SEMIOTIC PERSPECTIVES ON THE TEACHING AND LEARNING OF MATHEMATICS SERIES

A Cultural-Historical Perspective on Mathematics Teaching and Learning

Wolff-Michael Roth & Luis Radford

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A Cultural-Historical Perspective on Mathematics Teaching and Learning

By

Wolff-Michael Roth
University of Victoria, Canada

and

Luis Radford
Université Laurentienne, Ontario, Canada

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Preface

Eighty years ago, L. S. Vygotsky complained that psychology was misled in studying thought independent of emotion. This situation has not significantly changed, as most learning scientists continue to study cognition independent of emotion. Situated in activity theory – as developed by A. N. Leont'ev and Klaus Holzkamp – we investigate in this book the mutually constitutive nature of cognition and emotion. Activity theory not only stipulates the relation between individual and culture in very different ways than any other theory, but also emphasizes the constitutive role of emotions in knowing and being.

To investigate the mutually constitutive nature of cognition and emotion we draw on data from our longitudinal research program about the teaching and learning of algebra in elementary schools. We show (a) how emotions are reproduced and transformed in and through activity and (b) that in assessments of students about their progress in the activity, cognitive and emotional dimensions cannot be separated. Second, our analysis exhibits three main features: (a) the irreducible connection between emotion and cognition mediates teacher-student interactions; (b) the zone of proximal development is itself a historical and cultural emergent product of joint teacher-students activity; and (c) as an outcome of joint activity, the object/motive of activity *emerges* as the real outcome of the learning activity. We use the results of this study to propose (a) a different conceptualization of the zone of proximal development, (b) activity theory as an alternative to learning as individual/social construction, and (c) a way of understanding the material/ideal nature of objects in activity. This leads us to outline a subject's participation in activity and its connection to all the other activities in which a subject engages in the course of its everyday life. We conclude with a proposal for a cultural-historical science of mathematical learning.

In contemporary research, intellect – thought, cognition – and affect tend to be different domains of study; the latter, at best, is thought of as a factor. An example of such thinking is the 'hot cognition' approach, which postulates that 'classroom contextual factors', 'motivational factors', and 'cognitive factors' influence learning and conceptual change from the outside. The problem with this line of theoriz-

ing is that factors remain external to the sensing subject, and 'being externalities … are indifferent towards each other, and lack the necessity for one another that ought to lie in the relation between an *outer* and an *inner*' (Hegel 1979: 236). This situation has been discussed as problematic: The separation of intellect and affect 'as subjects of study is a major weakness of traditional psychology, since it makes the thought process appear as an autonomous flow of "thoughts thinking themselves", segregated from the fullness of life, from the personal needs and interests, the inclinations and impulses of the thinker' (Vygotsky 1986: 10). This segregation is a problem because it does not allow us to understand the *immediate* – i.e., unmediated – influences that thought and affect have on each other. Segregating intellect and affect closes the door on understanding *why* the learners do what they do because there is no directionality or propensity inherent in thought that would give it a 'desire' to transform itself.

The lineage of research following Vygotsky has given rise to a different conception of the intellect-affect relation where the fullness of life, reality, is reflected psychologically as a primary sensuousness, comprising both cognition and emotion as irreducible, mutually constitutive moments. Understanding cognitive processes in learning without separating them from the affective – i.e., making the affective and the cognitive two manifestations of the same process – was a project pursued and developed in the Leont'ev-Holzkamp lineage of cultural-historical activity theory.[1] In contrast to the standard interpretation of activity theory, this line of work focuses on (a) the subject of activity in relation to society and (b) consciousness as a superordinate category to which cognition and emotion are subordinated. Vygotsky had asked for 'unit analysis' of a 'dynamic system of meaning in which the affective and the intellectual unite' (Vygotsky 1986: 10). An analysis of the unit reveals that there is a 'transmuted affective attitude toward the bit of reality' to which an idea refers.

In this book, we develop a conception of teaching and learning mathematics that is very different from two available standard conceptions. On the one hand, there is the Piagetian individual who *re*discovers through his/her own actions the rationality of mathematics; on the other hand, there is the (deficient) individual as empty vessel, who comes to be filled with the knowledge that culture makes available. Newer conceptions combine the two but nevertheless substantialize the individual and the collective (culture) and place them in an interactional relation. The purpose of this book is to articulate the role of emotion in teaching-learning activity, where it, as an index for the subjective valuation of the current state of the activity, is both reproduced and transformed. Affect, as Vygotsky points out, is essential in an understanding of knowing as a process that transforms itself. Similarly, we show that the very object/motive that drives the learning activity is accessible to students

[1] Much of this work is not available in English. But there exists a good introduction to Holzkamp's theory (Tolman 1994) and an edited volume presents major contributors to this theory (Tolman and Maiers 1991). The best-known member of this lineage publishing in English probably is Ole Dreier (e.g., 2008). Mutual references in a number of works between K. Holzkamp (e.g., 1993) and J. Lave (e.g., 1993) also suggests an affinity between the works of the two scholars.

only as an outcome of the activity, which, when it involves the teacher, also allows the zone of proximal development to emerge.

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Toward a Science of the Subject

> From the standpoint of the traditional bourgeois psychological approach to consciousness only what 'is found' in consciousness, or 'belongs' to it, is subject to study, i.e. separate psychological phenomena and processes and their mutual relations and connections. (Leontyev 1981: 223)[1]

In Western approaches to the mind, psychological processes, thinking, and consciousness have been generally conceived of as entities somehow lodged in an individual 'interior space'. This idea of an 'interior space' is not new. It was articulated by philosophers such as René Descartes and Gottfried Leibniz in the 17th and 18th centuries. To give but one example, let us note here that Leibniz contended that 'our ideas, even those of sensible things, come from within our soul' (Leibniz 1705/1949: 15). The contemporary concepts of the mind, thought, and consciousness as something individual are indeed elaborations of ideas that arose at the dawn of the Western modern period, right after the Renaissance. These were continued by later philosophers such as Giambattista Vico and Immanuel Kant. With more or less pronounced nuances, they have been translated into educational and psychological research. Within this context, tests and interviews serve the purpose of revealing to us what is going on *in* the students' heads. They are supposed to reveal what is found in *there*: thinking, psychological processes, self-appraisals, and even consciousness – hence, the exact kind of psychological approach to the mind that Leont'ev criticizes in the opening quote. Leont'ev found misleading in the individualistic approach the fact that it extracts consciousness, thinking, and psychological processes from the individual's mode of life and considers it abstractly. By referring to the individual's mode of life Leont'ev had in mind something that is much more than a collection of purely individual self-determining acts. His main point in fact was that our modes of life 'are built up in any set of sociocultural conditions' (Leontyev 1981: 224) so that rather than providing the external conditions for inner development they are consubstantial of the individual's modes of life.

[1] The name of Alexei N. Leont'ev is spelled differently on different publications (speeled also Leontyev or Leontiev, in English, and Leontjew, in German). In the text, we use the dominant spelling 'Leont'ev', but we use the spelling from the book cover in references.

This is why Leont'ev argued that the study of the subject has to be broadened, and, in fact, that it has to be reconceptualized.

Many mathematics educators have pleaded for a new form of understanding the question of mathematical thinking and learning. There is a plea to attend to the social and cultural contexts in which students think and come to know. Some attempts have been made in order to conceive of the student as a member of her social group. Yet, often, the distinction between the individual and the collective tends to be maintained: the student's cognition and the social are kept apart (Radford and Roth 2010; Roth and Radford 2010). They remain two poles that can be explored from a psychological (that is, individual) perspective or a social one (Cobb and Yackel 1996). Apart from the problematic dualistic approach to the individual and the social, the question how the specifically cultural-historical nature that characterizes the knowledge of any era is not resolved. This approach, as Mikhail Bakhtin (1981) recognized for the study of literature, does not allow us to understand the historical evolution of writing genres, which cannot be explained if we look at the genres themselves but which requires the study of popular language itself *in the way it is realized in everyday practical activity*. Similarly, this approach does not allow us to understand the historical evolution of mathematical genres, a suitable understanding of it requiring the investigation of the historical-cultural context at large (Radford 1997). For Leont'ev – as well as other researchers working from a strictly cultural-historical activity theoretic perspective – individual development inherently means cultural development. This is why the relationship between the individual and the sociocultural cannot be investigated by means of a dualistic approach.

How does this development (both at the individual and the cultural level) occur? It is in the answer to this question that the specificity of Leont'ev's approach resides. To answer the question, Leont'ev draws on historical dialectical materialism. In so doing, he offers a new way in which to theorize the question of the subject (its consciousness, its psychic processes, its personality) in ways that are not dualistic in nature. Leont'ev's answer is this: development occurs in and through relations with others in the pursuit of collectively motivated *activity*. From this point of view, the psyche is a culturally and historically evolved form of reflection. Hence something that can exist through two *mutually constitutive* terms: an 'I' and an 'Ego' (a complex that includes subjects and the symbolic and material reality that surrounds them). Thus, we agree with Leont'ev when he says that 'any psychic reflection is the result of a real connection, of a real interaction of a living, highly organized, material subject and the material reality around him' (Leontyev 1981: 225). Psychic reflection, consciousness, mind, or abstract knowledge cannot exist or 'arise without the subject's activity. It cannot help depending on activity, cannot help being subordinated to the subject's life relations realized by activity' (ibid.: 225). These statements and the implications that unfold from them constitute the fundamental ideas that we articulate in this book.

Cultural-historical activity theory has become an important lens for learning scientists to conceptualize phenomena of their interest.[2] As the name of the theory suggests, *activity* is the central organizing category in activity theory. It is defined as 'the nonadditive, molar unit of life for the material, corporeal subject. In a narrower sense (i.e., on the psychological level) it is the unit of life that is mediated by mental reflection. The real function of this unit is to orient the subject in the world of objects' (Leont'ev 1981: 46). Activity, therefore, is thought as a 'system with its own structure, its own internal transformations, and its own development' (ibid.: 46). It is something real that we observe, not something that we make up and hypothesize in our minds.

The concept of *activity* is difficult to write and think about in English in part because it conflates two concepts that are distinct in the languages in which the theory was originally conceived. Thus, throughout this book, we use the English term in the sense of the German/Russian term *Tätigkeit/deyatel'nost'*, a system that contributes to satisfying collective needs as part of the division of labor in society, rather than in the sense of *Aktivität/ aktivnost'*, being busy with something (Roth and Lee 2007).[3] This definition has important consequences for the way in which the relation between individual and society, individual and collective consciousness, and individual and collective cognition and emotion are understood and theorized. One lineage of activity theory, which has made it from its Soviet origins to the West via the work of Yrjö Engeström (1987), emphasizes structural-systemic (static) dimensions of activity. These structural dimensions are made salient in drawings of 'mediational' triangles. Although interesting, here we do not pursue this line of work. It appears to us that its emphasis on the systemic and structural elements that organize activity limits the understanding of *inter*subjective processes and the subject's perspective on activity.[4] The Leont'ev–Holzkamp lineage that we continue here in this book emphasizes the subject and (individual, collective) *consciousness*, that is, it theorizes persons within the structures of societal practice. In the following we articulate – at greater length because it is less known – the activity theory of the Vygotsky–Leont'ev–Holzkamp lineage.

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[2] See, for example, Bartolini Bussi and Mariotti 2008, Jaworski and Potari 2009, Matos 2010, or Williams 2009.

[3] We ground our reading in the German versions of Leont'ev and Marx, which does the original more justice than the English translation. For example, the Russian and German versions distinguish between two very different nouns, *Tätigkeit* (*deyatel'nost'* [деятельность]) and *Aktivität* (*activnost'* [активность]), both of which are rendered in English as 'activity'. The Russian and German versions distinguish phenomena that are societal (*gesellschaftlich, obshchestvennoĭ* [общественной]) from those that are social (*sozial, sozial'n* [социальн]), but the English version renders both as 'social'. In English, we find the word 'meaning' that translates *znachenie* (значение)/*Bedeutung* even though the Russian/German equivalents refer to an objective phenomenon at the cultural-historical level – something that is neither culturally transcendent (as the Kantian *things-in-themselves*) nor reducible to the personal sense (*Sinn, smisl* [смысл]) that students produce as they engage in classroom activity. Our specific word choices have been made such as to promote the very different reading of Leont'ev's work that the German version allows.

[4] For critical reviews of the strengths and weaknesses of the theory see Roth and Lee 2007; Roth et al. 2009.

Activity

All contemporary cultural-historical theories that include the category of *activity* ground themselves in Marx and Engels and their conceptualization of what makes humans different from other forms of life. Although every historical analysis of present-day culture must take into account biology and the natural conditions that provided the context for anthropogenesis, Marx and Engels focus their attention on the origin of the distinction between what will become humans and other animate forms. The dividing line is a particular form of joint activity: food productive activity. Humans begin to distinguish themselves 'as soon as they begin to *produce* their food, a step that is conditioned by their corporeal organization' (Marx/Engels 1958: 21). But this joint activity that individuals make possible also produces material life: 'By producing their food, humans indirectly produce material life itself' (ibid.: 21). That is, human beings no longer are subject to their life conditions but they transform these conditions and therefore transform life itself. The production constitutes 'a specific kind of activity of individuals, a specific way to exteriorize life, a *life form* specific to them' (ibid.: 21). This form of life not only is reproduced and transformed in activity but also shapes who and what individuals are: 'Individuals are in the way they externalize life. What they are falls together with production both with *what* they produce and *how* they produce it' (ibid.: 21). The nature of the individual, which is the topic of psychology, therefore is a function of the material production of and for life.

Activity, as a category in psychology, has been introduced and presented to psychology as one of three main concepts in a book originally entitled *Deyatel'nost', Sosnanie, Ličnost'* and translated into English as *Activity, Consciousness, Personality*. The book conceives of activity as 'a process, which contains those inner moving contradictions, differentiations, and transformations that produce the psychic, which is a necessary moment of the proper motion of activity in its development' (Leontjew 1982: 17–18).[5] It was intended to 'introduce to psychology those analytic units that carry within them the psychic reflection in its inseparability from those moments of human activity that produce and mediate it [psychic reflection]' (ibid.: 18). That is, activity is a *process*. This process contains inner contradictions, differentiations, and transformations that *produce the psychic* aspects of everyday life. These psychic aspects are a *necessary* moment of activity and responsible for the *development* thereof. Moreover, psychic reflection is *in-*

[5] The noun *moment* in dialectical materialism generally and in cultural-historical activity theory specifically refers to an identifiable structure – e.g., tool, subject, rule – that cannot be understood independently of the consideration of the whole. A moment therefore is not an element, because different elements can be assembled to produce an atom. Two moments are interdependent because both are *manifestations* of the whole; they cannot be added up because they do not constitute independent quantities. Even the website of the Finnish Center for Activity Theory contains this error, referring to the *moments of activity* – i.e., subject, object/motive, tools, division of labor, community, and rules – as 'elements'. Vygotsky (1986) adamantly rejects analysis in terms of 'elements' and asks for 'unit analysis'. In cultural-historical activity theory, *activity* is this minimal unit.

separable from real, practical activity that both produces and mediates the production of psychic reality in the human being.

The concept of 'psychic reflection' frequently is related to the idea that underlying cultural-historical activity theory is a mirror conception of the conscious mind. Nothing could be farther from the truth. In fact, Leont'ev rejects the mirror view and points out – thereby actually coming very close to the mirror neuron research of modern neurosciences – that every afferent activity (from sensory surfaces to the central nervous system) during perception is accompanied by efferent activity (from central nervous system to the sensory surfaces); and inversely, every efferent activity is associated with afferent activity. That is, during concrete activity – whether of a material or an ideal (mental) kind – the inner and outer worlds are intimately connected and irreducible to each other. They are but manifestations of activity that sublates – does away with and keeps – the distinction.

Leont'ev's category of activity poses tremendous problems for traditional psychology.[6] This is so because Leont'ev's category of activity and the related category of consciousness aim at allowing an understanding of the 'real *transitions* that connect the psychic of the concrete individual with societal consciousness and its forms' (Leontjew 1982: 18). Societal consciousness is that which presents itself in the form of ideologies (understood as the various contrasting, conflicting implicit and explicit systems of ideas). Leont'ev agrees with other scholars, including Bakhtin, that ideologies shape the individual's consciousness of real life and social relations, without of course determining it in any causal sense.

Activity is a process in a system of relations that realizes the societal nature of human beings – it is the locus where 'the subject and the social world are connected in such a way that both are re-produced and changed' (Dreier 2008: 22). Activity therefore is a unit that cannot be reduced to inner (cognitive) or outer (material) processes. Thus, 'the production of ideas, representations, of consciousness is first immediately tied to the material activity and material intercourse of people, language of real life. . . . Consciousness cannot ever be anything other than conscious being, and the Being of humans is the real life process' (Marx/Engels 1958: 26).[7] It is not consciousness and (constructivist) thought that has given rise to human life but, rather, communal human life has given rise to consciousness and thought.

There are then two major dimensions of activity theory: (a) Human activity has instrumental (tool) structure in the satisfaction of primary and secondary needs and (b) activity is implicated in the mutual relations with other human beings. Activity mediates not only the relation with the natural world but also the relation with other human beings. There is an in-principle oneness of outer and inner activities that constitute the mediating processes relations of humans and their world. 'It is

[6] Cultural-historical activity theorists reject all forms of psychologism and subjectivism – including those typical of constructivism – that place the 'fundamental value . . . at the head' (Bakhtin 1993: 60). Both 'subjectivism and psychologism are direct correlatives of objectivism' (ibid.: 29).

[7] Marx/Engels draw on the opportunity of the German language to write a powerful aphorism. That is, the word for 'consciousness', *Bewußtsein*, is composed of the same words as 'conscious Being', *bewußtes Sein*.

not consciousness that determines life', write Marx/Engels, 'but life determines consciousness' (ibid.: 27).

Leont'ev takes up these ideas in his psychology of the human being. He suggests activity to be the smallest unit that allows us to understand thought, consciousness, emotions, personality, subjectivity, and so on. He concludes that 'the activity of the individual human being constitutes a system integrated into the system of societal relations. Outside of these relations there is no human activity' (Leontjew 1982: 84). Activity theory therefore constitutes a systemic-analytic approach to individual thinking and consciousness. The concrete sensual nature of *sympractical* (i.e., joint practical) activity not only is the source of the concrete nature of inner reflections of activity in consciousness but also the source of the concrete nature of needs, emotions, and feelings. It is precisely for this reason that 'any higher psychological function was external; this means that it was social; before becoming a function, it was the social relation between two people' (Vygotsky 1989: 56). In and with the category of activity, we therefore no longer have the divide between the two realms that Vygotsky complains about. Rather, the two are but different, irreducible moments within the same phenomenon: both are inner reflections of concrete human sympractical motive-oriented activity.

Levels of Activity

For Leont'ev the chief difference in activities is to be found in the difference of their objects or motives. An object/motive (fishing, for instance) is what endows the activity with a particular *intent*. But activities involve also actions and specific contextual methods and means to carry out these operations. Actions become subordinated to *goals*, which of course are related to the object/motive but are not equal to it. 'The actions that realize activity are initiated by its motive but are directed toward the goal' (Leont'ev 1982: 103). Thus, to continue with the example of fishing, the actions of an individual may be directed to preparing the equipment for fishing; the actions of another individual may be directed to finding the bait. The goals are different, yet they are related to the same object/motive (fishing). There is still another aspect of activity that needs to be emphasized: the concrete basic constituents that make it possible to carry out the actions – something that Leont'ev calls *operations*.

This view of human activity rests hence on three interrelated levels: (a) the level of object/motive of activity, (b) the level of goals/actions, and (c) the level of operations. At each level there is a coupling of elements that accounts for a non-dissociable relationship between the subject and the activity in which it participates: activity is related to objects/motives, as actions are related to goals, as operations are related to conditions. We hence see that the first level connects activity to collective motives, which impart the activity with a certain conscious, collective teleology or end. At the second level we find actions that are oriented to realize conscious goals. At the third level we find operations that are stimulated by the

current conditions. The levels are mutually constitutive. Thus, activities are concretely realized by goal-directed actions, but goals are formed, and actions initiated only because there already is a motive and activity. That is, the sense of an action can be determined only from its relationship to the activity such that the same action may have a very different sense when the activity is changed (e.g., Roth et al. 2004). The same action therefore has a different sense when it is produced in another activity system: Taking a ball in one's hand and throwing it toward the goal is what hand-ball players are expected to do but the same action is punishable in the context of soccer. In the absence of an object/motive, an action does not make or have a determinate sense.[8] The category of object/motive therefore highlights a reflexive moment of activity, where the consciousness of the subject reflects both the outer, material productive work and the development of the psychological, inner consciousness. It emerges in the course of development:

> The object that is able to satisfy a need [initially] is not outlined sharply in the need state of the subject. Ahead of its first satisfaction, the need does not 'know' its object, it has to be discovered first. Only through the discovery is the need rendered objective and the perceived (imagined, thought) object obtains its stimulating and activity-orienting function, that is, it becomes the motive. (Leontjew 1982: 181–182)

Actions and operations, too, stand in a mutually constitutive relation. Actions are realized through the enchainment of unconscious operations, but the operations are called forth by the goal-directed actions.

The structural approach to activity – with the pervasive triangles some scholars tend to draw – fails to capture one of the fundamental ideas of Marxian thought: the purpose of a theory of human activity must be to understand and capture the *dynamic* of life, not its structures. Just as grammar does not capture the dynamic aspect of a living language, which implies that it changes, the structure of activity theory as it is often employed neither represent nor allow us to understand why cultural historical activities continuously change. First Georg Hegel and then Karl Marx realized that to model change and movement, we have to have a fundamental unit that is itself change. Unfortunately, in the Western ways of theorizing, learning is defined as transition. Although this idea of transition evokes change and time, in the end it is a change from knowledge stage/structure at time t_1 to knowledge stage/structure at another time t_2. Most assessment approaches are based on the idea that knowledge can be assessed at some point in time so that the question whether learning has occurred can be assessed as the difference between the two assessments. The point of cultural-historical activity theory is different. Here, change is the fundamental unit, which means that this unit contains an internal *contradiction* (understood in a dialectical sense). It is *difference in itself* rather than difference *between* two identifiable states, one of which is transformed into the

[8] In the terminology of cultural-historical activity theory, 'the object is the true motive of activity' (Leontjew 1982: 102). We therefore denote the pole of activity opposite to the subject, which provides activity with its collectively defined orienting moment, as 'object/motive'.

other.[9] This is precisely why some scholars suggest that learning is the problem in the structural approach, whereas in the activity theoretic approach, '"knowledge" becomes a complex and problematic concept' (Lave 1993: 12). In real living labor, the inner contradiction exists between the current state of affairs and the anticipated future state, the anticipated product toward which the activity is moving, realized by means of the concrete actions that the subjects of the activity produce.

In summary, therefore, we are not interested in investigating an ideal conception of activity; we are not interested in activity *in the abstract*. Rather, we are interested in investigating real, living human activity as it presents itself. When we look at any human action, it always already realizes some form of activity. Understanding this activity is our job and business. This activity is flux itself; something is *happening*; we do not see something static, which, following some event, changes into something else. But rather, we are confronted with continuous flux. Cultural-historical activity theory is an attempt to understand and describe this flux *as flux*, not as a transition between two static states. If we were to attempt the latter, then we would have no mechanism *internal* to our phenomenon and we would have to explain *why* things are in flux, why change is occurring. This would require us to introduce an external force – akin to the motor of a movie projector that brings the contents of the reel 'to life'. Our task is precisely the other way around: Everything around us is changing, including the language we use, culture, thought, even if we are not thinking about it. There is an inner force to life itself that makes living things change. Therefore, if there were anything that requires an explanation, then it would be the presence of static structures.

Vygotsky wants to understand thought *process*, not as something autonomous, 'segregated from the fullness of life, from the personal needs and interests, the inclinations and impulses of the thinker' (Vygotsky 1986: 10). He suggests that this requires a special form of analysis, *unit analysis*, which is capable of capturing a *dynamic system*. The attempt is to have a theory where the change process is *inherent* rather than imposed from the outside, a theory where change, development, and learning are the norm. For Vygotsky, therefore, thought, language, and the relation between these two processes are *the result of* developmental processes; that is, process is the beginning of this way of theorizing.

Activity changes activity – as a whole and in any of its irreducible parts. Irreducible here is the same as saying that if one part is taken away, then there is a different activity altogether. But as an integral part of activity, any part changes with the activity and all the relations change as well. A simple analogy may help. As a river flows, not only its bed changes but also any neighboring parts of the river: it is in constant change, changing itself in the very instant of its Being. Vygotsky conceives of the integral unity that ties together thought and language in the

[9] Within classical Western forms of thought, thinking *difference in and for itself* is difficult because it no longer allows us to make the logical statement '$p = p$'. Difference in and for itself cannot be modeled by the difference between p and $\neg p$, because both p and $\neg p$ are self-identical, whereas *difference in and for itself* constitutes the non-self-identity of a thing with itself. In a strict sense, this idea of difference would have to be written as '$p \neq p$', which goes against all classical logic though it is consitent with dialectical logic. We elaborate this idea further in the section entitled 'Contradictions'.

same way. Not only are thought and language processes that mutually affect each other but also their relation, itself a process, changes. Activity through and through is process, from the global dimension of the unit as a whole to its tiniest identifiable but inseparable moment. Like a river, activity *is* a flux and *is in* flux at the same time. It is in flux even if we may not notice it. Thus, we could show that even the most boring task in a fish hatchery – using a scoop to throw 200 kg of feed into a pond to nourish the salmon smolt in it – changes the person doing the job both physically and mentally (s/he has a better understanding of the task), changes the fish, changes the stock of feed, and so on (Lee and Roth 2006). This becomes an important aspect of our analyses in the later chapters of this book, as it means that we may not assume either that a student is a constant aspect or that learning is only occurring when we somehow 'measure' it using a test. This is so because material processes constantly occur, entailing real changes in the world. Thus, chemical energy is consumed as a student sits, writes, talks, or simply is; material resources such as graphite and ink are used as students use their pens and pencils. As students' bodies change, their physiological, structural, muscular, hormonal make-ups, and so on change, as do their momentary emotive states. Finally, the awareness of getting or not getting closer to the end results, some material product, finds its reflection in the changing emotional state of the individual subjects.

The Material Plane: A Subject Perspective on Human Activity

As note above, we do not follow here the approach to cultural-historical activity theory that emphasizes its identifiable components ('elements'). We rather focus on the subject of activity and its relation to object/motive of activity and the way in which it realizes collective intersubjective consciousness in a concrete way monitored by and reflected in emotion. However, the reader must not think of the subject or object/motive as separate from the activity. Who the relevant subject is and what its object/motives are can only be determined in the concrete analysis of concrete, real-life instants of human activity, that is, living praxis of people at work. Our purpose, therefore, is to bring to the fore real life and real praxis, in the way humans live, feel, and experience it.[10]

Subject

In our elaboration and expansion of the Vygotsky-Leont'ev-Holzkamp line of cultural-historical activity theory we are mainly concerned with understanding cogni-

[10] On the difference between living/lived mathematical work and verbal accounts of mathematical work and on the related difference between ethnomethodological and other forms of research see Roth 2009c, 2011b.

tion generally and teaching-learning particularly *as they occur in real life*. This life can be comprehended only concretely, in the form of what *is* rather than what *ought* to be. The world that makes this life is unitary, unique, and experienced concretely: 'it is a world that is seen, heard, touched, and thought, a world permeated in its entirety with the emotional volitional tones of the affirmed validity of values' (Bakhtin 1993: 56). Here, the role of the individual subject and what is apparent to it in consciousness in sympractical activity is of primordial importance. What is apparent to the subject is important because this constitutes the condition of its decision-making and its being rather than what theorists might see in the situation. 'This work is given to me, from my unique place in Being, as a world that is concrete and unique. For my participative, act-performing consciousness, this world, as an architectonic whole, is arranged around me as around that sole center from which my deed issues or comes forth' (ibid.: 57). The approach to activity from the position of the individual subject, however, is not the same as the one chosen in psychological and subjectivist approaches. Thus, 'subjectivity at the level of the sensual reflection must not be understood as its subjectivism but rather as its subjectivity that belongs to an active subject' (Leontjew 1982: 59).

In cultural-historical activity theory, the subjects of activity are not the Piagetian/constructivist individuals that make discoveries and construct knowledge on their own; subjects are subjects of *collective* activity. In the course of participating in cultural-historically formed relations with others, individuals become cultural-historical beings through unending processes of *subjectification* (Radford 2008a), that is to say, processes of *be*coming through cognitive, emotional, ethical, political reflexive and critical differentiations, and identifications. Of paramount importance in the making of the subject – in the formation of this unique in-flux subject that is continuously *be*coming – are those cultural-historical significations it engages in and in which it finds itself immersed. Cultural-historical significations are those generalized forms in which the individual appropriates the generalized and reflected/refracted human experience (Leontjew 1982). For example, when Aurélie – a fourth-grade student to whom we shall come back later – says 'I don't understand. And I will never understand' (turn 029), she is describing a subjective experience that is nonetheless articulated in a form that is consonant with and understandable to others. Such a description is possible within her culture, and is understood by other members of the culture. It therefore is not really simply describing the experience of an individual; her description is not subjective. It is a generalized form of experience that Aurélie opts for and articulates here. She thereby subsumes her singular experience in a generalized expression. It is important to note in this respect that this expression embraces an inner contradiction in that a *generalized* expression also is a *particular* expression, both describing and not describing the real lived experience of a student at the instance. Similarly, if we point with the index finger and say, 'This is a pine tree', then there is an inner contradiction, because we use the name of a general concept – we can point to many entities PINE TREE and our utterance is true – and use it for a particular entity. The same applies to mathematical expressions such as 'This is a circle'. Expressions such as the one Aurélie uses to describe her personal sense have come to her from

culture, which, as language, changes in time. Each expression therefore is cultural and historical simultaneously, so that the way in which we express ourselves, and our personal sense inherently is cultural-historical. Students and teachers in different times and cultures will articulate their experiences in different ways. Their subjectivities, therefore, also are different and entirely mediated by their culture and its historical condition at the instant that is analyzed. Thus, it was only after some time – *Le Petit Robert*, a standard French dictionary suggests 1952 – that French people began using the adjective 'cool' for people and things. Thus, prior to that people would not describe others – or feel and describe themselves – as 'cool'. That is, the way in which reality is reflected for us is a function of culture and time and the *inherently shared* resources that culture makes available to articulate oneself. Thus, 'it is not so much that the expression adapts itself to our internal world but that our internal world adapts itself to possibilities of our expression, to its possible ways and orientations' (Bakhtine [Volochinov] 1977: 130).

The difference between (culturally relative) objective, collective significations and individual sense is captured in the relation of the universal (general) to the particular rather than in the contrast between the logical and psychological. *Individual sense* therefore is a concrete realization of *collective signification*, which, as a general (universal), exists only in and through all concrete realizations and the possibilities that these enable. It is only in and through collective cultural significations that the world can become an object of individual consciousness, itself enabled by those significations. Significations are mediated by language, which constitutes a practical consciousness for others and constitutes one of the main contents of collective consciousness. As such, linguistic signification 'becomes the "real consciousness" of individuals, objectifying in itself the subjective sense of the thing reflected for them' (Leontyev 1981: 226). Signification is the generalization of a collective experience of reality, crystallized and fixed in the sensuous semiotic vehicles used as part of communication. That is, 'signification does not lie in the word or in the mind of the speaker or in the mind of the interlocutor. Signification is the effect of *the interaction of speaker and receiver, which imposes itself on the material of a sonorous complex*' (Bakhtine [Volochinov] 1977: 146–147, original emphasis).

