JANETTE BOBIS, JOANNA HIGGINS, MICHAEL CAVANAGH
AND ANNE ROCHE

PROFESSIONAL KNOWLEDGE OF PRACTISING TEACHERS OF MATHEMATICS

Key words: professional development; pedagogical knowledge; expertise; pedagogical frameworks.

INTRODUCTION

Teachers' knowledge of mathematics has become a central focus of educational researchers and policy makers with conceptions of teacher knowledge continuously being transformed. Intuitively, we have known for some time what research now provides an evidence base for—that "teacher knowledge matters" (Sullivan, 2008b, p. 2). But exactly *what* knowledge matters more, and *why,* are more significant and vexing questions for researchers and educators to address. Consequently, attention has moved beyond looking solely at what knowledge teachers possess to *why* different types of knowledge are important and *how* that knowledge is acquired, studied and impacts on the quality of instruction.

While historically unquestioned in importance, it has become politically as well as educationally necessary to provide an evidence base as to why knowledge of mathematics content by itself is insufficient for effective teaching of mathematics. For instance, in a recent report commissioned for the Go8 universities on mathematics entry requirements for Australian primary teacher education programs, it was found that many accreditation bodies now required entrants to have studied mathematics to the final years of secondary school (G. Brown, 2009). The report recommended that knowledge of mathematics content should become a major focus of primary teacher education programs. Mathematics educators and researchers are aware that while such recommendations help to emphasise the importance of specific content knowledge, they can also be damaging when a full picture of teacher knowledge in all its complexity is not portrayed. Accordingly, theorising and research surrounding teacher knowledge has escalated, resulting in expanded notions of some aspects of teacher knowledge and the emergence of new conceptual frameworks informing and fuelling research on teacher knowledge (e.g., Chick, 2009a, 2009b; Hill, Ball, & Schilling, 2008).

This chapter provides a critical review of research and theoretically informed perspectives on knowledge in mathematics education and development of practising teachers published by Australasian researchers from 2008–2011. Previous four-yearly reviews published by MERGA have dealt with the

B. Perry et al. (Eds.), Research in Mathematics Education in Australasia 2008–2011, 313–341.

professional learning of practising teachers of mathematics (Anderson, Bobis, & Way, 2008), and as a consequence, have considered teacher knowledge. However, never before has there been an entire chapter specifically devoted to this topic—an indicator of the increased attention teacher knowledge has attracted in the past few years. While there is some inevitable overlap of content and issues relevant to the study of pre-service teachers' knowledge of mathematics, it is beyond the scope of this chapter to address that body of research. Research relevant to pre-service teachers is discussed elsewhere in this volume.

Our review has five sections. We first consider the situated nature of teacher knowledge, thus reflecting the growing recognition by researchers that knowledge for teaching mathematics is not only mediated by sociocultural contexts, but also by teachers' beliefs, their conceptions of mathematics and the confidence they have in their own mathematical knowledge. The second section introduces various frameworks for researching teacher knowledge and includes the emerging notion of what many researchers now refer to as the mathematical knowledge for teaching. We then examine the various domains of teacher knowledge that have most recently dominated research in the field. This includes investigations of specific content areas of mathematics, the expanding domain of pedagogical content knowledge and knowledge of curriculum. The fourth section considers the mechanisms and processes by which teacher knowledge is acquired. It also critically reviews approaches used for researching the knowledge of teachers of mathematics. Finally, the chapter distils the information emanating from this body of literature and suggests how it can inform emerging research agendas, policy debates, continuing teacher education and, most critically—classroom practice.

THE NATURE OF TEACHER KNOWLEDGE

The situated nature of teacher knowledge has come to greater prominence among Australasian researchers in recent years. During the period under review (2008–2011), there has been a growing recognition that teacher knowledge is filtered through the social and cultural context of teaching and mediated by teachers' beliefs, their conceptions of mathematics, and their confidence in their own mathematical knowledge.

In his introduction to a plenary panel discussing the possible role(s) of theory in the context of mathematics teacher education, David Clarke (2009) emphasised the situated nature of teacher knowledge, and in particular attended to teaching as a culturally situated activity. With such a perspective in mind, Owens and Kaleva (2008) addressed the issue of how primary school teachers in Papua New Guinea (PNG) could use their cultural knowledge to improve their students' understanding of measurement. They used everyday examples of mathematical applications drawn from indigenous communities around PNG to help teachers understand how their cultural knowledge can be used in mathematics instruction by communicating to students the mathematical thinking behind the activities, thus making tacit teacher knowledge more explicit.

Effective teachers require knowledge of content and knowledge of teaching (Sullivan, 2008a). However, teacher knowledge is also closely interrelated to beliefs about mathematics, how the subject is best learned, and how it should be taught. Since beliefs are also influenced by the contexts in which teachers work, recent research has examined teacher knowledge and beliefs and how they impact on their teaching practises. For instance, Goos (2009) highlighted the relationships between teacher knowledge and beliefs, professional contexts and professional learning experiences. She proposed a sociocultural framework for investigating teacher learning in terms of the integration of technology into secondary classrooms. Combining the results of semi-structured interviews, a mathematical beliefs questionnaire and a series of lesson cycles, Goos suggested that the degree of alignment between teachers' knowledge and beliefs and professional contexts may provide insights into how teachers at different stages of their careers created professional learning opportunities in schools. Although the role of teachers' beliefs is beyond the scope of this chapter, the interconnected nature of teacher knowledge and beliefs is becoming more widely recognised (e.g., Beswick, Callingham, & Watson, 2011). Teachers' knowledge of mathematics and their classroom practices depend to a large extent on their beliefs about the nature of mathematics, how it is learned, and the role of the teacher.

Barton (2009) theorised on mathematical knowledge for teaching in a MERGA conference keynote address. He suggested that knowing about mathematics includes teachers' attitudes and orientations towards mathematics, which he described as the way teachers hold their mathematics, the way they know mathematics, and their relationship with mathematics. According to Barton, teachers must develop a rich vision and a carefully considered personal philosophy of mathematics while remaining receptive to the ideas of others, particularly the diverse and developing views of their students. But teachers should not hold too rigidly to their views to ensure they remain active learners of mathematics.

Teachers' Confidence in Their Own Mathematical Knowledge

An important theme emerging from studies of teacher knowledge is the influence teachers' confidence in their own knowledge has on their instructional decision-making and ultimately on student learning. Sullivan, Clarke, Clarke, and O'Shea (2009) discussed teacher confidence in terms of their ability to identify children's conceptual level on a trajectory of learning. They compared how three primary school teachers converted the same rich task into classroom learning activities by investigating how the lessons reflected each teacher's instructional goals. The researchers found that the teachers acted as they intended but their ability to appreciate the mathematics involved in the task directly influenced the types of learning opportunities they provided for students. The potential of the task was reduced by two of the teachers, and the researchers attributed this to the teachers' lack of confidence in their own mathematical ability to solve the task rather than any lack of familiarity with implementing problem-based learning activities. In contrast, the confidence of the third teacher allowed her greater freedom to explore the task in her lesson, resulting in more interesting student responses and apparently better learning. The researchers concluded that teachers' mathematical confidence shaped the potential of the task as a learning opportunity for students.

In a recent study, Beswick et al. (2011) (see also Watson, Brown, Beswick, & Wright, 2011) reported on a three-year professional development program with 62 middle school teachers. The research aimed to assess aspects of teachers' knowledge previously identified by Shulman (1986), and Ball, Thames and Phelps (2008), but was extended to include teachers' confidence to teach mathematics and their beliefs about teaching and learning mathematics. Findings revealed a close connection between teacher knowledge, confidence levels, and beliefs about the nature of mathematics learning and teaching. The study found that while building teachers' confidence to use mathematics and promote student understanding of mathematics was desirable, its development alone was not necessarily an indicator of competence.

Teachers' confidence in their knowledge of mathematics can be especially important when a new syllabus is implemented. Warren (2008/09) described a cyclic model of professional development, *Transformative Teaching in the Early Years Mathematics* (TTEYM), to guide novice teachers towards becoming expert in teaching unfamiliar content in a new Patterns and Algebra strand. The model was grounded in the notion of a community of practice and adopted a socio-constructivist perspective. Six Year 1 teachers worked in pairs to design and implement classroom activities for students. Warren found that the teachers' growing understanding of the patterns and algebra content gave them greater confidence to experiment in the classroom. Furthermore, confidence about teaching seemed to be strengthened by the opportunities for teachers to compare their teaching with other participants in the TTEYM project. Warren also noted that the strong connection between teachers' improved mathematical knowledge and the ways they made connections between mathematical concepts, used a variety of mathematical representations, and encouraged more meaningful classroom discussion.

