4

Developmental Possibilities in/from Activity

> [A]ll cultural development has three stages: development in itself, for others, and for oneself (e.g., a demonstrative gesture – at first it is simply a failed grasping movement aimed at an object and designating an action; then the mother understands it as an instruction; and, finally, the child begins to point). (Vygotsky 1989: 56)

In the constructivist literature, there is an abundance of descriptions that attribute development to the conscious constructive effort of the acting subject – often described as a reflective abstraction of actions from the material into the ideal realm or as a conscious reconfiguration of one mental structure into another. Such descriptions make it appear as if the individual pulls itself up on bootstraps and comes to produce, all on its own, the forms of knowledge that others in a culture already attained. It is not surprising to read that Piaget invokes the idea of recapitulation of phylogeny by ontogeny.[1] A closer look at the introductory quote to this chapter shows that Vygotsky takes a very different stance. In his description of cultural development, a movement receives the sense of an action of a particular kind *first* by the culturally competent individual *before* this sense comes to be actualized by the child. In the example Vygotsky provides, there first is a random movement. The child does not know its cultural signification; it does not (yet) know to point. Rather, the parent who sees the child move understands it as a pointing gesture and, in acting toward the child, transforms it as such. It is in and through the social interactions with the parent that the child comes to understand that by means of such movements things are being pointed out. Its movement, though initially arbitrary, immediately takes *cultural* signification rather than being developed bottom up and recapitulating cultural history. As we note in the previous

[1] The author of *Piaget's Conception of Evolution: Beyond Darwin and Lamarck* notes: 'We might say that the study of the development of cognitive structures in children was for Piaget a surrogate investigation. And, by invoking the idea that "ontogeny recapitulates phylogeny", he hoped to draw conclusions about the development of human knowledge that would be applicable to both individual cognitive development and to the growth of knowledge in the history of science' Messerly (1996: 2).

chapter, the higher psychological function is a social relation first, which, in the present example, transforms the movement into a corporeal pointing gesture with cultural signification and which the child then comes to understand in this particular signification. Moreover, 'pointing' first is external before the child can anticipate its own pointing gesture that it may realize by means of vocal sounds, hand/arm movements, or in other ways by means of body movements.

We also note in the previous chapter that the object/motive of the activity is not and cannot be apparent to Mario and his peers. They are asked to engage in this task. And the only reason they have to do so is *trust*. Trust is both rational and relational. It is rational in the sense that there is always the promise that by engaging in the activity there will be the potential to enlarge one's possibilities of room to maneuver and control conditions (a control that includes possibilities for deeper understandings but also for oppositions, disagreements, oppositions, and subversions). This, however, is not enough for trust to really operate. More than rational, trust is relational. Trust is relational in the sense that it entails an attitude (a positive one) towards others – a letting-oneself-go to reach and to be reached by others. Trust is exposure. In activity there is the idea of doing something for the sake of being together, of finding pleasure in being with others, and trying to help and be helped in the attainment of the object/motive of activity, even if we cannot see it yet. In the course of the activity, Mario performs (saying, writing, counting, pointing) but the significance of these performances can only arise when the collective object/motive has revealed and concretized itself on the subjective plane. Here, too, we then have the learner first produce performances without knowing what they signify or that they signify anything at all. The relevance of particular actions comes to be shown first by the teacher and ultimately, with the disclosure of the object/motive to Mario that comes in and through his actions, he may come to realize not only what he has been working toward but also the significance of his actions in their relation to the object/motive in the process of revealing itself to him. For this to happen, it does not suffice for Jeanne to tell him the result: that to find the amount of money after week n equals $n \times 3 + 6$.

We do know – from our experience as teachers and from reading the literature – that students apply equations without *understanding* why they do so. In this case, there has not been teaching-learning activity properly speaking. For cultural learning objects (mathematical and others) can become objects of consciousness as objects of activity only, that is to say, as objects of joint ethical, cognitive, and emotional actions. The object/motive, therefore, is not to get Mario to use the equation to calculate the amount of money in the piggybank for various weeks the number of which is sufficiently large so that counting it out becomes unfeasible. In one sense, Mario has understood what the task requires him to do, and he has exhibited this understanding in placing the correct number of chips in each of the five goblets in front of him. In part, this may be one of the sources for his frustration, because he has correctly identified the amount of money that goes into each goblet. He cannot know what Jeanne wants from him, and her questions are difficult to understand *unless* the interlocutor knows that she is working toward a generalization whereby the repeated addition of *chips* is transformed into the repeated addi-

tion of *numbers*, which in turn is transformed into a multiplicative rule. For him to understand the significance of her question means understanding that she is attempting to make salient in his consciousness the repeated addition as multiplication.

In the previous chapter, we note that to come unstuck, our two protagonists need to act (move); and these actions, once successful (which is assessable only after the fact), will have created an opening, a space for development, which is equivalent to the movement of the activity toward its intended product. This is – with respect to Mario – not the mere use of the formula but for him to realize the equivalence between his repeated addition of $3 over and above the amount that is in the previous goblet, on the one hand, and the multiplication that models the repeated addition, on the other hand. This developmental space does not just exist in the way that this is often described in the research literature where the simple co-presence of an institutionally designated 'teacher' interacts with the institutionally designated 'learner' under the auspices of a kind of legal didactic contract. We already note in the previous chapter that both participants need to learn: one learns mathematics and the other one pedagogy (and often mathematics, too, for the teacher learns to see things from a different mathematical perspective). In the process, each may come to understand the other part as well. As part of this learning, or to allow this learning to occur, they need to produce the space in which development occurs. This space – which, following Vygotsky, is denoted by the term *zone of proximal development* – does not just exist like a box into which the participants step. It is something that the participants have to produce all the while they produce the mathematical context and contents that this lesson is to teach and that is to be learned.

