MONIQUE CHAPPET-PARIES, ALINE ROBERT AND
JANINE ROGALSI

4. STABILITY OF PRACTICES: WHAT 8TH AND 9TH GRADE STUDENTS WITH THE SAME TEACHER DO DURING A GEOMETRY CLASS PERIOD?

In this chapter, we will present a comparative study examining excerpts from two geometry classes[1] taught by the same teacher.[2] The classes involve students in two different grades at the same junior high school. The first class contains 8th graders (*quatrième* in France, students age 13-14) and the second 9th graders (*troisième*, age 14-15). The study focuses on ordinary teaching practices at a relatively privileged establishment. The class periods we will examine cover the first non-self-evident exercises given to students after lessons (in the previous class period) on two of the most important theorems studied in junior high: The Pythagorean theorem (8th grade), and Thales' intercept theorem (9th grade). In both cases, these exercises are given to students as in-class problems, and follow the in-class correction of a simpler exercise that was given as homework.

Our goal is to make progress on two research topics. The first concerns teaching practices, their stability for a given teacher, and, more specifically, the identification of intra-personal regularities, or "practice invariants." To understand these invariants, imagine if we were to enter another class taught by the same teacher. What, beyond personal characteristics (voice, gestures, etc.), could tell us that this was the same teacher?

The second topic, which we will touch upon only briefly, is that of the ultimate consequences of these invariants on students' activities.

As discussed previously, these analyses of in-class teaching practices fall within the framework of studying the five identified components of teaching practices, which can lead to several levels of work. This study is primarily focused on directly observable components (cognitive, mediative – cf. Robert & Rogalski 2005, and to a certain extent, personal), which are tied to in-class actions and which we will examine on local and micro levels.

Our study examines excerpts from two classes of similar makeup led by the same teacher. By studying two classes with similar student populations, we intend to neutralize any social or personal components. To lessen as much as possible the influence of the "institutional" parameter, we are focusing on two geometry class periods, and specifically on students' in-class work on two exercises that are given almost immediately following the corresponding lesson. In both cases, these exercises are the second given during the class period, and the first exercises on the subject to be somewhat complex. Students work on solving these problems during the classes' second half-hour.

F. Vandebrouck (ed.), Mathematics Classrooms: Students' Activities and Teachers' Practices, 91–116.

Our work is based on (transcribed) videos of class periods and on an interview with the teacher, called D.

We will define three types of progressively finer analyses:

– An analysis of assigned tasks, of how these tasks unfold in class, and of possible student activities, at a local level (first section);
– An analysis of selected periods of interaction between the teacher and students and their corresponding linguistic actions (second section);
– An analysis of linguistic markers (third section).

The analysis of tasks and of their realizations in class enables us to determine students' possible activities and to identify areas of potential regularities in teachers' practices.

The analysis of interactions reveals the manner in which the teacher, sentence by sentence, guides the progress of the didactic project while simultaneously guiding students' understanding. This analysis also allows us to more precisely define the invariants discussed above.

The analysis of linguistic markers allows us to identify patterns in teacher interactions with students (Robert & Rogalski, 2005). We will compare the nature and classification of these patterns in the two classes to complete the analyses.

This study, focusing on excerpts from just two class periods, can clearly serve only as an introduction to practice stability analysis. However, our hypothesis is that our results, though based on only a few class periods, can be taken as representative of the stability we seek.

ANALYSIS OF ASSIGNED TASKS, THEIR UNFOLDING IN CLASS, AND POSSIBLE STUDENT ACTIVITIES DURING THE TWO EXCERPTS

We will describe and compare the tasks assigned to students, the realization of these tasks, and student activities, following the methodology given below. Next, we will conduct a global analysis to identify possible invariants.

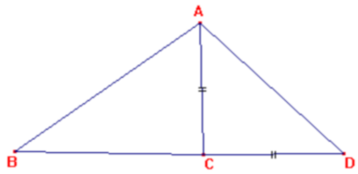
Analysis of the two exercise tasks

The 8th grade exercise

The lessons preceding this class period discussed the Pythagorean theorem, as well as the converse property (that any triangle for which the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs is a right triangle).

After a simple exercise on applying the theorem's converse, the teacher assigns the following exercise:

1) Construct the following figure, with BC=6,4cm
AC= CD=4,8cm
AD=6,8cm and BA=8cm
2) Are points *B*, *C*, and *D* collinear? Justify your answer.



The given figure is ambiguous, as we cannot tell if *C* is on *BD*. It is not explicitly stated that *B* and *D* are on either side of *C*, but all students will take it for granted that they are. We note that a figure in this type of geometry problem should be accurate enough to inspire the strategy to follow, but it should also inspire doubt, so that students will begin to work on the problem. This double role, described by Perrin-Glorian, is found in figures illustrating other geometry exercises.

Table 1. Tasks and task analysis in the 8th grade "Pythagoras" exercise.

A1. (Partial) recognition of the knowledge to be used and the way to do it A2. Introduction of intermediates. A3. Combining of multiple strategies or concepts. A4. Introduction of steps. A5. Introducing results from previous questions. A6. Choice.	Students are asked to complete a multistep reasoning process (A4) with multiple successive changes of viewpoint (A3), including: Going from looking for collinearity to identifying a 180-degree angle, from investigating one angle to seeing it as the sum of two, and from finding the measure of these angles to finding whether the triangles are right triangles. Note that there are three possible cases, as neither, one, or both of the triangles could have a right angle. If neither is a right triangle, students at this level will not be able to answer the questions. We can, however, count on the didactic contract to exclude this possibility. Note also that no comparison of the length of *BD* to the sum of the lengths of *BC* and *CD* is mentioned or implied. Two isolated tasks are included in these steps: the respective investigations of the natures of triangles *ABC* and *ABD*. The sides opposite the potential right angles are easily identifiable as the longest sides, so choosing the strategy of applying the Pythagorean theorem (or its converse) requires only a single adaptation: the choice of the "legitimate" theorem (A1). The required calculations involve integers or decimals with one digit after the decimal point, so calculator use is appropriate.

We note that the order of tasks as given in the exercise may actually be the reverse of the order in which they are completed by students.

The 9th grade exercise (ages 14-15)

The lesson that preceded this class covered Thales' intercept theorem. This theorem concerns the parallelism of two lines that cross a pair of intersecting lines, and equates this potential parallelism to the presence of equal length ratios.

After a simple exercise on applying the converse of the theorem, the teacher assigned the following exercise:

Triangle *EFG* has EF = 5, EG = 7, and FG = 9 (all units in cm). Point *M* lies on *EF* with EM = x. Point *N* is placed on *EG* such that lines *MN* and *FG* are parallel.
Express EN and MN in terms of x.
Find x such that the perimeter of trapezoid *MNGF* is equal to 19.8.

Below is a list of tasks corresponding to this exercise, taking into account the curriculum and the level of the class. Tasks marked in italics will not be analyzed.

Table 2. List of tasks and analysis of tasks in the 9th grade "Thales" exercise.

