Mathematics Education Research Group of Australasia

Research in Mathematics Education in Australasia

2008–2011

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INTRODUCTION

MATHEMATICS EDUCATION RESEARCH GROUP OF AUSTRALASIA (MERGA)

MERGA is a professional association for those interested in mathematics education research in Australasia. MERGA is an association that aims to:

- promote, share, disseminate, and co-operate on quality research on mathematics education for all levels particularly in Australasia;
- provide permanent means for sharing of research results and concerns among all members through regular publications and conferences;
- seek means of implementing research findings at all decision levels to the teaching of mathematics and to the preparation of teachers of mathematics; and
- maintain liaison with other organisations with similar interests in mathematics education or educational research.

MERGA has an annual conference. It also has a regular schedule of publications. These include refereed conference proceedings, two journals—*Mathematics Education Research Journal* and *Mathematics Teacher Education and Development*, a four-yearly review of mathematics education research in Australasia, books arising from Special Interest Groups, and some sponsored monographs. Electronic newsletters are distributed to members, and there is a moderated list for announcements as well as a web-based discussion forum for members. Further information concerning MERGA can be found at the group's website (www.merga.net.au).

SCOPE OF THE REVIEW

The review is the eighth such four-yearly review of research in mathematics education that has been commissioned by MERGA. Beginning with the *Summary of research in mathematics education in Australia* (Briggs, 1984) which was published to coincide with Australia's hosting of the fifth International Congress on Mathematical Education (ICME), the publication has grown into an important critique and celebration of Australasian mathematics education research that is eagerly anticipated by MERGA members and many other mathematics education researchers across the world.

B. Perry et al. (Eds.), Research in Mathematics Education in Australasia 2008–2011, 1–11.

This review, entitled *Research in Mathematics Education in Australasia 2008–2011*, uses the same definition of Australasian mathematics education research as the previous one did:

> The editors have defined "Australasian research" as research conducted in Australasia, about the Australasian context, or by Australasians. Australasia comprises: Australia, New Zealand, Papua New Guinea, and the Pacific Islands closely allied to Australia and/or New Zealand. (Forgasz et al., 2008, pp. 1–2)

The primary purpose of *Research in Mathematics Education in Australasia 2008–2011* is to highlight significant findings, demonstrate links among research, identify trends and foreshadow possible future research directions. Only research which has been published (books, book chapters, peer-reviewed journals, peer-reviewed conferences, research reports for funding bodies) during 2008–2011 has been considered. Space precludes the reporting of all Australasian mathematics education research published during the four-year period designated and chapter authors have made decisions about the selection of publications on which they have reported.

EDITORS OF THE REVIEW

The editors for the *Research in Mathematics Education in Australasia 2008–2011* were chosen from expressions of interest submitted to the MERGA executive. The successful editorial team consists of experienced and early career mathematics education researchers drawn from the Research Institute for Professional Practice, Learning and Education (RIPPLE), a leading research centre within Charles Sturt University. All editors are members of MERGA.

THE PROCESS OF WRITING CHAPTERS IN THE REVIEW

Two complementary processes were used to choose lead authors and author teams for each of the chapters:

- some MERGA members were approached directly by the editors to seek their willingness to lead chapter author teams; and
- a general call for chapter author teams was made via the MERGA Vice-President (Publications).

In either case, it was suggested by the editors that chapter author teams should include a mix of mathematics education research experience, gender and country of origin. All authors needed to be members of MERGA. The result of this recruitment drive was 15 author teams made up of 50 individuals spread across all of these variables.

Each author team (except for those writing the first and last chapters) developed a draft of their chapter by early 2011. Each of these 13 chapters was sent to two reviewers for assessment of their suitability for the review. The editors consolidated these reports and sent them to the author teams who had the task of revising the chapters by the end of October. Two of the editors met with representatives of most of the author teams at the MERGA conference in July, 2011 to follow up on the suggestions made by the reviewers. Final drafts of these 13 chapters were submitted by the end of October, 2011. Drafts of the first and concluding chapters were submitted during November, 2011 and were reviewed independently by members of the editorial committee. Revisions were undertaken by the author teams as necessary. All chapters were copy-edited by Dr Rosemary Farrell between November, 2011 and February, 2012.

COMPONENTS OF THE REVIEW

Research in Mathematics Education in Australasia 2008–2011 is broken into four major sections:

- Contexts for Mathematics Education
- Mathematics Learning and Teaching
- Teachers
- The Future.

Contexts for Mathematics Education

The importance of contextual aspects in mathematics learning and teaching is emphasised by both the size of this section and its diversity. Chapters around the contexts of mathematics learning have featured strongly in previous MERGA research reviews and this is reflected again in the current review. Although there is not specifically a chapter on the politics of mathematics education, as has been the case in a number of the previous MERGA research reviews, politics abounds in many of the chapters in this section (and, indeed, in other chapters). Similar comments can be made about issues around gender in mathematics learning and teaching. For the first time in this series of reviews, there is no specific chapter on gender as it is highlighted in many of the chapters. In both cases, such developments can be seen very positively as these topics are being incorporated into other challenges in mathematics education in ways that highlight their influence in these areas.

The section begins with a reflection on the previous review: *Research in Mathematics Education in Australasia 2004–2007* in which Clarke and her colleagues highlight what had been seen as the strengths and deficiencies in the research reported in the previous review and ask readers to use these to measure changes reported in the current review. This chapter challenges readers to consider how the political and social contexts of Australasian mathematics education research have changed since 2004–2007 and to reflect on whether the research reported in the current review has met the challenges arising from these changes. While not every reader will agree with some of the conclusions made by Clarke and her colleagues, there can be no doubt that this chapter does set down some criteria against which the impact of the current review can be measured.

In what is the third chapter on affective issues in mathematics education to appear in MERGA research reviews, Lomas and his colleagues report a lessening emphasis on the study of beliefs and an increase in matters of identity in mathematics education research, as well as continuing interest in a diverse range of research into affective aspects of mathematics education. There appear to be increases in the amounts of research at the primary school level, coupled with an increase in observational approaches to data gathering. While there are some strong developments reported in this chapter, the continuing challenge for the authors is the relative lack of strong theoretical frameworks informing both research and practice in the affective domain.

The third chapter of this review considers a very broad area of mathematics education research centred on various aspects of the social contexts in which people learn mathematics. Atweh, Vale and Walshaw (2012, p. 39) suggest that the chapter is based on the premise

> students' experience the education of mathematics differently, based on their learning opportunities and achievements that depend on the social context of their families and the schools they attend. Often such 'background' factors are associated with disadvantage, marginalisation, disengagement, and exclusion from the study of mathematics. There is also a heavy economic, social and political cost for the students individually, their communities and the broader society.

While the chapter reports a great deal of research in the areas of ethics, gender, diversity, rurality and socioeconomic status, it does note a move towards research using the more encompassing construct of social justice. It also notes the importance of globalisation in current and future research in the area. The chapter concludes with a stark reminder that, in spite of the large body of work in the many areas covered by the chapter, inequality still exists and continuing effort and commitment are required.

The next two chapters of the review build on the work of Atweh and his colleagues by considering two specific groups of students: Indigenous students and exceptional students. While there are some overlaps across these, and other, chapters in terms of the research cited, the editors see this as a strength of the review. Considering different research outputs from multiple perspectives means that their relevance to various aspects of mathematics education research can be tested and critiqued.

Meaney and her colleagues provide a strong critique of the substantial and increasing amount of mathematics education research relating to Indigenous students that has been reported in the 2008–2011 period. While critical of the positioning of Indigenous people in much of this research, the authors have considered the available research from theoretical, methodological and practice-based perspectives. They have celebrated research that has emphasised the strengths of Indigenous people and have been quite critical of the reliance on tests such as the National Assessment Program – Literacy and Numeracy (NAPLAN) in Australia. There is need for much more research around Indigenous students' learning of mathematics but focused on the strengths of these students rather than on their continued poor showing on tests developed from a Western paradigm.

Borrowing from Gervasoni and Lindenskov (2011), Diezmann and her colleagues consider the characteristics of learning environments in which exceptional children—those who are gifted and those who are experiencing learning difficulties—can thrive. The authors consider both groups of exceptional children separately and critique the research pertaining to each. For gifted children, matters such as identification, educational provision, role of adults and cultural perspectives are canvassed. For children experiencing learning difficulties, the challenges of identification and labelling are enormous. Consequences of these challenges and attempted solutions are discussed in the chapter. Approaches to teaching and learning for children experiencing learning difficulties are critiqued with a contrast being drawn between socio-constructivist and direct instruction approaches. Some warnings are given concerning the possibility of over enthusiasm in the interpretation of some evaluation results. The research on ability grouping and its impact on exceptional children is discussed, with one conclusion being the need for further research focused on pedagogy within the groups as well as the structural aspects. The chapter concludes with a plea for further research that considers how exceptional children can thrive in their mathematics education.

The final two chapters in the *Contexts for Mathematics Education* section of the review—*Technology in mathematics education* and *Assessment beyond all: The changing nature of assessment*—also canvass some of the same research, particularly in terms of the impact of ICTs and web-based technologies on assessment. However, their perspectives are quite different.

The technology chapter is organised into four sections: Learning Contexts and Curricular Design; Learners, Learning, and Digital Technology; Teachers, Teaching, and Digital Technology; and Gender, Affect, and Technology. In each of these sections, a comprehensive summary of the extant research is provided. Generally, the research reports the benefits of the use of technology in mathematics education. However, there are a number of warnings about students' and teachers' abilities and knowledge and curriculum traditions perhaps constraining the potential of the technologies to enhance mathematics learning and teaching and indeed change the nature of the mathematics being taught and learnt in Australasian schools. Geiger, Forgasz, Tan, Calder and Hill (2012, p. 133) suggest "that the inclusion of technology in mathematics education may not be fulfilling its promise of revolutionising the way mathematics is taught and learnt". The Gender, Affect and Technology section has some overlap with the Lomas, Grootenboer and Attard chapter, but considers the research through a different lens. The varied results reported in terms of gender and other affective matters point towards the need for much more research in this area. The chapter concludes with a list of such 'future' research, including the need for larger scale studies in areas such as the impact of digital technologies on learning trajectories, social interaction and learning communities, and understanding and attitudes to mathematics learning.

In many ways, the chapter on the assessment of mathematics learning is a direct 'sign of the times' in Australasian, and particularly Australian, mathematics education. There has not been such a chapter in previous reviews but the development of several regimes of both national and international high-stakes testing of students and increased standards-based accountability for teachers has made such a chapter a high priority. The chapter is organised into sections dealing with the national assessment agenda, classroom assessment, curriculum content and related assessment items, and assessment of both the content and pedagogical content knowledge of teachers. Lowrie and his colleagues, whilst recognising the potential of national assessment for mathematics education researchers, counsel against building a reliance on such assessment in spite of its political—and, therefore, potential funding—importance. Research is presented questioning the nature of the assessments being undertaken and the dangers that may be lurking for particular learners. Paradoxically, research in classroom assessment of mathematics learning has developed and evaluated many innovative approaches to assessment for learning that have assisted teachers at all levels from preschool to secondary school. Along with this has come an increased interest in curriculum content and items designed to assess this content. Important work is reported concerning ways in which small changes in the design of test items can change apparent student outcomes. The chapter concludes with suggestions for future research in what will inevitably become a key area of Australasian mathematics education research.

Mathematics Learning and Teaching

Five chapters in the review have been grouped under this heading. The first four deal with 'levels' of education and the learning and teaching of mathematics at each of the levels. The fifth chapter is anomalous in that it is the only content specific chapter in the entire review.

The strong tradition of early childhood mathematics education research in Australasia continues to be represented through the chapter from MacDonald and her colleagues. While noting a slight reduction in the quantum of research output, they highlight much quality research that is destined to have a higher profile than in the past because of national political and educational interest in both Australia and New Zealand in the importance of early mathematics learning to future success. The chapter is divided into three key sections: context; pedagogy; and content. The first section highlights the extensive research effort over the review period in the area of mathematics education for young Indigenous learners. In both Australia and New Zealand, this area has been a key focus politically, educationally and through funding. Results have been mixed in quality and impact, as might be expected in such a burgeoning field. While there are many contextual differences, there is much that New Zealand researchers can learn from their Australian counterparts and vice-versa. The use of technology in early childhood education is a key field of endeavour, as are play, assessment of mathematics learning and the professional development of early childhood educators. Research into mathematical content and young children has been restricted generally to number, algebra and measurement. The chapter concludes with an extensive list of research required over the next four years, including: enhancement of current research with Indigenous children, educators, families and communities through the development of more appropriate methodologies; the impact of new curricula on the mathematics learning of young children; and the impact of assessment regimes, including school entry assessment on young children's mathematics learning.

An early decision by the editors of this review, partly motivated by the current emphasis in both Australia and New Zealand on new, national curricula, was to conceptualise the chapters dealing with the school years and mathematics education not into the traditional 'primary' and 'secondary' groupings but into 'pedagogy' and 'curriculum'.

The chapter *Powerful pedagogical actions in mathematics education* considers Australasian mathematics education research under three main themes: creating powerful learning environments; selecting tasks and models that promote deep learning; and knowing and using pedagogical knowledge. There has been a great deal of research about creating powerful learning environments, particularly in terms of the construction of positive, culturally appropriate relationships between teachers and students and among students. The importance of relevant interactions among all the players and with the curriculum, are highlighted. In these interactions, language clearly plays an important role, as do questioning, generalising and the development of sound argumentation strategies. The selection of rich and authentic tasks for use in children's mathematics learning, along with problem solving and modelling derived from these tasks, are critical pedagogical actions. The research reviewed has highlighted the links between powerful pedagogical actions and teachers' content and pedagogical content knowledge. Not all teachers would appear to be sufficiently knowledgeable in these areas and more work needs to be done. This is particularly the case in the use of digital technology (see Geiger et al., 2012). Gervasoni and her colleagues conclude their chapter by suggesting that there needs to be extensive continuing collaborative research about relationships between culturally responsive pedagogy and powerful mathematics learning.

The chapter *Mathematics curriculum in the schooling years* provides a survey of the Australasian research undertaken in what will become a burgeoning area for mathematics education research over the next few years. The opportunities provided by the development and introduction of new curricula in both New Zealand and Australia should stimulate much innovative work. The current chapter begins by considering both the mathematics and numeracy lenses on school learning. Key research and reports are canvassed as background to the development of mathematics curricula in both Australia and New Zealand which is described in some detail. Critique of the curriculum content in Australian schools and of the level of expectation raised by mathematics textbooks commonly used in schools suggests that there are ongoing challenges around what has become known as the 'shallow teaching syndrome'. Summarising the research on the implementation of mathematics curricula in Australasia leads Anderson, White and Wong (2012, p. 240) to conclude that "curriculum reform through the written or intended curriculum does not necessarily lead to reform in the enacted curriculum via new teaching practices". The need for ongoing research is emphasised.

Tertiary mathematics education research has featured in a number of the previous reviews. In the chapter *Growth and new directions? Research in tertiary mathematical science education*, Barton and his colleagues explore a number of themes that have been canvassed in previous reviews and others that are new. One key theme is that of transition between school and university and the implications for mathematics education of that transition. While the contexts are quite different, there are many similarities between this section and the corresponding section in MacDonald, Davies, Dockett, and Perry (2012), emphasising the generalisations that can be made across many educational transitions. The teaching of specific mathematics topics, particularly linear algebra, mathematical modelling and calculus, at the tertiary level has received some consideration over the review period, as have the more generic issues of undergraduate mathematics and quantitative skills in science and engineering degrees. There is also substantial quality research in tertiary statistics education.

The numbers of tertiary students undertaking courses in mathematics is of universal concern and New Zealand and Australia were part of the International Mathematical Union (IMU) and International Commission for Mathematical Instruction (ICMI) *Pipeline Study* investigating this matter. Other research related to this study is also reviewed in the chapter. In terms of general pedagogical research at the tertiary level, it is perhaps surprising to see an amount of work being undertaken into lecturing and little in the area of technology use in tertiary mathematics education.

The area of tertiary mathematics education research is complex in terms of its diversity, who does it—mainly mathematicians, and what impact it has on practice. MERGA reviews have considered it over a number of years and this chapter continues to think critically about research into an area of practice that is critical in the development of future mathematicians and mathematics teachers.

The strength of the research in statistics education in Australasia has resulted in a separate chapter reviewing this area in the 2008–2011 review. This chapter, entitled *Uncertainty in mathematics education: What to do with statistics?* reflects both the curriculum and research pressures that have been exerted in Australasia and beyond as statistics education strives for its place in mathematics education. By considering the synergies and tensions represented by statistics education, Callingham, Watson and Burgess critique the Australasian statistics education research published in the review period resulting in commentary on differing aspects of mathematics and statistics and their teaching. The authors make a distinction between statistics and mathematics education on the basis of the importance of context and use this to explore pedagogical content knowledges in statistics education. The chapter concludes with implications from the synergies and tensions for research, policy and practice.

The *Teachers* section of the current review comprises two chapters, one dealing with pre-service teacher education and the other with the professional learning of practising teachers. Both of these chapters have their counterparts in many of the previous reviews, reflecting the ongoing interests and expertise of Australasian mathematics education and teacher education researchers.

The political, social and economic contexts in which pre-service teacher education is undertaken is highlighted by Anthony, Beswick and Ell in the chapter *The professional education and development of prospective teachers of mathematics.* They critique Australasian research on recruitment of teachers, knowledges of teaching, transition to teaching, teacher education practices and researching of mathematics teacher education by mathematics teacher educators. While they acknowledge and celebrate the high quality of the research under review, the authors agree with Ball and Even (2009) that there was still much to be done, particularly in the areas of researching mathematics teacher education practice, knowledges and supports required for quality mathematics teacher education and the assessment of teachers' learning.

The political, social and economic imperatives raised by Anthony and her colleagues are continued by Bobis, Higgins, Cavanagh, and Roche in their chapter which focuses on research into knowledge in mathematics education and development in practising teachers. This sharp focus not only allows the critique of a substantial number of Australasian research publications but it also celebrates the increased attention being given to teachers' knowledge and its importance in the educational endeavour. The chapter considers four substantive areas: nature of teacher knowledge; frameworks for researching teacher knowledge; domains of teacher knowledge; and acquisition of this knowledge. Future research needs are outlined including cultural aspects of teacher knowledge, the extension and development of frameworks for researching teacher knowledge and research into the links between teacher knowledge of mathematics and student learning outcomes.

Research in Mathematics Education in Australasia 2008–2011 continues the trend begun in the 2000–2003 and continued in the 2004–2007 review by asking one of Australasia's eminent mathematics education researchers to write a future-looking chapter which reflects on the overall review and move the field forward into the next quadrennium. In this review, the editors are honoured that the 2009 recipient of the Felix Klein medal, Professor Gilah Leder, has written the 'into the future' chapter.

For each of the chapters in the review, Leder summarises, critiques and offers her view on what might be 'the next steps' in each of the areas. In her conclusion, she consolidates these thoughts and then considers a number of areas of endeavour which, she believes, are not so well represented in Australasian mathematics education research.

CONCLUDING COMMENTS

For the editors of this review, it has been a pleasure to interact so closely with the output of their mathematics education research colleagues. While there are many benefits to the authors in writing chapters for the review, the major benefit of their substantial and careful work lies in the increased access researchers, students, policy makers and practitioners have to the research that has been critiqued. Past reviews have been praised for their impact on doctoral studies, ongoing research programs, policy and practice both within Australasia and beyond. Providing the chance to celebrate Australasian research and make it more easily available is something about which MERGA should be very proud.

Readers of this review are assured of quality critiques of important research in mathematics education. While the research does have an Australasian flavour through its authors and/or its settings, its impact goes far beyond this geographical region. Of course, care needs to be taken in adapting research methodologies, methods, analysis and results to fit different contexts and different times. Nonetheless, the review will provide a solid basis in many of the most important and popular fields of current mathematics education research. We commend the review as both an excellent starting point for thinking about readers' own research projects and a celebration of a fine tradition of Australasian mathematics education research.

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The editors wish to acknowledge the financial and in-kind support provided by both the Mathematics Education Research Group of Australasia (MERGA) and the Research Institute for Professional Practice, Learning and Education (RIPPLE) at Charles Sturt University. In particular, Colleen Vale, MERGA Vice-President (Publications) for most of the production period for the review, has been generous and firm in her ongoing support to the editors. Michel Lokhorst from Sense Publishers has been at the ready whenever questions needed to be answered and has continued his enthusiastic support for the review throughout the production period. The 50 chapter authors and 26 reviewers deserve particular credit for their work, their willingness to accept the time pressures involved and their good graces in reacting positively to critiques of the chapter drafts from the editors. Finally, the editors wish to acknowledge the work of Dr Rosemary Farrell who took on the task of ensuring the entire publication was appropriately and consistently formatted, proofread and generally made Sense.

As editors, we are proud of what we have achieved in delivering this latest MERGA review. We have worked hard to get to this stage and are, of course, responsible for any deficiencies, errors or gaps that remain. We could not have reached this point without a great team effort from everyone involved. We hope you enjoy the result.

REFERENCES

Anderson, J., White, P., & Wong, M. (2012). Mathematics curriculum in the schooling years. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 219–244). Rotterdam, The Netherlands: Sense.

Atweh, B., Vale, C., & Walshaw, M. (2012). Equity, diversity, social justice and ethics in mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 39–65). Rotterdam, The Netherlands: Sense.

Ball, D. L., & Even, R. (2009). Strengthening practice in and research on the professional education and development of teachers of mathematics: Next steps. In R. Even & D. L. Ball (Eds.), *The professional*

education and development of teachers of mathematics: The 15th ICMI Study (pp. 255–259). New York, NY: Springer.
Briggs, J. (Ed.). (1984). *Summary of research in mathematics education in Australia.* Brisbane, QLD: MERGA.
Forgasz, H., Barkatsas, T., Bishop, A., Clarke, B., Keast, S. Seah W. T., & Sullivan, P. (2008). *Research in mathematics education in Australasia 2004–2007.* Rotterdam, The Netherlands: Sense.
Geiger, V., Forgasz, H., Tan, H., Calder, N., & Hill, J. (2012). Technology in mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 111–141). Rotterdam, The Netherlands: Sense.
Gervasoni, A., & Lindenskov, L. (2011). Students with "special rights" for mathematics education. In B. Atweh, M. Graven, V. Walter, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 307–323). Dordrecht, The Netherlands: Springer.
MacDonald, A., Davies, N., Dockett, S., & Perry, B. (2012). Early childhood mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 169–192). Rotterdam, The Netherlands: Sense.

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CONTEXTS FOR MATHEMATICS EDUCATION

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REFLECTIONS ON THE MERGA RESEARCH REVIEW 2004–2007

INTRODUCTION

The tradition of the MERGA four-year review provides evidence of sustained and evolving research within the Australasian mathematics education community. As part of the editing team for the previous review (Forgasz et al., 2008) we appreciate the opportunity to reflect on that review and to provide some personal insights into recent developments.

The chapter headings from the 2004–2007 review were:

– The development of young children's mathematical understanding
– Learning mathematics in the middle-years
– University learners of mathematics
– Adults returning to study mathematics
– Mathematics education and Indigenous students
– Research into the teaching and learning of applications and modelling in Australasia
– Teaching and learning with technology: Realising the potential
– Characteristics of effective pedagogy for mathematics education
– Sociocultural perspectives in mathematics teaching and learning
– The affective domain and mathematics education
– Gender and mathematics: Theoretical frameworks and findings
– Research on the pre-service education of teachers of mathematics
– Teachers as learners: Building knowledge in and through the practice of teaching mathematics

While many of these are similar to those from previous reviews, the inclusion of a specific chapter on teachers as learners reflects the increased interest in researching teacher professional learning. There was also a reduced focus on content specific chapters.

In his concluding chapter to the 2004–2007 review, Ken Clements (2008) noted various reasons why not all areas of Australasian mathematics education research are covered in any review. Clements identified three particular areas that were not covered in the previous review:

– Australian and New Zealand performances on recent international comparative performance studies;

B. Perry et al., (Eds.), Research in Mathematics Education in Australasia 2008–2011, 15–22.

- whether mathematics education research has generated important and measurable improvements in mathematics curricula and mathematics teaching and learning; and
- the role of theory in mathematics education research.

A comment on each of these areas seems appropriate.

On a regular basis, Sue Thomson and her colleagues at Australian Council for Educational Research provide detailed reports of Australia's performance in PISA and TIMSS (e.g., Thomson, De Bortoli, Nicholas, Hillman, & Buckley, 2011; Thomson, Wernert, Underwood, & Nicholas, 2008), including on a range of affective and other measures. Since 2008, the Australian Curriculum and Reporting Authority have provided detailed reports on NAPLAN performance across Australia. Mathematics education researchers, who are particularly interested in the relative performance of identifiable sub-groups in the population or researching affective factors, increasingly use the data to support the rationales for their research domains and/or specific research studies. When relative performance is of interest, some of the following variables alone or in combination are examined: gender, socioeconomic status, Indigeneity, geographical location, and ethnicity (as measured by language background). These types of data are consulted by those focusing on state and territory differences as well as those interested in relative performance of students in specific content domains of mathematics. While much of this information is canvassed in various chapters in the current review, future editors might consider a chapter dedicated to an overview of government and other significant reports on performance, as well as other critical issues related to mathematics education or, alternatively, a return to a chapter on the politics of mathematics education.

A less straight forward dimension of our work to assess is Clement's second area, the impact of mathematics education research on curricula and mathematics teaching and learning. Specific funding for evaluations is scarce, particularly in the longer term. Often evaluations are included in government funded projects such as interventions, but studies of whether any benefits obtained in the short term are sustained over time are less often supported financially. Currently, in Australia, the program emphases and opportunities are to examine and explore the impact of the curriculum on children's learning, on teachers' practice, and on pre-service education and professional development. It is to be hoped that the preliminary work that has been undertaken in the review period will be extended into the future (see chapters by Anderson, White, and Wong; Anthony, Beswick, and Ell; Bobis, Higgins, Cavanagh and Roche in this review).

As Clements also highlighted, there was a lack of theoretical discussion in many of the chapters included in the 2004–2007 MERGA review and authors also identified a lack of theoretical development in various fields. This again is partially explainable through a dearth of research funding for theoretical and/or philosophical pursuits in our field. Research based in theoretical frameworks that are old is not necessarily poor research; nonetheless such work can, and should be building on the theories that underpin them. Often, however, researchers do not capitalise on the new dimensions or variables that their research has uncovered.

Challenging researchers to do this and to write in ways that herald that their research has resulted in a new or revised theoretical model or framework will strengthen the field.

IMPACT OF THE RESEARCH POLICY ENVIRONMENT

In the introduction to the 2004–2007 review, the editors identified the changing political context in which mathematics education researchers found themselves (Forgasz et al., 2008). They predicted a further tightening of accountability measures for research funding and research activity developments in Australia and New Zealand. They identified four dichotomies and their complementarities which they characterised as:

– A decrease in creative and idiosyncratic research versus an increase in programmatic research
– A decrease in individual research versus an increase in group or team research.
– A decrease in funding for basic research versus an increase in funding for practice-oriented projects
– A decreasing concern with the quantity of research versus an increasing concern with the quality of research

Particularly following the voluminous and stressful work that universities undertook in the Excellence in Research in Australia [ERA] exercise, and the on-going demands of the Performance-Based Research Fund in New Zealand, the previous editors' predictions appear to have come into being. The dichotomies and complementarities listed above remain and, if anything, are more clearly apparent during the 2008–2011 period. Readers of this review may want to consider the impacts of recent and current research policy frameworks on Australasian mathematics education research.

UNDER-REPRESENTED AREAS AND RECENT DEVELOPMENTS

Authors and editors of the 2004–2007 review identified areas that remained under-researched and under-theorised, following the trend that the authors of previous reviews had established. Wood (2008) described mathematics learning and teaching at university as being in a state of flux. No research had been reported on university students' attitudes toward the difficulty, the cognitive competence and the perceived value of university statistics courses. To some extent, this has been addressed by Barkatsas (in press) who reports that female Australian university students demonstrated increased confidence in their competence as learners of tertiary statistics. Other research on tertiary statistics education has been reported in this review in the Barton, Goos, Wood, and Miskovich chapter.

In the previous review period there was little published research in the Australasian mathematics education literature focusing on children with special needs. The work on early years intervention, particularly within the context of systemic-based projects, has continued but there appears to be little work within mathematics education research about students with physical disabilities or students with learning disabilities. It is estimated that approximately 3% to 8% of the school-age students have mathematical disabilities. In their meta-analytic study, Swanson and Jerman (2006) reported that "mathematical disabilities (MD) are as common as reading disabilities (RD) and that a similar deficit may contribute to the co-occurrence of MD and RD in some children" (p. 249). It has been many years since children with various physical and learning disabilities have been integrated into mainstream schooling, yet knowledge on effective teaching approaches to adopt with them is sparse. Working with our colleagues with expertise in special education should be encouraged.

The impact of gifted and talented students' education on attitudes toward the gifted and the self-perceptions of gifted are other areas that continue to be under-researched.

In the current review, research on the mathematics learning of both children with learning difficulties and gifted children is considered by Diezmann, Stevenson, and Fox in their chapter 'Supporting exceptional students and the learning of mathematics'. The fore-grounding of this research is encouraging.

A final area in which there has been limited writing in recent times is in the philosophical realm of mathematics education. Is the mathematics we teach defensible in the political, economic, and technological world of students? Are the contexts we use for problem-solving and investigations ethically and morally sound? Are the teaching and learning approaches equitable and culturally sensitive? These philosophical elements are also deserving of Australasian mathematics education researchers' time and attention.

BUILDING ON AND DEVELOPING PREVIOUS WORK

Reflecting the trends noted in the previous review, Australasian research, adopting the sociocultural perspective to mathematics learning and teaching, has continued to utilise approaches that were not hitherto employed much, if at all, in the region. It is an area of relative strength that continues to develop. Thus, for example, Prescott and Cavanagh (2008) reported on their employment of situated learning perspectives (Lave & Wenger, 1991), while McMurchy-Pilkington, Bartholomew and Greenwood (2009) made use of the notion of space of learning (Johnston, 2002) in their study with Māori students and their parents. Brown and Redmond (2008), on the other hand, interpreted the professional activities of a few teachers using the construct of teacher agency (Pickering, 1995). The approaches used in these studies reflect a maturing sociocultural research agenda in Australasia as researchers built on and expanded previous frameworks of investigations.

The current review period also saw the continuation of the research activities of Galligan (2008) and Goos (2008) in adopting Valsiner's theory of human development, applying it to deepen our understanding of the mathematics skill development of adult learners, and of the professional development of teacher educators respectively.

The apparent focus on the adoption of novel research approaches and perspectives might have reinforced Seah, Atweh, Clarkson and Ellerton's (2008) observation in the previous review that

> the literature abounds with interesting 'mapping' exercises, but studies which peel back the layers of respectability and accepted theories, to lay bare realities and fundamental ways of finding research-based strategies which make a difference in the mathematics classroom, are few and far between. (p. 242)

However, Brown and Redmond's (2008) study investigated teachers' agency during professional conversations and provided an example of a research overlap between mapping the field and finding research-based strategies. It represents a study in which socioculturally-based pedagogical strategies were identified through a design experiment (Schoenfeld, 2006) that began with no pre-conceived notion of how teachers negotiated their dance of agency (Pickering, 1995) between disciplinary agency and human agency.

On the other hand, research into the values aspects of mathematics pedagogy has developed in ways which broaden the mapping-the-field approach. The Australian-coordinated Third Wave Project, initiated in 2009 to document what students from 11 different countries and regions valued in effective mathematics lessons, extended our understanding of students beyond Australian classrooms (e.g., Kor, Lim & Tan, 2010; Law, Wong & Lee, 2011; Seah, 2011). Data collected from students across the countries/regions enable cross-cultural comparisons to be made, which potentially deepens our understanding of what students value in multicultural Australia.

INFORMING POLICY AND PRACTICE

In addition to the increased accountability within the tertiary sector in relation to research, schools and systems have also seen the impact of accountability policies. A significant series of events since the previous MERGA review included the consultation around the Shape Paper that established the principles for the Australian Curriculum: Mathematics, followed by the consultations associated with the development of the content descriptions. It is interesting to consider the extent to which the reviews of MERGA research influenced both the consultations and the outcomes. Reviews are an important opportunity to synthesise evidence that can inform policy and practice as well as provide synthesis for those within the field. This notion is canvassed in the current review chapter by Anderson, White and Wong.

While mathematics education researchers were well represented at each of the consultations associated with the curriculum development through appointments to advisory committees, the types of debates and discussions that occur at MERGA meetings, and in journals, were not prominent. It is relevant at this time to reflect on why this appears to be the case.

It is possible that the nature and content of the consultations is an artefact of the highly consultative process used to develop the curriculum. Substantial input was sought from teachers and others around successive drafts, piloting in schools across the nation, mapping of the drafts against the various state curricula, and many other steps beside. The advantage of this process is that a curriculum could be developed which was as familiar as possible to many teachers. The disadvantage is that the writing was informed by many contributions. In other words, there is a tension between seeking consensus and maximising coherence. In this process, the 'voice' of the mathematics education researchers was hard to hear. Perhaps this suggests that if mathematics education research has messages, then these messages need to be disseminated to the broader teaching community, as well as using the four-yearly reviews to endeavour to influence policy makers.

Perhaps there could be more emphasis by authors of MERGA research reviews on contributing to policy development. Indeed, it might be appropriate for some debate about the extent to which our research does seek to influence policy and practice.

Hattie's (2009) 800 meta-analyses highlight one further issue that arises from the consideration of the impact of research on policy. The respective meta-analyses had synthesised results where changes in student learning were measured. These measurements are made by comparing experimental and control groups, or by pre- and post- treatment comparisons. But in all cases the studies that contributed to Hattie's influential findings on schooling, learning, pedagogy, and even structures were informed by results that included measures of student achievement. On balance there seem to be too few of such studies reported and reviewed in the MERGA reviews. Are we likely to see changes as a result of stronger accountability frameworks including NAPLAN? Will they be changes for the better?

As previous editors, we look forward to studying the content of this review to see how our field has developed, to find out more about new and emerging areas, as well as how we have built on our strengths. While we have expressed concerns about the limitations and focused on possible gaps, it is important to acknowledge the strong contribution of these reviews both to the historic record but also to building our research and our discipline.

REFERENCES

Barkatsas, A. (in press). Students' attitudes, engagement and confidence in mathematics and statistics learning: ICT, gender, and equity dimensions. In H. Forgasz & F. D. Rivera (Eds.), *Advances in mathematics education: Equity and diversity in mathematics education.* Dordrecht, The Netherlands: Springer.

Brown, R., & Redmond, T. (2008). Reconceptualising agency through teachers talking about a sociocultural approach to teaching mathematics in the classroom. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 101–108). Brisbane, QLD: MERGA.

Clements, M. A. (2008). Australasian mathematics research 2004–2007: An overview. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, B. Keast, W. Seah, & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004–2007* (pp. 337–356). Rotterdam, The Netherlands: Sense.

Forgasz, H., Barkatsas, A., Bishop, A., Clarke, B., Keast, B., Seah, W., & Sullivan, P. (Eds.). (2008). *Research in mathematics education in Australasia 2004–2007*. Rotterdam, The Netherlands: Sense.

Galligan, L. (2008). Using Valsiner. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 211–217). Brisbane, QLD: MERGA.

Goos, M. (2008). Towards a sociocultural framework for understanding the work of mathematics teacher-educator-researchers. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 235–241). Brisbane, QLD: MERGA.

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta analyses relating to achievement*. New York, NY: Routledge.

Johnston, P. M. G. (2002). Disrupting/contesting/creating space: Challenging students to think Māori education. *Delta, 55*(1/2), 43–60.

Kor, L. K., Lim, C. S., & Tan, S. F. (2010). The third wave: What Malaysian Chinese primary school pupils value in an effective mathematics lesson: A pilot study. In Y. Shimizu, Y. Sekiguchi & K. Hino (Eds.), *Proceedings of the 5th East Asia Regional Conference on Mathematics Education* (pp. 95–102). Tokyo, Japan: SEAMS.

Lave, J., & Wenger, E. (1991*). Situated learning: Legitimate peripheral participation*. Cambridge, United Kingdom: Cambridge University Press.

Law, H. Y., Wong, N. Y., & Lee, L. N. Y. (2011). The third wave studies of values in effective mathematics education: Developing students' mathematical autonomy in classroom learning. *Mathematics Educator, 13*(1), 70–84.

McMurchy-Pilkington, C., Bartholomew, H., & Greenwood, S. (2009). Achieving in mathematics contested spaces and voices. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 355–362). Palmerston North, NZ: MERGA.

Pickering, A. (1995). *The mangle of practice: Time, agency, and science*. Chicago, IL: University Press.

Prescott, A., & Cavanagh, M. (2008). A situated perspective on learning to teach secondary mathematics. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 407–412). Brisbane, QLD: MERGA.

Schoenfeld, A. H. (2006). Design experiments. In P. B. Elmore, G. Camilli, & J. Green (Eds.), *Handbook of complementary methods in education research* (pp. 193–206). Washington, DC & Mahwah, NJ: AERA and Lawrence Erlbaum Associates.

Seah, W. T. (2011). Effective mathematics learning in two Australian primary classes: Exploring the underlying values. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 129–136). Ankara, Turkey: PME.

Seah, W. T., Atweh, B., Clarkson, P., & Ellerton, N. (2008). Sociocultural perspectives in mathematics teaching and learning. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W. T. Seah & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004–2007* (pp. 223–253). Rotterdam, The Netherlands: Sense.

Swanson, H. L., & Jerman, O. (2006). Maths disabilities: A selective meta-analysis of the literature. *Review of Educational Research, 76*(2), 249–274.

Thomson, S., De Bortoli, L., Nicholas, M., Hillman, K., & Buckley, S. (2011). *Challenges for Australian education: Results from PISA 2009*. Melbourne, Australia: ACER.

Thomson, S., Wernert, N., Underwood, C., & Nicholas, M. (2008). TIMSS 07: *Taking a closer look at mathematics and science in Australia*. Melbourne, Australia: ACER.

Wood, L. (2008). University learners of mathematics. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W. T. Seah & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004–2007*, (pp. 73–97). Rotterdam, The Netherlands: Sense.

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THE AFFECTIVE DOMAIN AND MATHEMATICS EDUCATION

Key words: identity; self-efficacy; beliefs; attitudes; anxiety; motivation; methodological approaches.

INTRODUCTION

This is the third chapter on affective issues to appear in MERGA reviews of research in mathematics education and as such reflects the ongoing importance of affective issues to the mathematics education research community. The first two chapters (Grootenboer, Lomas, & Ingram, 2008; Schuck & Grootenboer, 2004) noted a continuing move away from studies on attitudes to projects on beliefs and the consideration of a broader range of affective aspects. In the current review period, 2008–2011, there is a lessening focus on beliefs, a growing focus on identity, and an even spread of studies on other affective aspects.

While there has been some work internationally on evolving understanding and description of affective concepts (e.g., Zan et al., 2006) this does not seem to be reflected in Australasian studies. Given the continuing relative paucity of work in Australasia on clarifying concepts, the use of Australasian or international frameworks, or the development of new theoretical frameworks, there has been little evolution of definitions in the four key aspects of affect considered in the first review chapter (Schuck & Grootenboer, 2004). As a consequence these continue to be used here and are restated below:

> Beliefs are positions held by individuals that they feel to be true and their nature cannot be directly observed but must be inferred from actions. Although there are a variety of definitions of attitudes the common elements to these definitions are that attitudes are learnt and are evident in responses to a situation or object, and are seen as positive or negative. Emotions or feelings are described in terms of their transitory and unstable nature arising as an affective response to particular events/contexts whereas values are seen as criteria by which choices or assessments, in terms of desired/desirable outcomes or behaviours, are made. (Schuck & Grootenboer, 2004, p. 257)

There is work starting to theorise on the concept of identity including affective aspects and the impact of philosophical paradigms on the formation of identity. In contrast, there is little evidence of a more holistic view of the affect domain as a continuum, along with the cognitive, as proposed by Leder and Grootenboer

B. Perry et al., (Eds.), Research in Mathematics Education in Australasia 2008–2011, 23–37.

(2005), being used as a theoretical framework. This view was presented in the second review chapter (Grootenboer et al., 2008) as a prompt to the mathematics education research community to give greater consideration to theoretical frameworks and the place of individual studies within such.

The following sections consider Australasian mathematics education research on the affective domain. It discusses identity (or self-concept), self-efficacy, beliefs, attitudes, anxiety, motivation, and methodological approaches.

TOPICS/FINDINGS

In the broad area of affect in mathematics education, research has focused on a number of aspects, with about a third of the studies centred on identity and about a fifth on self-efficacy. Other affective aspects with around five studies each are beliefs, attitudes, anxiety and motivation.

Identity

The concept of identity has continued to be explored by Australasian researchers in mathematics education. Indeed, two of the keynote speakers at the annual MERGA conferences (Kemmis, 2008) have had a theme of ‘identity’ imbuing their address and associated papers. Apart from Ingram (2008a) who focused on secondary students, this research has primarily been undertaken with pre-service teachers (Tobias, Serow, & Schmude, 2010; Walshaw, 2009) and in-service teachers (Grootenboer & Ballantyne, 2010; Walshaw, 2010). Tobias et al. (2010) focused on ‘critical moments’ in pre-service teachers’ mathematical histories that influenced their ‘self-concept’. Grootenboer and Ballantyne (2010) examined mathematics teachers’ negotiation of their pedagogical and mathematics-based identities. The study of Ingram (2008a) was interesting in that it examined methodological issues in researching affective issues in learning mathematics through the lens of identity.

Unlike some aspects of the affective domain, research around the notion of identity has received particular theoretical attention. This was first noted in the previous review (Grootenboer et al., 2008), and has been continued with a particular emphasis on post-structural and psychoanalytical theory. Significantly, Walshaw continued her work examining mathematics education and identity primarily through psychoanalytical theory. In particular, she examined the negotiation of identity by pre-service teachers during their practicum (Walshaw, 2009), mathematics education researcher identity and mathematics teacher identity, reflective practice, and teacher development (Walshaw, 2010). Together, these articles and conference papers, along with her work prior to 2008, provide a rigorous theoretical foundation for continued research into the affective domain in mathematics education, and in particular the concept of identity. Despite advances in theoretical consideration of identity as a concept, it is still ill-defined. This requires more attention to the meaning taken in individual studies, and given the lack of statements on any theoretical position in many papers, this is problematic.

Self-efficacy

There were a small number of research reports that focused on the concept of mathematical self-efficacy. The qualitative study of teacher-researchers by Redmond and Sheehy (2009) focused on senior school mathematics teachers' sense of agency as they used the 'collective argumentation' pedagogical approach. Carmichael and Hay (2008) reported on the development of an instrument to measure middle-school students' self-efficacy vis-à-vis statistical literacy, and McConney and Perry (2010) examined the relationships between mathematics achievement, school socioeconomic status, and student self-efficacy with 15-year-old students through a secondary analysis of 2003 *Trends in International Mathematics and Science Study* (TIMMS) data. While these studies are interesting in and of themselves, there does not appear to be any themes of particular note across all the studies.

Beliefs

A continuing trend as seen in the last two MERGA reviews is a focus on beliefs about mathematics teaching and learning (Grootenboer et al., 2008; Schuck & Grootenboer, 2004). The studies conducted in the period 2008–2011 have predominantly focused on pre-service and practising teacher beliefs, with few studies focusing on student beliefs as in previous reviews. Researchers who have continued to address the area of beliefs are, for example, Beswick (2008, 2009, 2011a, 2011b), Beswick and Dole (2008) and Grootenboer (2008).

The focus of several studies was on the beliefs of pre-service teachers. Lo and Anderson (2010) conducted a study based on beliefs about mathematics teaching and learning in Hong Kong. Participants ranged from first to fourth year primary teacher education students intending to become specialist mathematics teachers. Lo and Anderson found beliefs were supportive of contemporary reform focused approaches and became stronger as students progressed through their course. In another study on pre-service teacher beliefs, Grootenboer (2008) focused on belief change during the course of study and found the responses fell into three categories: (a) non-engagement, where the focus was on passing the course; (b) those who formed a new set of contextualised beliefs based on their tertiary experience; and (c) those who engaged in belief change but found this challenging in terms of classroom practice.

A study by Bennison and Goos (2010) built on their previous work (Goos, 2009; Goos & Bennison, 2008) using their zone theoretical framework. It explored changes in Queensland secondary mathematics teachers' pedagogical beliefs towards the use of technology in the mathematics classroom as a result of participation in professional development. Bennison and Goos (2010) reported that teachers who participated in technology related professional development were more likely to influence technology integration in a positive manner.

In a longitudinal study that followed people from their time as pre-service teachers through to five years later as practising teachers, Beswick and Dole (2008) explored initial changes in beliefs and whether those new beliefs were maintained. During the initial phase of data collection the pre-service teachers described positive changes in beliefs that were attributed to the pre-service mathematics course and were in line with the course aims. After five years of teaching, the authors reported that teachers' beliefs remained positive but were accommodated to the extent that the teachers perceived their beliefs as always having been positive.

Beswick (2008) evaluated a brief professional learning program aimed at improving the teaching repertoires of primary school teachers of mathematics. The evaluation found that teachers held differing beliefs about appropriate goals and methods of mathematics teaching for students with mathematics learning difficulties. These beliefs were susceptible to change when facilitated through professional learning with the teachers most likely to change their beliefs being those who volunteered to participate in professional learning.

The consistencies and inconsistencies between teachers' professed and attributed beliefs remains a topic of study with, for example, Jorgensen, Grootenboer and Niesche (2009) examining this issue in a remote Indigenous education setting using a survey and lesson observations to collect data. Evidence from classroom observations highlighted discrepancies between teacher beliefs and actual practices, a finding comparable to those from other studies (e.g., Grootenboer, 2008; Sherley, Clark, & Higgins, 2008) where classroom observations were one method of data collection. Areas of mismatch found were inclusiveness, "group work, connectedness or applied contexts, and multiple pathways" (p. 284). For example, in the survey the participating teachers professed a commitment to group work as an effective and useful pedagogical approach in mathematics education, but during the lesson observations there was little evidence of students working in groups. The authors argued that the tension inherent in any mismatch should be seen as providing opportunity for professional growth, and their espoused beliefs should be seen as aspirational.

Other studies on beliefs focused on specific aspects of teaching mathematics. In an investigation of teachers' beliefs about mathematics as a discipline and the work of mathematicians, Beswick (2009) explored data from one teacher who was found to have positive beliefs about the nature of mathematics as a discipline, while not fully understanding the nature of mathematicians' work. Beswick claimed that any attempts to influence teachers' practices should address both beliefs about school mathematics and the discipline itself. Additional studies focused on teachers' beliefs in regard to textbook use (Jamieson-Proctor & Byrne, 2008) and teachers' beliefs in regard to five-year-old children beginning school and mathematics (Sherley, Clark, & Higgins, 2008).

However, there is an international consensus developing (e.g., Leatham, 2006; Liljedahl, 2008) that many identified discrepancies between espoused beliefs and practices arise from methodological limitations, and the ways in which data is interpreted often without adequate theoretical underpinnings. Beswick (2011a, 2011b) discussed teachers' various belief systems about aspects of mathematics and its teaching, and suggested these act as a matrix from which practice arises.

This more holistic view may help explain apparent discrepancies. Further work by Beswick, Callingham and Watson (2011) extended this approach in their measurement of teachers' knowledge to include teacher beliefs about mathematics teaching and learning and their confidence to use and teach certain topics. Their results showed that this gave rise to a single construct measured by the instrument, implying that the affective aspects were integral to the teachers' knowledge.

Attitudes

As in the previous review (Grootenboer et al., 2008), there have been few studies focused on attitudes towards mathematics. Young-Loveridge (2010) investigated how pre-service teachers' attitudes towards teaching mathematics may be different to attitudes towards mathematics. Data was collected from a range of pre-service teachers via a mathematics test and questionnaire. However, findings in this study were limited, as students who chose not to participate also displayed a lack of confidence in, and negative attitudes towards, mathematics. Meaney and Lange (2010) also included data from a mathematics test taken by pre-service teachers and a questionnaire that explored their affective responses to being tested on primary school mathematical knowledge. The results suggested that the way mathematical knowledge was tested may have been detrimental in terms of its influence on future teaching practices, by consolidating procedural rather than encouraging conceptual understanding. In both studies the authors indicated some concern over the level of mathematical knowledge of pre-service teachers.

A variety of research into student attitudes was reported, many of which incorporated teaching interventions focusing on specific areas of mathematics. Norton and Windsor (2008) reported on a case study in which primary students in Brisbane developed more positive attitudes as a result of using concrete materials in an algebra intervention. However, several students stated that once the concept was understood, using materials actually slowed down their calculations. Similarly, in Jennison and Beswick's (2010) study, Year 8 Tasmanian students indicated that the use of concrete materials and practical activities in a fraction intervention improved understanding and hence promoted positive attitudes towards mathematics. The study also highlighted interrelationships between attitudes, anxiety and understanding of mathematics. Afamasaga-Fuata'i (2009) reported that for some students intervention during the later stages of schooling (Year 10) may be too late to reverse entrenched negative attitudes.

Within a larger longitudinal case study on engagement, students' attitudes towards mathematics following transition to secondary school were explored by Attard (2010). This followed an earlier report of the same group of students' attitudes during their final year of primary school (Year 6). Attard found that although the Year 6 participants were aware of certain negative attitudes from peers and some parents, this was not an influence on their own positive attitudes. During the first year of secondary school (Year 7) the students' attitudes appeared to become more negative towards mathematics as a result of difficulties forming positive pedagogical relationships with their secondary mathematics teachers.

The impact of pedagogical relationships on attitudes towards mathematics has been explored in several studies (Attard, 2011; Averill, 2011; Sullivan, Clarke, & O'Shea, 2010). A commonality between each of the studies was the finding that student learning was enhanced by the positive attitudes of teachers who were able to plan and modify tasks to suit specific student needs. In addition, Averill's focus on teacher care highlighted the impact of cultural awareness within a multicultural school context.

The influence of others was also a theme in Leder and Forgasz's (2010) research into the public's perceptions of gender issues and school mathematics. The authors suggested an ongoing need to explore the views of all 'critical' others in the lives of students who may have some influence on their educational and career directions.

Motivation

During the period 2008–2011 some studies have emerged focusing on the construct of motivation. Carmichael (2010) wrote about the concept of 'interest' and its motivational influence on learning mathematics within the context of developing statistical literacy. In a mixed method study involving 425 secondary students, Carmichael (2010) found that students preferred to learn about statistical literacy within contexts outside the mathematics classroom. It was suggested that this finding was related to negative attitudes to mathematics rather than an interest in statistical literacy. Carmichael and Hay (2008) argued that interest development will be the result of a complex interplay of classroom influences and individual factors, such as "students' knowledge of statistics, their enjoyment of statistics and their perceptions of competency in relation to the learning of statistics" (p. 109).

In a study of practising teachers, Anderson (2008) investigated reasons why 109 teachers attended professional development events, and the type of knowledge they valued. Anderson found that motivations for participation ranged from personal growth and recognition to a desire to learn new ideas for implementation of the mathematics curriculum.

Other Affective Aspects

Other studies on the affective domain have included focuses on mathematics anxiety, optimism, and the effects of seating arrangement on students' affect. Two studies using bibliotherapy with pre-service and practising teachers were reported (Wilson, 2009; Wilson & Thornton, 2008) that build on the work of Wilson (2007). Wilson and Thornton (2008) found the use of guided reflections about school students' learning difficulties were powerful in assisting the participants to overcome their own anxieties about mathematics. Wilson (2009) had the same results when five practicing primary teachers participated in professional learning. Wilson and Thornton's (2008) work has implications for pre-service teachers in particular, with the potential to change attitudes, thus helping to prevent them lapsing into teaching styles based on their school experience, however, negative that may have been.

A number of studies on affect focused on low-attaining students. Williams (2008, 2009, 2010) built on her earlier work on optimism (or flow) and used her *Engaged to Learn* pedagogy in whole-class, small group and individual student projects. Williams found that as a result of creative mathematical activity, students' experiences of flow situations were of benefit in terms of confidence building, therefore increasing students' optimism in problem solving situations.

In a pilot study on affect, Ferguson (2009) investigated how teachers' use of particular mathematical tasks impacted on low-attaining primary students. The study was conducted within the larger *Task Types and Mathematics Learning* (TTML) project, and participants were two low-attaining Year 5 students. Ferguson reported that through the use of challenging tasks and the TTML pedagogical approach participants were able to maintain positive affective responses.

A study by Sullivan, Clarke, and O'Shea (2010) examined students' descriptions of their ideal mathematics lesson. Data were derived from a larger survey designed to gather responses on aspects of mathematics lessons and tasks from a cross-section of students in Years 5 to 9 in Victoria. Findings from this study highlighted the similarities between students' responses and literature on effective pedagogy. Following a two-year longitudinal study which investigated students' seating arrangements, Ingram (2008a) showed that students' feelings during mathematics, and how they learn the subject, is related to who they sit near in mathematics classrooms.

Critique of the Topics/Findings

In considering the foci and findings of the studies that have been published on the affective dimension of mathematics education in Australasia in the period 2008–2011, we identified some themes and issues that appeared noteworthy. These are briefly outlined and discussed below.

In the previous review (Grootenboer et al., 2008) the authors noted the preponderance of studies that were primarily descriptive, and in general contained a limited amount of theorising. This appears to have changed little in the ensuing four years, although Walshaw (2010) has provided a continued and robust theoretical analysis through a psychoanalytical framework. Others have also engaged in significant theoretical work vis-à-vis their findings (e.g., Goos & Bennison, 2008; Williams, 2008, 2009, 2010) and it is important to note that this has been reported across a number of publications. It seems likely that the avenues for reporting research (e.g., conference papers, journal articles) provide limited space for significant discussion on different theoretical underpinnings or the development of theoretical positions and frameworks, and hence authors are developing this through a program of publication.

There seems to be a reasonable amount of research around the mismatch between stated and enacted beliefs, and interventions for changing teachers' beliefs. However, there is still scope for studies about how teachers' beliefs influence mathematical pedagogy, and indeed, how they impact students' beliefs and other outcomes. Also, given the consistently reported discrepancy between teachers' beliefs and practice in regards to mathematics education, it is perhaps timely to explore avenues for reconciling these differences by seeing the espoused beliefs as aspirational for practice.

METHODOLOGICAL ISSUES

In this section we have focused on methodological issues because of their importance when researching the affective domain in mathematics education. The reliance on inference, largely based on what people are willing and able to share (Grootenboer et al., 2008), has continued throughout this review period with high proportions of self-reporting data collection methods evident, tempered by a significant increase in studies including observational data.

Attempts to group research papers by methodology in the Australasian literature over the review period was once again difficult because of a multiplicity of descriptors used by researchers, and in some conference papers in particular insufficient or incoherent detail. Philosophical assumptions and methodological frameworks were sometimes absent or only partially developed with just the methods and instruments used described to varying extents. There are many reasons why this may be the case, including the lack of comprehensive theoretical frameworks in the affective domain, but clarity on these issues is essential for the studies to have more than a one-off presentation value.

In the 2008–2011 period, excluding a small number (10) of non-empirical position and review papers, almost 14% of the studies were quantitative (26% in the 2008 review), 67% were qualitative (38% in the 2008 review) and 19% were mixed method studies (36% in the 2008 review) (Grootenboer et al., 2008). The figures suggest a significantly increased focus on qualitative studies and a corresponding decrease in both quantitative and, in particular, mixed methods. This last decrease is in seeming contrast to the growth in mixed methods compared to the review period (2004–2007). However, if research in this domain is seen as lying on a continuum between qualitative and quantitative (Grootenboer et al., 2008), then there are a number of studies in this review period that *tend* to be more qualitative and are likely to have been categorised as such. The overall number and proportion of studies with a principle focus on affective factors has remained relatively stable over the review period.

The size of samples considered in the studies reflects a feature of qualitative research which frequently focuses on small samples: around 60% of the studies were categorised as small scale with less than 60 respondents (e.g., Ferguson, 2009 [n=3]); around 14% as medium scale with 60 to 100 respondents (e.g., Williams, 2008 [n=86]); and around 25% as large scale with more than 100 respondents (e.g., Carmichael, 2010 [n=425]). However, in the case of a number of the mixed method studies there was a large quantitative survey followed by a smaller qualitative aspect such as interviewing (e.g., Lo & Anderson, 2010 [survey n=152 and interview n=19]).

Of the studies that focused solely on one group of participants, around 45% considered school students; around 20% teachers; and about 33% pre-service student teachers. These figures suggest a more even balance between school students and teachers as the focus of study than in the past two reviews. Of the studies that focused solely on one sector, around 50% considered primary aspects; around 15% secondary; and 15% tertiary/teacher education. Unfortunately, an early childhood aspect is represented in only one study (Beswick & Dole, 2008) evaluating a mathematics course for primary and early childhood pre-service student teachers, which does little to address a glaring gap in the literature.

The lack of studies on tertiary mathematics lecturers continues to be a concern, although mathematics education lecturers and their affective impact on pre-service student teachers has been examined in a series of papers by Klein (2008a, 2008b; Klein & Smith, 2009) and Walshaw (2009, 2010) through the lens of post-structuralism and psychoanalysis.

There was one study (Leder & Forgasz, 2010) examining the wider community's concerns with mathematics and affectivity that focused on determining parents' perceptions of gender differences with regard to school mathematics. The paucity of studies in early childhood, tertiary mathematics, and the wider community indicate that little progress has been made in investigating these areas since the last review.

The last four yearly review indicated more student-focused research in primary and secondary classrooms, but queried whether it could be considered classroom-based as there was little observational data evident (Grootenboer et al., 2008). In the current review period, however, there are a number of studies based primarily around observational data (e.g., Mornane, 2009; Sherley, Clark, & Higgins 2008; Williams, 2008, 2010; Wilson, 2009). In addition, there were a similar number (e.g., Ferguson, 2009; Grootenboer & Ballantyne, 2010; Ingram, 2008a, 2008b) which included observational data as one type of data alongside two or three other types. While Jennison and Beswick (2010) used five types of data: (a) survey, (b) pre- and post-test results, (c) pre- and post-interviews, (d) student journals, and (e) video observations. The growth in studies dealing with 'rich' data sets including classroom observation is encouraging. A similar number of student-centred studies were done through surveys only (e.g., Bennison & Goos, 2010; Beswick, 2008; Tait-McCutcheon, 2008) and a smaller number used both surveys and interviews (e.g., Norton & Windsor, 2008; Young-Loveridge, 2010). There was also a small group of studies that used interviews only (e.g., Meaney & Lange, 2010) or interviews and focus groups (e.g., Attard, 2010).

The most frequent survey instruments were Likert-scale or open-ended questionnaires, and interviews tended to be semi-structured as in the previous review. Specific methodological techniques were stated in some studies. For example, *Bibliotherapy* in Wilson and Thornton (2008) and Wilson (2009); *Discourse Analysis* in Brown and Redmond (2008), and Redmond and Sheehy (2009); *Learner's Perspective Study* in Williams (2008, 2010); *Multi Levelling Modelling* in Martin and Marsh (2008), and *Secondary Analysis* in McConney and Perry (2010). Some techniques were identified as having specific strengths, such as the *Learners' Perspective Study* methodology which allows the researcher to capture dialogue and actions that may not be captured using other methods.

There have been few advances made to research designs, data collection, and data analysis methods although Sullivan, Clarke, and O'Shea (2010) argued for narrative-based descriptions of behaviours as a way forward in affect research. The issue of attempting to clarify individuals' understanding and shared understandings of a question and the interpretation of that by researchers (Grootenboer et al., 2008) is still of concern. A pilot study by Sexton (2010) attempted to address this by the use of concept cartoons that typified traditional and constructivist teaching environments. The cartoons gave a detailed description of the two environments reflecting key aspects of each underlying approach to which the respondents reacted rather than relying solely on how individuals perceived the two types of teaching environments. The cartoons were used as a stimulus from which student and teacher participants' beliefs could be compared. Sexton's findings indicated that concept cartoons are useful in assessing affect, and further research in this area would be beneficial. One 'new' method of analysis used was Rasch measurement models in conjunction with Likert scales. This was used to analyse various elements 'mathematics teacher knowledge' (Beswick, Callingham, & Watson, 2011), including teacher's confidence and beliefs, using a teacher-profiling instrument. The analysis revealed a single underlying 'knowledge' construct allowing for a more holistic conceptualisation. The use of Rasch models to explore data sets for underlying constructs, which might combine a number of facets, could be a useful way to search for underlying constructs that 'unify' facets currently only treated separately.

In an effort to explore inconsistencies in terminology and measurement of affective factors in the learning of mathematics, Cretchley (2008) conducted a review of research that focused on four sets of research instruments and argued for two distinct broad primary areas of interest: self-concept; and intrinsic motivations for learning mathematics.

The increase in the collection of observational data in situ is encouraging as it allows for social factors. However, observational data on its own is insufficient to create a 'true' representation; it needs to be accompanied by other forms of data collection, for example, students' and/or teachers' perceptions, to create rich data sets.

There continues to be a relative absence of action research or interventionist type research in this review period. The focus of affect research needs to move beyond reporting and address some of these concerns. Putting our increasing knowledge about the affective domain into practice seems an important strategy for researchers to adopt.

CONCLUDING COMMENTS

Over the last four years, there has been a further increase in the number of research reports (to around 45%) dealing with school student perspectives and a shift toward primary focused studies (around 50%). There is now a predominance of student focused studies, with a reduction in the number of pre-service teacher and classroom teacher studies. This continues the trends reported in Grootenboer et al. (2008) and may be a response to the call for more research work with primary, although this is not evident for early childhood.

While links between beliefs and practice continue to be of interest, relatively few studies advance any theoretical considerations that might underpin causal relationships between the two. Implications in most of the studies are rarely developed beyond a local frame of reference and indeed the testing or development of theoretical frameworks is still not well represented. The need for theory development to underpin research continues to be a need within Australasian affect research.

Changes in the methodological approaches used have seen an increase in qualitative studies with lesser numbers of quantitative and mixed method studies being represented. However, the reality of this change is difficult to determine with any degree of precision due to the multiplicity of descriptors, the lack of a common terminology and frameworks alongside a lack of detail within some papers. The increase in the number of studies using multiple types of data including observational data to create rich data sets reflects the complexity of the affective domain where many factors may interact and determining causal relationships will always be a challenge. The types of instruments used have remained relatively constant with the main new development being an instrument looking at aspects of affect in statistics.

The trend toward studies containing observational data is an encouraging indicator of a shift of focus to what actually happens in the classroom rather than a reliance on self-report data. In previous review periods the prevalence of self-report data tended to restrict affect research to individual and group perceptions. For belief/practice investigations to move to more fertile ground it is important that observational data of teachers and students in classroom environments continues to be both a focus in itself and an integral part of research projects using multiple types of data.

REFERENCES

Afamasaga-Fuata'i, K. (2009). Innovative problem solving and students' mathematics attitudes. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 1–8). Palmerston North, NZ: MERGA.

Anderson, J. (2008). Teachers' motivation to attend voluntary professional development in K-10 mathematics. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 51–58). Brisbane, QLD: MERGA.

Attard, C. (2010). Students' experiences of mathematics during the transition from primary to secondary school. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 53–60). Fremantle, WA: MERGA.

Attard, C. (2011). The influence of teachers on student engagement with mathematics during the middle years. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics

Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 68–74). Adelaide, SA: AAMT & MERGA.

Averill, R. (2011). Teaching practices for effective teacher-student relationships in multi-ethnic mathematics classrooms. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 75–80). Adelaide, SA: AAMT & MERGA.

Bennison, A., & Goos, M. (2010). Learning to teach mathematics with technology: A survey of professional development needs, experiences and impacts. *Mathematics Education Research Journal, 22*(1), 31–56.

Beswick, K. (2008). Influencing teachers' beliefs about teaching mathematics for numeracy to students with mathematics learning difficulties. *Mathematics Teacher Education and Development, 9*, 3–20.

Beswick, K. (2009). School mathematics and mathematicians' mathematics: Teachers' beliefs about mathematics. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 153–160). Thessaloniki, Greece: PME.

Beswick, K. (2011a). Knowledge/beliefs and their relationship to emotion. In K. Kislenko (Ed.), *Current state of research on mathematical beliefs XVI* (Proceedings of the MAVI-16 conference 2010, pp. 43–59). Tallinn, Estonia: Tallinn University.

Beswick, K. (2011b). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics*. doi:10.1007/s10649-011-9333-2

Beswick, K., Callingham, R., & Watson, J. M. (2011). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education*. doi:10.1007/s10857-011-9177-9

Beswick, K., & Dole, S. (2008). Recollections of mathematics education: Approaching graduation and 5 years later. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 67–76). Brisbane, QLD: MERGA.

Brown, R., & Redmond, T. (2008). Reconceptualising agency through teachers talking about a sociocultural approach to teaching mathematics in the classroom. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 101–108). Brisbane, QLD: MERGA.

Carmichael, C. (2010). *The influence of the mathematics class on middle school students' interest for statistical literacy*. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 133–138). Fremantle, WA: MERGA.

Carmichael, C., & Hay, I. (2008). Middle school students' interest in statistical literacy. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 109–116). Brisbane, QLD: MERGA.

Cretchley, P. C. (2008). Advancing research into affective factors in mathematics learning: Clarifying key factors, terminology and measurement. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 147–154). Brisbane, QLD: MERGA.

Ferguson, S. (2009). Teachers' use of mathematics tasks: The impact on the mathematics learning and affective responses of low-attaining upper primary students. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 185–192). Palmerston North, NZ: MERGA.

Goos, M. (2009). Reforming mathematics teacher education: Theorising teacher's use of technology. In C. Ng & P. Renshaw (Eds.), *Reforming learning: Concepts, issues and practice in the Asia-Pacific region*. New York, NY: Springer.

Goos, M., & Bennison, A. (2008). Surveying the technology landscape: Teachers' use of technology in secondary mathematics classrooms. *Mathematics Education Research Journal, 20*(3), 102–130.

Grootenboer, P. (2008). Mathematical belief change in prospective primary teachers. *Journal of Mathematics Teacher Education, 2008*(11), 479–497.

Grootenboer, P., & Ballantyne, J., (2010). Mathematics teachers: Negotiating professional and discipline identities. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 225–233). Fremantle, WA: MERGA.

Grootenboer, P., Lomas, G., & Ingram, N. (2008). The affective domain and mathematics education. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W. Tiong Seah, & P. Sullivan (Eds.), *Review of mathematics education research in Australasia 2004–2007* (pp. 255–269). Rotterdam, The Netherlands: Sense.

Ingram, N. (2008a). Who a student sits near to in maths: Tension between social and mathematical identities. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 281–286). Brisbane, QLD: MERGA.

Ingram, N. (2008b, July). *The importance of length, breadth and depth when studying students' affective responses to mathematic through the lens of identity*. Paper presented at International Congress for Mathematics Education (ICME 11), Topic Study Group 26. Monterrey, Mexico: ICME.

Jamieson-Proctor, R., & Byrne, C. (2008). Primary teachers' beliefs about the use of mathematics textbooks. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 295–302). Brisbane, QLD: MERGA.

Jennison, M., & Beswick, K. (2010). Student attitude, student understanding and mathematics anxiety. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 280–288). Fremantle, WA: MERGA.

Jorgensen (Zevenbergen), R., Grootenboer, P., & Niesche, R. (2009). Insights into the beliefs and practices of teachers in a remote indigenous context. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 281–288). Palmerston North, NZ: MERGA.

Kemmis, S., (2008). Praxis and practice architectures in mathematics education. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 17–28). Brisbane, QLD: MERGA.

Klein, M. (2008a). How humanism can foster mediocrity in early years mathematics education: A poststructuralist comparison. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 311–316). Brisbane, QLD: MERGA.

Klein, M. (2008b). Preservice teachers and numeracy education: Can poststructuralism contribute? In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 317–322). Brisbane, QLD: MERGA.

Klein, M., & Smith, K. (2009). Being numerate for teaching: The indivisibility of learning landscape, participation and practice. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 299–306). Palmerston North, NZ: MERGA.

Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education, 9*, 91–102.

Leder, G., & Forgasz, H. (2010). I liked it till Pythagoras: The public's views of mathematics. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 328–335). Fremantle, WA: MERGA.

Leder, G., & Grootenboer, P. (2005). Affect and mathematics education. *Mathematics Education Research Journal, 17*(2), 1–8.

Liljedahl, P. (2008). Teachers' insights into the relationship between beliefs and practice. In J. Maab & W. Schloglmann (Eds.), *Beliefs and attitudes in mathematics education: New research results* (pp. 33–44). Rotterdam, The Netherlands: Sense.

Lo, W. Y., & Anderson, J. (2010). Beyond the curriculum: The mathematical beliefs of pre-service primary teachers in Hong Kong. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 657–644). Fremantle, WA: MERGA.

Martin, A. J., & Marsh, H. W. (2008). Academic buoyancy: Towards an understanding of students' everyday academic resilience. *Journal of School Psychology, 46*(1), 53–83.

McConney, A., & Perry, L. B. (2010). Socioeconomic status, self-efficacy, and mathematics achievement in Australia: A secondary analysis. *Educational Research, Policy and Practice, 9*, 77–91.

Meaney, T., & Lange, T. (2010). Pre-service students' responses to being tested on their primary school mathematical knowledge. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 399–406). Fremantle, WA: MERGA.

Mornane, A. (2009). The adolescent students' views of factors influencing their learning. *The International Journal of Learning, 16*(5), 221–229.

Norton, S. & Windsor, W. (2008). Students' attitude towards using materials to learn algebra: A year 7 case study. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 369–376). Brisbane, QLD: MERGA.

Redmond, T., & Sheehy, J. (2009). Reconceptualising agency in a senior mathematics classroom. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 451–458). Palmerston North, NZ: MERGA.

Schuck, S., & Grootenboer, P. (2004). Affective issues in mathematics education. In B. Perry, C. Diezmann, & G. Anthony (Eds.), *Research in mathematics education in Australasia 2000–2003* (pp. 53–74). Flaxton, Australia: Post Pressed.

Sexton, M. (2010). Using concept cartoons to access student beliefs about preferred approaches to mathematics learning and teaching. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 515–522). Fremantle, WA: MERGA.

Sherley, B., Clark, M., & Higgins, J. (2008). School readiness: What do teachers expect of children in mathematics on school entry? In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 461–466). Brisbane, QLD: MERGA.

Sullivan, P., Clarke, D., & O'Shea, H. (2010). Students' opinions about characteristics of their desired mathematics lessons. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 531–538). Fremantle, WA: MERGA.

Tait–McCutcheon., S. L. (2008). Self-efficacy in mathematics: Affective, cognitive, and conative domains of functioning. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 507–513). Brisbane, QLD: MERGA.

Tobias, S., Serow, P., & Schmude, M. (2010). Critical moments in learning mathematics: First year pre-service primary teachers' perspectives. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 805–812). Fremantle, WA: MERGA.

Walshaw, M. (2009). Exploring the identity of a pre-service teacher: Communal processes during the practicum. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 555–562). Palmerston North, NZ: MERGA.

Walshaw, M. (2010). The researcher's self in research: Confronting issues about knowing and understanding others. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of*

mathematics education (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 587–593). Fremantle, WA: MERGA.

Williams, G. (2008, July). *Links between optimism-building and problem solving capacity.* Paper presented at International Congress for Mathematics Education (ICME 11), Topic Study Group 26, Monterrey, Mexico: ICME.

Williams, G. (2009). Engaged to learn pedagogy: Theoretically identified optimism building situations. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 595–602). Palmerston North, NZ: MERGA.

Williams, G. (2010). *Symbiosis between creative mathematical thinking accompanied by high positive affect, and optimism*. In M. Pinto & T. Kawasaki (Eds.), *Optimising student understanding in mathematics* (Proceedings of the 34th conference of the International Group for the Psychology of Mathematics Education, pp. 297–304). Belo Horizonte, Brazil: PME.

Wilson, S. (2007). My struggle with maths may not have been a lonely one: Bibliotherapy in a teacher education number theory unit. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 815–823). Adelaide, SA: MERGA.

Wilson, S. (2009). *'Better you than me': Mathematics anxiety and bibliotherapy in primary teacher professional learning.* In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 603–610). Palmerston North, NZ: MERGA.

Wilson, S., & Thornton, S. (2008). 'The factor that makes us more effective teachers': Two pre-service primary teachers' experience of bibliotherapy. *Mathematics Teacher Education and Development, 9*, 21–35.

Young-Loveridge, J. (2010). Two decades of mathematics education reform in New Zealand: What impact on the attitudes of teacher education students? In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 705–712). Fremantle, WA: MERGA.

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63*(2), 113–121.

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EQUITY, DIVERSITY, SOCIAL JUSTICE AND ETHICS IN MATHEMATICS EDUCATION

Key words: equity; diversity; social justice in mathematics education.

INTRODUCTION

The basic research premise of this chapter is that students experience the education of mathematics differently, based on their learning opportunities and achievements that depend on the social context of their families and the schools they attend. Often such 'background' factors are associated with disadvantage, marginalisation, disengagement, and exclusion from the study of mathematics. There is also a heavy economic, social and political cost for the students individually, their communities and the broader society. Along with international efforts to increase the quality of the mathematics experience for school students, concerns about making mathematics education accessible to *all* students continues to provide a major focus for much research in the discipline, and a challenge for policy statements and initiatives as well as classroom practice.

The first section considers research that deals with the theoretical analysis of 'equity, diversity, social justice and ethics in mathematics education' that in many studies bring complexities to the 'areas of concern' for researchers, practitioners and policy makers. This is followed by a section on each of the major areas of research. Some are well established in the literature such as gender, language and culture, and socioeconomic considerations while other areas of concern such as rural education and global collaboration issues are more recent additions. In many studies these areas of concern overlap emphasising the complexity of equity, diversity and social justice for researchers, practitioners and policy makers.

THEORETICAL CONSIDERATIONS

In addition to the continual concerns about inequitable access and participation in mathematics and attempts to remedy exclusion and disadvantage in the field, recent literature in mathematics education in Australasia also reflects an increase in publications dealing with theorising the associated constructs and the search for epistemological approaches to investigate them. In this section, we examine four relevant themes illustrated in the recent published literature in the region.

The first theme is illustrated by the writings of Atweh and his colleagues (Atweh, 2007, 2009; Atweh & Brady, 2009) on issues related to multiplicity of discourses

B. Perry et al. (Eds.), Research in Mathematics Education in Australasia 2008–2011, 39–65.

associated with issues of inequality and disadvantage. Atweh and Keitel (2008) pointed out that the social justice agenda is often discussed in the mathematics education literature in conjunction with the constructs of equity and diversity. Although the terms equity and diversity are at times used interchangeably, their usage differed in the context of the disadvantage under consideration (e.g., gender is usually discussed in terms of equity while language issues are often constructed in terms of diversity). Atweh and Keitel also argued that group status aims were an important difference between the overlapping aims of both agendas. While equity projects aimed at reducing group differences (e.g., in differential achievement and participation), and hence ultimately aimed to abolish such differences, diversity discourse aimed at enhancing group differences and status.

As Gates and Jorgensen (2009) noted, the discourse of social justice is relatively more recent in mathematics education literature, although social justice concerns in the field are long-standing as demonstrated by the traditions that investigate issues of gender, low socioeconomic background, language and ethnicity. In an attempt to relate the constructs of equity and social justice, Burton (2003), from the United Kingdom, argued that there was a "shift from equity to a more inclusive perspective that embraces social justice" (p. xv). Atweh (2007) discussed theories of social justice, as elaborated by feminist writer Fraser (1995), that construct it as consisting of two dimensions, corresponding roughly to agendas of equity and diversity, namely, *distribution* and *recognition*.

The second theme related to theorising social justice is found in the introduction by Gates and Jorgensen (2009) to two Special Issues on social justice and teacher education of the *Journal of Mathematics Teacher Education*. Drawing on the work of Pierre Bourdieu, the authors explored the notion of social justice at the intersections of practice, habitus and field. With respect to their attempt to reach an understanding of the concept of social justice the author noted:

> The first challenge is perhaps to come up with a definition of social justice with which we can all agree ... Social justice is a relative concept; what is unjust to some, is not unjust to others; whether we consider something is socially unjust or relationally unjust will likewise differ. (Gates & Jorgensen, p. 165)

The authors presented a three level model to understand the different ways different authors deal with the agenda of social justice, although they acknowledged that this was done "at the risk of oversimplifying the problem of definition" (p. 166). First, the authors pointed to what they called, *moderate* forms of social justice that focused on concerns of 'fairness and equity'. The authors argued that this form of social justice reinforced the status quo as it did not challenge social conditions, giving rise to inequity of educational opportunities. At the second level, *liberal* forms of social justice "recognise[d] structural inequalities and ... address[ed] those in some way" (p. 176). The target of this approach was how to make the classroom socially just within the existing unjust social structures. Gates and Jorgensen (2009) added:

> Hence the classroom becomes a political arena and politics is produced at the level of the individual in a small community. For example, it would see the politics of gender relationships and identities as constructed within classrooms. (p. 176)

Within this level, the authors then placed research on social justice from a post-structural perspective. At the third level, *radical* forms of social justice are used in an attempt to address the social structures that cause injustice by directly trying to expose and change them. Of course, such an approach is more demanding of the social justice activist.

The third theme discussed here, perhaps in one sense in contrast to the arguments developed by Gates and Jorgensen, relates to post-structural critique of traditional approaches to understanding and remedying social injustice. Walshaw (2010) argued that concerns about the lack of equitable participation in mathematical experiences by certain individuals and groups of people are not new. However, she points out that "inequities in mathematics classrooms and in other mathematics educational institutions persist even when structural barriers are removed" (p.17). In particular, basing the understanding of inequity of participation on group identity (whether socioeconomic, cultural, linguistic or any other category) is a construction of identity as a unitary and fixed construct. The author argued that such discourse "lack[ed] the analytic power to change existing formations" (p.17). She pointed to recent epistemologies of identity which posit it as multiple and fluid, hence it is "not reducible to one of its manifestations" (p. 2). Rather than dealing with the issues of inequality as abstract generalised constructs or leading into the trap of inaction in the face of such a dilemma, Walshaw argued that these understandings of identity were crucial for grounding "ethical practical action" (p.1) that is emancipatory for the different subjects traditionally excluded from experiencing the power of mathematics in their lives. Such an approach understands social change not as a result of a mere removal of barriers of social participation but "through making more visible the ways in which commonplace daily social relations are rearticulated" (p. 17). In another context, Walshaw (2011) utilised these post-structural constructs to place identity as the cornerstone for understanding both quality and equity in mathematics education.

This shift in understanding of social justice from fairness and equity to "ethical practical action", and from focusing on structures giving rise to disadvantage to interactions between subjects within overall discourses of power lead us to the fourth theme. Atweh (2011), using the construct of ethics, argued for an approach to mathematics education that focused on quality and equity. The post-ontological philosophical writings of Levinas have been influential in the re-introduction of ethics within philosophy by establishing ethics as the "First Philosophy". Atweh and Brady (2009) argued that the agendas of ethics and social justice were complementary and provided two reasons why ethics complements social justice. First, social justice issues were often constructed as concerns relating to the participation of social groups in social activity 'enjoying their fair share' of social benefits. Such a construction has less to do with the outcomes of a particular individual, unless they are due to being a member of a specific social group.

Further, the social justice discourse is often silent on issues relating to the interaction between two people (e.g., people from the same social group). Ethics, on the other hand, is concerned with a face-to-face encounter and interaction between people. Secondly, a focus on ethical responsibility establishes social justice concerns as a moral obligation, rather than charity, good will or convenient politics. Based on a presentation from the Key Panel at the International Congress in Mathematics Education in Monterrey, Mexico in 2008, Atweh (2011) reconstructed the two international agendas of quality and equity in mathematics education on the construct of ethical responsibility. Atweh and Brady (2009) described Socially Response-able Mathematics Education as a means to reform teaching of mathematics in middle-school.

In the following sections the research on social justice that relates to particular groups of mathematics students is located within these four theoretical considerations of social justice.

GENDER

Vale and Bartholomew (2008) reviewed Australasian studies that reported the re-emergence of gendered differences in mathematics achievement, favouring males with a decline in participation by females in tertiary entry level secondary mathematics. The studies reviewed provided evidence of persistent differences in positive affect also favouring males. They argued that most of the research exploring gender issues was underpinned by liberal feminist theory or deficit theory since "these differences were understood to be located within individuals" (p. 287). Previous studies exploring pedagogy often essentialised girls and stated that teaching from a 'care' perspective had greater appeal for both girls and boys. In the past, research with respect to gender has been concerned with equity and distributive agendas of social justice along with a few studies that took a liberal and ethical approach to social justice. In the period since, these themes in equity and social justice prevailed. Researchers have continued to monitor the gender gap and have sought explanation for the re-emergence and widening of the gap in achievement and participation. Many of these studies have adopted a post-structural critique and explored aspects of identity and gendered mathematics. Another approach has been the investigation of education policy and its impact on pedagogy and curriculum which addressed one or more of the dimensions of social justice.

The Gender Gap: Distribution Dimension of Equity

Forgasz (2008a, 2010) and Vale (2010) discuss trends in the gender gap for achievement, participation and affective factors in Australia since the mid-nineties to reveal a widening gap favouring males in achievement in primary and secondary mathematics, and participation at the senior secondary level. They included findings from recent TIMSS and PISA studies (Thomson & De Bortolli, 2008; Thomson, Wemert, Underwood, & Nicholas, 2008) as well as from national testing

(MCEETYA, 2008). Similar trends were observed for New Zealand though significant differences favouring males have been present since 2000 amongst 15-year-olds for PISA (OECD, 2007) and fewer significant differences among 8-year-olds for the achievement variables measured by TIMSS (Mullis, Martin, & Foy (2008). The 2009 PISA study of 15-year-old students also found gender differences favouring males in Australia (Thomson, De Bortolli, Nicholas, Hillman, & Buckley, 2010) and New Zealand (OECD, 2010).

Forgasz and Leder, in various studies (Forgasz, 2008a, 2008b, 2010; Forgasz & Leder, 2010), focused attention on the gender gap among the highest achievers in the PISA study of 15-year-olds (Thomson & De Bortolli, 2008; Thomson et al., 2010) and provided further evidence of the gender gap from studies of Victorian Year 12 VCE students and participants in the Australian Mathematics Competition, a competition for high achieving students in junior, middle and senior secondary school.

A study of 76 Victorian government schools in low socioeconomic communities that were engaged in reforming mathematics teaching to improve learning outcomes for their students also found gender differences in mathematics achievement favouring males for students in all primary years and females for secondary students (Vale et al., 2011). The numeracy intervention programs where more females than males participated did not arrest the gender differences, as growth in achievement was higher for the male students than the female students.

These studies, which considered the distributive dimension of equity, showed that gender differences were clearly evident in the primary years and indicate that further attention and research needs to involve teachers' awareness of gender as a factor related to students' perceptions, participation and achievement in mathematics.

Gendered Mathematics: Post-Structural Critique or a Liberal Approach to Social Justice?

Forgasz (2008a) and Vale (2010) reported on the persistent findings from a range of studies that showed male students were more confident, positive and interested, with a higher level of enjoyment and expectation of success in mathematics than females at all age levels. Collaborating with international researchers they reviewed studies of gendered perceptions and pedagogies of mathematics classrooms and settings where students used digital technologies (Forgasz, Vale, & Ursini, 2010). Leder and Forgasz (2008) discussed the way in which the media interpreted findings about the gender gap. They argued that the media took an uncritical stance, distorted the facts, and so contributed to the perpetuation of gender stereotyping.

In the period under review researchers investigated perceptions and experiences of mathematics of students in the middle-years, Year 12 students, and high achieving students, while other researchers went further and investigated identity in the gendering of mathematics. Carmichael and Hay (2009) surveyed 366 middle-year students and found that girls preferred statistics learning embedded in statistical surveys whereas boys preferred problem solving contexts, especially those involving sports. They acknowledged that teachers needed to cater for these different preferences. However, teachers need to be mindful not to 'essentialise' girls' and boys' learning preferences. This issue is apparent also in the study of Year 12 students in low socioeconomic schools by Helme and Teese (2011). They found that girls taking the least demanding mathematics subject (Further Mathematics) were more dissatisfied with their learning experiences than boys taking this subject and students taking more demanding mathematics subjects. Girls were less likely than boys to perceive that mathematics was relevant to their future and more likely than boys to perceive that the teacher did not understand how they learnt. As well, girls were more likely than boys to perceive that the pace of learning was too fast and to be less confident in their expectation of success. Helme and Teese argued that "despite decades of research in gender differences and strategies making mathematics content and pedagogy more responsive to the needs of girls, this study reveals that there is still more to be done" (p. 356).

What can we learn from high achieving girls and women with mathematics careers? Studies by Leder and Forgasz (2010b) and Harding, Wood and Muchata (2010) of high achieving students indicated a return to research methods common in the 1980 and 1990s to explore liberal approaches to social justice and affirmative dimensions of equity. Leder and Forgasz (2010b) surveyed the medallists of the Australian Mathematics Competition some years after they had won their medals. They found that competition success 'opened-doors' for the male medallists but the female medallists didn't gain particular benefit from their success and were less likely than males to pursue mathematical careers. Ultimately, for the female medallists, the mathematics environment did not hold as much appeal as those of their other academic interests.

Harding et al. (2010) presented seven case studies of women who completed doctorates in mathematics and mathematics education later in life to find out why women entered these courses later in life than males. They found that intellectual curiosity and academic or research challenges arising from their work prompted women to pursue mathematics learning and research later in life.

The study by Forgasz and Mittelberg (2008) highlighted the situated nature of gender and identity with respect to mathematics. Despite gendered attitudes about mathematics, Australian students perceived mathematics to be gender neutral, while students in other countries with a significant gender gap in achievement, in this case Arab and Israeli students, also believed mathematics to be a male domain. Walls (2010) conducted a longitudinal ethnographic study to illustrate the social construction of feminine/masculine identities and corresponding gendered mathematical identities. She tracked the experiences and preferences of toys, leisure activities, mathematics learning, work experiences and career aspirations of a group of 10 children (four girls and six boys) from different schools from 7-years-of-age through to completion of their secondary education. Her findings showed how the parents' gendered experiences of school and career and their attitudes towards mathematics were reproduced in their children's preferences for leisure activities, reflections on learning mathematics experiences and aspirations for work. As young children, the boys were more positive than girls about their mathematics experiences and while both boys and girls developed and expressed negative attitudes about mathematics during their secondary schooling the boys sustained a belief that studying mathematics in their final year(s) of schooling was useful to them. Walls argued that students' perceived mathematics as 'masculinising' and boys take mathematics to be an "empowering signifier of their schooling" whereas a significant proportion of girls do not.

Leder and Forgasz (2010a, 2011) took up this theory of reproduction of gendered perceptions of mathematics and investigated the public's perception of mathematics. They wondered whether a public information or advertising campaign, as was conducted during the 1980's (*Mathematics Multiplies Your Choices*) was needed to confront the re-emergence and widening of the gender gap in mathematics and conducted a survey of 103 adults. Perhaps surprisingly they found that the majority of respondents were positive about mathematics, believed that they were good at mathematics (especially in their primary years of schooling), and agreed that students should continue to study mathematics after it was no longer compulsory. Almost all respondents thought that both boys and girls should study mathematics; those that believed there was as a gender difference in ability to do mathematics were more likely to believe that boys were better at mathematics than girls. Leder and Forgasz argued that public awareness about issues of gender and mathematics needed to be raised.

Education Policy: For or Against Gender Justice?

Vale (2010) sought explanations for the turnaround in the trend toward gender equity by examining shifts in education policy and mathematics curriculum. She traced Australian government policy from the 1980s to 2000s and discussed the positive and negative ways feminist theories influenced education policy and the policy for women in Australia. She described how affirmative and transformative approaches to gender mainstreaming in education were easily discarded by a change of government predisposed to feminist backlash ideology. Vale advocated for an ethical stance on social justice and argued that researchers have successfully drawn attention to poor outcomes for marginalised and disadvantaged students in Australia, and now a plan for action for gender justice for girls in mathematics is needed.

ETHNIC AND LANGUAGE DIVERSITY

Traditionally, diversity has been invoked "as an 'explanation' for the students' performance in mathematics" (Civil, 2011, p. 18). The move now is "away from deficit views" (p. 19) towards an understanding that reconciles "the identities that [students] are invited to construct in the mathematics classroom" (Cobb & Hodge, 2002, p. 249) with their participation in the practices of home communities, local groups and wider communities within society (see Atweh, Graven, Secada, &

Valero, 2011). Although there is much willingness across the research community to understand those contributions with a view towards providing equitable access to quality mathematics education across a wide range of diversities, “there is also an urgent need to provide guidance as to how this might occur” (Gervasoni & Lindenskov, 2011, p. 319).

Exploring the relationship between a classroom setting in a remote community context, and the students within those settings, Treacy and Frid (2008) looked at the counting approaches of Years 1 to 11 students. They noted that while Western mathematics is generally taught in Australian schools and is the primary means by which many people create an understanding of their environment, the ways in which Aboriginal people make sense of and organise their environments is distinctly different. Students in the study were provided with both standard counting tasks and a task that involved gathering a culturally familiar resource (maku) for a number of individuals in a picture. The students chose to draw on Western methods to answer the standard counting tasks, but used culturally-specific methods to solve the maku task. A number of other studies have investigated the challenges of teaching mathematics in diverse contexts. In a study by Edmonds-Wathen (2011), exploring the spatial concepts in Iwaidja, an Indigenous language spoken in the Northern Territory, children tended to use different spatial frames of reference from those typically used by English speakers. Clearly, teachers need to pay attention to the different needs and strategies that result from different home environments.

In their study on teachers’ professional learning in the Kimberley, Gervasoni et al. (2011) showed that Aboriginal Teaching Assistants played a critical role “in helping school communities in the Kimberley provide high quality learning environments for students and their families” (p. 306). Howard, Cooke, Lowe, and Perry (2011) pointed out that Australia’s Indigenous people “are the most educationally disadvantaged group” (p. 365) in the country. A number of programs, such as *Mathematics in Indigenous Contexts* (MIC) and *Wii Gaay,* designed to address disparity, have been developed and implemented to enhance outcomes of specific groups of students. A more recent program, *Make It Count* (2009–2012), has been implemented nationally with the potential to develop partnerships between the school, the family, and the community for long-term change. In Western Australia, the impact of *Make It Count* on teachers was explored in relation to best practice in teaching Indigenous children. Hurst, Armstrong, and Young (2011) reported that these practices used: oral discussions and drawings to communicate ideas; game playing to teach key concepts; and natural resources as well as rhyme, rhythm and movement.

Specific teacher-student relationships have been shown to strongly influence academic performance of minority group students. In a study of 100 Year 10 mathematics lessons involving six teachers and their classes, Averill (2011) found that teachers who demonstrated ‘essential caring teacher behaviours’ contributed to the enhancement of equitable access to mathematics learning. In a Māori medium education setting, Hawera and Taylor (2011) found that Māori values, language and culture provided a context for an enhanced engagement with mathematics for

Years 5 to 8 children. The influences helped children develop a broader view about the nature of mathematics, enhanced whānau (family) involvement in children's mathematics learning, and connected children's learning experiences with the mathematics in their community.

Social class, like ethnicity, differentiates students and is a marker of proficiency. From their research, Mills and Goos (2011) illustrated the ways in which teachers were able to enhance student proficiency in low socioeconomic areas. One of the research schools was a small inner-city primary school and the other a remote Indigenous community school. Students at both schools had a history of poor performance in mathematics. Mills and Goos looked closely at the effects of high quality pedagogies on students and the use of open-ended investigations. At the city school, the principal's and teachers' willingness to change and a desire to improve teaching practice contributed to improved student performance. At both schools the principals' interest in effective instructional practices initiated a shared sense of purpose amongst the staff. Both principals were able to generate enthusiasm and enhance teachers' belief in their own capabilities.

Meaney, Trinick, and Fairhall (2009) explored how projected beliefs in capabilities influenced a group of Māori-medium school teachers' level of engagement at a national English-medium mathematics teachers' conference. Invariably, inequitable social structures at the conference impacted on the teachers' feelings of belonging, and their professional experiences at the conference sessions. Greater evidence of collaboration and a shared sense of purpose amongst the teachers at the conference might have resulted in higher levels of capacity building.

Language

Language plays a central role in building bridges between students' intuitive understandings and the mathematical understandings sanctioned by the world at large. As Bose and Choudhury (2010) and Ilany and Margolin (2010) noted, language constructs meaning for students as they move towards mathematically acceptable modes of thinking and reasoning. In a study on pre-service teachers' analyses of middle-school English Language Learners' (ELLs) ideas of measurement, Fernandes (2011) found that the teachers recognised the importance of incorporating language goals into mathematics lessons, but that they needed to develop their expertise in doing this, particularly when working with ELLs students. Working from the premise that the language that students use derives from the language used by their teacher, Ilany and Margolin (2010) developed an instructional model to assist students to make sense of and solve mathematical word problems. They found that their nine-stage model enabled students to forge links between natural language and the language of the discipline, while sensitising them to the particular nuances of mathematical language.

For many researchers, inviting dialogue in the classroom is a socially responsible pedagogical practice. However, in some settings this invitation not only brings significant barriers, it also raises serious ethical issues for teachers. Jorgensen

(2010) reported on a project in which the overarching aim was to implement reform pedagogies in remote Aboriginal communities. Instructional practices relating to student discussion, explanation, justification and sharing of ideas were imported into the culture of the Kimberley communities on the understanding that such interactions would be beneficial, both mathematically and socially, for students. Jorgensen found that these practices were not able to be successfully implemented. Teachers in these settings reported that these pedagogical approaches violated many cultural norms. The implications of these findings to the role of traditional pedagogies of drill and practice are not made clear.

Given that Australia and New Zealand are characterised by considerable ethnic and cultural diversity, challenges for teaching raise significant social justice issues within mathematics education. This is made particularly acute in that mathematics "uses culturally laden language to express problems whose interpretation requires sophisticated linguistic and cultural competence" (Arkoudis & Love, 2008, p. 74). However, as a number of studies have revealed, there are very real pedagogical difficulties in integrating mathematical content with English language learning. As Bautista Verzosa (2011) found in her study with second grade Filipino children solving additive word problems in English, "mathematical difficulties were uncovered, but only when linguistic difficulties were minimised through the provision of linguistic scaffolds" (p. 21). Similarly, language-related misconceptions were reported by Jaffar and Dindyal (2011) in their study on post-secondary students' understanding of the limit concept.

Bose and Choudhury (2010), Arkoudis and Love (2008), and Parvanehnezhad and Clarkson (2008), for example, have all studied the tensions that arise in multilingual classrooms between mathematics and language. Arkoudis and Love (2008) looked at these tensions as experienced by one teacher and eight of her students. The students were Chinese international students in Australia, enrolled in the senior school subject, Specialist Mathematics. All the students had studied in Australia for around one and a half years with the expressed purpose of gaining entry into university. For their part, the goal-focused students prioritised their mathematics skills rather than English for developing understanding. Drawing on the notion of imagined communities, Arkoudis and Love (2008) argued that the international students' identities, at odds with those of local students, limited their participation in class. Specifically, the international students' identities were structured around an imagined future community rather than the present classroom community of practice.

Parvanehnezhad and Clarkson (2008) explored the ways in which a group of Iranian students used their home language as a resource to develop mathematical understanding by switching between the two languages when doing mathematics. They found that 14 of the 16 students tended to switch language while solving mathematical problems. Perhaps predictably, an increasing item difficulty amongst the word problems in the research led to higher use of language switching.

In some multi-lingual classrooms, teachers explicitly acknowledge cultural heritage by switching between the language of instruction and the learners' main language in order to advance students' understanding. Bose and Choudhury (2010) found evidence of language switching (code switching) for bilingual students, particularly when students could not understand the mathematical concept or when the task level increased. Code switching involved words and phrases as well as sentences and tended to enhance student understanding. The location of the study undertaken by Bose and Choudhury (2010) was in Mumbai (Bombay) at a camp held for lower achieving Grade 6 students over a period of two months for one and a half hours each week. While the first language of the teacher and students was Hindi, the official language of instruction, in keeping with the common practice, was English. Code switching occurred as the teacher and students switched between languages and tended to enhance student understanding. From a social justice perspective the practice empowers students and "helps in breaking the authoritative approach of mathematics teaching" (p. 99).

In Papua New Guinea, vernacular languages and Indigenous knowledge-based systems are emphasised in curriculum policies for the first three years of schooling. English is gradually introduced in the years that follow. Muke and Clarkson (2011) examined how eight teachers used multiple languages to teach mathematics in Year 3 classes. They found that when the teachers used the available languages, it was with a view towards making English more accessible to the students. Matang (2008) investigated the influence of primary school students' first language and traditional counting systems on their early number development in Papua New Guinea. Students' mathematical tasks were taken from the *Count Me in Too* project (NSW Department of Education, 2001). It was found that, generally speaking, the 125 children in the study learned more quickly and made fewer errors in task solution when they used traditional counting systems and learned in their home language.

Since students with limited English proficiency value hearing their peers use mathematical language, the researchers recommended that to assist in overcoming potential and real language difficulties, more competent bilingual students might be encouraged to support less able peers to solve mathematical problems. Home language exchange amongst students, Niesche (2009) argued, was a resource by which students were able to negotiate mathematical meaning. In her study of recent immigrant 7th grade students from Mexico into the United States context, Civil (2011) found that when students were given the option to explain their mathematical thinking in Spanish, their home language, it provided the researchers "access to very rich and lively mathematical discussions, which in turn gave [them] a window into their thinking about mathematics" (p. 21).

However, home language exchanges between peers in the remote Aboriginal classrooms researched as part of the Kimberley project did not assist peers mathematically. Peer interactions, Jorgensen (2010) found, were not typically focused on advancing student understanding. For their part, teachers were challenged by not knowing what the students were talking about. Friction between family groups in these settings often carried over into heated discussions within the classroom, resulting in the adoption of a more disciplinarian teacher stance.

RURAL AND REMOTE COMMUNITIES

International studies as well as the national testing programs in Australia and New Zealand have reinforced the significance of disadvantage for students in rural communities. Students in Australian rural communities do not perform as well as metropolitan students and achievement is related to the degree of remoteness and size of community, as many studies in this section demonstrate. The most recent PISA study (Thomson et al., 2010) found that the gap in mathematical literacy for 15-year-old rural students in remote Australian locations is almost one-and-a-half years of schooling behind their metropolitan peers. The gap between provincial students and metropolitan students is less but statistically significant. The most recent Australian national assessment program reported that there were 10% fewer students in remote locations than metropolitan who reached the national minimum standard at each year level assessed (MCEEDYA, 2010). The margin is up to four times greater for very remote students. Findings are similar in New Zealand and around the world (Williams, 2005).

Mathematics achievement of students attending schools outside metropolitan areas is also related to socioeconomic status, Indigenous status, language background and gender (McConney & Perry, 2010). The geographic patterns of socioeconomic status and other demographic factors are not common and provide further evidence of the complexity and diversity of regional and rural school communities. School or student factors such as teacher preparation and approaches, classroom climate and students' self-efficacy also contributed to the mathematics achievement of rural students (De Bortolli & Thomson, 2010; Panizzon & Pegg, 2007). Furthermore, the issues for schools in rural communities were subject to the influence of transient and fluctuating populations, immigration, rural economic circumstances, and seasonal conditions such as climate and natural disasters (Pegg, 2009).

In 2004 the National Centre of Science, Information and Communication Technology and Mathematics Education for Rural and Regional Australia (SiMERR) was established at the University of New England to bring social justice for the education of rural students to the attention of educators and policy makers, to collaborate with communities, education authorities and organisations, and to undertake strategic research in the field to improve outcomes for students. MERGA recognised this emerging field of sociocultural and social justice research in mathematics education when it approved a special issue of *Mathematics Education Research Journal* in 2011 focusing on rural issues in mathematics education.

In this section we review the research literature resulting from the much stronger recognition of the needs of rural students and teachers that has emerged in 2008–2011. We begin by reviewing theoretical perspectives for research involving rural schools and school communities and relate these theories to the themes of social justice, the focus of this chapter.

Diversity Perspective of Social Justice for Rural Education

According to Howley, Howley, and Huber (2005) equity oriented initiatives in rural school communities, aimed at addressing the needs of marginalised or excluded students and closing the gap in achievement outcomes, are often based on the presumption of deficit and a shallow understanding of poverty and culture. Corbett (2009) agreed, challenging the "set of inter-connected assumptions about educational success and failure, assumptions which end up … painting people who remain in rural places as somehow deficient" (p. 2). He argued that formal education for students in rural communities is about disconnecting with place and 'learning to leave'. Corbett described standardised curriculum and traditional pedagogies as 'urbanisation of the mind' and argued for recognition and valuing of difference through 'place-based pedagogy'.

Identity and place are strong themes in the theory of researchers working with Indigenous communities. Wallace and Boylan (2009) brought the 'rural lens' metaphor and the themes of 'challenging deficit theory' and 'understanding place' to the attention of Australasian researchers. Using a rural lens means that strategies are developed from within to sustain and strengthen social, cultural, economic and community attributes and capacity rather than be imposed from outside. Wallace and Boylan sought to challenge the deficit perspective that teachers, educators and policy makers have of rural society, schools, communities and the conditions in which they will work. They argued that 'place' is important in rural contexts because:

> Place recognises that uniqueness, value and relevance that the history, cultural value system, language, social infrastructure, the impact of the environment and the economic realities have on shaping the local community in ways that define it as different to other places. (p. 25)

Place-based education is about connecting with local concerns and traditions, including relevant place-based experiences and driving educational decision-making from within. The rural lens is consistent with the social justice strategy of 'transformative-recognition' since initiatives involve local mutual critical collaboration, develop agency, and lead to shared and contextualised learning. It has been deliberately adopted by some mathematics education researchers (e.g., Ell & Meissel, 2011), and implied or imbedded in the work of others (e.g., Connor, Auld, Eakin, Morris, & Tilston, 2010; Goos, Dole, & Geiger, 2011).

