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# 8. IN SEEKING A HOLISTIC TOOL FOR ETHNOMATHEMATICS

*Reflections on Using Ethnomodeling as a Pedagogical Action for Uncovering Ethnomathematical Practices*

Throughout history, people have explored other cultures and shared knowledge often hidden behind ideas, traditions, practices, and customs. This cultural dynamism has enriched these cultures, including Western culture. In this regard, the Greek foundations of European civilization are strongly influenced and impacted by Egyptian civilization (Powell & Frankenstein, 1997). Currently, however, there still exists a world-wide acceptance of supremacy of Western logical perspectives in math science in academic arenas; contemporary globalized non-Western societies place enormous value on capitalistic Western-oriented science and mathematics.

The literature on mathematics education, including texts and teaching materials and methods, is based on the scientific and mathematical concepts rooted in this dominant Western tradition that many of us are accustomed to thinking of when we *do* mathematics. Most of the examples used in teaching academic mathematics derive themselves from problems and contexts from within a Western cultural paradigm. Because mathematics is an expression of human development, culture, and thought, we have come to believe that this Western monocultural perspective in mathematics education causes problems for many members of non-Western cultures.

Mathematics forms an integral part of the greater cultural heritage of humankind and is the central premise of our work. What this heritage consists of is equally dependent upon time, sociocultural context, and place. In this regard, ethnomathematics has demonstrated how mathematics is made of many diverse and distinct cultural traditions, not just those emerging from the Mediterranean: it supports the premise that teaching/learning of mathematics should include, and place equal importance upon, those originating from indigenous and non-Western contexts.

Mathematical thinking is influenced by the vast diversity of human characteristics that include: language, religion, worldview, and economical-social-political activities. In concert with these, human beings have developed logical processes related to our universal need to quantify, measure, model, understand, comprehend, and explain. All have come to shape and operate within different socio-historical contexts. Each cultural group has developed, often unique, ways of

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incorporating mathematical knowledge; and has often come to represent given cultural systems, especially in ways that members of cultural groups quantify and use numbers, incorporate geometric forms and relationships, and measure and classify objects.

For all these reasons, each cultural group has developed unique and often distinct ways to *mathematize*[1] their own realities. Western-academic scientific arrogance often presents an overt disrespect of, and an outright refusal to acknowledge, a cultural identity (Zaslavsky, 1996). These *cultural particularities* should neither be ignored nor should they be disrespected when individuals from different cultures attend school. Equally important is the search for alternative methodological approaches. As Western mathematical practices have been adopted worldwide, it is necessary to recognize mathematical ideas from different cultures that are either ancient or traditional practices.

One alternative methodological approach is *ethnomodeling*, which may be considered a practical application of ethnomathematics that also adds the cultural perspective to modeling concepts. When justifying the need for a culturally situated view on mathematical modeling, our sources are rooted in the theory of ethnomathematics. We argue, as well, that recognizing cultural differences in mathematics reveals new perspectives on scientific questioning methods. Research of culturally situated modeling ideas addresses the problem of mathematics education in non-Western cultures by bringing the cultural background of students into the mathematics curriculum, and connects the local-cultural aspects of the school community to the teaching and learning of mathematics (Rosa & Orey, 2010a). On the other hand, local views used in mathematical modeling could be used also in global collaborations, thus potentially widening views of mathematics in others. This pedagogical approach is needed in mathematics education because it is a major factor in broadening the modeling process as well as an ethnically fairer view that can help to bridge the mathematical achievement gap of the students. This alternative approach helps in promoting intellectually innovative ideas in the area of modeling by deepening and widening the Western understanding of mathematics.

In this chapter, we present a number of arguments to justify the need to strengthen the research field of modeling by adopting a cultural perspective in problem solving methods, conceptual categories, structures, and the models used in representing mathematical ideas and practices developed by distinct cultural groups. We refer to this pedagogical approach as ethnomodeling.

## ETHNOMATHEMATICS AS A HOLISTIC APPROACH TO MATHEMATICS EDUCATION

In the past decades, ethnomathematics has come to evoke a worldwide discussion that raises fervent arguments both for and against. These discussions clearly show the uncertainty, on the side of the proponents as well as the opponents, of what the definitions and goals of ethnomathematics are, and have brought forth many misconceptions and fears related to this research paradigm. In an early article entitled "Ethnomathematics and its Place in the History and Pedagogy of Mathematics", *ethnomathematics* is defined as the study of scientific and, by extension, technological phenomena in direct relation to social, economic, and cultural backgrounds (D'Ambrosio, 1985). If described in terms of the development of science in general, ethnomathematics may be considered as a corpus of knowledge established as systems of explanations and *ways of doing* mathematics, which have been accumulated through generations in distinct cultural environments (D'Ambrosio, 1998).

Ethnomathematics as a research paradigm is much wider than traditional concepts of mathematics and ethnicity as well as current definitions of multiculturalism. *Ethno* may be that which is related to distinct cultural groups identified by cultural traditions, codes, symbols, myths, and specific ways of reasoning and inferring. Accordingly, ethnomathematics may be considered as the way that various cultural groups mathematize objects or phenomena and it examines how both mathematical ideas and practices are processed and used in daily activities. In the educational context, ethnomathematics can be described as the arts or techniques developed by diverse students to explain, to understand, and to cope with their own social and cultural environments.