Language is the vehicle of consciousness. In fact, 'language is a practical consciousness-for-others and, consequently, consciousness-for-myself' (Vygotsky 1986: 256). All consciousness therefore is connected to language generally and words particularly. In practical use, in any instant that we may analyze videotapes recorded in and as (classroom) interaction, words constitute aspects of consciousness. As such, 'the word is a thing in our consciousness . . . that is absolutely impossible for one person, but that becomes a reality for two' (ibid.: 256). In any concrete analysis, we must not take the word as a property or the reflection of the inner life of the person uttering it. This is so because 'the word addresses itself to an interlocutor; it is a function of the person of this interlocutor' (Bakhtine [Volochinov] 1977: 123). The word therefore will not be the same when the interlocutor is of a different social group, when 'he is inferior or superior in the social hierarchy, according to the more or less tight social links that he might have with the

speaker (father, brother, husband, etc.)' (ibid.: 123). When we look at any actual exchange, therefore, we *must not attribute it solely to the speaker*. In the following exchange, when Mario utters 'Tresa, you're on camera', it is not just his inner self, his subjectivity that is expressed in the utterance. Because it is addressed to Thérèse, the function of the utterance is irreducibly a function of the social interaction.

```
025  M   <<p>tresa, you=re on camera; >
026      (1.19)
027  T   <<len>i=know, i=m not writing anything. >
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Moreover, the understanding of the utterance *in this situation* is to be taken from the response, which is an expression of the situation that Thérèse makes available to Mario and all other members to the setting and anyone overhearing, such as those who watch the video camera that is recording the lesson (Roth 2009c, 2011b).

The subjective reflection of objective reality can be understood only as the product of those relations and mediations that emerge and form in the course of human history; any subjective reflection is but a concrete realization of a culturally possible reflection. Every higher order cognitive function and structure is therefore the result of interactions – which we understand here to be interaction rituals (Collins 2004) – with others; every one of these functions and structures is the result and reflection of outer, material, sympractical activity. Thus, inner, ideal activity cannot be separated from outer, material activity; the two are mutually constitutive processes. 'Outside of these relations (and outside the societal consciousness), the existence of an individual psyche – in the form of conscious reflection, in the form of conscious processes – is impossible' (Leontjew 1982: 127–128). 'Meanings', in the (radical, social) constructivist tradition, are the psychological product of individual constructions, a 'product of the association and generalization of impressions in the consciousness of the individual subject, the results of which are attached to words' (ibid.: 123).[11] This contrasts the cultural-historical activity theoretic perspective, where concepts are the result of the objectification (i.e., the process of becoming active and critically conscious) of historically achieved significations ('meanings'). In individual development (ontogenesis) critical reflexive processes of objectification occur as part of the child's activity in communication with others in its surroundings. Objectification is not a simple appropriation of

[11] After having established a referential theory of language, Wittgenstein spent the remainder of his life dismantling this perspective. Categories such as 'meaning' belong to a referential theory of mind. The author therefore is quite explicit in rejecting the category as one useful in understanding language and mind. Thus, he suggests that the 'philosophical concept of meaning has its place in a primitive idea of the way language functions. But one can also say that it is the idea of a language more primitive than ours' (Wittgenstein 1958: 3). Pragmatist philosophers tend to abandon the term 'meaning'. Thus, 'I urged at the end of the entry on IDEAS that there is no place in science for ideas, and under KNOWLEDGE that there is no place in the theory of knowledge for knowledge. Now we find me urging that there is no place in the theory of meaning for meaning' (Quine 1987: 131). Richard Rorty and Donald Davidson are other philosophers who do not have use for the category 'meaning', for in their approach, there is no difference between learning language and finding one's way around the world.

significations. The coming in contact with historical significations renews and transforms these historical significations. Without this renewal, cultures would be static and lifeless. The significations are *integral* to and characteristic of sensuous sympractical activity rather than entities attached to words that are owned by individuals. Or, to state it in yet another way, by reflecting the concrete life of individuals words *accrue to* always already existing, cultural-historically formed significations in and through sensuous practical activity.

Linguistic significations ('meanings') are idealizations of real, concrete relations in the world; in communication, these significations structure and become integrated into individual consciousness. That is, in individual consciousness societally achieved significations obtain a second life. The difference between the collective significations and those of the individual is captured in the differentiation of (collective) signification and 'personal sense'. The difference between the two may be given in the example of school grades, which have a particular, objective function in the activity system of schooling, the signification of which is understood by all students. But, for the individual student, a grade may constitute the possibility or obstacle to entering a career whereas for another, it may be a form of an ego-boost. Individual signification (personal sense) and objective, collective signification cannot be studied independently, for the latter depend on the concrete realization through the former, and the former are enabled by the possibilities inherent in the latter. Thus, individual significations are inherently societal-historical and culturally objective in nature. They refract the political, ethical, economical, social and cultural variants, conflicts, and oppositions of the world we live in. In concrete sympractical activity, individual significations return to the sensual objectivity of the multifarious world and its contrasting and often incommensurate ideologies.

Object/Motive and Motivation

Practical (material) activity is oriented to transform existing materials into some outcome. The difference between the current state of affairs and the anticipated outcome is reflected psychologically in consciousness as the motive of activity. Leont'ev refers to Marx/Engels in his definition of the object[12]: *The object of activity is its true motive*. As mentioned previously, motive is related to activity as goal is to action. The motive of activity is concretized as the transformation of existing materials into an outcome (product). This is referred to as the dual appearance of the object, in the material world and in consciousness (Leontjew 1982). The two aspects of activity, its inner and outer form, constitute a *single* unit. This unit, ac-

[12] The trouble arises to a large extent because English does not distinguish between material object (Ger. *Objekt,* Rus. *objekta* [объекта]) and an object that can have both ideal and material nature (Ger. *Gegenstand*, Rus. *predmet* [предмет]). Thus, in instances where Leont'ev uses both words – *objekta* and *predmet* – the English translation simply drops one, whereas the German translation retains both.

tivity, *is irreducible in principle*. This is so because when we look at and analyze any concrete activity, humans are involved in transforming something into something else. They do so *in order to* achieve something, and this *in-order-to* is as much an aspect of concrete reality as the *for-the-purpose-of,* the *what-with,* the *who/what-for*, and the *for-the-sake-of-which* that characterize everyday circumspect attention to the world as it offers itself to the subject of mundane activity (Heidegger 1977). The motive concretizes the *orientation* of activity toward its specific outcome. In standard psychology, objective significations ('meanings') are concretized in a personal sense, whereas for Leont'ev, sense concretizes itself in significations. Personal senses and cultural signification have different origins, are differently grounded, and follow different laws. Sense is produced in and through life, not by significations. Sense, because it is a relation of the person to the world, can be fostered in its emergence but it cannot be taught (told).

There is some inconsistency in the literature with respect to the use of 'object'. For some, the term only designates the material, object-sensory aspect of practical activity, distinguishing it from the 'ideal object', which is the object reflected in consciousness during activity (e.g., Davydov 1990). Others use the term 'motive of activity' to denote its ideal dimension, its 'inner' reflection, thereby bringing into play the motive forces that underlie the continuous change of activity. These motive forces are the results of inner contradictions – or, expressed in other words, because the category of activity theorizes living, inherently transformative processes, there are inner contradictions that also describe the transformative forces. Therefore, activity, the unit (of analysis) that comprises current materials and future anticipated outcomes, contains inherent contradictions of two kinds: between the material reality and its ideal reflection in consciousness and between current and future material/ideal states. As a way of avoiding the reduction to the material or ideal dimensions of activity, we use another way of denoting this category sometimes used in the literature: 'object/motive'.

Object/motives reflect collective interest, the interests of the collective, and therefore are general. They reflect generalized needs satisfied in and through the network of collective activities. '"Motivation" comes about as the emotional regulative of "autarchic" learning by exploration and is the orientation of activity via learned anticipation of a situation with higher (compared to the present) emotional value to be reached by means of activity' (Holzkamp 1983: 298). Thus, motivation is not a separate analytic category; rather, it constitutes the emotional dimension of the difference between present and future orientation in activity. The motivated nature of an activity does not depend on the anticipation of concrete results but on the subject's enhanced (material, cognitive, spiritual, etc.) quality of life that can be achieved by means of the activity. The motivated nature of activity is the result of considerations that concern the totality of action-embedding connections (significations) from the perspective of the individual; in this sense, it constitutes the '"*emotional aspect" of thought*' (ibid.: 299). Motivation is the '*emotional-anticipatory aspect* of the *real action planning and execution*' (ibid.: 300). Together with the higher emotional valuation of the anticipated quality of life at the

end of the activity, the agential subject also has to anticipate the emotional value of the way by which the activity is realized, that is, future effort and risk.

The question therefore is not whether or not a student is 'motivated' to engage in and complete the mathematical task that the teacher posits for the day. The real question is which activity students engage in, and, therefore, which object/motives they take up and pursue. Thus, a student who orients toward getting good grades does not actually have to take up the object/motive of knowing algebra some point down the road. Grades may be achieved by other means as well, including copying homework and copying from others or from notes during an exam. Leont'ev provides a compelling description of the role that the object/motive plays in the orienting activity and, thereby, in bringing about particular kinds of actions.

In the pioneer palace of Kharkov, organizers offered a workshop on building model airplanes.[13] Although the children were very interested in building the models, very few actually showed interest in understanding the theoretical aspects of flying that are relevant in a *conscious* construction of the planes. There were posters and knowledgeable adults, but the children were only oriented toward building beautiful models, leaving aside any considerations of what makes a plane fly or why the wings might be in a particular orientation. The psychologists were interested in organizing the task such that the students, on their own, would see the advantage in accessing theoretical information for advancing their own interests. This was achieved by framing the task as one of building model airplanes that were capable of flying a given distance. The students took up this new object/motive just as they had the earlier one. But as soon as they tried their models, they found out that the models they had built did not cover the desired distance. At that point, to expand their possibilities for redesigning the model planes, they did indeed read the available posters and books or asked available personnel. That is, the children engaged in learning loops not because the adults had told them to do so but because they anticipated an expansion of their own room to maneuver toward the ultimate object/motive they had taken up – building a plane that would fly the given distance. In addition to the significant increase in attending to theoretical issues from a few minutes to nearly half an hour, the total number of children signing up for the workshop also increased from an average of about 6 or 7 to an average of over 40 children per day.

In concrete activity, the ultimate outcome aimed at does not yet exist. It can therefore regulate activity only when it presents itself to the subject as an image that makes it possible to compare the current state with its starting materials and its intermediate forms: 'The psychic reflection of the target product has to exist for the subject in such a form that it can *work* with this image, can modify it under the existing conditions' (Leontjew 1982: 123). Consciousness, too, is the subjective product, the transformed appearance of the societal relations that are realized through human activity in an objective material world. This has consequences for

[13] Leont'ev writes about these experiments in an appended chapter 7 that follows his concluding chapter 6. This appended chapter, entitled 'Psychological Questions of the Consciousness of the Learning Process' (Leontjew 1982), though referenced very infrequently in the literature, actually contains a lot of material that ought to be of interest to educators.

theorizing learning activity, where learners, because of the very nature of the activity, cannot know the object/motive: The object/motive itself has to be the outcome of the learning activity so that others – e.g., teachers – have to take on the regulative function that in other productive human activities exist in the known object/motive.

This aspect of the object/motive places particular constraints on what we can expect to happen in mathematics classrooms. Leont'ev suggests that the target product has to 'exist for the subject in a form that it can *work* with this image'. Now, when we expect a student to learn algebra, which they do not yet know, what image can we expect the student to have? When a student is to solve some novel problem, what is the concrete image that the student can have of the outcome – which s/he does not yet know because knowing it is precisely the reason for the curriculum – that exists in a way so that s/he 'can *work* with this image'? This precisely is a contradiction in learning activity. The present book is an opportunity for us to articulate the contradiction and how it is resolved in practice. In anticipation of chapter 4, we suggest that an integral part of the learning activity is for students to recognize the object/motive in their own actions. That is, the object/motive *emerges* in the course of, and therefore also is the *product* of, the activity. Because students cannot recognize this on their own, there is a central role for the teacher in the function as the representative of the current cultural-historical conditions. But it will not be that the teacher can *tell* students what the object/motive is of *their* activity; rather, it is through joint actions with the teacher and other students that the object/motive of the current activity emerges for any particular student from his/her actions.

The Ideal Plane: Reflecting Concrete Reality

The fundamental difference between activity theory and other contemporary theories consists in the way human thought is theorized. Thought is considered as a cultural and historical evolved form of reflection, that, although it always remains enmeshed in sensation and perception, goes beyond the sensed and the perceived. Signs in general and language in particular endow the particular with general attributes that overcome the 'here' and 'now', that is to say, the limits of local spatial-temporal experience. Thus, 'generalization is a verbal act of thought and *reflects reality in quite another way than sensation and perception reflect it*' (Vygotsky 1986: 6, emphasis added). As a result, human thought does not merely mirror the external world in front of us, but refracts it in myriads of subtle and complex manners that seem to be beyond the reach of other species.

One way in which living activity is reflected is in the conscious awareness of the subject; the other way is in the form of emotion. Unit analysis 'demonstrates the existence of a dynamic system . . . in which the affective and the intellectual unite' (Vygotsky 1986: 10), and this includes material reality, for 'every idea contains a transmuted affective attitude toward the bit of reality to which it refers'

(ibid.: 10). Consequently, all unit analysis includes material sympractical activity, on the one hand, and the two manners that reality is reflected, namely consciousness and emotion, on the other hand. 'The true nature of psychic sensual images exists in their object character, in that these images are produced in activity, which relates the subject with the external, object-world' (Leontjew 1982: 134).

Consciousness

What is it that matters to our deliberations about what we do next? What matters precisely is what is salient to us in our consciousness. It is not an abstract world, a world described by all physicists after Galileo; it is not some abstract thoughts and concepts that are supposedly in our mind. What we do next is determined by what is salient to each of us in our conscious awareness of the actual, practical conditions and circumstances. It is this aspect of human reality that is of importance to a cultural-historical activity theoretic perspective. Thus, the 'psychic reality that immediately opens itself for us is the subjective world of consciousness' (Leontjew 1982: 122). In all its immediacy, consciousness reflects the world as it is given to the individual. As Marx/Engels (1958) suggest, conscious Being and being conscious are but two aspects of the same sensibly sensuous human life.

An often-used example of the difference between theoretical cognition and practical consciousness is that of hammering. Because the 'totality of useful things is always already discovered *before* the individual useful thing' (Heidegger 1977: 69), the hammer, which is a tool used *for some purpose* and *in order to* achieve something, does not appear in consciousness in the way rationalists and cognitive scientists tend to theorize it. The hammer is *not* represented. Rather, the hammer is subordinated to the activity at hand. 'The less we just stare at the entity called hammer, the more actively we use it, the more original our relation to it becomes and the more disclosed it is encountered as what it is, as useful stuff' (ibid.: 69). That is, the hammer as hammer is not apparent in consciousness. Rather, it is the thing or state that is to be achieved. Thus, our association with the hammer, as with other useful things, 'is subordinate to the manifold of references of the "in-order-to"' (ibid.: 69). What is apparent in consciousness and what a (practically knowledgeable) person is attending to is placing a nail to hang up a picture or to fasten a loose board in the chicken coop. Active attention to the hammer occurs only when something goes or has gone awry. We then notice that the hammer we have been using is too heavy or too light, that its handle is broken or that there is a splinter in the handle that hurts the hand. What we do next depends precisely on what appears in consciousness.

Activity theory differs from other theories of knowing and doing concerning the true nature of the focal concepts in the consciousness of the subject. Consciousness, rather than knowledge, becomes of primary interest because 'consciousness, as relation, is not characterized by comprehension, not by the knowledge of the significance of the subject matter, but by the personal sense that the subject matter

obtains for the child' (Leontjew 1982: 279). Confusing the two aspects produces intellectual formalism. Traditional psychology has not taken consciousness as its central problem. In this field, consciousness is a derivative of knowledge rather than the relation of humans with the world. Kantian/Piagetian approaches conceive of consciousness in terms of formal knowledge. But this creates the problem of the relationship between formal knowledge and the world. Activity theory, on the other hand, theorizes consciousness in activity as the *relation* of a person to the world: 'The inner movement of the individual consciousness is produced by the movement of the objective activity of the person' (ibid.: 150). In this, our approach also differs from others who claim to ground their work in cultural-historical activity theory yet continue to focus on knowledge as the starting point for understanding the way in which individuals act in the world. Human consciousness therefore is neither a plane nor a space filled with images and processes, but is the inner movement part of the total movement of activity that concretizes the real life of the embodied, living individual in society.

Consciousness plays a much greater role in cultural-historical activity theory, than in any other theory. Ontogenetically speaking, individual human consciousness is possible only when there is collective, societal consciousness. Without collective consciousness, individual consciousness does not develop beyond the realms of the sensorial and perceptual as shown by the various cases of the so-called 'wild children' (Newton 2002). In society, language and material culture are the substrate for consciousness at two levels: the individual and the collective (Vygotsky 1986). Individual and collective consciousness are mutually constitutive so that there can be nothing available to the consciousness of the individual that is not already a possibility configured in collective consciousness – as, for instance, possible conceptual or artistic emerging combinations from established ones that nonetheless need to be articulated in intelligible ways to produce a resonance in the forms of known social canons: it is precisely in this way that the well-known sociologist Norbert Elias (1993) explains Mozart's genius. Reciprocally, creativity supposes individual consciousness as the possibility for new forms of collective consciousness. Consciousness is thus the place where individual subjectivity and collective subjectivity come to be irreducibly intertwined, for '[t]he appearances of reality can become conscious only by means of "ready-made" significations appropriated from the outside – the knowledge, concepts, perspectives that the individual obtains in intercourse, in individual and mass communication' (Leontjew 1982: 149). That is, rather than being merely taken-as-shared, an understanding of reality is inherently enabled by collective cultural-historical forms, most important among these language (considered, of course, not as a simple 'tool' but as a conveyer of forms of life).

In activity theory, the role of consciousness changes. It is the reflection/ refraction of the world in the course of human engagement in sympractical activity. When consciousness is thought of in terms of knowledge (structures), then the role of emotions cannot be understood. For example, the effect of experience on emotions has to be understood in terms of experiences that require interpretations in terms of knowledge before it can affect emotion (the 'telegram effect'). The prob-

lem changes when consciousness is understood as the relation of the person to the world, as that which reflects his/her real life, his/her being in the world. This requires an entirely different approach from the one that considers Gnostic problems only, a change from the lifeless treatment of thinking processes.

By its very nature, *con*sciousness is something that we share with others. The etymology of the word points us to knowing (Lat. *sciēre*, to know) that we have in common, with others (Lat. *con*-, with). Vygotsky therefore insists on the fact that consciousness *never* is the consciousness of an individual, who is always but an 'inhabitant of the social edifice of ideological signs' (Bakhtine [Volochinov] 1977: 31). Consciousness realizes itself in the form of *participative (unindifferent) thinking* (Bakhtin 1993) that interaction participants make available to each other. If we are interested in understanding the unfolding of an event as an irreducible aspect of social life, we must focus on the various forms of signs that speakers make available for others as much as for themselves. These signs comprise words, pointing and iconic gestures, body positions and orientations, prosody (speech intensity, pitch, speech rate), rhythms, and so on – that is, anything that people use as a resource in the conduct of social life that both reproduces and transforms the activity at hand.

Emotion

The function of interest, boredom, inclination, remorse, *exhilaration*, or *frustration* is to signal to the subject the personal sense of events that occur in and as part of its real life. These affective forms constitute valences of the events as the individual subject experiences them and the activity that they constitute. Emotions reflect the relations between motives (needs) and the success in – or the possibility for a successful realization of – the corresponding activity of the subject. It is not the intellectual reflections on these relations that matter but the unmediated, sensuous, lived experience thereof. They emerge prior to any rational valuation of the activity. Emotions are relevant at the level of activity, not at the level of operations or actions. The same actions and operations may receive emotional colorings as a function of the emotion such that a successful action may be colored negatively and hardship may be charged positively – as long as it is perceived as getting the subject closer to realizing the motive (e.g., hardship and training for athletes in the face of the pay-offs that come with a victory).

Psychology and the learning sciences have tremendous difficulties in explaining the fact that consciousness is active and engaged rather than a disinterested calculating mechanism – in the way a central processor is in a computer. The computer processor works because of outside forces, whereas the human mind moves by itself. Affect and cognition tend to be theorized as external to each other, affect

often as a factor that diminishes cognition.[14] Kant (1956) did his part to expel affect from cognition, arguing that strength of the emotion constitutes the weakness of the mind. True strength and inner freedom come when the mind becomes the supreme master, subjugating all the affects to rule over them. In more recent constructivist thought, too, emotion is something external to cognition: emotion is to cognition what fuel is to the motor (Piaget 1981). After several decades of doing without emotion as a category, suggestions emerge only now in the constructivist conceptual change movement, that affect has to be included as another factor that mediates cognition and learning. In cultural-historical activity theory, however, affect generally and emotions specifically are theorized such that they have become *integral to cognition*, as a second form in which reality is reflected in the (individual, collective) subject. This was achieved as part of a categorical reconstruction of the human psyche on evolutionary grounds.

This reconstruction posed as its main question the origin of the relation of affect and cognition (Holzkamp 1983; Leontyev 1981).[15] It was recognized that affect is tied to life itself so that the question about affect has to be reconstructed from the beginning of organic life itself. Holzkamp and Leont'ev posit some originary situation where a one-cellular organism floats in brine with sufficient food to sustain it. To initiate anything like cognition, a number of internal and external conditions have to exist that lead to contradictions. On the outside, changes have to occur whereby the brine no longer contains plentiful food but has gradients. The organism needs to be able to 'experience' these gradients as a lack, which constitutes a state of *negative valence* to its life. Moreover, the organism needs to have an originary sensibility, which allows it to sense the food gradient or some other signal, such as light, associated with the food gradient. The organism has to have some mobility, which initially is random and arbitrary. The 'cognition' required for *voluntary, intentional* movements to be produced toward greater food availability presupposes that the organism correlates the movement, which it has to have some sense of, with increase in available food as mediated by the signal (e.g., light). The organism has to be able to assess the increase in food availability in some form of 'satisfaction', some measure of 'improvement' of its situation. That is, some form of mechanism has to exist that provides a negative valuation when there is a lack of food and a positive valuation if the motive of activity is realized and needs are satisfied. The final link required is that between (a) the intentional movement toward food gradients and (b) an associated change in the organism's valuation of the situation from negative (original state) to positive (final state). Here then, 'affect' (sensibility, valuation) and 'cognition' (movement mediated by sensibility) become part of one and the same mechanism. The organism moves, mediated by the signal, because doing so 'promises' a pay-off in terms of higher food availability and better conditions, and the organism *experiences* the change from negative to positive

[14] Besides cultural-historical activity theory, phenomenological philosophy treats affectivity (emotion) as a phenomenon *integral to* cognition, cognition and emotion as two sides of the same coin (e.g., Henry 2000; Sheets-Johnstone 2009).

[15] A short version of the way in which this reconstruction is conducted and its results can be found in Roth 2009a.

valuation, which also has to leave a 'trace' so that the organism 'remembers' the relation between originally arbitrary movement and change in the organism's condition.[16] In this approach, therefore, gnostic and affective moments are combined in an originary sensibility. Both moments are reflections of the situation and both are integral to the changes therein through active, intentional movement, of which there is a trace so that the organism immanently knows that it can move and bring about change.

In their reconstruction, Leont'ev and Holzkamp show how from these original conditions, given a combination of internal and external developments, changes, and developmental contradictions, there is a development from single-celled organisms to the hominid species from which Homo sapiens emerged in anthropogenesis. At this point, the control over life conditions is transferred from the individual to the collective, as the division of labor and the active and intentional production of food, the social organization, the handing-down of practices, and so on is selected in evolutionary processes to become the dominant form of life for this species. That is, emotions 'emerge in objective situations and "mark" in their own language these situations and individual objects, and sometimes enter these by chance or indirectly' (Leontjew 1982: 190). Changes in activity are reflected in changed affective tonalities of the situation as a whole and of individual objects. It is important to retain that in relation to the analysis of activity, 'the objectivity of activity generates not only the objective character of the images but also the objectivity of the needs, emotions, and feelings' (ibid.: 90). Here, emotions are 'the result of and the "mechanism" of its movement' (ibid.: 188). These reflect the relationship between object/motives and the levels of success that are anticipated as the outcome of a set of actions that concretely realize the activity. They therefore constitute, besides consciousness, a second mode in which the activity is reflected in the subject. Their import derives from the role they play in orienting activity rather than the action. Thus, as mentioned previously, a particular action may be associated with negative valuation (e.g., athletes put up with hardships on the way toward a victory) as long as the anticipated outcome of activity is associated with a positive valuation.

Affective valuation and movement are integral moments of the same phenomenon. It relates, now at a human level, 'a given lived-experience *to me* as the one who is actively experiencing it' (Bakhtin 1993: 36). This self-relation of the individual who not only acts but also experiences the action and its result has a 'sensuous-valuational and volitional – performative – character' (ibid.: 36). The different moments, that is, the sensuous-valuational and the volitional (intentional) form, are given in/as, a unity.

The emotional valuation (assessment) of relations with the environment is the basis and first step of any cognitive process, that is, of thinking and acting, questioning the existing relations. Emotional valuation reflects knowledge of these relations. But while they mediate orientations and goals of action, they tend to remain

[16] The resulting *immanent* memory arises from a self-affection of the living/lived body, which phenomenological philosophers refer to as 'flesh' (e.g., Henry 2000).

subconscious and unaddressed in the conscious orientation toward and selection of future actions (Holzkamp-Osterkamp 1978). That is, actions are not the result of cognition, but inherently mediated by emotional valuations that arise from, and reflect/refract the assessment of the current relations and mediate selection of goals and actions that move the activity further along. Emotions and volition are integral and mutually constitutive moments of the same unit so that it comes as little surprise when cultural-historical activity theorists use the adjective emotional-volitional to characterize the relation of the subject to its activity. Thus, 'everything that is actually experienced is experienced as something given and as something-yet-to-be-determined, is intonated, *has an emotional-volitional tone*, and enters into an effective relationship to me within the unity of the ongoing event encompassing us' (Bakhtin 1993: 33, emphasis added).

> The emotional-volitional tone, encompassing and permeating once-occurrent being-as-event, is not a passive psychic reaction, but is a certain ought-to-be attitude of consciousness, an attitude that is morally valid and answerably active. This is an answerably conscious *movement* of consciousness, which transforms possibility into the actuality of a realized deed (a deed of thinking, of feeling, of desiring, etc.). We use the term 'emotional-volitional tone' to designate precisely the moment constituted by my self-activity in a lived experience – the experiencing of an experience as *mine*: I think – performed a deed by thinking. (ibid.: 36)

The emotional-volitional tone is an integral aspect of the *movement of consciousness*, which, for Bakhtin as for Leont'ev, reflects the transformation of the current state into a future state already present in consciousness as anticipation. We also note the importance that the 'experience as *mine*' has to the thinking of Bakhtin, which will require our research to take into account lived experience and an adequate method to access, describe, and theorize it.

The emotional-volitional tone is central to activity. It is 'an inalienable moment of the actually performed act, even of the most abstract thought' (ibid.: 33). Moreover, and precisely in the way that Leont'ev frames the issue, the philosophers states that the 'function of the object within the unity of the actual event encompassing us is *its actual, affirmed value,* i.e., is *its emotional-volitional tone*' (ibid.: 33). That is, everything experienced has an emotional-volitional tone, most importantly, the object of the activity, which is its true motive.

From the above-said, we should therefore expect that (a) there is an emotional-volitional tone in every mathematical situation that we might study in school classrooms, (b) this emotional-volitional tone is changing in/with activity (an outcome, result), and (c) the emotional-volitional tone is itself a 'mechanism' of the movement of activity. In the episode made available and analyzed across chapters 2, 3, and 4, this is precisely what we describe; in those chapters we theorize the relationship between activity, learning, consciousness, and emotion.

Contradictions

> In the same way as concepts, our sensual generalized images contain movement and therefore contradictions; they reflect the object in its manifold relations and mediations. (Leontjew 1982: 73)

The Vygotsky-Leont'ev-Holzkamp lineage of cultural-historical activity theory has been created to capture the different moments of human life in terms of dynamic, living processes. This is apparent in the opening quote to this section, in which movement is attributed to (ideal) concepts and to the sensual generalized images that accompany concrete, material, and external activity. Consciousness, which constitutes an affective reflection of inherently dynamic activity, therefore has (to have) the same flow-like qualities – or it *could not be* a reflection. Static concepts *cannot* reflect a dynamic phenomenon. Flow and movement, therefore, if they are denoted, inherently require internally contradictory concepts. This is so because they need to capture the 'between-ness' of movement. Thus, to describe the historical changes in the market system, which 'fuels' its own changes, Marx/Engels required a concept that captured the movement of commodities. The concept that fulfills these demands is *value* (Marx/Engels 1962). It expresses itself in the use-value and exchange-value of a commodity. Thus, when we look at any barter trading action, a particular commodity (e.g., a piece of cloth) *simultaneously* constitutes use-value and exchange-value. The cloth is of exchange-value to the weaver, but of use-value to the tailor. It expresses itself as such not because seller (weaver) and buyer (tailor) have different perspectives on this commodity but rather because value itself has to be thought of as an internally contradictory category that can express itself one-sidedly in two different ways (Il'enkov 1982). To understand inner contradictions and the movement with which they are associated, we must not think of the commodity (object) *abstractly*, that is, independent of concrete activity. If we did that then we would not be able to understand movement, for why would a piece of gold make anything move?[17] To understand, we need to think that there is a natural phenomenon of movement and to reflect this movement, we need a concept that itself contains movement, that is, a concept that is not identical with itself.

We introduce inner contradictions here, because without them we cannot think activity in movement, movement in activity. But when we look at any mathematic lesson, we note that life does not stand still. Even if the teacher were to say 'stop everything now' and all students freeze, life would still not stand still. This standing still would be part of and therefore concretely realize the mathematics lesson as

[17] Michael Hoffmann, a philosopher with special expertise in Plato and Kant, asked the question in this way and thereby alerted us to the problematic way of understanding dialectical materialism. Inner contradictions, as idealizations, do not move anything. Concepts that are to represent movement rather than stasis, however, must contain this movement itself; that is, they must have to contain at a minimum two states at once *and* the transition between the two. That is, they must, in short, be non-self-identical.

an unfolding process.[18] To understand this movement, we require categories that reflect it. For activity theorists, *activity* is this category. But any human activity that we may observe – farmers producing grain, bakers producing bread, fish hatcheries producing fish, or schools reproducing and transforming cultural knowledge – is in movement, is life and therefore movement itself. The category of activity, to reflect this movement, has to be an internally contradictory one. Contradictions and self-movement are two integral and mutually constitutive moments of thinking activity specifically and human life more generally.

Inner contradictions are the most central but also most misunderstood category of cultural-historical activity theory. In Western scholarship, the category often is reduced to a logical contradiction between two terms or to a breakdown of some instrument or tool as part of the activity. But this is not what Marx/Engels and following them Vygotsky, Leont'ev, Bakhtin, or Evald Il'enkov have in mind. Logical contradictions can be removed; if something is broken, it can be fixed. An inner contradiction of the kind that is central to the category of activity, however, is *endemic* and *cannot* be removed. It is, as Marx/Engels suggest, tied to the evolution of the division of labor and everything else it has entailed – language, culture, consciousness, and so on.

> It is completely irrelevant what consciousness does on its own, what we get from all this garbage is one result: these three moments, productive power, societal condition, and consciousness can and do come into a contradictory relation, because with the *division of labor* comes the possibility, indeed the reality, that the ideal and material activity – that pleasure and labor, production and consumption, fall to different individuals. (Marx/Engels 1958: 32)

Through division of labor, contradiction is also tied to the relation between the universal and particular, for example, the general interests of society and the particular interests of the individuals.

> The problem of the relation of the universal to the individual arises . . . not only and not so much as the problem of the relation of mental abstraction to the sensually given objective reality but as the problem of the relation of sensually given facts to other sensually given facts, as *the object's internal relation to the object itself*, the relation of its different aspects to one another, as the problem of internal differentiation of objective concreteness within itself. On this basis and as a consequence of it, it arises as the problem of the relation between the concepts expressing in this connection the objective articulated concreteness. (Il'enkov 1982: 75–76)

This is a very dense paragraph that requires us to unpack it for its theoretical and practical relevance to be seen in its entirety. The relation between the universal and the individual (particular) is an important aspect of thinking inner contradic-

[18] Physically and physiologically, all human bodies assembled in a classroom burn energy and thereby change. Even if they remain quiet, particular individuals continue to think, and, because of the brain activity, continue to change.

tions. Thus, to draw on the example Marx/Engels often use, if we think of any individual person, we are confronted with the fact that s/he is both (a) a concrete realization of the human species, that is, the general in its concreteness, and (b) a particular human being. The same inner contradiction exists if we were to denote an object CIRCLE by pointing to it while producing the sound /'sɜ:k(ə)l/ ('circle').[19] Here, the category name, denoting the general, is applied to denote a particular. Now Il'enkov suggests in his quotation that this pointing to something else is possible only because of '*the object's internal relation to the object itself*'. Any individual person is both a particular and a (concretization of the) general; any CIRCLE and /'sɜ:k(ə)l/ ('circle') is both a particular and a (concretization of the) general. It is only because of the internal relation to itself that one object *also* may denote another, such as when a sound /'sɜ:k(ə)l/ or ink trace ('circle') comes to denote an object CIRCLE. To be able to refer to something else, any signifier has to be able to signify itself. As a result, '"the signifier of the signifier' is the movement of language itself" (Derrida 1967: 16). In any actual situation that we may analyze, a signifier (e.g., a word, a gesture, an intonation) points to itself at the same time as it points to something different. This double relation is enabled by the self-relation of the signifier in the same way that *value* stands for movement only when it is related to itself such as it incorporates an inner contradiction (Roth 2011).

Analysis of Activity

In this book, we take a theoretical perspective that is concerned with consciousness and the cognitive and emotional awareness social actors make available to each other. Our analysis is intended to provide an *ethnographically adequate account* of the perspective on activity from the viewpoint of the actors as these make it available to one another (McDermott et al. 1978). We do so because the *internal dynamic* that drives the observed situation is not explained by drawing on hidden parameters. Quite the contrary is the case. Social actors, the subjects of activity, have grounds (reasons) for acting in the way they do, and they exhibit to each other whatever is required to pull off an event as that which it is. When required, they make available reasons for their actions even thought these might not have been in their conscious awareness. Thus, we do not *interpret* individual utterances as having this or that sense. Rather, we understand ourselves to be social actors who overhear the conversation of our research participants (Garfinkel and Sacks 1986). None of the participants in mathematical activity can see any hidden contents of the minds of others. What they act upon and react to is what the respective other makes available to them (Livingston 1986).

[19] The notation for the sound is from the International Phonetics Alphabet. This alphabet therefore produces a guide for pronunciation independent of any language. It is used in most dictionaries around the world.

In the same way as speech act theory and conversation analysis, dialectical materialist (Marxist) approaches orient us to social interaction as the site of interest for understanding psychology. Thus, 'social psychology first is the ambient milieu of *speech acts* of all kinds, and it is in this milieu that all forms and aspects of the uninterrupted ideological creation is bathed: the conversation of the hallways, the exchanges of opinion in the theater or concert, in the different social meetings . . . the inner dialogue, and self consciousness' (Bakhtine [Volochinov] 1977: 38–39).

Concretely, we do not treat an utterance as a question unless there is evidence that another actor in the setting is treating it as such. This is why the punctuation in our transcripts do not mark grammar but aspects of prosody, which are interactional resources available to the participants. Thus, it may well be that a statement that has the grammatical structure of a statement nevertheless is treated as a question – likely based on the prosodic cues. Thus, in the following excerpt, we observe a sequence typical for a question–response pair *even though* Jeanne's utterance 'How much ought there be already' (turn 158) drops as in a constative rather than as in a question. (The translations of the two transcripts that follow can be found in the appendix and in chapter 3 and 4, respectively.)

```
158  J:  =combIEN devraitIL déjà y avoir.
159  T:  u:h:
160  M:  douze
```

In turn 160, Mario 'responds', providing a number that is consistent with the number of chips in one of the goblets. On the other hand, in the following sequence, a first turn is both grammatically and intonationally shaped as a question, but it is followed by another turn with rising intonation. A long pause in the verbal 'channel' develops, while the teacher Jeanne moves her fingers to another point in Mario's worksheet. He then produces two more rising speech segments, 'plus three? plus three?' In her turn at talk, Jeanne produces a drawn out 'yes'.

