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Problem Solving

Problem solving is an interesting area, and there exists a lot of psychological research, most notably, for example, the studies that the Gestalt psychologist Wolfgang Köhler conducted with chimpanzees. Most psychologists treat the phenomenon as a 'mental process' of 'problem finding', 'problem shaping', and 'solution finding'. Every now and then we can find some 'insight' thrown into the mix of concepts to explain a feature of the phenomenon, which occurs precisely when psychologists cannot really explain what has happened with their representational models. However, for a long time I felt that the application of these theories is rather limited because it misses essential aspects of the process. For example, in 8 years of studying scientists at work in their laboratories and in field research, I never found the kind of processes of 'hypothesis formation' and 'reasoning' that any popular textbook on cognitive psychology will feature. Most notable, for me, is a brief conversation that I have had with Marlene Scardamalia, an applied cognitive scientist who has done a lot of research on writing. I once spend some time in her research lab and field sites when she admitted to me that despite having conducted research on the topic of writing for two decades, her model could not describe my own writing process. At the time, I thought that this was not boding well for her model. Perhaps these models on writing and problem solving are useful when gross reductions are made from the original living-lived experience of dealing with troublesome situations – which for cognitive psychologists are of the simplest kind, like the Tower of Hanoi. I know that these models do not work well, especially after having observed experienced expert scientists in troublesome situations. What I observed led me to write an article answering the question 'What do scientists do when they do not know what they are doing?' Rather than applying rational models of problem solving, these scientists were 'groping in the dark', doing this and that precisely because they could not know what a proper next move would be.

What gross simplification looks like can be seen from Jean Lave's research on the use of fractions in 'best-buy problems' in supermarkets, outside supermarkets with selected items, and in paper-and-pencil format. In the supermarket, participants use many different, legitimate, and appropriate strategies to find out whether an item is a best buy or not. In paper and pencil format – where the best buy is reduced to a question such as 'is a 400-gram pack of cereal for $1.58 a better buy than a package (of the same or different brand) offered at $3.18 for a $750-gram package?' – some processing of numbers is inherently asked for. But in the supermarket while doing the weekly shopping, the type of brand, package size, shelf life, use-best-before dates, rate of consumption at home, the degree to which different family members like the brand, and many other considerations come into play, which mediate the ultimate cost. (Spoiled food means the item costs more in the long run.) The very contexts and constraints that make problem solving interesting, highly complex, but also suitable to innovative approaches are eliminated beforehand when researchers ask participants to do paper-and-pencil tasks. In fact, the gross reduction may go even further by limiting those doing paper-and-pencil tasks to paper and pencil, while disallowing the use of pocket calculators or other devices or methods that people use in their everyday settings.[1]

In schools and at universities, problem solving often is not asked for, because students are made to copy notes from the chalkboard and memorize it for examinations. This has effects on what and how people go about dealing with the problematic situations that they are asked to solve or have agreed to participate in. For example, in one of my studies, eighth-grade students had worked on a 10-week unit of ecology, where they framed questions that they wanted to answer about a 20–30-m^2 plot of wooded land on the school property. At the end of the study, we tested their competency to analyze data that another eighth grader had generated. Later we asked science teachers in training – all had a bachelors or Masters degree in science – to respond to the same task. It turns out that there were a statistically significant higher number of mathematically (statistically) more advanced approaches within the group of eighth-grade students than among the university students. We concluded that the eighth graders simply were more familiar with the open nature of the problematic task than the university students; we did not conclude that they were smarter or better at problem solving and data analysis.

There is a lot that we can learn about problem solving when we abandon the traditional (psychological) discourse and investigate what the experience is like and attempt to understand the invariants in problem solving. What I am interested in by working through the two situations in this chapter – both of which I had found problematic enough to engage with – are not the particulars, that is, that I used a mathematical software in one instance or a motorized screwdriver in the second instance. Rather, I am interested in working out the behavioral invariants that teach us something about problem solving *generally*. This means that we need to systematically interrogate these particular instances of problem solving presented here to extract the general, invariant properties. These, then, will assist us in describing other problem-solving situations that we might encounter elsewhere and, more importantly, that others encounter in the diverse situations of life. It is therefore unimportant whether the solution 'I' have come up with is the 'right' one, the only one, or the most economical one. What the first-person approach attempts to unearth and excavate are the invariants in the forms of phenomenalization that occur in the process.

[1] One method consists in asking other people: my wife will ask me whether the 750-gram yoghurt on sale is a better deal than the same yogurt in the 1.75-kilogram package. But asking another person is disallowed in the testing situation, though a perfectly legitimate approach in the supermarket.

In this chapter, I show the first-person method at work in two very different contexts. The first pertains to two mathematical puzzles that I was given. It became a problem because of my engagement with them. It is therefore of interest to our present purposes of describing invariants of problem solving; because there are two of these 'problems', we can actually ascertain the behavioral invariants across the two contexts. In presenting this, I actually revisit a problem that I have written about repeatedly but without truly working out the possibilities of a first-person approach. Moreover, in other instances where I describe this 'problem', I am concerned with other aspects of this episode, whereas in this chapter, the methodical aspect constitutes the important matter at hand. The other context pertains to an issue that I had with a pocket door in my home, which required what turned out to be a difficult form of repair.

Methodical Note In the instances presented here, I had kept ample notes and drawings or photographs to document what I 'file' under keywords such as 'phenomenology of learning', 'phenomenology of problem solving', or 'phenomenology of invention'. There tend to be two places where I keep such materials. The first is in my research notebooks, which contain dated pages and keywords above the heading line below which I keep my notes. The other place is on my computer hard drive, where I create folders the name of which index its contents. Like the notebooks, the folders are organized in a historical order, because I tend to remember when and where events have happened and when and where I write something. Historical organization, therefore, works for me. For example, when looking for the materials for the second case study, I remembered approximately when the event had occurred and that I had kept electronic images. Because I also have an approximate idea about when I purchased my different cameras, I quickly located the images in a folder labeled 'phenomenology' and, from the date the photos were saved, I could quickly identify the appropriate research notebook and find the entry under the same data as the photographs.

School Mathematics 'Problems'

Many puzzles constitute ill-defined problems – unless one is already familiar with the particular type of puzzle of which this new one is an instant. Interesting puzzles are those that do not come with *holds*, where I understand 'hold' from a first-person perspective, as a form of relation that I have with the situation at hand. I am thinking about a foothold or handhold that I would have while hiking in the mountains over difficult terrain. In this section, the experiences of two such instants are featured together with a first-person analysis. Such school 'problems' are designed to weed out those students who can versus those who cannot do them, that is, they have an evaluation scheme ready against which any individual achievement – and even how it has been done – will be judged. Just writing an answer tends to be illegitimate and students generally are asked 'to show the way how they got their answer'. However, some of these 'problems' may be taken up much in the same way that crossword puzzles, Sudoku puzzles, and the like are taken up, that is, in an 'authentic' way, as something truly problematic. This 'being taken up' is an interesting phenomenon in its own right because there is some affection, some form of allure that makes us pick up a 'puzzle' even though we do not have any guarantee of success. This affection cannot be understood solely by looking at the individual, as there is a pull that an object of consciousness exercises on the person (Husserl 2001). In the following, I illustrate the first-person approach at work in the context of two mathematics-related puzzles. In this situation, it is not the engagement with the mathematics problem that is invariant and generalizes to others, but the initial allure and engagement, which becomes an extended engagement until some point where it comes to a halt because some form of satisfaction is achieved. Again, coming to a halt and the particular interactions with the emotional response and state is more likely to generalize than being un/successful or the particular feeling that the person has. Because their affective pull was so great that they really grabbed me, these puzzles became problematic in their own right, though the precise nature of the problematic would have to be established. What is it – from the perspective of the acting subject – that is problematic in a particular situation? Thus, it may not actually be what the designer of the 'problem' wanted it to be.