A1. (Partial) recognition of the knowledge to be used and the way to do it. A2. Introduction of intermediaries. A3. Combining of multiple strategies or concepts. A4. Introduction of steps. A5. Introducing results from previous questions. A6. Choice.	Create a figure with a variable point. This first step is not explicitly indicated. The numeric data provided do not preclude a construction based on true lengths and proceeding by measuring. Placing *M* needs an adaptation (A1). Recognize that Thales' theorem must be used with the given figure. To use it, adapt the statement of the theorem as given in 8th grade. In effect, the length EM must be replaced by the variable x. This is not a simple use of the theorem (A1). *Perform an algebraic transformation on quotients involving numbers and letters to set in fractions. This task must be performed twice, independently, constituting work in a second framework (A3).* *Express the perimeter of a trapezoid, the definition of which is not given (but assumed to be known), by an algebraic expression derived from previous steps (A2).* *Write and solve an equation in* x *(unknown) of the type* cx = ax +b *(algebraic work).* *Verify that the solution is geometrically acceptable (not explicitly indicated).*

Some ambiguities may appear. The figure is not described strictly according to the order of its construction: *N* is given to be on side *EG* before *MN* and *FG* are revealed to be parallel. Students are accustomed to this type of text; nevertheless, could this result in differentiation among students? In addition, students at this level have not yet begun to frequently encounter the word "express."

The steps are mostly indicated in the problem statement, with the exception of the first and the last. The questions are not completely independent, but there is no preliminary conjecture or intermediary to introduce. Work can begin quickly.

Comparison of problem statements of exercises on using the Pythagorean theorem (8th grade) and Thales' theorem (9th grade) in terms of intended tasks.
The two problem statements differ with respect to ways the students have to adapt the relevant theorems to solve them. In the 9th grade exercise, adaptations are tied to contextualizing Thales' theorem through the recognition of the appropriate solving strategies and the integration of an algebraic work in a geometric task. The change of framework is indicated, with the algebraic portion treated almost independently. The only intermediate calculation is very guided. The theorem, while unmentioned in the problem statement, serves as a tool for a calculation that

is directly tied to a figure emblematic of the theorem. The theorem itself was previously encountered in 8th grade.

By contrast, for the 9th grade exercise, students must first undergo an adaptation tied to a complete reasoning process, with multiple steps and numerous changes of viewpoint. Only then can students place the Pythagorean theorem (and its converse) in context. In this case, the context involves a figure that clearly features at least two triangles. At this point, the only adaptation required is to twice choose the appropriate theorem from three possible choices. These choices were given immediately prior to this exercise,[3] and were used in isolation in the previous exercise. Again, the theorems function as tools, and are not cited in the problem statement. We see that their use is not obvious for students, despite the possible effects of the didactic contract.

The 8th grade exercise thus involves levels of action that require more initiative from students than is required by the 9th grade exercise. We can predict that few students will be able to solve the exercise by themselves.

How will the teacher organize the lesson to take these differences into account? And how will this translate into the class structure and into patterns of interactions with students?

Comparison of classroom events, teacher assistance, and student activities during the two excerpts

General characteristics of work

With the exception of board writing, the work done by students in the two classes is analogous. Students work at their desks, sitting by themselves. There, they work individually, or discuss strategies and share results as a group. Students raise their fingers to answer a question, and recopy correct answers written on the board. In 8th grade, these answers are dictated by students to the teacher to write. In 9th grade, students write at the board under strict supervision (except for the calculations). The board plays the same role in both classes (as a model).

Chronology and nature of work (overall)

Again, these exercises lasted approximately a half-hour and concluded the class period.

Table 3 compares the respective lengths of time allotted by the teacher to the "Pythagoras" and "Thales" exercises. (This table is only a rough indication of the lengths of time. It does not give exact seconds, and does not take into account transitions between activities, which lasted up to around 20 seconds.)

Despite the differences in tasks, we can note substantial structural similarities in the 8th and 9th grade classes in terms of organization, the breakdown of work, and the length of different subtasks assigned by the teacher.

Table 3. Time allocated to activities in the two classes. Percentages are calculated out of the total.

	Pythagoras (8th)	**Thales (9th)**
Organization of the work		First question : in two steps (1) beginning, (2) end
Work on the figure	6 min. (37%)	Construction: 2′30″ (17%)
Finding a strategy	Individual work followed by group work: 8′30″ (30%)	Group work (1): 5′30″ (37%)
Finding the solution	Individual work: 4′30″ (16%)	Individual work, in two separate periods: 2 min. (13%)
Recopying the solution	9 min. (32%)	(1) 2 min. (2) 3 min. (33%)
Total	28 min.	15 min.

The teacher first requires students to recopy the problem statement and the figure (if given). In both cases, he requires them to in some way "enter" the problem statement. In 8th grade, the required description of the associated figure prolongs the length of this stage.

In both cases, the teacher then assigns as a subtask the determination, as a group, of the strategy to follow. In the 8th grade class, students first try to find a strategy by themselves, and then share their ideas during the group phase. The percent of time spent on the group strategy stage is comparable in the two classes. The teacher then lets students work individually on solving the problems. In the 9th grade class, there are two separate periods of individual work. Again, the amount of time allotted to the activities is similar in the two classes. The teacher concludes both classes by providing a model of a correct answer, which is either dictated by students and written by the teacher, or written by a student under strict guidance. The drafting of this model answer occupies a third of the exercise time in each class.

Assistance

In both classes, the assistance provided to students is primarily procedural, and usually consists of identifying a step of the task to be solved. Assistance is particularly common at the beginning of each question or sub-question. For example, the teacher begins by asking students to construct the figure, and then divides the first and second questions into subtasks: "*Now then, of course, you should first draw the figure*," or "*Now then, what is it like? Arthur, describe the figure for us*." The teacher then engages students in finding a strategy (which consists of *dividing* the main task into subtasks). During the strategy finding stage, the teacher uses incomplete responses from students, and throws them back, slightly modified, to ask finally the precise question with the waited answer. In this way, he constructs a path for the strategy. He then summarizes the strategy, and lists the steps on the board.

Some informational assistance is given in response to questions on the definition of the perimeter and on recognizing an equation.

Finally, the teacher's assistance on the writing up of the solution is both procedural and constructive, as he explains in a somewhat general manner what to justify and how to do so. In particular, he notes how to contextualize the theorem and where to place the justifications.

In both classes, constructive assistance is present during exercise correction, with a link between old and new knowledge established through reminders and/or repetitions. This type of assistance is reserved for partial results, such as why fractional representations are preferred over decimal representations, when to use the Pythagorean theorem or its converse, how to explain the answer, remembering this type of exercise that mixes algebra and geometry, etc. By contrast, in both classes, no constructive help is given concerning the global solving method of beginning by drawing the figure, finding a strategy, etc.

Possible activities – A minima

We find two types of possible activities during the exercises: *a minima* activities, for students who wait for indications from the teacher before beginning, and *a maxima* activities, for those who can directly embark on the strategy suggested by the teacher.