```
200  J:  questce que tu vas écrire ici?
201  M:  trois?
202      (2.59) ((Jeanne moves finger to the cell on his left))
203  M:  <<p>plus trois? plus trois?>
204  J:  oUI:: ((he writes))
```

We can gloss this as Jeanne asking a question 'What are you going to write here' (turn 200) and Mario producing a tentative response, 'Three?' (turn 201) 'plus three?' (turn 203), which Jeanne confirms as correct, 'Yes' (turn 204). In Jeanne's finger movements, Mario can recognize the response delivered so far as not yet sufficient, and as soon as he produces two more additions, the evaluation is made known. Readers familiar with this form of analysis recognize what is known by linguists as triadic or IRE sequence, short for teacher *i*nitiation, student *r*esponse, and teacher *e*valuation.

Following the same logic, we do not identify 'episodes' unless particular segments of activity come to stand out because they are marked as such by one of the participants in the setting. Thus, for example, a significant stretch of the activity may be started off by a student, who notes that he understands what they have to

do, and may be ended by the same student who, after engaging in his activity, states that he does not understand and seeks the teacher's help. Or a segment is defined by the instance a teacher comes to help a student until the moment when she states that the student understands and leaves him to continue on his own. Here, it is the teacher who starts off and closes the lesson segment.

Throughout these chapters, we present the translations of the original transcription. This presents particular challenges, as the prosodic cues are somewhat deceiving given that French words, even if they look the same, tend to be pronounced differently and with different stresses. For example, the term dollar, which is part of the mathematical task that the children solve, would be pronounced in English as /'dɒlə(r)/, stress on the first syllable, whereas its French pronunciation is /dɔ'laʀ/ with the stress on the second part of the word. In addition, whereas the English stresses the first syllable (see stress sign), in French, which is a prosodic language, the last syllable is emphasized. Thus, if in her presentation of how to calculate something, the teacher stresses the second part of the word 'dollar', a possible English equivalent would be to stress the first part. This is important because the rhythmical and prosodic aspects are important interactional resources and may have important functions in bringing about desired responses. In our translation of the transcription, we have made every effort to provide the best English equivalent. Our reading of the transcript is based on the French version, which, in addition to the full English transcription, has been provided in the appendix.

2

Reproduction and Transformation of Affect in Activity

In the preceding chapter we suggest that in the Vygotsky-Leont'ev-Holzkamp version of cultural-historical activity theory articulated here, cognition cannot be understood independently of emotion. This is so because the latter constitutes an holistic expression of the subject's current state with respect to the object/motive and the subject's sense of the likelihood of success in realizing the object/motives it has subscribed to. That is, the activity, stimulated by the object/motive, continually transforms the situation at hand, including, as we show here, the emotion expressed and thereby made available to others. Affect is not a static, trait-like characteristic of the subject. Rather, emotion, the sensual valuational reflection of activity in the acting subject, is continuously reproduced and transformed together with the cognitive and material results that emerge from the hands and minds of the subjects. Affect is in movement together with the activity as a whole, of which it is one of the manifestations. That is, in this chapter, then, we show that affect is an irreducible moment of activity, which, like the activity itself, is in and brings about the (self-) movement. The category of activity was created precisely to capture movement; the analysis focuses on inherent change (becoming) rather than on how things are in and for themselves.

In the following sample episode featuring Aurélie, Mario, and Thérèse, we exhibit and theorize this continual production of cognition and emotion, both of which are thought to be reflections/refractions of the living activity. In the process, the subjects make thematic and available to each other and to themselves expressions of the emotional and cognitive reflection of activity. These expressions are resources that are employed in and therefore mediate the movement of the activity itself.

Since the beginning of the study in September 2007, regular meetings have been held involving the teacher, the researchers, and the research assistants at one school in Ontario, Canada. The meetings have taken place either at the school or at the university to discuss the mathematical content of the tasks, the design of the

Problème 4 :

Pour son anniversaire, Marianne reçoit une tirelire contenant 6$. Elle décide d'épargner 3 $ par semaine. À la fin de la première semaine elle se dit : « J'ai 9 $! ».
Questions :
a) Modélise le problème à l'aide des boîtes et des jetons jusqu'à la 6e semaine
b) Remplis la table de valeurs suivante :

Numéro de la semaine	1	2	3	4	5	6
Montant épargé ($)	3+6	3+3+6	3+3+3+6	3+3+3+3+6	3+3+3+3+3+6	3+3+3+3+3+3+6
Ou	3×6	2x3 +6	3x3 +6	4x3 + 6	5x3 +6	6x3 + 3

Fig. 2.1. Problem 4 was to be solved by the fourth-grade students. Presented is the copy of Mario's worksheet at the end of the lesson.

tasks and forms of interaction to be promoted in the classroom. Though experimental, the tasks were designed to meet the requirements of the provincial curriculum.[1] Among the curricular topics, one that has gained prominence is modeling. In the following, we focus on one of the lessons in a fourth-grade class (9–10 years) revolving around the topic of modeling situations by means of algebraic concepts. More specifically, at the heart of the present and subsequent chapters is Problem 4 (Fig. 2.1)[2], from which the fragments that we present below are drawn. Problem 4 includes two main tasks about the modeling of a saving process. The students have been provided with clear plastic goblets and chips to accomplish the first of the two tasks.

This first fragment – constituting the first 21 lines of the raw transcript, 46 turns in augmented transcript presented here – may be glossed in a summarizing way by saying that Mario moves from having an idea about what they have to do, through its articulation, to the eventual halt in the activity and the statement that he does not understand. In the course of this fragment, he moves from expressing confidence to frustration. Aurélie tells her peers that she does not understand, and, even though her worksheet comes to be filled, expresses frustration. Thérèse both completes the task and exhibits confidence throughout. How can we understand this changeover, which itself is the result of the students' activity? We suggest that the engagement in the activity produces a negative emotional response and a recognition that they do not understand so that an initially available positive emotional

[1] The provincial curriculum can be downloaded from the website of the Ontario Ministry of Education (2005): http://www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf.
[2] The task translates as:
Problem 4:
For her birthday, Marianne receives a piggybank containing $6. She decides to save $3 each week. At the end of the first week she says to herself, 'I have $9!'
Questions:
a. Model the problem until the sixth week using goblets and chips
b. Fill the following table of values

stance turns into a negative one. Affect here is a reflection, from the perspective of the subject of activity, of the distance between the current state and the anticipated – even if not articulated – object/motive of the activity.

The intent of the task is for students to begin by placing the number of chips into their goblets that correspond to the amount of money Marianne has in her piggybank at the end of each week. But rather than transferring the total number of chips in each goblet to the table of values, students are to note the repeated additions of $3 to the piggybank (see table in Fig. 2.1). To achieve this, the table of value specifies for the first cell +6, inviting the students to add a 3 to achieve the representation 3 + 6 corresponding to the $9 in goblet #1. Similarly, the +6 in the second cell is intended to encourage students to represent the repeated addition 3 + 3 + 6 rather than note the '12' corresponding to the 12 chips in goblet #2. The intent of the third row in the table is to have students write a shorthand representation for the contents of the cell above, which means that they might move from the repeated addition 3 + 3 + 6 to the more efficient multiplicative/additive structure 2 x 3 + 6. By filling the table, fewer terms are embedded in each cell, which embodies the curricular intent to allow the emergence of the pattern (# of weeks) x 3 + 6. To provoke this emergence, the worksheet shades the number of weeks in yellow in the entire first row and for weeks 2 to 5 in row 3. Subsequent tasks on the same worksheet are designed to lead the students to the generalization as they go from calculating the amount of money in the piggybank for weeks 10, 15, 25, and finally, an arbitrary number of weeks above 100. The table therefore constitutes an artifact that embeds a cultural-historical form of thinking about the saving process (Radford 2000). It highlights the theoretical content of the algebraic generalization, where repeated additions are conceptualized as a multiplication – a crucial step towards the *conscious awareness* of the algebraic structure of the sought-after model.

In the terminology of activity theory, the *object* of the classroom activity is learning to reflect algebraically about patterns. In the course of the activity, the object appears in its ideal (abstract) and material (concrete) form. In the material or concrete plane, its ideality is exhibited through particular instances. Yet, the particular instances do not exhaust the object to which they refer. This is why the object of the activity cannot be the production of the algebraic expression 6 + 3n or any other linear expression, like 1 + 2n, etc. In turn, the particular instances appear under the form of a *goal* to be reached – the production of a model featuring an algebraic structure of the saving process. Objectifications, that is to say, the collective processes through which individuals seek to attain the goal and the object of activity are entailed by cognitive and emotional transformations that arise as efforts to deal with the inherent contradictions of activity. These transformations are marked by the *motive* of the activity that is materialized in the form of affective orientation of the individuals as they produce understandings and non-understandings with clear emotional valences. In the next section we explore the question of emotional valences in light of the production of non-understandings.

How Activity Produces Negative Emotional Valence and Expressions of Not Understanding

For cultural-historical activity theorists, activity, rather than the individual subject, constitutes the smallest useful unit of analysis: Without other manifestations such as tools, division of labor, rules, and community, we cannot understand and theorize the events that we see on the videotapes. Emotions, therefore, need to be analyzed at the level of activity rather than at the level of the individual. Emotion, like consciousness, is an inner reflection of the material activity as a whole rather than a mere biological and physiological state of the human subjects involved. It is therefore as part of the unfolding activity that emotions are both reproduced and transformed. Over the course of the following three sub-sections, we exhibit the events in the course of which the nature of the emotions expressed changes from positive to negative, and from negative, to positive. Emotions are an assessment of the current state of the activity in respect to the outcome to be achieved.

'Now I Understand. You got it Wrong'

The three students begin by counting out the number of chips that they place into each of the five goblets. They count out 6 blue and 18 red chips for a pile that ends up next to the fifth goblet and corresponds to the final cell in their table of values (Week 6). At this point, Mario asks, 'What now?' Thérèse points to the table of values on Aurélie's work sheet and they begin the task to fill it. But there is a debate, because the latter points out that they do not have the same as he does. Aurélie and Mario repeatedly ask Thérèse what she is doing. Aurélie has already repeatedly expressed frustration and has rebuffed a student from another table who wanted to help: 'But we don't have the same thing that you have'. She continues, 'but look', we already have done this', while pointing at the table of values on her sheet. Mario tells her, 'Ali, just add on the side'. She asks Thérèse about the numbers highlighted in yellow on the worksheet and then announces all they had to do was 'add three and three'.

Fragment 2.1 takes up the last of these questions that Mario directs to Thérèse (turn 001). There is a long 15.11-second pause that follows during which Mario gets back to his sheet. We can see Mario moving his fingers up and down between two consecutive rows of the table of values. Aurélie pounds the desk with her fist, then throws herself backward against the backrest (Fig. 2.2), throws her hands up in the air, and then lets them drop into her lap (turn 002). Thérèse, who has been filling her table of values leans back and breaks the silence, utters a very long, drawn-out 'okay' (turn 003) continuing to gaze at her worksheet (turn 003). There is a pause, and then Mario produces an interjection of surprise, 'Oh, oo' and then says, 'Now I understand' (turn 005).

Fig. 2.2. Aurélie (left) has disengaged after pounding on the table (turn 001).

Fragment 2.1

```
001  M:  <<all>resa> what are you DOing? ((Aurélie leans
         backward, Fig. 2.2, Thérèse writes on the oriented
         toward her; English in the original))
002      (15.11) ((Mario orients to his sheet, Finger moves up
         and down between rows, pounds on table, throws herself
         back))
003  T:  okay:::::::
004      (1.40)
005  M:  <<f>oh oo> now i understand. you did it wrong! (1.49)
         .hh the first wEEK (0.78) she has how much; (0.21)
         ((He points to the goblet of Week 1)) n:IN:E. (0.89)
         we write n:IN:E (1.19) the second week (0.43) she has
         how much? we write it (0.24)
                    th[ird (0.35) how much            ] ((A
         still leans back))
006  A:  <<plaintive> [we havent even finished the fir]st
007  T:  no no no ((She laughs))
008      (0.74)
009  A:  <<plaintive> [we havent even finished] the first
         [And like it doesnt make sense]>
010  M   [look tresa, (0.58) look the  ] first s: (0.44) the
         first week, (.) she has nine. ((points to Week 1, Fig.
         2.3)) (1.10) second wEEK, she has:: (2.00) elEVen
         (0.63) wait no. (1.09) ((he points towards week 2))
         twELve. (0.74) third wEEK, she has (2.18) FIFteen
         (0.75) ((physically establishes relation between
         goblets and cell in table of values [Fig. 2.3]))
         (.)               [we write (0.32) that.  ]
011 A:   <<plaintive> what [are you doing thérèse .] ((hits
         table, rests head on table, Fig. 2.4)) ((3:01))
```

Mario further suggests to Thérèse that she has done something wrong and then articulates what needs to be done all the while doing it (turn 005). Placing his left arm and hand such that his index finger comes close to the goblet marked ‘1’, he says, ‘the first week . . . she has how much?’ He continues, ‘Nine’. He orients to

Fig. 2.3. Aurélie continues to be disengaged, Thérèse (center) writes, and Mario explains to her his understanding of the task (turn 010).

his worksheet, points to the first cell with the index finger of the left hand and then writes (right hand) while saying, 'we write nine'. There is a pause, during which he orients to the second cell in the table, and says, 'the second week he has how many, and you write it'. He continues, 'the third week, how many' and then moves his hand pointing to two more cells in the table exhibiting its sequential nature from left to right. In a plaintive intonation (high, strongly falling to the end), Aurélie suggests in a plaintive voice, 'We haven't finished the first, and further, that doesn't make sense' (turn 006).

Thérèse, who up to this point has apparently been listening but stared into the air, turns to Mario who rises from his worksheet to turn and gaze at her, when she says 'no' three times (turn 07). In a plaintive voice, Aurélie repeats what she has said before, 'We haven't even finished the first' and then continues, 'then, like this doesn't make sense' (turn 009). Neither Mario nor Thérèse appear to react to what Aurélie has said or how she has said it. Instead, simultaneous with the second part of Aurélie's utterance, Mario begins his explanation again. 'Look Thérèse, look, the first week, she has of it nine. Second week, she has . . . eleven . . . wait no . . . twelve. . .' (Fig. 2.3). He moves his sheet onto Thérèse's table, close to her. He continues, 'Third week she has of it . . . fifteen . . . and we write that' (turn 010). Aurélie rises from her lounging position, pounds the desk, then asks, 'What are you doing Thérèse?' (turn 011) with apparent frustration in her voice, then places her head on the folded arms on her desk (Fig. 2.4). At this point, Thérèse has completed four cells of the first row of the table of values and the entire second row (see statement of Problem 4). Mario, although he has verbally articulated how to fill the cells of the second table row, has not yet begun filling it in. Aurélie has just begun with the first cell.

Fig. 2.4. Aurélie, head on table, has disengaged from seeking a solution to the problem (turn 011).

In this first segment from the episode, Mario announces to his peers an insight, declares that Thérèse has done badly, and then explains twice what they have to do. His intonation – based on the correlates between prosody and emotion identified in psychological research (Scherer 1989) – expresses firmness and confidence. During his explanation, his gestures make an embodied link between the goblet-chip model (left hand index finger) and the worksheet in front of him (right hand pencil). While he explains, Aurélie repeatedly makes statements about the status of their work, her intonation expressing complaints, and says that this does not make sense. She pounds the table repeatedly, and throws herself back against the backrest, slouching for a while in disengagement.

As their sympractical activity unfolds, Mario exhibits confidence, and when Thérèse responds negatively to his first explanation, Mario does it over again, this time providing the actual number of chips for goblets 1, 2, and 3. She has finished her table of values and, following Mario's first explication, confidently says 'no' repeatedly, shaking her head sideways in apparent disagreement. Aurélie, on the other hand, increasingly exhibits frustration and disengagement from the activity. In the turn before the present fragment, she has already indicated that she will go on to the next because, and she continues in English, 'I have no clue what she is doing'. Thérèse appears confident. At the end of the fragment, Aurélie asks Thérèse again what she is doing.

There are three aspects to Aurélie's expressions. She makes statements about the status of the task and describes the situation as not making sense. These are cognitive expressions, ways of articulating forms of experience to others using words. They pertain to what she knows (does not know) the task to be, what to do next, and statements about understanding. Second, her intonation and other voice parameters – which tend to be produced unconsciously – express emotional valuations, here, of the negative type. Third, she makes two types of bodily expressions that can be seen and heard as expressions of emotion: she pounds on the table and she throws her body backward against the backrest. In fact, she is not simply producing these expressions sequentially, but the plaintive voice, expression of emo-

tion, simultaneously articulates speech sounds that encode cognitive statements. In this situation, the difference between emotion and cognition is undecidable. The same vocal material expresses both emotion and cognition: it has conscious and non-conscious aspects simultaneously.

Aurélie as a whole becomes an expression of the sensuous-valuational and volitional character of activity. She wants to engage in the task, complete and understand it, but at the same time, the sensuous-valuational aspects are expression of the distance between where she is and where she has to get. Wanting to understand and complete the task and the prospects of getting there are co-expressed reflections of the current state of the activity as Aurélie concretizes it in and with her actions.

'What are You Doing. . . I Don't Understand. And I Will Never Understand'

Following Aurélie's question to Thérèse of what she is doing, there is a pause, then an interjection (turn 013). Mario asks Thérèse what she is doing, and the latter suggests following another interjection, 'just copy me' (turn 015). Overlapping her, Mario indicates the intention to speak, but then stops, as Aurélie, in a plaintive intonation, suggest, 'We have no clue what you are doing, so' (turn 017), but Thérèse produces another series of repetition of interjections (turn 019). There is a pause, during which Thérèse turns her worksheet so that Aurélie can read it, and then she produces another interjection (turn 019). Aurélie has placed her head on her folded arms on the table (Fig. 2.4). As the camera zooms in, Thérèse addresses Aurélie by name, as if calling her and inviting her to participate, and then tells Aurélie that the camera is 'watching' her (turn 023). That is, Thérèse makes apparent to any bystander (including the analyst) that she is aware of Aurélie's disengagement and that this fact can be seen on camera.[3]

Thérèse then begins to fill in the first figures into Aurélie's worksheet and, after a 6.45-second pause, Mario in turn suggests to Thérèse that she, now filling out Aurélie's sheet, is on camera, to which Thérèse responds in a low voice and in a slow and deliberate manner that she knows and that she does not write anything (turn 027). During the pause that follows, Mario turns, leans far back, and looks around the classroom. He raises his hand (Fig. 2.5). His whole body is, following Merleau-Ponty (1945), an expression; teachers understand such expressions as those of students seeking help. There is another pause before Aurélie suggests that she does not understand and that she will never understand (turn 029). Mario has returned his gaze to his worksheet still holding up his right hand, but elbow on his

[3] It is evident in situations like this that the participants themselves make available to each other what they are conscious of and what they attend to. The researcher does not have to attempt to get into the head of the participants, who make available anything and everything required to each other for mutual and *participative understanding* of the situation.

Fig. 2.5. Mario raises his hand, turns toward the classroom; the teacher will eventually come and thereby acknowledges the gesture as a call that he has a question (turn 028).

desk. In this second part of the fragment, we observe further expressions that are simultaneously emotional and cognitive reflections of the activity from the perspective of the acting subjects. Aurélie's intonations are plaintive and lamenting while she repeatedly addresses Thérèse, complaining that she does not know what she does.

Fragment 2.2

```
011 A:  <<plaintive> what [are you doing tresa.  ] ((hits
        table)) ((3:01))
012     (2.69)
013  A: um chums.
014  M: <<p>what are you> DOing.
015  T: <<p>aw chuggy just [copy me. >   ] ((English in
        original))
016  M:                <<p>[okAY so first] [of all. > ]
        ((turns to Thérèse; English in original))
017  A:                       <<lamenting> [we have no] idea
        what youre dOIng sO> ((very high pitch, 570 Hz max,
        3:09, both A & M oriented toward T))
018     (1.33)
019  T: dan dan dan dan ((she moves the chips away from her
        page and toward))
020     (4.14)
021  T: <<confident>(qwi::::?) (gret?)>
022     (1.73)
023  T: <<f>aLI::;> cameras wATching you. ((3:21, Thérèse
        fills up the table for Aurélie))
024     (6.45)
025  M  <<p>tresa, youre on camera; >
026     (1.19)
027  T  <<len>i=know, i=m not writing anything. >
```

Fig. 2.6. Aurélie stares at her hands placed on the worksheet, while explaining in a plaintive voice that she does not understand and that she will never understand (turn 029).

```
028      (3.41) ((Mario raises his hand, turns around))
029  A:  <<plaintive>i dont understAND; and I will nEVer
         understand.> ((Stares at her hands placed on the
         worksheet, Fig. 2.6)) (3:38)
030      (0.84)
```

Aurélie has placed her head on her hands on the table, while Thérèse, confident throughout this part of the segment, fills out Aurélie's table of values. That is, Thérèse exhibits a recognizable act of helping. In her actions, she exhibits for Aurélie and for the analysts her helping stance. Her actions realize a request for (provision of) help interactional pair. The 'request for help' is articulated in multiple ways. In other words, these emotional expressions in the intonation and the inactivity (frustration?) are produced simultaneously with the cognitive content about the state of their tasks (not knowing what Thérèse does), about not understanding, and, very importantly, about never being able to understand. Emotion therefore constitutes an index of the possibility Aurélie anticipates to have about obtaining control over the activity and achieving a successful outcome: realizing the object/motive. When there is a high to perfect likelihood that success will not be attainable, then the emotional valuation will be negative and there is less likely for it to pursue the activity. There is no reason to do so, for the prospect is that the activity will not lead to an expansion of control and room to maneuver.

Mario has begun to look around, as if searching for the teacher. He has raised his hand, but, after some time without response to the raised hand, returns his gaze to the worksheet. These may be seen as the first signs of uncertainty. Whether his action interactionally is realized as a request – by providing the requested help as a response – remains to be seen. Given our cultural experience with children in schools, we may anticipate particular responses to be exhibited if the request for help remains unanswered.

'This is Dumb. I Don't Understand'

The third part of this instant of classroom life begins with Thérèse's instructions to Aurélie to do 'three plus six' and, mixing the two languages, 'to write whatever' (turn 031). Thérèse then turns around and begins to talk to the group behind at the next table. Nobody speaks during an extended period of time (49.52 seconds). During this period, Aurélie and Mario are writing, where Aurélie every now and then takes a look at Thérèse's worksheet (where she might be copying). Mario intently gazes at his worksheet, finger on table of values, moving up and down between the rows. He turns toward Thérèse, then turns about and gazes toward the other parts of the classroom. Near the end of the pause in speaking turns, Mario again raises his hand, continuing to look around. Thérèse breaks the silence saying 'Oh my god' and then, after another pause, turns back to the table and leans toward Aurélie. The latter pounds her fist onto the desktop (in apparent frustration), to which Thérèse responds by saying, 'We are all mixed up' (turn 038). Aurélie overlaps her, repeating in apparent frustration (intonation drops from much higher than normal pitch to very low toward the end) what she has said before, 'I don't understand' while pointing to her sheet (turn 039). Following a 2.46-second pause, Thérèse produces another confidently expressed interjection. A further long speaking pause unfolds. During this pause, Mario drops his hand (turn 042). During the same speaking pause, Aurélie pounds the desk again, throws herself against the backrest (turn 042). Mario gazes back at his sheet while Thérèse is writing something. Mario repeatedly shakes his head (sideways) in ways that we can observe in situations where the needs of someone (standing in line) are not addressed, which culturally competent people tend to understand as expressions of frustration. Thérèse continues to write, Aurélie places her feet on the bookshelf of her desk, and Mario raises his head again, looking around the classroom.

Fragment 2.3

```
031  T:  here (0.30) you have to do (1.41) three plus six
         (0.60) yup. (1.79) <<len, p>y=write whatever>
         ((Thérèse turns around and speaks to members of Group
         4 about other things))
032      (1.29)
033  A:  yeAH?
034      (49.52) ((Ali writes, Mario raises hand and Thérèse
         talks about something else))
035  T:  ah my god.
036      (8.70)
037  A:  ((pounds on the table))
038  T:  <<p>kay we are all mi[::xed up>        ]
039  A:                       [i dont understand] ((points to
         her page, Fig. 2.7a))
040      (2.46)
041  T:  <<confidently>uh hu:::; uh huh. >
042      (25.56) ((M drops his hand)) ((Ali pounds table again,
         throws herself back against back of seat)) ((4:57,
         Mario gets back to the task, A leans back, Fig. 2.7b))
043  J:  <<f>yes.> (0.52) whAT is the ques[tion.]
```

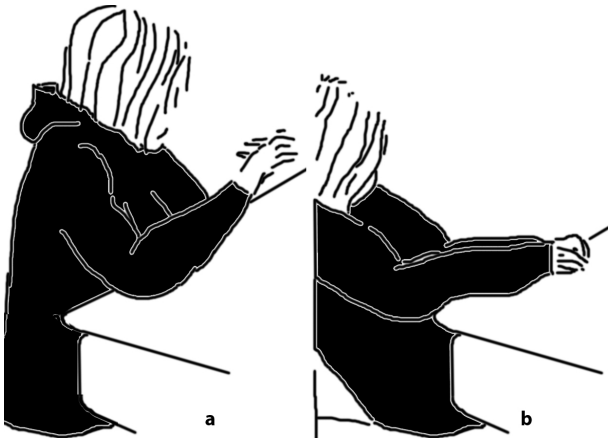

Fig. 2.7. **a.** Aurélie throws up her hands as she reiterates saying that she does not understand (turn 039). **b.** She throws her body backwards, visibly disengages with the task (turn