Policy, Programs and Resourcing for Rural Schools

Pegg (2009) reported that researchers and educators believed the underachievement of students in Australia's rural schools needed to be addressed in an integrated way and that educational renewal and reform in rural and regional Australia must be more broadly supported by policy and programs for development. Earlier Panizzon and Pegg (2007) reported the findings of a survey that compared the issues and needs of rural and regional teachers with those of urban teachers. Teacher shortages, lack of opportunities for professional learning, in particular time-release for participation, ICT resources and support staff were high on the list of issues for rural teachers. Teaching higher order thinking skills was their most pressing professional learning need, while teachers from schools where at least 20% of students were Indigenous requested support for teaching in context. These issues were taken up by those designing professional learning programs discussed below.

Following up on the issues of ICT resources, Loong, Doig, and Groves (2011) conducted a survey of 700 rural and urban students on their use of ICT for in-school and out-of-school mathematics learning. Few differences between rural and urban students emerged suggesting equity of access to ICT is not a problem. Where differences were found, in almost all cases, rural students were found to be more frequent users of the technology. These findings challenge any perceptions that rural students, schools and communities are technologically deficient.

Improving Teaching and Learning for Rural Students

Studies reporting on research of teaching and learning in rural locations typically involved multiple settings including projects across states and education systems. Watson and Stack (2008) described the way in which collaboration among education researchers, teacher organisations, schools and the education systems under the SiMERR umbrella were conducted to improve teaching and learning for rural students in Tasmania. Their paper is a meta-analysis of the 14 projects initiated by SiMERR 'hub' members, teachers or academics; four of these projects focussed specifically on mathematics. Watson and Stack noted success for most of these projects in the short-term but raised two significant issues: sustainability and scaling-up of these projects; and, foreshadowing Pegg (2009), they called for broader based systemic programs to support new rural teachers and the need to further engage parents and community. However, since Watson and Stack described the SiMERR programs as 'interventions', these projects may be interpreted as reactive and deficit focussed.

In contrast, two of the projects briefly described by Pegg and Krainer (2008) included a more proactive approach. The first gathered data on the attributes of schools in regional Australia which recorded outstanding achievements in mathematics; the second provided teachers with the expert advice and support to initiate professional learning or innovations. Comparing the various reform initiatives across different countries, Pegg and Krainer identified collaboration, communication and partnership as crucial elements.

Panizzon and Pegg (2007) sought to improve student learning by encouraging secondary teachers to review their assessment practices through their participation in a professional learning program. The program, which was conducted over two years, was designed to enable teachers to interpret student responses using the SOLO taxonomy to provide for more effective scaffolding of students' learning. Teachers nominated curriculum areas to trial these approaches and hence participated to some extent in the design of the program. The program providers also visited teachers in between sessions to provide further support to individual teachers in the program and their school colleagues. The authors noted the value of the two-year term of this program for sustaining changes in teachers' practices. It is not known whether this knowledge enabled better mathematical connections with students' rural identity.

The study by Gervasoni, Parish and colleagues (2010) also struggled to provide a rural lens. They provided compelling evidence of the success attributed to a numeracy intervention approach for children in the early years (*Extending Mathematical Understanding*) through the appointment of a school mathematics coordinator to lead a whole school approach to mathematics curriculum, assessment and intervention. Part of the success of the project at the case study school was the engagement of parents. The authors argued that the project had "enhanced ... the capacity of the entire school community ... to learn mathematics successfully" (p. 208). But what is required for these outcomes to be sustained and scaled up to include students in all year levels and other schools in the region?

Rather than bringing an urban model of professional learning to schools, Beswick and Jones (2011) set out to design and implement a teacher-centred approach to a professional learning program for primary and secondary teachers of mathematics in a cluster of three remote schools in Tasmania. The program was negotiated with principals and based on teachers' responses to a questionnaire about their professional learning needs and included individual and small group coaching or mentoring as well as after school seminars or forums. These took place on location. The school principals liked the flexibility of the program as it fitted in with the schools in terms of timing and teachers' expressed needs. However, perhaps because of the brevity of the program or its timing in the first week of the school year, the program failed to build a collaborative culture that could sustain reflective practice or begin to generate place-based pedagogy.

The professional learning program designed by Goos, Dole, and Geiger (2011) was more successful in this regard. Their program was teacher-centred but focused explicitly on developing capacity for numeracy teaching in context and with a critical orientation through the design and implementation of problems and investigations in secondary mathematics classrooms. Goos, Dole, and Geiger chose to focus their discussion on the design features of the program to build teacher agency, but could have focussed instead, or as well, on authenticity in rural teaching and learning and the personal connection the students made with this problem.

A study designed from within a proactive capacity building project for secondary school teachers of mathematics in a cluster of schools in regional New South Wales that set out to establish collaborative relationships, is described by Connor et al. (2010). The mathematics teacher leaders from four schools reviewed aggregated data about their region to develop a common purpose and focus for their praxis inquiry that they shared with their school colleagues. This project illustrates the importance of teachers defining the problem and focus of their collaboration to generate collegiality, a blame free environment, authoritative ideas and democratic empowerment to achieve sustainable collaborative practices.

Collaboration among primary teachers from a cluster of five schools in rural New Zealand was a significant feature of the reform project studied by Ell and Meisell (2011). The cluster was significant because it had a strong self-determination agenda having been initiated and sustained by teachers rather than outside experts. The cluster chose to focus on basic facts and all teachers in the cluster of schools worked in a group to design and implement action plans. Ell and Meisell documented the strategies investigated by the teachers and measured improvement in student achievement. The strategies included changes to school organisation, a focus on the test items, developing particular teaching strategies or a focus on knowledge in context. The most progress was made by students in the school taking action to make connections between basic facts and problems in context.

Researchers set out to work with rural schools and communities to improve teaching and learning for students and to meet the needs of teachers in rural and regional schools. However, a rural lens that challenged deficit thinking and included place-based pedagogy was rarely stated explicitly in the theoretical frameworks of these studies. Perhaps this is because learning to leave still dominates thinking when it comes to education in rural and remote locations.

SOCIOECONOMIC FACTORS

Even though research around the world has consistently pointed to the crucial role that socioeconomic factors plays in determining access and outcomes of educational experiences in mathematics, only a small number of research studies were reported in the Australasian literature dealing with these issues directly. This pattern is consistent with previous MERGA reviews. However, this observation should be moderated by the fact that many other studies reported in this chapter deal with issues that overlap with socioeconomic factors (e.g., gender, rural education and linguistic background). In this section, we identified two quantitative studies that dealt with evidence of the relationship of socioeconomic factors to participation and achievement in mathematics education and three qualitative studies that dealt with intervention programs in low socioeconomic schools.

McConney and Perry (2010) presented a detailed analysis of the PISA 2006 data in 15-year-old students to examine in detail the patterns of relationship between SES and mathematics and science literacy. In addition to the student background, the study examined the socioeconomic background of the school in which students attend. The authors noted that PISA's measure of student-level SES was a composite index of the following: highest parental occupational status; highest parental educational attainment (years of education); and economic and cultural resources in the home based on a questionnaire that the students completed. The reported findings indicated that the SES of individual students mattered considerably in science and mathematics literacy performance. Further, the SES measure of the school similarly was related to students' achievement in mathematics and science literacy. In their conclusion the authors declared:

> Our findings show that where one goes to school in Australia makes a significant difference for all students' mathematics and science performance. This is inequitable because it means that a student's achievement is heavily influenced by his or her family's ability to afford a good school. Moreover, our findings show that achievement gains are sharpest in middle-high and high SES schools. Yet access to these schools in Australia is restricted. (p. 446)

Similar results were reported by Ainley, Kos, & Nicolas (2008) who noted that:

> Two of the largest differences among specified groups of Australian students concerned socioeconomic background and Indigenous status. The difference in the [mathematical] literacy scores between students in the lowest and highest quarters of the distribution of socioeconomic background … 78 points. (p. 6)

Thornton and Galluzzo (2010) reported a study at the Catholic Archdiocese of Canberra/Goulburn as part of the Commonwealth Government Literacy and Numeracy Pilots in low SES schools. In this project, professional development sessions were conducted with considerable time devoted to discussing the fundamental concepts of mathematics that have been shown to be both troublesome and essential for further understanding. Teachers were not given set procedures to use with their interventions; rather they were asked to respond to students at their point of need. Many of the teachers involved were familiar with the *Reading Recovery* program which was chosen as a way of structuring the intervention. For each lesson, teachers were asked to plan using a template based on a combination of ideas from Reading Recovery and the concepts of brain based learning. Evidence from this project pointed to modest cognitive effects but strikingly positive affective results as reported by classroom teachers. However, the authors pointed out that one inhibiting factor to mainstreaming such interventions was that they were expensive to run.

Gervasoni, Parish and their colleagues (2010) reported on a collaborative project between 42 school communities under different Catholic Education Offices, and the Australian Catholic University. In this project, classroom teachers administered a one-on-one interview-based mathematics assessment using the *Early Numeracy Interview.* Similarly, teachers had access to a specialist teacher to assist in the use of these data to guide instruction and curriculum development at individual, class and whole school levels. The authors concluded that "this collaborative and rigorous approach for designing highly effective learning environments is having a positive impact on mathematics learning and instruction" (p. 202).

A final study conducted by Vale, Weaven, Davies, and Hooley (2010) was also a component of the Federal Government's Pilot program and concerned student-centred approaches (SCA)—one element of the multi-faceted approach implemented by the Victorian government. They investigated interpretation and implementation of SCA provided through personal accounts of practice by teachers and instructional leaders. Differentiated and targeted teaching, based on various student assessment data, were the dominant interpretations implemented in diverse ways in classrooms and schools. A major improvement in the practice of many teachers included more focussed lessons that connected mathematical ideas and included the explicit use of language to model mathematical thinking and explanation of that thinking.

GLOBAL COLLABORATIONS

Social justice concerns in the Australasian region in the period of the review are not restricted to social groups within the countries represented. In an increasing globalised world, Australia and New Zealand have an increasingly important role in international contacts and collaborations. This includes international conferences, international students, publications and collaborative research. The publication of a MERGA supported book on Internationalisation and Globalisation in Mathematics and Science Education (Atweh et al., 2008) has allowed a few Australasian mathematics education researchers to raise issues relevant to social justice on the global scene.

In particular, Neyland (2008) used the discourse of ethics to look at the role of mathematics education in a globalised world. He noted that mathematics education had been a tool of cultural imperialism. One of the patterns of globalisation is new and growing social stratification, resulting in increased bureaucratic domination of the poor. Neyland concluded that the foci likely to be useful to avoid the negative effects of globalisation on mathematics education in poor countries included: (a) conceiving of education as a public good in service of the world community; (b) using mathematics as a corner stone for the development of participatory democracy; and (c) presenting mathematics in programmes of work that emphasise its humanistic qualities and its basis in human ideas.

The chapter by Atweh and Keitel (2008) utilised the elaboration of social injustice by Young (1990) as markers of social injustice in international collaborations. The authors concluded that international contacts in education may be said to be *exploitative* if the knowledge of one social group is advanced at the expense of another group. Similarly, if the research questions and methodologies of some countries dominate international research at the expense of issues of concern of other nations, then the latter can be said to be *marginalised*. Economic situations in many less industrialised nations limit the capacity of educators from those countries to take an active and equal role in international academic activities and hence can lead to a sense of *powerlessness*. Further, the non-critical transfer of curricula and research results from one country, with a certain perceived higher status, to another can be said to be a form of cultural *imperialism*. Finally, the tying of international aid and development monies to the impositions of agendas, policies and priorities developed in Western countries can be regarded as a form of *violence* on less affluent nations.

The chapter by Southwell, Phanalasy, and Singh (2008) discussed some pertinent observations based on the authors' involvement in projects in three countries: Laos, Malaysia, and Maldives. While the issues encountered differed in the three counties, the authors identified the crucial role of appropriate communication between local educators and foreign consultants. In the majority of international development projects, local educators are expected to communicate with the international team in a foreign language. Further, the theories of learning developed in the foreign language needed to be translated by the local educators to be used by local teachers without having a strong base of research and publications in their own language. A related issue is that often the ignorance of the foreign consultants of the local philosophies of education and social problems give rise to the adoption of a globalised mathematics education curriculum and pedagogy.

In another context, Atweh et al. (2007) discussed a collaborative project between an Australian university and the government of The Philippines. The project employed the construct of capacity building in its design and implementation, and was designed through collaboration between leading academics from The Philippines with two Australian counterparts. The authors argued that collaboration does not necessarily imply an equal amount or the same type of contribution. Parity of esteem (Grundy, 1998) should be the guiding principle by which collaboration is judged. In developing projects of this scale, each participant has their own expertise and knowledge, which is often complementary to the others' contributions.

Finally, two studies reported in the period of the review discussed the issue of conducting cross country research. Cao, Forgasz, and Bishop (2008) discuss the challenges and difficulties that researchers face in the process of designing and administering a survey to be used in cross cultural settings, and how cultural factors can influence researchers' activities and research results. Some of the problems identified included: designing a survey that was intended to apply to two cultural contexts; choosing a topic of equal importance in both countries; choosing the right format of the instrument; the appropriate number of choices in response formats of the Likert scales; the adequateness of the survey content; and the precision in the translation of the questionnaire. Similar problems are identified by Davis, Seah, and Bishop (2009) who were involved in a doctoral research project for an Australian institution conducted in Ghana. In order to obtain ethical clearance for the project, the Australian university required a letter of approval from the educational authorities in the country. The educational officials in Ghana were reluctant to issue that letter because it did not match their own procedures. Interestingly the university insisted, and the official had to change his stance in order for the project to proceed. Similarly, school teachers and principals were often suspicious of the need to sign the letters of consent even to the extent that some schools had to withdraw from the study.

CONCLUSION

It would be an onerous task to attempt to reach definite conclusions from the diversity of research studies reported here. The theoretical frameworks, research questions, target samples, and methodologies vary considerably from one study to another. Rather, we will make some observations on the status of the research in this area and raise some of its implications for policy and practice including challenges for its own future directions.

First, we note that the literature reported here includes engagement with theoretical constructs used in the research and reflected in the policy statements it supports. There seems to us to be a movement from the disparate agendas such as equity, diversity and inclusion to a more comprehensive and perhaps unifying construct of social justice. Likewise, a few authors are beginning to understand the agenda of social justice in terms of ethics. How future research and policy in mathematics education may benefit from these developments, remains to be seen.

Second, we note a diversification of the social justice agendas in terms of groups of people traditionally marginalised in the discipline. As the many authors noted, factors of gender, language and culture, and socioeconomic status still play a decisive role for many students in access to, and participation and achievement in, mathematics. However, research in social justice in Australasia has begun to investigate new marginalisation issues such as rural education and globalisation. We commend this trend. It demonstrates that social justice concerns are more wide spread than a handful of agendas. Arguably, there are social justice concerns behind every action we take as mathematics educators, not to mention actions that we do not take. Perhaps the discourse of ethics may lead to raising questions of social justice in situations where we have not raised it before, such as in mono-lingual, mono-cultural and high achieving settings.

Third, by and large the literature on social justice in mathematics education has considered one or more of what can be called 'background' factors of marginalisation or disadvantage in the study of mathematics. Many authors have warned against the threats of essentialising students' differences and blaming the victim for explaining educational exclusion. However, we note, with an amount of disquiet, that factors related to physical, emotional and mental disabilities have not received the same level of attention from researchers in Australasia and arguably neither are they widely represented in the international literature in mathematics education.

Finally, as the literature reviewed above demonstrates, even after years of concerted policy and action to remove inequalities in mathematics education, they still persist. This is not to say that progress has not been made and that the patterns of inequality are the same. However, it draws our collective attention to maintain the vigilance and resolve to keep up with research that uncovers injustices and finding ways to deal with them. Research and action towards achieving social justice varies, as the 'intervention' studies reported above demonstrate. In this context we raise the question: is social justice in mathematics education a utopian ideal to achieve? In other words can we solve problems of social injustice once and for all? Commenting on several international projects designed to achieve equity for different social groups in mathematics education, Atweh (2011) raised the point that the road to equity has 'no highway and no destination'. However, it is a road we are compelled and committed to travel.

REFERENCES

Ainley, J., Kos, J., & Nicolas, M. (2008). *Participation in science, mathematics and technology in Australia*. Melbourne, Australia: ACER.

Arkoudis, S., & Love, K. (2008). Imagined communities in senior school mathematics: Beyond issues of English language ability. *Journal of Asian Pacific Communication, 18*(1), 71–90.

Atweh, B. (2007). International interactions in mathematics education. In U. Gellert & E. Jablonka (Eds.), *Mathematisation and demathematisation: Social, philosophical and educational ramifications* (pp. 171–186). Rotterdam, The Netherlands: Sense.

Atweh, B. (2009). What is this thing called social justice and what does it have to do with us in the context of globalization. In P. Ernest, B. Greer, & B. Sriraman (Eds.), *Critical issues in mathematics education* (pp. 111–124). Charlotte, NC: Information Age.

Atweh, B. (2011). Equity and quality in mathematics education as ethical issues. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 563–76). Dordrecht, The Netherlands: Springer.

Atweh, B., Balagtas, M., Bernardo, A., Ferido, M., Macpherson, I., & Salana, J. (2007). Ripples of change: Capacity building and curriculum reform through action research. In B. Atweh, M. Ubina-Balagatas, A. Bernardo, M. Frido, & I. Macpherson (Eds.), *Ripples of change: A journey in preservice teacher education reform in the Philippines* (pp. 2–23). Manila, Philippines: Commission of Higher Education.

Atweh, B., Borba, M., Barton, A., Gough, N., Keitel, C., Vistro-Yu, C., & Vithal, R. (Eds.). (2007). *Internationalisation and globalisation in mathematics and science education*. Dordrecht, The Netherlands: Springer.

Atweh, B., & Brady, K. (2009). Socially response-able mathematics education: Implications of an ethical approach. *Eurasia Journal of Mathematics, Science & Technology, 5*(3), 267–276.

Atweh, B., Graven, M., Secada, W., & Valero, P. (2011). *Quality and equity agendas in mathematics education*. Dordrecht, The Netherlands: Springer.

Atweh, B., & Keitel, C. (2008). Social justice in international collaborations. In B. Atweh, M. Borba, A. Barton, N. Gough, C. Keitel, C. Vistro-Yu, & R. Vithal (Eds.), *Internationalisation and globalisation in mathematics and science education* (pp. 95–112). Dordrecht, The Netherlands: Springer.

Averill, R. (2011). Teaching practices for effective teacher-student relationships in multiethnic mathematics classrooms. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 75–81). Adelaide, SA: AAMT & MERGA.

Bautista Verzosa, D. (2011). Children solving word problems in an imported language: An intervention study. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 98–106). Adelaide, SA: AAMT & MERGA.

Beswick, K., & Jones. T. (2011). Taking professional learning to isolated schools: Perceptions of providers and principals, and lessons for effective professional learning. *Mathematics Education Research Journal, 23*(2), 83–105.

Bose, A., & Choudhury, M. (2010). Language negotiation in a multilingual mathematics classroom: An analysis. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 93–100). Fremantle, WA: MERGA.

Burton, L. (Ed.). (2003). *Which way social justice in mathematics education?* London, United Kingdom: Praeger.

Cao, Z.; Forgasz, H., & Bishop, A. (2008). Surveys in different cultures: A case study from China and Australia. In B. Atweh, M. Borba, A. Barton, N. Gough, C. Keitel, C. Vistro-Yu, & R. Vithal (Eds.), *Internationalisation and globalisation in mathematics and science education* (pp. 303–320). Dordrecht, The Netherlands: Springer.

Carmichael, C., & Hay, I. (2009). Gender differences in middle school students' interests in a statistical literacy context. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the

32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 97–104). Palmerston North, NZ: MERGA.

Civil, M. (2011). Lessons learned from the Center for the Mathematics Education of Latinos/as: Implications for research with non-dominant, marginalised communities. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp.11–25). Adelaide, SA: AAMT & MERGA.

Cobb, P., & Hodge, L. L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. *Mathematical Thinking and Learning, 4*(2&3), 249–284.

Connor, V., Auld, B., Eakin, P., Morris, K., & Tilston, T. (2010). The praxis of building capacity in mathematics and science in a rural, non-government system of schools: Voices of teacher leaders. *Education in Rural Australia, 20*(2), 35–51.

Corbett, M. (2009). Rural schooling in mobile modernity: Returning to the places I've been. *Journal of Research in Rural Education, 24*(7). Retrieved from http://jrre.psu.edu/articles/24–7.pdf

Davis, E., Seah, W. T., & Bishop, A. (2009). Institutional gaps in education research procedures between developed and developing country. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 137–144). Palmerston North, NZ: MERGA.

De Bortolli, L., & Thomson, S. (2010). *Contextual factors that influence the achievement of Australia's Indigenous students: Results from PISA 2000–2006*. Melbourne, Australia: ACER.

Edmonds-Wathen, C. (2011). Locating the learner: Indigenous language and mathematics education. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 217–225). Adelaide, SA: AAMT & MERGA.

Ell, F., & Meissel, K. (2011). Working collaboratively to improve the learning and teaching of mathematics in a rural New Zealand community, *Mathematics Education Research Journal 23*(2), 169–187.

Fernandes, A. (2011), Mathematics pre-service teachers learning about English language learners through task-based interviews. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 235–243). Adelaide, SA: AAMT & MERGA.

Forgasz, H. (2008a) Stars, compass, and GPS: Navigating currents and charting directions for mathematics education research on gender issues. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 5–15). Brisbane, QLD: MERGA.

Forgasz, H. (2008b). Gender, socioeconomic status, and mathematics: Performance among high achievers. In O. Figueras, J .L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 25–32). Morelia, Mexico: Cinvestav-UMSNH.

Forgasz, H. (2010, April). *Re-emerging gender gap in Australia. Gender equity: Are we there yet? Research from Australia, Iceland & the U.S.* Paper presented at the annual research pre-session of the National Council of Teachers of Mathematics [NCTM], San Diego, CA.

Forgasz, H., & Leder, G. (2010, April/May). *Gender and high achievers in mathematics: Who and what counts?* Paper presented at the annual conference of the American Education Research Association, Denver, CO. Retrieved from http://convention2.allacademic.com/one/aera/aera10/index.php?click_key=1&cmd=Multi+Search+Search+Load+Publication+For+Extra&publication_id=385225&PHPSESSID=de1403fa91c1d8dd2c5b11c985d3f05d

Forgasz, H., & Mittelberg, D. (2008). Israeli Jewish and Arab students' gendering of mathematics. *ZDM Mathematics Education, 40*, 545–558.

Forgasz, H. J., Vale, C., & Ursini, S. (2010). Technology for mathematics education: Equity, access, and agency. In C. Hoyles & J-B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain* (pp. 385–404). New York, NY: Springer.

Fraser, N. (1995). From redistribution to recognition: Dilemmas of justice in a post-socialist society. *New Left Review*, July-August, 68–93.

Gates, P. & Jorgensen, R. (2009). Foregrounding social justice in mathematics teacher education. *Journal of Mathematics Teacher Education*, *12*, 161–170.

Gervasoni, A., Hart, A., Croswell, M., Hodges, L., & Parish, L. (2011). Insights from Aboriginal teaching assistants about the impact of the bridging the numeracy gap project in a Kimberley Catholic school. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 306–314). Adelaide, SA: AAMT & MERGA.

Gervasoni, A., & Lindenskov, L. (2011). Students with 'special rights' for mathematics education. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 307–323). Dordrecht, The Netherlands: Springer.

Gervasoni, A., Parish, L., Upton, C., Hadden, T., Thurkenburg, K., Bevan, K. … Southwell, J. (2010). Bridging the numeracy gap for students in low SES communities: The power of a whole school approach. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 202–209). Fremantle, WA: MERGA.

Goos, M., Dole, S., & Geiger, V. (2011). Improving numeracy education in rural schools: A professional development approach, *Mathematics Education Research Journal, 24*(2), 129–148.

Grundy, S. (1998). Research partnerships: Principles and possibilities. In B. Atweh, S. Kemmis, & P. Weeks (Eds.), *Action research in practice: Partnerships for social justice in education* (pp. 37–46). London, United Kingdom: Routledge.

Harding, A., Wood, L., & Muchata, M. (with Edwards, B., Falzon, L., Gudlaugsdottir, S., Huntley, B., Knowleds, J., Miller-Reilley, B., & Steyn, T.) (2010). Try and catch the wind: Women who do doctorates at a mature stage in their lives. In H. Forgasz, J. R. Becker, K-H. Lee, & O. B. Steinthorsdottir (Eds.), *International perspectives on gender and mathematics education* (pp. 1–11). Charlotte, NC: Information Age.

Hawera, N., & Taylor, M. (2011). Māori medium children's views about learning mathematics: Possibilities for future directions. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 340–348). Adelaide, SA: AAMT & MERGA.

Helme, S., & Teese, R. (2011). How inclusive is Year 12 mathematics? In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 349–357). Adelaide, SA: AAMT & MERGA.

Howard, P., Cooke, S., Lowe, K., & Perry, B. (2011). Enhancing quality and equity in mathematics education for Australian indigenous students. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 365–378). Dordrecht, The Netherlands: Springer.

Howley, C. B., Howley, A. A., & Huber, D. S. (2005). Prescriptions for rural mathematics instruction: Analysis of the rhetorical literature. *Journal of Research in Rural Education*, *20*(7), 1–16. Retrieved from http://jrre.psu.edu/articles/20-7.pdf

Hurst, C., Armstrong, T., & Young, M. (2011). Making a difference for Indigenous children. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 373–381). Adelaide, SA: AAMT & MERGA.

Ilany, B-S., & Margolin, B. (2010). Language and mathematics: Bridging between natural language and mathematical language in solving problems in mathematics. *Creative Education, 1*(3), 138–148.

Jaffar, S. M., & Dindyal, J. (2011). Language-related misconceptions in the study of limits. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 390–397). Adelaide, SA: AAMT & MERGA.

Jorgensen, R. (2010). Group work, language and interaction: Challenges of implementation in Aboriginal contexts. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 740–743). Fremantle, WA: MERGA.

Leder, G., & Forgasz, H. (2008) Mathematics education: New perspectives on gender, *ZDM Mathematics Education, 40*, 513–518.

Leder, G. C., & Forgasz, H. J. (2010a). I liked it till Pythagoras: The public's view of mathematics. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 328–325). Fremantle, WA: MERGA.

Leder, G. C., & Forgasz, H. J. (2010b). Gender and high achievers in mathematics: Who and what counts? In H. Forgasz, J. R. Becker, K-H. Lee, & O. B. Steinthorsdottir (Eds.), *International perspectives on gender and mathematics education* (pp. 315–339). Charlotte, NC: Information Age.

Leder, G. C., & Forgasz, H. J. (2011). The public's views on gender and the learning of mathematics: Does age matter? In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 736–743). Adelaide, SA: AAMT & MERGA.

Loong, E., Doig, B., & Groves, S. (2011). How different is it really? Rural and urban primary students' use of ICT in mathematics. *Mathematics Education Research Journal, 24*(2), 189–212.

Matang, R. (2008, July). *Enhancing children's formal learning of early number knowledge through Indigenous languages and ethnomathematics: The case of Papua New Guinea mathematics curriculum reform experience.* Paper presented at the 11th International Congress on Mathematical Education, Monterrey, Mexico: ICME.

McConney, A., & Perry, L. (2010). Science and mathematics achievement in Australia: The role of school socio-economic composition in educational equity and effectiveness. *International Journal of Science and Mathematics Education, 8,* 429–452.

Meaney, T., Trinick, T., & Fairhall, U. (2009). 'The conference was awesome': Social justice and a mathematics teacher conference. *Journal of Mathematics Teacher Education, 12*, 445–462.

Mills, M., & Goos, M. (2011). Productive pedagogies in the mathematics classroom: Case studies of quality and equity. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 479–491).Dordrecht, The Netherlands: Springer.

Ministerial Council on Education, Early Childhood Development and Youth Affairs [MCEECDYA]. (2010). *National report: Achievement in reading, writing, language conventions and numeracy 2009*. Canberra, ACT: Author. Retrieved from http://www.nap.edu.au/_Documents/National%20 Report/NAPLAN_2009_National_Report.pdf

Ministerial Council on Education, Employment, Training, and Youth Affairs [MCEETYA]. (2008). *National Assessment Program Literacy and Numeracy: NAPLAN summary report (Achievement in* *reading, writing, language conventions and numeracy).* Canberra, ACT: Author. Retrieved from http://www.naplan.edu.au/verve/_resources/NAPLAN_Summary_Report.pdf

Muke, C., & Clarkson, P. (2011). Teaching mathematics in the Papua New Guinea Highlands: A complex multilingual context. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 540–547). Adelaide, SA: AAMT & MERGA.

Mullis, I. V. S., Martin, M. O., & Foy, P. (with Olson, J. F., Preuschoff, C., Erberber, E., Arora, A., & Galia, J.). (2008). *TIMSS 2007 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades.* Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

New South Wales [NSW] Department of Education & Training (2001). *Count me in too*: *Professional development package.* Sydney, Australia: Author.

Neyland, J. (2008). Globalisation, ethics and mathematics education. In B. Atweh, M. Borba, A. Barton, N. Gough, C. Keitel, C. Vistro-Yu, & R. Vithal. (Eds.), *Internationalisation and globalisation in mathematics and science education* (pp. 113–128). Dordrecht, The Netherlands: Springer.

Niesche, R. (2009). The use of home language in the mathematics classroom. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 704–707). Palmerston North, NZ: MERGA.

Organisation of Economic Co-operation and Development [OECD]. (2007). *PISA 2006. Science competencies for tomorrow's world.* Paris, France: Author.

Organisation of Economic Co-operation and Development [OECD]. (2010). *PISA 2009 results: What students know and can do*. Paris, France: Author.

Panizzon, D., & Pegg, J. (2007). Enhancing student achievement in mathematics: Identifying the needs of rural and regional teachers in Australia. In J. Watson, & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 581–590). Adelaide, SA: MERGA.

Parvanehnezhad, Z., & Clarkson, P. (2008). Iranian bilingual students reported use of language switching when doing mathematics. *Mathematics Education Research Journal, 20*(1), 52–81.

Pegg, J. (2009). Lessons learnt: Implications of four large-scale SiMERR projects for rural education in Australia. In T. Lyons, J. Y. Choi, & G. McPhan (Eds.), *Innovation for equity in rural education* (Proceedings of International Symposium for Innovation in Rural Education, ISFIRE, pp. 33–44). Armidale, Australia: University of New England.

Pegg, J., & Krainer, K. (2008). Studies on regional and national reform initiatives as a means to improve mathematics teaching and learning at scale. In K. Krainer, & T. Woods (Eds.), *The international handbook of mathematics education, Vol. 3: Participants in mathematics education* (pp. 255–280). Rotterdam, The Netherlands: Sense.

Southwell, B., Phanalasy, O., & Singh, M. (2008). Globalisation/localisation in mathematics education: Perceptions, realism and outcomes of an Australian presence in Asia. In B. Atweh, M. Borba, A. Barton, N. Gough, C. Keitel, C. Vistro-Yu, & R. Vithal. (Eds.), *Internationalisation and globalisation in mathematics and science education* (pp. 509–523). Dordrecht, The Netherlands: Springer.

Thomson, S., & De Bortolli, L. (2008). *Exploring scientific literacy: How Australia measures up (The PISA 2006 survey of students' scientific, reading and mathematical literacy skills)*. Melbourne, Australia: ACER.

Thomson, S., De Bortolli, L., Nicholas, M., Hillman, K., & Buckley, S. (2010). *Challenges for Australian education: Results from PISA 2009*. Melbourne, Australia: ACER.

Thomson, S., Wemert, N., Underwood, C., & Nicholas, M. (2008). *TIMSS 07: Taking a closer look at mathematics and science in Australia*. Melbourne, Australia: ACER.

Thornton, S., & Galluzzo, G. (2010). One on one numeracy intervention: A pilot project in low SES communities. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 555–562). Fremantle, WA: MERGA.

Treacy, K., & Frid, S. (2008). Recognising different starting points in Aboriginal students' learning of number. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 531–537). Brisbane, QLD: MERGA.

Vale, C., (2010). Gender mainstreaming: Maintaining attention on gender equality. In H. Forgasz, J. R. Becker, K-H. Lee, & O. B. Steinthorsdottir (Eds.), *International perspectives on gender and mathematics education* (pp. 111–146). Charlotte, NC: Information Age.

Vale, C., & Bartholomew, H. (2008). Gender and mathematics: Theoretical frameworks and findings. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W. T. Seah, & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004–2007* (pp. 271–290). Rotterdam, The Netherlands: Sense.

Vale, C., Davidson, K., Davies, A., Hooley, N., Loton, D., & Weaven, M. (2011). Using assessment data: Does gender make a difference? In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 736–743). Adelaide, SA: AAMT & MERGA.

Vale, C., Weaven, M., Davies, A., & Hooley, N. (2010). Student centred approaches: Teachers' learning and practice. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 571–578). Fremantle, WA: MERGA.

Wallace, A., & Boylan, C. (2009). Reviewing the rural lens in education policy and practice. *Education in Rural Australia, 19*(2), 23–30.

Walls, F. (2010). Freedom to choose? Girls, mathematics and the gendered construction of mathematical identity. In H. Forgasz, J. R. Becker, K-H. Lee, & O. B. Steinthorsdottir (Eds.), *International perspectives on gender and mathematics education* (pp. 87–110). Charlotte, NC: Information Age.

Walshaw, M. (2010). Poststructuralism and ethical practical action: Issues of identity and power. *Journal for Research in Mathematics Education, 41*, 3–19.

Walshaw, M. (2011). Identity as the cornerstone for quality and equitable mathematics experiences. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Quality and equity agendas in mathematics education* (pp. 91–104).Dordrecht, The Netherlands: Springer.

Watson, J., & Stack, S. (2008, November/December). The SiMERR experience in Tasmania. In P. Jeffery (Ed.), *Changing climates: Education for sustainable futures* (Proceedings of the Australian Association for Research in Education Conference). Brisbane, Australia: AARE. Retrieved from http://www.aare.edu.au/08pap/abs08.htm

Williams, J. H. (2005). Cross-national variations in rural mathematics achievement: A descriptive overview. *Journal of Research in Rural Education*, *20*(5). Retrieved from http://jrre.psu.edu/articles/20-5.pdf

Young, I. M. (1990). *Justice and the politics of difference*. Princeton, NJ: Princeton University Press.

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INDIGENOUS STUDENTS AND THE LEARNING OF MATHEMATICS

Key words: social context; Indigenous studies including Māori; language.

INTRODUCTION

In the last four years, there has been significant growth in the amount of research that has investigated issues to do with Indigenous students learning mathematics. This research includes some exciting studies which document how the skills and knowledge that Indigenous students bring to their mathematics learning have been utilised as an affordance for that learning. Other research continues to position Indigenous students as being 'abnormal' because their achievement in mathematics on standardised tests is not the same as that of non-Indigenous students. There are many undesirable consequences of research which begins with such an assumption. In this chapter, we document the research which has been undertaken in this area in Australasia in the last four years and outline how the limitations from assumptions about Indigenous students affect the research findings and pedagogical implications.

For this chapter, we adopted the same definition for Indigenous students that we have used in the previous MERGA review (Meaney, McMurchy-Pilkington, & Trinick, 2008). We perceived Indigenous students to be those who are indigenous to the land in which they are learning mathematics. Therefore, we looked at research about Indigenous students living in Australia, New Zealand, Papua New Guinea and the Pacific. Book chapters, articles, PhD theses and reports that dealt specifically with these students have been reviewed. We have not reported on Indigenous students who were living outside of their country of origin such as Pasifika students in New Zealand. Although there has been a significant increase in Indigenous students learning mathematics in Australia, material from Papua New Guinea and the Pacific was limited. With only a few researchers in some of these countries working in mathematics education, this research is very dependent on them continuing to work in this area. For example, the untimely death of Rex Matang was a blow to his many colleagues around the world but also had an impact on the research from Papua New Guinea. The completion of Patricia Paraide's PhD in 2010, highlights the small number of researchers from these parts of the Australasian region who are currently pursuing further study options in mathematics education. If, in the future, we are to see more research from these areas then more opportunities

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need to be provided for early career researchers to complete further study and to attend conferences, such as MERGA, where their research can be heard and valued.

CONCERNS

Several articles were of concern to us because of the assumptions that they seemed to be working from and the effect that these had on the reported findings and pedagogical implications. These assumptions include the impact of socioeconomic considerations, assessment results as valid indicators of achievement, and differences between language and dialects. As described in the previous review (Meaney et al., 2008), there is a need to be very careful about how Indigenous students are portrayed as learners and users of mathematics. The stories that researchers produce contribute to the identities that students build about themselves. For example, Brown (2008) identified that “negative attitudes, values and misconceptions formed about Aboriginal and Torres Strait islander people are shaped around the concept of scientific thought” (p. 94). For her, the myth of the ‘childlike primitive’ has “promote[d] the idea that Indigenous students are childlike and simplistic in their thinking” (p. 94) and there was a need for this to be challenged. From a Papua New Guinean perspective, Paraide (2008) described how in colonial times, Indigenous knowledge was not just considered primitive but as a hindrance to Western learning. These assumptions are destructive to any attempts to work with Indigenous people on how to gain the most value from developing mathematical ideas.

Poverty and Economic Disadvantage

The intersection between poverty and ethnicity is rarely articulated in the research articles gathered from the last four years. Brown (2008) painted a particularly bleak picture of Indigenous students’ mathematics learning in order to set up the need for them to engage in mathematical modelling. Unfortunately, correlations between mathematical achievement and having a study area with a dictionary at home are converted into a cause and effect relationship, which suggests that if Indigenous students just have access to these study spaces then their mathematics achievement would improve. Even if the relationship is one of cause and effect, which is highly questionable, there is no discussion of poverty which does seem more relevant.

In Trinick and Stevenson’s (2010) review of the data from *Poutama Tau* numeracy project for teachers in Māori immersion schools, it was clear that students who came from poorer socioeconomic areas did not make as much improvement as those from higher socioeconomic areas, even when the distinctions between socioeconomic areas were not seen as being that large in themselves.

Sharon Cooke, an Aboriginal educator commenting on a range of issues to do with improving the quality of mathematics education for Indigenous students, stated that many Aboriginal families live in poverty but this often goes unrecognised as having an impact on their learning—"How is 'poverty' addressed in teacher education programs and the development of appropriate curriculum and teaching strategies? How do the teachers cope with being told all the time about the differences between Aboriginal and non-Aboriginal kids?" (Howard, Cooke, Lowe, & Perry, 2011, p. 369). Fifteen years ago,Jeannie Herbert (1995) wrote similarly:

> I have been asked to explore the intersection of gender, race and disadvantage. I cannot do that because I prefer not to associate the word 'disadvantage' with examining issues in the context of Indigenous people. It seems to me that the term 'disadvantage' is often used to categorise Aboriginal and Torres Strait Islander people. While there is no denying that many Indigenous families are disadvantaged—by poverty, by long-term unemployment and by racist attitudes of the wider community, they are not disadvantaged by being Aboriginal or Torres Strait Islander! Many teachers view Indigenous people as disadvantaged because they are Aboriginal or Torres Strait—part of the 'indigenous problem'. (p. 9)

It seems a pity that in the intervening years, more researchers have not recognised that the poverty many Indigenous families experience is a result of long-term institutional racism and the role of non-Indigenous people in overcoming this racism. One way that this institutional racism manifests itself is through the assumption that school mathematics is the only valuable mathematics that Indigenous students should know and their performance in tests of this is what defines them in regards to their ability to contribute to society.

Assessment and Life Chances

We contend that a continual focus on Indigenous students' poor achievement in national assessments is likely to produce in teachers, policy makers, the general public and Indigenous students themselves a belief that Indigenous students cannot learn or utilise mathematics in their everyday lives. Klenowski (2009) suggested that assessment results were used by politicians and others to present a particular view of the world and who was successful within it.

> Invalid uses of large-scale tests should be avoided because there are ethical and social justice issues at stake. The data from such international comparisons and the purposes for which they are used must be treated with prudence. (p. 82)

Lange and Meaney (2011) provided a discussion of how public discourse around National Assessment Program—Literacy and Numeracy (NAPLAN) contributed to students, including Australian Indigenous students, being considered disadvantaged. In New Zealand, Boustead and Strathdee (2008) discussed the public discourse surrounding boys' performance in schooling. Although their own research suggested that Māori boys progressed more slowly in secondary school than most other boys, they also commented on the need to better understand the contextual factors that affect achievement. The impact of public discourse around mathematics learning on students should not be underestimated. In research in New Zealand, Hāwera and Taylor (2008) showed that students in Māori-immersion schooling perceived mathematics performance to be strongly linked to intelligence. The implication is that those students who do poorly in mathematics tests are likely to see themselves as less intelligent.

It is worrying that Indigenous students perform poorly in tests such as PISA compared with their non-Indigenous peers (Thomson & De Bortoli, 2008). However, extrapolating from Herbert (1995), being Indigenous does not make a person a poor performer in mathematics and thus disadvantaged. Rather as Klenowski (2009) stated:

> If students have not developed certain skills or have not had access to certain knowledge because of their background, gender or indigeneity, then they are at a disadvantage when those skills or that knowledge are valued and assessed in high-stakes tests. (p. 83)

Yet, about half of the research articles that we reviewed started by mentioning the poor performance of Indigenous students in mathematics, usually in relationship to national testing as exemplified by NAPLAN in Australia. On the whole, there was very little questioning of these tests as being valid determiners of students' mathematical capabilities (Baturo, Cooper, Michaelson, & Stevenson, 2008; Howard et al., 2011), although some researchers had critiqued aspects of this testing regime. For example, Edmonds-Wathen (2010) noted that NAPLAN "conceives of mathematics and numeracy as a single entity, independent of culture, language and the different situations in which students use their mathematics" (p. 321). Klenowski (2009) queried the use of multiple choice and short answer questions when alternative methods of assessment may match better not only teacher aims, but also indigenous ways of learning. Teachers of Indigenous students in urban schools in Western Australia noted that their students seemed more able to show their mathematical skills and understandings when working with an educator than they did when completing NAPLAN (Hurst & Sparrow, 2010). They indicated that test literacy was a concern, as did Baturo et al. (2008). Although this was within a context where Indigenous students' conceptual understanding of mathematics was the focus, there is a risk that in order to improve students' results, teachers will teach to the test, especially when those tests are high stakes for teachers and schools.

The continual citing of achievement results from tests such as NAPLAN gives credence to the belief that such tests assess valuable mathematics. By implication, this devalues other mathematical competencies, such as the spatial awareness needed in remote communities. By not testing this mathematical knowledge and skills, remote Indigenous children's backgrounds and foregrounds are devalued. As Klenowski (2009) stated:

> It is important in terms of equity to consider the choice of knowledge and skills selected for the assessments. To achieve equity the curriculum needs to include valued knowledge and skills consisting of different kinds of cultural knowledge and experience, reflective of all groups, not privileging one group to the exclusion of others. (p. 83)

Klenowski (2009) also reiterated Wiliam's (2008) concerns about the effect of translation on the validity of international test items in the Programme for International Student Assessment (PISA). Consistent with Klenowski's critique, Edmonds-Wathen (2011) described a numeracy item from the 2010 NAPLAN test which required Indigenous students, who were additional language learners of English, to understand and make use of the term 'between'. She cited several associations of applied linguists including teachers of English as a second language who stated that the test item assessed students' language fluency rather than their mathematical knowledge.

Acceptance of national tests as valid determiners of students' skills and competencies in mathematics needs to be questioned, even when the results appeared positive for Indigenous students. Within the New Zealand context, the Ministry of Education has been lauding the results of Māori students in Māori-immersion education (Wang & Harkness, 2007). However, an in-depth analysis of the results indicated that performance in external, end-of-high school examinations showed the opposite outcome. Meaney, Trinick, and Fairhall (2011a) illustrated how the very tight timeline for production of the exams has led to a poor translation of the questions from English into *te reo Māori* (the Māori language). Interviews with students who had just completed the bilingual exams described how they had to work between the two languages, although no extra time was allocated for them to do this. In that chapter, Meaney et al. (2011a) suggested that the rhetoric of the Ministry of Education around student success in Māori-immersion schooling was contributing to silencing potential discussions about how to improve the exams so that students from Māori-immersion schools had the best possible opportunities to show what they knew and could do.

Some researchers suggested a link between poor test results and poor life opportunities. Within comments such as "lack of competency in mathematics reduces life chances and being innumerate can be profoundly disabling in every sphere of life including home, work and professional pursuits" (Warren, Cooper, & Baturo, 2009, p. 213), modal verbs such as 'can be' lose their significance. Consequently the cause and effect relationship is accepted as unchallengeable, yet there is little research that shows whether such outcomes are in fact true, especially when competency in mathematics is linked to test results. For example, in all walks of life, there are adults who have difficulties with percentages, yet they live rich and fulfilling lives. There is certainly a need for the research to move on from the standpoint that poor numeracy outcomes are the main factor in preventing Indigenous adults from entering a range of well-paid occupations.

Homogeneity amongst Indigenous Groups

A further issue is the labelling of Indigenous students as though they form a homogenous group. Indigenous students differ to the same degree as other students labelled by their ethnicity (Warren, Baturo, & Cooper, 2010b; Jorgensen, Grootenboer, Niesche, & Lerman, 2010). A focus on a student's indigeneity means that other more important features of a situation could be ignored. For example, in the data produced between 2002 and 2008 by the *Poutama Tau* numeracy professional development intervention for Māori-medium education, Trinick and Stevenson (2009) found that although students' proficiency in *te reo Māori* had some impact on their numeracy performance, the biggest influence was the teacher. Klenowski (2009), citing Wiliam (2008), also highlighted that teacher quality had a greater impact on student performance than any other factor but the variability from this is hidden in cohort comparisons.

For example, discussing Indigenous students as a whole group, Brown (2008) suggested that they were unmotivated to learn mathematics—"low performance and motivation to engage in Western mathematics practices appears to begin in primary school" (p. 94). Yet the evidence used to support this statement is on students' performance only and no evidence is provided about their motivation. At the end of an intervention Warren, Cooper, et al. (2009) were able to show that Indigenous students at one remote school enjoyed participating in mathematics as much as their non-Indigenous peers. Howard et al. (2011) indicated that for Indigenous students to engage in mathematics, teaching practices needed to be stimulating rather than relying on work sheets, particularly as many of these students came from a culture that has tended to be visual and oral rather than print based. Stories about Indigenous students being unmotivated to learn mathematics adds to the deficit discourse around these students. Interpreting research reports and their suggested implications is difficult when details of the backgrounds of Indigenous students and their learning environment are not provided.

Language and Dialects

Another related concern when reading the research reports was an apparent lack of understanding about the differences between language and dialects. Although definitions differ between linguists and the distinctions are not exclusive, on the whole it is considered that languages are not mutually intelligible. Therefore, in Australia it said that there are 50 Indigenous languages still used (Klenowski, 2009). The sounds, the words and the grammatical constructions will be specific to each language. Although there may be some sharing between two languages, there is not enough overlap for speakers of each language to consistently understand each other. *Te reo Māori* is the Indigenous language used throughout Aotearoa/New Zealand, yet there are dialectical differences, which identify the tribal affiliations of the speaker. These differences are not great enough to mean that speakers are unable to understand each other.

In Australia, there are many Indigenous languages that are mutually unintelligible. As well, Creole languages such as Kriol and Tok Pidgin are languages in their own right in that they can perform all the functions of a language. Often they have developed from the intermingling of two historically different languages such as English and an Indigenous language. Although they may contain English vocabulary, generally the grammatical constructions come from the Indigenous language and so Kriol would not be intelligible to English speakers. Unlike Kriol, Aboriginal English is a dialect of English as it can be understood generally by Standard Australian English speakers, although there are differences in phonetics and vocabulary. This difference and its impact on mathematics education programmes was summarised by Watson in 1988.

> There is a major shift in the semantic structure in that gap on the scale between Kriols and Aboriginal English. The coding which underlies Aboriginal languages and Kriols differs profoundly from that which forms the basis of the dialects of Aboriginal English, and other English language dialects ... This difference will be important in mathematics education in those communities that are bilingual with respect to Aboriginal languages/Kriols and English. (p. 257)

Although the terms Kriol and Aboriginal English are used interchangeably (Niesche, 2009; Baturo et al., 2008), Watson (1988) identified the importance of understanding the difference when considering how to support students to gain fluency in the mathematics register of Standard Australian English so that it becomes a tool for learning and thinking mathematics. As Warren and de Vries (2009) stated:

> While Standard Australian English is the discourse of the school, teachers need to create a bridge for young Indigenous students between Aboriginal English and Standard Australian English as they grapple with new language, new concepts and vocabulary presented for numeracy. (p. 162)

In the following sections we review a large number of articles and where necessary refer back to the concerns. Given that more interest and funding is being provided to undertake research with Indigenous communities, especially in Australia, it is imperative that these concerns are addressed explicitly in research.

PEDAGOGICAL PRACTICES THAT SUPPORT INDIGENOUS STUDENTS' LEARNING

A number of research studies have commented on teaching practices and their alignment with Indigenous ways of learning. Warren et al. (2010b) stated that in Australia much of what is written about Indigenous students' learning styles is based on the work done by Stephen Harris in the 1970s (Harris, 1980). Yet as they highlighted, since this time cultural practices may have changed:

> Thus it is conjectured that Harris' work requires revisiting because to simply adhere to his conceptions of the way in which Aboriginals learn is to deny the possibility that Aboriginal cultures and therefore Aboriginal learning styles have failed to change or develop over the last 20 years, a culturally reductionist perception. (Warren et al., 2010b, p. 167)

They saw later research into Indigenous learning styles as indicating that there was an affinity between these learning styles and teaching practices that advocated for supporting socio-constructivist mathematics learning. For example, from their own research, they considered the teachers' use of group work to be in alignment with socio-constructivist principles. Yet, they also considered that the teachers may have chosen to use group work to cater for the diversity of ability levels and behaviour management than because they recognised it as a method for supporting students' mathematics learning. In regards to her work in the Kimberleys, Jorgensen (2009) discussed how group work could support deep learning in mathematics but only if "group work was structured so as to enable learners to talk, debate, contest, clarify, etc their understandings as they engaged with mathematical tasks that were cognitively demanding" (p. 700). She saw group work as resonating with Indigenous ways of knowing and working. Yet she did not document the research that she based this assumption on except to acknowledge Indigenous cultures as being oral.

Warren et al.'s (2010a) research with Indigenous students in three schools was based on information supplied by teachers and it queried Harris' suggestion that Indigenous students learnt by imitation and observation. However, their discussion did not acknowledge that there may be differences in learning styles within a cohort of Indigenous students. Nor did they examine how contexts may contribute to one type of learning opportunity being more appropriate than another. As noted in the section on concerns, urban Indigenous students may learn differently to their peers in remote communities because of the different circumstances in which they live and the activities that they engage in. Certainly some teaching practices such as setting high standards for student achievement are relevant to all students (Hurst & Sparrow, 2010; Meaney, Trinick, & Fairhall, 2011b; Warren, Cooper, et al., 2009), but expecting all students of a particular Indigenous group to learn in the same way is detrimental to those students' learning. For example, Te Maro, Higgins, and Averill (2008) found that both teachers and Māori students felt that manipulatives were helpful for learning mathematics, but they also indicated that labelling all Māori students as being kinaesthetic learners could limit their learning opportunities.

Meaney et al. (2011a) problematised the idea of there being one set of Māori pedagogical practices. Using data collected in one school over a period of more than 12 years, they identified those practices that were used in the school and then interviewed students about their perceptions of their learning. They suggested that in order for pedagogical practices to be improved, it is important that current practices are documented and discussed. As suggested by Warren et al. (2010b), the reification of any set of practices as being appropriate for one group of students can be detrimental to the possibilities for improving opportunities for learning by this diverse group of students. Therefore, it is important that more research is conducted which involves working with and listening to Indigenous students' preferences for their learning.

Contextualising Mathematics and Ethnomathematics

There seems to be an acceptance that contextualising mathematics into real-world situations is likely to support Indigenous students becoming more successful mathematics learners and ultimately test performers (Brown, 2008; Warren, Cooper et al., 2009). Citing the work of others, Ewing (2009) suggested that Indigenous groups were 'high-context' in that they adopt "a holistic, top-down approach to processing information which is situated in the environment within which they interact" (p. 131). As an example, students involved in a block laying course were easily able to see connection between the mathematics they were learning and out-of-classroom opportunities for using it (Cooper, Baturo, Duus, & Moore, 2008). Parents of students in Māori-immersion schooling were able to articulate both the mathematics that their ancestors had engaged in as well as the mathematics that they wanted their children to be able to do in their future lives (Meaney, Trinick, & Fairhall, 2011b). Yet, Hawera and Taylor (2008) found that many students in Māori-immersion schooling almost exclusively related out-of-school mathematics to shopping situations. Although students found contexts engaging, Warren, Cooper et al. (2009) criticised the contexts chosen by the non-Indigenous teachers in their research as "mirror[ing] a very White consumer-centred world" (p. 179). There are two issues present in these discussions. The first is that there is a need for research to determine if contextualisation is valuable for Indigenous students. The second is that if contextualisation is found to be valuable, then teachers may need professional development in how to achieve contextualisation as currently research indicates that contextualisation is rarely utilised.

Warren, Cooper et al. (2009) suggested that contextualising is more about having students perceive their culture as being linked to success in mathematics.

> Contextualization ... means ensuring that students associate mathematics achievement with Indigenality [sic] and that Indigenous staff members and community have a prominent role in the school and mathematics classroom. (p. 223)

Related to the issue of contextualisation is that of ethnomathematics. In considering the place of ethnomathematics in mathematics classrooms, Dickenson-Jones (2008) discussed four different positions for ethnomathematics: (a) replacing academic mathematics; (b) supplementing it; (c) becoming a springboard into academic mathematics; or (d) considering it when planning for learning. All of these positions appear not to consider formal academic mathematics as an example of ethnomathematics in its own right (Meaney, Fairhall, & Trinick, 2008) and that both academic mathematics and mathematical ideas of different cultures can be equally valued. Dickenson-Jones (2008) requested that some thought be given to the outcomes that may result when cultural practices were written into a Western curriculum. In order to preserve aspects of two different knowledge systems, there are many issues to consider. Mathematics ideas can become transformed when they are relocated from a traditional context.

In the various reports, much has been written about the teaching strategies that can support Indigenous students' learning. Yet, little of it is supported by research that does more than reiterate teachers' beliefs about how their students learn. Jorgensen et al. (2010) showed that teachers often espoused beliefs about good teaching practices for their remote Indigenous students but rarely incorporated them into their actual teaching practices. Consequently professional development that is focused on the implementation of good teaching practices is needed. However, this needs to highlight the possibilities of differences both between cultural groups and individual students. As Baturo et al. (2008) stated "[m]any teachers seemed to be of the mind set of just wanting classroom activities or resources rather than wanting to understand the theories behind the ideas" (p. 64). Research needs to evaluate the effectiveness of different teaching approaches and how they can be adjusted to suit specific groups of Indigenous students. Working with students will provide insights into how their culture can be incorporated as part of good teaching practices. Good teaching which does not acknowledge and make use of students' Indigenous cultures may improve students' performance in tests but may also distance them from their home community. On the other hand, teaching which expects students to learn in traditional Indigenous ways may deny them opportunities to be full participants in the modern Western world. Without research undertaken with Indigenous students and their communities, it is unlikely that an appropriate balance in the sorts of learning opportunities will be achieved.

LANGUAGE OF INSTRUCTION

In many research articles, the issue of students' fluency in the language of instruction is mentioned. For example, in relation to developing number understandings, Hāwera and Taylor stated "being able to articulate a mental computation strategy is deemed beneficial for students' learning in mathematics" (p. 23). Yet the choice of the language of instruction for most Indigenous students is a political one (Meaney et al., 2011a). Good reasons can be provided for the language of instruction to be either an international language such as English or the Indigenous language of the students (Barton, 2008). Edmonds-Wathen (2011) described the situation in the Northern Territory where English is the only language that can be used in schools for the first four hours of schooling each day, even when students are not fluent in this language. In Māori-immersion schools, *te reo Māori* is often students' second language but to use English with these students in their mathematics learning would go against the principles for setting up these schools (Meaney et al., 2011a).

The decision about which language to use can change over time. For example, Niesche (2009) suggested in relation to Indigenous students in the Kimberley that students could discuss the mathematics activities in their home language, either an Indigenous language or Kriol, but report back to the whole class in Standard Australian English. However, classroom teachers were reluctant to take up this suggestion. Warren et al. (2010a) also found that teachers acknowledged that their students used a language different to that of Standard Australian English, but made no adjustment to their teaching to accommodate these differences. When multilingual students enter mathematics classrooms and their fluency in other languages goes unrecognised and underutilised, then their learning is likely to be impeded (Barton, 2008).

The incorporation of Indigenous languages in the teaching of mathematics requires much consideration. In an endeavour to develop Fijian mathematics vocabulary to support the learning of Fijian students, Bakalevu (2008) stated that although her language is expressive with a comprehensive grammar it is not always possible to make direct translations because mathematics is conceptualised differently between languages. For example, differences have been found in regard to position and space (Owens, 2010), measurement (Bakalevu, 2008), number (Bakalevu, 2008; Niesche, 2009), time and space (Dickenson-Jones, 2008), volume and mass (Owens & Kaleva, 2008).

Although the choice of language of instruction is often a political one, albeit sometimes taken at the local community level, it is usually teachers who are left to implement these decisions with differing levels of professional development and support. There appears to be a lack of research documenting how teachers support Indigenous students to gain fluency in the mathematics register within the language of instruction. In Papua New Guinea, there is a huge variety of languages (Muke & Clarkson, 2011a). Schools are encouraged to use the local language in the first three years of school. When students moved into Year 3 and primary school English is to be introduced, if it has not been done beforehand, so that by the time students enter Year 6 all teaching is in English. After examining sixteen Year 3 classes in the highlands, Muke and Clarkson (2011b) found that about half of the teaching was done in Tok Pidgin, the common lingua franca, with the remaining teaching time split equally between Wahgi, the local language, and English. The teachers recognised that the students would be able to understand and respond when the teaching was done in a language with which they felt comfortable. At the same time, the teachers were concerned that students would not improve their English unless they used it more frequently. Often it seemed that although English was used only a quarter of the time, the other languages were utilised to introduce new English terms and expressions, thus privileging this language (Muke & Clarkson, 2011a).

The research of Meaney, Trinick, and Fairhall (2009a) provided the most extensive documentation of how teachers adapt their teaching to support students' language learning in mathematics classrooms. They also documented the strategies that teachers employed when they themselves are also learners of the mathematics register in the language of instruction (Meaney, Trinick & Fairhall, 2011c). In the Pacific, many teachers do not teach in their first language and a better understanding is needed of how support can be provided to these teachers.

THE IMPORTANCE OF STRONG RELATIONSHIPS

Much research that focussed on Indigenous students learning mathematics has mentioned the need for the schooling system to foster good relationships between stakeholders. Repetition of this advice appears in research from the last four years. For example, students valued mathematics teachers who encouraged rather than discouraged them (Cooper et al., 2008; Te Maro et al., 2008). For the students in Cooper et al.'s (2008) research, relationships extended to being comfortable to ask for help from family and friends. It also meant that they saw learning as a joint activity that had value because it gave something back to their communities.

There is a need for teachers to be aware of the key features of Indigenous students' culture. In Australia, Hurst and Sparrow (2010) discussed the work of Perso (2009) in developing teachers' cultural competency. In New Zealand schools which taught in English, the use of the Māori language within mathematics lessons was perceived by teachers as being the main culturally responsive way to support Māori culture in a school-wide approach (Te Maro et al., 2008). A non-Indigenous teacher without an appropriate awareness of the impact of cultural differences may label Indigenous students as deficient when they do not match the teacher's expectations of 'normal' behaviour (Howard et al., 2011).

In New Zealand, implementation of the Treaty of Waitangi requires good relations between Māori and non-Māori. The Treaty signed in 1840 between the British Crown and Māori chiefs sets out the joint responsibility for caring for Māori resources including language and culture (Meaney, Trinick, & Fairhall, 2009b). If the Treaty is to be implemented appropriately, then there is a need for both Māori and non-Māori to work together. However, as Averill et al. (2009) documented in a paper on their connected projects to develop pre-service teachers' ability to implement the Treaty of Waitangi, implementation was not easy for non-Māori. The final project involved interviewing three beginning teachers who were deemed the ones who were most likely to implement the principles of the Treaty of Waitangi in their mathematics teaching. All three teachers found it difficult to overcome the established school cultures when attempting to implement culturally responsive mathematics teaching. A lack of awareness of the needs of Māori-medium teachers at a mathematics teacher conference was also documented in a study by Meaney et al. (2009b) in relationship to the Treaty of Waitangi. For Māori language and culture to be supported in mathematics lessons there is a great need for strong relations between Māori and non-Māori who are involved at all levels of mathematics education. This is likely to lead to more open debates about how established teaching practices can be changed so that mathematics teaching becomes more culturally responsive.

Although the need for strong relationships is reiterated in a variety of ways in research carried out in the last four years, more work needs to be done to determine how this can be achieved.

Community and Parent Involvement in Indigenous Students' Learning of Mathematics

It is often acknowledged that one of the most important sets of relationships is between mathematics educators and the Indigenous communities with whom they work. This is because elders who have the respect of their community are usually the knowledge keepers (Owens, 2010). Consequently, when Indigenous cultural ways of knowing are researched or incorporated into mathematics teaching, this needs to be done in collaboration with the elders and families of Indigenous students. Thus, links are developed with students' experiences, culture and home language and this supports their engagement with the curriculum content (Ewing, 2009).

In revising the national curriculum written in the Māori language, McMurchy-Pilkington (2008) described a co-evolving process whereby the New Zealand Ministry of Education fostered dialogue among Māori educators that included elders and the community at both horizontal and vertical levels. There are indications that the Ministry was inclusive of Māori and that more control has been passed over for resourcing, decision making and management of meaning than had happened in the past.

In remote communities in Australia, Indigenous Teaching Assistants (ITA) are recognised as providing a link between schools and communities (Siemon, 2009). In many of these communities, teachers are often new graduates from urban areas who have a lot of goodwill but little understanding of the situations in which they are teaching (Warren et al., 2010b; Howard et al., 2011). In the last four years, more work has been done in gaining ITAs' perspectives about their role (Gervasoni, Hart, Croswell, Hodges, & Parish, 2011) and working with them so that their role in the classroom is more directly linked to students' learning (Siemon, 2009). Warren et al. (2010a) argued for the role of ITAs to be less of an assistant, controlled by the teacher, and more of a partner with the teacher to develop culturally inclusive mathematics teaching. The change in power relations requires support not just from teachers, but also principals and the education system. It will be interesting to see how projects such as this develop over the next four years. As Siemon (2009) wrote:

> This points to the need to engage more openly and reflectively on how and why we act in certain ways and the impact of this on other members of the community. While this prompts the same sort of questions that were raised under the issue of level of support above, it also raises the more general question of how identity and agency operate within activity systems to marginalise and/or position community members in ways which restrict or enhance their participation in the social practices of the community? (p. 231)

Building relationships with students' parents and caregivers is equally as important. Perceptions about the lack of interest of Indigenous parents in their children's education can arise if teachers do not see parents acting in the ways that non-Indigenous parents do (Ewing, 2009). However, as shown by Warren, Cooper et al. (2009), in one school, Indigenous parents were providing homework help to their children just like non-Indigenous parents were. In describing the *Mathematics in Indigenous Contexts* programme, Howard et al. (2011) discussed how parents had supported students' learning at home as well as at school because of the collaborative relationships developed with schools. In New Zealand, Te Maro et al. (2008) described some of the initiatives used by one English-medium school to foster home-school relationships for Māori students. Hurst and Sparrow (2010) advocated that schools seek more involvement from parents even where they lived 30 km from the school with no individual transport. If parental involvement was valued, then it was up to the school to find ways to ensure that it was achieved.

School-community relationships need to be strong if Indigenous students are to gain the most from their schooling. For these relationships to be developed then the establishment of trust between teachers and community members is a good foundation. However, this may not be sufficient, especially in communities where there is a high turnover of teaching staff. Change also needs to occur in how school systems view and support the role of Indigenous staff and encourage Indigenous communities to see schools as 'their' places.

TEACHING MATHEMATICAL TOPICS TO INDIGENOUS STUDENTS

On the whole there is a scarcity of research studies which make connections between Indigenous students and the learning of specific mathematical topics, except for number for which there is much research about Indigenous students and their learning. This may be due to the emphasis placed on number understandings by a Western education system and perceptions that Indigenous students have little contact with Western concepts of number in their home circumstances, a form of 'cultural deprivation' (Warren, de Vries, & Young, 2008). Consequently, the focus has been on 'catching children up' rather than on starting with the strengths that children bring to school.

Number

There has been much discussion about number understandings that Indigenous students bring to school. Within this discussion there has been little mention of the contexts that shape these understandings and how they could be connected to opportunities for learning in schools. Paraide (2008), in discussing the numbering system in the Tolai community stated:

> I will add here that the Tolai complex counting is very much related to how the people lived for generations and used it for their particular needs such as recording, transporting and commercial purposes. The essence of how this counting system works cannot be comprehended easily from a Eurocentric point of view. (p. 76)

For remote Indigenous Australians, the assumption has been that a lack of counting words in their home languages was contributing to them arriving at school with limited facility for engaging with Western mathematical understandings. However, research by Butterworth and Reeve (2008) queried this assumption by finding that "quantification and computation may not depend on these words per se" (p. 456). They found that children whose home languages did not include counting words used a variety of strategies to match different contexts, to recall amounts including by reproducing spatial arrangements.

The issue is not that traditional Indigenous cultures did not determine amounts of things but rather that quantifying may have been done in ways that are different to Western cultural norms and thus remain unrecognised by Westerners. There also remains an issue of assuming that all Indigenous groups share the same set of cultural experiences when clearly this is not the case. Warren, de Vries, and Cole's (2009) research on subitising found "a tentative conclusion is that the ability of Indigenous students varies from context to context, and that for some groups of Indigenous students, the ability to subitise is indeed superior but for others it is not" (p. 51). It is problematic if all Indigenous students are considered to share similar mathematical experiences, understandings and skills.

In Australia, test results are used to indicate that Indigenous students begin school behind their non-Indigenous peers with suggestions that such gaps continue throughout their school lives (Warren & de Vries, 2009). Often the extrapolations come from loose connections with overseas research on students from low socioeconomic backgrounds because there is no research on this in Australia. Warren and de Vries (2009) conducted a study, with a very small number of students, about the value of formal teaching of mathematics to 4-year-olds. Unsurprisingly, they found that students involved in the project could respond to questions with mathematical knowledge. The researchers suggest that more Indigenous children would benefit from formal mathematics education being introduced to them at an earlier age. Warren and de Vries (2009) advocated that "learning from older peers, along with explicit teacher-directed learning, all within a play-based environment, provide the most effective context for pre-prep Indigenous students for developing early numeracy understandings" (p. 8). Such a response only goes so far in recognising the complexity of the situation and does not allow for other possibilities such as providing teachers of young children with better skills for working with the diverse range of mathematical knowledge and skills that they find in their classrooms.

In New Zealand, many Māori-immersion schools have continued to be involved in a major numeracy professional development project, *Poutama Tau*. Several studies (Trinick & Stevenson, 2010, 2009) used data from 2002 to show that students whose teachers are involved in this project are progressing through the stages of the Number Framework. When Year 4 students involved in the Numeracy Project were assessed on a paper-and-pencil test, Assessment Tool for Teaching and Learning (asTTle), they were shown to be close to the mean for Māori-medium students on the number strand but below on the algebra strand (Trinick, 2009). Other mathematical topics were not assessed. Year 7 students performed above the mean in Number and close to the mean for Algebra. This suggests that although the focus on number in the early years may have contributed to a poorer performance in algebra at Year 4, this was overcome by the time that students were completing primary school and about to enter high school when algebra becomes more of a focus. Although only a small study, Hāwera and Taylor (2009) showed that students who attended schools participating in *Poutama Tau* used a similar range of strategies for solving problems requiring the use of addition, subtraction and multiplication as students from schools not involved in *Poutama Tau*. They also found that students at three out of the four schools were unable to attempt the multiplication problem. These results suggest that the usefulness of *Poutama Tau* needs further investigation. It may be that students would learn a variety of different strategies for solving problems without their teachers being involved in

Poutama Tau and even if their teachers are involved then the students' difficulties with multiplication may not necessarily be overcome. Researchers working in this area (Hāwera & Taylor, 2009; Trinick & Stevenson, 2010) acknowledged the need for further research to understand inconsistencies in the results which would contribute to improving students' numeracy understandings.

Probability

As discussed in an earlier section, Bakalevu (2008) described some of the difficulties of talking about Western concepts of mathematics in Indigenous language. In *te reo Māori*, probability has been one of the most difficult areas to develop mathematical language (Meaney et al., 2011a). Although Maangi, Smith, Melbourne, and Meaney (2010) provided suggestions on how to introduce probability concepts to young children, many of the ideas are based on how the ideas would be taught to young children. This seems to be an area where a better understanding of the interaction with traditional cultural understandings needs more exploration.

Space and Geometry

Some cultures contain many words for a mathematical idea or concept and for degrees of the concept, for example, positioning and space, which can highlight "how limited a Eurocentric view of mathematics can be" (Owens 2010, p. 460). Owens suggested that the richer traditional Indigenous space systems be incorporated into mathematics classrooms. Connections that can be made between these systems and the Western system would be advantageous, especially for Indigenous students. Edmonds-Wathen's (2010, 2011) research on the spatial forms of reference found in one Australian Indigenous language, Iwaidja, seemed extremely promising. It is not just about understanding the differences between Iwaidja and English ways of talking about location and direction but showing what sorts of links are likely to be most promising is supporting Indigenous students to be bicultural.

CONCLUSION

In the last four years, the amount of research on Indigenous students learning mathematics has increased, especially in Australia where funding has soared. Much of the research proposes promising ideas for how Indigenous students' culture can be the supportive base from which mathematics learning can continue to develop. In particular the recognition of the potential from re-evaluating the role of Indigenous Teaching Assistants is likely to allow for more links to be made between home and school (in both directions). This will support students to view developing mathematical understanding as a strength in their identity as a bi-cultural member of their home community and that of the wider world.

However, increased funding can be considered as a two-edged sword in that it is often tied to evaluating programmes that will increase Indigenous students' achievement. In itself this seems like an extremely appropriate outcome, but the ability to ascertain whether this outcome has been achieved is tied explicitly to increases in test results. As discussed in an earlier section, the assumption that these tests assess valuable mathematical knowledge needs to be queried. One issue is that these tests fail to acknowledge, for the most part, Indigenous mathematical knowledge. Yet Indigenous students may need this knowledge to remain strong participants in their community's culture. If this 'other' mathematical knowledge is at best ignored and at worst disparaged then it is unlikely that Indigenous students will gain Western mathematical knowledge and still remain strong in their culture.

Another result of linking funding to increases in test results is that many research papers about Indigenous students learning mathematics begin with statistics about their underachievement. Consequently, these students continue to be positioned as abnormal and only achieving normality by gaining similar results of their non-Indigenous peers. Poor achievement in highly valued tests is a concern but it is even more of a concern if these results are used in ways that continue to disadvantage Indigenous students.

In other parts of Australasia, research around issues to do with Indigenous students learning mathematics seems to depend on a small number of researchers. Apart from one paper from the Pacific, we could find no other information about mathematics education in this region of Australasia. Research in Papua New Guinea also is done by just a handful of researchers, making this research precarious. In New Zealand, there has been a significant amount of research in Māori-immersion schools, some of which has been supported by Ministry of Education funding of the *Poutama Tau* project. However, only 16% of Māori students attend Māori-immersion schools (Averill et al., 2009) and there appears to be a very limited amount of research occurring in schooling where English is the medium of instruction.

This review illustrates that much work has been done in the last four years in relation to Indigenous students learning mathematics. Nevertheless, there is a continuing need for more research, especially which includes Indigenous researchers. Since the last review the number of Indigenous researchers has increased. Yet, the majority of research reported was done by non-Indigenous researchers. There is a need for research communities such as MERGA to consider ways to support Indigenous researchers in all parts of the Australasian area whose funding and/or support networks are limited. In order for social justice to be fulfilled, this cannot be conceived of as a problem for other people, but one that MERGA itself must tackle head on.

REFERENCES

Averill, R., Anderson, D., Easton, H., Te Maro, P., Smith, D., & Hynds, A. (2009). Culturally responsive teaching of mathematics: Three models from linked studies. *Journal of Research in Mathematics Education, 40*(2), 157–186.

Bakalevu, S. (2008, July). *Coping in multicultural and multilingual mathematics classes*. Paper presented at International Congress for Mathematics Education (ICME 11), Topic Study Group 26. Monterrey, Mexico: ICME. Retrieved from tsg.icme11.org/document/get/609

Barton, B. (2008). *The language of mathematics: Telling mathematical tales*. New York, NY: Springer.

Baturo, A., Cooper, T., Michaelson, M. T., & Stevenson, J. (2008). Using national numeracy testing to benefit Indigenous students: Case studies of teachers taking back control of outcomes. In M. Goos, R. Brown, & K. Makar, (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, p. 59–66). Brisbane, QLD: MERGA.

Boustead, T., & Strathdee, R. (2008). The gender ethnic divide: Mathematics and English progress of Pakeha, Māori, and Pasifika students. *New Zealand Journal of Educational Studies*, *43*(1), 33–49.

Brown, K. (2008). Employing mathematical modelling to respond to Indigenous students' needs for contextualised mathematics experiences. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 93–99). Brisbane, QLD: MERGA.

Butterworth, B., & Reeve, R. (2008). Verbal counting and spatial strategies in numerical tasks: Evidence from Indigenous Australia. *Philosophical Psychology*, *21*, 443–457.

Cooper, T., Baturo, A. R., Duus, E., & Moore, K. (2008). Indigenous vocational students, culturally effective communities of practice and mathematics understanding. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 377–384). Morelia, Mexico: Cinvestav-UMSNH.

Dickenson-Jones, A. (2008). Transforming ethnomathematical ideas in Western mathematics curriculum texts. *Mathematics Education Research Journal, 20*(3), 32–53.

Edmonds-Wathen, C. (2010). The everyday language of mathematics: Investigating spatial frames of reference in Iwaidja. In M. Pinto & T. Kawasaki (Eds.), *Optimising student understanding in mathematics* (Proceedings of the 34th conference of the International Group for the Psychology of Mathematics Education, pp. 321–327). Belo Horizonte, Brazil: PME.

Edmonds-Wathen, C. (2011). Locating the learner: Indigenous language and mathematics education. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 217–225). Adelaide, SA: AAMT & MERGA.

Ewing, B. (2009). Torres Strait Island parents' involvement in their children's mathematics learning: A discussion paper. *First Peoples Child & Family Review, 4* (2), 129–134.

Gervasoni, A., Hart, A., Croswell, M., Hodges, L., & Parish, L. (2011). Insights from Aboriginal teaching assistants about the impact of the bridging the numeracy gap project in a Kimberley Catholic school. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 306–314). Adelaide, SA: AAMT & MERGA.

Harris, S. (1980). *Culture and learning: Tradition and education in North East Arnhem Land*. Darwin, Australia: Professional Services Branch, Northern Territory Department of Education.

Hāwera, N., & Taylor, M. (2009). Māori and mathematics: Nā te mea he pai mō tō roro! (Because it is good for your brain). In Ministry of Education (Ed.), *Te Poutama Tau evaluation report: Research findings in Pāngarau for years 1–10* (pp. 36–48). Wellington, New Zealand: Ministry of Education.

Herbert, J. (1995). Gender issues for Aboriginal and Torres Strait Islander girls—Exploring issues for Aboriginal and Torres Strait Island boys. *The Aboriginal Child at School*, *23*(2), 9–16.

Howard, P., Cooke, S., Lowe, K., & Perry, B. (2011). Enhancing quality and equity in mathematics education for Australian Indigenous students. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 365–378). New York, NY: Springer.

Hurst, C., & Sparrow, L. (2010). The mathematical needs of urban Indigenous primary children: A Western Australian snapshot. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 272–279). Fremantle, WA: MERGA.

Jorgensen, R. (2009). The use of home language in the mathematics classroom. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 700–703). Palmerston North, NZ: MERGA.

Jorgensen, R., Grootenboer, P., Niesche, R., & Lerman, S. (2010). Challenges for teacher education: The mismatch between beliefs and practice in remote Indigenous contexts. *Asia-Pacific Journal of Teacher Education*, *38*(2), 161–175.

Klenowski, V. (2009). Australian Indigenous students: Addressing equity issues in assessment. *Teaching Education*, *20*(1), 77–93.

Lange, T., & Meaney, T. (2011, February). Becoming disadvantaged: Public discourse around national testing. *Proceedings of the 7th Congress of the European Society for Research in Mathematics Education* (n.p.). Rzeszów, Poland: CERME.

Maangi, H., Smith, H., Melbourne, P., & Meaney, T. (2010). Pēnei i au te āhua? Is this me? Teaching young children mathematics in Kura Kaupapa Māori. In R. Averill, & R. Harvey (Eds.), *Teaching primary school mathematics and statistics: Evidence based practice* (pp. 223–236). Wellington, New Zealand: NZCER.

McMurchy-Pilkington, C. (2008). Indigenous people: Emancipatory possibilities in curriculum development. *Canadian Journal of Education, 31*(3), 614–638.

Meaney, T., Fairhall, U., & Trinick, T. (2008). The role of language in ethnomathematics. *Journal of Mathematics and Culture*, *3*(1). Retrieved from http://nasgem.rpi.edu/index.php?siteid=37&pageid=543

Meaney, T., McMurchy-Pilkington, C., & Trinick, T. (2008). Mathematics education and Indigenous students. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W-T. Seah, P. Sullivan, & S. Willis (Eds.), *Research in mathematics education in Australasia 2004–2007*, (pp. 119–139). Rotterdam, The Netherlands: Sense.

Meaney, T., Trinick, T., & Fairhall, U. (2009a). Learning how to represent mathematics on paper. *Australian Primary Mathematics Classroom, 14*(1), 21–27.

Meaney, T., Trinick, T., & Fairhall, U. (2009b). 'The conference was awesome': Social justice and a mathematics teachers' conference. *Journal of Mathematics Teacher Education, 12*, 445–462.

Meaney, T., Trinick, T., & Fairhall, U. (2011a) *Meeting language challenges in an Indigenous mathematics classroom*. New York, NY: Springer.

Meaney, T., Trinick, T., & Fairhall, U. (2011b). One size does NOT fit all: Achieving equity in Māori mathematics classrooms [Special issue]. *Journal of Research in Mathematics Education, 42*(5), 536.

Meaney, T., Trinick, T., & Fairhall, U. (2011c, September). *Teaching mathematics in a second language*. Paper presented at the International Commission on Mathematical Instruction (ICMI) Study 21 Conference, São Paolo, Brazil.

Muke, C., & Clarkson, P. (2011a). Teaching mathematics in the Papua New Guinea Highlands: A complex multi lingual context. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 540–547). Adelaide, SA: AAMT & MERGA.

Muke, C., & Clarkson, P. (2011b, September). *Teaching mathematics in the land of many languages*. Paper presented at the International Commission on Mathematical Instruction (ICMI) Study 21 Conference, São Paolo, Brazil.

Niesche, R. (2009). The use of home language in the mathematics classroom. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 704–707). Palmerston North, NZ: MERGA.

Owens, K. (2010). Changing our perspective on space: Place mathematics as a human endeavour. L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 454–461). Fremantle, WA: MERGA.

Owens, K., & Kaleva, W. (2008, July). *Indigenous Papua New Guinea knowledges related to volume and mass*. Paper presented at the International Congress on Mathematics Education (ICME 11) on The Role of Ethnomathematics in Mathematics Education. Retrieved from http://dg.icme11.org/document/get/311

Paraide, P. (2008). Number in the Tolai culture. *Contemporary PNG Studies*, *9*, 69–77.

Perso, T. F. (2009). *Pedagogical framework for cultural competency*. Retrieved from http://makeitcount.aamt.edu.au/Resources/Cultural-competency

Siemon, D. E. (2009). Developing mathematics knowledge keepers—Issues at the intersection of communities of practice. *Eurasia Journal of Mathematics, Science & Technology Education*, *5*(3), 221–234.

Te Maro, P., Higgins, J., & Averill, R. (2008). Creating strong achievement gains for Māori students in English-medium mathematics classrooms. *Poutama Tau evaluation report 2007: Research findings in Pāngarau for years 1–10* (pp. 49–61). Wellington, New Zealand: Ministry of Education.

Thomson, S., & De Bortelli, L. (2008). *Exploring scientific literacy: How Australia measures up. The PISA 2006 survey of students' scientific reading and mathematical literacy skills*. Canberra, Australia: OECD Programme for International Student Assessment. Retrieved from http://research.acer.edu.au/ozpisa/2

Trinick, T. (2009). Te Poutama Tau student performance in asTTle. In Ministry of Education (Ed.), *Poutama Tau evaluations 2008: Research findings in Pāngarau for years 1–10* (pp. 13–21). Wellington, New Zealand: Ministry of Education.

Trinick, T., & Stevenson, B. (2009). Longitudinal patterns of performance: Te Poutama Tau. In Ministry of Education (Ed.), *Poutama Tau evaluations 2008: Research findings in Pāngarau for years 1–10* (pp. 1–12). Wellington, New Zealand: Ministry of Education.

Trinick, T., & Stevenson, B. (2010). Evaluation of Te Poutama Tau: Māori-medium numeracy project 2003–2009. In Ministry of Education (Ed.), *Findings from the NZ numeracy development projects 2009* (pp. 72–87). Wellington, New Zealand: Ministry of Education.

Wang, H., & Harkness, C. (2007). Senior secondary students' achievement at Māori medium schools—2004–2006 fact sheet. Wellington, New Zealand: Ministry of Education.

Warren, E., Baturo, A. R., & Cooper, T. J. (2010a). Power and authority in school and community: Interactions between non-Indigenous teachers and Indigenous Teacher Assistants in remote Australian schools. In J. Zajda, & M. A. Geo-Jaja (Eds.), *The politics of education reforms: Globalisation, comparative education and policy research 9* (pp. 193–207). New York, NY: Springer.

Warren, E., Baturo, A. R., & Cooper, T. J. (2010b). Pedagogies to support Indigenous students' mathematical learning in rural and remote Queensland classrooms. In J. Zajda (Ed.), *Globalization, education and social justice* (pp. 161–184). New York, NY: Springer.

Warren, E., Cooper, T. J., & Baturo, A. (2009). Bridging the educational gap: Indigenous and non-Indigenous beliefs, attitudes and practices in a remote Australian school. In J. Zajda, & K. Freeman (Eds.), *Race, Ethnicity and Gender in Education* (pp. 213–226). New York, NY: Springer.

Warren, E., & de Vries, E. (2009). Young Australian Indigenous students' engagement with numeracy: Actions that assist to bridge the gap. *Australian Journal of Education*, *53*(2), 159–175.

Warren, E., de Vries E., & Cole, A. (2009). Closing the gap: Myths and truths behind subsitisation. *Australiasian Journal of Early Childhood*, *34*(4), 46–5.

Warren, E., de Vries, E., & Young, J. (2008). The impact of early numeracy engagement on four-year-old Indigenous students. *Australian Journal of Early Childhood*, *33*(4), 2–8.

Watson, H. (1988). Language and mathematics education for Aboriginal-Australian children. *Language and Education, 2*, 255–273.

Wiliam, D. (2008) International comparisons and sensitivity to instruction, *Assessment in Education: Principles, Policy & Practice, 15*(3), 253–57.

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SUPPORTING EXCEPTIONAL STUDENTS TO THRIVE MATHEMATICALLY

Key words: learning difficulties; giftedness; catering exceptionality.

INTRODUCTION

Providing an appropriate education for exceptional students in mathematics is mandated in education policy in Australasia (Australian Curriculum, Assessment and Reporting Agency (ACARA), 2010; Ministry of Education, 2009; 2011) but is a challenge for teachers and schools. 'Exceptional students' refers to two distinct populations, namely those who are gifted in mathematics and have the capability to perform very highly compared to age peers and those who experience learning difficulties in mathematics and may underperform (Diezmann, Lowrie, Bicknell, Faragher, & Putt, 2004).

One indicator of the effectiveness of education for diverse student populations and a reality check on the rhetoric of 'a quality education for all' promoted by Australasian education authorities is the performance of exceptional students on international tests. In general, Australian and New Zealand students perform significantly above the Organisation of Economic Co-operation and Development (OECD) average on the Programme for International Student Assessment (PISA) and the Trends in Mathematics and Science Study (TIMSS). However, an examination of Australasian results reveals three concerns. First, in the 2007 TIMSS assessment, 30% of Year 4 and Year 8 Australian students achieved at or below the lowest benchmark in mathematics overall (Thomson, 2010). According to Thomson (2010), "these data indicate that there is a substantial proportion of students exhibiting poor levels of mathematical understanding in Australian schools at all year levels" (p. 77). Second, although New Zealand students performed reasonably well against the international benchmarks on the 2007 TIMSS, the results showed relatively high disparities in achievement for diverse students (Caygill & Kirkham, 2008). Walshaw (2011) argued that such trends in systemic underachievement "provide a sobering counterpoint to claims of equitable learning opportunities" (p. 91) for diverse students in New Zealand schools. Third, although the 2009 PISA results show that the proportion of Australian and New Zealand students ranked at Level 5 or 6 (two highest levels) was above the OECD average, there was a statistically significant drop of 4% in Australian students' results from 2003–2009 (Thomson, De Bortoli, Nicholas, Hillman, & Buckley, 2011). This decline adds to the concern that Australia is not doing enough to prepare adequate numbers of students with the high level proficiency needed to

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train for mathematically-oriented careers, including teaching (Ainley, Kos, & Nicholas, 2008). The performance of New Zealand students on the higher proficiency levels was statistically similar over this six-year period (Telford, 2010). Thus, the international mathematics test data indicated that the performance of gifted students in Australia and students with learning difficulties (LD) in Australasia needs attention.

According to Gervasoni and Lindenskov (2011), underperformance, quality learning and the teaching environment are inextricably linked: "[Students can] underperform in mathematics due to their exclusion from quality mathematics learning and teaching environments necessary for them **to thrive mathematically**" (p. 307) (emphasis added). Underperformance is not restricted to students with learning difficulties but also affects some gifted students (Al Hmouz, 2008). To understand better educational environments in which exceptional students thrive, we examine research on the gifted and those with LD. We then discuss the issue of ability grouping in relation to all exceptional students. We conclude with a comment on the progress of research since the last review on exceptional students in mathematics (Diezmann et al., 2004).

CONTEMPORARY ISSUES IN EDUCATING MATHEMATICALLY GIFTED STUDENTS

In this critique of research, we explore five broad themes: (a) who are the gifted and how do they achieve; (b) understanding the mathematically gifted; (c) educational provision for the mathematically gifted; (d) the roles of teachers and parents of gifted students; and (e) cultural influences on performance and motivation.

Who Are the Gifted and How Do They Achieve?

The defining characteristics of mathematically gifted students are their interest in mathematics combined with advanced reasoning and problem-solving abilities (Bicknell, 2009). These characteristics are part of the Intellectual Domain in Gagné's (2009) *Differentiated Model of Giftedness and Talent* (DMGT). Consistent with Gagné's view, we use the term 'gifted' rather than 'gifted and talented' when referring to students who have the potential to achieve highly in mathematics. According to Gagné, giftedness is an innate ability in one or more domains (e.g., intellectual), which is supported or hindered by environmental factors (e.g., teachers), intrapersonal factors (e.g., motivation), and chance factors, that optimally develops into demonstrated talent. Despite its wide acceptance in Australasia, Gagné's DMGT along with other theories of giftedness have been critiqued because "the theories were unable to provide a clear relationship between gift and achievement, rendering the conceptions (or theories) of giftedness incapable of being verified" (Phillipson & Callingham, 2009, p. 33). A contemporary theory that purports to address these concerns is Ziegler's (2005) *Actiotope Model of Giftedness* (AMG) which "describes an individually focused systems approach to achievement eminence together with a focus on the continual development of action repertories" (Phillipson & Callingham, 2009, p. 33). The term 'actiotope' refers to how individuals respond to and influence the environment and evolve over time in working towards excellence and *Achievement Eminence* (AE). According to Ziegler (2005) and Phillipson and Callingham (2009), AE commences with six key mathematical antecedents of action repertoires, namely numerosity, use of number words, additive and subtractive expectations, ordinal numbers and magnitude representations. In turn, these six action repertoires involving memory, computation, logical reasoning, spatial cognition, creativity, and intuition lead to AE. In addition to the action repertoires, Phillipson and Callingham argued that there is a need to consider the contribution of affect within the subjective action space which includes motivation and goals, beliefs about and attitudes towards mathematics, and self-efficacy and self-esteem. Although Gagné and Ziegler differed in their explanation of the mechanism for achievement, proposing either *development* or *evolution* respectively, they concurred that appropriate education is of paramount importance for the achievement of the gifted.

Understanding the Mathematically Gifted

The identification of mathematically gifted students is the first step in their educational provision. However in a case study of 15 New Zealand students from Year 6 to Year 8, Bicknell (2009) found that despite the documentation of identification processes in their school policies, teachers in the study schools did not have a sound understanding of how to identify gifted students. Bicknell's finding is significant in that these teachers worked within a context in which information *was* available about giftedness and there *was* a policy mandate for identification and provision. However, in a study of 239 high school teachers, Chesserman (2010) found that teachers with the most positive attitude towards the gifted were female and had training and responsibility for the gifted.

Theorists posit that with appropriate educational provision, mathematical giftedness will develop or evolve and manifest as *talent* (Gagné, 2009) or *Achievement Eminence* (AE) (Phillipson & Callingham, 2009). One indicator of talent or AE during school years is the award of medals in national and international competitions. Leder (2008) conducted a survey of respondents 79 (74 males; 5 females) medallists in the Australian Mathematics Competition (1978–2006) who were born between 1960 and 1994. Survey questions probed their work habits, motivations, careers and backgrounds. Over 60% of medallists identified mathematics as their favourite subject with their actual or intended career in mathematics or a related field. Medallists' liking for mathematics appeared to be closely associated with engagement in challenging intellectual tasks and for some an interest in working with like-minded peers (Leder, 2008). These findings mirror views in the international gifted literature that the hallmark of quality provision for the gifted includes substantial challenge and opportunity for working with capable peers.

Educational Provision for Gifted Students

Educational provision for gifted students involved some form of curriculum differentiation which is the "measures taken by educators to provide for high ability students" (Kronborg & Plunkett, 2008, p. 19). These measures typically referred to modifications to the process, product, content or learning environment. Kronborg and Plunkett (2008) argued that the concept of differentiation is not well understood by educators and there is a lack of guidance in the literature. Additionally, determining the effectiveness of differentiation has been problematic "due to many forms and interpretations of 'differentiation'" (Kronborg & Plunkett, 2008, p. 19). Our review endorses the issues raised by Kronborg and Plunkett, identifying few studies of curriculum differentiation published in the review period on disparate topics. These studies related to technologies, acceleration, and mathematics competitions, which are discussed here, and ability grouping, which is discussed later.

Technologies abound but there is a dearth of research on the appropriateness of using technologies to support gifted students' learning. Using electronic delivery of content, Maxwell (2008) investigated "whether gifted students learn more effectively under guided discovery design or example based instruction" (p. iv). The participants in the study were a total of 155 gifted and non-gifted students in Years 7, 8 or 9 at three Australian metropolitan high schools. This study comprised a series of three experiments focused on general problem solving ability to investigate the hypothesis that "as students advance from novice state to expert in particular domains of learning, ... students would benefit from worked example instruction to more efficient learning in guided discovery mode" (Maxwell, 2008, p. iv). This hypothesis was rejected indicating that there was no expertise-reversal effect as postulated in cognitive load theory. This finding questions the efficacy of educational strategies for gifted students that do not involve substantial teacher interaction and support (Maxwell, 2008). Thus, there is a need for research to determine the optimal type and timing of support and interaction and the role that technologies can play in this process.

Acceleration is often proposed as a key strategy for students whose mathematical capability is substantially above their age peers and is variously endorsed and practised in Australasia. Typically, acceleration involves studying work designed for older students. Hannah, James, Montelle, and Nokes (2011) conducted a large scale study of 400 New Zealand secondary school students who completed a University of Canterbury (UC) course, Maths199 Advancing in Mathematics (AIMS) that was designed for high-performing secondary students. Upon successful completion of this course at school, the students gained credit towards a UC degree. To test the efficacy of acceleration on success at university, Hannah et al. compared the academic performance of a group of 99 accelerated students who completed AIMS successfully with a group of non-accelerated students. Each accelerated student was matched randomly with a non-accelerated student who had achieved identical grades in the equivalent course to AIMS in the following year. Overall, at university, accelerated students tended to choose a broader range of areas of study, specialise earlier and take additional mathematics courses. In terms of academic performance, accelerated non engineering students outperformed their non accelerated peers. By contrast, accelerated engineering students underperformed compared to their non accelerated peers. The higher workload of engineering students was the explanation proposed for the achievement differences between engineering and non-engineering students (Hannah et al., 2011). Additionally, Hannah et al. reported that despite similarity in grade point averages in their early years of tertiary studies, accelerated students outperformed their non-accelerated peers in scholarship awards. They proposed that this might be because accelerated students were more independent, ambitious or aware of scholarship options. Based on the findings of their study, Hannah et al. argued that enrichment (extending the curriculum laterally) as well as acceleration should be considered as a strategy for expertise development in gifted tertiary students. However, research on the effectiveness of enrichment needs to be investigated before it is considered a viable alternative. Unlike acceleration, which utilises existing content and organisational structures, enrichment requires supplementary content produced or taught by knowledgeable teachers and structural modifications (e.g., withdrawal from class). A secondary school study (Kronborg & Plunkett, 2008; Kronborg, Plunkett, Kelly, & Urquhart, 2008) which included accelerated content is discussed shortly in relation to ability grouping. Although no research studies were identified on the acceleration of primary school aged students, it is a relatively uncommon practice.

Mathematics competitions are popular across primary and secondary schools. According to Bicknell (2008), the roles of mathematics competitions included (a) opportunities for the identification of gifted students through performance; (b) personal achievement and comparison to others; (c) showcasing talent; (d) external motivation from selection, certifications and awards; and (e) learning about the nature of competitions. Bicknell investigated students, teachers and parents' perceptions about mathematics competitions. The study considered 15 competition participants aged between 10 and 13 years-of-age, 15 of their parents, and 13 teachers who had taught these students over a two-year period. Bicknell reported that all favoured competitions but for different reasons. Teachers "felt that gifted students **thrive** on mathematics competitions ... but would not, however, use them for a student who found them threatening" (emphasis added) (Bicknell, 2008, p. 18). Students generally favoured mathematics competitions and liked team competitions and doing the same competitions in subsequent years because they were familiar with the expectations. Parents reported on students' enjoyment in participating in mathematics competitions and pride in their achievements. In summary, for participants in mathematics competitions, there is the benefit of working on challenging tasks within a natural ability grouped cohort. Leder (2008) reported that competition medallists valued extracurricular competitions as learning experiences above schoolwork including opportunities to work with like-minded peers. Leder's (2008) and Bicknell's (2008) studies highlight the value of mathematics competitions for students who have been engaged in preparation for mathematics competitions over a sustained period. However, research is needed on the effectiveness of participation in a one-off competition.

The Roles of Teachers and Parents of Gifted Students

Recent research indicated that both teachers and parents play a particularly important role in gifted students' learning in four ways; perceptions of intelligence, fostering of interests, providers of education and support, and advocates.

First, whether or not teachers and parents hold an entity (intelligence is fixed) or incremental (intelligence is malleable) view of intelligence impacts achievement (Thomas, 2008; Zhao & Singh, 2010). Thomas (2008) investigated these factors in a study across four disciplines including mathematics and reported that teachers' viewpoints were impacted by a student's gender: "Teachers attribute male success to ability and failure to lack of effort, and female success to effort and failure to a lack of ability" (p. 11). The perception that intelligence is perceived as more malleable for males than females needs to be further researched.

Second, teachers can influence gifted students' career decisions through the interests they foster. Watters (2010) investigated the mediation of teachers in the career decision-making of gifted students through surveys of 200 of the highest achieving students in one educational jurisdiction and follow-up interviews with 20 students. The most influential attributes of teachers identified by students of the Sciences were having a connection to the students (69%), the content (54%) and teachers' knowledge (38%) (Watters, 2010). The importance of the content that is presented and the teachers' knowledge raises a concern for the adequacy of appropriately qualified mathematics teachers.

Third, parents play multiple roles in educating and supporting their children. In her study of 15 New Zealand case study students (Years 6 to 8), Bicknell (2009) identified a range of critical roles parents played as "motivators, resource providers, monitors, mathematics content advisors, and mathematical learning advisors" (p. i). Realistically, the parents' capacity to provide this kind of support for their mathematically gifted children would to some extent depend on their own education and career. There does appear to be a link between high mathematics performance and parents' tertiary education. Leder's (2008) study of mathematical medallists suggests that the tertiary education of at least one parent was advantageous for high level mathematics achievement. It is interesting to note that the qualification did not need to be in mathematics or an aligned discipline to be of benefit.

Finally, both teachers and parents are likely to play a critical role when gifted students' transition from one school to another. In a study of 15 gifted students, Bicknell (2009) identified school transfer as a point of vulnerability in the provision for gifted students particularly if the reception school adopts the "practice of *tabula rasa* or fresh start" (p. 244). She proposed that "schools should at least take the information from the sending school in to (sic) consideration as part of the multiple method approach to identification [of gifted students]" (p. 244). The role of parents as advocates for their children during such transitions needs investigation.

CULTURAL INFLUENCES ON EXCEPTIONAL STUDENTS

International mathematics testing has highlighted achievement differences across countries with Chinese students from China, Taiwan and Hong Kong outperforming students from many Western countries. Using a cross-cultural case study, Zhao and Singh (2010) investigated why Chinese-Australian (C-A) students outperform their Anglo-Australian (A-A) peers in mathematics. The study was conducted in a public primary school which had an enrolment of over 800 students, a high proportion of whom (40%) identified as coming from a Chinese background. The data comprised observations of classroom practice, student work samples, examination papers and interviews with staff and parents. Zhao and Singh's key finding was that "Chinese-Australian students had high motivation for mathematics learning which were encouraged by their parents accordingly. The Chinese-Australian parents had high expectations for their children's mathematics learning and established high academic demand for them" (p. 83). An explanation at least in part for the C-A students' motivation and the C-A parents' expectations are the similarities and differences between the C-A and A-A parents' perspectives on three educational issues (Zhao & Singh, 2010). First, both C-A and A-A parents agreed that English was important in the primary years; however, only the C-A parents believed that learning mathematics was important in primary schooling. Second, both C-A and A-A parents supported homework and reported checking it; however, only C-A parents were supportive of using after-school tutoring or a coaching school which was opposed by A-A parents. C-A parents also coached the children themselves. Third, both C-A and A-A parents recognised that it was difficult for students to be admitted to Opportunity Classes which cater for a limited number of children in Years 5 and 6. C-A parents encouraged their children to work towards admission, seeing it as a pathway for entry to a Selective High School (NSW and WA), university admission, and future job prospects. In contrast, some A-A parents told their children not to be concerned about admission because only a limited number of children were accepted while other parents were not aware that such classes existed. The knowledge of how a particular cultural perspective impacts positively on students' motivation and achievement suggests that research is needed to investigate whether the expectations and support of C-A parents are transferable to other cultural groups and whether there are other cultural supports or inhibitors for motivation and achievement.

CONTEMPORY ISSUES IN EDUCATING STUDENTS WITH LEARNING DIFFICULTIES IN MATHEMATICS

Students with Learning Difficulties—Who Are They?

The term 'learning difficulties in mathematics' is often applied to students who have a "history of persistent difficulty and lack of success in school learning in this subject area" (Graham, Bellert, & Pegg, 2007, p. 172). Current prevalence levels estimate between 6–8% of school-aged students experience difficulties in learning basic mathematical concepts and skills (Finnane, 2008).

Since the publication of the previous MERGA research review chapter (Diezmann et al., 2004) on mathematical learning difficulties (MLD), learning difficulties (LD) continues to be a topic "beset by definitional difficulties" (Knight, Bellert, & Graham, 2008, p. 172) both for reasons relating to confusion over terminology, and the heterogeneous nature of LDs. Compared to other countries, Australia, and up until recently, New Zealand (Liberty, 2009), have placed far less emphasis on the formulation of precise definitions for categorising students according to particular levels of learning needs and characteristics (Gunn & Wyatt-Smith, 2011). As a result, there has been inconsistency in the definitions used throughout Australian and state territory education systems (with the exception of Queensland). Over the last decade, there has been no clear distinction made between 'learning difficulties' and 'learning disabilities' in policy discourse. The term 'learning disabilities' has been generally applied to students who are visually or hearing impaired, or have Down Syndrome or other intellectual and physical impairments (Gervasoni & Lindenskov, 2011).