On Hospitals and Birth of Boys

I have had an opportunity to investigate puzzle solving some time ago, when a graduate student of mine gave me what turned out to be a mathematical puzzle. At the time I do not know how to do it or how to go about it in the way teachers know how the word puzzles they assign have to be solved. I do not even know initially how to start and what kind of mathematics, if any, would assist me in answering the question at the end of the text. That is, the way the puzzle is framed tells me something about the kind of situation in which one might be asked to respond to something like it and that there likely exists a standard or standardized 'solution' against which I would be judged, if the situation were accordingly (e.g., in school, in a psychological laboratory). We already note a first invariant: There is a sense about the kind of situation that we can appropriately locate and of which we can provide descriptions as likely contextual factors. In the present instance, the way in which the story about the two hospitals is told and the question that follows it – 'In which of the two hospitals were there more such days?' – allows a characterization of the 'problem' to be of a certain kind. This kind of 'problem' differs significantly from the problematic situations that we face in everyday situations, such as the one described in the next section, where the problem itself is at issue – i.e., we do not even know what the problem is let alone how to solve it – and where existing solutions may define the nature of the problem.

After beginning to think about the contents of the text, I become so intensely absorbed (as I realized afterward) that I understand only subsequently what has happened to me. This is the kind of absorption I describe in chapter 8 and articulate methodological issues concerning its investigation. When I eventually do reflect on the event, I immediately notice the physical metaphors that I have used to describe

Fig. 11.1 Distribution of births in two hospitals with different numbers of average births per day. The drawing constituted a first hold.

what has happened to me. Although the puzzle phenomenalizes itself to me first in the form of a text, and although it appears to require some form of mathematics, my sense at the time is that I am exploring a space, and in my doing – using a computer modeling software, making diagrams, writing formulas and watching their results – it is as if I am exploring a physical terrain, seeking and creating holds, thereby creating space for me within which to operate, and eventually coming up with a reasonable answer. In the following, I narrate the events in the manner of this metaphor, which may actually differ for different individuals. This is so because 'space' and movement through space is a particular way in which I have experienced complex situations, such as making sense of data in research situations. Here, too, the entire dataset pertaining to a project and the associated literature constitute for me something like a library in which I move about to pull what I need off the shelves and from drawers. But let us turn to the text that my graduate student has handed to me, and which goes something like this:

> In a certain town there are two hospitals, a small one in which there are, on the average, about 15 births a day and a big one in which there are, on average, about 45 births a day. The likelihood of giving birth to a boy is about 50%. (Nevertheless, there were days on which more than 50% of the babies born were boys, and there were days when fewer than 50% were boys.) In the small hospital a record has been kept during the year of the days in which the total number of boys born was greater than 9, which represents more than 60% of the total births in the small hospital. In the big hospital, they have kept a record during the year of the days in which there were more than 27 boys born, which represents more than 60% of the births. In which of the two hospitals were there more such days? (a) In the big hospital there were more days recorded where more than 60% boys were born. (b) In the small town there were more days recorded where more than 60% boys were born. Or (c), the number of days for which more than 60% boys were born was equal in the two hospitals.

After reading the text, the image of two distributions emerges into my mind, an image that I quickly sketch on a piece of paper (Fig. 11.1). It represents the number of births in each of the two hospitals, which average 15 and 45. I have no idea at the time whether this diagram is or will be of any help, but it is one of the things that allow me to explore the implications of the text. When I check my watch, I realize that it is late and that I must get on my bicycle to ride home.

We already note in this beginning that there is a particular way in which the 'problem' is cast in my actions. The textual 'there are, on the average, about 15 births a day' is translated into graphical form with a particular form of curve – insiders know it as 'a Gaussian' – that is symmetrical around the 'average' of 15, clearly marked by a vertical line. The number of births in the other hospital is similarly represented. Now we can probably assume that the form of representation, the diagram, is not a universal response to the original text. This transformation is very likely a particular response, though a culturally possible and legitimate one; but in itself it will not be an invariant. Whereas we might observe some form of 'transformation' across the performances of other individuals, the one that has surged into my consciousness is not the one that we would find in every person's response. My background and training in physics, applied mathematics, and statistics may be good candidate reasons for explaining that this diagram rather than some other translation has occurred. Also noteworthy in this context – something that will repeatedly return – is the fact that I cannot give any reason why this diagram emerged into my consciousness rather than something else. The text in itself does not provide a foot- or handhold for this approach. It does not ask for a visual representation, diagram, or anything else. I do not know why the particular (Gaussian) curves forced themselves upon me, because other drawings that might have possibly emerged might have just included the vertical lines marked with the average number of births in each. In fact, when I checked Google using the beginning sentence of the text, I found the 'problem' with varying numbers posed in about the same way. The solutions, if they are offered, do not (tend to) include diagrams.

Traditional psychologists might want to suggest that there was an association made between the term 'on average' and the Gaussian curves. But then the question has to be who made this association? I know it was not I. I know that I did not make anything. An image imposed itself upon me. If psychologists want to create an appropriate theory to explain the experience I have had then they need to explain how it is that the images *gives itself* to me rather than I, the acting subject, constructing it or pulling it off the shelves of my long-term memory.

We note as an intermediate result: (a) there is a particular *manner in which the situation description is translated into a diagrammatic description, (b) there is a* translation process*, (c) the translation phenomenalizes itself through* givenness *rather than intentional selection from a shelf of strategies or possibilities, and (d) the translation does not appear* as *translation but as another form of the possible*.[2]

On my way home, while riding my bicycle, the puzzle returns. Away from the desk and without pen, another image comes to my mind, this time reducing the number of children I have to deal with: the number of families with three children and the distribution of boys and girls, which I envision in the form of zeros and ones (Fig. 11.2). I think: *So the possibilities of gender distribution in a family with*

[2] As a person who fluently communicates in three languages, I experience moments where I engage in translation, *consciously seeking* equivalents of something that I read in French but want to render in English. But when communicating in French or German, I do not *consciously look* for expressions, even though English is my dominant language. These are two different modes of being.

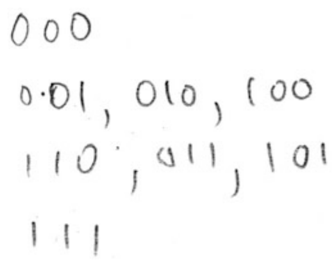

Fig. 11.2 Thinking about the puzzle while being away from my desk, another image comes to my mind, this one of suitable size to be envisioned in mind and operated upon it: The number of boys and girls in families with three children.

three children are three boys and zero girls, then two boys and one girl, one boy and two girls, and zero boys and three girls. In my mind's eye, I generate the image and count – literally, by envisioning the possibilities and counting them off – the number of triplets resulting from the previous operations. There are one, three, three, and one combination(s) in each of the four rows, respectively. That is, the possibility of having two boys and one girl or two girls and one boy is three times as high as having three boys or three girls. The distribution of triplets resembles my earlier paper-based drawing: it is 'like' 'a Gaussian' turned on the side.

Something noteworthy occurs in the preceding description: I write 'the distribution resembles my earlier paper-based drawing' even though I have been visualizing rather than actually drawing an image. I check off this 'mental' image as if I had a real image in front of me. This points to the fact that some aspect involved here is precisely the same as the one that has occurred in the situation with the actual drawing. Some aspect of the visual apparatus involved in looking at a diagram is the same as while generating the diagram virtually are the same. There is an immanent form of knowing that involves the organism – i.e., 'me' – in the same manner in both situations. In chapter 2, we already encounter what the eyes do when they count lines. Here we find in the description that in my mind's eye, I was seeing the distribution and counting. This tells us that visualizing is not something special but rather a form of doing what otherwise is done in the presence of the image. At this point I am reminded of something that I have read: 'the psychological nature of humans – the totality of societal relations, shifted to the inner sphere, having become functions of personality and forms of its structure' (Vygotskij 2005: 1223). That is, reading images has been a societal relation first before it becomes a form and function of individualized thinking. In fact, talking about the images, drawing them in the way they appeared (Fig. 11.2), and reproducing them as part of a description for how I have been going about it shows that there is something general, inherently intelligible and shareable about them that is not singular to me.