We assume that all students draw the figure and then try to find a strategy. For 8th graders, this search may beunclear, while for 9th graders, it may be incomplete. We cannot know for sure that they proceed in this sequence, nor even that they begin under the suggested method. Some of the intended adaptations may have escaped them, without this omission having any perceptible effect on their final work. Thus, at the moment of solving, many students may have completed the calculations (but nothing more).

They have, however, been able to recopy a completely solved example from the board and hear the teacher's explanations.

They will then have had access to isolated activities, each involving a single mathematical concept, but will not have been able to link them.

Comparison of proposed mathematical activities and possible activities according to the a priori task analysis

The commonalities found in Table 6 are more closely linked to the nature of the work provoked by the teacher and to the sequence of activities than to the mathematical content involved.

The consistency thus comes from the organization of the series of activities proposed to students, in which only the overall nature of the work (form and type, length of approximately five minutes) is imposed by the teacher.

Table 4. Tasks and activities for both exercises (with the less studied elements in italics).

	"Pythagoras" (8th grade)	"Thales" (9th grade)
***A priori* tasks given in the problem statement**	Use the Pythagorean theorem and its converse as steps in an overall reasoning process to determine if an angle is 180 degrees and if points are collinear. Recognize the methods of applying each property.	Use Thales' theorem to complete algebraic calculations (combining geometry and algebra). Recognize the methods of applying each theorem.
Mathematical activities proposed by the teacher[4]	Understand the problem statement and draw the figure. Find a global strategy with viewpoint changes (alignment → 180° angle → two right angles → two right triangles) (A4, A3). Solve. Show that *ABC* has a right angle at *C* (Pythagorean converse) (A1). Show that *ACD* does not have a right angle at *C* (contrapositive) (A1). Conclude by evaluating angle *BCD*.	Understand the problem statement and draw the figure. Find a strategy. Recognize that Thales' theorem is required and that EM should be replaced by *x* (A1). Solve. Complete two independent phases of algebraic work involving numbers and letters (A3). *Find the perimeter of a trapezoid (presumably a known task). Use the previous calculations to express it as an algebraic expression (A2).* *Write and solve a first-order equation (algebraic work). Verify that the solution is geometrically acceptable (not explicitly indicated).*
Possible student activities	Draw the required figure (SIT): – Describe as a group. – Draw individually. Try to solve the problem, possibly without success. (Individual and group work.) Listen to the correct method for completing all three steps. Note the three steps and treat them successively as simple, isolated tasks. Individual work. *Calculate AB^2 and $BC^2 + AC^2$ (SIT).* *Recopy the completed example of the above.* *Calculate $AC^2 + CD^2$ and AD^2 (SIT)* *Recopy the completed example of the above.* *Calculate angle BCD.* *Recopy the final example.*	Draw the required figure (A1). Try to solve the problem, possibly without success. (Group work.) Listen to the described strategy. Use Thales' theorem geometrically (A1). Individual work (1). Recopy the example. Begin the algebraic work (A3). Individual work (2). Recopy the example. *Listen to the group strategy discussion and begin to calculate the perimeter (find the missing lengths).* *Individual work.* *Correction, recopying.*

By contrast, the mathematical subtasks that determine the specific possible activities (notably the *a minima* activities) differ in nature, with the order of adaptations inversed between the two classes. In 8th grade, students pass from A4 to A1, while in 9th grade the sequence is from A1 to A3 and A2. Once the strategies are established, the 8th grade students are more likely than the 9th graders to continue to work on the sequence of simple, isolated tasks (SIT) that follow from their theorems. This can lead to variations in the students' knowledge development.

We realize that finer analyses will be necessary for evaluating this hypothesis. How, in particular, does a teacher's speech contribute to this consistency?

In terms of the mediative component of the teacher

At the very beginning, the teacher adds a subtask to the list of intended tasks: constructing the figure – we cannot know how long it would have taken students to come up with this step without help. This allows students to enter the exercise and the teacher to explain the concepts in play.

A period of trying to find a strategy is immediately imposed by the teacher as a "general" strategy. The teacher ensures that the presentation of the strategy to be followed is relevant to all students. This presentation occurs after a period of individual work. By having students share their thoughts, the teacher can reconcile a wider variety of ideas, holding onto ones that can help make progress toward a strategy. Any viewpoint changes are mentioned as part of the reasoning process, but, unlike changes of framework, are not highlighted. Students complete the (indicated) calculations during an additional period of individual research, and a detailed correct example on the board (the model) concludes each question.

We can say that this teacher introduces a number of systematic work patterns. The words "habit, habitual" appear frequently, both as actual words spoken repeatedly and as aspects of students' activities, which repeat. Thus, in this class it is habitual to draw the figure, to identify hypotheses and a conclusion before beginning, and to work as a group at the teacher's request to find the specific methods to use before beginning. This, we have seen, can take a variable amount of time. Each time, the teacher provides a corrected model on the board, possibly written by a student.

The teacher also provides substantial guidance to students. He does not let them follow their own initiative for long. Nor, with the exception of two or three students, does he make use of their ideas for finding a strategy. Only the quickest students will be able to develop their own methods before beginning to solve the problem. By contrast, D gives all students a certain amount of autonomy once the tasks have been laid out.

First assessment

The analyses above are associated with what we call the cognitive and mediative components of D's practice, during each of the excerpts studied. We noted important similarities in classroom activities, despite differences in the tasks' possible student activities. To what extent can consistency hypotheses based on only two excerpts be valid?

To answer, we will examine the personal component of D's practices (for certain elements).

We obtained some supplementary information on this component though a questionnaire completed by the teacher on the use of the board in 9th grade. The questionnaire was completed after the teacher watched a video of his own class (Beziaud et al., 2003).[5]

The responses to the questionnaire showed that the teacher considered the 9th grade class period under study to be a typical one. He described his practices as fairly stable, and did not imagine there to be possible alternatives to the choices he had made.

The teacher stated that his goal was to supervise students fairly closely and to encourage student-teacher interactions, both with the class as a whole and with individual students. He also stated that he chose what was to appear on the board carefully, preferring the amount of time spent on writing on the board to remain short.

These elements, apart from the two class periods under study, support the existence of practice invariants.

The questionnaire also allows us to partly deduce the manner in which D claims overall to manage his constraints and leeway. The time spent on an activity is dictated by the progression of the curriculum, which must be completed. The teacher is there to help students, to reassure them, to encourage them, and to allow them certain autonomy, but within a framework that is defined strictly enough that even the most "fragile" students can find something to do.

It thus seems valid to us to identify this teacher's "intervention logic" as a kind of recombination of the mediative and personal components. The fact that we can engage in this reconstitution is proof of the desired stability and explains the consistency suggested above, in the case of the first non-trivial exercises given after a geometry lesson on one of the curriculum's "big theorems." The teacher chooses to give problem statements that are different in terms of how they call upon their theorems, and analogous in terms of the management system they enable. From the teacher's point of view, the exercises allow some students to take initiative and others to work on simple isolated tasks.

In this type of class period, regardless of the task details, students' work is first established as a group. This process consists of listing at least the first subtasks, which then become isolated if not simple (cf. SIT, chapter 2). This listing of subtasks more or less transforms the activity on the corresponding tasks. To develop the list of subtasks, the teacher modifies and completes students' responses to open-ended questions. The students do not have control of the preliminary investigation.