From a research perspective, the current complexity surrounding the definition of 'learning difficulties in mathematics', as in other subject areas, has implications for the future development of evidence-based practice. The lack of differentiation between the terms 'learning difficulties' and 'learning disabilities' makes comparisons between studies difficult (Gunn, 2007). Attempts by Australasian mathematics education researchers to synthesise their research findings with US studies on MLD has proven problematic, with the terms 'learning difficulties' and 'learning disabilities' applied to different cohorts of learners across the two regions. While there have been shifts towards using more specific terms to identify different types of MLDs, researchers argued that definitional issues still prevailed. For example, Peard's (2010) analysis focused on the increasing prevalence of the use of the term 'developmental dyscalculia' in the literature to describe a specific learning disability in mathematics that affects the processing of numerical and quantitative concepts. Peard reported that while dyscalculia is a genuine condition that caused severe learning problems, there is little established consensus as to the precise meaning of the term. Dyscalculia is often applied to what would otherwise be identified as a type of MLD, thus Peard (2010) concluded that "the prevalence of dyscalculia is much lower than that reported in some of the literature" (p. 106).

Aside from terminology issues, some researchers related definitional difficulties to the heterogeneous nature of MLD. As Bellert (2009) has observed in relation to mathematics education in Australia, "the general population of students with LD is a heterogeneous group of individuals each with their 'own' story of underlying difficulties and past learning experiences" (p. 173). Within this population, Gunn and Wyatt-Smith (2011) noted a variation in causality, with some MLDs likely to be responsive to a more supportive environment while others less responsive: "A continuum of causality ranges from those difficulties perceived as almost exclusively biological (small in number) in nature and only marginally responsive to environmental factors, to those that appear to be more socially determined and shaped" (p. 18). Given the variety of explanations of factors associated with LD, there is little consensus as to the causes. While the position that no single theoretical perspective can describe all aspects of MLD has gained increasing credence in the field, concurrently researchers are challenged to find coherence across the literature to advance theoretical understandings, and to provide evidence for effective interventions. As Mulligan (2011) has argued, "the research basis for establishing root causes of mathematics learning difficulties lacks the necessary scope and depth and interdisciplinary perspective that may be essential for establishing consensus about research direction and application" (p. 20). As one case in point, Finnane (2011) drew attention to the need for further research to establish indicators of early learning problems in mathematics.

Given the definitional difficulties surrounding LDs in mathematics education, we have elected to use a combination of descriptors—*mathematics learning difficulties*, *learning difficulties*, *learning disabilities*, *special education needs*, *underachievement*, *low achievement*, *at risk* and *dyscalculia*—so as to encompass a broad range of studies in this examination of recent research on the topic. Our selection of literature reflects the current preference among Australasian mathematics education researchers, and indeed those in other fields (Special Education and Learning Difficulties), to use the generic term 'learning difficulties' to investigate high incidence learning problems in mathematics.

Effective Practices for Teaching Students with Mathematics Learning Difficulties

A number of recent Australasian studies reported on the beneficial effects of a range of teaching and learning practices on the mathematical learning performance of low-achieving students. The findings mainly relate to research on early identification and intervention, and research on effective instructional approaches for students with MLD.

Early identification and intervention in mathematics difficulties. Several researchers (Ellemor-Collins & Wright, 2009; Mulligan 2011; Wright, 2008) have reported on the effectiveness of various assessment instruments and intervention programs in early mathematics. Much of this research has foregrounded the development of teaching strategies for early number and arithmetic as necessary prerequisites for the future success of all students in mathematics education.

Mathematics Recovery (MR) (Wright, 2008) is a longstanding early intervention mathematics program which has been implemented in Australian schools over the last two decades. Analysis of the extensive database collected by Wright and colleagues since the program's implementation has generated key insights into early difficulties that children experience in number learning and contributed to the ongoing development of a series of intervention strategies. Reporting on anecdotes from first-grade students experiencing difficulties in number learning, Wright (2008) highlighted the potential of approaches to assessment and instruction, and approaches to teacher development in MR to advancing children's early number knowledge. While Wright's analysis is confined to a case study, the detailed account given of one child's progression of learning through the program has broader applicability. It described how to identify the difficulties that children encountered in early number learning as well as well-researched practices to address them. It also affirmed the importance of assessment in the formulation of instructional goals for effective intervention in mathematics learning.

As an extension of *Mathematics Recovery*, Ellemor-Collins and Wright (2009; 2011) have implemented the *Numeracy Intervention Research Project* (NIRP) with the aim of developing pedagogical tools for intervention in number learning with low-attaining 3rd- and 4th grade children. The NIRP has developed an experimental learning framework for teaching whole number knowledge across five key domains: (a) number words and numerals, (b) structuring numbers 1 to 20, (c) conceptual place value, (d) addition and addition 1 to 100, and (e) early multiplication and division. The project involved the professional development of 25 teachers, interview assessments with 300 students, and intervention with 200 of those students. Through adopting a teaching experiment methodology, the researchers generated longitudinal data on students' construction of mathematical knowledge within an interactive teaching context (Ellemor-Collins & Wright, 2011). Initial findings highlighted the importance of progressing students' knowledge of structuring numbers 1 to 20 to support development from counting to facile non-counting strategies. In relation to a second domain from the learning framework, Ellemor-Collins and Wright proposed conceptual place value (CPV) as an instructional sequence more suitable than conventional approaches to teaching place value. The CPV focused on developing students' mental strategies and structuring of multi-digit numbers through incrementing and decrementing numbers by ones, tens and hundreds in the instructional setting of base-10 materials. Comparisons between students' performances on pre- and post-assessments indicated significant gains made in students' knowledge of multi-digit numbers as the result of CPV tasks. Future research could extend to, as suggested by Ellemor-Collins and Wright (2009), investigating students' learning of structuring numbers in other domains, namely addition and subtraction and multiplicative reasoning.

Gervasoni's (2008) work associated with the *Extending Mathematical Understanding* (EMU) program in Australia provides further research-based insights into early identification and intervention for the numeracy development of under-performing students in number learning. The EMU program was designed to provide early identification and initial intensive support to assist 6-year-old children in Grade 1 not achieving success in learning mathematics within the regular classroom and to provide ongoing intervention for students who underperform in Grades 2 to 6 (Gervasoni & Lindenskov, 2011). The program's implementation has centred upon the professional development of teachers to work as specialist teachers in providing intensive instruction and feedback that specifically targets the learning needs of individual learners. Reporting findings from the program, Gervasoni foregrounded the diversity of children's mathematical knowledge of numbers and the need for customisable programs that address learning needs. Further, Gervasoni noted from analysis of students' learning across four number domains (Counting, Place Value, Addition and Subtraction, Multiplication and Division), that knowledge in any one domain should be not assumed as a prerequisite for knowledge construction in another domain. Gervasoni suggested the need for teachers to provide students with concurrent learning opportunities across all domains, rather than limiting experiences to one domain until mathematical proficiency has been gained in another. Given the critical importance of teachers' expertise to the program's implementation, further analysis needs to consider the impact of teacher knowledge in enhancing students' early number learning.

While much of the focus has been on investigating problems with number learning in early mathematics, mathematical pattern and structure (Mulligan, Mitchelmore, English, & Robertson, 2010) has also been researched as possible markers predictive of future MLDs. Mulligan and colleagues (Mulligan & Mitchelmore, 2009; Mulligan et al., 2010) described pattern as any predictable regularity involving number, space or measure; and structure, as the way in which various elements are organised and related. In examining how children develop an *Awareness of Mathematical Pattern and Structure* (AMPS), Mulligan et al. have developed an assessment interview, the *Pattern and Structure Assessment* (PASA) and implemented the *Pattern and Structure Mathematics Awareness Program* (PASMAP) as an intervention focused on young children's structural development of mathematics. Reporting on PASMAP, Mulligan and colleagues made reference to a series of empirical studies (Mulligan & Mitchelmore, 2009; Mulligan et al., 2010) on the program highlighting AMPS as a valid construct for recognising early MLDs. The researchers reported on a two-year longitudinal study investigating the effectiveness of PASMAP in four schools (two in New South Wales and two in Queensland), with 316 Kindergarten (non-compulsory year of school) students participating in the evaluation throughout 2009 and in 2010, and 303 students retained to participate in follow-up assessment in the first year of formal schooling. In each of the four schools, two Kindergarten teachers implemented the PASMAP and two implemented their standard classroom program. The analysis of interview data showed that students participating in the PASMAP program had higher levels of AMPS than those in the regular program, made connections between mathematical ideas and processes, and formed emergent generalisations. What was less clear, however, was whether more advanced examples of structural development could be directly attributed to PASMAP given that students in regular programs were also able to elicit structural responses without having participated in the intervention. On this point, the researchers noted that further analysis of the impact of PASMAP would need to consider the influence of individual teacher effect and school-based approaches on the program's implementation in classroom practice.

The above studies foreground both the depth of research and also some significant gaps in the literature on early identification and intervention for students with MLDs. While there continues to be a growing number of interventions trialled in schools to support students experiencing LDs in early mathematics, at the same time there is a growing recognition of the need for the reported benefits of particular instructional programs to be validated beyond the immediate settings of those research studies. The effectiveness of intervention programs in other settings and the impact of that context on students' learning performance are salient concerns in need of further investigation in mathematics education research. The case for more detailed attention being given to understanding the particular characteristics of learners and local school settings as influences impacting on program implementation has been strongly made in the field thus far by Mulligan and colleagues (2011). Extending the focus on context will enable researchers to generate more in-depth insights into "understanding where, when and how interventions are found to be effective" (Elkins & Wyatt-Smith, 2011, p. 350). With the continuing trend towards evidence-based practice in educational policy, it is imperative that mathematics education researchers are able to validate claims of the success of interventions and the transferability of these programs across different learning contexts through empirical evidence.

Research on Effective Instructional Approaches for Students with MLD

The merits and limitations of different instructional approaches for teaching students with MLD have been the focus of research-based debate. Two, clear theoretical orientations have emerged in the literature; constructivist teaching approaches, also referred to as activity-based or problem-based mathematics, and direct instructional approaches.

Constructivist teaching approaches. The shift in mathematics education away from conventional teacher-directed instruction towards constructivist approaches has been well-documented over the last decade (Wyatt-Smith, Elkins, Colbert, Gunn, & Muspratt, 2007). Constructivist learning principles remain influential in shaping current mathematics curricula (ACARA, 2010; Ministry of Education, 2007), yet little research has been published in recent years reporting on the effectiveness of constructivist approaches for improving learning outcomes for low-attaining students.

Ferguson and McDonough (2010) argued that constructivist-based approaches provided explicit instruction to assist students with LD acquire specific mathematical knowledge and skills. They reported on a case study which investigated the scaffolding practices of two upper primary teachers and the impact of this pedagogy on two low-attaining students in each class. This study was conducted in two Victorian primary classrooms with a student cohort identified as underperforming in mathematics. Two aspects of scaffolding were discussed, "one-to-one discussions between the teacher and students and the teacher's use of manipulatives" (p. 178). Reporting on data from classroom observations and teacher interviews, Ferguson and McDonough (2010) highlighted the beneficial effects of scaffolding conversations that teachers had with students in reinforcing conceptual understandings and demonstrating the use of more efficient cognitive strategies in mathematics.

This review indicates that while constructivist problem-based approaches are recognised as a valid method for teaching primary mathematics in current curricula (ACARA, 2010), little empirical evidence has been generated from research to substantiate its use as an instructional approach for teaching students with LD. Further research which examines the efficacy of constructivist-based learning practices in providing explicit instruction to support students with MLD acquire specific knowledge and skills is clearly needed. This significant gap in the literature needs to be addressed, particularly in view of the growing body of research critiquing constructivist-based approaches on the basis of there being no evidentiary base to support current claims as to the validity of this method in improving learning outcomes for students experiencing LD.

Direct instruction approaches. A number of studies have reported on the efficacy of direct instructional approaches for teaching students with MLD in presenting findings about the success of particular classroom-based interventions. In Australia, the *QuickSmart* mathematics program has been a significant intervention study promoting the “effectiveness of cognitive strategy of direct instruction and strategy instruction” (Bellert, 2009, p. 172) in helping low-achieving students gain fluency in basic mathematical skills. The *QuickSmart* program has been the subject of a series of evaluation studies (National Centre of Science, Information and Communication Technology, and Mathematics Education for Rural and Regional Australia (SiMERR), 2009), and research publications (Knight, Bellert, & Graham, 2008). This program was developed by a team of researchers based at SiMERR, to support students in their middle-years of schooling (Years 5 to 9) experiencing difficulties in reading and mathematics (Bellert, 2009).

Graham and Pegg (2008, 2010) provided a synopsis of the design and implementation of the *QuickSmart* program, while at the same time reporting on research findings drawn from school-based data collected over an eight-year period. *QuickSmart* is described as a “teacher or teacher aide-directed program” (Graham & Pegg, 2010, p. 11) focused on increasing the fluency of middle-school students in basic numeracy skills. Based on research evidence generated from the program’s implementation, Graham and Pegg argued that students with LD “learn best through explicit and systematic instruction that provides ample opportunities for fundamental knowledge and skills to become firmly established through guided practice and corrective feedback” (p. 11). They pointed to comparative data generated from pre- and post-testing as showing significant improvement in number knowledge and effective strategy use by students in the intervention program. Bellert’s (2009) small-scale study assessed the impact of the *QuickSmart* intervention and reported on the program’s success in narrowing the gap between the performance of 12 low-attaining, middle-school students with that of average-achieving peers in measures on response speed and accuracy in recalling basic mathematical facts.

Like *QuickSmart*, the teaching strategies and assessment devices employed in the *Building Accuracy and Speed in Core Skills (BASICS)* Intervention Mathematics Program have focused on improving the automaticity and accuracy of students’ recall of basic mathematics facts as obstacles to higher-order thinking. According to Byers (2009), “optimal aspects of instruction from direct, constructivist and contextually-based instruction” (p. 11) were incorporated into the design of the program so as to address the specific needs of ‘at-risk’ students. Byers reported on students who participated in one school-based implementation of the program from the start of the school year, highlighting that nearly one quarter of them made a successful transition from intervention to the core mathematics program.

In summary, research has shown that pedagogical practices associated with direct instruction produce positive learning outcomes for students with MLD.

Specifically, results from mathematics intervention programs have demonstrated gains in students' fluency and automaticity in basic number skills. Although the reported literature here provides empirical evidence as to the success of direct instruction methods, this review, as with others (Purdie & Ellis, 2005), highlights the relative paucity of available research on such interventions. Aside from research publications associated with the previously mentioned programs, Australian mathematics education researchers still rely heavily on international meta-analyses of interventions and studies (e.g., Gersten et al., 2009) to validate claims that direct instruction benefits low-attaining students in mathematics. This review points to the need for further research on two fronts. First, the case for more detailed attention be given to the particular characteristics of learners and their learning settings on program implementation, as with early mathematics programs, can be equally applied to interventions designed for middle schooling. Current reporting of findings from direct instruction programs will need to extend beyond conventional quantitative analysis of the learning performance of students in research settings to consider the impact of contextual factors on program implementation. Not only further but indeed different kinds of research will be needed to address whether direct instructional programs which have thus far reported considerable success in improving learning outcomes for low-attaining students in mathematics, will be just as effective in other learning settings (such as schools with significant populations of students with English as an Additional Language). Second, there is limited research to substantiate the increasing argument for using a combination of different approaches for teaching students with MLD. With the exception of Byer's (2009) study, reviewed earlier, no other current literature was found to provide evidence of the success of a 'balanced approach' of constructivist practices and direct instruction methods in improving learning outcomes for low-attaining students in mathematics.

ABILITY GROUPING AND EXCEPTIONAL STUDENTS

Ability grouping is a controversial issue in the education of exceptional students with proponents arguing the benefits for gifted students and opponents countering with the detrimental impacts on low-achieving students. This controversy extends beyond Australasia with the UNESCO publication *Fundamentals of Educational Planning: Methods of Grouping Learners at School* (Dupriez, 2010). This document highlights the complexity of the issue with an inherent difficulty in attributing causality for student performance to ability grouping alone. Hence, in our review of recent Australasian studies on ability grouping, we consider some of the contributing variables that could account for the perceived effectiveness of ability grouping.

A study by Kronberg, Plunkett and colleagues (Kronborg & Plunkett, 2008; Kronborg et al., 2008) compared gifted students' and their teachers' perceptions of learning opportunities in ability grouped mathematics class to non ability grouped mathematics classes at a Victorian secondary girls' school. However, the differences between the two types of mathematics classes extended beyond ability grouping. Gifted students in the ability grouped classes experienced an Extended Curriculum Program (ECP) that provided "exposure to qualitatively different curricula which provided complexity and challenge beyond the regular curriculum in each subject with dedicated extension classes" (Kronborg & Plunkett, 2008, p. 22). Additionally, the teachers of ECP students had additional professional training from the coordinator, who had postgraduate qualifications in gifted education (Kronborg & Plunkett, 2008). Further, a notable difference between the learning environments of the ECP and non-ECP teachers reported by Kronborg and Plunkett (2008) was the differential use of recommended teaching strategies for the gifted. For example, 58% of ECP teachers asked open-ended questions compared to only 7% of non-ECP teachers. Using online surveys from 39 teachers and interviews with 16 heads of department, Kronborg and Plunkett reported unanimous agreement among teachers and heads of departments about the effectiveness of the ECP for gifted students. Support for the ECP was further endorsed by higher measures of students' attitudes (n=58) towards learning and higher levels of confidence to achieve in mathematics (Kronborg et al., 2008). Some gifted students had a point of comparison between types of class (ECP or non-ECP) with some of them in ECP for mathematics but in mainstream classes for other subjects and vice versa. Overall, the results of this study indicate that students and teachers endorsed the ECP for gifted students within a context of ability grouping and professional support for ECP teachers. Hence, the curriculum and teachers' knowledge (e.g., teaching strategies) should not be overlooked as variables in subsequent ability grouping studies. A further variable which might have impacted the results was the interaction between types of educational provisions experienced by the gifted. Apart from the ECP, gifted students had access to mentoring, extension programs, and extra-curricular opportunities (Kronborg & Plunkett, 2008). Understanding the impact of the curriculum, teachers' knowledge and educational provisions on the effectiveness of ability grouping is essential. Kronborg et al. (2008) also advocated for research on ability grouping to explore students' motivational orientation and attitudes towards academic achievement.

A further study on ability grouping of gifted students was conducted by Chessor and Whitton (2007/2008) who investigated the effects of grouping on academic self-concept and mathematics performance. Over 600 primary students participated in this study; in the experimental group (n=250) in Opportunity Classes (Years 5 and 6 in New South Wales public school system), or in the comparison group (n=384: 197 in mixed ability settings and 187 in streamed settings). Self-concept and mathematics achievement was measured at two time intervals using the Self-Description Questionaire-1 (Marsh, 1987) and the Progressive Achievement Tests in Mathematics (Australian Council for Educational Research (ACER), 1984) respectively. Chessor and Whitton reported that the experimental and comparison groups commenced with similar mathematics self-concepts. However, over time the scores of the experimental group increased significantly in contrast to the comparison group. The experimental group achieved consistently higher mathematics self-concept scores and significantly higher mathematics achievement scores than the comparison group over time, and Chessor and Whitton concluded that Opportunity Classes have particular merit for mathematically gifted students. The impact of streaming on high or low achieving students could not be discerned as results for these students were reported for the comparison group which included streamed and mixed ability classes. Hence, further research is needed to isolate the effects of these organisational structures on self-concept and achievement for the gifted.

Consistent with the international literature, the studies by Chessor and Whitton (2007/2008) and Kronborg, Plunkett and colleagues (e.g., Kronborg & Plunkett, 2008) indicated that part-time and full-time forms of ability grouping can have benefits for gifted students. Forgasz's (2010) survey study of 44 teachers' views of streaming practices in Years 7 to10 in Victorian schools further supports this view. Additionally, she reported that teachers were overwhelmingly in favour of streaming for all students because "streaming caters well for the needs of students of different abilities" (p. 74). To conclude, there is evidence of benefit or perceived benefit of ability grouping for gifted students in particular learning contexts and possibly some non-gifted students.

Similar to the international literature, recent Australasian studies report some disadvantages of ability grouping. Forgasz (2010) reported limited criticism of streaming by teachers related to (a) whether the needs of the middle achieving students were being met, (b) the reliance on a single test as the selection criteria, (c) the logistical difficulty of getting students placed into the appropriate groups, and (d) streaming students who have just commenced high school with variable primary school experiences. Additional disadvantages reported for low achieving students included the assignment of the least qualified teachers to lower ability classes (Kilgour & Rickards, 2008) and the difficulties for students moving out of lower streamed classes, if those classes have not covered the same content as more advanced classes (Clarke & Clarke, 2008). However, many of these disadvantages are created by school practices and hence can be ameliorated for example by not putting the least qualified teacher on the lower achieving class. One disadvantage raised by Forgasz (2010), that is less easy to remedy and under-researched, is the issue of equity related to the impact of variables typically associated with less than optimal achievement or participation in mathematics such as; gender, socioeconomic background, language or ethnic/Indigenous background. Hence, students with these characteristics might underperform and be inappropriately placed in particular ability groups.

In conclusion, despite recent Australasian research on the impact of ability grouping much remains unknown. However, due to the widespread use of ability grouping including streaming in Australasia (Forgasz, 2010; Walls, 2009) and teachers' commitment to it (Forgasz, 2010; Kronborg & Plunkett, 2008), there is a need to investigate the factors that impact positively or negatively on exceptional students' learning in ability grouped settings. Such research needs to be comprehensive with a focus on pedagogy and not solely grouping practices (Jackson & Brown, 2009), teaching strategies (Kronborg & Plunkett, 2008), equity issues (gender, socioeconomic, language and ethnic/Indigenous background)

(Forgasz, 2010), whether teachers and students hold incremental or entity views of intelligence (Thomas, 2008; Walls, 2009) and detrimental effects of grouping (Clarke & Clarke, 2008; Forgasz, 2010; Macqueen, 2008) including unintended consequences (Dupriez, 2010).

CONCLUSION

In order for exceptional students to thrive mathematically, a rigorous research base is needed to inform policy and guide practice. Consistent with the recommendation of the previous MERGA review on exceptional students (Diezmann et al., 2004), there has been substantial research on unique issues for the gifted and students with LD. In relation to students with LD, we now have multiple ways to identify students' learning difficulties in mathematics and interventions to support their progress. However, future research could focus on expanding these programs *beyond* Number to other strands, *beyond* young children to older students who continue to struggle with mathematics through schooling and *beyond* the localised contexts of the majority of these studies. In relation to the gifted, considerable progress has been made on understanding the roles of teachers and parents in their achievement and the importance of the challenge of tasks for these students' interest and motivation. However, less clear are the advantages of accelerated or extended curriculum programs because typically there have been other mitigating factors including teacher expertise that could influence outcomes, and hence, warrant research. Notwithstanding the progress made in developing the literature base on exceptional students, there appears to be little progress in knowing under what conditions ability grouping is a viable educational provision for both gifted students and students with LD. Such research is urgent because various forms of ability grouping are widely practised in Australasian schools.

In conclusion, notwithstanding the scale of the economies of Australia and New Zealand compared to larger economies such as the USA, China and India, we need to invest in research that establishes how exceptional students can **thrive.** Australasian students with LD *must* become numerate and mathematically gifted students *must* have opportunities to develop their capability to the fullest. Hence, PISA and TIMSS are useful to benchmark and monitor progress by Australia and New Zealand relative to other nations.

REFERENCES

Ainley, J., Kos, J., & Nicholas, M. (2008). *Participation in science, mathematics and technology in Australian education* [Monograph 63]. Melbourne, Australia: ACER.

Al Hmouz, H. (2008). *A comparison of gifted high, moderate, and low achievers in their motivation, self-regulation, motivational goals, goal orientations, and attitudes toward their school and teachers* (Unpublished doctoral dissertation). University of Wollongong, Australia.

Australian Council for Educational Research. (1984). *Progressive achievement tests in mathematics.* Melbourne, Australia: ACER.

Australian Curriculum, Assessment and Reporting Authority (ACARA). (2010). *The Australian Curriculum: Mathematics.* Retrieved from http://www.australiancurriculum.edu.au/Mathematics/Implications_for_teaching_assessment_and_reporting

Bellert, A. (2009). Narrowing the gap: A report on the *QuickSmart* mathematics intervention. *Australian Journal of Learning Difficulties, 14*(2), 171–183.

Bicknell, B. (2008). Gifted students and the role of mathematics competitions. *Australian Primary Mathematics Classroom, 13*(4), 16–20.

Bicknell, B. (2009). *Multiple perspectives on the education of mathematically gifted and talented students.* (Unpublished doctoral dissertation), Massey University, Auckland, New Zealand.

Byers, T. (2009). The BASICS intervention mathematics program for at-risk students, *Australian Mathematics Teacher, 65*(1), 6–11.

Caygill, R., & Kirkham, S. (2008). *Mathematics: trends in Year 5 mathematics achievement 1994 to 2006, New Zealand results from three year cycles of the Trends in International Mathematics and Science Study (TIMSS).* Wellington, New Zealand: Ministry of Education.

Chesserman, A. M. (2010). *Teacher attitudes and effective teaching practices for gifted students at Stage 6.* (Unpublished doctoral dissertation). The University of New South Wales, Sydney, Australia.

Chessor, D., & Whitton, D. (2007/2008). The impact of grouping gifted primary students on self concept and achievement. *TalentEd, 25*(2), 7–22.

Clarke, D., & Clarke, B. (2008). Is time up for ability grouping? *Curriculum Leadership Journal, 6*(5). Retrieved from http://cmslive.curriculum.edu.au/leader/default.asp?id=22535&issueID=11280

Diezmann, C., Lowrie, T., Bicknell, B., Faragher, R., & Putt, I. (2004). Catering for exceptional students in mathematics. In B. Perry, G. Anthony, & C. Diezmann (Eds.), *Research in mathematics education in Australasia 2000–2003* (pp. 175–195). Brisbane, Australia: Post Pressed.

Dupriez, V. (2010). *Fundamentals of educational planning: Methods of grouping students at school.* Paris, France: UNESCO. Retrieved from http://www.iiep.unesco.org/fileadmin/user_upload/Info_Services_Publications/pdf/2010/Fund_93.pdf

Elkins, J., & Wyatt-Smith, C. (2011). Breaking down the silos: The search for an evidentiary base. In C. Wyatt-Smith, J. Elkins, & S. Gunn (Eds.), *Multiple perspectives on difficulties in learning literacy and numeracy* (pp. 349–357). Dordrecht, The Netherlands: Springer.

Ellemor-Collins, D., & Wright. R. J. (2009). Structuring numbers 1 to 20: developing facile addition and subtraction. *Mathematics Education Research Journal, 21*(2), 50–75.

Ellemor-Collins, D., & Wright, R. J. (2011). Developing conceptual place value: Instructional design for intensive intervention. *Australian Journal of Learning Difficulties, 16*(1), 41–63.

Ferguson, S., & McDonough, A. (2010). The impact of two teachers' use of specific scaffolding practices on low-attaining upper primary students. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 177–184). Fremantle, WA: MERGA.

Finnane, M. (2008). Addressing verbal memory weaknesses to assist students with mathematical learning difficulties. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 195–202). Brisbane, QLD: MERGA.

Finnane, M. (2011). Promoting an understanding of mathematical structure in students with high functioning autism. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 244–252). Adelaide, SA: AAMT and MERGA.

Forgasz, H. J. (2010). Streaming for mathematics in years 7 to 10 in Victoria: An issue of equity? *Mathematics Education Research Journal, 22*(1), 57–90.

Gagné, F. (2009). Building gifts into talents: Detailed overview of the DMGT 2.0. In B. MacFarlane & T. Stambaugh (Eds.), (2009). *Leading change in gifted education: The Festschrift of Dr. Joyce Vantassel-Baska* (pp. 61–80).Waco, TX: Prufrock Press.

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research, 79*(3), 1202–1242.

Gervasoni, A. (2008). Insights about identifying and assisting children who have difficulty learning mathematics. In L. Lindenskov & C. S. Vianna (Eds.), *Topic Study Group 7: Activities and programs for students with special needs* (pp. 1–4). Monterrey, Mexico: ICME.

Gervasoni, A., & Lindenskov, L. (2011). Students with 'special rights' for mathematics education. In B. Atweh, M. Graven, V. Walter, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 307–323). Dordrecht, The Netherlands: Springer.

Graham, L., Bellert, A., & Pegg, J. (2007). Supporting students in the middle school years with learning difficulties in mathematics: Research into classroom practice. *Australasian Journal of Special Education, 31*(2), 171–182.

Graham, L., & Pegg, J. (2008). *Addressing the needs of low-achieving mathematics students: Helping students 'trust their heads'.* Proceedings of the annual conference of the British Educational Research Association. Edinburgh, Scotland: Heriot-Watt University. Retrieved from http://www.leeds.ac.uk/educol/documents/174898.doc

Graham, L., & Pegg, J. (2010). Hard data to support the effectiveness of *QuickSmart* numeracy. *Learning Difficulties Australian Bulletin, 42*(1), 11–13.

Gunn, S. J. (2007). Literature review. In C. M. Wyatt-Smith, J. Elkins, P. Colbert, S. J. Gunn, & A. Muspratt, *Changing the nature of support provision—students with learning difficulties: Interventions in literacy and numeracy project (InLaN)* (Vol. 2). Canberra, Australia: DEST. Retrieved from http://www.dest.gov.au/sectors/school_education/publications_resources/profiles/effective_teaching_learning_practices_stud_learn_difficult.htm#authors

Gunn, S. J., & Wyatt-Smith, C. M. (2011). Learning difficulties, literacy and numeracy: Conversations across the fields. In C. Wyatt-Smith, J. Elkins, & S. Gunn (Eds.), *Multiple perspectives on difficulties in learning literacy and numeracy* (pp. 17–48). Dordrecht, The Netherlands: Springer.

Hannah, J., James, A., Montelle, C. & Nokes, J. (2011). Meeting the needs of our best and brightest: Curriculum acceleration in tertiary mathematics. *International Journal of Mathematical Education in Science and Technology, 42*(3), 299–312.

Jackson, F., & Brown, R. (2009). Exploring whether multiple intelligences facilitate 'Valuing and working with difference' within mathematics classrooms. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32rd annual conference of the Mathematics Education Research Group of Australasia, pp. 357–364). Palmerston North, NZ: MERGA.

Kilgour, P., & Rickards, T. (2008). Student perceptions of classroom environments in streamed middle secondary mathematics classes in Australian Christian schools. In D. Fisher, R. Koul & S. Wanpen (Eds.), *Science, mathematics & technology education: Beyond cultural boundaries* (Proceedings of the 5th international conference on Science, Mathematics & Technology of Education, Udon Thani, Thailand, pp. 298–305). Perth, Australia: Curtin University of Technology.

Knight, B. A., Bellert, A., & Graham, L. (2008). General classroom strategies to enhance the inclusion of students with learning difficulties. In C. Forlin & J. M. Lian (Eds.), *Reform, inclusion and teacher education: Towards a new era of special and inclusive education in the Asia-Pacific region* (pp. 169–186). Hoboken: Taylor & Francis.

Kronborg, L., & Plunkett, M. (2008). Curriculum differentiation: An innovative Australian secondary school program to extend academic talent. *The Australasian Journal of Gifted Education, 17*(1), 19–29.

Kronborg, L., Plunkett, M., Kelly, L., & Urquhart, F. (2008). Student attitudes towards learning in differentiated settings. *The Australasian Journal of Gifted Education, 17*(2), 23–32.

Leder. G. (2008). High achievers in mathematics: What can we learn from and about them? In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 195–202). Brisbane, QLD: MERGA.

Liberty, K. (2009). Reforming learning for children with learning differences in New Zealand. In C. Ng & P. Renshaw (Eds.), *Reforming learning: Issues, concepts and practices in the Asia-Pacific region* (pp. 293–317). Dordrecht, The Netherlands: Springer.

Macqueen, S. (2008). Between-class achievement grouping for literacy and numeracy: academic outcomes for primary students. *Proceedings of the annual conference of the Australian Association for Research in Education, Brisbane*. Retrieved from http://www.aare.edu.au/08pap/mac08085.pdf

Marsh, H. W. (1987). *SDQ 1 manual and research monograph.* San Antonio, TX: The Psychological Corporation and Harcourt Brace Jovanovich.

Maxwell, E. (2008). *An investigation of appropriate instructional design to match the ability of the learner.* Unpublished doctoral dissertation), University of New South Wales, Sydney, Australia.

Ministry of Education. (2007). *The New Zealand curriculum for English-medium teaching and learning in years 1–13.* Wellington, New Zealand: Learning Media Limited.

Ministry of Education. (2009). *Gifted and talented students: Meeting their needs in New Zealand schools.* Wellington, New Zealand: Learning Media Limited.

Ministry of Education. (2011). *Special education policy guidelines*. Retrieved from http://www.minedu.govt.nz/NZEducation/EducationPolicies/SpecialEducation/FormsAndGuidelines/SpecialEducationPolicyGuidelines.aspx

Mulligan, J. T. (2011). Towards understanding the origins of children's difficulties in mathematics learning. *Australian Journal of Learning Difficulties, 16*(1), 19–39.

Mulligan, J. T., & Mitchelmore, M. C. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21*(2), 33–49.

Mulligan, J. T., Mitchelmore, M. C., English, L. D., & Robertson, G. (2010). Implementing a Pattern and Structure Mathematics Awareness Program (PASMAP) in Kindergarten. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 796–803). Fremantle, WA: MERGA.

Peard, R. (2010). Dyscalculia: What is its prevalence? *Procedia-Social and Behavioural Sciences, 8*, 106–113.

Phillipson, S. N., & Callingham, R. C. (2009). Understanding mathematical giftedness: Integrating self, action repertoires and the environment. In L. V. Shavinina (Ed.), *International handbook of giftedness* (pp. 671–698). Dordrecht, The Netherlands: Springer International.

Purdie, N., & Ellis, L. (2005). *Literature review. A review of the empirical evidence identifying effective interventions and teaching practices for students with learning difficulties in Years 4, 5 and 6.* Melbourne, Australia: ACER.

SiMERR (National Centre of Science, Information and Communication Technology, and Mathematics Education for Rural and Regional Education) (2009). *QuickSmart intervention research program data 2001–2008: Full report.* Armidale, Australia: University of New England.

Telford, M. (2010). *PISA 2009: Our 21st century learners at age 15*. Wellington, New Zealand: Ministry of Education.

Thomas, D. (2008). *What attributions do teachers ascribe to student academic achievement?* (Unpublished doctoral dissertation). University of New South Wales, Sydney, Australia.

Thomson, S. (2010). Mathematics learning: What TIMSS and PISA can tell us about what counts for all Australian students. In C. Glascodine & K. Hoad (Eds.), *Teaching mathematics? Make it count* (Proceedings of the annual conference of the Australian Council for Educational Research, pp. 75–80). Melbourne, Australia: ACER

Thomson, S., De Bortoli, L., Nicholas, M., Hillman, K., & Buckley, S. (2011). *Challenges for Australian education: Results from PISA 2009: The PISA 2009 assessment of students' reading, mathematical and scientific literacy*. Melbourne, Australia: ACER.

Walls, F. (2009). *Mathematical subjects: Children talk about their mathematical lives.* Dordrecht, The Netherlands: Springer.

Walshaw, M. (2011). Identity as the cornerstone of quality and equitable mathematical experiences. In B. Atweh, M. Grave, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics* (pp. 91–103). Dordrecht, The Netherlands: Springer.

Watters, J. J. (2010). Career decision making among gifted students: The mediation of teachers. *The Gifted Child Quarterly, 54*(3), 222–238.

Wright, R. J. (2008). Mathematics Recovery: An early intervention program focusing on intensive intervention. In A. Dowker (Ed.), *Mathematical difficulties: Psychology and intervention* (pp. 203–223). London, United Kingdom: Academic Press.

Wyatt-Smith, C. M., Elkins, J., Colbert, P., Gunn, S. J., & Muspratt, A. (2007). *Changing the nature of support provision—students with learning difficulties: Interventions in literacy and numeracy project (InLaN)*. Canberra, Australia: DEST.

Zeigler, A. (2005). The actiotope model of giftedness. In R. J. Steinberg & J. E. Davidson (Eds.), *Conceptions of giftedness* (2nd ed.) (pp. 422–43). New York, NY: Cambridge University Press.

Zhao, D., & Singh, M. (2010). Why do Chinese-Australian students outperform their Australian peers in mathematics: A comparative case study. *International Journal of Science and Mathematics Education, 9*(1), 69–87.

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TECHNOLOGY IN MATHEMATICS EDUCATION

Key words: digital technologies; gender; pedagogical and affective aspects of technology.

INTRODUCTION

The role of digital technologies in enhancing learning and teaching has been a subject of interest to mathematics educators for at least the past three decades. International interest in this area has been paralleled in Australasia as is evidenced by chapters on this topic in previous MERGA research reviews (e.g., Goos & Cretchley, 2004; Thomas & Chinnappan, 2008). Earlier reviews have provided a 'time stamp' for the forms of technology that have gradually been implemented in educational settings, including review chapters concerned with computer and calculator use in school classrooms. The influence of internet technology to promote learning and teaching of mathematics has been more recently researched and reviewed. The limited variety of technologies available in earlier times encouraged previous authors to structure their reviews around the influence of specific digital tools. Thus, discussion of the influence of technology on the teaching and learning of mathematics tended to be organised around how computers, calculators, or the internet could be used by teachers to enhance the development of students' mathematical capacities.

The landscape of digitally enhanced approaches to teaching and learning has changed dramatically since 2006. The availability and power of computers and handheld digital devices continue to increase, along with expectations from students, parents, and teachers that these tools should be incorporated into the day-to-day activity of classrooms. This expectation is also reflected in documents developed by curriculum authorities where, for example, nearly all states and territories in Australia require the incorporation of some form of digital technology into the learning, teaching, and assessment of mathematics at the senior levels of schooling. Another influence is the rise of a much greater variety of technologies including, for example: interactive whiteboards; learning management technologies such as Moodle; virtual learning environments; further developments in Computer Supported Collaborative Learning (CSCL); and the potential of social networking technologies, such as mobile phones, which have begun to be explored from the perspective of educational opportunity.

This proliferation of new technologies led us to structure this review differently to previous authors. We have endeavoured to discuss research under broader

B. Perry et al. (Eds.), Research in Mathematics Education in Australasia 2002–2011, 111–141.

categories in light of the influence of digital technologies on mathematics education. As the core business of education is about the activities of learners and teachers, each of these aspects is discussed in separate sections. In addition, because what students and teachers do in technology rich mathematics classrooms is greater than the sum of individual activities, the ways in which digital tools can change learning environments are considered with particular emphasis on, the potential of connectivity to enhance purposeful and productive interaction. The design of learning environments that integrate the use of technology into teaching and learning must necessarily be the concern of curriculum developers. Thus, in a section of this chapter, research on learning environments and curricular design is examined. Finally, as the potential of technology to enhance learning and teaching rests on equitable access to digital tools and positive attitudes towards and beliefs about the potential of technology to assist the development of mathematical understanding, a section is included on research related to these aspects.

Thus, this chapter is organised under the following section headings:

- Learning Contexts and Curricular Design
- Learners, Learning, and Digital Technology
- Teachers, Teaching, and Digital Technology
- Gender, Affect, and Technology

While there is some overlap in the research studies under these categories, they provide lenses that allow for the critical review of research into digital technologies in mathematics education.

LEARNING CONTEXTS AND CURRICULAR DESIGN

This section explores how the use of digital technologies both shape and are shaped by teaching and learning practices as well as influencing policy implementation at a systemic level. The foci of the research examined were:

- the ways digital tools are used with the intent of changing the nature of the learning context or environment where teachers and students interact; and
- the impact of system-wide curriculum and assessment reform in relation to the use of technology.

Learning Contexts

While internationally there has been considerable research activity related to the creation of virtual learning environments designed to promote specific aspects of mathematics learning (e.g., Confrey et al., 2010; Hoyles et al., 2010), there appears to be little of this type of research reported by Australasian authors, although several have discussed the potential of collaborative learning environments that are mediated by digital tools.

Beatty and Geiger (2010) provided an overview of the role of technology in mathematics education from the perspective of social theories of learning. Through an historical analysis of the literature, they argued that there is a discernible growing interest in the use of technology to promote more collaborative approaches to learning and teaching. They developed a typography for the roles of digital technologies within this modality, including technologies designed for:

– both learning mathematics and collaboration;
– learning mathematics but not specifically for collaboration;
– collaboration but not necessarily learning mathematics; and
– neither learning mathematics nor collaboration.

Beatty and Geiger (2010) further noted that as new technologies were developed and refined, they supported new ways in which learning communities communicated and interacted. This led to the formation of new and different types of collaborative communities.

A complementary perspective is offered by Gadanidis and Geiger (2010) who traced the changes from discipline specific computer-based software through to Web 2.0 based learning tools. They argued that the increasing interest from mathematics educators in tools that mediated collaborative teaching and learning practices represented a confluence of developments in the definition of knowing and doing mathematics. This implies a shift in mind-sets from thinking about using technology to thinking with technology. The authors proposed that learning mathematics in a Web-based social environment allowed for possibilities such as viewing mathematics as an activity that was performed, rather than knowledge that was passively acquired. This view is consistent with Hughes' (2008) observation that the Web was becoming a performative medium. Gadanidis and Geiger (2010) concluded that thinking about mathematics as performance represented a new paradigm for knowing and doing mathematics that has implications for learning, teaching, and assessment.

Goos (2009a) also offered a socially orientated perspective on the use of digital technologies to enhance learning in mathematics through the development of a theoretical framework for analysing relationships between factors influencing teachers' use of digital technologies in secondary mathematics classrooms. In developing this framework, Goos built on Valsiner's (1997) zone theory of child development. She made use of case studies of both novice and experienced teachers to illustrate how interactions between teacher knowledge and beliefs, professional contexts and professional learning experiences relate to teachers' learning.

In a study of 14- and 15-year-olds using an animated Web-based exploratory environment – Java Maths Worlds – to mediate student learning, Herbert and Pierce (2008) reported that students were able to transfer understanding of rates of change from a 'model of' the situation to a more abstract 'model for' the situation. This had implications for the ways the environment facilitated learning, and hence through its use by students, how understanding might evolve. Meanwhile, Yeh (2010), reporting on a study of primary-school students' explanations of movement in 3D virtual spaces (using VRMaths), found that it provided young children with new ways of thinking and engaging with 3D geometry. The learners' potential to perceive and interact with geometrical elements in novel ways highlighted the potential to transform the experience of learning mathematics.

These positive reports on the potential of Web-based tools to enhance teaching and learning need to be tempered by the reality that both primary and secondary teachers of mathematics have little experience with online learning design. In a report involving online learning in general subject areas, Baker (2010) noted that although teachers in Australia were generally familiar with constructivist theories, they had difficulty applying the theories to designing online activities. This finding suggests a need for further research into what skills teachers need to develop in order to become online lesson developers.

Interactive White Boards (IWB) have increasingly become common fixtures in both primary and secondary school classrooms. They have been installed on the basis of claims that the IWB has the potential to mediate more engaging and interactive learning opportunities for students. Serow and Callingham (2008) studied the use of the IWB in primary schools focusing on the teaching strategies of three teachers. It was reported that students were more motivated and engaged and that the IWB enhanced their interest in exploring mathematical tasks. However, while recognising the potential of the IWB to enhance learning situations, Zevenbergen and Lerman (2008) examined the use of IWB in upper–primary classrooms and concluded that the opportunities for rich mathematical thinking were 'often lost' as teachers pursued prescribed lesson pathways, often inhibiting student dialogue and engagement. They observed that teachers' approaches were restricted to quick introductions to lessons and teacher-led whole class teaching which fostered shallow learning. Their contention was that high expectations of students, a shift of focus to the students, and facilitating high level questioning and thinking, were ways to enhance student learning.

In their study of Grade 1 students engaging in spatial tasks through programming a simple robotic tool, Highfield, Mulligan, and Hedberg (2008) identified transformational geometry and measurement as learning areas that were enhanced. Further analysis in an associated study highlighted the complex inter-relationships between children's representations, speech, gestures, and actions that would not have been apparent if those facets of learning had been analysed in isolation. The students used gestures to describe their representations and reflect on their thinking (Highfield & Mulligan, 2009).

Curricular Design

While many jurisdictions in Australasia either mandate or strongly encourage the use of digital technologies in the learning, teaching and assessment of mathematics, there has been a limited amount of research in this area.

In a pilot study, Tan (2009) employed Geiger's (2005) framework to analyse Singaporean students' use of graphic calculators. Within this framework, four metaphors—'master, servant, partner and extension-of-self'—were used to describe increasing levels of sophistication and integration into students' use of technology when working on mathematical tasks. Since the revision of its senior secondary/pre-university curriculum in 2006, the use of graphic calculators was expected in the Singaporean senior secondary national examinations for all mathematics subjects. Tan found the framework to be relatively reliable with factor analysis supporting the existence of three distinct factors, two corresponding to the first two metaphors of the original framework, and a third based on a construct developed from the other two metaphors.

In Victoria, where a Computer Algebra System (CAS) course was piloted in parallel with a graphics calculator course at senior secondary level, large scale studies reported the positive influence of CAS on student achievement with little evidence for the diminishing of 'by-hand' skills (Evans et al., 2008; Leigh-Lancaster, Les, & Evans, 2010). In successive studies of the results of two cohorts of students (approximately 7000 to 8000) with respect to common assessment items in the final year of the parallel implementation of the Victorian Mathematical Methods Units 3 and 4 and Mathematical Methods (CAS) Units 3 and 4, Evans et al. (2008) and Leigh-Lancaster et al. (2010) analysed the performance of both cohorts and found that students in a CAS environment achieved at a comparable level to those in a non-CAS environment when assessed on non-CAS examination items. This implied that students who studied mathematics courses where CAS enabled technologies were available, performed equally well in all areas as those who learned these skills without CAS.

LEARNERS, LEARNING AND DIGITAL TECHNOLOGY

Research reported in this section demonstrates that digital technologies have created opportunities for learning to occur in distinctive ways. They allow the learner to engage with mathematical phenomena differently than with pen-and-paper. They can promote interaction and discussion and shape learners' mathematical thinking. Hence, they influence the learning trajectories and understanding that emerges in a distinctive manner. This section considers Australasian research that examines the influence of digital technologies on the learning of mathematics.

Actual Learning Trajectories

A learning trajectory is a pedagogical construct that can be interpreted in several ways (Clements & Sarama, 2004). There is a distinction between the intended or hypothetical learning trajectory that identifies and characterises potential instructional routes through planning processes and activities, and the actual learning trajectory which indicates the learning pathways followed by learners as they interact with and through mathematical tasks (Sacristan, 2010). The actual learning trajectory is shaped by various sociocultural influences and discourses. A range of theoretical constructs in the literature examined the influence of digital technologies on the mathematics learning process. How such trajectories are shaped or influenced by the introduction of technology into the interaction of a learner with mathematics, in particular the effect of digital tools on modes of discourse and ways of reasoning, has been a topic of interest to researchers (e.g., Yeh & Nason, 2008). This form of participatory discourse also underpinned Geiger's (2009) use of the 'master, servant, partner, extension-of-self' metaphor for modes of technological use in varying classroom situations. Tan (2011), utilised the same metaphor to examine differences between Victorian and Singaporean students' use of CAS. She found that Victorian students appeared to have the greater fluency with sophisticated calculators. She also reported that males demonstrated greater mastery of the calculators than females. The affordances of various technologies influenced both the engagement with the task and the nature of student interaction (Sacristan et al., 2010). This examination of the approaches learners take when engaging with mathematics through digital technologies, also considers the ways learning trajectories differ as a result of the affordances and constraints offered by digital pedagogical media, as well as what the characteristics of that engagement might be.

Affordances of Digital Technologies

In much of the Australasian literature on technology in mathematics education, the terms 'affordances' and 'constraints' are used. Gibson (1977) defined affordances as the attributes of the learning setting that provided the potential for students' learning. The learning setting included the support structures, such as the tools, information sources, visual cues, and prompts. Thomas and Chinnappan (2008) differentiated these two elements by proposing that "affordances speak about the potential for action, while constraints impose the structure for that action" (p.166). They also utilised the term 'obstacle' as something that prohibited the occurrence of the environmental properties that might have facilitated an affordance. Brown (2006) described affordances as the potential relationships between the user and the environment.

While affordances relate to the particular technology and learning context, there are some that appear to be more generic across a range of situations (Calder, 2009a; Sacristan et al., 2010). Sacristan et al. (2010) situated the affordances within the interrelationships between pedagogical, contextual, and technical aspects, and the learner's perspective. The facilities of digital technology to allow the user to interact with multiple representations, or to receive immediate, non-judgmental feedback, are potential affordances which could influence actual learning trajectories (Sacristan et al., 2010). These were also among the affordances identified in research specifically related to spreadsheets (Calder, 2010). Other affordances discussed by Calder were: (a) the interactive nature of the environment; (b) the modelling of real life situations through visual-graphical representations; (c) the capacity to transform large amounts of numerical data; and (d) the propensity to give visual feedback.

In other studies, the affordances of specific technologies have been examined. Kissane and Kemp (2008) highlighted the affordances of the graphics calculator for exploring various calculus concepts such as differentiation using first principles, limits and asymptotes, derivatives, optimisation, and convergence of a series. Verenikina, Herrington, Peterson, and Mantei (2010) categorised computer games according to game characteristics that supported higher order thinking and problem solving skills for young children. They studied young children using some of the games to investigate the affordances and constraints provided by the software.

The focus of some complementary studies has been on constraints and how these have been managed. Loong (2009) described the difficulties encountered by a teacher conducting a series of interactive web-based lessons and how the teacher set up supporting structures to negotiate between the affordances and constraints. Galligan, Loch, McDonald, and Taylor (2010) described how tablet PCs and related technologies have been used in a university mathematics teaching and learning contexts, and the successes and challenges faced by both students and teachers in lecture (one-to-many), tutorial (one-to-few), and consultation (one-to-one) contexts.

In their discussion of the ways mathematical knowledge and practices emerge from access to digital technologies, Olive and Makar (2010) attended to several affordances. They contended that the use of digital technologies presents a fresh model for the interaction between the student, the mathematical knowledge, and the pedagogical instrument. The interactive, dynamic ways of processing mathematical phenomena, coupled with affordances such as the feedback provided through technology environments, opened up opportunities for new learning pathways and contributed to student learning. The affordances of the interactive programming language, *Scratch*, were found to facilitate mathematical thinking (Calder & Taylor, 2010). A study of primary-aged students creating mathematical games with *Scratch*, Calder and Taylor found that understanding of particular angle measurements and spatial movements was enhanced. *Scratch* was also a motivational and productive environment for facilitating mathematical thinking through creative problem-solving processes.

Affordances of technology that facilitate collaborative approaches to learning have also been considered. Beatty and Geiger (2010) discussed how these affordances offered an opportunity for the interchange and critical analysis of student ideas. They warned that the affordances of particular technologies in certain situations might inhibit learning, as it is not just the design of the technology that brings about positive changes in learning but also the nature of the interaction between the learner and the digital tool. This interaction is strongly influenced by the pedagogical practices employed by the teacher in orchestrating students' individual as well as collaborative and co-constructive use of a specific digital tool. Pierce and Stacey (2009a, 2009b) utilised a lesson study cycle to investigate affordances when students used CAS technology. They reported that this particular opportunity, if not linked clearly to the purpose of the task, could lead to a loss of focus and motivation.

The influence of a digital environment's affordances on teachers and teaching was considered by Goos and Soury-Lavergne (2010). Informed by the contribution of Brown (2006), they integrated affordances with zone theory and interpreted this relationship with instrumental genesis (Artigue, 2002). They noted that the extent to which the affordances were realised in a learning situation depended on the intentions of the teacher and the students. While the digital medium exerted influences on students' approaches and by inference on their learning and attitudes, it was their existing knowledge that guided the way the technology was used and, in a sense, shaped the technology.

While there has been much research activity aimed at examining the use of digital technologies in mathematics education, most has been situated in primary, secondary, or tertiary education settings. In their review of the use of technology in mathematics education in early childhood settings, Highfield and Goodwin (2008) reported that while the affordances of technology with older students are relatively prevalent, early childhood education research on technology in mathematics education "is scant and so judgements about potential affordances in mathematics instruction are, to a large extent, purely speculative" (p. 259). These authors examined literature relating to early mathematics learning and technology in previous meta-analyses and research published between 2003 and 2007, in five significant mathematics education research journals. They found that there was "a shortage of research pertaining to young children's mathematics in the selected journals, which is even more pronounced when technology is the modality for learning" (p. 263). The possible reasons put forward by the authors for the absence of such studies were that related research might be published in other journals such as early childhood journals, or that there was reluctance from the early childhood field to embrace technology. Alternatively, it could be that the use of technology is only beginning to be explored in the early childhood context and that mathematics, as numeracy and problem solving, is currently subsumed under general early childhood studies.

Nonetheless, there have been reports on the use of educational technology for early childhood ICT literacy (Jones, 2008), and efforts in teacher education programs in an Australian university to incorporate technology into early childhood education (Gibson, 2010). In Jones' investigations of three classroom activities, it was noted that although technology was used by students to create their own multimedia artefacts, "what does not appear to be happening is the highlighting by teachers of the overt integration of knowledge, understandings and skills from several curriculum areas in order to create more meaningful artefacts" (p. 2947). Gibson found that the graduates from a Master of Teaching program in Early Childhood were generally comfortable with using technology themselves, but were not ready to integrate technology in early childhood learning settings. These conclusions seem to support Highfield and Goodwin's (2008) arguments, and to call for more research to be conducted in this field.

Learning With and Through Digital Technologies

Computer Algebra Systems (CAS) has been a focus of a number of studies, predominantly with senior secondary students using hand-held devices. Recent research has involved students working in technology-rich environments where CAS-enabled graphic calculators might be available among a range of technology options, rather than the sole digital technology available (e.g., Geiger, 2009).

The ways CAS can facilitate mathematical modelling in the senior secondary school was considered by Geiger, Faragher, Redmond and Lowe (2008) and Geiger, Faragher and Goos (2010). They reported on the productive student-student and student-teacher interactions that ensued when a CAS device produced an unexpected output when used to solve an equation that students had developed to model a situation set in an environmental context. This output provoked students to reflect on the alignment of their mathematised model with the real world situation they were examining, and to rethink their preconceptions and assumptions about both concepts and processes. Geiger and colleagues (2008; 2010) also reported that these provocations presented opportunities for teachers to identify misconceptions and to evoke interactions leading to enhanced understanding, and the reconciliation of misconceptions.

In the teaching of university undergraduate mathematics courses, Oates (2009) identified significant tensions between some curriculum content and access to graphics calculators with CAS capability. He suggested, based on earlier research, that content areas such as algebraic manipulation might become redundant, with the focus being more on richer conceptual understanding. Oates (2009) also speculated that changes to the order of topics might place the exploration of differential equations modelling real-life problems ahead of more abstract concepts such as rates of change. He further argued that the value of technology was in the opportunity it provided for learners to develop a range of investigative strategies as they engaged in problem solving, rather than focusing on the use of repetitive instrumental techniques that could more easily be completed through the use of a digital tool. In support of this notion, Pierce and Stacey (2009b) explored how lesson study might enhance Year10 students' understanding in algebra in a TI-Nspire environment. They found that digital technology gave greater opportunities for students to explore problems from fresh perspectives. In particular, they found that pre-prepared activities, including screens that focused on particular mathematical concepts, were valuable in focusing students' mathematical thinking. By way of contrast, Thomas (2009) reported that students predominantly used CAS for procedural tasks rather than interacting conceptually. In this case, students learnt to apply techniques to particular problems rather than developing understanding of the underlying mathematical principles. Thomas (2009) recommended that it was more important to emphasise the mathematics in the learning, and not focus on the technology.

Research studies have also dealt with the use of other mathematics analysis software (MAS) (Pierce & Stacey, 2010), a generic term used to describe a wide range of digital tools, including, for example, graphic calculators, CAS, and dynamic geometry software. Various MAS have been designed to facilitate mathematical learning across a diverse range of conceptual areas. Forbes and Pfannkuch (2009), in their discussion of how students develop statistical thinking in secondary school, advocated the use of software such as *Tinkerplots* and *Fathom* to provide innovative ways for students to build statistical concepts. They also recommend that students use web-based data to explore meaningful contexts and develop statistical thinking. In the same vein, Calder (2009b) reported using spreadsheets to investigate open-ended problems and found that this could shape a learner's approach to investigations that enhance mathematical thinking. In a study of primary school students, Calder (2011b) reported that unexpected visual output (visual perturbances) stimulated mathematical discussion, and the posing and refining of informal conjectures and theories. Elsewhere Calder (2009c) reported that spreadsheet based digital environments allowed learners to move more readily from initial exploration to informal conjecturing and generalisation, that the noticing of mathematical relationships was enhanced, and that students used visual referents in their reasoning.

In contrast to international trends in research, there have been limited studies into the use of dynamic geometry systems to enhance instruction and assist students' conceptual development in mathematics. In a study of secondary students' understanding of the relationships among quadrilaterals, Serow (2008) concluded that the use of dynamic geometry software resulted in deeper conceptual understanding. Findings on the use of specifically developed tools have also been reported. Yeh and Nason (2008), for example, examined adult prisoners' understanding of fractional number when digital technology was the medium for the learning. They reported on the use of an ICT tool designed for mixing colours which facilitated the participants' understanding of ratio and fractions. A semiotic framework for mathematical meaning making had informed the design of the ICT tool. It was concluded that when multiple semiotic resources were utilised, the mathematical ideas could be better understood.

The internet is a rich repository for resources and interactive applets, as well as a source of data (e.g., Forbes & Pfannkuch, 2009). French (2010) outlined and analysed the process undertaken with pre-service teachers who developed WebQuests and discussed how mathematical learning might emerge from such activity. He maintained that WebQuests with mathematical topics enriched students' understanding of, and skills for working with, key mathematical concepts and that this enabled them to explore a diverse range of historical and cultural contexts. The internet has also allowed learning communities to evolve beyond the constraints of the classroom, and helped facilitate student-centred inquiry (Calder, 2011b).

TEACHERS, TEACHING AND DIGITAL TECHNOLOGY

In the chapters related to teaching with technology in the past two MERGA reviews, the development of a research focus on investigating the pedagogical use of digital tools (Forster, Flynn, Frid, & Sparrow, 2004) to the emergence of technology related pedagogical frameworks and related teacher professional development (Thomas & Chinnappan, 2008) was documented. In this section on teaching with technology, the work reported in previous MERGA reviews is extended. Current studies on approaches to teaching with technology, pedagogical frameworks, and models for teacher education are presented and discussed under the following headings:

– Teachers' Use of Technology
– Pedagogical Frameworks and Approaches
– Technological Pedagogical Content Knowledge
– Developing Teacher Expertise in Technology Integration

Teachers' Use of Technology

While technologies can be categorised in terms of hardware (e.g., computers, handheld devices, tablet PCs, iPads) or software (e.g., dynamic geometry software, spreadsheets, computer algebra systems), the increasing complexity and synergetic interactions of technologies mean that digital learning environments include a host of complex systems that are at the teacher's disposal. Interactive white boards (IWB), for example, enable teachers to combine the use of multiple resources from the internet. Learning management platforms, such as Moodle, can be used to mediate synchronous and asynchronous interaction between learners and teachers, as well as act as sources of information. Localised computer software, like spreadsheets and calculator emulators, can be used to support specifically targeted aspects of learning.

Several studies involved large numbers of secondary school participants:

– Goos and Bennison (2008a) studied Queensland teachers' attitudes, beliefs and confidence about using technology in teaching and learning mathematics, professional development and other factors affecting technology. Data were gathered from 485 teachers in 127 schools.
– Thomas, Hong, Bosley, and delos Santos (2008) reported on their ten year longitudinal survey data from secondary mathematics teachers in New Zealand in 1995 and 2005, focusing on teachers' use of calculators. In the 1995 survey, the questionnaire on calculator and computer use for teachers had a school information section for mathematics coordinators or heads of department to complete; 339 teachers from 90 schools replied. In the 2005 survey, the distinction between scientific and graphics calculators were made, and 465 teachers from 193 schools participated. Their paper discussed the extent of calculator use and teachers' attitudes and perceptions towards calculator use.
– Pierce and Ball (2010) collected responses from 92 secondary mathematics teachers in a state-wide survey in Victoria. In this study they reported on the mathematics software teachers were using, their purpose, and concerns related to the use of technology for teaching mathematics.
– Hudson, Porter, and Nelson (2008) and Hudson and Porter (2010) surveyed 114 public secondary school mathematics teachers in New South Wales regarding their use of computers, their beliefs about technology and mathematics learning, and their perceptions about professional development.

Although conducted at different times and in different regions, some general conclusions can be drawn from these large scale studies. Generally, there seemed to be a limited range of software available in schools (Goos & Bennison, 2008a), although in most there was access to spreadsheet software (97.6% overall). Perhaps this was due to the pervasiveness of Microsoft Office packages within which Excel is the bundled spreadsheet application. This could also be the reason why Hudson and Porter (2010) found training on Excel, compared to training on other software such as the internet or mathematics specific applications, was associated with teachers' computer use for teaching. Mathematics specific software was not as commonly available. Goos and Bennison (2008a), for example, found that less than two-thirds of the schools surveyed reported having graphing software, 28.6% had dynamic geometry software, 17.9% had statistical programs, and 17.9% had computer algebra systems. In their study, this lack of access translated into low teacher use, with only 30–55% of teachers reported as using computers sometimes or frequently, and between 10–30% using the internet sometimes or frequently. Attempting to establish the level of the use of digital tools by teachers is compounded by the degree of uptake of different types of technologies. Pierce and Ball (2010), for example, reported that there was greater teacher use of graphing software, spreadsheets and tables (from computers and calculators), compared to other software such as dynamic geometry, statistical programs and symbolic algebra software.

While it was widely reported that calculators were being used in schools, personal access to advanced calculators such as graphics calculators (GC) and Computer Algebra System (CAS) enabled calculators was more limited. Goos and Bennison (2008a) reported 24.7% student graphics calculator ownership, and Thomas et al. (2008) reported 27.1% student graphics calculator and 0.2% CAS calculator ownership. A large number of schools (73.0%) relied on class sets.

Despite the call in many national and regional curriculum policies, teachers continued to raise the issue of access to technology as a major obstacle to using digital tools in mathematics lessons. This included access to computers and computer laboratories (Hudson et al., 2008), and to advanced calculators (Pierce & Ball, 2010; Thomas et al., 2008). Personal access to advanced calculators was viewed as important for students in order to become familiar with the tool, especially in schools where there were constraints in obtaining computer laboratory access (Goos & Bennison, 2008a; Pierce & Ball, 2010).

It was also noted that the use of digital tools by teachers was higher when technology was incorporated into formal curriculum and assessment structures. For example, in both Queensland and Victoria, the senior secondary mathematics syllabus requires the use of digital technologies. This has resulted in higher proportions of senior secondary than junior secondary mathematics teachers using technology (Goos & Bennison, 2008a; Pierce & Ball, 2010). Additionally, Pierce and Ball (2010) reported that senior secondary teachers tended to limit their use of digital tools to syllabus requirements, whereas junior secondary teachers used a wider range of technology for a variety of purposes.

Pedagogical Frameworks and Approaches

In considering the affordances and constraints of mathematical digital tools from various perspectives, a number of researchers have built upon findings of empirical studies and existing models of teaching and learning to develop frameworks for the pedagogical use of technology in mathematics education. Pierce and Stacey (2008; 2010), for example, have developed a pedagogical map for mathematics analysis software (MAS). According to Pierce and Stacey (2008, 2010), functionalities of MAS provide opportunities for change in curriculum, assessment and pedagogy. In their view, the affordances of digital tools enabled changes at three levels: (a) teacher-assigned mathematical tasks; (b) classroom interactions; and (c) the area of mathematics taught.

Using Pierce and Stacey's (2008, 2010) pedagogical map as a framework, Pierce, Stacey, and Wander (2010) investigated the impact of handheld CAS technologies on what was termed the didactic contract (Brousseau, 1997). A didactic contract is "about reciprocal responsibilities and expectations of the teacher and students with respect to mathematical knowledge" (Pierce et al., 2010, p. 684). Through observations of Year 10 classes involving 12 different teachers who were new to MAS, it was found both teachers and students in all classes believed that the teacher had the responsibility to teach technological skills. Teachers, however, viewed mathematics as their main focus, whereas students viewed technological skills as the main focus of the observed lessons. The mismatch in expectations in this situation meant that, for effective learning of both mathematics and the use of technology, the didactic contract required renegotiation.

Pedagogical frameworks were also developed for specific types of mathematical software. Serow (2008) used van Hiele's (1986) five phase framework for using technology to facilitate students' mathematical cognitive development—information, directed orientation, explicitation, free orientation, and integration—to investigate a teaching sequence using dynamic geometry software. She found the framework to be effective in structuring sequences of activities for teaching geometry. Serow also reported that students were on-task throughout the activities and their discussions changed from using informal mathematical language to more formal and complex modes of expression.

The use of TI-Nspire for simulation and linking representations was investigated by Pierce and Stacey (2008) using a lesson study methodology. They identified four key principles for designing lessons which focus on linking multiple representations (symbolic, graphic, and numeric):

- focus on the main goal for that lesson (despite the possibilities offered by having many representations available);
- identify different purposes for using different representations to maintain engagement;
- establish naming protocols for variables that are treated differently when working with pen and paper and within a machine; and
- reduce extraneous cognitive load.

Frameworks for technology rich pedagogical approaches might be beneficial for teachers who prefer structured ways of thinking about how to integrate technology into their teaching, but might also lead to routine lesson sequences that become boring for students and teachers (e.g., Tanner, Jones, Beauchamp, & Kennewell, 2010)

Technological Pedagogical Content Knowledge

Researchers have gained new insights into teachers' use of digital tools by extending Shulman's (1986) notion of pedagogical content knowledge (PCK) as the intersection between content knowledge and knowledge of pedagogical methods to incorporate the use of ICTs into a technology active PCK framework.

In the previous MERGA review, Thomas and Chinnappan (2008) presented the emerging notion of pedagogical technology knowledge (PTK). In the mathematics education context this included teachers' knowledge of the principles, conventions, and techniques required to teach mathematics in technologically rich ways. Teachers' PTK enabled them to set instructional directions regarding the selection and use of technological tools to mediate students' learning. One aspect of PTK involved an understanding of the process of instrumentation (Rabardel & Waern, 2003; Verillon & Rabardel, 1995), where teachers' actions and decisions transformed a digital tool into an instrument suitable for a specific learning task. The instructional directions set by a teacher influenced how students saw the technology as a learning tool that shaped their learning trajectories.

Both in mathematics education research (e.g., Galligan et al., 2010; Holmes, 2009) and in Australasian general teacher education studies which include mathematics education (e.g., Bate, 2010; Redmond & Mander, 2009), a number of researchers have referred to the concept of technological pedagogical content knowledge, similar to PTK. Technological pedagogical content knowledge was formerly called TPCK by Mishra and Koehler (2006) and recently abbreviated as TPACK by Schmidt et al. (2009/2010). The TPACK framework describes the various synergetic interactions between three types of knowledge: (a) technological; (b) pedagogical; and (c) content knowledge (see Figure 1).

Schmidt et al. (2009/2010) defined TPACK as "the knowledge required by teachers for integrating technology into their teaching in any content area" (p. 125). PTK and TPACK provided frameworks in which the use of digital educational technologies and learning environments can be described in relation to teachers' characteristics. However, there are limitations in how such frameworks can be used to describe and analyse teachers' classroom practices. On one hand, both PTK and TPACK offered broad descriptions for categorising teachers' knowledge and skills into intersecting technological, pedagogical and content components; on the other hand, they do not provide a fine enough lens through which to examine exactly what technology enhanced knowledge is necessary for teaching and learning within specific mathematics topics. Other frameworks have attempted to address this gap, for example, the pedagogical map by Pierce and Stacey (2010); the phased approach to teaching and learning through dynamic geometry software developed by Serow (2008); and the model for numeracy which integrates the use of digital tools among other elements of teaching and learning mathematics reported by Geiger, Dole, and Goos (2011).

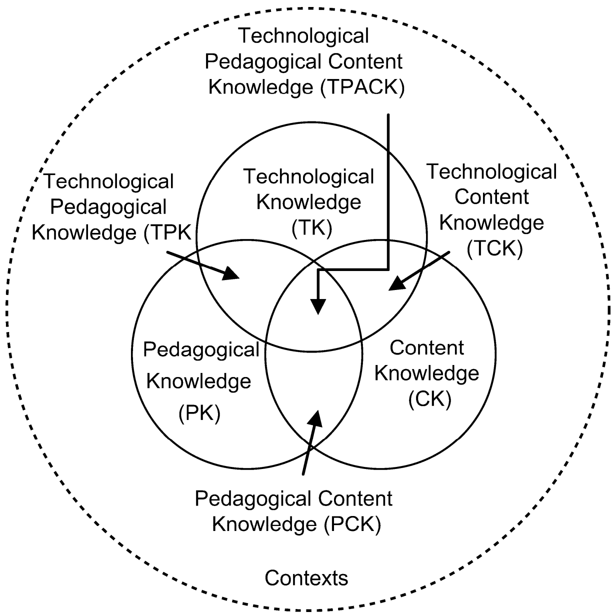

Figure 1. Components of TPACK framework (Schmidt et al., 2009/2010, p. 125).

How teachers develop technology enhanced approaches to pedagogy, and how the knowledge required to implement such pedagogy successfully is acquired are still open questions in need of further research. Recent studies showed that teachers' content knowledge and beliefs about the nature of learning affected how they viewed and used technology in the classrooms. For example, Lange and Meaney (2011) investigated pre-service teachers' use of internet resources for learning mathematics; Stillman and Brown (2011) explored teachers' views about the integration of technology into mathematical modelling activity; and Cavanagh and Mitchelmore (2011) concluded that the process of developing PTK is a gradual one—evolutionary rather than revolutionary. While these studies provided some insight into the development of teachers' approaches to technology enhanced mathematics pedagogy, more needs to be done to better understand the state of teacher's mathematical content knowledge (CK), pedagogical knowledge (PK), and technological knowledge (TK), as well as how these strands of knowledge are accessed during practice. Such research needs to attend to whether CK, PK, and TK should be developed concurrently through a teacher education program, or if there is an optimal sequence for the integration of the three components (Holmes, 2009).

The increasing complexity and variety of technological systems (hardware, software, and network/internet) makes it difficult to provide a clear and stable definition of what constitutes technological pedagogical content knowledge. This knowledge has to take into account the effects of the complex technologies available for specific mathematics content, assessment, and curriculum, and classroom management practicalities. Further research needs to be undertaken on testing existing frameworks in order that they better reflect current classroom practices.

Developing Teacher Expertise in Technology Integration

Consistent with studies into teachers' preferences in professional learning, and findings from studies related to the development of teacher expertise in technology/mathematics integration, teachers want useful and timely professional development that targets the use of technology for teaching specific content and for assisting different types of learners (Goos & Bennison, 2008a; Hudson et al., 2008; Thomas et al., 2008). However, while professional learning opportunities are necessary, these are not sufficient to fully develop teacher expertise in technology integration. Even when a technology-rich lesson is interesting and engaging to students, it might not bring about deepened mathematical understandings (Scott, Downton, Gronn, & Staples, 2008). Generally, research findings indicate that professional development programs should consider aspects such as TPACK, teacher confidence and attitudes, social support in school (e.g., support from mathematics coordinator, principal, and educational technology champion), and structural support (e.g., access to technological resources and support, time for planning and professional development).

As a way of attempting to describe and understand teachers' professional learning in technology active mathematics learning, Goos & Bennison (2008b) applied Valsiner's (1997) zone theory. This theory is an extension of Vygotsky's Zone of Proximal Development (ZPD), which encompasses a learner's potential for intellectual development, by the inclusion of two further zones: the Zone of Free Movement (ZFM) which represents the constraints within the school environment which limit teachers' access to, and interactions with, technology; and the Zone of Promoted Action (ZPA) which represents an individual's formal and informal opportunities to learn. Table 1 outlines the elements of each zone related to teacher development with the use of technology.

In particular, Goos and Bennison (2008b) explored the interactions between the ZPD, ZFM and ZPA in shaping teachers' professional identities in orchestrating technology-rich mathematics teaching. Longitudinal case studies of two beginning teachers and two experienced teachers were conducted and analysed using zone theory.

All four teachers were considered innovative in their technology use and held positive attitudes and beliefs about mathematics and the role of technology in mathematics learning (ZPD). For three teachers, there were strong overlaps between their ZPD and their school professional contexts (ZFM), which were well resourced and supportive of technology integration. For the fourth teacher, who was the head of the mathematics department, there was limited access to technology resources and "a culture of lethargy within the mathematics department

Table1. Factors affecting teachers' use of technology (Goos & Bennison, 2008b, p. 4)

Valsiner's Zones	***Elements of the Zones***
Zone of Proximal Development	Mathematical knowledge Pedagogical content knowledge Skill/experience in working with technology General pedagogical beliefs
Zone of Free Movement	Students (perceived abilities, motivation, behaviour) Access to hardware, software, teaching materials Technical support Curriculum & assessment requirements Organisational structures & cultures
Zone of Promoted Action	Pre-service teacher education Professional development Informal interaction with teaching colleagues

with few teachers interested in learning how to use technology" (Goos & Bennison, 2008b, p. 11). This teacher championed for changes in his department, such as obtaining class sets of graphics calculators through loan schemes and lobbying for more funds for technological resources, to align his ZFM to his ZPD. The four teachers were varied in their preferences for ZPA, ranging from self-directed learning, professional networks and formal professional training. Goos and Bennison (2008b) concluded that the four teachers had differing overlaps between the three zones, but theorised that the interaction in overlapping 'regions' is what enabled teachers to grow and develop their professional practice in technology integration.

Patahuddin (2008) used Goos' zone theory to analyse teachers' use of internet technology in teaching primary mathematics. The participants in this ethnographic study were an experienced practitioner who was a high level user of the internet (HUI) for professional growth and a beginning teacher who was a low level user of the internet (LUI). The HUI teacher held constructivist and student-centred views of learning mathematics (ZPD). The LUI teacher preferred a more teacher-centred approach, and placed greater emphasis on computational skills than on mathematical thinking and problem solving. Based on the two cases, Patahuddin (2008) concluded that teachers' views and interpretations of ZFM were more significant than the school environment.

In other studies, the emphasis has been on the development of teachers' TPACK. Beswick and Muir (2011), for example, used Beauchamp's (2004) five stage hierarchical model for the adoption of interactive whiteboards (IWB) as a framework in a professional development program for teachers to reflect on changes to their pedagogical use of IWB. The focus on this digital tool acted as a catalyst for collaborative lesson planning and reflection, and the teachers integrated the online resources and IWB purposefully into their teaching. In another study by Holmes (2009), teacher educators designed the course to maximise pre-service secondary mathematics teachers' TPACK in order to assist these teachers in learning to use IWB effectively. The study revealed that the pre-service teachers were able to integrate the use of IWB into their existing pedagogical and mathematical content knowledge.

In studying secondary mathematics teachers with no prior experience of using technology in teaching and with minimal professional development about technology, Cavanagh and Mitchelmore (2011) investigated how three secondary mathematics teachers taught with an online learning system over four school terms. Based on the analysis of teachers' PTK development, they proposed four sequential teacher roles: Technology Bystanders, allowing students to do work on their own; Technology Adopters, using technology to support teachers' pre-existing pedagogies; Technology Adapters, becoming more student-centred and teaching through rather than with technology to promote students' learning; and Technology Innovators, using technology to creatively encourage and support students' mathematical development and promote student inquiry. These researchers also highlighted the need for time to develop teachers' TK, and move teachers from technology bystanders to adopters, before they can develop PTK and move to become technology adaptors and innovators.

Mitchell, Stanelis, and Travers (2010) mapped ICT-based professional learning for Australia's digital strategy for education. Seven criteria for assessing effectiveness and quality of professional learning programs were identified: (a) focus on student learning; (b) engage staff in professional learning teams; (c) support a school-wide systemic approach to improvement; (d) involve staff in learning about and applying ICT pedagogy and skills; (e) occur in the context of consistent and rich external policies, guidelines, resources, research and networks; (f) address specialised learning for sectional interests such as leadership, pre-service teachers, levels of schooling and subject areas; and (g) incorporate specific content knowledge. These criteria are consistent with the research findings and frameworks discussed in this section that focus on the same issues in mathematics education contexts. In their recommendations, Mitchell et al. emphasised the need to build the capacity and skills of teacher educators and school leaders in incorporating technology into teaching and learning. While the Australasian mathematics education research community is increasingly focusing on the importance of teacher educators' learning (Goos, 2009b; Goos, Chapman, Brown, & Novotna, 2011), future research should also include studies on teacher educators' ICT practices in teacher education programs and, perhaps, teacher educators' own TPACK.

GENDER, AFFECT AND TECHNOLOGY

Besides access to technologies, teachers' technology use was found to be affected by their beliefs towards the nature of mathematics and the teaching and learning of mathematics; their confidence and skills in using technology; their involvement in professional development programs, and school support for technology use (e.g., time, resources, and ongoing support). The findings from several studies varied in the extent that each of these factors influenced teachers' use of technology, possibly due to the different contexts and methodologies used.

In a comparison of teacher use and non-use of computers, Hudson, Porter, and Nelson (2008) found that a significantly higher number of teachers who did not use computers indicated that a lack of lesson plans was a barrier. Additionally, female teachers seemed to be less confident than male teachers in using technology (Pierce & Ball, 2009; Thomas et al., 2008). It is clear that these factors play major roles in teachers' decisions about what technology to use, and when and how to use the technology. Despite this, non-cognitive issues associated with the use of various technologies in the mathematics classroom have received limited attention during the four year review period.

Most of the research on gender, affect, and technology for mathematics learning has been conducted at the secondary level of schooling. While some qualitative cases studies were reported, survey techniques were the prevalent research approach adopted.

Attitudes, Beliefs and Confidence

Research findings on students' and teachers' attitudes towards and beliefs about technology for mathematics education are summarised here.

Students: The results of a survey of 97 secondary school students who had used the Web in their mathematics classes were reported by Loong (2010). Attitudes towards mathematics were gauged from the Fennema-Sherman Mathematics Attitudes Scales; items related to the internet were researcher generated. Only about a third of the respondents agreed that there was value in using the internet for learning mathematics. Younger (Grade 8) participants valued the internet more highly than older students (Grades 10, 11, & 12), and many felt that there was little mathematical knowledge gained from assignments involving Web-based information retrieval and data collection. The findings from the study suggested that the lack of links to assessment might explain older students' less positive views of the value of using the internet for mathematics learning.

Teachers: Both barriers and enablers of technology use in mathematics classrooms can include affective factors (Pierce & Ball, 2009). Pierce and Ball administered a Technology Perceptions Survey Australia-wide, and reported the responses of 92 secondary mathematics teachers. Most teachers agreed that using technology would improve student motivation, assist students in gaining deeper mathematical understanding, and make mathematics more enjoyable. An item which questioned whether technology was too expensive for students to access, attracted the strongest agreement among the statements associated with barriers to the adoption of technology for mathematics learning—approximately a third of the respondents agreed. The results also suggested that female teachers may be less confident than male teachers about using technology, even though the majority in both groups saw value in using technology for teaching mathematics.

Goos and Bennison (2008a, 2008b) reported findings from a survey of 485 mathematics teachers' use of computers, graphics calculators and the internet in Queensland secondary schools. The relationships between technology use and teachers' pedagogical knowledge and beliefs, access to technology, and professional development opportunities were explored. Two surveys were designed based on instruments used in previous Australasian studies and on international research on factors known to influence mathematics teachers' use of technology. Teachers who frequently used graphics calculators were found to be more likely than others to agree that technology was beneficial to students' mathematical learning. Many teachers were undecided about some of the benefits of using technology to support mathematics learning, and more than a third felt it was time consuming to teach students how to use this technology. Longitudinal case studies of four teachers were also conducted and it was found that while access to technology was an important enabling factor, teachers in well-resourced schools did not necessarily embrace technology, while teachers in poorly resourced schools could be inventive in exploiting available resources to improve their students' understanding of mathematical concepts. This highlights the significance of teachers' beliefs, their institutional cultures, and the organisation of time and resources in their schools.

Elsewhere in this study, Bennison and Goos (2010) reported on the teachers' developmental experiences and needs for technology-related professional development. Teachers who had not participated in professional development on the use of technology for teaching mathematics, were found to be more likely than others to be undecided as to whether the use of technology made sophisticated mathematical concepts accessible to students or improved students' attitudes towards mathematics. The researchers concluded that professional development participation was related to greater confidence with technology, and more positive beliefs about technology use were beneficial for students' learning of mathematics.

Mathematics was not the main subject area of interest for Neal and Davidson (2008) who examined how teachers integrated Tablet PCs into their subject domains in secondary schools. Two large secondary schools in Melbourne were involved through a mixed methods approach. Tablet pen use was most commonly seen in the Arts, Mathematics, and LOTE, and it appeared that the type of teacher, rather than any particular subject domain, was the main indicator for the inclusion of the pen as a resource. Even though the Tablet pen was consistently used by students in one of the mathematics classes observed, students indicated that the teacher's use of the Tablet PC and pen did not support effective learning. One teacher used restrictive teaching sequences, and students were very aware of the teacher's lack of Tablet PC expertise. Patahuddin (2008) reported on uses of the internet for teacher professional development and the teaching of mathematics. Drawing on two case-studies, an analysis was undertaken of personal and contextual factors supporting or inhibiting mathematics teachers in making use of the internet for professional development or mathematics teaching. The findings demonstrated that resources alone did not guarantee successful teaching, and that preferred teaching approaches and pedagogical beliefs may hinder optimal use of the internet.

Gender Issues

Gender was an important variable in several studies where attitudes towards technology use for mathematics learning and mathematics achievement were examined.

Gender, technology, and attitudes: Barkatsas, Kasimatis, and Gialamas (2009) investigated the complex relationships between students' mathematical confidence, confidence with technology, attitudes towards learning mathematics with technology, affective engagement, and behavioural engagement. Gender and grade level were variables of interest. In this study the mathematics and technology attitudes scale (MTAS) (Pierce et al., 2007) was administered to 1068 Year 9 and Year 10 students from 27 state co-educational schools in Athens, Greece. Compared to girls, boys expressed more positive views towards mathematics and the use of technology in mathematics. In addition, high achievement in mathematics was associated with high levels of mathematical confidence, high confidence in using technology, and a strongly positive attitude towards learning mathematics with technology. Shamoail and Barkatsas (2011) also administered the MTAS scales to Years 10 and 11 students attending Catholic secondary schools in Victoria. Among those attending co-educational schools, males had higher levels of confidence for mathematics and for technology (CAS calculators) than females. For students attending single-sex schools, males again were more confident than females about mathematics, but there was no difference in confidence with technology. However, the males were found to have higher levels of affective engagement with mathematics than females.

Gender, technology, and achievement. Forgasz (2008) discussed findings from various studies on gender patterns in mathematics achievement, participation rates, and on the effects of technology on mathematics learning outcomes. Based on the weight of evidence on gendered patterns favouring males, it was argued that the data indicated a reversal of the narrowing of the gender gap that had been observed in the past decade. In a similar vein, Forgasz and Tan (2010) compared the patterns of enrolment and achievement in the two parallel intermediate-level Year 12 mathematics subjects offered in the Victorian Certificate of Education [VCE]: Mathematical Methods and Mathematical Methods CAS. The parallel offerings spanned a period of transition from graphics to CAS calculator use in the subject. Forgasz and Tan (2010) aimed to determine whether boys' and girls' achievements in the two subjects differed and if the difference in the type of calculator used might be implicated. For both subjects, a higher proportion of males than females received the grade of A+ for the three assessment tasks included in the overall assessment of the subject. This provided evidence of gender gaps (differences in the percentages of male and female students achieving the grade) in favour of males. It was also noted that there had been a decline in enrolments in the subjects over the years and that the decline was greater for females than males.

Research Overviews

Australasian researchers were well-represented at the 17th ICMI study on technology and mathematics learning held in Hanoi, Vietnam in 2007. In the book resulting from that conference, there were two chapters on non-cognitive factors associated with technology use for mathematics learning. Forgasz, Vale, and Ursini (2010) examined issues of equity, including gender, access, and agency. They argued that there appeared to be some disparity in research findings on the relationship between technology use and gender differences in mathematics achievement, with findings from Mexico showing that girls excelled in their mathematical learning with technology, while Australian findings tended to support boys being advantaged. The authors claimed that the availability of resources for mathematics learning with digital technologies varied according to the economic status of countries. More research was called for to identify factors contributing to the gap between low and high mathematics achievers in relation to the use of digital technologies. Assude, Buteau, and Forgasz (2010) identified factors influencing the integration of digital technology in mathematics. By examining the issues from various standpoints (social, political, economic and cultural; mathematical and epistemological; school and institutional; classroom and didactical) the authors concluded that access to technology was a factor that encouraged many mathematics teachers, but also served as a barrier to others. Institutional and didactical factors, such as access to hardware, professional development needs, and technical support appeared to outweigh personal factors such as confidence in preventing teachers from using technology in their mathematics teaching. Many changes, the authors concluded, were needed in order to integrate digital technologies into the teaching of mathematics, the effects of which might not be seen for some years.

CONCLUSIONS AND FUTURE DIRECTIONS FOR RESEARCH

While there is clear evidence that digital technologies influence classroom contexts and environments, little appears to be known about how to best support teachers in designing online or proximate technology based activities to best enhance student learning. The IWB is a particularly salient example. While this technology is becoming increasingly available, there appears to be little Australasian research that identifies how best to use it to promote deep mathematical knowledge and understanding, or to enhance genuinely collaborative approaches to learning. Pedagogical frameworks for mathematics analysis software, such as that of Pierce and Stacey (2008, 2010), may help set directions for research, but how teachers' best learn and implement TPACK is still an open question. This seems particularly relevant to the use of technology to enhance learning in early childhood contexts where few studies have been reported. Another area that received scant attention is teacher educators' practices in assisting pre-service teachers to make use of digital technologies in the classroom. This issue is related to teacher educators' own TPACK. More needs to be known about the best ways for teacher educators to help future teachers prepare for technology rich mathematics classrooms.

There have been a number of empirical and theoretical studies into how Web-based tools can mediate collaborative approaches to learning. Research recognises a movement towards more social ways of living and acting within a digitally drenched society. The findings are both encouraging and challenging, although methodologically this area needs far greater conceptualisation as it appears that most research is conducted within paradigms developed before the advent of socially oriented Web-based tools. An aspect of research design that needs further attention is the unit of analysis within research designs. In a multi-user virtual environment, or in a collaborative, technology rich classroom, the boundaries between humans and digital agents are often blurred. Thus, it is inappropriate to make judgements about a person's knowledge or capabilities independent of the digital technologies that mediated their new understandings or capacities. In these circumstances, Borba and Villarreal's (2006) notion of humans-with-media, a unit of analysis in which the product of human interaction with digital tools is accepted as an integrated whole, might prove to be a more appropriate view of the learning enterprise. This perspective has given rise to new approaches to conceptualising research about digital technologies, especially in relation to research design. At the same time, more research is needed into how individual learners know and come to know mathematics within collaborative environments. This will also demand the development of new methodological approaches.

The sociocultural concepts of affordances and constraints have received considerable attention in research into both teaching and learning mathematics with technology. The analysis of data using theoretical frameworks based on these concepts has yielded penetrating insights into the circumstances that promote or inhibit attempts to enhance mathematical learning through the use of technological tools. While some researchers have attempted to address how constraints can be managed, more research is required into how affordances that support technology rich mathematics learning can be optimised.

Compared to the previous four year review period, the extent of research into non-cognitive issues associated with the use of technology in mathematics teaching and learning appears to have diminished. It also seems that the inclusion of technology in mathematics education may not be fulfilling its promise of revolutionising the way mathematics is taught and learnt. Teachers' pedagogical knowledge and beliefs appear to be salient factors affecting whether or not teachers embrace technology in the mathematics classroom. Both of these factors are related to teachers' self-efficacy in relation to technology – an area that is currently under researched. Research on gender, attitudes, mathematics achievement, and the role of technology also raises concerns about the effect that technology may be having on females' participation and achievement in mathematics. As the role of technology in the mathematics classroom grows, research needs to keep pace to ensure that technology is utilised effectively and appropriately, and that its inclusion in mathematics education is not disadvantageous.

The following areas have emerged as requiring further, ongoing, research:

– how digital technologies influence actual learning trajectories and the associated understanding in distinctive ways;
– the ways dialogue, justification and conjectures emerge and are developed within digital environments;
– how social interaction and learning communities are transformed through learners engaging digital pedagogical media and how this influences learning;
– the ways student-centred inquiry learning, where students utilise the internet, affect both understanding and attitudes to mathematics learning; and
– how to incorporate technology equitably into the mathematics classroom.

It should also be noted that much of the research into how digital technology influences teaching and learning has been based in intensive small scale studies. While studies of this nature are important for developing an understanding of the issues in the field, additional studies of greater scale and scope are needed to strengthen evidence for claims emanating from current and future research.

REFERENCES

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, *7*(3), 245–274.

Assude, T., Buteau, C., & Forgasz, H. J. (2010). Factors influencing implementation of technology-rich mathematics curriculum and practices. In C. Hoyles, & J-B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain* (pp. 405–419). New York, NY: Springer.

Baker, R. (2010). *Pedagogies and digital content in the Australian school sector*. Retrieved from http://www.thelearningfederation.edu.au/verve/_resources/Pedagogies_Report.pdf

Barkatsas, A., Kasimatis, K., & Gialamas, V. (2009). Learning secondary mathematics with technology: Exploring the complex interrelationship between students' attitudes, engagement, gender and achievement. *Computers and Education*, *52*, 562–570.

Bate, F. (2010). The role of ICT in the pedagogical transformation of primary teachers: Dream, aspiration, reality. In *Proceedings of World Conference on Educational Multimedia, Hypermedia and Telecommunications 2010* (pp. 2301–2306). Chesapeake, VA: AACE. Retrieved from http://www.editlib.org/p/34959

Beatty, R., & Geiger, V. (2010). Technology, communication and collaboration: Rethinking communities of inquiry, learning and practice. In C. Hoyles, & J-B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain* (pp. 251–284). New York, NY: Springer.

Beauchamp, G. (2004). Teacher use of the interactive whiteboard in primary schools: Towards an effective transition framework. *Technology, Pedagogy and Education*, *13*(3), 327–348.

Bennison, A., & Goos, M. (2010). Learning to teach mathematics with technology: A survey of professional development needs, experiences and impacts. *Mathematics Education Research Journal*, *22*(1), 31–56.

Beswick, K., & Muir, T. (2011). Interactive whiteboards as potential catalysts of pedagogic change in secondary mathematics teaching. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton(Eds.), *Mathematics: Traditions and [new] practices.* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 115–123). Adelaide, SA: AAMT & MERGA.

Borba, M. C., & Villarreal, M. E. (2006). *Humans-with-media and the reorganization of mathematical thinking: Information and communication technologies, modeling, visualization, and experimentation*. New York, NY: Springer.

Brousseau, G. (1997). *Theory of didactical situations in mathematics: Didactique des mathématiques*, 1970–1990 (N. Balacheff, M. Cooper, R. Southerland, & V. Warfield, Eds. & Trans.). Dordrecht, The Netherlands: Kluwer Academic.

Brown, J. (2006). Manifestations of affordances of a technology-rich teaching and learning environment (TRTLE). In J. Novotna, H. Moraova, M. Kratka, & N. Stehlíkova (Eds.), *Mathematics in the centre* (Proceedings of the 30th conference of the International Group for the Psychology of Mathematics Education, pp. 241–248). Prague, Czech Republic: PME.

Calder, N. S. (2009a). How digital technologies might influence the learning process. In R. Averill & R. Harvey (Eds.), *Teaching secondary school mathematics and statistics: Evidence-based practice* (pp. 131–143). Wellington, New Zealand: NZCER Press.

Calder, N. S. (2009b). Visual tensions when mathematical tasks are encountered in a digital learning environment. In M. Tzekaki, M. Kaldrimidou, & C. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 249–256). Thessaloniki, Greece: PME.

Calder, N. S. (2009c). Processing mathematical thinking through digital pedagogical media. In C. Bardini, P. Fortin, A. Oldknow, & D. Vagost (Eds.), *Proceedings of the 9th International Conference of Technology in Mathematics Teaching* (pp. 1.1–1.5). Metz, France: ICTMT.

Calder, N. S. (2010). Affordances of spreadsheets in mathematical investigation: Potentials for learning. *Spreadsheets in Education*, *3*(3), 1–21.

Calder, N. S. (2011a). Investigating mathematics through digital media: Cognitive visual perturbations. *The International Journal for Technology in Mathematics Education*, *18*(3), 15–24.

Calder, N. S. (2011b). *Processing mathematics through digital technologies: The primary years.* Rotterdam, The Netherlands: Sense.

Calder, N. S., & Taylor, M. (2010). Scratching below the surface: Mathematics through an alternative digital lens? In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 109–116). Fremantle, WA: MERGA.

Campbell, C., & Macnish, J. (2010). Pre-service teachers using The Le@rning Federation's digital resources. In J. Harrington & B. Hunter (Eds.), *Proceedings of world conference on educational multimedia, hypermedia and telecommunications 2010* (pp. 47–51). Chesapeake, VA: AACE. Retrieved from http://www.editlib.org/p/34617

Cavanagh, M., & Mitchelmore, M. (2011). Teaching secondary mathematics with an online learning system: Three teachers' experiences. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 158–165). Adelaide, SA: AAMT & MERGA.

Clark-Wilson, A. (2010). Emergent pedagogies and the changing role of the teacher in the TI-Nspire Navigator-networked mathematics classroom. *ZDM—The International Journal of Mathematics Education*, *42*(7), 747–761.

Clements, D. H., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning*, *6*(2), 81–89.

Confrey, J., Hoyles, C., Jones, D., Kahn, K., Maloney, A., Nguyen, K., ... Pratt, D. (2010). Designing software for mathematical engagement through modelling. In C. Hoyles & J.-B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain* (pp. 19–46). New York, NY: Springer.

Evans, M., Jones, P., Leigh-Lancaster, D., Les, M., Norton, P., & Wu, M. (2008). The 2007 common technology free examination for Victorian Certificate of Education (VCE) mathematical methods and mathematical methods computer algebra system (CAS). In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 331–336). Brisbane, QLD: MERGA.

Forbes, S., & Pfannkuch, M. (2009). Developing statistical thinking: Teaching and learning. In R. Averill & R. Harvey (Eds.), *Teaching secondary school mathematics and statistics: Evidence-based practice* (pp. 93–128). Wellington, New Zealand: NZCER Press.

Forgasz, H. J. (2008). Stars, compass, and GPS: Navigating currents and charting directions for mathematics education research on gender issues. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 5–15). Brisbane, QLD: MERGA.

Forgasz, H. J., & Tan, H. (2010). Does CAS use disadvantage girls in VCE mathematics? *Australian Senior Mathematics Journal*, *24*(1), 25–36.

Forgasz, H. J., Vale, C., & Ursini, S. (2010). Technology for mathematics education: Equity, access and agency. In C. Hoyles & J-B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain* (pp. 385–403). New York, NY: Springer.

Forster, P., Flynn, P., Frid, S., & Sparrow, L. (2004).Calculators and computer algebra systems. In B. Perry, G. Anthony, & C. Diezmann (Eds.), *Research in mathematics education in Australasia 2000–2003* (pp. 313–336). Flaxton, Australia: Post Pressed.

French, S. (2010). Mathematical WebQuests: Inquiry-based learning to enrich mathematical understanding. In R. Averill & R. Harvey (Eds.), *Teaching primary school mathematics and statistics: Evidence-based practice* (pp. 275–290). Wellington, New Zealand: NZCER Press.

Gadanidis, G., & Geiger, V. (2010). A social perspective on technology-enhanced mathematical learning: from collaboration to performance. *ZDM—The International Journal of Mathematics Education*, *42*(1), 91–104.

Galligan, L., Loch, B., McDonald, C., & Taylor, J. A. (2010). The use of tablet and related technologies in mathematics teaching. *Australian Senior Mathematics Journal*, *24*(1), 38–51.

Geiger, V. (2005). Master, servant, partner, and extension of self: A finer grained view of this taxonomy. In J. P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Research theory and practice* (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, Melbourne, pp. 369–376). Sydney, NSW: MERGA.

Geiger, V. (2009). The master, servant, partner, extension-of-self framework in individual, small group and whole class contexts. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 201–208). Palmerston North, NZ: MERGA

Geiger, V. (2011). Factors affecting teachers' adoption of innovative practices with technology and mathematical modeling. In G. Kaiser, W. Blum, R. Borromeo-Ferri, & G. Stillman (Eds.), *Trends in the teaching and learning of mathematical modeling* (pp. 305–314). New York, NY: Springer.

Geiger, V., Dole, S., & Goos, M. (2011). The role of digital technologies in teaching numeracy learning. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 385–392). Ankara, Turkey: PME.

Geiger, V., Faragher, R. & Goos, M. (2010). CAS-enabled technologies as 'agents provocateurs' in teaching and learning mathematical modelling in secondary school classrooms. *Mathematics Education Research Journal*, *22*(2), 48–68.

Geiger, V., Faragher, R., Redmond, T., & Lowe, J. (2008). CAS enabled devices as provocative agents in the process of mathematical modelling. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 219–226). Brisbane, QLD: MERGA.

Gibson, I. (2010). Changing landscapes: Rethinking technology practice in teacher education for the 21stcentury from an early childhood context. In D. Gibson & B. Dodge (Eds.), *Proceedings of Society for Information Technology & Teacher Education International Conference 2010* (pp. 1435–1442). Chesapeake, VA: AACE. Retrieved from http://www.editlib.org/p/33560

Gibson, J. J. (1977). The theory of affordances. In R. Shaw & J. Bransford (Eds.), *Perceiving, acting, and knowing: Toward an ecological psychology* (pp. 67–82). Hillsdale, NJ: Lawrence Erlbaum.

Goos, M. (2009a). A sociocultural framework for understanding technology integration in secondary school mathematics. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 113–120). Thessaloniki, Greece: PME.

Goos, M. (2009b). Investigating the professional learning and development of mathematics teacher educators: A theoretical discussion and research agenda. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 209–216). Palmerston North, NZ: MERGA. Retrieved from http://www.merga.net.au/documents/Goos_RP09.pdf

Goos, M., & Bennison, A. (2008a). Surveying the technology landscape: Teachers' use of technology in secondary mathematics classrooms. *Mathematics Education Research Journal*, *20*(3), 102–130.

Goos, M., & Bennison, A. (2008b). Teacher professional identities and the integration of technology into secondary school mathematics. *Proceedings of the annual conference of the Australian Association for Research in Education*, Brisbane). Retrieved from http://www.aare.edu.au/08pap/goo08968.pdf

Goos, M., Chapman, O., Brown, L., & Novotna, J. (2011).The learning and development of mathematics teacher educators-researchers. Working Session 5. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 173–180). Ankara, Turkey: PME.

Goos, M., & Cretchley, P. (2004). Computers, multimedia, and the internet in mathematics education. In B. Perry, C. Diezmann, & G. Anthony (Eds.), *Research in mathematics education in Australasia 2000–2003* (pp. 151–174). Flaxton, Australia: Post Pressed.

Goos, M., & Soury-Lavergne, S. (2010). Teachers and teaching: Theoretical perspectives and issues concerning classroom implementation. In C. Hoyles and J-B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain* (pp. 311–328). New York, NY: Springer.

Herbert, S., & Pierce, R. (2008). An 'emergent model' for rate of change. .*International Journal of Computers for Mathematical Learning*, *15*(1), 231–249.

Highfield, K., & Goodwin, K. (2008). A review of recent research in early mathematics learning and technology. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 259–264). Brisbane, QLD: MERGA.

Highfield, K., & Mulligan, J. (2009). Young children's embodied action in problem-solving tasks using robotic toys. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 169–176). Thessaloniki, Greece: PME.

Highfield, K., Mulligan, J., & Hedberg, J. (2008). Early mathematics learning through exploration of programmable toys. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 169–176). Morelia, Mexico: Cinvestav-UMSNH.

Holmes, K. (2009). Planning to teach with digital tools: Introducing the interactive whiteboard to pre-service secondary mathematics teachers. *Australasian Journal of Educational Technology*, *25*(3), 351–365.

Hoyles, C., Kalas, I., Trouche, L., Hivon, L., Noss, R., & Wilensky, U. (2010). Connectivity and virtual networks for learning. In C. Hoyles & J-B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain* (pp. 439–462). New York, NY: Springer.

Hoyles, C., & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In A. J. Bishop, M. A. Clements, C. Keitel, J. Kirkpatrick, & F. Leung (Eds.), *Second international handbook of mathematics education* (pp. 323–349). Dordrecht, The Netherlands: Kluwer Academic.

Hudson, R., & Porter, A. (2010). ICT use to improve mathematics learning in secondary schools. In D. Gronn & G. Romeo (Eds.), *ACEC2010: Digital diversity*. *Conference proceedings of the Australian Computers in Education Conference*. Melbourne, Australia: ACEC. Retrieved from http://acec2010.info/proposal/970/ict-use-improve-mathematics-learning-secondary-schools

Hudson, R., Porter, A., & Nelson, M. (2008). Barriers to using ICT in mathematics teaching: Issues in methodology. In J. Luca & E. Weippl (Eds.), *Proceedings of World Conference on Educational Multimedia, Hypermedia and Telecommunications* 2008 (pp. 5765–5770). Chesapeake, VA: AACE. Retrieved from http://www.editlib.org/p/29181

Hughes, J. (2008). The performative pull of research with new media. *International Journal of Qualitative Methods*, *7*(2), 16–34.