Ethnomathematics embraces the mathematical ideas, thoughts, and practices developed by all cultures (Barton, 1996). From this perspective, a body of anthropological research has come to focus on both the intuitive mathematical thinking and the cognitive processes that are largely developed in various cultural groups. Ethnomathematics may also be considered as a program that seeks to study how students have come to understand, comprehend, articulate, process, and ultimately use mathematical ideas, concepts, procedures, and practices that may solve problems related to their daily activity. Ethnomathematics is not only the study of mathematical ideas; it also studies anthropology and history. This form or context of study of the history of mathematics assists in identifying the cultural and mathematical contributions of different cultures across the world and translates them into academic mathematics that Western researchers and educators are able to understand. Seen in this context, the focus of ethnomathematics consists essentially of a serious and critical analysis of the generation and production of the mathematical knowledge and intellectual processes, the social mechanisms in the institutionalization of knowledge and the diffusion of this knowledge (Rosa & Orey, 2006). In this much more *holistic*[2] context of mathematics that uses an anthropological perspective to include diverse perspectives, patterns of thought, and histories, the study of the *systems*[3] taken from reality help students to come to reflect, understand, and comprehend extant relations among all of the components of the system. Additionally, ethnomathematics may be defined as the intersection of cultural anthropology, academic mathematics, and mathematical modeling, which is used to help students to translate diverse mathematical ideas and practices found in their communities (Fig. 8.1).

Characterized by our very humanness, all individuals have the capability to develop mathematical concepts that are rooted in the universal human endowments of curiosity, ability, transcendence, life, and death. An awareness and appreciation

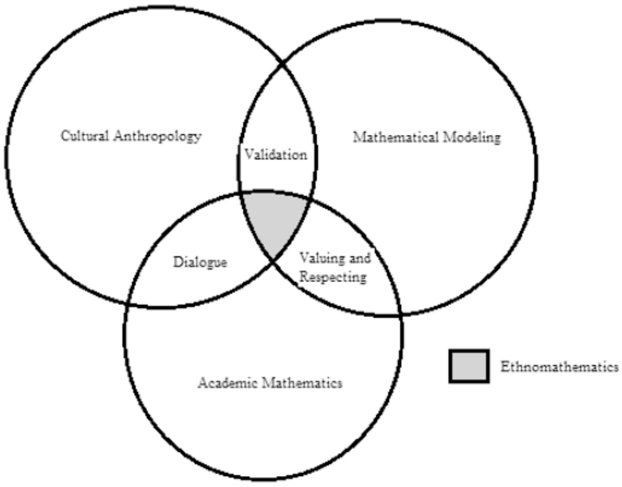



**Fig. 8.1** Ethnomathematics as an intersection of three research fields

of cultural diversity can be seen for instance, in our clothing, methods of discourse, our belief systems all combining to influence our own unique worldviews that allow us to understand each aspect of daily life.

The unique cultural background of each student represents, as well, a set of values and unique ways of seeing the world as it is transmitted from one generation to another. Principles of anthropology that are relevant to the work of ethnomathematics include the essential elements of culture such as language, economy, politics, religion, art, which most certainly influence the daily mathematical practices of diverse groups of students. Since cultural anthropology gives us tools that increase our understanding of the internal logic of a given society, detailed anthropological studies of the mathematics of distinct cultural groups most certainly allow us to further our understanding of the internal mathematical logic of diverse groups of students (Rosa & Orey, 2008). On the other hand, in ethnomathematics research, the terms *primitive* and *illiterate* are used in different and inconsistent ways.[4]

Ethnomathematics places a reliance on the idea that each cultural group developed its own ways, styles, and techniques of doing certain tasks, and responses to the search of explanations, understanding, and learning, which are named *systems of knowledge* (D'Ambrosio, 1998). All of these diverse systems use forms of *inference, quantification, comparison, classification, representation, measuring,* and *modeling*. Western science is a system of knowledge, yet there are other systems of knowledge with the same aims. The other systems of knowledge use other forms of inferring, quantifying, comparing, classifying, representing, measuring, and modeling, but are not simple-minded or primitive in comparison to academic-Western-mathematics.

However, from the claim that no single scheme has absolute explanatory power, it does not follow that *all* schemes are equally valid (Code, 1991). It is our view that comparisons between different systems of knowledge are, to some extent, groundless due to the fact that they have arisen in different environments to meet different demands. In this regard, not all logics and assumptions are equally effective in understanding and dealing with a certain situation, which implies that Western logic and its associated assumptions are not always as useful as we assume in all situations (Fasheh, 1982). Western academic science owes its success to the ability of scientists to regularly select problems that can be solved with conceptual and instrumental techniques close to those already existing. This science does not aim at novelties of fact or theories and, when successful, does not expect to find them.

Clearly, when a paradigm denies the validity of a large class of problems, it leaves out a number of socially and scientifically important problems and solutions. This nature of normal science considerably limits the possibilities of multicultural research and education, and hence the rules and findings of other cultural contexts. The terms *paradigm*, *normal science*, and *scientific revolution* and defined *normal science* may be used as analysis based on past scientific achievements, and acknowledged by peers as a foundation for further practice of that particular science (Kuhn, 1962). The reluctance to adopt a new paradigm might be due to the fact that any new paradigm often requires scientists to redefine or even discard their earlier work, as reviewed by their peers. In adopting new paradigms, some older problems may in fact be declared entirely *unscientific*, whereas others that were previously nonexistent or trivial might become the very archetypes of significant and new scientific and mathematical achievement.

Research does not aim to produce major novelties, both conceptual and phenomenal. Even those projects whose goal is paradigm articulation do not aim at the *unexpected* novelty, but reinforce the existing paradigm (Kuhn, 1962). We believe that ethnomathematics may be speeding up the process of paradigm re-evaluation and finding unexpected novelties in science in general. This means that ethnomathematics may work as a catalyst, introducing and encouraging new paradigms such as ethnomodeling (Rosa & Orey, 2010a) and ethnocomputing (Eglash, Bennett, O'Donnell, Jennings, & Cintorino, 2006) to challenge the prevailing ones. Concurrently, there exist several culturally-bound paradigms within a discipline such as mathematics. None of these are better or worse than the others, they just have arisen to meet the different needs of different cultural groups.