Still on my bicycle, I begin another example with the intent to make a comparison with the three-children example. This time, I try six children. As I try envisioning the sequences – {0,0,0,0,0,0,0}, {1,0,0,0,0,0}, {0,1,0,0,0,0} – I realize that the task of envisioning, enumerating, and counting the frequencies in each row is too complex to be held in mind and that I really need to wait until I have paper and pencil available. Here, using a second situation that is similar to the first is a more active selection than that has occurred for the first image. I try six, but do not know why six rather than some other number. The six is given to me, and I cannot provide a reason for this: why not four or five? But I do try only to realize that the size of the resulting situation exceeds what I can deal with then and there on my bicycle. That is, whereas the approach might have worked with a sheet of paper, where I could have listed all possibilities in the manner that I have done for the small hospital, with six children my capacity to imagine all possibilities has been exceeded. I realize this: *If I had anything like a conception of a* 'problem space', *then I would have eliminated the possibility of using six children. The fact that I abandon this case only after working with it for a time tells us that whether a possible move will yield anything at all cannot be established beforehand, as the landscape of the problem becomes available only in an unfolding manner*.

Something else comes to my mind, likely a consequence of the earlier image with the three-children families (Fig. 11.2). That is, this first (fleeting, ephemeral) image has created a hold that now leads me to another thing the implication of which I begin to work out. I thereby create even more holds. The thing I imagine at this instant takes the form of two parentheses enclosing two numbers but, I clearly remember my high school teacher's advice – this is neither a fraction nor a vector, two other and distinct mathematical quantities:

$$\binom{1}{3} = \qquad (11.1)$$

Something has happened here, sharing, in the process of phenomenalization, similarities with what has happened before. The opening sentence of the preceding paragraph already sheds light on the nature of phenomenalization: it is a given. I have noted at the time that 'something else *comes to* my mind'. In this sentence, 'something else' is in the subject position. *It* is the agent that 'comes'. It comes to *me* in the manner someone else might come to visit me. I am the welcoming host in the former as I am in the latter case. The image of this strange notation (11.1) is not the result of my agential search and construction but the result of a process that I denote as *donation*.

After the image has appeared to me, I (consciously) remember having seen it before and also, vaguely, remember that it had to do with probabilities. Since my high school days, I have completely forgotten what this notation means or how to work with it.[3] It is an entity, perhaps a possible hold, but I cannot hold on to it right

[3] I was not doing particularly well in mathematics at the time. Though I became something like an applied mathematician later, I struggled in high school mathematics, frequently unable to visualize and concretize what the mathematics 'really' 'meant'. Some researchers may be interested in researching the kind of frustration I experienced in not being able to 'see', for example, how to figure out the distance between two straight lines in three-dimensional space. But here I am not interested in 'my' 'feelings' unless I can analyze them in a manner that gives rise to invariants that are suitable to describe the possibility of experiences of others as well. What is invariant is the existence of an emotional coloring, which may lie somewhere across the entire spectrum from negative to positive with intermediate states where the emotional nature disappears from consciousness.

then and there on the bicycle to use it for my purposes. I do not pursue thinking about it.

Later that day, having arrived at home, I also have a new resource for creating further holds: my computer and a mathematical modeling program – MathCAD – that I have used with students from the elementary grades to the high school level. The interesting aspect of this program, as probably of other modeling programs as well, is its capacity to 'play around', and getting a response upon acting. I know from experience – though I am not making this salient at the time to think about it but rather go to the computer and work with it – that these action/response cycles allow me to evolve holds, and eventually to the framing of a problem/solution pair that responds to the issue at hand. After I start up MathCAD, I look through its manual to see whether I can find something similar to the image depicted in (the unsolved) equation (11.1). I cannot find anything that resembles the vague image in my mind. However, another image then appears and takes hold in my mind's eye. I play a willing host to it:

$$\frac{1!}{1!(3-1)!} = \qquad (11.2)$$

It is evident from the numbers in this strange equation that it is related to the image of the distribution of boys and girls in three-children families. That is, this image creates a hold, and based on this hold, further possible holds emerge, though some do not appear to be such – e.g., when I cannot find an equivalent to (11.1) in the MathCAD manual. Although I do not remember how this expression (11.2) was used in my high school mathematics class, I do remember all of a sudden that the exclamation mark is denoted by the term *factorial* and it mathematically meant that you had to multiply all integers up to it – for example, 3! = 1 x 2 x 3. I quickly type the fraction into MathCAD and, without even looking or thinking about the keys, simultaneously hold down [⌘][=], which I 'know' without reflecting upon it, recalculates all equations currently visible on the screen. The screen now shows

$$\frac{1!}{1!(3-1)!} = 0.5. \qquad (11.3)$$

In this situation, my action has a result. Typing an expression and then asking the computer to calculate it produces a number, which, qua change, becomes a new possible hold that allows me to come to grips with the expression itself. At least, it is a beginning hold on the expression. Without making thematic what I am doing or the objects that I manipulate, I first change the numerator to 2, then to 3 and each time press the keys [⌘][=] which yield the following two results:

$$\frac{2!}{1!(3-1)!} = 1 \qquad (11.4)$$

$$\frac{3!}{1!(3-1)!} = 3 \qquad (11.5)$$

It then occurs to me – note the passive formulation! – that I have three children and perhaps the '3' in the numerator should stay and I need to change the '1' in the denominator of (11.5). When I do that I obtain 1, 3, 3, and 1 when I use 0, 1, 2, and 3 in place of *n* in the equation

$$\frac{3!}{n!(3-n)!} = . \tag{11.6}$$

In this sequence of events, the new approach 'occurs to me'. I do not know *whence* it has come from. After the fact, it is always possible to generate what looks like a causal explanation. Someone might be tempted to say that the problem lies in (11.3), which is a number that does not appear in the image that originally appears to me (Fig. 11.1). Although the results '1' (11.4) and '3' (11.5) do appear in that image, the kind of symmetry that the figure displays is not obtained in the three equations. Zero – the possibilities to have a boy when a family has three children are 0, 1, 2, or 3 – could have been entered, too, but it would yield the same as 11.3, as 0! is 1. *At best, there is a vague sense that this approach* does not *work.*

I now test equation (11.6) by entering 0, 1, 2, and 3 for 'n', which gives me as results the answers 1, 3, 3, and 1, respectively. Here, then, a hold has appeared to me, allowing to be 'discovered', as much as I have created a hold. The ephemeral image of a fraction and numbers with exclamation marks, the sense of which has escapes me at that moment, and using the number '3' consistent with the three-children family, has yielded a series of results that are consistent with the image I have had while riding home from the university. It is a hold because two different images and processes have yielded the same result, which may be due to an underlying pattern. In a sequence of actions, the nature of which I have not been able to assess at the moment, new structures emerge, and these structures provide new holds.

We can retain this: When it becomes apparent that two different *approaches yield* the same *result, there is a sense of being on the 'right' track. That is, independent of the two different ways I come to use, the* invariant *is that after arriving at the same result, confidence is gained that the* general thinking *underlying the approaches is appropriate.*

I pursue the inquiry, and, encouraged by these latest results, return to the puzzle. I begin to type rapidly:

$$x := 0..15 \qquad y := 0..45$$
$$f(x) = \frac{15!}{x!(15-x)!} \qquad g(y) := \frac{45!}{y!(45-y)!}. \tag{11.7}$$

I have been using MathCAD for such a long time that I know how to define a variable (x, y) and I know without having to reflect upon it that I can create a function of each of these variables. Functions can be plotted, and creating a plot, too, is something that I have been doing frequently. Creating variables, functions, and graphs are holds that I do not need to think about but that are to hand much like a crevice is to hand in rock climbing without requiring the person to think much about whether or not it is promising. Once I plot the two functions f and g with respect to their defining variables, I realize that the maxima are of different height and I decide to 'normalize' them, which means, divide the function by its highest value, which, in the present situation, is at 8 and 23, respectively. Once I divide the functions in equation (11.7) by the values of the functions at 8 and 23, that is, $f(8)$