So if we think for an instance of this zone of proximal development as a box that does not preexist, then we are immediately confronted with a contradiction. As we see in chapter 3, Jeanne does not have the immediate answer to the question of how to help Mario. After more than 120 turns, Mario states that he does not understand, expresses frustration and perhaps annoyance, and, speaking with frustration in her voice, Jeanne says that what she has done is to help him. She is trying, but finds out in/with Mario's comment that thus far her effort has failed. That is, there has not yet been a space within which development could occur where Mario comes to do something he was not able to do on his own prior to Jeanine's arrival. Jeanine, too, has not yet had a space that would allow her to develop pedagogically. The space is actually something that arises in and from their societal relation and cannot be conceived apart from it. They have to develop this space together without knowing beforehand what it might look like and how to create it. The creation of this space, which allows them to develop and move closer to the object/motive of the activity, is itself an integral aspect of the learning-teaching (obuchenie) activity. Because neither one knows what actions will make this developmental zone, it can only emerge without that this emergence could be anticipated. This means that the participants come to realize consciously the possibilities that lie in their actions a posteriori. If it were not in this manner, Jeanne would have immediately acted such that Mario would have known. That she has

Fig. 4.1. Jeanne Has oriented to Thérèse (back) and Aurélie (left front) asking whether they are following ('are we looking'); Mario stares at his worksheet (turn 169).

done a lot without getting anywhere is a direct sign that the nature of the appropriate pedagogy itself has to emerge from their performances. Just as Vygotsky notes in the introductory quote of this chapter, in each cultural development there are actions first, then an attribution of social signification next, and finally the emergence of the realization of the signification on the part of the learner.

Emergence of a Developmental Zone

Up to this point, Jeanne and Mario have engaged in interaction but the situation has not come unstuck. Although they have acted and thereby realized the activity, it has not gotten them closer to the object/motive. This final part of the episode follows Jeanne's formulation that what she has been doing is trying to help him understand. Thus, even though it might not have looked like this to him, and even though he might not have experienced it as actually helping him, Jeanne *has actually tried* helping him to get unstuck. There is a brief pause. Jeanne appears to be exasperated. She looks around, takes a deep breath and announces, 'look well' (turn 169). There is a further pause as Jeanne first addresses the two girls, invites them to attend ('are we looking, Trèse?'). Jeanne has turned her attention to the two girls (Fig. 4.1). But then she lets them continue on their own as they suggest that they have to do the remainder of the task (turns 170–178).

Fragment 4.1

```
169  J:  <<p>you dont understand that> its what i=m trying to
         help you understand (2.40) lOOK well (3.50) are we
         lOOKing (0.65) trèse?
170      (0.40)
171  A:  hu? (0.22) done.
172      (0.40)
173  J:  you have answered everything? ((Oriented toward
         Aurélie, Fig. 4.1))
174      (1.01)
175      have you done c d and e?
176      (0.91)
177  A:  n::o:. (1.44) weve to do c d and e ((pounds desk top))
178      (0.33)
179  M:  two times three plus six that equals to nINE.
```

At this point, she exhibits being attuned not only to Mario, to whom she has just announced that she is trying to help him, but also to the more general situation of the activity from her perspective, which is a responsibility for the other students as well. But as Mario begins to speak, her whole upper body turns in his direction, exhibiting her orientation to the obuchenie episode with him.

'You Don't Understand. This is What I Try to Help You Understand': A First Objectification

Mario begins, 'Two times three plus six that equals to nine' (turn 179). Jeanne then engages Mario in a way that we might gloss as 'helping him to add up the numbers for the second goblet'. She places her finger on the first cell of the worksheet (Fig. 4.2) and then repeats the first three words Mario has uttered 'two times three' and then asks him to give the result, 'is what' (turn 181). Jeanne's use of Mario's words is an important form of inter-subjective tuning. It is part of what Arzarello and Paola (2007) term a 'semiotic game'. Here, the utterance has the falling intonation of a constative but the grammatical structure of a question, 'is what'. In fact, the repetition constitutes a confirmation of what Mario has said, but the appendix 'is what' asks about some result to come before the second part of Mario's utterance. Mario says with an insisting voice, as if he has already provided the answer, 'six'. Jeanne continues undisturbed, 'plus' (turn 184). There is a little pause and Mario then utters in a slow, drawn out manner 'six'. Jeanne continues, 'equals to', and moves her finger down in the table toward Mario. There is a pause until Mario utters '12' (turn 189). He then asks (mixing French and English), 'Where du write it now?'

Fragment 4.2a

```
179  M:  two times three plus six that equals to nINE.
180      (1.13)
181  J:  tWO times thrEE is what. ((places rH index on the
         first cell, will not remove it for a while, Fig. 4.2))
```

Fig. 4.2. Jeanne has placed her index finger on the cell corresponding to the week she is currently speaking about. Mario's gaze is oriented toward the place on his sheet where the index finger is placed (turn 181).

```
  182  M:  <<insisting>s:sIX:.>
  183      (0.18)
  184  J:  plus
  185      (0.65)
  186  M:  s::IX::.
  187  J:  =equals to, ((moves index up down))
  188      (1.02)
→ 189  M:  tWEL:v:e ((fills something into his table, 439>210
           HZ)) (2.96) Where do we write twelve now.
  190      (0.43)
```

In Mario's question, he exhibits his orientation toward the result of the calculation. The task states that at the end of the first week, 'Marianne says to herself "I have $9"'. Following the twofold addition of three plus the original $6, Mario now says that there are $12 in the piggybank. This is what Marianne would have said at the end of the second week, 'I have $12'. This is also the number of chips in the second goblet, the one marked '2'. Jeanne, however, points to the table on the worksheet in front of Mario. So he now asks where he has to write the 'twelve'. The fact that he asks makes salient the trouble spot. He has calculated the amount – it is apparently the correct amount, but he does not know where to write the number or what to do with the result.

But Jeanne responds, 'You don't write it. You are done' (turn 191). And she continues, 'So what does it say?', moving her index finger up and down between rows (Fig. 4.3). She responds, 'three plus six'. She resumes while pointing to goblet 1, 'So. In the first week, there are already $6. Then you add $3. You already have your $6, you add another $3. So three plus three, because three plus three is six'. She continues, 'Third week, how many do you have to add to your piggybank?' During the ensuing pause, Mario displays a questioning look (turn 192). Jeanne asks slightly transforming the question, 'How many three dollars will you have?' (turn 193). A long pause develops. Mario breaks it by asking in turn, 'How

Fig. 4.3. Jeanne points with her left-hand index to the inside of goblet 2 while talking about week 2 and pointing to the cell corresponding to week 2 in the same way as she does it for the other weeks she is talking about (turn 191).

much money (change) you will have?', but Jeanne repeats this time *emphasizing* the articulation of the number 'three' (turn 196). Another longer silence develops, which Jeanne breaks saying 'three dollars' three times, while pointing to a corresponding figure '3' on Mario's worksheet. Emphasis in discourse, like the one Jeanne is resorting to here, is a subtle way to have Mario noticing or objectifying the mathematical structure. Like rhythm, emphasis and structural question repeating, are semiotic means of objectification (Radford 2002). Along with the pointing gestures all these semiotic resources come to form what we have called elsewhere a semiotic node, that is to say, a segment of the teaching and learning activity where knowledge is been objectified (Radford 2009a).

