

8. STUDENT ACTIVITIES WITH E-EXERCISE BASES

INTRODUCTION

In this chapter, we examine the use of specific Internet resources: electronic exercises bases. We call an electronic exercises bases (EEB) for mathematics an Internet resource developed for mathematics teaching and learning purposes, consisting of mathematics exercises following a certain classification, and such that each exercises is associated to an environment, which includes different types of suggestions, aids, tools (graphs, calculators, etc.), lesson reminders, as well as explanations, answer analyses or complete solutions. In most of these products, the exercises have parameters which are randomly generated. This allows students to work on the same exercise several times. In such a case, the structure of the newly proposed exercise remains the same but the variables (e.g. numerical values, functions) differ.

Such resources, whether free or not, exist for all school levels and are more and more common. They can differ largely depending on their didactical structure, the type of accepted answer, or their type of implemented interactivity. Nevertheless, although the study of the use of technologies in mathematics learning is a fertile and expanding field of study, only few studies are specific to the use of EEB. Most of the articles dedicated to mathematics and digital technologies deal with “open environments” (microworlds or Computer Algebra Systems). These studies usually aim at conceiving and testing several didactical engineering, in which the “antagonistic milieu,” within the meaning of Brousseau (1997), includes the technological tool and is resistant to students’ actions, producing retroactions which help them to construct new knowledge. On the contrary, EEB constitute “allied milieu” designed to help learners. Moreover, using an EEB does not present any major technical difficulty. Many EEB have originally been designed for private use by students. Therefore, the question of handling technological tools is less complex in the cases of EEB and arouses less questions of instrumental genesis developed in the case of open environments (Artigue, 2002). The results of research are thus not transferable from one technology to the other. The question for the researcher studying the use of EEB in the classroom is to qualitatively analyze the use of resources in ordinary classrooms and to derive information about the activity of the students and teachers using these tools. This concern corresponds to the general question of this book, in which we try to analyze the teaching and learning of mathematics as they are, and not as they could or should be.

Positive consequences of the use of EEB have already been observed in certain research studies. Ruthven and Henessy (2002) have for example carried out an extensive study about the use of technologies in mathematics teaching in England. They observe that drill and practice products, which are particular EEB, allow a work adapted to the rhythm of each student, as well as an increase of the motivation of these students. However, it seems important to pursue more precise investigations about student activity using these tools, in order to determine the contributions, limits and constraints of using EEB in mathematics classrooms.

We directly import the model of double regulation of activity introduced in chapter 1 to analyze students' activity on EEB. In fact, the scenarios of use of these resources expect students to repeat several times the same exercise, with variant exercise statements, or to solve a series of similar exercises. The actual activity of the students thus produces results, mainly feedbacks from the software, which modify the initial situation on the EEB: we can talk about productive¹ activity of the students and functional regulations of their activity. The student actions and the software retroactions are particularly observable with EEB because students repeat several times the exercises. The evolution of the productive activity results for a given exercise, in the course of regulation loops, can hence be observed and interpreted or not in terms of constructive students' activity (during the average time of action). The question of the long-term learning, and the study of the effects of learning through the EEB in a paper-pencil environment, is more complex and therefore our results are necessarily limited.

Based on our observations, we conduct *a priori* analyses of the situations proposed to students, and in regards to chosen episodes, we analyze, in particular, the tasks prescribed to the observed students. For the mathematical analyses of tasks, we retain the tools developed in chapter 2. In particular, we wonder whether the tasks are direct applications of explicit mathematical knowledge or, on the contrary, if there are adaptations and/or recognitions of knowledge to be made. We also take into consideration the software environment of the tasks, that is all the external hints or instrumental factors that could be of help, or not, in completing the tasks. We finally specify personal data about the observed students even if we often only have few elements on that matter. The results of the productive activity of the subjects are observed through the answers entered by the students into the computers. In particular, the software retroactions, as well as the aids given by teacher, if any, provide us with data regarding the modifications of the situations in the regulation loops (chapter 1).

In the second section, we give a first example of situation analysis that consists of analyses of tasks and software environments for EEB exercises suggested to students. In the third and fourth section, we study, using examples, how the situations influence the students' activity. In particular, we show that the expected activity is not always the activity developed by the students. We also show how difficult it is for the students to regulate their activity while facing the software without teacher intervention. Finally, in the fifth section, we conclude about the favorable conditions for a reasoned use of the EEB with the students.

EXAMPLES OF TASKS AND SOFTWARE ENVIRONMENTS ANALYSES

From the point of view of the mathematical knowledge at stake, the methodology to analyze these tasks is the one presented in chapter 2. We distinguish in particular the task of direct application of knowledge from all the other tasks which are described as complex.² The analysis of the tasks depends from the scenario in which these tasks intervene. For example, the fact that the implemented tasks are old or new for the student, with a level of knowledge which is “available” or “indicated” (see chapter 2), is an information which must be taken into account. Some elements of the scenario are implemented in the resources whereas other items are left at the discretion of the teacher.