Bobis (2009, 2010) used survey and interview data to examine the influence of primary teachers' knowledge of the *Count Me In Too* numeracy program for primary schools in New South Wales. A key aspect of the program, the Learning Framework in Number (LFIN), is used to describe children's early number learning. Bobis investigated 28 primary school teachers from three schools, explored their perceptions about their knowledge of the LFIN, their confidence to use the framework to assess children's mathematical development, and the extent to which they could use this knowledge to plan appropriate instruction. Teachers tended to rate their confidence low while their ability to assess and plan instruction was high. Bobis expressed a concern that this lack of confidence might have a detrimental effect on their instructional decision-making. She also noted that some teachers rated themselves low in their understanding of the LFIN because they appreciated how much more they needed to learn in order to use it effectively.

Other researchers have found that teachers were more likely to want to learn about mathematical content if they were made aware of the gaps in their current knowledge. Anstey and Clarke (2010) reported on a program, the *Teaching and Learning Coaches Initiative*, which provided support to Victorian government schools to improve students' learning outcomes in mathematics. The researchers invited 15 numeracy coaches to participate in monthly forums as well as 16 days of professional development focusing on the topic areas of fractions and algebra. Questionnaire and interview data were used to investigate the coaches' changing perceptions of their learning needs over the six-month study. The results indicated that the coaches' priority for mathematics content knowledge strengthened over the year of study. Anstey and Clarke noted from the results that, the focus on content knowledge helped participants identify what they did not know, thus increasing their goals to further develop their content knowledge.

All these studies highlight the links between mathematical confidence, subject-matter knowledge, and the impact on their instructional decision-making. Additionally, they highlight the fact that mathematical knowledge of teachers is a relative construct. That is, a teacher may rate their level of knowledge quite highly when compared to their immediate colleagues, but quite low when exposed to a more knowledgeable other. The influence of teachers' mathematical self-concept on their knowledge for teaching mathematics is a worthy area for further exploration to more fully understand the nature of teacher mathematical knowledge.

FRAMEWORKS OF TEACHER KNOWLEDGE

In this section, we outline the various attempts to identify frameworks of teacher knowledge. We first describe in some detail, background work in Australasia and overseas (particularly in the United States). We do this for two reasons; it enables the discussion of more recent Australasian work to be situated, and most Australasian researchers draw upon this earlier work in establishing their own frameworks or in using existing frameworks.

The Importance of Teacher Knowledge and the Need for Frameworks of Knowledge

For many years, it has been accepted that the teacher is the crucial variable in student achievement in mathematics, as with most other subject areas, accounting for about 30% of the variance in student achievement (Hattie, 2002). Similarly, it has been recognised for a long period that, in particular, teacher knowledge is key (Fennema & Franke, 1992). Mason and Spence (1999) described teachers' knowledge as dynamic and evolving and noted the importance of knowing as it requires "relevant knowledge to come to the fore so it can be acted upon" (p. 139). It is here that knowledge and practice intersect/interact and the knowledge can prove to be useful or otherwise. Much of the content knowledge that teachers have is not accessible. Brophy (1991) argued in relation to content knowledge that:

> Where (teachers') knowledge is more explicit, better connected, and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways and encourage and respond fully to students' comments

> and questions. Where their knowledge is limited, they will tend to depend on the text for content, de-emphasise interactive discourse in favor of seatwork assignments, and in general, portray the subject as a collection of static, factual knowledge. (p. 352)

However, it has only been since Shulman's seminal paper in 1986, that there has been serious consideration of the various components of teacher knowledge, and the contributions each of these make to the act and art of teaching. As will be discussed in this section, the act and art categories of teaching have been important in discussions of the components which can be developed, and are essential for effective teaching, as well as in establishing both ways of assessing teacher knowledge, and in exploring the impact of various professional learning programs on such knowledge.

Shulman (1986, 1987) argued that the acceptance of two distinct categories (subject matter knowledge and pedagogical knowledge), was simplistic and that the art of teaching could be more appropriately encapsulated by the term *pedagogical content knowledge* (PCK)—the intersection of pedagogical knowledge and content knowledge. He emphasised the many aspects of PCK, which he saw as including "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations— ... the ways of representing and formulating the subject that make it comprehensible to others" (1986, p. 9).

However, Shulman (1987) also discussed knowledge of curriculum; learners and their characteristics; educational contexts; and educational ends, purposes and values. In a personal communication (quoted in Boaler, 2003), Shulman noted that his model needed more emphasis on teacher action in practice, and teacher learning.

Ball and her colleagues (Ball et al., 2008) noted that in addition to these components, Shulman's categorisation was theoretical and not empirical. They claimed that while this was helpful at the time, further research was needed to establish a research-based categorisation and proposed the model shown in Figure 1.

Ball et al. (2008, pp. 399–403) defined the various components in their model as:

- *Common Content Knowledge:* Mathematical knowledge and skill used in settings other than teaching.
- *Horizon Knowledge:* An awareness of how mathematical topics are related over the span of mathematics included in the curriculum.
- *Specialised Content Knowledge:* Mathematical knowledge and skill unique to teaching.
- *Knowledge of Content and Students:* Knowledge that combines knowing about students and mathematics.
- *Knowledge of Content and Teaching:* Knowledge that combines knowing about teaching and mathematics.
- *Knowledge of Content and Curriculum:* Such knowledge relates closely to Shulman's curricular knowledge.

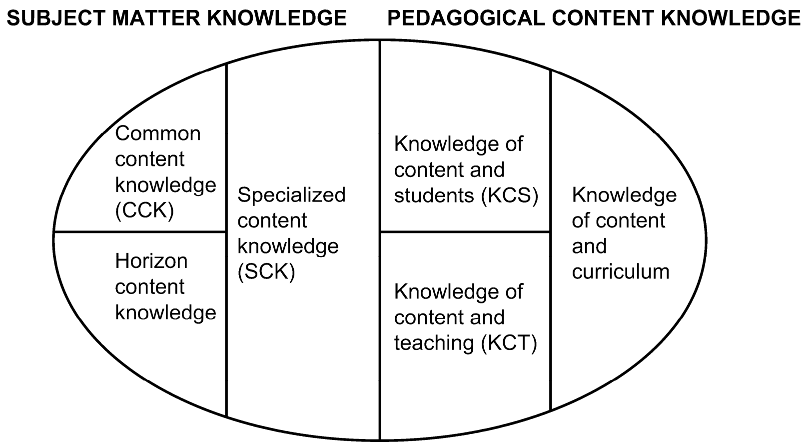

Figure 1. Framework of mathematical knowledge proposed by Ball et al. (2008, p. 403).

In much of their recent work, Ball and colleagues have used the term 'mathematical knowledge for teaching' to encompass those areas of their framework that are unique to the role of the teacher (Hill, Rowan, & Ball, 2005). Of course, any categorisation is unlikely to assume that the components or categories are mutually exclusive. As Ball et al. (2008) noted, "we recognise the problems of definition and precision exhibited in our current formulation" (p. 404).

A framework for teacher knowledge developed by Rowland, Turner, Thwaites and Huckstep (2009) was a result of an investigation into "how different kinds of primary mathematics teachers' content-related knowledge 'played out' in the classroom" (p. 26), by observing trainee teachers. This framework called the *Knowledge Quartet,* included four dimensions: foundation, transformation, connection and contingency. Although the development and use of this framework was primarily for "productive discussion of mathematics content knowledge between teacher educators, trainees and teacher mentors, in the context of school based placements" (Rowland, Huckstep, & Thwaites, 2005, p. 256) and therefore not strictly relevant to this chapter, it provides another lens through which to observe and describe practising teachers' mathematical knowledge for teaching.

Australasian Research Involving Frameworks of Teacher Knowledge

Several researchers in Australasia have taken-up the theme of categorisations of knowledge in recent years. Chick's (2009a, 2009b, 2010) recent work has focused on teachers' capacity to choose or design suitable examples, recognising what is afforded by these, and knowledge of how to adapt a given example to better suit an intended purpose. This built upon earlier work by Chick and her colleagues (Chick, Baker, Pham, & Cheng, 2006), who had proposed a framework for Pedagogical Content Knowledge that they used to investigate teacher knowledge of decimals and the teaching of decimals, through a questionnaire and interview protocol. This framework contained components under three broad categories: (a) Clearly PCK, (b) Content Knowledge in a Pedagogical Context, and (c) Pedagogical Knowledge in a Content Context. Although not published in the years spanned by this review, this framework was used in more current studies, and is therefore included here as Table 1.