One reason why ethnomathematics has emerged as a new paradigm in mathematics education may be because these different culturally-bound paradigms have not had the opportunity to really confront each other before and thus have not been able to interact.

For many, many years Western institutions have been educating many people from the emerging nations who return to work in their country of origin, and who integrate what they learned and experienced abroad with their own reality at home.

Presently, in a global scientific community where new publications are available on-line, where blogs and resources are universally available, paradigms from different cultures meet and interact continuously, and new ideas are emerging. It is only the all-extensive Western influence that prevents new viewpoints from emerging if they are not fully-fledged and totally revolutionary. Since science and mathematics do not, in our opinion, belong only to Europeans and North Americans, they are currently in a phase of adaptation as members of different cultures of distinct individuals participate globally. In this context, ethnomathematics was created and developed out of a non-Western reality.

## ETHNOMODELING: A DEFINITION

The prefix *ethno* in ethnomathematics is today accepted as a very broad term because it refers to the social, cultural, political, economic, and environmental contexts of the members of distinct cultural groups that form our contemporary and globalized society (D'Ambrosio, 1990). It includes language, jargon, myths, symbols, and code of behaviors that were developed in these contexts. The meaning of *mathema* is to explain, to know, to understand, and to do daily activities such as ciphering, measuring, classifying, inferring, and modeling, while the meaning of the suffix *tics* is derived from *techné* and has the same root as technique.

In the context of ethnomodeling, *ethno* refers to differences in cultures that are mainly based on language, history, religion, customs, institutions, and on the subjective self-identification of individuals and professional groups. However, these cultural differences can also arise from differences based on ethnic or nationality oppressions. These are the social, economic, political, environmental, and cultural backgrounds that define a group as a cultural entity. In this regard, individuals develop a sense of cultural identity, which "is adapted and changed throughout life in response to political, economic, educational, and social experiences" (Gollnick & Chinn, 2002, p. 21). An awareness of individuals' cultural identities provides the foundation for how they define themselves in terms of how others view them. Thus, in the educational context, educators need to view students as cultural beings, embrace their diversity, acknowledge their previous knowledge, and validate their cultural identity in order to create classrooms that model tolerance and appreciation of students' differences.

For example, teachers can help students feel comfortable with their cultural identity and assist them with their learning by a using a pedagogical approach that embraces diversity and validates their cultural background and previous knowledge. Gay (2000) defined this approach as "using the cultural knowledge, previous experiences, frames of reference, and performance styles of ethnically diverse students to make learning encounters more relevant to and effective for them" (p. 1). This is one of the most important goals of ethnomodeling as pedagogical action for the teaching and learning of mathematics.

In the pedagogical action of ethnomodeling, teaching starts by teachers getting to know their students on a personal level, building teaching around their interests

when possible, and showcasing their talents and creativity in order to use them as teaching tools (Bennett, 2003). These three areas allow the creation of a classroom environment that is learner-centered and promote the academic success of the students. In our opinion, the underlying principles of ethnomodeling are:
– Students must experience academic success.
– Students must develop and maintain their cultural competence.
– Students must develop a critical consciousness through which they may challenge social injustice.

In this context, educators empower students to succeed in mathematics by providing a learning environment that respects their cultural backgrounds, embraces their diversity, and celebrates their differences.

Since individuals of different cultural groups have different views on the relationships related to spiritual values, the individual and the group, the citizen and the state, they also have differing views on the relative importance of rights and responsibilities, liberty and authority, and equality and hierarchy. These differences between cultures are a product of centuries and they will not disappear rapidly because they are far more fundamental than differences among political ideologies and political regimes. In addition to these categories, in the ethnomodeling perspective, culture is expanded to include the cultures of differing professional groups and age classes as well as social classes and gender.

In other words, *ethno* in ethnomodeling refers to members of a group within a cultural environment identified by cultural traditions, codes, symbols, myths, and specific ways used to reason and to infer. In this context, *modeling* is considered as a combination of *mathema* and *tics*, in which *mathema* means to explain and understand the world in order to transcend, manage, and cope with reality so that the members of cultural groups can survive and thrive while *tics* refers to the techniques such as *counting, ordering, sorting, measuring, weighing, ciphering, classifying, inferring,* and *modeling*. The *mathema* develops the *tics* within the context of *ethnos* because it consists of daily challenges faced by individuals, larger problems faced by humanity, and endeavors of humans to create a meaningful world. The search for solutions for specific problems that help the development of mathematics is always imbedded in a cultural context (D'Ambrosio, 2001). In this regard, in order to understand how mathematics (tic*s*) is created, it is necessary to understand the problems (*mathema*) that precipitate it by considering the cultural context (*ethnos*) that drives them. This is the main objective of ethnomodeling.