From the point of view of the interface with the software, the characteristic elements of the tasks are the type of expected answer (multiple choice, numerical value, geometrical drawing, and so on), the aids proposed by the software (in particular the occasional corrections), and more generally the software environment of the exercise which can facilitate or complicate the solving of the tasks. This makes the analysis of the tasks more complex for the researcher than in the traditional environment. Below is an illustration of this complexity through an exercise from EEB Euler.³

It's an exercise which involves old knowledge from grade 9 at the indicated level; no recognition of knowledge at stake is necessary. Knowledge is explicit with the statement of the exercise. The exercise given is “Given an orthonormal coordinate system, move the points A and B such that the line (AB) represents the function defined for all x by $f(x) = 7/2 - x/8$.” Hence, the task consists in moving two points on the gridline so that the line passing through those two points becomes the curve representing a linear function randomly given. So the work is on the transition from the algebraic registry to the graphical registry (Duval, 1995).

nouveau aide guide notions brouillon clavier écran socio

EULER Placer deux points à coordonnées entières appartenant à la représentation graphique d'une fonction affine ressource 222

Le plan est muni d'un repère orthogonal.
Déplacez les points A et B de telle façon que la droite (AB) représente la fonction affine définie pour tout nombre x par $f(x) = \frac{7}{2} - \frac{x}{8}$.

Valider

Figure 1. Exercise from the EEB Euler.

The task is however not immediate since there are two sub-tasks for a grade 9 student: the first is finding the coordinates of the two points of line (AB) using its equation stemming from the algebraic expression of the given function f . The second sub-task is to move the points A and B on the screen until they reach the correct position.

The software environment brings difficulties because the presence of the line (AB) on the graph, from the beginning of the exercise, disrupts the student's perception of the expected task. Indeed, the task is to move points A and B which are already given, whereas in a traditional environment, the task consists in placing these points. Moreover, we can only move these points onto positions with integer coordinates. This constitutes a major difficulty related to the task environment and this complicates it since it doesn't allow the student to place all the points found by calculation. Students must test their calculation in order to find points A and B with integer coordinates. Finally, the points that can be placed must have abscissas and ordinates of values between -5 and 5 , which is another difficulty for the students while they look for the points. In a paper-pencil environment, we can extend the graphical representation to place the points with abscissas or ordinates outside $[-5, 5]$. Here, this is not possible. The interest of the software is yet considerable. On one hand, it offers the possibility of repeating the exercise with random variables. The students can thus repeat several times the exercise with new lines and practice until they succeed. On the other hand, the retroaction in the case of a mistake indicates that the student's answer is wrong and gives the function represented by the line (AB) suggested by the student, as is shown in the example in Figure 2.

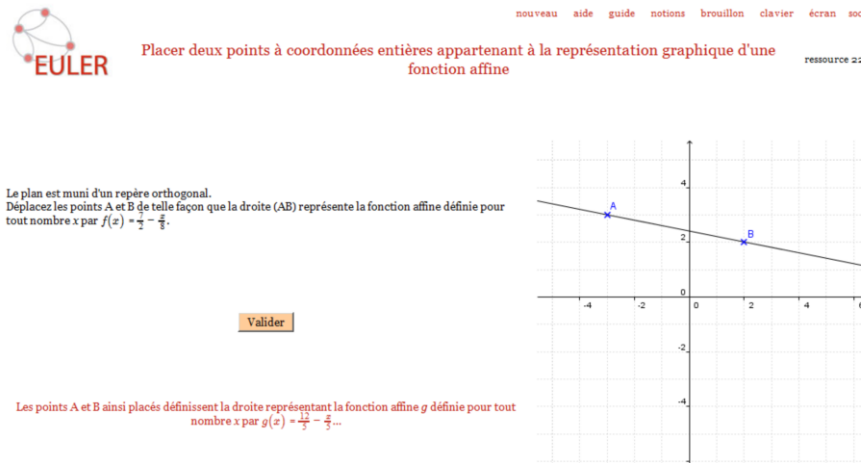


Figure 2. Graphical feedback for the exercise.

The software message is “*The points A and B that you placed define the line representing the function g defined by $g(x)=12/5-x/5$.*” This feedback allows students to reflect on their propositions and understand their mistakes. This could allow rectifying their answers but they are unfortunately not entitled to a second trial.

Analyses of teaching practices with EEB developed by Cazes, Gueudet, Hersant, and Vandebrouck (2006) show that such quite immediate exercises seem to be necessary for the learning of the students. However, due to their over simplicity when they are solved in a paper-pencil environment, such exercises are rarely proposed in class, particularly in classical solving sessions at university level. This simple example gives an idea of the work possible through EEB, but only a thorough exploration of each of the websites would allow us to discover the numerous possibilities offered by these resources. In particular, we have observed that tasks are made possible thanks to the work on the EEB, with new associated activities; whereas new activities about tasks similar to the paper-pencil ones can be created. The sections below are dedicated to the study of examples of the actual activity of students on EEB, in a grade 10 classroom on one hand, and in higher education on the other hand.