Table 1. A framework for pedagogical content knowledge (after Chick et al., 2006)

PCK Category	***Evident when the teacher ...***
Clearly PCK	
Teaching Strategies	Discusses or uses general or specific strategies or approaches for teaching a mathematical concept or skill.
Student Thinking	Discusses or addresses student ways of thinking about a concept, or recognises typical levels of understanding.
Student thinking – Misconceptions	Discusses or addresses student misconceptions about a concept.
Cognitive Demands of Task	Identifies aspects of the task that affect its complexity.
Appropriate and Detailed Representations of Concepts	Describes or demonstrates ways to model or illustrate a concept (can include materials or diagrams).
Explanations	Explains a topic, concept or procedure.
Knowledge of Examples	Uses an example that highlights a concept or procedure.
Knowledge of Resources	Discusses/uses resources available to support teaching.
Curriculum Knowledge	Discusses how topics fit into the curriculum.
Purpose of Content Knowledge	Discusses reasons for content being included in the curriculum or how it might be used.
Content Knowledge in a Pedagogical Context	
Profound Understanding of Fundamental Mathematics (PUFM)	Exhibits deep and thorough conceptual understanding of identified aspects of mathematics.
Deconstructing Content to Key Components	Identifies critical mathematical components within a concept that are fundamental for understanding and applying that concept.
Mathematical Structure and Connections	Makes connections between concepts and topics, including interdependence of concepts.
Procedural Knowledge	Displays skills for solving mathematical problems (conceptual understanding need not be evident).
Methods of Solution	Demonstrates a method for solving a mathematical problem
Pedagogical Knowledge in a Content Context	
Goals for Learning	Describes a goal for students' learning.
Getting and Maintaining Student Focus	Discusses or uses strategies for engaging students.
Classroom Techniques	Discusses or uses generic classroom practices.

The great complexity of teaching and teacher knowledge is emphasised by the fact that Chick (2007) and Chick and Pierce (2008), subsequently studied and reported on *just one* of the 18 components of PCK she had identified earlier—teachers' use of examples that highlight a concept or procedure. Few would disagree that this is a very important component of a mathematics teacher's role. However, it should be noted that in discussing this one component, many overlaps with other components from her framework and the frameworks of others were evident, and reported by Chick. Their framework and subsequent studies add to the knowledge and research in this area as they "investigate more specific aspects of PCK" (Chick, 2007, p. 7) than the more general aspects of PCK.

Chick (2009a) defined example as "a specific instantiation of a general principle, chosen in order to illustrate or explore that principle" (p. 26). She reported on a study involving observations of the choice and use of examples by two Year 6 primary teachers in Victorian schools, as they each taught two lessons on the topic of ratio. Both teachers knew their students' mathematical capabilities well enough to choose tasks with appropriate cognitive demand, which Chick related to Ball's categories of "knowledge of content and students", and "knowledge of content and teaching" (see Figure 1). Chick noted that despite the small number of teachers and lessons observed, the observations nevertheless provided "a stimulus to the external observer to question what knowledge is desirable and what role alternative examples might play" (p. 29). She also proposed that issues around constructing examples, identifying their affordances, and using them to best effect in the classroom might be more explicitly addressed in pre-service and in-service programs.

As part of a questionnaire and interview protocol intended to elicit information on the PCK of secondary mathematics teachers, Chick (2009b) presented a page from a current Year 8 mathematics textbook to 35 teachers from three schools, which included examples intended to illustrate the distributive law. Teachers were asked to identify positive and negative aspects of the way the distributive law was presented, and discuss how they would use these explanations. A follow-up interview with 33 of the teachers sought their opinions of the given page and asked them to elaborate on any 'issues' they noticed with textbooks and their usual explanations to students about the distributive law. Chick found that teachers' responses to these questions revealed much about their PCK for teaching algebra. In particular, she grouped their responses within three of her themes proposed in 2006: (a) knowledge of alternative explanations,(b) knowledge of structure and connections, and (c) knowledge of students' thinking. Somewhat disturbingly, many teachers' responses indicated a personal commitment to the 'fruit salad algebra' approach—long recognised as problematic (MacGregor, 1986)—even after they had been made aware of its inappropriateness for instructional purposes.

While exploring the teacher knowledge required to effectively teach ratios, Chick (2010) outlined a questionnaire item and interview protocol that investigated "the extent to which teachers can recognise a typical misconception associated with ratio understanding and what strategies they have for addressing it" (p. 145). Forty secondary teachers from three schools completed a questionnaire and were interviewed on key topics for Years 7 to 9. She acknowledged the complexity of the knowledge required for effective teaching and proposed similar issues regarding examples, as had arisen in her earlier work (Chick, 2009a).

Roche and Clarke (2009, 2011) proposed their own framework of PCK. Their purpose was to use the framework to develop survey items that could be used to assess teachers' PCK in mathematics. The teachers were involved in a two-year professional learning program (*Contemporary Teaching and Learning of Mathematics*, CTLM). Questionnaires of items were administered to teachers at the first professional learning session of a given year (February) and the last session (October or November). The results were used to assess any teachers' improvement in PCK over time. The Roche and Clarke (2009, p. 469) framework contains the following components:

– *Pathways:* Understanding possible pathways or learning trajectories within or across mathematical domains, including identifying key ideas in a particular mathematical domain.
– *Selecting:* Planning or selecting appropriate teaching/learning materials, examples or methods for representing particular mathematical ideas including evaluating the instructional advantages and disadvantages of representations or definitions used to teach a particular topic, concept or skill.
– *Interpreting:* Interpreting, evaluating and anticipating students' mathematical solutions, arguments or representations (verbal or written, novel or typical), including misconceptions.
– *Demand:* Understanding the relative cognitive demands of tasks/activities.
– *Adapting:* Adapting a task for different student needs or to enable its use with a wider range of students.

The authors stressed that the framework was not intended to be exhaustive, and clearly it is not as broad as Chick's. They also stated, while taking into consideration the PCK frameworks of other researchers, that the components were specifically chosen to correspond with some of the key skills and teaching characteristics that were being addressed in the CTLM professional learning program.

Similar to Ball and colleagues (Hill et al., 2008), Roche and Clarke (2009) used classroom scenarios to elicit teachers' PCK, by providing, for example, a mathematical operation, and asking teachers to create a story problem, which would involve the use of the particular operation. Ninety-two teachers from 11 primary schools were asked to name the two forms of division, provide a simple representation and story problem for each, and explain which form would best help to make sense of dividing a whole number by a decimal, in this case, $8 \div 0.5$. The teachers had completed six full days of professional learning on number, working mathematically, and early algebraic thinking, during which the topic of division was just one aspect addressed. Roche and Clarke identified this task as falling largely within the two components *Pathways* and *Selecting*, within their PCK framework. Teachers found these tasks particularly difficult, with 75% of teachers having difficulty making sense of the example, $8 \div 0.5$. Related work was also reported in Clarke, Roche, and Downton (2009). Of course, unlike the work of Chick and her colleagues, Roche and Clarke were unable to triangulate the teachers' responses to the questionnaire items with classroom observations.

Watson, Callingham, and Donne (2008a, 2008b) focused on three components of PCK: teachers' content knowledge; its reflection of their students' content knowledge; and their PCK in using student responses to devise teaching intervention, in order to measure teachers' PCK in statistics. Forty-four middle-years' teachers of mathematics from three Australian states were presented with four typical incomplete or inappropriate student responses to statistics tasks, and invited to suggest strategies for remediating students' inappropriate responses to proportional reasoning tasks, set in the context of chance and data. They found that teachers' PCK was not generally strong in these areas, with a lack of discrimination between different student responses. In particular:

> There was a general lack of PCK at the point of matching content knowledge with knowledge of students as learners. Knowing what questions to ask of students, or what cognitive conflict to generate, without directly telling them the answer, appears to be a difficulty for these teachers. (Watson et al., 2008b, p. 568)

The authors recommended that professional development programs may need to focus more clearly on developing targeted intervention regarding students' levels of understanding.

In extending their work, Watson, Callingham, and Nathan (2009) greatly enhanced the quality of their data collection, by incorporating interviews with 40 middle-years teachers of mathematics from three Australian states. The teachers were asked questions relating to student responses to a pictograph task, including (a) the identification of the big statistical ideas in the problem, (b) examples of appropriate and inappropriate responses, and (c) opportunities that the problem would provide for their teaching. The framework which emerged from the teachers' responses had four 'non-hierarchical components': (a) Recognises Big Ideas, (b) Anticipates Student Answers, (c) Employs Content-specific Strategies, and (d) Constructs Shifts to General, in what the authors described as an attempt to contain and clarify some of the "nebulous components of PCK" (Watson et al., 2009, p. 569). In this way, Watson et al. (2009) contributed further to the development of PCK frameworks, by presenting components of PCK highly specific to the task at hand.

