interactive orality – with moderate teacher and student use of mathematical terms: Tokyo 1 and 3, and Berlin 2.

Since the characterisation of each classroom is based on detailed analysis of at least five lessons per classroom, and the private language use of about ten students in each classroom, the patterns of language use outlined above should be quite robust as characterisations of the practices of each classroom. As acknowledged earlier, the nature of the mathematical language employed will reflect the topic taught in each classroom. However, each topic (with the possible exception of Melbourne 3) required a variety of technical mathematical terms, sufficient to provide evidence of a classroom emphasis on spoken mathematics or not.

To repeat the point made in the companion chapter: It is really only through international comparative studies such as this one that we can make such comparisons between classrooms so fundamentally different in their practices. The teachers in the LPS project were recruited on the grounds that the local mathematics education community endorsed their practice as competent. Given this selection criterion, it is reasonable to assume that we have documented competent mathematics teaching as this was conceived at the time of data generation in each city. Despite within-city variations, the mathematics classrooms from some cities do seem to share sufficient common features to suggest that they draw on a common tradition of practice.

Since it is the use of mathematical language that is the focus of this analysis, student facility in the use of mathematical language to describe the activities and content of particular mathematics lessons seems an appropriate outcome to examine. Given the popular (Western) advocacy of student participation in mathematical dialogue in the classroom, the classrooms studied in Seoul provided an interesting testing ground for this advocacy, since they represent the antithesis of this practice. The consistency of language use across the three Seoul classrooms suggests a well-established tradition of practice, even if contemporary curricular reforms require that this tradition be supplanted by a more discursive pedagogy. It has to be considered as feasible, therefore, that the Korean national success on international tests of mathematical performance (for example in the TIMSS study, reported in Beaton & Robitaille, 1999) was achieved through classroom practices like those documented here.

CONCLUSIONS

The Asian classrooms in this study varied in their practice from no spoken mathematics by students (Seoul), through almost entirely public spoken mathematics by students (Shanghai), to spoken mathematics by students in both public and private classroom settings (Tokyo and Singapore). Differences in outcome in terms of facility with spoken mathematics (as displayed in interviews) may reflect differences in aspiration (rather than simply differences in success) – different cultures valuing different types of mathematical performance. What is essential is that our theories of learning should not unwittingly incorporate culturally-specific assumptions about the nature of classroom practice and about valued outcomes. Instead, our theories should anticipate application in culturally-differentiated settings and be sensitive to the constraints and affordances that culture places on practice.

To summarise: Students in the mathematics classrooms in Seoul had few opportunities to speak in class (either privately or publicly) and seldom employed spoken mathematics. Students in the Hong Kong classrooms were publicly and privately vocal, but made very little use of spoken mathematical terms in either context. Students in the mathematics classrooms in Shanghai were guided through the public orchestrated rehearsal of mathematical terms by their teachers, but seldom spoke to each other in private during class time except when explicitly asked by the teacher to conduct group or peer discussions. Students in the mathematics classrooms in Tokyo and Singapore participated orally in both public and private discussion and employed mathematical terms to a significant extent in both. By comparison, the students in Melbourne classroom 1 were highly vocal in both public and private contexts, and made more frequent public use of mathematical terms than any of the three Japanese classrooms, but less frequent use of mathematical terms in their private conversations. These different combinations of oral interactivity and mathematical orality suggest distinct pedagogies.

The essential question is, of course, whether or not students are advantaged in terms of their mathematical achievement and understanding by classroom practices that afford the opportunity to develop facility with spoken mathematics. The post-lesson interviews provide some evidence of a connection between classroom mathematical orality and student learning outcomes. This evidence suggests that those classrooms that promote student spoken use of mathematical terms do develop in those students the capability to use mathematical terms to describe their mathematics classrooms and their mathematics learning. If we use the term "mathematical orality" to signify this fluency in spoken mathematics, then our analysis suggests that, if mathematical orality is promoted in the classroom, whether in the public or the private domain, then students can develop this facility. The question of whether such mathematical orality can be associated with some higher form of mathematical understanding requires further consideration, both empirically and theoretically. It is our hope that the analyses reported in this and the preceding chapter will provide the basis for further work on this important issue.

This research also has significance for the development of theory. The contemporary advocacy of student spoken mathematics in classroom settings is prompted by research conducted in Western classrooms. The analyses reported in this and the preceding chapter can be interpreted as problematising such unqualified advocacy. Since the research cited to justify such advocacy is entirely Western, it is possible that the prescribed instructional practices might only be practicable in "Western" classrooms. As proposed in the preceding chapter, interpretation and application of the Western advocacy of spoken mathematics should be subject to three considerations: (i) The advocated practices may be non-viable in a culture dissimilar to that in which the research studies were conducted; (ii) The advocated practices may target outcomes that are not valued in school systems different from those studied; and (iii) The theories of teaching/learning by which such advocacies are rationalised may themselves be culturally-specific. Contrast such advocacy with evidence of belief in the capacity of active listening (rather than oral participation) to promote student learning (Li, 2004; Remedios, Clarke, & Hawthorne, 2008).

The results of our analyses of classrooms in Singapore and Tokyo suggest such practices are at least feasible in some non-Western settings. Research is currently being undertaken into the cultural-specificity of the constructs (particularly pedagogical terms) from which our theories of teaching/learning are constructed and through which they are expressed. It is our hope that research in the classrooms of competent mathematics teachers around the world might lead to an expansion in the instructional repertoire of all teachers and to a more inclusive reconstruction of the theories by which accomplished mathematics teaching and learning are conceived.

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NOTES

[i] Lexical density here refers to the relative concentration within sampled utterances of technical terms drawn from the mathematics lexicon.

[ii] As this example shows, it was not always possible to interview the student immediately after the lesson. The majority of interviews occurred on the same day as the relevant lesson, but sometimes it was necessary to delay an interview over a weekend.

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CHAPTER FOUR

Students at the Front: Examples from a Beijing Classroom

INTRODUCTION

As we know, the front is where the teacher's desk, blackboard and/or a projection screen are located. In this chapter we focus on classroom episodes with "Students at the Front" as an event in which "a student presents information publicly in written form, sometimes accompanied by verbal interaction between the student and the teacher or other students about the written work; other students may attend to this information or work on an assignment privately" (Jablonka, 2006, p. 108). In China's mathematics classes, "Students at the Front" events can be categorised into two types: (i) writing down the procedures for the solution on the blackboard; and (ii) giving oral descriptions of the approaches, a highly valued practice in the new-century curriculum reform. Based on the video recordings of a sample of 6 lessons in a eighth grade mathematics class in Beijing, the authors carried out an analysis of the activity "Students at the Front". The findings presented in the chapter illustrated the various forms of activity involved in the "Students at the Front" event. We explained the nature of interactions within the event by considering norms for presentations of problems, the extent of oral explanation, and the form of teacher-student exchanges.

The results reported in this chapter reflected the latest trend in China's mathematics education. In 2001, "The Standard of the Full-time Compulsory Education to the Mathematics Courses (Trial version)" was promulgated, which indicated the official beginning of the nation's new mathematics course reformation. The new mathematics curriculum policy has made significant changes in the basic values, development mechanism, development process, implementation system, and support system for curriculum in China. Students now have more opportunities to engage in classroom discussions, give comments, and ask questions (Liu, Wang, Sun, & Cao, in press).

To ensure that students take a major role in mathematics classes with real participation in class and full expression of thoughts, it is important for the teachers to organise the "Students at the Front" activity. In aligning with the mathematics curriculum reform it has become increasingly important for teachers to encourage students to share their own thinking procedures for solutions with the rest of the class. As a result, great changes have taken place and nowadays in the classes given by excellent teachers, students enjoy opportunities to speak publically in class (Cao, Liao, & Wan, 2008).

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 53–64.

There have been some previous studies concerning "Students at the Front" in mathematics classes. Jablonka (2006) conducted a comparative study of students' behaviour at the front of the classroom (on the board or in front of the teacher's desk) focusing on forms and functions in six mathematics teachers' classes from Germany, Hong Kong, and the United States. In these six classrooms, Jablonka found that students were hardly initiated into 'talking mathematics'. She argued that the classroom practices do not afford public student argumentation for different reasons (p. 120). In another study, by Begehr (2006), research on students' oral behaviour in mathematic classes in Germany investigated the scope of verbal actions. Begehr found that the German teachers "outtalked" their students, without being aware of it and the students' verbal participation was restricted to "disjointed fragments" (p. 180).

This chapter analysed 6 lessons selected from the video recordings of 12 lessons taught by a mathematics teacher (BJ1) in Beijing. Through analysing the coded teaching video sets, the research investigated the following aspects of the "Students at the Front" event: (i) information about the lesson events, including features such as the type of student activity, percentages of the events durations of the total teaching time, the frequencies of occurrence of the events, and the individual time for each student in the "student at the front" activity; (ii) analysis of the types and characteristics of the "Students at the Front" events; (iii) problems found in students' performances in the activity and effective methods for the teachers to promote higher learning efficiency, which can improve the relationship between teachers and students and create good classroom atmosphere through the activity "Students at the Front".

RESEARCH DESIGN AND METHODOLOGY

The overall LPS research design as set out in the Appendix of this book was adopted by the study. Sequences of at least ten lessons were recorded in the classroom of three teachers who were selected as representatives for the normal level of all teachers. In Beijing, efforts were made to ensure that the three classrooms were in demographically different parts of the city. Three video records were generated for each lesson (teacher camera, focus student camera, and whole class camera), and this video record was supplemented with post-lesson video-stimulated interviews with two students after every lesson and with the teacher three times during the period of data generation. This combination of classroom video material plus teacher and student interviews constituted the primary data source for the analyses reported in this chapter.

For the purpose of this chapter we studied six lessons (5^{th}-10^{th}) of one teacher, BJ1. For the six lessons of BJ1 that we analysed, "Students at the Front" was identified according to the description: In the classroom teaching, the activity "Students at the Front" starts from the moment a student leaves his/her own seat to go to the front, where (s)he writes on the blackboard or gives an oral presentation concerning certain teaching content, knowledge points or problems in front of the whole class, and ends when the student is back to his/her own seat. It was noted that the tasks for the "Students at the Front" may vary – some required students to answer a complete question, while others required students to figure out certain steps of a solution.

Object of Study and Relevant Basic Information

Analysis was undertaken of the video recordings of the fifth to the tenth lessons out of the twelve consecutive lessons in natural settings given by an eighth grade mathematics teacher (BJ1) in Beijing. Teacher BJ1 was an experienced teacher who had previously taught classes ranging from the seventh to the twelfth grade and participated in many professional development activities. The focus of the lesson sequence was about knowledge of quadrilaterals for the eighth grade. The topics of the lessons are listed in Table 1.

Table 1. Contents of the fifth to the tenth lessons given by Teacher BJ1

Lesson	*Content of courses*
L5	Theorem and Property of Median of Triangle
L6	Rectangle & Square(1): Property of Rectangle
L7	Rectangle &Square(2): Decision Theorem of Rectangle
L8	Rhombus
L9	Rhombus & Square
L10	Special Quadrilaterals: Internal Relations among Parallelogram and Rectangle, Square, Rhombus

The teaching objectives of the lessons were: (i) to enable students to know different aspects of learning geometry; (ii) to gain a general perspective on the knowledge of lines, surfaces, and cubes; and (iii) to appreciate that learning mathematics could be an approach for improving one's analytical capabilities.

RESEARCH PROCEDURE

The first step of the analysis was to use the *Studiocode* software to code events in the video and generate relevant statistics. These statistics were then combined with the text records of the classes and after-class interviews with students to carry out the qualitative and quantitative analyses. The analysis included the following steps: (i) observe the lesson videos, consult relevant documents and set the primary codes; (ii) use the primary codes and the *Studiocode* software to carry out quantitative analysis of the video material. This process includes modifying the codes and establishing final codes; (iii) use the final codes to conduct quantitative analysis of the coded material, gather relevant statistics and then combine the statistics with the classroom record and the relevant video records to carry out qualitative and quantitative analyses of the classroom videos.

In relation to the "Students at the front" lesson event, it was discovered that there were two types of representative behaviour evident in the data: (i) blackboard presentation, meaning that students do in-class exercises on the blackboard with the procedures written down, and (ii) oral presentation, in which students stand at the front and explain their own approaches, understanding or thinking procedures to the rest of the class with occasional writings on the blackboard as support.

The analysis of students' presentations was based on frameworks used in Begehr's (2006) and Jablonka's (2006) research. For the coding, we looked into the verbal communication between the teacher and the students in each "Students at the Front" event, identifying such features as selection of the presenting student, the number of problems addressed in this mode, the type of problems, any evaluation of the presentation by the teacher and/or the class, the length of time taken by each and all students participants, and any consequent actions that could be associated with the event.

Through primary analysis, it was found that most students 'at the front' were selected by the teacher, while a few volunteered to give presentations, and sometimes the teacher utilised the analogous strategy of displaying students' in-class working via the projector. Sometimes several students gave presentations on the same problem, while at other times different students talked about different problems. The evaluation of the students' presentations could arise from: teacher's comment, peers' comment, and teacher-student mutual comment. The content of any particular evaluation could vary. Table 2 shows the codes in evaluating students' presentations.

Table 2. Codes used for the student oral presentation

Codes		*Codes Explanation*
Presentation Source (PS)	PS1	The teacher appoints a student to give a presentation on a problem.
	PS2	Students volunteer to give the presentation.
	PS3	Students volunteer to supplement the current presentation.
Presentation Type (PT)	PT1	One student talks about one problem.
	PT2	Several students talk about the same problem.
Presentation Content (PC)	PC1	Students explain approaches to a certain problem.
	PC2	Students elaborate specific steps and reasons for their solution and describe the procedure.
Presentation Mode (PM)	PM1	Oral speech only.
	PM2	Oral speech together with written procedures, drawings and marks on the blackboard.
Comment Type (CT)	TC	The teacher comments on the presentation.
	SC	Students comment on each other's presentation.
	TSC	The teacher and the students comment on the presentation together.

Teacher's Responses (TR)	TR1	The teacher approves of the student's presentation in simple ways such as nodding, applauding and saying "good".
	TR2	The teacher points out the incorrect or incomplete points in the presentation in simple ways such as shaking head, saying "that isn't correct", or "that isn't good".
	TR3	The teacher interrupts the presentation without letting him/her finish the speech or task.
	TR4	The teacher supplements the presentation, completing and improving.
	TR5	The teacher corrects the student's mistakes during presentation.
	TR6	The teacher further elaborates on the student's presentation.
	TR7	The teacher encourages and guides the student according to his/her presentation
	TR8	The teacher speaks instead of the student when (s)he encounters difficulty in presentation.
	TR9	The teacher gives no direct reaction to the student's presentation and continues the teaching.

The following is a sample coded transcript from L10 using the student oral presentation coding (Table 2):

Teacher	[PS2]Which two angles? Who will come to the front? (A student stands up) Ok, Fan Xiaoshu, please. Which two angles?
Teacher	Here, use this and speak at the front. Draw it out on the blackboard by yourself, do it on your own. Which two angles are equal?
Student A	[PM2][PT1] These two.
Teacher	[TR7] Right! Why?
Student A	[PM2][PC2] These two are the same, and equal angles lead to equal sides.
Teacher	[TSC] Hmm, he said these two are the same, right? (Students: Yes.) But he didn't give the justification. Lack of justification. What did you say?
Student A	[PC1] Just use the parallel interior alternate angles.
Teacher	[TC][TR6] Which parallel interior alternate angles? Which? Come to the front, yes, come here. Aha, I see you are anxious to speak at the front… Aha, he is so eager to speak out here. Yes, just come here and point out where he has made a mistake.
Student B	[PS3][PC1] Just now we said this equals this, so that's an interior alternate angle.
Teacher	[TC][TR7] Point it out clearly, which and which forms an interior alternate angle.
Student B	[PC2][PM2] This and this, and this and this are equal. And this is a bisector. These two angles are equal, so these two are equal, and equal angles lead to equal sides.
Teacher	[TSC] Is it OK? Right?
Students	Right.

STATISTICS AND ANALYSIS

The researchers used the *Studiocode* software to gather statistics for the videos of six coded lessons.

Table 3. Frequency of presentation source in BJ1's classes

PS	*L5*	*L6*	*L7*	*L8*	*L9*	*L10*	*AVG*
PS1	2	2	0	0	3	3	1.67
PS2	1	0	2	1	0	0	0.67
PS3	0	0	2	0	1	0	0.50
TOTAL	3	2	4	1	4	3	2.83

From Table 3 it is apparent that while students volunteered to do the presentations or offered supplementary presentations, more than a half of the presentations were still directed by teacher appointment (average frequency of PS1: 1.67, of PS2 & PS3: 1.17). It should be noticed that the pattern of participation was influenced by the lesson objective. For example, in Lesson 8, in order to introduce a new concept, the teacher taught for most of the time, only setting aside time for one presentation. However, in Lesson 7, the second lesson of rectangle and square, since the students were comparatively familiar with the content being taught, more time was used for students presentations.

Table 4. Length of time (minutes) of different types of presentations in BJ1's classes

PT	*L5*	*L6*	*L7*	*L8*	*L9*	*L10*	*AVG*	*% of Lesson Time*
PT1	2.08	0.00	2.20	4.50	1.02	2.09	1.98	4.95%
PT2	7.45	2.56	6.02	0.00	2.41	1.31	3.29	8.23%
TOTAL	9.53	2.56	8.22	4.50	3.43	3.40	5.27	13.18%

From Table 4 we can see that the average time of students' presentations accounted for 13.18% of the total length of lesson time which is 6 hours, since the length of a typical single lesson is 40 minutes. Also shown is that the time of cooperative presentations (PT2) lasted almost twice as long as the time of solo presentations (PT1), especially in Lesson 5 and Lesson 7, where PT2 lasted 7.45 minutes and 6.02 minutes respectively, taking up more than 15% of the length of time of a single class.

Most notably, there were no student-only comments during student presentations. The main form of comment was teacher-student mutual comment, taking up 61.75% of the total comments based on students' presentations, however, this accounts for only 4.4% of the total lesson time. From Table 5 we can see that in L5, L6, L9 and L10, teacher-student comments obviously outnumbered the teacher-only comments, especially in L6 where there was only TSC. However, in L7 the teacher-only comments dominated and in L8 there were only teacher's comments. It appeared that this was determined by the content being taught, which corresponds to the characteristics of lessons shown in Table 1.

Table 5. Length of time (minutes) for comment types used in student presentations

CT	*L5*	*L6*	*L7*	*L8*	*L9*	*L10*	*AVG*	*% of Comment Time*	*% of Class Hour*
TC	0.17	0.00	3.28	0.97	1.97	0.17	1.09	38.25%	2.73%
TSC	4.52	1.08	1.13	0.00	2.27	1.54	1.76	61.75%	4.40%
TOTAL	4.69	1.08	4.41	0.97	4.34	1.71	2.85	100%	7.13%

Only five types of responses were found in the data: TR1, TR4, TR5, TR6, and TR7. Within the six lessons there were no incidences found where the teacher negated the student's ideas (TR2), interrupted the student (TR3), spoke over the student (TR8), or gave no comment (TR9). Among the five types of response codes recorded, the most prevalent was TR7, as shown in Table 6, which related to encouraging and guiding students in their presentations.

Table 6. Length of time (minutes) on teacher's different responses to students' presentations

TR	*L5*	*L6*	*L7*	*L8*	*L9*	*L10*	*AVG*	*% of Comment Time*	*% of Class Hour*
TR1	0.14	0.00	0.00	0.20	0.21	0.11	0.11	3.89%	0.28%
TR4	0.67	0.00	0.00	0.00	0.43	0.51	0.27	9.54%	0.67%
TR5	0.60	0.00	1.23	0.07	0.00	0.26	0.36	12.72%	0.90%
TR6	0.00	0.00	0.35	0.41	0.45	0.20	0.24	8.48%	0.59%
TR7	2.84	1.09	2.57	0.39	2.51	0.71	1.60	56.54%	4.00%
TOTAL	4.25	1.09	4.15	1.07	4.60	1.79	2.83	100%	7.06%

However, whilst TR7 responses were the preferred form, it is noted that they comprised only 4% of the lesson time as shown in Table 6. Overall, the distribution of response types suggested that the teacher feedback was geared to provide correction, guidance and encouragement; she never interrupted or negatively criticised students' presentations. In other words, the teacher valued very much each student's presentation.

SUMMARY OF FINDINGS

Our analysis of classroom events involving student presentations "at the front" in six mathematics lessons of one teacher BJ1 is summarised as follows.

There is Rich Variety in the Types of Students' Presentations at the Front

The most common scenario was that the teacher selected certain students to give the presentations, while students also competed for the chance to do a presentation or to supplement others' presentations. The number of student presentations was close to three (2.83) times per lesson, regardless of how the presenter was selected. The overhead projector served as an important tool for the students to do the presentations.

The format of the student presentation typically began with the Teacher BJ1 asking a question with students volunteering to present their solutions or ideas at the front. When there was no volunteer, BJ1 appointed some students to do the presentation. If students failed to offer complete solutions or their answers needed to be supplemented and further explained, or when other students had different ideas, BJ1 usually encouraged other students to give supplementary presentations as seen in the following transcript:

Teacher	…these two triangles? Who can help him? Who has got any idea? Great! You! Come on!(BJ1-L05)
Teacher	(to the rest students) Do you have anything else to add?(BJ1-L07)
Teacher	So you got it? Well, you please! (BJ1-L08)

One or More Students Give Presentations Concerning One Problem

On completion of a presentation, Teacher BJ1 often encouraged other students to share their different ideas. Allowing as many students as possible to express their own thoughts encouraged discussion about the best idea and enabled each student's ideas to be known. Sharing ideas of several students took 8.23% of the total lesson time (about 3.3 minutes per lesson), almost twice as long as presentations (4.95%) in which one student talked about one problem.

Students' presentations are important in that they provide an opportunity for the teacher to know the extent to which the students have grasped the knowledge of the subject matter, as well as find out their existing problems and barriers in applying their knowledge. In addition, knowing the students' cognitive thinking means that the teacher can more effectively guide and help them. To support this point, the following is a sample transcript from Lesson 7 in which three students at the front were working to find the area of a parallelogram. The teacher, instead of telling the students the answer, encouraged them to ask and listen to each other, express their opinions to their classmate's answers.

Teacher	Can you work out its height?
Student B & C	Yes.
Teacher	OK. Maybe you can ask them.
Student A	(to B) How to work out its height?
Teacher	(to B) Please. (to A) You can write down what he said.
Student B	DE is known, and then MN can be obtained. So we can get the area of this parallelogram.
Teacher	Good! But I am not very sure. Which one?

```
Student B       This one. Parallelogram AMND.
Teacher         Sure? Is this parallelogram our target?
Student C       No!
Teacher         Please tell us your opinion.
```

The Main Form of Students' Presentations at the Front is Oral Explanation of the Approaches

From the statistics of content of all the presentations from BJ1's lessons (see Tables 3-6), it can be seen in all the coded lessons that the total time spent by students *describing* procedures (PC2) was 7.57 minutes, while 24.05 minutes was required for *explaining* the approaches (PC1).This shows that teacher BJ1 placed great emphasis on developing students' thinking and preferred to ask students to present their thoughts on certain knowledge points or typical examples. By careful attention to the students' own expression of their ideas, their existing problems can be found and any difficulties in mastering the knowledge of the subject matter can also be known. Hence, the teacher can guide and help the students to overcome any barriers in their thinking process, as well as promoting their initiative in learning.

In terms of the form of presentation, Teacher BJ1 encouraged students to write down their approaches and procedures while speaking, placing emphasis on the students' abilities of thinking, speaking, and doing. An example can be seen in the previous transcript when the teacher said to one of the students, A: "You can write down what he said."

Sometimes, however, a student's presentation was entirely in oral form as captured in the following transcript:

```
Teacher   This angle. Plus this angle and you can get 90 degrees, is
          it this angle? Ok, tell us please.
Student   Yes.
Teacher   You mean the angle EGD?
Student   No, it's the angle inside.
Teacher   The one inside? You mean this angle?
Student   Exactly, the exterior angle of the triangle AEG.
Teacher   Excellent, it is exactly this exterior angle.
```

Here, it is clear that the student's presentation involved answering closed-type questions posed by the teacher.

In other cases, a student's presentation comprised both written and oral form as illustrated in the following transcript:

```
Teacher    Come on share with us your ideas.
Student    EF is parallel to AD. (pointing to the drawing on the
           blackboard) These two angles are equal, and this angle
           equals to the sum of the two angles, so this angle and that
           angle are equal. Therefore, they are parallel.
Teacher    Can you understand?
Students   No.
Teacher    Please draw it out using the yellow chalk and tell us why
           the two angles are equal.
```

```
Student    (Drawing lines on the blackboard) this angle is the
           exterior angle of the triangle, and this big angle equals
           the sum of this angle and this angle.
Teacher    Can you understand this?
           (The rest of the students nod.)
Student    And since this is parallel to this, we can know this angle
           equals this angle.
Teacher    Can you understand now?
Students   Yes!
```

In L5, L6, L7, and L8 both written and oral forms were applied; in L10, only the oral form was adopted; and in L9, 2.15 minutes were spent on both forms, while 1.27 minutes was spent on the oral form only. In all the coded classes, time spent in oral form only was about 4 minutes with the rest spent in applying both forms.

The Main Evaluation Form is Teacher-Student Mutual Comment

The majority of BJ1's evaluative responses took the form of mutual comments from both teacher and students (61.75% of the total comment time) and there were no student-only comments.

Across the six-lesson sequence comments offered by BJ1 within the students presentations were mainly guidance and encouragement, (56.54% of the total comment time). Free of interruptions or negative critique there was a sense that each student's presentation was valued. As making positive comments is an art, the teacher should be good at finding students' strengths in learning activities and provide positive comments in time. In this way, the students' learning potential can be tapped and confidence boosted. Positive evaluation can increase students' self-esteem and confidence, while negative evaluation can lead to the opposite effect.

When the students were giving their presentations, Teacher BJ1 stayed at the front and maintained her interaction with the students and the rest of the class. She offered positive reaction to the correct and reasonable points in the students' presentation with "yes," "right," nodes and smiles. Meanwhile, she did not forget to interact with the rest of the class, including explaining important or difficult points to the students and asking the rest of the class whether they understood the speaker. When a student's presentation was not correct or complete, the teacher usually asked other students to correct or supplement, thereby ensuring full participation from the students. When the student finished the presentation, the teacher further summarised or explained the knowledge points for the other students to better understand and grasp the knowledge. She put great emphasis on guiding and encouraging the students to think (TR7), which accounted for 56.54% of the total comment time. Generally, Teacher BJ1 guided the students to think on their own and to come up with the approaches themselves.

DISCUSSION

In another study by Cao (2011), involving lessons taught by the same teacher from Beijing (BJ1) and another teacher from Shanghai (SH2), it was found that while

Teacher SH2 attended mostly to the procedure and result of students' problem solving, Teacher BJ1 placed more emphasis on students' thoughts about the solution. Our current analysis of the six lessons taught by Teacher BJ1 showed that the time spent by the teacher in presenting the procedure of problem solving was 7.57 minutes in total for the six lessons, while the time spent in presenting thoughts and approaches was 24.05 minutes over the six lessons. In a comparable analysis of Teacher SH2 we found that he mostly asked the students to write down the complete procedure and steps on the blackboard without asking them to share their thoughts and ideas. SH2 preferred to give lots of comments after the students' blackboard presentation thereby placing greater emphasis on the teacher's comments and summary. His evaluation, which emphasises the procedures and steps of the solution and helps students develop more established ability in automatically solving the problems took up 68.72% of the total assessment time (about 6.7 minutes per lesson, average lesson time 40 minutes).

As the education reform in China moves forward, the idea of the student-centered classroom will gain more significance. The *Mathematics Course Standards for Compulsory Education* points out that "the teaching activity is a process in which the teacher and the students actively participate, communicate, interact and mutually develop mathematical knowledge. An effective teaching activity requires the integration of students' learning and teacher's teaching, with the students as the main body of math learning and the teacher as their organiser, leader and partner" (Ministry of Education of the People's Republic of China, 2011). The case of Teacher BJ1 offers an encouraging model for our future classroom teaching activities.

One basic goal of mathematics education is to develop correct self-expression and communication skills. Learning mathematics requires not only solving mathematics problems but also being able to discuss, communicate, and express one's own ideas. Learning to share one's ideas with others is an important skill for everyone today. Our analysis suggests that Teacher BJ1's practice of providing open questions for the students and her push for students to discuss and share their thoughts at the front, affords an important way for students to express their own thoughts and ideas.

Students' initiative in learning is stressed to help them establish their own knowledge concepts. The task of the teacher is to help the students construct their own knowledge rather than implanting them with knowledge, for only when the students construct their own understanding can they develop interest in learning. Moreover, the students' confidence can also be boosted when they get the chance to express their own thoughts at the front, leading to higher efficiency in learning.

The teacher needs to provide some proper guidance after the student's presentation, because the teacher's encouragement and guidance can help the students think more thoroughly about the subject matter knowledge being developed. The teacher's praise and rewards after the presentation can further increase the student's confidence and interest in learning. Therefore, the teacher should seize every opportunity to guide and encourage the students, who can thus actively explore and build their own knowledge system leading to an improved cognitive structure of mathematics learning.

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CHAPTER FIVE

Participation of Students in Content-Learning Classroom Discourse: A Study of Two Grade 8 Mathematics Classes in Singapore

INTRODUCTION

Cazden (2001) pointed out that, in contexts such as schools, "one person, the teacher, is responsible for controlling all the talk that occurs while class is officially in session – controlling not just negatively, as a traffic officer does to avoid collisions, but also positively, to enhance the purposes of education" (p. 2). Herbel-Eisenmann (2009) noted the two main functions of talk and distinguished between discourse for content-learning purposes and discourse for social purposes. According to Herbel-Eisenmann, discourse for content-learning brings the learning of content to the foreground and moves the social control to the background. For example, a statement like "This function is a linear function" is mainly about the mathematics being studied (p. 30), while a statement like "Please put your notebooks away so we can go to lunch" serves more strongly a social control function (p. 30) and therefore may be classified as discourse for social purposes.

As part of the Learner's Perspective Study (LPS) in Singapore, we have studied sequences of lessons of three competent mathematics teachers at the eighth grade level. In our past studies (Kaur, 2008, 2009; Seah, Kaur, & Low, 2006) we have found that the lessons of these teachers were

- guided by very specific instructional objectives;
- the examples used during whole class demonstration were carefully selected and systematically varied in complexity from low to high;
- teachers actively monitored student's understanding during seatwork, as they moved from desk to desk guiding those with difficulties and selecting appropriate student work for subsequent whole class review and discussion; and
- reinforced student understanding of knowledge expounded during whole class demonstration by detailed review of student work done in class or as homework.

In addition, in the classes of these teachers, students attached importance to their teacher's explanations which were simple and logical; demonstration of mathematical procedures – showing them the "method" or concrete representation of a concept with the use of a manipulative; introduction of new knowledge – knowledge they were being exposed to for the first time; instructions that guided them in their work and the use of real-life examples that helped them appreciate the use of mathematics in life. As part of seatwork, students attached importance to

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 65–88.

individual work during class time that provided practice and an opportunity to check for own understanding; group work during which they experienced teamwork spirit and peer support and the material (mainly in print form) given by the teacher to engage them in practice of concepts and skills they had learned. As part of review and feedback they attached importance to review of prior knowledge which helped to bridge past knowledge with the present and also in the construction of new concepts using past knowledge; student presentations which resulted in the use of student work to highlight mistakes and demonstrate alternative approaches and feedback given to students individually during class time and also through grading of written assignments.

However, our past studies have not focussed on the nature of the classroom discourse in the classes of these teachers. Therefore to understand the nature of discourse for content-learning purposes a study of teacher-student discourse in the classrooms that is specific to public talk and content-learning purpose was undertaken and reported in this chapter.

In this chapter we provide an analysis of the data for two teachers in the study which is guided by the following research questions: During content-learning classroom discourse

(i) how often do students get an opportunity to engage in public talk?
(ii) what are the characteristics of teacher-student public talk?
(iii) what are the teachers' orientation of discourse (conceptual or calculational)?
(iv) do students initiate any public talk with their teachers or peers? If so, what was the purpose of the talk?

LITERATURE REVIEW

Showing and telling or explaining the ideas to be learned is often the predominant approach to teaching mathematics in most Singapore classrooms both in the primary and secondary schools. This does not appear to be unique to Singapore schools as showing and telling appear to have been traditional practices in classroom teachings for generations and continues to dominate classroom practice (Pimm, 1987). In classrooms where this takes place the discourse is teacher dominated and teachers may engage students in some dialogue according to their planned 'next step'. However, often little use is made of students' contribution as the nature of contribution sought from the students is not for deliberation but rather confirmation of their understanding.

Alternatives to showing and telling involve reviewing and restructuring (Anghileri, 2006) which aid development of students' own understanding of mathematics. Reviewing relates to interactions where the teacher encourages experiences to focus students' attention on pertinent aspects of the mathematics involved and restructuring involves teachers making adaptations to modify the experiences and bring the mathematics involved closer to students' existing understanding (p. 41). This approach would facilitate a student-centred discourse where the teacher would take on the role of a facilitator. Some significant actions in such classrooms would be students' explaining their thinking with justifications, teachers asking probing questions and rephrasing students' talk and negotiating meanings.

To study teacher-student oral interactions specifically during content-learning in mathematics lessons is certainly significant but the challenges to do so are also present. Stein (2007) noted that classroom discourse can be difficult to assess as classroom talk is dynamic. Hufferd-Ackles, Fuson, and Sherin (2004) created a framework to describe and evaluate the process a class goes through when discourse is introduced. The framework depicts growth in a math-talk learning community in two ways: the movement through four developmental levels from a traditional mathematics classroom in Level 0 to a classroom embracing meaningful collaborative math-talk in Level 3 and the growth that occurred within each of the four components from Level 0 to Level 3 which include (a) questioning, (b) explaining mathematical thinking, (c) source of mathematical ideas, and (d) responsibility for learning (see Hufferd-Ackles, Fuson, & Sherin, 2004, pp. 88-90). Stein (2007) adapted the framework, shown in Table 1, to assess discourse level in a mathematics classroom.

Table 1. Levels of discourse in a mathematics classroom

Levels	*Characteristics of Discourse*
0	The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher's questions.
1	The teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.
2	The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another's work. Students answer the questions posed to them and voluntarily provide additional information about their thinking.
3	The teacher facilitated the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.

According to Thompson, Philip, Thompson, and Boyd (1994) there are two contrasting teachers' orientations in classroom discourse. They characterised them as conceptual orientation and calculational orientation. A teacher with conceptual orientation is one whose actions are driven by the ways of thinking he/she wants the students to develop, students' engagement that can orient the students' attention in productive ways and insistence that students are intellectually engaged in tasks and activities. The questions conceptually orientated teachers often ask their students that allow them to view their arithmetic in a noncalculational context like the following:

- "What are you trying to find when you do this calculation?
- "What did this calculation give you?"

A teacher with calculational orientation is driven by the application of calculations and procedures for "getting answers". Although such teachers do not focus only on computational procedures, there is a tendency to speak exclusively in numbers and numerical operations language. They place emphasis on identifying and performing procedures and have an inclination to remediate students' difficulties with calculational procedures often disregarding the context in which the difficulties might have occurred. The questions a teacher with calculational orientation often asks his/her students tend to be computational in nature such as:

- "Why did you subtract 7 from 38?"
- "How come you multiplied 7 and 3?"

Both orientation of teachers' classroom discourse involve the teacher posing questions to which students' answer.

Alternatively, students too may pose questions to their teachers and peers. These questions serve different functions such as confirmation of an expectation, resolution of an unexpected puzzle, and filling a recognised knowledge gap (Biddulph & Osborne, 1982). The type of questions shows the gap or discrepancy in the students' knowledge or a desire to extend knowledge in some direction. Besides helping students learn, student questioning can also guide teachers in their work. Questions also reveal much about the quality of students' thinking and conceptual understanding (White & Gunstone, 1992).

Wong and Quek (2010) in their work on promoting student questions in mathematics lessons claim that most lessons are about one or more of the following four aspects: meaning, method, reasoning, and application. As such a variety of questions may be asked by students about each of these aspects. An example is as follows:

> Suppose the teacher has just spent about 15 minutes explaining congruency between triangles ABC and XYZ. The students may not have understood certain parts of the explanation and want to ask some focussed questions. Below are some possible questions.
>
> *Meaning*: How is the symbol "≡" different from the equal sign?
>
> *Method*: Do we have to strictly keep to the order of pairing A with X, B with Y, and C with Z?
>
> *Reasoning*: Why do congruent triangles have the same area?
>
> *Application*: When do people use congruent triangles in real life?
>
> (Wong & Quek, 2010, p. 2)

Analysing questions posed by students during content-learning discourse may shed light on what the student is focussing on during the learning of mathematics.

METHODOLOGY

Method

The study in Singapore adopted the research design as set out in the Learner's Perspective Study (LPS) (Clarke, 2006). A total of three mathematics teachers recognised for their locally-defined 'teaching competence' participated in the study. These teachers are from a pool of teachers deemed as "experienced and competent", where experience was a measure of the number of years they have taught mathematics in secondary schools and competency was a composite measure of their students' performance at examinations and their performance in class in the eyes of their students. The teachers were nominated by their respective school leaders and the LPS research team in Singapore followed up on the nominations and interviewed the teachers. A strict requirement for participation in the study was that the teacher had to teach the way she / he did all the time, i.e. no special preparation was allowed. Three teachers who met the requirements agreed to participate in the study.

Video-records of 13 consecutive lessons (three during the familiarisation stage and ten as part of the study) for each teacher were collected using three cameras. The Teacher camera captured the teacher's actions and talk during the lesson. The Student camera focused on a group of two students, known as the "focus group" and captured their actions and talk during the lesson. Each group of pupils was only videotaped once. The Whole Class camera captured the whole class in action. The source of data for this chapter is the whole class video records and their transcriptions for ten lesson sequences of Teacher 1 (T1) and Teacher 3 (T3).

Subjects

Although three teachers participated in the LPS in Singapore, in this chapter the lessons of only two teachers, T1 and T3, are studied. T1 is from school 1 (SG1) and T3 is from school 3 (SG3). T1 is a female with 21 years of teaching experience. There were a total of 37 students in her class; 15 boys and 22 girls. The students' Primary School Leaving Examination aggregate scores were in the range of 245 – 267 with mean score of 250 and median score of 249. T3 is a male with 15 years of teaching experience. There were a total of 40 students in his class; 25 boys and 15 girls. The students' Primary School Leaving Examination aggregate scores were in the range of 188 – 253 with mean score of 207 and median score of 206. Students in the class of T1 were of higher ability than those in the class of T3.

Data Analysis

The video recordings and transcripts of all the ten lessons for T1 and T3 were viewed and studied respectively to annotate segments of lessons which we refer to as episodes during which i) students were given an opportunity to engage in public talk by their teachers, and ii) students initiated public talk. Having identified the episodes, the duration of each episode in minutes was recorded. This was done as the number of episodes did not provide a good means of representation as the duration of lessons in SG 1 were typically 60 minutes while those in SG 3 were 30 minutes in duration.

The characteristics of the discourse during episodes in which students were given an opportunity to engage in public talk were examined and coded according to Stein's adaptation of the Hufferd-Ackles, Fuson, and Sherin (2004) framework for level of discourse shown in Table 1, and ii) teacher's orientation of classroom discourse following Thompson's et al. (1994) characterisation of conceptual orientation and calculational orientation.

In the process of analysis for level of discourse we found that the descriptors for levels 0 and 1 were adequate for the purpose but several episodes were beyond level 1 but definitely not at level 2. Hence we created level 1^+ , the description of which is shown in Table 2.

Table 2. Revised levels of discourse in a mathematics classroom

Levels	*Characteristics of Discourse*
0	The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher's questions.
1	The teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.
1^+	**The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another's work. Students give short answers to the questions posed to them.**
2	The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another's work. Students answer the questions posed to them and voluntarily provide additional information about their thinking.
3	The teacher facilitated the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.

The episodes, during which students initiated public talk, were also studied for the purpose of the talk. The questions posed by the students were examined using the four categories: meaning, method, reasoning and application proposed by Wong and Quek (2010).

DATA AND FINDINGS

In this section the data and findings are presented in order of the research questions presented in the chapter.

How Often Do Students Get an Opportunity to Engage in Public Talk?

Table 3 shows the number of episodes per lesson during which the teachers engaged their students in public talk. As the duration of the lessons were not the same for both teachers it was not appropriate to make any comparison of the number of episodes. We therefore computed the length of time per lesson during which students were engaged in public talk. Table 4 and Figure 1 show the data for T1 and T3 according to the duration of teacher-student public talk.

Table 3. Number of episodes when students were engaged in public talk

	Number of episodes	
Lesson	T1	T3
L01	15	3
L02	9	2
L03	6	0
L04	3	2
L05	7	2
L06	1	1
L07	8	1
L08	4	7
L09	3	3
L10	3	1
Total	59	22

Table 4. Duration of time students were engaged in public talk by T1 and T3

Lesson	Duration in minutes			
	T1		T3	
	Lesson	Students engaged in public talk (%)	Lesson	Students engaged in public talk (%)
L01	54.58	19.59 (35.89)	32.75	7.21 (22.02)
L02	51.95	23.92 (46.04)	34.87	1.34 (3.84)
L03	54.62	12.52 (22.92)	33.42	0.00 (0.00)
L04	60.30	8.28 (13.73)	69.57	3.37 (4.84)
L05	53.00	13.40 (25.28)	37.58	4.67 (12.43)
L06	48.48	2.42 (5.00)	31.50	3.97 (12.60)
L07	54.27	12.57 (23.16)	28.80	0.63 (2.19)
L08	53.83	5.42 (10.07)	67.92	9.61 (14.24)
L09	47.00	7.93 (16.87)	40.32	3.42 (8.48)
L10	54.53	8.86 (16.25)	33.85	0.83 (2.45)
Total	532.46	114.91(21.58)	410.58	35.05 (8.54)

From Table 4, it is evident that students in the class of T1 had more opportunity to engage in public talk with their teacher (21.58%) as compared to the students in the class of T3 (8.54%). During lesson 6 of T1 and lesson 3 of T3 students wrote a mathematics test and hence as shown in Figure 1, the opportunity to engage in public talk by the students in the class of T3 was none and the lowest compared to other lessons of T1. With the exception of lessons 6 and 8, the percentage of time students were engaged in public talk in the class of T1 was always higher than that in the class of T3.

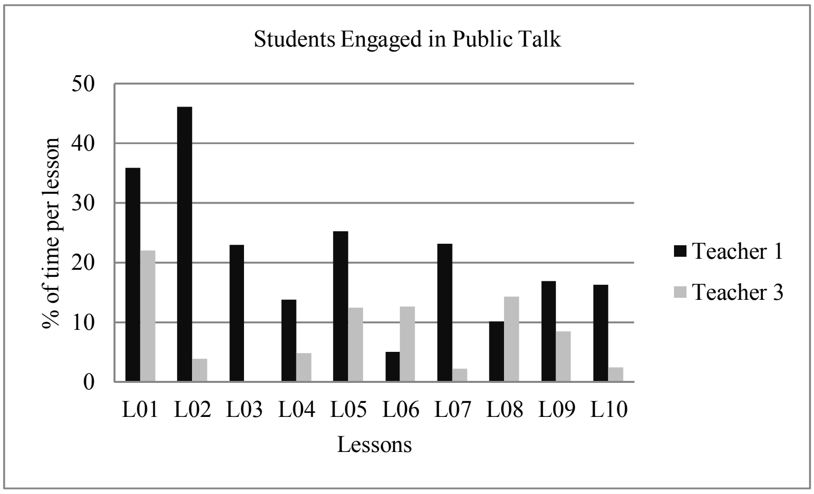


Figure 1. Percentage of time per lesson students were engaged in public talk

What Are the Characteristics of Teacher-Student Public Talk?