Next, the time given to students for individual work allows them to attempt and even complete at least the first of these subtasks. The teacher circulates among students and occasionally publically uses volunteers' indirect assistance.

Finally, once a certain number of students have finished working, a carefully completed example solution on the board gives students who recopy it into their notes a model to follow.

During the development of the example solution, there is little reference made to individual work. There is no overall assessment of strategies, or reference to subtasks or to methods used. There are comments on how this exercise differs from others or on how to write the example. "Constructive" assistance does not involve the global strategy.

The 8th grade/9th grade comparison leads us to ask if the consistency in the in-class activities might cause a more difficult task to be even more divided into isolated simple tasks. Perhaps the teacher compensates for the difficulty of a task by subdividing activities? Would students who work *a minima* have sufficient information to allow them to return on their own to the exercises completed in class?

There are alternatives: choosing other ways of working, or a different organization of the sequence of activities to engage weaker students in developing (even partially) the overall reasoning processes (A4).

Furthermore, we wonder if, implicitly, the teacher is delegating certain aspects of learning geometry to these strict procedural habits. The teacher behaves almost as though these types of routines could be transferred to students without being explicitly taught. The stages of drawing the figure, determining the hypotheses and the conclusion, finding a strategy and/or method, and writing out the answer are each distinct, and are always completed in the same order, with the same process. This is correlated to the reduced role of constructive assistance.

Question to pose at this stage:

– Can we find other invariants in teachers' speech? How do they fit into the already noted invariant organization of the sequence of activities proposed to students?
– What influence do these invariants have on student activities?

We have seen, for example, that more complex tasks lead to a more substantial subdivision, and that a certain number of elements remain implicit or absent. All students appear to be working, with some even reporting "success" on the mathematical task. Are there, nevertheless, misunderstandings or missing links in some students' mathematical work?

ANALYSIS OF STUDENT/TEACHER INTERACTIONS

We will first focus on the teacher's speech during interactions with students, to look for potential similarities.

This third analysis will supplement our *a priori* analysis of tasks and possible student activities, and enrich our detailed understanding of the way the teacher considers his students, interprets their work, and keeps them working on the mathematical activity, stage by stage. These local analyses therefore have global goals.

Note that in this study we are only analyzing interactions aimed at the whole class (which may nevertheless involve only a single student directly). We are also only considering interactions that involve more than two exchanges. Each interaction studied is initiated by a question from the teacher or from a student, and each interaction ends once the desired response has been given and the teacher is satisfied that everyone has heard it.

This choice of which interactions to study is supported by the fact that only these interactions represent a true negotiation between the teacher and the students, and only they are indispensible, from a didactic point of view, to in-class events. In particular, student/teacher interactions initiated by the teacher are opportunities for him to mark a step in the progress of his plan for the class period[6]. These interactions help us to understand what information students receive. They also provide us with information about the division of work between the students and the teacher. In other words, we can identify regularities or similarities in the manner in which the teacher, sentence by sentence, guides the progress of the didactic project as well as students' understanding, or in how he contributes to students' mathematical activities during these interactions. How does the teacher enlist students into the activity, and then keep them there? What autonomy do students have? How does the teacher contribute, at different moments, to the knowledge adaptations that were expected based on the *a priori* analysis? How does he intervene into the difficulties encountered by students, or keep track of what students have done, particularly during the presentation of the correct response? How does he handle contributions from students, and particularly from strong students? Does he revisit the methods and potential choices? Does he reassemble the subdivided steps? What type of help (procedural, constructive) does he provide?

The identification of linguistic actions from the transcripts will reveal the role of the teacher's speech during in-class interactions. It will also enrich our analysis of student and teacher activities. We will first present our methodology, and then the comparisons we found by using this methodology in the two class periods studied.

Methodology: Tools for analyzing linguistic actions in the teacher's speech

The teacher's linguistic actions allow us to identify the choices in speech that may contribute to the development of students' activities.

We use the term "linguistic actions," with its connections to language and context, to indicate various considerations. These considerations cause us to attribute a different linguistic action to a phrase depending on the circumstances of its utterance. A single phrase can also correspond to multiple linguistic actions.

For us, a linguistic action is a quadruple with four components. These components are the episode, the syntax type (question or statement), the content (mathematics, meta, etc.), and the speech's function.

The first component, the episode, is identified following the *a priori* analysis, and is characterized by students' work on a task or subtask. The linguistic interactions defined above are analyzed within the context of this episode.

For the syntax type, we identify questions posed by the teacher. These questions contribute to students' participation in the task, and provide information regarding how the teacher takes students into consideration.

When the content of the teacher's speech is "meta," it concerns his own interventions. This can include indications of method, elements of structuring class time or reminders, or placing work in a larger context. Meta speech helps us reconstruct the teacher's intentions. For example: "*Remember, you can find the sum of two squares directly with the calculator. You can write down the intermediate results, or you can do it directly*."

Finally, we analyze the functions of speech, as identified by Bruner for the processes of tutoring and the support provided by an adult when helping a child (Bruner, 1983; chapter 1). For Bruner, these functions define the manner in which the teacher contributes to students' work step-by-step, while trying to support their activities. Does he talk to them about what they have done, correct them, encourage them, or do something else?

Our functions are defined in terms of the work involved in leading a mathematics class. They are designed to allow characterization of the multiple forms of support possible in a class.

Below are the functions of speech that we have identified:

– Participation functions:

Engagement: "*Let's go*."

Repeating information.

Calling for attention: "*Now then, pay attention*."

Encouragement.

Sharing student responses. (Student: "*Variable*." Teacher: "*Variable*, *x is a variable*. *The point M varies, then x varies from what to what?*")

– Other functions, identified by comparing adaptations that students must make of their knowledge, the state of their work, and teacher remarks:

Identification of student work. The teacher considers student productions or questions: "*Now then, to answer Raphael, who just asked if we should write the hypotheses or the conclusion…*"

Information. The teacher provides or requests information regarding the knowledge in play. For example, he may ask for or provide results, theorems, etc.: "*EFG is a triangle such that, then I'll give you…EF = 5, EG = 7, FG = 9, and all units are in centimeters*."

Evaluation. The teacher gives his opinion only on the validity of students' responses, without other commentary.

Structure. The teacher punctuates students' work by placing them in a larger context: "*Now then, J. B., for the second step, tell me what should be done*."

Orientation. The teacher orients students' work without giving everything away: "*We don't really know its true location on EG, huh. In other words, it's a point?*"

Justification. The teacher engages in the justification process: "*Now then, why do we begin with AB squared? Why not one of the others?*"

Assessment. This indicator can refer to a recap or a reflection: "*We put the point M somewhere, and MN is parallel to FG. Now we apply Thales' theorem and write it up like we did in the earlier exercise*."

The teacher can express multiple functions in a single discussion, as in the example below:

Student: "*Well, we'll say we're using Thales' theorem*."

Teacher: "*There you go. We're going to use Thales' theorem because we evidently have straight lines?*"

The teacher evaluates the student's contribution while sharing it. He poses a question that orients the student toward a mathematical justification. The word "evidently" adds meta content to this discussion because it may refer to a habit or remind students of something.