In further work, Watson and Nathan (2010) interviewed the same cohort of teachers as those of Watson et al. (2008a, 2008b), "with the aim of extending the detail and richness of teachers' PCK" (p. 610). Forty teachers were presented with a newspaper article reporting a phone-in survey about the legalisation of marijuana. Teachers' PCK were assessed based on responses to questions about the big ideas underpinning the task, potential student appropriate and inappropriate answers, and suggestions from teachers on how they would intervene in relation to the three student answers. Most teachers (70%) could distinguish between appropriate and inappropriate responses. Only 10%, however, displayed a clear understanding of student reasoning. The authors noted that around half of the teachers demonstrated a capacity to assist the development of student understanding, but seemed less able to situate sampling within the wider context of statistics. The authors concluded that “the framework of four components of PCK ... provide[d] the researchers with a comprehensive way of describing teachers’ ability to explore the problem of sampling in their classrooms” (Watson & Nathan, 2010, p. 616).

Bobis, Papic, and Mulligan (2009/10) investigated teachers’ knowledge ‘in action’ in two pre-school centres (one rural and one regional) in New South Wales. Using the data sources of still photography, video footage, and interviews with teachers, the researchers coded the mathematical learning experiences provided by early childhood practitioners, and sought to describe the components of knowledge evident in what they saw. The framework and components of ‘knowledge of’ (Hill et al., 2008) revealed evidence of ‘knowledge of’ content and students, content and teaching, specialised content knowledge, and knowledge at the horizon. They concluded that “the ability of preschool practitioners to plan developmentally appropriate experiences that foster the advancement of mathematical concepts and processes of young children is dependent on a complex combination of both mathematical subject matter and pedagogical content knowledge” (p. 95). They urged that early childhood practitioners receive ongoing professional learning support and quality educational resources, and recommended further research into aspects of their mathematical knowledge.

DOMAINS OF TEACHER KNOWLEDGE

As part of the work on categorisations of knowledge discussed in the previous section, studies in Australasia have focused on describing specific content areas of mathematics, proposing the expansion of domains of pedagogical content knowledge and knowledge of the curriculum. These studies include different aspects of the field such as those that contest the specific categorisation, either by arguing for an expansion of the category, challenging the emphases of specific categories, or proposing new categories of teacher knowledge. The interconnected and complex nature of knowledge was discussed earlier in this chapter. We begin this section by reviewing recent research about content knowledge, pedagogical content knowledge and mathematical knowledge for teaching including curriculum knowledge. We end this section by reviewing studies that consider new aspects to domains of knowledge.

Categorising Teacher Knowledge

Categorising teacher knowledge remains a challenge for mathematics educators with much debate around the importance and complexity of the issues. As noted earlier, but important to reiterate, definition and precision in categorising knowledge as well as the interconnections between domains (Ball et al., 2008) are ‘ever present’ factors under consideration in the Australasian research reviewed here. Much of the work has been dominated by a focus on the expanding domain of pedagogical content knowledge and knowledge of the curriculum as can be seen in the composition of the frameworks discussed in the previous section and the studies that follow. Sullivan (2008b), in discussing why teacher knowledge matters, suggested that a useful approach was through articulating characteristics of effective mathematics teaching. He highlighted three perspectives—mathematics knowledge, mathematics knowledge for teaching, and knowledge of pedagogy—and used teachers' answers about a particular mathematics question to illustrate the challenge and complexity of describing the knowledge that mathematics teachers needed in order to be able to teach. He identified two sides to the debate about the characteristics of effective mathematics teaching; one side that argued for discipline-based learning to be intertwined with "physical, personal and social dimensions", and a second side that took "a more explicitly mathematical perspective with attention to the principles, patterns, processes, and generalizations that have conventionally formed the basis of the mathematics curriculum" (p. 2). He concluded by suggesting that the teacher knowledge debate should not be about traditional versus reform mathematics, nor about the level and purpose of mathematics, but be about the knowledge teachers needed to teach mathematics well, which he conceded was complex and multidimensional, but something that was important for mathematics educators to continue to work on. Sullivan's work illustrates the interconnected nature of the categories through combining the three perspectives on knowledge in an exemplar from practice. His challenge to attend to the depth and scope of debate about domains of knowledge underscores the importance of continuing to develop the field of teacher knowledge.

Content Knowledge

Content knowledge is one of the original broad categories of teacher knowledge considered essential to effective teaching. Historically, this knowledge was conceptualised in relation to the discipline and gained through university study with the level of the degree being indicative of the level of content knowledge. Shulman's (1986, 1987) work disrupted this view and has not only prompted different categories of teacher knowledge, but also the expanded delineation within categories such as content knowledge, with the interconnections between categories becoming as important to teaching as the category itself. A recent study by Beswick et al. (2011) makes a strong argument for treating teacher knowledge as a uni-dimensional construct. They used written survey evidence from a teacher knowledge profile instrument with 62 Australian middle school teachers at the beginning of their participation in a three-year professional learning programme to assess different aspects of teacher knowledge. Applying a partial credit Rasch model, they found that seeing teacher knowledge as a single construct made up of multiple aspects is possible and suggested that the various facets of teacher knowledge develop together. However, they acknowledged the complexity of teaching mathematics both in its execution and in identifying the knowledge teachers drew on. Furthermore, recent work about content knowledge, related to teacher knowledge of trajectories or frameworks for student thinking, is a good example of the blurring of the edges between categories of teacher knowledge. As Bobis (2009) observed, it is not just the content knowledge of teachers, but the quality of teachers' understanding of key points in student learning and their ability to design instruction to promote student understanding in relation to these key points that can make an ultimate difference to student learning. Several studies (Bobis, 2009, 2010; O'Keefe & Bobis, 2008; Sullivan, Clarke, Clarke, & O'Shea, 2009) discussed teacher knowledge of student thinking in terms of its importance in teaching. As noted in the previous section, specific frameworks, such as those referred to by Bobis, and proposed by Roche and Clarke, are underpinned by learning trajectories.

Work by White (2010), with a specific focus on low attaining students, similarly employs the notion of trajectories of student thinking by drawing on the *Counting On* number framework. The dual intent in White's study of improving student outcomes and developing teacher knowledge and practice is akin to the strategic objectives of the NSW *Count Me in Too Project* and the New Zealand *Numeracy Development Project*. Evaluation reports of government initiatives to improve teacher knowledge are one of the few places where there are attempts to link teachers' pedagogical content knowledge with student outcomes. While the political framing of this type of work often precludes opportunities to incorporate and generate nuanced views of teacher knowledge, the impact on teacher professional knowledge of interventions, such as the Australian National Curriculum, provides important opportunities to study teacher knowledge.

Some recent studies of teacher knowledge have investigated specific areas of mathematics content (e.g., J. P. Brown, 2009; O'Keefe & Bobis, 2008; Yeo, 2008). O'Keefe and Bobis (2008) investigated teachers' perceptions of the content knowledge of measurement and teacher knowledge of student growth of understanding measurement concepts. The study used self-report data from in-depth interviews of four primary school teachers from three schools. It had a dual focus on primary teachers' perceptions of their knowledge and understandings of length, area and volume alongside teachers' understanding of the development of students' growth of measurement concepts and processes. Rather than explicitly ask teachers what they did and did not know about length, area and volume, the interviewer invited teachers to describe what they considered to be the important concepts, knowledge and skills necessary to understand these aspects of measurement. The study found that teachers struggled to articulate their knowledge of measurement concepts and children's trajectories of learning and concluded that teachers' knowledge was often implicit possibly due to the fact that teachers are not usually required to articulate this kind of knowledge. The study was also useful in exposing issues in relation to measurement that require further exploration. Similarly, in a study of five Grade 4 area and perimeter lessons conducted by a Singaporean beginning teacher, Yeo (2008) referred to the challenges faced by teachers when required to articulate their content knowledge. Together, these studies highlight the importance in teacher professional development of providing opportunities for teachers to discuss and reflect upon their own knowledge of mathematics content.

Anderson (2008) investigated teachers' motivations for attending voluntary professional development courses to examine the particular types of knowledge that teachers sought and valued from such courses. She invited 109 participants from four six-week professional development courses to complete a survey and indicate their motivation for attending. Anderson was particularly interested in identifying any differences in the knowledge required by primary and secondary school teachers of mathematics. She found that while many teachers wished to develop their mathematical content knowledge, almost all of these comments came from primary school teachers. However, Anderson noted that this is not surprising given that secondary school teachers have studied more mathematics in their teacher training.