Table 5 shows the duration of teacher-student public talk according to the different levels of discourse per lesson for T1 and T3. It also shows for the three levels of talk its' percentage with respect to the duration of talk in the sequence of the ten lessons.

From Table 5, it is evident that the students in the class of T1 engaged in more public talk at level 0 (11.33%) and level 1 (9.01%) as compared to the students in the class of T3 (level 0 – 4.26% and level 1 – 1.81%). However, the students in the class of T3 spent twice as much time for level 1^+ (2.46%) when compared to the students in the class of T1 (1.24%). It is also apparent from the Table that when we consider only the teacher-student public talk time for the ten lessons collectively, both T1 and T3 spent about the same time, i.e., approximately 50% of the time for Level 0 of the discourse. However, the proportions of time spend on the other two Levels, 1 and 1+, were significantly different. T1 spend about 40% on Level 1 and less than 10% on Level 1+, while T3 spend about 20% on Level 1 and about 30% on Level 1+.

Table 5. Duration of teacher-student public talk by level of discourse for T1 and T3

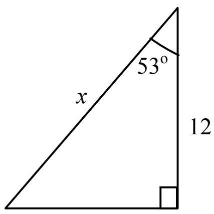
Lesson	% of time per lesson					
	Level 0		Level 1		Level 1+	
	T1	T3	T1	T3	T1	T3
L01	16.12	11.54	12.13	10.47	7.64	-
L02	20.35	1.18	21.02	2.06	4.68	-
L03	18.22	-	4.71	-	-	-
L04	13.73	2.90	-	1.94	-	-
L05	25.28	-	-	-	-	12.43
L06	-	12.60	5.00	-	-	-
L07	7.59	-	15.57	2.19	-	-
L08	2.32	8.39	7.75	0.71	-	5.05
L09	-	3.52	16.87	-	-	4.96
L10	7.26	-	8.99	2.45	-	-
Total	11.33	4.26	9.01	1.81	1.24	2.47

Level	% of total time for all 10 lessons	
	T1	T3
Level 0	52.50	49.88
Level 1	41.75	21.20
Level 1+	5.75	28.92

Level 0 of teacher-student discourse. At this level of teacher-student discourse the teacher mainly asked the students closed questions and students gave short answers. The teacher affirmed the accuracy of the answers and explained the underlying mathematical ideas almost always. Both teachers T1 and T3 spend almost half of the teacher-student public talk time during the sequence of ten lessons each engaging students in this type of discourse. Table 6 shows examples of teacher-student discourse at the level.

Table 6. Episodes of level 0 teacher-student discourse in the classes of T1and T3

Teacher/ Lesson/ Episode	Mathematical Content	Teacher's Questions	Student/s' Responses
T1 L02 Ep 04	$2.8 \times 10^4 + 3.2 \times 10^5$	Look at these two powers of ten, which is bigger? 10^4 or 10^5	10^5 (chorus)
	$2.8 \times 10^4 + 3.2 \times 10 \times 10^4$	10^5 is bigger. Now this is what we'll do. This is smaller, we put down 2.8×10^4. Okay we use 10^4. Now as for this one it becomes $3.2 \times 10 \times 10^4$. We break down 10^5 into 10×10^4. So that this and this are the same.	

	$2.8 \times 10^4 + 32 \times 10^4$	So I will have $2.8 \times 10^4 + 32 \times 10^4$. Okay. What's your next step?	Add, add (chorus)
	$2.8 \times \underline{10^4} + 32 \times \underline{10^4}$	Both (underlined 10^4) are the same right: Add?	Add (ten to the power of four) (chorus)
		2.8 + 3.2 right?	32 (chorus)
	34.8	Is that the answer? What is missing?	No The 10^4 (chorus)
	34.8×10^4	Good. Okay, but this is not in standard form. Is it in standard form?	No (chorus)
	3.48×10^5	No, so I must convert to standard form. So my final answer is…	
T3 L08 Ep 04		Okay now, (called on a student) you must tell me, which ratio you're going to use now? Whether you're going to use tangent, sine or cosine?	Er… cosine (individual student)
	$\cos 53^o = 12/x$ $x = 12\cos 53^o$	Cosine? Is (student's name) correct?	Yes (chorus)
	$x\cos 53^o = 12$ $x = 12/\cos 53^o$ $= 19.939$ $= 19.94$ units	(Student name) Don't do. Is (student name) correct? What did (student name) say?	Cosine (individual student)
	$x = \cos (12/53)^o$	Cosine? So let's check. Where is the, what's this side?	Hypothenuse (chorus)
		What is this side?	Adjacent (chorus)
		Is it opposite?	Adjacent (chorus)
		It's adjacent. Okay so we have a A, we have a H. yes or no? A and H. Now you look at the .. consult this lady again. Toa Cah Soh okay? So we have A and H. So which one must we use? Cosine right.	Yes (chorus)

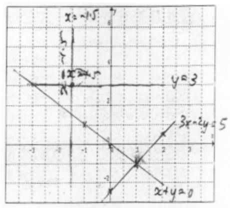
From Table 6, it is evident that during level 0 of teacher-student discourse in both classes the teachers mainly asked questions to check on students' understanding. The responses from the students were short and always in chorus form unless the teacher specifically asked a student to respond. The teachers explained further when students were unable to give expected answers.

Level 1 of teacher-student discourse. At this level of teacher-student discourse the teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. T1 spend about 40 % and T3 about 20% of the teacher-student public talk time during the sequence of ten lessons each engaging students in this type of discourse. Table 7 shows examples of teacher-student discourse at the level.

Table 7. Episodes of level 1 teacher-student discourse in the classes of T1 and T3

Teacher/ Lesson/ Episode	Mathematical Content	Teacher's Questions	Student/s' Responses
T1 L07 Ep 01		[Teacher calls a group to present their answers following group work activity in the class]	
	John's pay →: 100% Cut by 15% and left → 100% - 15% = 85% Increased by15% $= \frac{115}{100} \times 85 = 97.75$	Okay, would you like to present your solution.	John's pay is 100%. Then it is cut by 15% and left is 85%. Then the pay increased by 15%, then now he will get 97.75% of his original pay.
		So does he get more or less? Before the	Get less than 2.25%
		Yes, he got less less by 2.25% right? Good. Now this is one way of solving.	
		[Teacher calls on another group to show their solution]	
	If John's pay is 100*x*, John's pay in the certain year is	We have another way by the other group. Can you show us? They made use of X.	If John's pay is 100x, John's pay in the certain year is

	John's pay after increase $100x > 97.75x$ Ans: Less		John's pay after the 15% increase is. As a result the answer is less.
		Okay very good. Okay I am very impressed.	
T3 L04 Ep 02	$3x - 2y = 5$ $-2y = -3x + 5$ $y = \frac{-3x+5}{-2}$ $y = \frac{3}{2}x - \frac{5}{2}$	[Student name] can you see this line? Is it sloping upward or downward, this line? From left to right?	Downward (individual student)
		Huh? Downward. You're guessing. Why?	Minus two and a half (individual student)
		Minus two and a half? This give you the, this give you … what is this? You still cannot remember. Okay now you look at this equation here. Okay I want you to focus on this equation. Where is your **m**?	Three over two (chorus)
		Shh, I'm asking [student name]. Can you please stand up? Where is your **m** here?	Three over two (individual student)
		Stand up. **M** is three over two. What does **m** represent?	*y*-intercept (individual student)
		m is your *y*-intercept ah? Are you telling … what does **m** represent?	Gradient (individual student)
		Gradient. Where's your **c**? What's the **c** here?	Five over two (individual student)
		Are you sure it's positive five over two? What does c-represent?	*y*-intercept (individual student)
		y-intercept . Sit down. Okay so over here, okay if **m** is positive, so it's upward sloping or downward	Yes (chorus)

		sloping? **m** represent the gradient right? Positive means I told before is upward sloping means what, it’s going this way, left to right it’s going upward this way right?	
		Okay and where’s the y-intercept? Negative? Five over two or two and a half. Okay so basically when you look at your, your uhm grid here, okay you’re going to plot the points to draw a straight line and this straight line is a positive gradient sloping upward intercept the *y* axis at minus two point five. So what does it show, it shows that.. it’ll be something like this, okay two point five is here. And the line will go this way up because it’s a sloping upwards.	Five over two (chorus)

In Table 7, T1 in L07 asked two groups of her students to share their thinking about the same task with the rest of the class. The teacher-student public talk introduced the class to two ways in which the task given to the students as part of their group work activity could be solved. The direct questions asked by the teacher when the groups presented clarified students’ thinking. T3 in L04 clarified students’ knowledge about the gradient intercept form of an equation of a straight line using a worked example and a graphical representation. Both T1 and T3 in the episodes shown in Table 7, focussed on engaging students to clarify their thinking by asking them direct questions at appropriate junctions.

Level 1⁺ of teacher-student discourse. At this level of teacher-student discourse the teacher asks students open-ended questions to elicit student thinking and asks students to comment on one another’s work. Students give short answers to the questions posed to them. T1 spend about 5 % and T3 about 30% of the teacher-student public talk time during the sequence of ten lessons each engaging students in this type of discourse. Table 8 shows examples of teacher-student discourse at the level.

Table 8. Episodes of level 1$^+$ teacher-student discourse in the classes of T1and T3

Teacher/ Lesson/ Episode	Mathematical Content	Teacher's Questions	Student/s' Responses
T1 L01 Ep 15	$\frac{3.6 \times 10^4}{10^3}$ $= 3.6 \times (10^4 \div 10^3)$ $= 3.6 \times 10^{4-3}$ $= 3.6 \times 10$ $= 36$	Alright I asked him to come forward and show the working. What he did was, he notice that it's $10^4 \div 10^3$, so he simplify first. Alright. He takes the power 4 – 3.	
	$\frac{3.6 \times 10^4}{10^3}$ $= \frac{36000}{1000}$ $= 36$	I notice some pupils do it this way. Now both way are acceptable, but which one do you think , er, which one would you prefer?	First one (chorus)
		Why? Why the first one?	More meaningful (chorus)
		Because what happen if I give you $\frac{3.6 \times 10^7}{10^5}$? Then you end up writing a lot of zeros do you agree?	Yes (chorus)
		Okay. So it'll be easier if you simplify, alright, the base first.	
T3 L05 Ep 01	$x^2 + (x + 1)^2 = (x + 2)^2$ $x^2 + x^2 + 1 = x^2 + 4$	So $x^2 + x^2 + 1 = x^2 + 4$ Do you have this like that? Is it correct? Who said yes?	Yes (chorus) Yes (chorus) Huh? Yes. (chorus) Wrong, wrong (chorus)
		Yes right or wrong? [Student name] you shake your head. So why is it wrong? Correct what $(x + 1)^2 = x^2 + 1$ correct or not? Wrong?	Wrong Plus $2x$

		What should it be?	$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$
		You say $(x + 1)^2$ if you expand this thing out what will you get?	$x^2 + 2x + 1$
	$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$	Do you hear what [student name] said? Okay now this is the common mistake that many of you will make. Okay when you expand it out… okay you should have another term.. $2x$. And this one $x^2 + 2$ AB. Remember your 2AB so $2 \times x \times 2$ you have $4x + 4$	

From the examples in Table 8, it is evident that at Level 1^+ of teacher-student discourse the teachers in both classes asked open-ended questions such as "Which one would you prefer? Why? Why the first one?" and "So, Why is it wrong?" to elicit students' thinking on the work presented by the students on the board during classwork. But the students in both schools only managed to give short answers without explaining their answers further and the teachers also did not probe them further.

What Are the Teachers' Orientations of Discourse (Conceptual or Calculational)?

Table 9 shows the duration of teacher-student public talk according to the orientation of discourse per lesson for T1 and T3. It also shows for both the orientations its' percentage with respect to the duration of talk in the sequence of the ten lessons. From the table it is apparent that the orientation of T3's discourse was predominantly calculational. He spent almost 100% of the time for the teacher-student talk in his class in this orientation. However, this was not the case for T1. About two thirds of her class time during teacher-student discourse was in the calculational orientation while the other third was in the conceptual orientation. Table 10 shows examples of episodes that illustrate conceptual orientation and Table 11 shows examples of episodes that illustrate calculational orientation.

Table 9. Duration of teacher-student public talk by orientation of discourse for T1 and T3

Lesson	% of time per lesson			
	Orientation of teacher-student talk			
	Conceptual		Calculational	
	T1	T3	T1	T3
L01	13.61	-	22.28	22.02
L02	14.34	2.06	31.70	1.78
L03	-	-	22.92	-
L04	-	-	13.73	4.84
L05	16.26	-	9.02	12.43
L06	5.00	-	-	12.60
L07	4.55	-	18.61	2.19
L08	0.46	-	7.75	14.24
L09	2.83	-	14.04	8.48
L10	8.97	-	7.28	2.45
Total	6.73	0.18	14.85	8.36
Orientation	% of time for all 10 lessons			
	T1		T3	
Conceptual	31.19		2.11	
Calculational	68.81		97.89	

Table 10. Episodes of teacher-student discourse with conceptual orientation

Teacher/ Lesson / Episode	Teacher's Questions	Student/s' Responses
T1 L05 Ep 07	Alright. Look at these two pictures. I'm sure you know what's the name of this figure right? What is it called?	Square *(individual student)*
	Good. And what about the one on the right?	Rectangle *(individual student)*
	A rectangle. Are they similar?	No *(chorus)*
	Why not? They have equal corresponding angles.	Corresponding sides are not *(individual student)*
	Are they similar?	No *(individual student)*
	Why not?	They don't have the same … They don't have the same ratio for the corresponding sides. *(individual student)*
	Yes. The ratio – the corresponding ratio of the corresponding sides are not equal okay?	
T3 L02 Ep 02	Why I don't do that over here in Pythagoras Theorem. I didn't bother to put plus and minus	Not possible *(individual student)*

Not possible? Why not? [Student name] You know why? What is your c? What does c represent? The small letter c what does this represent in the question?	Line *(individual student)*
The line. Can the line be a negative or not?	No *(chorus)*
Can length be a negative?	No *(chorus)*
No right? So why you bother to put plus and minus? You know that it can it must be C must be always positive value. Are you following what I'm trying to tell you?	Yes *(chorus)*

From Table 10, it is apparent that both teachers, T1 and T3 used questions such as "Why are they not similar?" and "Why don't I do it here?" to illicit conceptual knowledge of their students and also place emphasis on the process of student learning.

Table 11. Episodes of teacher-student discourse with calculational orientation

Teacher / Lesson / Episode	Teacher's Questions	Student/s' Responses
T1 L01 Ep 04	Is this correct? Can you tell me what is the answer for this? Is this correct by the way?	No *(individual student)*
	Yes or No?	No, they can't be. *(individual student)*
	No. Why?	Plus *(chorus)*
	Good, it is plus. What should the correct answer be?	Eleven thousand *(individual student)* Seven thousand *(individual student)*
	What is the correct answer? Yes. Sorry?	Eleven thousand *(chorus)*
	Eleven thousand. Okay. Eleven thousand. Do you know how we get eleven thousand?	Yes *(chorus)*
	Good. Alright. Eleven thousand. So be very careful ah. You can add the power if its multiplication and the base are the same.	
T3 L08 Ep 07	[Student name] What do you think? Which ratio would you use to find ***X***?	Cosine (individual student)
	Use cosine? Why do you use cosine	Twelve (individual student)

because X is opposite and then what? You're using fifteen or twelve? Twelve? Okay. Now in this question if many information are given, you can use cosine like what [student name] has suggest. Okay or you are going to use fifteen you can use sine.

From Table 11, it is apparent that both teachers used direct questions to get numerical answers from their students when they were thinking aloud the steps of tasks they engaged their students to solve during demonstration. Teachers were contented when students provided the correct numerical answers and did not quiz them any further.

Do Students Initiate any Public Talk with Their Teachers or Peers? If So, What Was the Purpose of the Talk?

Students did initiate public talk with their teachers. Table 12 shows the number of episodes and the duration of time per lesson during which students' initiated student-teacher discourse as part of the public talk during lessons.

Table 12. Student initiated content-learning discourse

Lesson	Episodes			
	T1		T3	
	Number	Duration in minutes (%)	Number	Duration in minutes (%)
L01	2	4.32 (7.91)	0	-
L02	0	-	1	1.25 (3.58)
L03	2	2.40 (4.39)	0	-
L04	3	6.58 (10.97)	0	-
L05	1	0.65 (1.23)	0	-
L06	0	-	0	-
L07	3	1.37 (2.52)	2	0.62 (2.15)
L08	1	2.05 (3.81)	2	2.00 (2.94)
L09	2	2.83 (6.02)	0	-
L10	0	-	0	-
Total	14	20.20 (3.79)	5	3.87 (0.94)

From Table 12, it is apparent that in both classes student initiated public talk occurred infrequently. In the class of T1, over a sequence of ten lessons, students initiated talk on 14 occasions for a total duration of 20.20 minutes, i.e. 3.79 % of the time. In the class of T3, over a sequence of ten lessons again, students only initiated talk on 5 occasions lasting a total duration of 3.87 minutes, i.e. 0.94% of the time. On all the occasions, students initiated talk with their teachers only. Table 13 shows representative episodes of the different purposes for which students initiated public talk during the ten lesson sequences of T1 and T3.

Table 13. Episodes of student initiated content-learning discourse in the classes of T1 and T3

Teacher / Lesson / Episode	Students' Questions	Teacher's Responses
T1 L03 Ep 07	Can draw model? (individual student) Secondary school cannot use model (individual student)	Can you can do. You can use any method.
		(talks to the whole class) Okay somebody asked me this question "Can we draw model?" Yes, by all means go and draw model. And then some of you say but I thought in secondary school we cannot draw model. No, if the method works, why not? Go ahead … Alright some of you may want to use table
	Bar model (individual student)	Ah you can draw bar model can. Algebra also can yes. .Now not necessary we have to use algebra to solve all the time. Alright, for certain types of question model may be easier.
T1 L04 Ep 01	The question is illogical (individual student)	What illogical? Why do you say it's illogical?
	Because they say that the total cost of producing 600 copies of the magazine so each copy is … so how can … but the answer given is ….. 600 copies (individual student)	One magazine got 32 pages, one copy yeah? So you must have 600 copies of magazines
	Yeah but the answer given is 600 plus 32 pages (individual student)	Okay. You read the typing is one page $3 right? So 32 page will be $96 correct?
	Yeah. That they say it's typing.	Okay wait. I think I see your point. Can I borrow your

		calculator? So you $96?
	Plus 2 after that	Yeah plus 6 times 18.5. Yeah then you get 207 correct.
	Correct	Okay? One copy is $3. 32 sorry one page is $3 so 32 pages will be? $96 correct?
	Yes	Then for every 100 copies is $18.50 so 600 copies is18.50 times 6.
T3 L02 Ep 03	I don't know why the answer for this one cannot be negative.	This one? Why is it cannot be negative? That's what I'm trying to explain to you why it cannot be negative.
	Don't understand mah	That's what I'm trying to explain to you all just now, I didn't bother to put plus minus, **C** cannot be negative because I just asked [student name] what does **C** represent here [student name] ? What does **C** represent in the question?
	Side	Yeah the side. It's the length of the longest side in the right angle triangle right or not? Can the length be a negative value?
	No	Can or not?
	Student shake his/her head	Cannot right? A length of a side of a polygon it cannot be a negative value so I don't bother to put plus minus. That's the reason why.
T3 L07 Ep 01	What happen if the answer is one? What happen if exactly one? The ratio is one? (individual student)	The ratio is negative?
	No the ratio is one.	The ratio is one? Yeah lah the ratio can be one what, there. I can

	go up to one. When it's one to up to one what does it mean? It means that the opposite and the adjacent are the same length. Do you agree?
Oh okay.	Yeah it's the same length what so something the same length over equal to one isn't it? Alright it can be equal to one. Alright possible.

From Table 13, it is apparent that students in both classes initiated public talk for various reasons. In episode T1-L03-Ep 07, the student asked the teacher if he could use the method of drawing models to find the solution of an algebraic problem. In

Episode T1-L04-Ep 01, the student raised a concern about a likely error in a textbook question that the teacher had asked the class to work on. In both the episodes T3-L02-Ep 03 and T3-L07-Ep 01, students sought further clarifications about the concepts the teacher had explored during the lessons.

DISCUSSION

The data and findings presented in this chapter will be discussed in this section according to the research questions investigated.

During Content-Learning Classroom Discourse How Often Do Students Get an Opportunity to Engage in Public Talk?

It was found that in the two grade 8 mathematics classes of the competent teachers of the LPS in Singapore there was an apparent lack of teacher-student public talk. Over the ten lesson sequence in the class of T1 from SG 1, students were engaged in discourse by their teacher for 21.58% of the time. Similarly, T3 in SG 3 engaged his students for only 8.54% of the time. As the teacher was responsible for controlling all the talk that occurred while the class was officially in session, it is apparent from the above findings that the lessons of both T1 and T3 were dominated by teacher talk. Both T1 and T3 during teacher talk expounded mathematical concepts and problem-solving skills mainly through the use of examples (Seah, Kaur, & Low, 2006). Students were generally not engaged in co-constructing knowledge with their teachers. Both teachers spend considerable amounts of time explaining concepts and illustrating them (Kaur, 2009).

During Content-Learning Discourse What Are the Characteristics of Teacher-Student Public Talk?

In both classes of T1 and T3, the level of content-learning discourse during teacher-student public talk did not reach levels 2 and 3 as in Stein's adaptation of the Hufferd-Ackles, Fuson, and Sherin (2004) framework. The discourse was only at levels 0, 1 and 1^+. Level 1^+ was created by the researchers as they found several episodes of teacher-student public talk that was beyond level 1 and not at level 2. This shows that all the teacher-student content-learning discourse in both the classes of T1 and T3 merely focussed on teachers asking the what, which and how questions to evaluate student understanding of knowledge they were expounding through worked mathematical examples thereby clarifying the conceptual knowledge they were disseminating. It may be said that the talk centred around showing and telling or explaining, typifying traditional teaching (Pimm, 1987).

Examining more closely the percentage of teacher-student public talk time, it was found that both T1 and T3 spend about half (50%) of the time at level 0 of the discourse. At this level, the teacher mainly asked the students closed questions and students gave short answers. While T1 spend about 40% on Level 1 and less than 10% on Level 1+, T3 spend about 20% on Level 1 and about 30% on Level 1+. It is apparent from the episodes presented in the chapter that T3 addressed some common misconceptions that his students were developing during the course of the lesson. He also reframed from giving them the answers, but rather engaged them in thinking through it. In both classes, the teacher-student discourse at level 1^+ demonstrated that teachers were asking open-ended questions but lacked probing for reasons or justifications of answers students provided to their questions. Hence there was a lack reviewing and restructuring to develop students' own understanding of mathematics (Anghileri, 2006). It may be speculated that the actions on the part of the teachers may be due to the objectives of their questions, often dip-stick approaches for assessing student understanding or perhaps lack of time or expertise to engage students in dialogic talk.

During Content-Learning Discourse What Are the Teachers' Orientations of Discourse (Conceptual or Calculational)?

It is apparent from the data presented, that in both the classes of T1 and T3 there were both conceptual orientation and calculational orientation during the teacher-student public talk as part of the content learning discourse. However, in the class of T1 almost twice as much time was spend on calculational orientation than on conceptual orientation while in the class of T3 98% of the time was devoted to calculational orientation and a mere 2% to conceptual orientation.

Given that the students in the class of T1 were of higher ability than those in the class of T3, it appears that T3 placed a lot more emphasis on "doing it right" via the calculational orientation of teacher-student content-learning discourse in his class. It may also be speculated that in both the classes the marked emphasis on calculational orientation may be partly derived from assessment requirements as often teachers tend to teach to the test. Mathematics tests generally at national levels in Singapore test procedural/calculational knowledge. There is no doubt that sound conceptual knowledge can help one to weather all sorts of test questions but often give a finite duration of time, teachers tend to take a safe trajectory by ensuring that procedures and calculation techniques are honed well in their students.

During Content-Learning Discourse Do Students Initiate Any Public Talk with Their Teachers or Peers? If So, What Was the Purpose of the Talk?

In both the classes of T1 and T3 students initiated public talk with their teachers and peers rather infrequently. In the class of T1, over a sequence of ten lessons, students initiated talk on 14 occasions for a total duration of 20.20 minutes, i.e. 3.79 % of the time. In the class of T3, over a sequence of ten lessons again, students only initiated talk on 5 occasions lasting a total duration of 3.87 minutes, i.e., 0.94% of the time. On all the occasions, students initiated talk with their teachers only. The purpose of the talk was to clarify doubts about any preferred methods of solution, seek further explanations on concepts they had difficulty with and to draw the attention of the teacher to some irregularities in textbook questions. It is apparent that the questions students asked had to do with the meaning and method aspects of learning (Wong & Quek, 2010). This finding shows that students were concerned with getting the ‘content right’ and the ‘how to do it’. In addition, the very limited initiation of talk by the students perhaps sheds some light on the culture of learning in the classes of T1 and T3 that may be worth exploring further in a future study.

CONCLUSION

The findings in this chapter have shed light on the nature of teacher-student content learning discourse in two grade eight classrooms in Singapore. The data presented in this chapter cannot be used for generalisation of classrooms in Singapore. Nevertheless, we can say that in the classes of two competent teachers who participated in the LPS in Singapore the content-learning discourse was dominated by teacher talk and student listening. Student-teacher interaction for the most part, were related to the teacher’s assessment of students’ progress in understanding the demonstrated problem solution methods and this attributed to the calculational orientations of most episodes of the discourse. The apparent lack of student-initiated public talk was a consequence of the instructional organisation of the lessons in repeated rounds of teacher demonstration, seatwork, and whole class review of student work and common misconceptions. Lastly both teachers and students were focussed on getting the meaning and method correct for the content knowledge during the lessons.

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CHAPTER SIX

Martina's Voice

INTRODUCTION

In Swedish classrooms, the use of the instructional mode called student independent work has been encouraged since the 1990s. In this mode, students are expected to plan and work on their own on different tasks independent of other students and to a large extent independent of the teacher. The research team of the Learner's Perspective Study (LPS) (Clarke, 2006) has documented this form of instruction in Swedish classrooms. Video records confirm that typically the teacher provided short introductions at the beginning of the lessons and the students then worked on textbook tasks at their own pace. During independent work time, teachers moved around to interact with individual students.

This chapter explores the patterns of interactions, mathematical practices, and the quality of reasoning by focusing on the participation of one student throughout a sequence of lessons, in the form of a case study of one classroom. It is envisaged that giving voice to a student will further our understanding on what it means, in terms of possibilities for learning, to be student in such a classroom and that how a student participates and takes responsibility in the learning of mathematics. The focus of analysis involves a collection of phenomena or patterns drawn out from the participation of one student. The interpretive framework for analysing group and individual student's participation by Cobb, Stephan, McClain, and Gravemeijer (2001) aided the data analysis. Also, there is an attempt to argue the findings based upon the known theories of Vygostky's (1962) zone of proximal development and that of Brousseau's (1997) adidactical situations.

Based on video records of a sequence of ten lessons of one Swedish classroom, the voice of Martina (coded name) was selected as the case. In eight of the ten lessons, Martina's voice was evident in the form of mathematical utterances, either intended for her teacher, classmates or the whole class, or just merely thinking aloud. Also, her accounts of participation were captured in the post lesson interview as well as on her written outputs such as test results and answer sheets to the mathematical tasks taken from the textbook. From our analysis of these data sets our study aimed to establish patterns of Martina's mathematical talk as she participated in this type of instruction where she was progressing at her own rate and also consider how these patterns relate to selected learning theories. Two main research questions were posed:

(i) What are the mathematical practices that emerged from analysing the participation of one student in a mathematics classroom where the instructional

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 89–113.

mode is dominated by student independent work and the mathematical tasks were largely taken from the textbook?
(ii) What can be deduced about the quality of reasoning and its development that emerged from the participation of this particular student in such a mathematics classroom?

The first question considers Martina's participation from the social perspective, while the second question focuses more on the psychological perspective. We argue that consideration of both of these aspects is needed to enhance our understanding of the student's mathematical practices and the quality of her reasoning and its development. This is different from most studies on mathematical practices where collective learning in the classroom is typically the focus. In this paper, by studying just one student over several lessons we aim to provide insight into how a student could participate in such a class as a mathematics learner. Based on the voice of a single student, the proposed learning model adds to previous LPS studies on classroom interactions such as those conducted by Emanuelsson and Salhström (2008) and Gallos (2006). Moreover, our exploration of the development of mathematical reasoning within the topic of algebra in the middle schools contributes to an important area of research in mathematics education (Cobb, Stephan, McClain, & Gravemeijer, 2001).

SOME RELATED STUDIES

Studies on the mathematics learner's perspective are valuable for their contribution to our understanding on how students participate and take the responsibility to learn mathematics. Several studies based on LPS classrooms have provided a focus on the learners. In reviewing those that consider the role of the learner, Gallos (2006) explored how students can support each other to learn mathematics if given the opportunity to talk privately in a mathematics class. In another LPS study, Bergem and Klette (2010) found that students who were left on their own experienced difficulties relating their mathematics knowledge to the real-life tasks they were working on. In contrast, a study by Williams (2010) reported that those students doing self-created tasks developed deeper understanding than those who did tasks as set by the teacher. In comparing these two studies it is apparent that the expectations from students on the given tasks were different, so it would be difficult to directly compare findings between these two studies. What is common to these three studies is that the teacher directed the progress and format of the lessons, that is, the teacher provided similar tasks for every student. In contrast, in the Swedish classroom of the study reported in this chapter it is more likely that individual students were doing different tasks in any one lesson; hence the progress is more of the responsibility of the learners.

The instructional mode based on independent student work has been employed in Sweden for almost two decades. The study of Emanuelsson and Sahlström (2008) argued that Swedish classrooms allowed for more elaborated ways of students' participation that influenced the whole class discussions compared to their American counterparts. This could partly be due to the fact that the student independent mode of instruction delegates more responsibility for learning to the students. However, an analysis of the Swedish 2003 TIMSS results led Hansson (2010) to question the extent to which the teacher is guiding and supporting students in their progress towards more complex knowledge within the Swedish classroom environment. She claimed that an increase in the use of student independent work as a mode of instruction would likely result in a decrease in student performance. Even though the performance of Swedish learners in 2003 and 2007 TIMSS was above the international mean, a number of countries performed much better than Sweden (Skolverket, 2008). And like most TIMSS test takers, the items requiring application of reasoning skills were found to be difficult too by students from Sweden. Following one student's participation in a series of mathematics lessons could support these previous studies and provide a clearer picture of the student participation patterns, focusing on her mathematical practices and reasoning skills.

RESEARCH STRUCTURE

Focused on one student, this study utilises a non-emergent case study. The analysis of mathematical practices and reasoning was partly based on the socio-mathematical norms and on the social and psychological perspectives advanced by Cobb et al. (2001). The analysis of the reasoning patterns draw on constructs of categorisation provided by Brousseau (1997) during adidactical situations, on the strands of mathematical proficiency (National Research Council, 2001), and the categorisation of cognitive domain utilised in TIMSS (Mullis et al., 2008). The analysis of the development of reasoning was organised around the four goals described in the official teacher's guide (Carlsson, Hake, & Öberg, 2002). These four goals are: (i) draw and name points on the coordinate system; (ii) work with proportional relationships, for example comparing prices; (iii) work with relationships comprising a fixed and variable part; and (iv) interpret different types of linear relationships.

From the three classrooms that participated in the LPS in Sweden we selected one classroom. The purposive sampling of the case student was directed to a student whose independent work was visibly used as an instructional mode. The class was taught by a 32 year-old male teacher and comprised 26 students. This class was videotaped for 15 consecutive lessons, but because the first five lessons were designed to be familiarisation periods, only data from the sixth to the fifteenth lessons were used for analysis. In addition, data from student and teacher interviews conducted post lesson and copies of the textbook and the student's notes and test result were included in the data set. The choice of Martina as the case student was decided upon based on the fact that she had the most interactions on the video data set. The name of Martina, as well as of other students, are coded names assigned by the Swedish research team. Also, minor changes to the transcripts, such as deleting time and some pauses, were made, for these were assumed to be of not much relevance to the present study.

Based on the research questions, the variables of interest were the mathematical practices, quality in reasoning and its development. The Studiocode software aided the organisation and initial coding of the qualitative data. Lesson transcripts were the main data source, sometimes used in conjunction with video review.

By the nature of this research and its limitations, we do not claim that the findings are generalisable to all mathematics classrooms in Sweden, or most Swedish students' mathematical practices or reasoning skills. However, this study provides an example of how to investigate student's mathematical reasoning and practices in such a classroom where students progress at their own rate supported by textbook problems. In the following section we provide exemplars of the aspects under investigation with discussions as to how these were analysed and present associated findings.

SOCIOMATHEMATICAL NORMS IN CLASS

Within the classroom, the dominant use of the student independent work mode meant that students appeared to take significant responsibility for their own learning process. They were required to complete textbook tasks, drawing on the support of the teacher as and when needed. In this way, it was common to see students working on different tasks at a particular time hence working individually was more prevalent although there were also occasions when students worked in groups. Most often group work was prompted by directives from the textbook so as to engage in a group activity or game, or arose from a desire to discuss or compare their solutions with their seatmates. Self-responsibility also meant that some students skipped some tasks, sometimes of their own volition and sometimes in consultation with the teacher. Also, it was noticed that students often read the tasks aloud, tried working on the tasks, and then compared the answers that were provided at the end of the book or compared with their seatmates or confirmed with the teacher about the accuracy of the answers. Within such a learning environment, we move now to discuss the sociomathematical norms that were evident.

On Mathematical Solutions

In looking at what counts as acceptable mathematical solutions we analysed how the teacher and students valued presented mathematical solutions – as different, as insightful or elegant solutions, or as efficient.

Students' mathematical solutions vary. It is evident that students could provide varied ways of arriving at the same answer. An example was in Lesson 6 where students' conversation was about finding the price for certain weights of sweets, based on the table of values. The corresponding graph, taken from the textbook (Carlsson, Hake, & Öberg, 2002) is shown in Figure 1.

Weight (hg)	Price (kronor)	Coordinates (x, y)
0	0	(0, 0)
1	6	(1, 6)
2	12	(2, 12)
4	24	(4, 24)
10	60	(10, 60)

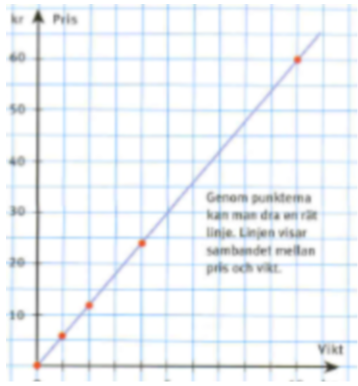

Figure 1. The table of values and corresponding graph (Textbook, p. 175)

Viktoria	It says on this one, one hectogram, is six kronor.
Johan	Five.
Viktoria	No, eighteen kronor! Eighteen.
Johan	Why eighteen?
Viktoria	Because one hectogram six, two hectograms twelve, and twelve plus six. One plus two is three. Twelve plus six is eighteen.
Johan	Three, three hectograms.
Viktoria	Yes, right. Let's do it this way then. Four plus ... (unintelligible) thirty six ...forty two kronor.
Johan	Then you just take eight times six times eight.
Viktoria	(Laughter)
Johan	Well, it must be, look.
Viktoria	Yes, that's what I'm saying.
Johan	Six times eight is forty eight, not forty two.
Viktoria	Why six times eight?
Johan	It's this that we are now (points at the task in the textbook), seven up here. And it goes all the way up.

It is apparent in the transcript that while Viktoria was using the table of values Johan was using the graph. In the post lesson interview Johan confirmed it by the comment – "and it goes all the way up" – referring to the graph. Despite the fact the book directed students to use the table of values to solve this task, the alternative method appeared quite acceptable to Johan. However, in discussions about specific mathematical operations to use, Viktoria's repeated addition method to compute the cost of 3 hectograms of sweets at 6 kronor (Swedish currency) per hectogram was challenged by Johan who argued for a more efficient solution by using multiplication or reading off from the graph.

Students were encouraged to provide different solutions. One explanation as to why students at times deviated from the required text book method could be that the teacher often prompted students to use another method in order to check their answers, to think of other solutions, or to think of a more sophisticated solution. One example was in Lesson 11 centred on a teacher illustration of two proportional relationships expressed in graphical form. Students were asked to suggest possible equations for the two line graphs, with one graph obviously steeper than the other. One equation suggested for the steeper graph was y = 90x. When the teacher asked for some numbers that could replace x, one student suggested 90. In calculating 90 x 90 a range of answers including 180, 1080, 1,800 and 8,100 were offered. The teacher then asked for elaboration on how they arrived at the answer recording suggested methods on the board (see Figure 2).

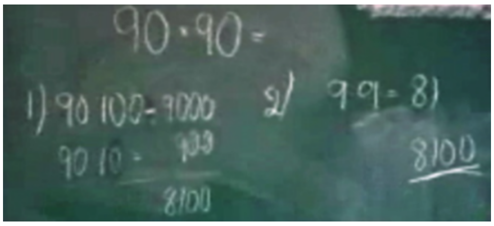

Figure 2. The boardwork for different solutions for finding the product of 90 and 90

The first solution method involved the distributive property, that is, 90(100-10) gives 90(100) – 90(10) = 8,100. The second method applied the associative property, that is, (9x10) (9x10) = (9x9) (10x10) = 81(100). However, in both cases, the discussions did not make any reference to specific mathematical properties. An attempt to provide a third method was as follows:

Teacher	Is there anyone who had a different way, to work this out?
Faro	That was the easiest.
Martina	No.
Teacher	Then I have to ask you, how come eighty percent of the class answered one thousand eight hundred, eh, and a lot of other things.
Johan	[Teacher's name], I know one too. Can I say it?
Teacher	Yes.
Johan	A hundred times a hundred minus one thousand and nine hundred.
Teacher	(Starts to writes on the board)
Viktoria	But are you joking, or?
Johan	It's the same thing!
Teacher	Yes, but yeah, what fun! (Then rubs out what he just wrote on the board.) Yeah, but now we'll go back.

From here, it was obvious that Johan had thought hard to find another solution, although viewed as a joke to one of the students. Although the teacher accepted it without explanation, considering it as fun, he quickly shifted back to the main topic. If elaborated, the mathematical expression 100(100) – 1,900 may well have proved the basis for generating a rich mathematical discussion, including discussions concerning efficiency of solutions. On other occasions, however, the teacher was observed to support students to think about efficient methods in addition to multiple methods as seen in the following interaction:

```
Martina     Will they be wrong if solved in other ways?
Teacher     Er, they won't be wrong, if you can.
Martina     Is it better to use an equation?
Teacher     I think it's a little plus.
```

These episodes affirm that in this class there is both acceptance and encouragement for students to use different solution methods for the same problem, some of which could be more efficient or elegant than the others, or just merely for the sake of having a different solution.

On Acceptable Mathematical Explanations

In this class, the student mathematical explanations – whether they be short or elaborate; conceptual or computational; disproving or substantiating a mathematical statement – enabled students' mathematical solutions to become visible and hence could support the discussions.

Short answers were typically generated from closed-type questions such as asking the coordinates of the points, answered by yes or no, or questions on basic operations such as in the following episode from Lesson 13. Here two students needing to find the answer to 28/40, were questioned on factors common to 28 and 40. Below was part of their conversations.

```
Teacher     What's seven times four?
Emma        Twenty-eight.
Teacher     What's ten times four?
Emma        Forty.
Teacher     Both are in fours table, what can we do then?
Emma        Yes, yes, yes, yes, yes ... then it makes seven-tenths.
Teacher     And how much is that then?
Emma        Zero point seven.
```

It is evident here that the teacher asked mainly closed-type questions and the students provided short and direct answers.

While students mainly gave short mathematical answers during class discussions, there were instances when they elaborated on their answers. An example to this was in Lesson 14 where Anton and Filip were trying to convey to the teacher the accuracy of their answers to this item:

What information can you draw out from the diagrams A, B and C? Figure out by yourself first then compare your ideas with your classmate.

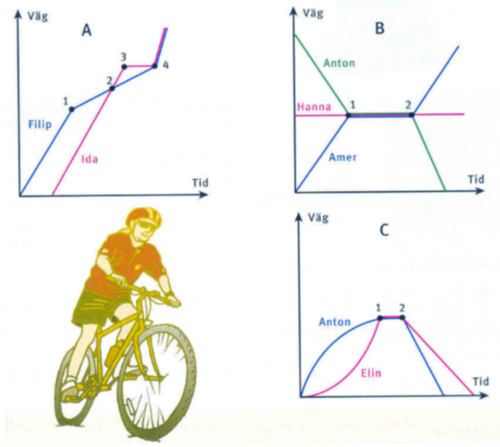

Figure 3. The item on interpreting distance-time graphs (Textbook, p. 182)

When the teacher monitored their work, Filip started to explain to the teacher his 'stories' to items A and B with minimal input from Anton. The teacher responded with short phrases such as "yes", "yeah, I agree" and "exactly yes". There was no evidence of the teacher disagreeing with the discussion.

Teacher	Are you following this (to Anton)?
Anton	Yes, yes.
Teacher	Yes, ok, good.
Filip	You can do the next one.
Anton	No, I don't want to do that one.
Filip	Yes, but you were right.
Anton	Yes, but yeah. Anton cycles really fast and Elin a bit more slowly, but then Elin speeds up and Anton slows down.
Teacher	Yes, right.
Anton	So then they get to a place where they like to stop and talk or something like that. Then Anton cycles off really fast and Elin goes a bit more, slower there.
Teacher	Yes, right, good.

It is apparent here that when students were provided with a mathematical task that could be interesting for them to pursue and was of a form that required more elaborate explanations, they could do so. In this example, encouragement from both the classmate and teacher also appeared to support the sharing of mathematical explanations.

It was also noted that during independent work, some students found a mismatch between their answers and the answer provided in the book. This is where the teacher support became more essential in guiding a particular student to be clarified about the mismatch. This was what happened when Beata was working on the task identified as challenging for it has an asterisk mark. The task is about locating the third vertex of a triangle lying on the y-axis. The coordinates of the two base vertices are given and the area of the triangle is the same as the area of one of the rectangles, that is 30 square units. Beata's working on this item and part of the explanation to the teacher follows:

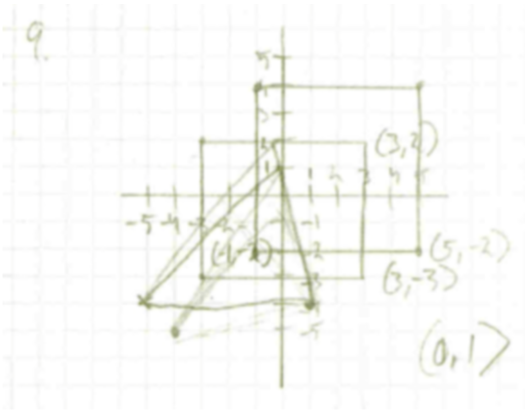

Figure 4. The work of Beata

Beata	The third one should be on the y-axis, in other words that one there.
Teacher	Hmm.
Beata	And then the area should be just as big as that one there, so it's, six squares times. No. One, two, three, four, five, six. Yes, six squares times five squares. In other words the area is thirty squares. But how do you do, do you do it?
Teacher	And that, there's supposed to be a triangle.
Beata	Yes.
Teacher	Um, it's wrong here, um.
Beata	And that point should lie on, that one there.
Teacher	Negative five.
Beata	Negative four.
Teacher	Negative five, negative four is there.
Beata	Yes, yes!
Teacher	Because then, it will be, then you have a base, that's a lot easier.
Beata	So thanks, yes. Exactly!
Teacher	Isn't it?
Beata	Yes, it was much better.
Teacher	Can you fixed it then?
Beata	Yes, I think so.