```
044  M:                                    [its  ] ^this :::
         (0.38) <f>um[::>] ((hands move downward, restrains not
         to pound on table, gazes at sheet, Fig. 2.8))
045  T:              [aur]élie sit properly (55:00)
046  M:  look this is (.) dUMb, <<p>i dont understAND. >
         ((487>217Hz))
```

Mario turns back to his worksheet noting something, then turns both hands upside, moves lips as if saying 'quoi' (what?) (just before turn 043). He looks up, and just at that moment, we can see the teacher Jeanne approach stating, 'Yes . . . what is the question?' (turn 043). With this, Jeanne exhibits a response to Mario's request for help, or rather, in approaching the table and asking 'what is the question', she formulates for us her understanding of the nature of Mario's preceding actions to be a question. 'It's this', Mario responds, and then produces an interjection. The palms of his hands open toward the ceiling, his arms move up and down as though he is containing himself with a lot of effort (Fig. 2.8). The intonation falls from high (480 Hz) rapidly to a much lower pitch value (300 Hz). The mean pitch is 396 Hz, up from 280 Hz, F1 mean is up from normal 500 Hz to 787 Hz.[4] All of these are consistent with the research that shows correlations of these parameter changes with despair/disgust and irritation (Scherer 1989). Jeanne then addresses the way in which Aurélie is sitting and articulates it as an improper way of sitting during this task: 'Aurélie, sit properly' (turn 045). Mario continues with expressions that provide intellectual assessments of the situation: 'look this is dumb, I don't understand' (turn 046).

[4] The pitch, or F0, is the main and lowest contributing frequency of the voice. F1, F2, . . . are the next (higher) contributors to the voice. Psychological and sociological research have shown significant correlations with emotions of the first two frequencies, F0 (pitch) and F1 (e.g., Scherer 1989).

Fig. 2.8. Mario expresses spending energy containing frustration, as if taking something and shaking it between his hands (turn 044).

Signs indicating that Aurélie and Mario do not understand mark this third part of the fragment. Their frustration is 'written all over the situation'. The emotional expressions include the pounding of the desk, leaning backward, looking around the classroom with raised hand, and the shaking of hands while articulating the fact that he is not understanding. The fragment does begin with the marking of an insight and the subsequent articulation of what the task is about. From the perspective of a person 'in the know', he is absolutely correct. Yet Mario apparently seeks the teacher's help substantiated in her addressing him with 'Yes, what is the question?' Jeanne has recognized that Mario has a question, and she articulates this understanding for us. The teacher also lets Aurélie – and everyone overhearing – know that her current way of sitting is not appropriate, and she asks the student to sit in the way one is expected to sit. Mario has filled the first row of his table of values, which is one of the goals communicated on the worksheet, and, despite successfully doing so (as judged from the outside), has become increasingly frustrated. That is, in the unfolding of his activity, as he realized his activity in a concrete way, Mario also changed his emotional tonality from confidence to frustration. As the activity is concretely realized in the material outcomes of Mario's actions, it also produces a negative affect. The coincidence of the affective expressions with the cognitive expressions is observable throughout this fragment. In fact, the difference between the two is undecidable, as they are produced simultaneously, in the case of the verbal productions even in the same medium (sound). One part of the sound material is heard as expression of cognitive content, the other as emotional content.

The instant is an expression of an inner contradiction, the co-presence of the current state and an object/motive that is not yet realized. The contradiction is reflected in consciousness, and expressed in both cognitive and emotional terms. Because cultural-historical activity theory is a dialectical approach, inner contradictions of the activity are understood as drivers of change: they are expressions of

change itself. Thus, contradiction is regarded 'as a necessary form of development of knowledge, as a universal logical form' (Il'enkov 1982: 234). That is, this contradiction is a necessary but, as we see, not sufficient condition for learning to occur.

The Relation of Emotion, Cognition, and Practical Activity

In this lesson fragment involving Mario and his two peers, the object/motive of activity does not and perhaps cannot emerge from their engagement because what they are conscious of (in what they make available for each other) is not that from which the generalization can (more easily) emerge. They count, they are busy with filling the required number of chips into the goblets, and they fill the upper row of the table of values. But this is not the object/motive of the activity. Mario announces his recognition of this fact in the expression 'I don't understand [Je ne comprends pas]'. It is precisely the momentary abandonment and the intonations that allow us to perceive the emotional quality of Mario's and Aurélie's current state, their frustration, their disorientation, and their questions. Each announces his/her assessment of his current cognitive state, 'I don't understand'. There is a gap between what they know and the object/motive of the learning activity, and this gap is so *large* that their current actions do not get them any closer; in fact, they cannot even establish how far away or how close they are to the object/motive. The contradiction that exists here is that Mario has already stated how to fill the table, already is on the way of realizing one of the goals toward the completion of the activity, but his emotional valuation is negative. Also of importance is the fact that Aurélie expresses extreme frustration although in the course of this fragment, her worksheet comes to be filled. That is, completion of the worksheet is not a sufficient criterion for completing the activity. Her frustration is the expression of the emotional valuation of the distance between where she is and the object/motive of the activity; this valuation goes hand in hand with the cognitive assessment: 'I don't understand. I will never understand'. We can appreciate here that the question for Aurélie is not just to get the table filled. She *wants* to understand, and not only to please the teacher.

In cultural-historical activity theory, 'the particularity of emotions is that they reflect the relation between the motives (needs) and the success or the possibility of a successful realization of the corresponding activity of the subject' (Leontjew 1982: 145). That is, 'emotional valences arise from emotional valuations of sensorially or cognitively comprehended object properties with respect to the 'appropriateness' for the reduction of certain negative state value and *change with the changes of the cognition of the corresponding objects*' (Holzkamp-Osterkamp 1976: 49). Emotions are the product and the mechanism of the motion of the activity. In Aurélie's and Mario's instance, we observe their emotive reaction as a result of the fact that despite their efforts, the motive of the activity does not reveal itself. We can also see in Thérèse's expressions of confidence the expressions of positive

valuations, as she has completed the important third row of the table of values with the corresponding values (3 x 1 + 6, 3 x 2 + 6, . . .). As the first part of the episode progresses, this becomes increasingly evident, and initially the girl, then Mario, indicates not to understand ('I will never understand'); both demonstratively stop their engagement. It is only at the very end of the events analyzed here (see next section), the positive valuation occurs as the motive progressively reveals itself, leading to a positively valued emotional state, clearly available to the onlooker in his comportment and the satisfied cognitive assessment of his current state by means of the utterance 'I understand'.

To become a learning *motive*, it is insufficient that the learner be conscious of the difference between his/her current prior knowledge and the learning object. S/he also needs to experience *directly the insufficient and partial nature of his/her current articulation of the learning object*. This experience necessarily introduces an emotional-motivational component: 'The obstructions to the realization of actions implies – as a limitation of control/life quality – a certain emotional sense of insufficiency, "frustration", disquietude, fear, and the likes as undisclosed premises with respect to the grounds and possibilities for overcoming the obstacles to action' (Holzkamp 1993: 214). It is this realization that serves as the emotional valuation at the source of the actions that diminish the gap between the current knowledge and the learning object. And the successful disclosure of the object/motive of the learning activity in its entirety – its objectification – is marked in terms of a positive emotional quality. Thoughts do not think themselves, and they do not *inherently* push themselves to learn and develop (Vygotsky 1986). It is only when there is an inherent emotional quality to knowing and thinking that we can understand why someone *wants* or *should want* to learn. But learning activity is easy to understand if successful disclosure of the initially unknown learning object leads to increases in control and action possibilities, increases that are associated with positive emotional valuations. It is precisely here that emotionality obtains an orientational function in activity with respect to the acquisition of knowledge (Holzkamp-Osterkamp 1978).

Evaluative feedback occurs by means of an emotional tone, which has a complex quality, and 'condenses all particular evaluations automatically into a unitary execution of action, on the basis of which alone goal-directed action is possible' (Holzkamp-Osterkamp 1991: 104). The emotional valuation of the conditions constitutes the first step of cognitive processes, including those that are involved in learning. 'The emotional reaction, generally a more or less diffuse feeling of "ease" or "unease" evoked by the complex situation, serves to inform and correct the conscious goal- or task-oriented exchange with the environment' (ibid.: 105).

Some educators might think that the teacher should have simply given the students the instruction to copy the number of chips into the equivalent table cell and given them the formula that could have led him to fill each of the cells in the third row of their table of values. But this would have been a mechanical acquisition of a lifeless fact that Mario might remember but that he would have less likely been able to use. It is possible, writes Leont'ev, to acquire factual knowledge in mathematics or physics in such a way that it remains dead and unused until life itself

awakens it – if the student does not forget the facts in the meantime. But 'if the subject matter content is not to be acquired in a formal manner, then we must not just "sit through" the lesson during learning, but we have to live it through' (Leontjew 1982: 281). This living through a productive process changes the person, who, in productive activity objectifies himself in the product of his labor and is subjectified as he becomes conscious of the outcomes of production, subjectifies the thing and activity in the form of the inner reflection and object/motive (Marx/Engels 1983). 'The inner (the subject) operates on the outer and thereby changes itself' (Leontjew 1982: 174). The lessons themselves have to become part of the significations for students generally and Mario here particularly. This signification arises from the motive of his activity, which is, as activity among other activities, constitutive of the totality of his life. It is precisely in the real life of the child that motives develop. The purpose of the lesson is not just to fill the worksheet, to get the numbers right in each cell. The purpose is for Mario and his peers to become conscious of the object/motive of their activity, which discloses itself in the course of the activity. It is not just the fact of the entries taking the form $3n + 6$ that matters but the child's consciousness of *his* activity and the role the object takes. Consciousness of his activity is possible only when the child actually brings about the activity, not when he is presented with the fact that the content of the cells take a particular form.

For Leont'ev, the transformation of the materials and means – that is, the text of the task into the goblet-chip model, and the table of values into the $3n + 6$ and into the consciousness of this product as it relates to the activity as a whole – *constitutes* the object/motive. The subject's awareness of how close it is to achieving the motive expresses itself affectively: positive emotive valuation when the object/motive is realized, negative emotive valuation when the object/motive remains out of reach. The thing subsequently produced is $3n + 6$. This is the *goal* of the activity and, at the same time, is only the material side of the double nature of the *object*. The formula $3n + 6$ is a material instance of the ideal object of the activity, which is *thinking algebraically about patterns*. The object only exists in this dual nature, and this would not exist if the teacher had told him that what he had to do was to fill the bottom row of the table according to the formula. This would have allowed Aurélie and Mario to fill the table of values in a routine, mechanical (thoughtless), and alienated manner. But this cognitive motive does not fulfill itself; rather, there has to be some *reason*. This reason is not the Kantian legislative and schematizing reason of human actions. It is rather one of the cultural and historical possible reasons that opens up possibilities for *thinking and feeling* marked by resonance in social forms of knowing. It is a reason out of which a sense of belonging is made apparent to the students. It lies in a positively valued subjective experience of an increase in control over life conditions, and room to maneuver and express oneself in a field of potential actions, agreements and disagreements.

In activity there is a primary sensuousness that contains cognitive and affective moments. As constitutive moments of sensuousness, the two moments cannot be understood independently but they are mutually constitutive and subordinate to the sensuousness, a psychic reflection of material activity. The vocal track, too, is a

means to articulate cognitive content (language) and emotional valence simultaneously. Mario does not just say that he does not understand, but the various prosodic parameters all are consistent with despair/frustration as shown in research on the correlates of affect and prosody (Scherer 1989). Because they are available to others, including the teacher, they also become resources in the interactional setting. These moments are created in sensual practical activity and are a reflection of the material world. Cognition reflects the object-content aspect of the conditions; emotions constitute valuations of the current conditions with reference to the object/motives that the ongoing activity is to achieve. *In* and *as result of* practical activity – which may entirely concern ideal entities – the 'affectogenic character of entities may be changed' (Leontjew 1982: 190). The same entity, in Leont'ev's case, a bear, may be the source of fear, during an unexpected encounter, or joy, in the case of a bear hunters waiting for their game. The emotional tonality of actions reflect the object/motives of activity, even when these are not present in consciousness, so that object/motives are never separate from consciousness, that is to say, from *objectifying* processes.

Emotions are expressed in sound as much as cognitive content is: Both are part of the same expressive material and therefore should not be considered as functioning independently (Merleau-Ponty 1945) but as two moments that each reflect the same situation but only partially and one-sidedly (Leontjew 1982). It is their embodiment and their physical co-presence that allows participants to make and have access to emotional valuations of each other: In this way that they shape the interaction rituals in and through which participants create society at the microscale (Collins 2004). The co-expression of cognition – Mario knows that he does not know – and emotion – Mario, as Aurélie, expresses frustration – is available to others, here Jeanne the teacher, who act upon these expressions. Jeanne and Mario are not mindless machines (computers) passing information (signals) between each other, they are corporeal human beings with emotions that they, too, make available by a variety of means including their body positions, body orientations, gestures, and prosody. Jeanne and Mario jointly orient to and collaborate in the production of sympractical activity, and this joint orientation is constitutive of their *participative (non-indifferent) understanding* (Bakhtin 1993). But theoretical cognition alone cannot explain the events we followed so far. Only activity as a whole gives us an understanding of the actually observed events. Cognitive content, too, may be articulated for others by nonverbal means, such as when a person nods to suggest agreement, and even hand gestures. In fact, hand gestures may articulate both, an affirmation that a response was appropriate (see gesture) even in the absence of words and a particular emotional orientation to the situation. Thus, just before the end of a subsequent fragment, Jeanne will make a two-handed gesture that might be glossed as 'You got it, so what was the problem'. We come to this and similar expressions on the part of the teacher in the following two chapters.

3

Learning as Objectification

In the going educational and psychological literature, learning is conceived of and theorized as the transition from one state of knowledge at some initial point in time, $K(t = t_1)$, to a second state of knowledge at a subsequent point in time, $K(t = t_2)$. Others prefer to write/speak about an initial conception $C(t = t_1)$ and the change to another conception $C(t = t_2)$ at a later point in time that is the consequence of teaching/learning actions and involves a more or less radical restructuring. An apparent problem that psychologists and teachers alike also make thematic is students' apparent unwillingness to learn or change conceptions. To understand the situation, these researchers then see themselves forced to introduce another category, motivation. It allows thinking about what a teacher might do to motivate students in such a way that they do what they do not want to do on their own (learn, change conceptions). As suggested in chapter 1, cultural-historical activity theory approaches the problem in a very different manner.

In cultural-historical activity theory, learning is thought of as a as transformation. This transformation is marked by a general movement produced by the subject's encounters with diverse and often conflicting cultural forms of being, knowing, and feeling and the unfolding subjectivity that is continuously produced and updated in the course of those encounters. We can see from the outset how different cultural-historical activity theory conceptualizes learning from other theories. Learning is not about an individual changing concepts from within (as in conceptual change); nor is it about developing more and more powerful cognitive mental structures as the individual tries to adapt to the environment (as in constructivism). Notwithstanding Piaget, there are no ahistorical and acultural universal mechanisms of knowledge production that could account entirely for the way human cognition functions. As Vygotsky argued, the lines of biological and cultural development merge. As a result, the higher forms of cognitive activity (visual, tactile and oral perception, memory, thinking, or symbolizing) are transformed in a way that the biological and the cultural are no longer separable. The cultural phonetic sounds to which the newborn is exposed during the first eight months define progressively the confines of those sounds that will constitute its phonetic repertoire

(Werker and Tees 1984). And those of us who have learned a second language as adults know very well how difficult – if not impossible – is to produce sounds beyond our first language repertoire.

In cultural-historical activity theory the idea of transformation that lies at the heart of learning captures the inseparability of the biological and the cultural and can be summarized as follows. At birth, we all arrive in a world that is already replete with concrete and conceptual objects. The world in front of us is not the Adamic world of untouched nature but a historical world which, through objects and practices, conveys significations and forms of reasoning – aesthetic, ethical, mathematical, scientific, and so on. In this world, the child interacts with others. Because according to Vygotsky (1989), any higher-order psychological function *is* a social relation first, the child's development (ontogeny) is a function of its societal relations with others. Now, precisely because the forms of reasoning that we encounter in the world have been forged and refined through centuries of cognitive activity, and are a result of conflicting ontological, aesthetic, economical, and political views, they are not natural. They are cultural. They are sedimented forms of complex historical sympractical activities and as such appear far from trivial for the students.

Within this context, learning can be theorized as those transformative processes through which students gradually become acquainted with historically constituted cultural significations and forms of reasoning and action. Those processes are termed *processes of objectification* (Radford 2002). They entail a moment of poēsis: a moment of 'bringing-forth' something to the realm of attention and understanding. Poēsis is a creative moment of disclosure – the event of the thing in consciousness.

Learning leads not only to the renewal and transformation of cultural forms of being, knowing, and feeling, but also, from the individual's perspective, to the creation of room to maneuver and to gain control over conditions within the activity at hand and the larger project of individual life more generally, which interconnects the various activities in which an individual participates in the course of his/her life (e.g. Holzkamp 1993). Intentional learning is *not* the conscious object/motive of the activity. It cannot be the object/motive because it is unknowable from the perspective of the student given that, qua learning, s/he *does not yet know* and therefore is asked to learn. If at all, the object/motive of the activity may reveal itself to the student. When a person has taken up the subject position in an activity, he or she has also taken up its object/motive. The conscious subject may realize that there are obstacles on the way to realizing the object/motive – i.e., concretizing the intended outcome – in which case s/he might engage in a form of action that expands his/her possibilities toward realizing the object/motive. This objectifying movement oriented towards new forms of action, possibilities, and control, a movement that is simultaneously cultural and subjective *is* learning. It is inherently associated with a positive valuation. Such learning does not have to be motivated: the subjective-cultural movement constitutes a self-motivating moment of activity. In chapter 2, we observe Mario expressing a lack of understanding and his request for assistance marks a *desire* to overcome this obstacle so that he can work toward

realizing the task specifically and discover the associated object/motive more generally. This is different from Aurélie, who also requested help but who accepts it in a form that does not allow her to understand or to independently end the task and realize its object/motive. Mario's desire to overcome the obstacles – or his *will to knowledge*, to use Foucault's (1971) expression – does not amount to looking for adherence to social forms of doing. If that would be the case, he could have patiently waited for the teacher to conduct a general discussion and see the solution appear on the blackboard. As we interpret it, Mario perceives an opportunity in the creation of new possibilities, which, as his actions express, is anticipated in his seeking of understanding rather than in merely copying from the worksheet of another person.

Creating Action Possibilities

In the previous chapter, we observe how Mario expresses both emotion – frustration – and a reflection on a cognitive state (expressed through the utterance 'I do not understand'). We also observe that the teacher, in approaching and beginning to interact with him, realizes that Mario is stumped, while also realizing his concomitant emotional states. Jeanne 'knows' Mario as a particular 'emotional' individual – though this is not available in this episode itself but through our weekly debriefing conversations with her. There is therefore a mutual understanding of the current need state in which Mario finds himself. This collective attention to need states is of particular relevance to cultural-historical activity theory and the integral relation it theorizes between the individual and the collective. Thus, action possibilities are no longer determined in terms of what the individual can do but – because of the extent and quality of the societal relations with fellow humans – in terms of collective possibilities (Holzkamp-Osterkamp 1978). The consideration of the emotional valuation of objective conditions on the part of others is taken into account and becomes of special signification for the individual, because in the overlapping evaluations of objective conditions is expressed the emotional connectedness as subjective valuation of the exponentiation of action possibilities in the collective. With Jeanne's arrival at the table and her interactions with Mario in particular, learning possibilities did not automatically expand. In fact, as we show here, the continued sympractical activity initially does not lead to appropriate teacher actions and does not open up new possibilities for student action. Knowledge objectification is not realized. The negative result is reflected in the negative cognitive and emotional evaluations.

'Okay. . . What Did You Have to Do?': Attempting to Get Unstuck

Mario and Jeanne have related and perhaps complementary problems. On the one hand, Mario does not understand the purpose of the activity. Coming to understand this object/motive is in fact the raison d'être of the classroom activity. Jeanne could have started the classroom activity by verbally articulating the object for the students, saying something like 'Today the object of our activity will be to learn to think algebraically about patterns' and, of course, this would not mean much to the students. In terms of understanding, the students would have practically gained nothing. At this point, Mario's task at hand is to engage in the sympractical activity so that this object/motive discloses itself in the course of the objectifying process.

On the other hand, Jeanne, as she articulates later (turn 169), is in the process of helping Mario to understand. Her problem is finding the appropriate pedagogy that will do the trick. Her problem is to launch an objectifying process where room is created for a joint work. The two have to work together, as Jeanne will be able to find the pedagogy that works for him all the while exhibiting that she is helping, even though success cannot be guaranteed; and Mario has to assist Jeanne in exhibiting what he knows and understands and what he does not. This collaborative work is possible only because they already share a great deal of intersubjectivity, of common ground, on the basis of which they can knowledgeably engage each other. In fact, every word that one or the other is going to utter implies the other's willingness to understand. Each word has to straddle the current speaker and listener – or they would fail to understand one another.

In the unfolding activity, emotionality, too, is reproduced and transformed. Initially, in the first two parts of this fragment, negative valuation is available in the expressions of frustration. This valuation therefore is reproduced from the end of Fragment 2.3 where we observe it for the first time. It is only at the very end of this second fragment that Mario will exhibit a positive emotional valence. At this point, the object will have revealed itself in the poetic moment of objectification, and the gap between the state to be achieved and the current state will be reduced.

Mario does not just offer a description of his current cognitive state, but also invites and even asks for help. The request is initially declined as Jeanne begins by asking whether they are discussing (the problem) within the group. Mario says with frustration in his voice that Thérèse 'just left' and that 'they have already written stuff' (turn 049). After a while, Mario adds that he does not like it. Jeanne responds 'but yes' 'because they are in the process of helping you' (turn 053). 'But how', Mario answers in frustration (turn 054). In response, Jeanne begins a first explanation (see arrow). We can gloss these events in this way. Jeanne initially refers Mario back to the group, but, given his negative response, she begins a teaching sequence. There is both a cognitive and an emotional response in stating that he does not like it and in the intonation that expresses his discouragement. The need for her engagement arises from the interaction with Mario, in response to his expressed emotional needs. If they had been purely cognitive, she could have referred Mario to work with his peers.

Fragment 3.1a

```
046  M:  look this is (.) dUMb, <<p>i dont understAND.>
         ((487>217Hz))
047  J:  =are you having a group discussion?
048      (0.16)
049  M:  no. tresa is just gone so from:: (0.53) lOOK (0.72)
         like (.) they alrEADy wrote thi:::ngs ((Frustration))
050      (1.20)
051  A:  <<f>^ma[da:me.>]
052  M:         [i like ] i dont like. ((discouraged))
053  J:  but YES because they are GUIding you.
054  M:  like, ^how:: that.
055      (0.11)
056  J:  kay (.) first week (0.84) wHY (0.16) ˇwOUld (0.75)
         there be? (0.91) why would there be:::sIX (0.61)
         dollars in the piggybank.
```

Jeanne begins by asking why there should be $6 during the first week (turn 56), but Mario responds that there are 9 (turn 060). There are longer pauses in her delivery, with repeated elements of the utterance, as if she were seeking the appropriate question. Mario says that there are 9 (turn 060), and Jeanne begins to overlap, asking whether there really are $6 (turn 061), while closely inspecting the first cell in the table. She insists, 'there are three plus six' (turn 64). Between Jeanne and Thérèse, they work out that the deposit during the first week is 3 + 6 = 9. Jeanne insists on the $3: 'Why do you think the three is in yellow?' (turn 069). Both Mario and Thérèse suggest that they do not know (turns 71 and 73, respectively). Jeanne insists asking where the three are coming from (turn 076), and, when the responses are negative (Thérèse) or about something that has nothing to do with the task ('the wedding thing'), she asks, 'but what precisely are the three dollars?' (turn 080). Thérèse begins by saying that these are the $3 that she saves (turns 082, 088); Mario overlaps her saying, 'the ones she takes each week' (turn 085). Although Thérèse notes that she understands – she does have the searched-for responses on her worksheet and these are 'like' (unstated, but perhaps consistent with what Jeanne has said) – Jeanne states a resolute 'Okay', and then asks that they re-read the problem (turn 094).

Fragment 3.1

```
→ 056  J:  kay (.) first week (0.84) wHY (0.16) ˇwOUld (0.75)
           there be? (0.91) why would there be:::sIX (0.61)
           dollars in the piggybank.
  057      (1.04)
  058      for the first week. (.) what did you ge::t (0.37) to
           do ((she takes the goblet of week 1))
  059      (0.75)
  060  M:  becau::se (0.30) ((points to goblet 1))
           [but there is nine ] <<dim> [the first week]>
  061  J:  [there is, you know] <<crsc>[is it really] six
           dollars? ((points to and looks closely at M's first
           cell))
  062  M:  no:n?
  063  T:  no. yea.
  064  J:  =ITs three plus six:.
```

Fig. 3.1. Mario has shaken his head and now gazes with a questioning look at his teacher (turn 071).

```
  065  T:  ive said <<whispering>[that is what ive said]>
  066  J:                          [why three plus six:. ]
  067      (0.34)
  068  T:  because it equals to the deposit of the first week she
           has nine.
  069  J:  it EQuals to nine the first week. (0.78) wHY is the
           thrEE in yellow? whydyou think? ((Index finger on
           number in first column))
  070      (0.19)
  071  M:  um um, um ((shrugs shoulders, shakes head 'no',
           questioning look, Fig. 3.1))
  072      (0.20)
  073  T:  <<all>i dun[no]>
  074  M:             [be]cause we are supposed to write it?
  075      (0.44)
  076  J:  WHEREe does the thREE come from?
  077  T:  donno?
→ 078  M:  <<f>a:=u:> (0.24) u:: (0.17) u: dududu: wedding thing
           there?
  079      (0.76)
→ 080  J:  but ((exasperation, turns head away from Mario))
           (0.14) the three dO:LLars? is wHAT exACtly? ((Mario,
           who has looked at her, grimaces in desperation, brings
           his hands up and covers face, Fig. 3.2 [6:11]))
  081      (1.61)
  082  T:  its its:: its [the three] do::llars there that
           s::he::.
  083  M:                [u::h:    ]
  084      (0.48)
  085  M:  she takes [each] week.
  086  T:            [ss: ]
  087      (0.38)
  088  T:  aves ((Jeanne moves head to side over shoulder, gives
           him 'a look'))
```

Fig. 3.2. Mario brings his hands up and covers his face, his whole body becoming an apparent gesture of frustration (turn 080).

```
  089      (0.45)
→ 090  M:  like i dont understa:nd. ((reacting to teacher look,
           points to his worksheet, Fig. 3.3 460>229 Hz))
  091      (0.59)
  092  J:  <<all>okay.>
  093  T:  <<len>i understand .h:: ive it l[ike   ].
  094  J:                                  [reREAd] the problem.
           (0.33) lets reREAd the problem. wHAT does it tell us
           to do here it?
```

We see both Mario and Jeanne produce emotional expressions, their overall valuations of the current state of affairs. Mario has introduced, partially in English, 'the wedding thing' (turn 078; he may be confusing the anniversary event with a wedding one), and there is nothing at present that has to do with a wedding. Jeanne produces signs of exasperation in the adversative conjunction 'but', the exhalation, her turning of the head away from Mario, and then the third iteration of a question about the 'three dollars'. Mario erects, grimaces, and then covers his face with both hands (Fig. 3.2). When he responds with the description of a fact (still in his normal voice parameters) that has no equivalent in the story problem, Jeanne gives him a look, and Mario responds beginning with a causal conjunctive ('like', in French 'comme'), 'because I don't understand' (turn 090, Fig. 3.3), his voice parameters are consistent with the despair that our cultural competence allows us to hear and see.

In this Fragment 3.1, they do not come closer to the object/motive of the activity, as Jeanne does not succeed in doing what she apparently intends. The objectifying process has not started yet. The current interaction amounts to the creation of the conditions for the birth of objectifying process. However, at this point, there is no guarantee that such a process will start. Jeanne has started with a resolute 'kay' (turn 056) and begins another attempt with an equally resolute 'okay' (turn 092). In both instances, the beginning of a teaching sequence follows Mario's description

Fig. 3.3. In response to the teacher's look, while saying with apparent frustration 'I don't understand', Mario takes his hands off the worksheet and turns them upward toward the ceiling (turn 090).

of the situation as not understanding; and this lack is also expressed in emotional terms. Jeanne, too, produces expressions of emotional valuations, here exasperation and facial expressions that are correlates of emotion (Ekman et al. 1987). That is, in their engagement of each other, they produce the activity and, as they realize they are not getting any closer to Mario's understanding of the object/motive, valuation of this distance is available in the emotional expressions. These are integral product and resources of activity, for it is precisely because they are still away from understanding that they continue and attempt to restore understanding by further engaging each other.

For Jeanne, this part is not just a failed attempt. Given that she has not had the opportunity to overhear the students in their prior conversations, she does not know exactly where they are with respect to activity. Although she fails, the segment also constitutes an opportunity for her to better understand the situation generally and what forms of knowing Mario exhibits specifically. In fact, without engaging with the students, or without at least attending to their conversations, she does not have the information required for tuning to, and understanding, the students. Engagement, then, also means search for an appropriate pedagogy, which, in its first attempt, fails – a fact that Mario clearly states.

This is not just a failed attempt: It is in fact an integral part of the activity, which involves identifying new goals when obstacles pose themselves (as recognized by the subject of activity, here Mario and Jeanne). At this point in the episode, we cannot know how this mutual engagement will contribute to understanding, but insofar as they begin another attempt, we see their anticipation that there is still the possibility to succeed. For Jeanne, this entails finding a pedagogy that will move both of them into an objectifying process, which consists not merely in having Mario filling the worksheet but in understanding cognitively and affectively the relevance of the result achieved with respect to the actions that preceded it. For both of them, this means further engagement – with the possibility that they still do

not get closer, a fact that would receive negative valuation, and a negative emotional response. This engagement is of an *ethical* nature: it is a *call* that has to be *answered*. In a previous work, we have called this ethical engagement *togetherness* (Radford and Roth 2010), an invisible ethical relationship that glues the participants in joint activity and makes activity more than additive actions and deeds: togetherness makes activity a real unity.

'Let's Re-Read the Problem': A Second Attempt at Getting Unstuck

To get unstuck, the activity itself has to produce the pedagogy necessary to move Mario along. It is not such that the pedagogy is on the outside of the activity, getting Mario unstuck so that he can re-enter the activity to continue. The very disclosing of where the issue lies is an integral part of the activity, and the understanding may emerge at any one point in the process. Teaching and learning no longer are separate processes, for in teaching Jeanne also has to learn what is impeding the progress and in learning, Mario has to assist his teacher understanding just what his problem is. Teaching is learning and learning is teaching. Teaching and learning are the two ways in which Vygotsky's concept of *obuchenie* [обучение] manifests itself in the way value manifests itself as use-value and exchange-value during the give-and-take of the actual, concrete barter trade. Like Marx/Engel's value, obuchenie captures the movement of the teacher-student as well as the student-student and teacher-teacher self-relations. In fact, for Vygotsky (1978), any higher order cognitive function has been a social relation before. Similarly, 'The structures of the world themselves are present in the structures (or, better, cognitive schema) that the agents put into play to comprehend it' (Bourdieu 1997: 180). Obuchenie is this social relation, and, as such, it cannot be reduced to its individual members but has to be understood as an overarching category that manifests itself in teaching and learning.

'Re-Read the Problem' The activity continues with Jeanne's request that Mario read the problem again, followed by a re-iteration that *they* re-read the problem, and an invitation to articulate what it tells them to do (turn 094). Why might the teacher invite him to re-read the problem? Mario accepts the invitation and begins, 'For her birthday', then stops for almost a second, staring at his worksheet, then uttering with rising intonation, 'what?' He grimaces, his hands turn palms open against the ceiling, as if her were saying, 'what's going on here?' (turn 096). Jeanne confirms, 'Yes, for her birthday' (turn 098). That is, we have here a statement/question–confirmation sequence, which reveals Mario's question about the context in the word problem, which he previously characterized as being related to a wedding (turn 078). Jeanne and Thérèse assist Mario in reading the text by articulating some of the words that he pronounces slowly or incorrectly. After the first two sentences, Jeanne summarizes, 'So she receives a piggybank', and continues by offering a question, 'How much money does she have in her piggybank?'

Fig. 3.4. Mario, in the apparent attempt to explain, moves his left hand forward until it is above goblet 1; but he withdraws as his teacher, who has overlapped his speech, continues to talk (turn 118–119).

(turn 108). Again, we see here a clear attempt to create the beginning of an objectifying process.

Fragment 3.2a

```
094  J:                                    [reREAd] the problem.
         (0.33) lets reREAd the problem. wHAT does it tell us
         to do here it?
095      (0.58)
096  M   for her anniversary- (0.97) what? ((grimace, hand
         gesture, open toward ceiling, 'what's going on here?')
097      (0.50)
098  J:  ˇyes (0.25) for her ANnivERsa:ry
099  M:  <<len>marie-na:in (0.52) receives (0.15) a piggyb
100      (0.15)
101  J:  a piggy[bank ]
102  T:         [piggy]bank
103  M:  <<p, len>a piggybank?> (.) containing six dollars
         (0.80) she deCIdes (0.15) ah tos[ave][(.)ave]
104  T:                                    [sa:][:v::e:]::
105  J:  SA:Ave,
106  M:  save three dollars (0.50) per week.
107      (0.35)
108  J:  so. (0.35) she receives a piggybank; how mUCH money
         does she have in her piggy [bank]?
```

At this point, they have established the fact that there are $6 in the piggybank to which $3 are added each week. Jeanne asks how much money there is in the bank, and Mario responds with a facial expression as if his teacher had asked of him something self-evident, 'six', and then continues, 'plus three equals nine' (turn 112). There is an exchange over how much is added and then Jeanne points to goblet 1 uttering 'so' in a constative rather than questioning fashion (turn 117). 'We are nine', Mario suggests (turn 118) moving his left arm and hand forward so that the hand comes to hover over goblet #1 (Fig. 3.4). At the same time, Jeanne articu-

lates 'three plus six' articulatorily stressing the first numeral (turn 120). In fact, we have a mini-IRE sequence here, for we can hear Jeanne ask a question, which is confirmed as such when Mario says 'nine', which is the number of chips in goblet #1, the one in question and the one that Mario reaches out for (and thereby designates as the currently relevant and salient one).

Fragment 3.2b

```
108  J:  so. (0.35) she receives a piggybank; how mUCH money
         does she have in her piggy [bank]?
109  M:                              [six ] ((facial expression
         as if teacher had asked the 'self-evident'))
110      (0.54)
111  T:  [six dollars]
112  M:  [plus three] equals nine.
113      (0.22)
114  J:  so each week, she saves (0.13) how much money.
115  M:  three dollars
116      (0.68)
117  J:  three dollars (0.11) so: ((points to cell 1))
118  M:  we [are nINE? ] ((holds 'pick-up' hand over goblet 1,
         as if wanting to grab it, Fig. 3.4))
119  J:     [three plus] six. ((continues pointing))
120  A:  <<plaintive>are we sup[posed to do this?>] ((Points to
         Mario's page))
```

Although Mario has provided the correct response, it is not the correct response in this obuchenie (i.e., teaching-learning) relation in respect toward the anticipated outcome that is to overcome the obstacles that interfere with understanding. Jeanne's emphasized articulation of 'three' constitutes the reiteration of the previously asked-for weekly saving, which now is augmented by 'six'. Three plus six *does* adds up to nine, but this is not the sought-for signifier for the contents of the first goblet, though it *also* is a correct one. The knowledgeable person – Jeanne and the reader – is aware that the algebraic pattern for calculating the goblet contents for any given week requires the realization of the repeated addition of $3, one such amount per week, so that the total amount added by week *n* equals to *n* x $3. This repeated addition does not become salient if Mario uses the signifiers 9, 12, 15 and so on for the contents of the goblets rather than the signifiers 3 + 6, 3 + (3 + 6), and 3 + (3 + (3 + 6))). From the latter, there appears to be a much shorter step to generalizing the total sum to *n* x 3 + 6 then from the sequence 9, 12, 15, and so on. The object of activity (thinking about patterns in an algebraic manner) appears refracted differently in the participants: while for Jeanne, the object appears refracted in the materiality of the expressions 3 + 6, 3 + (3 + 6), and 3 + (3 + (3 + 6)), for Mario it appears as a total. There is a dialectical contradiction here in the way the object/motive of the activity shows itself to the participants. The didactic problem is to invite Mario to consider the saving process not as totals, but as a process of successive additions and, later on, to see the successive additions as multiplications. This shift of attention requires a transformation in the manner in which actions can be perceived. Realizing or becoming aware of these new forms of seeing the saving process is what objectification is about.

Fig. 3.5. Mario, with an intonation as if the teacher was asking him the self-evident snaps his fingers of the left hand against goblet 2 and utters 'twelve' (turn 128).

In this part of the episode, therefore, Jeanne has asked and Mario completed a re-reading of the problem. Having thereby asserted the conditions of the task, Jeanne asks for an articulation of the steps taken. From the perspective of Mario, she is asking the self-evident, as he has already provided the answers before Jeanne asked the question: Each goblet contains the number of chips corresponding to the number of dollars specified in the task. He has placed $6 in the first goblet and, as instructed in the task, added $3 that Marianne has added at the end of the first week. But undistracted and undisturbed by the expressions, as if these had not occurred, Jeanne continues engaging with Mario.

'Second Week': Positive Evaluation They then move to the second week, as Jeanne picks up the corresponding goblet, and asks, after Thérèse and Mario already offer to begin responses, 'How much does she have already in the piggybank' with a stress on the 'she' (turn 126). Mario completes a question–response pair, 'twelve', with some exasperation in his voice, which marks her question as having asked the self-evident. He simultaneously snaps with his fingers while hitting the goblet (Fig. 3.5) as if he were saying, 'of course there are twelve in this goblet'. After all, he had been counting them out and placing them there. But Jeanne asks the question again, 'How much money does she already have in the piggybank' (turn 130), but intonationally different, as she emphasizes 'already'; and Mario responds with a literally exasperated (frustrated) 'What?' (turn 132). A repetition is not just saying the same thing, but is produced with the previous utterance as background, therefore articulating something different all the while leaving it the same (Bakhtine [Volochinov] 1977). Here, Jeanne and Mario have enacted a question–response turn, but the teacher repeats the utterance. This repetition therefore can be heard as an evaluation that the preceding response to its first iteration is incorrect. There is a contradiction: Mario expresses in his question 'What?' a non-understanding and an assessment simultaneously. Jeanne repeats the question again, emphasizing, as in the previous iteration of the question, the adverb 'al-

ready' (turn 134). Thérèse says 'six', but, when Jeanne repeats 'the second we–', articulates an interjection of surprise, 'Oh' (turn 140). Mario, who had begun a first response with an 's' sound (as in *six* [Fr.], six [Eng.]), then self-corrects, 'No', and he says, 'nine' (turn 137). He repeats this response, which Jeanne, by a constative statement of the same word while nodding, confirms. She then explicates, 'the six she started with and the three dollars'.

Fragment 3.3a

```
120  A:  <<plaintive>are we sup[posed to do this?>] ((Points to
         his page))
121  J:                        [second week       ] ((she takes
         the goblet of the second week))
122      (0.89)
123  T:  <<p>[yes I think]>
124  M:      [we:=ave:;  ]
125      (0.17)
126  J:  how much does sHE have already in the piggybank.
         ((continues to hold goblet 2))
127      (0.18)
128  M:  twELV::e. ((rapid confirming gesture toward goblet,
         Fig. 3.5, intonation of exasperation, as if she has
         asked the 'self-evident'))
129      (0.42)
130  J:  well the sECond week, how mUCH does she have AL:REady:
         ((nods with each emphases))
             [in the piggybank.] ((still holds gobler 2))
131  A:  <<p>[me [i=m done      ]
132  M:          [whAT  h?      ] ((frustration))
133      (0.63)
134  J:  how mUCH money does she have ALrEAdy in her piggybank?
         ((still holds goblet 2))
135      (0.55)
136  T:  <<p>[six ]>
137  M:      [a s:] (0.53) no[n. nINE:  ]
138  J:                      [the second] wee ((holds goblet 2,
         on 'wee' points to it with other index finger))
139      (0.38)
140  T:  <<f>O:H:.>
141      (0.22)
142  M:  nINE
```

As teachers, *we* (authors and readers) know what Jeanne is aiming at, even though this is not yet evident to the students. In fact, Jeanne's actions would be unnecessary if Mario and Aurélie knew what she is attempting to make salient. And what she does has arisen from the obvious realization that the two students do not conceive of the contents of each goblet as the repeated addition of $3 for each week. In a sense, the way in which the activity unfolds, the requirement of the task to have a different goblet representing each week changes from what would have been the lived experience, the one that Jeanne *actually* wants them to articulate and make salient. If Jeanne had operated with one and the same goblet, then Mario could have added $3 for each week. Jeanne's question 'how much does she already have in the piggybank' might have been recognized as descriptive of the content of

the one piggybank. To expand her own possibilities of teaching, Jeanne has to bring this repeated addition out and make it salient from a state of the activity, where the addition is enfolded and hidden. It has to be inferred from the contents of the first and second, second and third, and so on goblet. The entire sequence of turn pairs from turn 134 to turn 142 constitutes the work of making salient that *after* the first week there are $9 in the piggybank. It is to this amount that another $3 are added at the end of the second week to yield the result that Mario has already signified in uttering 'twelve', but which hides rather than reveals and exhibits the addition that he has conducted at the beginning of the task when he counted out 3 chips for each week and added them to what he knew was contained in the goblet standing for the preceding week.

The same turn sequences actually show surprise on the part of Thérèse when Jeanne repeats pointing to and asking about the second week. Mario is giving the anticipated response and, following Thérèse's interjection marking surprise, repeats this answer. Jeanne now has sufficient evidence at least from these two students that the $9 at the end of week 1 are salient. She nods and repeats Mario's utterance, thereby confirming it. That it is confirming can further be taken from the fact that she actually continues, which she would not have done had she noticed and oriented to some form of trouble. Instead, she rearticulates the process of arriving at the $9 contents of the first goblet, 'the six that she started with and the three dollars' (turn 143). Thérèse responds by uttering 'three', which is accepted in the constative utterance 'one more three dollars' that Jeanne produces. Mario insistently says 'it's twelve', and Jeanne confirms 'it's twelve' (turn 150). Jeanne then continues to the next goblet, picks up the one for the third week and says, 'how much money is there in' and then self-corrects 'already in the third week'.

Fragment 3.3b

```
142  M:  nINE
143  J:  nINE. ((nods)) (0.64) she had ((places goblet 2)) the
         sIX that sHE started with, ((demonstrative la gesture
         to left)) (0.19) and the three dollars ((rH index
         pointing into goblet 1)) (0.58) do (0.23) how mU::CH
         (0.24) do we ((rH index points into goblet 2)) add
         here.
144      (0.80)
145  T:  [three.]
146  M:  [what  ] (big? [one?)]
147  J:                 [one  ] more thrEE DOLLars.
148      (0.18)
149  M:  <<insisting>ITs <<f>tWEL:v:e.
150  J:  its <<f>tWELve ((confirming, nods deeplu, open rH
         gesture, palm upward)) (0.94) so (.) how much money is
         there in, how much money (0.92) ((she lifts the third
         week)) (2.50) how mUCH money is THEre (.) IN, (0.29)
         already in the third week ((raises goblet, jingles it,
         places it back))
```

There is a potential source of confusion in that the question 'How much money does she have in her piggybank?' may be answered both by stating the sum and by stating the repeated addition. It is only the second, the representation of the re-

peated addition that was required in the process of filling the goblets that leads to the emergence of the pattern and the algebraic formula $3n + 6$. At this point (turn 143), then, she has confirmed that Mario has provided a response that unfolds the total amount into two components. The point has been to bring out how much there was *already* in the piggybank in the second week. That is to say, to notice or to objectify an essential feature of the manner in which the amounts of money can be expressed. She articulates the composition of the $9 in Mario's response as being composed of the $6 plus $3 saved at the end of the first week, which she denotes with a gesture to the goblet marked '1'. She then points to the second goblet to ask what will be added to it.

It is perhaps not surprising that Mario would be confused. When Jeanne asks him how much there is in the piggybank in week one, the correct response was '9', which Jeanne explicates as being derived from '6 + 3'. In response to the first iteration of the question how much she already has in her piggybank, Mario responds by saying 'twelve'. As previously, he has stated the amount that is in the piggybank modeled here by the goblet. But as the event unfolds, it becomes clear that this is not the sought-after response. As the emphasis of 'already' in the two repetitions of the question that follow suggests to the knowledgeable hearer, Jeanne is after the amount that in the second week already is in the piggybank, and to which, as her final utterance makes clear, $3 are to be added. This would yield the $12 in Mario's response. Jeanne then moves onto the third week, asking the structurally identical question again, this time concerning the 'third week' and lifting the goblet numbered with the '3' (turn 150). Resorting to structurally identical questions is indeed part of the repertoire of the teachers' objectifying processes (for other examples, see Radford 2010).

Another potential problem for understanding lies in the change of the signifier for the piggybank. That is, although Jeanne first holds up the goblet marked '2' and then the goblet marked '3', the two goblets, though materially and markedly different are intended to refer to the same piggybank that appears in the story. As before, there are two signifiers, functioning like two different though similar words, referring to the same signified but at a different point in time. Whether this is apparent to students is not revealed in the situation. It is evident to the knowledgeable adult that the particular representation is to capture the dynamic of the situation, but, because the learners are confronted with two static representations, the movement has to be inferred. And this inference is precisely the point of trouble, the one that is addressed in the current obuchenie activity. As much as Mario is to learn and realize the repeated addition, Jeanne has to find a form of interaction that allows Mario to become aware of the repeated addition. But Jeanne cannot just know what is in Mario's mind. To learn, she has to interact with Mario, who, in and with his responses, teaches Jeanne about what he knows, whether she was successful in bringing about a realization, and what she might have to do to get the blocked understanding back in movement again. But the road is rocky and slippery, and there is no easy way to get from lack of understanding to understanding. The activity itself has to produce the obuchenie situation as much as its content, which is Mario's understanding of what the task requires and Jeanne's finding of

an appropriate pedagogy. They keep on slugging; and that the road is thorny and obstructed rather than clear can be found in further expression of frustration.

'How Much Money is There Already During the Third Week' Jeanne asks in the same way as she has asked for the second week; she even holds the corresponding gobbler as she had done for the goblet labeled '2' (turn 134). In the articulation, therefore, we find a repetition, of the same structure of the repeated addition of $3 from week to week. Thérèse is the first to respond, 'fifteen', thereby naming the number of chips in the goblet. Mario, however, responds 'twelve', and repeats this answer when Jeanne repeats the question (turn 160). Jeanne responds by asking, 'Why? It is composed of what?' (turn 162), to which Mario responds in turn with apparent frustration in his voice, open-hand gesture toward the worksheet, while uttering 'What? But look' intonationally stressing parts of the verb (turn 164). In these repeated expressions of frustrations we find the apparent sensuous-valuational expression for the status of the activity from the perspective of Mario, who utters questions in response to questions. We could gloss his utterance as 'What are you asking me? Take a look at the worksheet.'

Fragment 3.4

150	J:	its <<f>tWELve ((confirming, nods deeplu, open rH gesture, palm upward)) (0.94) so (.) how much money is there in, how much money (0.92) ((she lifts the third week)) (2.50) how mUCH money is THEre (.) IN, (0.29) already in the third week ((raises goblet, jingles it, places it back))
151		(0.79)
152	T:	um um u::m.
153		(1.12)
154	M:	u:[m::]
155	T:	[fifteen]
156		(0.30)
157	M:	<<p>tWELve. >
158	J:	=how mUCH should thERe already be.
159	T:	u:h:
160	M:	twelve
161		(0.21)
162	J:	wHY. ITs composed of what.
163		(0.68)
→ 164	M:	what well lOOK ((frustrated, hands stretched out, palm up, toward worksheet, Fig. 3.6a))
165		(0.27)
→ 166	J:	twelve dOLLars contAINs the::? ((Mario places head in hand, arm resting on table [Fig. 3.6b])) (1.48) six dOLLars that we start wITH?(0.46) and how mUCH money in the other two weeks beFORe? ((Jeanne places right palm on goblet 1 & 2, sticks left finger for '$6'))
167		(2.01)
→ 168	M:	what? (1.56) that makes- (0.80) i dont understANd (.) thOUgh. ((460>228 Hz)) ((Places both elbows on desk, head into his hands, Fig. 3.7))

Fig. 3.6. a. Mario's frustration is apparent from his intonations and gestures (turn 164). **b**. Gazing at his worksheet, Mario seems to abandon as Jeanne asks him again about the contents of the goblet (turn 166).