## PLACING AN ETHNOMODELING PERSPECTIVE INTO THE MATHEMATICS CURRICULUM

Since we share our culture with others in our own cultural group and communicate our culture to the members of other cultural groups in an ever-evolving response to the circumstances and challenges of our worlds, we engage our world through the manufacture of artifacts, the practice of behaviors, and the development and adherence to values and beliefs. In this regard, culture is defined as the ideations, symbols, behaviors, values, knowledge, and beliefs that are shared by a community. However, the essence of a culture is not its artifacts, tools, or other tangible cultural elements, but the way or ways in which the members of cultural groups interpret, use, and perceive them. Cultural artifacts may be used in different cultures in very different ways as well as for different purposes. According to this perspective, most of the interesting problems are practically solved using different kinds of heuristics as well as different types of cultural artifacts. On the other hand, culture may be defined as a cultural group's program for surviving in, and adapting to, its environment. In this regard, the cultural program consists of knowledge, ideas, concepts, values, beliefs, symbols, and interpretations shared by the members of cultural groups through communication that help them to make sense out of the world around them. García Coll et al. (1996) propose the concept of adaptive culture to describe how distinct cultures, in response to historical and current demands such as societal racism, classism, and sexism develop their own "goals, values, and attitudes that differ from the dominant culture" (p. 1896).

In the educational context, students proceed through a series of cognitive, affective, and social processes; unique ecological circumstances faced by students from diverse cultural groups result in developmental adaptations that may be necessary for the cultural group's survival and maintenance of self-esteem. It is important to understand the context in which cultural practices, belief, technique, ability, and competency have developed because they affect the ways in which students develop cognitively, affectively, and socially. Notions of culture are always a concern of educational policymakers. It has already been stated that school curriculum has been described as one version of cultural training. This poses serious structural demands on curriculum policy as well as its practice. The central questions in educational policy and practice concerning mathematics curriculum have surrounded the development of a curriculum that caters equally for a diverse student population.

What is different from the traditional view of modeling in the mathematics curriculum is how we define ethnomodeling as a combination of:

- The organized structures and models used to represent information (data structures).
- The ways of manipulating the organized information (algorithms).
- The mechanical and linguistic realizations of these ways, models, and structures as well as their application in distinct cultural groups and society.

Rather than changing the content of science itself, one of the goals ethnomodeling is to concentrate on the form, that is, the outward appearance of modeling. In other words, the aim of ethnomodeling is not to question the foundations of modeling but concentrate on the creative way mathematical ideas and concepts are *developed, presented, acquired*, and *accumulated*. We believe that instead of being another paradigm itself, the study of ethnomodeling aims at encouraging the search for novel ideas (creativity), their examination, adoption, and application to solve problems faced daily by the members of any cultural group.

## ETHNOMATHEMATICS AND MATHEMATICAL MODELING

Historically, models that arise from reality have been the first paths towards providing abstractions of mathematical concepts. Ethnomathematics that uses the manipulations of models of reality and modeling as a strategy of mathematical education uses the codifications provided by others in place of the formal language of academic mathematics. Within this context,[5] mathematical modeling is a methodology that is closer to an ethnomathematics program. Ethnomathematics may be defined as the intersection between cultural anthropology and institutional mathematics, which utilizes mathematical modeling to interpret, analyze, explain, and solve real world problems or mathematize existing phenomena.

Investigations in modeling have been found to be useful in the translation of ethnomathematical contexts by numerous scholars in Latin America[6] who study the mathematical practices and ideas found in diverse traditions and contexts. These studies also become an important tool used to describe and solve problems arising from specific systems such as cultural, economic, political, social, environmental, which brings with it numerous advantages to mathematics learning.

Considering ethnomodeling as the intersection of cultural anthropology, ethnomathematics, and mathematical modeling (Fig. 8.2) "[u]sing ethnomodeling as a tool towards pedagogical action of the ethnomathematics program, students have been shown to learn how to find and work with authentic situations and real-life problems" (Rosa & Orey, 2010b, p. 60). Outside of an ethnomathematics related research paradigm, it is known that many scientists search for mathematical models that can translate their deepening understanding of both real-world situations and diverse cultural contexts.

## ETHNOMODELING

Ethnomodeling is the process of elaboration of problems and questions that grow from real situations that form an image or sense of an idealized version of the *mathema*. The focus of this perspective essentially forms a critical analysis of the generation and production of knowledge (creativity), and forms an intellectual process for its production, the social mechanisms of institutionalization of knowledge (academics), and its transmission (education). From this perspective, the holistic disposition aims at creating an understanding of all components of the system as well as the interrelationships among them.

The use of modeling as pedagogical action for an ethnomathematics program values previous knowledge and traditions by developing student capacity to assess and translate the process by elaborating a mathematical model in its different applications. This can be done by starting with the social context, reality, and interests of the students and not by enforcing a set of external values and curriculum without context or meaning for the learner. This process may be characterized as "ethnomodeling" (Bassanezi, 2002), and ethnomathematics then is "practiced and elaborated by different cultural groups … [that] involves the

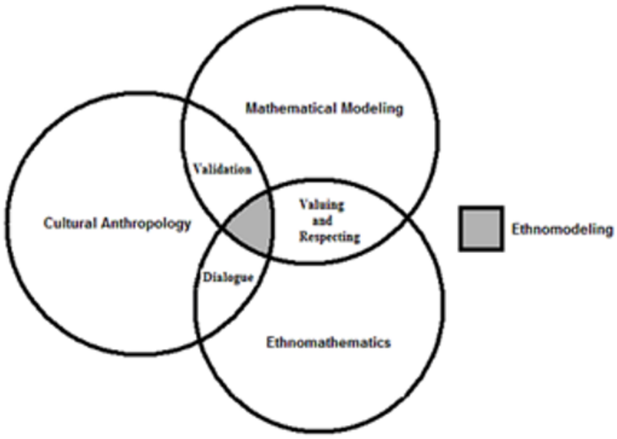



**Fig. 8.2** Ethnomodeling as an intersection of three research fields

mathematical practices … present in diverse situations in the daily lives of members of these … groups" (p. 208).