Here, Beata's conceptual understanding about areas of triangles and rectangles was evident. Instead of counting the total number of squares, the area was computed using a formula. While she was right in computing the area of the rectangle, it is apparent that she miscalculated the area of the triangle (missed to divide it by two). Hence in her figure the third vertex is at (0,1). While Beata said she could fix the error, there was no evidence of how she finally fixed the mismatch. Nevertheless, it was apparent in this class that while students mainly checked the accuracy of their answers against the answers provided on the last few pages of the book they did still seek the support of their teacher especially when a mismatch occurred.

It was evident from these discussions that despite the dominance of short mathematical explanations, the students were able to provide much more elaborate explanations or arguments to support their mathematical statements. Moreover, these mathematical explanations helped students to solve problems in different ways.

In summary, sociomathematical norms associated with acceptance of different mathematical solutions and mathematical explanations were evident from students' interactions and at times supported by their written outputs. It was also apparent that there was encouragement for students to provide different mathematical solutions that involved different representations and different calculational strategies from which efficient or elegant or just an insightful way of solving a problem could be discerned. In addition, it was also apparent that supporting students to come up with different strategies for mathematical solutions made demands on students to communicate their mathematical ideas with mathematical explanations. Collectively these experiences appear to support both students' problem solving skills as well as their communicating skills. To understand these mathematical practices further we now look in-depth at our case study with Martina.

MARTINA'S VOICE

In this section we consider Martina's interactions with her teacher and peers, focusing on her mathematical practices and her use of mathematical tools applied within her classroom and the quality of her mathematical reasoning.

The Patterns of Interactions

Data across the lesson sequences provided evidence that Martina regularly interacted with the teacher, her classmates, and on occasions with herself by thinking aloud.

Interactions with the teacher could be classified as either confirmation of the accuracy of her answers or clarification of the tasks. For example, in Lesson 6, Martina ask "[Teacher's name] can you just check to see if I got this right? I've done some sort of dots here now … that one kilo fifteen kronor, two kilos thirty, five kilos seventy five". It is evident here that despite Martina being able to do most of the tasks independently, she still sees the need of support from the teacher.

In addition to seeking support from the teacher Martina was frequently observed to be discussing with her seatmate Beata and also with a wider range of classmates. As she mentioned in the interview, interactions involved updates on progress or help seeking conversations as seen in the following episode from Lesson 7:

Dino	Martina, what exercise are you on?
Martina	I'm on thirty-five.
Dino	[Expletive], I'm on eight!
Martina	Yes, but I'm doing you know, I did this here before.
Madeleine	How did you work out eighteen?
Martina	(...) Are you on eighteen? Five mil, five mil, then you check on five, the five here because it's zero, ten, and then that line there, that you know sort of about four.
Madeleine	Ahaaa!

As well as illustrating that Martina was able to help Madeleine on task 18 from the book, this episode also serves to confirm that the students were progressing at different rates. In this case, Martina was beginning the tasks intended for the last goal of the four goals (see page 3 for the list of goals). She actually had done with all the basic tasks, including the unit test during Lesson 10, just the fourth day after they had started on the chapter. She was advised by the teacher to work on the blue sections. It could not be established though as to why the teacher suggested to Martina to work on the blue pages instead of the red pages. In the foreword of the textbook it is stated that the blue pages are only for those students that required more training. It was apparent that Martina did not require more training on basic concepts as compared to her classmates for her answers on the red course pages (documented in Lesson 12) were mostly correct. She worked on extra tasks on linear functions from a book provided by the teacher in Lesson 13 and that she could even easily provide help to her classmates. What became obvious here is that, during independent work there could be students that could show engagement in doing more mathematical tasks than what are required.

It is apparent that Martina was comfortable interacting with either the teacher or her classmates and that she was able to act as an alternative to the teacher in explaining some mathematics to her classmates. These interactions contributed to the establishment of social norms that sanctioned working with classmates to discuss solutions to mathematical problems.

The Emerging Mathematical Practices

The mathematical practices that are discussed here are those focused on Martina's emergent ways of mathematical reasoning and forms of communicating her mathematical ideas that have been captured over sequence of lessons, as they relate to the social perspective.

An examination of Martina's interactions provides evidence of her application of mathematical reasoning to support her problem solving efforts. One example of this was in Lesson 6 where Martina was trying to find the time it would take to drive 5 mil (50 km), given the distance-time relationship as shown:

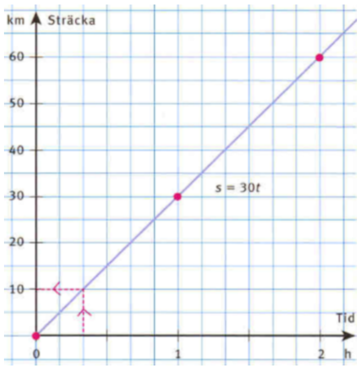

Figure 5. The graph and equation accompanying the task (Textbook, p. 178)

She appeared to clarify her reasoning with the teacher in this conversation:

Martina	Is it like the distance equals, 30, times ... Speed,
Teacher	Thirty, yes.
Martina	Times the time?
Teacher	Yeah, exactly. 'Cause if you take the time then, if you drive in an hour, then you will have driven, 30 kilometres.
Malin	Times t, yeah, exactly yes. Yes, ahhhhh!
Teacher	Do you see?
Martina	Wait.
Teacher	Hmm, there's another one.
Martina	Thirty kilometres, yeah.
Teacher	We've done it in Physics before, it's not so stupid really.
Martina	Yes, but it's thirty kilometres per hour, that's for sure. Yeah, yeah, yeah, yeah, I got it then.

:
.

Teacher	If you had got the distance and the time, and you want to find out the speed, then you cover that over ...
Martina	Then I divide that, ah right, ok.

Despite access to the equation and graph, it appears that Martina still wanted to confirm the accuracy of her answer through the authority of the teacher. It was not clear though if the use of equation that would require division (probably, t = s/30) was a preference over reading it off from the graph, or if it was in fact used to check the accuracy of her answer derived in the first instance from the graph.

In looking at ways of communicating ideas, the ways in which Martina provided mathematical arguments were considered. One example is in Lesson 6 where Martina discussed a proportionality problem with the teacher. In this task she needed to draw a diagram to show the relationship between weight and price of bananas that cost 15 kronor per kilogram. Part of the conversation follows:

Martina	Can I do a point here if I know that the weight goes here, so you can just draw a line from here to there?
Teacher	Exactly. If you had, if you've got three points then you can draw this one as long as you like.
Martina	Yes, but you only really need one point, I mean.
Teacher	Well, yes, ok, if you know that the price is going up proportionally.
Martina	If you know, I do know, I ...
Teacher	Yes, you can do it like that, the advantage is that you can go in, yeah, if I've got one hundred kronor how much can I spend.
Martina	Okay, yeah, okay.

Here, we see that Martina challenged the statement of the teacher that three points are needed to draw the line. She argued that one point would be sufficient if you knew the proportionally constant. As such, it appears that Martina used a more advanced idea of drawing a line, that is, the point-slope idea than plotting three points as suggested by the teacher.

In another instance, involving a more general discussion, Martina again was seen to be comfortable to challenge the status quo. This occurred in Lesson 10 where Martina responded to the teacher and classmates' assessment that fractions are boring with a quip: "It's were not really too bad" and added that "we have to do it, you just need to teach us".

Martina also appeared to be willing to challenge her classmates' mathematical arguments, involving clarification, as seen in the following episode from Lesson 12. The task at hand involved Beata wanting to draw a line showing the relationship between length of time on talking over the telephone and cost according to the given rate of 150 kronor per month plus 3 kronor per minute. A part of their arguments was as follows:

Beata	Hundred fifty plus three, times, there the minutes. And then as how much these becomes, so we took hundred twenty times three, three hundred sixty plus a hundred fifty.
Martina	No! But, these, you shall, it won't work.
Beata	Well, they would, I am sure it would work.
Martina	You shall.
Beata	I have written these here instead and it worked.
Martina	I do not understand what you have done.
Beata	Instead, and make those.
Martina	Instead, and make these.
Beata	I, I have done those, exactly. I have done those, and done those fixed. I have not written those without, I have written those as three hundred sixty instead for three times hundred twenty.
:	
.	
Martina	Well, okay. Yes, yes. I understand now, I understand now. I was a bit slow. Okay.

It appeared that Beata was trying to compute the cost for talking for two hours, where the one hundred and twenty that she referred to would be in minutes. But Martina rejected her mathematical explanation. Nevertheless when Beata insisted that it would work Martina admitted to not understanding what had been done and sought further elaboration. At the conclusion of a long conversation between the two of them Martina was convinced and admitted to have been quite slow in understanding it. Here, we see the overlapping of the psychological and sociological perspectives as derived from their interactions.

The Quality of Reasoning

In this section we draw on the psychological perspective to look into the mathematical tools and symbols used by Martina while engaging in mathematical tasks.

As noted earlier Martina's use of the point-slope form of drawing the line rather the use of plotting several points provide evidence for the use of sound mathematical reasoning skills. These reasoning skills were also evident when Martina responded to regular requests for help by her classmates. For example, in the following episode Martina explicitly models mathematical reasoning when helping Marika find the relationship between the cost and the number of liter of juice. The formula was to be derived from the graph below and what followed is part of their conversations:

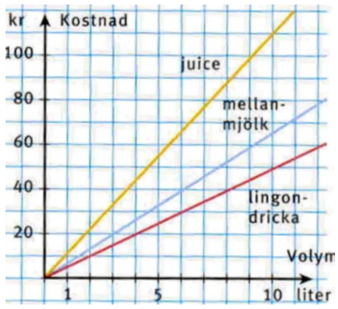

Figure 6. The graph showing costs of different drinks per liter (Textbook, p. 176)

```
Martina     One step in that direction, one step in that direction.
            One step, do you remember, what he said?
Marika      Yes.
Martina     So the cost equals
Marika      It makes ten, cost, no, the relationship.
Martina     Do you remember that you're supposed to include x and
            stuff.
Marika      Yeah yeah yeah yeah yeah, I get it I get it I get, cost,
            I did one there so I've got to do one there.
:
Martina     So cost equals?
Marika      I don't know.
Martina     Litre, if we take litre.
Marika      Yes.
Martina     Look, look at ten ... How many do you get then?
Marika      One hundred ten.
Martina     One hundred ten ... Yes, then you get for, one litre, you
            get, it costs eleven, eleven kronor.
```

Marika	Yeah.
Martina	So the cost is the number, of litres times eleven ... x times eleven.
Marika	X times eleven.
Martina	Eleven x.

Here we see that Martina prompted Marika to recall the class discussion. However, after Marika failed to find the answer, Martina provided leading questions. She introduced mathematical reasoning by saying that the problem is not simply about tracing "one step in that directions, one step in that direction" claiming that it is difficult to do one step at a time on the grid to find the slope. Rather, she suggests that her method of using 10 litres to start with is more efficient for it is where the point of intersection between volume and cost can be more accuracately read off and the relationship established from there.

However, on a subsequent occasion, Martina was observed to revert to a less sophisticated approach to solve a relationship problem. This happened in Lesson 10 where the task was to draw a similar diagram (see Figure 7) showing the relationship between length (längd) and cost (kostnad) of two ski boards R and S based on the relationships concerning the speed (fart), width (bredd), age (ålder) and turning ability (svängbarhet).

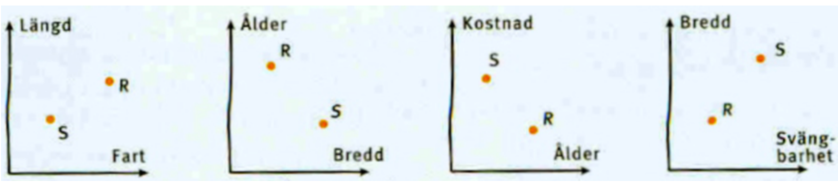

Figure 7. The graphs where to base the next task on length and cost (Textbook, p. 193)

For this task, her reasoning skills became apparent in the following conversation that involves Martina seeking help from the teacher:

Martina	I've done this, and I got it, like that. And the others I didn't get it.
Teacher	Wait, wait, wait, wait. Which is it now?
Martina	This.
Teacher	(Read from the book) Draw a similar diagram which shows the relationship between speed, and turning ability for skis... Oh, I've got to read the whole thing.
: .	
Martina	If you just, like that it's right. I measured so that it would be exact. Yeah, yeah, right, yeah, yeah. But it's the same R.
Teacher	So fast, yes exactly for the speed. 'Cause you that R, yes for the speed is higher for R. And now you're saying that S is
Martina	Yeah, okay.
Teacher	Yeah, I don't know that yet but, the cost.
Martina	'Cause I measured and it should be like that

Teacher	But they shouldn't be the same there, should they?
Martina	The cost for S, yes, I should be higher ... but I measured, it's correct.
Teacher	The cost for, S is actually higher than R.
Martina	How did I measure then?
Teacher	Isn't it?
Martina	Yes, yes, it doesn't matter where it is really. It can be there or
Teacher	Nah, because we don't have, because we don't have, any, scale.
Martina	I can put it here?
Teacher	Yes.
Martina	Okay.

As the analysis here focuses on the quality of reasoning, we describe what she did and what she could have done. For this task, Martina said that she measured so that she could be exact about the answer. This could mean that she measured the position of the coordinates of the points. By measuring, she focused on the relative positions of the dots on the coordinate plane rather than the relationships of the variables involved. She could have focused on the first and third graphs and figured out which graph showed a relationship between length and cost. The diagram that she drew for this task was not documented. It could have been interesting to see if she had a similar one as the answer in the book, or if it deviated from the one in the book with, the cost on the horizontal axis and length on the vertical axis. This could have been another basis for discussing the concepts of independent and dependent variables

The previous episodes illustrate that Martina has the mathematical knowledge and reasoning skills that enabled her to do most of the tasks either by herself or with support from the teacher. Of interest to our investigation is looking more closely at how these skills were supported and developed. For example, in Lesson 7, she had to identify which graph (see Figure 8) would match shop advertisements A, B, and C for developing photographs.

Shop A: free development and 5 kronor/copy
Shop B: 80 kronor development and copies
Shop C: 50 kronor for development and 1.50 kronor/copy

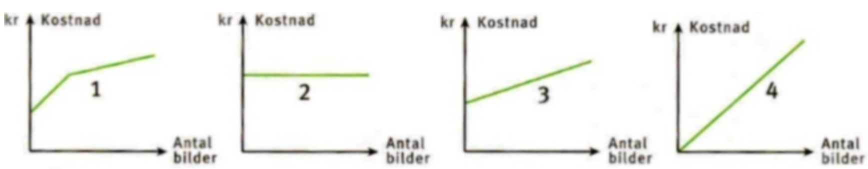

Figure 8. Possible graphs for Shops A, B and C (Textbook, p. 181)

On this task she sought clarification from the teacher about Shop B's advertisement as follows: "When they write like this, 80 kronor development and copies, what do they mean by that? 80 kronor for development and copies, or 80 kronor for development 80 kronor for copies?" The teacher replied that, "if I'd seen one like this, I'd thought that it was development and copies that cost 80 kronor, that you got both for 80 kronor". Martina confirmed adoption of the teachers' explanation in the post lesson interview:

```
Martina      Exactly, and then B then, 80 kronor development and
             copies, what did I write there? Two. I wrote, ehm, yes
             then then, yes there (laughter).
Interviewer  A two.
Martina      Yes.
Interviewer  Since?
Martina      Well, because, yes, it'll be, it'll be like a fixed, you
             pay 80 kronor for both development and copies, so it is
             a sort of fixed cost. You see the cost is always 80
             kronor.
```

Building on her enhanced interpretation, Martina continued her seatwork, successfully completing the question. However, in the post-lesson interview Martina's response to a request to create an advertisement to match the first graph – "the first ten are more expensive, and then it gets cheaper, the more pictures you have the cheaper it gets … if it gets fifty, first, then it may costs a kronor later then, or something like that" – suggested that her skills at interpretation were fragile. And in Lesson 9 Martina's efforts to solve the problem – "A person pays 290 kronor for 80 apple trees and pear trees. One apple tree costs 4 kronor and one pear tree costs 3 kronor. How many apple trees and pear trees did he buy? – indicated ongoing difficulties. Before she answered this item she partly discussed it with the teacher:

```
Martina     Here, there are eighty pear trees too, or is it plus a
            certain number of pear trees?
Teacher     No. Eighty.
Martina     Apple?
Teacher     Apple trees and pear trees, so eighty trees altogether.
Martina     Altogether, oh, okay.
```

The solution of Martina to this item was this:

```
Total money: 290 kr
Number of trees: 80 pcs
Apple   4 kr
Pear    3 kr
He buys 40 apples and 20 pears so it is 60 trees and the cost is 220
kr. Then missing 70 kr and 20 trees. Then he buys ten of each
variety.
Answer: He bought 50 apple trees and 30 pear trees.
```

It seemed that she used the "guess and check" method to solve this problem despite the teacher's encouragement that she utilise equations. It could be that she found this problem more easily solved by her method or that she lacked confidence to interpret and formulate equations for such a situation.

Finishing all the required class tasks early meant that in Lesson 13 Martina was set to do tasks, this time taken from another book. The conversation she had with the teacher revealed that the problem was about functional relationship between braking distance, speed and friction when driving a vehicle. As Martina had successfully completed the required set tasks on functional relationships it was expected that this task would not be a problem. However, Martina's unfamiliarity with the constructs of dependent and independent variables contributed to her difficulties associated with formulating a relationship. In seeking help she asked the teacher this: "What does it mean dependent and independent variables?" Her conversation with the teacher on this took about three minutes. Part of their conversation follows:

Teacher	And the braking distance depends on how much speed you have, so those two are dependent on one another, and the friction is independent, it, it's just like there like a, you can put it like this. Look k braking distance let's say.
Martina	Okay, yeah, but I get it, I think I get it, yes.
Teacher	Is the friction zero point twenty six?
Martina	Yes.
Teacher	Times the speed, this depends on that but that is always zero point twenty five.
Martina	Okay.
Teacher	So it's independent and those two are dependent.
Martina	Yeah, ok. Like this?
Teacher	Yeah.
Martina	But which it's two dependent then?
Teacher	Which is dependent?
Martina	There are two dependent then, then it's both that one and that one.
Teacher	Yes, two independent, those two are independent of one another, no those are dependent of one another.
Martina	Those are dependent.
Teacher	Those two are dependent on one another.
Martina	It's wrong in that case.
Teacher	Yeah, it must do 'cause those two are, well, you don't talk about dependent and independent too often.

It appeared that the obstacle to formulating an equation on this task is related to Martina's conceptual understanding of dependent and independent variables. Despite previous success on a series of problems involving distance-time relationships, time-cost for phone calls, volume-cost for different kinds of drinks, etc., it is clear that the concepts of independent and dependent variables that could have been drawn out had not been given emphasis. The teacher appeared to confirm this when he said that these dependent and independent variables are not being talked about that often. Thus, it can be deduced that Martina's skill to formulate relationships was hindered by her lack of conceptual understanding of independent and dependent variables at this stage.

In summary, Martina's voice contributed to the sociomathematical norms in the class. Her interactions with her teacher and classmates involved discussions that focused on solutions to mathematical problems. She was willing to support the teacher in explaining some mathematics to her classmates as well as to challenge their reasoning. As to her emerging mathematical practices, it was evident that she used multiple representations of mathematics such as equations and graphs. It was clear that Martina valued the authority of the teacher but it could not be established if this was a way of updating the teacher on her progress. In communicating ideas, she appeared not to hesitate to challenge the mathematical statements made by the teacher such as using the more elegant point-slope idea of graphing a line rather than plotting several points. But she was also open to accepting convincing arguments from her classmates humbly. Despite these instances showing that she could provide a more sophisticated solution to a problem, it was difficult to establish a clear developmental trajectory of her reasoning skills. When we thought progress was forward, she would sometime fold back to a more basic method such as measuring the lengths rather than interpreting graphs relationally. Her difficulty on interpreting and formulating linear relationships arose when the graphs did not provide scales or that the situations required application of pre-requisite knowledge such as dependent and independent variables. In relation to this, it is important then to look back at the goals and see how these compared to what were attained and then relate these to some theories of learning. The next section shall discuss this aspect.

LOOKING BACK

In this section we reflect on Martina's attainment of the goals on learning mathematical relationships as set in the textbook and relate these findings to some theories of learning.

At a glance, it appeared easy to claim that Martina went beyond the four learning goals: (i) draw and name points on the coordinate system; (ii) work with proportional relationships, for example comparing prices; (iii) work with relationships comprising a fixed and variable part; and (iv) interpret different types of linear relationships. Martina successfully completed all required work (and extra tasks) within the expected timeframe, seeking minimal clarification and help with those tasks associated with the third and fourth goals. However, when the analysis focused on her mathematical practices and reasoning skills, it showed that her progress was characterised as a series of "jumps" and "dives" into these practices and skills. Although it is expected that a student may use a more basic mathematical idea to solve a certain problem, in some situations folding back to a less appropriate one leads us to question the stability of the higher skill acquisition. To discuss this point, here are some cases.

Martina's successful completion of a range of problems involving linear equations suggested achievement of the third goal (refer to the list on page 3). However, this assessment was questioned when she used the guess and check method rather than formulating equations for situations in a subsequent test item.

Also, our observations to support an assessment that Martina had reached the fourth goal, that of interpreting different types of relationships, was challenged by her subsequent use of a more basic skill of measuring lengths rather than interpreting relationships based on the given graphs. Moreover, when she approached the task on braking distance she appeared confused about independent and dependent variables. Her uncertainty was unexpected based on the evidence that she could solve a mathematical problem using different methods and that her reasoning skills were quite adequate. Two main ideas seemed to emerge here. The first one pertains to the conceptual understanding of independent and dependent variables. It was not explicitly mentioned in the first goal, yet it could have been taken up as part of discussions around the "concepts of x and y on the coordinate axes" and on the "use of these designations for axes consistently" and then apply to proportional situations which was the focus of the second goal. The second idea pertains to her reasoning skills. Given that the task on braking distance was similar in structure to the one shown in Figure 6 – one that Martina had successfully done – was there a missed opportunity to support the development of skills by the explicit creation of multiple 'stories' from the same graph or identifying similarities among mathematical tasks?

How then can these opportunities – missed or taken – for the learning be partly explained by learning theories? As student independent work predominates in the class, it is unarguable that students were given much time to do mathematics by themselves. Hence, Brousseau's (1987) theory of didactical situations in mathematics is thought of. In his theory, a didactical contract between the teacher and the students could be negotiated in such a way that students could do mathematics by themselves during adidactical situations where the teacher provides well designed mathematical tasks involving patterns for actions, communications, formulations and validations, all with minimal interference from the teacher. This appeared to have happened in this particular class, but with some limitations. One of these limitations was that the teacher did not design the tasks; rather these were mainly from the textbook. In this case we question the appropriate balance of tasks that required students to arrive at formulation versus tasks that require validation of patterns. Most tasks required recall of facts and procedures such as plotting the points or reading off points on the coordinate plane; less tasks required mathematical formulation and validation involving justifying mathematical conjectures. Moreover, answers, especially those that could be a springboard for further discussions, such as students' misconceptions, different solutions or some other reasoning skills were not presented to the whole class. As such, opportunities for students to communicate their ideas and make arguments and validations were limited. Also, some students sought much time from the teacher to aid them to arrive at the answers, contradicting the theory that it should be minimal interference from the teacher.

Nevertheless in the case of Martina, Brousseau's (1987) theory appeared to fit more appropriately. Martina showed acceptance of the didactical contract to do the tasks, did most of the tasks by herself with minimal support from the teacher; and applied most of the reasoning skills expected of her as measured by the goals set in the textbook. We suggest then, that the theory of didactical situations may be less useful when applied to a whole class setting where student independent work is widely used. However, in such a classroom it appears that the theory has merit as a basis for looking into individual student's attainment of the different levels of mathematical reasoning that could be related to the attainment of goals.

Another theory that could be used to describe individual learning is that of Vygostky's (1962) zone of proximal development that refers to the range of tasks that a learner could do independently and those done with the help of the teacher or a more able person. In the case of Martina, the involvement of the teacher and the prevalent use of the textbook were the most visible supports for her learning. There were evidences that Martina needed the support of the teacher to move to the next level, such as the time when she had to understand first about independent and dependent variables before she could proceed on doing the task.

While interactions with the teacher and peers were valued, it was clear that within this independent learning environment, Martina regarded the textbook as the main source of support. With it, she took the responsibility to go through the pages and work on the mathematical tasks at her own pace and in her own way. We see this independence in her decision to skip those pages that involved working with a partner to play a mathematical game. Moreover, in response to an invitation by her seatmate to play a game she replied that "we'll do it later, it's you know it's the same as Sinking Ships, sort of … so you can, no, skip to that there" pointing to the next page. Martina's appeared more focused towards moving to next level once she had acquired particular knowledge and skills, in this case about plotting points on the coordinate plane. We could suggest that Martina moves to the next zone by mainly using the textbook and with minimal support from the teacher. It was not implicitly mentioned by Vygotsky that a book could be an alternative to the teacher or to a more able person yet it appeared that this was the case for Martina. However, her development of reasoning skills could be debatable based on these evidences alone.

CONCLUSION

The community within this mathematics class, organised chiefly around student independent work, generated specific sociomathematical norms. One of these norms pertains to those interactions between the teacher and the student/s or between students to arrive at different mathematical solutions, be they more efficient or sophisticated or just for the sake of having a different solution. Also, it was evident that the acceptable mathematical explanations could be an elaborate one or just a short or a guessed answer, either computational or conceptual, and could be a way of disproving or substantiating a mathematical statement.

Examination of Martina's patterns of interactions revealed that she was at ease interacting with either the teacher or her classmates. Martina had established some social norms such as working with classmates to discuss her solutions to mathematical problems and also a contributor to the sociomathematical norms set in class. Moreover, a more in-depth look at her participation in class revealed that her ways of reasoning were based on her mathematics knowledge such as using equations and graphs, for this particular topic, although at times she attempted to use a calculator or sought the help of the teacher. It was also evident that she could challenge the mathematical statements made by the teacher or that of her classmates, as well as accept her limitations, too, and from here the overlapping of the psychological and sociological perspectives as derived from the interactions became more apparent.

From the recorded interactions and work outputs of Martina, it was possible to discuss her quality of mathematical reasoning as well as its development. At times Martina provided a more sophisticated and conceptual approach to mathematical tasks and at times she folded back to using less elegant solution methods. From these examples we concluded that her development of reasoning skills was complicated to describe, especially on attainment of the fourth goal, that is to interpret different types of linear relationships.

For Martina, the textbook goals appear to have been easily met. She was able to finish most of the tasks required from the textbook and used another book on the eight day while the rest of her classmates were still working on textbook tasks. However, task completion rate appeared to be an overly simplistic way of assessing learning outcomes. A closer look at her quality of reasoning and its development challenged surface assessment about the attainment of the goals.

Linking Martina's case to theories of learning such as those by Brousseau and Vygotsky, it was clear that Martina was able to participate in the learning environment exhibiting a taken-as-shared didactical contract between the teacher and Martina and as well as with other students that they would work independently on tasks from the textbook at their own pace. In addition, it was apparent that aside from the teacher or a more able person, for her a textbook was the dominant learning support to assist movement to the next zone of learning development.

Returning to the research questions, it can be claimed that within this mode of instruction several sociomathematical norms emerged. When considering one student's participation in this class, we see that Martina used mathematical tools and symbols in her ways of reasoning and communicating mathematical ideas. In completing the tasks from the book she made progress towards the four goals. However, as mentioned previously, stability of learning associated with the four goals was questioned when the quality of reasoning and its development were taken into consideration. Nevertheless, from this study we could deduce two main points: that independent work as a mode of instruction exists in Sweden and to a certain extend learning took place in such a mode of instruction.

SOME IMPLICATIONS

It is apparent that this study has generated some answers to the research questions, yet some uncertainties have also become evident. This section discusses some reflections regarding some of these findings.

Despite the identification of sociomathematical norms in this class such as what counted as different mathematical solutions or acceptable mathematical explanations, it was clear that the learning environment dominated by independent seatwork acted to constrain opportunities for the teacher or students to engage and learn from whole class mathematical discussions. Examination of answers more likely involved student checking of the textbook answers or requests to the teacher to check accuracy. The opportunity for students to accept accountability and claim ownership for presented mathematical solutions to the whole class as well as learning these different solutions from other students appeared to have been missed in this class. Within such an independent learning environment, a modification could be made for the class to choose few tasks that students found difficult to do or those tasks rich in solution methods as the basis for whole class discussions. Here the students could share how different their solutions are from the others and the teacher could provide more encouragement or support when needed. It could also be an opportunity for students to pose more problems thereby enhancing their skills on problem posing and creating more new mathematics. These are areas for consideration for further research on how these whole class discussions could be carried out in such a learning environment.

It was evident that Martina was able to express her mathematical ideas to the teacher and her classmates, be these explaining mathematical solutions or arguing or supporting mathematical statements made by others. These were the instances where her ways of reasoning, ways of communicating ideas and the quality and development of reasoning became more apparent and which was the basis for describing her attainment of goals set. Adherence to the LPS design meant that the students in focus changed with every lesson. A much more in-depth study of an individual student within a sequence of lessons could enhance our understanding.

It was apparent that there were important findings that emerged from studying the participation of one student. Yet, it has some limitations. A student, especially one that is considered good in mathematics, provides only one story of learning. There is often an expectation for a good student to attain the goals more easily or even quickly learn whatever the goals. When looking into one student with low or average mathematics ability it may be that the attainment of goals is less predictable. Furthering our understanding of how learning is occasioned for these students within independent learning environments must be a priority.

For this class, the students' main mode of learning mathematics was working independently on prescribed textbook tasks. Although there were evidences to support the claim that students can learn mathematics from this mode of instruction, questions remain as to what the textbook has to offer in terms of opportunities to enhance students' mathematical reasoning and communication skills. The findings provide a confirmation that a textbook could support mathematics learning significantly or could even be an alternative to attain the student's zone of proximal development (Vygostsky, 1962). Such close examination of the classroom data enables us to track mathematics content and skills that were more useful in attaining the goals and which ones required further improvement. Our focus on Martina's experience has provided further insight into understanding of a learning model where an individual student participated as a learner in a class where the mode of instruction was dominated by student independent work. Yet, it is also clear that this study generated some questions to explore in the future.

NOTE

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CHAPTER SEVEN

What Do Students Attend to?
Students' Task-Related Attention in Swedish Settings

INTRODUCTION

What aspects of tasks on mathematical relationships do Swedish students attend to when interacting with the teacher? In this chapter three types of mathematics task-related issues that students focus their attention on in Swedish classrooms are analysed. Previously attention and motivational factors such as interest and student engagement have been objects of both theoretical and empirical research. However the main focus of existing studies are attitudes or psychological states, often dichotomised and not related to mathematical content. We are investigating what students attend to in tasks on mathematical relations. The overall purpose is to find out how students direct their attention towards this content matter during student-teacher interaction, and how this knowledge can be useful in mathematics classrooms. Our aim is to provide a student voice, focusing on what students attend to in student-teacher interaction when dealing with tasks on mathematical relations and how it can be linked to the concept of interest. Through our analysis of video recorded lessons from one grade eight class in a Swedish school we discovered three categories of task-related attention: (i) Relevance of a task, (ii) Solving a task and (iii) Validating a task. In this chapter, episodes will serve as empirical evidence of three types of task specific attention in student-teacher interaction.

BACKGROUND

Interest and Learning

The background to this study is ongoing research on the interactive view of motivational factors, such as interest and student engagement in mathematics. On the curriculum level, one of the official aims in Swedish school is to develop interest towards mathematics. Research related to the concept of interest in education has evolved from being a trivial, everyday term for internal/external state of affect, to empirical studies on interest towards content specific situations (Bikner-Ahsbahs, 2003; Dewey, 1913; Mitchell, 1993; Nilsson, 2009). There is empirical evidence to support the importance of interest in relationship to learning mathematics. The relationship between interest and learning was established through multilevel structural equation modelling and resulted in a reciprocal

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 115–131.

relationship between interest and learning (Ma, 1997). This reciprocity can be illustrated, as shown in Figure 1.

Interest ←→ Learning

Figure 1. Reciprocity established by Ma (1997)

In other words, learning affects interest and interest affects learning. The antecedence of interest and learning leads us to the question of how interest is evoked. This we want to approach by starting to analyse what students attend to in a task. It can be done on a classroom level, approached through analysis of mathematics lessons. The intentions of an empirical approach to interest lead us to initiating a study of task related attention. That is, attention directed towards subject matter in mathematics originates from this pedagogical dilemma. Also, it is important to stress that the concept of interest in studies of teaching and learning has certain features that are unique, not shared by motivational research. For one, interest is connected to the content- related area of learning rather than motives or goals (Ma, 1997). Interest is expressed during tasks and activities in classroom practice. On that basis, interest has been studied beyond the dichotomy of inner/outer state or motives and attitudes.

Further on, interest is introduced in relationship to knowledge and the process of learning mathematics in institutional environments. In other words, rather than focusing on the *motive,* the direct involvement in the learning situation is analysed in order to gain insight in the process of interest construction. When it comes to an empirical approach on interest, it can be searched for in the Gaps of Knowledge (GOK). This view originates from the Informational Gap Theory where interest is described as a perceived focus of attention on specific knowledge gaps, which become exposed to an observer during interaction (Silvia, 2006). This perspective is grounded on epistemological assumptions stated in the following way:

- Knowledge the student is aware of having
- Knowledge the student is unaware of having
- Knowledge the student is aware of not having
- Knowledge the student is unaware of not having

The gaps of knowledge are described as the difference between the wanted knowledge and the knowledge one is already aware of. What can be gained from this view? In this study attention is seen as a pathway towards the development of interest, constructed in the gaps of knowledge. GOK serves as a metaphor to conceptualise the bridge between the knowledge the student is aware of and the knowledge that the student desires. This view on knowledge indicates that there are constructions not yet experienced by the student; that the student is aware or unaware of (Marton & Booth, 1997, p. 7). In this study the GOK model is presented in order to relate student interest construction to students' knowledge. This epistemological standpoint is helpful when distinguishing between episodes in which students try to learn or try to receive social recognition. The use of this model will help us to gain insight in interest related to specific tasks that are a part of classroom interaction. It can help to interpret what students attend to, and how to choose sequences relevant as units of analysis.

Task-Related Attention

The first step towards an approach on the conditions for learning mathematics is to look at what students attend to. Attention as an object of study is also process oriented, “not a thing, at least in the sense of some thing to which you can point” (Mason, 2004). Mason gives a view on attention as a dynamic process, an act where a student’s focus has a certain direction. When looking at what students attend to, it is, according to Mason (2004) equally important to take into consideration *how* they attend.

In classroom interaction, the manner in which the students attend to a specific task can manifest through students engaging in a certain activity or task. This type of engagement has been investigated in different classroom behaviours (Helme & Clarke, 2001). In videotaped data and interviews Helme and Clarke used cognitive engagement (CE) to analyse how students engage on different levels. The results of their study found that students engaged on an individual level using a variety of forms including: verbalising thinking, resisting interruptions, externalising gestures, and giving feedback when working in pairs. Students who cognitively engaged in subject matter also completed utterances of the teacher or other peer students, exchanged ideas and suggestions and justified their argument and solutions. In other words, engagement was established as a term for student involvement and actions, beneficial for their cognitive development. Further, Helme and Clarke give insights into the quality of the engagement of a student through excerpts of conversation between a student and the teacher and the student and another peer. This type of research is a possible point of departure for investigating conditions for learning. Therefore, in order to research conditions for learning we need to analyse students when interacting in a certain way, focusing their attention on mathematical activity, expressing interest and cognitive engagement. Indicative factors can be used as a tool to pinpoint how the students behave when they attend to mathematics.

In this study we take a stance by linking motivational factors such as interest and student engagement to task-specific attention. We will contribute with insights into the ways the students attend to specific features of a task in their interaction with the teacher with mathematical relations in focus.

AIM AND RESEARCH QUESTIONS

The overall aim of this chapter is to propose a student-oriented view on interest during classroom interaction. It is a study where attention is linked to specific features of different tasks during naturalistic classroom interaction. In our study, we include the concept of task specific attention as a part of student interest development. Students’ focus of attention and what this attention is directed towards when dealing with mathematics is investigated. In other words, interest is manifested in a student's way of attending to a certain task. This provides a point of departure for an approach and analysis of empirical data.

The research questions addressed in this study concern students dealing with tasks within a specific topic area in mathematics, namely mathematical relations: *What aspects of a mathematics task do students attend to when interacting with the teacher? And how is interest co-constructed in such situations?* Hence, we aim to outline students' way of attending since attention can be observed.

METHOD

The main focus of this chapter is student voice during classroom interaction. Interaction has on a group level been described as "a collective pattern in how human beings understand and behave" (Emanuelsson, 2001, p. 23). Based on this conclusion we turn to a way of understanding interest as an interactive process inside the classroom. The object of study is the process of interaction, primarily on students' focus of attention relative to subject specific circumstances. In our study this point of departure provides an opportunity to give students a voice without decontextualising learning situations, without neglecting the specific turns that might be of importance in the process: "In focusing on the interaction itself as a unit of study, creating a more active image of the human being and rejects the image of the passive, determined organism" (Cohen, Lawrence, & Morrison, 2007, p. 404). This study highlights the active image of the student in the classroom context, without neglecting the teacher. Student-teacher interaction is chosen as a focus of analysis, because this form of interaction frequently occurs during Swedish lessons. As numerous studies show, the Learner's Perspective Study (LPS) data provides a rare opportunity to analyse development through students' actions in detail, as well as to follow this interactive process in naturalistic settings (Clarke, Emanuelsson, & Jablonka, 2006). An analysis based on a continuous lesson set can lead towards an overview on the theme of interest and provide suggestions for further methodological decisions and possibly further data collection. In this particular study a video analysis involved a sequence of 10 lessons from the LPS data in school SW1. The aim was to investigate if students' interest construction is visible for an observer, what students are interested in and if students' reflections on their actions are compatible. Episodes from one lesson (SW1L10) were selected for further analyses. This lesson was chosen because the theme of the whole lesson was individual work in textbook, especially rich in student-teacher interaction with content matter in focus.

The analysis was done in several steps: First individually, by choosing episodes from different parts of the lesson, illustrating students attending to a task. In step two we coded suggestions for categories in order to describe what students attend to. In the third step of the analysis a team of experienced researchers validated episodes and discussed the strengths and weaknesses of suggested categories. Revisions of the categories and coding of sequences were made. In the final step, the chosen episodes were organised into excerpts by the writers. Categories that were generated from this material can be recognised as a result of a qualitative approach that serves to "penetrate the situations in ways that are not always susceptible to numerical analysis" (Cohen et al., 2007, p. 407). Visual resources made it possible to go back and forth in the data, scrutinising it in collaboration with different researcher groups' various insights and perspectives.

This data is a unique high quality data set where ethical aspects are taken in consideration at all the stages of data gathering. In this secondary data analysis attempt we spent time reducing extensive amounts of recordings and transcripts to a manageable number of sequences. Therefore we chose to take specific precautions when handling and analysing recorded material. For one, the avoidance of material downloads to unprotected computer sources and the risk of spreading confidential material. Analysing recordings from the original source, in this case a server, highly protected by individual passwords, was a suitable solution to this ethical issue. As a part of the analysis and validation process, sequences were presented in working groups, courses and conference participants. In such cases of scrutiny the permission from students who participated in those sequences needs to be thoroughly controlled; there was permission to use the material. In some cases, permission was given for research but not for conference presentations and discussions. Those students were eliminated from the analysis. Only students who agreed to be full participants are included and referred to anonymously in the transcripts. One important precaution had to do with the teacher's role. When analysing a teacher's actions it is important to keep a sensitive, honouring way of expressing oneself in the analysis. It is important not to become normative in a sense that values the teacher's performance, but instead interpret what actions mean. These considerations correspond well with the legal requirements of confidentiality and utility according to Swedish recommendation, that individuals are protected from identification and not exploited for non-scientific purposes. Since this study is of an explorative character, not all ethical aspects are expected to be obvious from the beginning. By following general guidelines research participants', teachers' and students', personal integrity is not neglected.

RESULTS

In the following section we present three episodes, illustrating categories of what student's attended to during student-teacher interaction. The episodes are:

(i) Relevance of a task
(ii) Solving a task
(iii) Validation of a task

A common feature of all the episodes is that the student is initiating the interaction by approaching the teacher with a question or a comment. Hereby the results are presented in form of episodes, each supported by a set of excerpts, where students' attention is visible in student-teacher interaction. In order to interpret the presented data, we have revised lesson transcripts and impose the following key symbols:

, or . pause;
… unfinished sentence;
(…) speech impossible to detect;
[…] events or behaviour outside speech;
// simultaneous speech

(i) Relevance of a task: "What do you need this for?" In the first episode, the student who is instructed to work individually tries to understand the relevance of the tasks on mathematical relationships.

Excerpt L10:1

04:54:16	Student	Where's everything? [Leafs through the maths book] What's it called?
04:57:24	Teacher	Yes yes. What do you mean?
04:58:19	Student	(...)
04:59:09	Teacher	Yes //yes, you draw a line, yes you do. [Nods]
05:00:20	Student	//Yes but ... It's so ... It's like no one ... It ... I don't know how to explain ... Well ... It's so ... It's like ... Yes, am I going to have any use for being able to draw lines ... I mean I
05:19:25	Student	I understand ... well I sort of understand stuff like this ... I(...)But why should I? And I can show where this point is (...) compared to that one.
05:20:07	Teacher	Yes well yes//But that's really good ... But that's good, yes but that's really good then
05:29:24	Student	//But I don't want to, to keep on doing this for like 20 pages and stuff and carry on there and then there's even ... more (here)
05:34:11	Teacher	Yes but can you, can you, like, do it?
05:36:17	Student	Yes I think so?
05:37:19	Teacher	Yes well I think that you should do move on and stuff because in the red section ...
05:40:28	Student	Yes. [Nods]
05:42:18	Teacher	(...) then there'll definitely be things you can't do. It'll ... It'll be a bit more there (...) so it'll be a bit different.

The student in Excerpt L10:1 is upset and involves the teacher in a conversation about the tasks suggested by the teacher. The student starts by approaching the teacher, who is standing next to her, and questions the relevance of the tasks. In this episode the teacher listens actively, responding by nodding and confirming the student's concern (04:59:09, 05:40:28). He tries to direct the student's attention to the purpose of dealing with the tasks, by bringing up hierarchical structure of mathematics as a subject, where prior knowledge is important in order to deal with coming tasks. The student requires information about the long-term relevance of the topic and at the same time seeks justification for mathematics as a school subject.

Excerpt L10:2

05:49:28	Student	What do you *need* this for?
05:51:15	Teacher	What you need this for? Yes, well the thing, the thing, the thing is ... that it is good for ... It's that you will be able to read graphs and understand what they mean ...
05:58:06	Student	Hmm
05:58:29	Teacher	... and it's not always so very simple. If you can do it and read and understand the difference between the pear and apple tree right here, or pears and apples, that's good.

The teacher hesitates when it comes to explaining the practical implications of the task (05:51:15). The student expresses that she will attend to the tasks that are relevant. The teacher now tries to provide a meaningful explanation. He does so by suggesting procedural purpose of mathematics (05:58:29). His first explanation of these tasks' relevance to the student is to be able to interpret graphs in different situations and to become skilful when dealing with future tasks.