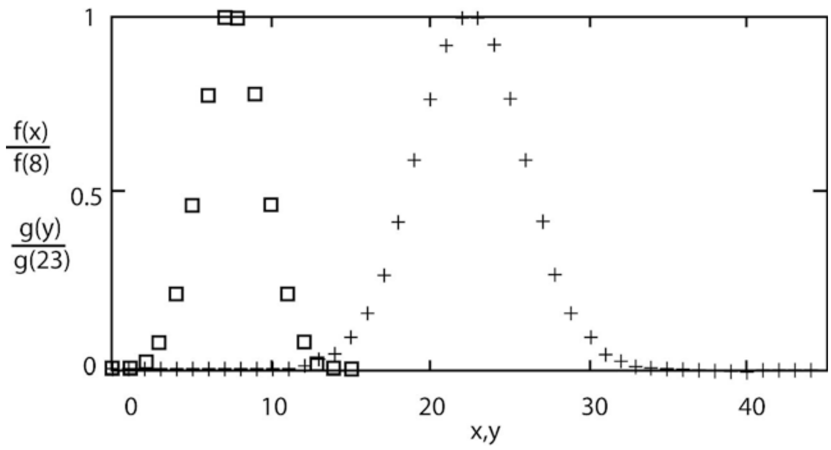

Fig. 11.3 The two graphs represent the boy/girl mixes in a hospital with 15 and 45 births, respectively.

and *g*(23), I obtain two graphs (Fig. 11.3). The resulting figure is similar to the first ephemeral image but looks right – I am not looking for the distribution of children around the mean values but for the distribution of the boy/girl mix, which has its maximum at seven boys and eight girls and eight boys and seven girls in the case of the 15-children hospital.

Now, after having created and tested action possibilities – in typing equations and making the computer calculate them, creating functions and plots and making the computer graph the former – holds have come to exist for grabbing purposes. Possibilities for sense emerge. *Sense here means that a real situation I can understand and envision is modeled appropriately when I use mathematical representations that I may not entirely understand or know the behavior of. Whereas I have learned to navigate successfully the real world around me, I am often not so sure about mathematical objects, the mathscapes that they give rise to, and the kinds of footholds that they provide*. A sense of sense: my two distributions express precisely what the earlier envisioned and pencil-noted distribution of boys and girls in a three-children family expresses. The possibilities for having about equal numbers of boys and girls are most likely, and the possibilities of having 15 boys or 15 girls – and, equivalently in the other hospital, 45 boys or 45 girls – virtually are nil.

That makes sense! But I still have not answered the question. I continue my pursuit.

The puzzle is asking me about the number of days when there are nine or more boys in the small hospital and, equivalently, 25 or more boys in the larger hospital, each representing 60 percent of the total average births. I look at the plot (Fig. 11.3) and note that the widths of the two functions at their half-heights are different. Not only does the question now make sense but also, it has tuned my gaze to see something in the representation that I have not been immediately attuned to. One hold has created another hold, or rather, the two holds have emerged into my consciousness simultaneously (I merely play the unprejudiced host). The different widths make sense because of the question, and the question now makes sense given the different distributions. At this time I can imagine that others might be led off track by the fact that 9 relates to 15 as 27 relates to 45 to state that the probabilities are the same in the two hospitals.

I know that the distributions tell me the probability to have a certain number of boys in the mix, each probability also representing a day. So I need to know how much area under each distribution is contained from the 60 percent mark to the upper end of the distribution. This means I have to add up all the values *f*(9) + *f*(10) + . . . *f*(15) and divide the sum by the total number of cases, that is, *f*(0), *f*(1) + . . . *f*(15). The associated vague sense is that of finding the area underneath each graph from 9 and 27, respectively, and to divide it by the total area. Vague sense here also means that I do not exactly now what I am doing but rather trying out different ways to see whether these are productive. Because I am dealing with integers, I am thinking of summing the values rather than integrating the functions, which I would have to do if I were dealing with continuous functions. So I type the division of the two sums into MathCAD

$$\frac{\sum_{9} f(x)}{\sum_{x} f(x)} = . \qquad (11.8)$$

I am thinking that I need to sum *from 9*, which I type into the empty box of the sum, i.e., my hold; I type *x* in the empty box of the lower sum, because I have defined *x* to run from 0 to 15 (equation 11.7). I hit press the [⌘][=] key combination. But, *Oh my!*, I get a message 'must be range' tagged to the upper sum. I stare at the screen. There is a moment of no-thought.[4] It is as if the hold I have found gave in, was a no-hold, an outcrop that caved in as soon as I hung my weight from it. *Must be range!* I ponder what the sense of it is, and then think, *9* is not a range, it is a number. Create *a new range that runs from 9 through 15*, I barely have the time to think when my hands create a new variable *z* similar to *x* and *y* (equation 11.7) but running from 9 to 15. I then enter the new variable – which does have a range – into the slot of the sum in the denominator. I hit the [⌘][=] key combination again. Low and behold, I get a result that I can ponder:

$$z := 9..15$$
$$\frac{\sum_{z} f(z)}{\sum_{x} f(x)} = 0.304 \, . \qquad (11.9)$$

The result is 0.304, which means that in about 30 percent of the days in the small hospital, staff sees nine or more boys being born. I check the distribution, and the result looks about right, about 30 percent of the total area under the first curve lies in the tail from 9 to the end (i.e., 15). I try the same for the other hospital, generating a new variable *w* that runs from 27 to 45. The result of the calcula-

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[4] Those psychologists who use think-aloud protocol as a method would encourage the participant in this situation to 'say out loud what you think', when in fact I am not thinking anything. There is just a big black emptiness.

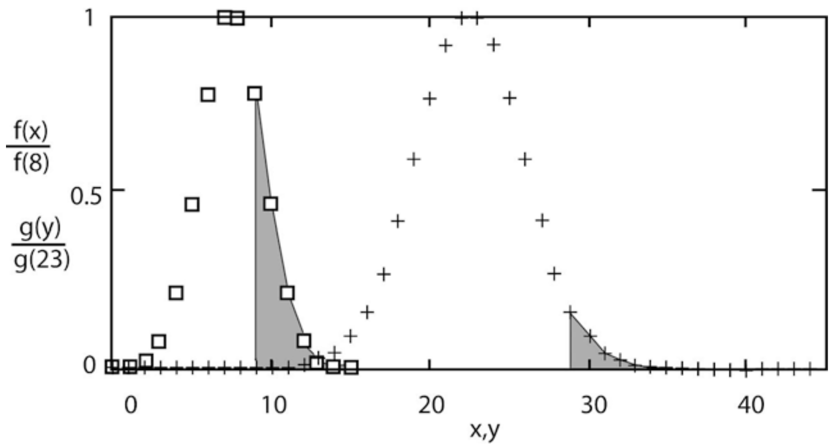

Fig. 11.4 The black tails represent the fraction of days in which there are 9 and 27 boys or more, respectively, born in the small and large hospital.

tion equivalent to (11.9) is 0.116, which means that on close to 12 percent of all days, staff in the larger hospital sees 27 or more boys out of the 45 total births. This brings me back to the image of the town with its two hospitals and a sense emerges in me that I have solved the puzzle. Into my mind emerges the image of a black-colored tail of the left curve that has a relative area of 0.304, which is larger than that of the right curve with its relative area of 0.116. I go back to the plot of the curve, print it out, and color the two areas. *Yea, I got it!* (An emotional response, an invariant, the particular of which depends on this situation, one of [possible] success.) The second grey area, the one corresponding to the larger hospital, is less than half the size of the larger grey area, corresponding to the smaller hospital (Fig. 11.4).

When I now look back at the entire episode, I know that it happened to me as much as that I was responsible for it. There may have been 'dispositions' that I am not consciously aware of. Much of the process involved passivity, from a first-person perspective, which is reflected in the associated language that makes use of the passive voice, which differs from the language normally used in psychology, education, and the learning sciences, where *transitive action verbs* tend to be the norm. As soon as there are intransitive verbs and passive formulations, the dominance of agential repertoires comes to be questioned (deconstructed). Slow readings of first-person accounts of problem solving allow us to throw much of the existing research into relief, seriously undermining the learning theories and concepts. In the present situation, the varying images, moves, and representations *came* to me: I *did not* intend or intentionally construct them. It is as if the entire episode occurred to me *despite* myself, taking me as a host to realize its occurrence. But this description also means that there is something more general in what I experienced, as something exceeding me: culture concretizing itself in the particulars of this case. It happened and I was as much a willing host as I was the agential subject who engaged with the task. Now our task as researchers is to get back to culture, to interrogate *this case* for the cultural invariants that it harbors apart from the contextual particulars.