In considering ethnomodeling as a tool to uncover and study ethnomathematics, teaching is much more than the transfer of knowledge because teaching becomes an activity that introduces the creation of knowledge (Freire, 1998). In our opinion, it is necessary for school curriculum to translate the interpretations and contributions of ethnomathematical knowledge into systemized mathematics because students will be able to analyze the connection between both traditional and non-traditional learning settings. Ethnomodeling offers a tool for developing a cultural approach in developing mathematics abilities in learners, and aims at developing skills in learners for observing mathematical phenomena that have their roots in distinct cultural settings.

The results may then lead to new viewpoints into mathematics education, which can be used to improve the cultural sensitivity in teaching modeling. The new viewpoints, thus generated, clearly benefit Eurocentric[7] mainstream education, in addition to promoting academic competence of learners from different culture. We characterize this approach as the transformative pedagogical nature of ethnomodeling because of its emphasis on diversity of knowledge, especially mathematics and science that are typically believed to be acultural. From our standpoint, ethnomodeling can be seen as an active force that impacts mathematics education and also studies a dynamic subject of change. In other words, ethnomodeling arises from the culture and adapts to changes in the culture.

## EXAMPLES OF ETHNOMODELING

Mathematical modeling uses mathematics as a language for understanding, simplification, and resolution of real-world problems and activities. Data gleaned from these studies are used to make forecasts and modifications pertaining to the objects initially studied. In this regard, one of the traditional definitions of a mathematical model is a body of symbols and mathematical relationships that represent the studied object, which is composed of a system of equations or inequalities, algebraic expressions, etc. that are obtained through the establishment of a relationship between considered essential variables of analyzed phenomena (Bassanezzi, 2002). It is the systematic study of algorithmic processes, theory, analysis, design, efficiency, implementation, and application that describe and transform information. This definition of the Western mathematical modeling includes all data structures, which form a part of both *theory* and *design*; algorithms, which deal with analysis and efficiency; mechanical and linguistic realizations, which deal with implementation; and applications that naturally apply the mathematical ideas and concepts to solve problems.

Academic mathematics stands to benefit by coming to value a wider diversity of mathematical conceptions. For example, the importance of a non-traditional view on mathematics has a bearing on the emergence of the new types of problems related to artificial intelligence. A characteristic of these new problems is that they cannot be solved using syllogistic, that is, classical Aristotelian logic, but need multivalued logic, often called *fuzzy logic*, which is the logic that underlies inexact or approximate reasoning (Zadeh, 1984). Multivalued logic is used in attempts to formalize human-like processes that are culturally bound. The Hindu, Chinese and Japanese cultures have contributed to the development of fuzzy logic more than Western science because, in these cultures, there is a greater acceptance of a truth-value that is neither perfect truth nor perfect falsehood (Zadeh, 1984). Some ethnomathematical examples may naturally come across as having a mathematical modeling methodology (D'Ambrosio, 2002). In the 1989–1990 school year, a group of Brazilian teachers studied the cultivation of wines brought to Southern Brazil by Italian immigrants in the early twentieth century. This was investigated because the cultivation of wines is linked with the farming activity of the members of a certain cultural group from that region in Brazil. This wine case study is an excellent example of the connection between ethnomathematics and mathematical modeling through ethnomodeling.

## DEFINITION OF ETHNOMODELS

In general, a model is a representation of an idea, a concept, an object, or a phenomenon (Gilbert, Boulter, & Emmer, 2000). We, however, define ethnomodels as cultural models that are pedagogical tools to facilitate comprehension of systems that are taken from the reality of the said cultural groups. Thus, ethnomodels can be considered as external representations that are precise and consistent with the scientific and mathematical knowledge that is socially constructed and shared by members of specific cultural groups. The primary objective for the elaboration of ethnomodels is then to *translate* the mathematical ideas, concepts, and practices developed by the members of distinct and diverse cultural groups. There follow a few examples of ethnomodels.

*Measuring Land*

Knijnik (1996) reported the mathematical thinking underlying demarcation of land by the members of the Landless Peoples' Movement[8] (Movimento dos Sem Terra – MST) of Southern Brazil. The activity on demarcation of land activity is about the method of *cubação* of the land, which is a traditional mathematical practice of the members of the movement. Flemming et al. (2005) defined the term *cubação* of the land as the solution of "problems of the measurement of land using diverse shapes" (p. 41). Thus, the practice of *cubação* of the land as a pedagogical proposal to elaborate activities for the teaching and learning of mathematics shows the importance of the contextualization of problems in the learning environment of ethnomodeling through the elaboration of ethnomodels.

Knijnik (1996), for example, presented the following problem to the landless people to calculate the area of quadrilaterals (Fig. 8.3). The mathematical knowledge of the landless people can be represented by a model that transforms "the shape of the given land into a [rectangle] of 138 meters x 102 meter with an area of 14076 square meters" (Fig. 8.4).

The model of this mathematical practice can be explained by the following ethnomodel:

– Transform the shape of the irregular quadrilateral into a rectangle whose area can be easily determined with the formula $A = b \cdot h$.

– Determine the dimensions of the rectangle by calculating the mean of each pair of opposite sides of the irregular quadrilateral. Base = (152 + 124)/2 = 138 m. Height = (114 + 90)/2 = 102 m.

– In order to determine the area of this irregular quadrilateral, it is necessary to determine the area of the rectangle. $A = b \cdot h = 138 \cdot 102 = 14076 \text{ m}^2$.

Regarding this problem, there is another ethnomodel proceeding from the mathematical knowledge of the landless people. Flemming et al. (2005) reported that the irregular shaped quadrilateral parcel presented in this example can also be transformed in to "a square with sides of 120 meters, therefore with an area of 14400 square meters" (p. 42). In this regard, it is possible to observe that adding the lengths of the sides of the quadrilateral, and then dividing it by four, the number of sides, produces the value 120.