Excerpt L10:3

06:09:12	Student	Hmm
06:09:25	Teacher	But it's a really ... It's a really simple diagram (this one)... But I ... I ... It's ... it's good if you're practicing this because it's ... It's I think important for everyone to be able to do. If you've got a graph in a newspaper you need be able to understand what the graph is. And later we're going to talk a bit about
06:25:10	Student	Hmm but that's what I'm doing. It's about (...)

The student argues with the teacher; she thinks that she is already able to work with the graphs (06:25:10). The teacher instructs her to provide an area of application from everyday life for the student to relate to (06:09.25). Also, at this point of the interaction the teacher signals that the conversation is over, by making an attempt to leave. The student resists this action and continues to express her frustration over plotting graphs.

Excerpt L10:4

06:29:03	Teacher	Yes but that's good. [Nods]
06:29:04	Student	Yes [tries to leave]
06:29:17	Student	[to T] Hey, you!
06:29:22	Teacher	[turns to the student]Then I think you should carry on with it a bit longer.
06:32:16	Student	[sigh]
06:33:02	Teacher	Yes.
06:33:19	Student	But there are millions of pages!
06:35:10	Teacher	No, there aren't millions of pages.
06:36:16	Student	Yes there are (...)
06:39:27	Teacher	Yes.
06:40:24	Student	And then it keeps going, there, it's just that there are millions of pages. I mean, how can one even think up so like many lines? [laughter] I don't get it.

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06:46:19   Teacher   [laughter, looks down at the student, repeats
                     with a comforting voice] Mmm. How can one ...
                     How can one think up so many lines.
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Here we see the student expressing unwillingness to carry out the tasks on mathematical relationships. Unsatisfied with the given answer, she carries on arguing. She does not let the teacher leave and move on to the next student (06:29:17). She is still attending to finding out the relevance of similar tasks.

Excerpt L10:4 ends on a positive note with laughter and smiles. In order to focus her attention on mathematical relationships, this student seeks confirmation of relevance within the tasks. Although she shows her understanding of the topic as she interprets it, about drawing lines and comparing points plotted in a graph (05:00:20) it is the procedure and not the concepts she is determined to avoid (05:29:24). In the exchange we see how the teacher justifies this topic (06:09:25). The student returns to that issue later in the interview, stating that she can proceed with any task as long as she knows the purpose: What do we need this knowledge for? When is it applicable? To summarise these excerpts, it can be said that interaction involving the clarification of the relevance structure, both practical but also considering abstract sides of mathematics, can be a part of the process. The selected episode (L10:1-4) illustrates the student's repeated questioning of tasks that involve plotting graphs indicate that interest can be co-constructed with the aspect of relevance in focus. The teacher tries to convince the student with examples of practical implication in everyday life, such as "to read and understand curves in newspapers." In the beginning the student is upset, questioning the relevance of dealing with this content matter. The student reveals her way of understanding mathematical relations while reflecting on her own knowledge. When she claims to already be capable of understanding representations in diagrams and graphs, it becomes visible that she doubts her own ability to "draw lines." She says that she does not want to attend to a procedure on a topic she claims to master. At the same time, in her interaction with the teacher she shows insecurity, indicating there might be gaps of knowledge (05:36).

Arguments that the teacher suggests for letting the student continue doing something she already claims to know is that it is necessary; that solving simpler tasks constitutes basic knowledge and is a condition for solving more complex tasks. There will be, according to the teacher, new challenges later in the chapter, in the red section. However, we see that the student is interested to pursue repetition only if she can see a relevance of the tasks. In other words, the student pays attention to the relevance of content matter, and she does so in a passionate way. By questioning the relevance she begins to construct interest through the interaction with the teacher.

(ii) Solving a task: "The bigger the x-value, the steeper the graph" Next scenario aims to capture a type of unit where the student communicates a wish to clarify mathematics strategies within a specific task. In the text, the task (see Figure 2) is marked with an asterisk (*), which means it is on a higher level than ordinary tasks.

Which of the following relationships in the diagram to the right a) are parallel lines b) start in origin.	K=100x K=6x K=20+6x K=20+2x

Figure 2. Representation of the task the student is working on

Excerpt L10:5

47:55:05	Student	... are parallel [reads the task]
47:55:26	Teacher	Yes
48:17:15	Student	Yes ... They can't be parallel because they
48:20:24	Teacher	Why is ... Why can't they be parallel?
48:22:17	Student	Well because it's ... It changes ... yeah.
48:22:23	Student#2	(...) A lot higher up
48:26:15	Teacher	Good, you've come a long way. What is this? What sort is this? Who ... What determines how much it is changed or how, how much it slopes?
48:33:13	Student#2	That!
48:34:27	Teacher	Yes, exactly, the one next to x there.
48:36:21	Student#2	Yes.
48:37:18	Teacher	The bigger x you have, what happens then with a graph?
48:41:01	Student#2	//Mhm
48:41:07	Student	//Mmm
48:42:12	Teacher	It goes up more quickly, doesn't it ... Hmm ... How quickly does that one go up?
48:47:26	Student	Quite quickly?
48:48:23	Teacher	Yes because for one step on the X-axis it rises ... six steps on the Y-axis ... and on that one then ... so for one step on the X-axis it also rises ... six ... steps on the Y-axis
49:06:15	Student	(maybe) [says it in English]
49:06:27	Teacher	Yes but look here.
49:07:14	Student#2	(maybe) [says it in in English]

The first student in excerpt L10:5 initiates a teacher-student conversation by raising her hand to seek help determining which lines are parallel and pair those together. At the same time, a peer student becomes involved by joining the conversation. In order to solve the task, the students focus their attention on the teacher's questions. As the progress of the conversation becomes more teacher-driven, the student's attention is directed towards answering the teacher's questions. The teacher explains how the shape of the line changes depending on the value of x. At first the student seems to be insecure and guessing how the student can find a satisfactory solution to the task (48:47:26). There is doubt in the students' comments, indicating that the students have not understood and are possibly trying to guess the answer or say what is expected of them in the conversation (48:47:26; 49:06:15; 49:07:14).

Excerpt L10:6

49:08:12	Student	But what are you doing now, do I have to *look?*
49:24:17	Teacher	One two three ... Eeer, here ... That's zero ... It goes from the origin
49:31:11	Student	Hmm
49:31:17	Student#2	Hmm
49:32:05	Teacher	Does this one go from the origin?
49:32:27	Student#2	Yeah
49:33:14	Student	Mmm (nodding)
49:33:22	Teacher	Does this one go from the origin?
49:34:06	Student#2	No
49:34:15	Student	Nah
49:34:26	Teacher	Does this one go from the origin?
49:35:17	Student#2	//no
49:35:21	Student	//no

In excerpt L10:6 we observed the rapid flow of teacher's questions followed by students' answering more and more simultaneously. Directly after every explanation, the students' attention is caught again. Here attention becomes visible in the data when the speech overlaps – the two students start to form a unity in answering question (48:41:07; 49:35:21). As the episode continues, the co-construction of interest in the form of overlapping speech becomes frequent, in fact every time the teacher receives an answer.

Excerpt L10:7

49:35:27	Teacher	No. That's good. Now we'll get an incline on this one and how much it inclines, it's, if I know that this is x six, then I know that for one step it inclines six
49:45:00	Student	//hmm
49:45:05	Student#2	//hmm
49:45:09	Teacher	Two steps then, it's going to rise, 12.
49:47:29	Student	//hmm
49:48:04	Student#2	//hmm
49:51:09	Teacher	*This* one on the other hand ... It actually starts on 20 ... And for each x it's only going to rise two
50:02:21	Student#3	[Approaches T by patting him on the back, tries to get his attention]
50:04:05	Student	Yes but that one also starts on 20
50:05:23	Teacher	That also starts on 20
50:06:13	Student	You've drawn that
50:07:23	Teacher	But was it that ... aha
50:09:26	Student	But was it that ... aha
50:12:06	Teacher	So that starts there as well but it's going to have ... the same ... So those two are going to be parallel ... Because they have the same "incline coefficient" - is what it's called
50:26:05	Student	//yeeees?
50:26:11	Student#2	//yeeees?
50:29:19	Teacher	The one with the x is called ... It says *how* much
50:32:06	Student	Hmm
50:32:17	Teacher	Each step of x is called because it says ... How much is step of x. How much y is going to rise and if they rise the same then they

		are always going to be parallel.
50:38:26	Student	Yes exactly.

In particular, in L10:7 the teacher is active; he tries to shift the attention towards the main question. The structure of the conversation is presenting the procedure to the student in chronological order, emphasising the important knowledge. In contrast to the first example, here the focus of attention is not on finding relevance; the teacher does not define or try to specify the meaning or application of parallel lines. Rather, he aims to communicate the key message: the larger the x-value, the steeper the graph. Also, in the beginning the student gets acknowledgement for her pre-knowledge and at the end support for her understanding of the concept. Up to this stage both students confirm that they understand what the teacher was trying to say (50:26:05; 50:26:11). It seems as if the student has understood how to determine if two lines are parallel (50:38:26).

Excerpt L10:8

50:40:14	Teacher	Do you understand?
50:41:05	Student	Yep.
50:42:25	Teacher	So those two are parallel.
50:44:01	Student	Okay.
50:44:21	Student#2	Okay.
50:46:01	Teacher	Are those two parallel?
50:48:29	Student	Can never be.
50:50:28	Teacher	Exactly, they can never be, right, this one here only increases by two for every step that one slopes a hundred.
50:54:08	Student	Then it should be two.
50:54:15	Teacher	[Nods]

In L10:8, the teacher is trying to test if the student can apply what they have learned about the parallel lines. During this conversation, both students confirm that they have learned what the teacher was trying to explain about parallel lines (50:40:14-50:44:21). However it is possible that students confirm or give a positive reply because it is expected of them as a part of interaction. In the next step the teacher choses to let the students show that they can apply their knowledge in a new example on the same theme. The teacher poses a question to see if the student can determine if two other lines are parallel (50:46:01). When the student gives a correct answer and also provides details on the new task, the teacher accepts the answers (50:54:08, 50.50.28) and it can be concluded that learning has been taking place. Rephrasing the answer and adding to the earlier explanation is a strategy this teacher uses to find out if the students learned (50:50:28).

The two students are engaged in the same task. Student number one is interacting with the teacher and the other student, initially passive in the conversation, is engaged as an active listener (observed to be concentrating and taking notes). The teacher is looking at her as well, including her in the conversation. Gradually, the dialog develops into a group interaction and both students are simultaneously engaged in the conversation. That aspect becomes visible in the teacher's gestures, initially directed to one student, but later turning to the other student, having eye contact and in that sense making a verbal and physical attempt to include the second student in the interaction (50:44:10, 50:44,11).

(iii) Validation of a task: "Can you work it out using this unit anyway?" The student sits at the back of the classroom and solves a task and validates the solution by comparing it to the suggested answer from the back of the book. The task involves a journey plotted on distance time graph (see Figure 3).

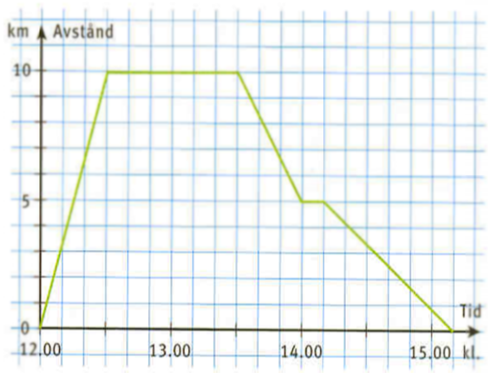

Figure 3. Graph of the journey in the task

The question reads: Which speed does the traveller have when she a) bikes to the beach, b) bikes home, and c) walks home. Next to the task there is a framed formula in red, describing the relationship between speed, distance and time.

Speed = Distance/Time

Figure 4. A representation of the illustration in the textbook

Excerpt L10:9

```
15:35:15  Student   That's not right at all? [laughter]
15:36:12  Teacher   All right what is it that is not right?
15:37:00  Student   It's wrong in the answers [in the book].
15:38:13  Teacher   Is it wrong in the answers [in the book]?
15:39:11  Student   Yes [laughter]
15:41:13  Teacher   (...) have cycled ... to the beach. What is
                    ... it is? It, that one [points on the task]
15:45:09  Student   (...) yes
15:47:14  Teacher   Or there?
15:47:29  Student   Hmm, ten kilometres.
15:49:12  Teacher   Yes they cycled ten kilometres in distance in
```

		the time.
15:52:23	Student	30 minutes.
15:56:00	Teacher	30 minutes.
15:57:27	Student	And then, then it says that it should be 20.
16:00:08	Teacher	Yes it says that yes, that's what I think as well.
16:00:17	Student	(...) ... yes but what is it that's wrong then?
16:03:09	Teacher	What have you worked out then?
16:04:21	Student	Yeah, kind of that.
16:05:27	Teacher	Yes, what? What sort of unit is it then?
16:07:14	Student	[sniggers]

When the student discovers that the answers do not match, she displays doubts and consequently approaches the teacher. The teacher is walking towards her and there is a student sitting next to her. This student is laughing and asks the teacher to resolve the matter. Here the student wants to point out what she believes is a mistake in the answer at the back of the book. The student notices that the teacher initially supports the answer to the task in Figure 4 suggested in the book (16:00:08). When the reason for the mistake is pointed out to the student a reaction is provoked (16:07:14).

Excerpt L10:10

16:07:26	Teacher	What ... What is this?
16:09:19	Student	Kilometres.
16:10:19	Teacher	Right. And that?
16:12:15	Student	Minutes.
16:13:08	Teacher	You've calculated a speed for how ... This many kilometres per minute.
16:17:20	Student	Yees ... Then it has to be ...
16:19:12	Teacher	... how many hours is that?
16:21:15	Student	A half.
16:27:18	Teacher	What's ... Ten?
16:28:01	Student	Oh, right.
16:30:28	Teacher	Kilometres per hour ... and you've calculated kilometres per minute.
16:32:26	Student	Yeah. Yeah
16:35:13	Teacher	But the most common is usually kilometres per hour.
16:36:25	Student	Mmm hmm
16:38:07	Teacher	So. But *that* is right.
16:39:12	Student	Yes.
16:39:29	Teacher	...*that*, the speed is right, except that kilometres per minute is an uncommon unit.
16:44:22	Student	What ... But can you ... work it out in this [using this unit] anyway?
16:45:29	Teacher	Yes you can work it out in this [using this unit] as well.

Attention in excerpt L10:10 is directed towards the correctness of the answer produced by the student and the one printed in the answer section in the book. To the student, those answers appear not to correspond, and surprisingly the answer the student truly believes is correct is her own (16:38:07). This is a case of validation and reflection, arguing not only about the validity of the answer but also questioning the correctness of the suggested answer in the book. The student directs her attention towards the numerical answer. This indicates student's strong confidence in herself and her own mathematics ability (15.37.00; 15.38.13; 15.39.11). In other words, the student is confident enough to think the suggested answer at the back of the book is wrong. The teacher interacts with the student having the answer in focus but at the same time clarifying why the answer in the book differs from the answer of the student. He tries to help her in the evaluation of the units in her answer (16:35:13). That is, the teacher helps her to shift attention towards the units. Despite the student's strong conviction of the correctness of her answer, she takes an accepting role throughout the conversation, but opposes at the end of the interaction sequence (16:38:07). She expresses the wish to make the teacher accept the correct part of her answer instead of adjusting it to the given answer at the back of the book (16:44:22). This student shows interest in attempts to validate the correctness of her answer in relation to mathematical correctness rather than expectations or the importance of having suitable units (16:45:29).

DISCUSSION

"Human behaviour is too complex to permit accurate predictions of what a given person will do in a specific situation, but by looking across people and situations we can see patterns of regularity" (Kilpatrick, 1993, p. 27). Looking across the interaction during video sequences in SW1, it can be argued that the interaction that takes place is *authentic* and recognisable from the perspective of researching teachers. Communicative validity is an important consideration when it comes to the episodes on classroom level (Booth et al., 1999).

The results show an alignment between task-related attention and student interest in content matter, based on the three types of task-related attention that emerge in student-teacher interaction. In the first one, attention can be seen in the process of a student questioning the relevance of assigned tasks. Specifically, the student questions the purpose of plotting a graph or points on a graph when she already feels she can understand the concept of mathematical relations. From this case, we argue that interest constructed during a discussion on the relevance of the task is a possible segue for the student to both solving the task and seeing the meaning of it. Importantly, this type of attention can seem unfamiliar in an international comparison, since the student is resisting and questioning the instruction of the teacher. However, as Clarke et al. (2006) points out, it is a part of Swedish classroom discourse for the student to question the relevance of the task and the teacher to engage in the matter.

In the episodes analysed in this chapter, examples from an every-day context but also the hierarchic nature of mathematics as a subject served as justifications for the student to study mathematical relationships. This shows that if the teacher attempts to clarify the relevance structure of a task, the student will have a specific reason for dealing with content matter and have an opportunity to become interested. The second type of task-specific attention examined focused on the students' own reasoning. In this case, the teacher had a clear idea of what he wanted to convey about the parallel lines. We argued that the teacher co-constructed interest by taking the initiative and leading the conversation, while the students joined in, supporting each other's answers to the questions that followed the teacher's reasoning. The third category of task-specific attention was illustrated by an episode where the student validated her answer. This episode shows how it is possible for the student in Swedish settings to express interest by reflecting over an answer. In this case, the student evaluated the answer in relation to the content of the task. The student looked at the problem as a whole and determined the value of her answer and the reason for it being incorrect through interaction with the teacher. This category, in contrast to the first one, emphasises the student's acknowledgement of the teacher as an authority, superior to both herself and the book.

What do our results imply? The results of this video analysis suggest the possibility to evolve a theoretical framework of motivational processes such as interest, beyond the psychological state of an individual, with support from empirical data in different forms of naturalistic classroom interaction. In all three categories the attention is expressed by the students, but the role of the interaction with the teacher on details in content matter stands out. This study could signify the importance of task-related attention of the students during a mathematics lesson and evolve into a study where interest construction is a condition for learning. In future research it would be fruitful to make comparative studies on students' interest construction in relation to different types of classrooms interaction, that is keeping the subject specific areas in mathematics constant and varying the type of interaction observed.

CONCLUSION

All mathematics classrooms have certain features in common and yet each classroom is unique in its own way. Student voice during classroom interaction in SW1 is familiar, but at the same time contributes with new insights. For example, questioning the relevance of a specific task or the topic of mathematical relationships is important to consider when teaching mathematics. Also, a student confident enough to rely on the teacher's answer rather than the one suggested in the book, points towards the importance of the teacher's role.

It can be concluded that task-related attention approached as a pathway to student interest, supports previous studies where interest is a condition for learning. In order to learn, students need to attend to the mathematics in the task. In this chapter we showed episodes of students' attention being directed towards different aspects of a task in the interaction between the teacher and student(s).

We showed what aspects of a task students attend to and the ways the interactive co-construction of meaning demands both student and teacher participation. In that sense, conditions for learning mathematical content matter can be approached by observations in coming research. For example, it can be investigated how interest or student engagement as constructs will become visible in student participation.

Most important gains from this study are different aspects of task-related attention that are crucial during classroom interaction. By giving examples of student voice during common Swedish classroom interaction we acknowledge students' perspective and at the same time capture the importance of the teacher's actions. According to the results of our study, interest can be approached at a classroom level, starting with determining the focus of attention constructed in the gaps of knowledge between what is known to the student and the knowledge desired. Hopefully, in future research this classroom-oriented approach can be set in relation to students' knowledge and have potential in inquiry and instructional practice.

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CHAPTER EIGHT

Students and Their Teacher in a Didactical Situation: A Case Study

INTRODUCTION

Giving students the space to actively participate in the introduction of new knowledge through their own independent discovery is one of the demands of pedagogical theory and curricular documents. For example, Czech official pedagogical documents demand that pupils develop their problem solving competence by "making use of the acquired knowledge to discover/identify various ways of a problem solution" (Framework Education Programme for Basic Education, 2007, p. 12). Prerequisite to such approach is providing the space in which the pupil may apply informal knowledge. Informal knowledge is often subconscious, chaotically connected, and unclearly formulated. If it is to be used, the teacher must be able to listen to his/her students' voices and make it the basis for the construction of a knowledge network (Kaur, 2009). It seems that this is more difficult in mathematics than in other subjects, as mathematical knowledge has a rigorous structure. Our case study demonstrates that a competent teacher who believes in the appropriateness of this approach may use it to activate and motivate her students.

The theoretical background to our considerations is Brousseau's Theory of didactical situations (TDS); namely the concept *a*-didactical situation and the role of students in it. The organisation of an *a*-didactical situation as such (Brousseau, 1997) involves listening to students' voices. This can be observed in the whole *a*-didactical situation, in the situation of action, but much more distinctly in the situations of formulation and of validation. Students not only (for themselves) draw some conclusions from the activities they are involved in but they also share them with their classmates and the teacher. It is the organisation of the situation that makes them formulate their ideas, not explicit summons by the teacher.

A-DIDACTICAL SITUATION AND ITS PHASES

In our previous work (Novotná & Hošpesová, in press) our focus was on the development of TDS. We explored the institutionalisation phase in *a*-didactical situation and the role of the teacher in it. In this chapter we would like to

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 133–142.

investigate the role of students. We use several concepts from TDS (for more details see e.g., Složil, 2005).

Brousseau (1997) formulated the concept of a *didactical situation;* a system in which the teacher, student(s), milieu and restrictions necessary for creation of a piece of mathematical knowledge interact "to teach somebody something". The educator "organises a plan of action which illuminates his/her intention to modify some knowledge or bring about its creation in another actor, a student, for example, and which permits him/her to express himself/herself in actions" (Brousseau & Sarrazy, 2002, p. 3). In a special case, *a-didactical situation,* the educator enables the student(s) to acquire new knowledge in the learning processes without any explicit intervention from him/her. It is possible to distinguish three phases of an *a-didactical situation*:

– *Situation of action* – its result is an anticipated (implicit) model, strategy, initial tactic
– *Situation of formulation* – its result is a clear formulation of conditions under which the situation will function
– *Situation of validation* – its result is verification of functionality (or non-functionality) of the model

In our data analysis we focused on the different roles played by the teacher and the students in the different phases of an *a*-didactical situation. Our work led us to ask several questions, which we want to focus on in this text:

– How is an *a*-didactical situation initiated? Is it always planned in advance? Do sometimes students bring it about?
– What is the role of teachers and students in exploring the situation?

The data processed in this chapter were obtained by video recording of 10 consecutive lessons of mathematics in the 8th grade (students mostly aged 14). The teacher was an experienced educator with 30 years of teaching practice. The lessons were given in a middle sized school in Mnichovo Hradiste in January 2010. The data format is based on the LPS design (Clark, 2006). The lessons were video recorded using three cameras. One camera focused on the teacher, the second camera recorded the whole class and the third camera monitored a selected pair of students. This pair was different in every lesson. In the course of the 10 recorded lessons almost all pupils became members of the monitored pair. In addition to lesson recordings, post-lesson interviews (based on the video recording) with the teacher and the selected pair of students were carried out immediately after each lesson. The recorded sequence of lessons dealt with the solution of system of equations.

THE TEACHER AS THE INITIATOR OF THE *A-DIDACTICAL SITUATION*

In our set of data the effort to create an *a-didactical situation* was evident in all lessons. The incentive was almost in all cases on the teacher's side. Her statements in the lessons and in the post lesson interviews clearly show that she had prepared the situation deliberately. For example, she stated at the beginning of the second lesson [CZ 3-L02, 00:03.27][i]: "Today we will continue … solving the task from the end of the last lesson. And let's see what will happen; what we'll discover; if we will manage to figure it out or solve something so that we won't have to guess the solution any more, as we did yesterday."

In the CZ3 lessons the *a-didactical situation* was started by students' independent activities as they worked individually, in pairs, or in groups on teacher assigned problems. The students were able to solve the problems, but without any previously learnt and practiced algorithms. The solution of the problems was based on the students' real life experience or on application of previously acquired knowledge or experience. Let us now look at several examples.

The sequence of the lessons was designed around one unifying concept (systems of two linear equations with two unknowns) to which the teacher kept referring. She decided to start from the solution of word problems using the trial and error strategy. She posed several word problems which led to a linear equation with two unknowns (in lessons 1 and 2). The knowledge of the context allowed the students to solve the problem without actually knowing the mathematical procedure. In the next step the teacher used this non-mathematical context to introduce systems of equations and different solving methods:

– [CZ3-L01]: Divide 3 l of water into cups sized 0.5 l and 0.2 l so that the cups are full to the mark. You must use all the water and cups of both sizes. Once you have a solution, you can use the cups and water over there to check correctness of your solution.
– [CZ3-L02]: A task from your skiing course. When you were on the skiing course in Janov, Veronika and Lucka went to the shop to buy some goods for themselves and for others. When counting and distributing chocolate bars and packets of nuts they found out that the shop assistant only gave them the total cost of two bars of chocolate and three packets of nuts, which was 49 CZK. Find out the price of a bar of chocolate and a packet of nuts.
– [CZ3-L07]: You will remember that in one of the previous lessons we bought nuts and chocolates. Let's now try different purchases. For example: 6 bars of chocolate and 9 packets of nuts cost 147 crowns. 6 bars of chocolates and 4 packets of nuts cost 92 crowns. Can we now say what the price of a packet of peanuts and a bar of chocolate is?

The students were asked to solve the problems on their own. Then they showed the different solutions on the blackboard. In most cases the teacher supported the discussion by questions asking for reasons, justification, and opinions. Her original idea was that the students would use their everyday life experience for solving this problem. However, it turned out that the teacher's and the students' perception of the situation differed. The teacher explained in the post-lesson interview that her intention of introducing pouring out water related to: "Hyperactive children … When they can do something manually, it is very useful for them. What was crucially important was how they selected the unknowns. Correction of wrong mathematisation – that's the point of discovery for some of the children." [CZ3-L01, post-lesson interview with the teacher, 00:12:40]: The students who commented on the same lesson said:

[CZ3-L01 post-lesson interview with the student 1, 00:00:34]:

```
Student1    The pouring out of water—all of us know that, but it was
            good to see it. It was not difficult today.
Exp         Was it useful that you had the chance to try it out?
Student1    It wasn't boring.
Student1    I found it simple to say which cup is big or small. If it
            is x or y. I did not enjoy it all the time but sometimes
            I'm more tired.
Exp         If there were greater numbers, would you enjoy it more?
Student1    If it's too easy, I don't want to think about it. I
            understood all of it, how it should be. I discovered the
            formula later.
```

[CZ3-L01 post-lesson interview with the student 2, 00:00:34]:

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Student2    It started with the trial. It was good that we could see
            it practically. But I didn't enjoy it, because we only
            did one thing.
```

However, the progress does not necessarily have to be smooth. Sometimes a student's voice brought in an inappropriate answer, sometimes a student did not answer at all despite the teacher's expectations. At that point the teacher needs much self-control to give students the chance to be heard as illustrated in the following extract from lesson 4.

Illustration

[CZ3-L04, 00:32:32]:
The teacher's intention was to support students to construct and solve of equations with one unknown (the two equations express the same unknown) and to the comparison of the "right sides". She wrote on the blackboard: $x = 3 + 2y$, $x = 9 - 3y$. The explanation went on as follows:

```
Teacher     Can you construct a valid equation for one unknown? ...
            Let's think about it together. Can anybody see it? We
            have two equations: x equals 3 plus 2 ypsilon, and the
            second: x equals something diferent, 9 minus 3 ypsilon.
            What must hold for equalities? If the left sides equal,
            what does it mean for the right sides of the equations?
            Any ideas? Peter?
Peter       3 plus 2 y equals 9 minus 3 upsilon.
Teacher     What do you say, Thomas? Could we write it like this? ...
            Yes? No? ...
Thomas      I don't think so.
Teacher     Why?
Thomas      If I substitute 2, so in one (equation) I get 7 and in
            the
            other 3.
Teacher     Hm. When we substitute 2 for y, are both equalities
            right? If we substitute 2 for y, do we get here the same
            x as here? [She points at the original equation on the
            blackboard.]
Students    Yes.
Teacher     So this is not what satisfies both equations. See? So 2
            was not well chosen. Veronika?
Veronika    If it should have the same solution it must be equal.
Teacher     Exactly. If both equations must have the same solution,
            the same number for x in the first and the second
            equation, so they must be equal and the second x must
```

therefore be equal to its counterpart. Solve one equation for the unknown *y*.

DIFFERENT ROLES OF THE TEACHER AND STUDENTS IN SITUATIONS OF FORMULATION OF CONCLUSIONS OF STUDENT INDIVIDUAL WORK

This section focuses on the situation of formulation when the relevant information is transmitted from one student who knows it to other students in a group. The analyses concern its forms and quality, as well as other students' reactions in situations when conclusions are transmitted by students. It is compared to similar situations when the information is transmitted by the teacher.

In this section, the following terminology is used: The person who formulates the conclusions and explains them to the others is called the *transmitter*, and those who get the information are called *receivers*. Students have both roles, that of a transmitter and a receiver.

Illustration

This extract comes from the 7th lesson. In the final part of the 6th lesson, students were divided into groups of four. Each group was given 4 problems A, B, C, and D with each member of the group being responsible for one of those problems. Then students left their "home groups" and met in four "expert groups" – in each group one of the four problems was solved collectively. The "expert groups" were given two tasks: to solve the assigned problem correctly and to learn how to explain the correct solution to all members of their "home group". The activity of explaining in "home groups" was scheduled for the beginning of the 7th lesson.

The following extract is a recording of the work in one "home group". The students are labelled S1, S2, S3 and S4. The problem discussed is B (transmitter S1). This problem involved the same system of linear equations as in problem A

$$3x - y = -3$$
$$2x + y = -2$$

but this time it was to be solved by substitution (solving one equation for one of the unknowns and substituting its value into the other equation). This episode follows the presentation of the solution to Problem A (transmitter S2, system of equations solved by comparison, i.e., by eliminating the same unknown from both equations, setting the two expressions equal to each other and then solving this equation). In the beginning S2's explanation was understood by the group. However, when they got to the equation $0 = -5y$ they remembered that there was a problem with division by 0 and did not know what to do with it. They failed to solve the problem until the teacher gave them a hint.

The group continued with S1's solution to problem B.

S1-1 Look how clear my solution is. Copy it and it will all be solved.

S3-1 Could you explain this? [S3 points at the equation where *x* is substituted by $(-3 + y)/3$]

S1-2	Oh, I forgot how I did it. Wait I'll rememeber. Substitution method, it means that ... Yes, clear. Look, this *x* is this [S1 circles the expression *x* = (-3 + *y*)/3] and you put this *x* here, then in fact you have it three times. S3 does not understand.
S1-3	If *x* was for example 2, then you ... I am explaining it to you.
S4-1	Don't explain, don't explain.
S1-4	You won't understand it. No, I will explain it to you when you don't understand. This here is *x*. This here is *x*. So in fact 3 times this *x* here. We only substitute in this equation.
S2-1	And what is this?
S1-5	As you have this, you know, you will only write down this. Do you understand?
S2-4	No.
S1-6	You calculate ...
S4-2	And why do you have it three times?
S1-7	Well, because here is the 3. Look. If you had 2, then you would have 3 x 2 – *y* = -3. Only *x* is not 2 but all this. Therefore you write there all this. Could you tell me why you don't understand it? To begin with you simply calculate how much *x* is. [Towards S2 who presented the solution of Problem A.] As you did it here [she points out the method of comparison].
S2-2	You said that there could as well be 2.
S1-8	No, I didn't say that. Look, you know how to find what *x* is from this equation, what *x* equals. But this *x* equals (-3 + *y*)/3. So our *x* equals this and I substitute this in that equation. Therefore the 3 is in fact this and I put there this *x*. So this is three times this. I substitute it in the equation, calculate it, and here is the result. [All the time when talking, she is pointing in the right places in her notation.]
S2,3,4	It is clear now.

The episode illustrates the following properties of the situation of formulation.

1. Active role of the transmitter and the receivers. The student who is in the position of the transmitter is very active in the whole episode. Although she has a clear idea what the correct procedure is and understands why it is correct, the transfer to his/her classmates is far from smooth. The receivers are active in their role. Their refusal to passively accept what the transmitter presents means that the discussion is very fierce with all participants heavily involved.

If we compare this to the situation when the teacher is the transmitter, the difference is mainly on the receivers' side. In case of transmission from the teacher, the students are much less active in trying to express their doubts than when the transmitter is one of the students. In the above transcribed episode, the transmitter had to answer questions 7 times. In a similar episode when the correct solution was presented by the teacher, only two questions were posed by students.

2. Formulations and reformulations; eliminating obstacles. When the first description of the procedure was not grasped by the other students, the transmitter tried to proceed in a way that is used by the teacher in similar situations – she tried to find reformulation of what was presented. Similarly to the teacher she tried to show an analogy to the situation with a concrete number. Although this procedure works when used by the teacher-transmitter,[ii] here it looked to be less productive, sometimes even counter-productive (see e.g., S2-2).

We offer two reasons for this outcome. One is the lower level of the language used by the student-transmitter. Her explanation was mostly based on what had been written in the model solution in the "expert group", she did not rewrite the calculation step-by-step, accompanying this rewriting by an accurate description of what she was doing in each step. As a consequence, the transmitter's discourse appears unclear to the receivers. When compared with the teacher's behaviour, the student-receivers grasped the teacher's accurate explanation much faster and more smoothly.

The other reason is linked to part of didactical contract evident within the classroom. As part of their expectation that the teacher provides students with clear and reliable information, the students trust that the teacher's explanation is correct, a trust which may not necessarily hold for a student-transmitter.

3. Originality of student-transmitter's techniques of explanation. In the analysed episodes, student-transmitters tried to apply the techniques that the teacher was using in mathematics lessons. This can be explained by the quality of the teacher's interventions during mathematics lessons. The students are well aware of the utility and good results of the teacher's techniques and therefore try to use them whenever they face the need of intervention.

4. Motivational potential of discussions in groups without the teacher's direct intervention. In the experiment, the use of students as transmitters was assessed by students as very useful. This is illustrated by the following extracts from post-lesson interviews with two students after the 7th lesson.

Interview with S3 (I denotes the interviewer)

```
I           What was interesting on group work?
S3          Well, everybody can express his/her ideas. Everybody
            calculates in a different way, so.
I           But you can do it also in the whole class discussion,
            can't you?
S3          Yes, that's true. But when it's in groups, it's more. I
            don't know, I think we're discussing it more.
...

S3          In one case none of us knew how to calculate it, we found
            it strange. But later we grasped it.
I           And do you think that it helped you that you could
            discuss it together?
S3          Yes, here definitely yes.
```

Interview with S1 (I labels the interviewer)

```
I           What do you personally find good on group work?
S1          Well, that the lesson is somehow livelier and we aren't
            just sitting and looking, but we can at least discuss
            with the others.
```

```
I           O.K., livelier, I understand, but is it also important
            from the perspective that you for example discover
            something when you're discussing?
S1          Yes, we have more ideas about it.
I           And it helps to find the solution to the problem.
```

5. Facing failures. In group discussion, students listen to other students' voices. They also learn that sometimes it happens that their effort to solve a problem may not be successful, that they may fail in the activity. This is a situation they will be facing repeatedly in their life and they must treat this situation not as an endpoint but as a stimulus to look for other solution strategies, using the lesson they have learned from the unsuccessful attempt. Of course this can also happen when the transmitter is the teacher. But natural school hierarchy influences how students see their failures face-to-face with the teacher. Although the didactical contract may have some effect on this hierarchy, it is still true that students feel more at ease if they fail within peer groups rather than when the teacher is involved. The advantage of the activity based on discussion among students is that after a failure they usually do not cease trying to find another way leading to the correct solution.

DISCUSSION AND SOME CONCLUSIONS

Illustrations of situations which were used in this text clearly show that it is impossible to study students' and the teacher's voices separately. The situation may be compared to the situation of an orchestra with a conductor and musicians. The roles both of the conductor and the individual musicians are clearly indispensable. The role of the teacher strongly resembles the role of the conductor. And even when the situation in the class looks like a concert without a conductor, it is never really so.

To follow in the line of the previous metaphor: in some cases the student can play the role of the conductor to her/his classmates (this role is referred to in the previous text as the transmitter). However, when this happens we see that the course of the concert can change. The "musicians" are much more open when expressing their doubts and ambiguity and if they do not understand the situation they ask for further explanations. They are not influenced by the unerring authority that the teacher represents for them. The student transmitters are more likely to try several versions of explanations using language that is more comprehensible to peers in which may in fact promote deeper understanding. However, overall the transmitter's role is influenced by the didactical situation in the classroom. S/he does not create a new didactical situation.

Within the group activities and report back, the teacher's role is crucial even if it is not always explicit. Even when it is the student's activity which is in the central position, the student must not be let down. As part of preparing that substantial and stimulating learning environment for the students the teacher must make the decisions on how the problem will be presented to the students, what forms of representation will be used, how much space the students will be offered for discussion of the problem, and which student strategies will be supported.

The teacher in this case study was exceptionally sensitive to students' voices in all their possible forms. Not only did she work with students' suggestions on how to solve a given problem, but also she reacted without hesitation to the unforeseen situations arising in consequence to other influences than mathematics. Her reactions do not merely reflect experience of a teacher of mathematics; they are also motivated by her deep knowledge of her students and behaviour of the class. The teacher reacted to her students' voices not only verbally but if necessary also by changes in the intended lesson plan. This was transparent in all the observed lessons and the post lesson interviews.

To conclude we may say that facilitating students' individual discoveries (a-didactic situations in school practice) makes strenuous demands on a teacher's competences, especially in the area of psychology, pedagogy, content knowledge, but also in the area of class management.

NOTES

[i] The transcripts from the classroom are labelled as follows: CZ 3 (3rd Czech collection of data based on LPS design), L02 (2nd lesson), time of the start of the episode.

[ii] In the 3rd lesson, students were asked to express radius r from the formula for circumference $l = 2\pi r$. Students suggested several formulas for r. Following a short discussion three were singled out as possible. The conversation proceeded as follows:

```
T If I put there concrete numbers would it help? What do you
  think?
S Yes.
T Let's try it. Well, let's say what we know. We know the
  circumference, let us choose 15 cm for l. Find the radius r of
  such a circle.
```

After having solved this problem with concrete numbers, students were able to decide which of the formulas on the blackboard was correct.

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CHAPTER NINE

Developing Mathematical Proficiency and Democratic Agency through Participation – An Analysis of Teacher-Student Dialogues in a Norwegian 9th Grade Classroom

INTRODUCTION

A central line of argument within mathematics education has been that learning mathematics provides individuals with tools to make considered choices, and that developing mathematical proficiency is beneficial because it informs human individual actions. In line with this argument, it is claimed that a mathematic literate population will contribute to society's political, ideological, and cultural maintenance and development, and as such, strengthen a nation's democratic processes (Niss, 1996).

Fostering democratic citizens is an important overarching educational goal in many countries and training students in communicational processes is considered to be one of the ways of achieving such a goal (L 97).[i] Communicational processes means providing opportunities for participation in social interactions, for sharing thoughts with others and listening to others share their thoughts.

Within the social sciences, having the ability to base one's actions on deliberate choices is expressed through the concept of *human agency* (Bandura, 2001). Building on the concepts and arguments from Bandura and Niss, in this chapter we will use the term *democratic agency* to denote the capacity to make decisions and to take actions in relation to social, cultural and political issues. In the classroom, taking part in the ongoing discussions, making judgments, formulating arguments and listening to fellow students are likely to be important elements in students' development of democratic agency.

Participating in classroom discussion, being trained in communicating one's own ideas and reflecting with fellow classmates is also seen as central to developing mathematical learning with understanding (e.g., Hiebert et al., 1997), which is said to be crucial for the development of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001). Communicating and thinking together about mathematical ideas and problems thus is likely to be critical for both the development of democratic agency and mathematical proficiency.

In Nordic curricular documents (L 97; K 06; Skolverket, 2011) there are multiple statements which emphasise the importance of developing both students' democratic agency and their mathematical proficiency. These competences are also closely linked, that is "an active democracy needs citizens that can … understand and critically evaluate quantitative information, statistical analysis and economical

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 143–160.

prognosis."[ii] Our objective in this chapter is to discuss the challenges that a mathematic teacher faces when trying to comply with this two-fold demand of the curriculum. How can students be educated/taught in order to develop both their mathematical proficiency and their democratic agency? What characterises the communicational processes in the classroom? To what degree does the teacher use student utterances to stimulate and develop mathematical proficiency and democratic agency?

Our analysis is based on teacher-student dialogues in a specific lesson in one Norwegian LPS classroom (NO2-LO1). We will also refer to statements from the follow-up teacher interview and to findings from analyses conducted on all of the thirty-eight 9th grade mathematics lessons constituting the Norwegian LPS-sample. How the teacher secures broad student participation and handles students' initiatives will be central issues in our discussion, but we will also comment upon the tasks selected by the teacher and discuss if these tasks support the development of students' mathematical proficiency.

THEORETICAL PERSPECTIVES

Socio-Cultural Theory

Within socio-cultural theory, learning is viewed as becoming a participant in a certain discourse comprising the totality of communicative activities practised within a given community (Sfard, 2000, 2006). Van Oers (2006) maintains that "learning in an activity theory approach is the extension or improvement of the repertoire of actions, tools, meanings and values that increases a person's abilities to participate autonomously in a socio-cultural practice" (p. 24).

Lave and Wenger (1991) have used the expression "legitimate peripheral participation" to account for the processes of learning by which a newcomer successively moves from a peripheral to a full participation in communities of practice. In the process of participating in collective activities Renshaw (1996) states that the opportunity to use speech with others is central to conceptual development. With regard to the learning of mathematics, Sfard (2006, p. 166) claims that:

> … the idea of mathematics as a form of discourse entails that individual learning originates in communication with others and is driven by the need to adjust one's discursive ways to those of other people.

Communication and collaborative activities as important tools for the learning of mathematics, have within mathematics education also been linked to the idiosyncratic cultural and historical aspects of this particular field of theoretical and practical knowledge. Cobb (2000) sees mathematics as a complex human activity and, leaning on Dörfler (2000), states that the task facing the teacher is that of supporting and organising students' induction into the practices that have emerged during the discipline's intellectual history. Yackel and Cobb (1996) have introduced the concept *socio-mathematical norms* to denote the normative aspects of classroom action and interaction that are specific to mathematics. They claim that these norms are interactively constituted, that they regulate mathematical argumentation, and influence learning opportunities for both the students and the teacher. The teacher is seen as a representative of the mathematical community and students' mathematical communication and reasoning viewed as acts of participation in communal practices that are established through the ongoing interactions in the classroom.

Mathematical Competence and Mathematical Proficiency

Two of the most influential descriptions of the concepts *mathematical proficiency* and *mathematical competence* have been provided by Kilpatrick et al. (2001), and Niss and Jensen (2002), respectively. The work of Kilpatrick's group has been used as a basis for informing the educational authorities in the U.S. Department of Education for the improvement of quality and usability of educational research (Ball, 2003). The work conducted by Niss and Jensen has been central to the refinement of the concept of mathematical literacy in PISA (OECD, 2009). The eight competences listed in the PISA Mathematics Theoretical Framework as constituting different aspects of mathematical literacy are as follows:

- Mathematical thinking and reasoning
- Mathematical argumentation
- Modelling
- Problem posing and solving
- Representation
- Symbols and formalism
- Communication
- Aids and tools

As noted by both Niss and Jensen (2002), and in the PISA framework (OECD, 2009), there is substantial overlap between these competences, and students are likely to draw on more than one of the competencies when solving mathematical problems. Of particular interest for this chapter are the connections between "Communication," "Representation," and "Symbols and formalism." Even though communication of mathematics does not necessarily involve the use of specific mathematical "tools," (e.g., symbols), if a goal is to develop students' communicative competences in mathematics, it is important to give them opportunities to be involved in mathematical discourses where mathematical symbols and representations are being used. This is closely connected to the central tenets within socio-cultural theory, where dialogues and participation in subject informed discourses are considered to be activities of key importance for student learning. For a student to be able to improve his/her participation in meaningful mathematical discourses, it is necessary to be introduced to mathematical symbols and formulas and to have opportunities to partake in classroom discussions applying subject specific concepts and categories.

Mathematical proficiency is a concept closely related to mathematical competence, as defined by Niss and Jensen. According to Kilpatrick et al. (2001), mathematical proficiency is made up of five strands:

– Conceptual understanding – comprehension of mathematical concepts, operations, and relations;
– Procedural fluency – skill in carrying out procedures flexibly, accurately and appropriately;
– Strategic competence – ability to formulate, represent, and solve mathematical problems;
– Adaptive reasoning – capacity for logical thought, reflection, explanation, and justification;
– Productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

As with mathematical competence, Kilpatrick et al. (2001) stress that these five strands are interrelated, representing "different aspects of a complex whole" (p. 116). They go on to argue that mathematical proficiency is a multi dimensional concept and that it cannot be achieved by giving attention to just one or two of these strands. Rather, they claim that the five strands provide a framework for discussing important notions related to mathematical proficiency, like knowledge, skills, abilities, and beliefs.

Democratic Agency

Carlgren, Klette, Myrdal, Schnack, and Simola (2006) argue that a particularly important trait of the Nordic comprehensive school system, characterised by being unified, unstreamed, and open for all students, is *individualisation* which includes the idea of the "active child" as well as attention to the needs of each child. As a result of the large school reforms throughout the 20th century, students from all socio-economic groups and with different socio-cultural backgrounds were given educational opportunities. According to Carlgren et al., teachers at this time adopted individualisation as the best way to accomplish differentiation within the mixed ability classrooms. In recent years, individualisation of working methods has been particularly widespread in Swedish and Norwegian classrooms. As a result, Carlgren et al. argue that the "idea of the educated citizen seems to have been replaced by the separated individual responsible for his/her own life" (p. 303). Furthermore, Carlgren at al. claim that individualisation is expected to strengthen each student's belonging to his/her community and his/her ability to be actively involved in civic activities. In this respect the practice of individualisation can also be considered as a tool for the development of students' democratic agency. This belief in stimulating personal active involvement can be linked to the central tenets of social-cognitive theory. Bandura (2001) contends that the power to shape actions for specific purposes is the key feature of personal agency. He argues that the challenge in collaborative activities is "to melt diverse self-interests in the service of common goals and intentions collectively pursued in concert" (p. 7).

In the Norwegian core curriculum, there are several statements aimed at defining the overarching goals for the national compulsory school system. Many of these statements stress that education must contribute to the development of personal agency as a pre-requisite for participating fully in a democratic society. These statements are closely connected to central and fundamental thoughts within western political philosophy related to beliefs in democratic institutions and civil rights. Statements related to a fundamentally scientific worldview and to the training of abilities that will secure high proficiency levels in academic subjects can also be found:

> Education in this … tradition entails training in thinking – in making conjectures, examining them conceptually, drawing inferences, and reaching verdicts by reasoning, observation and experiment. Its counterpart is practice in expressing oneself concisely – in argument, disputation and demonstration.

With reference to the last part of this statement, this is largely in line with constructivist and socio-cultural learning theories where participation is considered to be a crucial factor for students' learning (Yackel, 1995; Cobb, 2000; Van Oers, 2008). It also links the training of academic skills to key aspects of democratic agency. Students should not only get the opportunity to learn academic subjects, like for instance mathematics, but they should also be trained in actively using and communicating their knowledge socially.

Giving the students the opportunity to express their thoughts through participation in classroom discussions is strongly recommended in the Norwegian curriculum. Competency in expressing oneself verbally (oral skills) is one out of five "basic skills" in the national curriculum, (the other ones being "writing skills," "digital skills," "arithmetic skills" and "reading skills").

A prerequisite for developing "oral skills" in mathematics seems to be that the instructional formats used in the mathematics lessons are not dominated by individual work only, but also by formats supporting oral communication. Later in this chapter we will present findings related to the analysis of instructional formats across all the 38 mathematics lessons included in the Norwegian LPS-study. In the method section of this chapter, the analytical dimensions applied will be further explained, but some considerations related to the term whole class instruction will now be introduced.

Whole Class Instruction

Individual seat work and whole class instruction have been the traditional cornerstones of mathematics lessons. In TIMSS it is documented that on average about 70% of activities in mathematics classrooms in the participating nations consisted of these two activities (Mullis, Martin, Gonzalez, & Chrostowski, 2003). In TIMSS individual seat work is defined as "working on problems with or/and without teacher guidance," while whole class instruction is categorised as "listening to lecture-style presentations and/or reviewing homework."

To what degree students are given opportunities to actively participate in classroom discourse seems to be a particular interesting aspect of variations related to whole class instruction. As argued in the introduction of this chapter, communicational processes are seen as essential both for the development of democratic agency and mathematical proficiency.

Several studies have reported a high degree of teacher dominance and little student involvement during whole class instruction (e.g., Stodolsky, 1988; Hiebert & Wearne, 1993). In the analysis of public talk in the mathematics classrooms that were included in the TIMSS 1999 Video Study, the ratio between teacher and student talk – as measured in number of words spoken – was found to be quite high in all countries, varying between 16:1 in Hong Kong SAR, to 8:1 in the USA (Hiebert et al., 2003). However, lately some classroom studies have reported findings which indicate higher levels of student participation. Clarke and Xu (2008) compared patterns of utterances in mathematics classrooms in six nations. They report some interesting differences both related to frequency, to mathematical content of utterances, and to opportunities for student participation. Emanuelsson and Sahlström (2008) analysed and compared teacher-student dialogues in a U.S. and a Swedish mathematics classroom and report several deviations in patterns of student participation and in possibilities for influencing the content of the conversations during whole class instruction. In the U.S. classroom the teacher generally exerted a strict control of the discourse and the activities. In the Swedish classroom students' utterances seemed to influence the discourse and the teacher's instruction to a large extent. However, Emanuelsson and Sahlström (2008) argue that a consequence of high levels of student participation in the observed classroom was that "… the mathematics gets lost in the tangle of talk" (p. 218).

METHODS

The analysis in this chapter is based on empirical data collected by the Norwegian research group as part of the Learners Perspective Study (LPS) (Klette, 2009), and the characteristic design of the LPS study has been followed closely (see Clarke, Keitel, & Shimizu, 2006). Our present analysis is anchored in video data from one particular lesson in one particular mathematics classroom, and in data from the follow-up interview with the teacher. In addition to this we will present selected findings from a quantitative analysis performed on all the 38 lessons of the Norwegian LPS-study. This is done to provide an overview of some central traits characterising the teaching of mathematics in these classrooms, especially with regard to dimensions such as "main instructional format," "forms of communication" and "teacher-student relation." It is argued that information regarding these dimensions is relevant as it affords a background for the discussion of the selected classroom episodes with regard to the main themes of this chapter, challenges related to developing mathematical proficiency and democratic agency for all students.

A Quantitative Three Level Analysis of All Mathematics Lessons

All the video captures from the 38 lessons were analysed in the software program Videograph (Rimmele, 2002) by the use of theory-based categories developed by the Norwegian research team (Klette et al., 2005; Ödegaard, Arnesen, & Bergem, 2006). This analysis was carried out on three different levels, but only the categories relevant for the present analysis will be presented in this chapter. The coding was initially done in intervals of one second only, but later aggregated on the basis of one-minute intervals. This means that the code that dominated each minute "got" this minute. The way the coding was conducted makes it possible to present findings on different levels: one-lesson level; classroom level; or an all-comprehensive level, i.e., aggregating the results from all 38 lessons. In this chapter only a few aggregated results from the whole study will be presented.

At the first level of analysis the lessons were coded with regard to the following instructional format categories:

– Whole class instruction
– Individual seat work
– Group work

In the second level of analysis the characteristics of whole class instruction were further investigated. Several sub codes for this main category were applied, and the ones relevant for our analysis were the following:

– Dialogical Instruction: Use/mobilise students' knowledge for instructional purpose.
– Task Management: Teacher gives verbal/non-verbal instructions regarding assignments and class projects (grouping, material resources).
– General Messages: general messages and comments of classroom business.

It should be noted that *Dialogical Instruction* here implies that the students are actively involved and not only listening to the teacher's exposition.

At the third level of analysis subject specific categories were applied. Of particular interest for the issues discussed in this chapter are the findings related to the category named *Features of Dialogue*, consisting of the following three codes:

– Student initiatives: Students make comments or ask questions that initiate class discussion.
– Teacher exposition: Teacher presents or explains something monologically.
– Teacher initiatives: Teacher asks questions in order to mobilise student knowledge.

The findings from the three level quantitative analyses will be presented and discussed in relation to our main theme in a later section, and constitute the background to our main qualitative analysis of classroom interactions.

A Qualitative Analysis of Teacher Instruction and Classroom Interaction and Communication

Our qualitative analyses of teacher instruction and teacher-student interaction and communication are, as previously mentioned, based on video captures of one particular mathematics lesson in one specific 9th grade classroom. However, the lesson was taught twice as the class was split, and episodes from both lessons will be presented. This class was part of a lower secondary school situated in a suburban area characterised as being socio-economically and ethnically diverse, but mainly middle class Norwegian. The teacher described the proficiency level in this class as average, as assessed by national standard examinations. The mathematics teacher was well qualified. She was a certified teacher with several years of teaching experience, and she had recently completed a one-year study in mathematics education.

The criteria for our selection of episodes, excerpts and quotations were based on their relevance for the issues discussed in this chapter: the development of mathematical proficiency; and democratic agency. In addition to episodes from the particular lesson(s), a few quotations from the follow up teacher interview will be presented, selected on the basis of the same criteria.

FINDINGS

Findings from the Three-Level Quantitative Analysis

Based on the first level of analysis of all the 38 video taped mathematics lessons, the dominant instructional formats were found to be whole class instruction and individual seatwork. Nearly 95% of the lesson time in mathematics was used on these two activities, quite evenly distributed between the two of them. The remaining 5% of lesson time consisted of group work.

The second level of analysis revealed that almost 60% of the time allocated to Whole Class Instruction could be categorised as *Dialogical Instruction*, defined as "Use/mobilise students' knowledge for instructional purpose" (see code definitions above). This finding indicates that, when the teacher presented new mathematical themes, the students were generally given broad opportunities to participate and to get involved in the classroom discussions. As we developed from the literature communicational processes – sharing thoughts with others and listening to others – is critical for both the development of democratic agency and mathematical proficiency. At a first glance then, our findings suggest that the Norwegian LPS teachers handle this aspect of classroom instruction with success; with students involved in the activities to a large extent. In terms of developing students' democratic agency, affording opportunities for participation seems to be crucial. However, securing the development of mathematical proficiency presupposes that the teacher orchestrates classroom activities which involve more than one strands of this concept (Kilpatrick et al., 2001). This will be discussed in the qualitative analysis of the teaching-learning processes in the classroom (see below).

Figure 1 presents the percentage of time used on classroom discussions in mathematics lessons, under the category "Features of Dialogue." The following three sub codes were used: student initiatives; teacher initiatives; and teacher exposition. It should be noted that it was not coded for frequency of initiatives, but for time used on discussing themes raised through student or teacher initiatives, or through teacher exposition. The latter code is here defined as the teacher speaking for a minimum of one minute. This was done to discriminate between the codes Teacher Exposure (TE) and Teacher Initiative (TI) with the intention of analysing to what degree the teacher would initiate classroom discussions through longer monologues (TE), or through quite short propositions (TI).

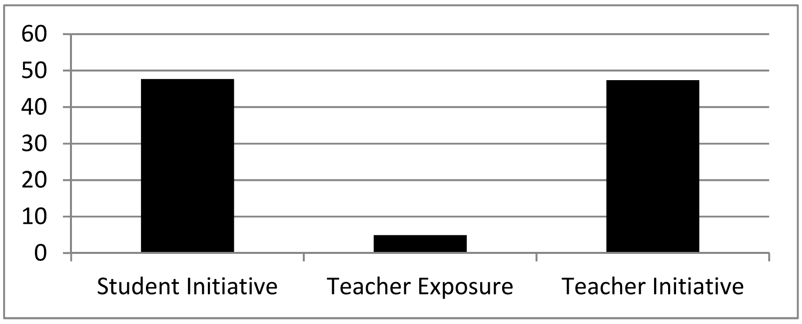

Figure 1. Features of dialogue; the percentage of time used on classroom discussions on the basis of student initiatives and teacher initiatives and the time used on teacher exposition

As revealed in Figure 1, a substantial amount of time in the Norwegian LPS classrooms was used on discussing issues raised through student initiatives. This finding indicates that the students' active involvement was given high priority, that students' opinions were valued and that students were being trained in communicating and participating in the mathematics lessons. All these are important pre-requisites for the development of democratic agency and mathematical proficiency. However, with regard to the development of mathematical proficiency, other strands of this concept also have to be activated. In the next section we will analyse particular episodes from one specific lesson from this perspective; investigating how the teacher responds to student initiatives and if other strands of mathematical proficiency come in to play.

Developing Democratic Agency and Mathematical Proficiency

As previously discussed, an important overarching educational goal expressed in the Norwegian curriculum relates to the development of democratic agency as evidenced by "critical abilities to attack prevailing attitudes, contend with conventional wisdom and challenge existing arrangements." Many student initiatives in our material can be related to this goal and a few illustrative examples are presented.

Example 1. The context of the first example is from an introductory lesson on equations where the teacher decided to use an apparently "very easy" text problem as her starting point. She wrote the problem on the blackboard and the teacher-student dialogue went like this:

Excerpt 1:

```
T       Per and Kari have five apples jointly. Per has two apples. How
        many has Kari?
Frank   Are we supposed to answer that?
Hanne   Are you kidding?
T       No kidding. We'll use very simple examples at this stage so
        that we understand how one can apply equations.
Hanne   This is childish.
T       This is childish, says Hanne.
```

In this dialogue the students challenge the teacher's use of the example to illustrate the theme of the lesson, an introduction to equations. It appears that they find the presented problem too easy and somewhat trivial – why use equations to solve this? Hence, they experience the teacher's choice of problem inappropriate. By referring to the question as "childish," Hanne indicates that she finds it inappropriate to be asked this kind of question and that it is not "problematic" for her, that is, she can solve this in her head and without using equations. In relation to the statements about democratic agency found in the curriculum, the students are clearly ready to express their opinions, even if this includes criticising the teacher's choice of illustrative examples and activities.

As the lesson continues another student, Alf, questions why one had to learn about equations at all:

Excerpt 2:

```
Alf     What is the point of doing equations? It's just; it's easier
        to write three plus two instead of dealing with this X.
T       Yes, but the reason we are doing this now is to learn, we will
        get more difficult exercises as we go on.
Alf     But what could we use it for?
```

Alf's comment initiated an extended classroom discussion (about 4 minutes) about why one should learn about equations. Many students participated eagerly in this discussion, expressing strong opinions about the value of learning about equations. Somewhat frustrated the teacher ended the discussion with the dictum that now they had to continue working on today's theme, "equations."

In this example we see how the teacher attempted to balance the students' rights to express their opinions, a central notion in the development of democratic agency, with the demands of the curriculum (to learn about equations). There are two main points here:

- The teacher's choice of example led to students doubting the purpose of equations.
- The authenticity of the example: it was not a "real life" example, but a contrived example (which did not work in this case).

This episode can also be related to Kilpatrick's concept of mathematical proficiency: the fifth strand, productive disposition, is referred to as the "habitual inclination to see mathematics as sensible, useful, and worthwhile." By questioning the use-value of equations, Alf challenged the teacher and his fellow students to come up with possible arguments "in favour" of learning equations. During the classroom discussion, and it should be noted that the teacher invited students to comment on Alf's challenge, mixed opinions were expressed. While some students agreed with Alf in the futility of learning about equations, other students argued that increasing one's mathematical knowledge, including knowledge about equations, could be useful in relation to future studies. The teacher added to this by emphasising that equations were an important theme in mathematics, and that the students were likely to value it once they had learned it. In the follow up interview the teacher commented on this episode:

> At the same time I want those students that would like to continue with maths and science in upper secondary and in their working life, that they … They are dependent upon equations.

So the teacher tries to argue both to the students in the classroom and in the interview the importance of viewing mathematics as "sensible, useful, and worthwhile." However, her choice of example to introduce equations does not seem to be in line with this argumentation, as it does not connect to the students' real life experiences and has little meaning for them, in particular in terms of learning about equations. In fact, it provokes student questions about the purpose of learning about equations.

This also links to Niss and Jensen's (2002) concept of mathematical competence. As pointed out by Blomhøj and Jensen (2007), it is the activity aspect of mathematical competence that is foregrounded by Niss and Jensen, meaning that a mathematical competent person should be ready to act with insight upon problems faced. The problem posed by the teacher in this lesson is not challenging for the students, and they are not invited to participate in a meaningful discussion involving mathematically relevant symbols and representations. Consequently, their communicative competences in mathematics were not stimulated and further developed.

However, this episode illustrates that the threshold for asking questions and challenging existing arrangements in this Norwegian LPS-classrooms was quite low. Students did not hesitate to critically comment on the teacher's statements or decisions and several incidents like the one presented were observed in other lessons in the Norwegian LPS material. Again, creating an open classroom climate where critique is tolerated and welcomed seems to be essential for the development of students' democratic agency. But these elements do not necessarily secure the development of mathematical competence/proficiency in the ways these skills are defined by Niss and Jensen (2002) and Kilpatrick et al. (2001) respectively.

Example 2. The interwining of two strands of mathematical proficiency – Productive disposition and Conceptual understanding is illustrated in the teacher's perception of the nature of mathematics, as formulated in the post lesson interview:

> … mathematics is a lot of numbers, and if you put it into words, or write or explain it to others, it might make things clearer. Then they get to see how to use equations for other things than just mathematics. But that is a method of problem solving. No matter what, if you have a problem you can set up an equation!

Later in this interview she explains how she intends to stimulate and widen the students' conceptual understanding of the concept "equations." She argues that in order to have the students learn more about equations they should apply it when working on various kinds of mathematical problems:

> We can't demand of the students to understand "equations" after just one week. They have to work on it. It has to be processed. We have to make use of equations in other topics, after we have learned about equations. When we start on volume for instance; then they are calculating volumes and stuff like that. Then they can apply equations on the formulas, and then they will understand a bit more of how to make use of equations.

Based on lesson observations it can be argued that the teacher provides opportunities for the development of students' conceptual understanding by using the following:

– Different representations (e.g., the scale drawing; apples; words; algebraic notation; numbers);
– Different explanations (students' and teacher explanations);
– Different solutions (e.g., pupils presenting their solutions on the board).

However, it is not clear that the teacher used these opportunities purposefully. Looking at her use of examples in the introductory lesson, these did not seem to trigger student curiosity of the concept of equation, nor did these examples seem to stimulate or deepen a conceptual understanding by clarifying important aspects of the concept. Indeed, when faced with a word problem, the students were guided by the teacher in following particular procedures. By remaining at a purely procedural level, the students were not given the opportunity to develop the conceptual strand of mathematical proficiency ("Conceptual understanding"). One could argue, however, that the teacher provided opportunities for developing procedural fluency, defined by Kilpatrick et al. (2001) as "skill in carrying out procedures flexibly, accurately and appropriately" (p. 5).

In summary, the way the teacher argued in the interview seem to indicate that she wanted to give the students a "handle" on equations by teaching them how to go about an easy word problem. However, given that they could solve the problem in their heads it did not make sense for pupils to utilise equations in the solution process. In addition, her teaching of the "equations" stayed at a rather procedural level, where students were told what to do and how.

Example 3. Another goal in the Norwegian core curriculum is formulated as:

> Skill in scientific thinking and working method demands the training of … the ability to wonder and to pose new questions;

The way this goal is expressed illustrates the close link between the training of scientific thinking and the development of democratic agency. Both these skills demand an open minded and a critical attitude. There are many examples in our video material of students asking questions that demonstrate the ability to wonder. As previously mentioned, the class that we have been focusing on was split in half during all mathematics lessons, and the episode we now present is from the lessen where the teacher taught 'equations' to the second group. The teacher followed more or less the same script in both lessons and after some initial talk about the use of algebra and equations, she again presented the easy word problem ("Per and Kari have five apples jointly. Per has two apples. How many has Kari?"). Here is the dialogue that followed:

Excerpt 3:

T	And then I write X instead and that equals five. Do you all understand that we can write it down in this way? X is the unknown, the answer we are going to find and it represents how many apples Kari has.
Peter	But what is the known?
T	What is the known? Yes, what is that? What is known in this? Nina?
Nina	How many apples Per has and how many they have jointly?
T	How many apples Per has, because that is hers (points at the board), and how many they have jointly, right Peter? That is five, and that is what we know and this is the unknown (points at the board again).
Peter	Don't we have to have a letter for that [the known] (pointing to the blackboard)?
T	No, we don't, because that is not something that is unknown.

Peter's first question seems very relevant in relation to the teacher's explanation. It is directly related to the teacher's use of the concept of "the unknown," a concept that is an integral part of a mathematical discourse about equations at beginners' level. Peter's spontaneous challenge to the teacher's choice of expression might seem naïve within a purely mathematical discourse, but forces the teacher to elaborate on the mathematical meaning of the concept of the "unknown" and relate it more clearly to the actual problem being addressed. The teacher first passed Peter's question on to the rest of the class, and got an adequate answer from Nina. Following up on Nina's response, the teacher revoiced (Franke, Kazemy, & Battey, 2007) to strengthen the explanation. However, Peter still wondered if the "known" did not qualify for having its own letter, when the "unknown" was granted one – a legitimate question in terms of mathematical thinking. Illustrating the student's ability to "wonder and pose new questions," it is an example of Peter's growing scientific attitude and skills related to democratic agency. However, in terms of developing mathematical proficiency, the opportunity to explain and elaborate on concepts such as "variable" (in connection to unknown) and "constant" was not taken up by the teacher. This potentially rich situation was not taken advantage of, nor exploited in terms of stimulating and developing students' conceptual understanding.

In terms of social norms in this mathematics classroom, it appears that these include opportunities for everybody to ask questions and contribute in class discussions. These are also important elements in developing students' mathematical proficiency, if they are linked to mathematical concepts and activities. Moreover, to develop a culture of "learning mathematics with understanding" (Hiebert et al., 1997), where student curiosity is triggered, we need to also develop socio-mathematical norms (Yackel & Cobb, 1996). That is, opportunities for learning must be grounded in discussions about mathematically significant concepts, and cognitively demanding questions and activities. However, this is not easily achieved. It requires appropriate teacher knowledge and careful lesson planning. The tasks and problems that are used as a basis for classroom discussions must be carefully selected. Boaler and Humphreys (2005) claim that teachers often seem to have a different approach; frequently introducing tasks that students can solve with a minimum of cognitive effort. As a consequence, the classroom discussions that follow will often be "unchallenging" and are not likely to contribute to the development of student mathematical proficiency.

Example 4. Another important principal in the Norwegian core curriculum is expressed in the following paragraph:

> Education shall contribute to the building of character that gives individuals the strength to take command of their own lives.

To give the students the opportunity to take command of their own lives presupposes a non-authoritarian relationship between teacher and students. Even if teacher and students often take on different roles there are many episodes in our video material that demonstrate an open classroom climate where everybody is respected. As an example of this, many students from N01 asked the teacher if they could come up to the front of the class and explain their work on the blackboard. In the spirit of egalitarianism the students could also decline to present their work on the blackboard, when asked by the teacher.

Excerpt 4:

T	Now we'll check the answer on one of the tasks. Who would like to come up here and check the answer? On this task: five plus two X equals 25? Is there anyone who would like to do it? Julie, would you like to have a go?
Julia	No

If the students refused to comply with the teacher's requests, the teacher would ask another student. These participation norms illustrate the egalitarian relationship between teacher and students, an important prerequisite for establishing an open classroom climate and for the development of democratic agency.

DISCUSSION OF FINDINGS AND CONCLUSIONS

In our study the teacher's views on how to develop student learning of "equation" guided her use of specific instructional strategies. These included the following: "simple" word examples as introductory activity; particular representations to "picturise" equations; and her ways of setting an "equation" question out on the board. We contend that these selected strategies are likely to develop mathematical competence or proficiency, as outlined by the literature (Kilpatrick et al., 2002; Niss & Jensen, 2002).

Moreover, it can be argued that in this classroom the kinds of task (for introducing equations) set the foundation for the teacher's instruction, and it is likely that a different kind of task would have led to a different kind of instruction and a different kind of classroom discourse. At the same time the learners, and amongst them individual pupils, also voiced their preferred ways of tackling the mathematical question posed, and it appeared that the task (used to introduce the theme "equation") was not challenging for students, and hence, did not stimulate meaningful mathematical discussions. In fact, the selected tasks did not seem to offer pupils sufficient opportunities to reflect and communicate; these tasks were too easy (in terms of finding the solutions), and they were not genuine authentic problems for these learners. It is known from the literature (e.g., Hiebert et al., 1996) that appropriate mathematical tasks are those that make the mathematics "problematic" for pupils; problematic in the sense that pupils regard the task as an interesting problem, for them, something worth finding out, "something to make sense" to them (Hiebert et al., 1997; Boaler & Humphreys, 2005). An important finding from the IPN Video Study (Seidel, Rimmele, & Prenzel, 2005) was that in classrooms with high-quality classroom discourse, students are more motivated and intrigued to find things out. In the present classroom many students reacted to the non-challenging task with raised voices and arguments indicating that they did not find learning about equations to be "sensible, useful and worthwhile". However, developing an inclination to see mathematics as sensible, useful and worthwhile is an important part of mathematical proficiency, as described by Kilpatrick et al. (2001).

The differences in views between the teacher and many students regarding the value of learning about equations provide evidence of relatively well developed skills in democratic agency amongst the students. They did not hesitate to speak up, to argue or to "take actions" in relation to the emerging issues. As such, even though this analysis is based on episodes from one lesson only, it illustrates that the students had previously been given opportunities for articulating and expressing their opinions. The social norms developed in this class clearly included opportunities for everybody to contribute to and participate in ongoing discussions.

This is also documented in other episodes: students could agree or refuse to come to the board when asked by the teacher; they could probe for deeper understandings ("what is the known"); or simply answer the teachers' questions. However, creating learning "communities of practice" within the mathematics classroom, based on democratic participation and mathematical proficiency, requires that other strands of mathematical proficiency come into action and that communicational processes are extended to include other patterns of dialogues between participants (e.g., pupils develop questioning for their peers, feedback to each other, etc.). Establishing an appropriate classroom culture, where problematic mathematical tasks are tackled, depends on learners participating and engaging themselves as members of the group, and learning opportunities for deeper mathematical understanding arise as different ideas and views are expressed. Thus, developing democratic agency should not be an add-on, or optional to developing a rich and fully functioning learning community in mathematics, but rather an integral part of a classroom that attempts to foster mathematics learning with understanding and mathematically proficient students.

Boaler and Humphrey (2005) argue that the effectiveness of teaching and learning situations depend upon multiple factors: the actual students involved; the curriculum materials and tasks; and also on the myriad of decisions taken by the teacher during the mathematics lesson. They claim that teachers traditionally have been offered general educational principles, abstracted from subject specific issues and that this leaves the teacher to translate these principles into actual practice. As our analysis of selected episodes have illustrated, this can be very challenging and demanding for the teacher, for example to carefully plan the classroom activities, in particular the tasks and examples used to introduce a new mathematical topic area. Ultimately, this indicates that an important challenge for teacher education and professional development is to provide opportunities for pre- and in-service teachers to discuss subject-related and didactical challenges, and at a detailed level. Video-based studies, like LPS, can offer opportunities for analysing classroom practices that are particularly apt for teacher learning.

NOTES

[i] L97 is the core curriculum for primary, secondary and adult education in Norway. While subject curriculums have been revised since 1997, the core curriculum has been kept unchanged.

[ii] On the website http://www.udir.no/K106/MATI-03/Hele/Formal/ the objective for mathematics (1-13) is formulated (2006 Norwegian mathematics curriculum).

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CHAPTER TEN

Matches or Discrepancies: Student Perceptions and Teacher Intentions in Chinese Mathematics Classrooms

INTRODUCTION

Efforts to pursue high-quality mathematics teaching have led to ever-increasing research interests in exploring the practices of mathematics classrooms in high-achieving countries, including China. There have been studies on how Chinese students learn mathematics (Fan, Wong, Cai, & Li, 2004), how Chinese teachers teach mathematics (Li & Huang, 2012), and the characteristics of effective mathematics teaching valued by Chinese teachers (Cai & Wang, 2010; Huang, Li, & He, 2010; Li, 2011). These studies have contributed to the understanding of mathematics teaching from different perspectives. Yet how Chinese students perceive their classroom learning and whether their perceptions are in line with teachers' intentions are largely unknown (Mok, 2006). Since learning is essentially the students' endeavour, exploring those classroom events which students perceive as important is crucial for enhancing our understanding of the learning processes. In particular, given the commonly recognised features of Chinese mathematics instruction such as well-structured lessons, polished teaching, well-disciplined classrooms (Leung, 1995, 2005; Stigler & Stevenson, 1991), and learner-trained learning (Cortazzi & Jin, 2001), what levels of consistency between teacher intended important events and students' perceived ones would be expected? This chapter provides an analysis of empirical classroom data to explore the relationship between student and teacher perceptions of important events.

The comprehensive data set from the Learner's Perspective Study (Clarke, Keitel, & Shimizu, 2006), which includes lesson plans, videotaped lessons, and post-lesson video-stimulated interviews, provides us with an opportunity to investigate this crucial relationship between the teacher's intended important events and the students' perceived important events. Our exploration is designed to address three issues. First, what are students' perceived important events within a mathematics lesson? Second, to what extent are the students' perceived important events in line with the teachers' intended important events? Third, do the matches between teacher intention and student perception, if any, contribute to achieving student learning goals?

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 161–188.

THEORETICAL CONSIDERATIONS

Chinese mathematics instruction has been explored sporadically over decades. The majority of studies have focused on the features of mathematics classrooms (Huang & Leung, 2004; Leung, 1995, 2005; Stigler & Stevenson, 1991). Recently, researchers have examined teachers' beliefs about effective teaching in order to gain a better understanding of the mathematics classroom (Cai & Wang, 2010; Huang et al., 2010). In addition, beyond examining mathematics classrooms, an effort to investigate the ways of improving mathematics teaching in China has been made (Li & Huang, 2012).

In this section, we first summarise the main findings of the characteristics of mathematics classrooms in China and Chinese teachers' beliefs about effective teaching. Then, based on these studies, we propose a framework for examining students' perceptions of classroom events and the relationship between teachers' intentions and students' perceptions.

Characteristics of Mathematics Instruction in China

Existing research has documented characteristics of teaching practices in China. In particular, a number of comparative studies at grades K-8 have revealed key features of Chinese mathematics instruction. When contrasted with U.S. mathematics lessons, researchers reported Chinese mathematics lessons as being more structured (Stevenson & Lee, 1995). The structure of the Chinese lessons has been described as occurring in four main stages: (i) revising work that students learned in the previous lesson; (ii) introducing and developing the topic of the lesson; (iii) demonstrating and discussing classroom exercises; and (iv) summarising and assigning homework (Huang & Wong, 2007; Leung, 1995; Lopez-Real, Mok, Leung, & Marton, 2004). Within the process of introducing and developing the lesson topic, researchers described the polished nature of the lessons (Stevenson & Lee, 1995), including an emphasis on lecture-dominated whole classroom instruction, careful explanations of new topics, and coherently constructed lessons (Huang & Li, 2009).

In addition to describing the nature of the Chinese lessons, researchers have sought to document the involvement of the students in the lessons. Through comparative studies of Chinese and U.S. classrooms, researchers documented that Chinese students were more involved in the mathematics tasks posed by the teacher (Stigler & Perry, 1988). Within the teacher-controlled class, student-centred features appeared (Lopez-Real et al., 2004; Mok, 2006) as the instruction emphasised students' engagement in mathematical reasoning and making mathematical connections (Huang & Li, 2009). These connections were formed among problems and exercises with variations. Students' practicing with systematic variations of problems was applied not only to form and consolidate new concepts and procedures but also to master knowledge and develop students' problem-solving ability (Huang, Mok, & Leung, 2006).

Effective Mathematics Instruction from Teachers' Perspectives

With key features of Chinese mathematics teaching identified, strengthening our understanding of these features can occur through consideration of the teachers' perspectives. Linked to the structured nature of lessons, researchers report that Chinese teachers value comprehensive and feasible learning goals along with the desire to develop knowledge coherently (Huang et al., 2010; Zhao & Ma, 2007).

In addition, Chinese teachers' perspectives on student involvement in the learning process support the previously described student-centred features (Huang & Li, 2009; Lopez-Real et al., 2004; Mok, 2006). Specifically, Chinese teachers indicated that they value engaging students in the instructional process and developing students' mathematical thinking and ability (Huang et al., 2010; Zhao & Ma, 2007). Li (2011) further enhanced these results by documenting elementary teachers' desire to emphasise students' participation and their understanding. This desire helps to explain the value teachers place on identifying and providing adequate treatment of difficult content points (Huang et al., 2010; Zhao & Ma, 2007). One way Chinese teachers support their students' learning is to emphasise abstract reasoning after using concrete examples (Cai & Wang, 2010).

A Framework for Examining Important Classroom Events

Collectively, these studies on the characteristics of mathematics instruction in China and the Chinese teachers' perspectives on effective mathematics instruction provide a portrait of classroom instruction that includes, among other things, setting and achieving comprehensive learning goals, developing students' mathematical knowledge and mathematics reasoning, and striving for a balance between teachers' guidance and students' self-explorations. This view is incomplete, however, without including the perspective of the student. To deepen our understanding of the student perspective, we looked to the work of Li, Chen, and Kulm (2009) for support.

In their study on lesson planning, Li et al. (2009) found that Chinese teachers paid considerable attention to dealing with three essential content points when designing their lessons. Referred to as the *Three Points*, these essential content points included *the lesson important point, the lesson difficult point, and the lesson critical point*. The *important point* refers to the most fundamental and important content identified in the curriculum standards or teachers' instructional goals. In contrast, the *difficult point* refers to content that is typically difficult for students to understand and master. In addition, the difficult point may include content that might easily lead to student mistakes or confusion. As such, the difficult point is not absolute, but rather is relative, depending on the students' learning situation. Finally, the *critical point* refers to any content that plays a decisive role in the student learning. The critical point is the teacher's consideration of how to help the student navigate the mathematical terrain, to eventually achieve the learning goals while avoiding or overcoming the pitfalls that might arise. Based on Li et al.'s three points, a successful teacher should help students understand and master *important points*, that is, achieve the learning goals, through overcoming *difficult points* and making use of *critical points*.

In examining the usefulness of the *Three Points* (Li et al., 2009), Huang, Rowntree, Yetkiner, and Li (2010) found clear alignment between lesson plans and classroom instruction in terms of instructional goals and treatments of the three points. Moreover, Yang and Ricks (2012) described a model widely used in school-based teaching research groups for improving teaching and developing teacher expertise through examining the three points. As such, the three points appear to provide a useful lens for examining Chinese instruction.

In the present study, we adopted a two-tiered framework for analysis. The first level referred to the structure of the lesson. At this level, we were interested in whether students realised the importance of the different stages within mathematics lessons and whether students' important events matched with teacher intended important events within lessons. The second level referred to the nature of the shared important events. We examined whether these events were closely related to addressing the three points, namely, the important point, the difficult point, and the critical point (Li et al., 2009). It was our assumption that students should realise the important stages of lessons as crucial for the smooth delivery of the lesson. More importantly, it is crucial for students to recognise the lesson's important events that are closely related to the three points.

METHOD

Classroom Context

Within the LPS (Clarke et al., 2006) we worked with data collected from three classrooms in Shanghai. For this chapter, we draw on one classroom's data set. The selected class was chosen because of the completeness of its data set, which consisted of 15 videotaped lessons, lesson plans, three teacher interviews, and 30 student interviews. The set of 15 consecutive lessons featured in the data set lasted approximately 45 minutes each.

The selected classroom featured Mr Zhang as the teacher. Mr Zhang, an experienced teacher, was recognised as an effective teacher by a group of local Chinese mathematics educators. His class included 55 seventh-grade students and utilised the unified official textbook in Shanghai.

Data Sources

From the data set associated with Mr Zhang's classroom, three primary sources of data were identified for use in this study - teacher interviews, student interviews, and lesson plans.

Teacher interviews. Mr Zhang was asked to identify what he considered to be an important lesson from the sequence of 15 lessons at three occasions (L3, L7, and L15). Interviews on each of the three selected lessons asked Mr Zhang to describe his intentions in the lesson, identify the important events, share his perception of what he did, thought, and felt during the important events, and provide a self-evaluation of the lesson. The main prompt questions included:

– What were your goals in that lesson (lesson content/lesson purpose)?
– In relation to your content goal(s), why do you think this is important for students to learn?
– Playing the video at normal speed, stop the video when you think the clips are important. Then, please comment on: Why did you say this is an important event? What were you thinking at that moment? How were you feeling at that moment?
– Would you describe the lesson as a good lesson for you? What has to happen for you to feel that a lesson is a "good" lesson?

Student interviews. For each lesson, two students from a videoed focus group participated in a post-lesson video stimulated interview (refer to Clarke et al., 2006). The student interviews were designed to solicit students' thoughts regarding their expectations of the lessons. In addition, students were invited to identify important events in the lessons, share what they did, thought, and felt during the lessons, and comment on the lesson overall. The student interview protocol included the following:

– What do you think that lesson was about (lesson content/lesson purpose)?
– Do you think this is the best way of learning mathematics?
– What did you want to learn from this lesson? Do you have a similar expectation for every lesson?
– Fast forward the videotape until you find sections of the lesson that you think were important. Play these sections at normal speed and describe for me what you were doing, thinking and feeling during each of these videotape sequences. You can comment while the videotape is playing, but pause the tape if there is something that you want to talk about in detail.
– Would you describe that lesson as a good one for you? What has to happen for you to feel that a lesson was a "good" lesson?

Lesson plans. Lesson plans corresponding to each of the 15 consecutive videotaped lessons were available for analysis. The lesson plans corresponding to the three lessons about which Mr Zhang was interviewed were analysed according to the Three Points framework.

Data Analysis

In this section we describe the analysis procedures. Descriptions of procedures will be separated according to the two-tiered framework previously noted. In tier one, the description will focus first on student interviews and then on the alignment between teacher intended important events and student perceived important events. Then, in tier two, the description will focus on the essential content point analysis of lesson plans.

Tier one. The tier one analyses focused on lesson structure and occurred in two phases. The first phase involved the development of an initial code system for the students' interviews based on open coding (Strauss & Corbin, 1990) of all 30 interviews across the 15 lesson sequence. Specifically, students' identification of important events were categorised according to the nature of the events. Next, an experienced Chinese teacher who had taught in middle and high schools independently coded student interviews of the first five lessons. The inter-coder reliability between the teacher and the researcher was 85%. For those statements where there was disagreement, the coders had extensive discussions to resolve all disagreements. After that, a final code table was created and the teacher completed coding the remaining interview data. The important events identified by students were then classified into categories in terms of pedagogical nature.

In phase two of the analysis, we were interested in the consistency between the teacher intentions and student perceptions of important events in the three focus lessons identified by the teachers as most important. The first author reviewed the teacher's interview transcripts and identified the important events indicated by the teacher. Next, the researcher identified instances where there were matches between the important events emerging from the teacher's interview data and those emerging from the student interview data. Identified matches were checked with video footage to be sure that the teacher and student(s) referred to the same events. If this was the case, a match was identified and described. Through the description, the goal was to identify the importance of these common events, if any. A match of teacher intention and student perception was defined as an important event to which both teacher and students referred in the same video clips. In some instances, the teacher and student(s) may have referred to the same event but in different time intervals from the clips. These instances were still noted as matches. Discrepancies were noted when either the teacher described an important event that was not recognised by the students or vice versa.

Tier two. In tier two of the analysis we were interested in whether the matched important events within lessons were related to dealing with the three content points: the important point, the difficult point, and the critical point (Li et al., 2009). To this end, the first author aimed to identify the three points stated by the teacher in the three lesson plans. In all lesson plans, the teacher explicitly stated the important point and the difficult point. The critical point, however, was implicitly reflected in the entire lesson plan. To keep the objectivity of the teacher's ideas, we elected to limit our analysis to the important point and the difficult point, which are coined as essential content points. Analysis proceeded to examine whether the matched events were related to the essential content points.

RESULTS

The findings are organised into three sections. First, the important events identified by students are presented and illustrated. Then, the matches and discrepancies of the teacher's intentions and students' perceptions are described. Finally, the extent to which these matches aligned with the essential content points (Li et al., 2009) is described. Throughout these sections, the video clips referred to by students are indicated in parentheses by lesson number and time within the lesson.

Students Perceived Important Events

As categorised in Table 1, the students recognised the following important events: introduction of new concepts or procedures, practicing new knowledge and skills, sharing students' work, and summarising key points.

Table 1. The category and example of important events

Category	Example
Introduction of new concepts and procedures	Introduce the procedure of solving systems of linear equations (L1, Daan, 9:42)
Practicing new knowledge and skills	Here, the teacher asked us to do classroom exercises individually. He circulated around. Sometimes, he came to our desk and helped us correct our mistakes; thus, we can increase our correct rate. (L2, Dabo, 24: 26)
Sharing of students' work	The teacher presented a correct answer and a wrong answer on screen. Thus, we can learn why the solution is wrong and how we can get a correct answer (L2, Dabo, 17:52)
Summary of key points	This is lesson summary. Mr. Zhang wrote the procedure of solving linear equation in two unknowns. It helps us to get a deep understanding of the procedures. (L7, Dove, 44: 12)

Note: L1, Daan, 9:42 indicates that in Lesson 1, the student Daan referred to an event in the video clip at 9 minutes and 42 seconds.

Introduction of new concepts or procedures. Twenty-eight of the 30 students noted the importance of introduction of new concepts or procedures a total of 47 times. The following responses of four students illustrate students' perceptions of events when new concepts or procedures were explored or introduced.

From Lesson 1, Daan identified the event of "introducing the procedure of solving system of linear equations" (1, 9:42) as important, noting that she was thinking of "how many solutions a linear equation may have." She further distinguished the importance of the "introduction of the concept of system of linear equation in two unknowns" (1, 11:41) and was thinking of how to find the methods of solving systems of linear equations at that moment.

In reference to Lesson 2, Dabo noted that when the "teacher taught us how to draw a Cartesian plane, number lines, and taught how to find the coordinate of a given point" (2, 12:36) this was important. In reflecting on this same lesson, Dana reported:

Dana	First, here, the teacher told us some concepts. For example, horizontal line is called x-axis; the vertical line is called as y-axis. The intersection point of these two axes is the original point. When you understand these concepts, you will be able to understand others, and understand the content of the lesson (2, 12:56).

As he reflected on lesson 7, Franc explained why the event was important and what he had learned:

Franc	It is important because the teacher was telling us. The teacher told us when elimination method by addition is used and when the elimination method by subtraction is applicable (7, 27:04).
INT	Then, when addition method should be used?
Franc	When coefficients of an unknown are contrary, then, add these two equations together; we can use addition method directly.
INT	When elimination by subtraction should be used?
Franc	If two coefficients of an unknown are the same, then, we can add two equations.
INT	Very good. What did you do at that moment?
Franc	I was thinking how to synthesise these points independently.

Based on the above conversation, Franc not only recognised the importance of introducing the new algorithm but also gave details of the algorithm.

Practicing new knowledge and skills. One third of the 30 students interviewed reported the importance of doing classroom exercises – on a total of 13 occasions. Some students focused on the forms of organising activities (individual or group), while others highlighted multiple solutions. In each instance, however, students referred to occurrences during the lesson time dedicated to practicing new knowledge and skill. As before, four students' responses will be shared to illustrate this category.

In lesson 2, Dabo reflected on the opportunity to complete individual work with the support of the teacher's guidance. "Here, the teacher asked us to do classroom exercises individually. He circulated around. Sometimes, he came to our desk and helped us correct our mistakes; thus, we can increase our correct rate" (2, 24:26). With regard to this same lesson, Ever pointed towards the importance of discussion in a group:

Ever	During the class, we were normally very intensive in learning; during the discussion, everyone was a little relaxed since interaction among students are more comfortable than that between teacher and students. When discussing with our peers, even though we said some wrong, we did not feel embarrassed. When answering the teacher's questions, we feel nervous because we worry about making mistakes. When discussing with classmates, it does not matter if we make mistakes since we are at similar level. We thus feel a little relaxed. Learning in

	a relaxed environment, without any press, will be more efficient (2, 23:33).
INT	Do you think that discussion in group is always better than teacher's telling?
Ever	Teacher's telling is important also. If students' discussion cannot make sense, the teacher tells us correct answers, and explains what is correct, what is wrong.

From Lesson 4, Easton further explained why group activities were important.

Easton	Here, the teacher asked us to discuss in groups. And let us to explain our answers. Thus, we know that two points determine a straight line. This concept, we feel very important (4, 27:40).
INT	Do you like the form?
Easton	Yes. Because we discuss in 4 students, we can consider many aspects and get more details.

Moreover, seven students mentioned the importance of finding multiple solutions through group activity. For example, Disney considered comparing different methods of finding a solution in order to find a simpler one. She said, "I was thinking that if there are several methods of solving a system of linear equations, there is a relatively simpler one" (12, 8: 47). She continued, "At that moment, I was thinking that eliminating x or y or z, because the coefficients are smaller, then computation will be easy."

Sharing students' work. Twenty-three of the 30 students mentioned the importance of teachers' explanations of and comments on students' solutions 39 times. Reported benefits from these events included checking solutions with those of the teacher and other students and getting different ideas through comparing different solutions. Statements from six students will be shared.

From Lesson 1, Dalia described why the teacher's explanation and comments were important.

Dalia	The teacher asked us which of these linear equations are linear equations in two unknowns or not? This is the main content of this lesson. It is important. Let us know that there are two unknowns with power of one in an equation. Based on these conditions, we can judge whether an equation is a linear equation in two unknowns or not (1,12:11).

She further explained, "The teacher asked us to discuss this problem. This means this is an important problem. Then, we discuss in-groups and tell the teacher our solutions. The teacher told which one is correct or not" (1, 13:05).

In addition, some students mentioned how they corrected their mistakes through comparing with peers and/or the teacher solution. For example, Dabo indicated, "The teacher presented a correct answer and a wrong answer on screen.

Thus, we can learn why the solution is wrong and how we can get a correct answer" (2, 17:52). Eva explained why she has to figure out her mistakes.

Eva	Here, I did wrong in one step; and later on, I corrected with teacher's help; we have to get a completely correct answer. Otherwise, if you did not correct your mistakes, you will make same mistakes in your review and test (5, 23:15).

Also students recognised that listening to the teacher's explanations could support their own understanding. As Dana expressed, "The teacher was commenting on students' solution. Sometimes, if I did not understand, I can listen to teacher comments. Then, it will deepen my impression and help me understand the content. So, I think this is important" (2, 34:30).

Some students felt this was an important event because they had worked the problem incorrectly and were able to correct their work through checking with the teacher's solution. For example, Eva explained:

Eva	I feel this is important. Because at the beginning, I drawn the figure as a segment as the student did. Later, the teacher explained that the figure should be a line; I felt it is important. So, I will not make similar mistake (4, 15:31).

Similarly, Laston explained, "The teacher presented answers. My plane coordinate is different from him. I did wrong. I decided to do it again after class. I feel this is important" (4, 46:26).

Students realised that presenting student work was a good opportunity to learn, as Eva explained.

Eva	Here, the teacher presents a student's answer (showing the whole process of solving the problem to us). Ask us to find mistakes or judge it's correctness. This is to provide opportunities to think independently. (5, 29:50)
INT	What did you feel when seeing classmates' work?
Eva	If it is correct, I will appreciate and happy as I did; if it is different from mine, I will think whose solution is wrong? If the teacher said the classmate's solution is wrong, I will figure out where is wrong. If I am wrong, I will figure out why I did wrong.

Summarising key points. Twenty-three students mentioned the importance of summarising the key points of solving problems and/or the whole lesson 25 times. Three student responses are provided to illustrate this category.

From Lesson 1, Daan stated, "Summarising key points is important and helps students deepen our impression" (1, 48:18). Likewise Dove stated, "This is lesson summary. Mr. Zhang wrote the procedure of solving linear equation in two unknowns. It helps us to get a deep understanding of the procedures" (7, 44:12). In

Lesson 2 Dana confirms these views and notes that the summary can be used independently to enhance learning:

Dana	This is lesson summary. It summarises what we have learned in this lesson. In this lesson, we learned the concept of coordinate plane. In addition, we learned how to draw Cartesian plane and relevant points. In the summary, what we have learned were summarised. Then, after class, we can study by ourselves, and do some homework (2, 42:14).
INT	What were you thinking and doing?
Dana	I was thinking that after the class I would read text; review the content, and complete homework.

Matches and Discrepancies between Teacher Intention and Student Perception

The teacher selected three lessons and explained their importance. The three lessons he identified were Lesson 3 – The Concept Lesson, Lesson 7 – The Algorithm Lesson, and Lesson 15 – The Problem-Solving Lesson. From Table 2, we noticed that for each of these lessons there were more matches of important events between the teacher's intention and the student's perception than there were discrepancies. We draw on lesson description and excerpts taken from the teacher and student interviews as illustrations of matches and discrepancies. In addition, we will describe the results of the analysis of the accompanying lesson plan in terms of the essential content points (Li et al., 2009). Finally, we will briefly describe lesson 15 in terms of the patterns among the matches, discrepancies, and essential content points identified in lessons 3 and 7.

Table 2. Number of matches and discrepancies of important events

Lesson	Number of matches	Number of discrepancies
L03-Concept	4	2
L07-Algorithm	4	2
L15-Problem solving	5	1

Lesson 3 – The Concept Lesson

The purpose of this lesson was to help students learn to plot points when the coordinates are given for a Cartesian coordinate plane and understand the one-to-one corresponding relationship between a point and its coordinates. This lesson included the following stages: introducing the new topic based on a review; introducing and practicing new knowledge; and summary and homework.

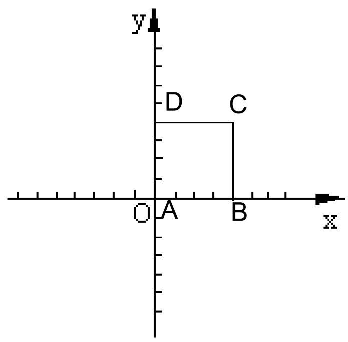
Brief lesson description. The teacher began the lesson by reviewing the concept of the Cartesian coordinate plane and illustrated how to find coordinates of given points through doing examples. Based on this review, students were asked to plot a point with the coordinates (3, 4), which was the topic of this lesson. Next, based on students' oral presentations of the task, the teacher formally explained the procedure of plotting a point with given coordinates. Then, the teacher raised some questions regarding why points with coordinates (3, 4) and (4, 3) are different and the characteristics of points located on the x-axis and the y-axis. After that, the teacher presented problems one-by-one with increasing difficulty for students to practice. These problems included:

1. Given coordinates of three vertices of a triangles of ABC, A (-5, 0), B (-1, 4), C (5, 0), draw the triangle in the Cartesian plane.
2. Given a square in Cartesian plane (right figure), find coordinates of the four vertices of a square (oral responses required).

2a. If the square is translated down 3 units along the y-axis, find the coordinates of vertices of the translated square;

2b. If the square is translated left 2 units along the x-axis, find the coordinates of vertices of the translated square;

2c. (Discussion) In previous problem, if given coordinates of two vertices of square ABCD, A (5,0), B (5, -4), can you find the coordinates of C and D?



Afterwards, the teacher summarised the key points of the lesson with students' input in response to his questions and highlighted the procedures of plotting a point. Finally, the teacher assigned homework from the text.

*Important event matches in the concept lesson***.** In comparing the important events by the teacher and the students, we identified four matches. These matches included: plotting a point given the coordinates of the point; discussing specific features of coordinates; finding coordinates of translated points; and finding coordinates of points in created figures. Each of these will be described in the following sections.

Plotting a point given the coordinates of the point. The teacher explained the importance of exploring how to plot a point from coordinates as follows:

T: At that moment, I presented the new topic through reviewing. Students were asked to find the point of given coordinates (3, 4). It is important to ask students to explore the position of the point individually. It will benefit students' mathematical learning. This is number one priority of learning (3, 8:21).

Students realised the importance of individual exercises, and public sharing. One student, Eddie, believed it was important when the teacher presented a student's work on the board for judgment and discussion.

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INT         This is a student's exercise. What did you feel?
Eddie       Check my answer with the solution presented on the board
            (3, 10:15).
INT         Ah, what did you do at that time?
Eddie       Did the same as the presented solution.
INT         The same answer as his, what did you feel?
Eddie       Very happy.
```