There is always the possibility that psychologists or learning scientists might say that I am a novice problem solver, even though they might have a hard time making the case based on other information from my life. Deficit perspectives appear more easily appropriate once the discourse already has established the lower-level performance of the participant. But even if I were only a novice problem solver, there are invariants in the behavior. It is precisely those invariants that the first-person approach is intended to identify rather than the particulars of my fitting the labels that psychologists and learning scientists have for categorizing the world. Besides, all we have to do to throw psychologists' frameworks into relief is put them into situation where the untenability of their own theories becomes apparent. For Marlene Scardamalia, it showed up has her inability to account for my writing, which means, her theoretical model is not invariant across *all* possible writers. All we have to do is look at the clumsiness with which many professors – including those of psychology, learning science, and education – go about particular aspects of their lives to show that (a) problem solving 'skills' do not transfer and (b) dealing with a real-world problematic is something other than running a rat in a maze that can be seen in its totality from a god's eye-perspective.

At that point, I have a sense of being done. I have climbed this rock – readers certainly take note of the similarity in the description with the sound experience I describe in chapter 4 on hearing – and it is no longer of interest. I do not know what a mathematician or mathematics teacher would say about what I have been doing. What I have done looks awfully complicated – like shooting flies with canons. I have the strong sense that a *real* mathematician would probably have a more elegant way of dealing with it, and wonder about how anyone could expect an ordinary teacher or student to solve this puzzle. Without further questioning myself, I remember having had a strong impression that this is too complicated for school mathematics and that what I have done is certainly unacceptable in school mathematics.

Postscript. While working on this chapter, I am doing a Google advanced search with the phrase 'In a certain town there are two hospitals'. Thirteen websites are listed, the first of which has the header 'Mr Danault – Word Problems, Age – 8 Questions, Multiple Choice'. The second and third result point to 'interactive quizzes' that appear on a site with 'teacherlink' in their URLs. I begin to wonder whether there is something in the problem as I have done and analyzed it for the present purposes. Surely I feel that there is a difference between what an eight-year-old child or even a high school student will do in an *understanding way* that I have failed to see. But I find one paper in which the author presents exactly the approach that has arisen for me in the end from my engagement with the task. I also find a paper that discusses research findings, according to which this task was given to students in grades 5, 7, 9, and 11 and to teachers in training. The studies show that with age, the proportion of individuals who respond 'inappropriately' increases. This is so because older individuals use ratios and proportions to say that the number of days is equal in the two hospitals because of the same ratios involved (9:15 = 27:45). That is, increasing (specialized) expertise in using ratios and proportions also become a constraint, and, in fact, decrease the level of (general) expertise.[5] Expertise is not only enabling but disabling as well.

Mathematics is a Sweet Fruit . . .

In 2000, I happened to come across an article in a German journal dedicated to *Kritische Psychologie* (critical psychology) and the first-person perspectives that its practitioners have developed based on the theories of the Russian psychologist Alexei Leont'ev, the 'father' of cultural-historical activity theory. The article is entitled 'Mathematik ist eine süße Frucht . . .' ('Mathematics is a sweet fruit . . .') (Busse 1999). The intent of the article is to show how certain practices allow students to frame and choose themes and hypotheses, to arrive at statements and conclusions, and to prove these outcomes. The author contrasts his findings with the ordinary way in which word problems are framed. Although 'word problems' are designed to provide a connection with the everyday reality of students, the manner in which these are constructed foster particular solution paths that hamper making connections with the reality described in the stem. This is similar to what the comparison between the activities of grocery shoppers had shown when they bought their groceries versus when they did word problems. Word problems, though these may be framed in terms of making a best-buy in the supermarket, lead the subject to engage in practices that disconnects them from rather than connects them to their everyday realities – i.e., school mathematics.

The danger for any social scientist is common sense, which is a form of ideology. Just because I think in a particular way, or because my culture presents particular forms of thought, this does not mean that what makes sense is scientifically tenable. The concepts we use to think with are themselves the outcomes of previous thought activity. Just because some concept is also used in the sciences – such as the concept of 'motivation' that psychologists have borrowed from everyday language without checking out its epistemological implications (e.g., Holzkamp 1983) – does not mean it is scientific. In fact, it may just be a preconstruction that has been naturalized. The point in slow reading ('deconstruction') then is to break not only with ordinary common sense but also with the way in which we tend to make sense. That is, '*we must also break with the instruments of rupture which negate the very experience against which they have been constructed'* (Bourdieu 1992: 251). The purpose of the method is to build models of understanding that also encompass our primary naiveté.[6] The following account ought to be taken as what it is: the quick notes that have been jotted down following a particular experience, in which the author has not reflected critically on the account itself but

[5] In psychological research, one can find numerous phenomena where development is in the form of a 'U', which means, as individuals get older their performance decreases before increasing again.

[6] The author has a stark warning for those who feel superior when they uncover flaws in the primary naiveté: 'I cannot refrain from saying here that the thrill of feeling smart, demystifying and demystified, of playing the role of the disenchanted disenchanter, is a crucial ingredient in a good number of sociological vocations . . . And the sacrifice that the rigorous method demands is all the more costly for that' (Bourdieu 1992: 251).

merely provides a quick description in commonsense terms – even though there is at least one paragraph that is explicitly denoted as 'comment'. This, then, is precisely what we need to interrogate to get at a better understanding of what such 'problem solving' involves rather than taking the account literally, in the way that is commonly done when scientists talk about what they have done subsequent to having made a scientific discovery. Here, the purpose of the analysis is to go beyond the literal account that assumes an account has to be true just because the person involved has told the story. That is, to extract anything of use from the following account we need to analyze it critically to extract invariants rather than taking it literally.

> For several years now, I have kept painstakingly recorded personal 'discovery activities' and critically analyzed them subsequently. My intent has been to come to a better understanding of learning than what traditional psychological theories tend to provide us with. Furthermore, most personal accounts of discovery work by scientists fall into 'Whig history' rather than being critical accounts of discovery as ongoing process. In the latter reports, what is missing is a rigorous inquiry into lived experience.
>
> Ich möchte euch ein Spiel mit 3 Ziffern zeigen, bei dem man Addition und Subtraktion braucht. Denkt euch drei verschiedene Ziffern zwischen 1 und 9. (Beispiel: 2, 3, 5) Bildet daraus eine dreistellige Zahl (Bsp. 325). Vertauscht die Reihenfolge der Ziffern, so dass eine neue Zahl entsteht (Bsp. 523). Zieht die kleinere Zahl von der größeren ab (Bsp. 523 – 325 = 198. Vertauscht im Ergebnis noch einmal die Reihenfolge der drei Ziffern (Bsp. 891) und addiert die letzten beiden Zahlen (Bsp. 198 + 891 = 1089).[7]
>
> Although I had not been mathematically inclined in school and, more importantly, have had considerable difficulties in making it unscathed through school (I repeated grade 5), I immediately found something interesting in this activity.
>
> As I read the problem, I mentally calculate the example provided in the text. (I intuitively know that the paper would not have been published had there been a problem with it.) Suddenly, I feel like writing a generalized solution to the task. I am not aware why this is happening, but immediately proceed to note on my scratchpad, and ultimately end up with the notes in Fig. 11.5.
>
> I first write a number, by converting the decimal positions into their full equivalent. That is, the digit in the hundreds position really stands for $x \cdot 100$, the digit in the tens position, y, encodes $y \cdot 10$, and so on if there were more than three digits.

[7] I want to show you a game with 3 numbers, which requires addition and subtraction. Think about 3 different numbers between 1 and 9. (Example, 2, 3, 5) Make a three-digit number with them (e.g., 325). Interchange the sequence of the digits to yield a new number (e.g., 523). Subtract the smaller number from the larger (e.g., 523 – 325 = 198). In the result, interchange again the sequence of the three digits (e.g., 891) and add the last two numbers (e.g., 198 + 891 = 1089).