Fragment 4.2b

→ 191	J:	((moves hand to right from Mario's perspective; throughout, Mario holds his head, gazes at his sheet)) you dont write it. ((moves index up and down between row 2 & 3, first cell)) (0.35) you are done (0.45) what does it say; (0.41) three plus six. (0.49) so. (0.29) first week, ((points to first goblet)) (0.51) there's already six dOLL:ars (0.21) and you add three dollars. (0.34) three dollars plus six (0.56) SECond week:; ((points to sedond goblet, Fig. 4.2)) (.) you already have three dollars ((points to '3')) (0.54) you already have your six dollars ((points to '6')) you add another three dollars. so thrEE plus thrEE you do three plus three (0.37) its six. (0.60) third week; how mANY threes are you going to add in your:: (1.03) piggybank?
192		(0.96) ((questioning look on Mario's face))
193		how mANY three dOLLars are you going to have.

```
194      (2.08)
195  M:  how much money are you going to have?
196  J:  how many thrEE dollars are you going to have?
197      (1.47)
198  J:  three dOLLars, three dOLL[ars  ], three dOLLars (0.23)
         ((points to the 2 '3's in week 2 and simultaneously
         points with left hand to the first, second, and third
         goblet))
199  M:                              [three]
200  J:  what are you going to write here?
201  M:  three?
202      (2.59) ((Jeanne moves finger to the cell on his left))
203  M:  <<p>plus three? plus three? >
204  J:  yES:: ((he writes))
```

It is precisely at this point – around turns 198–203 – that a first objectification is accomplished: new actions have become possible for Mario. After having articulated the series of additions in the second cell and adding one 'three dollar', Jeanne asks him what he will write in the cell for week 3. Mario articulates a questioning 'three' and, during the developing pause, Jeanne moves her index to the second week pointing to the two '3s' (turn 202). Mario responds by saying, in a low and subdued voice, with rising intonations after each unit, 'plus three? Plus three?'' Jeanne immediately exclaims, 'Yes'. This 'Yes' does not only confirm the questioningly stated response (as if saying, 'Is it plus three? And another plus three?'); her intonation also exhibits an emotionally positive nature. It is not just an emotionally positive response for Mario, with the potential of providing the emotional climate that will allow him to take further risks (after all, he was not certain about the response), but also a sign of the success Jeanne has experienced in eliciting a desired response.

To the knowledgeable reader it is apparent that the worksheet calls upon students to write the repeated addition in the cells of the second line and to write the equivalent multiplicative notation in the cells of the third. To aid students in becoming consciously aware of the transition from the multiple addition of $3 to the multiplication of $3 by the number of weeks, the cells starting with week 2 contain the sign for a multiplication 'x' (Fig. 2.1). But as the verbal interaction exhibits, Mario does not provide indications that the repeated addition is salient but rather the end result, which is the total amount in the piggybank at the end of each week. That the total amount is salient to him or the question about the total amount is once more apparent in turn 196, when Mario asks, 'how much money are you going to have?' in response to Jeanne's question 'how many three dollars are you going to have?' That is, whereas to the knowledgeable observer it is apparent that Jeanne is asking Mario to articulate the number of repeated additions, the same number that will subsequently enter into the multiplicative structure, the student asks whether she is asking him about the total amount. Jeanne repeats precisely the same words she has uttered before, but whereas the intonation was falling and therefore marking a constative in the first instance, it was rising toward the end as this is normal for a question. Also, her emphasis has changed from 'how mANY three dOLLars' (Fr. 'comBIEN de trois dollARs') to a stressed number 'how many

thREE dOLLars' (Fr. 'combien de trOIS dollARs'). That is, the emphasis has changed from the interrogative to the three, which thereby offers the opportunity to be heard as a group of objects that is identically added each week.

There is a pause much longer than what would be normal in a (telephone) conversation and much longer than the 0.7 seconds that research in the 1980s on wait time has shown that teachers leave to students. Jeanne then breaks the pause uttering 'three dollars' and then, while pointing to the two $3 figures on the worksheet in the cell of week 2, 'three dollars, three dollars' (turn 198). While she utters 'three dollars' three times, Jeanne places the left index finger onto the first, second, and third goblet as if it were placing the named amount. The movement produced by the hand-finger combination therefore bears an iconic relation to the action of placing the $3-saving that Marianne makes each week. Overlapping the second utterance of 'dollars', Mario says with a rising, that is, questioning intonation 'three?' Pointing to the third cell of the second row, Jeanne asks, 'What are you writing here?' (turn 200). Mario utters with rising (questioning) intonation 'three?', which we may hear as tentative, the offering of an uncertain response itself requesting an affirmation. There is a pause. Jeanne does not respond verbally, but taps with her right index finger on the '3' in the cell for the 'number of the week'. Mario responds, 'plus three plus three'. There is an affirming 'yes' (the pitch is rising to over 700 Hz and then returning to normal, see below), a sign that what Mario has said now is precisely what Jeanne wants and has been wanting him to do. He has provided for the first time not the total amount in the piggybank but rather, the amount in terms of repeated additions. That is, in the course of a first objectification, he has articulated the structure that is required before the next step – namely the transition to the multiplicative structure. To be achieved, this transition will need further objectification.

Before we pursue our analysis it is worthy to reflect on that what has made the reached objectification possible. If we go back to the turns 191–204, we notice that in the course of the verbal interaction, Jeanne uses linguistic terms to emphasize the repeated addition. For instance, she uses additive terms, such as 'to add'. However, the additive terms appeared embedded in a temporal dimension, marked by adverbs (e.g., 'already', 'then'). She says: 'So. In the first week, there are already $6. Then you add $3'. The temporal dimension brought forward by the adverbs serves to organize the appearance of the arithmetic signs in the emerging formula. Next, Jeanne moves to week 2 and then to week 3 and resort again to temporal adverbs to create the possibility for Mario to see the corresponding arithmetic expression, that is to say, '6 + 3 + 3' and '6 + 3 + 3 + 3'. But there are also additional elements into play. Pauses and tapping are two of them. Indeed, we see that just prior to the articulation of the repeated addition, which is immediately acknowledge by a 'yes', there is a long pause in which Jeanne also taps onto a cell of the table of values. When we look back over the transcript (e.g., turns 182–184 or 186–187), we observe that Jeanne confirms an answer or continues eliciting when the preceding utterance is correct. That is, confirming or continuing are ways in which preceding responses are marked as appropriate, whereas long pauses sometimes followed by reiteration of a question or production of a new question (e.g.,