While many tasks, which are well represented in EEB, seem adapted to be easily solved using a computer, definitely not all tasks can be completed using a computer. The teacher can, for example, choose to leave certain immediate applications for computerized work, so that the paper-pencil environment activity is centered on more complex tasks. We will come back to the work of the teacher in chapter 9.

EXAMPLES OF ACTIVITIES IN GRADE 10

The examples presented in this paragraph stem from observations conducted during the 2004-2005 and 2005-2006 school years in general and professional high schools. In each case, an observer is placed behind a student and notes all his/her visible actions. The methodology is the one detailed in chapter 2, with specificities related to the computerized work of the students. Only few episodes which are significant for our chapter are analyzed below, in order to directly access interesting results. These are observations of sessions organized in half-group, supervised by the teacher who is responsible of the class, who provides the students with individualized help. Each student-works on a computer to solve a series of exercises selected by the teacher. The two students observed during one session are called Alice and Fanny. These two students are good tenth graders, and they work during one session on the functions theme on the EEB MathEnPoche.⁴ The lesson has already been covered during the year but some new knowledge about functions is still ongoing learning. For each proposed situation, and for each student, we examine the expected activity, then their actual activity.

First situation proposed to Alice and Fanny, expected activity

The first exercise that they come across is a multiple choice questions type. It is a series of 5 questions which are immediate applications of knowledge about images and pre-images. The environment facilitates the activity since there are only two possible choices, like in the following question 1:

Question 1: Complete

We know that
2 has for image 1 by function f
Therefore

The point of coordinate $(\square \square)$ is on the graph of f .

Figure 3. First multiple choice questions with two possible choices (with English translation).

For example, Alice answers correctly 3 out of 5 questions. For the other two questions, she mixes up “image” and “pre-image,” receives a simple error message and rectifies her answer during the second trial: “*it’s enough to invert the answers!*” The same exercise is followed by 5 other analogous questions where now there are more choices, as shown in the following question 6:

Question 6: Complete

We know that
-5 has for image 2 by function f
Therefore

$f(\square) = \square$

\square has for pre-image \square by function f

The point of coordinate $(\square \square)$ is on the graph of f .

Figure 4. Other multiple choice questions with numerous possible choices (with English translation).

The questions are the same as before, but now there are six blanks to complete. The expected activity is not the same as in the previous question, elsewhere more than two writing registries are mixed, which constitutes an additional adaptation.

The strategy consisting of answering sort of randomly then eventually rectifying during the second trial does not work anymore, since the software does not indicate the error locations.

Alice's actual activity

Alice understands that she cannot simply rectify her answer during the second trial if needed. She thus looks at her lesson book before each answering and reads in a low voice the explanation about “pre-image” and “image.” Hence, she answers question 6 correctly. Again, she still gets mixes the two terms in questions 7 and 8, looks at her notebook again, and then corrects her answer. She makes the same mistake in question 9 and so she decides to call the teacher. He explains immediately. She solves the last question without any mistake.

Second situation proposed to Alice and Fanny, expected activity

During the rest of the session, Alice and Fanny work on finding graphical images and pre-images (exercise 8) and graphical solutions of equations of the type $f(x) = a$ (exercise 9). This is new knowledge for grade 10 but it has been studied previously in traditional sessions (knowledge in the process of acquisition).

In exercise 8, given a function defined through its algebraic expression and a representative curve, students must determine the image of a number and the pre-image(s) of another number if any.

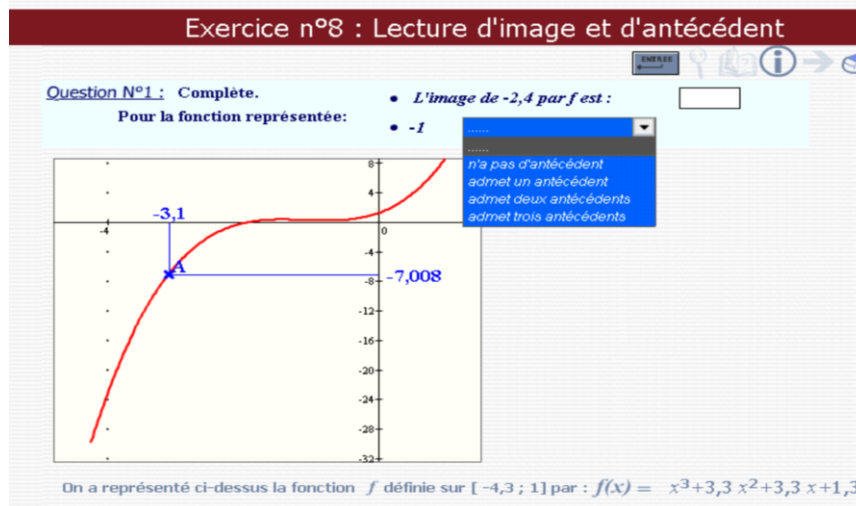


Figure 5. Exercice 8: Reading the image and the pre-image(s).