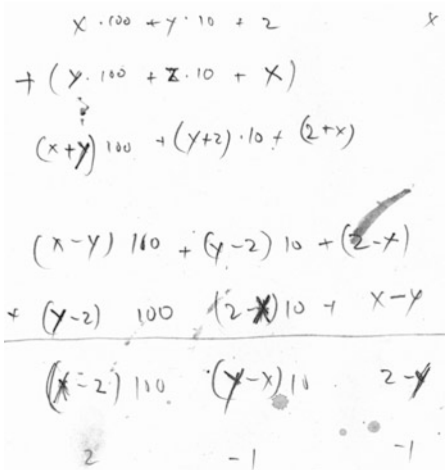

Fig. 11.5 The results of the first attempt to provide a general solution to the game with 3 numbers.

I permutate the generalized digits and subtract the two numbers, assuming that x > y. I note that the resulting differences (x – z), (y – x), and (z - y) are the differences of the permutations. I have the sense that there can't be a generalized solution because, depending on the permutation, different differences would be in the different positions. But this was not in any way articulated.

COMMENT: Even the choice of x, y, and z rather than any other letter is mediated by past experience. Actually, the set {a, b, c} would also have been a reasonable candidate. Setting up the digital representation in the form of an addition is something mediated by past experiences, though this was not salient to me at the moment when I did the problem and not even during my first attempts in analyzing what I had done.

With this sense that the permutated differences could appear anywhere, and filled with some unease, I begin to scribble down a few concrete cases (e.g., Fig. 11.6). Again, I change the order of the digits. My results are not at all 1089 as the text predicted it should be. There are two vague feelings present at this time. The first is about the article and that there might have been a case that the problem is not generalizable at all but that the author had given special instructions to the kids and these had done examples following the same pattern as the initial example. The other feeling was related to the failure to comprehend mathematical problems, the texts presenting them, while I was still in school.

I leave the problem and tend to my work. I am thinking about my failures in school mathematics, particularly the year that I could not calculate the amount of wallpaper needed to renovate a room given the dimensions of room, windows, and door. I also have the image of two lines in space whose distance I could not calculate as required in grade 11. I remember the frustra-

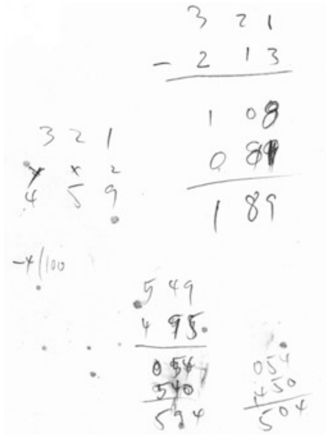

Fig. 11.6 After the initial attempt on a generalized solution does not yield a suitable result, I return to doing several practical examples.

tions I experienced in mathematics, and always being behind in my understanding.

Upon seeing the notes on my scratchpad on the following day, I do a few more examples, changing the position of the three digits (Fig. 11.7). I see from the example that there are different outcomes if different permutations are made. Thus, the 'correct' result only comes about when a particular permutation is implemented. I feel that the author must have asked the students to switch the first and last digit. I do not dwell on the question about the case of four and more digits that the author asks his students to do as an extension of the first task.

I do not get any further and leave for long day of work. On the way, as I am pushing myself hard on the bicycle, the problem returns to me. I wonder if in a four-digit number the outer digits are exchanged with each other and the inner ones. This could also be done for a five-digit number, with the middle digit remaining in its original position. Then all of a sudden I realize that this simply means that the number is written in 'reverse'. It then comes to me that 'vertauschen' may not mean, in this context, to permutate the digits, as this might have been meant in other mathematical settings, but 'to reverse'. Why did the author not use 'umkehren'?, I ask myself. I decide to check the word in a German-English dictionary.

When I return home that night, I immediately take to my scratchpad and begin to scribble my new attempt in the generalized solution (Fig. 11.8).

I begin this – as it later turns out – final session by noting a three-digit number in its generalized form ($x \cdot 100 + y \cdot 10 + z$). I reverse the order of the number, use '>' to note that '$x > z$'. I write down the result and note below it its reverse. As I look at my notes, I realize that addition would cancel my hundreds and my ones. I stare at the notes for a while, when it strikes me that $z - x$ will be a negative number. Even without thinking about the fact that negative numbers do not make sense in a digital representation, I begin to

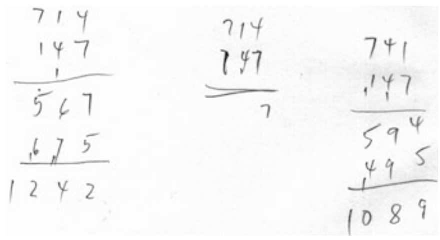

Fig. 11.7 A concrete worked out example of the game with 3 numbers, which yields the indicated result.

'take away' one hundred and divide it up into nine tens and 10 ones (in the last position). At first, I do not copy the 9 tens into the second line and end up with (10 – 1)·100 in the result (Fig. 11.8). I write 10 8 9 at the bottom, note that the addition gives me the correct result in the ones position. Then I think that adding two nines will give me the required 8, and note the forgotten 9·10 in the second line of the subtraction. As I add and carry over a one (in front of the first parenthesis, Fig. 11.8) I realize that this takes care of the –1 and that I have the required {10, 8, 9} sequence.

It is only at that point that it is clear to me that my first result could not have been appropriate, for if the result is always 1089, the generalized solution had to be independent of x, y, and z. It has not been evident to me throughout this process but only now that I have arrived at the final result.

Somehow the fact that I have the generalized result satisfies me more than knowing that any example that I can generate gives me the same 1089. The first question for mathematics education has to be, of course, 'Why this is so?', followed by the second question, 'What does mathematical curriculum have to look like to encourage such an attitude?' (Granted that such an attitude is what mathematics educators want to generate.)

Here at the end, I have a clear sense of satisfaction. I am pleased with the solution, even though it has only shown what I ought to have known all along. But I am filled with a sense of discovery, having found an answer, however trivial a mathematician might think that the problem is. I am able to close the problem and not to return to it. I know that I could now find the solution for any number of digits that the children had explored.

I do check the signification of 'vertauschen' in a German-English dictionary. It reads:

> **Vertauschen** v/t. exchange (*gegen, für, mit, um* for), (Handicraft, engineering) *usw*. A. interchange; (*Plätze*) change; (mathematics) substitute; (Rolle) reverse; → a. *verwechseln*.

I realize that among other things, I have discovered the signification of 'die Reihenfolge vertauschen' and therefore that of the activity for the children.

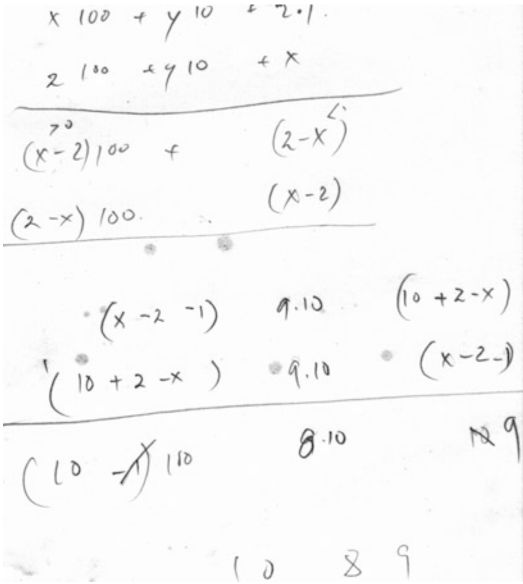

Fig. 11.8 The generalized solution proving that any 3 digits will yield the same result.

As we begin analyzing this *account*[8], we keep in mind that the event as a whole and in its part is a particular realization of the possible. It contains both aspects that are contingent and therefore particular to *this* case as well as aspects that constitute invariants.