```
169  J:  <<p>you dont understand that> its what i=m trying to
         help you understand (2.40) lOOK well (3.50) are we
         lOOKing (0.65) trèse?
```

Jeanne does not reject his answer, but asks how the $12 are composed. In so doing, Jeanne invites Mario to envision the creation of new possibilities of looking at the problem. The ethical commitment that she displays through her posture, utterances, attentiveness, and the very act of exposing herself to failure constitute a *call* that Mario, despite his frustration, is willing to answer. This ethical commitment creates social links that make the interaction far from authoritarian. She is exposed as much as Mario is. She begins by articulating the $6 they started out with and then, with rising intonation toward the end and an interrogative 'how many?' offers up another question (turn 166). There is a long pause, which Mario breaks with markers of disarray (Fig. 3.6b). There is an interrogative reinforced by rising intonation toward the end, an attempt in responding, 'this makes' that is preceded by a longer pause, and then a cognitive assessment, 'I don't understand' (turn 168). All prosodic indicators are consistent with what psychological research has identified as vocal correlates of despair (Scherer 1989): although his answer has been correct, the unfolding events have led him to a negative affective valuation. He does not understand where the line of questioning takes, and perhaps why these questions are asked, given that he has already provided the correct response from his perspective. The emotional assessment is a global one, as it also takes into account the questioning with respect to the overall object/motive that the obuchenie activity is supposed to reveal.

Across the extent of this fragment, Mario expresses what any culturally competent individual hears and sees to be frustration. His intonation and his gestures (Fig. 3.6a, 4.6b, 4.7) are consistent with his verbal assessment 'I don't understand' but also provide an emotional tone of frustration and despair. He does not know what to do, which is why he has called Jeanne, and he does not even understand

Fig. 3.7. Jeanne, right, uses gestures to orient and point; Mario for considerable stretches holds his heads with both hands, sometimes as if in desperation (turn 168).

her questions, as his repeated utterance of 'what?' with increasing intonation that follows her utterances grammatically formed as questions.

Jeanne articulates for Mario and everyone else overhearing the exchange – she has oriented previously to the camera, and, as clear in turn 169, also seeks the other students in this group to attend – that she is trying to help him to comprehend what he does not yet understand. That is, at this very instant she formulates (describes) for Mario what she has been doing so far ever since she followed his request for help. She says not merely that she wants him to understand, but more specifically that she wants him to understand his incomprehension. For her, too, their mutual engagement in this obuchenie activity has not brought them closer to their individual goals. It is not only that Mario's problem of incomprehension has not been addressed; Jeanne has also been unsuccessful in addressing the problem, and perhaps in understanding precisely what the problem is. But to get to this point, the two must have a minimal inter-comprehension on the basis of which they can contribute to the obuchenie activity. This engagement promises them to come to understand that their mutual efforts have not brought them closer to the goal of the task, the revelation of the object/motive of the obuchenie activity in the subjective experience of Mario.

It is important that we do not look at the observable expressions through a constructivist lens. If we were to do so, then the contents of the expression would be outer forms of internally pre-configured content. This content would be the result of mental structure, which brings about the content to be externalized by the various means available for doing so. But there are many analyses suggesting to us that this would be an inappropriate move. Thus, coming from very different theoretical backgrounds, both Merleau-Ponty (e.g., 1945) and Vygotsky (e.g., 1986) suggest that we actually find out what we are thinking *in* and *through* our expressions. That

is, we *find* our thoughts in the words we speak, and we find our emotions in the typical bodily expressions that go with these (e.g., prosody, body movements, body positions). The term 'expression' must not be taken as something pressed outside but as an articulation in which the subject can find its own position on the current situation. The various body movements and positions do not just present a position to the outside world; they *are* the taking of a position in a world always already shot through with significations. 'The phonetic gesture realizes for the speaking subject a certain structuration of experience, a certain modulation of existence, exactly as a comportment of my body invests – for others and for me – the objects that surround me with a certain signification' (Merleau-Ponty 1945: 225). Other scholars agree. Thus, 'the moment constituted by the performance of thoughts, feelings, words, practical deeds is an actively answerable attitude that I myself assume' (Bakhtin 1993: 37). Each performance is perfused by moments that we often attribute to different realms of experience, the intellectual, the emotional, the practical, and so forth. Bakhtin tells us that they are all different forms and manifestations of the same unity, an expression and a particular attitude toward the real living and lived situation as a whole: 'an emotional volitional attitude toward a state of affairs in its entirety, in the context of actual unitary and once-occurrent life' (ibid.: 37).

We are Going – But Where?

The events at the heart of this chapter begin when Jeanne is joining the group after Mario has clearly raised his hand, turning about and apparently looking for the teacher. That his comportment can be seen as such, and in fact was seen as such in that situation is evident from Jeanne's joining the group. There is a first attempt to get unstuck, but apparently without success. The fact that Jeanne makes Mario re-read the problem is an expression of her assessment that they are still stuck and that they have to go back to the beginning and re-read the problem. They re-read the problem and do figure out how much money there is after week one and what it is composed of. Jeanne then orients the effort to the second week, insisting on the articulation of the amount there already is separate from the amount added. She then moves on to the third, obviously repeating the structure of the orientation (pointing, holding goblet), pointing, and structure of the question. Knowledgeable readers recognize in the structure of the questioning that its point is the repeated addition of $3 to the goblet and the thereby increasing number of chips. But it is evident from Mario's expressions that this structure is not apparent in his consciousness. He verbally expresses a lack of understanding and also provides sensual-valuational expressions that mark the distance between their current state and the – from Mario's position unknown – endpoint of the task.

The process is a tricky one, as Jeanne and Mario engage in interactions that produce the obuchenie (teaching-learning) situation. That is, at a minimum – even if the object/motive of the activity is not yet clear to Mario – the interactions need to

be such that a recognizable event emerges. But it is not such that Jeanne is in the know. It is apparent that the first attempt to allow the emergence of an understanding of the task does not succeed: in and as integral part of the obuchenie activity, Jeanne has to search for a pedagogical approach that allows the situation to get unstuck. She cannot do it without Mario. Simultaneously, Mario wants to get unstuck, he wants to understand. Yet he cannot do so without Jeanne. Both have to engage the other so that together they expand their action possibilities. It is not that there is a flow of something from a teacher to a student. Together, in their turn taking, in the course of an objectification process, they produce the obuchenie situation all the while they get unstuck. That is, their sequential turn taking produces the process and the outcome simultaneously.

We can see that the turn-taking routines produce question–answer pairs in which the desired responses emerge. Thérèse and Mario do provide, as validated by Jeanne, the sought-for answers. And yet, Mario expresses frustration. Why might this be the case? From his perspective, not knowing the object/motive, he cannot know the sense of the actions (speech acts). Mario's interrogative terms that follow those that Jeanne utters as parts of recognizable questions exhibit that he does not know the pertinence of her question to the current task, and where this task is taking them. At the same time, his expressions are an integral part of the activity. Even though they do not appear to move anywhere, let alone closer to completing the task, they do move because the lack of success is as much part of the teaching-learning (or obuchenie) process that we need to understand and theorize as the eventual success. For Jeanne, this process also is a learning process, for it is in the social relation that she finds out and learns to assess what a student knows from his expressions. Together they articulate the current stand of the activity, produce a sufficient understanding of the obstacle to get moving, and they bring about movement (see chapter 4). In other words, in an objectifying process, the student and the teacher are transformed: both learn.

The activity does not exist apart from the effort of getting unstuck. Therefore, we observe movement to get moving again. Both forms of movement have to be understood at the collective level, which is irreducible. We cannot therefore ascribe non-movement to Mario ('because he does not understand'); but we cannot ascribe it to Jeanne either ('because she does not know how to lead [Mario] to see what is to be seen'). We cannot ascribe the fact that they come unstuck to either Mario or Jeanne, because it is the societal relation, the sequential turn-taking that we need to look at and understand.[1]

Getting unstuck is a form of movement to get moving again. They have to do it together, although Mario cannot know where they are heading and where they are to end up. It does not help that Jeanne knows where she would like them to end up. Success is only achieved when they get there *together*. But the point is not just to get the table of values filled, for there are faster ways to get this done. The point is

[1] The choice of the adjective 'societal' is purposeful, as the relation that Jeanne and Mario enact is institutional, and, because schooling is a societal activity, contributing to the reproduction and transformation of society, the relations themselves are societal (Bourdieu and Passeron 1979).

not for Mario to articulate the anticipated responses into the cells of the table. The point is for Mario to 'have an image of the object/motive', which is not only the articulation of a model for calculating the amount of money for week *n* (i.e., *n* x 3 + 6), but the personal sense for each of his actions. In the absence of the image, he cannot have a personal sense of each action, but without the actions, the activity does not even come to be realized. What the two therefore are doing over the course of this chapter is engaging in movement: the work of refining, increasing, and transforming their action possibilities so that they can work toward the completion of the activity, which includes the revelation (i.e., the objectification) on the part of Mario to come to know why he has done what he has done. At this point, Jeanne too will have achieved her part of the activity, which is to find a pedagogy that works to get Mario to the realization. Together, in and out of their societal relation, they come to realize success. The success of the activity looks differently from the different perspective, but these perspectives are but one-sided manifestations of the success of the obuchenie activity.

'Any higher psychological function was external', writes Vygotsky (1989: 56), and he continues, 'this means that it was social; before becoming a function, it was the social relation between people'. He summarizes his conclusions in a general form: 'the relation between higher psychological functions was at one time a physical relation between people. I relate to myself as people relate to me' (ibid.: 56–57). If we assume that both Mario and Jeanne were changed in the course of this episode, Mario becoming more knowledgeable in algebra and Jeanne becoming more knowledgeable in teaching algebra, then it is the result of the societal relation between the two. Having become more knowledgeable – i.e., having more room to maneuver and being in control over the conditions – not only is the trace that the societal interaction has left in the interaction participants; it also takes the form of the societal relation. Their forms of knowledgeability are of the relational type. If these traces come to be mobilized again, they inherently are of the social type. They are not singular and idiosyncratic but inherently intelligible.

It is precisely in the movement to come unstuck, of seeing something that until then had remained beyond reach, that the agential room to maneuver and control over the situation is transformed. This transformation constitutes learning. If Mario had simply filled the table as expected by simply copying the numbers from someone's table, there would not have been an opportunity to expand knowledgeability. Mechanical or senseless copying or reproduction cannot create transformation. It creates a mere formal imitation. For learning to occur, the object of activity has to become an object of consciousness. And in becoming an object of consciousness, consciousness changes, and transformation occurs.

At this point, however, although the situation seems to be moving, it is not yet. Objectification is only a possibility on the horizon. More interactional work is required, more reproduction and transformation of the social relation needs to be done until it leaves the kinds of traces that subsequently are recognized as higher psychological functions. Objectification and the obuchenie activity has not yet been successfully completed.

We can also see in this chapter how the emotion is itself an outcome of the activity all the while it is unfolding. Emotion is not a constant phenomenon, but, because it is a particular form of reflection of the material and ideal state of the activity, it is transformed at the same time, and, in fact, is part of what shapes subsequent actions that realize the activity at hand. That is, signs of frustration become resources for the subsequent actions of the other; and such signs are produced, as we see, by both participants in this interaction. Mario's understanding that he does not understand also is the result of the activity, which therefore is *not* some form of meta-activity in the way psychologists think about meta-cognition as a different form of cognition that accompanies the latter. This understanding and the emotional expressions *are* the obuchenie activity as much as the movement or non-movement that occurs with respect to the purely intellectual-mathematical moment. Our performative perspective makes the different aspects integral moments of the same phenomenon. It is only when we look at this phenomenon as an integral, irreducible whole that we can actually understand why it is moving in the way it does, and how the sensuous-valuational moments are related to the cognitive-volitional ones.

4

Developmental Possibilities in/from Activity

> [A]ll cultural development has three stages: development in itself, for others, and for oneself (e.g., a demonstrative gesture – at first it is simply a failed grasping movement aimed at an object and designating an action; then the mother understands it as an instruction; and, finally, the child begins to point). (Vygotsky 1989: 56)

In the constructivist literature, there is an abundance of descriptions that attribute development to the conscious constructive effort of the acting subject – often described as a reflective abstraction of actions from the material into the ideal realm or as a conscious reconfiguration of one mental structure into another. Such descriptions make it appear as if the individual pulls itself up on bootstraps and comes to produce, all on its own, the forms of knowledge that others in a culture already attained. It is not surprising to read that Piaget invokes the idea of recapitulation of phylogeny by ontogeny.[1] A closer look at the introductory quote to this chapter shows that Vygotsky takes a very different stance. In his description of cultural development, a movement receives the sense of an action of a particular kind *first* by the culturally competent individual *before* this sense comes to be actualized by the child. In the example Vygotsky provides, there first is a random movement. The child does not know its cultural signification; it does not (yet) know to point. Rather, the parent who sees the child move understands it as a pointing gesture and, in acting toward the child, transforms it as such. It is in and through the social interactions with the parent that the child comes to understand that by means of such movements things are being pointed out. Its movement, though initially arbitrary, immediately takes *cultural* signification rather than being developed bottom up and recapitulating cultural history. As we note in the previous

[1] The author of *Piaget's Conception of Evolution: Beyond Darwin and Lamarck* notes: 'We might say that the study of the development of cognitive structures in children was for Piaget a surrogate investigation. And, by invoking the idea that "ontogeny recapitulates phylogeny", he hoped to draw conclusions about the development of human knowledge that would be applicable to both individual cognitive development and to the growth of knowledge in the history of science' Messerly (1996: 2).

chapter, the higher psychological function is a social relation first, which, in the present example, transforms the movement into a corporeal pointing gesture with cultural signification and which the child then comes to understand in this particular signification. Moreover, 'pointing' first is external before the child can anticipate its own pointing gesture that it may realize by means of vocal sounds, hand/arm movements, or in other ways by means of body movements.

We also note in the previous chapter that the object/motive of the activity is not and cannot be apparent to Mario and his peers. They are asked to engage in this task. And the only reason they have to do so is *trust*. Trust is both rational and relational. It is rational in the sense that there is always the promise that by engaging in the activity there will be the potential to enlarge one's possibilities of room to maneuver and control conditions (a control that includes possibilities for deeper understandings but also for oppositions, disagreements, oppositions, and subversions). This, however, is not enough for trust to really operate. More than rational, trust is relational. Trust is relational in the sense that it entails an attitude (a positive one) towards others – a letting-oneself-go to reach and to be reached by others. Trust is exposure. In activity there is the idea of doing something for the sake of being together, of finding pleasure in being with others, and trying to help and be helped in the attainment of the object/motive of activity, even if we cannot see it yet. In the course of the activity, Mario performs (saying, writing, counting, pointing) but the significance of these performances can only arise when the collective object/motive has revealed and concretized itself on the subjective plane. Here, too, we then have the learner first produce performances without knowing what they signify or that they signify anything at all. The relevance of particular actions comes to be shown first by the teacher and ultimately, with the disclosure of the object/motive to Mario that comes in and through his actions, he may come to realize not only what he has been working toward but also the significance of his actions in their relation to the object/motive in the process of revealing itself to him. For this to happen, it does not suffice for Jeanne to tell him the result: that to find the amount of money after week n equals $n \times 3 + 6$.

We do know – from our experience as teachers and from reading the literature – that students apply equations without *understanding* why they do so. In this case, there has not been teaching-learning activity properly speaking. For cultural learning objects (mathematical and others) can become objects of consciousness as objects of activity only, that is to say, as objects of joint ethical, cognitive, and emotional actions. The object/motive, therefore, is not to get Mario to use the equation to calculate the amount of money in the piggybank for various weeks the number of which is sufficiently large so that counting it out becomes unfeasible. In one sense, Mario has understood what the task requires him to do, and he has exhibited this understanding in placing the correct number of chips in each of the five goblets in front of him. In part, this may be one of the sources for his frustration, because he has correctly identified the amount of money that goes into each goblet. He cannot know what Jeanne wants from him, and her questions are difficult to understand *unless* the interlocutor knows that she is working toward a generalization whereby the repeated addition of *chips* is transformed into the repeated addi-

tion of *numbers*, which in turn is transformed into a multiplicative rule. For him to understand the significance of her question means understanding that she is attempting to make salient in his consciousness the repeated addition as multiplication.

In the previous chapter, we note that to come unstuck, our two protagonists need to act (move); and these actions, once successful (which is assessable only after the fact), will have created an opening, a space for development, which is equivalent to the movement of the activity toward its intended product. This is – with respect to Mario – not the mere use of the formula but for him to realize the equivalence between his repeated addition of $3 over and above the amount that is in the previous goblet, on the one hand, and the multiplication that models the repeated addition, on the other hand. This developmental space does not just exist in the way that this is often described in the research literature where the simple co-presence of an institutionally designated 'teacher' interacts with the institutionally designated 'learner' under the auspices of a kind of legal didactic contract. We already note in the previous chapter that both participants need to learn: one learns mathematics and the other one pedagogy (and often mathematics, too, for the teacher learns to see things from a different mathematical perspective). In the process, each may come to understand the other part as well. As part of this learning, or to allow this learning to occur, they need to produce the space in which development occurs. This space – which, following Vygotsky, is denoted by the term *zone of proximal development* – does not just exist like a box into which the participants step. It is something that the participants have to produce all the while they produce the mathematical context and contents that this lesson is to teach and that is to be learned.

So if we think for an instance of this zone of proximal development as a box that does not preexist, then we are immediately confronted with a contradiction. As we see in chapter 3, Jeanne does not have the immediate answer to the question of how to help Mario. After more than 120 turns, Mario states that he does not understand, expresses frustration and perhaps annoyance, and, speaking with frustration in her voice, Jeanne says that what she has done is to help him. She is trying, but finds out in/with Mario's comment that thus far her effort has failed. That is, there has not yet been a space within which development could occur where Mario comes to do something he was not able to do on his own prior to Jeanine's arrival. Jeanine, too, has not yet had a space that would allow her to develop pedagogically. The space is actually something that arises in and from their societal relation and cannot be conceived apart from it. They have to develop this space together without knowing beforehand what it might look like and how to create it. The creation of this space, which allows them to develop and move closer to the object/motive of the activity, is itself an integral aspect of the learning-teaching (obuchenie) activity. Because neither one knows what actions will make this developmental zone, it can only emerge without that this emergence could be anticipated. This means that the participants come to realize consciously the possibilities that lie in their actions a posteriori. If it were not in this manner, Jeanne would have immediately acted such that Mario would have known. That she has

Fig. 4.1. Jeanne Has oriented to Thérèse (back) and Aurélie (left front) asking whether they are following ('are we looking'); Mario stares at his worksheet (turn 169).

done a lot without getting anywhere is a direct sign that the nature of the appropriate pedagogy itself has to emerge from their performances. Just as Vygotsky notes in the introductory quote of this chapter, in each cultural development there are actions first, then an attribution of social signification next, and finally the emergence of the realization of the signification on the part of the learner.

Emergence of a Developmental Zone

Up to this point, Jeanne and Mario have engaged in interaction but the situation has not come unstuck. Although they have acted and thereby realized the activity, it has not gotten them closer to the object/motive. This final part of the episode follows Jeanne's formulation that what she has been doing is trying to help him understand. Thus, even though it might not have looked like this to him, and even though he might not have experienced it as actually helping him, Jeanne *has actually tried* helping him to get unstuck. There is a brief pause. Jeanne appears to be exasperated. She looks around, takes a deep breath and announces, 'look well' (turn 169). There is a further pause as Jeanne first addresses the two girls, invites them to attend ('are we looking, Trèse?'). Jeanne has turned her attention to the two girls (Fig. 4.1). But then she lets them continue on their own as they suggest that they have to do the remainder of the task (turns 170–178).

Fragment 4.1

```
169  J:  <<p>you dont understand that> its what i=m trying to
         help you understand (2.40) lOOK well (3.50) are we
         lOOKing (0.65) trèse?
170      (0.40)
171  A:  hu? (0.22) done.
172      (0.40)
173  J:  you have answered everything? ((Oriented toward
         Aurélie, Fig. 4.1))
174      (1.01)
175      have you done c d and e?
176      (0.91)
177  A:  n::o:. (1.44) weve to do c d and e ((pounds desk top))
178      (0.33)
179  M:  two times three plus six that equals to nINE.
```

At this point, she exhibits being attuned not only to Mario, to whom she has just announced that she is trying to help him, but also to the more general situation of the activity from her perspective, which is a responsibility for the other students as well. But as Mario begins to speak, her whole upper body turns in his direction, exhibiting her orientation to the obuchenie episode with him.

'You Don't Understand. This is What I Try to Help You Understand': A First Objectification

Mario begins, 'Two times three plus six that equals to nine' (turn 179). Jeanne then engages Mario in a way that we might gloss as 'helping him to add up the numbers for the second goblet'. She places her finger on the first cell of the worksheet (Fig. 4.2) and then repeats the first three words Mario has uttered 'two times three' and then asks him to give the result, 'is what' (turn 181). Jeanne's use of Mario's words is an important form of inter-subjective tuning. It is part of what Arzarello and Paola (2007) term a 'semiotic game'. Here, the utterance has the falling intonation of a constative but the grammatical structure of a question, 'is what'. In fact, the repetition constitutes a confirmation of what Mario has said, but the appendix 'is what' asks about some result to come before the second part of Mario's utterance. Mario says with an insisting voice, as if he has already provided the answer, 'six'. Jeanne continues undisturbed, 'plus' (turn 184). There is a little pause and Mario then utters in a slow, drawn out manner 'six'. Jeanne continues, 'equals to', and moves her finger down in the table toward Mario. There is a pause until Mario utters '12' (turn 189). He then asks (mixing French and English), 'Where du write it now?'

Fragment 4.2a

```
179  M:  two times three plus six that equals to nINE.
180      (1.13)
181  J:  tWO times thrEE is what. ((places rH index on the
         first cell, will not remove it for a while, Fig. 4.2))
```

Fig. 4.2. Jeanne has placed her index finger on the cell corresponding to the week she is currently speaking about. Mario's gaze is oriented toward the place on his sheet where the index finger is placed (turn 181).

```
  182  M:  <<insisting>s:sIX:.>
  183      (0.18)
  184  J:  plus
  185      (0.65)
  186  M:  s::IX::.
  187  J:  =equals to, ((moves index up down))
  188      (1.02)
→ 189  M:  tWEL:v:e ((fills something into his table, 439>210
           HZ)) (2.96) Where do we write twelve now.
  190      (0.43)
```

In Mario's question, he exhibits his orientation toward the result of the calculation. The task states that at the end of the first week, 'Marianne says to herself "I have $9"'. Following the twofold addition of three plus the original $6, Mario now says that there are $12 in the piggybank. This is what Marianne would have said at the end of the second week, 'I have $12'. This is also the number of chips in the second goblet, the one marked '2'. Jeanne, however, points to the table on the worksheet in front of Mario. So he now asks where he has to write the 'twelve'. The fact that he asks makes salient the trouble spot. He has calculated the amount – it is apparently the correct amount, but he does not know where to write the number or what to do with the result.

But Jeanne responds, 'You don't write it. You are done' (turn 191). And she continues, 'So what does it say?', moving her index finger up and down between rows (Fig. 4.3). She responds, 'three plus six'. She resumes while pointing to goblet 1, 'So. In the first week, there are already $6. Then you add $3. You already have your $6, you add another $3. So three plus three, because three plus three is six'. She continues, 'Third week, how many do you have to add to your piggybank?' During the ensuing pause, Mario displays a questioning look (turn 192). Jeanne asks slightly transforming the question, 'How many three dollars will you have?' (turn 193). A long pause develops. Mario breaks it by asking in turn, 'How

Fig. 4.3. Jeanne points with her left-hand index to the inside of goblet 2 while talking about week 2 and pointing to the cell corresponding to week 2 in the same way as she does it for the other weeks she is talking about (turn 191).

much money (change) you will have?', but Jeanne repeats this time *emphasizing* the articulation of the number 'three' (turn 196). Another longer silence develops, which Jeanne breaks saying 'three dollars' three times, while pointing to a corresponding figure '3' on Mario's worksheet. Emphasis in discourse, like the one Jeanne is resorting to here, is a subtle way to have Mario noticing or objectifying the mathematical structure. Like rhythm, emphasis and structural question repeating, are semiotic means of objectification (Radford 2002). Along with the pointing gestures all these semiotic resources come to form what we have called elsewhere a semiotic node, that is to say, a segment of the teaching and learning activity where knowledge is been objectified (Radford 2009a).

Fragment 4.2b

→ 191	J:	((moves hand to right from Mario's perspective; throughout, Mario holds his head, gazes at his sheet)) you dont write it. ((moves index up and down between row 2 & 3, first cell)) (0.35) you are done (0.45) what does it say; (0.41) three plus six. (0.49) so. (0.29) first week, ((points to first goblet)) (0.51) there's already six dOLL:ars (0.21) and you add three dollars. (0.34) three dollars plus six (0.56) SECond week:; ((points to sedond goblet, Fig. 4.2)) (.) you already have three dollars ((points to '3')) (0.54) you already have your six dollars ((points to '6')) you add another three dollars. so thrEE plus thrEE you do three plus three (0.37) its six. (0.60) third week; how mANY threes are you going to add in your:: (1.03) piggybank?
192		(0.96) ((questioning look on Mario's face))
193		how mANY three dOLLars are you going to have.