turns 191–193 or 196–198) mark incomplete or insufficient responses. In turn 202, there is a pause that becomes longer and longer. Jeanne taps on a cell. Something else has to come, but it is not from Jeanne, who, in not taking the turn at talk allows Mario to continue. Tapping can be perceived as a further encouragement to produce something that goes into the cell referred to by means of the indexical pointing. And what is to come has to satisfy the preceding question, 'what are you going to write here?' But pausing and tapping are accompanied by other semiotic resources. As noted, Jeanne resorts to emphasis in word pronunciation and gestures. These various elements (word emphasis, pointing gestures, tapping, pausing, word emphasis, mathematical signs) play a fundamental role in the unfolding process of objectification. They are what we call *semiotic means of objectification* (Radford 2003). Their importance resides in the fact that they emphasize different aspects of the objectifying process. Thus, indexical gestures call *visual attention* to specific spatial key locations in the table and the formula; linguistic emphasis (pitch) call *aural attention* to what is been said, allowing the speaker to highlight some words and leave others in a relative background. Temporal adverbs call attention to *temporal attention*: they make it possible to imagine the *temporal unfolding* of events. Arithmetic signs capture quantitative aspects of the problem at hand. Rhythm – as presented in language, pausing, tapping, gesture, and repetition – provides analogical elements to highlight patterns to be attended (Radford et al. 2007). Semiotic means of objectification do not work additively. Their nature is systemic. Generally speaking, in isolation they have poor capacities to convey meaning. We could imaginarily suppress all but Jeanne's gestures in the previous fragment. We would hardly be able to make sense of what is going on. Semiotic means of objectification are powerful in building meanings and allowing objectification to occur because individuals use them in a coordinated manner. Each semiotic means of objectification puts forward a particular dimension of meaning (signification); the coordination of all these dimensions results in a complex composite meaning that is central in the process of objectification, as seen in the previous fragment. Usually, as the objectification proceeds, the coordination is refined more and more resulting in what we have termed a *semiotic contraction* (Radford 2009a). For instance, in turn 198, gesturing actions and speech go faster. Jeanne might have felt that she does not need to go again through the whole details as Mario was giving hints that the arithmetic additive expression was becoming apparent to him.

So, a complex sensuous or sympractical process mediated by an extraordinary array of coordinated semiotic resources (the semiotic means of objectification) provokes or makes the objectification possible. When Mario articulates two more 3s at the end of the fragment, which Jeanne immediately marks with a positive valuation, the structure that has now emerged verbally and in its written *re*presentation is that same structure that the mathematically knowledgeable person can already notice in Jeanne's previous productions. But this structure now emerges from the actions that Mario and Jeanne jointly produce in the objectification process. Because the objectification process involves Mario and Jeanne, *this* structure

cannot be reduced to one or the other. Objectification is a joint work in the emergence of the structure, in its revelation.

To understand the objectification, that is the revealing of the structure and its appearance in Mario's consciousness from an obuchenie perspective, that is, in the way Vygotsky understands teaching-learning unit, we need to focus on the relation. During development, what the participants do comes to be combined in one person. But it is not as if the learner constructs the function on his/her own following joint activity. The focus on the societal *relation* forces us to recognize that anything the learner will eventually do, s/he has *already done* as part of the relation. In addition, however, the part played by another person – here Jeanne who pointed, emphasized, south to shift attention, paused and who, by waiting, marked the answer as unfinished – comes to be added to the repertoire of the learner. Here, it is the attention attributed to the additional two occurrences of '3'. It is this attention that initially was there in Mario's behavior and that the social relation made possible to emerge. If Mario has learned, then we might expect him to produce the same with the remaining cells. But let us follow what happens next. What Mario learns *is not subjective*, his own, as it would be in constructivist theorizing, but it is social through and through. *All* psychological functions are societal rather than biological and they '*are internalized relations of social order, transferred to the individual personality*, the basis of the social structure of the personality' (ibid.: 58, original emphasis). But Vygotsky's articulation creates ambiguity when he writes about the transfer to the individual, when in fact scholars now realize that whatever function always is social, always articulated *qua* social relation. Even Vygotsky himself describes the reading of one's own notes, for example, in a diary, as a social relation: 'to read one's own jottings, to write for oneself, means to relate to oneself as to another' (ibid.: 58).

Vygotsky did not just dream up this way of thinking about human development, the 'principal driving force' of which is historically constituted societal interaction. Rather, his ideas fundamentally derive from the way in which Marx/Engels conceive not only of human beings but also about the relationship between individual and collective (society). Vygotsky (1989) makes repeated reference to Marx/Engels, especially to *German Ideology*, where the authors articulate their fundamental position on epistemology. In their sixth thesis on Feuerbach, the authors state that 'human nature is not an abstractum contained in the single individual. In its reality it [the individual] is the ensemble of societal relations' (Marx/Engels 1958: 6).[2] That is, the individual is a particular of the species *human being* only when it is a concrete realization of the latter – otherwise this human nature is an abstractum. This requires the individual to be and express humanness, which derives precisely from the particular forms of relations that is has in society with others. The individual is product and producer of societal relation, which, in

[2] The English translation of Vygotsky's original text replaces his (and Marx/Engel's) adjective *societal* (obschestvennix) by the adjective *social* (sozialnie), which Vygotsky also uses in the same text but at different places. From a critical (psychological) perspective, this is problematic, as the first adjective implies that development is shaped by society, including all its class-related inequalities, whereas the latter adjective does not imply the same.

the present instance, is the production of the cultural knowledge that we know as algebra.

Toward Independent Acting – A Second Objectification

At the moment that the additive pattern has emerged, there is a first sigh of relief. But following Jeanne's elated 'Yes', there is a pause, which eventually becomes longer that any one that the interactional relation has seen so far. Mario is writing the three '3s' into the third cell of the table of values. The next fragment begins when Jeanne says 'plus?' with a rising intonation, flagging it as a question, where the question pertains to the slot left open (given that their pattern was to add something). There is a pause, before Mario offers tentatively (rising intonation), 'six?' Jeanne confirms using a gesture that may be glossed as 'Yea, you got it' or 'This is it' (turn 209). She then continues by summarizing, 'Originally you have started with six.' The intonation is in the constative and confirming manner. Jeanne then continues, 'So instead of writing three plus three plus three, what would you be able to do?' Her right-hand index finger taps three times on the cell, slightly moving as she touches each of the '3s' in the cell corresponding to week 3. She moves the finger downward into the third row of the table and, while tapping on the '3 x' already written there, she utters 'Three times?' (turn 210). Here we can see again the subtle and complex coordination of gesture, pausing, tapping, words, word intonation, and mathematical signs that are mobilized in an attempt to create the conditions for the multiplicative-additive structure '3 x 3 + 6' to emerge. The manner in which these semiotic means of objectification are mobilized constitute what we have termed a *semiotic node* (Radford et al. 2003). In more precise terms, a semiotic node is a segment of semiotic activity where actions, gestures, words, mathematical signs, and other semiotic resources are coordinated to achieve knowledge objectification.