Flemming, Flemming Luz and Collaço de Mello (2005) confirmed Bassanezi's position that a model is efficient when we realize that we are only working with approximations of reality. Mathematically speaking, both methods presented by the members of the Landless People's Movement were an approximated calculation of the area an irregular quadrilateral that fully satisfied the necessities and the life histories of the participants of this specific cultural group.

*The Symmetrical Freedom Quilts*

Rosa and Orey (2009) affirmed that a quilt theme is a powerful way to integrate mathematics, art, history, and reading in an interdisciplinary approach. Using the

*Calculate the area of the land with a quadrilateral shape with sides 114, 152, 90, 124 meters. (p. 42)*

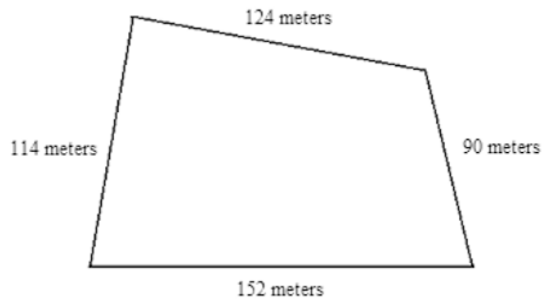



**Fig. 8.3** Problem on land demaracation for the landless people

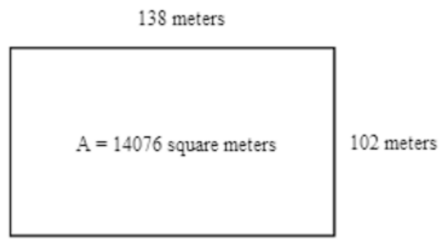



**Fig. 8.4** Solution proposed by the landless people

historical context of *Underground Railroad*[9] their lesson plans combined an ethnomathematical-historical perspective that allowed teachers to develop classroom activities and projects for students to better understand geometry, especially concepts of symmetry and transformations through ethnomodeling. One of the objectives of this project is to stimulate student's creativity and interest, because quilts may be considered as cultural and mathematical expressions of student's daily life. Thus, *Symmetrical Freedom Quilts* may be considered as links between mathematics, history, ethnomathematics, and the very tactile art of quilting.

Making quilt blocks is an intellectually engaging way to explore the concepts of symmetry. As quilts are typically made by piecing a number of square blocks (usually 4 and 9 blocks patches),[10] with each smaller block made with arrangement of triangles in a particular way- the craft lends itself readily to the application of symmetry. The Freedom Shoo Fly quilt shows how its blocks are symmetrical (Fig. 8.5).

*Shoo Fly*[11] is one of the simplest traditional Freedom Quilt patterns. Although *Shoo Fly* is a basic pattern, its versatility provides quilters with some wonderful opportunities for creative use of colors, fabrics, and stitching. *Shoo Fly* may be adapted to a variety of sizes. Blocks often measure 9 x 9, but variations such as 10 x 10 and 12 x 12 may also be used.

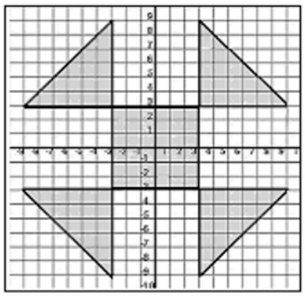


**Fig. 8.5** The Freedom Shoo Fly pattern.

A rotation turns the figure through an angle about a fixed point, called the center. The center of rotation is assumed to be the origin of the x-y coordinate system. A positive angle of rotation turns the figure counterclockwise, and a negative angle of rotation turns the figure in a clockwise direction.

Rotation is a transformation that is present in the *Shoo Fly* quilt block because it moves every point 90° counterclockwise around the origin of the x-y coordinate system. The mapping of this rotation is $R_{90°}(x,y) = (-y,x)$. In so doing, the coordinates of point *A* in its rotation around the x-y coordinate system are:

$$R_{90°}A(9,3) = A'(-3,9)$$
$$R_{90°}A'(-3,9) = A''(-9,-3)$$
$$R_{90°}A''(-9,-3) = A'''(3,-9)$$
$$R_{90°}A'''(3,-9) = (9,3)$$

Fig. 8.6 shows the rotation of point *A* around the x-y coordinate system.

The other mappings for rotation are:

– Rotation of 180°, that is, $R_{180°}(x,y) = (-x,-y)$. This is the same as the reflection in the origin of the x-y coordinate system.
– Rotation of 270°, that is, $R_{270°}(x,y) = (y,-x)$.

A rotation creates a figure that is congruent to the original figure and preserves distance (isometry) and orientation (direct isometry).

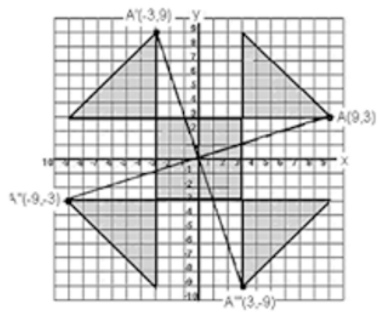



**Fig. 8.6** Rotational transformation of Freedom Shoo Fly pattern

*Modeling the Tipi*

Spatial geometry is inherent in the shape of the tipi and it was used to recall, indeed symbolize, the universe in which the Plains Peoples lived. The word *tipi* from the Sioux language refers to a conical skin tent or dwelling common among the prairie peoples. According to Orey (2000), the majority of Sioux tribes use the tripod foundation or three-pole foundation because it is stronger and offers a more firm foundation than a