```
194      (2.08)
195  M:  how much money are you going to have?
196  J:  how many thrEE dollars are you going to have?
197      (1.47)
198  J:  three dOLLars, three dOLL[ars  ], three dOLLars (0.23)
         ((points to the 2 '3's in week 2 and simultaneously
         points with left hand to the first, second, and third
         goblet))
199  M:                              [three]
200  J:  what are you going to write here?
201  M:  three?
202      (2.59) ((Jeanne moves finger to the cell on his left))
203  M:  <<p>plus three? plus three? >
204  J:  yES:: ((he writes))
```

It is precisely at this point – around turns 198–203 – that a first objectification is accomplished: new actions have become possible for Mario. After having articulated the series of additions in the second cell and adding one 'three dollar', Jeanne asks him what he will write in the cell for week 3. Mario articulates a questioning 'three' and, during the developing pause, Jeanne moves her index to the second week pointing to the two '3s' (turn 202). Mario responds by saying, in a low and subdued voice, with rising intonations after each unit, 'plus three? Plus three?'' Jeanne immediately exclaims, 'Yes'. This 'Yes' does not only confirm the questioningly stated response (as if saying, 'Is it plus three? And another plus three?'); her intonation also exhibits an emotionally positive nature. It is not just an emotionally positive response for Mario, with the potential of providing the emotional climate that will allow him to take further risks (after all, he was not certain about the response), but also a sign of the success Jeanne has experienced in eliciting a desired response.

To the knowledgeable reader it is apparent that the worksheet calls upon students to write the repeated addition in the cells of the second line and to write the equivalent multiplicative notation in the cells of the third. To aid students in becoming consciously aware of the transition from the multiple addition of $3 to the multiplication of $3 by the number of weeks, the cells starting with week 2 contain the sign for a multiplication 'x' (Fig. 2.1). But as the verbal interaction exhibits, Mario does not provide indications that the repeated addition is salient but rather the end result, which is the total amount in the piggybank at the end of each week. That the total amount is salient to him or the question about the total amount is once more apparent in turn 196, when Mario asks, 'how much money are you going to have?' in response to Jeanne's question 'how many three dollars are you going to have?' That is, whereas to the knowledgeable observer it is apparent that Jeanne is asking Mario to articulate the number of repeated additions, the same number that will subsequently enter into the multiplicative structure, the student asks whether she is asking him about the total amount. Jeanne repeats precisely the same words she has uttered before, but whereas the intonation was falling and therefore marking a constative in the first instance, it was rising toward the end as this is normal for a question. Also, her emphasis has changed from 'how mANY three dOLLars' (Fr. 'comBIEN de trois dollARs') to a stressed number 'how many

thREE dOLLars' (Fr. 'combien de trOIS dollARs'). That is, the emphasis has changed from the interrogative to the three, which thereby offers the opportunity to be heard as a group of objects that is identically added each week.

There is a pause much longer than what would be normal in a (telephone) conversation and much longer than the 0.7 seconds that research in the 1980s on wait time has shown that teachers leave to students. Jeanne then breaks the pause uttering 'three dollars' and then, while pointing to the two $3 figures on the worksheet in the cell of week 2, 'three dollars, three dollars' (turn 198). While she utters 'three dollars' three times, Jeanne places the left index finger onto the first, second, and third goblet as if it were placing the named amount. The movement produced by the hand-finger combination therefore bears an iconic relation to the action of placing the $3-saving that Marianne makes each week. Overlapping the second utterance of 'dollars', Mario says with a rising, that is, questioning intonation 'three?' Pointing to the third cell of the second row, Jeanne asks, 'What are you writing here?' (turn 200). Mario utters with rising (questioning) intonation 'three?', which we may hear as tentative, the offering of an uncertain response itself requesting an affirmation. There is a pause. Jeanne does not respond verbally, but taps with her right index finger on the '3' in the cell for the 'number of the week'. Mario responds, 'plus three plus three'. There is an affirming 'yes' (the pitch is rising to over 700 Hz and then returning to normal, see below), a sign that what Mario has said now is precisely what Jeanne wants and has been wanting him to do. He has provided for the first time not the total amount in the piggybank but rather, the amount in terms of repeated additions. That is, in the course of a first objectification, he has articulated the structure that is required before the next step – namely the transition to the multiplicative structure. To be achieved, this transition will need further objectification.

Before we pursue our analysis it is worthy to reflect on that what has made the reached objectification possible. If we go back to the turns 191–204, we notice that in the course of the verbal interaction, Jeanne uses linguistic terms to emphasize the repeated addition. For instance, she uses additive terms, such as 'to add'. However, the additive terms appeared embedded in a temporal dimension, marked by adverbs (e.g., 'already', 'then'). She says: 'So. In the first week, there are already $6. Then you add $3'. The temporal dimension brought forward by the adverbs serves to organize the appearance of the arithmetic signs in the emerging formula. Next, Jeanne moves to week 2 and then to week 3 and resort again to temporal adverbs to create the possibility for Mario to see the corresponding arithmetic expression, that is to say, '6 + 3 + 3' and '6 + 3 + 3 + 3'. But there are also additional elements into play. Pauses and tapping are two of them. Indeed, we see that just prior to the articulation of the repeated addition, which is immediately acknowledge by a 'yes', there is a long pause in which Jeanne also taps onto a cell of the table of values. When we look back over the transcript (e.g., turns 182–184 or 186–187), we observe that Jeanne confirms an answer or continues eliciting when the preceding utterance is correct. That is, confirming or continuing are ways in which preceding responses are marked as appropriate, whereas long pauses sometimes followed by reiteration of a question or production of a new question (e.g.,

turns 191–193 or 196–198) mark incomplete or insufficient responses. In turn 202, there is a pause that becomes longer and longer. Jeanne taps on a cell. Something else has to come, but it is not from Jeanne, who, in not taking the turn at talk allows Mario to continue. Tapping can be perceived as a further encouragement to produce something that goes into the cell referred to by means of the indexical pointing. And what is to come has to satisfy the preceding question, 'what are you going to write here?' But pausing and tapping are accompanied by other semiotic resources. As noted, Jeanne resorts to emphasis in word pronunciation and gestures. These various elements (word emphasis, pointing gestures, tapping, pausing, word emphasis, mathematical signs) play a fundamental role in the unfolding process of objectification. They are what we call *semiotic means of objectification* (Radford 2003). Their importance resides in the fact that they emphasize different aspects of the objectifying process. Thus, indexical gestures call *visual attention* to specific spatial key locations in the table and the formula; linguistic emphasis (pitch) call *aural attention* to what is been said, allowing the speaker to highlight some words and leave others in a relative background. Temporal adverbs call attention to *temporal attention*: they make it possible to imagine the *temporal unfolding* of events. Arithmetic signs capture quantitative aspects of the problem at hand. Rhythm – as presented in language, pausing, tapping, gesture, and repetition – provides analogical elements to highlight patterns to be attended (Radford et al. 2007). Semiotic means of objectification do not work additively. Their nature is systemic. Generally speaking, in isolation they have poor capacities to convey meaning. We could imaginarily suppress all but Jeanne's gestures in the previous fragment. We would hardly be able to make sense of what is going on. Semiotic means of objectification are powerful in building meanings and allowing objectification to occur because individuals use them in a coordinated manner. Each semiotic means of objectification puts forward a particular dimension of meaning (signification); the coordination of all these dimensions results in a complex composite meaning that is central in the process of objectification, as seen in the previous fragment. Usually, as the objectification proceeds, the coordination is refined more and more resulting in what we have termed a *semiotic contraction* (Radford 2009a). For instance, in turn 198, gesturing actions and speech go faster. Jeanne might have felt that she does not need to go again through the whole details as Mario was giving hints that the arithmetic additive expression was becoming apparent to him.

So, a complex sensuous or sympractical process mediated by an extraordinary array of coordinated semiotic resources (the semiotic means of objectification) provokes or makes the objectification possible. When Mario articulates two more 3s at the end of the fragment, which Jeanne immediately marks with a positive valuation, the structure that has now emerged verbally and in its written *re*presentation is that same structure that the mathematically knowledgeable person can already notice in Jeanne's previous productions. But this structure now emerges from the actions that Mario and Jeanne jointly produce in the objectification process. Because the objectification process involves Mario and Jeanne, *this* structure

cannot be reduced to one or the other. Objectification is a joint work in the emergence of the structure, in its revelation.

To understand the objectification, that is the revealing of the structure and its appearance in Mario's consciousness from an obuchenie perspective, that is, in the way Vygotsky understands teaching-learning unit, we need to focus on the relation. During development, what the participants do comes to be combined in one person. But it is not as if the learner constructs the function on his/her own following joint activity. The focus on the societal *relation* forces us to recognize that anything the learner will eventually do, s/he has *already done* as part of the relation. In addition, however, the part played by another person – here Jeanne who pointed, emphasized, south to shift attention, paused and who, by waiting, marked the answer as unfinished – comes to be added to the repertoire of the learner. Here, it is the attention attributed to the additional two occurrences of '3'. It is this attention that initially was there in Mario's behavior and that the social relation made possible to emerge. If Mario has learned, then we might expect him to produce the same with the remaining cells. But let us follow what happens next. What Mario learns *is not subjective*, his own, as it would be in constructivist theorizing, but it is social through and through. *All* psychological functions are societal rather than biological and they '*are internalized relations of social order, transferred to the individual personality*, the basis of the social structure of the personality' (ibid.: 58, original emphasis). But Vygotsky's articulation creates ambiguity when he writes about the transfer to the individual, when in fact scholars now realize that whatever function always is social, always articulated *qua* social relation. Even Vygotsky himself describes the reading of one's own notes, for example, in a diary, as a social relation: 'to read one's own jottings, to write for oneself, means to relate to oneself as to another' (ibid.: 58).

Vygotsky did not just dream up this way of thinking about human development, the 'principal driving force' of which is historically constituted societal interaction. Rather, his ideas fundamentally derive from the way in which Marx/Engels conceive not only of human beings but also about the relationship between individual and collective (society). Vygotsky (1989) makes repeated reference to Marx/Engels, especially to *German Ideology*, where the authors articulate their fundamental position on epistemology. In their sixth thesis on Feuerbach, the authors state that 'human nature is not an abstractum contained in the single individual. In its reality it [the individual] is the ensemble of societal relations' (Marx/Engels 1958: 6).[2] That is, the individual is a particular of the species *human being* only when it is a concrete realization of the latter – otherwise this human nature is an abstractum. This requires the individual to be and express humanness, which derives precisely from the particular forms of relations that is has in society with others. The individual is product and producer of societal relation, which, in

[2] The English translation of Vygotsky's original text replaces his (and Marx/Engel's) adjective *societal* (obschestvennix) by the adjective *social* (sozialnie), which Vygotsky also uses in the same text but at different places. From a critical (psychological) perspective, this is problematic, as the first adjective implies that development is shaped by society, including all its class-related inequalities, whereas the latter adjective does not imply the same.

the present instance, is the production of the cultural knowledge that we know as algebra.

Toward Independent Acting – A Second Objectification

At the moment that the additive pattern has emerged, there is a first sigh of relief. But following Jeanne's elated 'Yes', there is a pause, which eventually becomes longer that any one that the interactional relation has seen so far. Mario is writing the three '3s' into the third cell of the table of values. The next fragment begins when Jeanne says 'plus?' with a rising intonation, flagging it as a question, where the question pertains to the slot left open (given that their pattern was to add something). There is a pause, before Mario offers tentatively (rising intonation), 'six?' Jeanne confirms using a gesture that may be glossed as 'Yea, you got it' or 'This is it' (turn 209). She then continues by summarizing, 'Originally you have started with six.' The intonation is in the constative and confirming manner. Jeanne then continues, 'So instead of writing three plus three plus three, what would you be able to do?' Her right-hand index finger taps three times on the cell, slightly moving as she touches each of the '3s' in the cell corresponding to week 3. She moves the finger downward into the third row of the table and, while tapping on the '3 x' already written there, she utters 'Three times?' (turn 210). Here we can see again the subtle and complex coordination of gesture, pausing, tapping, words, word intonation, and mathematical signs that are mobilized in an attempt to create the conditions for the multiplicative-additive structure '3 x 3 + 6' to emerge. The manner in which these semiotic means of objectification are mobilized constitute what we have termed a *semiotic node* (Radford et al. 2003). In more precise terms, a semiotic node is a segment of semiotic activity where actions, gestures, words, mathematical signs, and other semiotic resources are coordinated to achieve knowledge objectification.

Let us return to Mario and Jeanne again. (Thérèse has been walking away and returning to the table, obviously having completed the task. She is turning to Aurélie, who is filling in the sixth cell of her table of values, speaking under her breath so that the table microphone cannot pick up what she is saying.)

Fragment 4.3a

```
204  J:  yES:: ((he writes))
205      (4.38)
206      plU:S::? ((points))
207      (0.96)
208  M:  s::sIX:?
209      (0.44) ((Jeanne moves rH to right, opens palm up, as
         if confirming 'this is it'))
210  J:  orIGinally you started with sIX. (0.39) so; instead of
         writing three plus three plus three, what would you be
         able to do. (0.27) three tIMEs?
211      (0.84)
212  M:  s::IX:?
```

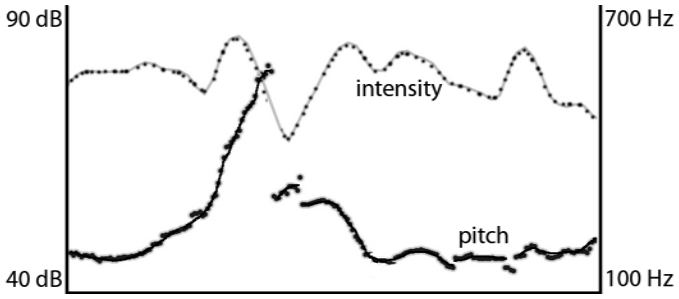

Fig. 4.4. Jeannine's pitch moves from about 180 Hz to 590 Hz and back down to her normal range while producing the 'excited' acknowledgment 'oui' (yes) (turn 215).

```
213        (0.88) ((J moves finger sideways repeatedly between
           two cells?))
214        no three times three
215   J:   yES::. (('excited' 'yes' [prosody in Fig. 4.4], makes
           the same rH movement to right, opens palm toward
           ceiling, Fig. 4.5)) (1.21)
           its just on the bottom its a [shortcut]
216   A:                                [madAMe: ]
```

There is a pause, before Mario responds tentatively (rising intonation), 'Six?' to which Jeanne responds by moving her index from week 2 to week 3 repeatedly. Mario continues now in a constative and in firm voice, 'three times three' (turn 214). Jeanne affirms, both with a clear 'Yes' the pitch of which moves way up and then returns, an intonation that seems to say, 'Yes, you got it'. Again, it is an emotionality that constitutes not only a response to the previous action, a way of creating a positive reception, but also an expression of Jeanne's valuation of her own success. She continues by explaining what he has been doing, 'It is just, on the bottom it is a shortcut' (turn 215; intonation in Fig. 4.4).

In turn 214, the multiplicative structure is emerging. But we know that we cannot analyze social interaction by confronting individual contributions. Rather, the minimal unit is the turn pair, which itself cannot be broken out of the activity as a whole – which, here, is an obuchenie (teaching-learning) activity. This turn is the response of an earlier one in which Jeanne utters a phrase that – though intonated as a constative – has the grammatical structure of a question 'what would you be able to do?' Then Jeanne utters 'three times', which we know to be the first part of the response, but which Mario has to discover as such. There is some further information demonstrating that this utterance is the first part of the response, because it is a repetition of the cell contents already available. It is to this cell content that the index finger is pointing. But the response initially is 'six' (turn 212). That is, if Jeanne has intended for a '3' to be uttered, this intention has not materialized. What matters to the unfolding event is that the response has been '6'. The next turn, which is typical of the *IRE* turn sequence that we find in schools, is an evaluation.

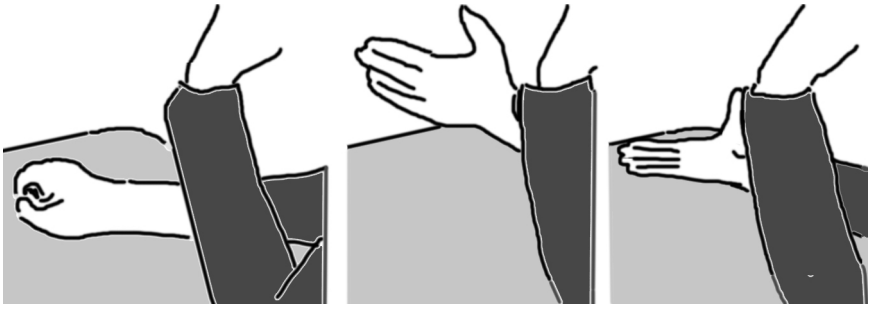

Fig. 4.5. Jeanne makes a hand movement that might be glossed as 'you got it' (turn 215).

Jeanne says nothing. There is a pause that unfolds. Simultaneously, Jeanne's index finger moves back and forth between the second and the third cell apparently attempting to provoke a different response. That is, the absence of a confirmation and the apparent solicitation of another response or an addition allow the understanding that the preceding response is not the one to be provided here. In responding 'no three times three', Mario confirms such a hearing. He has self-corrected an answer that in this sequence of turn was marked as incorrect and he provides the one sought for – which is marked as such by an excited 'oUI::' ('yES::'). The multiplicative structure, therefore, is the result of a societal relation; it is produced in the interaction between *this* teacher and *this* student. The 'three *times* three' is the response to 'instead of writing three plus three plus three, what would you be able to do'; in fact, from the perspective of a social relation as phenomenon *sui generis*, the former utterance concretizes the latter as a question. The two turns belong together, the first setting up the second as a response, the second confirming the first as a question. The tapping of the index finger constitutes a further sign that orients the participants toward the repeated threes in the second row and then to the beginning of a multiplicative structure '3 x' in the row below.

We can understand the actions performed by Jeanne as objectifying orienting moments of the societal relation. The following quotation exhibits the usefulness of Vygotsky's ideas for understanding teaching-learning of the kind that we observe here:

> If relationships among people genetically underlie psychological functions, then (1) it is ridiculous to look for specific centers of higher psychological functions or supreme functions in the cortex (or in the frontal lobes; Pavlov); (2) they must be explained not on the basis of internal organic relations (regulation) but in external terms, on the basis of the fact that man controls the activity of his brain from without through stimuli; (3) they are not natural structures, but constructs; (4) the basic principle of the functioning of higher functions (personality) is social, entailing *interaction* . . . of functions, in place of interaction between people. (Vygotsky 1989: 59)

The author suggests that the psychological functions are to be found in 'external terms', because 'man controls the activity of his brain from without through stimuli'. In interaction, the stimuli may be performed by another person. Whether these have bearing on the movement of the interaction is an empirical matter. In the present instance, we see that the various semiotic means of objectification, such as uttering the '3s' together with the indexical pointing to the '3s' on the paper and a subsequent index to the cell below where there is the beginning of the multiplicative structure '3 x' constitutes exactly the kind of stimuli Vygotsky is writing about. After the fact we can ascertain that they have been realized as stimuli as evident from the subsequent actions. It is this attention that occurs on the outside, because it is performed simultaneously by another person, Mario. *He* already attends to, looks at the '3s', listens to the repeated sound [tʀwa] ('trois', equivalent to [θri:], 'three'), and focuses on the beginning of the multiplicative structure already present in written form ('3 x') and performed verbally ([trwa fwa] 'trois fois', Engl. 'three times'). Mario, then, produces attention and salience, initially in response to semiotic means of objectification and, if these performative productions have left memorable traces, then he will produce attention and salience without the external stimuli produced in and as part of the societal relation with others.

We can see here that the normal framing of learning is not correct. It is not that something happens *between* Mario and Jeanne and subsequently, perhaps some time later, the same is happening *within* Mario. The same attention and salience that might allow Mario at some later time – whether he actually does it will have to be confirmed in our analysis of the events that follow – already is present at this instant. The important point to retain is, as Vygotsky suggests, that this attention and salience, the 'interaction of functions', is taking the 'place of interaction between people'. It cannot be fundamentally different, as any psychological function, *if it is a human characteristic*, must be so both at the collective (general) and the individual (particular) levels. If the realization that '3 + 3 + 3' (or [θri: plʌs θri: plʌs θri:] 'three plus three plus three') can be written as '3 x 3' is a *human* characteristic, here realized on the part of Mario, then it is necessarily (the result of) a societal relation. These realization are not, as Vygotsky maintains in the quotation, 'natural structures'; rather, they are cultural 'constructs'. Realizations and constructs can be reproduced at will precisely because there are collective possibilities, possibilities that arise from and for social interactions, and are always concretized by real living human beings. The reproducibility, which is a societal relation and social fact, is precisely what makes mathematics cultural-objective because it is grounded in the cultural-subjective performances of the individual (Husserl 1939).

The instance in which the performance of the multiplicative structure is greeted with excitement is further interesting because of the way in which this evaluation, itself a social fact arising from social interaction, is realized. We observe that the prosodic parameters that psychological studies have shown to express elation/joy all are present in Jeanne's speech (Scherer 1989). The mean pitch goes up strongly, here from 180 Hz to 590 Hz and then returns to about 180 Hz (Fig. 4.4). This also shows that the range varies substantially as does the contour. In addition, the F1 mean decreases, as observed in the experimental literature, from about 2,100 Hz to

between 1,270 and 1,170 Hz. The speech intensity goes from 76 dB to 83 dB, which corresponds to more than a quadrupling of the speech intensity, another change that is considerable as observed. In other words, the hand gesture (Fig. 4.5) together with the speech parameters all express and are manifestations of elation/joy – a positive emotional expression that comes precisely at the instant that Mario responds correctly and, hereby indicated, as the teacher wanted him to respond. It is an expression of the success of the preceding 'teaching moves'.

The 'oUI::' ('yES::') is, within the *IRE* turn-taking routine, only the final part of an irreducible pattern: by itself the utterance has no sense or function. The function is precisely as the third moment ('E') of the *IRE* structure. But we have just noticed that the multiplicative structure has arisen from the *IR* parts, in the to-and-fro of the question-response cycles, embedded in the objectifying process and mediated by an array of semiotic resources. In fact, this is not quite true, for we have not yet included the third moment of the irreducible pattern.[3] That is, the social interaction that produces the multiplicative structure *includes* the 'oUI::' as its constitutive moment – the poetic moment of objectification – bringing about the social evaluation of the performance *as* a cultural-historically typical performance rather than as something that has no sense. From the perspective of the learner, who does not yet know the structure, the assertion that some act (verbal utterance, physical movement) is a social act, an attribution as such as to be made: 'We become ourselves through others' (Vygotsky 1989: 56). In the same way that a mother confirms the child's hand-arm movement as a reaching gesture, Jeanne here confirms Mario's action as the culturally recognized and expected one. That is, the 'oUI::' is the initially external stimulus confirming the immediately preceding action as one that is cultural-historically recognized. It *is* the recognition as a societal relation, which itself is an aspect of the higher function of the multiplicative pattern once the (inherently cultural-historical) performance is completed in and through Mario.

Are further semiotic means of objectification (external stimuli) of the kind we observe above necessary or will the multiplicative structure emerge again in response to questions but without the particular stimuli that oriented attention and salience in the previous fragment? To find the answer, let us turn to the second part of Fragment 4.3. Jeanne now moves on to the fourth week (turn 218), asking Mario how many $3s he got. Her left-hand index finger points to goblet #4, then moves to the first goblet and then moves pointing to each of the intervening goblets back to goblet #4. There is a pause, but then he says 'four' writing the four 3s into the cell (turn 220). Just as he finishes, Jeanne provides a confirming 'kay' and, following a pause, asks, 'Instead of doing three plus three plus three plus three, what could you write here' while pointing to the third row cell of week 4. As she utters the first part of the turn, Jeanne moves her right-hand index finger back and forth across the four '3s' that Mario has written into the second-row cell of week 4 and, as she utters the last part, she moves the index finger to the third row of the table of values.

[3] We insist: Irreducible means that the *IRE* pattern stands and falls with all its parts. It is not an IRE pattern if any of its constitutive moments 'I', 'R', or 'E' were missing. We cannot look at a turn and say it is an 'I', 'R', or 'E' unless the structure as a whole is present; and this whole is present only if all the parts are present.

Fig. 4.6. Even evaluations communicated by means of gesture pertain both to the cognitive and the emotive aspects of activity reproduce and transform its emotive moments (turn 225).

Mario offers tentatively, 'four times three?' (turn 224), and Jeanne confirms with a two-handed gesture moving hands sideward and turning palms open toward the ceiling (Fig. 4.6), as if saying, 'You got it'. In this passage, Mario produces the expected answers in a quicker manner. The various signs that mediate the interaction (spoken words, gestures, rhythm, etc.) are displayed and articulated in a faster manner, constituting what we have termed elsewhere a 'semiotic contraction' (Radford 2008b). The semiotic contraction is a distinctive index of achievement of the objectification process.[4]

Fragment 4.3b