For each episode, we use this labeling system to characterize the functions in play in each linguistic interaction and in the set of interactions. We deduce from our analyses elements that involve taking students into account: questions, assistance, or support for students' work. This enables an initial approach to the class period excerpts, particularly in terms of "internal" regularities. It can also lead to other comparisons.

We used this methodology on the two phases analyzed below.

A comparison of linguistic actions in the two classes

To study the linguistic actions (and attribute the quadruples described above), we chose three episodes to study from the 8th grade class (Pythagoras). The first involves the description of the figure. The second, which includes two disjoint periods of time, focuses on the group efforts to find a solving strategy. The last is the presentation of the correct solution. The episodes are sufficiently long (more than two exchanges) to allow a true dialogue to be established in which the didactic stakes are perceptible.

For the 9th grade class (Thales) we analyzed interactions involving more than two exchanges, of which there were four: Finding a solving strategy for the first question, correcting the first question, finding a solving strategy for the second question, and finally correcting the second question.

In the appendix, we provide an extract of the complete analysis and the results for each exercise. These results provide the basis for what follows.

We are only comparing the linguistic actions in two analogous episodes in the two classes: development of a solving strategy and correction. For the 9th grade class, we are only considering the first exercise. We will try to identify similarities and differences in the episodes.

Comparison of functions of speech

Within the strategy development phases. In both classes, the teacher speaks much more often than the students. This is clear from the transcript.

With that said, students' **participation** is substantial, instigated through the questions asked or due to participation functions. The teacher systematically shares student results. This sharing often involves validation, which can then be modified with a commentary or question from the teacher, leading students to the intended results:

Teacher: "*So, this situation is fairly banal, huh. Given all that we've done, what is the only new thing, Bertrand?*"
Student: "*Uh … x.*"
Teacher: "*x, that is, the point M. What do you say about point M?*"
Student: "*Well, we don't know its real place on EG.*"
Teacher: "*We don't know its real place on segment EG. In other words, it's a point?*"

The **structuration function** is used frequently in both classes, but more in 8th grade than in 9th grade: "*For the first step, what will I do?" "For the second step, continue, Alexander*."

The **information** provided by the teacher in the 8th grade class primarily concerns mathematics: "*So, what, what are we going to look at? Well, if each is a right angle, and then we'll look at angle BCD. Okay? And it will either be flat or not*."

However, in the 9th grade class, there is also "meta" information that situates the proposed exercise in terms of students' knowledge ("*that theorem of Thales*") and in terms of old and new ("*Now then, the only thing that's going to be a little different from usual is?*").

In both classes, the mathematical information constitutes help that is apparently procedural.

In the correction phases. In this phase, the teacher again speaks much more than students.

In 8th grade, the exchange is marked by strong **participation** by students. This translates into questions posed at each teacher contribution, which engage students ("*Kurdis, can you give us the first step in detail?*") while **structuring** their reasoning ("*Now then, J. B., for the second step, tell me what we should do?*" "*Now then, how will we finish?*"). The questions can **orient** students toward the intended response ("*What can we conclude about BCD? That is isn't …?*"). This student participation through constant questioning is reinforced by the **sharing** function, which allows the teacher to share with the class the dialogue that he has established with the student at the board. The teacher frequently uses **structuration**. Different steps in the reasoning process are explicitly identified and are reassembled at the end. The teacher leads the process, and students' autonomy is weak. The teacher leads them step-by-step towards the intended answer while orienting their reasoning process.

Assistance is therefore more procedural in nature. However, the assessment that marks the end of the exchange can represent constructive assistance for some students: "*Now then, what is interesting in this exercise is that we have a single question and, ultimately, we applied the converse of the Pythagorean theorem: the result where the formula doesn't hold but we can conclude that the triangle is not a right triangle, and with some help after adding two angles we were able to get a conclusion*."

The teacher relies on students' work before indicating the desired write-up of the demonstrations: "*But I just explained, J. B., that the converse wasn't applicable when the equation was true*." The teacher's speech has mainly mathematics as its object; the meta content concerns possible solving methods.

In the 9th grade class, **participation** is less expressed by the speech's functions than in 8th grade. However, there are still numerous questions posed. The **information** in the exchanges primarily concerns mathematics; there is little meta information. Considerations of students take place orally, but are also based on what the teacher is able to observe in written work: "*Now then, remember when* *we're writing it up, that to apply Thales' theorem there is no 'if then.' Some, not many but two or three, wrote the exercise on the sheet with 'If the lines are parallel … '.*"

An explicit allusion to the exercise and a reminder on the write-ups can constitute constructive help for some students: "Remember that this is the general problem statement and we're applying it. So we know whether or not the lines are parallel." Reminders of new information were featured in the teacher's speech during the entire period and revealed his goals during the class.

Global results In almost every discussion in each episode of both classes, the teacher used questions to mobilize students.

During both classes, the teacher assessed, evaluated, and shared with students. Students were required to engage in tasks that were often subdivided. They were also required to mobilize their attention, and to collaborate with the teacher on justification and structure.

Student participation differed in the two classes, but this difference was in large part due to the task. Again, the 8th grade exercise required more adaptations than the 9th grade problem, and the 8th graders were perhaps less comfortable with solving such an exercise. The teacher needed to ensure that students understood the solving strategy. He therefore gave them time to put it in their own words after describing the three required steps. In 9th grade more than in 8th grade, a single word was often sufficient for students to complete the teacher's sentence. Furthermore, we note again that the correct answer was not presented in the same way in 8th grade as in 9th grade. In 9th grade, a student wrote on the board (occasionally prompted by questions by the teacher). By contrast, in the 8th grade class, the teacher wrote as dictated by students, who were therefore required to express themselves orally.

In comparison, we noted that this teacher had an overall stable use of functions during the strategy development phase, with little variation in details. We can note, however, more information, justification, and sharing functions in this phase in 9th grade, and more structuring in 8th grade. We hypothesize that these variations stem from the unfolding of events that are tied more or less strongly to the task, but that are certainly tied to students. The greater difficulty and lesser subdivision of the initial 8th grade task explains the greater presence of structuring, while their lower response frequency led to less sharing and assessment than for the 9th graders, who had more propositions to create.

For the correction phase, during which a correctly solved example was written on the board, we note stability in the use of structuration functions and in the direct involvement of students (beyond sharing). The sharing function is used more in 8th grade. We note again that the 8th grade exercise was more difficult, that students were less accustomed to it, and that the majority of them did not solve it. For 8th graders, the presentation of the correct response was also a time for students to solve and work on the problem. The teacher again engaged students during this phase and used their answers. In the 9th grade class, where many students were able to solve the problem, the correction phase served more as a time for evaluation or assessment.

Finally, a close study of the functions of speech allows us to identify another invariant: The use of sharing/evaluation/orientation functions that correspond to a light modification of student responses by the teacher to get closer to the desired response:

8th grade:

Student: "*We have a triangle ABD …*"
Teacher: "*We see a triangle … ABD …*"
Student: "*See if it's a right triangle*."
Teacher: "*Ah, we could know if it's a right triangle*."