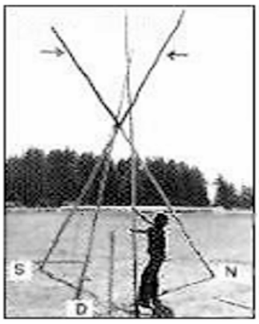
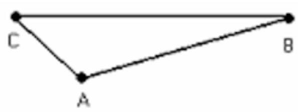


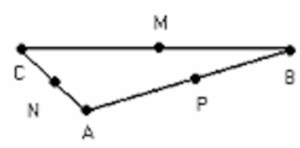

**Fig. 8.7** Setting up a tipi

quadripodal or four-pole tip foundation. An ethnomodel explains why a tripod is more flexible than a quadripod, or a four-legged structure. In this regard, imagine three points, A, B, and C that are not collinear. There are an infinite number of planes that pass through points A and B that contain the straight line AB. Only one of these planes also passes through point C therefore we can say that three points are not collinear if they determine one plane. This means that these non-collinear points exist on one plane and that three collinear points do not determine the only plane. This means that given any three non-collinear points, there is only one plane to which exist these same three points. This can be explained using the postulate for the determination of a plane. In other words, given any three non-collinear points, there is only one plane to which exists these same three points. For example, in the 4-legged table, it has the possibility of the extremity of one of the legs that do not belong to the same plane. A table that has 3 legs, therefore, is always balanced. Similar to a three-legged table, the structure of the tipi appears to be perfectly adapted for the harsh environment in which it was used. It had the advantage of providing a stabile structure, was lightweight and portable (Fig. 8.7).

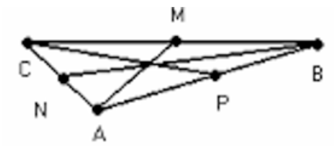
At the same time it withstood the prevailing winds and extremely variable weather of this region. Let us look at this information mathematically. The base formed by the tripod is ΔABC.




The midpoints of each of the sides of ΔABC are points M, N, and P.




It is possible to match each vertex of ∆ABC to the midpoint of each opposite sides that gives us the straight lines AM, BN, and CP.




These straight lines form three medians, which are the straight lines connecting the midpoint of each opposite side of the triangle and its vertex. The medians intersect at only one point called centroid. Archimedes demonstrated that medians of a triangle meet at its balance point or center of gravity, which is the centroid of the triangle. Native Americans place their fire and altar at this point in the tipi. Cartographers call this point the geographic center. The tipi cover is folded in half and the poles are laid together before tying them to form the tripodal or quadripodal frame, which forms the foundational base for the structure.

*Some Considerations about Ethnomodeling*

Ethnomodeling seems to be important especially in new fields of research such as artificial intelligence and fuzzy logic. However, ethnomodeling has been given a chance only in new research or it has led to new fields of research. Current academic-Western science does not give ethnomodeling of non-Western cultures much chance to introduce new views into old themes. Different cultures can and do contribute to the development of mathematical concepts and ideas and enrich them in the field of Mathematics Education. In addition to the development of mathematical modeling and education, ethnomodeling holds another equally important objective as D'Ambrosio (1997) recognizes that ethnomathematics has the common goal of equity and dignity. In this regard, the study of ethnomodeling may encourage the ethics of respect, solidarity, and co-operation across cultures.

## FINAL CONSIDERATIONS

The purpose of this chapter was to justify the research of cultural perspectives in ethnomathematics through ethnomodeling. The claim was that contemporary mathematics is dominantly Eurocentric and that this Eurocentrism has facilitated a divide that hinders the prospects of mathematics education in non-Western cultures. The motivation for a cultural approach was the assumption that adopting cultural perspectives into mathematical modeling would bring local issues into global discussion and thus help in meeting local needs.

We propose a sociocultural theory for mathematical modeling, suggesting that mathematics education is a social and cultural product and that there exist a dialectic relationship between mathematics, culture, and society. This claim is supported by the social constructivist theories in sociology and educational psychology as well as the idea of how scientific revolutions are structured. Moreover, we have presented an idea that mathematics is dominated by preferences of the West, and that this prevailing Eurocentrism poses a problem in mathematics education for non-Western cultures.

From these grounds, we have presented definitions for ethnomodeling and for the study of ethnomodeling as follows: *Ethnomodeling* stands for mathematical ideas and concepts that have their roots within a culture. The *study of* ethnomodeling is defined as the study of mathematical phenomena within a culture. Ethnomodeling differs from the traditional definition of modeling in that, whereas the traditional view considers the foundations of mathematics education as constant and applicable everywhere as such, the study of ethnomodeling takes the position that mathematics education is a social construction and thus culturally bound.

All variations of ethnomodeling have developed to meet the needs of a certain culture, which present their own consequences. First, the impact that culture has on mathematics represents a feedback loop whereby mathematics education is changing it and transforming it and culture is transformed by mathematics and science. In this process mathematics education and mathematical ideas that originate by members of cultural groups are constantly reshaping one another. This can be perceived as a transformation of both the native mathematical system and mathematics education through a cultural and global dynamic perspective, and according to D'Ambrosio (1990) this is an example of cultural dynamism. Second, there does not exist a standard for comparison between cultural groups, since every measure is subjective, and would only measure how well modeling works in a culture where it is applied. For example, Western standards such as efficiency and exactness are useful only in a very limited set of problems, and they may become insignificant if the legitimate problem field changes. Third, mathematics education can no longer ignore cultural considerations. The educators have to take into account the cultural and philosophical background of a society. Different cultures may have different perceptions of time and space, logic, problem solving methods, society, values, or which questions are considered legitimate.