Jones, A. (2008). Multimedia and multiliteracies in the early elementary years. In J. Luca & E. Weippl (Eds.), *Proceedings of World Conference on Educational Multimedia, Hypermedia and Telecommunications* (pp. 2942–2948). Chesapeake, VA: AACE. Retrieved from http://www.editlib.org/p/28783

Kennewell, S. (2001). Using affordances and constraints to evaluate the use of information and communications technology in teaching and learning. *Journal of Information Technology for Teacher Education*, *10*(1), 101–116.

Kissane, B., & Kemp, M. (2008). Some calculus affordances of a graphics calculator. *Australian Senior Mathematics Journal*, *22*(2), 15–27.

Lange, T., & Meaney, T. (2011). Preservice teachers learning mathematics from the internet. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 438–445). Adelaide, SA: AAMT & MERGA.

Leigh-Lancaster, D., Les, M., & Evans, M. (2010). Examinations in the final year of transition to mathematical methods computer algebra system (CAS). In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 336–343). Fremantle, WA: MERGA.

Loong, E. Y-K. (2009). Using web-based mathematical interactive exercises and exploratory investigations: The possibilities and pitfalls. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 339–346). Palmerston North, NZ: MERGA. Retrieved from http://www.merga.net.au/documents/Loong_RP09.pdf

Loong, E. Y-K. (2010). Web-based mathematics: Student perceptions. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 352–359). Fremantle, WA: MERGA.

Mishra, P., & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, *108*(6), 1017–1054. Retrieved from http://punya.educ.msu.edu/publications/journal_articles/mishra-koehler-tcr2006.pdf

Mitchell, P, Stanelis, N., & Travers, J. (2010). *ICT professional learning: National mapping of ICT-related professional learning*. Adelaide, SA Education Services Australia. Retrieved from http://www.deewr.gov.au/Schooling/DigitalEducationRevolution/DigitalStrategyforTeachers/Pages/Home.aspx

Neal, G., & Davidson, K. (2008). Contesting ideas of innovative teaching practice with tablet PCs. *Proceedings of the annual conference of the Australian Association for Research in Education*. Brisbane, QLD. Retrieved from http://www.aare.edu.au/08pap/nea08284.pdf

Oates, G. (2009). Relative values of curriculum topics in undergraduate mathematics in an integrated technology environment. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 419–426). Palmerston North, NZ: MERGA.

Olive, J., & Makar, K. (2010). Mathematical knowledge and practices resulting from access to digital technologies. In C. Hoyles and J-B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain* (pp. 251–284). New York, NY: Springer.

Patahuddin, S. M. (2008). Use of the Internet for teacher professional development and for teaching mathematics: Supports and inhibitors. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 399–405). Brisbane, QLD: MERGA.

Pierce, R., & Ball, L. (2009). Perceptions that may affect teachers' intention to use technology in secondary mathematics classes. *Educational Studies in Mathematics*, *71*, 299–317.

Pierce, R., & Ball. L. (2010). Secondary teachers' use of technology for teaching mathematics In S. Howard (Ed.), *Making a difference* (Proceedings of the 2010 conference of the Australian Association for Research in Education, pp. 1–13). Melbourne, Australia: AARE. Retrieved fromhttp://aare.edu.au/10pap/2214PierceBall.pdf

Pierce, R., & Stacey, K. (2008). Using pedagogical maps to show the opportunities afforded by CAS for improving the teaching of mathematics. *Australian Senior Mathematics Journal*, *22*(1), 6–12.

Pierce, R., & Stacey, K. (2009a). Lesson study with a twist: Researching lesson design by studying classroom implementation. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 369–376). Thessaloniki, Greece: PME.

Pierce, R., & Stacey, K. (2009b). Researching principles of lesson design to realise the pedagogical opportunities of mathematical analysis software. *Teaching Mathematics and its Applications*, *28*, 228–233.

Pierce, R., & Stacey, K. (2010). Mapping pedagogical opportunities provided by mathematics analysis software. *International Journal of Computers for Mathematical Learning, 15*(1), 1–20.

Pierce, R., Stacey, K., & Barkatsas, A. (2007). A scale for monitoring students' attitudes to learning mathematics with technology. *Computers and Education*, *48*(2), 285–300.

Pierce, R., Stacey, K., & Wander, R. (2010). Examining the didactic contract when handheld technology is permitted in the mathematics classroom. *ZDM—The International Journal of Mathematics Education*, *42*(7), 683–695.

Rabardel, P., & Waern, Y. (2003). Editorial: From artefact to instrument. *Interacting with Computers*, *15*(5), 641–645.

Redmond, P., & Mander, A. (2009). Constructing a pre-service teacher online learning community. In I. Gibson & D. Dodge (Eds.), *Proceedings of Society for Information Technology & Teacher Education International Conference* (pp. 2936–2942). Chesapeake, VA: AACE. Retrieved from http://www.editlib.org/p/31086

Sacristan, A., Calder, N. S., Rojano, T., Santos, M., Friedlander, A., & Meissner, H. (2010). The influence, and shaping, of digital technologies on the learning—and learning trajectories—of mathematical concepts. In C. Hoyles and J-B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain* (pp. 179–226). New York, NY: Springer.

Schmidt, D. A., Baran, E., Thompson, A. D., Mishra, P., Koehler, M. J., & Shin, T. S. (2009/2010). Technological pedagogical content knowledge (TPACK): The development and validation of an assessment instrument for preservice teachers. *Journal of Research on Technology in Education*, *42*(2), 123–149.

Scott, A., Downton, A., Gronn, D., & Staples, A. (2008). Engagement versus deep mathematical understanding: An early career teacher's use of ICT in a lesson. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 43–49). Brisbane, QLD: MERGA. Retrieved from http://www.merga.net.au/documents/RP522008.pdf

Serow, P. (2008). Investigating a phase approach to using technology as a teaching tool. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 445–452). Brisbane, QLD: MERGA.

Serow, P., & Callingham, R. (2008). The introduction of interactive whiteboard technology in the primary mathematics classroom: Three case studies. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 453–459). Brisbane, QLD: MERGA.

Shamoail, E., & Barkatsas, A. (2011). Students' attitudes towards handheld computer algebra systems (CAS) in mathematics: Gender and school setting issues. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the

23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 685–692). Adelaide, SA: AAMT & MERGA.

Shulman, L. C. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4–41.

Stillman, G., & Brown, J. (2011). 'Get down and get dirty in the mathematics': Technology and mathematical modelling in senior secondary. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 711–718). Adelaide, SA: AAMT & MERGA.

Tan, H. (2009). Development of an instrument for ways of using graphic calculators. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 531–538). Palmerston North, NZ: MERGA.

Tan, H. (2011). Students' ways of using handheld calculators in Singapore and Australia: Technology as master, servant, partner and extension of self. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 728–735). Adelaide, SA: AAMT & MERGA.

Tanner, H., Jones, S., Beauchamp, G., & Kennewell, S. (2010). Interactive whiteboards and all that jazz: Analysing classroom activity with interactive technologies. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 547–554). Fremantle, WA: MERGA.

Thomas, M. O. J. (2009). Hand-held technology in the mathematics classroom: Developing pedagogical technology knowledge. In R. Averill & R. Harvey (Eds.), *Teaching secondary school mathematics and statistics: Evidence-based practice* (pp. 147–160). Wellington, New Zealand: NZCER Press.

Thomas, M. O. J., & Chinnappan, M. (2008). Teaching and learning with technology: Realising the potential. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W-T. Seah, & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004–2007* (pp. 167–194). Rotterdam, The Netherlands: Sense.

Thomas, M. O. J., Hong, Y. Y., Bosley, J., & delos Santos, A. (2008). Use of calculators in the mathematics classroom. *The Electronic Journal of Mathematics and Technology (eJMT), 2*(2). Retrieved from http://www.freepatentsonline.com/article/Electronic-Journal-Mathematics-Technology/180749502.html

Valsiner, J. (1997). *Culture and the development of children's action: A theory of human development* (2nd ed.). New York, NY: John Wiley and Sons.

van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. New York, NY: Academic Press.

Verenikina, I., Herrington, J., Peterson, R., & Mantei, J. (2010). Computers and play in early childhood: Affordances and limitations. *Journal of Interactive Learning Research*, *21*(1), 139–159.

Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of thought in relation to instrumental activity. *European Journal of Psychology of Education, 10,* 77–103.

Yeh, A. (2010). Three primary school students' cognition about 3D rotation in a virtual reality learning environment. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 690–697). Fremantle, WA: MERGA.

Yeh, A., & Nason, R. (2008). Mixing colours: An ICT tool based on a semiotic framework for mathematical meaning-making about ratio and fractions. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 43–49). Brisbane, QLD: MERGA.

Zevenbergen, R., & Lerman, S. (2008). Learning environments using interactive whiteboards: New learning spaces or reproduction of old technologies? *Mathematics Education Research Journal*, *20*(1), 107–125.

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ASSESSMENT BEYOND ALL: THE CHANGING NATURE OF ASSESSMENT

Key words: national assessment; classroom assessment; assessment of mathematics concepts; professional development.

INTRODUCTION

This is the first time a chapter has been dedicated solely to assessment in MERGA's four yearly review. This fact, and the emerging status of assessment in Australia's educational research domain, dictates the structure and nature of the chapter. In previous reviews, assessment (as a process) was classified within student performance in the classroom and reflections on teachers' practices. Consequently, research about assessment was distributed across content chapters. However, we anticipate that such a chapter will have prominence in reviews to come. Indeed, Callingham's (2011b) keynote paper from the 34th annual MERGA conference argued for a need to re-assess mathematics assessment "and to reconsider the purpose, nature and use of assessment information" (p. 3). Hence, a chapter in this review is timely.

The stimulus for this chapter and the work of the research community has stemmed from a relatively new focus on high-stakes testing and the comparative nature of assessment across national and international boundaries. From a national perspective, the *National Assessment Program—Literacy and Numeracy* (NAPLAN) has impacted on the assessment culture in Australia in the sense that comparisons of student performance occur across state boundaries. From an international perspective, the influence of assessment instruments developed through the *Trends in International Mathematics and Science Study* (TIMSS) and *Programme for International Student Assessment* (PISA) initiatives have compared the performance of Australian students with students worldwide. Such examples have refocussed (no matter how subtly) classroom assessment practices and invariably the extent to which mathematics curricula will be developed in the future. Never before has the performance of Australian students been compared across states or discussed in relation to international benchmarks. Given the public nature of these comparisons, it is not only teachers who are being confronted with such issues; increasingly, this will impact on classroom practices.

In a high-stakes testing era, numerous assessment practices have been framed within an accountability regime which have resulted in some assessment reporting being quite quantitative in nature—and this focus has been awarded much attention by the media. In the past five years assessment in Australian schools has become a

B. Perry et al. (Eds.), Research in Mathematics Education in Australasia 2008–2011, 143–165.

national focus and as a result, raised the public awareness of assessment. This heightened awareness has resulted in different levels of accountability, public interest and the political desire for increased levels of transparency. This is in stark contrast to the underlying principles of quality assessment, which places the student at the centre of assessment practices. Furthermore, the research in this chapter and indeed, educational research over the last 30 years, has focused on the value, and need, to have a repertoire of assessment practices that are child centred and build upon what children know as opposed to what they **do not know**. This type of research remains highly valued and is highlighted in this chapter. However, it also needs to be recognised that the changed nature and increased levels of accountability have influenced the direction and representation of assessment.

From an international perspective, the past four years has brought increased attention on similarities and differences of students' mathematics performance across countries. Not only have studies highlighted differences in mathematics methods, techniques and adoption of ideas across countries, studies have also highlighted differences in teaching approaches and what is valued in school mathematics. Quantifying student performance through PISA and TIMSS studies has lead to assumptions about a specific country or region, yet only a small proportion of students are involved. Furthermore, much has been made of certain regions' performances on such tests, which can lead to assumptions such as, "if Asian countries are consistently successful on international measures of mathematics performance, then less-successful non-Asian countries would do well to adapt for their use the instructional practices of Asian classrooms" (Clarke & Xu, 2008, p. 964). However, work by Clarke and colleagues (Clarke, 2003; Clarke, Keitel, & Shimizu, 2006; Clarke & Xu, 2008) has highlighted assumptions such as:

> The performances valued in international tests constitute an adequate model of mathematics, appropriate to the needs of the less-successful country [and] that differences in mathematical performance are attributable primarily to differences in instructional practice, such as lesson structure (rather than to other differences in culture, societal affluence or aspiration, or curriculum). (Clarke & Xu, 2008, p. 964)

These descriptions of international comparisons (e.g., TIMSS) could well be aligned to Australia's present fascination with NAPLAN data. We are not suggesting that all international comparisons are fundamentally flawed. Clarke et al. (2006), for example, has allowed the research community to celebrate all forms of cultural and contextual diversity. The study of assessment has provided researchers with the opportunity to focus not only on classroom practice, but also look at practice(s) in different ways. This chapter goes some way toward highlighting diversity in student performance and classroom assessment practices.

This chapter is organised into four broad areas: First, we present a focus on the national assessment agenda and the research being undertaking in that arena. Second, we concentrate on the issues surrounding classroom assessment, specifically, promoting the learning environment and the tools and pathways used for classroom assessment. Third, the focus narrows to the concepts being assessed in classrooms with regard to curriculum content and the assessment items themselves. Fourth, we shift to a professional focus, looking at the assessment of teachers' mathematics concepts and Pedagogical Content Knowledge (PCK) and their classroom practices. Finally we present the conclusions and the implications for further research into assessment in mathematics education.

NATIONAL FOCUS

In 2008, a new era of testing and assessment in Australia was introduced with the advent of the NAPLAN. Previously, such responsibilities were the domain of the relevant states and territories, with as many as seven different kinds of tests being administered across several student age groups. Subsequently, this made it inappropriate to compare nationwide results and limited the accountability of teachers and schools. The introduction of the NAPLAN has changed this space. As stated by Australian Curriculum Assessment and Reporting Authority (ACARA) (n.d.):

> All Australian schools benefit from the outcomes of national testing, with aggregated results made available though comprehensive reports at the national and school level, accessible on-line. Schools can gain detailed information about how they are performing, and they can identify strengths and weaknesses which may warrant further attention ... Without the nationally comparable data about student performance that the National Assessment Program provides, states and territories have only limited information about the achievement of their students in relation to their peers. NAP data provide an additional suite of information, thus enhancing the capacity for evidence-based decision making about policy, resourcing and systemic practices.

At this point in time, New Zealand has not introduced a national assessment policy which publicly compares all students on standards or content. However, there are concerns among the New Zealand education community that the introduction of *The New Zealand Curriculum: Mathematics Standards for Years 1–8* (Ministry of Education, 2009) will be a catalyst for a national testing regime (Young-Loveridge, 2011). Despite these concerns, the Ministry of Education (2009) suggests using a variety of evidence including "self- and peer assessments, interviews, observations, and results from [existing] assessment tools" (p. 12) to assess student achievement against the standards. There was a paucity of research literature surrounding mathematics assessment emanating from New Zealand. Some is reported in the following sections.

From an Australian perspective, Lowrie and Diezmann (2009) explored the problematic nature of reporting national assessment data—particularly in instances when the teaching and learning experiences of any new curriculum are overly influenced by student results on such assessments. Moreover, they contested the structure and representation of specific items with these tests.

Connolly (2011) outlined the difficulties faced by the NAPLAN numeracy test designers as they attempted to develop items which (a) matched certain criteria,

(b) were applicable at a national level (not just within a state), (c) varied in complexity, and (d) considered the context and language of mathematics. He noted that more research needed to be undertaken in order to fully establish the validity of such tests and to define the intended purpose of this numeracy test. Prior the NAPLAN tests being implemented in 2008, Lowrie and Diezmann (2007; Diezmann & Lowrie, 2008a) had begun to challenge the design of mathematics assessment items. They noted the growing importance of graphics within mathematics test items and the impact this may have on student results. Their research revealed several implications for the classroom including the need for teachers to explicitly teach different forms of graphical representations; provide learning opportunities outside formal mathematics; identify specific terminology that may have more than one meaning; consider all graphical elements when decoding; and utilise a number of various graphical representations. Similarly, Diezmann (2008) analysed the types of graphics utilised in NAPLAN sample test items and found poor quality, atypical use of graphics, and a lack of consistency. Greenlees (2010) elaborated further on NAPLAN test item design and also argued the need for closer examination of assessment processes. This investigation found that the slight modification of the language used within some NAPLAN items influenced the sense making process of the students. It highlighted the need for the reliability of items within the NAPLAN to be well scrutinised and the need to assess what is being reported, particularly in this climate of intense accountability.

Nisbet (2011) analysed the NAPLAN from a mathematics content perspective, focusing particularly on probability. He found that probability items were under represented across the NAPLAN tests in all four grade levels. The items that were included were limited in their variation of probability reasoning constructs, measuring only a minimal level of students' probability knowledge. These, and the findings from above, reiterated the need for repeated analysis of test items and the test as a whole. Although research findings have highlighted some of the limitations of the tests and the items within the test, we should be mindful of the accountability and pressure teachers' face as a consequence of their implementation.

Dimarco (2009) acknowledged that a divide has emerged as "teachers struggle to follow through with high quality pedagogical interactions by using their professional judgment to build relevance for their students, whilst also adhering to the quality control measures of testing" (p. 675). In a small-scale research project, several teachers were given the opportunity to discuss teacher quality and the potential threats from the current coercive context in which they work. The project raised concerns "about how teachers can remain motivated and empowered to engage students in quality mathematics" (p. 676) within this current climate of accountability. Klenowski (2011) reinforced this struggle within the Queensland context, highlighting the need for teachers' assessment literacy to improve and for teachers to understand the new assessment environment.

However, it is not just teachers who are feeling disgruntled and uneasy about the increasing importance and value being placed on national assessment. Lange and Meaney (2011) highlight the public discourse surrounding the disadvantage in relationship to national testing of numeracy. Using press releases, online news articles and online public comments, Lange and Meaney (2011) showed how:

> [P]oliticians, parents, teachers and the general public discuss ideas around disadvantage in relationship to national testing of numeracy. Deficit language in these discussions identifies some children as being less likely to gain value from mathematics instruction. On the other hand, there is also a perception that poor results for individual schools contribute to their students being seen by the wider community as disadvantaged. (p. 1)

Their study highlighted the labelling that can occur as a result of national assessment, while others (Klenowski, 2009a; Klenowski, Tobias, Funnell, Vance and Kaesehagen, 2010; Morley, 2011) focused on other forms of disadvantage, especially that of Indigenous students in Australia. These researchers not only acknowledged the national but also the international trends of Indigenous students; specifically the large divide that remains in relation to performance. In general, they questioned the validity and fairness of high-stakes testing, particularly in light of the unique Indigenous culture and language. A key theme that emerged was an obvious need for teachers to accommodate, encourage and promote culture differences by valuing students' prior knowledge and experience. It was anticipated that the adoption of a culturally responsive pedagogy would broaden the curriculum and assessment practices to allow for different ways of knowing and thus provide equity and culture-fair assessment.

However, to put this theory into practice in the classroom, Klenowski et al. (2010) further explored the attitudes, beliefs and responses of Indigenous students to mathematics assessment with a particular focus on teacher knowledge. They found that:

> Underpinning the pedagogical and assessment approach is a broader view of how mathematics is taught in schools, one that encompasses students' understandings, dispositions, self-beliefs and acknowledges their personal view of the value of learning mathematics. Rich tasks (Luke, 2005) and open-ended questioning provide a basis for authentic problem solving to enhance personal and intrinsic motivation, perseverance and resilience. Students' attitudes to learning are directly affected by the value they place on learning and the success they believe they might have in reaching a satisfactory goal. (p. 15)

It was also recognised that the use of unfamiliar language and contexts disempowered not only Indigenous students, but the teachers of these students (Baturo, Cooper, Michaelson & Stevenson, 2008). In addition, negativity toward the test resulted in teachers finding it difficult to effectively use any information gathered from a national test to the benefit of the students in identifying strengths and weaknesses.

Other studies have focused on the use of assessment data in identifying trends. Hemmings and Kay (2009) noted the relationship among the *Literacy and Numeracy National Assessment* (LANNA) test scores in Year 7 to Year 10 School

Certificate English and Mathematics results. They found that Year 7 reading scores were the best predictor of Year 10 English results, and Year 7 numeracy scores were the best predicator of Year 10 Mathematics results. They also found a significant correlation existed between reading and mathematics performance, reinforcing the growing linguistic properties of mathematics today. Hill (2011) examined the NAPLAN numeracy performance of males and females as general cohorts in Grade 3 and Grade 9. She found that the mathematics performance of females was on the decline and that the gap between males and females still existed and was growing larger as they progressed through the grades. This research highlighted some of the invaluable information standardised tests can offer with regard to future student performance.

There has been a major shift in the way mathematics is being assessed nationally. Teachers, too, are searching for new ways to assess their students in an equitable and authentic manner to ensure results reflect a child's true mathematical understanding, while complementing the national testing performance data. This is evident through research being conducted in promoting the learning process, where assessment is embedded within the learning design. In addition, research is being conducted on the tools and instruments that are not only promoting learning opportunities but are specific in nature.

CLASSROOM FOCUS

Promoting the Learning Process

The Australian Council of Educational Research (ACER) sponsored the *Teaching mathematics? Make it count* 2010 conference. This national conference had unprecedented support by the teaching community, with the demand to attend so high that over 2000 teachers were placed on a waiting list. It is not by chance that such an event was so popular for classroom teachers given the undeniable impact all forms of assessment now have on current teaching practices. As reinforced by Callingham (2010):

> To make assessment count, the focus of professional learning for primary mathematics teachers might need to shift. Rather than developing teachers' mathematical content knowledge, changing pedagogical approaches through rich mathematical tasks, or applying models such as... Quality Teaching model[s], more productive professional learning might be focused on addressing students' specific, identified learning needs. (p. 41)

Similarly, Mohamad (2009) presented the challenges teachers face in this new climate, where traditional mathematics assessment culture is greatly challenged in tandem with the change of roles expected of teachers and students within this assessment process. As she stipulated, the predominant assessment culture has been deeply ingrained within a mathematics tradition of teaching and learning which has hampered assessment reform. Tellingly, some of the new political assessment regimes are moving back toward more traditional approaches as teachers are experimenting with various assessment practices. This has included the need to re-evaluate what Assessment for Learning (AfL) looks like in the classroom. As Klenowski (2009b) highlighted in the special issue editorial of *Assessment in Education: Principles, Policy & Practice*:

> The primary aim of Assessment for Learning (AfL) is to contribute to learning itself. This follows from the logic that when true learning has occurred, it will manifest itself in performance. The converse does not hold: mere performance on a test does not necessarily mean that learning has occurred. Learners can be taught how to score well on tests without much underlying learning. (p. 263)

In a mathematics context, Lauf and Dole (2010), for example, described preparation for a major external test (*Queensland Core Skills Test*) as a form of AfL. As part of the process, teachers were encouraged to particularly focus on student responses to open-ended tasks rather than the 'correctness' of the answers. Major findings from the study included a realisation of the difficulty of finding appropriate tasks and the limitation of dealing with student responses that were relatively narrow. In other examples of AfL, Vale et al. (2011) had teachers analyse student assessment data in order to inform teaching and found a subsequent need to change student centred teaching approaches to be more gender inclusive. Similarly, White and Anderson (2011) used school and student assessment data to focus on students' misconceptions and errors as topic areas for target teaching. They collaboratively developed teaching strategies to implement in the classrooms. However, there is a note of caution, as Pierce and Chick (2011) found that although teachers were able to interpret some aspects of student assessment data, many aspects were difficult to understand and as such, the teachers found little value in such reports to inform teaching. As Callingham (2010) pointed out, many teachers are experiencing considerable difficulty in identifying the next steps to take to develop students' understanding.

Other studies have highlighted changing practices in relation to assessment and the need to develop tasks that best accommodate new practices (e.g., the rich tasks described in Grootenboer (2009) and Jorgensen, Walsh, & Niesche (2009); and self-developed tasks for particular studies such as Treacey, Tiko, Harish, & Nairn (2010)). In addition, Treacy et al. (2010) highlighted the dual function of embedding classroom based assessment tasks into the learning process. They described a numeracy strategy that was trialled with 30 at-risk schools in Fiji. Teachers were introduced to Classroom Based Assessment and child centred pedagogy, which they used over a four-week period. Students showed considerable improvement in their mathematics knowledge and attitudes. Of equal importance, the teachers' move to child centred pedagogy and planning for their students' learning needs resulted in an increase in students' engagement with mathematics lessons. Also noted was an improvement in teachers' attitudes and enthusiasm towards teaching mathematics. In a similar vein, Lewis (2008) investigated School Based Assessment (SBA) as part of the National Certificate of Educational Achievement (NCEA) in secondary schools in New Zealand. His study revealed that certain mathematics topics such as geometry and trigonometry, measurement, and statistics and probability were better suited to SBA and that teachers were inclined to change their pedagogical practice to foster SBA.

Tools and Pathways Being Utilised

In accordance with the changing nature of mathematics assessment in the classroom, much research has focused on developing tools and instruments to better equip teachers with valid and reliable instruments. These instruments include writing tasks, quantifiable measures, self reflection and the use of technology as a tool for assessment. By empowering teachers with knowledge and understanding of how to effectively assess students' mathematical understanding, assessment is no longer viewed as a separate identity but rather incorporated in all learning situations. As Callingham (2008a) noted, "assessing the 'quality' of learning ... is better situated in the classroom, where teachers make judgements on a day-to-day basis about what their students know and can do" (p. 18). Subsequently, the instruments created valued the role students play in the assessment process and focus on such things as language and dialogue, self assessment, the many uses of technology as well as theoretical based assessment. All of these aspects acknowledge that if teachers want to gain a real insight into their classroom teaching and learning they must include their students in the assessment process.

Meaney, Trinick, and Fairhall (2009) highlighted the important use of language by utilising a writing task in the teaching and assessment process. They found that "writing explanations and justifications supports students to think mathematically and this can begin in the early years" (p. 21). Callingham (2008a) also explored ways where the teacher sets up a dialogue with the students, and provides feedback based on what the students do. This involved the students and teachers developing criteria and standards to 'estimate the quality' of mathematical learning. They concluded that this type of assessment needed to be dynamic as the dialogue has to be student and context specific and be utilised in conjunction with more formal and technical forms of assessment.

Fry (2011) also recognised the use of the PISA assessment framework as a meaningful way of assessing students as it valued multiple types of understandings rather than focusing and reporting on a narrow focus. In this inquiry:

> There were three levels of evidence to show how students were working: individually through the use of electronic learning journals, collaboratively by analysing poster sheets student groups worked on, and with a focus on differentiation as the teacher reflected and added anecdotal notes to their journal. (p. 277)

These results provided insight into ways to capture and analyse assessment opportunities in a primary classroom using mathematical inquiry.

Another tool developed to provide formative data for teachers was the *Written Strategy Stage Assessment Tool* (Lomas & Hughes, 2011). This tool was valuable in determining secondary student's numeracy levels for stage related teaching groups. With a measured consistency it had also been targeted as an effective tool for younger secondary students as well as for determining pre-service teachers' mathematics understanding. Similarly, Fitzallen (2008) developed a paper-based assessment to evaluate student prior learning designed around theoretical models of statistical thinking and reasoning with graphs. While providing an overview of student prior knowledge, the instrument design required further modification in order to address all aspects of the theoretical model.

Unlike other assessment instruments, self assessment measures the opinions and beliefs of the student and acknowledges the impact this may have on performance. The process of self reflection allows the students to assess their own understanding and monitor their progress; which has been found to be most beneficial for middle and low achieving students (McDonough & Sullivan, 2008). Similarly, Way (2009) noted the important use of self assessment, written reflection and online discussion for enhancing learning in pre-service teachers. Carmichael and Hay (2009) and Carmichael (2008) have developed instruments to measure middle school students' self-efficacy for, and interest in, statistical literacy. These instruments provided opportunities for students to self describe their beliefs and feelings about their ability in regards to statistical literacy. In these instances, emphasis is placed on students' self worth and underlying assumptions of their own understanding rather than a dichotomous response. The *Statistical Literacy Interest Measure* (SLIM) instrument was subsequently used by Carmichael (2010). However, Way (2009) concluded that further research is needed on the effectiveness of self assessment in promoting mathematics learning.

Research originating from New Zealand has focused on teachers' and students' conceptions of assessment. Brown (2011a, 2011b) and colleagues (Brown & Harris, 2011; Brown, Irving, Peterson, & Hirschfeld, 2009; Harris & Brown, 2008) developed two instruments to measure teachers' and students' self conceptions about assessment: the *Teachers' Conceptions of Assessment* (TCoA) survey instrument and the *Students' Conceptions of Assessment* (SCoA) inventory (both in various versions). With regard to the TCoA, the research has found teachers' had positive attitudes towards assessment for personalising their teaching and the students' learning but less positive responses towards assessment for compliance and reporting to outside entities. There is also the suggestion "that teachers develop or adopt conceptions of assessment that allow them to successfully function within their own policy or legal framework" (Brown, 2011a, p. 45). The SCoA research found students generally agreed that assessment (a) improved learning, (b) made schools and students accountable, (c) had a positive affect/benefit, and (d) was relevant to learning. These tools have undergone a number of changes and will continue to be a benefit to this area of research.

An area that requires further research is the use of technology in assessment. Although technology has been incorporated in the classroom for many years as a learning tool, its full potential as an assessment tool is yet to be recognised (Callingham, 2011a). Previously the use of traditional assessment processes has been sufficient as a way of assessing student's mathematical understanding (Callingham, 2010). However, with the increasing use of technology as a learning tool, teachers must change their assessment practices accordingly by using the technological tools with which their students are engaged.

> If *curriculum* is to say what students should, as a consequence of their learning, know and be able to do (concepts, skills, processes and the like) and *assessment* is the means by which judgments are made about progress and achievement, then a curriculum that sets expectations for the active use of technology as an enabling tool for working mathematically requires congruent expectations and practices for assessment. (Leigh-Lancaster, 2010, p. 43)

Afamasaga-Fuata'i and McPhan (2009) used software as an effective tool in the creation of concept maps as a form of assessment. This focus on conceptual interconnectedness provided the students with the opportunity to clarify their own understanding of the links and integration between concepts. These maps were also used to provide a snapshot of student understanding at a particular point in time as well as an evaluative tool for further teaching. It is interesting to note that the formations of such maps are similar to one of the graphical languages outlined by Diezmann and Lowrie (2009) that are being utilised within current assessment practices. Indeed this is one of many examples where research has targeted assessing a particular mathematical concept.

CONCEPTUAL FOCUS

Curriculum Based

As discussed in the previous section, a number of studies have focused on classroom based assessment, with a particular focus on mathematics content and conceptual understandings associated with such content. Unlike large-scale studies which tend to have a broad curriculum focus, the more concentrated classroom-based investigations target specific aspects of mathematics. Nevertheless, some of these investigations have drawn on readily available mathematics tasks and data to assess the performance of students in both whole class and small group situations. Diezmann and Lowrie (2008a) for example, investigated the extent to which different forms of representation in mathematics tasks influenced student performance across the last three years of primary school. In their research they distinguished between the contextual and informational aspects of graphics—where "information graphics are distinct from contextual graphics in that they represent mathematical information that supplements rather than complements the text or symbolic expression" (p. 647). In a longitudinal study of students performance in solving such graphics-based mathematics tasks, Lowrie (2008) and Lowrie and Diezmann (2011) reported distinct (and statistically significant) gender differences in favour of boys on map-based tasks and tasks that required the interpretation of information on either a horizontal (e.g., number line) or vertical (e.g., thermometer) axis. It is important to note that these performance differences were evident on moderate and difficult mathematics tasks but not on the easiest set of tasks. Lowrie and Diezmann (2011) argued such results are particularly concerning since these items are becoming much more prevalent in assessment tasks; and girls tend to not choose the most difficult mathematics subjects as they progressed through their schooling (Leder, Forgasz, & Vale, 2000).

Most of the research undertaken with a curriculum focus has concentrated around ideas associated with number sense and basic number skills. For example, one study described the assessment performance of primary aged students as they developed numeracy understandings. Rumiati and Wright (2010) assessed the 'number knowledge' of first and second grade Indonesian students within a Mathematics Recovery Framework. Findings revealed that most first grade students were in the 'advanced counting by ones' stage using a counting on and counting back strategy and by Grade 2 the students had a variety of approaches that were influenced by school and out-of-school experiences.

Other studies have advocated for assessment of children's numeracy understanding prior to school in order to enhance future mathematics development. Howell and Kemp (2009), in their preliminary study of young children's number sense, developed tasks which could be included in a questionnaire form suitable for assessing young student's performance. Mulligan, English, Mitchelmore & Welsby (2011) reported on the use of the *Pattern and Structure Mathematical Awareness Program* (PASMAP) that focused on children's patterning skills, structural relationship and big ideas in mathematics in the first year of primary school. This study produced a "valid and reliable interview-based measure and scale of mathematical pattern and structure that revealed new insights into students' mathematical capabilities at school entry" (p. 555). These results were further analysed by Mulligan, English, Mitchelmore, Welsby, & Crevensten (2011), who provided an overview of students' performance across items and descriptions of their structural development.

For older students, Pearn (2009) developed a pencil and paper number screening test to highlight students' speed and accuracy when recalling basic facts and the type of strategies they used when solving mathematics tasks. Ellemor-Collins and Wright (2008) developed an intervention program for low attaining third and fourth grade students. The term-long intervention program included activities which fostered mathematics attainment and provided strategies for developing computational efficiency for addition and subtraction facts. In an extended explanation of the research project, Ellemor-Collins and Wright (2009) described the instructional nature of the intervention program that was implemented by 25 teachers with approximately 300 students. They maintained that low-attaining students could progress to more independent thinking once students became familiar with structuring number patterns. Wong (2010) used mental computation processes to develop learning pathways which afforded teachers the opportunity to identify students' conceptual understanding of fractions and particularly concepts associated with equivalence. Other ways of assessing fraction knowledge have included the use of games (Lee, 2009) where, concepts associated with fractions were contained within a game scenario with the idea of forming staircases of 'fraction bricks'. In this study, several assessment tools such as pre- and postquizzes, and pre- and post-maths tasks were used to identify student feedback of the game and achievement in fractions.

From a problem solving perspective, Callingham (2008b) examined students' problem-solving skills and approaches by comparing the solution strategies of Grade 5 students in Hong Kong and Australia. Of note were differences in the students' metacognitive skills with Australian students generally finding it more difficult to move toward higher-order thinking. The Australian students tended to provide concrete examples of ideas, while the Hong Kong students thought more abstractly. Other research has examined, more closely, the merit and 'value' of specific assessment items.

Item Based

Whether featured within a national assessment or applied within a classroom focus, understanding mathematics assessment items provides teachers with an insight into their students' mathematical understanding. Diezmann and Lowrie (2008b) reported on student's ability to interpret graphically orientated mathematics assessment items. They found that the majority of student errors related to the interpretation of a graphic with a small proportion related to the text or calculations. The high proportion of graphical errors indicated a student's understanding of graphics is likely to compromise their mathematics performance. Similarly, Logan and Greenlees (2008) described the impact of test item design on students' understanding of assessment items. They found that modifications made to the graphical elements and the type of language used in a task impacted on student performance. The use of modifications also allowed Mousley (2009) to identify changes in low achievers' performance when manipulative objects rather than graphical illustrations were used in test questions. It was found that the difference between high achieving and low achieving groups was not in mathematical knowledge but the way in which the children coped with the representation. These studies highlight the need for test designers to carefully consider the graphic embedded within assessment items, especially when "children may be scored lower than they deserve because of the test format" (Mousley, 2009, p. 393).

PROFESSIONAL FOCUS

Teachers' Mathematics Knowledge

There has been renewed interest in issues associated with teachers' mathematics content knowledge and PCK. Although research has revealed the usefulness of assessment with regard to evaluating students' mathematical knowledge and understanding the complexity of student reasoning and examining particular concepts, it also has a place in regards to teacher professional practice. In particular, much work has focused on primary school and pre-service teachers. According to Meaney and Lange (2010) this has been the result of growing international concern. Through the use of interviews they found that many pre-service teachers lack confidence in their mathematical ability and consequently have concerns and apprehensions about teaching mathematics in the classroom. As part of their associated university course requirements, pre-service teachers needed to achieve 90% on a Year 7 Basic Skills Test. However, they found that:

> The way the test was organised resulted in the pre-service teachers reinforcing their views that what was important in mathematics was knowing the rules rather than understanding the concepts behind the rules, so that they could facilitate students' mathematical understanding. Performance rather than competence was seen as why they needed to pass the test. (p. 406)

This analysis of student responses "provided valuable insights into how these intentions were being thwarted by the circumstances in which the tests were being carried out" (p. 406). However, Afamasaga-Fuata'i, Meyer and Falo (2008) found real value in the use of diagnostic testing in pre-service teachers to gain an understanding of their content knowledge and the need for reform within the university system to improve "mathematics competency from year to year, towards mastery competence ideally before exit" (p. 48). In both instances, the research revealed a strong correlation between teacher confidence and their ability to maximise opportunities for engaging children in mathematics learning.

This form of diagnostic testing was also utilised by Galligan (2011) in determining pre-nursing student's academic numeracy. This consisted of a paper-and-pencil test to measure student competence and confidence as well as self-reflection of their mathematical knowledge. The implementation of these tests "have been refined to further engage students in being more critically aware of their own mathematics skills and the mathematics needed for the degree" (p. 295). Other professional learning programs, such as that highlighted by Watson & Beswick (2011), are aimed at improving teachers' mathematical understanding and teaching.

A common theme to emerge from the research was the important role that teacher's anxiety about mathematics played in the nexus between knowledge and practice. For example, Rayner, Pitsolantis and Osana (2009) examined the relationship between mathematics anxiety and the procedural and conceptual knowledge of fractions in prospective teachers. Using a paper-and-pencil test they were able to assess pre-service teachers' procedural and conceptual understanding of fractions. These results were then correlated with an anxiety rating scale. It was found that:

> Because of the negative relationship between mathematics anxiety and mathematical knowledge, it is possible that there may be a link between a teacher's mathematics anxiety and his or her ability to effectively use mathematical content knowledge during instruction. We propose that a teacher's weaknesses in mathematical content knowledge may not only hinder student performance, but may also be a source of the students' own mathematics anxiety. (p. 81)

Similarly, Roche and Clarke's (2009) administration of a questionnaire in regards to teachers' PCK of division concepts found that "careful thought needs to be given to how best to present it to teachers" (p. 472) with more teachers familiar with partitive division than quotitive. However, when assessing teachers' views on their geometry instruction and classroom learning environments, Ly and Malone (2010) received positive responses in regards to teaching confidence. These results were reflected in the classroom as teacher professionalism and confidence appeared to "enhance the students' understanding and ability to solve problems" (p. 373). Roche & Clarke (2011), however, acknowledged some of the difficulties of measuring teachers' mathematical PCK, such as:

> the limitations of pencil and paper items; designing items that (they) believed assessed faithfully a teacher's PCK for mathematics; creating rubrics that could be applied consistently; making choices about on which content to focus; and ultimately finding evidence of change over time, if it exists. (p. 665)

Nevertheless, the use of the PCK items improved both teachers' PCK and Common Content Knowledge. Subsequently, the use of assessment in researching teacher's mathematics content knowledge and PCK appeared to be invaluable in ascertaining challenges teachers are facing as well as identifying teacher strengths. It has also been useful in identifying successful assessment practices that can be utilised by teachers in their classrooms.

Teachers' Classroom Practice

The notion of assessment for learning has revolutionised the way assessment is being applied in the classroom. However, it has required teachers to rethink their use and understanding of assessment. As Pegg and Panizzon (2007/08) argued, the intent of the assessment for learning agenda has shifted the focus from the traditional view of assessment to one more closely aligned to understanding where students are conceptually situated and how they can be moved forward. Pegg and Panizzon (2007/08) highlighted the need for ongoing and sustained professional development to incorporate these changes, in particular the use of the Structure of the Observed Learning Outcome (SOLO) model as the theoretical framework. "Essentially SOLO is concerned with specifying 'how well' (qualitative) something is learned rather than 'how much' (quantitative)" (p. 67). However, this requires a major shift in the viewing of assessment from an activity that occurs at the end of a topic, to one that is used on a day-to-day basis to enhance student learning. They found that although the change was time consuming, teachers were positive and committed to the implementation of the program into their relevant schools.

Nonetheless, Callingham, Pegg and Wright (2009) noted "the promise of improved outcomes from assessment *for* learning has not been achieved on a large scale" (p. 81) with many teachers struggling to make it a reality in their classrooms. They noted the strong presence of external pressures such as parental expectations and examination and the impact these have on changing current assessment and reporting practices. Although teachers' responses revealed a real appreciation for the SOLO model, findings suggested that changes in teacher practice were mainly in the context of teaching rather than assessment. Tee Yong & Stephens (2011) also emphasised the need for classroom assessment to use thinking processes rather than a reliance on academic achievement alone. This was achieved through the implementation of the *Mathematical Thinking Assessment* (MaTA) Framework. However, like similar assessments, it was found to be time consuming and required further teacher training to be effective.

Other models of assessment have been recognised as valuable tools in assessing students' mathematical knowledge. Afamasaga-Fuata'i (2008) noted the use of Novak-type concept maps and Gowin's vee-diagrams as a means for pre-service teachers to evaluate teaching and learning in their classrooms. She concluded that such innovative and creative ways to assess and teach would be reflected in engaged and motivated students. Subsequently, Adie (2008) highlighted the use of online moderation meetings as having the "potential to support the collaborative professional development of teachers, and the formation of a common understanding of what denotes quality in student work in a standards based assessment system" (p. 1). It is believed that such a system promoted authentic forms of assessment in areas like problem solving and higher order thinking skills. Indeed, one of the most powerful and effective forms of classroom assessment, as outlined by Clarke, Clarke & Roche (2011), was the use of task-based, one-to-one interviews. They argued that the use of such forms of assessment built upon teacher expertise by enhancing teachers' knowledge of individuals and an understanding of typical learning paths in various mathematical domains. Similarly, the assessment component of the New Zealand *Numeracy Development Project* (NDP), the *Numeracy Project Assessment* (NumPA) used an oral, individual, task-based diagnostic interview to assess students' strategies and knowledge when problem solving (Young-Loveridge, 2011). Young-Loveridge contended that the use of such interviews with children in the early years of schooling was beneficial as there was no reading or writing for students and it allowed the teacher to engaged the student throughout the assessment process. The above research has highlighted the need to educate teachers on effective assessment strategies that are relatively easy to implement and are closely linked to curriculum requirements.

CONCLUSION

This chapter has described the nature of research on assessment and assessment practices within the Australasian research community. The description and analysis of this research has been positioned within four areas of focus; namely: national, classroom, conceptual and professional.

In terms of the national focus, an increasing number of studies has focused on aspects of the NAPLAN (introduced as a high-stakes test in 2008). The attention awarded to this form of assessment has moved in a number of directions. For example, some studies have considered the composition and structure of the items within the test, while others have looked at the impact on teaching and learning. Others have begun to consider NAPLAN measures from a student competency perspective. Now that Australian researchers have access to a 'standardised' measure of student progress across a significant span of a student's schooling, NAPLAN measures will become increasingly used to describe student and school performance. In the past three years, a high proportion of Australian Research Council projects have used NAPLAN data (or in fact the *My School* website where school results are published) to identify state-wide performance differences; high and low achieving schools within specific criteria; gaps in student knowledge in terms of content strands; and as baseline variables when comparing other aspects of mathematics performance. A consequence of such utility is likely to lead to less additional testing measures within and between schools—simply because there is now a measure researchers and teachers can utilise without having to undertake additional assessment. Although NAPLAN data can provide a benchmark and a potentially rich source of comparative data, this practice could be quite problematic and has the potential to lead to an over reliance on the national assessment agenda. Moreover, this concentrated focus on one measure narrows our understanding of **what children know**. We maintain that it is important to conduct research, and indeed question, the nature and design of the NAPLAN process.

Both the classroom and conceptual foci in the past four years have been resolute in mandating Assessment for Learning. Despite the most prominent change in assessment practices (in Australia at least), research continues to consider assessment from both the perspective of students and their teachers. Aspects of research include the appropriateness of assessment particularly with minority groups and the extent to which assessment practices consider the needs of all learners. Research has also focused on better understanding assessment tools and their use and especially within the brief of teacher and students attitudes and beliefs about mathematics and mathematics practices. Not surprisingly, work is also being conducted on students' conceptual understanding of mathematics—not only in terms of mathematics content but also in terms of the representation of mathematics assessment items and problem-solving processes. Given the increased accountability measures that are taking place across society, it is likely that more work in this area will be conducted in the next four years.

The professional focus on assessment has predominately considered the nature of teacher practices in the mathematics classroom. Given the direction of international research, where teacher practices are being scrutinised with increasing sophistication and detail, we also feel this area will continue to be a focus for Australasian researchers. We envisage that projects will investigate teacher practices at both macro and micro levels in an attempt to better understand the relationship between assessment practices and students' understanding and sense making in mathematics.

REFERENCES

Adie, L. (2008, December). *Changing assessment practices: The case for online moderation*. Paper presented at the Australian Association for Research in Education 2008 International Education Conference, Brisbane. Retrieved from http://www.aare.edu.au/08pap/adi08173.pdf

Afamasaga-Fuata'i, K. (2008). Students' conceptual understanding and critical thinking: A case for concept maps and vee-diagrams in mathematics problem solving. *Australian Mathematics Teachers, 64*(2), 8–17.

Afamasaga-Fuata'i, K., & McPlan, G. (2009). Concept maps as innovative learning and assessment tools in primary schools. In K. Afamasaga-Fuata'i (Ed.), *Concept mapping in mathematics* (pp. 87–113). New York, NY: Springer.

Afamasaga-Fuata'i, K., Meyer, P., & Falo, N. (2008). Assessing primary preservice teachers' mathematical competence. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 43–50). Brisbane, QLD: MERGA.

Australian Curriculum Assessment and Reporting Authority [ACARA]. (n.d.). *Benefits of the National Assessment Programme: Driving improvements*. Retrieved from http://www.nap.edu.au/About/Why_NAP/index.html

Baturo, A. R., Cooper, T. J., Michaelson, M. T., & Stevenson, J. (2008). Using national numeracy testing to benefit Indigenous students: Case studies of teachers taking back control of outcomes. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 59–65). Brisbane, QLD: MERGA.

Brown, G. T .L. (2011a). Teachers' conceptions of assessment: Comparing primary and secondary teachers in New Zealand. *Assessment Matters, 3*, 45–70.

Brown, G. T. L. (2011b). New Zealand prospective teacher conceptions of assessment and academic performance: Neither student nor practicing teacher. In R. Kahn, J. C. McDermott, & A. Akimjak (Eds.), *Democratic access to education* (pp. 119–132). Los Angeles, CA: Antioch University.

Brown, G. T. L. & Harris, L. R. (2011, April). *Level of schooling effects on student conceptions of assessment: The impact of high-stakes assessments on secondary students' beliefs*. Paper presented to the Classroom Assessment SIG at the annual meeting of the American Educational Research Association, New Orleans, LA: AERA.

Brown, G. T. L., Irving, S. E., Peterson, E. R., & Hirschfeld, G. H. F. (2009). Use of interactive-informal assessment practices: New Zealand secondary students' conceptions of assessment. *Learning and Instruction, 19*, 97–111.

Callingham, R. (2008a). Dialogue and feedback: Assessment in the primary classroom. *Australian Primary Mathematics Classroom, 13*(3), 18–21.

Callingham, R. (2008b). Perspectives gained from different assessment tasks on Chinese and Australian school students learning mathematics. *Evaluation & Research in Education, 21*(3), 175–187.

Callingham, R. (2010). Mathematics assessment in primary classrooms: Making it count. In C. Glascodine & K-A. Hoad (Eds.), *Teaching Mathematics? Make it count: What research tells us about effective teaching and learning of mathematics* (Proceedings of the Australian Council for Educational Research conference, pp. 39–42). Melbourne, Australia: ACER.

Callingham, R. (2011a). Assessing statistical understanding in middle schools: Emerging issues in a technology-rich environment. *Technology Innovations in Statistics Education, 5*(1), 1–12.

Callingham, R. (2011b). Mathematics assessment: Everything old is new again? In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 3–10). Adelaide, SA: AAMT & MERGA.

Callingham, R., Pegg, J., & Wright, T. (2009). Changing teachers' classroom practice through developmental assessment: Constraints, concerns and unintended impacts. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 81–88). Palmerston North, NZ: MERGA.

Carmichael, C. (2008). Middle school students' interest in statistical literacy. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 109–115). Brisbane, QLD: MERGA.

Carmichael, C. (2010). Students' frames of reference and their assessments of interest for statistical literacy. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 139–144). Fremantle, WA: MERGA.

Carmichael, C., & Hay, I. (2009). The development and validation of the students' self-efficacy for statistical literacy scale. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 89–96). Palmerston North, NZ: MERGA.

Clarke, D. J. (2003). International comparative studies in mathematics education. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick & F. K. S. Leung (Eds.), *Second international handbook of mathematics education* (pp. 145–186). Dordrecht, The Netherlands: Kluwer.

Clarke, D., Clarke, B., & Roche, A. (2011). Building teachers' expertise in understanding, assessing and developing students' mathematical thinking: The power of task-based, one-to-one assessment interviews. *ZDM: The International Journal of Mathematics Education.* doi:10.1007/s11858-011-0345-2

Clarke, D. J., Keitel, C., & Shimizu, Y. (Eds.). (2006). *Mathematics classrooms in twelve countries: The insider's perspective*. Rotterdam, The Netherlands: Sense.

Clarke, D. J., & Xu, L. H. (2008). Distinguishing between mathematics classrooms in Australia, China, Japan, Korea and the USA through the lens of the distribution of responsibility for knowledge generation: Public oral interactivity and mathematical orality. *ZDM: The International Journal of Mathematics Education, 40*, 963–972.

Connolly, N. (2011). Refining the NAPLAN numeracy construct. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 908–916). Adelaide, SA: AAMT & MERGA.

Diezmann, C. M. (2008). Graphics and the national numeracy tests. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 659–662). Brisbane, QLD: MERGA.

Diezmann, C. M., & Lowrie, T. (2008a). The role of information graphics in mathematical proficiency. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 647–650). Brisbane, QLD: MERGA.

Diezmann, C. M., & Lowrie, T. (2008b, December). *Upper primary students' errors in interpreting graphically-oriented mathematics assessment items.* Paper presented at the 2008 conference of the Australian Association for Research in Education, Brisbane. Coldstream, Australia: AARE.

Diezmann, C. M., & Lowrie, T. (2009). An instrument for assessing primary students' knowledge of information graphics in mathematics. *Assessment in Education: Principles, Policy and Practice, 16*(2), 131–147.

Dimarco, S. (2009). Crossing the divide between teacher professionalism and national testing in middle school mathematics? In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 673–676). Palmerston North, NZ: MERGA.

Ellemor-Collins, D. E., & Wright, R. (2008). Intervention instruction in structuring numbers 1 to 20: The case of Nate. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 179–186). Brisbane, QLD: MERGA.

Ellemor-Collins, D., & Wright, R. (2009). Structuring numbers 1 to 20: Developing facile addition and subtraction. *Mathematics Education Research Journal, 21*(2), 50–75.

Fitzallen, N. (2008). Validation of an assessment instrument developed for eliciting student prior learning in graphing and data analysis. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 203–209). Brisbane, QLD: MERGA.

Fry, K. (2011). Formative assessment tools for inquiry mathematics. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 270–278). Adelaide, SA: AAMT & MERGA.

Galligan, L. (2011). Measuring academic numeracy: Beyond competence testing. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 288–296). Adelaide, SA: AAMT & MERGA.

Greenlees, J. (2010). The terminology of mathematics assessment. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 218–224). Fremantle, WA: MERGA.

Grootenboer, P. (2009). Rich mathematical tasks in the *Maths in the Kimberley* (MITK) project. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 696–699). Palmerston North, NZ: MERGA.

Harris, L. R., & Brown, G. T. L. (2008, December). New Zealand teachers' conceptions of the purpose of assessment: Phenomenographic analyses of teachers' thinking. In P. L. Jeffery (Ed.), *Changing Climates, Education for sustainable futures* (Paper presented at the Australian Association for Research in Education [AARE] 2008 International Educational Research Conference). Brisbane, Australia. Retrieved from http://www.aare.edu.au/08pap/har08257.pdf

Hemmings, B., & Kay, R. (2009). LANNA tests and the prediction of Year 10 English and mathematics results. *Issues in Educational Research, 19*(1), 25–33.

Hill, J. C. (2011). Gender differences in NAPLAN mathematics performance. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 366–372). Adelaide, SA: AAMT & MERGA.

Howell, S., & Kemp, C. (2009). A participatory approach to the identification of measures of number sense in children prior to school entry. *International Journal of Early Years Education, 17*(1), 47–65.

Hurst, C. (2008). Using task-based interviews to assess mathematical thinking of primary school students. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 273–280). Brisbane, QLD: MERGA.

Jackson, F., & Brown, R. (2009). Exploring whether multiple intelligences facilitate 'valuing and working with difference' within Mathematics classrooms. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 357–364). Palmerston North, NZ: MERGA.

Jorgensen (Zevenbergen), R., Walsh, L., & Niesche, R. (2009). Reforming schools: A case study of New Basics in a primary school. *International Journal Leadership in Education, 12*(2), 115–133.

Klenowski, V. (2009a). Australian Indigenous students: Addressing equity issues in assessment. *Teaching Education, 20*(1), 77–93.

Klenowski, V. (2009b). Assessment for learning revisited: An Asia-Pacific perspective [Editorial]. *Assessment in Education: Principles, Policy & Practice*, *16*(3), 263–268.

Klenowski, V. (2011). Assessment for learning in the accountability era: Queensland, Australia. *Studies in Educational Evaluation, 37*(1), 78–83.

Klenowski, V., Tobias, S., Funnell, B., Vance, F., & Kaesehagen, C. (2010). Culture-fair assessment: Challenging Indigenous students through effortful mathematics teaching. In P. L. Jeffery (Ed.), *Changing Climates, Education for sustainable futures* (Paper presented at Australian Association for

Research in Education [AARE] 2008 International Educational Research conference. Brisbane, Australia. Retrieved from http://www.aare.edu.au/10pap/2532KlenowskiTobiasFunnelVance Kaesehagen.pdf

Lange, T., & Meaney, T. (2011, February). Becoming disadvantaged: Public discourse around national testing. *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education (CERME7)*. Rzeszów, Poland. Retrieved from http://www.cerme7.univ.rzeszow.pl/WG/10/CERME7_WG10_Lange.pdf

Lauf, L., & Dole, S. (2010). Assessment for learning tasks and the peer assessment process. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 320–327). Fremantle, WA: MERGA.

Leder, G. C., Forgasz, H., & Vale, C. (2000). Gender and mathematics: Changing perspectives. In K. Owens & J. Mousley (Eds.), *Mathematics education research in Australasia: 1996–1999* (pp. 305–340). Sydney, NSW: MERGA.

Lee, Y. L. (2009). Enhancement of fractions from playing a game**.** In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 323–330). Palmerston North, NZ: MERGA.

Leigh-Lancaster, D. (2010, August). The case of technology in senior secondary mathematics: curriculum and assessment congruence? In C. Glascodine & K-A Hoad (Eds.), *Teaching mathematics? Make it count: What research tells us about effective teaching and learning of mathematics* (Paper presented at the Australian Council for Educational Research conference, pp. 43–46). Melbourne, Australia: ACER.

Lewis, R. K. (2008). *School based assessment within a standards based assessment regime: A mathematics community's perspective*. (Unpublished doctoral dissertation). University of Otago, New Zealand.

Logan, T., & Greenlees, J. (2008). Standardised assessment in mathematics: The tale of two items. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 655–658). Brisbane, QLD: MERGA.

Lomas, G., & Hughes, P. (2011). Assessment of secondary students' number strategies: The development of a written numeracy assessment tool. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 473–481). Adelaide, SA: AAMT & MERGA.

Lowrie, T. (2008). A longitudinal study of student performance on items rich in graphics. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 651–654). Brisbane, QLD: MERGA.

Lowrie, T., & Diezmann, C. M. (2007). Solving graphics problems: Student performance in the junior grades. *Journal of Educational Research, 100*(6), 369–377.

Lowrie, T., & Diezmann, C. M. (2009). National numeracy tests: A graphic tells a thousand words. *Australian Journal of Education. 53*(2), 141–158.

Lowrie, T., & Diezmann, C. M. (2011). Solving graphics tasks: Gender differences in middle-school students. *Learning and Instruction*, *21*(1), 109–125.

Luke, A. (2005). *Report on rich tasks*. Brisbane, Australia: Education Queensland. Retrieved from http://education.qld.gov.au/corporate/newbasics/html/library.html#Report

Ly, K. R., & Malone, J. A. (2010). Teachers' perceptions of geometry instruction and the learning environment in Years 9–10 ESL classrooms. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 367–374). Fremantle, WA: MERGA.

McDonough, A., & Sullivan, P. (2008). Focusing Year 8 students on self-regulating their learning of mathematics. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 337–343). Brisbane, QLD: MERGA.

Meaney, T., & Lange, T. (2010). Pre-service students' responses to being tested on their primary school mathematical knowledge. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 399–406). Fremantle, WA: MERGA.

Meaney, T., Trinick, T., & Fairhall, U. (2009). Learning how to represent mathematics on paper. *Australian Primary Mathematics Classroom, 14*(1), 21–27.

Ministry of Education (New Zealand). (2009). *The New Zealand curriculum: Mathematics standards for years 1–8.* Wellington, New Zealand: Learning Media Limited.

Mohamad, R. (2009). Evolving mathematics classroom assessment cultures. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 379–386). Palmerston North, NZ: MERGA.

Morley, P. (2011). Victorian Indigenous children's responses to mathematics NAPLAN items. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 523–530). Adelaide, SA: AAMT & MERGA.

Mousley, J. A. (2009). Numeracy test item readability during transition from pre-school to school. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 387–394). Palmerston North, NZ: MERGA.

Mulligan, J., English, L. D., Mitchelmore, M. C., & Welsby, S. M. (2011). An evaluation of the pattern and structure mathematics awareness program in the early school years. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 548–556). Adelaide, SA: AAMT & MERGA.

Mulligan, J. T., English, L. D., Mitchelmore, M. T., Welsby, S. M., & Crevensten, N. (2011). Developing the pattern and structure assessment (PASA) interview to inform early mathematics learning. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 1022–1030). Adelaide, SA: AAMT & MERGA.

Nisbet, S. (2011). National testing of probability in years 3, 5, 7, & 9 in Australia: A critical analysis. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 575–581). Adelaide, SA: AAMT & MERGA.

Pearn, C. (2009). Highlighting the similarities and differences of the mathematical knowledge and strategies of year 4 students. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 443–451). Palmerston North, NZ: MERGA.

Pegg, J., & Panizzon, D. (2007/2008). Addressing changing assessment agendas: Impact of professional development on secondary mathematics teachers in NSW. *Mathematics Teacher Education and Development, 9,* 66–80.

Pierce, R., & Chick, H. (2011). Reacting to quantitative data: Teachers' perceptions of student achievement reports. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 631–639). Adelaide, SA: AAMT & MERGA.

Rayner, V., Pitsolantis, N., & Osana, H. (2009). Mathematics anxiety in preservice teachers: Its relationship to their conceptual and procedural knowledge of fractions. *Mathematics Education Research Journal, 21*(3), 60–85.

Roche, A., & Clarke, D. (2009). Making sense of partitive and quotitive division: A snapshot of teachers' pedagogical content knowledge. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing* *divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 467–474). Palmerston North, NZ: MERGA.

Roche, A., & Clarke, D. (2011). Some lessons learned from the experiences of assessing teachers' pedagogical content knowledge in mathematics. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 658–666). Adelaide, SA: AAMT & MERGA.

Rumiati, R., & Wright, R. (2010). Assessing the number knowledge of children in the first and second grade of an Indonesian school. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 493–500). Fremantle, WA: MERGA.

Tee Yong, H., & Stephens, M. (2011). Implementing a mathematical thinking assessment framework: Cross cultural perspectives. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 382–389). Adelaide, SA: AAMT & MERGA.

Treacy, K., Tiko, P., Harish, S., & Nairn, P. (2010). Now I'm teaching the children: Changing from assessment *of* learning to assessment *for* learning in Fiji. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 563–570). Fremantle, WA: MERGA.

Vale, C., Davidson, K., Davies, A., Hooley, N., Loton, D., & Weaven, M. (2011). Using assessment data: Does it make a difference? In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 736–743). Adelaide, SA: AAMT & MERGA.

Watson, J., & Beswick, K. (2011). School pupil change associated with a continuing professional development programme for teachers. *Journal of Education for Teaching*, *37*(1), 63–75.

Way, J. (2009). Reflection in self-assessed online discussion. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 571–578). Palmerston North, NZ: MERGA.

White, P., & Anderson, J. (2011). Teachers' use of national test data to focus numeracy instruction. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 777–785). Adelaide, SA: AAMT & MERGA.

Wong, M. (2010). Equivalent fractions: Developing a pathway of students' acquisition of knowledge and understanding. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 673–680). Fremantle, WA: MERGA.

Young-Loveridge, J. (2011). Assessing the mathematical thinking of young children in New Zealand: The initial school years. *Early Child Development and Care, 181*(2), 267–276.

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MATHEMATICS LEARNING AND TEACHING

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EARLY CHILDHOOD MATHEMATICS EDUCATION

Key words: contexts of mathematics learning; young Indigenous mathematics learners; mathematics and starting school; early childhood mathematics educators; assessment of early mathematics learning; content of early childhood mathematics education.

INTRODUCTION

During the review period, there has been unprecedented political interest in early childhood education in Australasia (taken to be education of and for children aged between 0 and 8 years old). In New Zealand a review of the implementation of the respected prior-to-school curriculum framework *Te Whāriki* (Ministry of Education [MoE], 1996) has been recommended. For schools, the *New Zealand Curriculum* (MoE, 2007) was introduced in 2007. In Australia, the *Early Years Learning Framework for Australia* (Department of Education, Employment and Workforce Relations [DEEWR], 2009) was implemented from 2010 and Phase 1 of the implementation of the *Australian Curriculum* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2010), including mathematics, has begun.

All of this interest in early childhood has provided some stimulus for early childhood mathematics education research in Australasia, building on the substantial work that was reported in the previous two MERGA reviews of research (Perry & Dockett, 2004; Perry, Young-Loveridge, Dockett, & Doig, 2008). However, the quantum of early childhood education research emanating from Australasia seems to have diminished since these earlier reviews, perhaps because of a substantial lessening of the work stimulated by the heavily supported systemic numeracy programs in both Australia and New Zealand.

The purpose of this chapter is to critique and celebrate the most significant of the Australasian early childhood mathematics education research that has been published over the review period 2008–2011 and to use this critique to look forward into the next review period with suggestions for future research. The chapter is divided into sections dealing with Australasian research of contexts, pedagogies and content for early childhood mathematics education.

CONTEXTS FOR EARLY CHILDHOOD MATHEMATICS EDUCATION

Much of the recent Australasian research in early childhood mathematics education considered elements of the context in which learning occurs. In this section we

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review research undertaken with Indigenous communities; as children make the transition to school and the connections among contexts that promote young children's mathematical learning.

Successful Approaches to the Mathematics Education of Young Indigenous Children

Successful approaches to the mathematics education of young Indigenous students continue to be a key issue in both New Zealand and Australia. In New Zealand, *Te Poutama Tau* (the Māori-medium component of the *NZ Numeracy Development Projects* [NDP]), underpinned by opportunities to develop the teaching of mathematics (pāngarau) in the medium of Māori, has continued to evolve. The focus of *Te Poutama Tau* is on improving student performance by improving the professional capability of teachers, and supporting the broader aims of Māori-medium schooling in the revitalisation of *te reo Māori*.

In their study of longitudinal patterns of performance of *Te Poutama Tau*, Trinick and Stevenson (2009) reported similar patterns of progress across years 2005–2008 for Years 2 to 8 students. Student progress was affected by a number of variables, including teacher competence, quality of time spent learning, and the quality and availability of support resources. The longitudinal data showed that students who initially performed at a higher stage on *Te Mahere Tau* (*The Number Framework*) maintained that advantage to at least Year 4.

Trinick and Stevenson's (2009) analysis from 2005–2008 suggested that students' ability to articulate their mental strategies was linked to their language proficiency in *te reo Māori* which impacted on their ability to communicate, extract meaning from mathematics statements and convey that meaning in spoken or written discourse. The importance of language proficiency was reiterated in their more comprehensive evaluation of *Te Poutama Tau*, from 2003–2009, where they reported differences in students' strategy components, particularly as students were required to verbalise their mental strategies (Trinick & Stevenson, 2010). Additionally Young-Loveridge (2008) postulated that many of the total immersion teachers were second-language learners themselves, raising some interesting linguistic issues around specialised vocabulary and discourse. With 46% of the New Zealand Year 1 population coming from backgrounds other than European, Peters (2010) suggested the importance of acknowledging research that focused on culturally appropriate pedagogy and assessment practices, and ways of building on valued learning from home. Efforts to achieve this were seen in the evaluation of *Te Poutama Tau*.

Currently 20% of Māori children attend Māori-medium schooling. Extensive analysis of NDP data of those 80% in mainstream education showed that NZ European students started school at higher stages of *The Number Framework* than Māori and Pasifika students. However, the gains from the NDP, as measured by effect sizes, for Māori and Pasifika students are very similar to those for NZ European students (Young-Loveridge, 2008). Furthermore, Young-Loveridge (2009) noted a clear advantage for Māori and Pasifika students attending higher decile schools, rather than lower decile schools, with the average effect size for the difference in gain around one third of a standard deviation.

Recent years have also seen the development and implementation of several large-scale Australian longitudinal studies of Indigenous children's engagement with mathematics. The four year *Make It Count* project aimed "to provide better mathematical outcomes for Indigenous children" (Hurst, Armstrong, & Young, 2011, p. 373) by developing "an evidence base of practices that improve Indigenous students' learning in mathematics and numeracy" (Australian Association of Mathematics Teachers, 2011). The *Make It Count* project had largely focused on improving teacher capacity for effectively engaging young Indigenous students in mathematics learning. Reporting on the Swan Valley cluster of the *Make It Count* project, Hurst et al. (2011) described initiatives implemented as part of the project and the resultant changes in practice. The first of these initiatives was the provision of professional learning in mathematics teaching for Education Assistants (EAs) and Aboriginal and Islander Education Officers (AIEOs). The second was the concentration of a school's Indigenous cohort within classes taught by teachers identified as culturally sensitive and empathetic towards Indigenous children (Hurst et al., 2011). These teachers "often spent a lot of time talking to their Indigenous children, dealing with social and emotional issues during their recess breaks or planning time and putting 'school stuff' to one side" (p. 378). Evidence from interviews with principals, teachers, EAs and AIEOs, and teacher questionnaires undertaken during the study suggested that:

> The mathematics professional learning for EAs and AIEOs contributed to the development of professional learning communities. As well, it is apparent that effective teachers of Indigenous children have particular qualities and use particular strategies that develop and enhance supportive and empathetic teacher-student relationships, and which will hopefully lead to improved numeracy outcomes for Indigenous children. (p. 381)

The *Bridging the Numeracy Gap in Low SES and Indigenous Communities* project (Gervasoni, Hart, Crosswell, Hodges, & Parish, 2011) emerged from the work undertaken in the *Early Numeracy Research Project* (ENRP) (Clarke, Clarke, & Cheeseman, 2006). The ENRP explored the numeracy abilities and experiences of young children in both prior-to-school and early school contexts.

The *Bridging the Numeracy Gap* project sought to build capacity and improve mathematics learning outcomes for children in low-socioeconomic status and Indigenous communities. The project involved 42 school communities across Victoria and Western Australia, including four schools in the Kimberley (Gervasoni, Hart et al., 2011). One aspect of the project explored the role of Aboriginal Teaching Assistants (ATAs) in the provision of high quality mathematics learning experiences for children, concluding that as "often the only permanent members of school staff, [they] play an essential role in building community connectedness and relationships between teachers and families" (p. 313). Acknowledging the importance of community connection to the provision of meaningful and relevant mathematics experiences for young children, Gervasoni, Hart et al. called for school communities to "draw upon the expertise of ATAs, invest in their professional learning, and acknowledge their critical role in building community connectedness and advocacy for Aboriginal students and their families" (p. 313).

The *Young Australian Indigenous Students Literacy and Numeracy* (YAILN) study investigated teaching and learning activities that support Indigenous children as they enter school (Warren, Young, & de Vries, 2008a, 2008b). The YAILN study involved collaboration between researchers, 120 children attending Prep (non-compulsory first year of school) and their teachers at five schools in North Queensland. The multi-tiered design consisted of four data gathering activities: (a) pre- and post-tests; (b) student portfolios; (c) classroom observations; and (d) teacher interviews. Results outlined several strategies for supporting the mathematical learning of young Indigenous students. In particular, the role of pre-Prep (two years prior to Year 1) was noted in promoting understanding of mathematics concepts and understandings. The authors concluded that "the students who had participated in pre-Prep not only possessed a better understanding of numbers to 5 but also the associated mathematical language used to access this understanding" (Warren et al., 2008a, p. 552). Results also suggested that direct teaching together with play-based opportunities generated contexts that promoted young Indigenous students' early mathematics learning.

The *Maths in the Kimberley* (MitK) project (Jorgensen, 2010; Niesche, Grootenboer, Jorgensen, & Sullivan, 2010) also investigated effective mathematics pedagogy for Indigenous students. While recognising the critical role of teachers in educational reform, this project trialled an innovative mathematics pedagogical model in six remote Indigenous communities in the Kimberley region, Western Australia. Extensive data, including questionnaires, classroom observations, interviews and student testing, were used to evaluate the model and its impact. Results have highlighted the importance of home language use in the classroom. As well, Jorgensen (2010, p. 743) has questioned the appropriateness of group work, suggesting that it may indeed be a "domain of Western/modern education" not necessarily suited in Indigenous contexts.

Another component of the MitK project explored the ways in which teachers in remote schools could connect the mathematical concepts they were teaching to the experiences of the students (Sullivan, Grootenboer, & Jorgensen, 2011). Giving the example of using coins in number operation tasks, Sullivan et al. highlighted the importance of incorporating contexts which are familiar to students to effectively engage young Indigenous students in learning mathematics.

As with the YAILN study, the MitK study emphasised the importance of allowing children to discuss their mathematical reasoning in their home language (Niesche et al., 2010). However, the MitK researchers were met with concerns from teachers about "not knowing what the students were talking about and whether they would remain on task" (Jorgensen, 2010, p. 742). Despite an initial reaction from the MitK research team "that 'loss of control' was not a good reason for absolving the use of home language" (Jorgensen, 2010, p. 742), they acknowledged that the flow-over of community issues and resultant loss of control presented a challenge for educators of Indigenous children. A key insight from the MitK project was the need to confront assumptions around good mathematics pedagogy for young Indigenous students (Jorgensen, 2010).

As the initial stage of the *Representations, Oral Language and Engagement in Mathematics* (RoleM) longitudinal study, McDonald, Warren, and de Vries (2011) investigated the nature of oral language and representations in the mathematics education of young Indigenous students. They found that an English as a second language (ESL) approach was employed by the majority of teachers in schools with high proportions of Indigenous students. They cautioned that this approach may result in interactions becoming linguistic exercises rather than a means to develop mathematical concepts. McDonald et al. suggested that teachers of young Indigenous students attended to a combination of oral language communication and rich mathematical representations.

The reviewed research contributes much to the interrogation of appropriate pedagogies and approaches for teaching and learning mathematics in Indigenous contexts. On the basis that much of this research considered school contexts, there is room for greater research focus on the mathematical knowledge and experiences of children in their prior-to-school contexts, including educational, family and community settings.

Mathematics as Part of the Transition to School

A range of research has considered the mathematical knowledge and understanding of young children as they start school. Consistent themes in this research include the importance of recognising and valuing learning that has occurred before children start school and the role of early childhood education in promoting mathematical learning.

MacDonald (2010b; 2011; MacDonald & Lowrie, 2011) conducted a three-year longitudinal study of young children's understandings of measurement at the start of school, concluding that these understandings resulted from children's informal engagements in a variety of contexts, prior to the commencement of formal schooling. Using a series of drawing tasks, MacDonald (2011) elicited children's understandings about measurement, and the contexts—prior-to-school and out-of-school—that influenced these. Conclusions from the research included recommendations about the value of mathematical drawing activities for assessing and extending children's understandings; recognition of the measurement learning that occurs in prior-to-school contexts; and reconsideration of the measurement curricula for children in the first year of school.

The participants in the *Competent Children* project, funded by the New Zealand MoE and the New Zealand Council for Educational Research, are now young adults. The project has gathered information on the development of 500 children in the Wellington region of New Zealand, from 1993 onwards. A report from this study (Wylie, Hodgen, Hipkins, & Vaughan, 2008) confirmed earlier results, that high quality interactions in the early childhood years continued to have positive benefits for cognitive outcomes (including literacy, numeracy and logical problem solving) and attitudinal competencies, to age 16.

In a description of the mathematical learning contexts of four New Zealand early childhood settings, Davies and Walker (2008) identified strengths in the child-directed focus of learning, integration of curriculum areas, play-based pedagogies and teacher commitment to extending children's existing understandings. Detailed narrative assessment addressed children's dispositions, while at the same time providing evidence of specific mathematics concepts being developed. However, no specific policies or approaches were in place to share this deep knowledge with schools. This proved to be problematic for families, who expected that the wealth of documentation from the early childhood setting, as well as their own knowledge, would be accessed by school teachers.

A further, year-long study (Davies, 2009) investigated existing transition practices, particularly around mathematics learning and teaching, between early childhood services and primary schools in a small town in New Zealand. The study considered five key aspects: (a) structural provisions for mathematics; (b) assessments of children's mathematical understanding; (c) transfer of information between sectors; (d) processes and provisions for transition; and (e) parental perceptions and expectations. Although they had been prepared by prior-to-school educators, portfolios of narrative assessments were not used by the new entrant teachers. In completing her investigation on the mathematical practices as children moved into two primary schools, Davies (2011) reiterated that the connections between early childhood and the school setting were very tenuous. Limited evidence was found of the New Zealand curriculum's suggestion that "children's learning builds upon and makes connections with early childhood learning and experiences" (MoE, 2007, p. 41). Recommendations from the Davies (2009, 2011) study noted that focusing on dispositions and key competencies could well initiate closer links and promoted reform of transition practices to ensure that "schools can design their curriculum so that students find the transitions positive and have a clear sense of continuity and direction" (MoE, 2007, p. 41).

The role of teacher beliefs about mathematics and how children learn mathematics was the focus of a small study of five New Zealand teachers (Sherley, Clark, & Higgins, 2008). Results highlighted the general lack of attention that teachers paid to the knowledge and skills that children had when they started school. There were also marked differences between what teachers said they believed when compared with what they actually did in the classroom: the stated constructivist practices were inconsistent with the transmission approaches noted in classroom interactions. The interplay of beliefs and practices was demonstrated as four of the teachers disregarded the stated curriculum in favour of practice based on their own beliefs.

Connections among Contexts in Early Childhood Mathematics

Children's development of mathematical ideas almost always begins with a connection between the idea and a relevant experience in their lives. Indeed, facilitating such connections is a key role of early childhood educators. Sawyer's (2008) analysis of two Year 1/2 teachers and their efforts to help students make mathematical connections—both between the children and their worlds and within mathematics—outlined some possible strategies to achieve such connections.

In their study of Year 2 and Year 3 students' performance on map tasks, Lowrie, Diezmann, and Logan (2011) explored connections between children's lived experiences (in terms of geographic locality) and their ability to decode maps. They found some difference between the performance of metropolitan and non-metropolitan students on a coordinate-map and a landmark-map task. They suggest that this may be the result of difference in exposure to map systems, thus highlighting the connection between the mapping tasks and the children's lived experiences in out-of-school contexts.

The use of picture books to stimulate mathematics learning has been investigated by van den Heuvel-Panhuizen, van den Boogaard, and Doig (2009). They provided examples of picture books that were not blatantly mathematical and suggested that these books could be used as scaffolds to mathematical learning. Recommendations were made as to how this might happen and what role the adult might play in the story reading and mathematical development.

In a completely different 'connection', Jorgensen and Grootenboer (2011) investigated the mathematics learning opportunities afforded by swimming lessons for under-fives. From careful observation, they concluded that the swimming school environment could help expose very young children to mathematical vocabulary through everyday discourses, such as swimming lessons.

Parents, and the home learning environment they help create, influence children's mathematics development. While attempts to assist parents to help in their children's mathematical development are not new, they are relatively sparse in this review period. Muir (2009) reported a study designed to investigate parents' perceptions of mathematics through an intervention in which the parents became actively involved in their children's mathematics development. Collaborative support from teachers and clear understandings of the purpose for certain strategies and activities were identified as critical to the effectiveness of this program.

A strong point made by both Sawyer (2008) and Jorgensen and Grootenboer (2011) was that equity issues needed to be considered when connections in mathematics education are being advocated. For example, swimming lessons were out of the financial reach of many families, meaning that access was limited to those who could afford them. While this is not a reason for not having the experiences available, it should ring alarm bells. There is much mathematics in children's worlds and early childhood mathematics educators need to ensure that all children have access to it.

PEDAGOGIES FOR EARLY CHILDHOOD MATHEMATICS EDUCATION

Many issues influence the teaching and learning of mathematics in early childhood. These include pedagogical issues—such as the approaches used to promote and assess mathematical learning—as well as issues related to the confidence and competence of the educators engaged in this endeavour. In this section, we review research exploring the use of technology and play in the mathematics education of young children; assessment in the early childhood years; and the importance of early childhood teacher education and professional development.

Technology in the Mathematics Education of Young Children

Despite the currency of issues surrounding the use of technology in young children's learning (Robbins, Jane, & Bartlett, 2011; Sweeney & Geer, 2010; Yelland, 2011), there appears to have been relatively few research reports published in the area of early childhood mathematics education and technologies during the review period. This reiterates the work of Highfield and Goodwin (2008) who reported few articles addressing early childhood mathematics education and technology in five of the leading international mathematics education research journals over the previous five years. It also reinforced the claim made in the early childhood education chapters of the two previous MERGA research reviews (Perry & Dockett, 2004; Perry et al., 2008).

Highfield (2010a; 2010b; Highfield & Mulligan, 2009) and Goodwin (2008a, 2008b) have continued in their work in the area, with Goodwin particularly studying the links between interactive multimedia and the representations of fraction concepts by children in the first three years of primary school. Her detailed intervention study in two first-year-of-school classes used a range of multimedia tools including interactive whiteboards and personal computers. While the study was small in scale, preliminary analysis of students' fraction representations using the SOLO taxonomy led to the conclusion that "multimedia tools afforded the intervention students the opportunity to engage with advanced mathematical ideas that exceed current teaching practices and syllabus requirements" (Goodwin, 2008b, p. 115).

Highfield has continued her work on robotic (techno) toys with children in preschool and the first years of school and has developed some innovative tasks, particularly in the area of problem solving. These included play experiences with the techno toys followed by mathematical problem solving tasks that involved the children in 'programming' the toy. Highfield suggested that using a multi-faceted approach through dynamic tasks "can promote rich mathematical thinking and sustained engagement" for young children (2010b, p. 27) and that techno toys can assist young children develop meaningful mathematical understandings and social skills (Highfield, 2010a).

Yelland and Kilderry (2010) reported a three-year study that considered how young children became numerate in contexts that were rich in information and communication technologies. The study was designed to ascertain the range and nature of tasks with which children engaged in their first three years of school and which contributed to their becoming numerate. It was undertaken in two Melbourne schools with 11 teachers and the children in their classes. Yelland and Kilderry (2010) developed a *Mathematical Tasks Continuum* from data generated in the first year of the study to explore the complexity of tasks across the four variables: (a) using mathematical concepts and processes; (b) applying mathematical knowledge; (c) opportunities for exploration; and (d) learning outcomes. One conclusion from the study was that most of the mathematics tasks met by young children within the mathematics curriculum were unidimensional – "characterised by simple sequences of activity that often have a single outcome, minimal opportunities for exploration and where mathematical concepts and processes are introduced via structured tasks" (Yelland & Kilderry, 2010, p. 97). On the other hand, other curriculum areas were more likely to use mathematical tasks that were multidimensional—"open-ended, integrated investigations that not only built on basic or introductory mathematical skills and concepts, but also provided students with multiple opportunities for exploration" (Yelland & Kilderry, 2010, p. 97). Often, these multidimensional tasks could be facilitated using appropriate technology as a stimulus and context in which the more complex learning about numeracy could flourish.

An interesting juxtaposition of older and newer technologies has been investigated by researchers from Western Australia. The advent of 'virtual manipulatives' or virtual representations of concrete materials such as pattern blocks or MAB via interactive white boards or as pictures in NAPLAN tests has reopened questions about the role of such manipulatives in mathematics learning. Swan and Marshall (2010) used virtual and physical manipulatives in a revision of an older study (Perry & Howard, 1997). While there were some key differences in the results of the two studies, particularly around the perceived need of the teacher respondents for professional development, Swan and Marshall (2010) confirmed conclusions from the earlier study, warning that while there were good reasons for using manipulatives in mathematics learning, their use did not guarantee success: the major benefit of the manipulatives comes from the discussion that goes on around them and explicit linking by the teacher to the mathematics they represent.

Play and Mathematics

While the mathematical content of young children's play has been established in national and international research (Ginsburg, 2006; Perry & Dockett, 2008), there have been few research reports relating to this area during the review period. In one study, Lee (2010) confirmed that the outdoor play of toddlers incorporated a wide range of mathematics and that toddlers were indeed competent and confident mathematics learners. Despite this, she cautioned that both integrated, play-based curriculum and adult input are required to make the most of these experiences.

The trend in recent years seems to be away from research exploring specific areas of play and mathematics, such as block play, to greater interrogation of what constitutes play and the connection between learning and play (de Vries, Thomas, & Warren, 2010; Hunting, 2010; Perry & Dockett, 2010, 2011). This research emphasised the importance of educators themselves having a sound understanding of mathematics, as well as the confidence to use it, as they engage with children.

Professional Development of Early Childhood Teachers

It is well known that the quality of teacher knowledge—both pedagogical and content—is positively correlated with children's mathematical learning outcomes. There is also evidence that some early childhood teachers do not have a strong grasp on mathematics and, in particular, do not understand the future trajectories for the mathematics developed by children in the prior-to-school years: "This not only makes it difficult for them to provide necessary scaffolding for young learners but it may also even lead to negative attitudes about the subject—attitudes that may be transferred to the children in their care" (Perry & Dockett, 2008, p. 97). This situation has ramifications for both initial teacher education and ongoing professional development of early childhood educators.

In New Zealand, Sherley et al. (2008) identified the importance of support for teachers to recognise children's prior knowledge and understandings, and to explore the interactions of beliefs and practices in teaching mathematics. As well, Johnston, Thomas, and Ward (2010) suggested that professional development with first-years-of-school teachers encouraged them to acknowledge children's existing knowledge and strategies, rather than emphasising the pedagogical importance of strategy over knowledge. They reported that practising known strategies assisted students to develop new number knowledge and facilitated the development of increasingly sophisticated strategies for solving number problems.

In a major study, *Mathematical Thinking of Preschool Children in Rural and Regional Australia* sponsored by the National Centre of Science, ICT, and Mathematics Education in Rural and Regional Australia (SiMERR), 12 early childhood mathematics education researchers from ten universities in Australia and New Zealand investigated the conceptions and views of preschool practitioners with respect to young children's mathematical thinking and development (Hunting et al., 2008; Perry, 2009/2010). Drawing on extensive interviews with 64 early childhood educators in three Australian states, the project gathered data, inter alia, on the mathematical knowledge of the preschool educators (Bobis, Papic, & Mulligan, 2009/2010) and on the support they felt they needed to facilitate the mathematics development of the children in their care (Pearn, Hunting, & Robbins, 2009/2010).

Bobis et al. (2009/2010) used videorecords and still photography to capture evidence of mathematical activities involving preschool children and their educators. They reported that the two practitioners in their study did display sound knowledge of relevant mathematics but were less able to extend the children's mathematical thinking and development. There were many potentially rich mathematical episodes that were missed by the practitioners. The authors called for the development of professional resources and professional development opportunities for early childhood educators.

Pearn et al. (2009/2010) reported on the stated needs of the 64 preschool educators in professional development in mathematics. In particular, the following two research questions were addressed:

– Where do you get information about suitable mathematical activities? and
– What kind of assistance would you find useful?

The most important source of information about mathematical activities was other people, particularly other early childhood educators. While university early childhood staff and consultants were mentioned, they were not seen as important sources of such information. Other sources were the media, particularly the internet, and, only occasionally, professional publications and meetings such as scheduled professional development opportunities. In terms of assistance that would be useful, there were again three main categories: (a) personal support, (b) resources, and (c) training. Networking with colleagues was the most frequently elicited personal support followed by the institution of a mentoring system. Overall, the need for people and resources (mainly time, although in more remote areas, also money for travel) to support professional development were seen as the key needs of those interviewed.

As part of her team's extensive work on the development of early algebra concepts, Warren (2008/2009) undertook a project designed to develop and implement a professional development model that supported teacher learning in this mathematical domain. The *Transformative Teaching in the Early Years Mathematics* (TTEYM) model is based on the principle that learning is cyclical and consists of four components: (a) knowing person; (b) collaborative planning; (c) collaborative implementation; and (d) collaborative sharing. Data generated at the conclusion of two cycles of implementation and 18 months after the completion of the project suggested that "TTEYM proved effective in assisting teachers to implement new curriculum that contained unfamiliar mathematics content knowledge and pedagogy ... its effectiveness was independent of the content knowledge being introduced" (Warren (2008/2009, p. 44). The TTEYM model deserves further investigation in other mathematical areas.

Perry (2011) has reported on the professional development aspects of his ongoing *Early Years Numeracy Pilot Project* which used an inquiry model of professional learning linked to the intensive development of numeracy leaders from preschool and the first years of school. Using interview and journal data, Perry (2011) chronicled the growth of four numeracy leaders as they led colleagues through the project. With particular reference to the artefacts of the project, the numeracy leaders showed that the intensive and extensive professional development that they experienced through the project had changed their skills, confidence and competence in leading change among their early childhood colleagues.

While professional development of early childhood educators has received some mention here, there is very little to report on researching initial teacher education specifically in early childhood mathematics. This is an area of much needed research.

Approaches to Assessment in Early Childhood Mathematics

Over several years, Howell and Kemp (2009, 2010) have explored number sense and its assessment among young children. Drawing on an earlier Australian Delphi study, an international study was undertaken to establish some consensus among early mathematics researchers about the elements of number sense (Howell & Kemp, 2009) and its assessment (Howell & Kemp, 2010). Assessment of counting, number principles and number magnitude on standardised measures, as well as receptive vocabulary, identified a broad range of number skills among children prior to starting school. The majority of the children demonstrated counting skills to at least 10 and an intuitive understanding of number magnitude, but without sound understanding of counting principles. These studies provided the basis for an ongoing research agenda investigating potential causal links between number sense and later mathematical performance.

The nature of assessment, rather than its specific content, has long been a contentious issue in early childhood education, with Meisels (2007, p. 35) describing young children as "unreliable test takers", affected by the nature of the test taking environments as much as the tests themselves. He argued further that the common practice of assessment at school entry assumes that children have had similar prior-to-school experiences and would be entering similar educational contexts. This view positions assessment at school entry as formative and diagnostic, rather than summative; that is, the start of an appropriate learning and teaching program, rather than a predictor of future success. A similar argument is offered by Young-Loveridge (2011) in her overview of assessment practice in New Zealand. While recognising the potential value of recently introduced National Standards, Young-Loveridge promoted the continued use of diverse opportunities for assessment of these standards. These included the individual diagnostic interview, with its associated advantages of limited demands on children's reading or writing ability, and opportunities for teachers to convey clear instructions and engage in diagnosis throughout the interview, as well as higher levels of child engagement and opportunities to build on teacher-child relationships. This was also evident in the smaller effect size when using interviews over written tests with children from minority groups (Young-Loveridge, 2008).

In a context where school-entry assessments are increasingly popular, there remains limited research addressing issues of the assessment of mathematics in the early childhood years across Australia and New Zealand. School entry assessments vary considerably, but each has a focus on numeracy. Most assessments are conducted once only. An exception is the *Performance Indicators in Primary School* (PIPS) which was used to assess what children had learnt over their first year of school (Wildy & Styles, 2008). With the current emphasis on national and international testing, there is much potential for the downward-push of testing regimes. Research focusing on the nature and role of mathematics assessment in early childhood will inform future trends for testing and assessment.

New Zealand's *Numeracy Development Projects* (NDP) continue to be analysed, resulting in regular reports of student data across all school years and research projects focused on various aspects of the NDP (MoE, 2008a, 2008b, 2009, 2010a). Analysis of data specific to the early school years identified benefits for children after a year engaging in NDP (Young-Loveridge, 2009).

Within the *New Zealand Number Framework* (MoE, 2006) a distinction was made between strategy and knowledge with importance placed on making progress in both, as "strong knowledge is essential for students to broaden their strategies across a full range of numbers, and knowledge is often an essential prerequisite for the development of more advanced strategies" (p. 2). Johnston et al. (2010) reported data from 3742 students (including 2117 from Years 0 to 3) over three consecutive years from 2006–2008. Results supported the notion that students require an initial body of knowledge to solve number problems using strategy and that this body of knowledge is accumulated rapidly during their first three years at school. These conclusions are supported by a comprehensive analysis of the NDP data undertaken by Young-Loveridge (2010). She concluded that student achievement in the early years of school fell some way short of the Ministry's numeracy expectations (MoE, 2007) and the mathematics standards (MoE, 2010b). For example, just over half (57%) of the students were able to count on (stage 4) by the end of Year 2. Results showed students appear to progress through the early (lower) stages on the *New Zealand Number Framework* far more quickly and easily than they progress through the later (upper) stages, thus reinforcing some of the findings by Johnston et al. (2010). Young-Loveridge (2009) postulated that students in the early years of school used counting strategies for longer than is desirable or necessary. She suggested that introducing ideas about the composition of numbers as wholes made of different parts may be of benefit.

Mathematics teaching and learning in the early childhood years is influenced by many factors. In recent years, attention has been directed towards the professional development of teachers, and away from areas such as the role of play in teaching and learning mathematics. In a context where standards and assessment are becoming increasingly important—both for teachers and children—there is great potential to examine the impact and implications of these.

CONTENT FOR EARLY CHILDHOOD MATHEMATICS EDUCATION

Three areas of content have dominated Australasian research over the review period: (a) number; (b) algebra; and (c) measurement. Important work has also been completed in the areas of data and modelling but very little has been addressed in geometry.

Number

Recent number research has challenged assumptions relating to children's understanding of multi-digit numbers, mathematisation and subitising. As part of the *Bridging the Numeracy Gap* project, Gervasoni and her colleagues (Gervasoni, Parish, Hadden et al., 2011; Gervasoni, Parish, Bevan et al. 2011) extended the *Early Numeracy Interview* (ENI) and growth points developed during the ENRP (Clarke et al., 2002) to explore the understandings of 2-digit and 3-digit numbers of children in Grades 2 to 4. Five additional tasks were included: (a) bundling; (b) 2-digit number line; (c) 3-digit number line; (d) 10 more; and (e) 10 less. Analysis of the performance of approximately 2,000 Grade 1 to Grade 4 students from the *Bridging the Numeracy Gap* cohort of 42 low-socioeconomic status communities across Victoria and Western Australia indicated that:

> These tasks distinguished students who were assessed as understanding 2-digit and 3-digit numbers respectively, but who in fact could not reliably identify numerals on a number line or state the total of a collection reduced or increased by ten. These additional tasks assist teachers to identify students who need further experience with multi-digit numbers to construct full conceptual understanding, and highlight the importance of teachers focusing instruction on interpreting quantities and developing a mental number line, and not simply reading, writing and ordering numerals. (Gervasoni, Parish, Hadden et al., 2011, pp. 321–322)

From the *Numeracy Intervention Research Project*, Ellemor-Collins and Wright (2008, 2009, 2011), stressed the importance of mathematical sophistication (mathematisation) in students' development of arithmetical knowledge. They proposed a framework of ten dimensions of mathematisation for arithmetic instruction and showed how this framework could be used by teachers. The framework has the potential to synthesise the important aspects of mathematisation for learning arithmetic.

Warren, de Vries, and Cole (2009) explored the conjecture that young Indigenous students possess an innate ability to subitise, superior to their non-Indigenous counterparts. Reporting on the results of a series of subitising tasks, Warren et al. concluded that, contrary to previous findings (Treacy & Frid, 2008; Willis, 2000), there was no significant difference between the Indigenous and non-Indigenous students' ability to subitise. From this, Warren et al. (2009) recommended intervention in the first year of school in order to increase both Indigenous and non-Indigenous children's ability to subitise.

Algebra

In the previous MERGA review, Perry et al. (2008) identified work on patterning, structure and early algebra as a significant 'new' field for Australasian early childhood mathematics education researchers. A significant Australian program of research in this area has been the *Pattern and Structure Mathematical Awareness Program* (PASMAP) (Mulligan & Mitchelmore, 2009). PASMAP, incorporating the *Pattern and Structure Assessment* (PASA), and *Early Mathematical Patterning Assessment* (EMPA) (Papic, Mulligan, & Mitchelmore, 2011) has made substantial contributions to educators' understandings of young children's development of patterning skills, spatial structuring, and multiplicative reasoning. Mulligan and colleagues have recently undertaken a comprehensive evaluation of PASMAP and PASA (Mulligan, English, Mitchelmore, Welsby, & Crevensten, 2011), exploring the effectiveness of PASMAP for children's mathematical development in a two-year longitudinal study. Comparing PASMAP with standard programs implemented by teachers of the first year of school in two Sydney and two Brisbane schools, Mulligan et al. (2011) noted that while students engaged with PASMAP demonstrated some advantages, students who had not engaged with the program were capable of demonstrating similar learning outcomes. They recommended that "further analysis of the impact of PASMAP on structural development must consider individual teacher effect and school-based approaches to evaluate the program's scope and depth of achievement" (Mulligan et al., 2011, p. 555).

Warren and Miller (2010a; 2010b), have explored understandings of patterning among young Indigenous children across tasks requiring children to copy, continue, complete and create repeating patterns. They noted that children found it easier to copy patterns than they did to continue and complete patterns. Children whose strategy for copying a pattern involved working from left to right performed at higher levels across all tasks, leading to the conclusion that "how a child copies a pattern provides insights into their ability to see the structure of the pattern as a whole" (Warren & Miller, 2010b, p. 600). Furthermore, they hypothesised "that 'seeing structure' of repeating patterns requires the identification of two components, identifying the rhythm of the pattern, and breaking this rhythm into the repeating component" (Warren & Miller, 2010b, p. 600).

In a related study, Warren, Miller, and Cooper (2011) considered how children aged 5 to 9 years grasped and expressed generalisations. As one aspect of the larger study, they examined Year 1 students' ability to identify function rules. Connecting this work to their other work on patterning, they hypothesised that the act of grasping generalisations entailed an understanding of the function or pattern, and translation of this to a process that efficiently reached accurate answers.

Measurement

Research encompassing children's understandings of mass and length has been identified for this review. Cheeseman, McDonough, and Clarke (2011) renewed explorations of ENRP data about young children's understandings of mass. They reported that by the end of the first year of school:

> Most students were able to compare masses, and three-fifths were able to use an informal unit to quantify a mass. By the end of Grade 1, virtually all students were able to compare masses, and 69% were able to quantify masses and were ready to move towards using standard units. By the end of Grade 2, over 40% were using standard units successfully, and the rest were ready to move towards that goal. (p. 178)

On the basis of these results, Cheeseman et al. (2011) have identified targets for the teaching of mass for children in the first three years of school, reflecting children's increasing understanding and the move towards standard units to quantify mass. Advice for teachers has emphasised the value of children's engagement in rich, hands-on experiences with mass measurement in the early years of schooling.

The prior-to-school years also offer many opportunities for children to generate understandings of mass and to use measurement attributes to make comparisons between objects. MacDonald (2010a) examined children's drawings of 'heavy' and 'light' objects, proposing that children developed theories about mass, based on their experiences, which informed their perceptions of, and decisions about, mass measurement. In further tasks, children's drawings indicated their competence in comparing similar and different objects, and at the most sophisticated level, comparison between more than two objects. Additionally, children were able to use appropriate measurement language in both a dichotomous and comparative manner.

In the same study, MacDonald and Lowrie (2011) explored children's understandings of length at the beginning and the end of the first year at school. Children's drawings of a ruler at each of these points indicated good understandings of length at the start of school, with these becoming more contextualised and sophisticated as the year progressed. McDonough and Sullivan (2011) also reported on the development of length understandings in early schooling, using ENRP data to identify key targets for the learning of length in the first three years of school. They suggested that learning to compare, learning to use a unit iteratively, and measuring using formal units were the most appropriate targets for children learning length measurement.

From her study of the home measurement experiences of a 6 to 7 year old girl over a 20 week period, Meaney (2009) questioned expectations that most interactions involved length. Rather, she reported regular discussions of time, particularly in the context of the child often being late for school. In this case, time, rather than length, was suggested as the context for introducing formal units of measurement. The recognition of context as an important factor in learning suggests that early school curriculum requires connection between home and school contexts.

Statistics and Probability

The paucity of statistical and probabilistic learning in early childhood is reflected in English's (2011) call for "a renewed focus on statistical reasoning in the beginning school years, with opportunities for children to engage in data modelling" (p. 226). The value of work in this area is reflected in English's (2010; 2011) use of data modelling to explore young children's abilities to identify diverse and complex attributes, sort and classify data, and create and interpret data representations. This work also emphasised the influence of task context on children's responses to data modelling activities, suggesting that task context appeared to "present both support and obstacles in the children's reasoning" (English, 2011, p. 231).

Building on her extensive opus in this field, Watson and colleagues (Watson, Skalicky, Fitzallen, & Wright, 2009) explored the practical application of statistics while modelling a manufacturing process in Years 1 and 3 classrooms. They not only showed that the children were capable of dealing with the data modelling involved in the activity but also that it exposed them to many other mathematical topics, technology and real life situations in which data were used.

CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

The above analysis shows that there has been much significant research in early childhood mathematics education in Australasia over the 2008–2011 period, and that this should be celebrated. The focus of this research has changed from that evident in earlier years, possibly reflecting political as well as educational agendas. The burgeoning work around the mathematics education of Indigenous children in both New Zealand and Australia, for example, is a response to both political and social justice priorities in these countries. Much of the work in specific content areas has at least 'one eye on' the new curriculum agendas. The area of mathematics learning as children move between educational sectors continues to attract attention. However, there are some contextual challenges remaining, particularly around collaborative work across these educational sectors.

Anthony and Walshaw (2009) noted there was limited cross-sector collaboration within the mathematics education community. They argued that understanding of effective pedagogies to enhance young children's mathematics learning would benefit from cross-sector research and recommended a research focus that bridged the early years divide to ensure a harmonisation of mathematics teaching across the early years.

Davies (2009; Davies & Walker, 2008) reiterated the call from Anthony and Walshaw and suggests that a future focus on dispositions and key competencies could well initiate closer links, resulting in early years of school programs aligning more closely with those from early childhood settings. Changes in both educational sectors will impact on the learning and teaching within those contexts. For example, what are considered appropriate pedagogies in the prior-to-school years seems to be changing, particularly around the way in which play now seems to be valued only if there is consequent learning, rather than as a worthwhile experience in its own right. While it is important that in mathematics education research, we forefront mathematics learning, it would also seem important to remember that there are other reasons why children might be encouraged to play.

Previous review chapters on early childhood mathematics education research in Australasia have concluded with some suggestions for future research. From the current analysis, the following would seem to be the key areas for consideration beyond 2011:

– continue the extensive work on Indigenous children's mathematics learning by consolidating findings; addressing the role of Indigenous knowledges and pedagogies in the learning of 'school' mathematics; and investigating the mathematics learning of Indigenous students in urban settings, prior-to-school, family and community contexts;

– continue to investigate ways in which there can be greater continuity of mathematics learning and teaching across the prior-to-school to school transition;
– build on the continuing investigations in mathematical content areas in early childhood and consider less-researched areas such as geometry and statistics and probability;
– consider the ramifications of new or revised curricula and standards in Australia and New Zealand on early childhood mathematics education, with particular reference to the impact of school curricula and practices on prior-to-school mathematics education;
– investigate the impact of school entry assessment on mathematics learning and teaching in the first years of school and in prior-to-school settings;
– address the needs and concerns of culturally and linguistically diverse children and families in early childhood mathematics education;
– research the use of ICT as mainstream pedagogies in early childhood mathematics education;
– develop and evaluate programs of initial teacher education and professional learning for practising educators that address their needs in early childhood mathematics education, particularly in rural and remote regions; and
– continue to investigate the role of families and communities in the mathematical learning of young children.

This chapter has shown that there is much to celebrate about the early childhood mathematics education research that has been undertaken in Australasia between 2008 and 2011. However, much more quality research in this field is needed, both to extend the areas of strength and to address the identified gaps. Perhaps this chapter will assist future researchers as they work to improve the mathematical wellbeing of young children and their families, communities and educators.

REFERENCES

Anthony, G., & Walshaw, M. (2009). Mathematics education in the early years: Building bridges. *Contemporary Issues in Early Childhood, 10*(2), 107–121.