A series of 5 consecutive questions of the same type is given. The considered functions are random polynomial functions of first, second and third degree, with

decimal coefficients having at most one digit after the decimal point, defined on an interval. The value of the image should be typed in a box. For the pre-image(s), a rolling menu (see [figure 5](#)) allows the selection of the answer. In this example above, the algebraic expression of f is $f(x) = x^3 + 3.3x^2 + 3.3x + 1.3$. The two questions are “The image of -2.4 by f is: ...” and “-1: doesn't have any pre-image; has one pre-image; has two pre-images; has three pre-images.” Depending on the choice made, one or several boxes are displayed to enter the value(s) of the pre-image(s). A point A which moves on the curve allows students to read the requested values. We note that when several pre-images exist, the tool does not allow visualizing them simultaneously. Students should not stop at the first found value. The expected activity is to move the cursor in order to graphically read the images and pre-images. Hence, the work only covers the graphical registry and immediately applies the knowledge about this registry. The choice of work registry is thus imposed and this registry, for this kind of questions, does not belong to the usual didactical contract of the students. Moreover, the environment which imposes the cursor manipulation complicates the proposed task as we will see.

Alice's actual activity

The first curve proposed to Alice in exercise 8 is that of the function $f(x) = x^2 - 4$. Alice must determine the image of 1.2 and the pre-image(s) of -7, if any. In compliance with the usual didactical contract, Alice calculates algebraically the image of 1.2. This approach is reinforced by the fact that the second degree polynomial is a polynomial that she can easily handle algebraically. In order to find the pre-image, Alice notices right away that there isn't any on the graph, since the value -7 is relatively far from the minimum of f . Alice validates her answer, and receives a congratulations message. She does not find the exercise very interesting and moves to exercise 9.

Fanny's actual activity

The first curve proposed to Fanny in exercise 8 is that of the function $f(x) = x^2 - 5,19$. She is asked to find the image of 0.2 and the pre-image(s) of -4.7, if any. Fanny also calculates algebraically the image of 0.2 but finds it hard to complete her calculation to find the pre-image of -4.7. Some adaptations emerge and are linked to the presence of a decimal number. The teacher walks nearby Fanny who calls her: “it's not clear to find the pre-image!” The teacher shows Fanny how to move point A to obtain a display of the coordinates of the points on the curve. Fanny immediately applies this instrumented method, answers correctly, and completes successfully the 5 exercises of the series, proceeding in the same manner, graphically looking for the pre-images as well as the images and using the cursor.

Third situation proposed to Alice and Fanny, expected activity

In exercise 9, the aim is to solve graphically a series of five equations of the form $f(x) = \alpha$, α being a decimal number with at most one digit after the decimal point,

and f defined by its graphical representation on an interval. Depending on the cases, there are 0, 1, 2 or 3 solutions.

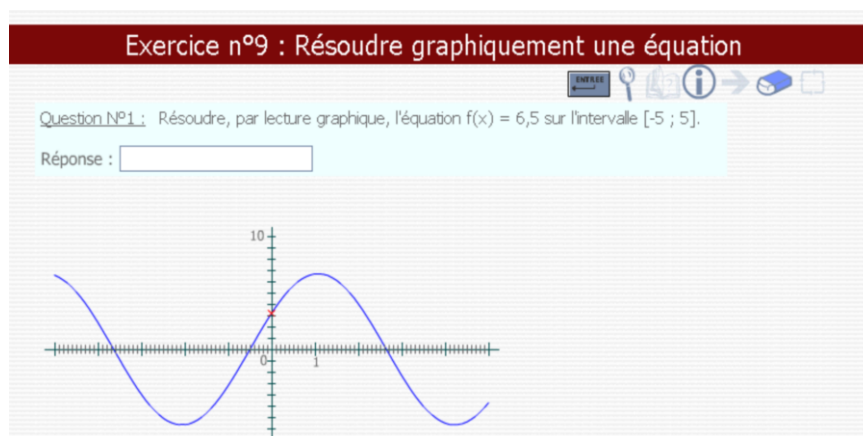


Figure 6. Exercise 9: Solve equation graphically.

In the example in Figure 6, the equation to solve graphically is $f(x) = 6.5$ in $[-5,5]$. Hence, students' activity is always to immediately apply knowledge still in acquisition. Note that the task is again made complicated by the computer environment. In fact, for each question, students must move the cursor on the curve, the cursor being originally placed at the origin. Moreover, the cursor coordinates are not displayed, and students have to read those coordinates over the axis.