In this match, both teacher and student have identified plotting the point as an important event. The teacher described it, noting the mathematical significance of the exercise. In contrast, the student described it in terms of the importance of checking his solution to the exercise.

Discussing specific features of coordinates of points at x-axis and y-axis. In his interview the teacher shared the following in reference to an important event:

```
T           The characteristics of coordinates of points at x-axis
            and y-axis; Students can find results based on
            observation and discussion. On one hand, it enhances
            students' collaborative learning. On the other hand, it
            helps students understand the features of specific points
            on x-axis and y-axis. So, I circulated around and
            participated in students' discussion in-group, and raised
            some questions for students to consider (3, 22:59).
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Students realized the benefit of group discussion. Eddie explained as follows:

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Eddie       When discussing with classmates, we should focus on the
            content, rather than talking off-task topics (3, 23:21).
INT         Ah, did you say something not related to the topic?
Eddie       No
INT         Why did you think it was important?
Eddie       If you did not discuss seriously when the teacher asked
            you to discuss, afterwards you would not be able to
            answer questions when the teacher called on you.
```

As both the teacher and student identified the same event within the video, this event was coded as a match. Within the event, however, the teacher realised that this was an opportunity to develop students' collaborative learning through arranging a group activity and giving students the necessary support due to the difficulty of the problem. From the student's perspective, Eddie noted that he should be engaged in the collaborative effort because the teacher may call on him randomly and he cares about being able to answer questions appropriately.

Finding coordinates of translated points. In reflecting on important events, the teacher noted the problem that required students to find coordinates of points after translations. His purpose was to "let students find coordinates of vertexes of shapes (triangles and squares); that means to integrate figures and coordinates of vertexes" (3, 25:28). Moreover, the teacher explained:

T In the second problem, students are asked to translate the figure and then they are required to further observe how the coordinates of vertexes change. It is also important to have preliminary understanding of translations in geometry, namely moving of geometric figures. That is to say that creating new problem through changing some conditions of a problem.

In identifying important events, Eddie stated, "Example 2 is very important because we need to plot the vertexes and find their coordinates. I paid close attention to what the teacher explained after completing correct plotting" (3, 25:39).

In this match, the teacher and student's reference to solving example 2 both acknowledged the importance of the content in the problem.

Finding coordinates of points in created figures. In his interview, the teacher also noted as an important event the problem that involved finding the coordinates of the two missing vertexes.

T At that time, I presented a problem [referred to 2c]: given coordinates of two vertexes (A and B) of a side of a square, students are asked to draw different squares and then find the coordinates of points C, D. First, I asked student to discuss in-group. Because this problem is a little bit difficult for seventh graders. Classification thinking method is required. That is say there are two possible positions. Usually, students just considered one situation, while neglecting the other. Based on this consideration of students' learning difficulty, we organized students' discussion in-group. On one hand, through discussion and exchanges, students can get a clear understanding. So, in the final discussion, some students did not consider completely. They just explained one situation. Through students' exchanges and supplementation, the problem can be solved completely (3, 27: 49).

With regard to the two positions, Eddie noted that two students presented different solutions, indicating that he was confused.

INT What did you hear?
Eddie I heard two solutions? (3, 37: 48)
INT What did you think about solutions?
Eddie I am hesitated to decide which one is correct. Later on, the teacher said that the two student solutions are correct.
INT Then, why do you hesitate to decide which one is correct ?
Eddie Because two students presented different solutions.
INT You are hesitated to accept two solutions. Did you consider two solutions are correct?
Eddie No.

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INT        Why not? Did you think there is only correct solution?
Eddie      Yes.
```

In this match, the teacher indicated the challenge the students would face in recognising the two solutions to the problem associated with the event. The student has also addressed the difficulty associated with more than one solution.

Important event discrepancies in the concept lesson. In their interviews, students described two important events that the teacher did not recognise. First, Ever described the event in which the teacher called on a student who was typically reluctant to answer the teacher's questions (3, 42:34). Ever was worried about the student whether she can answer the question correctly. With regard to this event, it may have been too specific to be noted by the teacher. Second, Eddie emphasised the importance of the lesson summary (3, 43:56) while the teacher stressed summaries after completing an exploratory activity. For this event, we hypothesised that this may be due to the routine nature of the summary.

Essential content points and the concept lesson. According to the lesson plan, the learning goals for the concept lesson included:

(i) find the position of a point when given the coordinates of the point in the Cartesian plane; and
(ii) develop preliminary understanding of the corresponding relationship between a point and its coordinates in the Cartesian plane.

The first instructional objective represented the important point for the lesson. The teacher also recognised that the lesson's difficult point was to find the coordinates of specific points within the context of geometrical figures.

Table 3 shows the relationship between the match analysis and the essential content points. As indicated in the table, the first two matches focused on highlighting the important point and achieving the first instructional objective. The third and fourth matches were aimed at overcoming the difficult point, and pursuing the second instructional objective implicitly. This analysis demonstrated that during the co-recognised important events, the teacher and the students jointly worked together to achieve learning goals through appropriately dealing with difficult points with a focus on important points.

Table 3. Relationship between the match analysis and the content points for lesson 3

Important Event Matches	Content Points
Plotting a point given the coordinates of the points	Important Point
Discussing specific features of coordinates of points at x-axis and y-axis	Important Point
Finding coordinates of translated points	Difficult Point
Finding coordinates of points in created figures	Difficult Point

Lesson 7 – The Algorithm Lesson

The goal of this lesson was to introduce the method of elimination by addition and subtraction in solving a system of linear equations in two unknowns. This lesson included the following stages: review through questioning; introduction of new content; classroom exercises; and summary and homework.

Brief lesson description. To begin the lesson, the teacher reviewed the elimination method of substitution (elimination unknown) and the fundamental mathematical method of substitution (transformation from two unknowns to one unknown). Next, the teacher introduced the elimination method by addition and subtraction through students' exploration of a word problem. Afterwards, the teacher deliberately presented a system of linear equations in two unknowns that could be effectively solved using the addition and subtraction method.

Based on a discussion of solving the problem, the teacher drew the students' attention to why this method could be used as well as the rationale for its use. The teacher then wrote the topic of this lesson as the elimination method by addition and subtraction on the blackboard. Some variation problems were assigned to students to identify which methods (addition or subtraction) could be used effectively. The teacher summarised the conditions of using addition or subtraction through questioning. Based on worked examples, the procedures of addition and subtraction elimination were explored and summarised. After that, several classroom exercises were assigned and discussed.

Finally, the following key points of this lesson were summarised: what is the elimination method by addition and subtraction? What are the characteristics of using addition or subtraction method? What are the similarities and differences between the substitution method and the addition and subtraction method? With regard to homework, in addition to several problems from the textbook, the teacher assigned one open-ended problem: given a linear equation ($2x + 3y = 8$), create another linear equation so that the system of linear equation could be solved by addition and subtraction methods.

Important event matches in the algorithm lesson. In comparing the important events described by the teacher and students, we identified four matches: working on a daily life problem toward the learning of the new topic; discovering the elimination method by addition and subtraction through self-exploration; synthesising the conditions of using elimination methods of addition or subtraction; and creating problems for further learning. Each of these will be described in the following sections.

Working on a daily life problem toward the learning of the new topic. In reflecting on this lesson, the teacher described the daily life problem created to introduce the new topic. He explained:

T Through reviewing what students learned in last class, I put forward a problem. Siu Ming's family intends to travel to Beijing during the national holiday by train, so they have booked three adult tickets and one student ticket, totalling \$560. After knowing this news, Siu Ming's classmate Siu Wong would like to go to Beijing with them. As a result they bought three adult tickets and two student tickets for a total of \$640, can you calculate how much does it cost for each adult and each student ticket? This daily life problem is actually a learning situation I created. On one hand, I motivated students' learning interest. They felt this is an interesting problem and they want to explore how to solve it. I asked them to find the solution using oral computation. Thus, as an introductory problem for learning elimination methods by addition and subtraction, students quickly found the solution that is the difference between 640 and 560. This result created the conditions of learning elimination methods by addition and subtraction. That means that discussing this problem lays solid foundation for further learning. So, I think this is an important problem. However, this problem is different from textbook. In the textbook, a problem of chicken and rabbits staying in the same cage [a classic Chinese ancient problem]. In the text, the problem was solved by using properties of equality directly. Through adding or subtracting two equalities, the one unknown is eliminated. Students may felt boring because it looks like the teacher force students to follow. In the problem I created, students have to try and actively participate in the learning process; it also motivate their learning interests (7, 1:39).

INT How about students' acceptance/understanding of the problem during the instructional process?

T Regarding this problem, based on students' responses to the problem, I can say that they are easy to find the answer. Because in the second condition, there is one more student ticket, so students naturally figured out the solution by subtraction. So, I believe that this problem plays solid foundation of learning elimination methods of addition and subtraction; it will naturally lead to learning the new topic.

Two students noted the importance of working on this problem. One of the students, Franc, shared the following:

Franc Because the teacher was telling us how to solve [system of] linear equations. (7, 7:32)

INT What did you do at that moment?

Franc I am thinking of whether there are other simpler methods of solving the equations.

The other student, Franks (7, 6:01), realised the importance of solving the problem through setting a system of linear equations. At that moment, he "was thinking about why did we need to set a system of linear equation? How to solve this system of linear equation? "

In this match, both the teacher and students recognised the daily life problem used to introduce the new topic as an important event. From the teacher's perspective, the problem was interesting to students and laid the foundation for exploring the mathematics. From the students' perspectives, this problem was important, and they reported trying to make sense of its solution process.

Discovering the elimination method by addition and subtraction through self-exploration. In his interview, the teacher discussed the principles of a particular teaching approach (called Qingpu experiment [青浦实验]), which has been developed based on more than 10 years of experiments, tests, and syntheses. These principles include creating a learning situation, doing an experiment, synthesising the exploration, practicing with variation, and feedback and summary (Gu, 1994). Following the first stage of creating a learning situation, that is, the daily life problem, the teacher further explained how he used four problems to help students to try and find the methods of elimination by addition and subtraction as follows:

T	After completing the word problem, we asked students to solve the system of linear equations in two unknowns, namely $3x + 2y = 8$ and $3x - 2y = 4$. Based on their experience in solving the word problem, students quickly found the method of subtraction because I used the elimination method by subtraction. In fact, based on my observation, majority of the student has found this method. But some students also come up with elimination method by addition. Essentially, this problem was aimed to provide student with a try and discovery opportunity. This is our second principle of Qingpu experiment – Exploratory activity (7, 8:45).
INT	In fact, this problem looks like an extension of previous word problems. With the scaffolding of previous problem, now students should have a deep impression. For example, in this problem, using subtraction elimination was more complicated than addition elimination. What is the purpose for you to arrange this problem?
T	My purposes include: first, this problem can be solved by addition elimination method, but a little bit complicated. If we subtract these two equations, then we got $4y = 4$. The solution is still simple; the purpose is to focus on methods of eliminations, rather than computation of numbers. I deliberately selected the coefficients of one unknown are the same while those of the other unknown are contrary numbers. As the teacher can eliminate unknowns using subtraction method, I intended them to discover whether addition method can be used to eliminate unknowns. In fact, some students found elimination method by addition.

The teacher further explained that after he assigned the problem, he circulated it around the class and gave students individual guidance as expressed below:

T	As a teacher, I should pay attention to all students overall, but we also care about individual students who

> need help. So, I circulated around and helped students to use what they have learned to solve this problem. Some students did not find how to solve this problem, and then I gave them specific guidance. But majority of them found they could use subtraction to eliminate an unknown (7, 10:02).

When we reviewed the student interview transcripts, we noted one student, Franc, who commented on the importance of using subtraction and /or addition method to eliminate.

Franc	Here, the teacher taught us subtraction method. [So, we can] solve the system of linear equations using subtraction. (7, 13:15)
INT	What did you do at that moment?
Franc	I was thinking.
INT	What did you think?
Franc	I was thinking why I did not figure out this method.
INT	What did you feel?
Franc	I felt I am so stupid.

In this match, both the teacher and student identified this opportunity for self-exploration. The teacher described his intent to support students in discovering the elimination method by addition. The student also learned the method from making mistakes, although he was frustrated about his mistake.

Synthesising the conditions of using elimination methods of addition or subtraction. The teacher explained that observation and synthesis is an essential step after exploratory activities, according to the Qingpu experiences approach. He said:

T	In the previous problems, which required students to answer orally, it was aimed to speak out whether they used addition or subtraction to eliminate unknowns. After that, students discussed another two problems. It solicited students to find the conditions for students to use addition or subtraction methods. In fact, I put forward problems, and students discussed purposefully and then summarised. Doing so, it is aimed to develop observation and synthesis. The synthesis ability is an important stage as described in *Learning to Teach* by Gu (1994). Thus, the knowledge can be integrated into the system. We asked students to discuss and synthesise (7, 24:15). … After students discussed, synthesised, I found that students' synthesis is not comprehensive. I gave hints and discussed with students to develop and complete summary. As these grade 7 students, they have not developed high synthesis ability, so teacher guidance is important (7, 26:38).

One student, Franc, also noted this event's importance, saying that "because the teacher told us that when addition method can be used to solve a system of linear equation and when subtraction method should be used to solve" (7, 27:04). Probed by the interviewer, the student further detailed that "when the coefficients of an unknown are contrary numbers, the two equations can be added directly; when the coefficients of an unknown are the same, then, the two equations can be subtracted." The other student, Franks, said "I watched the screen; I felt it is very important because there is a summary of the concept of elimination of addition and subtraction" (7, 27:02).

In this match, both the teacher and the students recognised the significance of the synthesis stage that followed the exploration.

Creating problems for further learning. When asked whether there were other important events in this lesson, the teacher highlighted a problem in the homework, which required students to create a problem: Given the equation, $2x + 3y = 8$, create a second new linear equation in two unknowns so that both equations could be solved by the methods of addition or subtraction to eliminate the unknown. In his interview, the teacher said:

T In this problem, I deliberately did not lead students to focusing on elimination method of addition and subtraction directly. What is my intention? I intend students to create some problems, which are related to the topics of next lesson: solving system of linear equations with multiple relationships of coefficients of unknowns. Students' feedback in next class showed that I achieved this goal. In students sharing, they presented various coefficients of the same unknown. They [coefficients of the same unknown] included the same, contrary numbers, and multiple and non-multiple relationship, or changing multiple relationships between coefficients after multiplying a number to equations. These problems consisted of the main instructional task in next class. Thus, it laid the foundation for further learning. Creating problems is challenging for students. It should enhance students' problem-solving ability because the students first need to create problems and then to solve them. Moreover, they need to apply learned knowledge when creating problems, rather than doing randomly.

One student, Free, in the following lesson (Lesson 8) appreciated the self-created problems. The student said:

Free We created this problem. We designed the problem and we felt very comfortable. Since the teacher asked us to design and solve the problem. It tests imagination ability. I feel that providing this problem by the teacher is very good. So, I feel that solving self-created problems helps us to develop imagination and creation ability. In the future, we can do something by ourselves (8, 9:11).

One of the salient features of Chinese mathematics classroom is delivering lessons coherently (Chen & Li, 2010). Laying the foundation for further learning is a critical element of instructional coherence. The teacher intended activity was well recognised by the students. This event featured in this match not only motivated the students but also provided a coherent learning experience for the students.

Important event discrepancies in the algorithm lesson. Two discrepancies occurred in this lesson. First, the students did not recognise the teacher intended strategy of highlighting the new topic. The teacher described how he deliberately designed the process of discovery learning from creating a learning situation, organising the exploratory activity, followed by observation and synthesis. After students made their own findings, the teacher wrote down the topic of the lesson on the board, which is a typical way to highlight the topic of the lesson. The teacher mentioned this as an essential event in the lesson. The students, however, did not recognise it as such.

In addition, the teacher emphasised the summary after the exploratory activities while the students did not note the importance of this event. Alternatively, the students appreciated the summary at the end of the lesson. The teacher believed that this was a good lesson because it reflected the essential features of the Qingpu experimental approach. He stated,

T	For example, [according to Qingpu experiment approach] step 1 is to create learning situations, I did; step 2 is to organise exploratory activities, I did; step 3 is to observe and synthesise, we asked student to synthesise and summarise; Step 4 is to [do exercises with variation]; Step 5 is to give feedback immediately. When I circulated around the class, I gave them individual guidance and feedback.

It is important to note that from the teacher' perspective, synthesis and summary were an internal component after any exploratory activity rather than the summary at the end of a lesson. Both Franks and Franc recognised the importance of the entire lesson summary. Franks specified the reason that "the teacher emphasised the similarities between elimination by addition and subtraction and elimination by substitution" (7, 38:42). Similarly, Franc said, "this is the most important part of the lesson because it is the summary of the entire lessons" (7, 37:30).

Essential content points and the algorithm lesson. In his lesson plan, the teacher identified that the instructional objective/learning goal was for students to develop a preliminary mastering of the elimination method for solving systems of linear equations. The difficult point was to apply the elimination method to solve systems of linear equations in different situations.

Table 4 displays the relationship between the previously identified important events and the essential content points for this lesson. The first three matches were aimed to achieve an important point associated with the learning goals with the first two matches designed to overcome the difficult points through exploring the contextual problem. The fourth match sets a platform for further learning in the next lesson. These common important events indicated that the teacher and students collaboratively developed students' understanding and mastering of the important point.

Table 4. Relationship between the match analysis and the content points for lesson 7

Important Event Matches	Content Points
Working on a daily life problem toward the learning of the new topic	Important Point & Difficult Point
Discovering the elimination method by addition and subtraction through self-exploration	Important Point & Difficult Point
Synthesising the conditions of using elimination methods of addition and subtraction	Important Point
Creating problems for further learning	----------

Lesson 15 – The Problem-Solving Lesson

The instruction featured in lesson 15 utilised organised, systematic activities based on joint efforts of both the teacher and students. Students engaged in an initial puzzle involving boxes followed by two problem-solving tasks related to solving systems of linear equations. Throughout the lesson, students worked in pairs and participated in group discussions of problem solutions. The lesson ended with a lesson summary.

A review of the interview transcripts with the teacher and students revealed five matches (see Table 5). In addition to the matches, one discrepancy was noted when the students mentioned the importance of solving a problem based on a previous problem's solution. The teacher did not indicate this to be an important event. With five matches and one discrepancy, the previously noted pattern of more matches than discrepancies was affirmed.

Table 5. Relationship between the match analysis and the content points for lesson 15

Important Event Matches	Content Points
Organise the puzzle activity of making of boxes.	--------------
Deal with students' misunderstanding of making boxes	Difficult Point
Solve problem 2	Difficult Point & Important Point
Share solutions to problem 2	Important Point
Summarise the key points of entire lesson	Important Point

According to the lesson plan, the instructional purpose of lesson 15 was to further develop students' mastery and application of methods of solving system of linear equations as well as to cultivate students' ability in thinking and solving problems from different perspectives. The teacher identified that the important point was to develop students' problem solving ability in using solving systems of linear equations. He indicated that the difficult point was in solving the initial puzzle and formulating a system of linear equations for the remaining problems. As the teacher intended, the puzzle and two problems were interconnected and progressively aimed to develop students' problem-solving ability.

Table 5 also presents the relationship between matches and the important points and the difficult points. The first match aimed to motivate and lay the foundation for the exploration of problems one and two. Although this scaffolding was important for students' success, the event itself did not align with one of the essential points. The second and third matches were focused on dealing with the *difficult point*. Also, the last three matches were aimed at approaching problems from multiple perspectives and developing students' problem solving ability, an important point. Based on these relationships between important event matches and essential content points, the students and teacher worked together in Lesson 15 to overcome difficult points and grasp the important points, finally aiming to achieve the instructional goals.

DISCUSSION AND CONCLUSION

The Chinese classroom has often been described as teacher-dominated, well-structured and well-disciplined (Leung, 1995; Stigler & Stevenson, 1991). Research has documented that Chinese lessons tend to be delivered coherently with transactions of classroom activities that run smoothly (Wang & Murphy, 2004). Moreover, recent studies have revealed that Chinese mathematics classes include student-centred learning approaches (Huang & Leung, 2004; Mok, 2006). For example, Huang and Leung (2004) reported that students were actively engaged in learning activities with teachers' skilled guidance in large classes. Some scholars coined this phenomenon as learner-trained learning: "students know the procedures and react promptly to teachers' cues" (Cortazzi & Jin, 2001, p. 128). For the most part, these studies have described classroom instruction and possible students' learning based on classroom observations of teaching and the teachers' perspectives.

In contrast, this study provides students' voices in addition to teacher's perspective regarding mathematics learning in a Chinese mathematics classroom. It is crucial for students to recognise the important events that the teacher intends to carry out within the lesson. With this recognition, the students are well prepared to engage in these shared important events. The significance of this shared recognition of important events lies within the potential relationship that exists between the shared important events and the essential content points [*Three Point*]. When the essential content points are dealt with appropriately, the learning goals are more likely to be achieved.

In recognition of these ideas, the present study examined the students' perceptions of important lesson events and the alignment of these with the teacher's intended events. In addition, we examined the relationship between these shared important events with the essential content points. In this discussion, we will highlight three important findings that emerged from our work.

Students' Perceptions of Important Events

Based on the analysis of student interviews, the students in this study recognised the importance of events commonly associated with Chinese classroom routines. These student-perceived important events included introducing new concepts or procedures, practicing new knowledge and skills, sharing students' work, and summarising key points.

When practicing new knowledge and skills, students appreciated group activities and the teacher's individual guidance. In addition, they valued the opportunities to learn from sharing of students' work with the teacher's comments. They also stressed the importance of summarising key points, regardless of whether the summarising occurred in the process of learning or at the end of a lesson. These findings suggest that the Chinese students perceived the typical Chinese instructional flow as described in literature (Huang & Wong, 2007; Leung, 1995). Furthermore, these findings provide evidence from the learner's perspective that supports the learner-trained learning approach (Cortazzi & Jin, 2001).

Consistencies between Teacher Intentions and Student Perceptions

In reviewing the relationship between the teacher's intended important events and the students' perceived important events, we noted that for each of the three lessons more matches than discrepancies existed. This pattern occurred across all three lessons despite the different focus each lesson represented, that is, concept development, algorithm development, and problem solving. However, in match cases the teacher and students often interpreted the importance of shared events differently.

Previous studies in Chinese classrooms (e.g., Leung, 2005; Stigler & Perry, 1988) have reported that, students engage in more mathematically challenging tasks and rigorous reasoning. While selection and implementation of tasks are critical elements of effective mathematics instruction, our finding of high consistency between teacher intention and student perception of important events provides an addition necessary for understanding the effectiveness of Chinese mathematics instruction. Specifically, because the students' important events match those of the teacher, the students are more likely to engage in the mathematical tasks at the cognitive level the teacher intends.

Alignment with Essential Content Points

In addition to examining the consistency between the important events identified by the teacher and students, we aimed to connect these important events to the essential content points, namely difficult points and important points as defined by Li et al. (2009). Our analysis revealed that during the shared important events the teacher facilitated and supported students in achieving the learning goals through appropriately dealing with the difficult points with a focus on mastering the important content points.

Two essential Chinese learning goals are to perfect learners morally and socially and acquire knowledge and skills for oneself (Li, 2009). Being a well-disciplined learner is one of the most important goals. Emphasis on students' mastering of basic knowledge and skills has enjoyed a long tradition in Chinese mathematics education along with the model of "Two Basics" teaching (Tang, Peng, Chen, Kuang, & Song, 2012; Zhang, Li, & Tang, 2004). Consequently, in Chinese mathematics classrooms, knowledgeable and respected teachers (Ma, 2010), and well-disciplined and trained students (Cortazzi & Jin, 2001) work together to pursue the shared learning objectives by focusing on essential content points.

Therefore, this study extends our understanding from the learner's perspective of why Chinese students can achieve excellent performance in various international mathematics assessments. Through joint efforts of the teacher and the students, the clearly set learning objectives can be achieved by overcoming difficult points with a focus on important points.

CONCLUSION

Classroom instruction in China cannot be described and interpreted by dichotomous theoretical frameworks such as student-centeredness and teacher-centeredness. It is imperative to develop more inclusive theories to study mathematics instruction (Clarke et al., 2007). Huang and Leung (2004) tackled the Chinese learner's paradox (Watkins & Biggs, 2001) by highlighting features of good teaching in Chinese classroom. With reference to classroom studies, they argued that students were actively engaged in learning mathematics through exploring variation problems with teachers' skilled guidance, even in large classes. This study provides a unique perspective toward further understanding Chinese mathematics instruction by including the perspective of the students.

In the Chinese mathematics classroom featured in this study, students were attuned to the lesson format and expectations, allowing the teacher and students to work together in carrying out mathematical tasks designed to achieve learning goals. Moreover, the students recognised those important events that the teacher deliberately designed to overcome learning difficulties and highlight important content points. Learning goals were aligned to worthwhile mathematical tasks that were designed to overcome lesson difficult points with a focus on lesson important points. The teacher and students shared their responsibilities in generating knowledge (Clarke & Seah, 2005).

The purpose of this case study was to provide a new perspective, namely the learner's perspective, to interpreting mathematics classroom learning in China, a country where students achieve superior performance on international assessments of mathematics. Our findings highlight the value of focusing on *Three Points* as a tool for examining classroom instruction. However, more empirical studies are needed to explore how using Three Points to guide lesson planning, lesson implementation, and post lesson reflection may improve classroom instruction and develop teachers' profession learning. Moreover, this study raises an issue: how to develop a classroom culture/norm where students and the teacher have high agreement in terms of instructional flow and important classroom events.

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CHAPTER ELEVEN

What Really Matters to Students? A Comparison between Hong Kong and Singapore Mathematics Lessons

INTRODUCTION

The original concept of giving students a voice was initiated by Rudduck who suggested that student voice approaches could offer a practical agenda for change in education at a variety of levels (Fielding, 2007). Following this idea, McGregor (2005) suggested students as 'experts' in schooling and argued that students had knowledge of the school which adults might not have. In this sense, students could hold different views regarding what is important in enhancing standards which could meet their individual needs. The Learner's Perspective Study (LPS), motivated by a strong belief that the characterisation of the practices of mathematics classrooms must attend to the learners' practice with at least the same priority as that accorded to the teacher's practice, is a comprehensive study that adopts a complementary accounts methodology to negotiate meanings in classrooms.

Drawing on student interview data from one Singapore class (SG1) and one Hong Kong class (HK1), the authors aim to compare student perspectives of their lessons in Singapore and Hong Kong respectively. In earlier studies, Kaur (2008, 2009) from Singapore and Mok (2009) from Hong Kong, independent analysis found that these students were very positive about their learning. In this present study we explore the pertinent features that are important from students' perspectives, and seek answers for the following two research questions:

(i) What are the similarities and/or differences between HK and SG students' perceptions about what is important in their mathematics lessons?

(ii) What are the similarities and/or differences between HK and SG students' perceptions about what are characteristics of good mathematics teaching?

In the following section we will give a brief literature review for why students' voices and perceptions are important for learning and a review of recent studies on student voices and perceptions of their learning. This will be followed by the background of methodology of the current study in the context of the Learner's Perspective Study and the method of analysis. The results will be reported in separate sections for the Hong Kong lessons (HK1) and the Singapore lessons (SG1) respectively, followed with a summary of comparison.

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 189–208.

LITERATURE REVIEW

Why Are Students' Voices Important for Learning?

The importance of allowing or encouraging students to voice their views and preferences has been widely recognised in contemporary education. Researchers have argued that giving students a voice on their learning will support their learning at different levels. The argument can be divided into two broad categories: (i) encouraging students' voice in the process of learning directly empowers their learning; and (ii) understanding students' perspectives provides valuable ideas for bringing about changes.

Encouraging Students' Voice for Empowering Students' Learning

Hargreaves (2006) suggested that through expressing their views, students could develop their responsibility, independence, confidence, as well as maturity. In particular, students could take greater responsibility in setting their own learning goals, controlling their learning progress, as well as managing learning resources. Cole (2006) noted that in such learning environments students would have greater choice and responsibility for co-constructing learning and reflecting on learning progress. For example, students would likely be more active in making decisions about how to undertake learning tasks.

Manefield, Collins, Moore, Mahar, and Warne (2007) suggested that student voice should include opportunities for reflecting on learning progress and assessment. In Mitra's (2004) project, students reported that they took more responsibility for homework and study when their voices were heard and appreciated. Moreover, Flutter and Rudduck (2004) found that giving students more opportunities to reflect on teaching and learning had a direct impact on students' meta-cognitive development and their approach to learning. Others (e.g., Oldfather, 1995; Rudduck & Flutter, 2000; Wallach, Ramsey, Lowry, & Copland, 2006) noted that students improved academically when students are given a greater voice in improving curriculum and instruction. Ranson (2000) further suggested that giving students a voice could enable them to explore self and identity, develop self-understanding and self-respect. Among nine main gateways identified by Hargreaves (2004) for personalising learning around students' needs, facilitating student voice was regarded as the most powerful one.

Understanding Students' Perspectives for Bringing about Changes

Mitra's (2004) study revealed that when students' voices were respected and valued, students' sense of agency increased. In particular, the students got opportunities to articulate their opinions which supported them to construct a new role as change makers in schools. When their power increased as decision makers, they also developed their leadership skills. Consistently, Fielding and Rudduck (2002) found that students became more aware of their important impact on school matters. Fletcher (2005) argued that meaningful involvement of students means to validate and authorise students to express their own ideas, opinions, knowledge and experience throughout education in order to improve schools. The OECD (2006) also advocated that school ethos should focus on students needs with the whole school taking time to find out the needs and interests of the students so as to make use of their voice to drive school improvement.

In addition to agency, Mitra (2004) remarked that students' sense of belonging and self-worth increased when they were more involved in school development. Students developed a sense of ownership and increased pride about their school. Lee and Zimmerman's (2001) study also noted a positive correlation between the promotion of student voice in school culture and school attainment. As Oldfather (1995) argued, student voice opportunities helped students to build awareness that they can make changes in schools not only for themselves but also for others.

From another perspective, Mitra (2004) and Hargreaves (2004) both argued that giving students a voice is not only to encourage them to play a more active role in their education, but also to establish a more open and trustful relationship between staff and students. The students in Mitra's study remarked that their teachers were likely to develop deeper understanding and respect of their difficulties in their lives via listening to their voices. Johnston and Nicholls (1995) indicated that students can actually help teachers do a better job to meet their needs via articulating how they learn best.

Though it is clear that student voice is an invaluable source in education, views and opinions of young people have until late been traditionally discounted as having less legitimacy than the views of adults (Dennehy, 2010). According to Dennehy, many critiques of student voice relate to its radical, inconceivableness, and unnecessariness. Consequently, most efforts to reform education have been based on adults' notions of how education should be conceptualised and practised. As noted by Cook-Sather (2002) "there is something fundamentally amiss about building and rebuilding an entire system without consulting at any point those it is ostensibly designed to serve" (p. 3).

Some Recent Studies Related to Students' Voices in the Context of Mathematics Learning

Recently, the notion of listening to students' voices has been increasingly advocated in both research and practical arena with several of these studies focusing on students' perceptions within mathematics education.

Fan and colleagues (2005) investigated the Singapore lower secondary students' attitudes toward mathematics and mathematics learning. Survey responses revealed that these students' views on mathematics and mathematics learning were positive in part; they regarded challenging problems and the application of mathematics in their adult life less positively.

Masinglia (1995) studied students' perception using mathematics outside the classroom. Students were interviewed before and after keeping a log sheet to record their mathematics activities in their daily life. Responses also indicated that there was a gap between mathematics and school mathematics in students' mind.

McDonough, Sullivan, and Harrison (2004) investigated student's perceptions of the extent to which their own efforts affected their mathematics achievement and their life opportunities. Based on a task based interview and survey of self confidence and rating on mathematic ability they concluded that students were confident in their own ability and perceived themselves as trying hard at the tasks. The students expressed a strong awareness of the importance of effort in approaching problems.

Although relevant research is still limited in this area, listening to students does help researchers and educators to have better and more thorough understanding about their needs. In this sense, student voice should become a primary source of data influencing our decision as educators and policy makers.

THE LEARNER'S PERSPECTIVE STUDY

The Learner's Perspective Study (LPS) adopts a complementary accounts methodology (Clarke, Keitel, & Shimizu, 2006) to negotiate meanings in classrooms. The complementary accounts methodology enables researchers to record the interpersonal conversations between focus students during the lesson and identify the intentions and interpretations of participants' statements and actions during the lesson through video-stimulated interviews. The Teacher camera captured the teacher's actions and talk during the lesson. The Student camera focused on a group of two students, known as the "focus group" and captured their actions and talk during the lesson. The Whole Class camera captured the whole class in action. A split-screen video record mixed on-site from the Teacher and Student camera images was used as a stimulus for students to reconstruct accounts of classroom events during the interviews. Two students from the focus group were interviewed separately after each lesson. Student artefacts (e.g., worksheet and homework) from the focus group were also collected after each lesson. The teachers were interviewed three times during the period of data collection. The interviews were based on a lesson the teacher had taught during the week and the video recording of the lesson was used as a stimulus for the teacher interview. In addition to the teacher interviews, the teacher completed two substantial questionnaires before and after video-taping as well as a shorter questionnaire after every videotaped lesson.

For each participating region of LPS, three mathematics teachers recognised for their locally-defined 'teaching competence' participated in the study. Each participant teacher was recorded for at least 10 consecutive lessons. In our analysis we compare the data of one Singapore school (SG1) with that of one Hong Kong school (HK1). From SG1, video-records of 13 consecutive lessons (three during the familiarisation stage and ten as part of the study) were collected. HK1 data was collected from 18 consecutive lessons comprising a complete topic.

THE METHOD OF ANALYSIS

The Student Interviews

The interviews were conducted immediately after school each day. A split screen video record, mixed on-site from the Teacher and Student camera images of the day's lesson, was used as a stimulus for the student interview. Prompts used by the interviewer for the student interviews included:

– Please tell me what you think that lesson was about?
– How, do you think, you best learn something like that?
– What were your personal goals for that lesson?
– After watching the videotape, is there anything you would like to add to your description of what the lesson was about?
– What did you learn during the lesson?
– What are the important things you should learn in a mathematics lesson?
– How would you generally assess your own achievement in mathematics?

As part of the interviews, the students were invited to stop the lesson videos at places where they saw something important and comment on what they were doing, thinking and feeling at that point during the lesson.

Analysis of Student Data: Development of Codes

The instructional practice of T1 from SG1, reported in detail in Kaur (2009), was divided into episodes of exposition, seatwork, and review. Exposition was characterised by whole class mathematics instruction aimed to develop students' understanding of mathematical concepts and skills; seatwork was the period during which students were assigned questions to work on either individually or in group; and whole class review of student work involved the teacher-led review of the work done by students.

Transcripts of the student interviews were coded using a grounded approach whereby the researcher extracted essential parts of the transcript of the student interview with reference to the relevant lesson video segments, and wrote the remarks for the reason/inference. A total of 19 student interviews and 10 lessons of SG1 were collected and analysed. Table 1 provides the itemisation of the 11 places where the student (SG1-1) stopped the lesson video (SG1-L01).

The segments of lessons paused by the students were analysed and classified according to instructional episodes: exposition, seatwork, review, or feedback. The segments were further classified into sub-categories that the students attached to importance. For SG1, there were 10 lessons and 19 student interviews altogether.

Table 1. Examples of the analysis of lesson segments that student (SG1-1) attached importance to in the lesson (SG1-L01)

Pause	Transcript - Lesson segment (Student: SG1-1)	Key sentences	Codes
1	SG1-1: Ah… this part. She teaches us the method. // The teacher teaches us the method of doing the… standard form and the power of ten.	The teacher teaches us the method of doing the… standard form and the power of ten.	Exposition – demonstrates a procedure, "teaches the method" or shows using manipulative a concept/ relationship (D)
2	SG1-1: Yeah, this part also. She is teaching the method lah. ("lah" is a colloquial term for okay.) //Int: So this part is important to you? SG1-1: Yeah.	She is teaching the method lah	Exposition – demonstrates a procedure, "teaches the method" or shows using manipulative a concept/ relationship (D)
3	SG1-1: Um this part where we do the question. Uh, because like you get to do it lah … And then you can show it to the class but I didn't get to show it lah.	Uh, because like you get to do it lah … And then you can show it to the class but I didn't get to show it lah.	Seatwork – student work in groups (GW)
4	SG1-1: Ah, this one. //Ah, she is explaining clearly about the (points). // Because I you get to solve the problem. // Because she teach us then I was thinking oh yeah hoh like that	Because she teach us then I was thinking oh yeah hoh like that	Exposition – teacher explains/ explains clearly (EC)
5	SG1-1: Yeah this part. [student presented the work on the board]// Ah we get to see the mistakes of other people so that we won't… do the same mistakes lah….//Because like um…the teacher will explain to you like more more detailed.	Ah we get to see the mistakes of other people so that we won't… do the same mistakes lah…. Because like um…the teacher will explain to you like more more detailed.	Review and Feedback – Teacher uses student's presentation or work to give feedback for class work or homework (SP)
6	SG1-1: Yeah, this paper. [paper refers to the worksheet given by teacher] Yeah. It gives us more practice and to identify the… um the ten to	Yeah. It (the worksheet) gives us more practice and to identify the… um the ten to what power ah.	Seatwork – instructional material used as a part of instruction

	what power ah.		(worksheet or other printing resource) (M)
7	SG1-1: Ah…this one. //Because ah the the teacher explains clearly how to how ah about one problem that I'm stuck on eh. Is the the ten.	Because ah the the teacher explains clearly how to how ah about one problem that I'm stuck on eh. Is the ten.	Exposition – teacher explains/ explains clearly (EC)
8	SG1-1: Ah this one…. Standard form… [Student making connections between what teacher is showing and what the student has seen in the textbook]. Um like you can write the the speed (of the violet wave). //In a shorter way eh	Um like you can write the the speed (of violet wave). //In a shorter way eh	Exposition – teacher introduces new Knowledge (NK)
9	SG1-1: Ah, this part we are doing the practice again lah… Practice on the book (work in a group of 3 students). //Yeah, because if … like at home is like sometimes not enough time to practice eh.	Yeah, because if … like at home is like sometimes not enough time to practice eh.	Seatwork – student work in groups (GW)
10	SG1-1: Eh, this one [teacher comments on the student's work on the board]. We get to see our answers and check our answers.	We get to see our answers and check our answers.	Review and Feedback – teacher uses student's presentation or work to give feedback for class work or homework (SP)
11	SG1-1: Ah, yeah, the the two ways which is important lah…. The two ways of answering the question. Solving the answer and putting the answer whether in standard form or… not. Yeah, … then you can like pick the easier one eh.	Solving the answer and putting the answer whether in standard form or… not. Yeah, … then you can like pick the easier one eh.	Review and Feedback – teacher uses student's presentation or work to give feedback for class work or homework (SP)

For exposition, the teacher's instruction that students attached importance to were:

– teacher explains/explains clearly (**EC**);

– teacher demonstrates a procedure, "teaches the method" or shows using manipulative a concept/relationship (**D**);
– teacher introduces new knowledge (**NK**);
– teacher gives instructions (assigning homework, how work should be done, when work should be handed in for grading, etc.) (**GI**); and
– teacher uses real-life examples during instruction (**RE**).

For seatwork, the student activities or thing students to which the students attached importance were:

– working individually on tasks assigned by teacher or making/copying notes (**IW**);
– students working in groups (**GW**); and
– instructional materials (worksheets or any other print resources) (**M**).

For review and feedback, the teacher's comments on the student work or review of previous knowledge that students attached importance to were:

– teacher reviews prior knowledge (**PK**);
– teacher uses student's presentation or work to give feedback for in class work or homework (**SP**);
– teacher giving feedback to individuals during lesson (**IF**); and
– teacher giving feedback to students through grading of their written assignments (**GA**).

A similar approach was applied to analyse the student interview data of HK1. There were 34 student interviews and 18 lessons in HK1 altogether. An example of the analysis for the student interview (HK1-16) is shown in Table 2.

Table 2. Examples of the analysis of lesson segments that student (HK1-16) attached importance to in the lesson (HK1-L09)

Pause	Transcript (Student: HK1-16)	Key sentences	Codes
1	HK1-16: This is because when student do their work wrongly, it is good that there is a teacher teaches them. That means if they do some steps wrong, or do it in a faster way for no reason, how can I explain that, I mean they skip some steps for no reason. They can ask Mr. X if they do not know how to do. Int: Yep. HK1-16: When it was written wrongly and it was done by you. You can correct it while Mr. X told you.	Teacher told student who was working outside on the blackboard where his mistake is	Review and Feedback – teacher uses student's presentation or work to give feedback for in class work or homework (SP)

2	HKl-16: Well, I was calculating. Int: You are calculating. You think this is important. // What are you thinking?// HK1-16: //Yes. HK1-16 //No. He had given us some homework and you might do the steps wrongly for no reason. You can calculate quickly if you follow those steps.	He had given us some homework and you might do the steps wrongly for no reason. You can calculate quickly if you follow those steps.	Seatwork – students working individually on tasks assigned by teacher or making / copying notes (IW)
3	HK1-16: //Let me think. When teacher was teaching the students, that means when some student did not know how to calculate, they went to ask. Mr. X would explain.	When teacher was teaching the students, that means when some student did not know how to calculate, they went to ask. Mr. X would explain.	Review and Feedback – teacher giving feedback to individuals during lesson (IF)
4	HK1-16: The most important part of the lesson is the time when Mr. X was talking. If you do not know what was he talking about, you did not know how to do.	Teacher explaining questions	Exposition – teacher explains / explains clearly (EC)

SINGAPORE: THE RESULTS FROM SCHOOL SG1

For SG1, 19 students were interviewed and all student interviews were analysed. Table 3 shows the number and categorisation for each of the lesson segments for SG1 student interviews.

Table 3. Numbers and categorisation of lesson segments for SG1 student interviews

	Instructional Practice (IP)			Total
	Exposition	Seatwork	Review & Feedback	
	EC / D / NK / GI / RE	IW / GW / M	PK / SP / IF / GA	
	15 / 11 / 10 / 1 / 2	4 / 7 / 3	2 / 18 / 1 / 2	
Total number of segments in the IP / Total number of segments	39/74 (52%)	14/74 (19%)	23/74 (31%)	74
Student head count*	15/19 (79%)	9/19 (47%)	14/19 (74%)	19

* The number of students who commented on the IP / The total number of students.

Legend:
EC – explains / explains clearly;
D – demonstrates a procedure: "teaches the method" or shows using manipulatives a concept / relationship
NK – introduces new knowledge;
GI – gives instructions (assigning homework / how work should be done / when work should be handed in for grading, etc.);
RE – uses real-life examples during instruction;
IW – students working individually on tasks assigned by the teacher or making/copying notes;
GW – students working in groups;
M – material used as part of instruction (worksheet or any other print resource);
PK – reviews prior knowledge;
SP – uses student's presentation or work to give feedback for in class work or homework;
IF – gives feedback to individuals during lesson;
GA – gives feedback through grading of written assignments.
* Student head count –Note if the student made comments on more than one instructional practice, the student was only counted once in the head count.

Findings of Student Data from SG1

When examining the data for SG1, it was found that at times a student attached importance to a specific classroom event for multiple reasons. An example is student 15 (SG1-15) who attached importance to one segment of the lesson for three aspects of the instructional practice (see Appendix I). This student stopped the video at the juncture where the teacher was giving the students pieces of paper. When asked "Why is this important to you?", she said that it was important because the piece of paper meant that they were going to work in groups and subsequently present their solutions to the class and this would mean that they will get to see several ways in which the mathematical task could be done. In addition she said that working in groups meant that she could "gain from her friend's knowledge. It appears that in the episode the student attached importance to three aspects of the instructional practice, namely material, group work and student presentations.

It is also apparent that the number of lesson segments that students chose to note as important varied from none to eleven (see Appendix I). By virtue of the methodology used for the data collection, that is, viewing of the video-record and selecting segments that were pertinent to them, it may be claimed that the lesson segments that the students chose to comment on were very personal; that is, the students were guided by their own mental framework.

From Table 3 we can see that students attached importance to a range of aspects of the instructional practice. Regarding teacher exposition the students valued clear explanations, demonstration of "how to do it", and also demonstration using manipulative a concept or a relationship. They also attached importance to the introduction of new knowledge by the teacher, instructions given by the teacher and use of real-life examples during instruction. As part of seatwork, the students valued individual work, group work and the instructional material (worksheet) given by the teacher to engage them in practice of the concepts and skills they learned during the lesson. The students also valued as part of review and feedback, the teacher's review of prior knowledge, student presentations which the teacher used to highlight common errors, misconceptions and alternative approaches, individual feedback given to students and also feedback through the grading of written assignments.

The five aspects which were selected as important most often by students were: explains/explains clearly, demonstrates a procedure, new knowledge, group work, and student presentation. A discussion of these five categories with accompanying excerpts of students' interview transcripts together with the authors' interpretations follows.

Exposition – explains/explains clearly

Episode 1