Australian Association of Mathematics Teachers [AAMT]. (2011). *Numeracy, mathematics and Indigenous learners*. Retrieved from http://makeitcount.aamt.edu.au/

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2010). *Australian curriculum*. Retrieved from http://www.acara.edu.au/default.asp

Bobis, J., Papic, M., & Mulligan, J. (2009/2010). The mathematical knowledge of preschool practitioners: Evidence from the field. *Journal of Australian Research in Early Childhood Education, 16*(2), 87–97.

Cheeseman, J., McDonough, A., & Clarke, D. (2011). Investigating children's understanding of the measurement of mass. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 174–182). Adelaide, SA: AAMT & MERGA.

Clarke, B., Clarke, D.M., & Cheeseman, J. (2006). The mathematical knowledge and understanding young children bring to school. *Mathematics Education Research Journal, 18*(1), 78–103.

Clarke, D., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, M., McDonough, A. ... Montgomery, P. (2002). *Early Numeracy Research Project (ENRP) Final Report*. Melbourne: Australian Catholic University.

Davies, N. (2009). Mathematics from early childhood to school: Investigation into transition. In Ministry of Education, *Findings from the New Zealand Numeracy Development Projects* 2008 (pp. 86–97). Wellington, New Zealand: Learning Media.

Davies, N. (2011). Supporting young children's mathematics learning as they transition to school. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 208–216). Adelaide, SA: AAMT & MERGA.

Davies, N., & Walker, K. (2008). Explorations of early childhood–new entrant transitions in mathematics. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 155–162). Brisbane, QLD: MERGA.

Department of Education, Employment and Workforce Relations (DEEWR), (2009). *Belonging, being and becoming: The early years learning framework for Australia*. Canberra: Commonwealth of Australia. Retrieved from http://www.deewr.gov.au/earlychildhood/policy_agenda/quality/pages/earlyyearslearningframework.aspx

de Vries, E., Thomas, L., & Warren, E. (2010). Teaching mathematics and play based learning in an Indigenous early childhood setting: Early childhood teachers' perspectives. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 719–722). Fremantle, WA: MERGA.

Ellemor-Collins, D., & Wright, R. J. (2008). From counting by ones to facile higher decade addition: The case of Robyn. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the joint meeting of PME32 and PME-NA XXX, pp. 439–446). Morelia, Mexico: Cinvestav-UMSNH.

Ellemor-Collins, D., & Wright, R. J. (2009). Structuring numbers 1 to 20: Developing facile addition and subtraction. *Mathematics Education Research Journal, 21*(2), 50–75.

Ellemor-Collins, D., & Wright, R. J. (2011). Unpacking mathematisation: An experimental framework for arithmetic instruction. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 313–320). Ankara, Turkey: PME.

English, L. (2010). Young children's early modelling with data. *Mathematics Education Research Journal, 22*(2), 24–47.

English, L. (2011). Data modelling in the beginning school years. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 226–234). Adelaide, SA: AAMT & MERGA.

Gervasoni, A., Hart, A., Croswell, M., Hodges, L., & Parish, L. (2011). Insights from Aboriginal teaching assistants about the impact of the Bridging the Numeracy Gap project in a Kimberley Catholic school. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 306–314). Adelaide, SA: AAMT & MERGA.

Gervasoni, A., Parish, L., Bevan, K., Croswell, M., Hadden, T., Livesey, C., & Turkenburg, K. (2011). Exploring the mystery of children who read, write and order 2-digit numbers, but cannot locate 50 on a number line. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 401–408). Ankara, Turkey: PME.

Gervasoni, A., Parish, L., Hadden, T., Turkenburg, K., Bevan, K., Livesey, C., & Croswell, M. (2011). Insights about children's understanding of 2-digit and 3-digit numbers. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practice* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 315–323). Adelaide, SA: AAMT & MERGA.

Ginsburg, H. P. (2006). Mathematical play and playful mathematics: A guide for early education. In R. M. Golinkoff, K Hirsh-Pasek, & D. Singer (Eds.), *Play=learning* (pp. 145–165). New York, NY: Oxford University Press.

Goodwin, K. (2008a). The development of a digital assessment of early fraction learning. In O. Figueras, J.L. Cortina, S. Alatorre, T. Rojano & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint Meeting of PME 32 and PME-NA XXX, pp. 263). Morelia, Mexico: Cinvestav-UMSNH.

Goodwin, K. (2008b). The impact of interactive multimedia on kindergarten students' representations of fractions. *Issues in Educational Research, 18*(2), 103–117.

Highfield, K. (2010a). Possibilities and pitfalls of techno-toys and digital play in mathematics learning. In M. Ebbeck & M. Waniganayake (Eds.), *Play in early childhood education: Learning in diverse contexts* (pp. 177–196). Melbourne, Australia: Oxford University Press.

Highfield, K. (2010b). Robotic toys as a catalyst for mathematical problem solving. *Australian Primary Mathematics Classroom, 15*(2), 22–27.

Highfield, K., & Goodwin, K. (2008). A review of recent research in early mathematics learning and technology. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions*. (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 259–264). Brisbane, QLD: MERGA.

Highfield, K., & Mulligan, J. T. (2009). Young children's embodied action in problem-solving tasks using robotic toys. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 273–280). Thessaloniki, Greece: PME.

Howell, S. C., & Kemp, C. R. (2009). A participatory approach to the identification of measures of number sense in children prior to school entry. *International Journal of Early Years Education, 17*(1), 47–65.

Howell, S. C., & Kemp, C. R. (2010). Assessing preschool number sense: Skills demonstrated by children prior-to-school entry. *Educational Psychologist, 30*(4), 411–429.

Hunting, R. (2010). Little people, big play, and big mathematical ideas. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 727–730). Fremantle, WA: MERGA.

Hunting, R. P., Bobis, J., Doig, B., English, L., Mousley, J., Mulligan, J., ... Young-Loveridge, J. (Eds.). (2008). Mathematical thinking of preschool children in rural and regional Australia: Research and practice. Final report to National Centre of Science, ICT, and Mathematics Education in Rural and Regional Australia (SiMERR). Bendigo, Australia: La Trobe University.

Hurst, C., Armstrong, T., & Young, M. (2011). Making a difference for Indigenous children. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 373–381). Adelaide, SA: AAMT & MERGA.

Johnston, M., Thomas, G., & Ward, J. (2010). The development of students' ability in strategy and knowledge. In Ministry of Education, *Findings from the New Zealand Numeracy Development Projects* 2009 (pp. 49–59). Wellington, New Zealand: Learning Media.

Jorgensen, R. (2010). Group work, language and interaction: Challenges of implementation in Aboriginal contexts. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of*

mathematics education (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 740–743). Fremantle, WA: MERGA.

Jorgensen, R., & Grootenboer, P. (2011). Early years swimming as new sites for early mathematical learning. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 398–405). Adelaide, SA: AAMT & MERGA.

Lee, S. (2010) Mathematical outdoor play: Toddler's experience. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 723–726). Fremantle, WA: MERGA.

Lowrie, T., Diezmann, C., & Logan, T. (2011). Primary students' performance on map tasks: The role of context. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 145–152). Ankara, Turkey: PME.

MacDonald, A. (2010a). Heavy thinking: Young children's theorising about mass. *Australian Primary Mathematics Classroom, 15*(4), 4–8.

MacDonald, A. (2010b). Young children's measurement knowledge: Understandings about comparison at the commencement of schooling. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 375–382). Fremantle, WA: MERGA.

MacDonald, A. (2011). Young children's representations of their developing measurement understandings. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 482–490). Adelaide, SA: AAMT & MERGA.

MacDonald, A., & Lowrie, T. (2011). Developing measurement concepts within context: Children's representations of length. *Mathematics Education Research Journal, 23*(1), 27–42.

McDonald, S., Warren, E., de Vries, E. (2011). Plotting the use of oral language and rich mathematical representations of teachers in the early years of formal schooling. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 209–216). Ankara, Turkey: PME.

McDonough, A., & Sullivan, P. (2011). Learning to measure length in the first three years of school. *Australasian Journal of Early Childhood, 36* (3), 27–35.

Meaney, T. (2009). Only two more sleeps until the school holidays: Referring to quantities of things at home. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the 6th Congress of the European Society for Research in Mathematics Education* (pp. 2617–2626) Lyon, France: CERME.

Meisels, S. J. (2007). Accountability in early childhood: No easy answers. In R. C. Pianta, M. J. Cox, & K. L. Snow (Eds.), *School readiness and the transition to kindergarten in the era of accountability* (pp. 31–48). Baltimore, MD: Brookes.

Ministry of Education (1996). *Te whāriki he whaariki matauranga: Early childhood curriculum.* Wellington, New Zealand: Learning Media. Retrieved from http://www.educate.ece.govt.nz/~/media/Educate/Files/Reference%20Downloads/whariki.pdf

Ministry of Education, (2006). *Book 1: The Number Framework – the Numeracy Development Projects*. Wellington, New Zealand: Author.

Ministry of Education, (2007). *The New Zealand Curriculum*. Wellington, New Zealand: Author.

Ministry of Education, (2008a). *Book 2: The Diagnostic Interview—The Numeracy Development Projects*. Wellington, New Zealand: Author.

Ministry of Education, (2008b). *Findings from the New Zealand Numeracy Development Projects 2007*. Wellington, New Zealand: Learning Media.

Ministry of Education, (2009). *Findings from the New Zealand Numeracy Development Projects 2008*. Wellington, New Zealand: Learning Media.

Ministry of Education, (2010a). *Findings from the New Zealand Numeracy Development Projects 2009*. Wellington, New Zealand: Learning Media.

Ministry of Education, (2010b). *Mathematics standards for years 1–8*. Wellington, New Zealand: Author.

Muir, T. (2009). At home with numeracy: Empowering parents to be active participants in their child's numeracy development. In R. Hunter, B. Bicknell, & T. Burgess (Eds.) *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 395–402). Palmerston North, NZ: MERGA.

Mulligan, J. T., English, L. D., Mitchelmore, M. C., Welsby, S. M., & Crevensten, N. (2011). An evaluation of the Pattern and Structure Mathematics Awareness Program in the early school years. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings from the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 548–556). Adelaide, SA: AAMT & MERGA.

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21*(2), 33–49.

Niesche, R., Grootenboer, P., Jorgensen, R., & Sullivan, P. (2010). The maths in the Kimberley project: An overview. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 732–735). Fremantle, WA: MERGA.

Papic, M. M., Mulligan, J, T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education, 42*(3), 237–268.

Pearn, C., Hunting, R., & Robbins, J. (2009/2010). What support do preschool practitioners need to foster young children's mathematical thinking? *Journal of Australian Research in Early Childhood Education, 16*(2), 63–75.

Perry, B. (2009/2010). Mathematical thinking of preschool children in rural and regional Australia: An overview. *Journal of Australian Research in Early Childhood Education, 16*(2), 1–12.

Perry, B. (2011). Early childhood numeracy leaders and powerful mathematical ideas. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 617–623). Adelaide, SA: AAMT & MERGA.

Perry, B., & Dockett, S. (2004). Mathematics in early childhood education. In B. Perry, G. Anthony, & C. Diezmann. (Eds.), *Review of mathematics education research in Australasia: 2000–2003* (pp. 103–125). Brisbane, Australia: Post Pressed.

Perry, B., & Dockett, S. (2008). Young children's access to powerful mathematical ideas. In L. D. English (Ed.). *Handbook of international research in mathematics education* (second edition) (pp. 75–108). New York, NY: Routledge.

Perry, B., & Dockett, S. (2010). What makes mathematics play? In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 715–718). Fremantle, WA: MERGA.

Perry, B., & Dockett, S. (2011). Playing with mathematics: Implications from the Early Years Learning Framework and the Australian Curriculum. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 624–630). Adelaide, SA: AAMT & MERGA.

Perry, B., & Howard, P. (1997). Manipulatives in primary mathematics: Implications for teaching and learning. *Australian Primary Mathematics Classroom, 2*(2), 25–30.

Perry, B., Young-Loveridge, J., Dockett, S., & Doig, B. (2008). The development of young children's mathematical understanding. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W. T. Seah, & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004–2007* (pp. 17–40). Rotterdam, The Netherlands: Sense.

Peters, S. (2010). *Literature review: Transition from early childhood education to school.* New Zealand: Ministry of Education. Retrieved from http://www.educationcounts.govt.nz/publications/ece/78823

Robbins, J., Jane, B., & Bartlett, J. (2011). Supporting technological thinking: Block play in early childhood education. In D. Corrigan, J. Dillon, & R. Gunstone (Eds.), *The professional knowledge base of science teaching* (pp. 223–241). Dordrecht, The Netherlands: Springer.

Sawyer, A. (2008). Making connections: Promoting connectedness in early mathematics education. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 429–435). Brisbane, QLD: MERGA.

Sherley, B., Clark, M., & Higgins, J. (2008). School readiness: What do teachers expect of children's mathematics on school entry? In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 461–465). Brisbane, QLD: MERGA.

Sullivan, P., Grootenboer, P., & Jorgensen, R. (2011). Considering student experience and knowledge of context in planning mathematics learning. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 225–232). Ankara, Turkey: PME.

Swan, P., & Marshall, L. (2010). Revisiting mathematics manipulative materials. *Australian Primary Mathematics Classroom, 15*(2), 13–19.

Sweeney, T., & Geer, R. (2010). Student capabilities and attitudes towards ICT in the early years. *Australian Educational Computing, 25*(1), 18–24.

Treacy, K., & Frid, S. (2008). Recognising different starting points in Aboriginal students' learning of number. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 531–537). Brisbane, QLD: MERGA.

Trinick, T., & Stevenson, B. (2009). Longitudinal patterns of performance: Te Poutama Tau. In Ministry of Education, *Findings from the New Zealand Numeracy Development Projects 2008* (pp. 27–38). Wellington, New Zealand: Learning Media.

Trinick, T., & Stevenson, B. (2010). Evaluation of Te Poutama Tau: Māori-medium Numeracy Project, 2003–2009. In Ministry of Education, *Findings from the New Zealand Numeracy Development Projects* 2009 (pp. 72–87). Wellington, New Zealand: Learning Media.

van den Heuvel-Panhuizen, M., van den Boogaard, S., & Doig, B. (2009). Picture books stimulate the learning of mathematics. *Australasian Journal of Early Childhood, 34*(2), 30–39.

Warren, E. (2008/2009). Early childhood teachers' professional learning in algebraic thinking: A model that supports new knowledge and pedagogy. *Mathematics Education Research Journal, 10,* 30–45.

Warren, E., de Vries, E., & Cole, A. (2009). Closing the gap: Myths and truths behind subitisation. *Australasian Journal of Early Childhood, 34*(4), 46–54.

Warren, E., & Miller, J. (2010a). Exploring four year old Indigenous students' ability to pattern. *International Research in Early Childhood Education, 1*(2), 42–56.

Warren, E., & Miller, J. (2010b). Indigenous children's ability to pattern as they enter kindergarten/pre-prep settings: An exploratory study. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 594–601). Fremantle, WA: MERGA.

Warren, E., Miller, J., & Cooper, T.J. (2011). Exploring young children's functional thinking. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 329–336). Ankara, Turkey: PME.

Warren, E., Young, J., & de Vries, E. (2008a). Indigenous students' early engagements with numeracy: The case of Widgy and Caddy. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 547–554). Brisbane, QLD: MERGA.

Warren, E., Young, J., & de Vries, E. (2008b). The impact of early numeracy engagement on 4-year-old Indigenous students. *Australian Journal of Early Childhood, 33*(4), 2–8.

Watson, J., Skalicky, J., Fitzallen, N., & Wright, S. (2009). Licorice production and manufacturing: All-sorts of practical applications for statistics. *Australian Primary Mathematics Classroom, 14*(3), 4–13.

Wildy, H., & Styles, I. (2008). Measuring what students entering school know and can do: PIPS Australia 2006–2007. *Australian Journal of Early Childhood*, *33*(4), 43–52.

Willis, S. (2000, October). '*Strengthening numeracy' reducing risk*. Paper presented at the ACER research conference. Brisbane, Australia: ACER.

Wylie, C., Hodgen, E., Hipkins, R., & Vaughan, K. (2008). *Competent learners on the edge of adulthood. A summary of key findings from the competent learners @16 project*. Wellington, New Zealand: Ministry of Education. Retrieved from http://www.educationcounts.govt.nz/publications/ece/2567/35076/35079

Yelland, N. (2011). Reconceptualising play and learning in the lives of young children. *Australasian Journal of Early Childhood, 36*(2), 4–12.

Yelland, N., & Kilderry, A. (2010). Becoming numerate with information and communication technologies in the twenty-first century. *International Journal of Early Years Education, 18*(2), 91–106.

Young-Loveridge, J. (2008). Development of children's mathematical thinking in the early school years. In O. Saracho & B. Spodek (Eds.), *Contemporary perspectives on mathematics in early childhood education* (pp. 133–156). Charlotte, N.C.: Information Age.

Young-Loveridge, J. (2009). Patterns of performance and progress of NDP students in 2008. In Ministry of Education, *Findings from the New Zealand Numeracy Development Projects 2008* (pp. 12–25). Wellington, New Zealand: Learning Media.

Young-Loveridge, J. (2010). A decade of reform in mathematics education: Results for 2009 and earlier years. In Ministry of Education, *Findings from the New Zealand Numeracy Development Projects* 2009 (pp. 15–35). Wellington, New Zealand: Learning Media.

Young-Loveridge, J. (2011). Assessing the mathematical thinking of young children in New Zealand: The initial years at school. *Early Child Development and Care, 181*(2), 267–276.

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POWERFUL PEDAGOGICAL ACTIONS IN MATHEMATICS EDUCATION

Key words: pedagogical actions; schooling; learning environments.

INTRODUCTION

A critical issue for all involved in primary and secondary mathematics education is how to ensure that all students learn mathematics successfully so that at the end of schooling they have the mathematical knowledge, skills, and confidence necessary to fully participate in further learning, employment, community life, and citizenship. This issue focuses attention on *pedagogy* which is the method or process of teaching, and generally refers to instructional strategies or actions (Ball & Bass, 2000). This chapter provides an overview of Australasian studies that have explored successful pedagogy within the context of mathematics education over the past four years (2008 to 2011). Our purpose is to highlight new insights that contribute to knowledge about what constitutes successful mathematics pedagogy for all students, to note any issues or tensions emerging from the findings, to identify any silences in the research agenda, and to recommend areas for future research.

In selecting the studies to review in this chapter, we have been inclusive of research methods, and considered both small and large-scale studies. Our central selection criterion has focused on Australasian studies where the findings are valid, reliable, and make an important contribution to knowledge about successful pedagogy in mathematics education. Our review suggests that over the past four years, Australasian research about successful mathematics pedagogies has concentrated on three important themes, and these form the framework for our review. These themes are: (a) creating powerful learning environments; (b) selecting tasks and models that promote deep learning; and (c) knowing and using pedagogical knowledge. This chapter discusses the key findings within this framework, and then considers the overall contribution of these studies to knowledge and understanding about successful mathematics pedagogy.

CREATING POWERFUL LEARNING ENVIRONMENTS

A key challenge for all school communities and teachers is creating high quality learning environments and communities in which students experience belonging, know that they are valued, and in which they are motivated to engage in the process of learning. Pedagogical actions that create a positive environment and

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community that engages all, motivates all, and enables all to learn mathematics successfully was an important theme in the research reviewed for this chapter.

Constructing Positive and Culturally Responsive Classroom Relationships

Relationships established by teachers with students, and the pedagogical repertoires they employ, are key deciding factors in how students engage in, and with mathematics. Researchers and teachers need to consider the cultural and political contexts of the communities in which they work, particularly when communities face multiple disadvantages. Attard (2011) provided evidence that student engagement increased in environments where the students had opportunities to construct positive relationships through interaction and dialogue. However, Hunter and Anthony (2011) argued for the need to find culturally specific actions and expectations to engage Pasifika students in mathematics. Their study, sited within a Pasifika community, illustrated the significant shifts in mathematical dispositions of former disinterested students when the teacher used powerful culturally responsive pedagogical practices to engage students. Similarly, Averill's (2011) study argued a need for all pedagogy to be underpinned by successful socially and culturally responsive pedagogical relationships. Averill's study, set within senior secondary classrooms, outlined ways teachers show their care within a model drawn from Indigenous perspectives. The teachers' pedagogical practices were linked to positive cognitive, social, emotional, physical, and dispositional aspects for the learners. Averill identified a paucity of culturally-linked models for mathematics teaching. This is a clear focus for further research by the MERGA community.

Scaffolding Successful Group Interactions

When teachers scaffold students to work in collaborative group situations their opportunities for interaction and engagement in a range of key mathematical practices increases. R. Hunter (2010) argued that when scaffolding practices are used as a pedagogical tool to mediate engagement of all participants in mathematics learning situations, then the learners are able to experience the practices of mathematicians. This is especially evident when problem solving groups are employed by the teachers and where students of varying mathematical expertise are grouped together to engage in opportunities to 'talk and do' mathematics. Importantly, Hunter's study showed that explicit pairing or grouping resulted in the students achieving more than they would independently. Likewise, Askew (2011) concluded that paired or group work that allows for collaborative emergence may result in more sophisticated, improvised, mathematical performance than could be achieved by individual students. Askew used a two year teaching experience with Year 2 students in England to illustrate that creative mathematics teaching and learning required a certain amount of unpredictability and that, particularly with regard to problem solving, learners' solutions have a certain quality of emergence that is similar to improvisational drama. This finding has implications for the pedagogical actions associated with the planning and implementation of lessons, and the designing of tasks.

Williams (2008a) contended that teachers need to deliberately consider group compositions and more specifically the optimism (ability to actively engage in exploring the mathematical territory) different members display. Williams illustrated that when group members were optimistic, rich learning occurs but when a group contains a non-optimistic member the learning is likely to be limited to familiar mathematics known to all the students. The zones of proximal development which are created are important for extending the learning and creating new knowledge if there is support from the group composition (Williams, 2008b).

Engaging Students in Relevant and Engaging Curricula

Other studies argued the need for changes in teachers' pedagogical actions. For example, over 600 Year 12 mathematics students in one of Victoria's poorest regions were surveyed (as a part of a larger survey) and asked to suggest how mathematics and its teaching could be improved (Helme & Teese, 2011). The students stated that they wanted teachers who could speak and explain clearly, and adapt their explanations to individual needs. Students requested that teaching methods be less dependent on textbooks, more interactive, and with a slower pace of teaching. The students also sought more enjoyable coursework, course content with stronger links to real life situations, and more *hands on* approaches to learning. The expressed desire for changes in pedagogy resonates strongly with the views of Aboriginal Teaching Assistants in the Kimberley (north-west Australia) about effective pedagogy for primary Aboriginal students in this remote location. The Aboriginal Teaching Assistants interviewed as part of the *Bridging The Numeracy Gap Study* (Gervasoni, Hart, Croswell, Hodges, & Parish, 2011) explained that successful pedagogy required the teacher to implement actions that (a) engaged students in a relevant and engaging curriculum that they enjoyed; (b) used teaching strategies that are culturally appropriate and involved visualising, modelling and practical experiences, with minimal use of worksheets and textbooks; and (c) made Aboriginal students and families feel part of the education system and highly involved in decision-making. Similar views were expressed by teachers interviewed as part of the *Make it Count Project* in the Swan Valley cluster (Hurst, Armstrong, & Young, 2011).

Kaur and Ghan (2011) explored the preferred pedagogical practices of mathematically low-attaining primary students in Singapore and found 98% of interviewees identified with being taught mathematics in classrooms where teacher-led whole class instruction was the norm. However, teacher-led whole class instruction was the preference of only 28% of students, while 40% identified that they preferred to work in groups on mathematical tasks with the help of manipulatives. These student preferences are further exemplified in Marshman, Pendergast, and Brimmer's (2011) study. These researchers illustrated how mathematical investigations were most effective in increasing engagement of students in the middle years. This stance is supported by Anthony and Walshaw (2009) who cautioned selecting one pedagogical approach over another but stated the strong support for an interactive and investigational approach.

Attending to the Literacy Demands of Learning Mathematics for Learners from ESL Backgrounds

Students' literacy abilities impact on mathematics learning. Culturally responsive pedagogy requires that teachers are aware of the literacy demands of tasks and include powerful pedagogical strategies to address these. A number of studies (e.g., Bautista, Mitchelmore, & Mulligan, 2009; Bautista & Mulligan, 2010; Bautista, Mulligan, & Mitchelmore, 2009) examined how disadvantaged Filipino students with limited English language, engage with English word problems. These researchers outlined how teachers must provide appropriate time for students to understand the problem situation, narrate the problems, support the narration with concrete tasks, support the understanding of structures that underpin the number operations, and encourage the use of representations that are meaningful to the students.

In contrast to expecting students to use English, Niesche (2009) contended that the use of home or community languages should be viewed as a valuable resource for teachers in Indigenous schools. In the context of the *Maths in the Kimberley* Project (set in Western Australia), Niesche argued that using a home language can help students negotiate meaning and mathematical concepts. The teachers' use of Kriol (largely a mix of local Aboriginal languages and English) demonstrated explicit valuing of Indigenous cultures and heritage. Edmonds-Wathen (2011) also highlighted the pedagogical importance of teachers understanding the influence of home languages on how instruction is designed. For example, Edmonds-Wathen found that when teaching location concepts, the frame of reference used in Indigenous Australian languages has implications for the order in which the curriculum is introduced. Teachers need to understand that the frame of reference for location in many Australian Indigenous languages is absolute. In contrast, terms such as north and south are viewed as more specialised in Australian curriculum documents and are not introduced early in schooling. This issue needs to be considered when designing instruction for Indigenous Australian language speakers. Furthermore, if teachers are serious about improving the performance of Indigenous students then using home languages should form part of essential pedagogical practice. However, McDonald, Warren, and de Vries (2011) cautioned that literacy instruction in mathematics needed to be considered carefully. In a study involving 40 teachers in Queensland Indigenous and multi-cultural schools, they found the teachers relied heavily on a literacy approach to mathematics instruction, rather than using rich mathematical representations to model concepts. They recommended that teachers' pedagogical actions include a combination of oral language communication in association with rich mathematical representations.

Promoting Productive Discourse and Collaborative Argumentation

Teachers supporting the construction of productive discourse is an ongoing research thread from the previous MERGA four yearly review. Exploring the type of pedagogical actions that create and promote student participation in mathematics discourse and argumentation were important features of several recent studies (e.g., R. Hunter, 2008a, 2008b; J. Hunter, 2009). In these studies the pedagogical actions of the teachers were critical in facilitating student participation in productive mathematical discourse. Key teacher actions included the construction of social and mathematical norms which demanded student engagement in collaborative interactions, active listening, questioning, and agreeing and disagreeing with mathematical reasoning. J. Hunter (2009) illustrated that these norms were for student development and use of argumentation and justification. Similarly, R. Hunter (2008a) showed the importance of teachers constructing collaborative partnerships with students as well as between students. These partnerships supported collective justification and generalisations within mathematical argumentation. As part of the partnership, teachers gradually inducted students in ways to participate in and use the discourse of inquiry and argumentation; they pressed the students to use specific questions and prompts, to engage in collaborative reasoning and the use of proficient mathematical practices. Central to these studies was the notion of teachers constructing intellectual partnerships and scaffolded conversations, which made student reasoning visible.

Problem-centred pedagogy using collective argumentation (CA) (Brown & Renshaw, 2000) promotes the importance of teachers enacting collaborative conversations. Redmond and Sheehy (2009) illustrated that not only were students in senior mathematics classes more engaged in the mathematical learning when CA was incorporated but also their sense of agency was enhanced. Enacting CA, supported students' increased participation in the task and with each other as well as in important mathematical practices. Through experiencing generalisability objectivity, consensus, and re-conceptualisation, they realised the value and applications of the mathematics they were using in class. Furthermore, Redmond, Sheehy, and Brown (2010) showed that when collaborative discourses supported and promoted the communication of mathematical ideas (as the upper secondary students engaged in mathematical modelling) their opportunities to use mathematical modelling practices, which paralleled those of mathematicians, became evident. Brown and Reeves (2009) extended the use of CA to explore the effects of students' participation in CA on their learning in mathematics beyond the classroom. They provided important evidence in the snapshot, of how the use of CA promoted active student participation and engagement in their learning. The students consistently reported engagement in practices such as discussing, sharing, and validating ideas as being positive to their learning.

Questioning and Prompting

Clearly the role of questions and prompts are significant in promoting productive classroom discourse. R. Hunter (2008b) showed how the importance of teachers scaffolding a diverse range of students to use specific questions and prompts promoted their engagement in a range of important mathematical practices. Importantly, as their repertoire of questions and prompts increased so to their skills in mathematical explanations, justifications and generalisations, and their ability to interrogate their own reasoning and the reasoning of their peers improved. Williams (2011) provided an alternative to the notion of questions in the form of a powerful pedagogical prompt which she referred to as *queries.* In her study she illustrated that *queries* from different sources (group member, self, expert other) during problem solving led to students identifying something they did not yet know before then intently continuing their spontaneous exploration. These *queries* did not contain mathematical input or hints or affirmations or contradictions but drew attention to something that required further elaboration as the students began to construct new understandings.

The types of questions teachers asked influenced the nature of the students' responses; however, developing appropriate questioning skills has its challenges. Muir (2008, 2009) found that, although teachers were willing to have extended exchanges with students, their questions were often limited to recall or seeking clarification. They rarely used probing questions that encouraged student explanation and justification. Muir concluded that teachers, despite current reforms and an emphasis on questioning opportunities, did not use student exchanges as an effective follow-up move to maximise conceptual understanding. Given the importance of classroom discourse and questioning on rich student engagement in mathematical practices, discussion and research needs to continue into how to support teachers to enact these core pedagogical practices.

Connecting and Provoking Generality

Extended classroom discourse with opportunities for students to justify reasoning is integral to making connections and generalising their mathematical reasoning. A number of studies (e.g., Anthony & Hunter, 2008; J. Hunter, 2010; R. Hunter & Anthony, 2008) illustrated that when teachers used specifically designed rich open-ended tasks and powerful pedagogical actions their students' justification strategies were extended. These researchers showed that the enactment of the discourse of inquiry and argumentation was as important as the task, and that when students triangulated their reasoning using numeric, verbal, and visual strategies it provoked generality. Additionally, Sullivan (2008) argued that open-ended tasks are central to students making mathematical connections. Sullivan outlined how students who were provided with opportunities to create examples for themselves, constructed patterns of responses. Similarly, Windsor (2010) noted that when students were encouraged to use generalisations constructed during algebraic thinking lessons, opportunities for shifting from a purely answer focused perspective of mathematics become evident. As Cooper and Warren (2008) argued, mathematical structure and abstraction was founded on the ways students generalised from particular examples to general rules, and from real world situations to abstract representations. From their extensive work in the longitudinal *Early Algebraic Thinking Project (*EATP), these researchers placed importance on teachers attending to content and pedagogy, and the intertwining of algebraic and arithmetic reasoning. Within their design experiment methodology, Warren and Cooper (2009) used a sequence of teaching experiments from which rich generalised reasoning emerged. Of importance were both inquiry-based discourse, and the use of multi-representations (Warren & Cooper, 2008).

Sawyer (2008) argued that aiding students to make connections between different pieces of mathematical knowledge, understanding how ideas interconnect, and building to a coherent whole were vital. A powerful pedagogical action promoted by Mason, Drury, and Bills (2007) was teachers explicitly modelling themselves generalising, valuing learners' attempts at generalising, while also allowing space for students to generalise for themselves. Mason et al. highlighted that a teacher's utterance of a generality might be experienced by some learners as: (a) a crystallisation of a semi-focused awareness; (b) a restating of the obvious; (c) bridging or filling in an awareness of which they were not yet aware, and so taking away a generative experience; or (d) nothing at all because it passes them by. Mason et al. argued that for some learners the utterance of a generality by the teacher "might be part of the wallpaper of the lesson, for others it has a transformative action, and for others it confirms an awareness" (2007, p. 53). Mason and his colleagues proposed the idea of Zone of Proximal Awareness. They explain that a "generality is just one kind of awareness that can come to someone as a result of engaging in an activity with cultural tools and using practices encouraged and displayed by a relative expert" (p. 53). This term, they argued, described "awarenesses that are imminent or available to learners, but which might not come to their attention or consciousness without specific interactions with mathematical tasks, cultural tools, colleagues, teacher, or some combination of these" (p. 53).

These findings highlight the need to consider the careful construction of learning environments that meet the needs of an increasingly diverse classroom membership. They also alert us to the fact that sometimes there is a mismatch between our vision for powerful mathematics learning environments and the lived experience of students and teachers. Overall, findings from these studies present a vision for effective pedagogical practice that includes the following pedagogical actions (a) building student-teacher pedagogical relationships including those which are socially and culturally responsive; (b) scaffolding effective student to student collaborations; (c) engaging students in relevant and engaging curriculum; (d) ensuring that the literacy demands of tasks are considered; (d) promoting productive discourse and collaborative argumentation; (e) questioning and prompting; and (f) connecting and provoking generality.

SELECTING TASKS AND MODELS THAT PROMOTE DEEP LEARNING

Task selection is a critical pedagogical action because it provides the context for learning and teaching. Indeed, many studies reviewed for this chapter explored the use and effectiveness of various mathematics tasks. O'Shea and Peled (2009) in a study associated with the *Task Types and Mathematics Learning Project* (TTML) categorised tasks in four ways according to whether the teacher: (a) used a model, example, or explanation that elaborated or exemplified the mathematics; (b) situated the mathematics within a contextualised practical problem to engage the students; (c) provided open-ended tasks to enable students to investigate specific mathematical content; or (d) provided tasks requiring interdisciplinary investigations. These categories provide a useful framework for further research.

Selecting Rich Tasks

The use of rich mathematical tasks is not new and a broad base of research supports their use in the successful teaching of mathematics. Powerful pedagogical actions associated with rich tasks include, for example, the appropriate selection of rich tasks and associated questioning and scaffolding practices, especially those related to appropriate recording, and extended discussions. Indeed, J. Hunter (2010) found this to be central to supporting young students to develop ideas related to algebraic notations and variables. Grootenboer (2009) elaborated on how "a rich mathematical task must cater for a range of students in terms of previous mathematical achievement and interest, and different ways of thinking, learning and working mathematically" (p. 698). Grootenboer outlined the characteristics of rich tasks and suggested they provided 'tools' for teachers to develop their pedagogy so that deep mathematical learning is promoted for learners. These characteristics included: (a) academic and intellectual quality; (b) group work; (c) extended engagement; (d) catering for diversity through multiple entry points and multiple solution pathways; (e) connectedness; and (f) multi-representations. The pedagogical action of providing multiple entry points for tasks was the focus of a study by Jackson and Brown (2009) that used Gardner's Multiple Intelligences (Gardner, 1999) as *points of entry* to facilitate some 'valuing and working with difference' within classrooms. Their findings supported Gardner's ideas that topics should be addressed in a lesson by using all points of entry to create a more equitable learning environment.

Implementing Problem Solving and Using Authentic Tasks

Problem solving is an essential component of mathematics and many theorists contend that students require regular opportunities to solve complex problems. Cavanagh (2008) illustrated the powerful impact on learning that occurred when a teacher began to incorporate a problem-solving approach into his pedagogical practice. Selection of appropriate tasks was a key feature of the problem-solving approach and as the teacher became more confident the tasks became open-ended and more conducive for supporting extended investigation and dialogue. Downton (2010) suggested the need for teachers to realise the relationship between the difficulty level of multiplication tasks and the sophistication of the strategy choice used by primary school students. She contended that when faced with challenging tasks, students have the capacity to draw upon and use higher levels of thinking to solve such tasks than observed when they engaged in simpler tasks. For this reason, Downton recommended that teachers pose, for some of the time, problems that extend student thinking beyond what normally is expected.

A reconceptualisation of problem solving in the school curriculum was proposed by Dindyal et al. (2009) as part of the *Mathematical Problem Solving for Everyone* (M-ProSE) study in Singapore. The authors built on Polya's fourth stage of 'looking back' and emphasised the extending and generalising of a problem. Carefully crafted mathematics problems using a more practical context were also designed to guide students systematically and meta-cognitively through the problem solving approach. In line with this thinking, Keng and Kian (2010) outlined how authentic mathematics learning embedded in financial learning concepts in a Singaporean secondary school led to the students constructing deeper knowledge and application of financial learning ideas. The teachers used real-world examples such as taxation and interest rates as contexts for mathematical discussion and problem solving.

Cavanagh (2008) suggested that the selection of appropriate tasks for problem solving is challenging. Sullivan, Clarke, and Clarke, (2009) extended this further and argued the importance of teachers converting mathematics tasks into powerful learning opportunities. Their analysis of teachers' responses to a task indicated that teachers needed common content knowledge as well as specialised content knowledge (in this case fractions) and knowledge of students, if they are to transform tasks into effective instruction and opportunities for deep learning. Likewise, the students needed to have mathematical confidence in solving the tasks themselves (Sullivan, Clarke, Clarke, & O'Shea, 2009). These researchers cautioned that the potential of a task may not be realised, despite a teacher's willingness and ability to draw on student ideas during problem solving, if they lacked the conceptual understandings inherent in the task.

Posing Tasks for 'Sowing the Seeds'

Through case study investigation, Anthony and Ding (2011) proposed two further successful pedagogies—the teacher posing a set of tasks for students to work on prior to new instruction, and 'sowing the seeds'. In the first case the teacher posed a series of tasks prior to instruction, these first acted as revision or consolidation activities, then gradually became more challenging. These 'springboard' tasks generated new ideas that were central to learning goals that were more fully developed later in the lesson, and/or revisited in subsequent lessons. Anthony and Ding claimed that a key feature of this instructional approach is that it supported the connection of students' existing knowledge to new knowledge. In the 'sowing the seeds' approach, the 'seeds' corresponded to the multiple layers of new knowledge and methods embedded in the intended curriculum. The teacher sowed seeds with a careful consideration of the distance between the actual and potential development of individual students, and these seeds assisted the teacher to plan a logical sequence of knowledge construction that built and linked to students' existing and emergent ideas.

Selecting and Using Models

Aligned with selecting tasks and problems, selecting models to illustrate and explain mathematical concepts is another important pedagogical action, and the subject of several studies, restricted however, to a few mathematical concepts.

Chick (2007) explained that the strengths and weaknesses of any model depended on its capacity to effectively represent the mathematical attributes of the concept. Choosing the best model for this task required both knowledge of different models and a consideration of what each offered. Appropriate classroom use of any chosen model depended on the teacher's recognition of the students' present levels of understanding and development of appropriate explanations, and her own ability to find ways to respond to students' uncertainties and questions. Further, Chick and Pierce (2008) outlined that teachers needed to hold sound content knowledge and pedagogical content knowledge around the learning context so that aspects of the model selected are integrated into the learning at every phase of the teaching.

Clarke, Roche, and Mitchell (2007) in their exploration of the teaching and learning of fractions, recommended that successful pedagogy depended on teachers understanding and presenting a wider range of sub-constructs of fractions. They recommended this be done during teaching and assessment, using a greater variety of fraction models, and available interview assessment tasks with their students. Interviews provided teachers with considerable insights into student understanding, common misconceptions, and forms a basis for discussing the 'big ideas' of mathematics and curriculum implications from what they have observed.

Other researchers have focused on teachers using specific models as powerful ways to support students constructing mathematical understandings. Young-Loveridge and Mills (2009) suggested that arrays were useful representations for enhancing students' understanding of multi-digit multiplication, provided there was a good match with the students' learning needs and prior understanding of multiplication. These researchers outlined how a group of students made substantial progress in their understanding of multiplication when scaffolded from single-digit to two-digit by two-digit multiplication through the use of arrays. Similarly, Afamasaga-Fuata'i (2009) examined the impact of two meta-cognitive tools (vee diagrams and reflective stories) on 32 Year 10 students' understandings, competence in problem solving, and attitudes to mathematics. The vee diagram and reflective prompts helped students systematically approach problems and supported their thinking, reasoning, justification, reflection, and communication. She also noted an improvement in some students' attitudes towards mathematics.

In summary, these findings illustrate that powerful pedagogical actions require that teachers are able to select and use rich tasks, problems and models to provoke student engagement in productive discourse, collaboration, and the construction of rich mathematical understandings. They also draw our attention to some of the difficulties that teachers may encounter when teaching mathematics. These findings both build on previous reviews and provide direction for continued research into the selection and use of models to support mathematics learning and teaching.

KNOWING AND USING PEDAGOGICAL KNOWLEDGE

This review has examined current Australasian research on how teachers construct powerful learning environments and select appropriate problems and models to use within them. In this next section we examine the research which outlines teachers' knowledge of pedagogical actions, and their understandings of mathematical knowledge, and how they use both to respond to student reasoning, and how teachers use technology as a pedagogical tool. These studies draw attention to the many challenges involved and reveal important areas for on-going research.

Learning to Use Mathematical Inquiry

Many research articles reviewed for this chapter argued a view of classrooms and tasks which support increased student inquiry, discussion, and argumentation. In such learning environments teachers are positioned as facilitators and thus need an additional set of skills. However, this is a challenging expectation. Makar (2011) conducted a three-year study to provide insight about teachers' experiences as they developed proficiency in using mathematical inquiry as a pedagogical action. The results of the research showed that implementing mathematical inquiry, while highly promising as a pedagogical practice, was challenging for teachers and required substantial time and resources to operationalise. The study highlighted that pedagogical improvement pathways shifted and turned in unexpected ways, with dips and plateaus broadly evident. Makar concluded that teacher educators, principals, and policy makers needed to expect rather than reject the non-linear nature of teachers' adoption and adaptation of new pedagogical actions, and not conclude that a dip in practice indicated the new pedagogical practice had gone into disuse. Instead, implementation dips needed to be acknowledged as a normal part of the process so that teachers were supported and encouraged to persist through them. This conclusion likely applies to the development and implementation of many new pedagogical approaches.

Knowing and Using Pedagogical Knowledge

The idea that teaching effectiveness in mathematics depended on more than disciplined content knowledge alone was proposed by Shulman (1986) when he introduced the notion of pedagogical content knowledge (PCK). Shulman identified key aspects of knowledge that contributed to PCK, including knowing what models and explanations supported learning, understanding typical student conceptions, and recognising what made a task complex or simple. PCK in mathematics education has continued to be a focus of research. Chick's (2007) view that everything a teacher did—planned lessons, implemented lessons, responded to what arises in the classroom, and interacted with students—involved one or more aspects of PCK is supported by Watson, Callingham, and Donne (2008). Watson and her colleagues illustrated that when teachers were shown student responses to proportional reasoning tasks and asked to provide suitable responses they did so in a generic manner. Their lack of PCK suggested both a restricted view of the students as learners and a lack of knowledge of appropriate questions to prompt cognitive conflict and therefore promote learning.

Roche and Clarke (2009, 2011) designed an instrument to measure PCK and determine whether professional learning improved this aspect of practice. An analyses of responses concluded that the process of measuring PCK is complex and challenging (Roche & Clarke, 2011). In the context of measuring PCK related to partitive and quotitive division, Roche and Clarke (2009) found that primary teachers lacked an understanding of quotitive division. Although this finding focused on only one aspect of PCK, it contributes to our understandings about the importance of PCK and the challenges for teachers in understanding and helping students to respond to both partitive and quotitive division problems. A productive area for further research would be determining which other aspects of teachers' PCK may need development. The findings will be important for planning appropriate professional learning opportunities for teachers.

Watson, Callingham, and Nathan (2009) explored teachers' PCK in statistics at the middle-school level. The task chosen for analysis was deemed to be basic to the foundations of a chance and data curriculum and was based on a pictograph. The study used a framework for a refinement of PCK incorporating four key components: (a) recognising the big mathematical ideas; (b) anticipating student answers; (c) employing content specific strategies; and (d) constructing shifts to generalising. These components reflected the authors' decision to adopt a more holistic approach and a different appreciation of PCK. This was a valuable contribution as we challenge and broaden understandings of PCK as a creative process. The study confirmed the significance of PCK and how teachers integrate their understanding to plan for effective instruction.

Barton (2009) acknowledged the importance of PCK but suggested that we cannot ignore the mathematical part of a teacher's knowledge and the influence this has on their pedagogical actions. Barton builds on Watson's (2008) ideas about 'mathematical modes of inquiry' and attempts to improve our thinking about Mathematical Knowledge for Teaching (MKfT). He focused attention on "HOW a teacher must know" (p. 4). Moving on from thinking about topics that mathematics teachers must know, he proposed some key components of MKfT for all teachers. These included: (a) having a vision of mathematics; (b) ideas of philosophy; (c) the relation of mathematics to society; and (d) a personal approach to the subject. This framework may be useful for researchers to explore.

Responding to Student Reasoning

Determining students' current knowledge and misconceptions and responding appropriately is an important pedagogical action. Teachers use this knowledge to plan instruction and select appropriate tasks, to respond and interact with students during lessons, and to customise tasks and instruction. The process of determining students' mathematical knowledge also contributes to teachers' understanding of typical student conceptions and misconceptions, a key component of PCK. These aspects all involve teachers listening and responding with 'on the spot' responses; important actions that all teachers require (Muir, 2008). Muir contended that teachers needed to listen carefully to students in order to identify and know when to respond to 'teachable moments'. She suggested that while teachers may be reluctant to interrupt the flow of a lesson, student comments often reflect erroneous mathematical thinking which needs addressing.

The usefulness of learning trajectories and growth point frameworks to guide both assessment and subsequent instruction were considered important in the previous review. This focus is continued in studies reviewed for this chapter (e.g., Cheeseman, McDonough, & Clarke, 2011; Gervasoni, 2011; Gervasoni et al., 2011).

Several studies sought to identify new or refined learning trajectories that may be used by teachers as powerful tools to plan and implement instruction in particular mathematics topics. For example, Wong (2010) explored the learning pathways for understanding fraction ideas, in particular the fraction constructs of area. Parish (2010) proposed a set of growth points that related to the learning of ratio. Parish argued that the points of growth could be used to support teachers to gain insight into student understandings of ratio and help them press students to gain deeper understandings of this rational number sub-construct. Cheeseman, McDonough, and Clarke (2011) proposed the use of a growth point framework and assessment tasks for measuring mass. They outlined the importance of teachers assessing students' understandings of mass measurement and then structuring learning opportunities to build on and extend those understandings. MacDonald (2011) demonstrated the importance of teachers assessing students' measurement knowledge from school entry. Importantly, MacDonald showed that many of the 5-year-old students in the study had more sophisticated knowledge about measurement than what is acknowledged in the current Australian curriculum. These students were required to complete a series of six drawing tasks relating to different measurement concepts, and provide a description of each drawing. These effective pedagogical actions provided ample insight into the knowledge the students had as well as potentially challenging the teachers' PCK related to young students' measurement learning.

Gervasoni (2011) provided further insight about the importance of interview-based assessment that is linked to a growth point framework, for informing instruction and challenging teachers' PCK. As part of a two-year study, classroom teachers assessed students using the whole number domains of the *Mathematics Assessment Interview* and determined the growth points reached by each student. The growth point data was independently coded and entered into a database for analysis. The findings highlighted the broad distribution of students' growth points in each domain and each grade level, and the wide distance between the lowest and highest growth points in each grade level and each domain. This demonstrated the complexity of classroom teaching and of meeting each student's learning needs. It is likely that teachers need to make individual decisions about the instructional approach for each student based on current assessment data, and that there is no pedagogical 'formula' that will meet all students instructional needs.

Kamol and Yeap (2010) proposed that teachers and curriculum developers use an algebraic thinking framework (that described the characteristics and development of algebraic learning of upper primary schools students) to support instruction and assessment. Stephens and Armanto (2010) suggested that textbooks may have the potential to support a learning framework because of the clear learning trajectories evident in some Japanese primary school mathematics textbooks. These researchers argued that these are important to Japanese students' development of relational ways of thinking related to the four number operations. In an alternative view Hartnett (2008) illustrated a method to help teachers capture student thinking about their reasoning in performing mental calculations. She showed how a framework of mental computation strategies, provided to both teachers and students, improved the students' abilities to make their reasoning visible when recording their solution strategies.

Several studies explored the effectiveness of various assessment strategies for gaining insight into how to plan instruction for students. Bautista (2011) proposed that any use of written tests should be supplemented with individual interviews, or informal conversations, because some students may have issues with language comprehension and use that conceal underlying mathematical difficulties that may therefore go unrecognised. The usefulness of one-on-one interviews for revealing students' knowledge was also emphasised by Mitchell and Horne (2011). They demonstrated the importance of observational listening as a pedagogical action and explained that the teacher prompting for further elaboration of the students' explanations was needed to determine whether the students were correct (correct answer and mathematically correct strategy) or incorrect (correct answer and mathematically incorrect strategy). They further suggested that the relationship of observational listening had to be maintained, without cueing the student into a directive listening exchange during which the teacher listened *for* a particular response from a student (sometimes an assumed response) rather than listening *to* the student's actual explanation. The importance of listening and responding appropriately to student reasoning is a powerful pedagogical action. How teachers learn to do this and what trajectories or frameworks of knowledge or assessment tools support them doing so, all require further research.

Using Technology as a Pedagogical Tool

Increasingly over the past decade, knowing how to teach with technology in powerful ways has become a focal point of research in Australasia. Cavanagh and Mitchelmore (2011) drew on the recently developed construct, Pedagogical Technology Knowledge (PTK). This construct encompasses teachers' recognition of the role of technology in learning and teaching and decisions about how to use technology to assist students learn mathematical concepts and processes. Cavanagh and Mitchelmore used PTK to document changes in teachers' practice when using an on-line learning system. They found the most effective pedagogical practices were when teachers played the role of technology innovators who recognised the affordances of technology to encourage and support students' mathematical development in novel ways, and promoted student-generated learning, inquiry, and reflection. Geiger (2009) also challenged teachers to look at students' use of technology in individual and collaborative classroom settings within a framework which consists of using technology as a *master*, a *servant,* a *partner,* and an *extension-of-self.* Geiger's study provided evidence for consideration to be given to the practice which best resonates with the teacher's pedagogical intentions. His findings illustrated that in different settings, from individual to small group, to whole class, the students were engaged in different learning formats using technology as an effective mediated collaborative practice. Geiger's framework is significant because it could "lead to more sophisticated technology rich pedagogies" (p. 201) in a variety of teaching contexts.

Productive student interactions and learning mediated by technology, was a focus of some papers in this review. Geiger, Faragher, and Goos (2010) argued that technologies have the potential to be used during all phases of the mathematical modelling cycle, not just at the solve juncture. Importantly, their study showed that technology provoked mathematical learning for secondary school students engaging in mathematical modelling, and fulfilled a role greater than that of computational tool. Likewise, Yeh (2010) found that technology supported students with their mathematical thinking. His study illustrated that primary school students had increased opportunities to think about and do mathematics with 3D geometry in new or novel ways. Productive student-to-student and student-to-teacher interactions were the outcome of a study by Geiger, Faragher, Redmond, and Lowe (2008). These authors outlined how the technology of Computer Algebra Systems (CAS) was an integral partner in the learning process. Through its use the students were pressed to re-evaluate original assumptions and adjust their initial problem-solving approaches. At the same time the teachers were given opportunities to address misconceptions and scaffold student reasoning towards solutions. Gaining valuable insights into students' thinking and learning was also the focus of a study by Highfield and Mulligan (2009). These researchers reported on the examination of young students' interactions while they programmed a simple robotic toy. Integrated analysis of speech, gesture, action, and representations were an effective method of gaining insight into mathematical thinking and affirmed the potential of robotic toys in syntonic learning. The importance and value of teachers seamlessly integrating digital tools with other elements of mathematics use in authentic contexts was highlighted by Geiger, Dole, and Goos (2011) through two cases based in primary schools.

Other studies recognised the motivational aspects of ICT. However, they also highlighted a need for further research into how various aspects of ICT can be productively employed as pedagogical tools. For example, Serow (2008) argued that if ICT is used to improve students' mathematical learning then further exploration of specific strategies that incorporated technology within pedagogical strategies is required. Serow's study, which used dynamic geometry software, illustrated the value of sequencing technological tasks within teaching phases to increase the complexity of student responses. Scott, Downton, Gronn, and Staples (2008) also recognised the increased engagement and positive outcomes available for students' mathematical learning when ICT was integrated in classroom pedagogy. But their research also illustrated the difficulties inherent in using ICT to deepen students' mathematical understandings. Their case study of two teachers integrating ICT and measurement showed surface aspects of lessons which appeared to be engaging and student focused, but the teachers did not probe deeper to scaffold mathematical connections. Scott and her colleagues suggested that teachers need to take a more critical stance towards interrogating the learning available in ICT and in particular they need to focus on students making connections between understandings.

Interactive whiteboards (IWB) were the focus of several studies. Like other technological tools the pedagogical approaches of teachers influence their use of IWB (Serow & Callingham, 2008). These researchers explored the teaching strategies of three teachers as they began to use IWB technology as part of their mathematical teaching. Their findings suggested that teachers' use of the large wall-mounted IWB tended to conform to teacher-centred approaches. In contrast, Beswick and Muir (2011) provided a brief professional learning program for several secondary teachers that supported the development of the teachers' pedagogy alongside their use of the IWB. They concluded that increased awareness of the potential of IWB to enhance student engagement and hence learning, and commitment to collaboration and improved teaching, can motivate experimentation with the technology such that technical competence and pedagogical change occur together. Likewise Tanner, Jones, Beauchamp, and Kennewell (2010) explored interactive whole-class technologies (IWCT). They argued that these tools, like IWB, have the potential to facilitate opportunities for more creative teaching and learning, including the exploration and sharing of ideas within whole class teaching. They explained that final products can be demonstrated through the use of IWCT but, more importantly, the thinking processes used to create such products can be made visible.

The Internet is often used by teachers to support mathematics teaching but there is still limited information about Web-based pedagogies. The Internet provides potential learning opportunities for engaging students in meaningful mathematical exercises and investigations. Loong (2009) argued that students enjoyed working on the Internet as it provided a motivating alternative pedagogy in mathematics. Over half the students in Loong's study felt that they learned mathematics faster and that the Internet helped them to better understand the mathematical concepts. However, Loong's study also showed that teachers often experience practical problems in using Web-based mathematical interactive exercises such as slow download times, logging on problems, and management issues.

Clearly, the findings in this section show the importance of teachers knowing about, and using a range of pedagogical actions in teaching mathematics. These include teachers listening to student reasoning, knowing what models and explanations support learning, and understanding students' conceptions and responding appropriately. Learning trajectories or growth points and formative assessment are tools which support teachers understanding of student reasoning and of how to customise instruction for individuals. The use of technology was considered, in its role as a pedagogical tool. This highlighted the challenges encountered by teachers in making sure that technology is used in ways which maintain focus on the learning process and not just the product. Research which examines technology as a pedagogical tool is still in its infancy and has potential for a lot more research.

CONCLUSION

This chapter has reviewed Australasian research that provided insight about powerful pedagogical actions in mathematics education. The research centred around three themes related to powerful pedagogical actions: (a) creating powerful learning environments; (b) selecting tasks and models that promote deep learning; and (c) knowing and using pedagogical knowledge. Most studies focused on particular aspects of these themes, but together they provide insight about the type of pedagogical actions that enable successful mathematics learning for all students.

The research has contributed to our knowledge of the powerful pedagogical actions that effective teachers draw on to create positive mathematical learning communities for all students. A number of studies addressed effective pedagogy for teaching mathematics in diverse classrooms. Such studies, in contrast to taking a deficit view, direct attention to how student learning opportunities and engagement increase when teachers use socially and culturally responsive pedagogical actions. Studies include utilising students' social relationships, cultural understandings, and home languages, within relevant and engaging curricula. These studies open a pathway for Australasian and international researchers to listen to, and work with, Indigenous communities in developing powerful pedagogical actions for all. Other studies reviewed for this section of the chapter extend previous international research on teachers scaffolding group interactions and engaging students in productive discourse. Particular focus was placed on student development of mathematical inquiry and argumentation: core mathematical discourse which is used to provoke students to provide mathematical explanations, justification, representations, and make connections and generalisation across the differing domains of mathematics. Further research about both the impact of using these pedagogical actions for learning, and the type of professional learning approaches that enable teachers to use these pedagogical actions is warranted.

Teachers demonstrate their pedagogical knowledge through their selection of rich tasks and models that cater for multiple entry points and provide a significant context for students to develop rich mathematical concepts, mathematical discourse, and mathematical practices. A hallmark of the reviewed studies was the need for tasks to be sufficiently problem-based to provoke collaborative and interactive group work within extended dialogue premised in the discourse of inquiry and argumentation. Studies of problems, tasks, and models suggest that these need to: (a) involve visualising, modelling, and practical experiences; (b) be challenging enough to extend learning; and (c) be engaging and authentic for students. Some studies reinforce the need for well thought through pedagogical planning to ensure that the richness of the conceptual aspects of the problems and models are realised. It is clear from the research that teachers need to hold not only adequate mathematical content knowledge but also knowledge of their students, and specialised domain knowledge (e.g., fractional knowledge) to ensure that their students are able to capitalise on the richness of problems and tasks. Studies of pedagogical planning with problems and models and their conversion to rich conceptual contexts were restricted to only a few mathematical concepts. This suggests an area that would benefit from a synthesis of current international research and extension of research to include a broader range of mathematics concepts.

Sound teacher knowledge holds an important role in many of the reviewed studies. But teacher knowledge not only includes content knowledge; it also includes pedagogical content knowledge. Pedagogical knowledge underwrites the actions teachers take to determine students' current mathematical conceptions and misconceptions, plan, select, and customise tasks and instructions, and make appropriate responses to teachable moments during lessons. Although this knowledge shaped teachers' pedagogical actions, it was the focus of few studies. Profitable future research might include how this knowledge develops, and any discontinuance between teachers' pedagogical actions and their pedagogical content knowledge. Some studies suggested the potential role that learning trajectories and growth point frameworks hold in assessing student learning and shaping teachers' pedagogical knowledge. However, there was little evidence of synthesis of findings across mathematical domains to determine, for example, whether there is sufficient elaboration of learning trajectories for all domains. Further, the studies reviewed focused more on determining the mathematics knowledge of primary school students rather than secondary school students or tertiary students, so there is potential to extend research further in this area.

Technology, in its role as a pedagogical tool, holds significant possibilities for future research. It is clear from the reviewed studies that teachers play a critical role in determining the way that technology is used in mathematics classrooms. Studies showed the significance of ICT as a motivational tool and how it can be a spring-board to prompt mathematical learning during all phases of mathematical activity. However, caution needs to be used because there are indications that without expert pedagogical actions students may only attend to surface features of the mathematics rather than making connections across broader understandings. Similarly, teachers influence the role IWB hold in a classroom as a pedagogical tool. Studies note their motivating features but recognise the potential these have to default to teachers using more transmission modes of teaching. These studies point to a need for the research community to further explore the affordances of these tools and determine the specific strategies which incorporate all the different forms of technology within effective pedagogical practice.

The studies we reviewed establish the critical role teachers play in creating positive learning environments through using powerful forms of pedagogy. The diverse range of research studies combine to offer an emerging view of successful pedagogy in mathematics education. In this review there is an advance in research which involves collaborative partnerships with researchers, teachers, and other members of the community and this opens windows for further exploration of how culturally responsive pedagogy can powerfully enhance learning for all students. The many papers reviewed in this chapter indicate how seriously Australasian research is focused on exploring this area of mathematics education. Given that the themes in this current review of the literature extend those outlined in the previous 2003–2007 MERGA review, we feel justified in suggesting that Australasian research may take the lead in the exploration of effective pedagogy internationally.

REFERENCES

Afamasaga-Fuata'i, K. (2009). Innovative problem solving and students' mathematics attitudes. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 33–39). Palmerston North, NZ: MERGA.

Anthony, G, & Ding, L. (2011). Instructional coherence: A case study of lessons on linear equations. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 40–49). Adelaide, SA: AAMT & MERGA.

Anthony, G., & Hunter, J. (2008). Developing algebraic generalisation strategies. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 65–72). Morelia, Mexico: Cinvestav-UMSNH.

Anthony. G., & Walshaw, M. (2009). *Effective pedagogy in mathematics*. Belley, France: International Academy of Education (IAE).

Askew, M. (2011). Unscripted maths: Emergence and improvisation. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 56–67). Adelaide, SA: AAMT & MERGA.

Attard, C. (2011). The influence of teachers on student engagement with mathematics during the middle years. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 68–74). Adelaide, SA: AAMT & MERGA.

Averill, R. (2011). Teaching practices for effective teacher student relationships in multiethnic mathematics classrooms. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 75–81). Adelaide, SA: AAMT & MERGA.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83–104). Westport, CT: Ablex.

Barton, B. (2009). Being mathematical, holding mathematics: Further steps in mathematical knowledge for teachers. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 3–9). Palmerston North, NZ: MERGA.

Bautista, D. (2011). Children solving word problems in an imported language: An intervention study. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research

Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 98–106). Adelaide, SA: AAMT & MERGA.

Bautista, D., Mitchelmore, M., & Mulligan, J. (2009). Factors influencing Filipino children's solutions to addition and subtraction problems. *Educational Psychology, 29*, 729–745.

Bautista, D., & Mulligan, J. (2010). Why do disadvantaged Filipino children find word problems in English difficult? In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 69–76). Fremantle, WA: MERGA.

Bautista, D., Mulligan, J., & Mitchelmore, M., (2009). Young Filipino students make sense of arithmetic word problems in English. *Journal of Science and Mathematics Education in Southeast Asia, 32*, 131–160.

Beswick, K., & Muir, T. (2011). Interactive whiteboards as potential catalysts of pedagogic change in secondary mathematics teaching. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 115–123). Adelaide, SA: AAMT & MERGA.

Brown, R. A. J., & Renshaw, P. D. (2000). Collective argumentation: A sociocultural approach to reframing classroom teaching and learning. In H. Cowie and G. van der Aalsvoort (Eds.), *Social interaction in learning and instruction: The meaning of discourse for the construction of knowledge* (pp. 52–66). Amsterdam, The Netherlands: Pergamon Press.

Brown, R., & Reeves, B. (2009). Students' recollections of participating in collective argumentation when doing mathematics. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 73–80). Palmerston North, NZ: MERGA.

Cavanagh, M. (2008). One secondary teacher's use of problem-solving teaching approaches. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions.* (Proceedings of the 31st annual conference of the Mathematical Research Group of Australasia, pp. 117–124). Brisbane, QLD: MERGA.

Cavanagh, M., & Mitchelmore, M. (2011). Teaching secondary mathematics with an online learning system: Three teachers' experiences. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 158–165). Adelaide, SA: AAMT & MERGA.

Cheeseman, K., McDonough, A., & Clarke, D. (2011). Investigating children's understanding of the measurement of mass. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 174–182). Adelaide, SA: AAMT & MERGA.

Chick, H. (2007). Teaching and learning by example. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice*: Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 3–21). Adelaide, SA: MERGA.

Chick, H., & Pierce, R. (2008). Issues associated with using examples in teaching statistics. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 321–328). Morelia, Mexico: Cinvestav-UMSNH.

Clarke, D. M., Roche, A., & Mitchell, A. (2007). Year six fraction understanding: A part of the whole story. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 207–216). Adelaide, SA: MERGA.

Cooper, T. J., & Warren, E., (2008). The effect of different representations on years 3 to 5 students ability to generalise. *Zentralblatt fur Didaktik der Mathematik, 40*(1), 23–37.

Dindyal, J., Leong, Y., Toh, T. L., Quek, K. S., Leong, Y. H., & Tay, E. G. (2009). Reconceptualising problem solving in the school curriculum. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 700–703). Palmerston North, NZ: MERGA.

Downton, A. (2010). Challenging multiplicative problems can elicit sophisticated strategies. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 169–176). Fremantle, WA: MERGA.

Edmonds-Wathen, C. (2011). Locating the learner: Indigenous language and mathematics education. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 217–225). Adelaide, SA: AAMT & MERGA.

Gardner, H. (1999). *The disciplined mind: What all students should understand*. New York, NY: Simon Schuster.

Geiger, V. (2009). The master, servant, partner, extension-of-self framework in individual, small group and whole class contexts. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 201–208). Palmerston North, NZ: MERGA.

Geiger, V., Dole, S., & Goos, M. (2011). In The role of digital technologies in numeracy. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education, pp. 385–392). Ankara, Turkey: PME.

Geiger, V., Faragher, R., & Goos, M. (2010). CAS-enabled technologies as 'agents provocateurs' in teaching and learning mathematical modelling in secondary school classrooms. *Mathematics Education Research Journal, 22*(2), 48–68.

Geiger, V., Faragher, R., Redmond, T., & Lowe, J. (2008). CAS enabled devices as provocative agents in the process of mathematical modelling. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematical Education Research Group of Australasia, pp. 219–226). Brisbane, QLD: MERGA.

Gervasoni, A. (2011). Exploring the whole number knowledge of children in grade 1 to grade 4: Insights and implications. In T. Dooley, D. Corcoran, & M. Ryan (Eds.), *Mathematics teaching matters* (Proceedings of the 4th conference on Research in Mathematics Education, pp. 168–178). Dublin, Ireland: St Patrick's College, Drumcondra.

Gervasoni, A., Hart, A., Croswell, M., Hodges & Parish, L. (2011). Insights from Aboriginal teaching assistants about the impact of the Bridging the Numeracy Gap Project in a Kimberley Catholic School. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 306–314). Adelaide, SA: AAMT & MERGA.

Gervasoni, A., Parish, L., Hadden, T., Turkenburg, K., Bevan, K., Livesey, C., & Croswell, M. (2011). Insights about children's understanding of 2-digit and 3-digit numbers. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 315–323). Adelaide, SA: AAMT & MERGA.

Grootenboer, P. (2009). Rich mathematical tasks in the *Maths in the Kimberley* (MITK) Project. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 696–699). Palmerston North, NZ: MERGA.

Hartnett, J. (2008). Capturing students' thinking about strategies used to solve mental computations by giving students access to a pedagogical framework. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematical Research Group of Australasia, pp. 251–257). Brisbane, QLD: MERGA.

Helme, S. & Teese, R. (2011). How inclusive is year 12 mathematics? In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 349–359). Adelaide, SA: AAMT & MERGA.

Highfield, K., & Mulligan, J. (2009). Young children's embodied action in problem-solving tasks using robotic toys. In M. Tzekaki, M. Kaldrimidou, & Sakonidis (Eds.) *In search for theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 169–176). Thessaloniki, Greece: PME.

Hunter, J. (2009). Developing a productive discourse community in the mathematics classroom. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 249–256). Palmerston North, NZ: MERGA.

Hunter, J. (2010). 'You might say you're 9 years old but you're actually B years old because you're always getting older': Facilitating young students' understanding of variables. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 256–263). Fremantle, WA: MERGA.

Hunter, J., & Anthony, G. (2008). The development of students' use of justification strategies. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions.* (Proceedings of the 31st annual conference of the Mathematical Research Group of Australasia, pp. 265–271). Brisbane, QLD: MERGA.

Hunter, R. (2008a). Facilitating communities of mathematical inquiry. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematical Research Group of Australasia, pp. 31–39). Brisbane, QLD: MERGA.

Hunter, R. (2008b). Do they know what to ask and why? Teachers shifting student questioning from explaining to justifying and generalising reasoning. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 201–208). Morelia, Mexico: Cinvestav-UMSNH.

Hunter, R. (2010). Coming to 'know' mathematics through 'acting, talking and doing' mathematics. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 264–27). Fremantle, WA: MERGA.

Hunter, R. K., & Anthony, G. (2011). Forging mathematical relationships in inquiry-based classrooms with Pasifika students. *Journal of Urban Mathematics Education*, *4*(1), 98–119.

Hurst, C., Armstrong, T., & Young, M. (2011). Making a difference for indigenous children. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 373–381). Adelaide, SA: AAMT & MERGA.

Jackson, F., & Brown, R. (2009). Exploring whether multiple intelligences facilitate 'valuing and working with difference' within mathematics classrooms. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 257–264). Palmerston North, NZ: MERGA.

Kamol, N., & Yeap, B. H. (2010). Upper primary school students' algebraic thinking. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 289–296). Fremantle, WA: MERGA.

Kaur, B., & Ghan, M. (2011). Learning experiences of Singapore's low attainers in primary mathematics. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research

Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 414–420). Adelaide, SA: AAMT & MERGA.

Keng, N. K., & Kian, L. H. (2010). Learning mathematics concepts through authentic learning. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 305–311). Fremantle, WA: MERGA.

Loong, E. Y-K. (2009). Using Web-based mathematical interactive exercises and exploratory investigations: The possibilities and pitfalls. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 339–346). Palmerston North, NZ: MERGA.

Makar, K. (2011). Learning over time: Pedagogical change in teaching mathematical inquiry. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 2–37). Adelaide, SA: AAMT & MERGA.

Marshman, M., Pendergast, D., & Brimmer, F. (2011). Engaging the middle years in mathematics. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 500–507). Adelaide, SA: AAMT & MERGA.

Mason, J., Drury, H., & Bills, L. (2007). Studies in the zone of proximal awareness. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice*: Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 42–58). Adelaide, SA: MERGA.

MacDonald, A. (2011). Young children's representations of their developing measurement understandings. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 484–490). Adelaide, SA: AAMT & MERGA.

McDonald, S., Warren, E., & de Vries, E. (2011). Plotting the use of oral language and rich mathematical representations of teachers in the early years of formal schooling. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 209–216). Ankara, Turkey: PME.

Mitchell, A., & Horne, M. (2011). Listening to children's explanations of fraction pair tasks: When more than an answer and an initial explanation are needed. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 515–522). Adelaide, SA: AAMT & MERGA.

Muir, T. (2008). 'Zero is not a number': Teachable moments and their role in effective teaching of numeracy. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematical Research Group of Australasia, pp. 361–376). Brisbane, QLD: MERGA.

Muir, T. (2009). Investigating teachers' use of questions in the mathematics classroom. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.) *In search for theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 161–168). Thessaloniki, Greece: PME.

Niesche, R. (2009). The use of home language in the mathematics classroom. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 704–707). Palmerston North, NZ: MERGA.

O'Shea, H., & Peled, I. (2009). The task types and mathematics learning research project. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 2–5). Palmerston North, NZ: MERGA.

Parish, L. (2010). Facilitating the development of proportional reasoning through teaching ratio. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 469–476). Fremantle, WA: MERGA.

Redmond, T., & Sheehy, J. (2009). Reconceptualising agency in a senior mathematics classroom. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 451–458). Palmerston North, NZ: MERGA.

Redmond, T., Sheehy, J., & Brown, R. (2010). Exploring the relationship between mathematical modelling and classroom discourse. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 485–492). Fremantle, WA: MERGA.

Roche, A., & Clarke, D. (2009). Making sense of partitive and quotitive division: A snapshot of teachers' pedagogical content knowledge. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 467–474). Palmerston North, NZ: MERGA.

Roche, A., & Clarke, D. (2011). Some lessons learned from the experience of assessing teacher pedagogical content knowledge in mathematics. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 658–666). Adelaide, SA: AAMT & MERGA.

Sawyer, A. (2008). Making connections: Promoting connectedness in early mathematics education. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematical Research Group of Australasia, pp. 429–435). Brisbane, QLD: MERGA.

Scott, A., Downton, A., Gronn, D., & Staples, A. (2008). Engagement versus deep mathematical understanding: An early career teacher's use of ICT in a lesson. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematical Research Group of Australasia, pp. 437–444). Brisbane, QLD: MERGA.

Serow, P. (2008). Investigating a phase approach to using technology as a teaching tool. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions.* (Proceedings of the 31st annual conference of the Mathematical Research Group of Australasia, pp. 445–452). Brisbane, QLD: MERGA.

Serow, P., & Callingham, R. (2008). The introduction of interactive whiteboard technology in the primary mathematics classroom: Three case studies. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematical Research Group of Australasia, pp. 453–459). Brisbane, QLD: MERGA.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, (15)*2, 4–14.

Stephens, M., & Armanto, D. (2010). How to build powerful learning trajectories for relational thinking in the primary school years. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 523–530). Fremantle, WA: MERGA.

Sullivan, P. (2008). Developing mathematical connections and fostering procedural fluency: Are they in tension? In O. Figueras, J.L. Cortina, S. Alatorre, T. Rojano & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 305–312). Morelia, Mexico: Cinvestav-UMSNH.

Sullivan, P., Clarke, D., & Clarke, B. (2009). Converting mathematics tasks to learning opportunities: An important aspect of knowledge for mathematics teaching. *Mathematics Education Research Journal, 21*(1), 85–105.

Sullivan, P., Clarke, D., Clarke, B., & O'Shea, H. (2009). Exploring the relationship between tasks, teacher actions, and student learning. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.)

In search for theories in mathematics education (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 185–192). Thessaloniki, Greece: PME.

Tanner, H., Jones, S., Beauchamp, G., & Kennewell, S. (2010). Interactive whiteboards and all that jazz: Analysing classroom activity with interactive technologies. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 457–554). Fremantle, WA: MERGA.

Warren, E., & Cooper, T. (2008). Generalising the pattern rule for visual growth patterns: Actions that support 8 year olds' thinking. *Educational Studies in Mathematics, 67*, 171–185.

Warren, E. & Cooper, T. (2009). Developing mathematics understanding and abstraction: The case of equivalence in the elementary years. *Mathematics Education Research Journal, 21*(2), 76–95.

Watson, A. (2008, March). *School mathematics as a special kind of mathematics.* Paper presented to Working Group 1 at the Symposium on the Occasion of the 100th Anniversary of ICMI. Rome, Italy. Retrieved from http://www.unige.ch/math/EnsMath/Rome2008/partWG1.html

Watson, J., Callingham, R., & Donne, J. (2008). Proportional reasoning: Student knowledge and teachers' pedagogical content knowledge. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematical Research Group of Australasia, pp. 563–571). Brisbane, QLD: MERGA.

Watson, J., Callingham, R., & Nathan, E. (2009). Probing teachers' pedagogical content knowledge in statistics: 'How will Tom get to school tomorrow?' In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 563–570). Palmerston North, NZ: MERGA.

Williams, G. (2008a). How group composition can influence opportunities for spontaneous learning. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematical Research Group of Australasia, pp. 581–588). Brisbane, QLD: MERGA

Williams, G. (2008b). Group composition: Influences of optimism and lack thereof. In O. Figueras, J.L. Cortina, S. Alatorre, T. Rojano & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 425–432). Morelia, Mexico: Cinvestav-UMSNH.

Williams, G. (2011). Queries without hints, affirming, or 'telling', that sustained spontaneous problem solving activity. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 795–803). Adelaide, SA: AAMT & MERGA.

Windsor, W. (2010). Algebraic thinking: A problem solving approach. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 665–672). Fremantle, WA: MERGA.

Wong, M. (2010). Equivalent fractions: Developing a pathway of students' acquisition of knowledge and understanding. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 673–680). Fremantle, WA: MERGA.

Yeh, A. (2010). Three primary school students' cognition about 3D rotation in a virtual reality learning environment. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 609–697). Fremantle, WA: MERGA.

Young-Loveridge, J., & Mills, J. (2009). Teaching multi-digit multiplication using array-based materials. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 635–642). Palmerston North, NZ: MERGA.

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MATHEMATICS CURRICULUM IN THE SCHOOLING YEARS

Key words: curriculum development; learning trajectories; numeracy; primary; secondary.

INTRODUCTION

The term 'curriculum' is used to describe the intended curriculum, the implemented curriculum or the attained curriculum. While the intended curriculum refers to the written or prescribed curriculum, the implemented curriculum refers to curriculum as teachers enact it in classrooms. Consequently, it is difficult to separate the enacted curriculum from teachers' pedagogical practice. Nevertheless, it is both worthwhile and timely to focus on research that specifically addresses curriculum with the recent development of the Australian curriculum for mathematics in the schooling years (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2010a; 2010b). Part of the intent of this chapter is to describe the background and processes used to develop this curriculum as well as point to recent Australasian research which informed its development. We anticipate—as do the Editors of the Review—that increased attention will be given to issues that surround curriculum in the coming years as classroom practitioners and researchers describe the changed *practices* that will inevitably occur with the introduction of a new curriculum. We acknowledge that New Zealand's national curriculum is not in such a transitional phase although the new curriculum has only been in full implementation since 2010.

In this chapter, we focus specifically on curriculum as the plan of learning about mathematics content to be taught in schools, or the intended curriculum. Our definition of curriculum is closely aligned to that of Clements (2007, p. 36) who defined curriculum as "a specific set of instructional materials that order content used to support pre-K to grade 12 classrooms". Additionally, our definition is broadened, particularly when referring to numeracy, to include workplaces as well as schools. Also reviewed here are recommendations for the school mathematics curriculum from reports into mathematics and numeracy, recent research into the development or review of school mathematics curriculum, as well as curriculum documents and resources such as syllabus and textbooks designed to interpret and map curriculum into plans for teachers. We conclude with a section on research into teachers' use of, and reactions to, new curriculum documents. Thus, this chapter not only serves as a review of recent Australasian research in this area, it also provides a contextual framework for forthcoming research agendas and directions.

B. Perry et al. (Eds.), Research in Mathematics Education in Australasia 2008–2011, 219–244.

SCHOOL MATHEMATICS AND NUMERACY

Some recent research specifically focuses on mathematics in the curriculum while other studies and reports refer to numeracy in the school curriculum. Both are included here with the terminology adopted from the authors of each paper. For clarification, we use the Australian Association of Mathematics Teachers (AAMT) (1997, p. 15) position describing numeracy as involving:

> ... the disposition to use, in context, a combination of: underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic); mathematical thinking and strategies; general thinking skills; [and] grounded appreciation of context.

Further, the AAMT (1997) statement indicates, "numeracy is not a synonym for mathematics, but the two are clearly interrelated. All numeracy is underpinned by some mathematics; hence school mathematics has an important role in the development of young people's numeracy" (p. 11).

The natural question which arises is what balance should exist in the mathematics curriculum between mathematics classified as numeracy and higher order mathematics. Ernest (cited in Sullivan, 2011) argued that, while aspects of specialised mathematical knowledge can be introduced, the emphasis in the school curriculum for the compulsory years should be on practical mathematics. Ainley, Kos, and Nicholas (2008) pointed out that fewer than 0.5 per cent of university graduates specialise in mathematics, and only around 40% of graduates are professional users of mathematics, while all school students need practical mathematics to function in society. The consensus seems to be that the priority in the compulsory years of schooling (in Australia this is to the end of Year 10) should be numeracy.

The National Numeracy Review

In the Australian context, the most recent comprehensive review of research-based evidence about good practice for promoting the learning of mathematics was the *National Numeracy Review Report* (Council of Australian Governments [COAG], 2008). The *Report* reemphasised the importance of a strong mathematics curriculum. This position was reinforced in the *Melbourne Declaration* (Ministerial Council on Education Employment, Training and Youth Affairs [MCEETYA], 2008) where literacy and numeracy were described as the cornerstones of schooling for young Australians. The *Report* refers to studies of workplace numeracy demands suggesting that from adult and workplace perspectives, the school curriculum should provide:

– a deep understanding of the real number system and its links with the metric system of measurement so that this knowledge is embodied rather than a series of disconnected and often incorrectly recalled facts;
– similar understandings for statistical, geometrical and algebraic thinking;
– experience grounded in practical situations of making contextualised judgements about levels of accuracy, reasonableness of answers, and when to approximate;

– experience in the use of artefacts—e.g., charts, tables, electronic databases, internet support, as well as working interactively and creatively with spreadsheets;
– experience working in inter-disciplinary (or inter domain) project teams to incorporate the range of generic competencies (e.g., problem solving, working in teams, communication, technology skills); and
– opportunities to work within realistic constraints.

Debates about the role of numeracy in the school curriculum and the role of mathematics teachers in developing students' numeracy abound. As noted by Thornton (2009), various stakeholders have competing positions, which highlights the difficulty in constructing mathematics curriculum. He used a textual analysis of two responses to the *National Numeracy Review Report* (COAG, 2008) prepared by the mathematics community and the mathematics education research community concluding that knowledge within these disciplines is based on different epistemic devices, and hence, debates surrounding mathematics education arise, at least in part, from differing ways of viewing how mathematical knowledge is constructed. Sullivan (2011) noted the ongoing debate within the Australian community on which aspects of mathematics were important, and which aspects were most needed by school leavers. He cited conventional discipline-based learning with practical perspectives on one side as opposed to an emphasis on specifically mathematical issues on the other.

Numeracy in the Workplace

Sullivan (2011) observed that teachers need to consider, among other things, preparing young people for the demands of employment and the general demands of adult life. The numeracy demands of work-readiness can inform the content of school curricula, and teaching approaches adopted. Sullivan and Jorgensen (2009), for example, reported cases in which students saw contextualised tasks as relevant, and therefore worthy of the effort needed to learn the numeracy.

In a large project investigating the mathematical practices in contemporary work places, Jorgensen and Zevenbergen (2011) determined considerable differences between the ways young people approach tasks compared to their older counterparts in 19 different workplace contexts. In particular, "technology has shifted the emphasis on what and how numeracy activities are undertaken" (Zevenbergen & Zevenbergen, 2004, p. 605). By examining the practices of young people in a range of contemporary workplaces including bricklayer, boat builder, hairdresser, motor mechanic, retail assistant, chef and others, they observed estimation and problem solving were an integral component of practices in all sites. The authors noted:

> The results of this study suggest that younger people often approach their work in unique ways that are often different from those taught and learned in school mathematics. They are more likely to approach tasks holistically, to use estimation, to problem solve, to use technological tools to support their work and thinking, to use intuitive methods, and to see tasks aesthetically. (Zevenbergen & Zevenbergen, 2004, p. 611)

Jorgensen and Zevenbergen (2011) described young workers in the retail industry as having different dispositions and new ways of working compared to their older counterparts. They are less likely to perform mental calculations but more likely to estimate and problem solve.

If one of the purposes of the school mathematics curriculum is to prepare students for life and work beyond school, then the curriculum needs to provide pathways to support effective transition to further study or the workforce. To support these new ways of working, the school mathematics curriculum may need to shift to 'new numeracies' (Zevenbergen & Zevenbergen, 2004) paralleling a similar change in thinking by literacy researchers, educators and curriculum developers (e.g., Freebody, 2007). This advice has implications for the curriculum in both the junior secondary as well as the senior secondary years of schooling when mathematics may not be a compulsory subject. For students who do not plan to continue to university study, completing a numeracy course in the senior years of schooling may be highly desirable. The possibilities for numeracy in the curriculum will be further investigated in the section on the development of the new Australian curriculum for mathematics.

Mathematics in the Senior Years of Schooling

The importance of the school mathematics curriculum is emphasised in the *National Numeracy Review Report* (COAG, 2008), particularly in relation to transitions from school to tertiary study. The *Report* notes the best transitions are achieved by those with high levels of mathematics (around 95% have good transitions), ahead of those with high levels of English (92%)—the higher the level the lower the chance of unemployment. Yet, since 1995 participation in Year 12 higher-level mathematics courses has fallen dramatically and recent reports suggest the rate continues to fall (Forgasz, 2006). The *Report* points to the fact that universities have dropped higher-level mathematics as a prerequisite for many courses and that because the curriculum in these courses is found to be difficult, there is often a lack of appropriately qualified teachers and adequate rewards for students for taking higher-level mathematics courses.

Ho (2010) reported on one strategy to address concerns about the dwindling demand for Advanced Mathematics in Western Australia where five schools collaborated to provide an otherwise unavailable opportunity for their students to study at this level. The results were that a small number of Year 11 students in these schools (18 out of about 1,000 in total) engaged with the course and enjoyed the experience. The main obstacle was transport to the classes and the feeling by the students that the transition from Year 10 to Year 11 was "too big for most to handle".

Another informative report resulted from considering why capable students were not choosing to take higher-level mathematics in the senior years of schooling. The *Maths? Why Not?* project (McPhan, Morony, Pegg, Cooksey & Lynch, 2008) drew principally on the perceptions of mathematics teachers and career advisers through online surveys, supplemented by student surveys and focus group discussions with students and mathematics teachers. Results found the four most important influences affecting students' engagement with mathematics were:

– self-perception of ability;
– interest and liking of mathematics;
– previous achievement in mathematics; and
– perceived difficulty of mathematics.

Recommendations for the curriculum to address these student-related influences included the need to:

– research problematic components of the curriculum and teaching;
– develop 'second-chance' programs that offer junior secondary students opportunities to consolidate their understanding at critical developmental points in their learning;
– develop learning units that explore and illuminate links between careers and mathematics; and
– establish incentives to encourage mathematics graduates into primary and secondary mathematics teaching.

Currently researchers at the Centre for Science, Information and Communication Technology and Mathematics Education in Rural and Regional Australia (SiMERR) based at the University of New England, Armidale, New South Wales, are developing a *Second Chance Algebra Learning Environment* (SCALE) to address some of the recommendations noted above (SiMERR, 2011).

Mathematics across the Curriculum

Another issue raised in the *Report* (McPhan et al., 2008) suggested that while mathematics can be taught in the context of mathematics lessons, teachers of subjects other than mathematics need to understand the mathematical demands of their subject area and address these in their teaching (Goos, Geiger, & Dole, 2011). The report recommended an across the curriculum commitment to mathematics and/or numeracy. Goos, Geiger, and Dole (2010) explored the nature of an across the curriculum commitment by conducting a numeracy audit of the published curriculum framework in South Australia (Department of Education and Children's Services, 2005), particularly focusing on the middle-years (Years 6 to 9). Recognising the "rapidly evolving nature of knowledge, work, and technology" (Goos et al., 2010, p. 211), they used the numeracy model shown in Figure 1 that incorporates mathematical knowledge, dispositions, tools, contexts, and a critical orientation to represent numeracy in the 21st century.

All learning areas in the intended curriculum were found to have distinctive numeracy demands in relation to each of the factors in the numeracy model. The audit was seen as an encouragement to teachers to promote numeracy in even richer ways in the curriculum they enact with students. However, encouraging and supporting all teachers to embed numeracy experiences in their lessons, particularly in the secondary school context, can be a real challenge and has implications for teacher education.

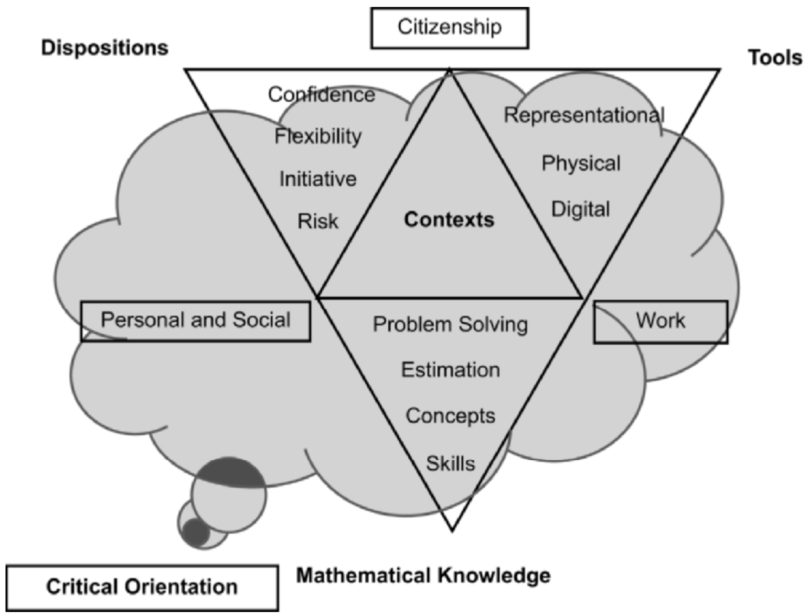

Figure 1. Numeracy model used for the audit.

Sullivan (2011) differentiated the challenges for primary teachers, who in Australian schools teach all subjects to their class, and secondary teachers who are specialists. For primary, he claimed it is mainly a matter of finding potential links across different curriculum areas. However, for secondary teachers, Sullivan argued that incorporating numeracy perspectives into subjects other than mathematics is more of a challenge for two reasons. First, teachers of other curriculum areas are sometimes not convinced a stronger focus on the numeracy aspects of their subject is necessary or helpful. Second, for many, their own mathematical knowledge and ability is inadequate to the task. Hence, if teachers of all subjects are to be expected to incorporate appropriate mathematics in their lessons, they are going to need adequate preparation.

White and Cranitch (2010) examined the impact of a unit called “Curriculum Literacies” in the final year of a Secondary Bachelor of Teaching/Bachelor of Arts course. The unit developed students’ personal skills and understanding of literacy and numeracy and their application to teaching in particular discipline areas. Findings showed the unit had positive effects on most students’ personal knowledge and pedagogy. However, these effects varied depending on content area with literacy generally viewed as more relevant and more easily integrated into lesson plans than numeracy.

This section has focused on two large-scale reports of school mathematics and numeracy, as well as the issues around the senior secondary curriculum and the notion of mathematics across the curriculum. The recommendations from the reports inform curriculum development in Australia but there have also been curriculum reforms in New Zealand, providing valuable insights into the development of the first national curriculum for Australian schools, which will be considered in the next section.

DEVELOPMENT OF CURRICULUM: NEW ZEALAND AND AUSTRALIA

Recent curriculum review and development has occurred in both New Zealand and Australia. The latest New Zealand curriculum was launched in 2007 after several years of review and development. The first national Australian curriculum for mathematics was released in December, 2010 for trial and review during 2011 with substantial implementation up to Year 10 to be completed in all states and territories by 2013.

New Zealand

The new national curriculum in New Zealand aims to address the sociocultural needs of its students by developing a vision and a set of principles for the whole curriculum, which acknowledge cultural diversity and inclusion. Cowie et al. (2009, p. 1) describe this curriculum development as different to an "earlier period of 'rolling revision' ... where curriculum was revised subject-by-subject with a haphazard timeline". The latest curriculum was developed over several years to accommodate extensive consultation and revision with publication in 2007 for full implementation in 2010.

The curriculum articulates a vision of "young people who will be confident, connected, actively involved, lifelong learners" (Ministry of Education, 2007a, p. 7). Associated with this vision are values, key competencies, learning areas and a set of principles which included high expectations, the principles of the Treaty of Waitangi, cultural diversity, inclusion, learning to learn, community engagement, coherence, and future focus. The mathematics and statistics learning area is structured into three strands of Number and Algebra, Geometry and Measurement, and Statistics. While there is no process strand, at the beginning of each level, the following statement in the mathematics achievement objectives clearly indicates the intention of providing such experiences for students:

> In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations ... (Ministry of Education, 2007b, p. 19).

Schools were encouraged to begin early implementation with case studies conducted to identify successful experiences (Cowie et al., 2009). Using document analysis, interviews and observations, seven common themes were identified across the case-study schools although all themes revealed enablers, constraints and tensions as teachers grappled with early adoption. The themes included starting somewhere, understanding the curriculum and how to implement it, school leaders as lead learners, the processes of change, changing pedagogy, engaging the community (including student voice), and aligning structures and supports. While further research has yet to be reported on the most recent case studies, Cowie et al. (2009, p. 49) suggested the need "to consider a series of *levels of action* within an *improvement infrastructure*" with two dimensions of whole-school and school/individual teacher where personal understandings of the curriculum become enacted in classrooms. The report does not examine the particular implementation of the mathematics achievement objectives.

Prior to the development of the latest curriculum in New Zealand, and as a result of their *Literacy and Numeracy Strategy*, curriculum reform was adopted through the implementation of *Numeracy Development Projects*, which began in 1999 to improve primary mathematics education (Higgins & Parsons, 2009). The projects were introduced across the phases of schooling and were accompanied by ongoing research and evaluation. Fundamental to the projects is the Number Framework, which helped to inform the Mathematics and Statistics learning area of the new curriculum (Holton, 2010). Through her review of the decade of reform in mathematics education in New Zealand, Young-Loveridge (2010, p. 15) noted "the absolute levels on the Framework attained by students were in many cases well short of the numeracy expectations for students at particular year levels stated in *The New Zealand Curriculum*". She conceded further research is required to explore whether the expectations in the new curriculum are too high or whether more professional development for teachers is required.

In 2009, the Ministry released the *Mathematics Standards for Years 1–8* (Ministry of Education, 2009), which specify the expected outcomes for students at the end of each year of schooling. Substantial support has been provided for teachers to assist implementation of the curriculum. Hipkins (2011) reported on schools in the *Curriculum Implementation Exploratory Studies* (CIES) project. She described how patterns were identified in the way learning networks developed over time within schools and how an awareness of the dynamics in these networks could assist other school leaders in collaborative professional learning.

These experiences from New Zealand curriculum reform and development had the potential to inform curriculum development in the Australian context. However, since a very different approach was used, rather than begin with a vision and principles for the whole curriculum, the Australian Government determined the national curriculum development would begin with four subject areas including mathematics. This approach has not been without its critics.

Australia

Background to the first national curriculum for Australia. For the first time in the history of Australian education and under the political banner of an 'education revolution', a national school curriculum has been developed for the years of schooling from Foundation (the first year of compulsory, formal school education) to Year 12. Following a failed attempt to introduce a common Australian template of Statements and Profiles for mathematics in the early nineties, the Federal Labour government in Australia began the development of a national curriculum for English, Mathematics, Science and History in 2007 as a first phase of Australian curriculum development. Other learning areas have followed, accompanied by the introduction of a set of seven general capabilities and three cross-curriculum priorities.

Until these recent developments, responsibility for curriculum development has resided in each of the eight states and territories of Australia, indeed "curriculum governance in Australia is allocated by the Constitution to the Australian State governments" but the Commonwealth government has not sought a more active role in curriculum policy until now (Yates & Collins, 2010, p. 89). This has presented a unique opportunity for collaboration and commitment to sharing resources for the benefit of all students in all locations throughout Australia; an opportunity which has been met with overwhelming support from teachers, parents, and other stakeholders. It makes sense to develop a common, quality curriculum that prepares students for the 21st century, particularly for a country with a population of only 22.7 million people.

The Federal government offered two main reasons for developing a national curriculum. The first was to ensure uniformity and consistency across states and territories for children who move each year—this would address duplication and enable the sharing of resources. The second was aimed at addressing the variations in retention rates and student achievement between state jurisdictions (Reid, 2009). Both of these reasons are technical rather than considering the more important issues about our values and beliefs (Kennedy, 2009). However, *The Shape of the Australian Curriculum* paper (National Curriculum Board [NCB], 2009a) presents the goals of schooling from the *Melbourne Declaration* (MCEETYA, 2008) as informing the development of the curriculum documents in order to develop successful learners, confident individuals, and active and informed citizens.

Visions of the Australian curriculum included the notion of curriculum as a "verb" rather than simply as a "noun" (Reid, 2005, p.11) so that "one way of thinking about curriculum ... is as the regularly updated minutes of an ongoing public conversation about what it means to be an Australian in the 21st century" (Reid, 2005, p. 36). Further, Reid grounded the curriculum in teaching "THROUGH knowledge/content FOR capabilities, rather than the teaching of subjects" (p. 51). Kennedy (2009, p. 5) argued, "above all, the school curriculum is a cultural construction" and it "is about the collective—what is best for everyone" (p. 8). Both Reid and Kennedy suggested we should begin with the bigger picture of curriculum and debate fundamental questions of access and equity, as well as consider what knowledge is important and for whom. However, the Council of Australian Governments (COAG) determined the initial curriculum development would be for the subjects English, Mathematics, Science and History. This has led to the development of each of these subjects in concert with the development of the general capabilities and cross-curriculum perspectives.

The first Australian curriculum for mathematics. The process of creating the first national curriculum for mathematics by the NCB began with the development of a *Framing Paper for Mathematics* that was released in

November 2008 for consultation with stakeholders. Based on feedback, the *Shape of the Australian Curriculum: Mathematics* (NCB, 2009b) was written to guide the creation of the Australian mathematics curriculum. The design indicated that the curriculum would include content for each year of schooling, and achievement standards presenting a continuum of typical growth. The *Shape Paper* outlined the aims, key terms and structure of the new curriculum. The structure included three content strands: Number and Algebra; Measurement and Geometry; and Statistics and Probability as well as four proficiency strands: understanding; fluency; problem solving; and reasoning (informed by Kilpatrick, Swafford & Findell, 2001).

Several important considerations were presented in the *Shape Paper* (ACARA, 2010b): equity and opportunity; connections to other learning areas; breadth and depth of study; the role of digital technologies; the nature of the learner; general capabilities; and cross-curriculum perspectives. Of particular note for the key consideration of equity and opportunity was the "unintended effect of current classroom practice [that] has been to exclude some students from future mathematics study" (NCB, 2009b, p. 9).

Three effects were raised as challenges to be addressed in the development of the Australian curriculum for mathematics. First, to engage more learners with mathematics based on concerns about the "syndrome of shallow teaching" from the *TIMSS Video Study* (Hollingsworth, Lokan, & McCrae, 2003). Second, ensuring the inclusion of all groups so that there are options available to continue the study of mathematics for as long as possible, and that the differential achievements among particular groups of students are considered and addressed. From the PISA 2006 results, these differences related to socioeconomic status, geographical location and cultural background (particularly between non-Indigenous and Indigenous students). Third, the challenge of creating opportunity requires "a commitment to ensuring that all students experience the full mathematics curriculum until the end of Year 10" (NCB, 2009b, p. 10). This third effect challenges the practice of 'streaming' or offering an alternative, limited mathematics curriculum for groups of students considered not able to learn the more challenging content offered in Years 9 and 10 in some state-based curriculum documents.

Included in the *Shape Paper* was a brief overview of the possible mathematics curriculum for the senior secondary, or typically the post-compulsory years of schooling. The proposal was to develop four types of courses ranging from an "applied study of mathematics" to a course "intended for students with a strong interest in mathematics ... intending to study mathematics at university" (NCB, 2009b, p. 9). The draft mathematics curriculum for K-10 was released for consultation in May, 2009 and the draft curriculum for Years 11 and 12 was released in July, 2010. Also during this period, the NCB became the statutory body, the Australian Curriculum, Assessment and Reporting Authority (ACARA) responsible not only for curriculum but also for associated accountability processes.

Through an extensive consultation process with the full range of stakeholders, feedback on the draft curriculum documents for K to 10 yielded questions about content, uniformity across states and depth of treatment. Differences between states and territories have been highlighted during these discussions, particularly differences in starting ages and the first year of secondary education (either Year 7 or Year 8). For some states (Queensland, South Australia and Western Australia), Year 7 is the last year of primary school so generalist teachers teach the mathematics curriculum. In all other states and the territories, Year 7 is the first year of secondary school where a specialist teacher usually teaches mathematics although this is being seriously challenged with the current shortage of specialist mathematics teachers throughout Australia (AAMT, 2010a).

Professional associations including the Mathematics Education Research Group of Australasia (MERGA) and the Australian Association of Mathematics Teachers (AAMT) were involved in the consultation and providing written responses to inform the development of the final curriculum document. The *MERGA Response to Australian Curriculum (Mathematics)* (MERGA, 2010a) affirmed the vision described in the *Shape Paper* but indicated the vision had not been realised. Several recommendations were presented and supported by evidence from the research literature. While many concerns were raised, of note were the inadequate representation of the proficiency strands in the content descriptions, the need for further reduction of content to allow for more problem solving and modelling, the poor sequencing of some content, and the need to further consider current research into the content strands. Similar issues were raised in the response to the draft document by the AAMT (2010a).

The *Australian Curriculum: Mathematics* was released online in December 2010 with opportunities for trialling and review during 2011 (ACARA, 2010a). The final document presents mathematics for Foundation to Year 10. The ACARA website notes:

> The term Foundation Year has been used as a nationally consistent term for the year of schooling prior to Year 1 for the purpose of the Australian Curriculum. It does not replace the equivalent terms used in states and territories—Kindergarten (NSW/ACT), Prep (QLD/VIC/TAS), Pre-primary (WA), Reception (SA) and Transition (NT).

In addition to the content descriptions and achievement standards, the website includes a glossary of key terms and elaborations for each of the content descriptions to assist teachers with interpretation and planning. There is also a preamble, which sets the scene for the content and outlines the aims and rationale for the curriculum.

Presenting the curriculum online allows for ongoing review from trialling (see http://www.australiancurriculum.edu.au/Home). This capability goes someway to addressing Reid's (2005) vision for curriculum with the possibility of ongoing public conversations. In addition to this website, teachers will have access to a website through their state and territory jurisdictions where resources will be tagged to the appropriate content in the curriculum, allowing for national sharing and collaboration on a much larger scale than has occurred in the past.

The Australian mathematics curriculum document makes it clear that this is the intended curriculum; it does not prescribe how the content should be taught. The content descriptions "describe the knowledge, concepts, skills and processes that teachers are expected to teach and students are expected to learn. However, they do not prescribe approaches to teaching" (ACARA, 2010a, p. 3).

In the curriculum document there is evidence that issues raised during the consultation have been addressed with a continued strong equity commitment to all students learning the mathematics content until the end of Year 10. There has been a further reduction in the amount of content and a review of the sequencing of concepts within the three content strands, with the content descriptions grouped into sub-strands. In addition, the embedding of the proficiency strands has been revised with the use of more 'actions' through the use of verbs at the beginning of content statements. The definitions of each of the proficiencies (see Table 1) highlight the types of verbs used to represent the actions recommended.

Table 1. The definitions for each of the proficiencies (ACARA, 2010a, p. 3)

Understanding	Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the 'why' and the 'how' of mathematics …
Fluency	Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods …
Problem Solving	Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations …
Reasoning	Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached …

To highlight the embedding of the proficiencies in the content descriptions for each year level, the following statement is presented at the beginning of each page followed by examples of each of the proficiencies.

> The proficiency strands Understanding, Fluency, Problem Solving and Reasoning are an integral part of mathematics content across the three content strands: Number and Algebra, Measurement and Geometry, and Statistics and Probability. The proficiencies reinforce the significance of working mathematically within the content and describe how the content is explored or developed. They provide the language to build in the developmental aspects of the learning of mathematics. (ACARA, 2010a, p. 41)

With the release of the first Australian curriculum for mathematics, an information sheet described the international references that have been drawn upon in developing the curriculum. In particular, reference is made to the use of the Singapore curriculum for comparison.