The first point we may note pertains to the title of the article that got it all started. It says, 'mathematics is a sweet fruit', which we might find confirmed in the present instance. But surely, mathematics is not sweet for every person. Even avid athletes will pursue a range of sports but never become affected by some others. If the preceding account were taken as a reification of the message and title of the article, little would have been gained and we would have fallen back to the general assumption of those already affected with and by mathematics – mathematicians, mathematics teachers, mathematics buffs – that what tastes sweet to them has to taste sweet to everybody else as well. But we may interrogate this account as an instance of becoming affected. What has it been about this riddle that has made me engage with it even though I tend to leave many other (mathematical) riddles untouched? The question therefore is this: 'are there laws of propagation of intentional awakening? The most privileged case here is where affection results in attentiveness, grasping, the acquisition of knowledge, explication. Then this lawful regularity would of itself pass over into the law of awakening or again would lead the attentiveness further, or which is to say, would lead thematic interest further' (Husserl 2001: 198–199). What is it that makes *this* riddle an object that affects me at the time (it may not have done so at another time), become prominent enough at

[8] In chapter 12, I work out the difference between living-lived work and experience, on the one hand, and accounts thereof, on the other hand.

the time to lead to the deep and sustained engagement that is evident from the written account that remained after I have stopped?

In the account we note that there is an immediate turning toward a generalized approach. That is, the account shows that the initial attempt is doing precisely *not* what the stem asks students to do: to work through concrete cases. Rather, the very first attempt is to develop a generalized approach that works for any three numbers that are used as digits of a three-digit number. It is also immediately evident that most people in the general population would not attempt doing this task using the letters x, y, and z. This approach, therefore, is not something that generalizes as an invariant across people. Nevertheless, the fact that it was chosen lets us know that acting in this manner likely is a possibility in a culture characterized by the disposition of presenting results in a generalized manner – scientists, mathematicians, and perhaps others. What we observe, however, is that a rather simple, elementary-level task of forming, adding, and subtracting three-digit numbers is taken to a different level where, for the subject involved, it *becomes* a challenge and which is sustained as a challenge over a couple of days. All I have to do is look out of the window to know that there are other instances where someone creates a challenge and then pursues it to the point of mastering it: the children on my street build their own equipment for developing new moves and skills for riding their skateboards. They may begin by copying something they have seen and then add on special features that make engaging with the equipment a challenge for them. In both types of instances, the mathematical riddle and the skateboard challenges, there is no real 'need' for doing what is being done. What is it then that gives rise to sustained engagement even when the practical outcomes are rather inconsequential to the everyday lives of the persons? It is not just engagement, for I might not have abandoned the activity but tried to model the four, five, and higher digit numbers as well. We see here the absence of the affection that characterizes the beginning.

In this account, affects appear in overt – explicitly described or referred to – and covert ways – such as when an initial unnamed affection leads to salience, interest, and engagement. The interest is not simply there. It is in and through reading the text that affection occurs. We subsequently see evidence of negative tonality, such as when the experiences of having failed fifth grade or the experience of not being able to figure out the nearest distance between two lines in space are articulated. Affection occurs because 'every sense-field forms for itself a unique, self-contained realm of affective tendencies, capable of forming organizing unities by means of association' (Husserl 2001: 199). That is, engagement with the chosen task engenders a change in the emotional tone. Yet despite the thoughts about failure in school mathematics classes, I return to the riddle on the next day, doing more of the things I had done when I left it. In the end, following what feels like a successful solution, there is a positive emotional tone reflected in the account: 'Somehow the fact that I have the generalized result satisfies me more than knowing that any specific example that I can generate gives me the same 1089'. Again, the change in emotional tone is brought about by the activity itself, despite the negative tones that have accompanied it throughout. We therefore see that the negative and positive emotional tones are produced in and through the activity, as a measure of the distance between the current state and the unspecified end. Solving problems may require patiently waiting for a solution in deliberately turning away from the problem. In this way, problem solving has a component of epoché, where the third stage means 'no attention'.

We also note the passive linguistic constructions, which I have used at a time to describe the events even though the role of passivity has not yet been clear or a focus of my research. The first such formulation actually constitutes the beginning: 'Suddenly, I feel like writing a generalized solution to the task. I am not aware why, but immediately proceed to scribble on my scratchpad and ultimately end up with the notes in Fig. 11.5'. The adverb 'suddenly' suggests that there has been no warning, preparation, or anticipation. The event arrives and I 'feel like writing a generalized solution' but 'I am not aware why'. That is, the intention to write the generalized solution has come to me; and I, a willing host, accept this offering and donation. Here, the intention toward an object is awakened by nothing other than the object itself. This, in classical thought, is a chicken-and-egg situation when conceived as the opposition of intention and intentional object. But it does not lead us to a contradiction if the fundamental category is that of change, which then allows us to understand the birth and death of intention and object at the same time. In the other example, on the bicycle, there is an episode in which a realization suddenly strikes me. I do not work on the puzzle or intend dealing with the problematic issues that have been arising for me, but I am 'suddenly' struck. Finally, there is a realization at the end that could not have been anticipated but required the entire process as antecedent: 'if the result is always 1089, the generalized solution had to be independent of x, y, and z. It has not been evident to me throughout this process but only now that I have arrived at the final result'.

Everyday Settings

In everyday life, we often encounter situations that need some response, some kind of action to get the mundane order of things back on track. We tend to find solutions, sometimes on our own, sometimes after consulting with a neighbor or with other individuals (e.g., in a hardware store). In other instances, we may call an individual craftsperson or a company to fix what is broken and thereby get us back on track. Such situations are ideal for finding out about 'problem solving' in naturalistic settings that tend to be so complex that the simplistic models developed on the topic are insufficient to account for and explain what happens. As with inventions or discoveries, however, it is dangerous and misleading to take the agents' own accounts literally, as if these depicted the events as a whole and in their development. Rather, after-the-fact accounts tend to be colored by the discovery process itself and by the ultimate outcome (Husserl 1980). Thus, we never have direct access to past events from the present now point. Rather, some present experience 'sinks' into the past such that – from subsequent now points – it is viewed through the events that have occurred since. We do not, however, reconstruct the original events by projecting backwards through the different ways in which it has appeared. Rather, the event is visible to me transparently through the ways in which it has appeared (Merleau-Ponty 1945). That is, I see the object as if it were unmediated by my intervening experience as if I were to see it through a set of

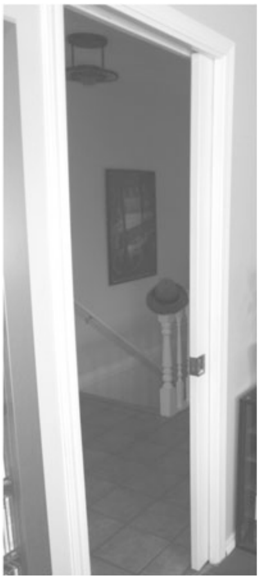

Fig. 11.9 A pocket door is only about 35 mm thick and slides in a pocket of 45 mm width.

(colored) spectacles without being aware that what I see is colored and is given shape by the glasses. This leads at least some social science researchers to suggest that practitioners, by principle, have no better access to their practice – events that they have lived through – than others (Bourdieu 1980). The other important dimension is that practitioners themselves make use of common language that stresses rules and regularities of behavior to explain what inherently follows very different principles.

In the following, I exemplify the first-person approach by drawing on a situation and event that occurred in my home: it required fixing, possibly with major structural work, and therefore has had a truly problematic nature. My kitchen is separated from the hallway by means of a door that slides into a pocket in the wall (Fig. 11.9). The door itself is 35 mm thick and slides into the pocket of 45-mm thickness. One day, we could no longer close it: Upon investigating the matter it appears to jam as it is pushed into the pocket. I unhang the door and it becomes apparent that the track has come loose inside the pockets. Two screws have come out completely and are about to fall to the ground; another one further to the door jam also is loose. Unsuspectingly, I go to get my power drill to fasten the screws again. But I cannot get the drill into the pocket. The pocket is so narrow that I cannot turn the screw-driver to fasten the first screw let alone reach the place where the other two screws. In fact, I get stuck pushing my arm too hard into the pocket. When I finally get myself out of the jam, I go to ask my neighbor, who has always been a handyper-son and who has installed the electrical wiring in the garden level floor of his house. He comes with an electrical hand drill (seen as part of the contraption in Fig. 11.10) and we succeed in getting the first of the three screws fastened. But the others are out of reach. After pondering for a while, he suggests that this means having to take off the plasterboard next to the door. He tells me about a tool that requires only damaging part of the wall. The thought of having to remove the wall and then plaster and repaint the entire hallway flashes through my mind's eye. My neighbor returns home leaving me to myself. The idea of having to re-do this entire part of the house just because I need to fasten two screws frightens me sufficiently to be staring at the door.