```
217      (0.42)
218  J:  your fourth week; (.) how mANY three dollars do you
         have.
219      (1.00)
220  M:  u:m:::  (1.73) fo. ((Fills table, Thérèse makes
         noises))
221      (9.48) ((writes 4 '3s'))
222  J:  <<pp>kay> (0.97) instead of doing three plus three
         plus three plus thrEE whAT are you going to wrITE
         here? ((Points to the row on the bottom of the table
         of values))
223      (0.66)
224  M:  uh:m:: (1.36) four times thrEE?
225  J:  ((2-handed gesture sidewards, opening palm upward:
         'you got it' [Fig. 4.6]))
226      (3.83)
227      i=think you understand now. uh?
228      (50.93) ((Mario slightly nods, writes, after 26
         seconds looks at Therese's worksheet, back at his
         own))
229 M:   <<confident>ME i understand now.>
```

[4] For similar examples, see, among others, Radford 2010 and Radford and Roth 2010.

In this situation again, we see the additive structure '3 + 3 + 3 + 3 + 6' and the corresponding multiplicative structures '4 x 3 + 6' – a correspondence that is instantiated by the index finger moving from the second to the third row – emerge *in and as* societal relation. For many readers struggling to understand how the structure *is* a societal relation, it may help to use the more familiar *IRE* structure, which is an irreducible cultural pattern of an institutional nature. The additive structure emerges *in and from* the initiation 'your forth week . . . how many three dollars do you have' (turn 218), the response constituted by the verbal 'fo[ur]' (turn 220) and the writing of '3 + 3 + 3 + 3' (turn 221), followed by the evaluation 'kay' (turn 222). Here *IRE* denotes the process of sequentially taking turns, where each turn is irreducibly embedded in and constitutive of a sequence with a preceding and a succeeding turn. It thereby is a social phenomenon that cannot be reduced to, yet requires the participation of, either Mario and Jeanne. There is an additional pointer to the additive structure: Once a person knows that the response is '4', s/he can see the hand gesture itself as an iconic index to the repeated addition. The hand pointing into each goblet moving from the first to the fourth articulates both the placement of something, the repetition, and the movement in time as the hand moves from right to left over the four goblets.

The multiplicative structure, too, exists as societal relation first. It is initiated in and by the utterance 'instead of doing three plus three plus three plus three, what are you going to do here' accompanied by the right-hand index finger placed on the third-row cell of week 4. The second moment of the three-part *IRE* structure is produced in turn 222, 'four times three'. Again, it is the third moment of the *IRE* social relation – realized across the turns 222, 224, and 225 – which finalizes the objectification of what subsequently will exist, if what we term by learning has occurred, as a psychological function.

Nearly 4 seconds later, she comments, 'I think you understand now. Uh?' (turn 227). Mario slightly, almost unnoticeably nods and continues to fill up his worksheet. Jeanne immediately raises her head gazing at a student who is coming around the table and then stops next to her and speaks to Jeanne. The teacher then gets up and walks to another set of desks. Some 26 seconds after the last utterance, Mario leans over in the direction of Thérèse's desk, gazes at her worksheet, then returns to work on his own (turn 228). Another 24 seconds later, he confidently utters, 'Me, I understand now' (turn 229). He has completed the cells for week 5 and 6 on his own. That is, the structures '3 + 3 + 3 + 3 + 3 + 6' and '5 x 3 + 6' now appear in rows 2 and 3 of the table; similarly, the structure '3 + 3 + 3 + 3 + 3 + 3 + 6' appears in the second-row cell of week 6, but there is a variance with the expected structure in the final cell: '6 x 3 + 23' (Fig. 2.1). We note that the complete structure is not yet produced without error, for in the instance of week 5, the final part '+ 6' (see underline) was already part of the cell. It is part of the structure (condition); when this part is absent (for the first time as we go from left to right in the table), the corresponding response is (still) incorrect. Given that the results of the second-row cell and the first part of the third-row cell have been completed as anticipated, we may say that the entire multiplicative structure has been objectified

at least in part from the societal relation and now is observable as a psychological function.

Social Relations, Obuchenie, and Developmental Possibilities

> We become ourselves through others. (Vygotsky 1989: 56)

Vygotsky notes that we become who we are through others. He makes direct reference elsewhere in the article to a note by Marx/Engels concerning Peter and Paul. The original source is actually a footnote which states that 'only through the relation to the human [*Mensch*] Paul as similar to himself does the human Peter relate to himself as human. In this, Paul also . . . appears to him as manifestation of the genus human' (Marx/Engels 1962: 67, note 25). To summarize, we do something like adding and multiplying numbers, as typical human actions, *as a consequence* of the institutional relations we entertain with others in society. Saving money and modeling the saving of money using algebra *are forms of human relations* and emerge *only* in and because of the kind of relations that we entertain within a cultural-historically contingent society. We can understand *cultural development* and *obuchenie* activity only when we properly relate the general – human society and the kinds of relations that constitute and are the results of it – and the particular, the concrete living human being whose actions concretely realize the societal activity. This, to us, appears to be both the essence of Vygotsky's thoughts about development and the most difficult aspect, because it requires dialectical thinking. In that form of thinking, the general is as concrete as the particular, for it always realizes itself in the particular, which always is the concretization (*object*ification) of a general pattern. If it were not in that way, an individual would never be able to produce signs that are inherently shared with others and communication would not be possible at all. There is no sense in speaking if the word is not always already a general feature of society and culture, which allows the addressee to hear and understand something of significance.

We note that additive and multiplicative structures are societal relations first. But in the early part of the obuchenie situation, the societal relation did not realize and produce these patterns. The possibility for the societal relation to produce these structures has itself to be created *in and through* societal relations. The structure exists *as* societal relation rather than *in* the relation, the way many Vygotsky interpreters including the most illustrious ones seem to read it.[5] We can gloss Jeanne's actions as repeated attempts at opening up this possibility. These attempts themselves are societal relations in which the 'non-algebraic' response, Mario's emotional articulations, and his description 'I don't understand' all are assessment forms that the intentions and the effects of Jeanne's actions have not coincided. Whether any subsequent initiation will bring about the societal relation that actu-

[5] For a critique of the positions Mike Cole or Jaan Valsiner take see Veresov, 2004, who suggests that there is a missing link in their interpretations of the general (genetic) law of cultural development.

ally makes the two mathematical structures emerge is not known – Jeanne would not have to make repeat initiations if she had known what was required. What is required is always unknown, because interaction always unfolds in unexpected ways. The obuchenie activity therefore is not simply one of selecting a teaching strategy and applying it. Rather, the appropriate strategy itself is an emergent outcome of the societal interaction (this, by the way, is one of the challenges in the design of educational software). Or, rather, in the way the mathematical structure has emerged for Mario, the pedagogical action has emerged for Jeanne. It, too, is a societal relation of the structure *I*(*RE*), where the teacher initiates and the student responds also is the evaluation of the preceding move as an appropriate or promising one. The 'correct' strategy, and one that can be taught to other teachers, is the result of a societal relation first. In this sense, therefore, *obuchenie* (teaching-learning) means, from Jeanne's perspective learning-to-teach-by-teaching. Unaware of this, Mario also produces an assessment together with the 'not-yet-algebraic' response. In this sense, his actions allow Jeanne to know and learn. Mario is both contributing to teaching and learning simultaneously. (Learning is occurring even and precisely when a 'wrong' response is provided, namely that *this* response is an inappropriate one; and this, too, initially is a social relation.)

Development frequently is conceived of as the 'socialization' of the child. In traditional, bourgeois psychology and sociology, the child is taken as an individual that is unsocialized and that is fashioned in the course of the process of socialization to become a socialized individual (Holzkamp 1979). Thus, for Piaget there are two channels of socialization, one in relation with peers, the other in relation with adults. 'The playfellow provides the opportunity for such social conduct as will determine the true socialization of intelligence. Conversely, where equality between playmates prevents questions and interrogation, the adult is there to supply an answer' (Piaget 2002: 262–263). That is, the child develops an intelligence of its own, and this 'wild' intelligence subsequently is socialized in and through social interactions with peers and adults. The preceding analyses suggest that the direction of the movement is opposite. There are forms of societal relations, here realized in and through the interactions Mario and Jeanne entertain. It is the individualization of these societal relations that constitutes the development we observe. This is precisely the position that is taken in cultural-historical activity theory: 'Development proceeds not toward socialization, but toward *individualization* of social functions' (Vygotsky 1989: 61).

We can think of mathematics as emerging in the same way that language does. Again, Vygotsky points us to Marx/Engels as the origin for his own thinking. These authors suggest: 'language *is* practical – also for other people existing – consciousness, thus, only in this way, also for me existing real consciousness, and language emerges, as consciousness, from the need, the genuine necessity of relations with other people' (Marx/Engels 1958: 30). As mentioned previously, consciousness of the individual is the consequence of collective consciousness. Consciousness can really exist for me only because it always already exists for other people as well. Being human means that that *my* consciousness *is* a concrete realization of *human* consciousness; being human means that my mathematical consciousness *is*

collective mathematical consciousness. Consciousness, however, is irremediably tied to societal relations. Development therefore is achieved when mathematical development is the individualization of societal relations rather than the socialization of some idiosyncratic 'constructions'.

The realization of the additive and multiplicative structures arises first in/as the societal relation; the conscious realization is enacted in the societal relation; because the 'psychic reality that opens up immediately before us is the subjective world of consciousness' (Leontjew 1982: 121), the conscious realization of the pattern *is the psychic reality* at this instant. That is, this aspect of mathematical consciousness emerges in and as societal relation. Development is said to have occurred when this form of consciousness emerges for the individual again, but now without the external stimulation that occurs in the societal relation. It is precisely for this reason that we need to be concerned with consciousness, as it constitutes the psychic reality that determines what the individual does and why. This is also why Vygotsky orients us toward 'man' rather than toward the 'brain', toward the real sensuous-valuational and volitional person rather than to the mental act. Vygotsky and Bakhtin articulate in almost identical ways the same concern when they write about scholars who focus on the mind at the expense of focusing on the person. The former says that the weakness of traditional psychology is that 'the thought process appears as an autonomous flow of "thoughts thinking themselves", segregated from the fullness of life, from the personal needs and interests, the inclinations and impulses of the thinker' (Vygotsky 1986: 10). The latter suggests that Kantian constructivism detaches judgment from real life in a way that 'the detached content of the cognitional act [seemingly] comes to be governed by its own immanent laws, according to which it then develops as if it had a will of its own' (Bakhtin 1993: 7). As a result of this abstractive detachment, 'we are simply no longer present in it [thought] as individually and answerably human beings' (ibid.: 7). But our episode from a fourth-grade mathematics classroom forces us to consider the human being, for, as we see at the very end, it is not just a brain that produces a (meta-) cognitive statement about itself, but it is a real person who is proud and elated about understanding: 'Me, I now understand'. It is a *person* that developed in and through the social relations it has entertained – not a brain that also acted. Thus, we need to focus on 'man', who 'regulates or controls his brain, the brain does not control man' (Vygotsky 1989: 71). This control first occurs by means of external (i.e., cultural) stimuli in societal relations and subsequently, by means of signs (e.g., memory), the stimulation of the brain is displaced within the person though, for Vygotsky and us, it necessarily remains societal.

In a strong sense, it does not suffice to use, as Vygotsky often does, the adjective social and to disconnect the discussion from the implications that come with the adjective societal. If school is to prepare for life in society – the Romans already noted *non scholae sed vitae discimus* [we learn not for school but for life] – then the relations must be societal. The individual is societal in nature if and only if the objectified relations always and already are societal. It is therefore not surprising that *serious* cultural-historical activity theoretic scholars, such as Leont'ev and Critical Psychologists, retain the use of the adjective *societal*. This immediately

allows us to connect up to critical scholarship in the area of mathematics education, for if the relations in which students participate during their school mathematics experiences are societal in nature, then this means that these also bear all the marks of class-related inequities in a given society. It is then possible, for example, to think that children coming from particular classes are less prepared to participate in the *actual* relations typical of classrooms with a middle class ethos and organization. That is, these children will immediately be disadvantaged because the kinds of relations they are familiar with are not the ones that they encounter in the typical classroom (Eckert 1989). It is not that algebra in itself is difficult or too abstract for and biased again under class and working class children. Rather, the kinds of societal relations that constitute algebra initially in the public sphere are not of the relations with which middle und upper class children are familiar and knowledgeably co-produce.

5

Re/Thinking the Zone of Proximal Development

> *The zone of proximal development . . . is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers.* (Vygotsky 1979: 86, original emphasis)

Vygotsky had a strong sense that the cultural development of the child is a function of its interaction with others – within such institutions as the family or school – on tasks that exceed its current capabilities. He writes, aphoristically, 'what a child can do in cooperation today, he can do alone tomorrow' (Vygotsky 1986: 188). Thus, he critiques the work of educators who adapted instruction to the level of the child. With such methods, we fail indeed to make use of the potential for development that arises from sympractical activity generally and sympractical obuchenie activity more specifically. Some mathematics educators might challenge the idea of offering algebra tasks to young children. However, our description and analysis of a particular classroom situation (chapters 2 to 4) is consonant with current research in early algebra[1]; it shows that development not only *is* possible but that it is also possible in a situation where a student might initially exhibit considerable frustration and, quite apparently, cannot go further on his own. As an outcome of the institutional relation that Mario and Jeanne produced, the former was enabled to complete the task on his own employing the mathematical structure that was a societal relation before. We may think of the process as a way in which the world (societal relation, setting) comes to be reflected in its part (Mario), the world reflected in a raindrop that nevertheless is an integral part of the world: a self-reflection. The problem of traditional psychology has been that it has only studied the outward manifestation of development but never the real processes, the real

[1] See, for example, Becker and Rivera 2008; Cai and Knuth 2011; Carraher and Schliemann 2007; Warren and Cooper 2008.

societal relations that produce the higher cognitive-psychological but always already societal functions (Leontyev 1981).

As the introductory quotation shows, Vygotsky appears to be focusing on learning as a function of a gradient where knowledge is enabled by a more capable individual and then goes to the less capable person. A more fruitful approach, however, is to recognize that collective, sympractical activity opens up room to maneuver for all participants and, therefore, that there is always developmental potential for any person participating in a collective activity. In fact, the preceding chapters suggest that we need to take a more symmetrical approach in thinking about and theorizing the zone of proximal development because the teacher also appears to learn.

Another important aspect that is often forgotten pertains to just what it is that students learn. As the term 'back to the basics' suggests, there are forms of educational thought grounded in a belief that learning and development occur bottom up as if a child had to reconstruct the entire history of human thought ('ontogeny recapitulates phylogeny'). Many educational theories play into the hands of those who believe in development as a decontextualized feature of the cognitive apparatus. The cultural-historical approach, however, orients us to think very differently. If development means that a child does on its own what it has done in societal relations before, and if social relations change together with culture, then the development of yesterday's child no longer is the same as that of today's or tomorrow's child. As we see in the preceding chapters, Mario does not work or make sense on his own; he is not the builder of his knowledge and consciousness independent of the collective in which he is a constitutive part. His activity implicates both the mathematical cultural entities in front of him and the societal relations that he entertains as part of being a member of this classroom. Both the mathematical entities that he manipulates and the living and lived relations are cultural-historical (societal). Who he can become arises from these relations, inherently cultural-historical. That is, in his activity, Mario takes up previous parts of the experience of humanity. All of his psychic processes *unavoidably* obtain a structure that contains the cultural-historical (societal) means and methods that are 'transmitted' to the individual during interaction rituals with others in the surrounding world. This process has to occur in outer form and therefore has to be inherently an objective, material process.

Developments in cultural-historical activity theory that occurred subsequent to Vygotsky's death actually help us to better appreciate thinking about the zone of proximal development to make it more appropriate than some box-like situation within which some transfer occurs from the more to the less capable one. As our analysis in chapter 4 shows, the zone of proximal development is itself a result of the sequentially ordered turn-taking embedded in a process of objectification, that is, of the sympractical (outer) activity. When Mario does the remaining cells of his table, producing the additive and multiplicative structures on his own, we see how the previously real (external) activity now shows up on the ideal plane. It is this relation between the real and ideal produced in and as part of the zone of proximal development that constitutes the advance of the theory over others (Mikhailov

2001). In this, the dialectical materialist heritage of cultural-historical activity theory is quite apparent. Thus, the ideas about the zone of proximal development derive directly from Marx/Engels' idea that life determines consciousness rather than the other way around. The zone of proximal development, which really denotes a form of institutional relation, constitutes a form of life. Whatever occurs in this form of life determines the consciousness of it, which precisely is the argument about the individual being nothing other than a unique concretization and therefore unique reflection of 'man in general'. Thus, 'consciousness initially is mere consciousness of the closest sensual environment and consciousness of the limited relation with other people and things external to the individual who is becoming conscious' (Marx/Engels 1958: 31).

There is another quite frequent mistake in thinking about the zone of proximal development, where learning is thought to be a transfer from without to within the individual, whereby a distinction is made between inner and outer (Zinchenko 2001). Moreover, even cultural-historical approaches have been marked by the confusion of the inner and the outer, the confusion between teaching and learning, and the problematic conflation of learning action with teaching action.[2] From the perspective of cultural-historical activity theory that we are developing here, however, such a distinction between the internal and the external makes no sense, for any activity *always already* implies two *irreducible* moments: material and ideal. These are the two moments of *one and the same level of* development, two moments of the same event (Veresov 2004). Culture is the ideal form. It constitutes the general possibilities of acting, which come to life in concrete, real human praxis.

Toward an Alternative

As noted above, the zone of proximal development is frequently thought of and applied in a one-sided manner that juxtaposes a more knowledgeable teacher or peer and a less knowledgeable learner. In our analysis of the lesson fragment involving Jeanne and Mario, it is not just the teacher who produces utterances that are heard as questions. Mario, too, produces utterances that have rising inflections and are interactionally treated as questions. The interaction ritual therefore is not just triadic (IRE): there is a dynamic of give and take. There is not just the typical Socratic dynamic of questions that unfold the truth at hand. The zone of proximal development arises from Jeanne's and Mario's joint practical, that is, sympractical activity. Teaching here means not just mechanical selection and application of some pedagogy. Rather, to find out what the appropriate pedagogical moves might

[2] Holzkamp (1993), who provides a consequential formulation of learning from the subject-centered perspective Leont'ev had started to outline with his focus on consciousness and personality, exhibits a number of shortcomings that beleaguer, for example, the work of Piotr Galperin and Wassili W. Dawydow and work inspired by the two scholars.

be Jeanne has to engage, she must dive into the activity without knowing whether she *can* be successful. It is in the course of this engagement that her own goals can emerge, for she cannot select actions unless she knows their relation to the required activity. These goals arise from her place in the division of labor in the total activity, whereas Mario's goals are characteristic of his place in the division of labor of the same total activity. As for Mario, the object/motive of *his* activity emerge, which pertain to his sense and understanding of mathematics, so for Jeanne the object/motive of *her* activity has to emerge. Collectively viewed, they are in the same interaction ritual but in fact purse two different but intertwined refracted forms of an object/motive. However, the refracted object/motive of Jeanne's activity does not depend on her alone, for her teaching is to assist Mario and allow the refracted object/motive of his activity to disclose itself through activity. (Mario cannot aim at the object/motive of the joint activity, as he does not know it. Jeanne does not know it either, as the object/motive pertains to *his* relation to the world.) Thus, their mutual refracted object/motives can only emerge from their transactions – thereby radically changing our conception of the nature of the *zone of proximal development* as realized here.

For Vygotsky, learning precedes development – in fact, it creates the conditions for development. Thus, 'an essential feature of learning is that it creates the zone of proximal development; that is, learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers' (Vygotsky 1979: 90). When these relations come to be enacted independently of others, and when the societal relations are produced by the child on his/her own, then development has occurred.

In the featured lesson fragments, we see that a zone of proximal development does not simply come into existence when a 'teacher' and a 'student' get together, the former 'helping' the latter. In fact, in the early parts of the interaction featured here, it does not look as if new forms of action have emerged for Mario. He does not exhibit developmentally more advanced actions over what he has shown prior to Jeanne's appearance in the group. Quite the opposite is the case: As they interact, Mario becomes increasingly frustrated rather than experiencing an increase in his control over the task and rather than coming any closer to the object/motive of the activity. If there is any phenomenon that can appropriately by classified as zone of proximal development, then it emerges from their mutual engagement that provides for a space that allows Jeanne to come closer to her goal of helping Mario. Jeanne's collaboration and the ensuing emergence of her are a refracted form of the activity's object/motive. Equally, Jeanne's collaboration is a prerequisite for Mario's refracted form of the object/motive to emerge. It is not clear beforehand whether they will be successful, but both exhibit to one another the willingness to engage the other. This is the kind of trust that we pointed to in a previous chapter and that relates to the ethical commitment Mario and Jeanne make. Their mutual orientation and participation exhibits a willingness to continue even when their overall assessment of the situation, as exhibited in the affective dimensions of their verbal, prosodic, and bodily productions.

At which point, we may ask, does the room for maneuver comes to be enlarged for Mario? At which point is he, by the *sym*practical activity, enabled to do more than he has done just prior to the interactions with the teacher? It is not during the first two fragments of their interaction, but emerges in Fragment 2.3 at line 189 when Mario asks where to write the '12'. Jeanne responds, 'You are not writing it'. Here we are at the exact dividing line between arithmetic and algebraic thinking. To focus on totals is to see the actions from an arithmetic viewpoint. To see the actions as actions *per se*, as actions that do not need to be carried out, is to see them from an algebraic viewpoint. Looking back, we can see that Mario has been focusing on the total sum of the money in the piggybank (chips in the goblets), and here he asks precisely the question to which Jeanne responds that it is not the '12', the sum, that comes to be noted; 'you are finished', she says, moving on to the next cell. Her question about the composition ('composed', turn 162, 'comprises', turn 166) has not had the effect to make Mario attend to writing down a representation of the process. Rather, he asks where to note the result – whereto he is to transfer the number of chips in the goblets. Jeanne then does what she has done before, but, using her pointing gestures, she continues with the third week, this time asking, 'how many three dollars do you have' (turn 193). And it is here that Mario responds 'three'. Now comes the crucial question: 'What do you write here?', she says while pointing to the corresponding cell (turn 200). And the tentative first part of the new action, 'Three?' to which Mario adds, following Jeanne's movement of the finger to the neighboring cell, 'plus three? plus three?' The utterance that follows – the 'social evaluation' of the preceding one (Bakhtine [Volochinov] 1977) – not merely affirms, but does so with an emotionally positively charged intonation. Mario has completed, for the first time, together with the teacher's pointing to a representation of the previous action, the action required to fill the cell. For the next cell, the action still emerges, tentatively, but less so than in the previous case, from the *sym*practical activity. It is at this point, then, that Jeanne notes that she thinks he understands. Mario indeed fills the next cell on his own, verifies what he has done by looking at Thérèse's worksheet, then finishes, whereupon he states aloud and firmly that he now understands. It is not the correct action itself that achieves the object/motive, but rather the fact of having objectified the object/motive, that is, the *becoming conscious* of the relevance of the goal-directed action with respect to the motive-directed activity as a whole.

In this situation, the zone of proximal activity does not come about mysteriously when the teacher sits down next to Mario. It does not exist initially, when she interacts with him and he with her. The zone of proximal development, that is, the point when Mario's action possibilities expand, when he produces new actions, is already under way for a while. It emerged in their sympractical activity, and from it, as the result of the sympractical activity. This means that Mario was as much responsible for its emergence as Jeanne. Both made it possible. Both have to subscribe to the same goal of making it possible. In fact, it is Mario who marks the need for the zone of proximal development repeatedly, in stating that he does not understand. He states the need for a structure that would allow him to do what he cannot do at the time.

Here we might say, in adapting Holzkamp (1993) to our context, that the emergence of the object/motive (see also chapter 6) and the control over what is to be learned (because of the creation of new action possibilities) in the process of overcoming the initially existing precariousness and frustration are but two different manifestations of the same process: sympractical activity. The object/motive and transformation and expansion of action possibility emerge together and, with them, the sense of precariousness and frustration subside and positive valuation becomes possible. The negative emotional tonality at the end of chapter 2 and during the initial two parts of their subsequent interaction is a reflection of the object/motive, here, a reflection of the experienced distance between the current situation and the only vaguely available object/motive to be revealed in and as result of activity.

The first two fragments, in fact, constitute a search for actions that would make the emergence of Mario's zone of proximal development possible. Jeanne does not produce the required actions, and her search requires Mario's participation. It is in the sympractical activity that Jeanne can find the actions required for Mario's room to expand the repertoire of his actions. When Jeanne attends to other students, gets up and leaves the group, she thereby provides a sign that her task has been completed, and simultaneously, Mario's action possibilities have expanded, that he has developed. In itself, her departure could also mean that she has abandoned him and now attends to other students' needs. But she explicitly provides an evaluation of her pedagogical activity: 'I think you understand now'.

At this point, also, Jeanne produces a sign that constitutes the affirmation of a connection between the societal, collective practice and the individual action. Mario has concretely brought about an action that is consistent with the object/motive of the collective activity. The accompanying psychic reflection is a realization of collective consciousness marking the coincidence of individual sense and objective, collective signification. This coincidence is in fact the concrete realization of the latter in the former mediated by the signs in which cultural significance is congealed. Algebra has been reproduced. It is this possibility of algebra to emerge from individual, subjective, corporeal human praxis that is reproduced; because anyone potentially can reproduce it in this way, the objective nature of algebra is also reproduced in and through subjective, living and lived labor. There is a tight interconnection between culture and the individual, a connection that is not achieved in theories where students and teachers produce 'taken-as-shared' actions and sense. The objectivity of mathematics, Husserl's (1939) geometry, our algebra, and in fact any cultural product (Merleau-Ponty 1960), requires more than taken-as-shared conventions. It requires the identity of object/motive, the cultural objectivity of the activity itself, which leads to the objective nature of the subject matter. What has to reveal itself to Mario is the fact that his personal sense of the activity is a concrete realization of collectively available significations. Like in the case of Mozart (Elias 1993), it is precisely when Mario's personal sense is expressible in and by means of collective significations that we have a reproduction of mathematics within the limits of its culturally objective nature (Radford 2006). It is precisely when his experience realizes collectively possible experiences, an

understanding mediated by the collective nature of language and signs, that the object/motive of the learning activity has revealed itself to Mario.

The zone of proximal development does not just emerge. The possibility of its emergence depends on whether the learner really is willing or in the position to take up the yet to be disclosed object/motive, and consequently the opportunity of expansive learning. To learn what the curriculum intends, the student actually has to take up and pursue the intended object/motive. Students may realize a task without taking up the object/motive, in which case they do not expand their action possibilities in the intended way, and do not learn what they are invited to learn. We see this clearly in the contrast between Aurélie and Mario. The former comes to complete the table of values, but her outer expressions continue to be frustrations, hitting (pounding) the desk surface, and verbal articulation of not comprehending. She does complete the task, but she does not engage in a way that allows the intended object/motive to emerge, whereas it does so from Mario's activity. Both have the table of values filled in the same way, both at least partially with the 'help' from another person. But whereas Aurélie copies what Thérèse has in her table, Mario produces the entries in and through his individual actions the cultural-historical appropriateness Jeanne has ratified in and through her evaluative contributions. In his case, the emerging object/motive does have 'sense-producing function', whereas in her case, the activity realizes a different object/motive, filling the table, and it does not have, in Leont'ev's terminology, 'sense-producing function'. These actions come to make sense when the sense of these actions emerges, when Mario becomes conscious of the fact that he is to note in each cell not the total sum of money in the piggybank but that he is to represent the repeated addition itself. Such a divergence as observable in the object/motive for Mario and Aurélie has been theorized in terms of the notions of *expansive* (transformative) and *defensive learning* (Holzkamp 1993). Expansive learning activity increases the action potential of the subject, whereas defensive learning only leads to the avoidance of negative repercussions (grades, punishment). Both expansive and defensive learning might lead to task completion and examination success. Copying, like cheating, is a practice that arises from and leads to defensive learning; it produces material outcomes (e.g., filled worksheets, correct exam responses) but does not lead to the intended learning outcomes that the curriculum specified.

Re/Thinking ZPD (Symmetrically)

> *The success of common developmental interests of parents and children, which is grounded in general interests, is an absolutely necessary condition* for the adequate *development of the child's individual subjectivity*. (Holzkamp 1979: 45, original emphasis)

In this quote, the development of the child is intimately tied to the developmental interests of the collective, including parents and children alike. Individual devel-

opment is realized in the pursuit of common interests. This orients us toward a more symmetric treatment of the zone of proximal development in which the developmental interests of all parties are involved and satisfied.

The notion of zone of proximal development has come to be used widely to theorize learning and learning opportunities. Unfortunately, following a simplified reading of its original definition and primary sense in the quotation that opens this chapter, the concept tends to be thought of in terms of the *opposition* of individuals. One of these individuals, a teacher or peer, is more capable than another individual, the learner. Somehow they engage in an 'inter-mental' or 'inter-psychological' plane, where the learner constructs knowledge from himself or herself on an 'intra-mental' or 'intra-psychological' plane. Vygotsky, following Marx/Engels[3], does not think of the higher psychological function as appearing *in* the societal relation – rather, the societal relation *is* the function. This view is at odds with the oppositional conceptualizations, which convey a substantialist approach that thinks of learning as knowledge assimilation and collectivity in terms of ensembles of individual actors interacting in self-interest. Their interaction is thematized through the dubious prism of the differences of what happens within the individual consciousness and what happens in collective consciousness – as if they could exist separately. Speaking is reduced to the individual, subjective intention of the speaker, who, in speaking, is considered to externalize ideas that have previously formed on the inside. The approach is substantialist in that it takes some prior situation, including the institutional positions of the participants in an interaction (i.e., teacher, student), and uses it to make causal attribution about the events that ensue. But such approaches are unsatisfactory given that there is insufficient attention to the co-constitutive nature of subjective consciousness and collective consciousness. The two forms of consciousness are co-constitutive because subjective consciousness always already realizes a form of collective consciousness; but collective consciousness exists in and as a possibility of subjective consciousness. More so, individualist and oppositional approaches convey notions of verbal expressions that are 'radically false' (Bakhtine [Volochinov] 1977: 122). Do we have to think of the zone of proximal development in terms of knowledge transmission and the underpinning opposition of a more and a less capable individual? Is it possible to think of this concept in terms of the unicity of interactional processes in which any moments (individual subjects) are *constitutive*, that is, cannot be thought of independently? In this chapter, we propose a different way to think about the zone of the proximal development in which asymmetries are possible because of the existing intercomprehension of interacting participants who become each other's teachers and students.

In the work of Vygotsky, who created the concept, we do find starting points for thinking about the zone of proximal development from a *symmetric* perspective. There is a real societal relation (Fig. 5.1), and it, as with all societal relations, en-

[3] 'Consciousness never can be something other than conscious being, and the being of humans *is* their real *life process*' (Marx/Engels 1956: 26). Here, Marx/Engels make the connection from consciousness as a form of being and being as a life process. Consciousness, therefore, is nothing other than the ideal reflection of real life process.

Fig. 5.1. There is a real social relation in real concrete activity. The ideal reflection of this concretely situated and anchored social relation *is* the first appearance of the higher function that emerges.

compasses more than one person. It is irreducible to the individual and constitutes a societal phenomenon sui generis. The symmetric perspective is grounded in a common world of historical significations and ways of life that we come to share since our birth and that form the basis of common implicit or explicit reference, common knowledge, assumptions, and so on. It is also grounded on the sharing of language. Thus, in a conversation – a word whose sense derives from the Latin *conversare* in the middle voice, that is, with active and passive aspects – speakers use words. But, any word spoken for the purpose of understanding is symmetrical, belonging to both speaker and listener. Thus, '[t]he word is a thing in our consciousness, as Ludwig Feuerbach put it, that is absolutely impossible for one person, but that becomes a reality for two' (Vygotsky 1986: 256). A conversation is a *con*versation only when the word is a reality for two – when 'each word has two sides. It is determined equally by the fact that it comes *from* someone as by the fact that it is directed *toward* someone. It constitutes precisely *the product of the interaction of speaker and listener*' (Bakhtine [Volochinov] 1977: 123, original emphasis). When we take a conversation as the unit, in which each word has two sides, any asymmetry within the unit, or between moments of the unit, has to be thought of differently. How, then, within this context, can we think about obuchenie (teaching/learning) situations differently than from the asymmetry of institutional positions of teachers and students? In the following, we develop our reflections concretely using an instant from our mathematics classroom, an excerpt from Fragment 4.2b.

The excerpted instant begins when Jeanne moves from the second to the third week of the saving process to be modeled in algebraic terms. She names the week to be considered and then offers up a question (turn 191).

Fragment 5.1 (excerpted from Fragment 4.2b)