Alice's real activity

In exercise 9, Alice does not figure out that she must use the movable cursor since she did not have the chance to do so during exercise 8. Moreover, in this exercise, it is not possible to proceed algebraically. She hence tries to estimate the answer and gives successively two coherent answers but not precise enough. So she comes across the aid window which does not help her. Indeed, the aid explains how to find the solution graphically whereas Alice's problem is that of handling the cursor. So she calls the teacher who shows her how to use the cursor. Alice then engages correctly in the expected activity. But she still can't validate exercise 9 because of three mistakes in three consecutive statements. These three mistakes have different causes:

- Alice forgets a solution;
- Alice doesn't see one solution which is at the border of the frame of the graph;
- Alice reads the coordinates incorrectly.

In the latter, Alice thinks that the precision of her answer is insufficient. She doesn't see that in fact she made a mistake while reading the given. The observer

explains to her this mistake. In the two other cases, the mistake is more serious, but Alice does not have the possibility to rectify it because of a poor manipulation of the software. Alice's activity is therefore not regulated since she does not understand her mistakes or she cannot correct them due to her manipulation errors. However, Alice seems to be able to globally complete the requested task.

Fanny's real activity

In exercise 9, Fanny spots the red cursor and manages to move it, like in exercise 8. Then, she starts the expected activity. In general, she finds the correct answers using the cursor. However, for question 3, she comes across a sinusoidal curve and the equation " $f(x) = 4.5$ " which has three solutions. Yet, Fanny forgets one of them and provides successively two wrong answers: -1.5 and 4.8 then -0.5 and 4.8. She is directed to the aid window, which gives her, like Alice, the method to graphically find an image or a pre-image. Since she knows how to solve the exercise, she says: "*I don't want these aids!*" The teacher passing by, she asks her: "*how can I delete this?*" The teacher points out the red button to close the aid window. Fanny clicks on it and the correction is displayed (something that Alice was not able to find). She looks at the displayed values in the correction having two decimals and this attracts her attention. She then attributes her mistakes not to the fact that one solution was missing, but to the fact that she did not find all the decimals since she did not use the magnifying glass.

Analysis of Alice and Fanny's actual activity

In the first questions proposed to Alice (the questions with two choices, [Figure 3](#)), the students can reach the expected result as of the second trial. All seems to be happening as if information allows them to develop a "two shots strategy" based on the feedback. This strategy is fostered by multiple choice questions with two options. It is only when this "two shots strategy" stops working and when the task gets complicated that Alice tries more in-depth work to rectify her mistakes. These examples illustrate the idea that students take into consideration the feedback only when they feel the need to do so. We can nevertheless hypothesize that Alice's confusion of "image" and "pre-image" is well corrected since she correctly tackles exercises 8 and 9 afterwards. The learning does not however happen on a short cycle of regulations: the correction is long and progressive. In particular, Alice needs to take several initiatives, the situation must become problematic for her (Alice only tried to understand when more than two registries were mixed, that is when there were 6 blanks to fill), and the teacher must intervene fast and advisedly.

In exercise 8, there is, since the beginning and for both students, a gap between the expected activity and the activity developed by the observed students. The task is the same as in the paper-pencil environment, but the expected activity is not the same. The students do not develop the expected activity since they do not understand that the EEB expects them to manipulate the cursor. Moreover, the algebraic expressions are provided by the software and this does not favor graphical work. Finally, the random expressions which Alice and Fanny come across are of second degree, which does not discourage them from looking for

answers algebraically. This would probably not be the case if the expressions had been systematically more complex or of third degree.

Lastly, the examples illustrate the software feedback, and if though they are taken into account by the students, they do not allow them to easily regulate their activity correctly. In the multiple choice exercise, the feedback is not enough to let Alice regulate her activity on her own. She needs her lesson notebook, then a conversation with the teacher, in order to successfully complete the task. The work which consists in interpreting information provided by the software is generally hard to do. In exercise 8, Alice suggests a correct answer and receive a retroaction validating her answer but without any explanation of the expected procedure. Thus, she has no indication allowing her to detect the gap that exists between the work she completed and what was expected from her. We can wonder about what made Alice gives up that exercise. She probably, subconsciously, has the impression to be missing something, but not enough to ask for the teacher's help. The activity in exercises 8 is hence not at all regulated. As a result, Alice cannot tackle correctly the activity in exercise 9, since she does not know that she can manipulate the cursor. Here as well, the aids proposed, reminding her that the mathematical method to solve equations, does not allow her to regulate her activity since her problem is now the manipulation of the cursor. Once more the teacher comes to the rescue and shows her how to use the cursor.

On the other hand, Fanny, possibly thanks to her critical look at the software work, calls the teacher as early as exercise 8 which quickly regulates her activity. So Fanny adapts to the situation in exercise 9 on her own. However, like Alice, she cannot regulate correctly alone her activity in exercise 9 and gets angry at the aids ("*I don't want these aids!*"). Reading the correction is not enough for her to understand that one solution is missing. This seems to be an ambiguity of the software: the answers are accepted with a 0.1 error margin, but in the correction, they are given with a 0.01 precision. In this situation also, because of her critical look at the software, Fanny prefers to say that the software is hard to use rather than question her own work.