All of a sudden, an image emerges: I am extending my arm with a skinnier ex-tension no wider than my flat hand and that reaches the battery-driven screwdriver far enough back to set the screws comes to me. I go to the basement riffling through the wood, then to the outside where there are all sorts of scrap wood un-derneath the deck. I have no clear idea what I need or what it is that I am looking for. I wonder about holding the screwdriver in place and, the image of its rotating out of position as soon as I begin to screw appears in my mind. I do not have a 'clear' idea about what an 'appropriate' piece of wood might look like. But, after

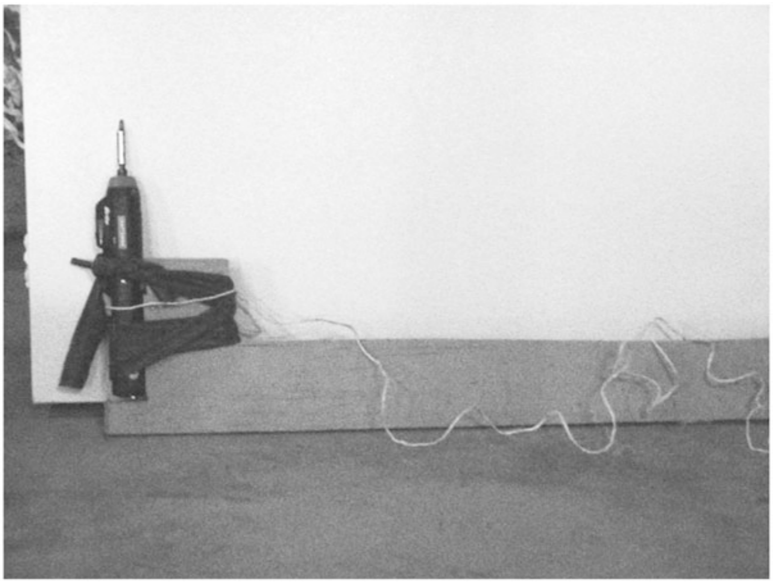

Fig. 11.10 This tool is an ad hoc creation for going into a narrow cavity to place and tighten a screw that would otherwise have required the removal of a wall.

finding nothing in the basement or underneath the deck, I know that I have something appropriate while seeing a piece of stock cut from the old deck that I had to replace because it had rotted. 'Appropriate' meant that I would not use 'good wood', at least not during the time of trying to come up with a suitable solution. I immediately take note that 'appropriateness' *emerges* at the very moment that I see the piece of board – like in the old saying, 'I know one when I see one'. I build a first contraption, but the screwdriver comes off almost immediately and there is insufficient space for holding it to the wood. Using an old piece of plywood, I cut it such that I can use an old inner tube to hold the screwdriver at a sufficient amount of length along the tool. When the string with which I pull the trigger does not stay in place, the idea comes to me that a screw might hold it. I end with a strange-looking contraption (Fig. 11.10).

When I am actually ready to try the tool, a worry creeps up within me about how to provide sufficient force from underneath the screwdriver to be able to hold it in place on the screw. The first thought that arises within me is that of using a household ladder as a fulcrum, but the one I have is not of the right height. An image surges: using my left hand as a fulcrum and the right hand and arm as the force. The contraption allows me to fix both screws firmly. I end this episode with a sense of relief – about not having to take the wall apart – and a sense of elation about having allowed the episode to come to a successful ending. I have a sense of discovery – which is captured in the keyword of the associated notebook entry that I produce after the fact: 'phenomenology of problem solving / discovery'.

Nothing in this experience and nothing in the account that I subsequently jot down rapidly and stenographically in my research notebook justifies a description of the kind that we may find about problem solving in standard textbooks of cognitive psychology or learning science. Even when this literature categorizes an approach as 'trial and error', the source of the trialed solution remains unexplained. There probably are many different ways to get the two screws back into place and appropriately tightened, but not all of these ways are equally palatable and some, like taking out part of the wall, would have been mere brute force and not a reflection of efficiency.

At the time, the notebook entry begins with what the essence of the event has been for me, and possibly a reason for the notes to be jotted down in the first place: 'emergence of idea, without my will; dialectic will | no will'. That is, although I also subsequently note the word 'système D' – a French expression that is used synonymously with the verbs *se demerder* ('get oneself out of shit') and *se débrouiller* ('get oneself out of the fog') – thereby making reference to agency, it is the passive aspects of receiving ideas as if from nowhere that dominate this experience and its account.[9] I also jot down the initial impression that the ideas that I hosted were not 'clear', did not constitute 'knowledge', but were rather vague. That is, even if I make use of the term 'image', it is, at the time, more something one might apperceive through a haze, a 'foggy idea' that requires concretization and working out. Sometimes there is no idea about what is needed, which leads to situations perhaps best characterized by 'taking a look' to see what there possibly might be. Or perhaps I am looking for an idea without knowing what it might look like but with the implicit hope or anticipation that I might recognize an idea once I see it or once it has come to me.

The episode might be material for describing how people 'make do' and how they act when they do not know what they are doing. In fact, this characterization does not do justice to the event. At the time, I know what I am looking for: a solution to my problem of getting the screws into place and fixing the track of the sliding door without necessarily having to take the wall apart. But there are no 'states in problem space' that I might investigate; and there is no 'searching of the problem space. Sometimes psychologists talk about 'means-end' analysis, where people are said to use the difference between the current state and the end state to break the bigger problem into smaller problems that eliminate part of the difference. The present account and initial reaction shows that there are no 'states' that are constructed in mind but rather a lot of passive acceptance of images and ideas that have come to me from elsewhere.

'Making do' also places in relief the old diction of 'thinking outside the box'. Thinking in terms of the concepts and images available to us precisely constitute a box; but these concepts and images are all that we have to think with. 'Thinking outside the box', if it is an appropriate descriptor of the events accounted for here, means precisely not thinking, or not thinking in the way we tend to theorize it. It is a form of intensely engaging with a problem and then taking a 'no attention' attitude so that possible candidate solutions may emerge that I can subsequently

[9] While working on this section of the chapter, I realize that I have used these French expressions almost an entire decade prior to writing an article in which I recommend thinking about learning along the life span in terms of these expressions even though I was not aware of this fact the second time around (Roth and van Eijck 2010).

evaluate for their usefulness to the issue at my hand. This situation, as the cases of the mathematical puzzles, has to be thought in terms of the category *event*, in which, because of its nature of saturated phenomenon, intuition is in excess of intention. This also means that there is an important place for the unforeseen – and, therefore, the uncontrolled – typical of phenomena that we also characterize by the adjective 'emergent'.

Conclusion

Researchers in psychology and education, as well as teachers, ask their participants (students) to engage in tasks that the former already have an answer to. Those 'owners' of the problem then evaluate others whether they have come to the same end state. If the 'subjects' do not arrive at these pre-determined end states, then negative assessments are made with respect to the ideal solutions (end states) that someone else has defined as the norm. It is an exercise in metaphysics, where real world constraints, interests, and needs are completely left to the side. The present investigation, a concrete instantiation of the first-person approach, shows that there is much more to problem solving than pushing pawns on a rather well-known chess board or moving rings from one peg to another in the Tower of Hanoi game. Most importantly, we notice that whereas there are images and other representations, these come to the agent as if from nowhere. These are not intended but given to the problem solver, who can accept or reject them as useful. In any event, problem solvers might want to investigate those images that come to them and which they accept to host as willing hosts in order to find out what there is to them and what they yield.[10]

[10] The French language would allow me to use an active passive construction, *Voire ce que cela donne* ('Looking what it gives [yields]'), where we can look at that which has given itself.