> In comparison to the Singapore mathematics curriculum, the *Foundation to Year 10 Australian Curriculum: Mathematics* content is introduced more slowly in the early and primary years to ensure students have the opportunity to develop deep understanding before moving on. By Year 10, the conceptual difficulty is similar to that described in the Singapore documents. The *Foundation to Year 10 Australian Curriculum: Mathematics* also has greater emphasis than the Singapore mathematics curriculum on building depth of mathematical understanding and includes the use of a variety of digital technologies to enhance the teaching and learning of mathematics. The *Foundation to Year10 Australian Curriculum: Mathematics* facilitates a deep knowledge of statistics and probability and includes practical application of mathematics including financial literacy. (ACARA, 2010a, p. 41)

It is unclear whether a mapping exercise comparing where content is placed in the curriculum actually takes into account research about placement of content. Yet when a government promises a 'world class' curriculum, one way to demonstrate high standards and expectations is to compare the curriculum with a country which performs well on international comparative studies such as the Trends in Mathematics and Science Studies (TIMSS).

General capabilities and cross-curriculum priorities. Seven general capabilities have been identified for all students to "succeed in life and work in the twenty-first century" (ACARA, 2010a, p. 8). They are:

– literacy;
– numeracy;
– competence in information and communication technology (ICT);
– critical and creative thinking;
– ethical behaviour;
– personal and social competence; and
– intercultural understanding.

The curriculum documents note that these capabilities are embedded in the content or elaborations where appropriate. When accessing the Australian curriculum online, it is possible to apply a filter to the content descriptions to identify where the capabilities have been embedded. For numeracy, the description contained in the updated version of the *Shape of the Australian Curriculum* (ACARA, 2010b, p. 19) is:

> Students become numerate as they develop the capacities, confidence and dispositions to use mathematics at school, at home, at work and in community life. In the context of schooling, numeracy is about students engaging with whatever mathematics they need within and across all learning areas.

In the Australian curriculum for mathematics, it is noted that "mathematics makes a special contribution to the development of numeracy in a manner that is more explicit and foregrounded than is the case in other learning areas" (ACARA, 2010a, p. 8). As discussed earlier in this chapter, this statement may reinforce the view frequently held by teachers of other secondary learning areas that the development of students' numeracy is not necessarily their responsibility. For each of the capabilities, continua of learning have been developed and distributed for feedback. It will be up to state and territory authorities to determine how these capabilities will be assessed and reported.

Three cross curriculum priorities are identified in the Australian curriculum:

– Aboriginal and Torres Strait Islander histories and cultures;
– Asia and Australia's engagement with Asia; and
– Sustainability.

Again, these priorities are embedded in the curriculum content descriptions depending on their relevance to each learning areas. For the mathematics curriculum, the importance of mathematical concepts to Aboriginal and Torres Strait Islander children is discussed with particular mention of the connections between representations of number, space and pattern in traditional communities. The curriculum also notes the contribution of Asian mathematicians to the development of mathematics, as well as the mathematical concepts associated with Asian games, art and architecture. For sustainability, students are able to use their skills of problem solving and modeling to investigate a range of critical issues associated with the impact of human activity on the environment. These notions also connect with content in other learning areas of the curriculum including English, Science and History.

The mathematics curriculum for the senior years of schooling. In July 2010, the draft mathematics curriculum for the senior years was released for consultation. Four differentiated courses were identified to cater for appropriate pathways beyond schooling: (a) Essential Mathematics; (b) General Mathematics; (c) Mathematics Methods; and (d) Specialist Mathematics. Essential Mathematics focuses on using and applying mathematics in real contexts such as workplaces and community settings. The design of this course was informed by the research of Zevenbergen and Zevenbergen (2004), and Jorgensen and Zevenbergen (2011) and aims to address the 'numeracies' needed for life and work beyond school. The courses are designed to become progressively more challenging with the Specialist Mathematics course aimed at students who require a strong grounding in mathematics for further study in university courses such as the physical sciences or engineering.

An extensive consultation process revealed many stakeholders were concerned about the declining numbers of students choosing to study higher levels of mathematics in the senior years of schooling and welcomed a suite of courses aimed at encouraging all students to continue to study mathematics. However, concerns were raised about whether these courses would engage all learners with mathematics in the senior years. The MERGA response (2010b) indicated there was a lack of alignment between the rationale, aim and content as well as with the intentions outlined in the *Shape Paper* (NCB, 2009b). In addition, the response indicated a lack of a clear purpose for each course, explicit reference to the appropriate use of technology in each course, and too much content to ensure the proficiencies could be enacted in classrooms through the mathematics content. The AAMT response (2010b) described similar issues and expressed a desire for clearer articulation with the mathematics curriculum for the earlier years. A revised senior years curriculum was released for further consultation during 2011.

This section has outlined the development of new mathematics curriculum documents in New Zealand and Australia. According to ACARA documentation in the Australian context, the curriculum writers of the first national curriculum for mathematics used current research in mathematics education to inform the placement and sequencing of content. In the recent past, there has been limited research into trajectories of content by Australasian researchers. However, the following section presents studies that have occurred since 2008 as well as research into the use of school mathematics textbooks by teachers.

CURRICULUM DOCUMENTS AND RESOURCES

Trajectories of Content in the School Curriculum

The idea of developing and testing theoretical learning trajectories for content areas in the mathematics curriculum is a popular approach to developing curriculum. Three large scale examples in Australasia have been *Count Me in Too* (NSW), the *Early Numeracy Research Project* (Victoria), and the New Zealand *Numeracy Development Project* (Bobis et al., 2005). All of these projects informed the development of curriculum documents in their respective jurisdictions. Other recent smaller examples have the potential to inform future curriculum development.

Callingham and Pegg (2010) noted that recent curriculum documents in Australia have been designed around outcomes and related standards, and that teachers need to provide opportunities for students to learn the content that will allow them to meet the expectations defined in the curriculum. They reported that after undertaking professional learning sessions about SOLO Taxonomy, mathematics teachers in six high schools hypothesised developmental pathways for several key mathematical ideas. They concluded that using the SOLO model allowed teachers to theorise levels of mathematical development which led to the development of learning activities aimed at meeting students' learning needs.

This research is an example of how theoretical models can provide a sound basis for developing learning trajectories of content to meet curriculum standards. Another example (White & Mitchelmore, 2010) outlines a ten year history of using the constructs of empirical abstraction (Skemp, 1986) to build learning trajectories for the content areas of angles, percentages, decimals and ratios based on identifying similarities in different contexts which have the desired mathematical concept underpinning them.

MacDonald (2010) presented data from a three-year study that explored the experiences with measurement that children have in prior-to-school and out-of-school contexts, and the ways in which children are able to represent these experiences. Children's responses to an open-ended drawing task, collected at the commencement of Kindergarten, were backward-mapped in relation to the draft Australian Curriculum's Measurement and Geometry strand for Kindergarten (or more recently, Foundation). The conclusion was that most of the measurement skills of the Australian Curriculum are being exhibited by children at the commencement of schooling, prior to any formal teaching about measurement taking place. This finding has serious implications for the appropriateness of the expected standard for the first year of schooling in the Australian Curriculum for mathematics and needs to be investigated on a broader scale with children from all states and territories.

Cheeseman, McDonough and Clarke (2011) used one-to-one interviews to collect data about the measurement of mass from 1806 children in the first three years of school. Generic growth points for measurement from the Early Numeracy Research Project were used as a framework for the investigation which found steady progression through the growth points with two key transitions—moving from comparing to quantifying mass, and moving from quantifying to using standard units of mass. From this, teachers need to be aware of the need to provide particularly rich experiences to aid progression.

Watson and Fitzallen (2010) theorised the development of graph understanding in the *NSW Mathematics Curriculum* (Board of Studies NSW, 2003). As described in the introduction:

> ... various graph types to be introduced at particular stages of the curriculum, only to be superseded by other graph types introduced later on. It is somewhat unfortunate e.g., that the pictograph, which is included in the mathematics curriculum for the early years of schooling, has traditionally been forgotten, whereas it is often used in media and older students need to be able to analyse such forms critically, particularly where 'area' is involved in representing quantity. (p. 6)

This highlights the need for curriculum developers to consider the current needs of students in relation to the media, particularly given their extensive use of digital media to seek information, and is worthy of consideration when developing new curriculum documents.

Further to the investigation of graph understanding, Fitzallen and Watson (2011) explored Year 5 and 6 students' graph creation and interpretation using the software *TinkerPlots*. The twelve students in the study were able to "pick up graphing skills and application of contextual understanding in meaningful ways" (p. 258) and they were able to deal with two attributes in relationships with scatterplots. The authors noted scatterplots were not specifically mentioned in the new Australian curriculum although their use was implied in the additional curriculum information provided in content elaborations from about Year 4. They highlighted the usefulness of statistics in developing numeracy across the curriculum and argued the use of such software packages can assist "rapid consideration of various representations of data sets" (p. 259). Their study supported the development of critical thinking in statistics through two notions of context—the context embedded in the graph itself and the context brought to the task through the students' own experiences of the world (Mooney, 2010).

Stephens and Amanto (2010) argued strongly for building closer links between children's understanding of numbers and number operations and the beginning of algebraic (relational) thinking in the primary school years. While the new Australian Curriculum for mathematics combines Number and Algebra into one strand, the authors claimed that Australian mathematics textbooks rarely give enough guidance for teachers to use good activities in the classroom to promote algebraic thinking. By contrast, Japanese mathematics textbooks introduce students to relational thinking about number sentences, starting from the first grade. Japanese textbooks give just as much attention to computation and correct calculation, but they have a clear learning trajectory, which progressively develops relational ways of thinking about the four number operations. This learning trajectory is continuous and systematic throughout the primary years. The influence of textbooks on teachers' interpretation and implementation of the intended curriculum should not be underestimated. The next section examines recent research into the types of tasks typically presented in textbooks commonly used by teachers and the use of textbooks by Australasian teachers.

Textbooks as Curriculum Support

Studies have shown that a set mathematics textbook chosen by teachers is the dominant resource in mathematics classrooms in Australia and many countries throughout the world (Shield & Dole, 2009). The textbook exerts a strong influence on the content taught, the sequence of the content, and the approaches to teaching. Contrary to previous studies, a small-scale study by Jamieson-Proctor and Byrne (2008) provided some evidence to suggest that the 34 Queensland teachers they surveyed appeared to make less frequent use of textbooks and were more discerning about the manner in which they used textbooks in their classrooms. While the degree of usage may vary, the evidence for textbooks being substantially used in Australasian schools in both primary and secondary is convincing. Research, therefore, into how the curriculum is implemented using textbooks is warranted.

Australian eighth-grade mathematics lessons were shown by the 1999 *TIMSS Video Study* to use a high proportion of problems of low procedural complexity, with considerable repetition, and an absence of deductive reasoning (Stacey, 2003). Vincent and Stacey (2008) re-investigated this 'shallow teaching syndrome' by examining the problems on three topics in nine Year 8 textbooks from four Australian states for procedural complexity, type of problem solving processes, degree of repetition, proportion of 'application' problems, and proportion of problems requiring deductive reasoning. They found there was broad similarity between the characteristics of problems in the textbooks and in the Australian Video Study lessons. However, there were considerable differences between textbooks and between topics within textbooks. In some books, including the best-selling textbooks in several states, there was a predominance of repetitive exercises of low procedural complexity.

Stacey and Vincent (2008) also investigated the nature of reasoning in schools by examining the modes of explicit reasoning in the explanations, justification and proofs of several topics in four textbooks. They concluded that all the textbooks attempted to explain ‘the rule’ but in a way that omitted essential parts of the reasoning. The main purpose appeared to be to move on quickly to the practice exercises.

Shield and Dole (2009) used a method of analysing textbooks with three middle-years mathematics textbook series. The method was based on connectedness, structure and context, and focused on mathematical ideas based on proportional reasoning. The analysis revealed a predominance of calculation procedures, with relatively few tasks and explanations to support conceptual understanding. There was little or no recognition of similar structures in different problem contexts.

Dickenson-Jones (2008) found that when mathematical ideas of different cultural groups are included in mathematics texts they could become part of the learning experience in various ways. In particular, when included in western classroom mathematics textbooks, the ethno-mathematical ideas become transformed. Dickenson-Jones described the development of a conceptual model that illustrates five different modes of transformation that may occur when Indigenous cultural practices are incorporated in mathematics texts. She claimed that an awareness of how cultural practices are transformed might allow teachers constructing their own curriculum materials to choose the most appropriate modes of transformation.

Heirdsfield, Warren, and Dole (2008) investigated textbook materials which had been designed in accordance with principles advocating a student-centred approach, conceptual understanding and the fostering of students’ thinking and mathematical communication. Observations were conducted in six primary teachers’ mathematics classrooms as they implemented a new textbook series. The observations were combined with interview data to explore the impact of the textbook upon teachers’ classroom practice. The results were mixed. When the textbook was regarded as a resource, the classroom practice was judged as high quality. Conversely, if teachers felt challenged by the material in the textbook they tended to follow it in a prescriptive manner, resulting in teacher-directed practice.

Research shows that textbooks are widely used and provide curriculum content in a comprehensive, systematic way. However, the studies cited here question whether the type of curriculum delivery that research into mathematical engagement and understanding advocates is being implemented. The research reported here confirms the vital role of the teacher in implementing the school mathematics curriculum and using resources.

TEACHERS' USE OF, AND REACTIONS TO, CURRICULUM DOCUMENTS

Curriculum documents provide direction about what to teach but also often contain suggestions about how to teach. Teachers' attitude to how much of the 'how' is desirable is a source of interest for research. Wilson and McChesney (2010) reported on how pre-service students engaged with the mathematics and statistics section of the New Zealand Curriculum (Ministry of Education, 2007a), when writing a yearly long-term plan in this curriculum area. They showed that teachers were more comfortable with specific guidelines in curriculum documents than with broader ones and generally wanted direction from the curriculum materials. During the implementation phase of the *Year 1–10 Mathematics Syllabus* in Queensland (Queensland Studies Authority, 2004), Lamb and Spry (2009) identified the need for both internal and external sources of support for teachers. Luke, Weir and Woods (2008) argued that not only direction about content but also the technical features of syllabus documents contributed to 'high quality/high equity' outcomes.

Clarke (2008) took another perspective when he described the teacher as 'curriculum maker' since it was the teacher's enactment of the curriculum which had the greatest impact on learners. He also acknowledged the need for awareness of curriculum guidelines and where mathematics topics will lead learners in subsequent years. But how detailed should curriculum guidelines be and what support is needed for teachers to enact the curriculum in meaningful ways in classrooms? Clarke (2008) suggested highly detailed curriculum materials do not necessarily serve teachers well.

> The form, size and style of curriculum documents developed for classroom teachers often provides insights into the ways in which the role of the teacher is perceived by the authors of such documents. Where teachers are seen as key players in curriculum implementation, such documents often take the form of general guidelines, upon which a teacher can place her/his stamp, as they work together with colleagues to adapt materials to the perceived needs of their students.
>
> On the other hand, developers of highly prescriptive materials possibly think of teachers as incompetent or lacking experience. These materials are derived from an era in which curriculum developers attempted to provide 'teacher-proof' materials in order to bypass the influence of the teacher on student learning. This approach, however, does not allow for the impact of context and culture on the way in which materials might be implemented in classrooms ... the notion of teacher-proof material is nonsense—not possible and I would argue, not at all desirable. (p. 136)

The first Australian curriculum for mathematics provides a broad framework of content descriptions for teachers with support offered through online resources. A recent paper by Edmonds-Wathen (2011) also supported the notion of a framework, particularly as it related to Indigenous language speaking students in remote Northern Territory schools since teachers in these areas needed to adapt curriculum documents for the specific needs of special groups of students.

He argued that by providing less detail, the curriculum is more inclusive, allowing teachers to make decisions based on local contexts.

Ability and willingness of teachers to engage with curriculum change has also attracted research interest. In her analysis of the reasons for 109 teachers in NSW attending voluntary professional development programs, Anderson (2008) categorised the responses using Shulman's (1987) knowledge components with pedagogical content knowledge, curriculum knowledge and knowledge of learners as most valued. The implementation of new curriculum documents provided opportunities for teacher professional learning and engagement with potentially new content to enable planning and programming. Stillman and Galbraith (2009) pointed to how mathematical modelling has been a distinctive part of the senior secondary curriculum in Queensland for two decades but deep understanding of mathematical modelling has eluded some teachers, sometimes leading to their not seeing the purpose of gentle nudges from monitoring panels to engage more with modelling. A 'softly, softly' approach to implementation has led to some progress on all fronts with approaches described as ranging from minimalist to very rich. Curriculum reform through the written or intended curriculum does not necessarily lead to reform in the enacted curriculum via new teaching practices.

CONCLUSION

In an era of continuing curriculum change in many countries, there are calls to provide students with experiences in school mathematics which will enable them to be prepared for the 21st century. The goals of the *Melbourne Declaration* (MCEETYA, 2009) required Australian schooling to promote equity and excellence and have an expectation that all students "become successful learners, confident and creative individuals, and active and informed citizens" (p. 7). These are highly desirable goals that have the potential to be realised through the implemented or enacted curriculum if teachers are presented with a clear, well-structured curriculum informed by research. The intended curriculum is only the beginning but it can lead to reform if teachers are then supported during the implementation phase in schools.

As noted in the *Shape of the Australian Curriculum: Mathematics* (NCB, 2009a), for many students, their "experience of mathematics is alienating and limited" (p. 9), particularly given the frequency of use of low complexity problems (Hollingsworth et al., 2003). If problem solving is to be promoted as an important component of the curriculum, the types of problems which are most desirable must be made explicit and must be included in textbooks. Curriculum developers recognise that providing problem-solving experiences is critical if students are to be able to use and apply mathematical knowledge in meaningful ways. It is through problem solving that students develop deeper understanding of mathematical ideas, become more engaged and enthused in lessons, and appreciate the relevance and usefulness of mathematics. The implementation of the new Australian curriculum and the proficiency strands provides an opportunity for researchers to investigate the efficacy of this approach.

Development of curriculum does not occur in isolation (Hollingsworth & Pearn, 2011). Alongside the development of the curriculum in Australia has been the development of the *National Professional Standards for Teachers* (Australian Institute of Teaching and School Leadership [AITSL], 2011). There are seven generic standards described at four career stages: graduate, proficient, highly accomplished, and lead. Under Standard 2 (know the content and how to teach it), Focus area 2.3 relates to "curriculum, assessment and reporting" (see Table 2). This aspect of the *Standards* highlights the importance of knowledge of the curriculum at all career stages. In the future, teachers of mathematics in Australian schools will not only need to develop an understanding of the new Australian curriculum for mathematics but they will also have to be able to demonstrate how they meet the new teaching *Standards*. These significant changes provide opportunities for new research into the implementation of new curriculum as well as into the use of teaching standards and how these impact on teachers' work in schools and classrooms.

Table 2. National professional standards for teachers, standard 2, focus area 2.3 (AITSL, 2011, p. 10)

Standard 2 – Know the content and how to teach it				
Focus area	***Graduate***	***Proficient***	***Highly Accomplished***	***Lead***
2.3 Curriculum, assessment and reporting	Use curriculum assessment and reporting knowledge to design learning sequences and lesson plans.	Design and implement learning and teaching programs using knowledge of the curriculum, assessment and reporting requirements.	Support colleagues to plan and implement learning and teaching programs using contemporary knowledge and understanding of curriculum, assessment and reporting requirements.	Lead colleagues to develop learning and teaching programs using comprehensive knowledge of curriculum, assessment and reporting requirements.

The research into curriculum documents reviewed in this chapter shows that 'not a great deal' of actual research has been reported since 2008 and highlights the need for more research into curriculum development and implementation in Australasian contexts. Reducing the content in the curriculum does have the potential for teachers to provide more opportunities in mathematics lessons to engage students in richer learning experiences, but the old question of what to remove remains a challenge since many teachers believe this is still a key issue for them. Researching the ways teachers manage and integrate content to engage students would provide valuable information for further curriculum development.

REFERENCES

Ainley, J., Kos, J., & Nicholas, M. (2008). *Participation in science, mathematics and technology in Australasian education.* [Monograph 4]. Melbourne, Australia: ACER.

Anderson, J. (2008). Teachers' motivation to attend voluntary professional development in K-10 mathematics. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 51–58). Brisbane, QLD: MERGA.

Australian Association of Mathematics Teachers [AAMT]. (1997). *Numeracy=everyone's business. The report of the numeracy education strategy development conference, May 1997*. Adelaide, Australia: Author.

Australian Association of Mathematics Teachers [AAMT]. (2010a). *AAMT response to the draft K-10 Australian curriculum: Mathematics.* Adelaide, Australia: Author.

Australian Association of Mathematics Teachers [AAMT]. (2010b). *AAMT response to the draft senior secondary Australian Curriculum: Mathematics*. Adelaide, Australia: Author.

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2010a). *The Australian curriculum mathematics*. Sydney, Australia: Author.

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2010b). *The shape of the Australian curriculum, Version 2.0.* Sydney, Australia: Author.

Australian Institute for Teaching and School Leadership. (2011). *National professional standards for teachers*. Melbourne, Australia: Education Services Australia.

Board of Studies NSW (2003). *Mathematics K-6 syllabus.* Sydney, Australia: Author.

Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, R., Young-Loveridge, J., ... Gould, P. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large scale cases. *Mathematics Education Research Journal*, *16*(3), 27–57.

Callingham, R., & Pegg, J. (2010). Using development frameworks to support curriculum outcomes. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 125–132). Fremantle: MERGA.

Centre for Science, Information and Communication Technology and Mathematics Education in Rural and Regional Australia (SiMERR). (2011). *Second chance algebra learning environment* (SCALE). Armidale, Australia: University of New England. Retrieved from http://www.une.edu.au/simerr/scale/

Cheeseman, J., McDonough, A., & Clarke, D. (2011). Investigating children's understanding of the measurement of mass. In J. Clarke, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and (new) practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 174–182). Adelaide, SA: AAMT & MERGA.

Clarke, D. (2008). The mathematics teacher as curriculum maker: Developing knowledge for enacting curriculum. In P. Sullivan & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp. 133–152). Rotterdam, The Netherlands: Sense.

Clements, D. H. (2007). Curriculum research: Towards a framework for research-based curricula. *Journal for Research in Mathematics Education*, *38*, 35–70.

Council of Australian Governments. (2008). *National numeracy review report*. Canberra, Australia: Commonwealth of Australia. Retrieved from http://www.coag.gov.au/reports/docs/national_numeracy_review.pdf

Cowie, B., Hipkins, R., Boyd, S., Bull, A., Keown, P, & McGee, C. (2009). *Curriculum implementation exploratory studies: Final report.* Wellington, New Zealand: Ministry of Education.

Department of Education and Children's Services (2005). *South Australian curriculum, standards and accountability framework*. Retrieved from http://www.sacsa.edu.au

Dickenson-Jones, A. (2008). Transforming ethnomathematical ideas in western mathematics curriculum texts. *Mathematics Education Research Journal, 20*(3), 32–53.

Edmonds-Wathen, C. (2011). Locating the learner: Indigenous language and mathematics education. In J. Clarke, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and (new) practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 217–225). Adelaide, SA: AAMT & MERGA.

Fitzallen, N., & Watson, J. (2011). Graph creation and interpretation: Putting skills and context together. In J. Clarke, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and (new) practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 253–260). Adelaide, SA: AAMT & MERGA.

Forgasz, H. (2006). *Australian Year 12 mathematics enrolments: Patterns and trends—Past and present*. Melbourne, Australia: ICEEM.

Freebody, P. (2007). Literacy education in school: Research perspectives from the past, for the future. *Australian Education Review 52*. Camberwell, Australia: ACER.

Goos, M., Geiger, V., & Dole, S. (2010). Auditing the numeracy demands of the middle-years curriculum. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 210–217). Fremantle: MERGA.

Goos, M., Geiger, V., & Dole, S. (2011). Teachers' personal conceptions of numeracy. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group of the Psychology of Mathematics Education, pp. 457–464). Ankara, Turkey: PME.

Heirdsfield, A., Warren, E., & Dole, S. (2007, November). *The use of a new mathematics textbook scheme—support or impediment*. Paper presented at the 2007 annual conference of the Australian Association for Research in Education, Fremantle, Australia. Retrieved from http://www.aare.edu.au/07pap/her07131.pdf

Higgins, J., & Parsons, R. (2009). A successful professional development model in mathematics. *Journal of Teacher Education, 60*(3), 231–242.

Hipkins, R. (2011). Learning to reinvent the school curriculum. Retrieved from http://www.nzcer.org.nz/system/files/learning-reinvent-school-curriculum.pdf

Ho, K. F. (2010). Year 11 advanced mathematics: Hearing from students who buck the trend. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 248–255). Fremantle, SA: MERGA.

Hollingsworth, H., Lokan, J., & McCrae, B. (2003). *Teaching mathematics in Australia: Results from the TIMSS 1999 video study*. Camberwell, Australia: ACER.

Hollingsworth, H., & Pearn, C. (2011). Towards excellence in mathematics teaching: Forging links between national curriculum and professional standards initiatives. In J. Clarke, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and (new) practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 957–965). Adelaide, SA: AAMT & MERGA.

Holton, D. (2010). Forward 2010. In Ministry of Education, *Findings from the New Zealand Numeracy Development Projects* 2009 (pp. 1–2). Wellington, New Zealand: Learning Media.

Jamieson-Proctor, R., & Byrne, C. (2008). Primary teachers' beliefs about the use of mathematics textbooks. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 295–302). Brisbane, QLD: MERGA.

Jorgensen, R., & Zevenbergen, K. (2011). Young workers and their dispositions towards mathematics: Tensions of a mathematical habitus in the retail industry. *Educational Studies in Mathematics*, *76*, 87–100.

Kennedy, K. (2009). The idea of a national curriculum in Australia: What do Susan Ryan, John Dawkins and Julia Gillard have in common? *Curriculum Perspectives*, *29*(1), 1–9.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Lamb, J., & Spry, G. (2009). Leading change in mathematics: The Queensland mathematics syllabus. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 307–314). Palmerston North, NZ: MERGA.

Luke, A., Weir, K., & Woods, A. (2008). *Development of a set of principles to guide a P-12 syllabus framework: A report to the Queensland Studies Authority*. Brisbane, Australia: Queensland University of Technology.

MacDonald, A. (2010). Young children's measurement knowledge: Understandings about comparison at the commencement of schooling. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 375–382). Fremantle, WA: MERGA.

Mathematics Education Research Group of Australasia. (2010a). *MERGA response to the Australian curriculum (mathematics)*. Adelaide, Australia: MERGA. Retrieved from http://www.merga.net.au/node/49

Mathematics Education Research Group of Austalasia. (2010b). *MERGA feedback: Draft senior years Australian Mathematics Curriculum.* Adelaide, Australia: MERGA. Retrieved from http://www.merga.net.au/sites/default/files/editor/documents/MERGA%20Senior%20Mathematics%20CurriculumFeedback300710FF.pdf

McPhan, G., Morony, W., Pegg, J., Cooksey, R., & Lynch, T. (2008). *Maths? Why Not?* Department of Education, Employment and Workplace Relations, Canberra, Australia. Retrieved from http://www.dest.gov.au/sectors/school_education/publications_resources/profiles/maths_why_not

Ministerial Council on Education, Employment, Training and Youth Affairs [MCEETYA]. (2008). *Melbourne declaration on educational goals for young Australians*. Retrieved from http://www.curriculum.edu.au/verve/_resources/National_Declaration_on_the_Educational_Goals_for_Young_Australians.pdf

Ministry of Education. (2007a). *The New Zealand curriculum*. Wellington, New Zealand: Learning Media.

Ministry of Education. (2007b). *Achievement objectives by learning areas.* Wellington, New Zealand: Learning Media.

Ministry of Education (2009). *Mathematics standards for years 1–8*. Wellington, New Zealand: Author.

Mooney, G. (2010). Preparing students for a data rich world: The case for statistical literacy. *Curriculum Perspectives*, *30*(1), 25–29.

National Curriculum Board. (2009a). *The shape of the Australian curriculum*. Canberra, Australia: Author.

National Curriculum Board. (2009b). *Shape of the Australian curriculum: Mathematics*. Canberra, Australia: Author.

Queensland Studies Authority. (2004). *Year 1–10 Mathematics Syllabus*. Brisbane, Australia: Author.

Reid, A. (2005). *Rethinking national curriculum collaboration: Towards an Australian curriculum. Report for Australian Government Federal Department of Education, Science, and Training*. Canberra, Australia: DEST.

Reid, A. (2009). Is this a revolution?: A critical analysis of the Rudd government's national education agenda. *Curriculum Perspectives*, *29*(3), 1–13.

Shield, M., & Dole, S. (2009). An analysis of middle-years school mathematics textbooks. In *Improving science and mathematics literacy: Theory, innovation and practice* (Proceedings for the 3rd International Conference of Science and Mathematics Education). Penang, Malaysia: CoSMed. Retrieved from http://eprints.qut.edu.au/28847/1/28847.pdf

Shulman, L. (1987). Knowledge and teaching: Foundations of the reform. *Harvard Educational Review*, *57*(1), 1–22.

Skemp, R. (1986). *The psychology of learning mathematics* (2nd ed.). Harmondsworth, United Kingdom: Penguin.

Stacey, K. (2003). The need to increase attention to mathematical reasoning. In H. Hollingsworth, J. Lokan, & B. McCrae (Eds.), *Teaching mathematics in Australia: Results from the TIMSS 1999 video study* (pp. 119–122). Melbourne, Australia: ACER.

Stacey, K., & Vincent, J. (2008). Modes of reasoning in explanations in year 8 textbooks. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 475–481). Brisbane, QLD: MERGA.

Stephens, M., & Amanto, D. (2010). How to build powerful learning trajectories for relational thinking in the primary school years. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 523–530). Fremantle, WA: MERGA.

Stillman, G., & Galbraith, P. (2009). Softly, softly: Curriculum change in applications and modelling in the senior secondary curriculum in Queensland. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 515–522). Palmerston North, NZ: MERGA.

Sullivan, P. (2011). Teaching mathematics: Using research-informed strategies. *Australian Education Review, 59*. Retrieved from http://research.acer.edu.au/aer/13

Sullivan, P., & Jorgensen, R. (2009). Education as creating opportunities: The contribution of numeracy. In *Future SACE school to work innovation program: Literacy & numeracy final report* (pp. 24–30). Adelaide, Australia: South Australian Department of Education and Children's Service and the future SACe Office.

Thornton, S. (2009). Speaking with different voices: Knowledge legitimation codes of mathematicians and mathematics educators. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 515–522). Palmerston North, NZ: MERGA.

Vincent, J., & Stacey, K. (2008). Do mathematics textbooks cultivate shallow teaching? Applying the TIMSS video study criteria to Australian eighth-grade mathematics textbooks. *Mathematics Education Research Journal, 20*(1), 82–107.

Watson, J., & Fitzallen, N. (2010). *The development of graph understanding in the mathematics curriculum: Report for the NSW Department of Education and Training*. Sydney, Australia: NSW DET.

White, P., & Cranitch, M. (2010). The impact on final year pre-service secondary teachers of a unit in teaching literacy and numeracy across the curriculum. *Australian Journal of Teacher Education*. *35*(7), 54–63. Retrieved from http://ajte.education.ecu.edu.au/ISSUES/vol357.htm

White, P., & Mitchelmore, M. C. (2010).Teaching for abstraction: A model. *Mathematical Thinking and Learning*. *12*(3), 205–226.

Wilson, S., & McChesney, J. (2010). "I always feel more confident when I know where things are going": How do pre-service teachers engage with mathematics curriculum documentation? In L. Sparrow, B. Kissane, & C. Hurst (Eds.). *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 649–656). Fremantle, WA: MERGA.

Yates, L. Y. N., & Collins, C. (2010). The absence of knowledge in Australian curriculum reforms. *European Journal of Education, 45*(1), 89–102.

Young-Loveridge, J. (2010). A decade of reform in mathematics education: Results for 2009 and earlier years. In Ministry of Education, *Findings from the New Zealand Numeracy Development Projects* 2009 (pp. 15–35). Wellington, New Zealand: Learning Media.

Zevenbergen, R., & Zevenbergen, K. (2004). Numeracy practices in contemporary work: Changing approaches. In I. Putt, R. Faragher, & M. McLean (Eds.), *Mathematics education for the third millennium: Towards 2010* (Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Townsville, pp. 605–612). Sydney, NSW: MERGA.

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GROWTH AND NEW DIRECTIONS? RESEARCH IN TERTIARY MATHEMATICAL SCIENCE EDUCATION

Key words: tertiary; approaches to teaching; mathematical content.

INTRODUCTION

Have the quantity and quality of papers on tertiary mathematics education in Australasia changed? Are we going over the same ground, or are we venturing into new areas and exposing a vista we had not previously seen? Our review of research found indications of improved quality as well as some repetitive work. We can say that new directions are being opened up. We hope they will provoke productive research in the next four years.

Compared with the last review (Wood, 2008), the numbers of refereed papers increased 12%, and shows the same natural cyclic variation. The biennial Delta conference on undergraduate mathematics teaching and the International and Australian Conferences on Teaching Statistics (ICOTS/OZCOTS) produce a two-year boom and bust cycle of papers in the tertiary area. This is to be expected, and attests to the power of these conferences to focus researchers' energies and promote publications.

This chapter, unlike the corresponding chapters in previous reviews, adds consideration of papers on statistics in tertiary education. The majority of the papers in this area deal with statistics as a service subject rather than post-secondary statistics majoring courses.

In mathematics, nearly half of the papers reviewed for this chapter were published in journals, compared with one quarter in the previous four-year period. Such figures probably reflect both an increasing quantity and quality of manuscripts, and in all likelihood, increased production that has been driven by the demand from university employers for fully refereed research outputs. The increase may also be stimulated by an increasing need to address pedagogical issues in the tertiary sector as governments and students demand higher quality teaching.

In the previous review, Wood (2008) noted that there was surprisingly little research being conducted on mathematics learning in universities, given the number of students involved and the importance of mathematics to the national economy. She suggested one reason might be that research on university mathematics learning was usually done by mathematicians who, although passionate about teaching and learning, were obliged to maintain a strong research

B. Perry et al. (Eds.), Research in Mathematics Education in Australasia 2008–2011, 245–265.

profile in their mathematical discipline. In these circumstances it was difficult for mathematicians to develop depth of knowledge in educational literature and methodologies. Thus, research on university mathematics education was not always informed by theories of learning, and the findings did not often contribute to building a body of literature in the field. Instead, papers were often descriptive and limited to reporting on a course taught by the author.

Has the focus and depth of research changed in the past four years? We found papers on similar themes occasionally came from one source, indicating a strong research group, but not necessarily a general Australasian interest in any particular area. The major theme to emerge across institutions is that of transition from school to university. Papers on technology remain regular, but there is an emerging interest in theoretical perspectives that was not seen before. Another new trend is papers written by younger mathematicians who are developing a strong research interest in mathematics education.

As in the past, there were several papers dealing with particular mathematical topics: ordinary differential equations; linear algebra; number theory; functions; calculus; and modelling are all given specific attention. However, two themes that were evident in the last review are not strongly represented in these four years. Assessment was a part of several papers, but not the major topic of attention, and language issues appear in only one paper we identified.

A developing international focus on lecturing as a pedagogical practice (e.g., Petropoulou, Potari, & Zachariades, 2011; Viirman, 2011) has led to some radical suggestions about approaches to teaching and learning at university in both mathematics and statistics. Given the lack of attention to university teaching practices in previous research, we may well ask whether we are on the edge of a new era.

TRANSITION

The transition from secondary school to university has attracted the attention of a variety of researchers in New Zealand and Australia. A theoretical model based on the idea of a rite of passage was developed and then enhanced by Clark and Lovic (2009) with a consideration of cognitive conflict and culture shock. The implications of this perspective are extensively discussed, including that change is inevitable and a certain amount of shock may be useful in the establishment of new practices such as correct language, the role of theorems, and some pedagogical practices. Bridging courses that have a stigma attached are strongly critiqued by this view.

The gap between school and university is further explored in a major government-funded research project in New Zealand focusing on affective issues. Hong et al. (2009) undertook extensive surveying to show that lecturers and high school teachers do not fully understand the other’s perspective on transition, and argue for improved communication between the two sectors. The number of students who make the transition from school to university mathematics is influenced by their mathematical preparation at school. McPhan and Pegg (2009) investigated attitudes of teachers and career guidance professionals with a view to understanding school students’ decisions about taking advanced mathematics in the senior secondary years. The attitudes are predominantly negative: Mathematics is hard, repetitive, and demanding of time. These are reinforced when students fail or numbers decline, creating a vicious circle of dissuasion from taking the subject that denies school students access to university mathematics.

A more mathematically detailed investigation of the transition was exemplified by Godfrey and Thomas' (2008) study of the way an equation is understood at school compared with university. Secondary students had a much more restricted view of equations, often requiring a process to be present. University students did show a more flexible view, including understanding of, for example, transitivity, but many still had not entered the 'formal' mathematical world.

Three studies examined mathematics results. Final school and first year university results were analysed by James, Montelle, and Williams (2008) as part of an on-going attempt to refine entrance criteria for advancing mathematics courses. They concluded that excellence in any area of the mathematical science is generally a good indicator of success, but refining particular grade entry points is not likely to improve student outcomes. A deficit approach towards the abilities of mathematics students was investigated by Jennings (2009) who commented on the effects of increasing diversity of student backgrounds. Rylands and Coady (2009) looked at the use of school and tertiary entrance results to guide entry to university programmes. School background, both explicit results and the type of course taken, were found to be good indicators of university mathematics success, leading them to conclude that universities must better accommodate the divergent mathematical backgrounds of students, and provide for those who have not taken appropriate courses.

Wood and Solomides (2008) extended the notion of transition to include the tertiary to workplace transition, and argued that attending to where the student is heading with their learning is more important than from where they have come. Therefore, rather than looking at the gaps in the students' knowledge, curriculum should adjust to the mathematical and professional needs of where a student is headed in their studies and career. Wood followed up this theme in two reports; the first considering the mathematical communication needs of recent graduates (Wood, 2011), and the other (Wood, 2010) using in-depth interviews of recent graduates to conclude that they required more mathematical computing than was taught at university.

Assessment for the transition to university stage of a student's life was part of a new model for undergraduate first year assessment developed and researched by Taylor (2008). Taylor found that defining three stages (transition, development, and achievement) in the first year helped student engagement across a variety of subjects, including mathematics.

Research on transition is being extended in three directions away from studies that simply specify the knowledge gap (often described as knowledge lack). One is exploring the cultural difference between school and university, another is investigating the way a university copes with knowledge diversity on entry, and the third is examining the trajectories of students through their degree and into the workplace.

TECHNOLOGY

Some of the research papers on technology were accounts of the introduction of a particular piece of software, hardware, or web-application into the teaching or learning of particular topics in tertiary mathematics. For example Adams et al. (2008) reported on the development of an electronic testing and tutoring system; Blyth and Labovic (2009) reported the development of an e-Learning system using MAPLE; Blyth again (2010) discussed digital ink for annotating lectures broadcast over Access Grid in a distributed learning model; e-Learning is the subject of Loch (2010, 2011); and Wiwatanapataphee, Noinang, Wu, and Nuntadilok (2010) used a PowerPoint MAPLE display to teach multivariate integral calculus. The use of technology was found in all cases to enhance learning, with the enhanced visual stimulus often being the inferred cause.

Tablet technology was explored by two groups. Yoon and Sneddon (2011) looked at Tablet PC technology to record lectures and make them available to students. A concern, expressed by mathematicians in their department when tablet technology was introduced, was that they would be counter-productive because they would lead to decreased lecture attendance, and that lectures were more powerful learning experiences. Yoon and Sneddon found, however, that mathematics grades were not associated with the missing of lectures. Those students intending, but failing, to watch videos did have lower grades. They also found that the proportion of students missing lectures and not watching videos was small. Tablet PC use by students in a variety of ways is explored in Galligan, Loch, McDonald, and Taylor (2010) and Loch, Galligan, Hobohm, and McDonald (2011). As a result of their studies, and the collaborative learning that resulted, the university decided to provide inexpensive tablet technology to large cohorts of students.

Kyng, Tickle, and Wood (2011) investigated the type and use of software by graduates in financial mathematics. Seventy-three graduates responded to a questionnaire asking what software they used and presented detailed reflections on their university study. An interesting outcome was that graduates reported spending 60% of their working day using spreadsheets.

Oates (2009, 2010, 2011) is the only person who appears to have undertaken a wider study of technology use, investigated curriculum issues, asked what integrating technology means, and described what can be accomplished. It seems that, despite the tertiary mathematics education community's knowledge of the high need for technology by mathematical science graduates, there is little coordination with respect to the impact of technology on content. We might add that Oates' work indicated little coordination with respect to the implementation of technology as well.

MATHEMATICAL TOPICS

More than twenty articles address a particular topic in university mathematics, the most common of which is linear algebra. All but one of the ten articles on this particular topic came from the PhD thesis of Sepideh Stewart (2008) (e.g., Stewart, 2009). The thesis applied Dubinsky's (1992) theory of learning mathematical concepts through Action, Process, Object, and Schema (APOS), in conjunction with Tall's (2004) three worlds of embodied, symbolic and formal mathematics, to examine the learning of linear algebra concepts by groups of first and second year university students. It revealed that those with more representational diversity had more overall understanding of the concepts. In particular the embodied introduction of the concept proved a valuable adjunct to their thinking. Britton and Henderson (2009) examined the importance of understanding symbols in learning linear algebra, and, from an analysis of errors, found that students understood the need for making a general argument, but responded to this by inappropriate symbol manipulation or using sequences of manipulation used in a different context.

Three papers on modelling focused on engineering students. Two investigated their awareness and attitudes to the topic (Klymchuk, Zverkova, Gruenwald, & Sauerbier, 2008; Lim, Tso, & Lin, 2009). The former used non-engineering (environmental) models in first and also in later year engineering courses, finding that they were received positively only by the more mature students. The latter taught mathematical modelling to Earth Science majors, and found that although it did not change attitudes to mathematics, it did enhance students' enjoyment of mathematics. Narayanan, Klymchuk, Gruenwald, Sauerbier, and Zverkova (2010) undertook a pilot study of several biomathematical models of infectious diseases, confirming that both students and lecturers responded well to modelling real data, although they would prefer the models to be sufficiently complex to match the situation.

Calculus remains a focus of attention for researchers, with several different approaches investigated: model-eliciting activities (Yoon, Dreyfus, & Thomas, 2010)—a preliminary case study to be followed up with more subjects; and application problems (Klymchuk, Zverkova, Gruenwald, & Sauerbier, 2010)—where students attributed lack of practice, not lack of understanding, to explain their poor performance; and counter-examples (Klymchuk, 2010). A paper on ordinary differential equations (Mallet & McCue, 2009) reported the successful trialling of a discovery approach; a paper on number theory (McAndrew, 2009) showed how divisibility problems often introduced as induction, can be approached using difference equations; and another on reasoning (Easdown, 2009) discussed several misconceptions and, echoing Britton and Henderson (2009), highlighted the understanding of syntax and symbols.

These individual studies contributed only marginally to our overall understanding of the detailed mathematical aspects of learning and teaching. In the area of understanding symbolism, the studies confirmed and extended our awareness of the problem, but did not advance our theoretical understanding.

Rather than investigating specific mathematical topics, some studies took a broader, curriculum, view. As part of a larger doctoral study of undergraduate students' understanding of key concepts in mathematics, Worsley (2011) interviewed mathematics lecturers at one university to discover what they considered to be the 'big ideas' in their courses. She found that lecturers differed in what they considered important for students to learn and in how they justified choices of concepts. Some lecturers identified abstract ideals such as critical reasoning, as being important. However, these general goals were not explicitly communicated to students in the course outline, which instead focused on the mathematical content to be mastered.

Looking beyond the mathematics curriculum, Belward et al. (2011) investigated the relevance of quantitative skills in university science degrees in Australia. They argued that although there is an increasing need for science graduates to develop quantitative skills, there is much confusion over how to help students appreciate the intimate relationships between science and mathematics. The initial stages of this investigation analysed public documents (e.g., university websites) to record the entry requirements to science degrees that deemed prior study of secondary school mathematics necessary, together with compulsory requirements for mathematics or statistics within the degree programme. Twelve of the 17 institutions in the study required mathematics for entry. Only eight institutions had a compulsory quantitative course in their BSc degree. Belward et al. suggested that this information may be interpreted to mean that the study of mathematics is unnecessary for a science degree.

The Australasian research mirrors, to a large degree, that occurring in Europe, although the theoretical bases are different (see Conclusion below). Another notable comparison is that the topic of proof is largely ignored in comparison to, for example, the high number of proof-related contributions to the 2011 Research in Undergraduate Mathematics Education (RUME) conference in America.

PIPELINE AND POLITICS

A number of papers addressed wider issues about the numbers of students undertaking university mathematical study and the possible causes for this. The *Pipeline Project* commissioned by the International Mathematical Union (IMU) and International Commission for Mathematical Instruction (ICMI) was based in New Zealand and included Australian and New Zealand data. Holton et al. (2009) reflected, in a preliminary study to the *Pipeline Project*, on the causes of fluctuations in mathematical science student numbers, with the way courses are taught being the only academic-controlled variable out of five major ones (the others are government decisions, state of the economy, job market; and university funding mechanisms). Despite the difficulties of obtaining long and reliable time-series of student numbers, eventually sufficient data were found for Australia and New Zealand. In Australia, the number of mathematical science graduates are slowly increasing, but declining in relation to population growth in that age-group. The numbers studying mathematically-related subjects remains steady. In New Zealand there is overall steady growth, even in relation to population growth. (Barton, Clark, & Sheryn, 2010; Barton & Sheryn, 2010; Thomas, Muchatuta, & Wood, 2009).

If improved teaching will help remediate lack of student numbers, what do we know about teaching and learning in universities? Cretchley (2009) undertook a survey of academics in science, technology and engineering, and concluded that (a) senior academics still perceive that there are far higher professional rewards for research activities than for learning and teaching, and they gain far more job satisfaction from research activities; and (b) academics at all levels still experience a lack of role models, support and reward for learning and teaching. She concluded that unless rewards and support for learning and teaching activities become comparable to those for research, and mainstream job opportunities become available for academics to focus on such activities, then the needed changes in academic behaviour will be marginal.

Other papers represented a new direction in tertiary research, and may help us understand the pipeline trends. Klinger (2008) investigated attitudes of undergraduate students or described the way tertiary students reported their mathematical experiences (Bartholomew, Darragh, Ell, & Saunders, 2011). The latter identified many positive attitudes, and the sense of belonging to an exclusive ‘club’ amongst those choosing mathematical courses.

The student perspective on lecturing has been addressed in an international study that included Australia (Wood et al., 2011). It used both in-depth interviews and large-scale surveys. Many students were unable to articulate how mathematics would be used in their future (whether further studies or their careers), and those that did cited procedural rather than conceptual uses of mathematics. Such attitudes have implications for their expectations of university courses.

Also in the review period, Rubenstein (2009) produced the National Strategy for the Australian Mathematical Sciences Institute. It specified challenges in providing adequate numbers of mathematical science graduates and teachers, and declining quality in education. The report advocated modification to current government strategies, noting that extra funding failed to get to mathematics and statistics departments and that fee reductions and curricular change have not had the desired effects. It proposed raising the profile of mathematics in the community, strategies to improve the quality of teachers, measures to ensure the quality of university mathematics, and more governmental support for infrastructure such as the Australian Mathematical Sciences Institute.

LECTURING

An emerging field of investigation later in the review period was the practice of lecturing. As noted in the international literature, there has been little research of the teaching practices of university mathematics lecturers. For a recent indication of international interest see Petropoulou, Potari, and Zachariades (2011), Viirman (2011), or the proceedings of Working Group 3 at CERME-7 (Pytlak, Swoboda, & Rowland, 2011).A major New Zealand study in the area has occurred in the review period (Thomas et al., 2011). It includes the work of Yoon, Kensington-Miller, Sneddon, and Bartholomew (2011), in which semi-structured interviews were used to investigate the social norms resulting in student passivity during lectures. Students were aware of the norms, but regarded the behaviour as allowing lecturers to complete content material. The explicit use of norms to promote interaction is suggested. Another aspect of the Auckland study used video to record, view, and promote discussion of lectures by small groups of colleagues in a professional development trial founded on the Knowledge, Orientation, Goals (KOG) model developed by Schoenfeld. Several papers have emerged from the study, for example Paterson, Thomas, and Taylor (2011) who focused on the way lecturers reach decisions.

In Australia, professional development of lecturing received attention by Wood et al. (2011), who reported on a collaborative research project aimed at investigating the type of professional development that Australian tertiary mathematics teachers need and their preference for delivery modes.

Schoenfeld's framework was also used by Hannah, Stewart, and Thomas (2011) who followed one lecturer through a detailed analysis of his interactions with students, relating it to pre- and post-recorded accounts of the lecturer's intentions in the lecture.

Particular lecturing techniques were investigated by Paterson and Sneddon (2011) who conducted an in-depth examination of team-based learning in a third-year discrete mathematics course, and by Tonkes, Isaac, and Scharaschkin (2009) who discussed the use of 'partially populated' lecture notes. However the characteristics of lecturing, and the need for lecturers to be aware (or to 'notice' in the language of John Mason (Lerman & Davis, 2009)) was highlighted by Klymchuk and Thomas (2011), who, in a comparative study of lecturers and teachers, found that both groups failed to 'notice' essential properties or conditions of the mathematical objects in the questions presented.

Associated with the developing interest in lecturing was an increasing attention to theoretical issues, both the use and adaptation of learning theories, in particular APOS and Tall's three worlds, to examine particular topics within lectures (Thomas & Stewart, 2011). Barton (2011) has developed a new framework based on the interaction between mathematics and the university environment by adapting Artigue's heuristic, epistemic and pragmatic value concepts. He used it to analyse his own lecturing. Begg (2011) used the metaphor of an axiom to propose an examination of our underlying assumptions in undergraduate teaching and learning. He proposed a set of his own, for example "undergraduates expect university to differ from high school" (p. 839) and "start where the learner is" (p. 840).

STATISTICS EDUCATION

Much statistics education is in the form of service teaching—either a supplement to other courses and majors (e.g., biology and psychology), or a component of professional training (e.g., for researchers, medical personnel and policy makers). Service teaching also occurs in mathematics, but to a much lesser extent than statistics. Almost all statistics teaching is in service units. Much of this is taught outside mathematics or statistics departments, and often such statistics is taught by numerate psychologists and scientists rather than by mathematicians or statisticians. There may be literature about learning statistics in these disciplines, but we have not reported on such literature in this chapter. The service aspect is reflected in the research literature in tertiary statistics education.

Examples of such literature included reporting of the successful multi-university Masters programme in biostatistics by Simpson, Ryan, Carlin, Gurrin, and Marschner (2009) who proposed it as a model for others to address biostatistics workforce shortages. Dhand and Thomeson's (2009) study of a scenario-based approach to biostatistics for veterinary students was reported as successful although with low numbers of respondents. Wilson and Bulmer (2008) described on-line tutorials to support learning of randomness concepts by engineering and health sciences students; and Luo, Vemulpad, and Bilgin (2008) described two statistical software packages used with chiropractic students, evaluating them through student responses. They found that WebSTAT did not appear to be superior to EcStat, at least from the chiropractic students' perspective. However, WebSTAT produced more comprehensive outputs and was therefore the preferred package.

At a more theoretical level, Kalinowski, Lai, Fidler, and Cumming (2008) undertook a mini meta-analysis of qualitative and quantitative methods in statistics education research, highlighting the added value of the former methods. They found that qualitative methods make a significant addition to quantitative in statistics education research. Cumming (2010) and Lai (2010) critiqued the use of hypothesis testing and dichotomous (reject/accept) orientations as opposed to estimation and meta-analytic thinking.

Other papers considered appropriate university curricula for specialist professional groups. For example, Black (2008) examined the standards and assessments in a certificate for state employees in New Zealand who use official statistics and Forbes (2009) further evaluated and developed this curriculum. An innovative study inferred from the statistical advice embedded in the APA Publication Manual that statistical understanding increases the chance that a research paper will be accepted for publication (Fidler, 2010). The statistical knowledge needed by the pharmaceutical, medical device and biotechnology sectors was reviewed by Badcock (2008) by considering the skills needed for the development of new drugs. With respect to assessment, conventional testing was considered inappropriate by Martin (2008) in his examination of a course in industrial quality control.

At a broader level, Low Choy, and Wilson (2009) argued that interviewing professionals reveals misconceptions that should be addressed during their university education. An overview of statistics curricula for modern professionals by Reid and Petocz (2008) distinguished between narrow and broad curricula as those that focus on statistical techniques or their use in a professional context respectively. A large Australian/Swedish study produced a model of professional learning based on students' views (Reid, Dahlgren, Dahlgren, & Petocz, 2010). The model intersects conceptions of professional learning and views of professional knowledge to investigate the ways in which students navigate the transition from tertiary study to professional work. It is described in more detail in a recent book (Reid, Abrandt Dahlgren, Dahlgren & Petocz, 2011).

Many university statistics courses are introductory, and therefore mirror some of the characteristics of school courses. Papers that addressed universal issues are not considered in this university-specific chapter but there is a growing body of work reporting only university environments. Several articles argued for and researched the use of (a) more humour (Neumann, Hood, & Neumann, 2009), examples (Gordon & Nicholas, 2008), statistical diversions (Sowey & Petocz, 2010), and more graphics (Pruzek & Helmreich, 2009); (b) promoting various technologies such as chat tutorials and on-line resources (McDonald, Loch, & Lloyd, 2010); and (c) the use of surveys to gather student data for analysis (Neumann, Neumann, & Hood, 2010).

New courses are described by researchers at various universities. Gordon, Finch, and Maillardet (2008) reported a general statistics course; Richardson (2008) focused on language interventions; David and Brown (2010) shifted the orientation of their course from 'how' to 'why'. In this vein, Wood and Petocz (2008) explained how they applied research findings to produce a 'second-generation' textbook that focuses on statistical thinking first and techniques second, and MacGillivray (2009) argued for a reform movement in statistics with a focus on data rather than on theory and 'recipes'. She produced examples for such teaching.

The increased focus in tertiary statistics courses on statistical thinking reflects a similar move at the school level, and has resulted in a current focus of research studies (e.g., Forbes & Pfannkuch, 2009).

Theoretical issues of learning are being researched in some depth. For example, Reid and Reading (2010) built on their Consideration of Variation Hierarchy to assist analysis of students' concepts of explained and unexplained variation; Kalinowski (2010) presented a taxonomy of misconceptions about confidence intervals; Pfannkuch, Regan, Wild, and Horton (2008) pursued more in-depth conceptual understanding by an analysis of the way statistical 'stories' are told and reasoning takes place, and Pfannkuch et al. (2011) provided an argument for a major change in the way inference is introduced at university level. They argued for a shift from mathematical approaches to computer-based approaches.

Bilgin and Crowe (2008) investigated learning strategies of cultural groups and undergraduates versus postgraduates, and found significant differences only for the latter with postgraduates using more deep learning strategies. Bilgin (2010) followed this up with a study showing no differences in approach from second to third year, nor on other characteristics such as country, gender, or degree course enrolment.

Only one paper was found that dealt with a particular topic in advanced statistics. Kachapova and Kachapova (2010) discussed two techniques for teaching linear regression (using a population regression model and a geometric approach). There are very few such papers in the statistics education literature because the focus has been less on the technical aspects of advanced statistics and more on the pedagogy (theoretical and practical).

Methodological issues in statistics education research were addressed by Petocz and Newberry (2010) who argued for conceptual analysis rather than the common qualitative methods. They applied conceptual analysis to statistics education in psychology, and exposed some research issues. They queried the lack of questioning by researchers of their own conceptual constructs used in their research; they asked whether psychology and statistics might usefully be de-aggregated and become occasional collaborators instead; and they suggested that conceptual analysis may help researchers become intimately engaged in, and changed by, their research rather than maintaining 'objective' status.

Tertiary statistics education is a growing field, mirroring the last two decades of development at school level. The research is broad in its scope, covering affective, cognitive, statistical and social orientations. It also exhibits depth, with the appearance, on the one hand, of papers that consider research methodology, and, on the other, of papers that build on individual studies to conclude that there is a need for major change.

Why is this field strong? We suggest that it can be attributed to the growing presence of groups of researchers within an institution or region. A critical mass of people thinking about these issues appears to produce higher quality research and theoretical development over a period of time. Indeed, Australasia seems to be taking a lead internationally if this is judged by the appearance of researchers from the region in executive or editorial positions amongst the statistical education research community worldwide.

MATHEMATICS IN ENGINEERING

One study investigated the science/engineering split in university mathematics education (Plank, James, & Hannah, 2011). Drawing on grade data from 1000 students, it concluded that both lower and higher ability engineering students are disadvantaged by being separated, either from non-engineering students, or in ability groups. This appears to be the only study looking at the separation of engineering mathematics, but confirms the academic disadvantage of ability grouping known from secondary school studies.

The review period included publication of Henderson and Broadbridge's (2009) report of a project that looked at engineering mathematics in Australia. A questionnaire was sent to the 32 Australian institutions offering engineering degree programs, and received 27 replies. They noted widespread agreement that a good grounding in mathematics is essential for engineers. The same changes that affect all aspects of the university (widening diversity, lowering entry standards, and increased competition for curriculum space) have created challenges for engineering mathematics. New techniques must be employed to engage and effectively educate the student body. The report explored methods of teaching and learning trialled in Australia and overseas, finding that it was essential to provide additional mathematics support for students, both to aid the transition from school to university and to encourage students to complete extra mathematics practice. Computer-aided assessment (both in-house and commercial software) also provided students with additional mathematics practice that could be easily and quickly monitored by staff. Group learning has been seen to be an effective way to incorporate the teaching and learning of professional practice skills within science subjects.

While there is widespread disagreement about which mathematics topics should be included in the reduced number of mathematics subjects for engineering students and which teaching methods are most effective, this problem is minimised in institutions where the engineering department and the mathematics department have a formal joint committee that communicates openly and decides on a compromise mathematics curriculum. Joint ownership of the curriculum also helps to provide engineering applications that have a strong motivation for the study of mathematics. The study found some institutions had dramatically improved their students' ratings of mathematics instruction.

Henderson and Broadbridge (2009) provided a coherent set of recommendations that built on innovations, from around the country, to provide strategies to address identified challenges. The project itself has benefited from an improved level of co-operation between mathematics educators and the engineering profession.

MATHEMATICS IN NURSING

Two sets of researchers are working with the quantitative requirements of nursing. Pierce, Stacey, Steinle, and Widjaja (2008), when working with students in practical contexts, identified decimal understanding as an issue that required attention. Galligan (2008) and Galligan, Loch, and Lawrence (2010) similarly focused attention on numeracy competencies of nursing students, with a theoretical orientation derived from the work of Valsiner and Vygotsky.

CONCLUSION

The chapter on university learners of mathematics in the previous four-yearly review concluded that mathematics learning and teaching was in a "state of flux" (Wood, 2008, p. 91). The research reviewed in that chapter reported contradictory findings: perhaps because there was little attempt to delve beneath the surface features of lectures to inquire into conditions that support student learning. Deficit models of student learning were often implicit in these studies. There was significant interest in teaching with technology, but investigations into the effectiveness of computer hardware and software tended to look at student achievement gains without exploring the nature of the learning that was promoted. Although it was encouraging to see growing interest in university teaching and learning of mathematics, research was being conducted in a piecemeal fashion without the findings being interpreted in the light of theory. Not surprisingly then, the main challenge identified by the last review was building an integrated and theoretically informed body of research.

At the beginning of this review we asked whether the quantity and quality of papers on tertiary mathematics education in Australasia had changed, and stated that our review found indications of improved quality as well as some repetitive work. We also said that new directions were being developed. There have been changes. The expected growth in output is there, with a corresponding increase in quality. Repetitive or isolated studies are a smaller proportion of the total. Statistics education is now a major player.

Are we venturing into new areas? Yes, the issue of transition is now a major focus, and the nature of lecturing or other delivery methods is being scrutinised as a phenomenon, rather than exemplars being described. The professional development of university staff in pedagogical aspects of the mathematical sciences is also a developing theme. Two areas that remain essentially unresearched are higher levels of mathematics and transition to employment.

The current review found that investigations of mathematical topics and mathematics learning are now more theoretically based. Learning theories developed from a consideration of the nature of mathematics (Dubinsky's APOS theory and Tall's three worlds) are part of several papers. Unlike the 2011 Conference on European Research in Mathematics Education (CERME) Working Group 14 on university mathematics education, there has been no introduction of European perspectives such as Chevallard's anthropological theory of didactics (Chevallard, 1999) or Brousseau's didactic situations (Brousseau, 1997). These have proved insightful tools for analysis by Europeans, and may become so in Australasia.

As research in tertiary mathematical science education is still an emerging field in Australasia, it is worth commenting on strategies that might stimulate further development and dissemination of findings. The previous four-yearly review suggested that grants awarded by the Australian Learning and Teaching Council (ALTC) could support mathematicians in conducting more rigorous research into teaching and learning. This has certainly been the case, although publications to date have mainly been in the form of reports and conference papers rather than refereed journal articles. There have been three major ALTC grants awarded during the review period. One is concerned with building leadership capacity for development and sharing of mathematics learning resources across disciplines and universities. A second is a national discipline-specific professional development program for lecturers and tutors in the mathematical sciences (Brown et al., 2011). The third is for a major intervention of the quantitative sciences in university science education (Matthews, Adams, & Goos, 2009). In New Zealand, the Teaching and Learning Research Initiative funding from the NZ Council for Education Research has also successfully been tapped by researchers in tertiary mathematics and statistics education. These grants have had a positive effect on research activity and output.

The previous review also pointed out that publication opportunities occurred mainly in refereed conference proceedings and journals associated with conferences. This is still true. In particular, some of the refereed papers submitted to the biennial Delta conference on undergraduate mathematics teaching are selected for publication in a Special Issue of the *International Journal of Mathematical Education in Science and Technology*. While this combination of refereed conference proceedings and 'Special Issue' journal provides a valuable outlet for new as well as experienced researchers, the concluding comment of the last review remains valid: There is room for publication in a wider range of journals and in more depth than is presently the case. It is to be hoped that the growing variety and depth of research over the last four years, documented in this review chapter, will lead to stronger publications in the near future.

REFERENCES

Adams, P., Alcock, J., Bulmer, M., Grotowski, J., Hong, M., Jennings, M., & Scharaschkin, V. (2008). *A new enabling technology for learning and teaching quantitative skills*. Melbourne, Australia: ALTC.

Badcock, C. (2008). Drug development—What statistical skills are required by whom and when. In H. L. MacGillivray & M. A. Martin (Eds.), *OZCOTS 2008* (Proceedings of the 6th Australian Conference on Teaching Statistics, pp. 123–127). Melbourne, Australia: ALTC & SSAI.

Bartholomew, H., Darragh, L., Ell, F., & Saunders, J. (2011). 'I'm a natural and I do it for love!': Exploring students' accounts of studying mathematics. *International Journal of Mathematical Education in Science and Technology, 42*(7), 915–924. doi:10.1080/0020739X.2011.608863

Barton, B. (2011). Growing understanding of undergraduate mathematics: A good frame produces better tomatoes. *International Journal of Mathematical Education in Science and Technology, 42*(7), 963–973. doi:10.1080/0020739X.2011.611911

Barton, B., Clark, C., & Sheryn, L. (2010). Collective dreaming: A school-university interface. *New Zealand Journal of Mathematics, 40*, 15–31.

Barton, B., & Sheryn, L. (2010). *The Pipeline Report*. Berlin, Germany: ICMI.

Begg, A. (2011). Mathematics 101: Reconsidering the axioms. *International Journal of Mathematical Education in Science and Technology, 42*(7), 835–846. doi:10.1080/0020739X.2011.608859

Belward, S., Matthews, K., Rylands, L., Adams, P., Coady, C., & Simbag, V. (2011). A study of the Australian tertiary sector's portrayed view of the relevance of quantitative skills in science. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 107–114). Adelaide, SA: AAMT & MERGA.

Bilgin, A. (2010). Non-statisticians learning statistics. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistics Institute. Retrieved from http://icots.net/8/cd/pdfs/contributed/ICOTS8_C147_BILGIN.pdf

Bilgin, A., & Crowe S. (2008). Approaches to learning in statistics: Cross cultural learning behaviour among university students. *Asian Social Science, 4*(3), 37–43. doi:10.5539/ass.v4n3P36

Black, M. (2008). Unit standard assessment involving official statistics: An evaluation of four assessment tools. In H. L. MacGillivray & M. A. Martin (Eds.), *OZCOTS 2008* (Proceedings of the 6th Australian Conference on Teaching Statistics, pp. 128–135). Melbourne, Australia: ALTC & SSAI.

Blyth, B. (2010). Digital ink and annotations in e-teaching of advanced mathematics. *MSOR Connections*, *9*(4), 18–22.

Blyth, B., & Labovic, A. (2009). Using Maple to implement e-learning integrated with computer aided assessment. *International Journal of Mathematical Education in Science and Technology, 40*(7), 975–988. doi:10.1080/00207390903226856

Britton, S., & Henderson, J. (2009). Linear algebra revisited: An attempt to understand students' conceptual difficulties. *International Journal of Mathematical Education in Science and Technology, 40*(7), 963–974. doi:10.1080/00207390903206114

Brousseau, G. (1997). *Theory of didactical situations in mathematics: Didactique des mathématiques, 1970–1990.* Dordrecht, The Netherlands: Kluwer Academic.

Brown, N., Bower, M., Skalicky, J., Wood, L. N., Donovan, D., Loch, B., ... Joshi, N. (2010). A professional development framework for teaching in higher education. In M. Devlin, J. Nagy and A. Lichtenberg (Eds.), *Research and development in higher education: Reshaping higher education Volume 33*. Melbourne, Australia: Higher Education Research and Development Society of Australasia, Inc. Retrieved from http://www.herdsa.org.au/wp-content/uploads/conference/2010/papers/HERDSA2010_Brown_N.pdf

Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en Didactique des Máthématiques, 19*(2), 221–266.

Clark, M., & Lovric, M. (2009). Understanding secondary–tertiary transition in mathematics. *International Journal of Mathematical Education in Science and Technology, 40*(6), 755–776. doi:10.1080/00207390902912878

Cretchley, P. (2009). Are Australian universities promoting learning and teaching activity effectively? An assessment of the effects on science and engineering academics. *International Journal of Mathematical Education in Science and Technology, 40*(7), 865–875. doi:10.1080/002073909 03199228

Cumming, G. (2010). Statistics education in the social and behavioural sciences: From dichotomous thinking to estimation thinking and meta-analytic thinking. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistics Institute. Retrieved from http://icots.net/8/cd/pdfs/contributed/ICOTS8_C111_CUMMING.pdf

David, I., & Brown, J. (2010). Implementing the change: Teaching statistical thinking not just the methods. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistics Institute. Retrieved fromhttp://icots.net/8/cd/pdfs/contributed/ICOTS8_C155_DAVID.pdf

Dhand N. K., & Thomeson, P. C. (2009). Scenario-based approach for teaching biostatistics to veterinary students. In C. Reading (Ed.), *Next steps in statistics education* (Proceedings from the 2009 IASE Satellite Conference). Durban, South Africa: International Association for Statistics Education. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications/sat09/8_3.pdf

Dubinsky, E. (1992).A learning theory approach to calculus. In Z. Karian, (Ed*.) Symbolic computation in undergraduate mathematics education, MAA Notes, 24,* (pp. 48–55). Washington, DC: MAA.

Easdown, D. (2009). Syntactic and semantic reasoning in mathematics teaching and learning. *International Journal of Mathematical Education in Science and Technology, 40*(7), 941–949. doi:10.1080/00207390903205488

Fidler, F. (2010). The American Psychological Association publication manual sixth edition: Implications for statistics education. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistics Institute. Retrieved from http://icots.net/8/cd/pdfs/contributed/ICOTS8_C156_FIDLER.pdf

Forbes, S. (2009). Creation and evaluation of a workplace based certificate in official statistics for government policy makers. In C. Reading (Ed.), *Next steps in statistics education* (Proceedings from the 2009 IASE Satellite Conference). Durban, South Africa: International Association for Statistics Education. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications/sat09/9_2.pdf

Forbes, S., & Pfannkuch, M. (2009). Developing statistical thinking: Teaching and learning. In R. Averill & R. Harvey (Eds.), *Teaching secondary school mathematics and statistics: Evidence-based practice* (pp.93–128). Wellington, New Zealand: Council for Educational Research.

Galligan, L. (2008). Using Valsiner. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 211–218). Brisbane, QLD: MERGA.

Galligan, L., Loch, B., & Lawrence, J. (2010). Building academic numeracy in a first year nursing course using an evaluative model of program development. In D. Coben & J. O'Donoghue, (Eds.), *TSG 8: Adult mathematics education: Papers from the Topic Study Group (TSG) 8, 11th International Congress on Mathematical Education* (pp. 165–181). Limerick, Ireland: ALM in association with NCE-MSTL.

Galligan, L., Loch, B. I., McDonald, C., & Taylor, J. A. (2010). The use of tablet and related technologies in mathematics teaching. *Australian Senior Mathematics Journal, 24*(1), 38–51.

Godfrey, D., & Thomas, M. O. J. (2008). Student perspectives on equation: The transition from school to university. *Mathematics Education Research Journal, 20*(2), 71–92.

Gordon, I., Finch, S., & Maillardet, R. (2008). Statistics as breadth: The Melbourne experiment. I: Content and delivery. II: Assessment. In H. L. MacGillivray & M. A. Martin (Eds.), *OZCOTS 2008* (Proceedings of the 6th Australian Conference on Teaching Statistics, pp. 143–155). Melbourne, Australia: ALTC & SSAI.

Gordon, S., & Nicholas, J. (2008). Why do statistics educators use examples to teach statistics? In H. L. MacGillivray & M. A. Martin (Eds.), *OZCOTS 2008* (Proceedings of the 6th Australian Conference on Teaching Statistics, pp. 97–103). Melbourne, Australia: ALTC & SSAI.

Hannah, J., Stewart, S., & Thomas, M. O. J. (2011). Analysing lecturer practice: The role of orientations and goals. *International Journal of Mathematical Education in Science and Technology, 42*(7), 975–984. doi:10.1080/0020739X.2011.610008

Henderson, S., & Broadbridge, B. (2009). Engineering mathematics education in Australia. *MSOR Connections, 9*(1), 12–17.

Holton, D., Muller, E., Oikkonen, J., Sanchez V., Oscar A., & Zizhao, R. (2009). Some reasons for change in undergraduate mathematics enrolments. *International Journal of Mathematical Education in Science and Technology, 40*(1), 3–15.

Hong, Y. Y., Kerr, S., Klymchuk, S., McHardy, J., Murphy, P., Spencer, S., ... Watson, P. (2009). A comparison of teacher and lecturer perspectives on the transition from secondary to tertiary mathematics education. *International Journal of Mathematical Education in Science and Technology, 40*(7), 877–889.

James, A., Montelle, C., & Williams, P. (2008). From lessons to lectures: NCEA mathematics results and first-year mathematics performance. *International Journal of Mathematical Education in Science and Technology, 39*(8), 1037–1050.

Jennings, M. (2009). Issues in bridging between senior secondary and first year university mathematics. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 273–280). Palmerston North, NZ: MERGA.

Kachapova, K., & Kachapova, I. (2010). Orthogonal projection in teaching regression and financial mathematics. *Journal of Statistics Education, 18*(1), 1–18. Retrieved from http://www.amstat.org/publications/jse/v18n1/kachapova.pdf

Kalinowski, P. (2010). Identifying misconceptions about confidence intervals. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistics Institute. Retrieved from http://icots.net/8/cd/pdfs/contributed/ICOTS8_C104_KALINOWSKI.pdf

Kalinowski, P., Lai, J., Fidler, F., & Cumming, G. (2008). Qualitative research: An essential part of statistical cognition research. *Statistics Education Research Journal, 9*(2), 22–34.

Klinger, C. M. (2008). Experience, the difference: Maths attitudes and beliefs in commencing undergraduate students and pre-tertiary adult learners. In T. Maguire, N. Colleran, O. Gill, & J. O'Donoghue (Eds.), *The changing face of adults mathematics education*, (pp. 197–206.) Limerick, Ireland: Adults Learning Mathematics—A Research Forum & University of Limerick.

Klymchuk, S. (2010). *Counterexamples in calculus*. Washington, DC: MAA.

Klymchuk, S., & Thomas, M. O. J. (2011). The influence of attention on mathematical knowledge of teachers and lecturers: A comparison. *International Journal of Mathematical Education in Science and Technology, 42*(7), 1011–1020. doi:10.1080/0020739X.2011.607246

Klymchuk, S., Zverkova, T., Gruenwald, N., & Sauerbier, G. (2008). Increasing engineering students' awareness to environment through innovative teaching of mathematical modelling. *Teaching Mathematics and its Applications, 27*(3), 123–130. doi:10.1093/teamat/hrn007

Klymchuk, S., Zverkova, T., Gruenwald, N., & Sauerbier, G. (2010). University students' difficulties in solving application problems in calculus: Student perspectives. *Mathematics Education Research Journal, 22*(2), 81–91.

Kyng, T., Tickle, L., & Wood, L. N. (2011). Graduates' use of technical software in financial services. In M. Lau & S. Sugden (Eds.), *Applications of spreadsheets in education: The amazing power of a simple tool* (pp.175–194). Bussam, The Netherlands: Bentham Science.

Lai, J. (2010). Dichotomous thinking: A problem beyond NHST. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistics Institute. Retrieved from http://icots.net/8/cd/pdfs/contributed/ICOTS8_C101_LAI.pdf

Lerman, S., & Davis, B. (2009). *Mathematical action and structures of noticing*. Rotterdam, The Netherlands: Sense.

Lim, L. L., Tso, T. Y., & Lin, F. L. (2009). Assessing science students' attitudes to mathematics: A case study on a modelling project with mathematical software. *International Journal of Mathematical Education in Science and Technology, 40*(4), 441–453.

Loch, B. (2010). What do on campus students do with mathematics lecture screencasts at a dual-mode Australian university? *Proceedings of 'Opening Windows on Mathematics and Statistics', the Continuing Excellence in the Teaching and Learning of Maths, Stats and Operational Research Conference* (CETL-MSOR 2009), (pp. 43–37). Milton Keynes, United Kingdom: The Maths, Stats & OR Network.

Loch, B. (2011).Screencasting for mathematics online learning – A case study of a first year operations research course at a dual-mode Australian university. In A. A. Juan, M. A. Huertas, S. Trenholm & C. Steegman (Eds.), *Teaching mathematics online: Emergent technologies and methodologies* (pp. 43–59). Hershey, PA: IGI Global.

Loch, B., Galligan, L., Hobohm, C., & McDonald, C. (2011). Learner-centred mathematics and statistics education using netbook tablet PCs. *International Journal of Mathematical Education in Science and Technology, 42*(7), 939–949.

Low Choy, S., & Wilson, T. (2009). How do experts think about statistics? Hints for improving undergraduate and postgraduate training. In C. Reading (Ed.), *Next steps in statistics education* (Proceedings from the 2009 IASE Satellite Conference). Durban, South Africa: International Association for Statistics Education. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications/sat09/4_3.pdf

Luo, K., Vemulpad, S., & Bilgin, A. A. (2008). Statistics learning: Making it easy for non-statisticians. In H. L. MacGillivray & M. A. Martin (Eds.), *OZCOTS 2008* (Proceedings of the 6th Australian Conference on Teaching Statistics, pp. 114–122). Melbourne, Australia: ALTC & SSAI.

MacGillivray, H. (2009). Examples from an introductory course in developing probabilistic statistical thinking: Part 1 & 2. *MSOR Connections, 8*(1), 7–10, and *8*(2), 2–5.

Mallet, D. G., & McCue, S.W. (2009). Constructive development of the solutions of linear equations in introductory ordinary differential equations. *International Journal of Mathematical Education in Science and Technology, 40*(5), 587–595.

Martin, P. (2008). Assessment of participants in an industrial training program. In H. L. MacGillivray & M. A. Martin (Eds.), *OZCOTS 2008* (Proceedings of the 6th Australian Conference on Teaching Statistics, pp. 136–142). Melbourne, Australia: ALTC & SSAI.

Matthews, K. E., Adams, P., & Goos, M. (2009). Putting it into perspective: Mathematics in the undergraduate science curriculum. *International Journal of Mathematical Education in Science and Technology, 40*(7), 891–902.

McAndrew, A. (2009). Mathematical induction, difference equations and divisibility. *International Journal of Mathematical Education in Science and Technology, 40*(8), 1013–1025.

McDonald, C., Loch, B., & Lloyd, M. (2010). Interaction in synchronous chat tutorials facilitate learning in introductory statistics. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistics Institute. Retrieved from http://icots.net/8/cd/pdfs/contributed/ICOTS8_C179_MCDONALD.pdf

McPhan, G., & Pegg, J. (2009). The option of selecting higher-level mathematics courses: Transitional tensions. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 363–370). Palmerston North, NZ: MERGA.

Narayanan, A., Klymchuk, S., Gruenwald, N., Sauerbier, G., & Zverkova, T. (2010). Mathematical modelling of infectious disease with biomathematics: Implications for teaching and research. In G. Kaiser (Ed.), *Proceedings of the 14th International Conference on the Teaching of Mathematical Modelling and Applications: ICTMA-14*. Hamburg, Germany: Horwood.

Neumann, D. L., Hood, M., & Neumann, M. M. (2009). Statistics? You must be joking: The application and evaluation of humor when teaching statistics. *Journal of Statistics Education, 17*(2). Retrieved from http://www.amstat.org/publications/jse/v17n2/neumann.html

Neumann, D. L., Neumann, M. M., & Hood, M. (2010). The development and evaluation of a survey that makes use of student data to teach statistics. *Journal of Statistics Education, 18*(1), 1–19. Retrieved from http://www.amstat.org/publications/jse/v18n1/neumann.pdf

Oates, G. (2009). Relative values of curriculum topics in undergraduate mathematics in an integrated technology environment. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 419–426). Palmerston North, NZ: MERGA.

Oates, G. (2010). Integrated technology in undergraduate mathematics: Issues of assessment. *The Electronic Journal of Mathematics and Technology, 4*(2), 1–13.

Oates, G. (2011). Sustaining integrated technology in undergraduate mathematics. *International Journal of Mathematical Education in Science and Technology, 42*(6), 709–721.doi: 10.1080/0020739X.2011.575238

Paterson, J., & Sneddon, J. (2011).Conversations about curriculum change: Mathematical thinking and team based learning in a discrete mathematics course. *International Journal of Mathematical Education in Science and Technology, 42*(7), 879–889. doi:10.1080/0020739X.2011.613487

Paterson, J., Thomas, M. O. J., & Taylor, S. (2011). Reaching decisions via internal dialogue: Its role in a lecturer professional development model. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 353–360). Ankara, Turkey: PME.

Petocz, A., & Newberry, G. (2010). On conceptual analysis as the primary qualitative approach to statistics education research in psychology. *Statistics Education Research Journal, 9*(2), 123–145.

Petropoulou, G., Potari, D., & Zachariades, T. (2011). Inquiring mathematics teaching at the university level. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 386–392). Ankara, Turkey: PME.

Pfannkuch, M., Regan, M., Wild, C., Budgett, S., Forbes, S., Harraway, J., & Parsonage, R. (2011). Inference and the introductory statistics course. *International Journal of Mathematical Education in Science and Technology, 42*(7), 903–913. doi:10.1080/0020739X.2011.604732

Pfannkuch, M., Regan, M., Wild, C., & Horton, N. J. (2008). Telling data stories: Essential dialogues for comparative reasoning. *Journal of Statistics Education, 18*(1), 1–38. Retrieved from http://www.amstat.org/publications/jse/v18n1/pfannkuch.pdf

Pierce, R., Stacey, K., Steinle, V., & Widjaja, W. (2008). Understanding decimal numbers: A foundation for correct calculations. *International Journal for Nursing Education Scholarship, 5*(1), 1–15. doi:10.2202/1548-923X.1439

Plank, M., James, A., & Hannah, J. (2011). Group by subject or by ability? Tertiary mathematics for engineering students. *International Journal of Mathematical Education in Science and Technology, 42*(7), 857–865.

Pruzek, R. M., & Helmreich, J. E. (2009). Enhancing dependent sample analyses with graphics. *Journal of Statistics Education, 17*(1). Retrieved from http://www.amstat.org/publications/jse/v17n1/helmreich.html

Pytlak, M., Swoboda, E., & Rowland, T. (2011). *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education*. University of Rzeszów, Poland: European Society in Mathematics Education.

Reid, A., Abrandt Dahlgren, M., Dahlgren, L. O. & Petocz, P. (2011). *From expert student to novice professional*. Springer, Dordrecht, The Netherlands.

Reid, A., Dahlgren, L. O., Dahlgren, M. A., & Petocz, P. (2010). Applying a model of professional learning to case studies in statistics education. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistics Institute. Retrieved from http://icots.net/8/cd/pdfs/invited/ICOTS8_6G2_REID.pdf

Reid, A., & Petocz, P. (2008). A tertiary curriculum for future professionals. In C. Nygaard & C. Holtham (Eds.), *Understanding learning-centred higher education* (pp. 31–49). Copenhagen, Denmark: CBS Press.

Reid, J., & Reading, C. (2010). Developing a framework for reasoning about explained and unexplained variation. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistics Institute. Retrieved from http://icots.net/8/cd/pdfs/contributed/ICOTS8_C169_REID.pdf

Richardson, A. (2008). Language support for statistics learners. In H. L. MacGillivray & M. A. Martin (Eds.), *OZCOTS 2008* (Proceedings of the 6th Australian Conference on Teaching Statistics, pp. 89–96). Melbourne, Australia: ALTC & SSAI.

Rubinstein, H. (2009). *A national strategy for mathematical sciences in Australia*. Melbourne, Australia: Australian Mathematical Sciences Institute.

Rylands, L. J., & Coady, C. (2009). Performance of students with weak mathematics in first-year mathematics and science. *International Journal of Mathematical Education in Science and Technology, 40*(6), 741–753.

Simpson, J. M., Ryan, P., Carlin, J. B., Gurrin, L., & Marschner, I. (2009). Training a new generation of biostatisticians: A successful consortium model. *Journal of Statistics Education, 17*(2). Retrieved from http://www.amstat.org/publications/jse/v17n2/simpson.pdf

Sowey, E., & Petocz, P. (2010). Enriching statistics courses with statistical diversions. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistics Institute. Retrieved from http://icots.net/8/cd/pdfs/invited/ICOTS8_4D3_SOWEY.pdf

Stewart, S. (2008). *Understanding linear algebra concepts through the embodied, symbolic and formal worlds of mathematical thinking*. (Unpublished doctoral dissertation).The University of Auckland, New Zealand.

Stewart, S. (2009). Understanding linear algebra concepts through APOS and the three worlds of mathematical thinking theories. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 169–176). Thessaloniki, Greece: PME.

Tall, D. (2004). Building theories: The three worlds of mathematics. *For The Learning of Mathematics, 24*(1), 29–35.

Taylor, J. A. (2008). Assessment in first year university: A model to manage transition. *Journal of University Teaching and Learning Practice, 5*(1), 19–33.

Thomas, J., Muchatuta, M., & Wood, L. N. (2009). Mathematical sciences in Australia. *International Journal of Mathematical Education in Science and Technology, 40*(1), 17–26.

Thomas, M. O. J., Kensington-Miller, B., Bartholomew, H., Barton, B., Paterson, J. & Yoon, C. (2011). Mathematics undergraduate teaching: Perspectives and interactions. *Proceedings of the Conference of the New Zealand Association for Research in Education.* Auckland, New Zealand: NZARE.

Thomas, M. O. J., & Stewart, S. (2011). Eigenvalues and eigenvectors: Embodied, symbolic and formal thinking. *Mathematics Education Research Journal, 23*(3), 275–296.

Tonkes, E. J., Isaac, P. S., & Scharaschkin, V. (2009). Assessment of an innovative system of lecture notes in first-year mathematics. *International Journal of Mathematical Education in Science and Technology, 40*(4), 495–504.

Viirman, O. (2011). Discursive practices of two university teachers on the concept of 'linear transformation'. In B. Ubuz (Ed.), *Developing mathematical thinking* (Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education, pp. 313–320). Ankara, Turkey: PME.

Wilson, R., & Bulmer, M. (2008). Joining the DOTS: Directed online tutorials for statistics. In H. L. MacGillivray & M. A. Martin (Eds.), *OZCOTS 2008* (Proceedings of the 6th Australian Conference on Teaching Statistics, pp. 77–80). Melbourne, Australia: ALTC & SSAI.

Wiwatanapataphee, B., Noinang, S., Wu, Y. H., & Nuntadilok, B. (2010). An integrated Powerpoint-Maple based teaching-learning model for multivariate integral calculus. *International Electronic Journal of Mathematics Education, 5*(1), 5–31.

Wood, G., & Petocz, P. (2008). One in a million: An alternative transformation of the first-year statistics course. *Asian Social Science, 4*(3), 43–47.

Wood, L. N. (2008). University learners of mathematics. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W-T. Seah, & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004–2007* (pp. 73–97). Rotterdam, The Netherlands: Sense.

Wood, L. N. (2010). Graduate capabilities: putting mathematics into context. *International Journal of Mathematical Education in Science and Technology, 41*(2), 189–198.

Wood, L. N. (2011). Practice and conceptions: Communicating mathematics in the workplace. *Educational Studies in Mathematics, 79*(1). doi:10.1007/s10649-011-9340-3

Wood, L. N., Mather, G., Petocz, P., Reid, A., Engelbrecht, J., Harding, A., ... Perrett, G. (2011). University students' views of the role of mathematics in their future. *International Journal of Science and Mathematics Education.* doi:10.1007/s10763-110-9279-y

Wood, L. N., & Solomonides, I. (2008). Different disciplines, different transitions. *Mathematics Education Research Journal, 20*(2), 117–134.

Wood, L. N., Vu, T., Bower, M., Brown, N., Skalicky, J., Donovan, D., ... Bloom, W. (2011). Professional development for teaching in higher education. *International Journal of Mathematical Education in Science and Technology, 42*(7), 997–1009.

Worsley, S. (2011). The big ideas in two large first level courses of undergraduate mathematics. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 839–845). Adelaide, SA: AAMT & MERGA.

Yoon, C., Dreyfus, T., & Thomas, M. O. J. (2010). How high is the tramping track? Mathematising and applying in a calculus model-eliciting activity. *Mathematics Education Research Journal, 22*(1), 147–157.

Yoon, C., Kensington-Miller, M., Sneddon, J., & Bartholomew, H. (2011). It's not the done thing: Social norms governing students' passive behaviour in undergraduate mathematics lectures. *International Journal of Mathematical Education in Science and Technology*. doi: 10.1080/0020739X.2011.573877.

Yoon, C., & Sneddon, J., (2011). Student perceptions of effective use of tablet PC recorded lectures in undergraduate mathematics courses. *International Journal of Mathematical Education in Science and Technology, 42*(4), 425–445.

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UNCERTAINTY IN MATHEMATICS EDUCATION: WHAT TO DO WITH STATISTICS?

Key words: differences and relationships between mathematics and statistics; statistics education.

INTRODUCTION

The use of the word 'uncertainty' in the title of this chapter is intended to convey both the dilemma in the question and the challenge that statistics presents to the deterministic foundations of mathematics. The growth in the field of statistics over the past century, especially in the light of advancing technology, has resulted in a downward curriculum thrust from tertiary education, to secondary education, to primary education.

In the *New Zealand Curriculum* (Ministry of Education, 2007) the relative importance of statistics has been elevated in a number of ways, the most obvious being the renaming of the learning area from Mathematics to Mathematics and Statistics. Frankcom (2008) indicated:

> The change was made to reflect the difference between deterministic (mathematical) and stochastic (statistical) thinking. The inclusion of statistics in the title of this teaching area reflects the increasing importance of using and interpreting data as part of critical citizenship. (p. 3)

The recent development of the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2011) has also emphasised the importance of statistics in a modern school curriculum. One content strand of the mathematics curriculum is titled "Statistics and Probability", in recognition of the increased emphasis on statistical knowledge in the 21st century, but there has been no apparent desire to rename the curriculum itself in Australia.

At the same time, however, the inclusion of statistics and probability as a content strand of the mathematics curriculum has ignited discussion about similarities and differences between the two disciplines of mathematics and statistics. Although the claims on curriculum space have existed in both Australia and New Zealand since the early 1990s, acceptance has not been universal. There has been considerable discussion in the press about the development of the Australian curriculum, with many articles criticising the perceived emphasis on statistics (e.g., Polster & Ross, 2010; Slattery & Perpitch, 2010). Examples of textbook series with little or no statistics (Brown et al, 2006) and strong views by

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mathematicians to reduce the quantity of statistics in the Australian curriculum (e.g., Dean, 2010) indicate that, at least in Australia, there is still resentment in some circles about the encroachment of this newcomer into the traditional mathematics club. Such discussions suggest that the curriculum values of mathematics and statistics may be different, leading inevitably to tensions for teachers and curriculum designers.

In this chapter we aim to explore the issues around the synergies and tensions between mathematics and statistics and the implications for mathematics education. The chapter is not intended to be an exhaustive review of the domain of statistics education research in Australia and the surrounding region. Nevertheless, there is a significant body of research from Australian and New Zealand researchers that is beginning to inform both curriculum development and the teaching of statistics, especially at the school level. The chapter, hence, begins with a brief consideration of the current state of statistics education research in the region to place the later discussion within a context. Following this examination, the chapter scrutinises research surrounding the differing perceptions of mathematics and statistics from the viewpoints of students and teachers. This scrutiny leads to a consideration of teaching statistics and mathematics and some observations on student outcomes. One of the major differences between statistics and mathematics is the place of context. It is particularly relevant at the school and introductory tertiary level, and is the focus of the next major section of the chapter. The challenge for teachers in integrating various aspects of content knowledge, context knowledge, pedagogical knowledge, and student knowledge in mathematics and statistics is considered under the general heading of pedagogical content knowledge. The chapter concludes with a discussion of the implications of the debate for mathematics education in Australasia, focusing on the synergies and tensions.

The chapter draws on recent published research from Australasia. Sources for this review come from primary, secondary, and tertiary education levels. In addition to the Mathematics Education Research Group of Australasia (MERGA) publications (MERJ, MTED and Annual Conference Proceedings), the authors considered the International Group for the Psychology of Mathematics Education (PME) proceedings and conducted a key word search in key mathematics and statistics education journals. In addition they relied heavily on the proceedings from two conferences: the 2008 Joint ICMI/IASE Study conference and the related book: *Teaching Statistics in School Mathematics—Challenges for Teaching and Teacher Education*; and the 8th International Conference on Teaching Statistics in 2010.

STATISTICS EDUCATION RESEARCH IN AUSTRALASIA

Australasian researchers have been particularly active in the field of statistics education and this work is well recognised internationally. As a discipline, statistics education is still developing, and much of the current research focuses on the development of students' understanding of specific statistical concepts. Ideas such as reasoning about variation, distribution, informal inference and the statistics investigation cycle have all been the subject of Australasian studies. Many of these studies mirror early mathematics education research with, for example, hierarchies of development being identified such as that for tertiary students' understanding of variation (Reid & Reading, 2008) or middle school students' probabilistic reasoning in the context of multiple dice (Watson & Kelly, 2009) as well as exploration of specifically statistical notions such as sample size (Bill, Henderson, & Penman, 2010) and distribution (Watson, 2009).

A key aspect of statistical thinking is that of inference, and here also Australasian researchers have made a large contribution in mapping the transition from informal to formal inference (e.g., Arnold, Pfannkuch, Wild, Regan, & Budgett, 2011; Makar, Bakker, & Ben-Zvi, 2011; Makar & Rubin, 2009; Watson, 2008; Wild, Pfannkuch, Regan, & Horton, 2011). Technology use has also been a focus, especially with the development of new, specially designed software packages important for both teaching and learning statistical concepts (e.g., Bill & Gayton, 2010; Fitzallen & Watson, 2010; Ireland & Watson, 2009; Watson & Donne, 2009), as well as its influence more broadly (Callingham, 2011). There has also been work focusing on the foundations of statistical literacy (e.g., Watson, 2011a, 2011b) and its necessity across a range of statistical fields (e.g., Forbes, Camden, Pihama, Bucknall, & Pfannkuch, 2011).

Many of these core ideas in statistics understanding are not yet recognised as part of the usual mathematics curriculum. The significant contributions made by Australian and New Zealand researchers have the potential to impact on future curriculum development, nationally and internationally. The work on the development of statistical inference, for example, is recognised in the UK (Wild et al., 2011). There are opportunities to strengthen links between mathematical modelling and statistics and some of the possibilities are discussed further in following sections.

PERCEPTIONS OF DIFFERENCES BETWEEN MATHEMATICS AND STATISTICS

What is the extent of the difference that is considered by some to exist between statistics and mathematics? Forbes and Pfannkuch (2009) stated categorically in the context of teaching statistics, "one is not teaching a branch of mathematics, but teaching a discipline that has its own independent intellectual method" (p. 94). Gordon and Finch (2010, p. 2) appeared to agree on designing a 'breadth' unit focusing on overriding statistical concepts without numerical detail in order to attract students "not strongly disposed towards mathematics" In each case the authors described statistics courses or units that they want to distinguish from mathematics courses or units. In contrast to this strong distinction, other authors speak only of 'mathematics' in contexts where general questions are asked of students. Horne (2009) asked Year 7 to Year 10 students what they thought of 'maths' with no sub-topics considered and Wood and Solomonides (2008) questioned students about their transitions into university and then into professions also with questions only based on 'mathematics', even though some of their students were majoring in statistics within mathematical sciences degrees. Whether these authors see mathematics and statistics as indistinguishable in the larger arena of their research questions is not clear.

Houston et al. (2010) asked tertiary students "what is mathematics?" They found that over half of the respondents had an instrumental view of mathematics as calculations or as a toolbox of applications. In this study statistics were again not explicitly included. Petocz and Reid (2010) found at least one tertiary student who equated mathematics and statistics in their study of what it means to be a statistician: "Like, just like count something, and find something wrong and something like that, just like math" (p. 277). This description was placed by Petocz and Reid in the lowest of six levels of conceptions of statistics, whereas their highest level of expansive descriptions corroborated Forbes and Pfannkuch's (2009) thinking. In contrast to this view associating tertiary statistics with mathematics, at the middle school level one teacher in the study by Watson and Nathan (2010b) commented that her students absolutely did not see mathematics and statistics as the same. They complained that they did not want to read, think and write about statistics because that was English, not maths—maths was 'sums'. The students just wanted to do straight mathematical computations with no complications. It would appear that the student quoted by Petocz and Reid had had very different experiences in his statistics classes than the middle school students who linked statistics with doing English.

In considering students' interest in statistical matters, Carmichael and Hay (2010) found changes in students' views as they progressed through the high school years. Younger students tended to be interested in statistics when novel approaches were used, such as activities using computers or involving chance, whereas older students appeared more interested when social contexts were involved. Similar findings were observed through students' feedback about changes to a first-year tertiary statistics course in New Zealand (David & Brown, 2010), where more active and relevant approaches to teaching increased students' confidence and course satisfaction ratings. It seemed that students at upper secondary and tertiary levels appreciated the social relevance of statistics.

Obviously the numbers and operations of mathematics are required in statistics, but how much more? One of the fundamental mathematical concepts employed in statistics is proportional reasoning. In a PISA 2009 item called "Robberies", students were required to interpret a media-based claim that required proportional reasoning in a statistical context. In Australia only 40% of students were able to respond at the highest level (Thomson, De Bortoli, Nicholas, Hillman, & Buckley, 2010). Similarly, Thomson (2009) reported that only 45% of Year 8 students could apply proportional reasoning in a probabilistic context in TIMSS 2007. She suggested that this was not altogether surprising when 35% of the students reported that they had never experienced probabilistic reasoning activities. These findings indicated that despite appearing in the mathematics curriculum, many students do not appear to experience statistical ideas.

In surveys with both students and teachers, Watson, Callingham, and Donne (2008) employed two proportional reasoning items. The social contexts provided were minimal—selecting a ball unseen from one of two boxes and considering the association of lung disease and smoking in a two-way table—placing the items within the mathematics or statistics arena, rather than other curriculum areas such as studies of society or health education. To achieve the highest level responses on both questions required students to be able to apply proportional reasoning to the problems presented. Difficulty with proportional reasoning appeared to hinder many students in achieving the highest level response on the probability problem of selecting a ball unseen from a box (26% in Grades 5/6 to 65% for Grades 9/10 achieved the highest level). On the harder two-way table problem very few students could effectively use proportional reasoning skills (only 2% in Grades 5/6 to 8% in Grades 9/10 achieved the highest level). Of concern, however, was that although well over half of the teachers recognised the mathematics in the problem, many could not suggest ways of remediating incorrect responses that went beyond telling the answer. Although focusing more on teachers' pedagogical content knowledge than on specific mathematical or statistical knowledge, Watson and Nathan (2010a) found that this two-way table problem about smoking and lung disease was seen by most teachers in their study as a mathematics problem involving the comparison of proportions rather than as a problem of the statistical association of two variables, suggesting a mathematical view of the problem rather than a statistical perspective. If the view of these teachers about statistics is common, the limitation of this perspective may also have impacted on students' understanding as demonstrated in the PISA and TIMSS reports.

Other researchers have considered structural differences between mathematics and statistics. Sharma (2008), for example, described some tensions between mathematics and statistics conventions, such as how in mathematics, when listing the elements of sets, it is not acceptable to repeat set elements, whereas when listing data elements in statistics, repetition, if it occurred, was essential. There was no question, however, that mathematical reasoning is an essential ingredient of statistical understanding, and that working within statistics could help to develop mathematical understanding. An example of this intersection between mathematics and statistics arose in a discussion of hat plots, a data summary representation from the *TinkerPlots* software (Konold & Miller, 2005) based on percentages of the data covered by the crown and brim of a hat over a stacked dot plot. Watson, Fitzallen, Wilson, and Creed (2008) reported that when asked to explain their hat plot representations, students developed confidence in their use of percent, which they had been taught elsewhere in the mathematics curriculum. In this instance there was a synergy between the mathematical and statistical understandings: Without per cent understanding students could not understand the hat plots but, in combining the two, the hats reinforced the meaning of per cent and the students were better able to understand the representation of the data.

Teaching Statistics and Mathematics in the School Classroom

Several researchers considered aspects of teaching statistics. Hay (2010) used a software program, *Leximancer* (Smith, 2009), with general questions from interviews about teaching statistics from 12 teachers who taught from Year 7 to Year 12. The software generated a concept map showing that teachers identified the engagement of students through doing statistics as critical. Interesting activities that led to active participation by students were seen as central to developing students' statistical literacy. Such views resonated with the findings of Carmichael and Hay (2010) and Arnold et al. (2011). In mathematics, as opposed to statistics, there was also evidence that providing students with appropriately attractive tasks, especially through an inquiry approach, led to improved understanding (Fielding-Wells & Makar, 2008; Makar, 2011). Unsurprisingly, pedagogy that encouraged engagement would seem to be similar in statistics and mathematics education, and learning statistics could reinforce mathematical understanding.

In another analysis of data from 40 teacher interviews addressing Pedagogical Content Knowledge (PCK) in the area of statistics, Watson and Nathan (2010b) found a range of perceptions about the differences between mathematics and statistics in the classroom. In considering the themes that emerged when teachers distinguished mathematics and statistics in the classroom, Watson and Nathan found responses clustered into three groupings: (a) teaching practices; (b) curriculum values; and (c) cognitive experiences. Half of the teachers made comments related to teaching practices. When teaching statistics there was more action and fun in the classroom, the visual aspect of learning was enhanced, and more collaboration took place. Over half of the teachers made comments favouring statistics over mathematics based on curriculum values: 'low' achievers had a greater chance to contribute positively and high achievers did not dominate in statistics; topics were more contextualised and contested, being less 'mechanical'; statistics was imagined as being cross-curricular whereas mathematics was not; statistics was practical, and more concrete than abstract. More than half the teachers commented positively about statistics under the theme of cognitive experience, referring to: the lively discussions which promoted critical thinking by students; diverse ways of looking at questions; students posing questions, giving them control they had not had before; and the student experience being more 'exciting' and the students more engaged. These findings reinforce those of Hay (2010). The overall impression conveyed by Watson and Nathan based on their analysis was that teachers saw mathematics as 'mechanical' and statistics as encouraging 'critical thinking'. It appeared that most of these teachers did not appreciate the potential of mathematics teaching to model critical thinking, although Afamasaga-Fuata'i (2008) indicated that Year 10 students could develop critical thinking about mathematics when provided with suitable tools.

Relationship between Mathematics and Statistics Outcomes

At the school level, statistics is generally taught and assessed within the mathematics subject area. The assumption is that learning outcomes in mathematics and statistics will be highly correlated, and that valid inferences may be made about students' statistical competence based on their mathematical ability. There are some reasons to question these assumptions.

As assessed through NAPLAN, statistics and probability are limited to procedural aspects such as reading data from graphs and tables (Nisbet, 2010, 2011). The skills required are largely mathematical, and do not cover aspects of statistics such as planning and conducting an investigation, data collection and analysis, or questioning claims about data. Carmichael, Callingham, Hay, and Watson (2010) found that in middle-years classrooms the impact of prior mathematical achievement on statistics outcomes was mediated by students' self-efficacy with respect to statistics. This finding differed from similar studies in mathematics education, where prior achievement was the best direct predictor of future success, and suggested that students may perceive their capacity to do mathematics differently from that in statistics.