Let us return to Mario and Jeanne again. (Thérèse has been walking away and returning to the table, obviously having completed the task. She is turning to Aurélie, who is filling in the sixth cell of her table of values, speaking under her breath so that the table microphone cannot pick up what she is saying.)

Fragment 4.3a

```
204  J:  yES:: ((he writes))
205      (4.38)
206      plU:S::? ((points))
207      (0.96)
208  M:  s::sIX:?
209      (0.44) ((Jeanne moves rH to right, opens palm up, as
         if confirming 'this is it'))
210  J:  orIGinally you started with sIX. (0.39) so; instead of
         writing three plus three plus three, what would you be
         able to do. (0.27) three tIMEs?
211      (0.84)
212  M:  s::IX:?
```

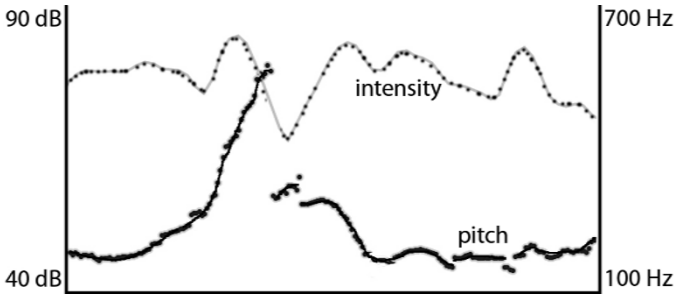

Fig. 4.4. Jeannine's pitch moves from about 180 Hz to 590 Hz and back down to her normal range while producing the 'excited' acknowledgment 'oui' (yes) (turn 215).

```
213        (0.88) ((J moves finger sideways repeatedly between
           two cells?))
214        no three times three
215   J:   yES::. (('excited' 'yes' [prosody in Fig. 4.4], makes
           the same rH movement to right, opens palm toward
           ceiling, Fig. 4.5)) (1.21)
           its just on the bottom its a [shortcut]
216   A:                                [madAMe: ]
```

There is a pause, before Mario responds tentatively (rising intonation), 'Six?' to which Jeanne responds by moving her index from week 2 to week 3 repeatedly. Mario continues now in a constative and in firm voice, 'three times three' (turn 214). Jeanne affirms, both with a clear 'Yes' the pitch of which moves way up and then returns, an intonation that seems to say, 'Yes, you got it'. Again, it is an emotionality that constitutes not only a response to the previous action, a way of creating a positive reception, but also an expression of Jeanne's valuation of her own success. She continues by explaining what he has been doing, 'It is just, on the bottom it is a shortcut' (turn 215; intonation in Fig. 4.4).

In turn 214, the multiplicative structure is emerging. But we know that we cannot analyze social interaction by confronting individual contributions. Rather, the minimal unit is the turn pair, which itself cannot be broken out of the activity as a whole – which, here, is an obuchenie (teaching-learning) activity. This turn is the response of an earlier one in which Jeanne utters a phrase that – though intonated as a constative – has the grammatical structure of a question 'what would you be able to do?' Then Jeanne utters 'three times', which we know to be the first part of the response, but which Mario has to discover as such. There is some further information demonstrating that this utterance is the first part of the response, because it is a repetition of the cell contents already available. It is to this cell content that the index finger is pointing. But the response initially is 'six' (turn 212). That is, if Jeanne has intended for a '3' to be uttered, this intention has not materialized. What matters to the unfolding event is that the response has been '6'. The next turn, which is typical of the *IRE* turn sequence that we find in schools, is an evaluation.

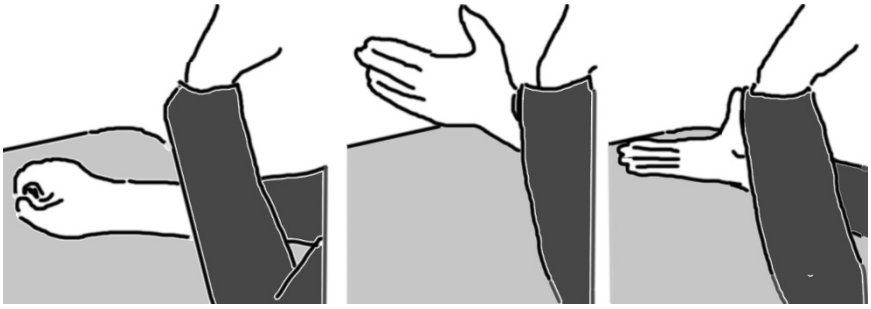

Fig. 4.5. Jeanne makes a hand movement that might be glossed as 'you got it' (turn 215).

Jeanne says nothing. There is a pause that unfolds. Simultaneously, Jeanne's index finger moves back and forth between the second and the third cell apparently attempting to provoke a different response. That is, the absence of a confirmation and the apparent solicitation of another response or an addition allow the understanding that the preceding response is not the one to be provided here. In responding 'no three times three', Mario confirms such a hearing. He has self-corrected an answer that in this sequence of turn was marked as incorrect and he provides the one sought for – which is marked as such by an excited 'oUI::' ('yES::'). The multiplicative structure, therefore, is the result of a societal relation; it is produced in the interaction between *this* teacher and *this* student. The 'three *times* three' is the response to 'instead of writing three plus three plus three, what would you be able to do'; in fact, from the perspective of a social relation as phenomenon *sui generis*, the former utterance concretizes the latter as a question. The two turns belong together, the first setting up the second as a response, the second confirming the first as a question. The tapping of the index finger constitutes a further sign that orients the participants toward the repeated threes in the second row and then to the beginning of a multiplicative structure '3 x' in the row below.