The examples of Fanny and Alice illustrate the importance of taking into consideration the instrumental aspect in the proposed situations. The more the resource environment gets sophisticated, the more the students have to articulate the software manipulation with the learning of mathematics. The examples always show that the role of the teacher is fundamental.

EXAMPLES OF UNIVERSITY ACTIVITIES (L1)

The example developed in this paragraph stems from observations conducted over the course of the school year 2004-2005 in a university. The student is called Charles, and we study his activity during a computerized solving session during which the students work on the EEB Wims⁵ and the teacher walks around providing individual help. Charles works alone on his PC and the session is focused on practicing previous knowledge or ones in the process of acquisition. The observation methodology is different from the one used in the previous

paragraph as students working with EEB Wims are logged, which allow to recover their activity traces through log-files. No one directly observes Charles' activity, but we can note the times of work of the student on a Wims exercise called *Joint*. Then, we examine a posteriori Charles' work on an exercise similar to the exercise *Joint*, proposed in a paper-pencil exam.

Situation proposed to Charles, expected activity

The exercise in [Figure 7](#) deals with knowledge about continuity and differentiability of functions of a real variable. An adaptation of knowledge is required from the students since they have to recognize that the given functions are of class C^1 on the considered intervals. Then, it is enough to compute the limits and the derivatives of the two given restrictions and to equal the results, which is a direct application of the algebraic knowledge about limits and derivations. They must write that the two limits and the two derivatives should be equal, which allows calculating a_1 and a_2 through an immediate calculation also. When the student gives an incorrect answer, Wims provides a retroaction of the type as presented in [Figure 8](#).

Exercise. Let $f(x)$ be a real function defined on the interval $[-0.5, 0.5]$, by the following formulas.

$$f(x) = \begin{cases} a_1 + a_2 x & \text{si } x < 0; \\ -3\exp(4x) & \text{si } x \geq 0. \end{cases}$$

Please find the values of the parameters a_1, a_2 such that $f(x)$ is continuous and differentiable to order 1.

Send your reply:
 $a_1 =$ $a_2 =$

Remarks.

1. The numerical precision required in your reply is of 0.005.
2. To do your computations, you can use the online tools: [function calculator](#), or [linear system solver](#) (which will open in another window).

[Change the function](#) .

Figure 7. Wims Joint exercise.

Software retroaction for the Joint exercise

It is not possible to solve this exercise by trial and error since only one answer is accepted. After the first student answer, Wims provides a retroaction. This retroaction is in the graphical register whereas the exercise statement is in the analytical register. It does not necessarily help the student in finding the correct answer but it does suggest another point of view. Nevertheless, the student cannot go back to propose another solution. Wims proposes directly another analytic function, built from a panel of reference functions and linear combinations.

STUDENT ACTIVITIES WITH E-EXERCISE BASES

Exercise. Let $f(x)$ be a real function defined on the interval $[-0.5, 0.5]$, by the following formulas.

$$f(x) = \begin{cases} a_1 + a_2 x & \text{si } x < 0; \\ -3\exp(4x) & \text{si } x \geq 0. \end{cases}$$

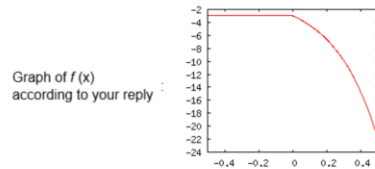
Please find the values of the parameters a_1, a_2 such that $f(x)$ is continuous and differentiable to order 1.

You have given the reply: $a_1 = -3, a_2 = 0$, hence

$$f(x) = \begin{cases} -3+0x & \text{si } x < 0; \\ -3\exp(4x) & \text{si } x \geq 0. \end{cases}$$

This reply is not correct. $f(x)$ is continuous but it is not differentiable.

Your score: 5/10.



[Another function](#)

Figure 8. Example of feedback for the Joint exercise.

Charles' actual activity

The log-file shows that Charles worked for 34 min on this exercise. He also worked on four consecutive statements of this exercise. In his first trial, he worked for 9 min and got the score of 5/10; this means that he only found the missing value a_1 (like in the previous error example, Figure 8). After reading the correct answer, that is the missing value a_2 , he did not try to understand where this value came from. Indeed, the log-file shows that he quickly restarted the exercise with a new statement. This attitude is not abnormal; students almost always restart the next exercise immediately. He worked for 13min and obtained the score 10/10. Then he restarted this exercise two additional times, worked each time for 5min, and obtained in both cases the score 10/10. So we can see that Charles did solve the exercise correctly, that is he regulated correctly his activity a priori. However, we can wonder why his calculations last 5 long minutes each time despite the fact that they should be immediate algebraic calculations? What does he do for the same exercise in the exam? Below is his exam paper. The statement is “ $f(x)$ is a real function defined on $[-0.5, 0.5]$ by the following formulas : $f(x) = -5 \exp(-5x)$ if $x < 0$ and $f(x) = a_1 + a_2 x$ if $x \geq 0$. Find the values of the two parameters a_1 and a_2 such as $f(x)$ is continuous and derivable of order 1.”