Another difference was identified by Callingham (2010), who found that in the context of statistics the well-documented plateau in mathematics performance as students transitioned from primary into high school (Anderson, 2008) was delayed until the second year of high school. Callingham suggested that the nature of the statistics component of the curriculum might play a part, rather than pedagogical considerations.

It would seem that although statistics and mathematics are related, success in one cannot guarantee achievement in the other domain. Several reasons might be implicated including the role of language and inference, the probabilistic thinking associated with statistics, and the place of context. Context is considered in the next section.

CONTEXT IN MATHEMATICS AND STATISTICS TEACHING AND LEARNING

In considering context, as well as other areas of the school curriculum and their relationships to mathematics and statistics, two potential types of difference occur. One relates to whether the context arises first, as it does in other areas of the school curriculum, whereas the other is associated with whether a mathematical or statistical concept is the initiating and instrumental idea in the learning experience. Within each of these scenarios the question then arises as to whether mathematics and statistics behave similarly. The similarities and differences of these situations are considered with reference to the use of context and the wider implications of that usage.

Uses of Contexts in Teaching Mathematics and Statistics

Starting in other areas of the school curriculum with a problem in context, the need may arise for either mathematics or statistics to help resolve the problem. The question is whether this relationship is different if it is mathematics or if it is statistics that provides the tool for the solution. This question is relevant given the cross-curriculum requirement for numeracy in the Australian curriculum (ACARA, 2011). The issue of numeracy across the curriculum includes statistical literacy and widens further when moving outside of the classroom to the broader society to which students belong. Mooney (2010) made the strong point, reinforced by Clark

(2010), that the concepts required for statistical literacy are best taught in real life contexts – contexts that arise in other areas of the school curriculum. No such claim has been found for mathematics in the Australian and New Zealand literature.

In contrast to beginning with context as a reason for learning, when focusing on the development of either a mathematics or statistics concept, it may be considered useful to introduce a context to elucidate the concept. The question then arises as to whether the use of context to clarify the mathematics or statistics is different for the two subjects. Pfannkuch and her colleagues outlined research that explored the interrelationship between statistical knowledge and contextual knowledge, making a distinction between the social or classroom contexts in which the data arise, and learning experience contexts. Data context is centred on the problem, and focuses on a social situation, whereas learning experience context includes the background that students bring to a task, and the physical and social environment in which those students operate. The learning experience context may be the same for mathematics as statistics but the data context is markedly different in its role in statistics (Pfannkuch, Regan, Wild, & Horton, 2010).

Pfannkuch (2011) described the role of context in mathematics as being introduced as a vehicle to lead ultimately to generalisation and abstraction and hence the context being dropped along the way. In contrast, in statistics context is an essential component of the learning experience—context is relevant throughout the whole experience of learning the associated statistical concept. Chick and Pierce (2008) reached the same conclusion when considering the affordances of media-based examples chosen to teach statistics. For mathematics, however, they questioned the "existence of real-world examples in the 'public' domain that are potentially 'good for teaching' " (pp. 327–8).

The importance of mathematical modelling was highlighted in the previous MERGA review where Stillman, Brown and Galbraith (2008) described a growing research agenda. They suggested that using modelling to "motivate the study of mathematics" (p. 142) is to place the modelling—and thus the mathematics—in a real-life context: "The goal is to equip students with skills that enable them to apply and communicate mathematics in relation to the solving of problems in their world" (p. 145). This comment appeared to place mathematical modelling at the school level as a motivator to learn mathematics, rather than seeing context as essential to the mathematics itself.

At the other end of the education ladder, English (2010, 2011) considered young children involved in data modelling. Her perspective agreed with Pfannkuch's (2011) in that it "involves choosing contexts in which stimuli for the desired mathematics learning are embedded" (p. 26). This mathematics, however, differed from what students were usually taught at the first grade level. Genuine problem situations were used as vehicles for students to construct their data modelling ideas rather than using standard textbook word problems "that constrain problem-solving contexts to those that often artificially house and highlight the relevant concepts" (p. 26). Similarly Brown (2008) suggested that mathematical modelling provided a contextualised experience that could enhance Indigenous students' mathematical learning through challenging, relevant problems.

Langrall, Nisbet, Mooney, and Jansem (2011) considered the role of 'context experts' in a middle-years classroom where the focus was on statistical investigation. They found that having a person in a group who understood something of the context in which the task was set, such as tennis or pop stars, helped engagement with the data. Only those students who had a grasp of the context were able to recognise the limitation of their findings, which is one aspect of inferential thinking in statistics (Makar & Rubin, 2009), emphasising the importance of social context in statistics. Langrall et al. concluded, "The use of such tasks also may reinforce, for students, the view of mathematics (via statistics) as a relevant, interesting, and motivating activity" (p. 65), suggesting that they saw statistics as offering a contextualised, motivating experience for students in much the same way as do proponents of mathematical modelling.

Although placing problems in context is an approach often advocated for teaching mathematics on the basis of relevance, there can be difficulties with this approach. Sharma (2008) found that students in her study displayed procedural knowledge of mean and median, but did not display conceptual understanding. Contextual knowledge, which is an integral part of statistical thinking, could be challenged by the diverse situational knowledge that students bring to the classroom. In Sharma's study, this commonly led to misinterpretations of data by the students. She warned that "it appears that learning for these students is situation specific, and that connecting students' everyday contexts to academic mathematics in a way that enhances meaning, is not easy" (p. 41).

Transfer of knowledge from one context to another is known to be a problem for students in mathematics, and it appears that statistics is similar. Fielding-Wells (2010) described how two problems that Year 6 students investigated were handled differently by the students, because the students were not able to envisage the statistical enquiry cycle for one problem. This was in spite of the teaching focus and development of the statistical enquiry cycle with the first problem. Although mathematics literature refers to similar issues about lack of transfer, for statistics this is critical, as context is not an optional addition to the problems and the enquiry process, but an integral part of understanding statistics. Fielding-Wells suggested that multiple iterations of activities are therefore needed "in order to develop linkages between aspects of statistics investigations across a range of contexts in order to adequately develop deep learning" (p. 6).

Implications of the Use of Context for the Curriculum

The use of context and the development of inferential thinking, which are peculiar to statistics, provide some challenges for the mathematics curriculum. Frankcom (2008), for example, recognised that one challenge was for curriculum planning to take account of the proportion of the teaching time needed for statistics, particularly by early to middle secondary school, when it should be about one-third of each year's work.

Pfannkuch (2010) described how a change in New Zealand's national curriculum (Ministry of Education, 2007) required a reconsideration of inferential statistics. The learning pathways for students to develop statistical inference needed to be carefully mapped, from junior secondary, where informal inference and the logic of inference held a central place, to the senior secondary level, where formal tests of statistical inference were found. In related work, Arnold et al. (2011) described how content changes to curriculum necessitated thinking about how to teach new content around inferential thinking with Year 9 students, in particular making claims about a population based on samples and comparison of boxplots. Statistical inferential thinking must be embedded in a suitable context, and the work of Langrall et al. (2011) suggested that contexts must be accessible to students, but also go beyond their school experiences.

Pfannkuch, Regan, Wild, and Horton (2010) recommended a curriculum that builds on students' classroom experiences to develop a "rich network of concepts and visual imagery over many years" (p. 7) in order to lead students from informal through to formal inference. Pfannkuch et al. went on and described some of the changes they saw as important as a result of the need to 'tell the story' in the data, which necessitates the use of context. They described how students and teachers in mathematics classrooms are not used to such an approach, which required coherent 'natural language' use to explain the thinking in contrast to the largely symbolic language of mathematics.

It is, however, essential in statistics education that this story telling is developed. Understanding the relationship between the specific context of the statistical problem, and the underlying mathematical procedures used to elucidate the problem, is an indispensable requirement for statistics education. Just as the mathematics education literature refers to the challenges of mathematical terms with specific meaning in contrast to the everyday use of such terms (e.g., Callingham & Falle, 2010), so does statistics education. In statistics education, however, the processes of statistical investigations that are embedded in context—from framing questions in natural language, through the data collection and analysis stages that may use specialised statistical language, to communicating the findings in written or oral language—mean that the statistics curriculum needs to take account of and accommodate the development of appropriate language around the use of investigations. Many textbooks, however, tend to separate the mathematical processes of descriptive statistics, such as computation of mean, median and mode, from inferential statistics and the descriptive aspects needed to communicate findings (Pfannkuch et al., 2010). Beyond school, communication was emphasised by Gibbons and MacGillivray (2010) who stated: "Although all disciplines need communication skills in their graduate capabilities, they are especially important for mathematics and statistics graduates, because of both the nature of these disciplines and the very diverse workplaces and careers open to such graduates" (p. 1).

Considerations of curriculum change, especially one requiring a paradigm shift in mathematics teachers as in the instance of statistics education, lead inevitably to questions about teachers' knowledge. Issues relating to teachers' content and pedagogical content knowledge are addressed in the next section.

MATHEMATICAL AND STATISTICAL PEDAGOGICAL CONTENT KNOWLEDGE

Interest in the ways in which teachers use their knowledge of mathematics and pedagogy to develop students' understanding of mathematics has increased over the past decade and Australasian researchers have made a significant contribution to this field. Work has included classroom focussed work such as that of Muir (2008) on teacher actions, Beswick, Callingham and Watson (2011) on the nature of teachers' knowledge for teaching mathematics and Norton (2010) on the mathematical content knowledge of one-year trained pre-service teachers.

Barton (2009), in his MERGA conference keynote address, considered issues related to mathematical knowledge for teaching. He discussed the importance of a teacher having substantial 'VPRO'—that is "Vision, Philosophy, Role for mathematics, and Orientation" (p. 7), arguing that teachers' approaches to teaching would be influenced by their VPRO. Two aspects of VPRO appeared to be particularly pertinent to statistics: Vision addresses the content of mathematics, and Role for mathematics considers the relationship of mathematics to other parts of the curriculum. As indicated earlier, there is some evidence that teachers see mathematics and statistics in a different light. If this is so, then there is likely to be a need to address explicitly the place of statistics in the mathematics curriculum, and across the broader curriculum, in professional learning and pre-service programs.

In relation to teachers' mathematical knowledge a body of research in both statistics and mathematics education exists. For example, Sullivan, Clarke, and Clarke (2009) considered teaching using mathematical tasks and found that many teachers had difficulty converting a potentially useful idea into a deep learning experience. Similar findings were reported by Chick and Pierce (2010) in relation to statistics. They described how initial teacher education students could not 'see' all the teaching opportunities that were offered by the use of real world data in examples and tasks. Through examining the lesson plans that were prepared by pre-service teachers, Chick and Pierce determined that consideration of context resulted in lesson plans that developed higher thinking levels in students. In considering the potential for successful implementation of a mathematics task that also had 'statistics-oriented affordances', they found that lower-level lessons were more mathematical in approach with some 'dubious use of statistical tools', whereas the higher-level lessons were focused more on the broader statistical aspects, including giving greater consideration to the context of the data and its implications. They concluded that teachers need to enhance their content knowledge and pedagogical content knowledge in order to make the most of the statistical affordances of classroom tasks. In a similar vein, Visnovska and Cobb (2010) stated that teachers needed to develop ways to encourage motivation and interest in the context of a problem, rather than expecting the context itself to be motivating to students, especially if the context is not within the students' experiences.

Burgess (2008) discussed the way in which some pre-service students approached an open problem from a mathematical thinking perspective compared with those whose approach showed evidence of statistical thinking. Whereas the former group used proportional thinking to calculate an answer, those who used statistical thinking had, for example, taken into consideration the context of the problem, possible variation in the data, and their reasoning was based on the statistical models (such as graphs or calculations) that they had used. This finding is similar to that for experienced teachers reported by Watson and Nathan (2010a). Such findings have implications for the development of teachers' knowledge, including appropriate uses of context in both statistics and mathematics.

Makar (2008, 2010) recognised the challenges associated with supporting teachers in their development of teaching approaches commensurate with curricula ideals. She suggested that the teaching of mathematical and statistical inquiry more often takes place in subjects other than mathematics, as the mathematics classroom is often 'inquiry-averse'. She called for statistical inquiry to be a focus for pre-service and in-service courses for teachers. Burgess (2008, 2011) used a framework developed from both statistics and mathematics education to consider the issues associated with teachers using an inquiry-based approach to teaching statistics. One of the implications for teacher development was that teachers needed to experience a complete statistical investigation cycle if they were to become successful teachers of statistics. In addition to emphasising the importance of teachers having thorough content knowledge of statistics, Burgess illustrated how pedagogical content knowledge (with both knowledge of students and knowledge of teaching) impacted on the effective teaching of statistics.

One approach to the dilemma of creating effective statistics teachers was to use an integrated curriculum. Tait-McCutcheon (2010) studied the changes teachers made to their planning and pedagogy when teaching statistics through other curriculum areas, during an extended professional learning program. As a result of the approach, teachers stated that statistics appeared to have more meaning for students and critical thinking was more likely to be displayed, resulting in higher measured achievement for students. The integrated approach required teachers to make considerable changes to their pedagogy. Tait-McCutcheon concluded, "The teachers felt they were better planned to teach through their increased understanding of statistics content, and more prepared to teach through their increased experiences with statistics pedagogies" (p. 62).

Issues of integration are not confined to the school classroom. At tertiary level, statistics is often taught as a 'service course', where statistics educators teach statistics to students in other disciplines. Using email interviews, Gordon, Petocz, and Reid (2009) asked statistics educators to identify issues around service teaching. One key aspect was lecturer knowledge, not only about the statistical ideas but also about the discipline context in which the statistics was being used. They also recognised the need for tertiary students to experience the complete cycle of a statistical investigation, similar to the views of Makar (2008) and Burgess (2011). The place of mathematical knowledge, however, revealed divergent views. Some respondents felt that statistics learners needed a strong mathematical background whereas others "contested the 'deterministic thinking' students may learn from studying mathematics, describing it as antithetical to the uncertainty and complex interaction of context and content surrounding statistical problems" (p. 36). Such findings reprise similar comments in the school sector.

The use of examples, or context, in tertiary statistics courses is similar to school classroom use. One category of use that appeared limited was that of examples generated by students themselves to help learning, such as the identification of misuse of statistics (Gordon & Nicholas, 2008). Watson (2011b) reported such a use of examples as an assessment tool in a pre-service teacher course intended to develop personal and professional numeracy. Implicitly, these approaches to teaching statistics, whether at school or tertiary level, were linked to teacher transformation. Petocz and Reid (2010) were explicit about this transformation, not just for teaching emerging statisticians but also "professionally competent and confident users of statistics" (p. 283).

The previous discussion indicates that although mathematics and statistics share some pedagogical issues, there are several aspects of statistics that need to be addressed specifically in teacher development programs, whether for practising or pre-service teachers. In particular, statistics are underpinned by an investigative cycle, and occurred embedded in context (Pfannkuch & Ben-Zvi, 2011). This aspect was emphasised by Martin (2010) in a discussion about the neglect of statistics in the vocational education and training (VET) sector. He proposed better industry-university links to provide improved service for employers and authentic projects to motivate VET students.

Arnold (2008) worked with secondary school teachers to develop their statistical pedagogy through an action research model. The teachers appreciated opportunities to undertake relevant activities in conjunction with others at workshops. In particular they wanted experiences with data handling that included identifying sources of variation, data cleaning and data recategorising. None of these kinds of ideas would be dealt with in a mathematics teaching workshop. Pfannkuch and Ben-Zvi (2011) also emphasised the importance of pre-service teachers experiencing the "games of statistics", where in Game 1 the data were treated descriptively as the population and in Game 2 the data were treated as a sample from which to make an inference about the population. The content and identified needs of teachers for statistics teaching were markedly different. Indeed one of the unique aspects of statistics pedagogy appeared to be the skill of developing an understanding in students of the statistical investigation cycle. Makar and Fielding-Wells (2011), suggested that, in learning to teach statistical investigations, a four stage model of Orientation, Exploration, Consolidation, and Commitment was useful. Although presented in the context of statistics, the model was couched in terms of inquiry and the four phases—"envisioning inquiry, exploring inquiry, refining inquiry, and embracing inquiry" (p. 354)—were applicable across any topic in mathematics that was the focus of an investigation. These synergies suggested that research on teaching statistics had a contribution to make across the mathematics curriculum, especially as mathematics widens its horizons to become more relevant to students through inquiry-based learning.

Reid, Dahlgren, Abrandt Dahlgren, and Petocz (2010) developed a model for learning in statistics at the tertiary level. In particular, the model provided a scaffold for examining the ways in which statistical knowledge was presented to students, and the subsequent outcomes. The ways in which statistics were taught during their undergraduate years affected final-year students' perceptions about the place of statistics in their professional lives. Such a finding suggested that how statistics is presented to pre-service teachers could impact on their views about its place in the curriculum and the appropriate pedagogy.

There may be differences between the kinds of knowledge needed by teachers for teaching statistics and for teaching mathematics, as indicated by the earlier discussion. In recent years there has been considerable interest in identifying and measuring what is variously called pedagogical content knowledge, mathematical knowledge for teaching and mathematics teachers' knowledge (Beswick, Callingham, & Watson, 2011). In statistics education there has been less formal development of this idea, although the previous discussion suggests that there is something different about the knowledge needed to teach statistics. Callingham and Watson (2011) identified a scale of statistical pedagogical content knowledge based on teachers' responses to student answers on surveys. Although this scale could be segmented into four levels, termed Aware, Emerging, Competent and Accomplished, it did not arise from an empirical consideration of statistical and pedagogical knowledge needed by teachers for teaching statistics. It does not, for example, consider the kinds of need identified by Burgess (2011), Gordon and Nicholas (2008) or Tait-McCutcheon (2010).

There are, clearly a number of issues leading to tension in consideration of the place of statistics in relation to mathematics in teaching, learning and in the curriculum, but also great synergies. The final section of this chapter canvasses some of these matters and suggests some routes for further research.

TENSIONS AND SYNERGIES BETWEEN MATHEMATICS AND STATISTICS

The growing interest in the place of statistics in the school curriculum has been mirrored by an emerging research agenda, to which Australasian researchers are contributing in full measure. Work reviewed in this chapter raises a number of further considerations for researchers. These considerations apply at all levels of education, primary, secondary and tertiary, as well as in related areas such as vocational education and training (VET).

It appears that there is a policy issue in the placement of statistics within the mathematics curriculum. Although it is not possible to learn statistics divorced from mathematics, whether the understanding of statistics is best developed within the mathematics curriculum appears open to debate. In some countries, such as England, statistics is taught as a separate subject in the later years of schooling, but there appears to be little consideration of this possibility in Australia, where statistics is subsumed by the mathematics curriculum. In New Zealand statistics has been linked firmly to mathematics in the renaming of the curriculum. The outcomes and implications of these three positions, separate from, embedded in or linked to mathematics, need to be clarified. Evidence of the impact of these approaches on students' learning outcomes in both mathematics and statistics, participation rates in statistics and mathematics courses, and implications for teaching and teachers would be helpful in informing future discussion.

There are also implications for teacher development. At the school level, teachers need to experience ‘doing’ statistics, and to develop the skills of instigating a statistical investigation, managing data collection and analysis, and making meaning from the data. Much good work has already started into the best ways of doing this, but more is needed. In particular, pre-service high school mathematics teachers should have experiences that take them beyond the generic mathematical skills of statistics. Few of these students enter pre-service teacher education with strong backgrounds in statistical investigations, and work with practising teachers suggests that it is the skills of conducting statistical investigations that teachers seek. Time in pre-service education, however, is very limited and strongly contested by other aspects of education. Shifting emphases in teacher education, both in content knowledge development and pedagogical approaches, with a focus on the similarities and differences between teaching statistics and mathematics could be fertile ground for researchers.

More work is needed into teachers’ and students’ beliefs about and attitudes towards mathematics and statistics and the extent to which these differ. There are some intriguing pointers to potential differences, and empirical studies are needed to illuminate this issue. The impact of beliefs on learning and teaching is well established and if teachers and students think differently about statistics compared with mathematics, pedagogical strategies for mathematics may be ineffective with statistics.

Finally, alongside mathematics education, deeper consideration is needed about statistical pedagogical knowledge and its similarities and differences from mathematics. Many questions arise from the studies reported in this chapter in relation to the ways in which teachers approach teaching statistics compared with mathematics, and the essential knowledge needed to be an effective teacher of statistics. In particular, studies that intentionally and explicitly address both domains are needed, in contrast to current work which tends to consider either mathematics or statistics teaching.

From the consideration of research reported in this chapter, it is clear that although strong synergies are evident, some tensions between mathematics and statistics are unresolved. Australasian researchers in both domains are making a considerable contribution to the debate.

REFERENCES

Afamasaga-Fuata’i, K. (2008). Students’ conceptual understanding and critical thinking: A case for concept maps and vee-diagrams in mathematics problem solving. *Australian Mathematics Teacher, 64*(2), 8–17.

Anderson, J. (2008, July). *Addressing the ‘dip’ in lower secondary mathematics learning through schoolbased projects: A case study of one school’s approach.* Paper presented at the 11th International Congress on Mathematical Education, Monterrey, Mexico: ICME. Retrieved from http://tsg.icme11.org/document/get/825

Arnold, P. (2008). Developing new statistical content knowledge with secondary school mathematics teachers. In C. Batanero, G. F. Burrill, C. Reading, & A. Rossman (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (Proceedings of the Joint

ICMI/IASE Study Conference 18 and IASE Round Table Conference). Monterrey, Mexico: ICMI, IASE, and ISI. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications.php?show=rt08

Arnold, P., Pfannkuch, M., Wild, C., Regan, M., & Budgett, S. (2011). Enhancing students' inferential reasoning: From hands-on to 'movies'. *Journal of Statistics Education, 19*(2). Retrieved from http://www.amstat.org/publications/jse/v19n2/pfannkuch.pdf

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2011). *The Australian Curriculum: Mathematics*, *Version 1.2*. Sydney, Australia: ACARA. Retrieved from http://www.australiancurriculum.edu.au/Home

Barton, B. (2009). Being mathematical, holding mathematics: Further steps in mathematical knowledge for teaching. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 3–9). Palmerston North, NZ: MERGA.

Beswick, K., Callingham, R., & Watson, J. (2011). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education*. Retrieved from Online First http://www.springerlink.com/content/th22781265818125/

Bill, A. F., & Gayton, P. (2010). Coin-sequences and coin-combinations presented as companion task. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistics Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications.php?show=icots8

Bill, A. F., Henderson, S., & Penman, J. (2010). Two test items to explore high school students' beliefs of sample size when sampling from large populations. In L. Sparrow, B. Kissane, & C. Hurst. (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia, pp. 77–84). Fremantle, WA: MERGA.

Brown, K. (2008). Employing mathematical modelling to respond to Indigenous students' needs for contextualised mathematics experiences. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia, pp. 93–99). Brisbane, QLD: MERGA.

Brown, P., Edwards, A., Evans, M., Gaudry, G., Hunt, D., McIntosh, J., & Pender, B. (2006). *ICE-EM mathematics secondary 1A*. Melbourne, Australia: University of Melbourne on behalf of the International Centre for Excellence for Education in Mathematics.

Burgess, T. (2008). Teacher knowledge for teaching statistics through investigations. In C. Batanero, G. F. Burrill, C. Reading, & A. Rossman (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (Proceedings of the Joint ICMI/IASE Study Conference 18 and IASE Round Table Conference). Monterrey, Mexico: ICMI, IASE, and ISI. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications.php?show=rt08

Burgess, T. (2011). Teacher knowledge of and for statistical investigations. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics–Challenges for teaching and teacher education: A joint ICMI/IASE study* (Ch. 19). Dordrecht, The Netherlands: Springer.

Callingham, R. (2010). Trajectories of learning in middle years' students' statistical development. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on the Teaching of Statistics, July, 2010, Ljubljana, Slovenia) Voorburg, The Netherlands: International Statistical Institute. Retrieved from www.stat.auckland.ac.nz/~iase/publications.php

Callingham, R. (2011). Assessing statistical understanding in middle schools: Emerging issues in a technology-rich environment. *Technology Innovations in Statistics Education, 5*(1), 1–11. Retrieved from http://escholarship.org/uc/item/3qr2p70t

Callingham, R., & Falle, J. (2010). The use of language in the mathematics classroom. In T. Le & M. Short (Eds.), *Language and literacy education in a challenging world* (Ch. 10). New York, NY: Nova Science.

Callingham, R., & Watson, J. (2011). Measuring levels of statistical pedagogical content knowledge. In C. Batanero, G. Burrill, & C. Reading (Eds.)*, Teaching statistics in school mathematics—Challenge* *for teaching and teacher education: A joint ICMI/IASE study* (Ch. 21). Dordrecht, The Netherlands: Springer.

Carmichael, C. S., Callingham, R., Hay, I., & Watson, J. M. (2010). Statistical literacy in the middle school: The relationship between interest, self-efficacy and prior mathematics achievement. *Australian Journal of Educational and Developmental Psychology, 10*, 83–93. Retrieved from http://www.newcastle.edu.au/journal/ajedp/current-issue.html

Carmichael, C., & Hay, I. (2010). Developmental changes in Australian school students' interest for statistical literacy. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg: The Netherlands: International Statistical Institute. Retrieved from www.stat.auckland.ac.nz/~iase/publications.php

Chick, H. L., & Pierce, R. (2008). Issues associated with using examples in teaching statistics. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 321–328). Morelia, Mexico: Cinvestav-UMSNH.

Chick, H., & Pierce, R. (2010). Helping teachers to make effective use of real-world examples in statistics. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications.php?show=icots8

Clark, M. (2010). Statistical literacy: A priority for students but also for policy makers. *Notepad-Issue 3*. Retrieved from http://austcolled.com.au/notepad/article/statistical-literacy-priority-students-also-policy-makers

David, I., & Brown, J. (2010). Implementing the change: Teaching statistical thinking not just methods. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from www.stat.auckland.ac.nz/~iase/publications.php

Dean, M. (2010). Queensland high school mathematics needs a back-to-thinking revision. *Teaching Mathematics*, March, 20–25.

English, L. (2010). Young children's early modelling with data. *Mathematics Education Research Journal, 22*(2), 24–47.

English, L. (2011). Data modeling in the beginning school years. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 226–234). Adelaide, SA: AAMT & MERGA.

Fielding-Wells, J. (2010). Linking problems, conclusions and evidence: Primary students' early experiences of planning statistical investigations. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications.php?show=icots8

Fielding-Wells, J., & Makar, K. (2008). Student (dis)engagement in mathematics. In P. Jeffery (Ed.), *Proceedings of the Australian Association for Research in Education Conference.* Retrieved from http://www.aare.edu.au/08pap/mak08723.pdf

Fitzallen, N., & Watson, J. (2010). Developing statistical reasoning facilitated by TinkerPlots. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications.php?show=icots8

Forbes, S., Camden, M., Pihama, N., Bucknall, P., & Pfannkuch, M. (2011). Official statistics and statistical literacy: They need each other. *Statistical Journal of the International Association of Official Statistics, 27*(3/4), 113–128.
Forbes, S., & Pfannkuch, M. (2009). Developing statistical thinking: Teaching and learning. In R. Averill & R. Harvey (Eds.), *Teaching secondary school mathematics and statistics: Evidence-based practice* (pp. 93–128). Wellington, New Zealand: NZCER Press.
Frankcom, G. (2008, July). *Statistics teaching and learning: The New Zealand experience*. Paper presented at the 11th International Congress on Mathematical Education, Monterrey, Mexico: ICME. Retrieved from http://tsg.icme11.org/document/get/489
Gibbons, K., & MacGillivray, H. (2010). Training for statistical communication in the workplace. In H. MacGillivray & B. Phillips (Eds.), *OZCOTS 2010* (Proceedings of the 7th Australian Conference on Teaching Statistics). Fremantle, Australia. Retrieved from http://opax.swin.edu.au/~3420701/OZCOTS2010/OZCOTS2010_paper_Gibbons.pdf
Gordon, I., & Finch, S. (2010). How we can all learn to think critically about data. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications/icots8/ICOTS8_7G4_GORDON.pdf
Gordon, S., & Nicholas, J. (2008). Why do statistics educators use examples to teach statistics. In H. L. MacGillivray & M. A. Martin (Eds.), *OZCOTS 2008* (Proceedings of the 6th Australian Conference on Teaching Statistics). Melbourne, Australia: Statistical Society of Australia.
Gordon, S., Petocz, P., & Reid, A. (2009). What makes a 'good' statistics student and a 'good' statistics teacher in service courses? *The Montana Mathematics Enthusiast, 6*(1&2), 25–40.
Hay, I. (2010). Teachers' perceptions of best practice in statistical literacy education. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications/icots8/ICOTS8_C158_HAY.pdf
Horne, M. (2009). Students provide information on affective domain and learning environment through interview. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education, pp. 185–192). Thessaloniki, Greece: PME.
Houston, K., Mather, G., Wood, L. N., Petocz, P., Reid, A., Harding, A., ... Smith, G. H. (2010). Is there life after modelling? Student conceptions of mathematics. *Mathematics Education Research Journal, 22*(2), 69–80.
Ireland, S., & Watson, J. (2009). Building an understanding of the connection between experimental and theoretical aspects of probability. *International Electronic Journal of Mathematics Education*, *4*, 339–370.
Konold, C., & Miller, C. D. (2005). *Tinkerplots: Dynamic data exploration*. [Computer software] Emeryville, CA: Key Curriculum Press.
Langrall, C., Nisbet, S., Mooney, E. & Jansem, S. (2011). The role of context expertise when comparing data**.** *Mathematical Thinking and Learning, 13*(1&2), 47–67.
Makar, K. (2008). A model of learning to teach statistical inquiry. In C. Batanero, G. F. Burrill, C. Reading, & A. Rossman (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (Proceedings of the Joint ICMI/IASE Study Conference 18 and IASE Round Table Conference). Monterrey, Mexico: ICMI, IASE, and ISI. Retrieved from www.stat.auckland.ac.nz/~iase/publications/rt08/T4P4_Makar.pdf
Makar, K. (2010). Teaching primary teachers to teach statistical inquiry: The uniqueness of initial experiences. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications/icots8/ICOTS8_3A3_MAKAR.pdf

Makar, K. (2011). Learning over time: Pedagogical change in teaching mathematical inquiry. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 27–39). Adelaide, SA: AAMT & MERGA.

Makar, K., Bakker, A., & Ben-Zvi, D. (2011). The reasoning behind informal statistical inference. *Mathematical Thinking and Learning,* 13, 152–173.

Makar, K., & Fielding-Wells, J. (2011). Teaching teachers to teach statistical investigations. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics—Challenges for teaching and teacher education: A joint ICMI/IASE study* (Ch. 26). Dordrecht, The Netherlands: Springer.

Makar, K., & Rubin, A. (2009). A framework for thinking about informal statistical inference. Statistics *Education Research Journal, 8*(1), 82–105. Retrieved from http://www.stat.auckland.ac.nz/serj/SERJ8(1)_Makar_Rubin.pdf

Martin, P. (2010). Evaluating statistics education in vocational education and training. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications/icots8/ICOTS8_8I2_MARTIN.pdf

Ministry of Education. (2007). *Draft mathematics and statistics curriculum.* Wellington, New Zealand: Author.

Mooney, G. (2010). Preparing students for a data rich world: The case for statistical literacy. *Curriculum Perspectives*, *30*(1), 25–29.

Muir, T. (2008). Principles of practice and teacher actions: Influences on effective numeracy teaching. *Mathematics Education Research Journal, 20*(3), 78–101.

Nisbet, S. (2010). National testing of data handling in years 3, 5 and 7 in Australia. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications/icots8/ICOTS8_2A1_NISBET.pdf

Nisbet, S. (2011). National testing of probability in years 3, 5, 7, & 9 in Australia: A critical analysis. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 575–581). Adelaide, SA: AAMT & MERGA.

Norton, S. (2010). How deeply and how well. How ready to teach mathematics after a one-year program? *Mathematics Teacher Education and Development, 12*(1), 65–83.

Petocz, P., & Reid, A. (2010). On becoming a statistician—A qualitative view. *International Statistical Review*, *78*, 271–286.

Pfannkuch, M. (2010). Inferential reasoning: Learning to 'make a call' in practice. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics, July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Available from http://www.stat.auckland.ac.nz/~iase/publications/icots8/ICOTS8_8B2_PFANNKUCH.pdf

Pfannkuch, M. (2011). The role of context in developing informal statistical inferential reasoning: A classroom study. *Journal of Mathematical Thinking and Learning*, *13*(1&2), 27–46.

Pfannkuch, M., & Ben-Zvi (2011). Developing teachers; statistical thinking. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics—challenges for teaching and teacher education: A joint ICMI/IASE study* (Ch. 24). Dordrecht, The Netherlands: Springer.

Pfannkuch, M., Regan, M., Wild, C. J., & Horton, N. J. (2010). Telling data stories: Essential dialogues for comparative reasoning. *Journal of Statistics Education, 18*(1), 1–38. Retrieved from www.amstat.org/publications/jse/v18n1/pfannkuch.pdf

Polster B., & Ross, M. (2010, March). New maths curriculum a feeble tool calculated to bore. *The Age*. Retrieved from http://www.theage.com.au/opinion/society-and-culture/new-maths-curriculum-a-feeble-tool-calculated-to-bore-20100303-pivw.html

Reid, A., Dahlgren, L. O., Abrandt Dahlgren, M., & Petocz, P. (2010). Applying a model of professional learning to case studies in statistics education. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications/icots8/ICOTS8_6G2_REID.pdf

Reid, J., & Reading, C. (2008). Measuring the development of students' consideration of variation. *Statistics Education Research Journal, 7*(1), 40–59. Retrieved from http://www.stat.auckland.ac.nz/~iase/serj/SERJ7%281%29_Reid_Reading.pdf

Sharma, S. (2008). Exploring Fijian high school students' conceptions of averages. *Pacific-Asian Education, 20*(2), 29–45.

Slattery, L., & Perpitch, N. (2010, March). Maths students in serious decline. *Australian.* Retrieved from http://www.theaustralian.com.au/news/nation/mathematics-students-in-serious-decline/story-e6frg6nf-1225838901032

Smith, A. (2009). *Leximancer*. [Computer software]. Brisbane, Australia: Leximancer. Retrieved from http://www.leximancer.com

Stillman, G. A., Brown, J. P., & Galbraith, P. L. (2008). Research into the teaching and learning of applications and modelling in Australasia. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W-T. Seah, & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004–2007 (*pp. 141–164). Rotterdam, The Netherlands: Sense.

Sullivan, P., Clark, D., & Clark, B. (2009). Converting mathematics tasks to learning opportunities: An important aspect of knowledge for mathematics teaching. *Mathematics Education Research Journal, 21*(1), 85–104.

Tait-McCutcheon, S. (2010) Significant statistics viewed with a contextual lens. *Mathematics Teacher Education and Development, 12*(1), 50–64.

Thomson, S. (2009). Informing mathematics pedagogy. *Research Developments, 22*(6). Retrieved from http://research.acer.edu.au/cgi/viewcontent.cgi?article=1076&context=resdev&sei-redir=1#search=%22mathematics%20pedagogy%20Australia%22

Thomson, S., De Bortoli, L., Nicholas, M., Hillman K., & Buckley, S. (2010). *Challenges for Australian education: Results from PISA 2009*. Melbourne: Australian Council for Educational Research.

Visnovska, J., & Cobb, P. (2010). Supporting shifts in teachers' views and uses of problem context in teaching statistics. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on Teaching Statistics July, 2010, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications/icots8/ICOTS8_C118_VISNOVSKA.pdf

Watson, J. M. (2008). Exploring beginning inference with novice grade 7 students. *Statistics Education Research Journal, 7*(2), 59–82. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications.php?show=serjarchive

Watson, J. M. (2009). The influence of variation and expectation on the developing awareness of distribution. *Statistics Education Research Journal, 8*(1), 32–61. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications.php?show=serjarchive

Watson, J. M. (2011a). Foundations for improving statistical literacy. *Statistical Journal of the International Association of Official Statistics*, *27*(3/4), 197–204.

Watson, J. M. (2011b). Personal and Professional Numeracy: A unit for pre-service teachers at the University of Tasmania. *Numeracy: Advancing Education in Quantitative Literacy, 4*(1), Article 2. doi:10.5038/1936-4660.4.1.2

Watson, J. M., Callingham, R., & Donne, J. (2008). Proportional reasoning: Student knowledge and teachers' pedagogical content knowledge. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 555–562). Brisbane, QLD: MERGA.

Watson, J. M., & Donne, J. (2009). TinkerPlots as a research tool to explore student understanding. *Technology Innovations in Statistics Education, 3*(1), 1–35. Retrieved from http://repositories.cdlib.org/uclastat/cts/tise/vol3/iss1/art1/

Watson, J. M., Fitzallen, N. E., Wilson, K. G., & Creed, J. F. (2008). The representational value of hats. *Mathematics Teaching in the Middle School*, *14*, 4–10.

Watson, J. M., & Kelly, B. A. (2009). Development of student understanding of outcomes involving two or more dice. *International Journal of Science and Mathematics Education*, *7*, 25–54.

Watson, J., & Nathan, E. (2010a). Assessing the interpretation of two-way tables as part of statistical literacy. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society* (Proceedings of the 8th International Conference on the Teaching of Statistics, July, 2008, Ljubljana, Slovenia). Voorburg, The Netherlands: International Statistical Institute. Retrieved from http://www.stat.auckland.ac.nz/~iase/publications/icots8/ICOTS8_5E1_WATSON.pdf

Watson, J. M., & Nathan, E. L. (2010b). Approaching the borderlands of statistics and mathematics in the classroom: Qualitative analysis engendering an unexpected journey. *Statistics Education Research Journal*, *9*(2), 69–87.

Wild, C. J., Pfannkuch, M., Regan, M., & Horton, N. J. (2011). Towards more accessible conceptions of statistical inference (with Discussion). *Journal of the Royal Statistical Society A, 174*(2), 247–295.

Wood, L., & Solomonides, I. (2008). Different disciplines, different transitions. *Mathematics Education Research Journal*, *20*(2), 117–134.

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THE PROFESSIONAL EDUCATION AND DEVELOPMENT OF PROSPECTIVE TEACHERS OF MATHEMATICS

Key words: teacher education practices; becoming a teacher; reflective practice.

INTRODUCTION

This critical review of Australasian research on the professional education of prospective teachers of mathematics, presented or published in the period from 2008–2011, covers a period in which teacher education has undergone ‘dynamic reform’ at a global level (Tatto, Lerman, & Novotna, 2010). In Australasia, teacher education programs have experienced a range of systemic, political, social, and economic pressures that have led to modifications in program and curricula structures and increased performativity requirements. These pressures are fuelled by the widespread belief that “improvements in student learning depend on substantial, large-scale changes in how we prepare and support teachers” (Ball & Forzani, 2009, p. 497). The motivating force behind this attention is the claim that teachers are ‘key’ to students’ opportunities to learn mathematics. In creating these opportunities to learn mathematics it is clear that “what mathematics teachers know, care about, and do is a product of their experiences and socialisation both prior to and after entering teaching, together with the impact of their professional education” (Even & Ball, 2009, p. 1). It is the experiences and socialisation associated with the education of prospective teachers of mathematics—the pre-service and induction phase—that are the focus of attention in this chapter.

Grossmann and McDonald (2008) argued that changes in the way we have come to look at teaching, shifting from a focus primarily on teacher characteristics to looking at teaching behaviours, teacher decision making, teacher knowledge, and teacher reflection and dispositions, necessitate that we “attend to preparing novices for the relational as well as the intellectual demands of teaching” (p. 185). Internationally, pre-service mathematics educators’ efforts to ‘prepare’ quality teachers of mathematics have focused on recruitment and retention of prospective teachers, development of new pathways to teaching, and renovations to the curriculum of professional education for teachers (Ball & Forzani, 2009). In this chapter we look at how research from Australasia has provided insight and direction in each of these areas. Taking the lead from the conceptual frameworks presented at the 15th International Commission on Mathematical Instruction (ICMI) study *The Professional Education and Development of Teachers of*

B. Perry et al. (Eds.), Research in Mathematics Education in Australasia 2008–2011, 291–312.

Mathematics (Even & Ball, 2009), and adapting the structure of the 2004–2007 MERGA review chapter (Goos, Smith, & Thornton, 2008) we have chosen to organise our analysis of Australasian studies as follows:

– research on recruitment of prospective teachers of mathematics;
– research that has sought to understand the process of becoming a teacher;
– research that has sought to probe the knowledges for teaching;
– research that interrogates initial teacher education practices and pedagogies; and
– research involving teacher educators researching on and within their own practice.

The conclusion to the chapter reflects upon the overall contribution of the studies to furthering the field against the recommendations posed by Goos et al. (2008) in the previous review period. Guided by the 15th ICMI study *Next Steps* for strengthening practice in and research on the professional education and development of prospective teachers of mathematics (Ball & Even, 2009), we frame a set of recommendations for further research directions.

We begin by outlining current structures of pre-service teacher education programs in Australia and New Zealand.

PRE-SERVICE TEACHER EDUCATION PROGRAMS IN AUSTRALASIA

Teacher education in Australasia takes many forms (Callingham et al., 2011). Programs vary from three or four years of tertiary study designed as fully integrated programs, or include an undergraduate degree with an add-on one-year Graduate Diploma or two-year Master of Teaching programs of professional education. Reflecting an international trend, the course structures, the extent to which mathematics content and mathematics pedagogy are integrated, the placement of school based experiences, and the amount of time devoted to studies relating to mathematics and/or its teaching varies widely in both New Zealand and Australian teacher education contexts. Leder (2009), in her comments on contributions to the 25th ICMI study, noted that internationally, pre-service teacher education appeared to be predominantly an undergraduate undertaking, but beyond that there was enormous variation.

In New Zealand, there is a requirement, enforced from 2011, that all approved initial teacher education (ITE) programs must demonstrate how graduates meet the *Graduating Teacher Standards: Aotearoa New Zealand* (New Zealand Teachers Council, 2007). These standards include statements related to professional knowledge and professional practice, and professional values and relationships. Likewise, this review period has seen the development of graduating standards for teacher education within Australia, culminating with the 2011 release by the Australian Institute for School Leadership and Teaching (AITSL) of generic teaching standards describing four career stages of teachers, including the phase immediately following graduation. Similar to the New Zealand graduating teacher standards, AITSL's graduate standards address professional knowledge, professional practice, and professional engagement. It is anticipated that the

National Professional Standard for teachers at the graduate career stage will be linked to national accreditation of teacher education programs in the near future (AITSL, 2011). In the context of increasing accountability, the project described by Callingham et al. (2011) involving seven Australian universities and aimed at providing useful tools for establishing an evidence base for ongoing improvement of mathematics teacher education is especially timely.

RECRUITMENT OF PROSPECTIVE TEACHERS OF MATHEMATICS

Concerns about recruitment of mathematics teachers focus on multiple issues, but most notably on quality and quantity (Stacey, 2008). Although there is agreement that mathematics learning is to a large extent dependent on teacher quality, debates about what aspects of teacher quality are central to improving learners' access to and participation in mathematics are ongoing (Borko & Whitcomb, 2008; Louden, Heldsinger, House, Humphry, & Fitzgerald, 2010). These debates are not confined to teacher educators. This review period is characterised by reported dissatisfactions with the quality of pre-service programs from the school sectors (see New Zealand Government, 2010; Goos, 2009).

In looking to address quality, concerns about the sufficiency of mathematics content knowledge, both at the recruitment and graduate phase of pre-service teacher education, is a central and ongoing issue across all sectors (Barton & Sheryn, 2009; Brown, 2009). Both the Australian and New Zealand graduating teacher standards express clear expectations that graduates must understand the content that they teach and be able to design and deliver lessons that meet curriculum, assessment and reporting requirements, as well as the individual needs of students. However, graduating requirements continue to be a concern in relation to prospective teacher entry levels of mathematics.

In the early years and primary sectors, research studies highlight pre-service teachers' lack of proficiency in content knowledge (e.g., in Samoa, see Afamasaga-Fuata'i, Meyer, & Falo, 2008; in New Zealand, see Young-Loveridge. 2010; in Australia, see Forrester & Chinnappan, 2010). Looking at the consequences of weak mathematics content knowledge, Livy (2010) probed the relationship between practice, mathematical content knowledge, and pedagogical content knowledge in the classroom setting. She observed, a practicum lesson taught by a second-year pre-service teacher who, like many (50%) in the course, had not yet passed the Mathematics Competency, Skills and Knowledge test. Using the "knowledge quartet" framework (Rowland, Turner, Thwaites, & Huckstep, 2009) to analyse how the pre-service teacher drew on her content knowledge, Livy concluded that 'gaps' in the teacher's mathematical content knowledge contributed to her failure to implement a Grade 3 subtraction lesson in a way that promoted students' mathematical understanding.

In New Zealand, requirements for entry mathematical knowledge levels have been strengthened in policy changes for initial teacher education programs (New Zealand Teachers Council, 2010) that take effect from 2011. In Australia, education authorities (e.g., New South Wales Institute of Teachers, 2011) also specify minimum entry achievement standards in mathematics and there are moves towards a national approach in this regard. In addition, AITSL (2011) have suggested that entrants into teacher education programs should be drawn from the top 30 per cent of the population, but the precise meaning of this requirement remains unclear. Concerns that state mandated entry requirements may not be sufficient to ensure that prospective teachers have appropriate mathematical understandings and attitudes, have led some universities to implement additional basic skills mastery requirements as part of their teacher education programs. However, research suggests that this intervention is not without its risks. Based on interviews with pre-services teachers, Meaney and Lange (2010) cautioned that mastery-testing regimes in one university supported the development of procedural rather than conceptual understanding.

In the secondary sector, initial teacher education also faces challenges concerning teacher mathematics knowledge. Anthony, Butler, and Rawlins (2011), reporting on a national study of prospective mathematics teachers in New Zealand, noted that nearly 20% of students studying mathematics methods courses had no tertiary qualifications in statistics, and 10% had only studied statistics papers as preparation for their mathematics methods courses; a finding that mirrored Stacey's (2008) claim that many secondary teachers of mathematics had qualifications related to users of mathematics in the service of other professions rather than mathematical majors. A related issue, raised by both Anthony et al. and Stacey, is the trend for an increasing proportion of career switchers entering secondary teaching. How might initial teacher education adjust to this diversity of prior experiences, and how might the use of mathematics in previous careers be instrumental in improving the mathematical learning experiences of students? To date, these questions do not appear to have been researched in the Australasian context.

While many studies raised questions about the appropriate level of teacher knowledge, Frid, Goos, and Sparrow (2009) cautioned that calls for a greater emphasis on subject matter knowledge in pre-service programs are only part of the picture. Studies concerning what types of knowledge are needed for effective teaching of mathematics, and alternative ways to support pre-service teachers' development of these, including beliefs and attitudes, are discussed later in this chapter.

THE PROCESS OF 'BECOMING' A TEACHER

Education is a unique enterprise because the "what we teach is also how we teach" (Liljedahl, 2009, p. 29). It has been well established that beginning teachers tend to teach as they were taught, bringing with them attitudes and beliefs about mathematics teaching that are contrary to the aims of the courses that they enter. As well as developing the knowledge required for them to teach mathematics, another major role of teacher education is often seen as influencing the beliefs and attitudes of pre-service teachers. In an examination of the influence of an ITE program, Beswick and Dole (2008) interviewed teachers five years after their graduation. They found that the teachers were most likely to retain knowledge from the ITE courses that were highly connected with their own beliefs and personal identity, that is, knowledge that had an emotional meaning to the individual. In looking at the trajectory of becoming a teacher, Beswick and Dole noted that for some teachers, the common preoccupations of beginning teachers—behaviour management and business of teaching (Goos, 2009; Prescott & Cavanagh, 2008)—had given way to more substantive issues that concerned meeting the learning needs of individual students.

Several studies have looked closely at how ITE may serve to influence prospective teachers' beliefs (e.g., Lo & Anderson, 2010; Smith, 2009). Importantly, Grootenboer's (2008) in-depth study of 43 pre-service teachers enrolled in a course designed to facilitate reflection on beliefs, draws our attention to the ethical issues inherent in mathematics teacher education that aims to change teachers' beliefs. In his study, participation resulted in some unexpected changes: Approximately one third of participants did not engage in reflection on their beliefs in any meaningful way, a second third engaged meaningfully but appeared to develop a new set of beliefs in addition to their existing beliefs, and the remaining third entered a process of profound belief change that they experienced as emotionally upsetting and frustrating. Grootenboer concluded that this last third of the participants, for whom the course was most effective, suffered a loss of confidence and feelings of preparedness, at least in the short term. On the other hand, those who did not actively engage were able to achieve high marks by writing assignments that gave their lecturers what they wanted to hear. The potential for pre-service teachers to respond in ways they know will be approved has long been acknowledged as a potential threat to the validity of beliefs research using self-report methods. Grootenboer's contribution is significant in presenting data that show that it also happens when pre-service teachers engage in course assessments.

A similar difficulty was highlighted by Beswick and Callingham (2011) in a survey concerning teacher knowledge and beliefs. They noted that pre-service primary teachers found it much easier to endorse statements aligned with a problem solving view of mathematics and student-centred views of mathematics pedagogy than to respond appropriately to questions demanding knowledge of mathematics or pedagogical content knowledge (PCK). Beswick and Callingham suggested that the relative ease of endorsing the belief statements could reflect a willingness to adopt rhetoric that aligned with messages from their university mathematics education without necessarily having fully understood the meaning or implications of the statements. In support of this contention, Beswick, Callingham, and Watson (2011) cited data from practising teachers, in which some less reform-oriented beliefs about mathematics teaching and learning were associated with more sophisticated PCK, suggesting a willingness to critique prevailing orthodoxies. This latter study used Rasch techniques to demonstrate that the belief and knowledge items worked together to measure a single underlying variable that they called teacher knowledge. Beswick (2011) argued that although beliefs have long been considered to constitute the most cognitive point on a hypothetical spectrum of affects (McLeod, 1992), these results give weight to the theoretical argument that the distinction between beliefs and knowledge is more a matter of the degree of consensus that a proposition attracts, which in turn is highly contextual, varying with culture and time. Importantly, this viewpoint opens up possibilities for research that truly integrates the two, a call that harks back to McLeod's (1992) seminal review of research on affect in mathematics education. The conceptual unification of knowledge and beliefs broadens the conception of teacher knowledge and paves the way for belief change to be viewed as learning and therefore approached in accordance with theories of learning, a view expressed earlier by Stacey (2008).

Studies on beliefs more often mention rather than systematically study emotions (e.g., Beswick & Dole, 2008; Prescott & Cavanagh, 2008). However, just as research that has challenged the beliefs/knowledge divide promises new possibilities, it is likely that closer attention to emotion and its entailment in belief change and learning more generally will be a profitable field for future research. We see a hint of the possibilities in a study by Wilson and Thornton (2007/2008) that utilised bibliotherapy as a tool for assisting pre-service teachers to confront and address negative experiences of mathematics learning. Wilson and Thornton located the power of bibliotherapy in the contemporaneous nature of their subjects' cognitive and emotional responses. They found that bibliotherapy addressed both cognitive and affective outcomes for pre-service primary teachers by (a) enhancing self-image as mathematics learners and doers, (b) helping them appreciate learning as the acquisition of deep knowledge and the teacher's role as providing rich and complex problems, and (c) developing a belief in the capacity of all students to learn mathematics. Wilson and Gurney (2011) continued to develop an action research-based model, supported by the bibliotherapy process, whereby pre-service teachers with mathematics anxiety could develop more positive teacher identities.

Increasingly, research on becoming a teacher incorporates the concept of identity. Walshaw (2008) argued that by incorporating cognition and emotion, identity had the potential to provide an overarching view of teacher development. In a critique of the commonly advocated approach to teacher change, namely reflective practice, Walshaw (2010b) drew attention to the necessarily emotional nature of identity formation and the fact that interviewees (or writers of reflections) are necessarily influenced by their beliefs about what the interviewer (or reader) wanted to hear. Drawing on a case study of a secondary teacher, Walshaw illustrated how this teacher was influenced by the constant need to 'close the gap' between his image of himself and the way he believed others (in this case the researchers) perceived him as well as to reconcile his current self with what he might be in the future. Using the theoretical framework provided by Lacanian psychoanalysis, Walshaw argued how such reflection, influenced by the individual's personal, emotional, and cognitive state, may in fact work to reinforce the status quo rather than to induce change. Walshaw's conclusion that "reflective practice is as regulatory as it is emancipatory" (p. 496) provides a caution for how we might choose to use reflections as instruments of change in our ITE programs.

Brandenburg (2008, 2009) introduced new modes of reflective practice inquiry in her primary education program. She replaced the traditional tutorial structure, with its predetermined content based on her own assumptions about pre-service teachers' prior knowledge and needs, with a series of roundtable sessions. These sessions provided a means to integrate inquiry into weekly learning experiences based on issues related to the pre-service teachers' own experience. Critical Incident Questionnaires provided a further structured approach to gathering snapshots of learning related to pre-service teachers' experiences and aspects of learning. Also focused on the impact of self and collaborative reflective practices, Way (2009) evaluated the potential of online discussions as a space for learning. Reporting on the use of a combination of prompted written reflections, online discussion, and self-assessment, Way provided exemplars of pre-service students' growing awareness of their emergent professional teachers' identities and responsibilities alongside their growth in pedagogical content knowledge. Way concluded that the online discussions proved to be a powerful learning environment when combined with self-assessment processes. Balatti and Rigano (2011) also reported on a course that involved weekly online pre-service teachers' reflections on their past school and current practicum experiences. Their analysis suggested that "how pre-service teachers think about their experiences of good teaching may be as relevant to teacher educators as the content of their narrative" (p. 87). Given the increased use of online learning environments, Way's (2009) claim that we needed more research to inform our "understanding of how individuals and groups interact and collaborate to build higher levels of reflection" (p. 578) must remain at the fore.

The practicum experience, a powerful and critical learning space for pre-service teachers, provided the context for several studies. Researchers (e.g., Goos et al., 2009; Walshaw, 2010a) have taken a number of theoretical stances to help make sense of pre-service teachers' experience of practicum. Central to these varied stances is the notion that pre-service teachers develop an identity as a teacher of mathematics as they engage in the act of teaching. What they encounter as they do this is a powerful influence upon their emerging teacher identity. Walshaw (2009, 2010a) used the work of Foucault to view the opportunities afforded in practicum. Her analysis highlighted discourse, power, governmentality, surveillance, normalisation, and dividing practices as notions that can help us capture and understand the complexity and significance of the practicum experience. Drawing on survey responses from primary pre-service teachers, Walshaw (2010a) made it clear that the practicum is more than an individual's journey. She described it as a "barely visible set of highly coercive practices" (p. 126).

A key concern with the practicum is its possible role in perpetuating classroom practices, rather than contributing to reform or transformation agendas. For example, Prescott and Cavanagh (2010) found evidence that seven supervising teachers in secondary settings adopted a traditional, technical orientation which encouraged pre-service teachers to imitate current practices. In contrast, Ashman and McBain (2011), reported on a unit of study that involved pre-service teachers working on an integrated school and university program within their final year of study, described how collaborative partnerships afforded by the program supported both the mentor and pre-service teachers to move towards a more balanced view of the worth of the university and classroom based learning. An important difference between this teaching experience and the more traditional practicum experience was the fact that the assessment of classroom practice utilised pre-service teachers' own reflective evaluations.

Using the idea of normalisation to describe how an experience in a school can limit a pre-service teacher's view of what counts as good teaching in mathematics, Walshaw (2009) argued that the 'reality' of the classroom that they experienced, both in person and through their associate's words and behaviours, may conflict with their emerging beliefs about mathematics teaching. Walshaw's research provided an alternative viewpoint, to try to explain the lack of fit between practices advocated by course work and actual teaching practice as a problem of the school settings. Using a case study of a secondary teacher's practicum experiences, described as "fraught with ambiguous and at times painful negotiations to produce a sense of self-as-teacher" (p. 560), Walshaw illustrated how "the practicum functions as part of the technology of surveillance and discipline—how it imposes conditions in schools which induce teachers into a particular pedagogical pattern" (p. 561). In another case analysis, this time drawing on psychoanalytic displacement theories from Lacan and Foucault, Walshaw (2010a) introduced the concepts of dividing practices and registers of identifications, relatively new theoretical tools in mathematics education.

Goos' (2008a, 2008b) research program looked closely at the process of becoming a teacher and applied Valsiner's (1997) Zone concepts of Proximal Development (ZPD), Free Movement (ZFM), and Promoted Action (ZPA) to represent the set of possibilities for teacher development. The ZFM represented environmental constraints within the teachers' professional context and the ZPA represented those teaching approaches specifically promoted by pre-service teacher education, or informal interactions with colleagues in the school setting. Each practicum experience differed in what was allowed and what was possible, shaping the pre-service teacher's practice as a mathematics teacher.

Goos and Geiger (2010) illustrated the strength of their adaptation of Valsiner's theory through a critical analysis of studies published in two Special Issues of the *Journal of Mathematics Teacher Education* (JMTE) on mathematics teacher and mathematics teacher educator changes. Looking across several studies, Goos and Geiger speculated that "there may be [an] important difference between the zone configurations of experienced and beginning teachers, perhaps related to the degree of autonomy and confidence they feel in being able to change their environment (ZFM) or how they perceive their environment" (p. 503). Anthony's (2008) analysis of induction programs within secondary school settings in New Zealand highlighted the complex learning systems negotiated by beginning teachers. Those mathematics teachers who found themselves in schools with a strong 'craft knowledge' culture, reported explicit pressure from both their more experienced colleagues and the parental community to abandon efforts to engage in ambitious pedagogies in favour of more traditional approaches. Differential school influence was also a feature of Frid, Smith, Sparrow, and Trinidad's (2009) study of teachers in transition. These researchers noted that school-related factors influenced beginning teachers' curriculum planning and their professional learning needs.

Overall, the studies concerning teacher affect and identity draw on an increasingly diverse set of theoretical frameworks. Most, but not all, interrogate pre-service teachers' accounts concerning beliefs and experiences in becoming a teacher. There remains considerable scope for observations of pre-service teachers' practice within the classroom practicum and integration of supervisors' experiences to be incorporated into future research.

TEACHER LEARNING

What pre-service teachers should learn within mathematics teacher education programs has come under increased scrutiny in an era of accountability and standards. In making decisions about what knowledges prospective teachers of mathematics require, the research studies within the review period reflect the international interest concerning dimensions of teacher knowledge (Liljedahl, 2009). In our review it is clear that interest in domains of teacher knowledge is particularly evident in early years and primary ITE programs.

Drawing on findings from the *Mathematical thinking of preschool children in rural and regional Australia: Research and practice* project (Hunting et al., 2008), Perry (2009) argued that quality teacher education for early years must place a greater emphasis on the content relevant to the mathematical thinking and problem solving of young children and infants, alongside the affective aspects of mathematics learning and teaching. At the primary level, Butterfield and Chinnappan's (2010) assessment of entry knowledge proficiency of 40 primary pre-service teachers across the four dimensions of teacher knowledge (Ball, Thames, & Phelps, 2008) led them to propose that the focus for professional learning needed to be more aligned to specialised content mathematics and the links to pedagogical content knowledge. Watson (2011) argued that in light of new demands for personal and professional numeracy proficiency, across the curriculum and in the interpretation and use of system data associated with local and national testing programs, that attention to critical numeracy need be a central concern of ITE. Other studies provided evidence for learning needs in more specific areas, for example, fractions (Forrester & Chinnappan, 2010; Livy, 2011). Chick and Pierce's (2008) study examined pre-service teachers' pedagogical content knowledge in the context of a statistics lesson planning task combined with an attitude survey. They found that students' lack of personal valuing of the rich data set on water supply—a topical issue—was clearly evident in their approach to the lesson-planning task. The researchers concluded that very few pre-service teachers were able to develop lesson plans that "involved sustained and effective use of the resource to bring out key statistical concepts" (p. 4). Despite designing hands-on tasks, the required thinking was shallow in nature reflecting the desire to be 'fun' at the expense of opportunities for conceptual challenge. Investigations of prospective secondary teachers' knowledge base led Brown, Stillman, Schwarz, and Kaiser (2008) to conclude that even with sound mathematical qualifications at tertiary level, some teachers were unable to "convey a complete image of proving at the lower secondary level" (p. 90).

While evaluation-type studies serve to highlight shortcomings in teachers' knowledge and frequently suggest additive solutions to initial teacher education curricula, some studies have considered alternative ways to improve teacher knowledge. For example, Forrester and Chinnappan (2011) and Harvey (2011) addressed identified weaknesses in prospective primary conceptual understanding of fractions by implementing model based teaching approaches. Concerned to develop pre-service teachers' knowledge about decimals and fractions, Widjaja and Stacey (2009) implemented a teaching experiment working with tasks incorporating concrete models such as linear arithmetic blocks (LAB). In detailing the development of knowledge of one participant, Widjaja and Stacey attributed the growth in part to the pre-service teacher's ability to extend the use of LAB from a representational tool to a thinking tool.

Other studies have focused on detailing growth across a program of study. Ell, Grudnoff, Aitken, Hill, and Le Fevre (2008) assessed 120 pre-service teachers' mathematics content knowledge and their responses to an example of student work, first at entry to, and then at exit from, a one-year primary teacher education qualification. They found that the pre-service teachers' ability to unpack and offer appropriate responses to the students' work improved significantly over the duration of the qualification. Pre-service teachers with higher levels of personal content knowledge tended to be more sophisticated in their response to the work sample when they entered the course, and this relationship was even stronger at exit from the program.

Pre-service teachers' use of resources to enhance their content knowledge has also been a feature of some studies. Lange and Meaney (2011) developed and trialled a web-based instruction model that provided pre-service teachers with a resource of internet websites that emphasised mathematical concepts. However, for some pre-service students, their beliefs about how mathematics was learnt, limited the value of the resource. They concluded from their study that, challenging the view that mathematics learning necessarily involved an expert, such as a teacher, showing a novice how to do the problem remained an urgent priority for teacher education. Wilson and McChesney (2010) were interested to understand how pre-service teachers engaged with curriculum material during their ITE program and how they transformed their knowledge of the curriculum for classroom teaching. Through questionnaires and interviews, students who were completing a course on issues related to developing and implementing mathematics programs in the primary classrooms discussed their expectations and experiences of using the curriculum document and supporting resources (e.g., nzmaths website) to support a long-term planning activity. Wilson and McChesney reported that the long-term planning task provided an authentic opportunity for students to develop both curriculum and content knowledge. However, they highlighted that little is known about how newly qualified teachers transform this knowledge into the classroom setting, nor how beginning teachers might continue to develop this knowledge during their induction phase.

Some studies have examined teacher professional learning as it develops across the continuum of initial teacher education and the classroom workplace. In advocating that initial teacher education must support teachers to develop as professionals who have capacities to break the cycle of tradition, Frid et al. (2009) conducted a two phase study involving a survey and interviews. Their aim was to understand the links between the pre-service learning and the graduate teachers' current practices, an aim that is particularly important in a time when teacher education is portrayed as a weak intervention (Feiman-Nemser, 2001). The researchers claimed that three factors emerged as key influences upon the reported mathematics teaching practices: (a) university learning, (b) the diversity of learners in the classroom, and (c) affordances or constraints within the school community. In looking to understand the impact of the university learning experiences in particular, the researchers concluded that the development of a mathematics teaching philosophy, the strength of pedagogical content knowledge, and the professional confidence were hallmarks of those teachers who were 'prepared' to "be thinking-acting-leading mathematics teachers" (Frid & Sparrow, 2009, p. 52). The concept of professional confidence adds a further dimension to the teacher learning literature that may usefully link with studies involving beliefs and affect.

INSTRUCTIONAL INNOVATIONS

In looking more closely at what it is that we do in initial teacher education programs, how we create a space for "learning the work of teaching" (Lampert, 2010, p. 21), we review those studies that have explicitly investigated the use of innovative pedagogies and associated instructional activities. Klein (2008) investigated ways for pre-service mathematics education students to find an 'at homeness' and satisfaction in ways of doing mathematics. The program attempted to develop the mathematics though an emphasis on pedagogy rather than the other way round. Although students responded positively to this learning experience, not all students came to know new ways of 'being' a learner of mathematics. In arguing that it remains challenging to know "how a program of study at university could successfully overwrite already constituted discursive alienation" (p. 322), Klein noted that it is important that researchers look further than the immediate effects of teaching and "track conscientiously the effects of our teaching on numeracy education in schools" (p. 322). Another intervention aiming to build pre-service primary teachers' confidence and abilities in teaching and learning mathematics utilised problem-based learning. In this pilot study, Schmude, Serow, and Tobias (2011) provided students with authentic scenarios of children engaged in mathematical tasks from which the pre-service students derived Learning Targets related to personal knowledge development. Students valued the collaborative approach to learning and the authentic teaching contexts. The researchers used their experience of this study to scale up the intervention to include an on-line learning environment.

Looking also to develop the mathematical identity of her students, Owens (2007/2008) trialled an intervention based on social learning theories (e.g., Wenger, 1998). Focused on the need for pre-service teachers to become confident, self-regulated mathematical problem solvers, Owens provided pre-service primary teachers with opportunities to work collaboratively on mathematical problems of their own creation drawn from their own local communities. According to Owens, these mathematical activities, combined with access to social interaction and technological tools, supported the development of pre-service teachers identities as self-regulated learners, their social identities within the group, and hence their identities as mathematical thinkers. Although Owens positioned confidence within the affective domain, her study points to important links between the development of confidence and experiences of success. In a later exploration of prospective teachers' confidence, Beswick, Ashman, Callingham, and McBain (2011) contrasted the view of confidence as emotive and contextual (Burton, 2004) with Graven's (2004) conceptualisation of confidence as inherent in learning. They pointed to a need for further research on the notion of confidence, its relationship with cognition and affect, and its role in prospective teachers' evolving identities.

Several studies have contributed to international calls for teacher education to focus more on "core tasks that teachers must execute to help pupils learn" (Ball & Forzani, 2009, p. 497) Bragg and Nicol (2008) trialled an innovative way for prospective teachers to learn the practice of selecting and posing good mathematics tasks. Their study explored the experiences of elementary pre-service teachers who developed open-ended tasks inspired by a self-selected set of digital images collected for the purpose of investigating mathematics. They found that although the experiences supported the pre-service teachers' confidence to create mathematical problems that were beyond the textbook and classroom walls, the activity was not without its challenges. Reflections on efforts to create open-ended problems exposed prospective teachers' uncertainties as to the nature of open-ended questions. Of note, was that images were more likely to depict school mathematics, rather than mathematics that connected to one's own interests and life experiences. Nicol and Bragg (2009) detailed the types of problems that pre-service teachers posed, what they noticed, and what they found challenging in the process. They concluded that requiring pre-service teachers to pose more than one open-ended problem from an image supported their learning about how to create, adapt, and extend problems. Utilising vee diagrams as part of assessment practices with a secondary mathematics education course was the subject of investigation by Afamasaga-Fuata'i (2008/2009). This study explored pre-service teachers' use of vee diagrams to make their thinking about problem solving visible and open to further reflection and refinement.

Watson and Sullivan (2008) also highlighted the importance of task selection. To inform prospective teachers about the range and purpose of possible tasks, while simultaneously providing them with opportunities to learn more about mathematics and the nature of mathematical activity, they proposed engaging teachers in a range of tasks each with a specific purpose (e.g., conceptual understanding, mathematical fluency, strategic competence, or adaptive reasoning). The tasks were embedded in a template to facilitate discussion of the key phases of a lesson. The templates, they claimed, are particularly useful with prospective teachers in that they "provide structures that can assist in their creation of mathematics lessons" (p. 114). Like others (e.g., Kazemi, Franke, & Lampert, 2009; Ball & Forzani, 2009), Watson and Sullivan's placement of typical task-types into lesson templates recognises the practical needs of teachers, and the need for more explicit instruction aligned towards the practical work of teaching.

Related to studies focused on enactment of core tasks, are studies that look closely on ways to provide "more appropriate ways to develop, fine-tune and coach novice teachers' performance over a variety of settings" (Kazemi et al., 2009, p. 120). Some of these studies have been discussed earlier in relation to the (re)design of practicum experiences. However, an explicit focus on learning the work of teaching was evident in a study conducted by McDonough and Sexton (2011). Their study involved 12 pre-service teachers who purposefully selected high-leverage routines (Ball, Sleep, Boerst, & Bass, 2009)—such as structuring purposeful tasks that enable a range of strategies and products to emerge or orchestrating whole-class discussions—as the focus of their learning. Learning was supported by a triadic school-university-pre-service teacher partnership involving an eight-week extended practicum. Within such a partnership the researchers noted that they also learnt more about their work as teacher educators, a topic of the next section.

TEACHER EDUCATORS RESEARCHING THEIR OWN PRACTICE AND THE PRACTICE OF ITE

As Goos (2009) has stated, "calls for reform in mathematics education are implicitly based on the assumption that well prepared mathematics teacher educators are available who can foster change in teachers' traditional beliefs and practices" (p. 209). However, the "notion of educating teacher educators for their professional task is relatively new and thus the practices that might support their learning and development are not well understood" (Goos, 2009, p. 214). This review highlights a small but significant group of studies that have contributed to the emerging field of professional learning of mathematics teacher educators.

Maher (2011), in a study involving four teachers seconded as mathematics educators in a university setting, provided a rare glimpse of the experiences of the emergent teacher educator. Although the respondents were all experienced classroom teachers they found the transition to the role of teacher educators challenging. Challenges included the need to link theory with practice, assessment process, teaching in an online environment, and not being part of the research culture. Maher concluded that more research into how to support teacher-educators' pedagogical content knowledge is needed.

Goos offered a sociocultural framework, based on an adaptation of Valsiner's (1997) Zone theory, to understand the work of mathematics teacher educators as researchers and as learners. Applying this framework to the learning of mathematics teacher educators focused attention on the knowledges needed by the teacher educator that included pedagogies of practice that "connect prospective teachers' learning in the university and practicum context" (Goos, 2009, p. 5). The framework also focused on the affordance and constraints of context. For example, Goos (2008c) claimed that as a teacher educator, her own Zone of Free Movement (ZFM) is constrained by the likes of access to technology resources, perceptions amongst colleagues that teacher education is low status work, and student characteristics, such as their mathematical knowledge. To compound this situation, in the period of review, tertiary education institutions within Australasia have experienced significant staffing, resourcing, and workload pressures that are common to the university environment.

Such a landscape presents few opportunities for teacher educators' professional development, or research associated with theorising their own practices. An exception is the process in which researchers from seven universities engaged to develop instruments for large scale study (see Callingham et al., 2011). Described by Beswick and Callingham (2011), and reflected upon by Chick (2011), the process exposed a range of dilemmas inherent in making judgments about the content and pedagogical approaches used to develop prospective teachers' PCK, and the extent to which these are contestable. More often, studies that have involved teacher educators reflecting on their own practice have tended to be conducted by small groups within an institution. For example, in seeking to develop a curriculum that would prepare teachers for transformative action, Lomas (2009) evaluated current practice within an institution by surveying prospective teachers' perceptions of the mathematics educators' practice in terms of constructivist orientations. Brandenburg (2008, 2009) used a self-study of her own practice to examine the ways in which the reflection and reflective practices activities, Roundtable Reflection sessions and Critical Incident Questionnaires (see earlier discussion of these activities), influenced the development of her pedagogy. For her, she argued that the space for learning about teaching—'the crucible of inquiry'—was created in the intersection of the subsets related to the teacher educator teaching and the prospective teacher learning about teaching.

CONCLUSION: LOOKING BACK AND MOVING FORWARD

The professional education of prospective teachers of mathematics continues to be situated within the context of ongoing concerns about the recruitment of quality teachers and associated policy changes regarding accountabilities of teacher education programs. Our critical review highlights exemplars of Australasian research that form part of the groundswell of studies that "help us learn more about what constitutes learning spaces for teachers and teacher educators" (Clandinin, 2008, p. 358). In looking at how these studies advance the field of teacher education, we considered how they have informed our thinking about learning the work of teaching, how they have informed changes in program design and learning spaces, and how they have informed our own learning about who we are as teacher educators and researchers and what we stand for. The reviewed studies collectively demonstrate not only the complexities involved in creating learning spaces in teacher education, be they in the prior learning experiences, a lecture room, the practicum experience, or situated in a distance learning environment, they also served to reaffirm the implications of the design of such spaces and the activities that are offered within such spaces, for prospective teachers' constructions of self, identity and learning.

Reflection on the recommendations from the previous MERGA review (Goos et al., 2008) suggests that we have moved positively in some directions but have still to make significant moves in others. We continue to pursue small scale qualitative studies, studies that for the most part are conducted by teacher educators with their own students. However, we note that those studies that move beyond a commentary of local issues, supported by the application of appropriate and sometimes bold theoretical frameworks, can offer wider implications. For example, Walshaw's (2009, 2010a) powerful analyses of individual student's practicum experiences alerts us to the need to attend more closely to the way in which people and systems shape prospective teachers' identities. Goos and colleagues (2008c, 2010) have developed new frameworks for analysing the process of becoming a teacher and teacher educator learning.

We have yet to see sustained evidence of an expansion of the field through large-scale studies, as recommended in the previous review (Goos et al., 2008). With the exception of the work in progress by Callingham et al. (2011), a study that offers a promising opportunity for a broader consideration of the effectiveness of initial teacher education, there remains a scarcity of studies involving cross institutional collaboration at national and international levels. As noted by Goos and colleagues, such collaboration is valuable from two standpoints. Firstly, it supports the "development of a more critical stance in research of our own teacher education practices and programs, by creating scepticism and distance" (p. 305). Secondly, collaboration opens up more potential for comparative studies on the organisation and the influence of teacher preparation on teacher learning and subsequent practice.

In looking at the recommendations from the 15th ICMI study, Ball and Even (2009) urged that we focus "*teachers' education on practice*—and the problem of doing it effectively" (p. 255). Some reviewed studies have made bold moves in trialling new pedagogies (e.g., Frid & Sparrow, 2009; Klein & Smith, 2009; McDonough & Sexton, 2011) that aim for more than teaching prospective teachers *about* effective practices, but look more closely at how teachers might learn the work of ambitious teaching (Lampert, 2010). In these studies, the focus has been on developing teachers' dispositions to engage students in serious academic work, and to be able and willing to use this knowledge in particular moments of practice. The challenge will be for future research to look more closely at how we might best establish a common practice within teacher education that enables prospective teachers to learn to enact key practices of ambitious mathematics teaching in principled ways, using knowledge of subject matter and knowledge of students appropriately (Kazemi et al., 2009).

Given that the improvement of teacher education depends on the quality of teacher educators, we endorse Ball and Even's (2009) recommendation that we have much to learn about the kinds of knowledges and supports that are needed for teacher educators' own growth. In this review we have seen a small but sustained program of research that is adding to this important area.

Ball and Even's third and final major recommendation concerns the growing need "for valid and reliable assessments of teachers' learning" (p. 257). To date, most research efforts have focused on assessments of teacher knowledges and beliefs within an institution. We lack studies which offer insights into how to assess the learning of knowledge *and* the learning of practice components of teacher education in a way that supports teachers' professional growth and inform our own practices. Whilst studies such as Ell et al. (2008) provide promise, there remains a pressing need for studies "that can track the impact [in terms of teacher and student learning] of programs over time while respecting the complexity of linking initial preparation to eventual outcomes" (Grossman & McDonald, 2008, p. 199).

Taken together, these recommendations suggest that there is much work to be done. Such work will benefit from collaboration across all sectors and should make strong connections with research on the continuing development of mathematics teachers in schools. As Walshaw (2009) noted, it is imperative that we engage in research that moves us forward if we are "to advance our understanding of how teacher education might become an asset rather than a (presumed) liability for sustained growth in the twenty-first century" (p. 561).

REFERENCES

Afamasaga-Fuata'i, K. (2008/2009). Secondary pre-service teachers' use of vee diagrams to analyse problems and illustrate multiple solutions. *Mathematics Teacher Education and Development, 10*, 15–29.

Afamasaga-Fuata'i, K., Meyer, P., & Falo, N. (2008). Assessing primary preservice teachers mathematical competence. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 43–49). Brisbane, QLD: MERGA.

Anthony, G. (2008, March). *Mathematics teacher education: The continuum from initial teacher education to the classroom*. Paper presented at the Symposium on the Occasion of the 100th anniversary of ICMI, Rome, Italy.

Anthony, G., Butler, P., & Rawlins, P. (2011). Secondary mathematics and statistics teachers: Who are our teachers of the future? *New Zealand Mathematics Magazine*, 23–37.

Ashman, D., & McBain, D. (2011). Teacher and preservice teacher beliefs about mathematics teacher education. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 50–58). Adelaide, SA: AAMT & MERGA.

Australian Institute for Teaching and School Leadership. (2011). *National professional standards for teaching*. Melbourne, Australia: Education Services Australia.

Ball, D. L., & Even, R. (2009). Strengthening practice in and research on the professional education and development of teachers of mathematics: Next steps. In R. Even & D. L. Ball (Eds.), *The professional education and development of teachers of mathematics: The 15th ICMI study* (pp. 255–259). New York, NY: Springer.

Ball, D. L., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60*(5), 497–511.

Ball, D. L., Sleep, L., Boerst, T., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *The Elementary School Journal, 109*(5), 458–474.

Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. *Journal of Teacher Education, 59*(5), 389–407.

Ballati, J., & Rigano, D. (2011). Preservice teacher perceptions of good mathematics teachers: What matters? In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 82–88). Adelaide, SA: AAMT & MERGA.

Barton, B., & Sheryn, L. (2009). The mathematical needs of secondary teachers: Data from three countries. *International Journal of Mathematics Education in Science and Technology, 40*(1), 101–108.

Beswick, K. (2011, June). *Knowledge/beliefs and their relationship to emotion.* Paper presented at the 16th annual Mathematical Views conference, (Article 2). Tallinn, Estonia.

Beswick, K., Ashman, D., Callingham, R., & McBain, D. (2011, November). *Teachers' and preservice teachers' confidence to teach primary school mathematics.* Paper presented at the annual conference of the Australian Association for Research in Education, Hobart, Australia.

Beswick, K., & Callingham, R. (2011, November). *Building the culture of evidence-based practice in teacher preparation: Instrument development and piloting.* Paper presented at the annual conference of the Australian Association for Research in Education, Hobart, Australia. Retrieved from Online First http://www.springerlink.com/content/th22781265818125/

Beswick, K., Callingham, R., & Watson, J. M. (2011). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher.* Retrieved from Online First http://www.springerlink.com/content/th22781265818125/

Beswick, K., & Dole, S. (2008). Recollections of mathematics education: Approaching graduation and 5 years later. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 67–73). Brisbane, QLD: MERGA.

Borko, H., & Whitcomb, J. A. (2008). Teachers, teaching, and teacher education: Comments on the National Advisory Panel's report. *Educational Researcher, 37*(9), 565–572.

Bragg, L., & Nicol, C. (2008). Designing open-ended problems to challenge preservice teachers' views on mathematics and pedagogy. In O. Figueras, J. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint Meeting of PME 32 and PME-NA XXX, pp. 201–208). Morelia, Mexico: Cinvestav-UMSNH.

Brandenburg, R. (2008). *Powerful pedagogy: Self study of a teacher educator's practice*. New York, NY: Springer.

Brandenburg, R. (2009). Assumption interrogation: An insight into a self-study researcher's pedagogical frame. In D. Tidwell, M. Heston, & L. Fitzgerald (Eds.), *Research methods for the self-study of practice* (pp. 195–211). New York, NY: Springer.

Brown, G. (2009). *Review of education in mathematics, data science and quantitative disciplines: Report to the Group of Eight Universities*. Retrieved from http://www.go8.edu.au/storage/go8statements/2010/Go8MathsReview.pdf.

Brown, J., Stillman, G., Schwarz, B., & Kaiser, G. (2008). The case of mathematical proof in lower secondary school: Knowledge and competencies of pre-service teachers. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 67–73). Brisbane, QLD: MERGA.

Burton, L. (2004). "Confidence is everything": Perspectives of teachers and students on learning mathematics. *Journal of Mathematics Teacher Education, 7*, 357–381.

Butterfield, B., & Chinnappan, M. (2010). Walking the talk: Translation of mathematical content knowledge to practice. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of*

mathematics education (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 109–116). Fremantle, WA: MERGA.

Callingham, R., Beswick, K., Chick, H., Clark, J., Goos, M., Kissane, B., Tobias, S. (2011). Beginning teachers' mathematical knowledge: What is needed? In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 900–907). Adelaide, SA: AAMT & MERGA.

Chick, H. (2011, November). *God-like educators in a fallen world*. Paper presented at the annual conference of the Australian Association for Research in Education, Hobart, Australia.

Chick, H. L., & Pierce, R. U. (2008). Teaching statistics at the primary school level: Beliefs, affordances, and pedagogical content knowledge. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education* (Proceedings for the ICMI study 18 and 2008 IASE Round Table Conference, CD-ROM, pp. 4). Voorburg, The Netherlands: ICMI, IASE & ISI.

Clandinin, D. J. (2008). Editorial: Creating learning spaces for teachers and teacher educators. *Teachers and Teaching: Theory and Practice, 14*(5&6), 385–389.

Ell, F., Grudnoff, A., Aitken, G., Hill, M., & Le Fevre, D. (2008). *Thinking for effective teaching in mathematics: How does it change in an initial teacher education programme?* Paper presented at the annual conference of the American Educational Research Association, Chicago, IL.

Even, R., & Ball, D. L. (2009). Setting the stage for the ICMI study on the professional education and development of teachers of mathematics. In R. Even & D. L. Ball (Eds.), *The professional education and development of teachers of mathematics: The 15th ICMI study* (pp. 1–9): New York, NY: Springer.

Feiman-Nemser, S. (2001). From preparation to practice: Designing a continuum to strengthen and sustain teaching. *Teachers College Record, 103*, 1013–1055.

Forrester, T., & Chinnappan, M. (2010). The predominance of procedural knowledge in fractions. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia on the Mathematics Education Research Group of Australasia, pp. 185–192). Fremantle, WA: MERGA.

Forrester, T., & Chinnappan, M. (2011). Two avatars of teachers' content knowledge of mathematics. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 261–269). Adelaide, SA: AAMT & MERGA.

Frid, S., Goos, M., & Sparrow, L. (2009). What knowledge is needed for effective teaching of mathematics. *Mathematics Teacher Education and Development, 9*, 1–3.

Frid, S., Smith, M., Sparrow, L., & Trinidad, S. (2009). An examination of innovations in mathematics, information and communication technology and science in a pre-service teacher education program. *International Journal of Innovation and Learning, 6*(5), 517–536.

Frid, S., & Sparrow, L. (2009). "You just have to take a bit of a risk sometimes": Breaking the 'cycle of tradition' in primary mathematics. *Mathematics Teacher Education and Development, 11*, 36–53.

Goos, M. (2008a). Research-teacher relationships in mathematics education. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 227–233). Brisbane, QLD: MERGA.

Goos, M. (2008b). Sociocultural perspectives on learning to teach mathematics. In B. Jaworski & T. Wood (Eds.), *The mathematics teacher educator as a developing professional* (pp. 75–91). Rotterdam, The Netherlands: Sense.

Goos, M. (2008c). Towards a sociocultural framework for understanding the work of mathematics teacher-educator-researchers. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 235–241). Brisbane, QLD: MERGA.

Goos, M. (2009). Investigating the professional learning and development of mathematics teacher educators: A theoretical discussion and research agenda. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 209–216). Palmerston North, NZ: MERGA.

Goos, M., Arvold, B., Berdnarz, N., DeBlois, L., Maheux, J., Morselli, F., Proulx, J. (2009). School experience during pre-service teacher education from the students' perspective. In R. Even & D. L. Ball (Eds.), *The professional education and development of teachers of mathematics* (pp. 83–91). New York, NY: Springer.

Goos, M., & Geiger, M. (2010). Theoretical perspectives on mathematics teacher change. *Journal of Mathematics Teacher Education, 13*(6), 4999–4507.

Goos, M., Smith, T., & Thornton, S. (2008). Research on the pre-service education of teachers of mathematics. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W. T. Seah, & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004–2007* (pp. 291–311). Rotterdam, The Netherlands: Sense.

Graven, M. (2004). Investigating mathematics teacher learning within an in-service community of practice: The centrality of confidence. *Educational Studies in Mathematics, 57*, 177–211.

Grootenboer, P. (2008). Mathematical belief change in prospective primary teachers. *Journal of Mathematics Teacher Education, 11*(6), 479–497.

Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal, 45*(1), 184–205.

Harvey, R. (2011). Challenging and extending a student teacher's concepts of fractions using an elastic strip. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 333–339). Adelaide, SA: AAMT & MERGA.

Hunting, R. P., Bobis, J., Doig, B., English, L., Mousley, J., Mulligan, J., ... Young-Loveridge, J. (Eds.). (2008). *Mathematical thinking of preschool children in rural and regional Australia: Research and practice. Final report to National Centre of Science, ICT, and Mathematics Education in Rural and Regional Australia (SiMERR)*. Bendigo, Australia: La Trobe University.

Kazemi, E., Franke, M., & Lampert, M. (2009). Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research group of Australasia, pp. 11–29). Palmerston North, NZ: MERGA.

Klein, M. (2008). Preservice teachers and numeracy education: Can poststructuralism contribute? In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 317–322). Brisbane, QLD: MERGA.

Klein, M., & Smith, K. (2009). Being numerate for teaching: The indivisibility of learning landscape, participation and practice. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research group of Australasia, pp. 299–306). Palmerston North, NZ: MERGA.

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education, 61*(1–2), 21–34.

Lange, T., & Meaney, T. (2011). Preservice teachers learning mathematics from the Internet. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 438–445). Adelaide, SA: AAMT & MERGA.

Leder, G. (2009). Initial mathematics teacher education: Comments and reflection. In R. Even & D. L. Ball (Eds.), *The professional education and development of teachers of mathematics: The 15th ICMI study* (pp. 135–138). New York, NY: Springer.

Liljedahl, P. (2009). Components of mathematics teacher training. In R. Even & D. L. Ball (Eds.), *The professional education and development of teachers of mathematics: The 15th ICMI study* (pp. 25–33). New York, NY: Springer.

Livy, S. (2010). A ‘knowledge quartet’ used to identify a second-year pre-service teachers’ primary mathematical content knowledge. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference on the Mathematics Education Research Group of Australasia, pp. 244–351). Fremantle, WA: MERGA.

Livy, S. (2011). We can order by rote but can’t partition: We didn’t learn the rule. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 464–472). Adelaide, SA: AAMT & MERGA.

Lo, W., & Anderson, J. (2010). Beyond the curriculum: The mathematical beliefs of pre-service primary teachers in Hong Kong. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference on the Mathematics Education Research Group of Australasia, pp. 657–664). Fremantle, WA: MERGA.

Lomas, G. (2009). Pre-service primary teachers’ perceptions of mathematics education lecturers’ practice: Identifying issues for curriculum development. *Mathematics Teacher Education and Development, 11*, 4–21.

Louden, W., Heldsinger, S., House, H., Humphry, S., & Fitzgerald, D. (2010). *Learning from teacher education: The impact of teacher education on knowledge of literacy and mathematics teaching*: Perth, Australia: University of Western Australia.

Maher, N. (2011). From classroom to campus: The perception of mathematics and primary teachers on their transition from teacher to teacher educator. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* ((Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 491–499). Adelaide, SA: AAMT & MERGA.

McDonough, A., & Sexton, M. (2011). Building preservice teacher capacity for effective mathematics teaching through partnerships with teacher educators and primary school communities. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 508–514). Adelaide, SA: AAMT & MERGA.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York, NY: Macmillan.

Meaney, T., & Lange, T. (2010). Pre-service students’ responses to being tested on their primary school mathematical knowledge. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 399–406). Fremantle, WA: MERGA.

New South Wales Institute of Teachers. (2011). *Graduate teacher standard.1.1.1 – 1.1.2 Subject content requirements.* Retrieved from http://www.nswteachers.nsw.edu.au/

New Zealand Government. (2010). *A vision for the teaching profession: Education workforce advisory group report to the Minister of Education.* Wellington, NZ: Author.

New Zealand Teachers Council. (2007). *Graduating teacher standards: Aotearoa New Zealand.* Wellington, NZ: Author.

New Zealand Teachers Council. (2010). *Approval, review and monitoring processes and requirements for initial teacher education programmes*. Wellington, NZ: Author.

Nicol, C., & Bragg, L. (2009). Design problems: What kinds of open-ended problems do preservice teachers pose? In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International

Group for the Psychology of Mathematics Education, pp. 255–232). Thessaloniki, Greece: PME.
Owens, K. (2007/2008). Identity as a mathematical thinker. *Mathematics Teacher Education and Development, 9*, 36–50.
Perry, B. (2009). Mathematical thinking of preschool children in rural and regional Australia: Implications for the future. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference the Mathematics Education Research Group of Australasia, pp. 658–661). Palmerston North, NZ: MERGA.
Prescott, A., & Cavanagh, M. (2008). A situated perspective on learning to teacher secondary mathematics. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting direction*s (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 407–413). Brisbane, QLD: MERGA.
Prescott, A., & Cavanagh, M. (2010). Roles of secondary mathematics supervising teachers during the practicum. In M. Pinto & T. Kawasaki (Eds.), *Optimising student understanding in mathematics* (Proceedings of the 34th conference of the International Group for the Psychology of Mathematics Education, p. 90). Belo Horizonte, Brazil: PME.
Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2009). *Developing primary mathematics teaching. Reflecting on practice with the knowledge quartet*. London, England: SAGE Publications.
Schmude, M., Serow, P., & Tobias, S. (2011). Improving self-confidence and abilities: A problem-based learning approach for beginning mathematics teachers. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 2 3rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 676–684). Adelaide, SA: AAMT & MERGA.
Smith, K. (2009). Bridging understandings, interest and identity gaps in a first year numeracy subject. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 669–672). Palmerston North, NZ: MERGA.
Stacey, K. (2008). Mathematics for secondary teaching. In P. Sullivan & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp. 87–113). Rotterdam, The Netherlands: Sense.
Tatto, M. T., Lerman, S., & Novotna, J. (2010). The organization of the mathematics preparation and development of teachers: A report from the ICMI study 15. *Journal of Mathematics Teacher Education, 13*, 313–324.
Valsiner, J. (1997). *Culture and the development of children's action: A theory of human development* (2nd ed). New York, NY: John Wiley & Sons.
Walshaw, M. (2008). Developing theory to explain learning to teacher. In T. Brown (Ed.), *The psychology of mathematics education: A psychoanalytic displacement* (pp. 119–137). Rotterdam, The Netherlands: Sense.
Walshaw, M. (2009). Exploring the identity of a pre-service teacher: communal processes during the practicum. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 555–562). Palmerston North, NZ: MERGA.
Walshaw, M. (2010a). Learning to teach: Powerful practices at work during the practicum. In M. Walshaw (Ed.), *Unpacking pedagogy: New perspectives for mathematics classrooms* (pp. 109–128). Charlotte, NC: Information Age.
Walshaw, M. (2010b). Mathematics pedagogical change: Rethinking identity and reflective practice. *Journal of Mathematics Teacher Education.13*, 487–497.
Watson, A., & Sullivan, P. (2008). Teachers learning about tasks and lessons. In D. Tirosh & T. Wood (Eds.), *Tools and processes in mathematics teacher education* (pp. 109–134). Rotterdam, The Netherlands: Sense.

Watson, J. M. (2011). Personal and professional numeracy: A unit for pre-service teachers at the University of Tasmania. *Numeracy: Advancing Education in Quantitative Literacy*, *4*(1), Article 2. Retrieved from http://services.bepress.com/numeracy/vol4/iss1/art2

Way, J. (2009). Reflection in self-assessed online discussion. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 571–578). Palmerston North, NZ: MERGA.

Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge, England: Cambridge University Press.

Widjaja, W., & Stacey, K. (2009). Growth of pre-service teachers' knowledge and teaching ideas about decimals and fractions: The case of Vivi. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 587–594). Palmerston North, NZ: MERGA.

Wilson, S., & Gurney, S. (2011). "My self-esteem has risen dramatically": A case-study of pre-service teachers' action research using bibliotherapy to address mathematics anxiety. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 804–812). Adelaide, SA: AAMT & MERGA.

Wilson, S., & McChesney, J. (2010). "I always feel more confident when I know where things are going": How do pre-service teachers engage with mathematics curriculum documentation? In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 649–656). Fremantle, WA: MERGA.

Wilson, S., & Thornton, S. (2007/2008). "The factor that makes us more effective teachers": Two pre-service primary teachers' experience of bibliotherapy. *Mathematics Teacher Education and Development, 9*, 21–35.

Young-Loveridge, J. (2010). Two decades of mathematics education reform in New Zealand: What impact on the attitudes of teacher education students? In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education.* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 705–712). Fremantle, WA: MERGA.

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PROFESSIONAL KNOWLEDGE OF PRACTISING TEACHERS OF MATHEMATICS

Key words: professional development; pedagogical knowledge; expertise; pedagogical frameworks.

INTRODUCTION

Teachers' knowledge of mathematics has become a central focus of educational researchers and policy makers with conceptions of teacher knowledge continuously being transformed. Intuitively, we have known for some time what research now provides an evidence base for—that "teacher knowledge matters" (Sullivan, 2008b, p. 2). But exactly *what* knowledge matters more, and *why,* are more significant and vexing questions for researchers and educators to address. Consequently, attention has moved beyond looking solely at what knowledge teachers possess to *why* different types of knowledge are important and *how* that knowledge is acquired, studied and impacts on the quality of instruction.

While historically unquestioned in importance, it has become politically as well as educationally necessary to provide an evidence base as to why knowledge of mathematics content by itself is insufficient for effective teaching of mathematics. For instance, in a recent report commissioned for the Go8 universities on mathematics entry requirements for Australian primary teacher education programs, it was found that many accreditation bodies now required entrants to have studied mathematics to the final years of secondary school (G. Brown, 2009). The report recommended that knowledge of mathematics content should become a major focus of primary teacher education programs. Mathematics educators and researchers are aware that while such recommendations help to emphasise the importance of specific content knowledge, they can also be damaging when a full picture of teacher knowledge in all its complexity is not portrayed. Accordingly, theorising and research surrounding teacher knowledge has escalated, resulting in expanded notions of some aspects of teacher knowledge and the emergence of new conceptual frameworks informing and fuelling research on teacher knowledge (e.g., Chick, 2009a, 2009b; Hill, Ball, & Schilling, 2008).

This chapter provides a critical review of research and theoretically informed perspectives on knowledge in mathematics education and development of practising teachers published by Australasian researchers from 2008–2011. Previous four-yearly reviews published by MERGA have dealt with the

B. Perry et al. (Eds.), Research in Mathematics Education in Australasia 2008–2011, 313–341.

professional learning of practising teachers of mathematics (Anderson, Bobis, & Way, 2008), and as a consequence, have considered teacher knowledge. However, never before has there been an entire chapter specifically devoted to this topic—an indicator of the increased attention teacher knowledge has attracted in the past few years. While there is some inevitable overlap of content and issues relevant to the study of pre-service teachers' knowledge of mathematics, it is beyond the scope of this chapter to address that body of research. Research relevant to pre-service teachers is discussed elsewhere in this volume.

Our review has five sections. We first consider the situated nature of teacher knowledge, thus reflecting the growing recognition by researchers that knowledge for teaching mathematics is not only mediated by sociocultural contexts, but also by teachers' beliefs, their conceptions of mathematics and the confidence they have in their own mathematical knowledge. The second section introduces various frameworks for researching teacher knowledge and includes the emerging notion of what many researchers now refer to as the mathematical knowledge for teaching. We then examine the various domains of teacher knowledge that have most recently dominated research in the field. This includes investigations of specific content areas of mathematics, the expanding domain of pedagogical content knowledge and knowledge of curriculum. The fourth section considers the mechanisms and processes by which teacher knowledge is acquired. It also critically reviews approaches used for researching the knowledge of teachers of mathematics. Finally, the chapter distils the information emanating from this body of literature and suggests how it can inform emerging research agendas, policy debates, continuing teacher education and, most critically—classroom practice.

THE NATURE OF TEACHER KNOWLEDGE

The situated nature of teacher knowledge has come to greater prominence among Australasian researchers in recent years. During the period under review (2008–2011), there has been a growing recognition that teacher knowledge is filtered through the social and cultural context of teaching and mediated by teachers' beliefs, their conceptions of mathematics, and their confidence in their own mathematical knowledge.

In his introduction to a plenary panel discussing the possible role(s) of theory in the context of mathematics teacher education, David Clarke (2009) emphasised the situated nature of teacher knowledge, and in particular attended to teaching as a culturally situated activity. With such a perspective in mind, Owens and Kaleva (2008) addressed the issue of how primary school teachers in Papua New Guinea (PNG) could use their cultural knowledge to improve their students' understanding of measurement. They used everyday examples of mathematical applications drawn from indigenous communities around PNG to help teachers understand how their cultural knowledge can be used in mathematics instruction by communicating to students the mathematical thinking behind the activities, thus making tacit teacher knowledge more explicit.

Effective teachers require knowledge of content and knowledge of teaching (Sullivan, 2008a). However, teacher knowledge is also closely interrelated to beliefs about mathematics, how the subject is best learned, and how it should be taught. Since beliefs are also influenced by the contexts in which teachers work, recent research has examined teacher knowledge and beliefs and how they impact on their teaching practises. For instance, Goos (2009) highlighted the relationships between teacher knowledge and beliefs, professional contexts and professional learning experiences. She proposed a sociocultural framework for investigating teacher learning in terms of the integration of technology into secondary classrooms. Combining the results of semi-structured interviews, a mathematical beliefs questionnaire and a series of lesson cycles, Goos suggested that the degree of alignment between teachers' knowledge and beliefs and professional contexts may provide insights into how teachers at different stages of their careers created professional learning opportunities in schools. Although the role of teachers' beliefs is beyond the scope of this chapter, the interconnected nature of teacher knowledge and beliefs is becoming more widely recognised (e.g., Beswick, Callingham, & Watson, 2011). Teachers' knowledge of mathematics and their classroom practices depend to a large extent on their beliefs about the nature of mathematics, how it is learned, and the role of the teacher.

Barton (2009) theorised on mathematical knowledge for teaching in a MERGA conference keynote address. He suggested that knowing about mathematics includes teachers' attitudes and orientations towards mathematics, which he described as the way teachers hold their mathematics, the way they know mathematics, and their relationship with mathematics. According to Barton, teachers must develop a rich vision and a carefully considered personal philosophy of mathematics while remaining receptive to the ideas of others, particularly the diverse and developing views of their students. But teachers should not hold too rigidly to their views to ensure they remain active learners of mathematics.

Teachers' Confidence in Their Own Mathematical Knowledge

An important theme emerging from studies of teacher knowledge is the influence teachers' confidence in their own knowledge has on their instructional decision-making and ultimately on student learning. Sullivan, Clarke, Clarke, and O'Shea (2009) discussed teacher confidence in terms of their ability to identify children's conceptual level on a trajectory of learning. They compared how three primary school teachers converted the same rich task into classroom learning activities by investigating how the lessons reflected each teacher's instructional goals. The researchers found that the teachers acted as they intended but their ability to appreciate the mathematics involved in the task directly influenced the types of learning opportunities they provided for students. The potential of the task was reduced by two of the teachers, and the researchers attributed this to the teachers' lack of confidence in their own mathematical ability to solve the task rather than any lack of familiarity with implementing problem-based learning activities. In contrast, the confidence of the third teacher allowed her greater freedom to explore the task in her lesson, resulting in more interesting student responses and apparently better learning. The researchers concluded that teachers' mathematical confidence shaped the potential of the task as a learning opportunity for students.

In a recent study, Beswick et al. (2011) (see also Watson, Brown, Beswick, & Wright, 2011) reported on a three-year professional development program with 62 middle school teachers. The research aimed to assess aspects of teachers' knowledge previously identified by Shulman (1986), and Ball, Thames and Phelps (2008), but was extended to include teachers' confidence to teach mathematics and their beliefs about teaching and learning mathematics. Findings revealed a close connection between teacher knowledge, confidence levels, and beliefs about the nature of mathematics learning and teaching. The study found that while building teachers' confidence to use mathematics and promote student understanding of mathematics was desirable, its development alone was not necessarily an indicator of competence.

Teachers' confidence in their knowledge of mathematics can be especially important when a new syllabus is implemented. Warren (2008/09) described a cyclic model of professional development, *Transformative Teaching in the Early Years Mathematics* (TTEYM), to guide novice teachers towards becoming expert in teaching unfamiliar content in a new Patterns and Algebra strand. The model was grounded in the notion of a community of practice and adopted a socio-constructivist perspective. Six Year 1 teachers worked in pairs to design and implement classroom activities for students. Warren found that the teachers' growing understanding of the patterns and algebra content gave them greater confidence to experiment in the classroom. Furthermore, confidence about teaching seemed to be strengthened by the opportunities for teachers to compare their teaching with other participants in the TTEYM project. Warren also noted that the strong connection between teachers' improved mathematical knowledge and the ways they made connections between mathematical concepts, used a variety of mathematical representations, and encouraged more meaningful classroom discussion.

Bobis (2009, 2010) used survey and interview data to examine the influence of primary teachers' knowledge of the *Count Me In Too* numeracy program for primary schools in New South Wales. A key aspect of the program, the Learning Framework in Number (LFIN), is used to describe children's early number learning. Bobis investigated 28 primary school teachers from three schools, explored their perceptions about their knowledge of the LFIN, their confidence to use the framework to assess children's mathematical development, and the extent to which they could use this knowledge to plan appropriate instruction. Teachers tended to rate their confidence low while their ability to assess and plan instruction was high. Bobis expressed a concern that this lack of confidence might have a detrimental effect on their instructional decision-making. She also noted that some teachers rated themselves low in their understanding of the LFIN because they appreciated how much more they needed to learn in order to use it effectively.