We can understand the actions performed by Jeanne as objectifying orienting moments of the societal relation. The following quotation exhibits the usefulness of Vygotsky's ideas for understanding teaching-learning of the kind that we observe here:

> If relationships among people genetically underlie psychological functions, then (1) it is ridiculous to look for specific centers of higher psychological functions or supreme functions in the cortex (or in the frontal lobes; Pavlov); (2) they must be explained not on the basis of internal organic relations (regulation) but in external terms, on the basis of the fact that man controls the activity of his brain from without through stimuli; (3) they are not natural structures, but constructs; (4) the basic principle of the functioning of higher functions (personality) is social, entailing *interaction* . . . of functions, in place of interaction between people. (Vygotsky 1989: 59)

The author suggests that the psychological functions are to be found in 'external terms', because 'man controls the activity of his brain from without through stimuli'. In interaction, the stimuli may be performed by another person. Whether these have bearing on the movement of the interaction is an empirical matter. In the present instance, we see that the various semiotic means of objectification, such as uttering the '3s' together with the indexical pointing to the '3s' on the paper and a subsequent index to the cell below where there is the beginning of the multiplicative structure '3 x' constitutes exactly the kind of stimuli Vygotsky is writing about. After the fact we can ascertain that they have been realized as stimuli as evident from the subsequent actions. It is this attention that occurs on the outside, because it is performed simultaneously by another person, Mario. *He* already attends to, looks at the '3s', listens to the repeated sound [tʀwa] ('trois', equivalent to [θri:], 'three'), and focuses on the beginning of the multiplicative structure already present in written form ('3 x') and performed verbally ([trwa fwa] 'trois fois', Engl. 'three times'). Mario, then, produces attention and salience, initially in response to semiotic means of objectification and, if these performative productions have left memorable traces, then he will produce attention and salience without the external stimuli produced in and as part of the societal relation with others.

We can see here that the normal framing of learning is not correct. It is not that something happens *between* Mario and Jeanne and subsequently, perhaps some time later, the same is happening *within* Mario. The same attention and salience that might allow Mario at some later time – whether he actually does it will have to be confirmed in our analysis of the events that follow – already is present at this instant. The important point to retain is, as Vygotsky suggests, that this attention and salience, the 'interaction of functions', is taking the 'place of interaction between people'. It cannot be fundamentally different, as any psychological function, *if it is a human characteristic*, must be so both at the collective (general) and the individual (particular) levels. If the realization that '3 + 3 + 3' (or [θri: plʌs θri: plʌs θri:] 'three plus three plus three') can be written as '3 x 3' is a *human* characteristic, here realized on the part of Mario, then it is necessarily (the result of) a societal relation. These realization are not, as Vygotsky maintains in the quotation, 'natural structures'; rather, they are cultural 'constructs'. Realizations and constructs can be reproduced at will precisely because there are collective possibilities, possibilities that arise from and for social interactions, and are always concretized by real living human beings. The reproducibility, which is a societal relation and social fact, is precisely what makes mathematics cultural-objective because it is grounded in the cultural-subjective performances of the individual (Husserl 1939).

The instance in which the performance of the multiplicative structure is greeted with excitement is further interesting because of the way in which this evaluation, itself a social fact arising from social interaction, is realized. We observe that the prosodic parameters that psychological studies have shown to express elation/joy all are present in Jeanne's speech (Scherer 1989). The mean pitch goes up strongly, here from 180 Hz to 590 Hz and then returns to about 180 Hz (Fig. 4.4). This also shows that the range varies substantially as does the contour. In addition, the F1 mean decreases, as observed in the experimental literature, from about 2,100 Hz to

between 1,270 and 1,170 Hz. The speech intensity goes from 76 dB to 83 dB, which corresponds to more than a quadrupling of the speech intensity, another change that is considerable as observed. In other words, the hand gesture (Fig. 4.5) together with the speech parameters all express and are manifestations of elation/joy – a positive emotional expression that comes precisely at the instant that Mario responds correctly and, hereby indicated, as the teacher wanted him to respond. It is an expression of the success of the preceding 'teaching moves'.

The 'oUI::' ('yES::') is, within the *IRE* turn-taking routine, only the final part of an irreducible pattern: by itself the utterance has no sense or function. The function is precisely as the third moment ('E') of the *IRE* structure. But we have just noticed that the multiplicative structure has arisen from the *IR* parts, in the to-and-fro of the question-response cycles, embedded in the objectifying process and mediated by an array of semiotic resources. In fact, this is not quite true, for we have not yet included the third moment of the irreducible pattern.[3] That is, the social interaction that produces the multiplicative structure *includes* the 'oUI::' as its constitutive moment – the poetic moment of objectification – bringing about the social evaluation of the performance *as* a cultural-historically typical performance rather than as something that has no sense. From the perspective of the learner, who does not yet know the structure, the assertion that some act (verbal utterance, physical movement) is a social act, an attribution as such as to be made: 'We become ourselves through others' (Vygotsky 1989: 56). In the same way that a mother confirms the child's hand-arm movement as a reaching gesture, Jeanne here confirms Mario's action as the culturally recognized and expected one. That is, the 'oUI::' is the initially external stimulus confirming the immediately preceding action as one that is cultural-historically recognized. It *is* the recognition as a societal relation, which itself is an aspect of the higher function of the multiplicative pattern once the (inherently cultural-historical) performance is completed in and through Mario.

Are further semiotic means of objectification (external stimuli) of the kind we observe above necessary or will the multiplicative structure emerge again in response to questions but without the particular stimuli that oriented attention and salience in the previous fragment? To find the answer, let us turn to the second part of Fragment 4.3. Jeanne now moves on to the fourth week (turn 218), asking Mario how many $3s he got. Her left-hand index finger points to goblet #4, then moves to the first goblet and then moves pointing to each of the intervening goblets back to goblet #4. There is a pause, but then he says 'four' writing the four 3s into the cell (turn 220). Just as he finishes, Jeanne provides a confirming 'kay' and, following a pause, asks, 'Instead of doing three plus three plus three plus three, what could you write here' while pointing to the third row cell of week 4. As she utters the first part of the turn, Jeanne moves her right-hand index finger back and forth across the four '3s' that Mario has written into the second-row cell of week 4 and, as she utters the last part, she moves the index finger to the third row of the table of values.

[3] We insist: Irreducible means that the *IRE* pattern stands and falls with all its parts. It is not an IRE pattern if any of its constitutive moments 'I', 'R', or 'E' were missing. We cannot look at a turn and say it is an 'I', 'R', or 'E' unless the structure as a whole is present; and this whole is present only if all the parts are present.

Fig. 4.6. Even evaluations communicated by means of gesture pertain both to the cognitive and the emotive aspects of activity reproduce and transform its emotive moments (turn 225).