```
SG1-17      This part ah?
Int         This part ah? Why is this part?
SG1-17      The teacher go through the answer ah we don't know at
            first then she go through already we know lor.
Int         Oh ... okay. So um, how did she help you to know the
            answer?
SG1-17      Er ... go through the ... go through the answer very
            clearly ah.
```

Here we see how Student SG1-17 argues that the teacher's clear explanation enabled him to understand the solution of a question he was unable to do otherwise.

Exposition – demonstrates a procedure: teaches the method or shows using manipulative a concept/relationship

Episode 2

```
Int         Why is this part important to you?
SG1-1       Um, she teaches us the method. The teacher teaches us the
            method of doing the ... standard form and the power of
            ten.
Int         Oh, so this part's important to you because it is the
            part where the teacher teaches you how to write the power
            of ten?
SG1-1       Yeah.
Int         Alright, what were you doing at that time?
SG1-1       Paying attention.
```

Student SG1-1 found this segment of the lesson important as he had to pay attention to the teacher demonstrating on the board the standard form notation and what the power of ten signified.

Exposition – new knowledge

Episode 3

SG1-1	Ah this one. Standard form.
Int	Why is this part important to you?
SG1-1	Um like you can write the the speed. /In a shorter way eh.
Int	What were you thinking at that time?
SG1-1	Ah... thinking about ... thinking ... thinking of the – I I read the book and then I saw the violet light thinking about (vio) the violet light wave (prism).

Here student SG1-1 makes reference to a book about the wavelength and speed of violet light. So when the teacher started teaching them the topic standard form, he found this segment of the lesson important as it helped him make sense of what he had seen in the book.

Seatwork – group work

Episode 4

SG1-4	I think this one is quite important.
Int	Why?
SG1-4	Because we are practising and at the same time we are having teamwork team spirit.
Int	Mm I see. So you enjoy doing in a group?
SG1-4	Small (workgroup) yes. Maybe like I don't know, then the rest of my friends know, then they can teach me. / because it's quite difficult for me as I can't really solve that question.
Int	Oh. So what do you do?
SG1-4	Ah I ask my friends. / Yeah. Then they work out on the paper then I like look through then look at the differences between their working and my working.

Student SG1-4 found working in small groups important as she is able to get help from her friends with her work.

Review – student presentation

Episode 5

Int	Okay. So this part is important to you?
SG1-3	Yeah.
Int	The part where the two students present different methods.
SG1-3	Yes.
Int	I see. So what were you doing at that time?
SG1-3	Um…trying to figure out which is the like…better way of finding out the answer.
Int	Alright so is… Ah… so which way is better?
SG1-3	None of those cause the teacher teach another new method whereby // I think it's easier.
Int	So what were you thinking at that time?
SG1-3	Um I'm impressed. [giggle]
Int	Oh you're impressed by the?
SG1-3	Miss SG!. [giggle]

For Student SG1-3 this lesson segment is important as he not only gets to see two different solution approaches used by his classmates but also a third method demonstrated by the teacher.

HONG KONG: THE RESULTS FROM SCHOOL HK1

For HK1, 18 lessons were recorded and 34 students were interviewed. Codes for the lesson segments valued as important are provided in Table 4.

Table 4. Number and categorisation of lesson segments for HK1 student interviews

	Instructional Practice (IP)			Total
	Exposition	Seatwork	Review & Feedback	
	EC/D/NK/GI/RE	IW/GW/M	PK/SP/IF/GA	
	19/29/2/4/2	21/4/0	2/20/2/0	
Total number of segments in the IP / Total number of segments	56/105 (53 %)	25/105 (24%)	24/105 (23 %)	105
Student head count*	30/34 (88%)	15/34 (44%)	16/34 (47%)	34

* The number of students who commented on the IP / The total number of students.

Findings of Student Data from HK1

The number of lesson segments that HK1 students noted as important in any one lesson varied from 1 to 7 (see Appendix II). Table 4 shows that majority of the 34 students (88%) selected events that were part of the teacher's exposition. In this category, segments regarded as important included teacher explanation (EC) and teacher demonstration of the method of solving problems (D).

Seatwork was deemed important too as it was selected by nearly half of the students (44%) as an important event. For the most part of seatwork, individual seatwork (IW-21 segments), was highlighted. It is also apparent that equally important was also the review and feedback category as 16 of the 34 students (47%) noted events within the review and feedback time as important. Responses reflected that the teacher's use of students' presentations to highlight common errors, misconceptions and alternative approaches were important. The most frequent segments were those when the teacher used the student's presentation or work to give feedback on class work or homework (SP-20 segments).

From Table 4, we can see that the four aspects of instructional practice which students attached importance were: explains/explains clearly (EC, 19 segments) and demonstrates a procedures (D, 29 segments) for exposition; individual work (IW, 21 segments) for seatwork; student presentation (SP, 20 segments) for review and feedback. To further illustrate what happened in these video segments, examples of the episodes for the instructional practice with different categories are shown as follows:

Exposition – explains / explains clearly

Episode 1

HK1-24	This part ... was important, because he explained why there was no solution ... the reason
Int	That's the shot in twenty-two minutes and thirty-one seconds. Can you tell me why there is no solution now?
HK1-24	Because both equation were x plus y but their answers were not ... the same.
Int	Yes. What were you thinking?
HK1-24	I was thinking why there was no solution, but I couldn't figure that out.

Student HK1-24 could not figure out the reasons for the case of "no solution" at first but he could provide the reason in the post-lesson interview after listening to the teacher's explanation during the lesson. He thus valued the teacher's explanation as important.

Exposition – demonstrates a procedure: teaches the method or shows using manipulative a concept / relationship

Episode 2

HK1-5	He asked us what should be multiplied by five open bracket negative x minus y close bracket square in order to get five open bracket x minus y close bracket to the power three. It is ...
Int	Is this question forty?
HK1-5	Yes, this is question forty. Mr. X was asking us question forty.
Int	Could you solve it before Mr. NX explained it?
HK1-5	Yes. He introduced an easier method, so that we could use it in future.
Int	Why is it easier?
HK1-5	He drew a shape and we can ...
Int	The rectangle?
HK1-5	We could multiply them like the way we do division. It's like dividing a polynomial.

Student HK1-5 found this segment of the lesson important because the teacher used the problem as a platform for teaching an easier method so that the student could use it to solve problems in the future.

Seatwork – individual work

Episode 5

HK1-8	We should be calculating ... the last ten questions given by the teacher.
Int	Why is it important?
HK1-8	Um ... it gives us the chance to do more exercises and practice more.

Student HK1-8 valued this segment of the lesson because of the opportunities to complete the assigned exercises for practice purposes during the lesson.

Review and feedback – student presentation

Episode 4

HK1-23	Em ... was more or less the same watching him to do it and doing it myself, but ... Mr. X has pointed out his mistake ...
Int	That is twenty-six minutes?
HK1-23	Yes ... pointed out our mistake.
Int	So? Tell me once more.
HK1-23	So ... that is ... he started to tell ... started to demonstrate his wrong steps ...
Int	Em ...
HK1-23	Yes ... even that student is quite good at mathematics; he got it wrong, too. So we have to be careful, watched ... listened.

Student HK1-23 chose this episode when the teacher pointed out the mistake in a student's work and students could learn from the teacher's correction.

Linked to an earlier analysis of the Hong Kong lessons that showed that the teacher had a directive role in teaching (Mok, 2009), the results of the students' perspectives presented in this chapter provide a further understanding of both what students in this class expect and value. The high attachment to expositional explanation and demonstration suggests that the students valued the teacher's explanation and demonstration as the foundation of their learning. Their reproduction of the skills/methods in their work showed their understanding of the content knowledge. Students used the review and feedback part of the lesson as a way to know the correctness of their work and avoid mistakes. Collectively, the students' perceptions of important aspects of the instructional practice reflected how the students learned mathematics in their lessons.

SIMILARITIES AND DIFFERENCES

For SG1, the findings of the student interview data show that collectively the students attached importance to several sub-aspects of the exposition, seatwork, and review and feedback parts of the lesson. As part of exposition, students attached importance to their teacher's explanations being simple and logical; demonstration of mathematical procedures which involved showing them the "method" or concrete representation of a concept with the use of a manipulative; introduction of new knowledge; instructions that guided them in their work; and the use of real-life examples that helped them appreciate the use of math in life. As part of seatwork, students attached importance to individual work during class time that provided practice and an opportunity to check for own understanding; group work during which they experienced teamwork spirit and peer support and the material (mainly in printed form) given by the teacher to engage them in the practice of concepts and skills they had learned. As part of review and feedback they attached importance to review of prior knowledge which helped to bridge past knowledge with the present and also in the construction of new concepts using past knowledge; student presentations which resulted in the use of student work to highlight mistakes and demonstrate alternative approaches and feedback given to students individually during class time and also through grading of written assignments.

The findings of HK1 students' interview data were similar to that of SG1 in many aspects. The students attached importance to segments of the lessons associated with the instructional practice of exposition, seatwork; review and feedback. Within the exposition section of the lesson most of the students thought that teacher's explanation and demonstration of mathematical procedures were important. Their keenness to follow the teacher's exposition to obtain better understanding of concepts or mastering methods was explicit. Seatwork was also important for HK1 students but the sub-categorisation fell mostly onto individual student work. Compared with SG1, there were relatively less opportunities for group work. The students appeared to believe that opportunities for carrying out practices and copying what the teacher demonstrated were both important. Similar to SG1, the importance of the review and feedback was linked to the use of students' presentation or work for giving feedback in class. The students regarded checking the answers, learning via corrective feedback as an important part of the learning process.

DISCUSSION: WHAT REALLY MATTERS TO STUDENTS?

The results in this chapter suggest that expository instruction with strong teacher guidance is not necessarily unwelcomed by students. The phenomena in both HK1 and SG1 suggest that the students liked their teacher and their mathematics lessons. The students valued the expository instruction that served the purpose of helping them to learn. Instructional practices regarded as important included were clear explanation, demonstration of methods, and strategies helping them to learn something new. The next thing that they saw as important was the opportunity to do work in the class and learning from either the teacher or other students. They appreciated the teacher's effort of corrective feedback for mistakes and peer support given either in seat work or public sharing in the lessons.

It seems that embedded in the education culture there is an expectation for seeking a "Virtuoso" to follow (Mok & Morris, 2001). However there are two sides of a coin. On the one hand, the students in the study welcome this for the good experience of learning something new by following the demonstration and explanation given by competent teachers. This harmonious match may suggest a possible explanation for why students in Asian classrooms may have good performances despite the unfavourable factors of large class size and high examination pressure. On the other hand, while the students' preference is dominated by teacher expository and instructional pedagogies, one needs to question to what extent that such a preference constrains opportunities for students making free exploration and taking an more active role in learning? Given that mathematics teachers are passing on values, habits and customs as well as knowledge and skills (Bishop, 1997), these findings cause us to reflect critically whether these practices are inducting the students into the culture of mathematics learning that we want for them.

APPENDIX I
NUMBERS AND CATEGORIZATION OF LESSON SEGMENTS FOR SG1 STUDENT INTERVIEWS

Student ID	No of segments	Instructional Practice		
		Exposition	Seatwork	Review & Feedback
		EC / D / NK / GI / RE	I W / GW / M	PK / SP / IF / GA
SG1-1	11	2 / 2 / 1 / 0 / 0	0 / 2 / 1	0 / 3 / 0 / 0
SG1-2	8	1 / 1 / 3 / 0 / 0	0 / 1 / 0	0 / 2 / 0 / 0
SG1-3	5	0 / 1 / 2 / 0 / 0	0 / 0 / 0	0 / 2 / 0 / 0
SG1-4	8	1 / 2 / 0 / 0 / 0	0 / 1 / 0	2 / 2 / 0 / 0
SG1-5	0	0 / 0 / 0 / 0/ 0	0 / 0 / 0	0 / 0 / 0 / 0
SG1-6	1	0 / 0 / 0 / 0 / 0	0 / 1 / 0	0 / 0 / 0 / 0
SG1-7	5	2 / 0 / 0 / 1 / 0	2 / 0 / 0	0 / 0 / 0 / 0
SG1-8	5	1 / 1 / 0 / 0 / 0	0 / 1 / 1	0 / 1 / 0 / 0
SG1-9	2	1 / 0 / 0 / 0 / 0	0 / 0 / 0	0 / 1 / 0 / 0
SG1-10	3	0 / 0 / 0 / 0 / 1	1 / 0 / 0	0 / 1 / 0 / 0
SG1-11	3	1 / 1 / 0 / 0 / 0	0 / 0 / 0	0 / 0 / 0 / 1
SG1-12	3	1 / 0 / 1 / 0 / 0	0 / 0 / 0	0 / 1 / 0 / 0
SG1-13	3	1 / 1 / 0 / 0 / 1	0 / 0 / 0	0 / 0 / 0 / 0
SG1-14	1	0 / 0 / 0 / 0 / 0	0 / 0 / 0	0 / 0 / 1 / 0
SG1-15*	3	0 / 1 / 1 / 0 / 0	0 / 1 / 1	0 / 1 / 0 / 0
SG1-16	2	0 / 0 / 0 / 0 / 0	0 / 0 / 0	0 / 1 / 0 / 1
SG1-17	3	1 / 0 / 0 / 0 / 0	1 / 0 / 0	0 / 1 / 0 / 0
SG1-18	7	2 / 1 / 2 / 0 / 0	0 / 0 / 0	0 / 2 / 0 / 0
SG1-19	1	1 / 0 / 0 / 0 / 0	0 / 0 / 0	0 / 0 / 0 / 0
Total	74	15 / 11 / 10 / 1 / 2	4 / 7 / 3	2 / 18 / 1 / 2

APPENDIX II
NUMBERS AND CATEGORISATION OF LESSON SEGMENTS
FOR HK1 STUDENT INTERVIEWS

Student ID	No of segments	Instructional Practice		
		Exposition	Seatwork	Review & Feedback
		EC / D / NK / GI / RE	IW / GW / M	PK / SP / IF / GA
HK1-1	1	1/0/0/0/0	0/0/0	0/0/0/0
HK1-2	5	0/0/0/0/0	2/0/0	1/2/0/0
HK1-3	6	2/1/0/0/0	1/0/0	0/2/0/0
HK1-4	6	1/1/0/0/0	3/0/0	0/1/0/0
HK1-5	3	1/0/0/0/0	0/0/0	0/2/0/0
HK1-6	3	1/2/0/0/0	0/0/0	0/0/0/0
HK1-7	1	1/0/0/0/0	0/0/0	0/0/0/0
HK1-8	2	0/1/0/0/0	0/0/0	0/0/1/0
HK1-9	3	1/1/0/0/0	1/0/0	0/0/0/0
HK1-10	1	0/0/1/0/0	0/0/0	0/0/0/0
HK1-11	2	2/0/0/0/0	0/0/0	0/0/0/0
HK1-12	6	0/4/0/0/0	1/0/0	0/1/0/0
HK1-13	3	1/0/0/1/0	0/0/0	0/1/0/0
HK1-14	3	0/0/0/0/0	2/0/0	0/1/0/0
HK1-15	3	0/2/0/0/0	0/0/0	0/1/0/0
HK1-16	4	1/0/0/0/0	1/0/0	0/1/1/0
HK1-17	2	0/0/0/0/0	1/0/0	0/1/0/0
HK1-18	3	0/1/0/0/0	1/1/0	0/0/0/0
HK1-19	4	0/3/0/0/0	1/0/0	0/0/0/0
HK1-20	6	1/0/1/1/0	1/2/0	0/0/0/0
HK1-21	2	1/0/0/1/0	0/0/0	0/0/0/0
HK1-22	2	0/2/0/0/0	0/0/0	0/0/0/0
HK1-23	1	0/0/0/0/0	0/0/0	0/1/0/0
HK1-24	5	0/1/0/0/0	4/0/0	0/0/0/0
HK1-25	2	1/0/0/0/0	1/0/0	0/0/0/0
HK1-26	7	0/5/0/0/0	0/0/0	0/2/0/0
HK1-27	1	0/1/0/0/0	0/0/0	0/0/0/0
HK1-28	4	0/1/0/0/0	0/0/0	0/3/0/0
HK1-29	1	0/0/0/1/0	0/0/0	0/0/0/0
HK1-30	4	2/2/0/0/0	0/0/0	0/0/0/0
HK1-31	4	1/0/0/0/1	1/0/0	0/1/0/0
HK1-32	2	0/0/0/0/1	0/0/0	1/0/0/0
HK1-33	1	0/1/0/0/0	0/0/0	0/0/0/0
HK1-34	2	1/0/0/0/0	0/1/0	0/0/0/0
Total	105	19/29/2/4/2	21/4/0	2/20/2/0

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GLENDA ANTHONY

CHAPTER TWELVE

Student Perceptions of the 'Good' Teacher and 'Good' Learner in New Zealand Classrooms

INTRODUCTION

What constitutes 'good' teaching and 'good' learning is a complex and controversial issue. Educational agencies in New Zealand, like those in other western countries, have called for synthesis of research evidence (see Anthony & Walshaw, 2007; Stanley, 2008; Ingvarson, Beavis, Bishop, Peck, & Elsworth, 2004; National Mathematics Advisory Panel, 2008; Sullivan, 2011) to inform policy and professional development initiatives aimed at improving the quality of teaching and learning outcomes. In many western countries, the goal of mathematics education has been characterised as mathematical proficiency – a proficiency that includes both cognitive and dispositional/participatory components (Anthony & Walshaw, 2007; Kilpatrick, Swafford, & Findell, 2001; Sullivan, 2011) for all. This is particularly so for New Zealand where recent PISA and TIMSS results (Caygill & Kirkham, 2008, Ministry of Education, 2004) reveal a high proportion of students situated at the lower levels of proficiency when compared with other participating countries.

To realise this intellectually and socially ambitious goal, researchers (e.g., Boaler & Staples, 2008; Cobb, Gresalfi, & Hodge, 2009; Kazemi, Franke, & Lampert, 2009) argue that we need more evidence about how teachers work at enhancing students' access to powerful mathematical ideas, alongside the development of powerful mathematical identities. Recent examples of Australasian classroom-based studies that utilise analysis that includes a dual focus on students' mathematical proficiency and identity include research on student engagement (e.g., Attard, 2011; Sullivan, Tobias, & McDonough, 2006), and research on equitable pedagogical practices (e.g., Hunter & Anthony, 2011; Thornton, 2006). It is also the approach of our current study of secondary mathematics classrooms. As part of the New Zealand component of the international Learner's Perspective Study (LPS) (see Clarke, Emanuelsson, Jablonka, & Mok, 2006), our project involves the exploration of the teaching learning nexus within a series of 10 consecutive lessons in each of three Year 9 (Grade 8) mathematics classrooms. Using the lens of identity enables us to include "the broader context of the learning environment, and all the dimensions of learners' selves that they bring to the classroom" (Grootenboer & Zevenbergen, 2008, p. 243).

In this chapter I explore how students' perceptions about good teaching relate to their views about what good learning and being a good mathematics learner look

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like. In particular, I use the construct of obligations and identity to look at how students' perceptions of good teaching and good mathematics learning are co-constructed within each of the micro-cultures within the respective LPS classrooms.

THE STUDY DESIGN AND CONTEXT

The study involved students and teachers from three Year 9 (equivalent to Grade 8) urban based secondary mathematics classrooms.

- NZ1 was part of large girls' secondary school with an affluent parent base. The students in the class were drawn from a top-band group of students. The content of the NZ1 sequence of lessons involved fractions.
- NZ2 was part of a medium size co-educational secondary school. Students were from a band of lower achievers in mathematics, drawn from families who had under-privileged backgrounds and with diverse ethnic affiliations. The topic across the 10 lesson sequence involved decimals.
- NZ3 was part of a large co-educational school that drew on a largely middle socio-economic sector of the community. Like NZ1, students in the NZ3 class were drawn from a banded group of high mathematics achievers. The lesson sequence involved solution of linear equations.

The principal data generation method involved teacher and student stimulated recall interviews conducted immediately after each of the lessons. Images from two cameras, located as unobtrusively as possible within the classroom, created a split-screen video record of teacher and student actions. Two students, interviewed after each lesson, were invited to comment on their experiences of a particular lesson. This data set was supplemented by researcher field notes and lesson artefacts of student work and teacher lesson plans. Also, each teacher participated in four post-lesson stimulated-recall interviews drawn from their 10 lesson sequence.

THEORETICAL FRAMEWORK

Sfard (2005), along with many other researchers, argues that exploration of teaching is best researched with an emphasis on the social context of learning. The idea that teaching and learning are located with a complex social web draws its inspiration from Vygotskian (1986) ideas and the work of activity theorists (Engeström, 1999). This body of work proposes a close relationship between social processes and conceptual development and is given a clear expression in Lave and Wenger's (1991) well-known social practice theory, in which the notions of 'a community of practice' and 'the connectedness of knowing' are central features. According to social practice theory, students' mathematical identities and proficiencies are developed within a complex web of relationships surrounding the organisation and facilitation of knowledge production. Importantly, the organisation through regularities of shared practice of these social systems shape the ways that students are expected, entitled, and obligated to participate (Gresalfi, Martin, Hand, & Greeno, 2009; Hunter & Anthony, 2011; Xu & Clarke, 2012).

Moreover, interactions between elements with the system shape the meanings that students make of particular acts and profoundly influence the students' construction of identity.

Students' evolving mathematical identities include the ways they "think about themselves in relation to mathematics and the extent to which they have developed a commitment to, and have come to see value in, mathematics as it is realised in the classroom" (Cobb et al., 2009, pp. 40-41). As such, it is deeply implicated in the nature of the pedagogical experience – be it perceived as quality or otherwise – within the classroom (Walshaw, 2011). In contrast to studies that seek teachers' views on quality mathematics teaching and learning (e.g., Balatti & Rigano, 2011; Perry, 2007; Wilson, Cooney, & Stinson, 2005), the analysis in this paper brings students' views into play. As such, it builds on New Zealand classrooms studies that affirm that students have well-formed and articulate views about themselves as a learner and the conditions that support their learning (e.g., Bishop, Berryman, Cavanagh, & Teddy, 2009; Hunter & Anthony, 2011; Kane & Maw, 2005).

My reading of the data set (from 10 students in each of NZ1, NZ2, and NZ3) led me to question how students' perceptions of their teacher and their ways of learning mathematics are implicated in their evolving mathematical identities. My investigation of how students' perceptions of their teacher and their ways of learning constitute part of their evolving mathematical draws on the interpretative scheme offered by Cobb, Gresalfi, and Hodge (2009). In order to produce situated accounts of the identities that students are developing as doers of mathematics in particular classrooms, they organised their around two central constructs: *normative identity* and *personal identity.*

Perceived as a collective or communal notion, normative identity concerns "the identity that students would have to develop in order to become mathematical persons in a particular classroom" (Cobb & Hodge, 2011, p. 188). In looking at the development of this affiliation, the analytic focus is on the obligations that students have to fulfil in order to be an effective and successful in the context of their classroom. These obligations include both general norms for classroom participation and obligations that are specific to mathematical activity. According to Cobb et al. (2009), key general obligations concern the *distribution of authority* and the ways that students are able to *exercise agency.* Specifically mathematical obligations include:

> (i) norms for what counts as an acceptable mathematical argumentation, (ii) normative ways of reasoning with tools and written symbols, (iii) norms for what counts as mathematical understanding, and relatedly, (iv) the normative prupose for engaging in mathematical activity. (Cobb & Hodge, 2011, p. 188)

Collectively, these mathematical norms regulate what counts as being mathematically competent within the classroom.

Personal identity focuses on who students are becoming in particular mathematics classroom (Cobb & Hodge, 2007). It concerns the "extent to which individual students identify with, merely comply with, or resist their classroom obligations, and thus with what it means to know and do mathematics in their classroom" (Cobb et al., 2009, p. 44).

A WAY INTO THE DATA

Given that we know that 'good' teaching is enacted in various ways by different teachers across different classroom settings with different students (Kaur, 2009; Clarke, 2012), it was hardly surprising to observe that each of the three New Zealand classrooms offered unique learning environments – in this case broadly classified on a continuum of reform (NZ3) to traditional (NZ1), with NZ2 in between. However, despite the outward differences in the classroom learning environment, it was readily apparent that students from each of the classes collectively identified their teacher as 'good'. How then were the differential ways in which teaching was enacted with effect within each classroom implicated in the normative and personal identities that students developed? I was interested to explore whether those teacher attributes and teaching and learning practices identified as most valued by students aligned to the shared understanding of what it means to do mathematics as it is realised in each of the classrooms. Analysis was organised around the following questions:

- What attributes of the teacher were valued?
- How did the students' perception of the 'good' teacher contribute to their understanding of mathematical obligations and collective normative mathematical identity?

Ten students' post lesson interviews (in most cases student completed two interviews) from each of the three classrooms were coded according to attributes mentioned in relation to a 'good' student and their perceptions of their teacher. In some instances coding occurred in discussion of their expressed expectations of behaviour as they watched the video of their lessons, but most coding was in response to the direct questions prior to watching the lesson: "What do your think makes good mathematics students?" and "What are the important things that a student should learn in mathematics?" and the post reflections of the lesson based on the questions, "Was this a good lesson for you?", and "What makes a good lesson for you?" Based on the sets of ten students' coded responses a composite profile for each teacher was composed as discussed in the following section on the good teacher.

In looking to understand how the students' perpections of 'good' teaching and teachers were implicated in the formation of normative identities within each of the classrooms, data from both student and teacher interviews were analysed. Responses concerning ways of acting as a mathematical learner within each of the classrooms lessons were used to support conjectures about the general and specific mathematical obligations within each classroom. Additionaly, each set of classroom videos, viewed chronologically, was coded for potential general and specific mathematical obligations based on the expected (as explicit mentioned by the teacher or student) and observed behaviours of the learner across discrete episodes involving whole class teaching, group work, and individual seatwork.

Those observed obligations as classified by the researcher were triangulated with those that were evident in the student and teacher post-lesson reflections about what it means to be a learner within the class. These findings are discussed in the the subsequent section on normative mathematical identities.

GOOD TEACHERS – GOOD TEACHING

All thirty students interviewed expressed satisfaction with their teacher, and in most cases, positively enthused about their teacher. Students were able to articulate different teacher activities to emphasise their teachers' effectiveness. In considering alignment, I took identification to be suggestive that the particular behaviour is seen as normative within the classroom. For instance, a student who identifies a good teacher as one who demonstrates solution methods on the board is likely to provide calculational steps similar to the teacher in response to requests for explanations of their thinking (Cobb et al., 2009).

For students in NZ1 the reason the teacher was regarded as 'good' is captured in the response: "she is really *helpful* and she teaches me *new things* and *she explains it really well*." In elaborating the attributes of helpfulness and clarity, students provided examples of specific teacher pedagogic actions included fostering a sense of care and belonging and generally 'looking out' for those students who might be struggling:

> I think our class is really cool, we are all very friendly towards each other … there are a few quiet people who don't really speak up that much but you give them a bit of encouragement and they are pretty sweet. (S1-post L1)

Students felt that the teacher was highly attuned to their mathematical progress – actions such as regular checking of homework, requesting answers from a range of students, and walking around the room checking and helping were noted as positive teacher actions:

> There are always a few people who put their hands up and she doesn't always pick them, she picks other people apart from them so everybody else gets a chance to explain what they are doing. (S1-post L1)

Students reported a sense of confidence and willingness to seek teacher help, be it in public and in one-on-one situations:

> Like you can ask her stuff if you don't understand it, and she'll be cool, she won't like just tell you something about it and then walk off like other teachers. (S2-post L1)

Explaining was associated clarity, and often happened in association with the note taking as a regular feature of the lessons:

> The teacher, she explains things so you can think, like so you know what she's talking about. (S7-post L5)

> I think it's how the teacher writes it all up on the board. It's really helpful. (S6-Post L5)

For some students having the teacher explain things was associated with more than 'following' – it was linked to developing understanding:

> When I learn something in like the years before I would like learn it and then I would just forget and this year I understand more…I like how she does all the things on the board, like she does all those questions on the board and then she answers them and then she shows us how to answer them yourself, like she draws diagram and stuff and show you how you got the answer. (S3-post L3)

The expectation that students were required to explain and discuss solutions (sometimes between peers) and other times in whole-class discussion was also viewed positively:

> Sometimes we discuss the answer, and we discuss how different people got the answer using different strategies. I think it helps because you can see other people's point of view, and it helps you think of different ways to get the answer, like different easier ways. (S3-post L3)

Students in NZ2 assessed their teacher as 'good' along two dimensions: his actions associated with *care* and on the ways in which he made mathematics *accessible.* Collectively, these students offered a range of pedagogic actions that endorsed the teacher's efforts to create an environment that was responsive to the diverse sociopolitical realities of the class (Walshaw, Ding, & Anthony, 2009). For example, students reported that perceptions of belonging were enhanced by opportunities to come to the front of the class to present answers, with the assurance that the teacher would not embarrass them in front of the class. They appreciated that the teacher provided a certain amount of freedom to make social arrangements that suited their ways of learning as exemplified in the following description:

> We have got the coolest class ever … I like to have my friends around me...I can't work by myself, I can't think straight … We don't have a seating plan and we can sit next to anybody we want and most of my other classes we aren't allowed to. (S8-post L8)

Students' sense of belonging was associated with feeling valued and having their concerns (including social issues) and mathematical contributions 'listened to':

> He actually listens to what we are saying because most teachers don't. They don't really care what we are saying and they think they are always right all of the time and they won't listen to our side of it. Mr X takes it on and thinks about it instead of just letting it go. (S8-post L8)

> We are allowed to speak out and in the other classes you have to put your hand up and that would take forever. In out class we can say it and he won't get angry. (S9-post L9)

Making mathematics accessible was typically expressed in comments endorsing mathematics as easy, fun, or cool. Several students noted that for the first time they were enjoying mathematics – "I am learning more things that I didn't know."

In contrast to students in NZ1, the students in NZ2 were less sure of how much the teacher knew about their mathematical progress. As one student noted, "He wanders around the class quite a lot so I guess he is looking at what you are doing and asking questions." In reality, a significant proportion of the teacher time wandering around the class involved monitoring student behavior.

Students in NZ3 believed that their teacher was 'good' because he helped students to understand the mathematics. Like students in NZ1, understanding was associated with the teachers' role as a 'good' explainer. They valued that the teacher appeared to not become frustrated nor expect his students to immediately understand mathematical concepts or know how to solve all problems:

> Yes this is a really good class and it helps you to extend what you already know and the teacher is really good at explaining stuff to you and helping you if you don't understand what is going on … If you don't understand what he is doing he won't get frustrated and stuff which is good, because when teachers get frustrated it just doesn't help you to learn anything more. (S6-post L7)

> He doesn't always worry about the answer it's more about how you worked it out. (S9-Post L9)

However, in valuing the teacher's ability to 'explain things' several students noted that the process of coming to understand required effort on their part; it was challenging and required struggling with the mathematics. In relation to explaining, the students liked that the teacher encouraged them to communicate their mathematical thinking. There was a sense that participation involved more than providing answers or explanations when asked; it was also about asking questions to further learning. As one student remarked:

> [It is important to] learn how to communicate with the teacher. I think because then you can ask him questions because then you don't feel bad when you ask some questions because you need to know a lot of things … I think some teachers don't [encourage you to communicate], they just say 'do this' and they don't really care about what you are doing and they don't look back as much. (S2-Post L1)

In contrast to the students in NZ2 who associated participation with a sense of belonging, for students in NZ3 participation was more strongly aligned to getting "on with our work; we don't just do our thing we get on with the work."

NORMATIVE MATHEMATICAL IDENTITIES

Given the claim that the student in each of these classes widely endorsed the teaching and teacher, it would be likely students would also affirm the normative identity as a doer of mathematics within each class. In this section I provide a summary of the respective classroom obligations as perceived by students and teachers, and observed within specific classroom episodes, to illustrate how students' perceptions of a good teacher are implicated in the collective normative identity (Cobb et al., 2009) within each of the classrooms?

In NZ1 students regarded their teacher as 'good' because she explained things well and helped them to learn new things. This perception was strongly connected to the expressed and observed ways of participating as a mathematics learner. Across the sequence of lessons general classroom obligations included:

- Being fully resourced in terms of equipment and homework completed;
- Solving problems linked to prior knowledge, with the view to being able to explain how one worked the problem out to the teacher; and
- Listening, taking notes and asking the teacher clarifying questions in order to understand the demonstrated methods.

Specifically mathematical obligations included:

- Seeing where the calculational steps involved in computational procedures come from with reference to concrete models (e.g., overlapping arrays to explain multiplication of fractions); and
- Being able to specify the calculational steps involved in a computation using a preferred strategy.

The following episode from NZ1-L03 [22:54–24:16] exemplifies those obligations associated with the student perception of the teacher as 'expert explainer'.

Teacher	Ok, everyone put their pens down and watch up at the board because what we are going to do now is go from drawing diagrams to coming up with a method so that we don't have to draw the diagram every single time. So can I have everyone with their pens down watching please? The reason that we are going to make sure we know how to do this is when we get up to the harder questions on adding and subtracting, multiplying and dividing fractions which we are getting up to in a couple of lesson, when they ask you to convert between one or the other I am going to expect that you know how to do that. So I'm taking you right back to the basics to see what you are doing and then we are going to develop a method. So for example 1 I have drawn a diagram but we don't want to draw a diagram every single time we do this. So if I draw 4¼ ok, is everyone happy with what I have drawn up on the board? We have got four wholes split up and we have got one left over. How many quarters is that? Altogether there's seventeen of them, agree? Now that's ok, so instead of drawing the diagram every time how can I go from getting 4¼ to know that it is 17/4? What do I do, Juliet?

Juliet	Well, in a ¼ there is four and there is 4 wholes, so 4 times 4 is 16 plus the other one is 17.
Teacher	Good, I like it, nice explanation. So she is basically saying that in each whole you have got four quarters. Look at the numbers that are up on the board. So she went 4 x 4 is 16 and then we added one.

In this episode, the obligation of listening and attempting to understand was strongly linked to requests to replicate and explain each step in a calculation. Understanding was associated with seeing the link between concrete or diagrammatic representations and the calculational procedure. In addition to closely attending to the teacher several of the NZ1 students reported that they learnt by watching or listening to more expert peers or the teacher. For example:

> [You can learn things by watching what your partner does] cause I have my way of working things out and Sarah like she's so good and a bit extra, further on, and shows me things in maths and that gives me something else to think about cause I've never been strong at maths and I always take the easy way, so now I'm learning harder ways or way more complicated ways and that's like extending my thinking. (S8-post L7)
>
> I just try and remember like what we were told that lesson? (S9-post L9)

Although the students in NZ1 reported that they were expected to interact and ask the teacher questions, observations of the classroom found that questions frequently related to procedures rather than conceptual understandings. Limited opportunity to exercise conceptual agency, meant that for some students, they were unsure of how to use mathematical tools to justify or refute particular claims or to assess their own progress:

Int	How do you get to know if your maths thinking is on the right track?
S10	I guess I just ask people around me, like if I've done a question I just ask them if it's right or what they got and just compare how we worked it out together.

In NZ2 the students described their roles as mathematics learners through a range of descriptors as follows: "listen and take everything in and talk at the appropriate times," "be cooperative," "be responsible when you are putting your hand up to talk," try your hardest one hundred percent," "pay attention and do everything right," "give them respect," and "do the work." In that their descriptions of engagement were strongly related to participation in social obligations, rather than descriptions of mathematical practices, we see a clear link to their perceptions a good teacher as one who supports and cares for them as individuals and one who creates a safe space for them to participate.

The general classroom obligations for students that were apparent across the lesson sequence included:

- Volunteering solutions to review questions and being prepared to write these on the board;
- Allowing time to make responses and respecting other students' contribution;

- Listening to and taking notes of the solution methods to computational problems demonstrated by the teacher; and
- Asking the teacher for help when stuck.

However, unlike NZ1 the mathematical obligations focused more keenly on:

- Being able to specify the calculational steps involved in a computation.

The following episode from NZ2-09 [09:03-10:15] exemplifies how the shared notion of care was enacted in the classroom in terms of general and mathematical obligations. In this episode, Student R [R] is complying with the teacher [T] request to write an answer on the board to the question 3·28 × 10. Part way through the episode, when it becomes obvious that Student R is struggling to provide the correct answer, the teacher chooses to discretely supply Student R with the correct answer.