In contrast to the Watson et al. (2008a, 2008b) studies discussed earlier about pedagogical content knowledge in statistics, Burgess (2009), in a study about statistical knowledge for teaching, based his work on Ball et al.'s (2008) *Teacher Statistical Knowledge*: content knowledge (common, specialised), and pedagogical content knowledge (knowledge of content & students, knowledge of content and teaching). These dimensions of statistical thinking included types of thinking such as (a) need for data, (b) trans-numeration, (c) reasoning with models, (d) integration of statistical and contextual, (e) investigative cycle, (f) interrogative cycle, and (g) dispositions. Using a sequence of four or five lessons videotaped from four upper primary school teachers, he selected and edited 'episodes of interest' for use in stimulated recall interviews scheduled for the same day as the lesson. The video and audio data were analysed against a teacher knowledge framework that had been formulated in relation to categories of teacher knowledge and components of statistical thinking. The profiles developed provided a useful way of identifying patterns of missed opportunities for each teacher to show aspects of teacher knowledge that needed development.

Pedagogical Content Knowledge

Of the original categories proposed by Shulman (1987) pedagogical content knowledge continues to spark interest from researchers intent on expanding understanding of the complexities about the knowledge used, and needed, to effectively teach mathematics. It is now generally accepted that there is an ongoing need to critique this construct as increasing numbers of studies argued for nuanced views of teacher knowledge, and perhaps more importantly the term 'pedagogical content knowledge' has become a descriptor in mandated curriculum and teacher assessment systems through its adoption by policy makers and implementers as a way to link student achievement to the quality of mathematics teaching and teacher knowledge. Barton (2009) in reflecting on the phrase "pedagogical content knowledge" suggested that while it is commonly accepted that it:

> includes knowledge about how mathematical topics are learned, how mathematics might best be sequenced for learning, having a resource of examples for different situations, and understanding of where conceptual blockages frequently occur, and knowing what misunderstandings are likely. Questions remain about how teachers best come by this knowledge, the extent to which it can be taught and the extent to which it depends on experience, and, inevitably, the hard question: what is the relation of this type of knowledge to student learning? (p. 4)

One study that examines the specific knowledge needed to promote student achievement is that of J. P. Brown (2009). Reporting on secondary school teachers' understanding of function she suggested factors that enable "teachers to perceive particular affordances of technology-rich teaching and learning environments (TRTLE's) and act on these to develop student understandings of functions and the development of higher order thinking?" (p. 65). The study involved seven experienced secondary mathematics teachers of Year 9 to 11 students in six schools who were part of a larger study about the use of technology in the teaching and learning of mathematics. Teachers completed "a concept map of function" which was considered as "somewhat representative of the teachers' understanding of function" (p. 66) rather than capturing all their knowledge. Rejecting a specific numerical scoring system as a way of identifying the essence of teacher knowledge, the maps were analysed according to (a) key notions related to the definition of function, (b) process or object view of function, and (c) identification of the importance of working within and across representations. Brown noted that it was of concern that none of the maps contained more than half of the key notions of functions noted by Tall (1992). Concern was also raised about the lack of teacher knowledge about "how different representations can contribute to making different aspects of a function transparent or the relationship more understandable" (p. 71). Brown postulated that the shortcomings identified in teachers' knowledge might not support the development of a deep conceptual understanding of functions by students, but did not include an analysis of student outcome data.

Vale and McAndrew (2008) designed and implemented a professional learning program based on the algebra and functions content of the Victorian senior secondary mathematics curriculum. The participants were unqualified secondary mathematics teachers who had no experience of teaching advanced senior secondary mathematics. Ten teachers from five government secondary schools completed mathematics and professional learning tasks during 21 three-hour seminars conducted fortnightly over one school year. Questionnaires, field notes and teacher portfolios were analysed qualitatively using codes derived from a PCK framework developed by Chick et al. (2006). The paper reported on case studies of three of the teachers to illustrate the mathematical and pedagogical learning attained by program participants. Vale and McAndrew found that developing teachers' content knowledge of senior mathematics also improved the participants' understanding of junior secondary mathematics content and pedagogy. The authors concluded that the ‘teachers as learners of mathematics’ model used in the program had the potential to help extend teachers’ knowledge.

Mathematical Knowledge for Teaching

Sullivan (2008a) provided a succinct review of Ball’s framework of mathematical knowledge for teaching (MKT). He argued that to give effective feedback to students, teachers needed all of these types of knowledge. He also suggested two other important aspects needed to be considered: teacher beliefs and a commitment of teachers to interact with students in situations that move beyond whole class teaching. He stressed that, “it all has to come together” (p. 433).

Teachers’ knowledge about how to represent mathematical ideas in ways that foster student understanding is an important aspect of MKT. Studies with this focus included investigations of teachers’ understanding and use of tasks, both in lessons as well as in textbooks. A number of papers that are part of a larger Australian project, *Task Types in Mathematics Learning*, reported on teachers’ insights into their choice of task types for teaching (Clarke & Roche, 2010; Sullivan, Clarke, & Clarke, 2009; Sullivan, Clarke, Clarke, & O’Shea, 2009; Zaslavsky & Sullivan, 2011). Similar to these task type studies, are others that focused on textbook examples including Stacey and Vincent (2009) and Ding, Anthony, and Walshaw (2009).

Also relevant here in terms of teacher confidence, is Sullivan, Clarke, Clarke, and O’Shea’s (2009) study concerning teacher knowledge of learning trajectories.Their study further illuminates Ball et al.’s (2008) components of ‘specialised content knowledge’ and ‘knowledge of content and of students’ while also incorporating notions of curriculum and teaching through the use of tasks. In another study, Sullivan, Clarke, & Clarke (2009) compared two groups of teachers’ ability to recognise the mathematical content in a task; one group participating in the professional development programme, *Task Types and Mathematics Learning* (TTML), with another group who were not. They conducted two surveys of primary and secondary teachers to examine how teachers converted mathematics tasks to learning opportunities. Using subcategories of the Hill et al. (2008) categorisation of teacher knowledge, they discussed responses to one particular item that sought teachers’ ideas on taking a fraction comparison task (which is larger 2/3 or 201/301?) and converting it into a mathematics lesson in the middle-years of schooling. Teachers’ abilities to identify the mathematical content of the task as “comparing fractions” varied and “raised the possibility that some of the teachers were not able to identify readily the focus or potential of this mathematical task” (p. 94). Further, they suggested those teachers without common content knowledge may have limited enactment of pedagogical content knowledge. They concluded “the responses call into question the sense teachers make of curriculum documents including syllabuses (i.e., the intended curriculum), when knowledge of content and curriculum is limited” (p. 102). The implications drawn from the study included the need for professional development programmes to focus on all six components of knowledge for teaching mathematics to ensure that greater numbers of teachers are able to translate a task into a worthwhile student learning experience.

In a later study, Clarke and Roche (2010), also drawing on the *Task Types and Mathematics Learning* project, investigated the insights of 16 middle school teachers into their choice of task types for use in their mathematics teaching. The focus of the study was to establish teacher knowledge of task types after two years involvement in a professional development programme. The study found that teachers' use of tasks did not vary across three types of models, incorporating contextual and open-ended scenarios. While teachers could articulate reasons for their choice, the choice and reasons varied considerably across the group. The teachers reported becoming more aware of task type and felt that they made better choices as a result of participating in the project and became more active in looking for opportunities to use all task types in their teaching including an increased use of contextual tasks. Teachers noted, as a result of the project, they were "now more aware of the range of task types and looked actively for opportunities to use all three task types" and were "able to select the task type that best suited the purpose or focus of the lesson and were more likely to choose tasks that catered for the range of abilities in their class" (p. 159).

As part of a larger New Zealand study (*Learners' Perspective Study*), Ding et al. (2009) also examined the use of classroom tasks. Using a teaching experiment methodology, they reported on teachers' choice and use of examples in solving number problems about fractions at the early secondary school level. Teachers used the teaching strategies and examples advocated as part of the *New Zealand Numeracy Development Project* at the secondary level. To establish teacher effectiveness in terms of mathematical content knowledge (MCK) and pedagogical content knowledge, Ding et al. (2009) established how their findings, based on observation data and video-stimulated recall interviews of "teachers' example-related practice" could be used in teacher education programmes (p. 425). Using teaching episodes the analysis highlighted potential affordances and limitations of the teacher's implementation of the examples in terms of student learning, and suggested alternative ways of implementing the examples to illustrate the importance of the understanding of the relationship of the instructional model and mathematical thinking patterns. The study made links to Chick's (2007) study about the implementation of examples where the mathematical potential (affordance) was not realised.