Other researchers have found that teachers were more likely to want to learn about mathematical content if they were made aware of the gaps in their current knowledge. Anstey and Clarke (2010) reported on a program, the *Teaching and Learning Coaches Initiative*, which provided support to Victorian government schools to improve students' learning outcomes in mathematics. The researchers invited 15 numeracy coaches to participate in monthly forums as well as 16 days of professional development focusing on the topic areas of fractions and algebra. Questionnaire and interview data were used to investigate the coaches' changing perceptions of their learning needs over the six-month study. The results indicated that the coaches' priority for mathematics content knowledge strengthened over the year of study. Anstey and Clarke noted from the results that, the focus on content knowledge helped participants identify what they did not know, thus increasing their goals to further develop their content knowledge.

All these studies highlight the links between mathematical confidence, subject-matter knowledge, and the impact on their instructional decision-making. Additionally, they highlight the fact that mathematical knowledge of teachers is a relative construct. That is, a teacher may rate their level of knowledge quite highly when compared to their immediate colleagues, but quite low when exposed to a more knowledgeable other. The influence of teachers' mathematical self-concept on their knowledge for teaching mathematics is a worthy area for further exploration to more fully understand the nature of teacher mathematical knowledge.

FRAMEWORKS OF TEACHER KNOWLEDGE

In this section, we outline the various attempts to identify frameworks of teacher knowledge. We first describe in some detail, background work in Australasia and overseas (particularly in the United States). We do this for two reasons; it enables the discussion of more recent Australasian work to be situated, and most Australasian researchers draw upon this earlier work in establishing their own frameworks or in using existing frameworks.

The Importance of Teacher Knowledge and the Need for Frameworks of Knowledge

For many years, it has been accepted that the teacher is the crucial variable in student achievement in mathematics, as with most other subject areas, accounting for about 30% of the variance in student achievement (Hattie, 2002). Similarly, it has been recognised for a long period that, in particular, teacher knowledge is key (Fennema & Franke, 1992). Mason and Spence (1999) described teachers' knowledge as dynamic and evolving and noted the importance of knowing as it requires "relevant knowledge to come to the fore so it can be acted upon" (p. 139). It is here that knowledge and practice intersect/interact and the knowledge can prove to be useful or otherwise. Much of the content knowledge that teachers have is not accessible. Brophy (1991) argued in relation to content knowledge that:

> Where (teachers') knowledge is more explicit, better connected, and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways and encourage and respond fully to students' comments

> and questions. Where their knowledge is limited, they will tend to depend on the text for content, de-emphasise interactive discourse in favor of seatwork assignments, and in general, portray the subject as a collection of static, factual knowledge. (p. 352)

However, it has only been since Shulman's seminal paper in 1986, that there has been serious consideration of the various components of teacher knowledge, and the contributions each of these make to the act and art of teaching. As will be discussed in this section, the act and art categories of teaching have been important in discussions of the components which can be developed, and are essential for effective teaching, as well as in establishing both ways of assessing teacher knowledge, and in exploring the impact of various professional learning programs on such knowledge.

Shulman (1986, 1987) argued that the acceptance of two distinct categories (subject matter knowledge and pedagogical knowledge), was simplistic and that the art of teaching could be more appropriately encapsulated by the term *pedagogical content knowledge* (PCK)—the intersection of pedagogical knowledge and content knowledge. He emphasised the many aspects of PCK, which he saw as including "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations— ... the ways of representing and formulating the subject that make it comprehensible to others" (1986, p. 9).

However, Shulman (1987) also discussed knowledge of curriculum; learners and their characteristics; educational contexts; and educational ends, purposes and values. In a personal communication (quoted in Boaler, 2003), Shulman noted that his model needed more emphasis on teacher action in practice, and teacher learning.

Ball and her colleagues (Ball et al., 2008) noted that in addition to these components, Shulman's categorisation was theoretical and not empirical. They claimed that while this was helpful at the time, further research was needed to establish a research-based categorisation and proposed the model shown in Figure 1.

Ball et al. (2008, pp. 399–403) defined the various components in their model as:

– *Common Content Knowledge:* Mathematical knowledge and skill used in settings other than teaching.
– *Horizon Knowledge:* An awareness of how mathematical topics are related over the span of mathematics included in the curriculum.
– *Specialised Content Knowledge:* Mathematical knowledge and skill unique to teaching.
– *Knowledge of Content and Students:* Knowledge that combines knowing about students and mathematics.
– *Knowledge of Content and Teaching:* Knowledge that combines knowing about teaching and mathematics.
– *Knowledge of Content and Curriculum:* Such knowledge relates closely to Shulman's curricular knowledge.

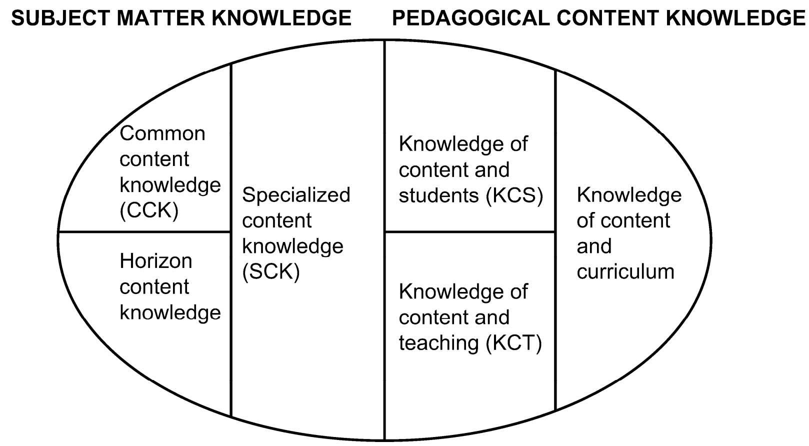

Figure 1. Framework of mathematical knowledge proposed by Ball et al. (2008, p. 403).

In much of their recent work, Ball and colleagues have used the term 'mathematical knowledge for teaching' to encompass those areas of their framework that are unique to the role of the teacher (Hill, Rowan, & Ball, 2005). Of course, any categorisation is unlikely to assume that the components or categories are mutually exclusive. As Ball et al. (2008) noted, "we recognise the problems of definition and precision exhibited in our current formulation" (p. 404).

A framework for teacher knowledge developed by Rowland, Turner, Thwaites and Huckstep (2009) was a result of an investigation into "how different kinds of primary mathematics teachers' content-related knowledge 'played out' in the classroom" (p. 26), by observing trainee teachers. This framework called the *Knowledge Quartet,* included four dimensions: foundation, transformation, connection and contingency. Although the development and use of this framework was primarily for "productive discussion of mathematics content knowledge between teacher educators, trainees and teacher mentors, in the context of school based placements" (Rowland, Huckstep, & Thwaites, 2005, p. 256) and therefore not strictly relevant to this chapter, it provides another lens through which to observe and describe practising teachers' mathematical knowledge for teaching.

Australasian Research Involving Frameworks of Teacher Knowledge

Several researchers in Australasia have taken-up the theme of categorisations of knowledge in recent years. Chick's (2009a, 2009b, 2010) recent work has focused on teachers' capacity to choose or design suitable examples, recognising what is afforded by these, and knowledge of how to adapt a given example to better suit an intended purpose. This built upon earlier work by Chick and her colleagues (Chick, Baker, Pham, & Cheng, 2006), who had proposed a framework for Pedagogical Content Knowledge that they used to investigate teacher knowledge of decimals and the teaching of decimals, through a questionnaire and interview protocol. This framework contained components under three broad categories: (a) Clearly PCK, (b) Content Knowledge in a Pedagogical Context, and (c) Pedagogical Knowledge in a Content Context. Although not published in the years spanned by this review, this framework was used in more current studies, and is therefore included here as Table 1.

Table 1. A framework for pedagogical content knowledge (after Chick et al., 2006)

PCK Category	***Evident when the teacher ...***
Clearly PCK	
Teaching Strategies	Discusses or uses general or specific strategies or approaches for teaching a mathematical concept or skill.
Student Thinking	Discusses or addresses student ways of thinking about a concept, or recognises typical levels of understanding.
Student thinking – Misconceptions	Discusses or addresses student misconceptions about a concept.
Cognitive Demands of Task	Identifies aspects of the task that affect its complexity.
Appropriate and Detailed Representations of Concepts	Describes or demonstrates ways to model or illustrate a concept (can include materials or diagrams).
Explanations	Explains a topic, concept or procedure.
Knowledge of Examples	Uses an example that highlights a concept or procedure.
Knowledge of Resources	Discusses/uses resources available to support teaching.
Curriculum Knowledge	Discusses how topics fit into the curriculum.
Purpose of Content Knowledge	Discusses reasons for content being included in the curriculum or how it might be used.
Content Knowledge in a Pedagogical Context	
Profound Understanding of Fundamental Mathematics (PUFM)	Exhibits deep and thorough conceptual understanding of identified aspects of mathematics.
Deconstructing Content to Key Components	Identifies critical mathematical components within a concept that are fundamental for understanding and applying that concept.
Mathematical Structure and Connections	Makes connections between concepts and topics, including interdependence of concepts.
Procedural Knowledge	Displays skills for solving mathematical problems (conceptual understanding need not be evident).
Methods of Solution	Demonstrates a method for solving a mathematical problem
Pedagogical Knowledge in a Content Context	
Goals for Learning	Describes a goal for students' learning.
Getting and Maintaining Student Focus	Discusses or uses strategies for engaging students.
Classroom Techniques	Discusses or uses generic classroom practices.

The great complexity of teaching and teacher knowledge is emphasised by the fact that Chick (2007) and Chick and Pierce (2008), subsequently studied and reported on *just one* of the 18 components of PCK she had identified earlier—teachers' use of examples that highlight a concept or procedure. Few would disagree that this is a very important component of a mathematics teacher's role. However, it should be noted that in discussing this one component, many overlaps with other components from her framework and the frameworks of others were evident, and reported by Chick. Their framework and subsequent studies add to the knowledge and research in this area as they "investigate more specific aspects of PCK" (Chick, 2007, p. 7) than the more general aspects of PCK.

Chick (2009a) defined example as "a specific instantiation of a general principle, chosen in order to illustrate or explore that principle" (p. 26). She reported on a study involving observations of the choice and use of examples by two Year 6 primary teachers in Victorian schools, as they each taught two lessons on the topic of ratio. Both teachers knew their students' mathematical capabilities well enough to choose tasks with appropriate cognitive demand, which Chick related to Ball's categories of "knowledge of content and students", and "knowledge of content and teaching" (see Figure 1). Chick noted that despite the small number of teachers and lessons observed, the observations nevertheless provided "a stimulus to the external observer to question what knowledge is desirable and what role alternative examples might play" (p. 29). She also proposed that issues around constructing examples, identifying their affordances, and using them to best effect in the classroom might be more explicitly addressed in pre-service and in-service programs.

As part of a questionnaire and interview protocol intended to elicit information on the PCK of secondary mathematics teachers, Chick (2009b) presented a page from a current Year 8 mathematics textbook to 35 teachers from three schools, which included examples intended to illustrate the distributive law. Teachers were asked to identify positive and negative aspects of the way the distributive law was presented, and discuss how they would use these explanations. A follow-up interview with 33 of the teachers sought their opinions of the given page and asked them to elaborate on any 'issues' they noticed with textbooks and their usual explanations to students about the distributive law. Chick found that teachers' responses to these questions revealed much about their PCK for teaching algebra. In particular, she grouped their responses within three of her themes proposed in 2006: (a) knowledge of alternative explanations,(b) knowledge of structure and connections, and (c) knowledge of students' thinking. Somewhat disturbingly, many teachers' responses indicated a personal commitment to the 'fruit salad algebra' approach—long recognised as problematic (MacGregor, 1986)—even after they had been made aware of its inappropriateness for instructional purposes.

While exploring the teacher knowledge required to effectively teach ratios, Chick (2010) outlined a questionnaire item and interview protocol that investigated "the extent to which teachers can recognise a typical misconception associated with ratio understanding and what strategies they have for addressing it" (p. 145). Forty secondary teachers from three schools completed a questionnaire and were interviewed on key topics for Years 7 to 9. She acknowledged the complexity of the knowledge required for effective teaching and proposed similar issues regarding examples, as had arisen in her earlier work (Chick, 2009a).

Roche and Clarke (2009, 2011) proposed their own framework of PCK. Their purpose was to use the framework to develop survey items that could be used to assess teachers' PCK in mathematics. The teachers were involved in a two-year professional learning program (*Contemporary Teaching and Learning of Mathematics*, CTLM). Questionnaires of items were administered to teachers at the first professional learning session of a given year (February) and the last session (October or November). The results were used to assess any teachers' improvement in PCK over time. The Roche and Clarke (2009, p. 469) framework contains the following components:

– *Pathways:* Understanding possible pathways or learning trajectories within or across mathematical domains, including identifying key ideas in a particular mathematical domain.
– *Selecting:* Planning or selecting appropriate teaching/learning materials, examples or methods for representing particular mathematical ideas including evaluating the instructional advantages and disadvantages of representations or definitions used to teach a particular topic, concept or skill.
– *Interpreting:* Interpreting, evaluating and anticipating students' mathematical solutions, arguments or representations (verbal or written, novel or typical), including misconceptions.
– *Demand:* Understanding the relative cognitive demands of tasks/activities.
– *Adapting:* Adapting a task for different student needs or to enable its use with a wider range of students.

The authors stressed that the framework was not intended to be exhaustive, and clearly it is not as broad as Chick's. They also stated, while taking into consideration the PCK frameworks of other researchers, that the components were specifically chosen to correspond with some of the key skills and teaching characteristics that were being addressed in the CTLM professional learning program.

Similar to Ball and colleagues (Hill et al., 2008), Roche and Clarke (2009) used classroom scenarios to elicit teachers' PCK, by providing, for example, a mathematical operation, and asking teachers to create a story problem, which would involve the use of the particular operation. Ninety-two teachers from 11 primary schools were asked to name the two forms of division, provide a simple representation and story problem for each, and explain which form would best help to make sense of dividing a whole number by a decimal, in this case, $8 \div 0.5$. The teachers had completed six full days of professional learning on number, working mathematically, and early algebraic thinking, during which the topic of division was just one aspect addressed. Roche and Clarke identified this task as falling largely within the two components *Pathways* and *Selecting*, within their PCK framework. Teachers found these tasks particularly difficult, with 75% of teachers having difficulty making sense of the example, $8 \div 0.5$. Related work was also reported in Clarke, Roche, and Downton (2009). Of course, unlike the work of Chick and her colleagues, Roche and Clarke were unable to triangulate the teachers' responses to the questionnaire items with classroom observations.

Watson, Callingham, and Donne (2008a, 2008b) focused on three components of PCK: teachers' content knowledge; its reflection of their students' content knowledge; and their PCK in using student responses to devise teaching intervention, in order to measure teachers' PCK in statistics. Forty-four middle-years' teachers of mathematics from three Australian states were presented with four typical incomplete or inappropriate student responses to statistics tasks, and invited to suggest strategies for remediating students' inappropriate responses to proportional reasoning tasks, set in the context of chance and data. They found that teachers' PCK was not generally strong in these areas, with a lack of discrimination between different student responses. In particular:

> There was a general lack of PCK at the point of matching content knowledge with knowledge of students as learners. Knowing what questions to ask of students, or what cognitive conflict to generate, without directly telling them the answer, appears to be a difficulty for these teachers. (Watson et al., 2008b, p. 568)

The authors recommended that professional development programs may need to focus more clearly on developing targeted intervention regarding students' levels of understanding.

In extending their work, Watson, Callingham, and Nathan (2009) greatly enhanced the quality of their data collection, by incorporating interviews with 40 middle-years teachers of mathematics from three Australian states. The teachers were asked questions relating to student responses to a pictograph task, including (a) the identification of the big statistical ideas in the problem, (b) examples of appropriate and inappropriate responses, and (c) opportunities that the problem would provide for their teaching. The framework which emerged from the teachers' responses had four 'non-hierarchical components': (a) Recognises Big Ideas, (b) Anticipates Student Answers, (c) Employs Content-specific Strategies, and (d) Constructs Shifts to General, in what the authors described as an attempt to contain and clarify some of the "nebulous components of PCK" (Watson et al., 2009, p. 569). In this way, Watson et al. (2009) contributed further to the development of PCK frameworks, by presenting components of PCK highly specific to the task at hand.

In further work, Watson and Nathan (2010) interviewed the same cohort of teachers as those of Watson et al. (2008a, 2008b), "with the aim of extending the detail and richness of teachers' PCK" (p. 610). Forty teachers were presented with a newspaper article reporting a phone-in survey about the legalisation of marijuana. Teachers' PCK were assessed based on responses to questions about the big ideas underpinning the task, potential student appropriate and inappropriate answers, and suggestions from teachers on how they would intervene in relation to the three student answers. Most teachers (70%) could distinguish between appropriate and inappropriate responses. Only 10%, however, displayed a clear understanding of student reasoning. The authors noted that around half of the teachers demonstrated a capacity to assist the development of student understanding, but seemed less able to situate sampling within the wider context of statistics. The authors concluded that “the framework of four components of PCK ... provide[d] the researchers with a comprehensive way of describing teachers’ ability to explore the problem of sampling in their classrooms” (Watson & Nathan, 2010, p. 616).

Bobis, Papic, and Mulligan (2009/10) investigated teachers’ knowledge ‘in action’ in two pre-school centres (one rural and one regional) in New South Wales. Using the data sources of still photography, video footage, and interviews with teachers, the researchers coded the mathematical learning experiences provided by early childhood practitioners, and sought to describe the components of knowledge evident in what they saw. The framework and components of ‘knowledge of’ (Hill et al., 2008) revealed evidence of ‘knowledge of’ content and students, content and teaching, specialised content knowledge, and knowledge at the horizon. They concluded that “the ability of preschool practitioners to plan developmentally appropriate experiences that foster the advancement of mathematical concepts and processes of young children is dependent on a complex combination of both mathematical subject matter and pedagogical content knowledge” (p. 95). They urged that early childhood practitioners receive ongoing professional learning support and quality educational resources, and recommended further research into aspects of their mathematical knowledge.

DOMAINS OF TEACHER KNOWLEDGE

As part of the work on categorisations of knowledge discussed in the previous section, studies in Australasia have focused on describing specific content areas of mathematics, proposing the expansion of domains of pedagogical content knowledge and knowledge of the curriculum. These studies include different aspects of the field such as those that contest the specific categorisation, either by arguing for an expansion of the category, challenging the emphases of specific categories, or proposing new categories of teacher knowledge. The interconnected and complex nature of knowledge was discussed earlier in this chapter. We begin this section by reviewing recent research about content knowledge, pedagogical content knowledge and mathematical knowledge for teaching including curriculum knowledge. We end this section by reviewing studies that consider new aspects to domains of knowledge.

Categorising Teacher Knowledge

Categorising teacher knowledge remains a challenge for mathematics educators with much debate around the importance and complexity of the issues. As noted earlier, but important to reiterate, definition and precision in categorising knowledge as well as the interconnections between domains (Ball et al., 2008) are ‘ever present’ factors under consideration in the Australasian research reviewed here. Much of the work has been dominated by a focus on the expanding domain of pedagogical content knowledge and knowledge of the curriculum as can be seen in the composition of the frameworks discussed in the previous section and the studies that follow. Sullivan (2008b), in discussing why teacher knowledge matters, suggested that a useful approach was through articulating characteristics of effective mathematics teaching. He highlighted three perspectives—mathematics knowledge, mathematics knowledge for teaching, and knowledge of pedagogy—and used teachers' answers about a particular mathematics question to illustrate the challenge and complexity of describing the knowledge that mathematics teachers needed in order to be able to teach. He identified two sides to the debate about the characteristics of effective mathematics teaching; one side that argued for discipline-based learning to be intertwined with "physical, personal and social dimensions", and a second side that took "a more explicitly mathematical perspective with attention to the principles, patterns, processes, and generalizations that have conventionally formed the basis of the mathematics curriculum" (p. 2). He concluded by suggesting that the teacher knowledge debate should not be about traditional versus reform mathematics, nor about the level and purpose of mathematics, but be about the knowledge teachers needed to teach mathematics well, which he conceded was complex and multidimensional, but something that was important for mathematics educators to continue to work on. Sullivan's work illustrates the interconnected nature of the categories through combining the three perspectives on knowledge in an exemplar from practice. His challenge to attend to the depth and scope of debate about domains of knowledge underscores the importance of continuing to develop the field of teacher knowledge.

Content Knowledge

Content knowledge is one of the original broad categories of teacher knowledge considered essential to effective teaching. Historically, this knowledge was conceptualised in relation to the discipline and gained through university study with the level of the degree being indicative of the level of content knowledge. Shulman's (1986, 1987) work disrupted this view and has not only prompted different categories of teacher knowledge, but also the expanded delineation within categories such as content knowledge, with the interconnections between categories becoming as important to teaching as the category itself. A recent study by Beswick et al. (2011) makes a strong argument for treating teacher knowledge as a uni-dimensional construct. They used written survey evidence from a teacher knowledge profile instrument with 62 Australian middle school teachers at the beginning of their participation in a three-year professional learning programme to assess different aspects of teacher knowledge. Applying a partial credit Rasch model, they found that seeing teacher knowledge as a single construct made up of multiple aspects is possible and suggested that the various facets of teacher knowledge develop together. However, they acknowledged the complexity of teaching mathematics both in its execution and in identifying the knowledge teachers drew on. Furthermore, recent work about content knowledge, related to teacher knowledge of trajectories or frameworks for student thinking, is a good example of the blurring of the edges between categories of teacher knowledge. As Bobis (2009) observed, it is not just the content knowledge of teachers, but the quality of teachers' understanding of key points in student learning and their ability to design instruction to promote student understanding in relation to these key points that can make an ultimate difference to student learning. Several studies (Bobis, 2009, 2010; O'Keefe & Bobis, 2008; Sullivan, Clarke, Clarke, & O'Shea, 2009) discussed teacher knowledge of student thinking in terms of its importance in teaching. As noted in the previous section, specific frameworks, such as those referred to by Bobis, and proposed by Roche and Clarke, are underpinned by learning trajectories.

Work by White (2010), with a specific focus on low attaining students, similarly employs the notion of trajectories of student thinking by drawing on the *Counting On* number framework. The dual intent in White's study of improving student outcomes and developing teacher knowledge and practice is akin to the strategic objectives of the NSW *Count Me in Too Project* and the New Zealand *Numeracy Development Project*. Evaluation reports of government initiatives to improve teacher knowledge are one of the few places where there are attempts to link teachers' pedagogical content knowledge with student outcomes. While the political framing of this type of work often precludes opportunities to incorporate and generate nuanced views of teacher knowledge, the impact on teacher professional knowledge of interventions, such as the Australian National Curriculum, provides important opportunities to study teacher knowledge.

Some recent studies of teacher knowledge have investigated specific areas of mathematics content (e.g., J. P. Brown, 2009; O'Keefe & Bobis, 2008; Yeo, 2008). O'Keefe and Bobis (2008) investigated teachers' perceptions of the content knowledge of measurement and teacher knowledge of student growth of understanding measurement concepts. The study used self-report data from in-depth interviews of four primary school teachers from three schools. It had a dual focus on primary teachers' perceptions of their knowledge and understandings of length, area and volume alongside teachers' understanding of the development of students' growth of measurement concepts and processes. Rather than explicitly ask teachers what they did and did not know about length, area and volume, the interviewer invited teachers to describe what they considered to be the important concepts, knowledge and skills necessary to understand these aspects of measurement. The study found that teachers struggled to articulate their knowledge of measurement concepts and children's trajectories of learning and concluded that teachers' knowledge was often implicit possibly due to the fact that teachers are not usually required to articulate this kind of knowledge. The study was also useful in exposing issues in relation to measurement that require further exploration. Similarly, in a study of five Grade 4 area and perimeter lessons conducted by a Singaporean beginning teacher, Yeo (2008) referred to the challenges faced by teachers when required to articulate their content knowledge. Together, these studies highlight the importance in teacher professional development of providing opportunities for teachers to discuss and reflect upon their own knowledge of mathematics content.

Anderson (2008) investigated teachers' motivations for attending voluntary professional development courses to examine the particular types of knowledge that teachers sought and valued from such courses. She invited 109 participants from four six-week professional development courses to complete a survey and indicate their motivation for attending. Anderson was particularly interested in identifying any differences in the knowledge required by primary and secondary school teachers of mathematics. She found that while many teachers wished to develop their mathematical content knowledge, almost all of these comments came from primary school teachers. However, Anderson noted that this is not surprising given that secondary school teachers have studied more mathematics in their teacher training.

In contrast to the Watson et al. (2008a, 2008b) studies discussed earlier about pedagogical content knowledge in statistics, Burgess (2009), in a study about statistical knowledge for teaching, based his work on Ball et al.'s (2008) *Teacher Statistical Knowledge*: content knowledge (common, specialised), and pedagogical content knowledge (knowledge of content & students, knowledge of content and teaching). These dimensions of statistical thinking included types of thinking such as (a) need for data, (b) trans-numeration, (c) reasoning with models, (d) integration of statistical and contextual, (e) investigative cycle, (f) interrogative cycle, and (g) dispositions. Using a sequence of four or five lessons videotaped from four upper primary school teachers, he selected and edited 'episodes of interest' for use in stimulated recall interviews scheduled for the same day as the lesson. The video and audio data were analysed against a teacher knowledge framework that had been formulated in relation to categories of teacher knowledge and components of statistical thinking. The profiles developed provided a useful way of identifying patterns of missed opportunities for each teacher to show aspects of teacher knowledge that needed development.

Pedagogical Content Knowledge

Of the original categories proposed by Shulman (1987) pedagogical content knowledge continues to spark interest from researchers intent on expanding understanding of the complexities about the knowledge used, and needed, to effectively teach mathematics. It is now generally accepted that there is an ongoing need to critique this construct as increasing numbers of studies argued for nuanced views of teacher knowledge, and perhaps more importantly the term 'pedagogical content knowledge' has become a descriptor in mandated curriculum and teacher assessment systems through its adoption by policy makers and implementers as a way to link student achievement to the quality of mathematics teaching and teacher knowledge. Barton (2009) in reflecting on the phrase "pedagogical content knowledge" suggested that while it is commonly accepted that it:

> includes knowledge about how mathematical topics are learned, how mathematics might best be sequenced for learning, having a resource of examples for different situations, and understanding of where conceptual blockages frequently occur, and knowing what misunderstandings are likely. Questions remain about how teachers best come by this knowledge, the extent to which it can be taught and the extent to which it depends on experience, and, inevitably, the hard question: what is the relation of this type of knowledge to student learning? (p. 4)

One study that examines the specific knowledge needed to promote student achievement is that of J. P. Brown (2009). Reporting on secondary school teachers' understanding of function she suggested factors that enable "teachers to perceive particular affordances of technology-rich teaching and learning environments (TRTLE's) and act on these to develop student understandings of functions and the development of higher order thinking?" (p. 65). The study involved seven experienced secondary mathematics teachers of Year 9 to 11 students in six schools who were part of a larger study about the use of technology in the teaching and learning of mathematics. Teachers completed "a concept map of function" which was considered as "somewhat representative of the teachers' understanding of function" (p. 66) rather than capturing all their knowledge. Rejecting a specific numerical scoring system as a way of identifying the essence of teacher knowledge, the maps were analysed according to (a) key notions related to the definition of function, (b) process or object view of function, and (c) identification of the importance of working within and across representations. Brown noted that it was of concern that none of the maps contained more than half of the key notions of functions noted by Tall (1992). Concern was also raised about the lack of teacher knowledge about "how different representations can contribute to making different aspects of a function transparent or the relationship more understandable" (p. 71). Brown postulated that the shortcomings identified in teachers' knowledge might not support the development of a deep conceptual understanding of functions by students, but did not include an analysis of student outcome data.

Vale and McAndrew (2008) designed and implemented a professional learning program based on the algebra and functions content of the Victorian senior secondary mathematics curriculum. The participants were unqualified secondary mathematics teachers who had no experience of teaching advanced senior secondary mathematics. Ten teachers from five government secondary schools completed mathematics and professional learning tasks during 21 three-hour seminars conducted fortnightly over one school year. Questionnaires, field notes and teacher portfolios were analysed qualitatively using codes derived from a PCK framework developed by Chick et al. (2006). The paper reported on case studies of three of the teachers to illustrate the mathematical and pedagogical learning attained by program participants. Vale and McAndrew found that developing teachers' content knowledge of senior mathematics also improved the participants' understanding of junior secondary mathematics content and pedagogy. The authors concluded that the ‘teachers as learners of mathematics’ model used in the program had the potential to help extend teachers’ knowledge.

Mathematical Knowledge for Teaching

Sullivan (2008a) provided a succinct review of Ball’s framework of mathematical knowledge for teaching (MKT). He argued that to give effective feedback to students, teachers needed all of these types of knowledge. He also suggested two other important aspects needed to be considered: teacher beliefs and a commitment of teachers to interact with students in situations that move beyond whole class teaching. He stressed that, “it all has to come together” (p. 433).

Teachers’ knowledge about how to represent mathematical ideas in ways that foster student understanding is an important aspect of MKT. Studies with this focus included investigations of teachers’ understanding and use of tasks, both in lessons as well as in textbooks. A number of papers that are part of a larger Australian project, *Task Types in Mathematics Learning*, reported on teachers’ insights into their choice of task types for teaching (Clarke & Roche, 2010; Sullivan, Clarke, & Clarke, 2009; Sullivan, Clarke, Clarke, & O’Shea, 2009; Zaslavsky & Sullivan, 2011). Similar to these task type studies, are others that focused on textbook examples including Stacey and Vincent (2009) and Ding, Anthony, and Walshaw (2009).

Also relevant here in terms of teacher confidence, is Sullivan, Clarke, Clarke, and O’Shea’s (2009) study concerning teacher knowledge of learning trajectories.Their study further illuminates Ball et al.’s (2008) components of ‘specialised content knowledge’ and ‘knowledge of content and of students’ while also incorporating notions of curriculum and teaching through the use of tasks. In another study, Sullivan, Clarke, & Clarke (2009) compared two groups of teachers’ ability to recognise the mathematical content in a task; one group participating in the professional development programme, *Task Types and Mathematics Learning* (TTML), with another group who were not. They conducted two surveys of primary and secondary teachers to examine how teachers converted mathematics tasks to learning opportunities. Using subcategories of the Hill et al. (2008) categorisation of teacher knowledge, they discussed responses to one particular item that sought teachers’ ideas on taking a fraction comparison task (which is larger 2/3 or 201/301?) and converting it into a mathematics lesson in the middle-years of schooling. Teachers’ abilities to identify the mathematical content of the task as “comparing fractions” varied and “raised the possibility that some of the teachers were not able to identify readily the focus or potential of this mathematical task” (p. 94). Further, they suggested those teachers without common content knowledge may have limited enactment of pedagogical content knowledge. They concluded “the responses call into question the sense teachers make of curriculum documents including syllabuses (i.e., the intended curriculum), when knowledge of content and curriculum is limited” (p. 102). The implications drawn from the study included the need for professional development programmes to focus on all six components of knowledge for teaching mathematics to ensure that greater numbers of teachers are able to translate a task into a worthwhile student learning experience.

In a later study, Clarke and Roche (2010), also drawing on the *Task Types and Mathematics Learning* project, investigated the insights of 16 middle school teachers into their choice of task types for use in their mathematics teaching. The focus of the study was to establish teacher knowledge of task types after two years involvement in a professional development programme. The study found that teachers' use of tasks did not vary across three types of models, incorporating contextual and open-ended scenarios. While teachers could articulate reasons for their choice, the choice and reasons varied considerably across the group. The teachers reported becoming more aware of task type and felt that they made better choices as a result of participating in the project and became more active in looking for opportunities to use all task types in their teaching including an increased use of contextual tasks. Teachers noted, as a result of the project, they were "now more aware of the range of task types and looked actively for opportunities to use all three task types" and were "able to select the task type that best suited the purpose or focus of the lesson and were more likely to choose tasks that catered for the range of abilities in their class" (p. 159).

As part of a larger New Zealand study (*Learners' Perspective Study*), Ding et al. (2009) also examined the use of classroom tasks. Using a teaching experiment methodology, they reported on teachers' choice and use of examples in solving number problems about fractions at the early secondary school level. Teachers used the teaching strategies and examples advocated as part of the *New Zealand Numeracy Development Project* at the secondary level. To establish teacher effectiveness in terms of mathematical content knowledge (MCK) and pedagogical content knowledge, Ding et al. (2009) established how their findings, based on observation data and video-stimulated recall interviews of "teachers' example-related practice" could be used in teacher education programmes (p. 425). Using teaching episodes the analysis highlighted potential affordances and limitations of the teacher's implementation of the examples in terms of student learning, and suggested alternative ways of implementing the examples to illustrate the importance of the understanding of the relationship of the instructional model and mathematical thinking patterns. The study made links to Chick's (2007) study about the implementation of examples where the mathematical potential (affordance) was not realised.

Stacey and Vincent (2009) focused on knowledge for teaching mathematics by examining examples of several topics in nine Australian eighth-grade textbooks. They developed a classification system incorporating seven modes of reasoning of "appeal to authority, qualitative analogy, concordance of a rule with a model, experimental demonstration, deduction using a model, deduction using a specific case, and deduction using a general case" (p. 274). In a content analysis, with a specific focus on the introductory text, the study found that while most textbooks provided explanations on most topics, some explanations were in preparation for practice exercises rather than as thinking tools that could be useful in other examples. If students needed to rely on teachers to elaborate on examples, Stacey and Vincent suggested, it was "unlikely" that they could "from the material provided" and that this highlighted "the often cited need for teachers to possess sufficiently strong mathematical knowledge and deep mathematical pedagogical content knowledge" (p. 286).

Using Shulman's categories, Clarke (2008) positioned a teacher as a 'curriculum maker' through a process by which a teacher begins with the intended curriculum as outlined in curriculum frameworks, and enacts it. He considered what kinds of knowledge a teacher might draw upon when being a curriculum maker by systematically working through each of Shulman's categories in a process of identifying constraints that may prevent a teacher from fully enacting this role. He suggested such aspects as the ability to "identify big ideas within a topic, sequence concepts within that topic, recognise and enhance connections between concepts, and match the curriculum to the developing understanding of students" (p. 133). Clarke concluded by discussing professional development to prepare prospective and practising teachers to be active curriculum makers.

Stacey (2008) in addressing the mathematics required for teaching in secondary schools, worked from a vision of good mathematics learning which valued working from reasons not rules, and being able to use whatever mathematics that had been learned for solving problems within and beyond mathematics. She proposed four aspects of teacher knowledge: "(1) knowing mathematics in a way that has special qualities for teaching; (2) having experienced mathematics in action solving problems; (3) knowledge about mathematics including its history and current developments; and (4) knowing how to learn mathematics" (p. 87).

Frid, Goos, and Sparrow (2008/9) provided a useful overarching comment on the importance of teacher knowledge in the context of teacher shortages and the emergence of teacher knowledge frameworks with specific reference to Chick's (2007) and Ball et al.'s (2008) frameworks. They reminded us that our focus needed to be on the complexity of teacher knowledge and its significance for teaching. In the spirit of this comment, Barton (2009) extended the thinking about mathematical knowledge by moving "through wider aspects of mathematical knowledge, through acting like a mathematician and creating a mathematical environment, to how a teacher holds mathematics" (p. 9). In this position paper, Barton reflected on the complexities of mathematical knowledge for teaching and suggested that further understanding of dimensions of mathematical knowledge for teaching (MKfT) is needed given the evidence that is in classroom research "we are far from capturing what it is a teacher does, why they do it, and what effect it might have on student learning" (p. 3). Barton's comments are important to framing an increasing focus on treating teacher knowledge as complex. In acknowledging theoretical models of mathematical knowledge for teaching, such as Ball et al.'s (2008), Barton suggested that they all focus on *what* the teacher must know, but what is also important is *how* a teacher must know.

SOURCES OF TEACHER KNOWLEDGE AND MODES OF INQUIRY

As evident from the preceding discussion, some key reasons for studying teacher knowledge are to explore what knowledge teachers possess (or do not possess), and to discover the most effective ways by which it is acquired. The intention is that such insights will inform programs of professional development and ultimately help to enhance teacher knowledge and student learning outcomes. Another related reason is to gain some measure of how successfully such mechanisms and processes, designed to improve teacher knowledge, have actually worked (e.g., Dole, Clarke, Weight, Hilton, & Roche, 2008; White, 2010). In reality, it seems that many aspects of teacher knowledge have been quite difficult to determine. We believe this is partly due to the complexity of teacher knowledge—a point reiterated by several researchers in the field (e.g., Chick, 2010; Frid et al., 2008/9; Roche & Clarke, 2009). It is also partly due to the fact that such knowledge not only comes from a wide range of, and sometimes ‘unexpected’, sources, but is mediated by multiple contributing factors—including a teacher’s beliefs (Sullivan, 2008a); their sociocultural contexts (Goos, 2009; Owens & Kaleva, 2008), and their level of confidence (Bobis, 2009, 2010; Sullivan, Clarke, Clarke, & O’Shea, 2009).

The actual processes by which teacher learning and development might occur were a focus of the previous MERGA review of research (see Anderson et al., 2008). Such processes continue to range from small-scale, individualised teacher professional learning opportunities (e.g., Muir, 2008; Muir, Beswick, & Williamson, 2010), to small groups of teachers (J. P. Brown, 2009) and large-scale programs of professional development (Higgins & Parson, 2009; White, 2010) involving off-site workshops, professional reading and/or classroom support. The ‘tools’, sources of knowledge or mechanisms employed to support changes in teacher knowledge are just as varied. For instance, Muir and colleagues (Muir, 2008; Muir et al., 2010) scaffolded teachers’ individualised reflections and action learning processes themselves. Higgins and Parsons (2009) identified three pedagogical tools that participants in the *New Zealand Numeracy Development Project* described as improving their mathematics knowledge and practice: (a) the number framework, (b) the diagnostic interview, and (c) the strategy teaching model—a model designed to explicitly teach problem-solving strategies. They argued that the power of the professional development model lies in the integration of these three tools that enabled teachers to deepen their professional knowledge.

Numeracy coaches (Anstey & Clarke, 2010) can also be viewed as a ‘tool’ or source of teacher knowledge, but as Gaffney and Faragher (2010) highlighted in their report on results of the *Leading Aligned Numeracy Development* (LAND) project, the success of any such mechanism for teacher development may depend on local contextual factors such as the effectiveness of school leadership. Gaffney and Faragher found that successful school leadership teams (including principals) were more able to sustain improvements in student mathematical achievement when their own PCK was well developed.

Researchers have extensively used students’ own responses to mathematical tasks, or the tasks themselves, as a source by which teachers can improve their knowledge for teaching mathematics. Horne (2008) used students' responses to interview tasks as a reflective tool that motivated teachers to extend their knowledge of student thinking strategies. Sullivan, Clarke, and Clarke, (2009) and Clarke and Roche (2010) are groups of researchers that have structured professional learning opportunities around teacher understanding of task types in mathematical learning. Similarly, Visnovska, Cobb, and Dean (2011) used 'other' teachers as a source of knowledge when groups of teachers were asked to collectively design a unit of work on statistics as part of a professional development program. Despite the variation in knowledge sources and tools employed by providers of professional development, each case required a scaffold by a more knowledgeable individual to actually make a discernible difference in teacher knowledge.

Modes of Inquiry

An ongoing and vexing issue for researchers studying teacher knowledge has been the search for inquiry methods that reveal information about teacher knowledge and how to adequately assess and examine it. Predominantly, the modes of inquiry into teacher knowledge in the review period 2008–2011 have been qualitative in nature. Our intention here is to provide some critical reflection on the array of methods used to study teacher knowledge.

The method of inquiry is mainly determined by the size of the cohort, with studies involving large participant numbers generally opting for written responses via surveys (e.g., Anderson, 2008). In cases where teachers' own perceptions about their knowledge were being sought, such as when Bobis (2010) asked teachers to rate their level of confidence regarding aspects of their knowledge needed to plan mathematics instruction, multiple-choice type answers were deemed effective. However, increasingly, Australasian researchers seem to be moving away from reliance on multiple-choice instruments to favouring open-response survey instruments often using follow-up methods involving a combination of either interviews and/or classroom observations. Roche and Clarke (2009) noted 'a tension' between collecting vast amounts of rich qualitative data from a relatively small number of teachers and collecting "less data from a larger number of teachers" (p. 473). They critiqued the work of Ball and her colleagues (e.g., Ball et al., 2008), considering the use of multiple-choice items as the sole indicator of teacher knowledge to be a major weakness. Instead, Roche and Clarke modified items on their questionnaire designed to assess PCK, requiring teachers to provide written justifications for their choices.

In their examination of teachers' abilities to respond to middle-year students' problems involving proportional reasoning, Dole et al. (2008) used a survey requiring teachers to provide written responses to a hypothetical scenario. They found that further work was required on the survey items to create a useful instrument. They also noted the necessity of combining interview and other data, including classroom observations, to determine a more complete picture of teacher PCK growth over the course of their professional development program. A similar realisation was made by Bobis (2009, 2010), when she employed a scenario as part of a survey to explore teachers PCK. Primary teachers were required to provide a written interpretation of a student's response to a mathematical task and suggested relevant teaching intervention strategies to address the student's needs. While only half of the 28 teachers involved in the survey component provided adequate levels of responses to the scenario, follow-up interviews with 22 teachers involving a similar scenario task, revealed that all but two teachers provided far richer responses, revealing much greater insights into their PCK than previously determined from the survey alone.

Further, Watson and Nathan (2010) moved beyond written survey responses in their study, intent on probing teachers' PCK in statistics. Reflecting on results and issues that had emerged from a previous study (Watson et al., 2008a, b) involving written survey responses to student answers on proportional reasoning tasks, Watson and Nathan (2010) decided to employ interviews "with the aim of extending the detail and richness of teachers' PCK" (p. 610). They argued that such an inquiry method allows PCK to be explored as a dynamic process which is more akin to the actual work of teachers.

Other methods used to gather information about teacher knowledge have included stimulated recall of video-recorded teaching episodes (Burgess, 2009; Chick, 2009a, 2009b; Muir, 2008) and the analysis of a range of teaching artefacts such as teaching plans and teacher reflections (Vale & McAndrew, 2008). With the move away from multiple-choice type surveys, to modes of inquiry that are far more revealing of teacher thinking, a need for sophisticated assessment rubrics that considered teacher responses more holistically has emerged (Roche & Clarke, 2009). To be effective, such rubrics will need to be finely tuned to detect differences in teacher knowledge levels and will most likely need to be content specific, depending on the PCK components under investigation.

J. P. Brown's (2009) investigation of secondary mathematics teachers' knowledge of function is one of a few studies in the review period that specifically sought to determine mathematical content knowledge. She used concept mapping because it was considered to provide some insight into teachers' organisation and structure of their knowledge about functions. While the focus of nearly every study reviewed for inclusion in this chapter was overwhelmingly on specific components of PCK, occasionally judgements of mathematical content knowledge were also determined from the same analyses. For instance, Watson and Nathan (2010) preferred to assess teacher interview comments on a continuum ranging from low to high levels of PCK as determined by the researchers. While they acknowledged that some basic mathematical content knowledge would precede development of the PCK components in question, they treated it as part of the wider concept of PCK rather than as a separate body of knowledge. Certainly, a move away from previous paper and pencil 'tests' of teacher content knowledge as the sole mode of inquiry to determine teacher quality, are a welcome development in investigations of teacher knowledge.

CONCLUSION

We can see a number of developments and issues emerging from the field of research concerned with teacher knowledge. First, the situated nature of teacher knowledge has certainly come to greater prominence among Australasian researchers in recent years. While we have seen a growing recognition that teacher knowledge is filtered through social and even political contexts, there has been little mention of 'cultural' influences on teacher knowledge, with one exception: Owens and Kaleva's (2008) research. Perhaps this is because the research reviewed was predominantly conducted by researchers from western cultural backgrounds, focused on mathematical content from western curricula and interpreted via frameworks of teacher knowledge developed by scholars based on western cultural views of knowledge. While beyond the scope of this chapter, it is important for the future to consider different cultural perspectives on teacher knowledge.

A second theme emerging from this review of research is the growing awareness of the influential role of certain affective characteristics on teacher knowledge. In particular, studies by Beswick et al. (2011), Bobis (2009, 2010), and Sullivan, Clarke, and Clarke (2009) highlighted the interplay occurring between teachers' beliefs and their knowledge, and the fact that teacher beliefs (such as beliefs and confidence about their own mathematical knowledge), can be a major regulator of teaching practices. As such, we have learnt that when studying certain types of teacher knowledge, affective factors cannot be ignored.

While the incredible complexity of teacher knowledge was acknowledged and confirmed by many researchers, we also sought to extend current conceptions of teacher knowledge, viewing it as 'relative'. Drawing upon the work of researchers such as Anstey and Clarke (2010), we saw how teachers' perceptions and ratings about their own knowledge varied depending on contextual factors, including the perceived knowledge of their peers or an increased awareness of new bodies of knowledge previously unavailable to them.

It is clear there has been an increasing focus on frameworks of teachers' knowledge by Australasian researchers in the period of the review. Perhaps the most obvious omission in this body of research is a debate or rationale for why (or, *if*) we need such frameworks. From a policy perspective, frameworks of teacher knowledge, such as that proposed by Ball and her colleagues, made it clear that expertise in mathematical content knowledge alone, is insufficient for effective teaching of mathematics. Hence, moves by politicians to short-circuit teacher education programs by fast tracking so-called 'outstanding graduates' to alleviate current teacher shortages (including mathematics), does not have a sound rationale for building a teacher's professional knowledge base. Furthermore, from a research perspective, frameworks can act as great drivers of research agendas aimed at deepening our understanding of teacher knowledge and how this knowledge enables certain teaching practices. Understanding teachers' thinking about their own knowledge and its perceived impact on teaching practices is paramount to improving the professional learning of teachers. For instance, in some cases, the research reported in this review has broadened the categories of knowledge identified by earlier work, particularly in respect of pedagogical content knowledge. In others, the focus has been on exploring the knowledge needed by teachers (in school and preschool settings) to teach mathematics more effectively. Still others have assessed the extent and kinds of knowledge possessed by teachers, through analysing data from observations, questionnaires, interviews, videotapes, and still photography, with many studies focusing on very specific content areas. Sometimes, the research uses (without seeking to extend) existing frameworks. In other cases, innovative components and frameworks of knowledge have been proposed. All this work has reinforced the growing view that it is the way in which a teacher's knowledge is structured and used that is so crucial in the effective teaching of mathematics.

There is growing pressure from educational stakeholders at all levels to establish evidence of the effects of teacher knowledge on student outcomes. Despite this, there has been little Australasian research to date that links teachers' knowledge with student achievement. What are the ways that teacher knowledge influences student outcomes in mathematics? Surely the pivotal reason for examining teacher knowledge to the extent evident in this review is to ultimately learn how to improve student learning.

Finally, we have seen a proliferation of mechanisms and tools by which teachers have been shown to acquire knowledge and the methods by which it is studied. However, what is missing is some documentation of the processes by which teachers learn without externally imposed intervention. Teachers can learn from their own practice but more systematic research is needed to understand the circumstances by which this occurs best. More importantly, we need to further explore the implications of different types and levels of teacher knowledge for their teaching practice and ultimately student outcomes.

REFERENCES

Anderson, J. (2008). Teachers' motivation to attend voluntary professional development in K-10 mathematics. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 51–58). Fremantle, WA: MERGA.

Anderson, J., Bobis, J., & Way, J. (2008). Teachers as learners: Building knowledge in and through the practice of teaching mathematics. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W. T. Seah, & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004–2007* (pp. 313–335). Rotterdam: Sense.

Anstey, L., & Clarke, B. (2010). Perceived professional learning needs of numeracy coaches. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 45–52). Fremantle, WA: MERGA.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59,* 389–407.

Barton, B. (2009). Being mathematical, holding mathematics: Further steps in mathematical knowledge for teaching. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 3-9). Palmerston North, NZ: MERGA.

Beswick, K., Callingham, R., & Watson, J. (2011).The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education, 14*(1), 1–27.

Boaler, J. (2003). Studying and capturing the complexity of practice—The case of the 'dance of agency'. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 2003 Joints Meeting of the International Group for the Psychology of Mathematics Education and the Psychology of Mathematics Education Group North America* (3–16). Honolulu, HI: University of Hawaii.

Bobis, J. (2009). Teachers assessing young children's mathematical development: How confident are they? In T. Dooley (Ed.), *Mathematics for all: Extending mathematical capacity* (Proceedings of the 3rd Conference of the Mathematics Education Ireland Association, pp. 20–33). Dublin, Ireland: MEI.

Bobis, J. (2010). The impact of a developmental framework in number on primary teachers' classroom practice. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 85–92). Fremantle, WA: MERGA.

Bobis, J., Papic, M., & Mulligan, J. (2009/10). The mathematical knowledge of preschool practitioners: Evidence from the field. *Journal of Australian Research in Early Childhood Education, 16*(2), 87–98.

Brophy, J. E. (1991). Conclusion: Teachers' knowledge of subject matter as it relates to their teaching practice. In J. E. Brophy (Ed.), *Advances in research on teaching* (pp. 349–364). Greenwich CT: JAI Press.

Brown, G. (2009). Review of education in mathematics, data science and quantitative disciplines. (Commissioned Report for Go8 Universities). Retrieved from http://www.go8.edu.au/government-a-business/go8-policy-a-analysis/2010/183-the-go8-review-of-education-in-mathematics-data-science-and-quantitative-disciplines

Brown, J. P. (2009). Concept maps: Implications for the teaching of function for secondary school students. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 65-72). Palmerston North, NZ: MERGA.

Burgess, T. (2009). Statistical knowledge for teaching: Exploring it in the classroom. *For the Learning of Mathematics, 29*(3), 18–21.

Chick, H. (2007). Teaching and learning by example. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 3–21). Adelaide, SA: MERGA.

Chick, H. L. (2009a). Choice and use of examples as a window on mathematical knowledge for teaching. *For the Learning of Mathematics*, *29*(3), 26–30.

Chick, H. (2009b). Teaching the distributive law: Is fruit salad still on the menu? In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia). Palmerston North, NZ: MERGA. Retrieved from http://www.merga.net.au/documents/Chick_RP09.pdf

Chick, H. (2010). Aspects of teachers' knowledge for helping students learn about ratio. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 145–152). Fremantle, WA: MERGA.

Chick, H., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical content knowledge for decimals. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Mathematics in the centre* (Proceedings of the 30th conference of the International Group for the Psychology of Mathematics Education, pp. 297–304). Prague, Czech Republic: PME.

Chick, H. L., & Pierce, R. (2008). Issues associated with using examples in teaching statistics. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 321–328). Morelia, Mexico: Cinvestav-UMSNH.

Clarke, D. (2008). The mathematics teacher as curriculum maker: Developing knowledge for enacting curriculum. In P. Sullivan & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp. 133–151). Rotterdam, The Netherlands: Sense.

Clarke, D. (2009). Theoretical perspectives in mathematics teacher education. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 85–93). Thessaloniki, Greece: PME.

Clarke, D., & Roche, A. (2010). Teachers' extent of the use of particular task types in mathematics and choices behind that use. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 153–160). Fremantle, WA: MERGA.

Clarke, D. M., Roche A., & Downton, A. (2009). Assessing teacher pedagogical content knowledge: Challenges and insights. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, p. 362). Thessaloniki, Greece: PME.

Ding, L., Anthony, G., & Walshaw, M. (2009). A teacher's implementation of examples in solving number problems. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 425–432). Thessaloniki, Greece: PME.

Dole, S., Clarke, D. M., Weight, T., Hilton, G., & Roche, A. (2008). Eliciting growth in teachers' proportional reasoning: Measuring the impact of a professional development program. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 163–169). Brisbane, QLD: MERGA.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147–164). New York, NY: Macmillan.

Frid, S., Goos, M., & Sparrow, L. (2008/9). What knowledge is needed for effective teaching of mathematics? *Mathematics Teacher Education and Development, 9*, 1–3.

Gaffney, M., & Faragher, R. (2010). Sustaining improvement in numeracy: Developing pedagogical content knowledge and leadership capabilities in tandem. *Mathematics Teacher Education and Development. 12*(2), 72–83.

Goos, M. (2009). A sociocultural framework for understanding technology integration in secondary school mathematics. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 113–120). Thessaloniki, Greece: PME.

Hattie, J. A. C. (2002). What are the attributes of excellent teachers? In B. Webber (Ed.), *Teachers make a difference: What is the research evidence?* (pp. 3–26). Wellington: New Zealand Council for Educational Research.

Higgins, J., & Parsons, R. (2009). A successful professional development model in mathematics: A system-wide New Zealand case. *Journal of Teacher Education, 60*(3) 231–242.

Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualising and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371–406.

Horne, M. (2008). Using educational research to inform mathematics teaching in a school. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 81–86). Morelia, Mexico: Cinvestav-UMSNH.

MacGregor, M. (1986). A fresh look at fruit salad algebra. *Australian Mathematics Teacher, 42*(3), 9–11.

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics, 38,* 135–161.

Muir, T. (2008). Principles of practice and teacher actions: Influences on effective teaching of numeracy. *Mathematics Education Research Journal, 20*(3), 78–101.

Muir, T., Beswick, K., & Williamson, J. (2010). Up, close and personal: Teachers' responses to an individualised professional learning opportunity. *Asia-Pacific Journal of Teacher Education, 38*(2), 129–146.

O'Keefe, M., & Bobis, J. (2008). Primary teachers' perceptions of their knowledge and understanding of measurement. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 391–397). Brisbane, QLD: MERGA.

Owens, K., & Kaleva, W. (2008). Case studies of mathematical thinking about area in Papua New Guinea. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Mathematical ideas: History, education and cognition* (Proceedings of the Joint meeting of PME 32 and PME-NA XXX, pp. 73–80). Morelia, Mexico: Cinvestav-UMSNH.

Roche, A., & Clarke, D. (2009). Making sense of partitive and quotitive division: A snapshot of teachers' pedagogical content knowledge. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 467–474). Palmerston North, NZ: MERGA.

Roche, A., & Clarke, D. (2011). Some lessons learned from the experience of assessing teacher pedagogical content knowledge in mathematics. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 658–666). Adelaide, SA: AAMT & MERGA.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8,* 255–281.

Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2009). *Developing primary mathematics teaching.* London, United Kingdom: Sage.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1–22.

Stacey, K. (2008). Mathematics for secondary teaching: Four components of discipline knowledge for a changing teacher workforce. In P. Sullivan, & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp. 87–113). Rotterdam, The Netherlands: Sense.

Stacey, K., & Vincent, J. (2009). Modes of reasoning in explanations in Australian eighth-grade mathematics textbooks. *Educational Studies in Mathematics, 72,* 271–288.

Sullivan, P. (2008a). Education for the knowledge to teach mathematics: It all has to come together. *Journal of Mathematics Teacher Education, 11*, 431–433.

Sullivan, P. (2008b). Knowledge for teaching mathematics: Introduction. In P. Sullivan, & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp. 1–9). Rotterdam, The Netherlands: Sense.

Sullivan, P., Clarke, D., & Clarke, B. (2009). Converting mathematics tasks to learning opportunities: An important aspect of knowledge for mathematics teaching. *Mathematics Education Research Journal, 21*(1), 85–105.

Sullivan, P., Clarke, D., Clarke, B., & O'Shea, H. (2009). Exploring the relationship between tasks, teacher actions, and student learning. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *In search of theories in mathematics education* (Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education, pp. 185–192). Thessaloniki, Greece: PME.

Tall, D. (1992). The transition to advanced mathematical thinking: Functions, limits, infinity, and proof. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 495–511). New York, NY: Macmillan.

Vale, C., & McAndrew, A. (2008). Deepening the mathematical knowledge of secondary mathematics teachers who lack tertiary mathematics qualifications. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 539–545). Brisbane, QLD: MERGA.

Visnovska, J., Cobb, P., & Dean, C. (2011). Mathematics teachers as instructional designers: What does it take? In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development*. Dordrecht, The Netherlands: Springer.

Warren, E. (2008/9). Early childhood teachers' professional learning in early algebraic thinking: A model that supports new knowledge and pedagogy. *Journal of Mathematics Teacher Education & Development, 9*, 30–45.

Watson, J., Brown, N., Beswick, K., & Wright, S. (2011). Teacher change in a changing educational environment. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers, Alice Springs, pp. 760-767). Adelaide, SA: AAMT & MERGA.

Watson, J., Callingham, R., & Donne, J. (2008a). Establishing PCK for teaching statistics. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (Proceedings of the Joint ICMI/IASE study conference 18 and IASE Round Table Conference, CD). Monterrey, Mexico: ICMI, IASE & ISI. Retrieved from http://www.ugr.es/~icmi/iase_study/Files/Topic2/T2P10_Watson.pdf

Watson, J., Callingham, R., & Donne, J. (2008b). Proportional reasoning: Student knowledge and teachers' pedagogical content knowledge. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 563–571). Brisbane, QLD: MERGA.

Watson, J., Callingham, R., & Nathan, E. (2009). Probing teachers' pedagogical content knowledge in statistics: "How will Tom get to school tomorrow?" In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 563-570). Palmerston North, NZ: MERGA.

Watson, J. M., & Nathan, E. L. (2010). Biased sampling and PCK: The case of a marijuana problem. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 610–617). Fremantle, WA: MERGA.

White, A. L. (2010). Counting On in the middle years. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 618–625). Fremantle, WA: MERGA.

Yeo, K. K. J. (2008). Teaching area and perimeter: Mathematics-pedagogical-content knowledge-in-action. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 621–627). Brisbane, QLD: MERGA.

Zaslavsky, O., & Sullivan, P. (2011). *Constructing knowledge for teaching secondary mathematics: Tasks to enhance prospective and practicing teacher learning.* New York, NY: Springer.

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THE FUTURE

GILAH LEDER

TAKING STOCK: FROM HERE TO THE FUTURE

> Molly multiplied 899 by 32 in her head. A small, light, happy calculation. It meant nothing. She multiplied in relief. A flood of numerals marched across her mind and swept away her misery. 7,676 by 296, she thought, marching down the stairs behind her brothers. (Carey, 1985, p. 97)

The first compilation of mathematics education research "undertaken in Australia or by Australians" (Briggs, 1984) was published in 1984, when a slim volume was produced "to commemorate the occasion of ICME5 as a gesture of intellectual sharing and international goodwill" (p. 2). The work reviewed was clustered under five headings: (a) an introduction sketching the context and conditions which spawned the birth of the Mathematics Education Research Group of Australia (MERGA), the aims of the organization and the scope of the publication; (b) a review of research presented at MERGA conferences in the years 1977–1983 and compiled in the organization's annual conference proceedings; (c) a review of Australian mathematics education research covering the same period but disseminated through other outlets; (d) an annotated bibliography of Australian research on girls and mathematics; and (e) a final chapter listing 'current research and development in Mathematical Education', covering the period 1982–1984, and categorised under various headings. The decision to produce another compilation of 'Australian' mathematics education research, not only in time for ICME6 but for each ICME conference since then, is indicative of the community's judgement about the value and impact of the exercise.

The 2008–2011 review of research is a far cry from the slim volume published in 1984 by MERGA itself. Instead there is a work of solid book length, professionally produced, published and marketed by a well known publisher with a strong international readership. In response to MERGA's increasing influence among its geographic neighbours, 'Australia' has been changed to 'Australasia'—comprising, as for the 2004–2008 review, "Australia, New Zealand, PNG, and the Pacific Islands closely allied to Australia and/or New Zealand" (Forgasz et al., 2008, p. 2).

Inspection of the current review confirms that many of the topics covered in the first of the four-yearly reviews have continued to attract considerable research attention, while interest in others appears to have waned. New topics have emerged since the 1980s and methods of exploration have diversified. But some things have remained constant. The material in the review is still compiled and edited by members of MERGA.

The chapters in the 2008–2011 review of research have been clustered under four main headings: Contexts for Mathematics Education, Mathematics Learning

B. Perry et al. (Eds.), Research in Mathematics Education in Australasia 2008–2011, 345–364.

and Teaching, Teachers, and The Future. These boundaries, and the boundaries of the comprising chapters, inevitably are not rigid but ambiguous and porous. Some papers and findings are thus discussed in more than one review chapter.

When appraising the contents of this volume, it is both useful and relevant to consider the broader educational climate during the period under review (similar trends can be traced outside Australia as well). Until 2007, Australian states ran their own numeracy and literacy testing programs. In 2008 national testing in these areas, the *National Assessment Program - Literacy and Numeracy* (NAPLAN), was introduced for students in Grades/Years 3, 5, 7, and 9, and has been administered annually since then. The aims, reporting, and value of the testing program continue to be debated. The development and implementation of an Australian curriculum, including for mathematics, have also engaged many in the mathematics education community and focused attention on aspects of content, teacher and student needs and behaviours, and national priorities.

To capture this challenging environment, albeit simplistically, I have bookended my vastly abbreviated synopsis of each chapter between one or more brief excerpts taken from formal national curriculum and policy documents and a summary of recommendations for future research made by each team of writers. For these official texts I have relied on curriculum-related documents published in Australia and New Zealand, the countries most visible in the research covered in the review. Although each of these documents has its own specific focus, there is also much overlap in their contents. There are directives, guidelines, wish lists, and snapshots of reality. Finally, my own reflections on future directions conclude this chapter.

CONTEXTS FOR MATHEMATICS EDUCATION

This section commences with a reflective chapter written by some of the editors of the previous MERGA review (Forgasz et al., 2008). The chapter explores the key concepts raised in the 2004-2007 review with the benefit of hindsight. I shall make no comment on this chapter beyond commending its connective purpose.

The Affective Domain

> Mathematics has its own value and beauty and it is intended that students will appreciate the elegance and power of mathematical thinking, experience mathematics as enjoyable, and encounter teachers who communicate this enjoyment—in this way, positive attitudes towards mathematics and mathematics learning are encouraged. (National Curriculum Board [NCB], 2009, p. 5)
>
> **Vision**. What we want for our young people [is to be]... . Positive in their own identity. Motivated and reliable. (Ministry of Education, 2007, p. 8)

The opening chapter in this section, written by Gregor Lomas, Peter Grootenboer, and Catherine Attard, covers research on affective factors and the learning and teaching of mathematics, subject matter also included in two previous reviews and of interest to the broader community. The authors have clustered the work surveyed under two headings: Topics/Findings and Methodological Issues. In the first category, studies on attitudes and, particularly, on beliefs dominate. Also included are a smaller but not insignificant number concerned with other affective aspects such as identity, self-efficacy, motivation, and a small number of pieces less readily tagged. In some studies researchers focused on teachers, both pre-service and practising, in many others, students—mainly at the primary and secondary levels—were the focus. As discussed in more detail in the section on methodology, data gathering was frequently via Likert scales, open-ended questionnaire items and semi-structured interviews and thus "largely based on what people (were) willing and able to share" (p. 30), and less often on the reporting and analysis of observational data. An increase in the number of qualitative studies, typically involving only a small sample, and a corresponding decrease in the prevalence of quantitative and mixed method studies was reported. Also noted, however, was the difficulty of comparing and synthesizing research findings given "the multiplicity of descriptors, the lack of a common terminology and frameworks alongside a lack of detail within some papers" (p. 33). With almost two-thirds of the work cited in the chapter being papers included in MERGA's annual conference proceeding, that is, contributions with severe space constraints, it is not entirely unexpected that Lomas et al. described the work they covered as: a "preponderance of studies that were primarily descriptive, and in general a limited amount of theorising" (p. 29). However, a lack of conceptual clarity and an inconsistency of definition for work in the affective domain appear to be a persistent issue. I also reported ambiguous and confusing use of terminology (Leder, 2007) in my extensive examination of research reports linking beliefs and mathematics education included in the 2007 annual MERGA and PME conference proceedings. Mason's (2004) evocative alphabetical listing of synonyms for terms used to discuss and examine the role of 'affect' in the teaching and learning of mathematics serves as a further example. Here is his listing for the first six letters:

> A is for attitudes, affect, aptitude, and aims; B is for beliefs; C is for constructs, conceptions, and concerns; D is for demeanour and dispositions; E is for emotions, empathies, and expectations; F is for feelings; ... (Mason, 2004, p. 347)

Next steps. Key recommendations made by the writers for future research include: achieving definitional clarity for affective concepts, developing new theoretical frameworks to understand causal links between espoused beliefs and observed practices; increased consistency in the methods and instruments used in qualitative studies, and continued collection and analysis of observational data.

Addressing Marginalisation and Disadvantage

> Australian governments, in collaboration with all school sectors commit to promoting equity and excellence in Australian schooling. This means that all

> Australian governments and all school sectors must provide all students with access to high-quality schooling that is free from discrimination based on gender, language, sexual orientation, pregnancy, culture, ethnicity, religion, health or disability, socioeconomic background or geographic location. (Ministerial Council on Education, Employment, Training and Youth Affairs [MCEETYA], 2008, p. 7)

> The New Zealand Curriculum applies to all English medium state schools (including integrated schools) and to all students in those schools, irrespective of their gender, sexuality, ethnicity, belief, ability or disability, social or cultural background, or geographical location. (Ministry of Education, 2007, p. 6)

Research on equity, diversity, social justice and ethics in mathematics education is reviewed in the chapter by Bill Atweh, Colleen Vale, and Margaret Walshaw. Some of the areas covered (e.g., gender and equity) have been of long standing and explicit interest to the MERGA education community. For others (e.g., ethics in mathematics education) previous considerations are more aptly described as embedded in the planning and execution of research rather than as a foregrounded consideration.

In contrast to the previous chapter, theoretical considerations played a prominent role in the work reviewed and thus in the writers' overview of relevant research—both in delineating its scope and in establishing a common framework for its presentation and interrogation of findings and recommendations. The full range of work surveyed is captured by key headings used in the chapter: Gender, Ethnic and language diversity, Rural and remote communities, and Global collaborations. Although each area has its unique issues, the authors' assertion that "the research on social justice that relates to particular groups of mathematics students (and is included in the chapter) is located within these four theoretical considerations of social justice" (p. 42) captures their, largely successful, attempt to optimise coherence throughout the different sections. As is evident from the quotations that head this chapter, many topics covered coincide with issues raised in recent documents outlining the aims of schooling and the delivery of the curriculum. But the impact of the body of work reviewed, and the foundational work on which it has built have, Atweh et al. imply, had only limited impact: "even after years of concerted policy and action to remove inequalities in mathematics education, they still persist" (p. 58).

Next steps. Key recommendations made by the writers for future research include: continued search for "a more comprehensive and perhaps unifying construct of social justice" (p. 57); and the acceptance and accompanying investigation of a broader range of marginalisation issues which should also include physical, emotional and mental disabilities.

Indigenous Students

> The development of partnerships between schools and Indigenous communities, based on cross-cultural respect, is the main way of achieving highly effective schooling for Indigenous students. (MCEETYA, 2008, p. 10)

> The New Zealand Curriculum is a statement of official policy relating to teaching and learning in English medium New Zealand schools. Its principal function is to set the direction for student learning and to provide guidance for schools as they design and review their curriculum. A parallel document, Te Marautanga o Aotearoa, will serve the same function for Māori medium schools ... Together, the two documents will help schools give effect to the partnership that is at the core of our nation's founding document, Te Tiriti o Waitangi / the Treaty of Waitangi. (Ministry of Education, 2007, p. 6)

In their chapter, Tamsin Meaney, Colleen McMurchy-Pilkington, and Tony Trinick examined the growing area of research about students "who are indigenous to the land in which they are learning mathematics ... [and] living in Australia, New Zealand, Papua New Guinea and the Pacific" (p. 67). Reference to such work is, predictably, also found in several other review chapters. Diverse views and areas of interest are embedded in the body of work examined. These are conveyed by the umbrella headings of Pedagogical practices that support indigenous students' learning, Language of instruction, The importance of strong relationships, and Teaching mathematical topics to Indigenous students. Collectively they capture the different perspectives that have shaped the various projects conceived, executed, and reported during the period under review. Although at the beginning of the chapter the authors indicate that they have identified "some exciting studies which document how the skills and knowledge Indigenous students bring to their mathematics learning have been utilised as an affordance for that learning" (p. 67), this is soon qualified. The planning and reporting of much research, they cautioned, still seems to be shaped by a tacit acceptance of theories of deficit which ignore or simplify often very complex settings and needs. Highlighted are false assumptions (a) of homogeneity rather than a recognition of diversity within groups of Indigenous students; (b) of heavy reliance on culturally biased assessment tasks including NAPLAN tests to describe students' mathematical proficiency; (c) of failure to take account of the impact on educational outcomes of factors such as, poverty and economic disadvantage and limited facility in English; and (d) differences in the norms accepted by the researcher and his or her Indigenous students.

Next steps. Recommendations made by the writers for future research include: extending the role of Indigenous Teaching Assistants; attracting more indigenous researchers to this area of work, by providing additional support from MERGA if needed; and applying increased caution and sensitivity in the reporting of the outcomes of new programs to assist the learning of indigenous students.

Supporting Exceptional Students

> The Australian Curriculum is based on the assumptions that each student can learn and the needs of every student are important. It enables high expectations to be set for each student as teachers account for the current levels of learning of individual students and the different rates at which students develop. (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2011, p.10)
>
> Advanced students can be extended appropriately using challenging problems within current topics. (NCB, 2008, p. 1)
>
> Students with disability can engage with the curriculum provided appropriate adjustments are made, if required, by teachers to instructional processes, the learning environment and to the means through which students demonstrate their learning. Adjustments to the complexity or sophistication of the curriculum may also be required for some students. (ACARA, 2011, p. 18)
>
> ERO [Education Review Office] noted that teachers' focus was most often on how to meet the learning needs of students at risk rather than on extending more able students. (Education Review Office, 2011, p. 15)

Carmel Diezmann, Melissa Stevenson, and Jillian Fox reconcile theory and practice succinctly in their review of research centred on exceptional students: "Providing an appropriate education for exceptional students in mathematics is mandated in educational policy in Australia ... but a challenge for teachers and schools" (p. 89).

The opening pages of the chapter focus on research about mathematically gifted students: (a) their characteristics; (b) identification; (c) the delivery and appraisal of special programs and provisions; (d) the contributions made by parents, teachers, and cultural expectations; and (e) meeting the needs of such students. Then the diverse needs of students with specific or more global learning difficulties in mathematics are subsequently described and explored. These report different intervention strategies and programs aimed at students in varying settings, with varying content, and different ages dominate early identification and intervention programs.

A discussion of the benefits and disadvantages of ability grouping for exceptional students—a broad umbrella covering many different options—completes the chapter. Here the emphasis is primarily on research involving high ability students. Factors that seem to enhance or impede the success of ability grouping in specific settings and circumstances are highlighted but the broader relevance or generalizability of these findings is far from clear. Indeed, as

Diezmann et al. argue pragmatically, for initiatives and pilot programs to be productive and to have a lasting impact, “it is imperative that mathematics education researchers are able to validate claims of the success of interventions and the transferability of these programs across different learning contexts through empirical evidence” (p. 100). Given the pervasiveness of explicit or implicit adoption of ability grouping, and the controversies surrounding such practices, it is an area ‘crying out’ for evidence-based research.

Next steps. Key recommendations made by the writers for future research to include further development of current research agendas to expand programs. For those with learning difficulties this should include an increased focus on older students; for gifted students, an examination of the relative benefits of accelerated and enrichment programs. More generally, the authors point to the need for “a rigorous research base ... to inform policy and guide practice” (p. 105).

Technology in Mathematics Education

> Rapid and continuing advances in information and communication technologies (ICT) are changing the ways people share, use, develop and process information and technology, and young people need to be highly skilled in ICT. While schools already employ these technologies in learning, there is a need to increase their effectiveness significantly over the next decade. (ACARA, 2011, p. 6)
>
> **Why study technology?**
>
> The aim is for students to develop a broad technological literacy that will equip them to participate in society as informed citizens and give them access to technology related careers. They learn practical skills as they develop models, products, and systems. They also learn about technology as a field of human activity, experiencing and/or exploring historical and contemporary examples of technology from a variety of contexts. (Ministry of Education, 2007, p. 32)

Research on the impact of digital technologies on the teaching and learning of mathematics is reviewed and summarised by Vince Geiger, Helen Forgasz, Hazel Tan, Nigel Calder, and Janelle Hill. While fully aware of the content and directions of previous MERGA reviews on this topic, changing emphases and the increased use of new technologies influenced the authors to adopt a fresh approach to the structure of the chapter. A solid body of work is referenced under the key headings of Learning contexts and curricular design; Learners, learning and digital technology; Teachers, teaching, and digital technology; and Gender, affect and technology. Explicit consideration in this chapter of gender and affect, factors also given considerable space in a previous chapter, serve as yet another reminder of the unavoidable porosity of the boundaries applied within and across the chapters in this volume.

The challenges faced by mathematics educators wishing to capitalise on the special features and demands of an evolving body of technology options are many and varied, and are broadly sketched by the headings listed above. Included are the different needs, backgrounds, and capabilities of students within and across grade levels, the resources available, the proficiencies and preferences of teachers, and the demands of the broader environment. Although important and of interest, the listing of successful programs and curriculum adaptations raises questions about their sustainability and likely success in different settings. The authors argue repeatedly that more research needs to be done if mathematics teaching and learning is to be enhanced optimally by the new, increasingly available, technologies.

Next steps. A multitude of areas are highlighted by the writers for future research, including: How teachers acquire and apply technological pedagogical content knowledge (TPACK), generally and in their use of specific tools and digital environments; and what instructional strategies and curriculum modifications best capitalise—equitably and for learners of different ages—on new learning pathways enabled by digital technology?

Assessment

> Assessment of student progress will be rigorous and comprehensive. It needs to reflect the curriculum, and draw on a combination of the professional judgement of teachers and testing, including national testing. (MCEETYA, 2008, p. 14)

> The primary purpose of assessment is to improve students' learning and teachers' teaching as both student and teacher respond to the information that it provides. With this in mind, schools need to consider how they will gather, analyse, and use assessment information so that it is effective in meeting this purpose. (Ministry of Education, 2007, p. 39)

Although touched on in earlier reviews, this is the first year that a full chapter has been devoted to research on assessment. According to authors Tom Lowrie, Jane Greenlees, and Tracy Logan, this move has been prompted by the "relatively new focus on high-stakes testing and the comparative nature of assessment across national and international boundaries" (p. 143).

Australia's performance in large scale international tests such as the *Trends in International Mathematics and Science Study* (TIMSS) and the *Programme for International Student Assessment* (PISA) have long been of interest to politicians and educational authorities and are reported, more or less faithfully, in the media as new findings are released. The introduction in 2008 of NAPLAN, the nationwide testing program for students in Grades/Years 3, 5, 7, and 9 added to the arsenal of measures along which students, and curricula, could be judged. From the outset, the test results attracted attention both within and beyond the educational community.

> The 2008 NAPLAN test data, and their breakdown by state, "sex, location, parental background and Indigenous status", were variously reported in the popular print media, with direct between-state comparisons of student performance considered of particular interest. With students throughout Australia sitting for the same test the uneven performance of students at different schools in Australia could no longer be masked. The tests gave parents and governments an unprecedented level of information and would enable better targeting of resources to schools and students in specific areas or years. Furthermore, it was pointed out in multiple articles, the high proportion of Indigenous students who failed to meet the numeracy (and literacy) benchmarks was now more obvious. (Forgasz & Leder, 2011, p. 213)

Again the scope of the research undertaken and reported over the last four years can be inferred from the headings used in the chapter, National focus, Classroom focus, Conceptual focus, and Professional focus. The researchers explored the advantages and disadvantages of the NAPLAN testing regime—within and beyond the classroom, diverse approaches to classroom based assessment, and trialled formative and diagnostic assessment. Examples of tests covering different content areas and response formats are reported and appraised. Teachers' mathematical content knowledge, *per se* or in association with its impact on instructional strategies, has also attracted research interest from the Australasian community. There is a clear imbalance in the reporting of findings from studies assessing what students know. In many there is a tendency to interpret documentation of what students do not know in terms of deficits, but there are also examples of assessment used productively to advance and support student learning. As the authors note, "it ... needs to be recognised that the changed nature and increased levels of accountability have influenced the direction and representation of assessment" (p. 144) undertaken in recent years.

Next steps. In their concluding section the authors predict that exploration of many of the research themes identified over the past four years is likely to continue. They point to the need for continued examination of the scope and content of the NAPLAN tests and of assessment instruments more generally, equity issues associated with assessment, and further investigation of the impact of assessment practices on student learning and understanding of mathematics.

MATHEMATICS LEARNING AND TEACHING

Early Childhood Mathematics Education

> The Australian Curriculum is aligned with the Early Years Learning Framework and builds on its key learning outcomes, namely: children have a strong sense of identity; children are connected with, and contribute to, their world; children have a strong sense of wellbeing; children are confident and involved learners; and children are effective communicators. (ACARA, 2011, p. 10)

> As students journey from early childhood through secondary school and, in many cases, on to tertiary training or tertiary education in one of its various forms, they should find that each stage of the journey prepares them for and connects well with the next. (Ministry of Education, 2007, p. 41)

Early in their review chapter Amy MacDonald, Ngaire Davies, Sue Dockett, and Bob Perry point to an "unprecedented political interest in early childhood education in Australasia (taken to be education of and for children aged between 0 and 8 years old)" (p. 169) but also note that "the quantum of early childhood education research emanating from Australasia seems to have diminished since (these) earlier reviews". A substantial body of work is nevertheless reviewed and described from different perspectives. These are the 'contexts', 'pedagogies', and 'content for early childhood mathematics education'.

Of particular interest in the section centred around context are references to the number, and outcomes, of longitudinal studies undertaken—some of which have enabled insights into transition to school issues—and reports of approaches to the mathematics education of Indigenous students that appear to have yielded successful outcomes. Clustered under pedagogy are investigations involving; the incorporation of technology in the teaching of young children, the role of play in mathematics, assessment, and teacher professional development.

The review of research where content is of particular interest touches on, primarily, number, algebra, and measurement, and to a lesser extent on statistics and probability. A range of disparate factors considered likely to influence performance in those areas has been explored—some of it as part or extensions of larger projects, some with a particular emphasis on Indigenous students, and some spawned by the researchers' own specific interests. The low level of mathematical background and poor attitudes to mathematics of significant numbers involved in the education of young students remain areas of major concern and, together with policy driven issues, may serve as partial determinants of the choice and scope of the research issues pursued.

Next steps. Key recommendations made by the writers for future research include: consolidating and extending "the extensive work on Indigenous children's mathematics learning" (p. 185); issues involved in optimizing students' mathematics learning as they move from a before-schooling to a school environment; and extending research on content beyond areas most popularly researched.

Mathematical Pedagogies

> There will be substantial opportunities and challenges for teacher learning in the implementation of the national mathematics curriculum. Structuring a curriculum in the way that is proposed in this document will create a need for adjustments to some aspects of professional learning for mathematics teachers. In particular the emphasis will be on teachers understanding the big ideas of mathematics, as articulated in the curriculum, and then making

> active and interactive decisions on ways to teach that curriculum. This includes greater emphasis on finding out what the students know, and also greater emphasis on ways of adapting activities to enable access for students experiencing difficulty, and to extend students who may benefit from richer activities. Such emphases are compatible with current approaches to facilitate school- and classroom-based teacher learning. (NCB, 2008, p. 15)

> Since any teaching strategy works differently in different contexts for different students, effective pedagogy requires that teachers inquire into the impact of their teaching on their students. (Ministry of Education, 2007, p. 35)

Charged with the task of reviewing key ingredients for achieving an optimum environment for the promotion of mathematics learning, Ann Gervasoni, Roberta Hunter, Brenda Bicknell, and Matthew Sexton cluster the material identified into three broad categories: "creating powerful learning environments", "selecting tasks and models that promote deep learning", and "knowing and using pedagogical knowledge" (p. 193). A high proportion of their extensive reference list—approximately three-quarters—comprises reports included in the annual proceedings of MERGA. Some of these publications, and indeed the issues raised in them, are also considered in the overviews of research reported in other chapters of the review. Detailed are a variety of trialled techniques and approaches, most focusing on one or a limited number of many factors or approaches listed as potentially important contributors to powerful pedagogical actions. The chapter's contents highlight the many similarities but also different nuances evident in the curriculum and instructional directives advocated by Australian and New Zealand authorities and adopted by teachers.

Next steps. Areas for further research are suggested throughout the chapter. All are embedded in the three themes under which the work reviewed has been clustered and resonate with areas identified in other chapters as warranting further research.

Mathematics Curriculum Taught in Schools

> The Australian Curriculum is a dynamic curriculum. The online publication of the curriculum facilitates ongoing monitoring and review as well as providing the opportunity to update the curriculum in a well-managed and effectively communicated manner ... Any updating will take into account review and evaluation data; new national and international knowledge and practice about learning, teaching, curriculum design and implementation; and contemporary research in discipline and cross-discipline areas. (ACARA, 2011, p. 25)

This chapter by Judy Anderson, Paul White, and Monica Wong incorporates, more than any other in this review, the edicts and recommendations published in recent Australian national and educational system documents. It is especially timely, they argue, given “the recent development of the Australian curriculum for mathematics in the schooling years” (p. 219). Insightful appraisals are given of the forces and circumstances which shaped the development and evolution of this curriculum.

Once again a substantial body of work is reviewed, in this chapter under the headings of School mathematics and numeracy, Development of curriculum: New Zealand and Australia; Curriculum documents and resources; and Teachers’ use of, and reactions to, curriculum documents. In this careful and comprehensive consideration of the formal documents, many of the research topics and endeavours summarised in other chapters surfaced. For example, the authors wrote: “Several important considerations were presented in the *Shape Paper* (ACARA, 2010b): equity and opportunity; connections to other learning areas; breadth and depth of study; the role of digital technologies; the nature of the learner; general capabilities; and cross-curriculum perspectives” (p. 228).

Next steps. With respect to curriculum development and implementation initiatives the authors noted, “‘not a great deal’ of actual research has been reported since 2008 and highlights the need for more research ... Researching the ways teachers manage and integrate content to engage students would provide valuable information for further curriculum development” (p. 239).

Tertiary Mathematical Science Education

> The senior years of schooling should provide all students with the high quality education necessary to complete their secondary school education and make the transition to further education, training or employment. (MCEETYA, 2008, p. 12)

> Students entering the senior secondary years of schooling will have had opportunities to develop ‘a solid foundation in knowledge, understanding, skills and values on which further learning and adult life can be built’ with a strong focus on literacy and numeracy skills. They will have ‘practical knowledge and skills in areas such as ICT and design and technology which are central to Australia’s skilled economy and provide crucial pathways to post-school success.’ (ACARA, 2009, p. 5)

Although not a new topic in the four-yearly MERGA review, the body of research focusing on the tertiary level of education is dwarfed by work centred on the primary and secondary levels of mathematics education. Once again a catalogue of the major headings used by the authors, Bill Barton, Merrilyn Goos, Leigh Wood, and Adel Miskovich provides a good indication of the range and scope of the research reviewed. They list research about Transition, Technology, Mathematical topics, Pipeline and politics, Lecturing, and

Statistics education (representing some 30 and by far the largest number of papers), Mathematics in engineering, and Mathematics in nursing (with only three contributions from two sets of authors cited). The authors' explanation for the relative strength of statistics education is worth noting: "We suggest that it can be attributed to the growing presence of groups of researchers within an institution or region. A critical mass of people thinking about these issues appears to produce higher quality research and theoretical development over a period of time" (p. 255).

Next steps. Several areas currently deemed under researched are identified by the authors, including teaching and learning issues in higher levels of mathematics and the transition to employment. Three important points made about future research directions and output, though not couched as specific recommendations, warrant attention. First the authors' statement, that for the purposes of their review chapter they chose to ignore the not insignificant number of small, single studies conducted at only one institution and not obviously linked to, or building on, previous work; and that "these individual studies contributed only marginally to our overall understanding" (p. 249) yet deserve recognition and dissemination via appropriate channels. Secondly, the importance of access to suitable publishing outlets for affirming an area of research. Thirdly, the way the allocation of major grants can stimulate research activity and productivity.

Mathematics Education and Statistics

> **Why study mathematics and statistics?**
>
> By studying mathematics and statistics, students develop the ability to think creatively, critically, strategically, and logically. They learn to structure and to organise, to carry out procedures flexibly and accurately, to process and communicate information, and to enjoy intellectual challenge. (Ministry of Education, 2007, p. 48)
>
> **4.2 Content strands**
>
> The content strands are the collected concepts and terms that form the basis of the curriculum. To maximise interconnections, coherence and clarity, the concepts and terms are grouped into developmental sequences that are termed strands. For mathematical and pedagogical reasons, it is proposed that the national mathematics curriculum includes three content strands: Number and algebra, Measurement and geometry, and Statistics and probability. (NCB, 2009, p. 5)

The authors of the previous chapter pointed to a welcome 'beacon of light', or less prosaically, the growing strength of research in statistics education. It may or may not be a coincidence that it is followed by Rosemary Callingham, Jane Watson, and Tim Burgess' review of research focusing on the "synergies and tensions between mathematics and statistics and the implications for mathematics education" (p. 268). Considered in turn are Statistics education research in Australasia, Perceptions of differences between mathematics and statistics, Context in mathematics and statistics teaching and learning, Mathematical and statistical pedagogical content knowledge, and the concluding section Tensions and synergies between mathematics and statistics. The appreciation and understanding by students of different ages of specific statistical concepts has continued to attract research attention. Teachers and students differ in their views of the overlap and schisms between statistics and mathematics and, it has been found, there is not necessarily a high correlation between student achievements in the two domains. Various research studies explored the role of context in the learning and teaching of mathematics and statistics. Also examined is the way in which teachers' pedagogical content knowledge influences how statistical concepts are, or could be, introduced and developed in classrooms, with examples set primarily but not exclusively in mathematics lessons.

Next steps. Among the areas suggested for further work are: clarification of the impact on students' learning in statistics and mathematics of statistics being taught as "separate from, embedded in or linked to mathematics" (p. 280); a greater focus on teachers' statistical pedagogical knowledge and instructional strategies; and overlap and differences in students' and teachers' attitudes and beliefs about statistics and mathematics.

TEACHERS

Education of Prospective Teachers of Mathematics

> All Australian governments, universities, school sectors and individual schools have a responsibility to work together to support high-quality teaching and school leadership, including by enhancing pre-service teacher education. (MCEETYA, 2008, p. 11)

Influenced by earlier work, Glenda Anthony, Kim Beswick, and Fiona Ell organised the material reviewed in five broad research areas: (a) the recruitment of prospective teachers of mathematics, (b) understanding the process of 'becoming' a teacher, (c) investigations into 'the knowledges' for teaching, (d) explorations of initial teacher education practices and pedagogies, (e) and research involving teacher educators researching on and within their own practice. Despite the dangers of oversimplifying their findings, brief summaries are respectively: (a) a great diversity is evident among pre-service teacher programs in Australasia in general, and with respect to its mathematics and mathematics education pre-requisite components in particular; (b) the charting of affect and identity equilibrium and permutations in and through specific course work and practicum experiences; (c) identification of levels of mathematical proficiency at entry and the putative acquisition of specific content during to the program; (d) single and generally short term trials and applications of "innovative pedagogies and associated instructional activities" (p. 301); and (e) few studies in which teacher educators researched their own practices.

Next steps. In each of the topics listed above there is room for further research. In particular, the writers, like those of other chapters in this volume, urge that a higher priority be given to the planning of large scale studies, the findings of which are both replicable and generalizable.

Professional Knowledge of Practising Teachers of Mathematics

> ERO also recommends that the Ministry of Education: explores ways to make more accessible, through Ministry of Education websites, research material that will support teachers and school leaders to extend their understanding about pedagogical practice, including engaging in teaching as inquiry. (Education Review Office, 2011, p. 39)
>
> The curriculum should allow jurisdictions, systems and schools to implement it in a way that **values teachers' professional knowledge** and reflects local contexts ... The curriculum should be established on a **strong evidence base on learning, pedagogy and what works in professional practice** and should encourage teachers to experiment systematically with and evaluate their practices. (NCB, 2008, p. 17, emphases added)

An overview by Janette Bobis, Joanna Higgins, Michael Cavanagh, and Anne Roche of the research conducted over the past four years, within the Australasian context, on teachers' knowledge completes the section on teachers. The chapter contents are clustered under the main headings of: The nature of teacher knowledge, Frameworks of teacher knowledge, Domains of teacher knowledge, and Sources of teacher knowledge and modes of inquiry. Much of the work reviewed consisted of single, small scale studies, although findings from larger studies, extensions and replications of earlier work are also found in this well embraced area of research.

Very briefly, knowledge for teaching mathematics is recognised to be a (still somewhat nebulous) function of sociocultural contexts, conceptions of mathematics, and perceived level of knowledge about mathematics. The authors reported findings from relevant research outside Australasia in some detail to provide a context for the work on frameworks of teacher knowledge. Both that body of work and data from investigations within the settings covered by this review confirm the intricacies embedded in teaching and teacher knowledge and still leave many aspects obscure and outside our grasp. In studies aimed at furthering our knowledge about the domains of teacher knowledge aspects of content knowledge, of pedagogical content knowledge, and of mathematical knowledge for teaching are tapped. Despite the substantial body of work in this area, captured under the last of the four headings listed above, "we are far from capturing what it is a teacher does, why they do it, and what effect it might have on student learning" (Barton, 2009, p. 3).

Next steps. Issues typically not yet taken up within the Australasian context, but noted by the authors as worthy of research investigations, are cultural perspectives on teacher knowledge; the impact of politically driven pressures to influence the timing and setting of teacher education programs; the putative link between teacher knowledge of and about mathematics and student learning outcomes; and how, what, and when teachers learn from their own experience without interventions from outside sources.

LOOKING TOWARDS THE FUTURE—A PERSONAL PERSPECTIVE

Reflections

A substantial body of research is captured in the chapters of this review. It encompasses the labours of a community of active researchers, with varied interests and diverse theoretical perspectives. Some of the issues explored in the period covered by this volume clearly resonate with questions and concerns particularly pertinent to the changing educational environment; others are more aptly described as continuing or renewed explorations of areas of long standing concern.

A common brief was given to all chapter authors. This read in part:

> Each chapter should be a critique and celebration of the Australasian mathematics education research in the field covered by the chapter and published between 2008–2011 inclusive. The chapters are not descriptions—comprehensive or otherwise—of the research, but critiques of it.

Perusal of the chapters reveals that implementation of this directive varied. Some chapter authors took considerable care to place the Australasian work in the relevant international research context, for others this appeared to be a lower priority. Although reference was made in many of the chapters to the preponderance of single, one-off studies, treatment of their findings differed. In some cases they were given equal weight with findings from more extensive and seemingly more representative research; in others there was explicit recognition of the lack of generalizability and the barely perceptible amount of information being added to the existing body of research. Opportunities to weave together the findings of small studies were often overlooked and the implications for apparently contradictory findings left unexplored. In other words, the instruction to produce a critique rather than a summary of research was variously interpreted. Despite these concerns, I echo the sentiments expressed by King and McLeod (1999) in their review of Sierpinska and Kilpatrick's publication of the fourth ICMI study: "there is much of value here ... [It] will be especially useful to researchers who are new to the field [and] ... senior researchers in mathematics education should find it useful" (p. 234).

What can be changed, improved, or added to enhance the content, directions, and boundaries of Australasian research and ensure that such work is compiled and captured, optimally, in the next four-yearly review?

Next steps

Structure and contents of the review. Given that tertiary institutions place a high priority on conducting research and the dissemination of its findings, the amount of material to be summarised, analysed, and synthesised is likely to increase. Some topics are reasonably self-contained; others teem with many interacting factors. Balancing the need for an extensive and comprehensive overview of Australasian research, affirming key areas of research through the allocation of their own chapter, and yet avoiding unnecessary duplication is no easy matter. Juggling the different requirements is a continuing challenge for editors of the four-yearly review.

Earlier in this chapter I commented on the importance of access to suitable publishing outlets for affirming an area of research. MERGA publications, including refereed conference proceedings, undoubtedly serve as quality outlets not just for Australasian research and researchers but also for those engaged in research elsewhere. But does every piece of research warrant equal emphasis? Highlighting commonalities of findings, reconciling or explaining apparently contradictory results, and linking reports of consecutive small studies into a more comprehensive unit, now done effectively in some chapters, should be a feature common to all.

Identification of questions as yet unanswered and recommendations for further research are, appropriately, found in every chapter. At times the recommendations are simply for further, unspecified research; at others specific areas are named. Many of these are congruent with my own personal interests and need no repetition. In the next and final section I point to some promising routes currently not obviously explored in Australasian research.

Research directions—what if? “The search for one’s identity”, wrote King and McLeod (1999) in the fourth ICMI study, “is typically thought of as an adolescent activity. Adolescents begin to define themselves in exercising independence from their parents, often by determining their values, goals, and career choices in relation to those of the parents” (p. 228). In mathematics education, the parents are most aptly represented by mathematics and psychology—influences still very much in the forefront of much of the research endeavours captured in this volume. Over the years, the theoretical perspectives, research methods, and determination of significance established in other fields have influenced educational research. These fields include, and with varying levels of impact, anthropology, cognitive neuroscience, ethnography, history, philosophy, sociology, and technology. What might be added to our knowledge about the teaching and learning of mathematics if researchers, facing apparently impenetrable barriers, actively and strategically drew on new lenses rather than following well trodden, but clearly not particularly productive, paths? What horizons might be extended, or barricades breached, through carefully planned research collaborations with those working principally in one of the fields listed above? Might such cross fertilization replace a chain of repetitive studies with more skilfully nuanced research questions, fresh methods, and different strategies to tackle important practical dilemmas crying out for more evidence based input? Such interdisciplinary partnerships are, incidentally and fortuitously, consistent with the urgings from 'higher' authorities for an increase in cross faculty (and implicitly cross discipline) research. Just two provocative examples are given below. Both point to new directions already evident or advocated in the wider international mathematics education research community. Both represent a crack in the doorway that can lead to new territories.

> Einstein famously said that his pencil was more intelligent than he was—meaning, that he could achieve far more using his pencil as an aid to thinking than he could unaided. There is a need to recognise that mathematical digital technologies are the pencils of today and that we will only fully exploit the benefits of digital technologies in teaching, learning and doing mathematics when it becomes unthinkable for a student to solve a complex mathematical problem without ready access to digital technological tools. (Clark-Wilson, Oldknow, & Sutherland, 2011, p. 4)

> One of the fields wherein cognitive neuroscience has been most successful in meeting research on learning and instruction is the domain of mathematics ... (examples) cover various domains of mathematics (number processing, arithmetic, geometry, algebra), a variety of methodological approaches (experimental design, longitudinal studies, training studies, pharmacological intervention, research on learning disorders) and several neuroimaging techniques. (De Smedt & Verschaffel, 2009, p. 5)

What new insights and robust findings might be achieved when sophisticated use of technology is applied, not only to solving complex mathematical problems but is incorporated in the planning, execution, and analysis of research? Could informed access to the tools and theories of other disciplines revive areas currently devoid of robust theoretical explorations and lead, for instance, to enhanced explanations and understanding of the putative causal links between espoused beliefs and observed practices—inside and beyond the mathematics classroom? Might it halt the current imbalance between qualitative, small sample research and large scale, quantitative explorations? Formal documents blandly state that "adjustments to the complexity or sophistication of the curriculum may be required for some students" (ACARA, 2011, p. 18). Might issues and tools typically outside mathematics education research lead to more fine-tuned and comprehensive investigations not just of the needs but also the strengths of exceptional students and ways of harnessing or meeting these? Individual preferences and expertise will influence which of the other areas highlighted throughout this volume and this chapter as crying out for new or better answers or seem ripe for cross-discipline cooperative research. Finding answers to difficult mathematics problems often requires a rewording or re-framing of the problem as initially posed. The message of this for mathematics education research is inescapable.

However, to keep the recommendation for a greater link between mathematics education and other disciplines in perspective, I add a final caveat borrowed from King and McLeod (1999), for like them, I consider

> research in mathematics education as an important and independent discipline with a history of its own and with its own contributions to make to the world... . While it continues to mature, research in mathematics education is influenced by many other fields, but it needs to proceed on an independent path, not a path chosen by others. (p. 234)

REFERENCES

Anderson, J., White, P., & Wong, M. (2012). Mathematics curriculum in the schooling years. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 219–244). Rotterdam, The Netherlands: Sense.

Anthony, G., Beswick, K., & Ell, F. (2012). The professional education and development of prospective teachers of mathematics. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 291–312). Rotterdam, The Netherlands: Sense.

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2009). *National curriculum in the senior secondary years. Position paper, Version 6.* Retrieved from http://www.acara.edu.au/verve/_resources/Curriculum_Design_Paper_.pdf

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2010). *The shape of the Australian curriculum, Version 2.* Sydney, NSW: Author.

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2011). *The shape of the Australian curriculum, Version 3.* Sydney, NSW: Author.

Atweh, B., Vale, C., & Walshaw, M. (2012). Equity diversity, social justice and ethics in mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 39–65). Rotterdam, The Netherlands: Sense.

Barton, B. (2009). Being mathematical, holding mathematics: Further steps in mathematical knowledge for teaching. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 4–10). Palmerston North, NZ: MERGA.

Barton, B., Goos, M., Wood, L., & Miskovich, A. (2012). Growth and new direction? Research in tertiary mathematical science education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 245–265). Rotterdam, The Netherlands: Sense.

Bobis, J., Higgins, J., Cavanagh, M., & Roche, A. (2012). Professional knowledge of practising teachers of mathematics. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 313–341). Rotterdam, The Netherlands: Sense.

Briggs, J. (Ed.) (1984). *Summary of research in mathematics education in Australia.* Brisbane, QLD: MERGA.

Callingham, R., Watson, & J., Burgess, T. (2012). Uncertaintly in mathematics education: What to do with statistics? In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 267–287). Rotterdam, The Netherlands: Sense.

Carey, P. (1985). *Illywhacker*. St Lucia, Australia: University of Queensland Press.

Clark-Wilson, A., Oldknow, A., & Sutherland, R. (Eds.) (2011) *Digital technologies and mathematics education.* Joint National Council of the United Kingdom. Retrieved from http://cme.open.ac.uk/cme/JMC/Digital%20Technologies%20files/JMC_Digital_Technologies_Report_2011.pdf

De Smedt, B. & Verschaffel, L. (Eds.). (2009). Preface. *Cognitive neuroscience meets mathematics education* (pp. 5–6). Retrieved from http://www.earli.org/resources/ASC/Program_EARLIASC_NeuroscienceMathEducation_Bruges2009.pdf

Diezmann, C., Stevenson, M., & Fox, J. (2012). Supporting exceptional students and the learning of mathematics. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 89–109). Rotterdam, The Netherlands: Sense.

Education Review Office (2011). *Directions for learning. The New Zealand curriculum principles and teaching as inquiry*. Wellington, New Zealand: Author. Retrieved from http://www.ero.govt.nz/National-Reports/Directions-for-Learning-The-New-Zealand-Curriculum-Principles-and-Teaching-as-Inquiry-May-2011

Forgasz, H., Barkatsas, T., Bishop, A., Clarke, B., Keast, S., Seah W. T., & Sullivan, P. (2008). *Research in mathematics education in Australasia 2004–2007*. Rotterdam, The Netherlands: Sense.

Forgasz, H. J., & Leder, G. C. (2011). Equity and quality of mathematics education: Research and media portrayals. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 205–222). Dordrecht, The Netherlands: Springer.

Gervasoni, A., Hunter, R., Bicknell, B., & Sexton, M. (2012). Powerful pedagogical actions in mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 193–218). Rotterdam, The Netherlands: Sense.

King, K. D., & McLeod, D. M. (1999). Coming of age in academe. *Journal for Research in Mathematics Education, 30*(2), 227–234.

Leder, G. C. (2007). Beliefs: What lies behind the mirror? *The Montana Mathematics Enthusiast*, [Monograph 3], 39–50.

Lomas, G., Grootenboer, P., & Attard, C. (2012). The affective domain and mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 23–37). Rotterdam, The Netherlands: Sense.

Lowrie, T., Greenlees, J., & Logan, T. (2012). Assessment beyond all: The changing nature of assessment. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 143–165). Rotterdam, The Netherlands: Sense.

MacDonald, A., Davies, N., Dockett, S., & Perry, B. (2012). Early childhood mathematics. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 169–192). Rotterdam, The Netherlands: Sense.

Mason, J. (2004). Are beliefs believable? *Mathematical Thinking and Learning, 6*(3), 343–351.

Meaney, T., McMurchy-Pilkington, C., & Trinick, T. (2012). *Indigenous students and the learning of mathematics*. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Review in mathematics education in Australasia 2008–2011* (pp. 67–87). Rotterdam, The Netherlands: Sense.

Ministerial Council on Education, Employment, Training and Youth Affairs [MCEETYA]. (2008). *Melbourne declaration on educational goals for young Australians*. Retrieved from http://www.curriculum.edu.au/verve/_resources/National_Declaration_on_the_Educational_Goals_for_Young_Australians.pdf

Ministry of Education. (2007). *The New Zealand curriculum*. Wellington, NZ: Learning Media.

National Curriculum Board [NCB]. (2008). *National mathematics curriculum: Framing paper*. Melbourne, Australia: Author. http://www.ncb.org.au/verve/_resources/National_Mathematics_Curriculum_-_Framing_Paper.pdf

National Curriculum Board [NCB]. (2009). *Shape of the Australian curriculum: Mathematics*. Barton, Australia: Author.

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