```
R     Can I put the zero on [privately to the teacher]? It's going to
      have the same three digits in it I think.
R     I don't know where the decimal point goes?
T     No you don't need a zero on the end actually. What about you
      think about this, alright what is three times ten?
R     Thirty.
T     Sorry?
R     Thirty.
T     So what do you think the answer might be if three times ten was
      thirty, what do you think the answer might be?
R     I don't know sorry...Thirty point.
T     No, no thirty. What is three times ten?
R     Three times ten is thirty.
T     What do you think three point two times ten might be?
R     I don't know.
T     What do you think point two times ten is?
R     Twenty seven.
T     [tells R the correct answer in a very quiet voice] The answer
      is thirty two point eight. Three, two, point, eight.
R     Like this (R writes 32.8 on the board)?
```

In this episode we see that the teacher efforts were directed to making Student R feel valued and comfortable in the mathematics classroom, more so than developing mathematical proficiency (Walshaw et al., 2009). Here the focus on creating a climate that fostered students' confidence in themselves as mathematics learners was enacted by lowering the cognitive challenge of tasks (Boston & Smith, 2009). This pattern was observed also in whole class teaching episodes where the teacher typically invited volunteers to give the 'next step' of the solution method and competence was demonstrated by giving the response expected by the teacher. The tendency to lower cognitive demands by simplifying tasks was favorably received by the students. In particular, students valued the way the teacher presented and explained the mathematics tasks in manageable parts, suggesting that this approach made it easier for them to work out:

> He tells us how to learn different ways and teaches us how to evolve our skill, so it's is easy because he does it step-by- step. … He takes us step-by-step instead of just saying write out this equation and then do it. (S2–post L3)

> The way he tells us the questions he gives it in an easier form, easier way to put it, so we can work it out easier. (S6-post L5)

In this class, authority was distributed to the teacher. The students determined whether they understood, or could follow, a procedure on the basis of whether the teacher endorsed their answers. For the most part, they were not expected to engage in the process of justifying how they knew that an answer was correct. When unsure, students relied on the teacher re-explaining: "He explains it more and if you don't get it he will explain it again." While this teacher's empathy with individual student's personal situations assisted in achieving his goal of creating an inclusive environment in which students willingly participated, for the most part students had limited opportunities to engage in deeper learning of mathematics.

In NZ3 the dominant classroom obligations related to a shared expectation that students actively 'struggle' with mathematical ideas as part of developing skills and understanding. General obligations included:

- Solving problems with the view to being able to explain and justify to peers and teacher how one worked the problem out;
- Asking clarifying questions in order to understand the student or teacher demonstrated methods; and
- Working collaboratively with peers.

Specifically mathematical obligations included:

- Understanding reasoning for solving problems, both the how and why a particular strategy works;
- Indicating and giving reasons for disagreement with other students' mathematical arguments; and
- Making connections between different representations and between different mathematical topics.

The expectation to struggle with problems sometimes meant attempting tasks that required students to transfer their prior knowledge to new learning and other times it meant working on a problem for an extended time. In each case, students were expected to exercise conceptual agency as is evident in the following episode occurring in NZ3-L09 [19:44-21:30]. This episode involves a whole class review of the problem $5x + 4 = 3x + 12$, following individual seat time to attempt the problem. In total, the class spent nearly 6 minutes discussing the solution process, beginning with a review of solving equations using a balance set of scales model in which you do the "same to both sides":

T J, with K's advice of doing the same to both sides, if we wanted to get rid of that plus four what would we have to do to the other side?

J We would have to do minus four on the other side.

T Minus four on the other side as well. Okay, so the strategy is to do the same to both sides and J is saying let's start off by getting rid of the plus four from the left hand side. In terms of the rest of what you said J there are actually two steps involved there. Does anyone know what they are in terms of the '*x*' terms? Let's work through it then come back to it shall we, let's follow J's advice. First, he is suggesting that we get rid of the plus four and I will draw it in red to show what we are doing to both sides. J has already suggested that we do that, the red is what we are doing to both sides. K said we need to do the same to both sides and you can see clearly that minus four and minus four is exactly the same. Then we simplify this, what does the left hand side give us H?

H 5*x*.

T Simply 5*x* and that is the whole point of what J's strategy was—no more numbers by themselves on the left. But we need to tidy up this right hand side, so what does that simplify to N?

N 3*x* plus 8.

T Good, where do we go from here....right J?

J Take away 3*x* from 5*x*.

T Why are you taking away 3*x* from 5*x*?

J Because they are like terms and you want only a number on the right hand side.

T Very good, so J has recognized two things—we only want a number on the right hand side because that is our ultimate goal and he has also recognised that we can take away 3*x* from 5*x* because they are like terms.

In this episode, "issues that emerged as topics of conversation also included the interpretation of instructional activities that underlie particular methods and strategies, and that constituted their rationale" (Cobb et al., 2009, p. 56). Students needed to explain the choice of each step in terms of not just the procedure but also in terms of the conceptual model of the balance. The teacher revoiced and modeled conceptual explanations to support students' development of conceptual agency. Such obligations were valued by the students:

> I like it at the end how he will go over it and doesn't just give us the answer he will do the working to show us how to get to the answer and that is important that you don't just get the right answer. It's the working of how you got to the answer because some people will not do it the right way or the correct process, so I think that is important. (S7-post L7)

When working at their desks, students were aware that the teacher's response to help-seeking would include requests that they reconsider part of their working to find out where they were making a mistake, or to look over a similar problem and work out how they might try the problem again. Help was directed at their understanding rather than at completion:

> If I don't understand something I know I can ask him, and he will stay with me until I understand it. And he might say something then give me a different example and then I can try and work it out and if I do then he knows I understand. (S2-post L1)

Students also expected that peers would be a source of help. Unlike in NZ2 where help involved being shown how to do it, help was more likely to involve a collaborative approach:

> Sometimes we will talk about our questions together and what we got for the answer and if we got different answers we would discuss how we got to that. (S7-post L7)

> Usually I sit with her and if we both get stuck on something we share, she'll do a little bit and then I will figure out the next bit and sometimes you need just a little push in the right direction. (S10-post L10)

Students were not worried about asking for help publically, or offering suggestions in class discussions that may or may not be correct:

> Yeah I don't have a problem doing anything like that because it is better to understand it than sit there and not know what is going on. (S7-post L7)

> Learn how to communicate with the teacher, I think because then you can ask him questions because then you don't feel bad when you ask him questions because you need to know a lot of things … to explain questions. (S2-post L1)

> Also the teacher will ask a range of different people for the answer or to explain so I think that is pretty good. (S7-post L6)

Help seeking practices, combined with obligations to justify activity in terms of making explanations comprehensible to others, meant that student learning focused on developing insight and understanding, rather than task completion or getting problems correct.

DISCUSSION

Regardless of the differences in students and the differences in learning environments between the three classrooms attributes associated with an ethic of care and teacher as explainer prevailed as descriptors of 'good' teaching.

In NZ1, the group of high-achieving students perceived that their teacher exhibited an ethic of care because she understood where they were mathematically, she created opportunities for all to contribute, and she wanted them to be confident and competent mathematically. Caring involved the teacher making sure that they were provided with the task and practice opportunities needed to progress and do well in tests within a learning environment that enabled them to work efficiently. In endorsing the teacher as explainer the students in NZ1 noted that her explanations were clear; they helped them see links between concrete and formal ways of doing problems. They liked the notes that were provided each lesson because they clarified what was required and how they should be doing each problem. In valuing these core attributes of the good teacher, students for their part suggested that being a good student is "someone who is good at numbers, and who can find alternative ways to work out a problem," "someone that does their homework, is willing to learn and doesn't just switch off all the time," "someone who is prepared, listens, just basically cooperates as well." A key obligation for students was that they needed to understand *how* the teacher did problems – "She says this is how you do it" – and thus they needed to indicate and seek more explanation if things were not clear. In meeting their obligation students in NZ1 valued the role of practice – as one student noted good students "practise everything and look over what they have done in class and at home." These comments endorsed the co-constructed general obligations of the class: that effort is needed, that you need to be prepared and work consistently in class and at home to meet the shared expectations of high achievement set by the teacher and student, and indeed the wider school/family community.

In NZ2, the group of low-achieving students perceived an ethic of care differently. They valued that their teacher 'understood' them as individuals, tolerated their need to express themselves in sometimes less than conventional ways, encouraged them to participate, and boosted their confidence as mathematics learners. For these students, care included respect for the individual as a social person as well as a mathematical learner. Although the students felt the teacher knew and cared about them as individuals, many were less sure if the teacher knew about them as a mathematics learner. For instance, Peter responded to the question of how the teacher might get to know him as a mathematics student with: "He knows my personality." In NZ2, the teacher's role as explainer, while perceived as central to the students, was also perceived and enacted differently to that of NZ1 and NZ3. Central was the view that the teacher explanations made things simple and easy for them to follow. As one student noted, because good learners "need to have skills and know ways to figure out questions" there was an appreciation that breaking learning activities into small parts helped them learn. In response to the teacher explanations student felt that their role as a good learners was to "listen to the teacher." Reliant on the teacher's ability to reduce the complexity of the mathematics task in hand, meant that there was a clear division of roles in relation to authority and agency within this class.

The group of high-achieving students in NZ3 interpreted their teacher's ethic of care as his wanting them to learn and understand mathematics. To enact this care the teacher established a learning community in which students could take risks both in group and whole class settings; where authority was distributed. In taking risks, students were obliged to communicate not just about what they knew, but also about what they didn't yet understand. These students valued the teacher as explainer, not because his explanations made mathematics problems easy to solve, but because the explanations challenged and deepened their understanding of mathematics; supporting students' conceptual agency. Completing problems was seen as an activity that engaged them in learning rather than as an end in itself. In questioning students about their role as a learner, they were more mixed in their response. While some expressed the need to know a lot of basic maths and work hard, several students were explicit in the need to communicate with the teacher, and to seek help to understand. However, what was common across all student responses was the need to be prepared to struggle with the mathematics, offering their own ideas, and attending to the mathematical thinking of others in the class.

CONCLUSION

In looking to understand why students from different classrooms, with different teachers, unanimously regarded *their* teacher as 'good', we see that the different attributes of 'goodness' – the range of teacher attributes that were most valued by students – were influenced by the diverse socio-political realities of the respective student cohorts. These realitities, in turn, influenced co-constructed normative identies within each classroom and the ways learners perceived the 'good' mathematics student. In each of the classes the students thought that the teachers were 'good' because they were caring and they explained things well. These endorsements, when linked to the general and mathematical obligations observed in each of the classrooms, closely matched students' understandings about what it means to do mathematics within their respective classrooms. It is claimed that the significant level of satisfaction, cooperation, and enjoyment expressed by students, appeared for the large part to align with the ways in which students identified (rather than resisted) with the classroom obligations associated with participating in these activities. These findings affirm that identity is rendered meaningful by particular groups and particular classroom practices (Walshaw, 2011).

What was also apparent in this analysis is that 'good' teachers and their 'good' learners co-construct unique learning communities. Each class learning environment comprised significantly different activities and associated mathematical practices that variously afforded or constrained students' opportunities to develop mathematical proficiency. Most notably, notions of 'good' were aligned with different levels and forms of teacher control and authority and levels of student agency. Understanding how students and teachers co-construct notions of 'good' practices of teaching and learning within the classroom offers possibilities of deepening our understanding of how students and teachers contribute to the ongoing regeneration of the normative identity as doers of mathematics. In the mathematics classroom, this is precisely because mathematics knowledge is created in the spaces and activities that the classroom community shares within a web of economic, social and cultural difference. Understanding how mathematical identities evolve and how students develop a sense of affiliation with mathematical activity as it is realised in their classroom must continue to be of primary concern in our quest to contribute to the improvement of equitable and culturally responsive learning and teaching.

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APPENDIX

The LPS Research Design

INTRODUCTION

The originators of the LPS project, Clarke, Keitel and Shimizu, felt that the methodology developed by Clarke and known as complementary accounts (Clarke, 1998), which had already demonstrated its efficacy in a large-scale classroom study (subsequently reported in Clarke, 2001) could be adapted to meet the needs of the Learner's Perspective Study. These needs centered on the recognition that only by seeing classroom situations from the perspectives of all participants can we come to an understanding of the motivations and meanings that underlie their participation. In terms of techniques of data generation, this translated into three key requirements: (i) the recording of interpersonal conversations between focus students during the lesson; (ii) the documentation of sequences of lessons, ideally of an entire mathematics topic; and, (iii) the identification of the intentions and interpretations underlying the participants' statements and actions during the lesson.

Miles and Huberman's text on qualitative data analysis (Miles & Huberman, 2004) focused attention on 'data reduction.'

> Even before data are collected … anticipatory data reduction is occurring as the researcher decides (often without full awareness) which conceptual framework, which cases, which research questions, and which data approaches to use. As data collection proceeds, further episodes of data reduction occur (p. 10).

This process of data reduction pervades any classroom video study. The choice of classroom, the number of cameras used, who is kept in view continuously and who appears only given particular circumstances, all contribute to a process that might better be called 'data construction' or 'data generation' than 'data reduction.' Every decision to zoom in for a closer shot or to pull back for a wide angle view represents a purposeful act by the researcher to selectively construct a data set optimally amenable to the type of analysis anticipated and maximally aligned with the particular research questions of interest to the researcher. The process of data construction does not stop with the video record, since which statements (or whose voices) are transcribed, and which actions, objects or statements are coded, all constitute further decisions made by the researcher, more or less explicitly justified in terms of the project's conceptual framework or the focus of the researcher's interest. The researcher is the principle agent in this process of data construction.

B. Kaur et al. (eds.), Student Voice in Mathematics Classrooms around the World, 227–241.

As such, the researcher must accept responsibility for decisions made and data constructed, and place on public record a transparent account of the decisions made in the process of data generation and analysis.

In the case of the Learner's Perspective Study: Research guided by a theory of learning that accords significance to both individual subjectivities and to the constraints of setting and community practice must frame its conclusions (and collect its data) accordingly. Such a theory must accommodate complementarity rather than require convergence and accord both subjectivity and agency to individuals not just to participate in social practice but to shape that practice. The assumption that each social situation is constituted through (and in) the multiple lived realities of the participants in that situation aligns the Learner's Perspective Study with the broad field of interpretivist research.

DATA GENERATION IN THE LEARNER'S PERSPECTIVE STUDY

Data generation in the Learner's Perspective Study (LPS) used a three-camera approach (Teacher camera, Student camera, Whole Class camera) that included the onsite mixing of the Teacher and Student camera images into a picture-in-picture video record (see Figure 1, teacher in top right-hand corner) that was then used in post-lesson interviews to stimulate participant reconstructive accounts of classroom events. These data were generated for sequences of at least ten consecutive lessons occurring in the "well-taught" eighth grade mathematics classrooms of teachers in Australia, the Czech Republic, Germany, Hong Kong and mainland China, Israel, Japan, Korea, The Philippines, Singapore, South Africa, Sweden and the USA. This combination of countries gives good representation to European and Asian educational traditions, affluent and less affluent school systems, and mono-cultural and multi-cultural societies.

Each participating country used the same research design to generate videotaped classroom data for at least ten consecutive mathematics lessons and post-lesson video-stimulated interviews with at least twenty students in each of three participating 8th grade classrooms. The three mathematics teachers in each country were identified for their locally-defined 'teaching competence' and for their situation in demographically diverse government schools in major urban settings. Rather than attempt to apply the same definition of teaching competence across a dozen countries, which would have required teachers in Uppsala and Shanghai, for instance, to meet the same eligibility criteria, teacher selection was made by each local research group according to local criteria. These local criteria included such things as status within the profession, respect of peers or the school community, or visibility in presenting at teacher conferences or contributing to teacher professional development programs. As a result, the diverse enactment of teaching competence is one of the most interesting aspects of the project.

In most countries, the three lesson sequences were spread across the academic year in order to gain maximum diversity within local curricular content. In Sweden, China and Korea, it was decided to focus specifically on algebra, reflecting the anticipated analytical emphases of those three research groups.

Algebra forms a significant part of the 8th grade mathematics curriculum in most participating LPS countries, with some variation regarding the sophistication of the content dealt with at 8th grade. As a result, the data set from most of the LPS countries included at least one algebra lesson sequence.

In the key element of the post-lesson student interviews, in which a picture-in-picture video record was used as stimulus for student reconstructions of classroom events (see Figure 1), students were given control of the video replay and asked to identify and comment upon classroom events of personal importance. The post-lesson student interviews were conducted as individual interviews in all countries except Germany, Israel and South Africa, where student preference for group interviews was sufficiently strong to make that approach essential. Each teacher was interviewed at least three times using a similar protocol.

Figure. 1 Picture-in-picture video display

With regard to both classroom videotaping and the post-lesson interviews, the principles governing data generation were the minimisation of atypical classroom activity (caused by the data generation activity) and the maximisation of respondent control in the interview context. To achieve this, each videotaped lesson sequence was preceded by a one-week familiarisation period in which all aspects of data generation were conducted until the teacher indicated that the class was functioning as normally as might reasonably be expected.

In interviews, the location of control of the video player with the student ensured that the reconstructive accounts focused primarily on the student's parsing of the lesson. Only after the student's selection of significant events had been exhausted did the interviewer ask for reconstructive accounts of other events of interest to the research team. Documentation of the participant's perspective (learner or teacher) remained the priority.

In every facet of this data generation, technical quality was a priority. The technical capacity to visually juxtapose the teacher's actions with the physical and oral responses of the children was matched by the capacity to replay both the public statements by teacher or student and the private conversations of students as they struggled to construct meaning. Students could be confronted, immediately after the lesson, with a video record of their actions and the actions of their classmates.

In the picture-in-picture video record generated on-site in the classroom (Figure 1), students could see both their actions and the actions of those students around them, and, in the inset (top right-hand corner), the actions of the teacher at that time. This combined video record captured the classroom world of the student. The video record captured through the whole-class camera allowed the actions of the focus students to be seen in relation to the actions of the rest of the class.

CLASSROOM DATA GENERATION

Camera Configuration

Data generation employed three cameras in the classroom – a "Teacher Camera," a "Student Camera" and a "Whole Class Camera." The protocol below was written primarily for a single research assistant/videographer, but brief notes were provided suggesting variations possible if a second videographer was available. In order to ensure consistency of data generation across all schools in several countries, the protocol was written as a low inference protocol, requiring as few decisions by the videographer as possible. One or two possible anomalous cases were specifically discussed – such as when a student presents to the entire class. However, the general principles were constant for each camera: The Teacher Camera maintained a continuous record of the teacher's statements and actions. The Student Camera maintained a continuous record of the statements and actions of a group of four students. The Whole Class Camera was set up in the front of the classroom to capture, as far as was possible, the actions of every student – that is, of the "Whole Class." The Whole Class Camera can also be thought of as the "Teacher View Camera." While no teacher can see exactly what every individual student is doing, the teacher will have a sense of the general level of activity and types of behaviors of the whole class at any time – this is what was intended to be captured on the Whole Class Camera.

Camera One: The Teacher Camera

The "Teacher Camera" maintained the teacher in centre screen as large as possible *provided that all gestures and all tools or equipment used could be seen* – if overhead transparencies or boardwork or other visual aids were used then these had to be captured fully at the point at which they were generated or employed in the first instance or subsequently amended – but did not need to be kept in view at the expense of keeping the teacher in frame (provided at least one full image was recorded, this could be retrieved for later analysis – the priority was to keep the teacher in view). The *sole exception* to this protocol occurred when a student worked at the board or presented to the whole class. In this case, the Teacher

Camera focused on the "student as teacher." The actions of the Teacher during such occasions should have been recorded by the Whole Class Camera. If the teacher was positioned out of view of the Whole Class Camera (eg front of classroom, at the side), then the Teacher Camera might "zoom out" to keep both the student and teacher on view, but documentation of the gestures, statements, and any written or drawn work by the student at the board should be kept clearly visible. Note: Although the teacher was radio-miked, in the simulated situations we trialled it was not necessary for the teacher to hand the lapel microphone to the student. The student's public statements to the class could be adequately captured on the student microphone connected to the Student Camera. The first few lessons in a particular classroom (during the familiarisation period) provided an opportunity to learn to "read" the teacher's teaching style, level of mobility, types of whole class discussion employed, and so on. A variety of practical decisions about the optimal camera locations could be made during the familiarisation period and as events dictated during videotaping.

Camera Two: The Student Camera

Where only a single videographer was used, the "Student Camera" was set up prior to the commencement of the lesson to include at least two adjacent students and was re-focussed in the first two minutes of the lesson during the teacher's introductory comments – during this time the Teacher Camera could be set up to record a sufficiently wide image to include most likely positions of the teacher during these opening minutes. Once the Student Camera was adequately focussed on the focus students for that lesson, it remained fixed unless student movement necessitated its realignment. After aligning the Student Camera, the videographer returned to the Teacher Camera and maintained focus on the teacher, subject to the above guidelines.

If two research assistants ("videographers") were available (and this was frequently the case), then it became possible for the Student Camera to "zoom in" on each student's written work every five minutes or so, to maintain an on-going record of the student's progress on any written tasks. This "zooming in" was done sufficiently briefly to provide visual cues as to the progress of the student's written work, but any such zooming in had to be done without losing the continuity of the video record of all focus students, since that would be needed for the subsequent interviews. Since it was Learner Practices that were the priority in this study, the continuous documentation of the actions of the focus students and their interactions (including non-verbal interactions) was most important. A copy of the students' written work was obtained at the end of the lesson. The video record generated by this camera served to display each student's activities in relation to the teacher's actions, the tasks assigned, and the activities of their nearby classmates.

Camera Three: The Whole Class Camera

The "Whole Class (or Teacher-View) Camera" was set up to one side of whichever part of the room the teacher spoke from (typically, to one side at the "front" of the classroom). All students should be within the field of view of this camera (it is necessary to use a wide-angle lens). Apart from capturing the "corporate" behavior of the class, this camera provided an approximation to a "teacher's-eye view" of the class. It was also this camera that documented teacher actions during any periods when a student was working at the board or making a presentation to the entire class.

Microphone Position

The teacher was radio-miked to the Teacher Camera. The focus student group was recorded with a microphone placed as centrally as possible in relation to the focus students and recorded through the Student Camera (use of a radio microphone minimized intrusive cables). The Whole Class Camera audio was recorded through that camera's internal microphone.

Fieldnotes

Depending on the available research personnel, fieldnotes were maintained to record the time and type of all *changes* in instructional activity. Such field notes could be very simple, for example:

00:00 Teacher Introduction
09:50 Students do Chalkboard Problem
17:45 Whole Class Discussion
24:30 Individual Textbook Work
41:45 Teacher Summation

Specific events of interest to the researcher could be included as annotations to such field notes.

Where a third researcher was available, in addition to the operators of the Teacher and Student cameras, this person was able to take more detailed field notes, including detail of possible moments of significance for the progress of the lesson (eg public or private negotiations of meaning). In such cases, the field notes became a useful aid in the post-lesson interview, and the interviewee could be asked to comment on particular events, if these had not been already identified by the interviewee earlier in the interview.

Student Written Work

All written work produced by the focus students "in camera" during any lesson was photocopied together with any text materials or handouts used during the lesson. Students brought with them to the interview their textbook and all written material produced in class. This material (textbook pages, worksheets, and student written work) was photocopied immediately after the interview and returned to the student.

INTERVIEWS

In this study, students were interviewed after each lesson using the video record as stimulus for their reconstructions of classroom events. It is a feature of this study that students were given control of the video replay and asked to identify and comment upon classroom events of personal importance. Because of the significance of interviews within the study, the validity of students' and teachers' verbal reconstructions of their motivations, feelings and thoughts was given significant thought. The circumstances under which such verbal accounts may provide legitimate data have been detailed in two seminal papers (Ericsson & Simon, 1980; Nisbett & Wilson, 1977).

It is our contention that videotapes of classroom interactions constitute salient stimuli for interviewing purposes, and that individuals' verbal reports of their thoughts and feelings during classroom interactions, when prompted by videos of the particular associated events, can provide useful insights into those individuals' learning behaviour. Videotapes provide a specific and immediate stimulus that optimises the conditions for effective recall of associated feelings and thoughts. Nonetheless, an individual's video-stimulated account will be prone to the same potential for unintentional misrepresentation and deliberate distortion that apply in any social situation in which individuals are obliged to explain their actions. A significant part of the power of video-stimulated recall resides in the juxtaposition of the interviewee's account and the video record to which it is related. Any apparent discrepancies revealed by such a comparison warrant particular scrutiny and careful interpretation by the researcher. Having relinquished the positivist commitment to identifying 'what really happened,' both correspondence and contradiction can be exploited. The interview protocols for student and teacher interviews were prescribed in the LPS Research Design and are reproduced below.

Individual Student Interviews

Prompt One:	Please tell me what you think that lesson was about (lesson content/lesson purpose).
Prompt Two:	How, do you think, you best learn something like that?
Prompt Three:	What were your personal goals for that lesson? What did you hope to achieve? Do you have similar goals for every lesson?
Prompt Four:	Here is the remote control for the videoplayer. Do you understand how it works? (Allow time for a short familiarisation with the control). I would like you to comment on the videotape for me. You do not need to comment on all of the lesson. Fast forward the videotape until you find sections of the lesson that you think were important. Play these sections at normal speed and describe for me what you were doing,

thinking and feeling during each of these videotape sequences. You can comment while the videotape is playing, but pause the tape if there is something that you want to talk about in detail.

Prompt Five: After watching the videotape, is there anything you would like to add to your description of what the lesson was about?

Prompt Six: What did you learn during that lesson?
[Whenever a claim is made to new mathematical knowledge, this should be probed. Suitable probing cues would be a request for examples of tasks or methods of solution that are now understood or the posing by the interviewer of succinct probing questions related to common misconceptions in the content domain.]

Prompt Seven: Would you describe that lesson as a good* one for you? What has to happen for you to feel that a lesson was a "good" lesson? Did you achieve your goals? What are the important things you should learn in a mathematics lesson?
[*"Good" may be not be a sufficiently neutral prompt in some countries – the specific term used should be chosen to be as neutral as possible in order to obtain data on those outcomes of the lesson which the student values. It is possible that these valued outcomes may have little connection to "knowing," "learning" or "understanding," and that students may have very localised or personal ways to describe lesson outcomes. These personalised and possibly culturally-specific conceptions of lesson outcomes constitute important data.]

Prompt Eight: Was this lesson a typical [geometry, algebra, etc.] lesson? What was not typical about it?

Prompt Nine: How would you generally assess your own achievement in mathematics?

Prompt Ten: Do you enjoy mathematics and mathematics classes?

Prompt Eleven: Why do you think you are good [or not so good] at mathematics?

Prompt Twelve: Do you do very much mathematical work at home? Have you ever had private tutoring in mathematics or attended additional mathematics classes outside normal school hours?

Prompts 9 through 12 could be covered in a student questionnaire – the choice of method may be made locally, provided the data is collected.

Student Group Interviews

Prompt One: Please tell me what you think that lesson was about (lesson content/lesson purpose) (Discuss with the group – identify points of agreement and disagreement – there is NO need to achieve consensus).

Prompt Two: Here is the remote control for the videoplayer. I would like you to comment on the videotape for me. You do not need to comment on all of the lesson. I will fast forward the videotape until anyone tells me to stop. I want you to find sections of the lesson that you think were important. We will play these sections at normal speed and I would like each of you to describe for me what you were doing, thinking and feeling during each of these videotape sequences. You can comment while the videotape is playing, but tell me to pause the tape if there is something that you want to talk about in detail.

Prompt Three: After watching the videotape, is there anything anyone would like to add to the description of what the lesson was about?

Prompt Four: What did you learn during that lesson? (Discuss)
[As for the individual interview protocol, all claims to new mathematical knowledge should be probed. BUT, before probing an individual's responses directly, the interviewer should ask other members of the group to comment.]

Prompt Five: Would you describe that lesson as a good* one for you? (Discuss) What has to happen for you to feel that a lesson was a "good" lesson? (Discuss) What are the important things you should learn in a mathematics lesson?
[*As for the student individual interviews, "good" may be not be a sufficiently neutral prompt in some countries – the specific term used should be chosen to be as neutral as possible in order to obtain data on those outcomes of the lesson which the student values].

Prompt Six: Was this lesson a typical [geometry, algebra, etc] lesson? What was not typical about it?

The Teacher Interview

The goal was to complete one interview per week, according to teacher availability. The Whole Class Camera image was used as the stimulus. In selecting the lesson about which to seek teacher comment, choose either (1) the lesson with the greatest diversity of classroom activities, or (2) the lesson with the most evident student interactions. Should the teacher express a strong preference to discuss a particular lesson, then this lesson should take priority. Tapes of the other lessons should be available in the interview, in case the teacher should indicate an interest in any aspect of a particular lesson.

Prompt One: Please tell me what were your goals in that lesson (lesson content/lesson purpose).

Prompt Two: In relation to your content goal(s), why do you think this content is important for students to learn?
What do you think your students might have answered to this question?

Prompt Three: Here is the remote control for the videoplayer. Do you understand how it works? (Allow time for a short familiarisation with the control). I would like you to comment on the videotape for me. You do not need to comment on all of the lesson. Fast forward the videotape until you find sections of the lesson that you think were important. Play these sections at normal speed and describe for me what you were doing, thinking and feeling during each of these videotape sequences. You can comment while the videotape is playing, but pause the tape if there is something that you want to talk about in detail.

In particular, I would like you to comment on:

(a) Why you said or did a particular thing (for example, conducting a particular activity, using a particular example, asking a question, or making a statement).

(b) What you were thinking at key points during each video excerpt (for example, I was confused, I was wondering what to do next, I was trying to think of a good example).

(c) How you were feeling? (for example, I was worried that we would not cover all the content)

(d) Students' actions or statements that you consider to be significant and explain why you feel the action or statement was significant.

(e) How typical that lesson was of the sort of lesson you would normally teach? What do you see as the features of that lesson that are most typical of the way you teach? Were there any aspects of your behavior or the students' behavior that were unusual?

Prompt Four: Would you describe that lesson as a good lesson for you? What has to happen for you to feel that a lesson is a "good" lesson?

Prompt Five: Do your students work a lot at home? Do they have private tutors?

OTHER SOURCES OF DATA

Student tests were used to situate each student group and each student in relation to student performance on eighth-grade mathematics tasks. Student mathematics achievement was assessed in three ways:

Student written work in class. Analyses of student written work were undertaken both during and after the period of videotaping. For this purpose, the written work of all "focus students" in each lesson was photocopied, clearly labelled with the student's name, the class, and the date, and filed. Additional data on student achievement was also collected, where this was available. In particular, student scores were obtained on any topic tests administered by the teacher, in relation to mathematical content dealt with in the videotaped lesson sequence.

Student performance to place the class in relation to the national 8th grade population. In Australia, Japan, Korea, China and the USA, this was done by using the International Benchmark Test for Mathematics (administered immediately after the completion of videotaping). The International Benchmark Test (IBT) was developed by the Australian Council for Educational Research (ACER) by combining a selection of items from the TIMSS Student Achievement test. In the case of this project, the test for Population Two was used, since this was in closest correspondence with the grade level of the students taking part in the LPS project. In administering the IBT, the local research group in each country constructed an equivalent test using the corresponding version of each of the TIMSS items, as administered in that country. In some countries, where this was not possible (Germany, for example), the typical school performance was characterised in relation to other schools by comparison of the senior secondary mathematics performance with national norms.

Student performance in relation to other students in that class. Since student-student interactions may be influenced by perceptions of peer competence, it was advantageous to collect recent performance data on all students in the class. Two forms of student mathematics achievement at class level were accessed, where available: (a) student scores from recent mathematics tests administered by the teacher, and (b) brief annotated comments by the teacher on a list of all students in the class – commenting on the mathematics achievement and competence of each student.

Teacher Goals and Perceptions

Teacher questionnaires were used to establish teacher beliefs and purposes related to the lesson sequence studied. Three questionnaires were administered to each participating teacher:

- A ***preliminary*** teacher questionnaire about each teacher's goals in the teaching of mathematics (TQ1);
- A ***post-lesson*** questionnaire (TQ2 – either the short TQ2S or the long TQ2L version – if the short version was used, the researcher's field notes provided as much as possible of the additional detail sought in the long version);
- A ***post-videotaping*** questionnaire (TQ3) (also employed by some research groups as the basis of a final teacher interview).

DATA CONFIGURATION AND STORAGE

Transcription and Translation

A detailed Technical Guide was developed to provide guidelines for the transcription and translation of classroom and interview, video and audiotape data. It was essential that all research groups transcribe their own data. Local language variants (e.g., the Berliner dialect) required a "local ear" for accurate transcription. Translation into English was also the responsibility of the local research group. The Technical Guide specified both transcription conventions, such as how to represent pauses or overlapping statements, and translation conventions, such as how to represent colloquialisms. In the case of local colloquial expressions in a language other than English, the translator was presented with a major challenge. A literal English translation of the colloquialism may convey no meaning at all to a reader from another country, while the replacement of the colloquialism by a similar English colloquialism may capture the essence and spirit of the expression, but sacrifice the semantic connotations of the particular words used. And there is a third problem: If no precise English equivalent can be found, then the translation inevitably misrepresents the communicative exchange. In such instances, the original language, as transcribed, was included together with its literal English translation. Any researcher experiencing difficulties of interpretation in analysing the data could contact a member of the research group responsible for the generation of those data and request additional detail.

Data Storage

To carry out serious systematic empirical work in classroom research, there is a need for both close and detailed analysis of selected event sequences, and for more general descriptions of the material from within which the analysed sample has been chosen. To be able to perform this work with good-quality multiple-source video and audio data, video and audio materials have to be compressed and stored in a form accessible by desktop computers. Software tools such as *Final Cut Pro* are essential for the efficient and economical storage of the very large video data files. Compression decisions are dictated by current storage and back-up alternatives and change as these change. For example, when the Learner's Perspective Study was established in 1999, it was anticipated that data would be exchanged between research teams by CD-ROM and compression ratios were set at 20:1 in order to get maximum data quality within a file size that would allow one video record of one lesson to be stored on a single CD. As a result, the complete US data set in 2001 took the form of a set of over fifty separate CDs. Later, it was possible to store all the data related to a single lesson (including four compressed video records) on a single DVD. The contemporary availability of pocket drives with capacities of 60 gigabytes and higher, has made data sharing both more efficient and cheaper. It is possible to store all the data from a single school in compressed form on such a pocket drive, making secure data transfer between international research groups much more cost-effective.

The materials on the database have to be represented in a searchable fashion. In Figure 2, the configuration of the LPS database is displayed as a stratified hierarchy of: Country (column 1), school (column 2), lesson (column 3), data source (column 4), specific file (column 5). Any particular file, such as the teacher camera view of lesson 4 at school 2 in Japan, can then be uniquely located.

Setting up data in this way enables researchers to move between different layers of data, without losing sight of the way they are related to each other. Further, data can be made accessible to other researchers. This is a sharp contrast to more traditional ways of storing video data on tapes, with little or no searchable record available, and with data access limited to very small numbers of people. At the International Centre for Classroom Research (ICCR) at the University of Melbourne, for example, several researchers can simultaneously access the full range of classroom data. This capacity for the simultaneous analysis of a common body of classroom data is the technical realisation of the methodological and theoretical commitment to complementary analyses proposed by Clarke (1998, 2001) as essential to any research attempting to characterise social phenomena as complex as those found in classrooms.

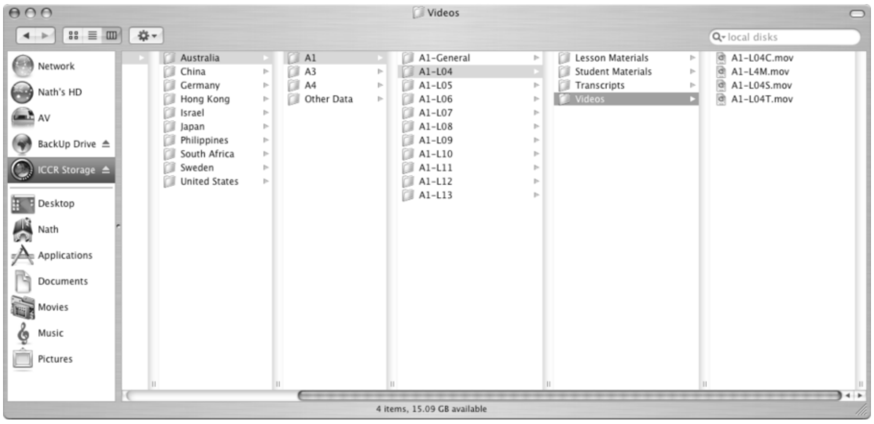

Figure 2. Structure of the LPS database at the ICCR circa 2004

ANALYTICAL TOOLS CAPABLE OF SUPPORTING SOPHISTICATED ANALYSES OF SUCH COMPLEX DATABASES

Research along the lines argued for above requires the development of software tools for analysing video efficiently. The reasons for this are, in short, that video editing software (such as *Final Cut Pro*) is not analytically resourceful enough, whereas qualitative analysis software (such as *Nudist* or *nVivo*) is not well enough adapted to video and audio work. Early examples of video analysis software (such as *vPrism*) have been hampered by problems arising from their project-specific origins, leading to a lack of flexibility in customising the analysis to the demands of each particular project or research focus.

Collaboration with the Australian software company, Sportstec, was carried out to adapt the video analysis software *Studiocode* for use with classroom video data. These adaptations were driven by specific methodological, theoretical and practical needs. For example, the commitment to the capturing and juxtaposition of multiple perspectives on classroom events was partially addressed with the onsite capture of the picture-in-picture display shown in Figure 1, but the need to 'calibrate' the actions of the focus students against the actions of the rest of the class required multiple viewing windows.

Figure 3 displays the key analytical elements provided within *Studiocode*: video window, time-line, transcript window, and coding scheme. The researcher has the option of analysing and coding the events shown in the video window, or the utterances shown in the transcript window, or both. The resultant codes can be displayed in timelines (as shown in Figure 3) or in frequency tables. Once coded, single lessons, events within single lessons, or combinations of lessons can be merged into a single analysis.

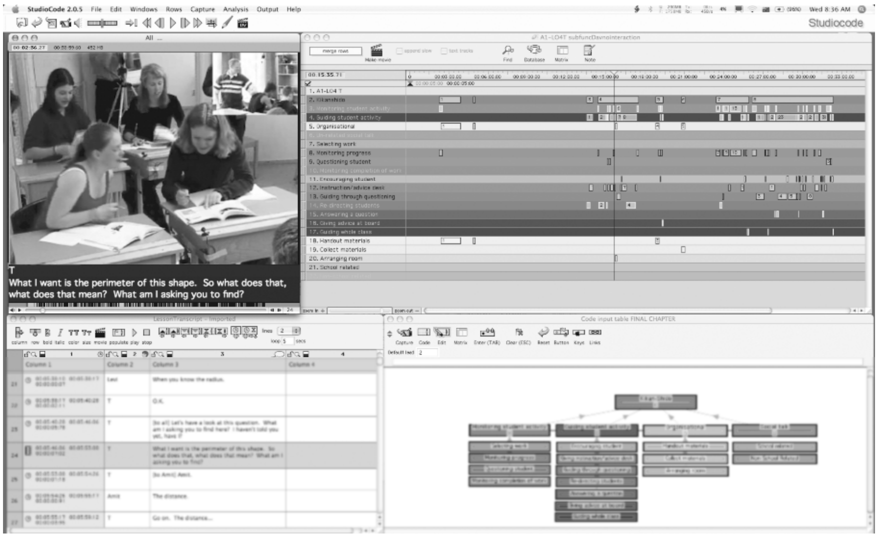

Figure 3. Sample analytical display (Studiocode) – video window (top left), time-line (top right), transcript window (bottom-left) and coding facility (bottom-right)

The continual addition of new countries to the Learner's Perspective Study community required that video data already coded should not need to be recoded when additional data (eg from a different country) were incrementally added to the database. Only the new data should require coding and the newly-coded data should be accessible for analysis as part of the growing pool of classroom data. This flexibility is ideally suited to a project such as the Learner's Perspective Study, with many collaborating researchers adopting a wide range of different analytical approaches to a commonly held body of classroom data.

The *Studiocode* software described above is only one of the many analytical tools available to the classroom researcher. Increasingly sophisticated public access software tools are being developed continually. Most of the chapters in this book and in the companion volume (Clarke, Keitel, & Shimizu, 2006) report specific analyses of different subsets of the large body of LPS classroom data. Each analysis is distinctive and interrogates and interprets the data consistent with the purpose of the authoring researcher(s). Analytical tools such as *nVivo* and *Studiocode* can support the researcher's analysis but ideally should not constrain the consequent interpretation of the data. In reality, all such tools, including statistical procedures, constrain the researcher's possible interpretations by limiting the type of data compatible with the analytical tool being used, by restricting the variety of codes, categories or values that can be managed, and by constraining the range of possible results able to be generated by the particular analytical tool.

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