Stacey and Vincent (2009) focused on knowledge for teaching mathematics by examining examples of several topics in nine Australian eighth-grade textbooks. They developed a classification system incorporating seven modes of reasoning of "appeal to authority, qualitative analogy, concordance of a rule with a model, experimental demonstration, deduction using a model, deduction using a specific case, and deduction using a general case" (p. 274). In a content analysis, with a specific focus on the introductory text, the study found that while most textbooks provided explanations on most topics, some explanations were in preparation for practice exercises rather than as thinking tools that could be useful in other examples. If students needed to rely on teachers to elaborate on examples, Stacey and Vincent suggested, it was "unlikely" that they could "from the material provided" and that this highlighted "the often cited need for teachers to possess sufficiently strong mathematical knowledge and deep mathematical pedagogical content knowledge" (p. 286).

Using Shulman's categories, Clarke (2008) positioned a teacher as a 'curriculum maker' through a process by which a teacher begins with the intended curriculum as outlined in curriculum frameworks, and enacts it. He considered what kinds of knowledge a teacher might draw upon when being a curriculum maker by systematically working through each of Shulman's categories in a process of identifying constraints that may prevent a teacher from fully enacting this role. He suggested such aspects as the ability to "identify big ideas within a topic, sequence concepts within that topic, recognise and enhance connections between concepts, and match the curriculum to the developing understanding of students" (p. 133). Clarke concluded by discussing professional development to prepare prospective and practising teachers to be active curriculum makers.

Stacey (2008) in addressing the mathematics required for teaching in secondary schools, worked from a vision of good mathematics learning which valued working from reasons not rules, and being able to use whatever mathematics that had been learned for solving problems within and beyond mathematics. She proposed four aspects of teacher knowledge: "(1) knowing mathematics in a way that has special qualities for teaching; (2) having experienced mathematics in action solving problems; (3) knowledge about mathematics including its history and current developments; and (4) knowing how to learn mathematics" (p. 87).

Frid, Goos, and Sparrow (2008/9) provided a useful overarching comment on the importance of teacher knowledge in the context of teacher shortages and the emergence of teacher knowledge frameworks with specific reference to Chick's (2007) and Ball et al.'s (2008) frameworks. They reminded us that our focus needed to be on the complexity of teacher knowledge and its significance for teaching. In the spirit of this comment, Barton (2009) extended the thinking about mathematical knowledge by moving "through wider aspects of mathematical knowledge, through acting like a mathematician and creating a mathematical environment, to how a teacher holds mathematics" (p. 9). In this position paper, Barton reflected on the complexities of mathematical knowledge for teaching and suggested that further understanding of dimensions of mathematical knowledge for teaching (MKfT) is needed given the evidence that is in classroom research "we are far from capturing what it is a teacher does, why they do it, and what effect it might have on student learning" (p. 3). Barton's comments are important to framing an increasing focus on treating teacher knowledge as complex. In acknowledging theoretical models of mathematical knowledge for teaching, such as Ball et al.'s (2008), Barton suggested that they all focus on *what* the teacher must know, but what is also important is *how* a teacher must know.

SOURCES OF TEACHER KNOWLEDGE AND MODES OF INQUIRY

As evident from the preceding discussion, some key reasons for studying teacher knowledge are to explore what knowledge teachers possess (or do not possess), and to discover the most effective ways by which it is acquired. The intention is that such insights will inform programs of professional development and ultimately help to enhance teacher knowledge and student learning outcomes. Another related reason is to gain some measure of how successfully such mechanisms and processes, designed to improve teacher knowledge, have actually worked (e.g., Dole, Clarke, Weight, Hilton, & Roche, 2008; White, 2010). In reality, it seems that many aspects of teacher knowledge have been quite difficult to determine. We believe this is partly due to the complexity of teacher knowledge—a point reiterated by several researchers in the field (e.g., Chick, 2010; Frid et al., 2008/9; Roche & Clarke, 2009). It is also partly due to the fact that such knowledge not only comes from a wide range of, and sometimes ‘unexpected’, sources, but is mediated by multiple contributing factors—including a teacher’s beliefs (Sullivan, 2008a); their sociocultural contexts (Goos, 2009; Owens & Kaleva, 2008), and their level of confidence (Bobis, 2009, 2010; Sullivan, Clarke, Clarke, & O’Shea, 2009).

The actual processes by which teacher learning and development might occur were a focus of the previous MERGA review of research (see Anderson et al., 2008). Such processes continue to range from small-scale, individualised teacher professional learning opportunities (e.g., Muir, 2008; Muir, Beswick, & Williamson, 2010), to small groups of teachers (J. P. Brown, 2009) and large-scale programs of professional development (Higgins & Parson, 2009; White, 2010) involving off-site workshops, professional reading and/or classroom support. The ‘tools’, sources of knowledge or mechanisms employed to support changes in teacher knowledge are just as varied. For instance, Muir and colleagues (Muir, 2008; Muir et al., 2010) scaffolded teachers’ individualised reflections and action learning processes themselves. Higgins and Parsons (2009) identified three pedagogical tools that participants in the *New Zealand Numeracy Development Project* described as improving their mathematics knowledge and practice: (a) the number framework, (b) the diagnostic interview, and (c) the strategy teaching model—a model designed to explicitly teach problem-solving strategies. They argued that the power of the professional development model lies in the integration of these three tools that enabled teachers to deepen their professional knowledge.

Numeracy coaches (Anstey & Clarke, 2010) can also be viewed as a ‘tool’ or source of teacher knowledge, but as Gaffney and Faragher (2010) highlighted in their report on results of the *Leading Aligned Numeracy Development* (LAND) project, the success of any such mechanism for teacher development may depend on local contextual factors such as the effectiveness of school leadership. Gaffney and Faragher found that successful school leadership teams (including principals) were more able to sustain improvements in student mathematical achievement when their own PCK was well developed.

Researchers have extensively used students’ own responses to mathematical tasks, or the tasks themselves, as a source by which teachers can improve their knowledge for teaching mathematics. Horne (2008) used students' responses to interview tasks as a reflective tool that motivated teachers to extend their knowledge of student thinking strategies. Sullivan, Clarke, and Clarke, (2009) and Clarke and Roche (2010) are groups of researchers that have structured professional learning opportunities around teacher understanding of task types in mathematical learning. Similarly, Visnovska, Cobb, and Dean (2011) used 'other' teachers as a source of knowledge when groups of teachers were asked to collectively design a unit of work on statistics as part of a professional development program. Despite the variation in knowledge sources and tools employed by providers of professional development, each case required a scaffold by a more knowledgeable individual to actually make a discernible difference in teacher knowledge.

Modes of Inquiry

An ongoing and vexing issue for researchers studying teacher knowledge has been the search for inquiry methods that reveal information about teacher knowledge and how to adequately assess and examine it. Predominantly, the modes of inquiry into teacher knowledge in the review period 2008–2011 have been qualitative in nature. Our intention here is to provide some critical reflection on the array of methods used to study teacher knowledge.

The method of inquiry is mainly determined by the size of the cohort, with studies involving large participant numbers generally opting for written responses via surveys (e.g., Anderson, 2008). In cases where teachers' own perceptions about their knowledge were being sought, such as when Bobis (2010) asked teachers to rate their level of confidence regarding aspects of their knowledge needed to plan mathematics instruction, multiple-choice type answers were deemed effective. However, increasingly, Australasian researchers seem to be moving away from reliance on multiple-choice instruments to favouring open-response survey instruments often using follow-up methods involving a combination of either interviews and/or classroom observations. Roche and Clarke (2009) noted 'a tension' between collecting vast amounts of rich qualitative data from a relatively small number of teachers and collecting "less data from a larger number of teachers" (p. 473). They critiqued the work of Ball and her colleagues (e.g., Ball et al., 2008), considering the use of multiple-choice items as the sole indicator of teacher knowledge to be a major weakness. Instead, Roche and Clarke modified items on their questionnaire designed to assess PCK, requiring teachers to provide written justifications for their choices.

In their examination of teachers' abilities to respond to middle-year students' problems involving proportional reasoning, Dole et al. (2008) used a survey requiring teachers to provide written responses to a hypothetical scenario. They found that further work was required on the survey items to create a useful instrument. They also noted the necessity of combining interview and other data, including classroom observations, to determine a more complete picture of teacher PCK growth over the course of their professional development program. A similar realisation was made by Bobis (2009, 2010), when she employed a scenario as part of a survey to explore teachers PCK. Primary teachers were required to provide a written interpretation of a student's response to a mathematical task and suggested relevant teaching intervention strategies to address the student's needs. While only half of the 28 teachers involved in the survey component provided adequate levels of responses to the scenario, follow-up interviews with 22 teachers involving a similar scenario task, revealed that all but two teachers provided far richer responses, revealing much greater insights into their PCK than previously determined from the survey alone.