We notice that Charles develops well the expected activity to find the missing value a_1 . On the other hand, to calculate the derivative to the left then to the right, he uses the limit of the rate of change, whereas it would be enough to apply the classical derivation formulas in the definition intervals. In other words, Charles uses a correct procedure but it isn't the fastest and most suitable one. This explains the considerably long time spent on this exercise for each trial during the Wims

session and which certainly penalizes Charles during the exam (since he is left with less time to solve the rest of the exam than what the examiner had scheduled).

Exercice. Soit $f(x)$ une fonction réelle définie sur l'intervalle $[-0.5, 0.5]$, par les formules suivantes

$$f(x) = -5 \exp(-5x) \text{ si } x < 0$$

$$f(x) = a_1 + a_2 x \text{ si } x \geq 0$$

Veillez trouver les valeurs des paramètres a_1, a_2 telles que $f(x)$ soit continue et dérivable d'ordre 1.

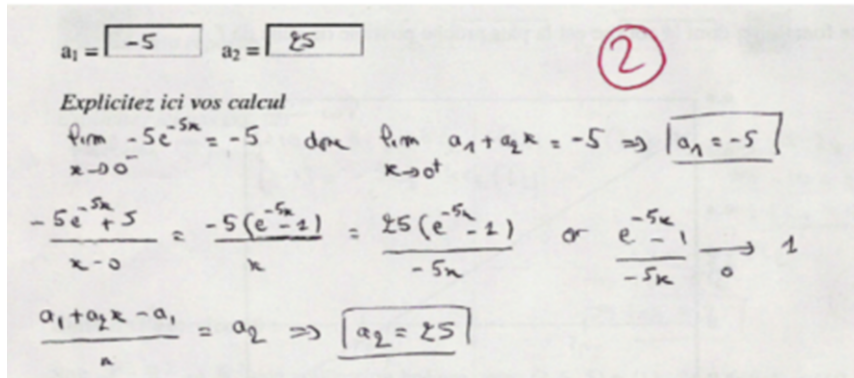


Figure 9. Charles exam, exercise Joint.

Analysis of Charles' activity

In this case, Charles is able to solve the exercise. A regulation of the activity has been made after the first trial where he was able to solve only partially the exercise. It is the software retroaction in the form of a note which allows Charles to regulate his activity. The log-file indicates that this regulation happens in a very short time. During the second trial, he provides a complete answer for the exercise. His only problem is that his solving strategy is not optimal. Unfortunately, at this time, neither the software retroactions, which connect two points of view, nor the presence of the teacher during the Wims session, allow Charles to improve his strategy. In particular, the software regulation "correct answer" as of the second trial, does not allow Charles to realize that his procedure is not optimal. He cannot regulate best, autonomously, his activity during the Wims session. This leads to two new activity loops, where Charles applies the same solving strategy without having any doubt. He always succeeds, but does not visibly increase his speed between the last two trials and by using a relatively long solving method (5 minutes).

CONCLUSIONS

We have described in this chapter results of the students' activity using EEB, which complete those developed in Cazes, Gueudet, Hersant, and Vandebrouck (2006). In the observed situations of EEB use, whether in high school or university, the EEB allow a strong individualization of the students' activity with respect to their work in sessions of traditional exercises (mainly in university solving sessions), even though the student always follow a work plan proposed at the beginning of the session par the teacher. The model of double regulation of the activity (chapter 1, Leplat 1997) allows us to analyze precisely the activity of the students with the EEB and to highlight in particular the regularities and differences between the students with respect to the organization of this autonomous activity.

Introducing task analyses reveals that the latter are often very close to the tasks that can be proposed by the teachers in traditional sessions. Nevertheless, the observations show that the students work much longer on a same exercise during the EEB sessions than during the traditional sessions. For example, in Charles' case, the log-file tracking the EEB activity shows that even on a task with immediate application of knowledge, Charles works for several minutes, restarting the exercise as many times as needed. In a paper-pencil environment or during a classical solving session, he would have only solved one example. This same log-file shows that in other situations, the students are not easily discouraged, in general, by exercises which require adaptations of knowledge. The situation is thus different from the traditional paper and pencil situation since the students have more responsibilities in their activity and can follow at their own rhythm the work plan proposed by the teacher. If they do nothing, then nothing happens, and so they are somehow obliged to work. In particular, there are rare moments of collective corrections where the students can just wait for the answers. However, managing the progress of the path, repeating or changing the exercise, activating an aid or a correction, choosing to take notes, all definitely contribute to the empowerment of the students but also seem to be a source of difficulty, especially for weak high school students. Furthermore, in certain cases, some of them prefer to continue succeeding in competing simple exercises rather than facing more difficult exercises, an observation we had previously noted in Cazes, Gueudet, Hersant, and Vandebrouck (2006).