Adopting ethnomodeling as pedagogical action for teaching and learning mathematics could serve several purposes. First of all, it presents a practical method where learners can learn to recognize mathematics in the context of their own local context. This would definitely produce educators, students, and researchers that have fresh, novel views on the issues of mathematics, science and engineering education.

The hardest part in the adoption of ethnomodeling is the pervasive view of the Western philosophy as the crowning jewel of scientific evolution. Current formal science is seen as good and final as such, which, in the light of history of science, is clearly an inaccurate conclusion. Much more probable is that future scientific revolutions will turn the direction of mathematics education towards directions that are unexpected, and so far even unheard of.

Ethnomodeling may be considered a young discipline and is still looking for its identity. However, this field is rapidly evolving in directions that are changing and difficult to anticipate. The boundaries of ethnomodeling are not strictly defined. Change and reform are actually a part of the nature of the scientific paradigm. This is why we believe that recognizing ethnomodeling does not give the mathematical methods of other cultural groups a Western stamp of approval, but recognizes that they offer important alternatives to problem solving and have always have been important to the development of humanity's overall mathematical knowledge.

Any study of ethnomathematics and mathematical modeling represents a powerful means for validating a student's real life experience, and gives them the tools to become critical participants in society. In so doing, educators should be empowered to analyse the role of what Borba (1990) refers to as a student's *ethnoknowledge*[12] in the mathematics classroom. There is no doubt that there exists a need to create a new role in mathematics instruction that empowers students to understand power and oppression more critically by considering the effect of culture on mathematical knowledge by working with students to uncover distorted and hidden history of mathematical knowledge.

This perspective forms the basis for significant contributions of a Freirean-based ethnomathematical perspective in re-conceiving the discipline of mathematics and in a pedagogical practice. The use of Freire's (1970) dialogical methodology is seen as essential in developing the curricular praxis of ethnomodeling by investigating the ethnomathematics of a culture in constructing a curriculum with people from other cultures to create curricula that enable the enrichment for all people's knowledge of mathematics. Seen in this context, we would like to broaden the discussion of possibilities for the inclusion of ethnomathematics and mathematical modeling perspectives that respect the social and cultural diversity of all people with guarantees for the development of understanding our differences through dialogue and respect. This is how ethnomodeling can empower students in this century against all kinds of domination and oppression.

## NOTES

[1] Mathematization is a process in which individuals from different cultural groups come up with different mathematical tools that can help them to organize, analyze, comprehend, understand, and solve specific problems located in the context of their real-life situation. These tools allow them to identify and describe a specific mathematical idea or practice in a general context by schematizing, formulating, and visualizing a problem in different ways, discovering relations, discovering regularities, and transferring a real world problem to a mathematical idea through mathematization.

[2] D'Ambrosio (1990) stated that a holistic context consists essentially of a critical analysis of the generation (creativity) of knowledge, and the intellectual process of its production. The focus on history analyzes the social mechanism and institutionalization of knowledge (academics), and its transmission through the educational process.

[3] A system is a part of reality considered integrally. It is a set of components taken from the reality, which analyses components and interrelationships between these components (D'Ambrosio, 1990).

[4] Ascher and Ascher (1986) defined ethnomathematics as the study of the mathematical ideas of non-literate people. In this regard, we prefer to use the term *non-literate* instead of *primitive*, *illiterate* or *uneducated* for a number of important reasons. First, we use it to emphasize the idea that primitive people are not uneducated, but that they use different ways of transferring knowledge from generation to generation (Negroponte, 1995). Second, we use it to disassociate from the evolutionary-biased term *primitive*. The presumption about the abilities of non-literate people or people whose thinking does not follow the logical rules of Western science is usually that they are "simpleminded, childlike, illogical, of lesser intelligence, or incapable of analytic thought" (Ascher & Ascher, 1986).

[5] For details, see D'Ambrosio (1993) and Bassanezi (2002).

[6] For details, see Bassanezi (2002), Biembengut (2000), Rosa and Orey (2007), and Rios (2000).

[7] Eurocentrism or privileging Eurocentric ideas has been discussed by many scholars such as Banks (1999), Joseph (1997), and McCarthy (1998).

[8] The Landless Peoples Movement (Movimento dos Sem Terra – MST) is one of the most important Brazilian social movements. It is a national organization, spread throughout 23 of 27 states of the country, involving about seven hundred thousand peasants who strive to achieve land reform and social changes in a country with very deep social inequalities. For details, see Knijnik (1996) and Monteiro (1998).

[9] The term *Underground Railroad* has come to us from a story of a farmer chasing a runaway who testified that the slave vanished on some kind of Underground Railroad. It was used to describe the network of abolitionists and safe houses that helped slaves escape to Ohio and Canada. Safe houses along the way were known as *stations*, those who guided the escapees were called *conductors* and the runaways themselves were called *passengers*. For details, see Rosa and Orey (2009).

[10] The 4 and 9 patches can be added in order to make different blocks, but not 16 nor 25. It would depend upon how many blocks we use and sizes in a quilt using the basic 4 or 9 patches within the block. Every block has variations, depending on size, fabric used, as well as how the block is turned and how many extra rows are added

[11] Shoo Fly pattern. See http://www.popularpatchwork.com/news/article.asp?a=8043 and http://blackhistory.owensound.ca/quilts.php

[12] Ethnoknowledge is acquired by students in the pedagogical action process of learning mathematics in a culturally relevant educational system. In this process, the discussion between teachers and students about the efficiency and relevance of mathematics in different contexts should permeate instructional activities. The ethnoknowledge that students develop must be compared to their academic mathematical knowledge. In this process, the role of teachers is to help students to develop a critical view of the world by using mathematics.

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