Mario offers tentatively, 'four times three?' (turn 224), and Jeanne confirms with a two-handed gesture moving hands sideward and turning palms open toward the ceiling (Fig. 4.6), as if saying, 'You got it'. In this passage, Mario produces the expected answers in a quicker manner. The various signs that mediate the interaction (spoken words, gestures, rhythm, etc.) are displayed and articulated in a faster manner, constituting what we have termed elsewhere a 'semiotic contraction' (Radford 2008b). The semiotic contraction is a distinctive index of achievement of the objectification process.[4]

Fragment 4.3b

```
217      (0.42)
218  J:  your fourth week; (.) how mANY three dollars do you
         have.
219      (1.00)
220  M:  u:m:::  (1.73) fo. ((Fills table, Thérèse makes
         noises))
221      (9.48) ((writes 4 '3s'))
222  J:  <<pp>kay> (0.97) instead of doing three plus three
         plus three plus thrEE whAT are you going to wrITE
         here? ((Points to the row on the bottom of the table
         of values))
223      (0.66)
224  M:  uh:m:: (1.36) four times thrEE?
225  J:  ((2-handed gesture sidewards, opening palm upward:
         'you got it' [Fig. 4.6]))
226      (3.83)
227      i=think you understand now. uh?
228      (50.93) ((Mario slightly nods, writes, after 26
         seconds looks at Therese's worksheet, back at his
         own))
229 M:   <<confident>ME i understand now.>
```

[4] For similar examples, see, among others, Radford 2010 and Radford and Roth 2010.

In this situation again, we see the additive structure '3 + 3 + 3 + 3 + 6' and the corresponding multiplicative structures '4 x 3 + 6' – a correspondence that is instantiated by the index finger moving from the second to the third row – emerge *in and as* societal relation. For many readers struggling to understand how the structure *is* a societal relation, it may help to use the more familiar *IRE* structure, which is an irreducible cultural pattern of an institutional nature. The additive structure emerges *in and from* the initiation 'your forth week . . . how many three dollars do you have' (turn 218), the response constituted by the verbal 'fo[ur]' (turn 220) and the writing of '3 + 3 + 3 + 3' (turn 221), followed by the evaluation 'kay' (turn 222). Here *IRE* denotes the process of sequentially taking turns, where each turn is irreducibly embedded in and constitutive of a sequence with a preceding and a succeeding turn. It thereby is a social phenomenon that cannot be reduced to, yet requires the participation of, either Mario and Jeanne. There is an additional pointer to the additive structure: Once a person knows that the response is '4', s/he can see the hand gesture itself as an iconic index to the repeated addition. The hand pointing into each goblet moving from the first to the fourth articulates both the placement of something, the repetition, and the movement in time as the hand moves from right to left over the four goblets.

The multiplicative structure, too, exists as societal relation first. It is initiated in and by the utterance 'instead of doing three plus three plus three plus three, what are you going to do here' accompanied by the right-hand index finger placed on the third-row cell of week 4. The second moment of the three-part *IRE* structure is produced in turn 222, 'four times three'. Again, it is the third moment of the *IRE* social relation – realized across the turns 222, 224, and 225 – which finalizes the objectification of what subsequently will exist, if what we term by learning has occurred, as a psychological function.

Nearly 4 seconds later, she comments, 'I think you understand now. Uh?' (turn 227). Mario slightly, almost unnoticeably nods and continues to fill up his worksheet. Jeanne immediately raises her head gazing at a student who is coming around the table and then stops next to her and speaks to Jeanne. The teacher then gets up and walks to another set of desks. Some 26 seconds after the last utterance, Mario leans over in the direction of Thérèse's desk, gazes at her worksheet, then returns to work on his own (turn 228). Another 24 seconds later, he confidently utters, 'Me, I understand now' (turn 229). He has completed the cells for week 5 and 6 on his own. That is, the structures '3 + 3 + 3 + 3 + 3 + 6' and '5 x 3 + 6' now appear in rows 2 and 3 of the table; similarly, the structure '3 + 3 + 3 + 3 + 3 + 3 + 6' appears in the second-row cell of week 6, but there is a variance with the expected structure in the final cell: '6 x 3 + 23' (Fig. 2.1). We note that the complete structure is not yet produced without error, for in the instance of week 5, the final part '+ 6' (see underline) was already part of the cell. It is part of the structure (condition); when this part is absent (for the first time as we go from left to right in the table), the corresponding response is (still) incorrect. Given that the results of the second-row cell and the first part of the third-row cell have been completed as anticipated, we may say that the entire multiplicative structure has been objectified

at least in part from the societal relation and now is observable as a psychological function.

Social Relations, Obuchenie, and Developmental Possibilities

> We become ourselves through others. (Vygotsky 1989: 56)

Vygotsky notes that we become who we are through others. He makes direct reference elsewhere in the article to a note by Marx/Engels concerning Peter and Paul. The original source is actually a footnote which states that 'only through the relation to the human [*Mensch*] Paul as similar to himself does the human Peter relate to himself as human. In this, Paul also . . . appears to him as manifestation of the genus human' (Marx/Engels 1962: 67, note 25). To summarize, we do something like adding and multiplying numbers, as typical human actions, *as a consequence* of the institutional relations we entertain with others in society. Saving money and modeling the saving of money using algebra *are forms of human relations* and emerge *only* in and because of the kind of relations that we entertain within a cultural-historically contingent society. We can understand *cultural development* and *obuchenie* activity only when we properly relate the general – human society and the kinds of relations that constitute and are the results of it – and the particular, the concrete living human being whose actions concretely realize the societal activity. This, to us, appears to be both the essence of Vygotsky's thoughts about development and the most difficult aspect, because it requires dialectical thinking. In that form of thinking, the general is as concrete as the particular, for it always realizes itself in the particular, which always is the concretization (*object*ification) of a general pattern. If it were not in that way, an individual would never be able to produce signs that are inherently shared with others and communication would not be possible at all. There is no sense in speaking if the word is not always already a general feature of society and culture, which allows the addressee to hear and understand something of significance.