And in 9th grade:

Teacher: "*What is the only new thing, Bertrand?*"
Student: "*Uhh … x*."
Teacher: "*x, that is, point M*."

Taking students into account

The teacher takes students into account at several levels:

- Directly in exchanges. The teacher may take students into account by varying his responses depending on whether they give the desired response. He may also take them into account by choosing the answers according to the moment the students give them.
- In answering students' questions.
- In reference to their work during the individual work time.

Some examples:

- In 8th grade, during the correction phase: "*Now then, I would like to insist on the placement of this sentence.* ***Corentin*** *did the same thing, but he put this sentence a little earlier. He stated right away that he had, that he was going to apply the converse of the Pythagorean theorem. Now then* ***Corentin****, what did I say to you? Did you understand what I said?*"
- In the same phase: "*There you go. We're not at all sure that we're going to apply the converse of the Pythagorean theorem, because at the beginning you don't know if the equality will hold or not. If it does, you'll say that it follows from the converse of the Pythagorean theorem, sure; but if it doesn't, we can't justifiably apply it. So it's really important that you do this in this order. Do* ***you*** *understand?*"
- Or in 9th grade: "***Fanny****, you have 2x/5 – 5x/5. That makes -3x/5*."
- 9th grade: "*It's not clearly false. Well, now then, here's the first question. So we have answers in function of x. Re member that 7x over 5, then, that can be written in different ways. There are some …* ***raise a finger those of you who wrote a decimal****, like* ***Ludovic****. What did you write?*"

Assessment: What analogues are there in the speech during interactions in both classes?

In both classes, the overall use of speech functions is similar, with a few variations. Most notably, there were more participation functions during the 8th grade episode than in 9th grade.

In particular, we see large analogues, adapted to the level of the class, in the teacher's responses to students' answers. For example, during interactions with students, the teacher always takes their responses and shares them, validates them, and modifies them in ways adapted to the class and the students. The teacher negotiates the desired response while remaining careful to maintain communication.

The teacher's role in class is to evaluate, share, and assess, while the student's role is primarily to resolve subdivided tasks. Students are all asked to participate and are encouraged by participation functions, which are more frequent in 8th grade than in 9th grade.

The teacher's speech thus adapts itself to the class and to the type of task: more calling for student involvement when they've been working on their own, more maintaining student attention in 8th grade when the solving work is taking place in real time during the correction phase (for example), and more controlled by the teacher in 9th grade when he is validating a model solution at the board.

This analysis is still missing elements that could further reinforce (or weaken) the mark of the teacher on the speech. For instance, we have not examined the use of personal pronouns (*we*, *one*), which could help the teacher place himself on the same side of the task as students. In fact, such a study, as yet done in Chappet-Pariès (2008) shows an analogous usage of the use of personal pronouns in the two classes

We borrow several practical tools used in research led by Trognon at the Ecole de Nancy (Gilly et al., 1999) to describe more precisely the illocutionary goals that indicate what speech content is trying to produce. Here, again, the choices of goals manifested during the exchanges are very close in the two classes (Chappet-Pariès, 2008).

Possible next steps include the treatment of other class periods to see what analogies prove persistent and to understand the impact on students. What do students understand, with which potential effects?

LINGUISTIC MARKERS IN THE TEACHER'S SPEECH

We will now compare the above analyses with a different approach to the teacher's speech. This approach relates verbal formulations to the organization of the teacher's contributions to students' in-class work. This organization is identified through verbal indicators we will call "speech markers." Speech markers are "particles," such as "good!" or "so" (when not used as a logical connector). They are grammatically optional and do not change a statement's truth-value. They have been studied in teaching activities, with initial work done in the teaching of English as a foreign language (Sinclair & Coulthard, 1975).

These markers have two functions. First, they mark the organization of verbalized content, and thereby play a role in speech coherence. Particles such as "now then" (*alors*) and "so" (*donc*) play this role. The second function of markers is to ensure the pragmatic structure of the interaction and mark the roles of the speaker. This function is played by terms such as "good" (*bien*) and "Okay" (*d'accord*). The markers are therefore a signal of the relationship between the student's statement and the teacher's reaction, between the teacher's statement and the desired student response (statement or action), or between the teacher's units of speech. They can also simply "punctuate" the teacher's public activity, such as writing on the board. They can mark the introduction of a new element in the teacher's speech, or return to a previous line of speech after an interruption by a student's action or by an observed action to which the teacher responds.

We will first present these markers as evidence of the organization of the teacher's speech. We will examine their use in the initial "draw the figure" episode in D's classes in the 9th grade "Thales" exercise and the 8th grade "Pythagoras" exercise. We will see how they constitute indicators of invariants in the organization of the speech. We will also identify variations, which we can then interpret in terms of the relationship between the mathematical content in play and students' ability levels.

Markers: a diversity of contributions in speech

Markers can introduce acts of speech (analyzed above) that place students in their role as students by using imperatively tensed verbs (or present tenses or infinitives with the same intent as imperatives). These acts of speech can also involve posing questions requiring a response. The teacher uses these markers to call for student participation.

– "***Now then***, *listen closely to what he says* …" (Pythagoras, 8th grade.)
– "***Now then***, *this says you draw a figure*." (Thales, 9th grade; present tense functioning as an imperative.)
– "… *and* ***then***, *it will vary from what to what?*" (Thales, 9th grade.)
– Markers also punctuate the progress of the class activity. They ensure that students are all working on the same goal at the same time.
– "***Now then***, *I'm going to write the third step here* …" (Pythagoras, 8th grade; announcement of an activity.)
– "***Now then***, *there are some who have finished* …" (Pythagoras, 8th grade; state of activity in the class).
– "***So***, *here we'll pick, sure, the first and the last relationship*." (Thales, 9th grade; commentary on current activity at the board.)

Different markers may specifically signal the end of an activity and the completion of a (sub) task. For example, "*There you go!*" (*voilà*), "*Okay*" (*d'accord*), and "*Good*" (*bon*) are examples of considering a result proposed by students. We see them function here within an interaction:

Teacher: "***Now then,*** *we're at our main exercise. How many results are there in this chapter?* […]"
Student: "*The Pythagorean theorem*."
Teacher: "*The Pythagorean theorem*."
Student: "*The converse of the Pythagorean theorem*."
Teacher: "*The converse of the Pythagorean theorem, and then, the one that doesn't really have an official name. Well, it's when the equality doesn't hold and we can conclude that the triangle isn't a right triangle.* ***Okay****?*" (Pythagoras, 8th grade.)

And this other one interaction:
Teacher: "*And these, these are what?*"
Student: "*Hypotheses*."
Teacher: "*Hypotheses.* ***Now, then,*** *pay attention.* ***So,*** *we will put the conclusion here,* ***okay****?*" (Thales, 9th grade.)

"So" (*donc*) as a marker can have a "conclusive" function, or can function by connecting previous activities to those that will follow, appearing in the introduction of a new unit of interaction.