Further, Watson and Nathan (2010) moved beyond written survey responses in their study, intent on probing teachers' PCK in statistics. Reflecting on results and issues that had emerged from a previous study (Watson et al., 2008a, b) involving written survey responses to student answers on proportional reasoning tasks, Watson and Nathan (2010) decided to employ interviews "with the aim of extending the detail and richness of teachers' PCK" (p. 610). They argued that such an inquiry method allows PCK to be explored as a dynamic process which is more akin to the actual work of teachers.

Other methods used to gather information about teacher knowledge have included stimulated recall of video-recorded teaching episodes (Burgess, 2009; Chick, 2009a, 2009b; Muir, 2008) and the analysis of a range of teaching artefacts such as teaching plans and teacher reflections (Vale & McAndrew, 2008). With the move away from multiple-choice type surveys, to modes of inquiry that are far more revealing of teacher thinking, a need for sophisticated assessment rubrics that considered teacher responses more holistically has emerged (Roche & Clarke, 2009). To be effective, such rubrics will need to be finely tuned to detect differences in teacher knowledge levels and will most likely need to be content specific, depending on the PCK components under investigation.

J. P. Brown's (2009) investigation of secondary mathematics teachers' knowledge of function is one of a few studies in the review period that specifically sought to determine mathematical content knowledge. She used concept mapping because it was considered to provide some insight into teachers' organisation and structure of their knowledge about functions. While the focus of nearly every study reviewed for inclusion in this chapter was overwhelmingly on specific components of PCK, occasionally judgements of mathematical content knowledge were also determined from the same analyses. For instance, Watson and Nathan (2010) preferred to assess teacher interview comments on a continuum ranging from low to high levels of PCK as determined by the researchers. While they acknowledged that some basic mathematical content knowledge would precede development of the PCK components in question, they treated it as part of the wider concept of PCK rather than as a separate body of knowledge. Certainly, a move away from previous paper and pencil 'tests' of teacher content knowledge as the sole mode of inquiry to determine teacher quality, are a welcome development in investigations of teacher knowledge.

CONCLUSION

We can see a number of developments and issues emerging from the field of research concerned with teacher knowledge. First, the situated nature of teacher knowledge has certainly come to greater prominence among Australasian researchers in recent years. While we have seen a growing recognition that teacher knowledge is filtered through social and even political contexts, there has been little mention of 'cultural' influences on teacher knowledge, with one exception: Owens and Kaleva's (2008) research. Perhaps this is because the research reviewed was predominantly conducted by researchers from western cultural backgrounds, focused on mathematical content from western curricula and interpreted via frameworks of teacher knowledge developed by scholars based on western cultural views of knowledge. While beyond the scope of this chapter, it is important for the future to consider different cultural perspectives on teacher knowledge.

A second theme emerging from this review of research is the growing awareness of the influential role of certain affective characteristics on teacher knowledge. In particular, studies by Beswick et al. (2011), Bobis (2009, 2010), and Sullivan, Clarke, and Clarke (2009) highlighted the interplay occurring between teachers' beliefs and their knowledge, and the fact that teacher beliefs (such as beliefs and confidence about their own mathematical knowledge), can be a major regulator of teaching practices. As such, we have learnt that when studying certain types of teacher knowledge, affective factors cannot be ignored.

While the incredible complexity of teacher knowledge was acknowledged and confirmed by many researchers, we also sought to extend current conceptions of teacher knowledge, viewing it as 'relative'. Drawing upon the work of researchers such as Anstey and Clarke (2010), we saw how teachers' perceptions and ratings about their own knowledge varied depending on contextual factors, including the perceived knowledge of their peers or an increased awareness of new bodies of knowledge previously unavailable to them.

It is clear there has been an increasing focus on frameworks of teachers' knowledge by Australasian researchers in the period of the review. Perhaps the most obvious omission in this body of research is a debate or rationale for why (or, *if*) we need such frameworks. From a policy perspective, frameworks of teacher knowledge, such as that proposed by Ball and her colleagues, made it clear that expertise in mathematical content knowledge alone, is insufficient for effective teaching of mathematics. Hence, moves by politicians to short-circuit teacher education programs by fast tracking so-called 'outstanding graduates' to alleviate current teacher shortages (including mathematics), does not have a sound rationale for building a teacher's professional knowledge base. Furthermore, from a research perspective, frameworks can act as great drivers of research agendas aimed at deepening our understanding of teacher knowledge and how this knowledge enables certain teaching practices. Understanding teachers' thinking about their own knowledge and its perceived impact on teaching practices is paramount to improving the professional learning of teachers. For instance, in some cases, the research reported in this review has broadened the categories of knowledge identified by earlier work, particularly in respect of pedagogical content knowledge. In others, the focus has been on exploring the knowledge needed by teachers (in school and preschool settings) to teach mathematics more effectively. Still others have assessed the extent and kinds of knowledge possessed by teachers, through analysing data from observations, questionnaires, interviews, videotapes, and still photography, with many studies focusing on very specific content areas. Sometimes, the research uses (without seeking to extend) existing frameworks. In other cases, innovative components and frameworks of knowledge have been proposed. All this work has reinforced the growing view that it is the way in which a teacher's knowledge is structured and used that is so crucial in the effective teaching of mathematics.

There is growing pressure from educational stakeholders at all levels to establish evidence of the effects of teacher knowledge on student outcomes. Despite this, there has been little Australasian research to date that links teachers' knowledge with student achievement. What are the ways that teacher knowledge influences student outcomes in mathematics? Surely the pivotal reason for examining teacher knowledge to the extent evident in this review is to ultimately learn how to improve student learning.

Finally, we have seen a proliferation of mechanisms and tools by which teachers have been shown to acquire knowledge and the methods by which it is studied. However, what is missing is some documentation of the processes by which teachers learn without externally imposed intervention. Teachers can learn from their own practice but more systematic research is needed to understand the circumstances by which this occurs best. More importantly, we need to further explore the implications of different types and levels of teacher knowledge for their teaching practice and ultimately student outcomes.

REFERENCES

Anderson, J. (2008). Teachers' motivation to attend voluntary professional development in K-10 mathematics. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 51–58). Fremantle, WA: MERGA.

Anderson, J., Bobis, J., & Way, J. (2008). Teachers as learners: Building knowledge in and through the practice of teaching mathematics. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W. T. Seah, & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004–2007* (pp. 313–335). Rotterdam: Sense.

Anstey, L., & Clarke, B. (2010). Perceived professional learning needs of numeracy coaches. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 45–52). Fremantle, WA: MERGA.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59,* 389–407.

Barton, B. (2009). Being mathematical, holding mathematics: Further steps in mathematical knowledge for teaching. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 3-9). Palmerston North, NZ: MERGA.

Beswick, K., Callingham, R., & Watson, J. (2011).The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education, 14*(1), 1–27.

Boaler, J. (2003). Studying and capturing the complexity of practice—The case of the 'dance of agency'. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 2003 Joints Meeting of the International Group for the Psychology of Mathematics Education and the Psychology of Mathematics Education Group North America* (3–16). Honolulu, HI: University of Hawaii.

Bobis, J. (2009). Teachers assessing young children's mathematical development: How confident are they? In T. Dooley (Ed.), *Mathematics for all: Extending mathematical capacity* (Proceedings of the 3rd Conference of the Mathematics Education Ireland Association, pp. 20–33). Dublin, Ireland: MEI.

Bobis, J. (2010). The impact of a developmental framework in number on primary teachers' classroom practice. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 85–92). Fremantle, WA: MERGA.

Bobis, J., Papic, M., & Mulligan, J. (2009/10). The mathematical knowledge of preschool practitioners: Evidence from the field. *Journal of Australian Research in Early Childhood Education, 16*(2), 87–98.

Brophy, J. E. (1991). Conclusion: Teachers' knowledge of subject matter as it relates to their teaching practice. In J. E. Brophy (Ed.), *Advances in research on teaching* (pp. 349–364). Greenwich CT: JAI Press.

Brown, G. (2009). Review of education in mathematics, data science and quantitative disciplines. (Commissioned Report for Go8 Universities). Retrieved from http://www.go8.edu.au/government-a-business/go8-policy-a-analysis/2010/183-the-go8-review-of-education-in-mathematics-data-science-and-quantitative-disciplines

Brown, J. P. (2009). Concept maps: Implications for the teaching of function for secondary school students. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 65-72). Palmerston North, NZ: MERGA.