We note that additive and multiplicative structures are societal relations first. But in the early part of the obuchenie situation, the societal relation did not realize and produce these patterns. The possibility for the societal relation to produce these structures has itself to be created *in and through* societal relations. The structure exists *as* societal relation rather than *in* the relation, the way many Vygotsky interpreters including the most illustrious ones seem to read it.[5] We can gloss Jeanne's actions as repeated attempts at opening up this possibility. These attempts themselves are societal relations in which the 'non-algebraic' response, Mario's emotional articulations, and his description 'I don't understand' all are assessment forms that the intentions and the effects of Jeanne's actions have not coincided. Whether any subsequent initiation will bring about the societal relation that actu-

[5] For a critique of the positions Mike Cole or Jaan Valsiner take see Veresov, 2004, who suggests that there is a missing link in their interpretations of the general (genetic) law of cultural development.

ally makes the two mathematical structures emerge is not known – Jeanne would not have to make repeat initiations if she had known what was required. What is required is always unknown, because interaction always unfolds in unexpected ways. The obuchenie activity therefore is not simply one of selecting a teaching strategy and applying it. Rather, the appropriate strategy itself is an emergent outcome of the societal interaction (this, by the way, is one of the challenges in the design of educational software). Or, rather, in the way the mathematical structure has emerged for Mario, the pedagogical action has emerged for Jeanne. It, too, is a societal relation of the structure *I*(*RE*), where the teacher initiates and the student responds also is the evaluation of the preceding move as an appropriate or promising one. The 'correct' strategy, and one that can be taught to other teachers, is the result of a societal relation first. In this sense, therefore, *obuchenie* (teaching-learning) means, from Jeanne's perspective learning-to-teach-by-teaching. Unaware of this, Mario also produces an assessment together with the 'not-yet-algebraic' response. In this sense, his actions allow Jeanne to know and learn. Mario is both contributing to teaching and learning simultaneously. (Learning is occurring even and precisely when a 'wrong' response is provided, namely that *this* response is an inappropriate one; and this, too, initially is a social relation.)

Development frequently is conceived of as the 'socialization' of the child. In traditional, bourgeois psychology and sociology, the child is taken as an individual that is unsocialized and that is fashioned in the course of the process of socialization to become a socialized individual (Holzkamp 1979). Thus, for Piaget there are two channels of socialization, one in relation with peers, the other in relation with adults. 'The playfellow provides the opportunity for such social conduct as will determine the true socialization of intelligence. Conversely, where equality between playmates prevents questions and interrogation, the adult is there to supply an answer' (Piaget 2002: 262–263). That is, the child develops an intelligence of its own, and this 'wild' intelligence subsequently is socialized in and through social interactions with peers and adults. The preceding analyses suggest that the direction of the movement is opposite. There are forms of societal relations, here realized in and through the interactions Mario and Jeanne entertain. It is the individualization of these societal relations that constitutes the development we observe. This is precisely the position that is taken in cultural-historical activity theory: 'Development proceeds not toward socialization, but toward *individualization* of social functions' (Vygotsky 1989: 61).

We can think of mathematics as emerging in the same way that language does. Again, Vygotsky points us to Marx/Engels as the origin for his own thinking. These authors suggest: 'language *is* practical – also for other people existing – consciousness, thus, only in this way, also for me existing real consciousness, and language emerges, as consciousness, from the need, the genuine necessity of relations with other people' (Marx/Engels 1958: 30). As mentioned previously, consciousness of the individual is the consequence of collective consciousness. Consciousness can really exist for me only because it always already exists for other people as well. Being human means that that *my* consciousness *is* a concrete realization of *human* consciousness; being human means that my mathematical consciousness *is*

collective mathematical consciousness. Consciousness, however, is irremediably tied to societal relations. Development therefore is achieved when mathematical development is the individualization of societal relations rather than the socialization of some idiosyncratic 'constructions'.

The realization of the additive and multiplicative structures arises first in/as the societal relation; the conscious realization is enacted in the societal relation; because the 'psychic reality that opens up immediately before us is the subjective world of consciousness' (Leontjew 1982: 121), the conscious realization of the pattern *is the psychic reality* at this instant. That is, this aspect of mathematical consciousness emerges in and as societal relation. Development is said to have occurred when this form of consciousness emerges for the individual again, but now without the external stimulation that occurs in the societal relation. It is precisely for this reason that we need to be concerned with consciousness, as it constitutes the psychic reality that determines what the individual does and why. This is also why Vygotsky orients us toward 'man' rather than toward the 'brain', toward the real sensuous-valuational and volitional person rather than to the mental act. Vygotsky and Bakhtin articulate in almost identical ways the same concern when they write about scholars who focus on the mind at the expense of focusing on the person. The former says that the weakness of traditional psychology is that 'the thought process appears as an autonomous flow of "thoughts thinking themselves", segregated from the fullness of life, from the personal needs and interests, the inclinations and impulses of the thinker' (Vygotsky 1986: 10). The latter suggests that Kantian constructivism detaches judgment from real life in a way that 'the detached content of the cognitional act [seemingly] comes to be governed by its own immanent laws, according to which it then develops as if it had a will of its own' (Bakhtin 1993: 7). As a result of this abstractive detachment, 'we are simply no longer present in it [thought] as individually and answerably human beings' (ibid.: 7). But our episode from a fourth-grade mathematics classroom forces us to consider the human being, for, as we see at the very end, it is not just a brain that produces a (meta-) cognitive statement about itself, but it is a real person who is proud and elated about understanding: 'Me, I now understand'. It is a *person* that developed in and through the social relations it has entertained – not a brain that also acted. Thus, we need to focus on 'man', who 'regulates or controls his brain, the brain does not control man' (Vygotsky 1989: 71). This control first occurs by means of external (i.e., cultural) stimuli in societal relations and subsequently, by means of signs (e.g., memory), the stimulation of the brain is displaced within the person though, for Vygotsky and us, it necessarily remains societal.

In a strong sense, it does not suffice to use, as Vygotsky often does, the adjective social and to disconnect the discussion from the implications that come with the adjective societal. If school is to prepare for life in society – the Romans already noted *non scholae sed vitae discimus* [we learn not for school but for life] – then the relations must be societal. The individual is societal in nature if and only if the objectified relations always and already are societal. It is therefore not surprising that *serious* cultural-historical activity theoretic scholars, such as Leont'ev and Critical Psychologists, retain the use of the adjective *societal*. This immediately

allows us to connect up to critical scholarship in the area of mathematics education, for if the relations in which students participate during their school mathematics experiences are societal in nature, then this means that these also bear all the marks of class-related inequities in a given society. It is then possible, for example, to think that children coming from particular classes are less prepared to participate in the *actual* relations typical of classrooms with a middle class ethos and organization. That is, these children will immediately be disadvantaged because the kinds of relations they are familiar with are not the ones that they encounter in the typical classroom (Eckert 1989). It is not that algebra in itself is difficult or too abstract for and biased again under class and working class children. Rather, the kinds of societal relations that constitute algebra initially in the public sphere are not of the relations with which middle und upper class children are familiar and knowledgeably co-produce.