We see in these last examples that the markers indicate the boundaries of units in which the teacher and students take turns speaking. These units are not necessarily limited to the well known triplet of "question from the teacher," "answer from the student," "evaluation by the teacher."

Finally, the markers can "punctuate" the continuous speech of a teacher: "***Now, then****, remember when we're writing it up, that to apply Thales' theorem there is no 'if then.' Some, not many but two or three, wrote the exercise on the sheet with 'If the lines are parallel…'* ***Now then,*** *remember that this is the general problem statement and we're applying it, so we know whether or not the lines are parallel ...*" (Thales, 9th grade). We note here that "so" is used as a logical connector relating to the current mathematical activity: "… *so* we know …"

"***Now, then****, what is interesting in this exercise is that we have a single question and, ultimately, we applied the converse of the Pythagorean theorem: the result where the equality doesn't hold but we can conclude that the triangle is not a right triangle, and with some help after adding two angles we were able to get a conclusion.* ***Okay****?* (Pythagoras, 8th grade.)

Speech markers as traces of the organization of the teacher's activity

The analysis of markers leads to defining "interaction units" bounded by introductory markers (particularly "Now, then") and conclusive markers (particularly "There you go," "Okay"). These units include a variable set of turns at speaking by the teacher and the students. The teacher's turns at speaking (and occasionally the students' as well) themselves contain one or more semantic units (the equivalent, for oral speech, of multiple clauses on the same content).

An initial analysis of speech markers for a 10th grade algebra teacher (Robert & Rogalski, 2005) has revealed the existence of patterns of interaction that are identifiable by markers. These interaction patterns contain an introductory marker, a set of treatments of the task object, an assessment of the activity, and a conclusive marker. (The assessment phase is optional, and can also take place after the conclusive marker). When such patterns are recurrent in a teacher's speech, they indicate an invariant organization of the teacher's activity.

Our analysis of D's speech markers during the Thales (9th grade) and Pythagoras (8th grade) problems shows the same organization for the initial phases, when the students need to make a figure according to the problem statement.

After the statement is read, an interaction unit begins:

"***Good, then****, this says you draw a figure*" (Thales, 9th grade).

"***Then*****, *****so,*** *you go in the exercise book; you paste this little piece of paper and you draw the figure*" (Pythagoras, 8th grade).

In both cases, the link with previous activities is marked, by "Good … this says" and by "so," respectively. The "then" marker introduces the task to complete: "*Draw a figure*." In both classes, there was an analogous closing marker several minutes later, with a number of contributions in between:

"*Everyone has had time to draw a figure?* ***It's going okay****?*" (Thales, 9th grade.)

"***Now that*** *everyone has had time to draw a figure* …" (Pythagoras, 8th grade.)

Inside this interaction unit are several subunits of interaction. The first involves analyzing the situation through questioning a selected student.

"***So****, this situation is fairly banal, huh. Given all that we've done, what is the only new thing? Bertrand!*" (Thales, 9th grade.)

"***Now then****, what is it like? Arthur, describe the figure for us!*" (Pythagoras, 8th grade.)

These interaction units are themselves concluded by once the answer to the question is given. This conclusion happens after several exchanges and a number of semantic units (analogues of clauses) from D. The interaction subunit remains enclosed within the main unit.

In 9th grade, the teacher wanted to explicitly highlight the presence of a variable (*x* as a number, *M* as a point), which constituted the introduction of an important new factor. This factor is made explicit in the closure of the main interaction unit: "***So*** *x is a variable, M varies*."

In 8th grade, it is important that the figure be described as composed of two separate triangles, and not as one "big" triangle (as the drawing in the problem statement could imply). The closure of the interaction unit is strongly marked: "***So*** *we describe it [the figure] as you said afterward; that is, two triangles.* ***So*** *you have ABC and you have ACD.* ***There you go****!*"

Beyond these invariants, the study of markers during this figure-constructing episode reveals a difference that is linked to students' ability levels in terms of the mathematical content in play. We therefore find in 8th grade a long, argumentative contribution from D that is aimed at involving students in analyzing the figure by taking "what we see" (in the figure, the sides of the two smaller triangles seem to form a side of another triangle) and distinguishing it from what we can deduce from the problem statement (which raises the question of the collinearity of three points on these sides). Involving Arthur (a student) in this analysis will require an interaction between what Arthur sees in the figure, and what is really there. The teacher's remark at this point is organized with multiple clauses, using both argumentative connectors (*because, if, yet, so*) and markers (*now then, so, there you go*), until the conclusion: "*We do not define the figure like that*." After delivering this long argumentative thread, the teacher restates the initial task: "***Now then**, first job, **so**, you're told to draw the figure*" (where the "so" marker brings us back to the figure).

We have also compared the role of the particles "now then" (*alors*) and "so" (*donc*) in D's remarks, as well as in the comments of four other 9th grade teachers, in class periods devoted to exercises or discussion sessions.

In general, "now then" predominates over "so" as a speech marker (appearing twice as frequently, with some variability). One teacher, however, used "so" 80% of the time. The particle "now then" appears most often functioning as a marker, and only very rarely in its function as a logical connector. "So," however, is consistently present as a logical connector, but with wide variability between teachers in the same grade. For teacher D, "so" is as much a logical connector as a marker, and is the connector used in approximately half of all logical connections, in both grades.

These data, though "surface" data, indicate a larger variability between teachers than within a single teacher's practice. The stability of a given teacher's practice is tied to the teacher's style, and is not only reflected in general invariants (the genre) of the mathematical activity.

DISCUSSION AND CONCLUSION

Given these three analyses, we ask: If we were to enter a class taught by this teacher, with his body hidden from view and his voice distorted, what would allow us to say, "This is the same teacher"?

A first type of teacher invariance concerns the global organization of in-class events (first analysis; first section).

The types of work are the same in both classes. Students' activities take analogous amounts of time, and the speech that accompanies these activities is also managed in the same way.

After an initial period spent on the figure and on the question in play, a second phase, which may take place immediately, is dedicated to listing the solving strategies. The teacher responds to the choices of the students who are called upon, and moderates the sharing of their answers. The third phase is more directed, and gives students time to work on their own, according to the plan that was designed in the previous phases. This third phase is followed by a very structured correction period in which a model solution is written on the board. In both classes, there is little constructive assistance from the teacher. There is more or less a kind of procedural assistance, actually of the same nature when it occurs.

A second type of teacher invariance relates to certain characteristics of the teacher's speech during interactions with students (second section). The functions of the speech are relatively stable. Some of these functions are more variable than others, and are associated with adaptations that the teacher implements as part of his goals for the class, while taking the task and the students into account. Broadly, for example, the role played by direct participation in a more difficult exercise appears to be compensated by sharing for in an easier exercise.

Moreover, consideration of students' work and student questions is very similar in the two classes. During the strategy-finding period, contributions from all students are regularly and systematically considered for evaluation and sharing. During the correction phase, the teacher refers to his observations of students during the individual work period. He worries about showing the details of the calculations to students who were not able to start working on the problem. However, students called upon in class are never allowed to describe a complete reasoning strategy other than the one intended. It is as if only one course of reasoning is acceptable and only one could lead to a correct result. In each class, the differentiation between students is apparent from the moment they are called upon, in the form and length of their exchanges. Occasionally, exchanges are initiated by students. In addition, the teacher addresses some "meta" responses in an aside to certain students.