Here, the results illustrate the valuation of the occasional productive activity using these tools, with often gaps between the expected activity and the students' observed activity. It could be that the task is not a direct application task, that the knowledge to be used is not explicit or sometimes the software environment complicates the task compared to the traditional environment. From the start of the exercise flow, a modification of the initial situation must be applied in order for the actual student activity to be in compliance with the expected activity. This modification is made thanks to the teacher, if s/he is present at the right time. This pertains to the problem of working in total autonomy with these tools, whenever we want to tackle slightly more complex tasks. Charles' case is a good example of the teacher not intervening at all, the completed activity is then deferred with

respect to the expected one. This gap can also be due to the denaturation of the task by the software environment, which can permit for example obtaining the correct answer without developing the expected mathematical activity (mainly in the case of the multiple choice with two choices, but we did identify more complex examples), or it can favor a more economical activity, of the type trial and error in particular. As for the results of the activity, we also found that the software retroactions to the students' actions are often not enough, too difficult to understand by the students to allow them to regulate alone and correctly their activity, and even not adapted to the actual activity. In fact, the retroactions can only be generated for the result of the activity and not the activity itself. It is therefore very hard to implement a priori retroactions which are relevant and adapted to the diversity of the students.

These difficulties, as soon as the tasks are not easy for the students, can generate ineffective activity loops (see chapter 1). The students can for example be satisfied by inadequate procedures by repeating several times an exercise, since these lead to a correct result, or even "very often" to the correct result. The allied environment can also reinforce certain "action logic" at the expense of a "learning logic": in the "action logic," the aim of the students is exclusively to obtain the answer expected by the software (valuing the productive activity at the expense of the constructive activity). It is the case of Alice when she responds to her two-options multiple choice questions. In other examples, the students can intentionally identify regularities in the correct answers displayed by the EEB, after several unfruitful attempts. These regularities can allow them to gradually infer the correct answer without fail (misappropriation of the EEB), without being able to know if it is based or not on a learning. In certain extreme cases, the gaps between the activities are not intentional and result in undesired learning.

The EEB seem to be at first well adapted for a students' work on technical exercises, that is to say exercises of immediate application of knowledge. This wasn't the case in exercises 8 and 9 for Alice and Fanny, nor for the Joint exercise proposed to Charles. The task analyses thus appear to be important in determining the exercises that are proposed to students with an EEB. In this sense, the EEB re-emphasize the importance of the technical exercises, which are important for the learning and which are often neglected in work sessions. The observations or the activity log-files provided by certain EEB show that this work is important, that students need time to successfully solve these technical exercises, and that the work is less repellent thanks to the technological potentialities of the EEB.

As soon as the tasks move away from the technical level (whether the application is not direct, or whether the knowledge are assumed to be available for students), it is more difficult to get from the students autonomous activity loops (the activity and its regulations) which are mathematically acceptable, in other words that the produced or returned mathematics be correct and consistent. This does not however mean that the EEB cannot be used in other ways than for simple or direct tasks. This means that there is a margin phenomena which, if placed too high in terms of task complexity, excludes weak students. The notion of ZPD (Vygotski, 1978, chapter 1) is particularly useful here in the sense that the students

essentially work autonomously. In particular, the EEB can emphasize the differentiation between students if the teacher is not specifically vigilant to the student difficulties. Learning is observed, on an average term, for students confronted to tasks which include adaptations that are accessible and for which they find immediate resources. It is the case of Alice who, after the multiple choice, and using her notes and with the help of the teacher, seems to have well understood the difference between image and pre-image. She was obviously able to reinvest this knowledge during exercises 8 and 9. However, we saw how she went from an “action logic” to a “learning logic” as soon as the exercise was not a questionnaire with two blanks anymore, but had six blanks, that is as soon as her “two shots strategy” stopped working and the task mixed more than two writing registries. This example thus shows that the distinction that we introduce between “action logic” and “learning logic” depends on the student of course, and on the situation s/he faces on the EEB as well. We go back to the initial idea of a situation that must include knowledge adaptations in order to hope for an intentional constructive activity and in particular for a mathematical learning. Quite often, it is the teacher, present and vigilant, who can react to the actual activity of the students and emphasize this learning. The teachers must hence develop a specific work for help and for the integration of the EEB use in the usual classroom practice. We will examine the activity of the teachers who use EEB in the next chapter.

NOTES

- ¹ The terminology of productive activity comes from the field of professional didactics but could be understood in a more naive way, in the sense that the student activity produces results numerical answers, implementation...
- ² For which we can specify the types of adaptations of knowledge at stake (A1 à A7). See chapter 2.
- ³ http://euler.ac-versailles.fr/baseeuler/recherche_fiche.jsp
- ⁴ <http://mathenpoche.sesamath.net/>
- ⁵ <http://wims.unice.fr/wims/>

Claire Cazes
Laboratoire de Didactique André Revuz
Université Paris Diderot

Fabrice Vandebrouck
Laboratoire de Didactique André Revuz
Université Paris Diderot