Burgess, T. (2009). Statistical knowledge for teaching: Exploring it in the classroom. *For the Learning of Mathematics, 29*(3), 18–21.

Chick, H. (2007). Teaching and learning by example. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 3–21). Adelaide, SA: MERGA.

Chick, H. L. (2009a). Choice and use of examples as a window on mathematical knowledge for teaching. *For the Learning of Mathematics*, *29*(3), 26–30.

Chick, H. (2009b). Teaching the distributive law: Is fruit salad still on the menu? In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia). Palmerston North, NZ: MERGA. Retrieved from http://www.merga.net.au/documents/Chick_RP09.pdf

Chick, H. (2010). Aspects of teachers' knowledge for helping students learn about ratio. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 145–152). Fremantle, WA: MERGA.

Chick, H., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical content knowledge for decimals. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Mathematics in the centre* (Proceedings of the 30th conference of the International Group for the Psychology of Mathematics Education, pp. 297–304). Prague, Czech Republic: PME.

Chick, H. L., & Pierce, R. (2008). Issues associated with using examples in teaching statistics. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 321–328). Morelia, Mexico: Cinvestav-UMSNH.

Clarke, D. (2008). The mathematics teacher as curriculum maker: Developing knowledge for enacting curriculum. In P. Sullivan & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp. 133–151). Rotterdam, The Netherlands: Sense.

Clarke, D. (2009). Theoretical perspectives in mathematics teacher education. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 85–93). Thessaloniki, Greece: PME.

Clarke, D., & Roche, A. (2010). Teachers' extent of the use of particular task types in mathematics and choices behind that use. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 153–160). Fremantle, WA: MERGA.

Clarke, D. M., Roche A., & Downton, A. (2009). Assessing teacher pedagogical content knowledge: Challenges and insights. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, p. 362). Thessaloniki, Greece: PME.

Ding, L., Anthony, G., & Walshaw, M. (2009). A teacher's implementation of examples in solving number problems. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 425–432). Thessaloniki, Greece: PME.

Dole, S., Clarke, D. M., Weight, T., Hilton, G., & Roche, A. (2008). Eliciting growth in teachers' proportional reasoning: Measuring the impact of a professional development program. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 163–169). Brisbane, QLD: MERGA.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147–164). New York, NY: Macmillan.

Frid, S., Goos, M., & Sparrow, L. (2008/9). What knowledge is needed for effective teaching of mathematics? *Mathematics Teacher Education and Development, 9*, 1–3.

Gaffney, M., & Faragher, R. (2010). Sustaining improvement in numeracy: Developing pedagogical content knowledge and leadership capabilities in tandem. *Mathematics Teacher Education and Development. 12*(2), 72–83.

Goos, M. (2009). A sociocultural framework for understanding technology integration in secondary school mathematics. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 113–120). Thessaloniki, Greece: PME.

Hattie, J. A. C. (2002). What are the attributes of excellent teachers? In B. Webber (Ed.), *Teachers make a difference: What is the research evidence?* (pp. 3–26). Wellington: New Zealand Council for Educational Research.

Higgins, J., & Parsons, R. (2009). A successful professional development model in mathematics: A system-wide New Zealand case. *Journal of Teacher Education, 60*(3) 231–242.

Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualising and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371–406.

Horne, M. (2008). Using educational research to inform mathematics teaching in a school. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 81–86). Morelia, Mexico: Cinvestav-UMSNH.

MacGregor, M. (1986). A fresh look at fruit salad algebra. *Australian Mathematics Teacher, 42*(3), 9–11.

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics, 38,* 135–161.

Muir, T. (2008). Principles of practice and teacher actions: Influences on effective teaching of numeracy. *Mathematics Education Research Journal, 20*(3), 78–101.

Muir, T., Beswick, K., & Williamson, J. (2010). Up, close and personal: Teachers' responses to an individualised professional learning opportunity. *Asia-Pacific Journal of Teacher Education, 38*(2), 129–146.

O'Keefe, M., & Bobis, J. (2008). Primary teachers' perceptions of their knowledge and understanding of measurement. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 391–397). Brisbane, QLD: MERGA.

Owens, K., & Kaleva, W. (2008). Case studies of mathematical thinking about area in Papua New Guinea. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 73–80). Morelia, Mexico: Cinvestav-UMSNH.

Roche, A., & Clarke, D. (2009). Making sense of partitive and quotitive division: A snapshot of teachers' pedagogical content knowledge. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 467–474). Palmerston North, NZ: MERGA.

Roche, A., & Clarke, D. (2011). Some lessons learned from the experience of assessing teacher pedagogical content knowledge in mathematics. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 658–666). Adelaide, SA: AAMT & MERGA.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8,* 255–281.

Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2009). *Developing primary mathematics teaching.* London, United Kingdom: Sage.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1–22.

Stacey, K. (2008). Mathematics for secondary teaching: Four components of discipline knowledge for a changing teacher workforce. In P. Sullivan, & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp. 87–113). Rotterdam, The Netherlands: Sense.

Stacey, K., & Vincent, J. (2009). Modes of reasoning in explanations in Australian eighth-grade mathematics textbooks. *Educational Studies in Mathematics, 72,* 271–288.

Sullivan, P. (2008a). Education for the knowledge to teach mathematics: It all has to come together. *Journal of Mathematics Teacher Education, 11*, 431–433.

Sullivan, P. (2008b). Knowledge for teaching mathematics: Introduction. In P. Sullivan, & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp. 1–9). Rotterdam, The Netherlands: Sense.

Sullivan, P., Clarke, D., & Clarke, B. (2009). Converting mathematics tasks to learning opportunities: An important aspect of knowledge for mathematics teaching. *Mathematics Education Research Journal, 21*(1), 85–105.

Sullivan, P., Clarke, D., Clarke, B., & O'Shea, H. (2009). Exploring the relationship between tasks, teacher actions, and student learning. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 185–192). Thessaloniki, Greece: PME.

Tall, D. (1992). The transition to advanced mathematical thinking: Functions, limits, infinity, and proof. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 495–511). New York, NY: Macmillan.

Vale, C., & McAndrew, A. (2008). Deepening the mathematical knowledge of secondary mathematics teachers who lack tertiary mathematics qualifications. In M. Goos, R. Brown, & K. Makar (Eds.),

Navigating currents and charting directions (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 539–545). Brisbane, QLD: MERGA.
Visnovska, J., Cobb, P., & Dean, C. (2011). Mathematics teachers as instructional designers: What does it take? In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development*. Dordrecht, The Netherlands: Springer.
Warren, E. (2008/9). Early childhood teachers' professional learning in early algebraic thinking: A model that supports new knowledge and pedagogy. *Journal of Mathematics Teacher Education & Development, 9*, 30–45.
Watson, J., Brown, N., Beswick, K., & Wright, S. (2011). Teacher change in a changing educational environment. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 760-767). Adelaide, SA: AAMT & MERGA.
Watson, J., Callingham, R., & Donne, J. (2008a). Establishing PCK for teaching statistics. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (Proceedings of the Joint ICMI/IASE study conference 18 and IASE Round Table Conference, CD). Monterrey, Mexico: ICMI, IASE & ISI. Retrieved from http://www.ugr.es/~icmi/iase_study/Files/Topic2/T2P10_Watson.pdf
Watson, J., Callingham, R., & Donne, J. (2008b). Proportional reasoning: Student knowledge and teachers' pedagogical content knowledge. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 563–571). Brisbane, QLD: MERGA.
Watson, J., Callingham, R., & Nathan, E. (2009). Probing teachers' pedagogical content knowledge in statistics: "How will Tom get to school tomorrow?" In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 563-570). Palmerston North, NZ: MERGA.
Watson, J. M., & Nathan, E. L. (2010). Biased sampling and PCK: The case of a marijuana problem. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 610–617). Fremantle, WA: MERGA.
White, A. L. (2010). Counting On in the middle years. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 618–625). Fremantle, WA: MERGA.
Yeo, K. K. J. (2008). Teaching area and perimeter: Mathematics-pedagogical-content knowledge-in-action. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 621–627). Brisbane, QLD: MERGA.
Zaslavsky, O., & Sullivan, P. (2011). *Constructing knowledge for teaching secondary mathematics: Tasks to enhance prospective and practicing teacher learning.* New York, NY*:* Springer.

AFFILIATIONS

Janette Bobis
Faculty of Education and Social Work
University of Sydney

Joanna Higgins
Faculty of Education
Victoria University of Wellington

Michael Cavanagh
School of Education
Macquarie University

Anne Roche
Mathematics Teaching and Learning Research Centre
Australian Catholic University