A third invariance concerns the similarity in the use of linguistic markers that structure speech (third section).

There are differences between grades in the phases of teacher interaction with the class. These differences indicate adaptations by the teacher based on students' reactions. We note variations in the number of students who are called upon at their desk or to write on the board. The length of each phase also varies between the two grades. In addition, the greater difficulty of the 8th grade exercise led the teacher to divide the problem into more simple and isolated tasks than in 9th grade. The teacher also included students more frequently in the correction phases and encouraged more sharing.

Our analyses have thus allowed us to show a real stability in the mediative component of this teacher's practice, both at the most global level (in-class events) and at the most micro level (linguistic markers). A local analysis reveals more variations (in procedural assistance and functions) that are determined by students' reactions, but no modification of the sequence of planned activities associated to the different tasks assigned in the two classes.

If we suppose that students' activities presumably occur multiple times in the year in analogous unfolding initiated by the teacher, there can be repetition effects that differ for different students. Based on the results above, we present some examples of possible such effects over the long term.

If the teacher suggests every time that students begin a geometry exercise by trying to find a strategy, will students all appropriate this step without constructive assistance?

Some types of tasks given by the teacher for short individual work periods encourage *a maxima* activities, which are visible in the work of some students. Will others students be always excluded?

The highly structured correction phase does not allow for questions from students who are still very far from solving the problem after working on their own. Will these students always have doubts as to the validity of their resolution? Will they be able to use the calculations they have recopied?

Local assistance is provided by the teacher to all students. Is this sufficient for them to learn?

Finally, some intended activities are only possible as long as the teacher's usual management style does not contradict the necessary course of action. For example, if there were never any long individual or group work periods, we might wonder if students were capable of coming up with the steps of a complex exercise by themselves.

In other words, does the stability of this teacher's practice contribute to all students' learning in the same way? And does this always play out in the same way for each student?

In addition, the invariant linguistic characteristics shown in the analyses of communication and speech (with priority given to the use of certain associations of functions) are tied, to a certain extent, to the personal component of the teacher. We can investigate more generally the relationships between the cognitive, mediative, and personal components of a single teacher.

The possibility that certain student activities are incompatible with certain practices has not been ruled out. The teacher cannot develop these activities without making changes that are all the more costly since his practices are stable. We can then wonder if this stability of practices, as studied for experienced teachers, can be modified, and how much expensive it may be.

A comment made by D at the end of the 8th grade class period led to a glimpse of the difficulty involved: "*Now, then, what is interesting in this exercise is that we have a single question and, ultimately, we applied the converse of the Pythagorean theorem: the result where the equality doesn't hold but we can conclude that the triangle is not a right triangle, and with some help after adding two angles we were able to get a conclusion. Okay? Now then, we could certainly imagine this exercise with intermediate questions. It would definitely have been simpler. You were all completely capable of finding what needed to be done each time*."

We can also ask what invariants are shared between teachers. This question leads to an examination of constraints and personal choices.

Learning more about the stability of experienced teachers' practices may allow us to better adapt professional development trainings, by more clearly outlining the links between tasks, intended activities, and adapted management.

APPENDIX

Example of the speech methodology analysis

Finding an overall strategy (9th grade)

Teacher's and students' speech	**Linguistic actions**
So, this situation is fairly banal, huh. Given all that we've done, what is the only new thing, Bertrand? *Uhh…x*	Meta. Information, question, participation
X, that is, the point *M*. What do you say about point *M*? Well, we don't know its real place on *EG*.	Sharing. Question, math, orientation
We don't know it's real place on segment *EG*. So in other words it's a point? *Unknown.*	Sharing. Question, math, orientation
Unknown. What other word could we… *We don't know where it is*.	Sharing. Question, math. Orientation
We don't know where it is, Marc? *Variable.*	Sharing. Question, math
Variable, *x* is a variable. The point *M* varies, so *x* varies from what to what*?* *From…well from 0 to 7.*	Sharing. Math, orientation, question, math, information
From 0 to 7, we can even write that at the beginning. They don't ask for that, huh. One time we did a problem where they asked for that. But we'll write right away that *x* goes between? *Zero and 7.*	Sharing. Math, information, structure, question, math
Zero and 7. Okay? *Zero and 5.*	Sharing. Question, getting attention.
Zero and 5? I wasn't paying attention to… *M* is on *EF* and EF, look, it's 5. Ah you switched them, pay attention. Everyone has had time to draw a figure? It's going okay?	Evaluation. Getting attention, information. Question. Other, getting attention.

This episode is dominated by participation, with a strong sharing component and numerous questions. The information from the teacher primarily concerns mathematics.

All episodes were analyzed in this way, which allows for rough quantitative evaluations. The overall results are presented below.

In the 9th grade class, in the first episode (work on the problem statement) we find a mix of structure, mathematical information, and meta speech. Participation is fairly week and is dispersed during the course of the exchange through a few questions and through contributions from students.

In the next episode, which takes place before the individual work phase, we note strong participation. The first part of this episode concerns primarily mathematical information, and the second integrates more meta content that situates the exercise relative to new and old elements.

At the beginning of the correction phase, there is less participation. Most of the information relates to mathematical justification. The teacher refers frequently to students' work.

In the 8th grade class, the teacher encourages student participation through questions and participation functions, which include a strong sharing element. The teacher speaks after students' responses to share their comments with the class, and then orients students' work towards a path that will more effectively lead them to the desired response. The episodes we analyzed are also marked by their strict structure. Different steps in the reasoning process are first identified in the search for a strategy, and then elaborated explicitly during the correction phase. Justifications are requested: "*Now then, why do we begin by AB squared? Why don't we begin with the others?*"

Above all, the speech concerns mathematics. Nevertheless, the teacher does pose several questions concerning the reasoning process: "*Now then, we will try to put several ideas on the board, without writing them out in full. Dominique, do you have an idea? What could we look at?*") and comments on the calculation: "*Remember, you can find the sum of two squares directly with the calculator. You can write down the intermediate results, or you can do it directly*."

NOTES

1. Each class had approximately 30 students.
2. The name of the teacher has been changed (we refer to him as "D").
3. Immediately before presenting the problem statement, the teacher had a student list the possible three theorems to be used: The Pythagorean theorem, its converse, and its contrapositive (which does not have a specific name in this class).
4. Key: SIT = Simple isolated task; A1 = recognition of methods of application; A2 = introduction of intermediary; A3 = combination of multiple frameworks; A4 = introduction of steps.
5. This was completed through an interview that strictly followed the questionnaire (private oral communication).
6. Interactions initiated by the student (which were rare in the observed class periods), if aimed at the whole class, were also indispensible for determining the progress of students' activities and were analyzed with this in mind.

Monique Pariès
Aline Robert
Laboratoire de Didactique André Revuz
Université de Cergy Pontoise

Janine Rogalski
CNRS, Laboratoire CHART
Université Paris 8