```
  191  J:  third week; how mANY threes are you going to add in
           your:: (1.03) piggybank?
→ 192      (0.96) ((questioning look on Mario's face))
  193      how mANY three dOLLars are you going to have.
  194      (2.08)
→ 195  M:  how much money are you going to have?
  196  J:  how many thrEE dollars are you going to have?
  197      (1.47)
  198  J:  three dOLLars, three dOLL[ars  ], three dOLLars (0.23)
           ((points to the 2 '3's in week 2 and simultaneously
           points with left hand to the first, second, and third
           goblet))
  199  M:                          [three]
  200  J:  what are you going to write here?
  201  M:  three?
  202      (2.59) ((Jeanne moves finger to the cell on his left))
  203  M:  <<p>plus three? plus three? >
  204  J:  yES:: ((he writes))
```

We might gloss this excerpt in a traditional way saying that the teacher Jeanne attempts to allow Mario to 'construct' the idealization of what he has done earlier, the repeated additions of $3 to the existing amounts in the piggybank, to the additive and multiplicative structures. Some readers might think that she provides a sort of 'scaffold' that allows Mario to do what he eventually does. But in reading this transcript in this manner we would neglect the active part that Mario plays in this event. He not only responds, does what one might attribute to Jeanne as wanting him to do, but in fact contributes to bringing about the particular teaching moves. Thus, for example, in turn 193, Jeanne repeats what she has said before. We ask, why might she be doing so? In fact, it is itself a response to something that is not said but nevertheless present as a signifier in the situation. As indicated in the transcript (turn 192), Mario produces a questioning look, which we might gloss as 'what are you saying?' (the signified). She responds to this question by repeating what she has said. Now she no longer intonates it as a question but as a constative. She articulates again *precisely* what she had said, and now is saying again. There is a long pause and then Mario offers what he has heard Jeanne to say: 'how much money are you going to have?' (turn 196). The intonation is a questioning one (pitch moves upward). He offers a hearing, and, in intonating the offer as a question, simultaneously asks whether *this* is what he was asked, 'how much money?'. Something is unclear, and he provides Jeanne with the resource that might assist in helping her understand *his* problem.[4]

[4] We may actually hear this sequence as part of a conversational *repair*, in which the interaction participants have to clarify what is being asked before the response becomes possible. This repair itself is produced as part of the ongoing activity and therefore is as much an integral part of it as the production of the answer itself. Moreover, the entire situation is discursively produced so that the talk is not just about the contents of question and answer but also about the making of this situation: Mario is an integral and constitutive moment rather than an auxiliary and incidental vessel to be filled with existing cultural knowledge.

When we take an approach to the analysis in which each word uttered in the transcript is a thing in the consciousness of both, then the analytic situation changes. In fact, we may say that not only does Jeanne guide Mario to the point of naming what goes into the cell, but Mario also guides Jeanne towards what she needs to do to assist him. Mario does exhibit considerable cultural competence, which allows the conversation to unfold in the manner we observe. For example, we can see from the unfolding episode that he knows that Jeanne is asking a question. What is problematic is the content of this question. That it is problematic, Mario exhibits at least twice, once with his questioning look (turn 192) and another time when he offers a possible hearing (turn 195). The *question* of what she wants is problematic, rather than the fact that she wants something from him. He allows her to know more than that he has simply not understood. His lack of understanding may have arisen from not listening or not hearing what she has said. But in such a situation he might have asked, 'What did you say?', thereby indicating that the problem is a failure to hear rather than a failure to comprehend. Symmetrically, in producing at least the first part of what comes to be the sought-for response (i.e., 'three plus three plus three'), Mario lets Jeanne know that she now has asked the appropriate question. His appropriate response constitutes the evaluation of the appropriateness of her question. That is, Mario is a teacher allowing Jeanne to find an appropriate manner to phrase her question at the very instant that she is attempting to allow him to articulate a proper response. In other words, Jeanne and Mario are each other's teacher and student; and they are so simultaneously.

Up to now, we have focused on Jeanne and Mario. But the words that they have oriented toward each other also have been produced for everyone else present. The arrangement has the organization of a theater in which the audience is allowed to follow and understand. In this theater, each word exists not only is for the benefit of the two main protagonists but also for the benefit of the generalized other, the other children constituting this group, the researchers present, and all those who will vicariously come to know about the event through the researchers' writings. The active participation of the audience is exhibited in the orientation that Jeanne exhibits to the two girls and the camera.

Our analysis shows that far from exhibiting an asymmetry, the zone of proximal development is an interactional achievement that allows all participants to become teachers and learners. In our analysis, each utterance has come to be paired with an evaluation. Not only does the participant with an institutional position of teacher evaluate, but so do the participants with the designated institutional positions of student (learner). Each word is the product of the relation – an objective social fact sui generis (Durkheim 1919) – which makes the turn pair the minimal unit of analysis. Each word (locution) is paired with a 'social evaluation'; and it is the 'social evaluation' that 'defines all aspects of the utterance, totally permeates it, but finds its most pure and typical expression in expressive intonation' (Bakhtin [Medvedev] 1978: 122). It is precisely because of the evaluative role of each utterance that the teacher can know that the student has or has not understood, and the student can know that he has or has not provided the appropriate response. In other words, it is the unfolding and unpredictable *connectivity* that is allowed by the so-

cial evaluation of utterances and intentions that ties together, in a reciprocal manner, the participants in a symmetric space of *inter*-action.

Asymmetries are possible because the symmetry constitutes a basis (ground) for asymmetrical teaching and learning roles to emerge, roles that reflect a division of labor in collective obuchenie activity. This approach is based on the idea that a word never belongs to the speaker only because it 'addresses itself to an interlocutor; it is a function of the person of this interlocutor' (Bakhtine [Volochinov] 1977: 123). The utterance, therefore, 'absolutely cannot be considered as individual in the narrow sense of the term; it cannot be explained in reference to the psychophysiological conditions of the speaking subject' (ibid.: 119). The utterance *is* shared by speaker and listener rather than 'taken-as-shared' by their separate minds; it reflects *inter*-comprehension rather than separate comprehension. The advantage of the symmetric approach to the zone of proximal development that we propose here is that it allows the question of the more capable subjectivity to emerge from the interaction, appropriate especially when the question of who is in the know cannot be established on the basis of the institutional positions that the individuals otherwise take. Both Jeanne and Mario take the role of teacher; and both take the role of learner. Who is in the know and who learns is a product interactionally and contingently achieved as participants engage with each other. That is, it is appropriate to think of the institutionally sanctified 'teacher' to be a 'learner' and of the institutionally designated 'student' to be the 'teacher'. This approach allows us to understand why and how teachers learn during the course of their professional experience: In each interaction, teachers can find out whether something they have done or said was or was not successful, and also whether their subsequent attempts in changing their actions/utterances bring about the appropriate response. To them, the institutional relation is one that they can 'objectify', in other words, that they will exhibit or offer up in subsequent situations with other students. This case is in fact very common in classrooms. In our classroom research we have often followed some teacher with a camera around the classroom, recording his/her interactions and observing how refined the teacher's actions and discourse become as the teacher goes from one group of students to another (e.g., Roth 1998a). Far from constituting a sole opportunity for the student to learn (e.g., subject matter), the zone of proximal development constitutes an opportunity for the teacher to learn too (e.g., subject matter pedagogy).

The reconceptualization of the zone of proximal development that we are suggesting rests hence in a form of intersubjectivity that is grounded in a common world of cultural-historical significations and ways of life that we come to share with others since our birth. As noted previously, this common world forms the basis of common implicit or explicit reference, common knowledge, assumptions, and so on. It is this common world of reference that makes intelligible for the teachers and the students the game of 'finding the contents of the piggybank' and all that this game entails. Intersubjectivity is grounded in this common subbasement. But there is more: Our shared complex language, with its intricate forms of reference, auto-reference, and expression, accounts for the symmetrical role that participants come necessarily to play in conversations. Yet all this is not enough

for learning to occur. What is still missing is what we observe in the episode: the willingness to tune ourselves to others, to commit to a common cause, and to engage in a manner that is other-oriented. Thus, in the conversation with Jeanne, Mario could have given up the discussion. Jeanne could have too. She could have called on another student. But she did not. She kept adjusting to Mario, as Mario kept adjusting to Jeanne, both oriented towards the respective other.

Of course, it would be a mistake to think that we enter in interaction with others as *tabulae rasae*. The teacher knew beforehand the multiplicative-additive formula '3*n* + 6.' It is part of the cultural and historical knowledge that the teacher ubiquitously and continuously draws on to organize her experience of the world. She may not be aware of the fact that pattern generalization was an intense area of research in Pythagoras' brotherhood, or in Diophantus' Alexandria, or the Renaissance. Yet this cultural knowledge of pattern generalization endows the teacher with a particular asymmetrical role in the fourth-grade interaction. It is this asymmetrical element to which Vygotsky refers in his definition of the zone of proximal development. But this asymmetry in itself is not sufficient to understand learning. The teacher cannot make the object of knowledge merely appear in the students' consciousness. As Vygotsky points out in *Educational Psychology* (a text written during the years when he taught in his hometown Gomel, Belarus), 'strictly speaking . . . [i]t is impossible to exert a direct influence on, to produce changes in, another individual' (Vygotsky 1997: 47). In the same text he complains that 'the old pedagogics . . . treated the student like a sponge which absorbs new knowledge' (ibid.: 48). The primary asymmetry that results from the societal distribution of cultural knowledge is drowned in a symmetrical space where the participants' consciousnesses connect. Such a connection requires the appearance of a form of intersubjectivity where the participants de-center themselves. Their respective consciousness seeks the respective other through words and corporeal actions and reactions, such as grasping, touching, and pointing. And it is only when the object of knowledge appears simultaneously in Jeanne's and Mario's consciousness that learning occurs.

Naturally, the semantic density of knowledge (the as additive-multiplicative algebraic structure as a theoretical construct) is not the same in each one of the participants. For the teacher, the conceptual object of the multiplicative pattern – i.e., *n* x 3 + 6) – may relate to many theoretical aspects (first degree polynomials, theorems, abstract definitions, etc.) that are not part of the discussion. Yet, as our episode suggests, a common conceptual ground is reached. The appearance of the object of knowledge in Mario's consciousness, that is to say, its objectification, is a gradual and lengthy process in the course of which the various conceptual layers of the object are disclosed – e.g., that instead of totals, we can also think in terms of repeated actions, like in '6 + 3+ 3' or as '6 + 2 x 3' or even more theoretically, as '6 + 3*n*' or 'a + b*x*'.

To sum up, conceptualizing the zone of proximal development in the manner we suggest here rests on a non-transmissive and non-authoritarian form of knowing and on a non-individualistic conception of the participants. As to the former, knowing is not theorized as the reception of already-made pieces of cultural-

historical knowledge. Knowing refers rather to the possibilities that become available to the participants for thinking, reflecting, arguing, and acting in a certain historically contingent cultural practice – here the practice of algebra. As to the latter, instead of conceiving of participants as self-contained agents having already preformed intentions and ideas, or as solipsistic actors that merely take knowledge and intentions as shared illusions of interaction, participants are considered as actively involved in the *co*-formation of an emerging intersubjective attunement that is made possible by language, forms of perception, and more generally, our biological, historical, and cultural heritage. The emerging intersubjective attunement is certainly beyond a 'pure' cognitive realm. As our classroom episode illustrates, it entails a tremendous load of mutual emotions and continuously adjusted corporeal positions in the space of discourse and inter-action.

There are various theoretical and practical implications here. From a theoretical viewpoint, the role of participants in a zone of proximal development entails a better understanding of language and interaction. The perspective articulated here resorts to a conception of language and interaction that is at odds with classical ideas of information processing approaches and individualistic psychologies. Our notion of zone of proximal development draws on a conception of language, corporeality, and other semiotic resources that recognize the multiple perspectives of participants while they are at the same time seen to offer a constitutive background for intersubjectivity and the attunement of the participants. Within this context, we need to better understand how participants draw from those resources to position themselves in zones of proximal development and to tune to others in conceptual and affective layers to collectively reach interactional achievement. We also need to better understand how participants deal with the various political forms of asymmetries (e.g., knowledge distribution, genre, and ethnicity) to orient to others in the symmetrical space of language and intentions. Language, we note above, ties us together. A word always exists for more than one consciousness. But at the same time, a word is *ideological*; that is, a word always belongs to a system of ideas: '*The word is the ideological phenomenon par excellence*' (Bakhtine [Volochinov] 1977: 31). A word hence reflects the social, political, and theoretical position of the person uttering it. What this means is that in the encounter of consciousness that the zone of proximal development brings together, there is also an encounter of ideologies and perspectives and potentials for their transformations. This is why the idea of learning as transmission is terribly misleading. As we suggest above, both Jeanne and Mario learned from each other. However, the most important aspect of the zone of proximal development is not the mutual benefits that participants obtain in achieved interaction. To think along those lines is still to remain in the waters of individualism, one that justifies interaction in terms of the profits that each one of the participants collects (Radford and Roth 2010). The most important aspect of the zone of proximal development is the emergence of a new form of collective consciousness, something that cannot be achieved if we act in solitary fashion.

From a practical viewpoint, we need to investigate the discursive, corporeal, and other actions that encourage participants to attend to others in a responsible and

committed way, and to understand how new knowledge, subjectivity, and new forms of collective consciousness become variously produced. More efforts have to be deployed to understand – through empirical examples – zones of proximal development not only as zones of agreements but also of tensions, disagreements, misunderstandings, conflict, and subversion.

The Subject's Perspective on Learning

Cultural-historical activity theory in the Leont'ev–Holzkamp lineage orients us to take into account the subject's perspective on the activity, in other words, what is and can be apparent in the consciousness of the subject. When we consider Mario's perspective, it is quite evident that the learning object, the algebraic modeling of a practical saving situation, cannot be apparent in his consciousness. This is what the learning activity is to produce as its outcome. What then, we have to ask, is it that is and can be apparent to the subject? In the present instance, Mario is confronted with an empty table of values, the description of a life-like situation, and some questions. The latter ask him to fill the table of value. However, as we follow him along, we recognize that at one point Mario realizes that his currently available action possibilities are insufficient to fill in the table. He even asks what to write while interacting with Jeanne. What will allow him to complete the task is an expansion of his action possibilities. But which actions will do the trick? He cannot know, for if he knew, he would be able to apply them. The *learning* problematic therefore will be one of expanding the action possibilities such that the task can be completed. Simply telling (ordering, instructing) Mario to do this or that (e.g., 'put "3 + 3 + 3" into the cell of week 3') will not allow him to understand the sense, for he still does not know the object/motive of the activity, which, as developed in chapter 6, determines the sense of the action. The action acquires its sense as Mario makes sense of the action. This 'acquisition of sense' or, rather, the attribution of sense, occurs through the other in sympractical activity – as in Vygotsky's (1989) example of the child who may make a hand movement but who learns the social signification of the movement as a gesture only through the relation to its parents. With the recognition that the currently available actions are insufficient to complete the task also comes the experience of being at the mercy of the situation, subject to the conditions, and frustration as its emotional reflection. To overcome being at the mercy and subject to the conditions requires an expansion of control over the condition and, with it, of the experienced quality of life. A learning theory that improves upon all other existing learning theories has to be able to articulate what it means to learn from the perspective of the learner and eliminate the going conflation of teaching action with learning action. As we have seen, activity itself *may* transform the level of control over the situation and therefore the quality of life, which Mario expresses at the end in the utterance 'Me, I now understand'. By means of his intonation, the utterance comes to be replete with satisfaction at being able not only to fill the table but to understand what is required to do so.

Learning situations tend to be theorized from a third-person perspective making the researcher describe the mental structures or practices that students enact. The question never is one from the students' perspectives, who participate in the classroom interactions as conscious beings. In flesh and blood, students are not abstractions, are not cognitive frameworks or abstract practices in action. They do what they do based on life as it is available to them in their consciousness. As sensual living beings, students engage in real relations with the material and social reality *as these present themselves in* their *consciousness* (rather than that of the researcher). Mario and Jeanne are not abstract beings, not computers placed into human bodies, but concrete persons caught up in classroom life. As real human beings, individuals act in the manner that is sensible and intelligible to them, which reflects previous interactions that they have had with others in equally concrete situations. It is precisely for this reason that Mario *can* understand an offering as a question and that Jeanne may offer up an utterance as a question rather than as a constative. It has been noted that Vygotsky's theory, as generally used, with its focus on the differential distribution of knowledge in a teaching/learning situation is not suited to characterize learning more generally, especially adult learning and its generally often autarkic nature (Holzkamp 1993). Our reframing of the zone of proximal development shows that learning is not limited to one party in the relation but is open to all participants. What this requires, too, is a better understanding of learning from the position of the subject in activity – which forces us to consider and take into account the consciousness of the acting subject. It has reasons for doing what it does, and these reasons determine actions rather than some mental frameworks not only hidden from researchers but also, and more important so, hidden from the acting subject him/herself. The nature of these reasons, because they can be articulated when necessary, is inherently cultural-historical.

In cultural-historical activity theory, the concrete subject is understood not as a constant moment of the activity but as a moment in continual flux, in which changes reflect the changes within the material-practical activity as a whole. This outer context arises from the conditions in which the activity is embedded and the changes that the subjects brings about. The changes are continuous, expressing themselves more or less predominantly in all aspects of the situation of which the acting subject is conscious. Thus, we can see Mario first engage intensively and, in and through his activity, then become more and more disaffected to the point where he manifestly expresses extreme frustration. Interestingly, it is in and through the same activity that the frustration subsides, leading to the emotionally positively charged statement, 'I understand' at the end of the episode. It is not the teacher who brings about this change: It is the engagement itself, the sensuous *sym*practical activity that enables and promotes the change.

The question we face is how to conceptualize and theorize the work that the subject accomplishes in articulating the object of learning – based on its own perspective. Learning is not just a mechanism that somehow unfolds. Rather, learners do what they do for *their* personal reasons that are nevertheless intelligible *generally*. Some students may be interested in learning because they experience it as an expansion of their control over life conditions and action possibilities – expansive

learning leading to more adequate forms of thought and reflections of reality (Il'enkov 1994). Other students do engage in learning activities to avoid negative consequences – defensive learning. Still others engage in learning activities to try to do something with others, to confront the unknown and the challenge, even if the object/motive and the risks are unclear and remain so for a considerable time. We cannot understand what the reasoning subject does in the learning activity – which is based on what is given to its consciousness – unless we begin with the grounds that found its actions. This, in turn, requires that the difference between the learning demands posed from the outside, by the teacher, and the learner's own subjective learning interests are articulated (Holzkamp 1993). This is so because the student does not *have to* respond to the learning demand so that the motive of his/her activity may actually be different than the one that the teacher wants to initiate.

Fundamental to the question about learning is how learners can *intend* what there is to learn and how to learn it, because that which is to be learned is precisely unknowable. If they already knew what they had to learn, then students did not have to engage in learning it. This is important to the question of directing and controlling one's activity, for if learners do not and cannot know the object/motive beforehand, they also cannot aim at it – an activity-theoretic articulation of the learning paradox (Bereiter 1985). This is also why 'a specific learning object, precisely as *learning* object, that is, in the context of a problematic of learning, is articulated necessarily in the beginning only in a reduced way – incompletely, superficially, in undifferentiated manner, and so on' (Holzkamp 1993: 212). In the emergence of the learning object/motive, learners will become conscious of the problematic nature of the access to the learning object. It is only in the initial actions that the learning problem – accessing and realizing the object/motive of the learning activity – can become concrete in and to consciousness. The initial actions only open the problem superficially, and the learner needs to continue in the pursuit of the object/motive so that it can become available to his/her consciousness in its entirety. For the learner to engage with the learning object in a conscious and directed way, the discrepancy between his/her current knowledge and the learning object/motive needs to become available in his/her experience. The individual needs to be able to experience that there is more to learn than what is available to them on the basis of their current knowledge and understanding. In other words, they have to experience the dialectical contradictions that are situated at the epistemological level of classroom activity.

In this sense, the cultural-historical activity theoretic approach that we are describing here comes close to other theories in mathematics teaching and learning. In particular, it comes close to Brousseau's theory of didactic situations. However it departs from it in the manner in which the resolution of the dialectical contradiction is conceptualized. In Brousseau's theory, the student interacts with a 'milieu' that has to provide him/her with the appropriate feedback to realize the contradiction and to overcome it. Although part of the milieu, the teacher has to recede into the background to let the student engage with the problem in a phase that Brousseau calls a-didactic. The a-didactic situation rests on the idea that knowledge has

to come from the student him/herself. The teacher cannot show it. For 'if [the student] accepts that . . . the teacher teaches her the result, she [the student] does not establish it herself and therefore does not learn mathematics; she does not make it her own' (Brousseau 1997: 41–42). His is a very different view of what we have been saying about the manner in which the student-teacher relationship is conceptualized in activity theory.

Let us note, nevertheless, that the epistemic claim made by the theory of didactic situations is very much consistent with the constructivist one. However, the theory of didactic situations takes a different route vis-à-vis its concept of knowledge (hence an ontological claim). Indeed, as it is well known, the knowledge that the student produces is merely *viable* for constructivism. There is not even the slimmest possibility to correlate it to a common cultural knowledge, for all knowledge, by being constructed in a strict manner by the individual itself, remains personal (von Glasersfeld 1989a). At the inter-personal or social level, the individual's knowledge appears only as a working hypothesis, the illusion of something taken-as-shared. This, of course, is the well-known problem of solipsism. In the theory of didactic situations, in contrast, students are supposed to generate by themselves something common, more specifically a knowledge (savoir) that can be related to cultural knowledge. And the epistemological dialectical contradiction is overcome by assuming that the situation (e.g., a well engineered mathematical problem) will necessarily make recourse to the target knowledge (Radford 2009b).[5]

From a subject-oriented approach to cultural-historical activity theory, the perspective of the learner as the conscious subject in the learning process becomes of primary importance. In this approach, traditional conceptions of the teaching-learning situation, which emphasize either the agency of the student (e.g., constructivism) or the agency of the teacher (e.g., traditional teaching), have to be rethought. The subject of the activity needs to reflect on its own acts, as reflected in consciousness, which tends to occur when a contradiction is sensed in the activity. The essential psychological role of reflection resides in the emergence of the object/motive of activity, which the subject, in successful instances of learning activity, discovers in its own acts. Thus, 'the development of mathematical cognitive acts has a distinctly developmental aspect. It bears not only on the origins of particular acts but also on the genesis of thought in general, which functions as an "ideal component of real activity of social man"' (Davydov and Andronov 1981: 24). The purpose of teaching is to allow the emergence of a relation between personal sense and collective signification, and this relation cannot be transferred or given. It cannot be intended, as the student is supposed to learn the object/motive of activity in and through his/her participation in it. The object/motive of activity, therefore, can only be disclosed/discovered from the concrete circumstances in which it is realized and materially embedded.

Some of the pedagogical work grounded in cultural-historical activity theory has been critiqued because it focuses too much on the agency of the teacher and

[5] 'Each item of knowledge can be characterized by a (or some) adidactical situation(s) which preserve(s) meaning; we shall call this a fundamental situation' (Brousseau, 1987: 30).

too little on the creative agency and subjectivity of the learner. Learning is equated with teaching and the knowable teacher is juxtaposed with the deficient student. Typical of this kind of research are learning sequences realized and analyzed in terms of the *zone of proximal development*. Thus, 'in the current exclusively "educational" reception of the concept . . . an interpretation is suggested, according to which substantial learning progress of the child over and above its current state is possible only via the support of the teacher, who is the real subject of the child's learning' (Holzkamp 1993: 418). The teaching/learning relation that we circumscribe by the term *obuchenie* comes to be shorted to *teaching*. However, our work presented here suggests that the form of teacher-student interactions in the zone of proximal development does not have to be conceptualized asymmetrically and that in fact an interaction ritual requires a fundamental symmetry for teaching/learning to occur. Our foregoing description and analysis exhibits the active participation of Mario in the societal relation – the one that subsequently exhibits itself as higher function. He not only tries to tune in to what Jeanne says, but also is an important subject who actively contributes to the interaction ritual. As a result of this active participation, the child comes to discover the object/motive of activity, which the teacher cannot transmit by telling. In fact, to be a learning activity, the student must engage so that the object/motive of the activity, embodied in the concrete materials and concretely enacted social relations, discloses itself to him in and through his activity. This is so because the object/motive of the learning activity is the personal sense, that is, the relation between individual actions and collective activity.

A focus on consciousness as the organizing concept of the learning process allows us to integrate two heretofore separately theorized phenomena: cognition and emotion, the latter having orienting and valuation function in the learning process. Consciousness is the 'medium of the intersubjective relation to the world' (Holzkamp 1991: 89). The problem that current research has not yet answered is this fundamental contradiction: to be a self-directed subject, the learner needs to compare the learning object with his/her current knowledge, yet is inherently not in the position to know what s/he *is supposed to learn*. In this situation, the independent learner needs to be able to articulate the learning object sufficiently so that the learning process can be planned, organized, started, and self-regulated. The presence of the knowledgeable teacher, however, changes this equation, for the teacher can assume these functions in the activity where the object is initially unknown to the student but progressively discloses itself to him in and through his own engagement.

6

The Dual Nature of the Object/Motive

Central to cultural-historical activity theory is the notion of the object, reflecting the real, material object in practical activity and its future transformation into a product/outcome (Leontiev 2005). Yet despite its centrality, the notion as presented by those working in the other lineage of activity theory tends to be inconsistent with its original conception. This is so because the concept as used tends to reduce the object/motive to the material dimension although Leont'ev and other activity theorists (e.g., Il'enkov 1994) emphasize its irreducibly dual nature that includes the material and the ideal. It is, to use an analogy, as if some physicists continued referring only to the particle nature of light rather than referring to a phenomenon that manifests itself sometimes as particle and sometimes as wave. This exclusive consideration of the material moment of the object/motive is a problem that Leont'ev' clearly articulates as being associated with materialism, which, in its naïve version, takes the object/motive, reality, as existing only 'in the form of the object . . . but not as human activity, not subjectively' (Leontjew 1982: 25). Such an approach, Leont'ev thought, leaves little room to account for the growth of intersubjectivity and the self. This is why in Leont'ev's activity theory, the material object has a different function than it has in materialist and (individual/social) constructivist theories.

In chapter 1 we note that each activity is defined by and oriented to a collective object/motive. It gives sense to the (learning) action. If there is no object/motive, then there is no sense to the action. In science laboratory tasks, many students tend to be frustrated – they do not know *why* they have to do what they are doing. In fact, the *relevance* (i.e., object/motive) is understood only when they know what the task is to let them learn. That is, in learning activity, the object/motive is not available to students. Referring to an unreferenced source in Hegel's work, Leont'ev suggests that 'the individual cannot determine the goal his action until he has acted' (Leontiev 2005: 8).[1] Prior to any object/motive that can orient, in fact

[1] It turns out that this is the case even for the most accomplished scientists with decades of experience. We observed biologists, who after 20 years of dissecting fish eyes, find out many hours after a dissec-

bring about activity – i.e., object/motive → process of activity – an original participation in activity is required so that a reverse process may emerge in which an ideation occurs of the kind '"activity" → "its subjective product"' (ibid. 8). It is in activity, therefore, that the object/motive can emerge for the first time as the subjective product of activity. This subjective product constitutes 'the ideal'. Thus, 'in order for a representation about a thing to emerge, it is essential to allow the beginning action, the object of which appears within a certain system of objective relationships and is independent of subjective states such as needs, feelings, affects, inclinations, and so on' (ibid.: 8). We also note in chapter 1 that the object exists twice. In fact, this situation needs to be more differentiated.

To produce goal-directed actions requires an image. This image appears twice: as an initial sensation (sensory representation) of the object and as an objectively existing form. As a result of the actions, the object is changed. But to serve as a goal, there has to be an image of the future state of the object, the transformed result. This image guides the action. But afterward, the altered object also exists twice: as objective form and as its original, subjective image. 'The result is a unique duplication of the life of this image, of this representation: (1) its existence in the form of a subjective image; and (2) its existence in the form of an objective object' (Leontiev 2005: 9). The real object and its transformation are the material (objective) equivalent, a mirror, of my subjective image. Leont'ev suggests that this transformation constitutes the mirror that allows us to become conscious of our own thoughts, because mind can become consciously aware of itself only through mediation. Thus, 'I cannot see my own representation, but seeing it embodied in something or someone else, I see it' (ibid.: 10). What we have said so far, however, is a necessary but not sufficient condition.

We see that there are several *inner* contradictions in this articulation, which are internal to the activity system. The images do not just exist in and for themselves, but they exist in duplicate, prior to and following the transformation. Both of these transformation processes, which is what we are really talking about here, are simultaneous inner contradictions. These, as we point out in chapter 1, are an integral aspect to the way in which activity systems are thought of and theorized. It is not that there is some reified contradiction that then puts itself in motion. Rather, there are processes of life[2] at material and ideal levels. Because we are looking at the activity as a whole rather than at the beginning and end states of the transformation *in and for themselves*, we have a unit of analysis that contains an inner contradiction *and* it consists of flow.

For consciousness to grasp the objective nature of the object, the latter has to exist in idealized form. The object has to exist in 'a system of relations in which it is able to play the role of mirror, transforming the representation of man into awareness. This object must be signified and live in a special form – in the form of

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tion that they had not done what they intended to do in/with/during the dissection (Roth 2009b). It is only after having acted that they found out what they had done.

[2] Life *inherently* means change and transformation. Something that is not changing, is not living: it is not changing and therefore dead. At its very heart, cultural-historical activity theory is interested in capturing real life, its flow and living transformations.

language' (ibid.: 10). Leont'ev does not think here of language in the abstract but of its concrete, material realization. The object begins to live only in the 'body of the word', where the term 'body' refers to the 'physical substance of the word'.[3] This process is already at work in Vygotsky's account of an involuntary movement that becomes a sign for the child: first there is the movement itself (material), which receives signification in the societal relation with the mother, and only then exists in ideal form for the child, as a manner of designating an object.[4] With respect to the learning of mathematics, we must now specify how the object/motive that initially cannot exist for students such as Mario emerges from the relations that exist in learning activity, relations to the objects, others, language, tools, division of labor, rules, and so forth.

Emergence of the Object/Motive

The first two goals of the present task (Problem 4) are (a) to model the problem with the help of goblets and chips and (b) fill the table of values. The students achieve the first goal as Mario counts out the required chips and places them in the goblets (Fig. 6.1). They then ask the question what next, at which point they focus entirely on the worksheet and the table of values. Mario tells Thérèse, while pointing to the first goblet, that there are 'nine' in it, and therefore that they have to write 'nine' into the first cell. Then he says that for the second week, 'she has how much, and we write it'. That is, in his account, it is the total amount in the goblet (piggybank). This is not the intended object/motive of the activity. The object/motive is to think about patterns in an algebraic manner. To reach this object/motive, to objectify it, what needs to become the object of their consciousness is the *repeated adding of the same number of chips*, that is, the repeated number of adding of the same number of dollars *and* the relational multiplicative *correspondence of the number of repetitions with the number of weeks*. In other words, what has to become the object of their consciousness is a particular instance of the general object/motive. This object/motive *materializes* here in the theoretical form of a additive-multiplicative structure. From a cultural-historical activity theoretic perspective, the place that the sequence takes in the structure of the activity is central. What Mario and his peers achieve in the first instance is the modeling of the situation and filling in the first row of the table of values, but the *repeated* addition and

[3] Why this is so has been the object of philosophical inquiries from Heidegger to Derrida. Fundamentally, consciousness is associated with the separation of Being (*Sein, Être*) and beings (*Seiendes, étants*), the latter being externalities with respect to the former. A 'word', which is an externality to the material sound and its modulation that realizes it, stands not for a thing (object) in itself (the object as *Being*) but for the way it manifests itself – the externality of the thing, the thing as a being.

[4] What Vygotsky does not discuss is how the material object that the child points to also has to exist twice: in itself and as externality, which allows the object to manifest itself differently. It is only because of this self-relation that an object can appear differently to different people – in the way Marx/Engels (1962) conceive of value.

Fig. 6.1. Thérèse and Mario are absorbed by the task of counting out the number of chips that correspond to the weekly amount of money in the piggybank.

the *coincidence* of the number of repeated addition and the number of weeks, required in the second step, has remained tacit. It is precisely this *coincidence* that emerges in and as the relation between Mario and the teacher in the course of a first process of objectification mediated by various semiotic resources. It exists *as* the relation of which Mario is an integral part. Because he participates in the relation, it subsequently may also exist at a second, ideal level, and therefore in consciousness. The student has become aware that this coincidence is the object of *his* activity. (The *coincidence* is available in the deictic gesture of the teacher.)

Mario has completed the first part of the task, modeling the problem with his chips and goblets. As a result, he is sitting in front of five goblets and one pile of chips, which contains 9, 12, 15, 18, 21, and 24 chips. Part b of the question asks him to continue completing the table of values. But it contains a '6' in both of the cells corresponding to week 1. A contradiction emerges between the nine chips he has repeatedly obtained – and articulates in the discussion with Jeanne – and the 6 that he already finds in the cell that he is to fill. Mario repeatedly gestures to the sheet, sometimes as if wanting to strangle something in place just above it (Fig. 3.6a), and he invites Jeanne to 'look' while pointing to the sheet.

For Jeanne it does not suffice to tell him something like 'put 3 + 6 in the first cell, then 3 + 3 + 6 into the second' and so on, and equivalently for the second row, 'Put 1 x 3 + 6 in the first cell, 2 x 3 + 6 into the second' and so on. If she does this, she still will not be addressing Mario's plea, 'I don't understand'. He might, like Aurélie, fill up the worksheet and continue to express his frustration, as the girl has been doing throughout this part of the lesson (see figures in chapter 2). She does produce a completed worksheet, but has perhaps resigned to her assessment, 'I do not understand and I will never understand'. The object/motive reveals itself when Mario comes to realize *why* he fills the top row with the sequence 3 + 6, 3 + 3 + 6. . . . The realization of the *why*, its fulfillment, amounts to the first objectification: It consists in endowing with meaning the sequence of mathematical signs 3 + 6, 3 + 3

+ 6, ... The cultural mathematical signification and Mario's personal sense meet. But an additional objectification is required to reach the additive-multiplicative structure (1 x) 3 + 6, 2 x 3 + 6, ... The object/motive of the *learning* activity is the conscious awareness of the necessary content of the cells – the encounter with the cultural mathematical signification.

To understand how the object of activity can emerge, one has to distinguish between that which has to become conscious and that which is actually conscious in the learning activity. This requires a distinction between the two lower levels in the triadic hierarchy constituting activity: actions and operations. Actions are oriented toward conscious goals, whereas operations are unconscious and determined by the conditions. The students take as their goal the adding of 3 to a given amount in the sequence ((6 + 3) + 3) + 3 . . .. To arrive at the abstraction – i.e., at the appearance of the activity at the ideal level – the students in this class have to become conscious of the operations and conditions that lead to a realization of the goal of their actions, which constitutes the learning activity as a whole.

Much of the trouble we observe in the episode appears to be generated between the total number of chips in each of the model goblets, which in fact depict a temporal sequence, and the articulation of the repeated additions. Mario has done these repeated additions when he first counts out 6 chips, then adds 3, to arrive at the representation of the piggybank after week 1. He then counts out 9 chips and adds 3 more, to arrive at a total of 12 chips for the second week. Rather than representing *this* process, which corresponds to repeated additions, he gives 9 and 12 as the responses for the first and second cell in row 1 of the table. He does so even though Jeanne rephrases the 9 as '6 + 3'. That this is the issue becomes clear only later. Jeanne does not explicitly articulate this issue for Mario, and may not be conscious of the problematic nature of the issue at the time. Initially, she articulates the composite nature of the sum totals. It is only later that she asks Mario what a number is 'composed' and 'comprised' of. Further advance seems to be evident (after the fact) when she articulates the nature of the cell content in terms of repeated addition, which she both articulates verbally and exhibits in specific deictic gestures. The relation between the first and second row to be filled is articulated as a 'shortcut', whereby 3 + 3 is transformed into 2 x 3, 3 + 3 + 3 is transformed into 3 x 3, and so on. Curiously, it is not this second part that appears to be the source of the trouble, but the transformation required from Problem 4a, and its results, into the representation of the entire process, the repeated additions that happens across progressive weeks.

The a definite progress announces itself across the turns 218–220. Jeanne asks, 'How many three dollars do you have?' and Mario responds, 'fou(r)'.

Fragment 6.1 (Excerpted from Fragment 4.3b)

```
  217      (0.42)
→ 218  J:  your fourth week; (.) how mANY three dollars do you
           have.
  219      (1.00)
→ 220  M:  u:m:::  (1.73) fo. ((Fills table, Thérèse makes
           noises))
  221      (9.48) ((writes 4 '3s'))
```

```
222  J:  <<pp>kay> (0.97) instead of doing three plus three
         plus three plus thrEE whAT are you going to wrITE
         here? ((Points to the row on the bottom of the table
         of values))
```

He notes four times the three (with + signs added), as the structure already prefigures the response for the cell below: he has four.[5] Although this combined expression could serve to signify four additions of $3, it also can be heard as four times $3. The subsequent exchange facilitates the emergence of this transition from the four additions that Mario is in the process of noting down to the multiplication. The translation from '3 + 3 + 3 + 3' to '4 x 3' is marked as a 'shortcut'. After some hesitation, this multiplication is offered in a tentative way.

Jeanne and Mario achieve this process, by means of which the object/motive reveals itself, on the basis of a completely shared language and other articulations that they have to understand and make sense of. It is in this manner that they pull off this event. This symmetry is required for the event to be able to take place (see chapter 5). Moreover, Jeanne cannot put the object/motive in words, because, within the subject-scientific approach to cultural-historical activity theory, it is only in Mario's own consciousness that the object/motive can reveal itself because of its relational character. The object/motive is an expression of *his* relation to the activity; it is Mario the person (i.e., not Mario as an abstract subject but Mario-in-activity) who has to make salient and articulate this relation. But prior to its emergence, Mario cannot intend the object/motive because he does not know it yet. In learning activity, in contrast to other human practical activities (farming, manufacturing, etc.), the object/motive has to reveal itself from the perspective of the subject.

Jeanne suspects that Mario has understood ('*I think* you understand now'), and leaves him on his own. There is only a slight response. It is after completing the table of values, once the teacher is gone, that Mario announces that he understands. That is, whatever his sense is and whenever it emerges, he announces it as such just after completing the cells corresponding to weeks 5 and 6.

This way of stating the problem is a direct consequence of the way in which activity theory frames the relation between individual and collective subjectivity. Thus, important to the way in which subjects engage is the way in which they relate to the collective object/motive in their own subjectivity, where the former is expressed in the form of personal goals. In other activities, the subject has already concretized the collective object/motive and proceeds to realize it in and through its activity by means of concrete, goal-directed actions. These goals and actions make sense because of their relation to the collective object/motives and activity. But in the absence of a concretization of the object/motive, as is the case for students in the process of learning, that is, of objectifying the object/motive, their actions do not and cannot make sense. They do not know whether what they have

[5] We do not know whether the four are actually for $3 units, as this might be heard in the request 'how many three dollars'. There is evidence from the sciences that children may operate on the numbers rather than at the number-unit unity (Roth 1998b). In the present instance, this would correspond to Mario operating on the numbers alone rather than on amounts of money.

done brings them closer to the revelation that is sought. They have no way of being certain that their responses are those that the teacher wants them to give. So they have to be tentative, living in uncertainty and, as available in the students' emotional responses (Aurélie, Mario), in a situation of precariousness. It is also for this reason that Mario offers up tentative solutions to a problem that he does not yet know in its full extent.

Much of the learning science literature does not concern itself with emotions and the role these play in learning. From an activity theoretic perspective, this does not make much sense, for affect (emotion) and cognition are inseparable, two faces (manifestations) of the same process – reflections of the concrete sympractical activity that contributes to the collective control human beings exercise over their environment. Learning activity is but part of the network of activities that constitute society, and can be understood only if considered as constitutive part of the human life form as a whole. In this episode, we observe the role emotions play in evaluating and directing activity. Affective responses bring the activity to a halt (after Fragment 2.3), and are continuously reproduced and transformed as the activity unfolds. Here, we note that the difference between cognitive and emotional expression is undecidable, as both make use of the vocal tract to realize themselves. Moreover, there are other expressions that make use of the body, torso, hand/arm (iconic, deictic, beat) gestures, and body orientations and positions.

Mathematical Consciousness as the Reflection of Concrete Mathematical Activity

> Consciousness cannot ever be anything other than conscious being, and the Being of humans is the real life process. (Marx/Engels 1958: 26)

Mathematical consciousness, to use the argument structure of Marx/Engels in the introductory quote to this section, is consciously doing mathematics. But the object of mathematical activity is different from the objects of other activities, where the subjects do not experience themselves as doing mathematics but pursue the object of their activity as part of which they may be using representations and forms of argument that share some resemblance with mathematical entities (Roth 2005). But different activities mean different object/motives and actions, just as different games – to use the analogy Wittgenstein (1958) proposes – imply different object/motives (though there might be a family resemblance between them) and rules by which to play. The people in the fish hatcheries observed by one of us (WMR) did not think about themselves as 'doing mathematics', though they used distributions, equations, numbers, graphs, mathematical models, and so on. Similarly, the observation of 'home and school practices involving calculation, the significant transformation occurred to make the task mathematical only when calculation *became the target of the exercise*' (Walkerdine 1997: 67, our emphasis). That is, even though some practices 'may have some form of calculation contained within them,

[they] are not the same as academic mathematical practices because the product of each practice is different – a calculation in one case and not in the other' (ibid.: 68). As a consequence, precisely because students are to learn *mathematics*, they also have to become conscious of the object/motive of the mathematical game. Thus, it is unlikely that children would learn *mathematics* if they were to engage in *fish hatching*, for the object/motives of the two activities are very different and, with them, the forms of consciousness (idealizations) that result from engagement. From the cultural-historical activity theoretic perspective, children become conscious of the object/motive as a consequence of participating in the mathematical activity, just as young children eventually become conscious of the grammatical rules only after they already speak the language. Without a language, there is nothing to know the grammar of, just as without the activity, there is no object/motive to be conscious of.

Drawing on Leont'ev's ideas, our study shows how, for Mario, the object/motive of the activity becomes a reality of his activity; that is, the object/motive emerges as the result of his activity that he completes in joint action with the teacher. It is not that the mathematical object, the formula $3n + 6$, is simply the result of his own actions, in the way Piagetian educational theory would understand it, but it is the result of contingent, cultural-historical forms of relations with cultural-historically shaped objects that have a cultural-historical character. So what Mario comes to think and how he comes to think is, in turn, cultural-historical through and through. His subjectivity is not something singular, the result of a monadic subject, but it is the concrete and unique realization (an *event – the event of Being,* to use Bakhtin's term) of a cultural-historically possible and collectively enabled subjectivity. The function of the concrete materials, therefore, is different from the function it has in other theories.

Even during those instances that the object/motive of activity has not yet revealed itself to Mario, it does exist and cannot be separated from his consciousness. Thus, 'even when the motives do not become conscious, that is, the person does not take account of what stimulates him/her to produce this or that action, these do find their psychic reflection but in a special form – in the form of the emotional coloring of the actions' (Leontjew 1982: 192). This is precisely what we describe in the preceding chapters. The emotional coloring that we observe manifests itself as frustration, perhaps despair. This emotional coloring disappears together with the appearance of the object/motive, which Mario acknowledges and manifests with his comment 'me, I now understand'.

It has been noted that visual materials and the place they take in the process of instruction 'are determined by the relationship between the learner activity in which these materials exist as the object of the immediate goal of actions to the activity that affords becoming conscious of that which is to be appropriated' (Leontjew 1982: 249). This relation between the two forms of activity may take one of three forms. The first relation is one in which the two activities coincide, where the real activity and the required activity are the same. Here, the learning object is most likely to emerge. Second, the first activity in which the students focus on the concrete materials may prepare for the second, the one in which the object is ab-

straction; in this case, the task of the teacher is to find the proper sequence of activities such that the ultimate one can take place. Finally, the two activities may not be connected, in which case the reliance on concrete materials is useless and in fact constitutes a distraction. Many materials that teachers use to 'motivate' their students may actually be of that kind. Leont'ev provides, in the appended chapter 7 of *Activity, Consciousness, Personality*, an example of such materials. He writes about a very conscientious teacher who wanted to interest and motivate his students in learning mathematics by preparing plates on which were depicted tanks and anti-aircraft canons (during WWII). He asks whether the nice drawings that the teacher had produced would elicit *mathematical* actions, and responds to his rhetorical question by saying, 'Of course not' (Leontjew 1982: 245). He suggests that the difficulty of the task of abstracting the mathematical dimensions from the psychic image students form is correlated with the richness of the outer image provided. Thus, 'it is easier for the child to count uninteresting pencils than interesting tanks' (ibid.: 246).

The psychological function of the materials, such as the goblets with colored chips in our study, is that they are outer scaffolds for the child's inner activities accomplished under the joint work with the teacher. These materials are *not* the real object of activity – the object/motive – but stand in for the object/motive. What the child learns is not to add three chips to the amount that it had in the previous goblet and has now placed again in the present goblet. The child is not learning arithmetic, or, rather, the object of the activity is not to learn arithmetic. Rather, the child participates in the activity of learning algebra, that the problem can be modeled by means of the formula $3n + 6$. The problem is not about concretization of conceptions, of the students' knowledge; the object/motive is generalization. 'Thus, the visual materials in these cases constitute a material, in and with the aid of which the object/motive to be appropriated is yet to be found' (Leontjew 1982: 244). This is precisely what we observe in the case of Mario, who, in saying 'Je ne comprends pas [I don't understand]' articulates for his teacher Jeanne, his peers at the table, and therefore for us, that the object/motive of the activity escapes him at that instance. It is in the course of his engagement, with the teacher, that the object/motive of his activity discloses itself to him. Because he does not know it, he cannot aim at the object/motive. But the collaborative work with the teacher does not *give* him the object/motive, nor is the object *transferred* externally to him in some way such as cough syrup is given to a child using a spoon. It is through the joint work with the teacher that he becomes conscious of the object/motive that reveals itself: the generalization from cell to cell in his table of values from $6 + 3 \rightarrow (6 + 3) + 3 \rightarrow ((6 + 3) + 3) + 3 \ldots$ where successive brackets indicate what is contained in the goblets for day 1, 2, and so on. His slight nod following the teacher's question, and his subsequent announcement (the teacher is already absent) 'Je comprends [I understand]' is precisely the instant when the object/motive of his activity has emerged for and become clear to him. The object/motive is not the concretization of concepts and ideas but the generalization thereof.

Teachers frequently use hands-on materials to 'motivate' their students. The design of learning materials may indeed involve considerations of their particular

attractiveness to the learner. Aurélie, Mario, Thérèse, and their peers may more likely engage in the challenges that the problems pose for them. But attractiveness and the attention that it entails is only the first part of the task from the activity theoretic perspective. Attractiveness and attention may lead to the false assumption that the presence of the curriculum materials in students' consciousness will lead to the intended learning. The abstraction of the algebraic properties is not enhanced or diminished by the particular properties of the materials used. In fact, the elaborations that such materials include may actually detract learners from engaging in the real activity, that is, in discovering the real object of their activity. 'The inner actions that are to be structured by the students require the abstraction from the materially objective content of the presentations, and this [abstraction] is the more difficult the richer it [content] is' (Leontjew 1982: 246). Rich theoretical contexts hence require the overcoming of prior conceptual understandings through theoretical abstractions that open up new possibilities to act, think, and reflect about the problem at hand – in our case a modeling process based in a historically and culturally constituted form of thinking that we call algebraic. The attainment of such a form of thinking, its objectification, rests on the possibility of realizing that the additive saving process can be modeled through a multiplicative structure that synthesizes the repeated additions. This book is precisely about the fact that this problem is not merely a cognitive or epistemological one. The attainment of rich theoretical contents are indeed embedded in activities, which unfold and evolve out of contradictions where affective components come to be indistinguishable from the cognitive ones. The inseparability of the cognitive and the affective is present not only in the students, but also in the teacher. This inseparability constitutes the ground of the societal relations and forms of knowing that arise in activity. Cultural-historical activity theory, as articulated by Leont'ev, offers a fertile perspective through which to investigate the irreducibility of cognition and emotion – a problem that is fundamental in the context of teaching and learning and of which, curiously, we still know very little.

Mathematics Classroom as a Microcosm of Society

Much of mathematics education focuses on what happens in mathematics lessons or in teaching experiments with very well defined tasks. Learning comes to be theorized independently of the societal relations, which, as we show in the preceding chapters, *are* (the origin of) the higher psychological functions. There is, however, an increasing although still marginal amount of research in the field of mathematics education that studies its political dimensions, for example, from the perspective of social justice.[6] Although a lot of work has been conducted from a decontextualized and decontextualizing perspective on mathematical learning, very

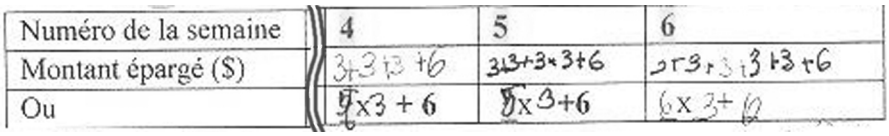
[6] See, for example, Alrø and Skovsmose 2002, Brown 2008, Skovsmose 2008, or Valero and Zevenbergen 2004.

little work and theory exists for the latter endeavors. It is especially unclear what mathematical learning has to do with anything social (Radford 2008c). However, one clue is provided in earlier chapters when we articulate higher psychological functions as having their origin in societal relations. In order to arrive at a fuller analysis, the place where learning activity occurs needs to be theorized in its relation to society at large. Cultural-historical activity theory provides us with the means of doing such analyses, an approach to which we sketch in the current section.

Doing school tasks is not the object/motive of what students do. From the perspective of society, school is the place where students become systematically engaged in cultural-historical practices and where students come in contact with forms of knowing and being in *this* society at *this* point in historical time. The school is the systemic place where cultural continuity is exposed. But this is not how cultural continuity and its transformation originally occurred. Schooling, as with all other cultural activities, has arisen in the course of an increasing division of labor. In early forms of a society, the upbringing of children and their preparation for a contribution to collective life occurs in the family and community – much like the production of food, the fashioning of tools, the production of cloth and clothing, and so on resided within the family unit. In the course of their histories, specific societies have come to develop special places where the education of future generations took place, coinciding with the emergence of a special profession. The object/motive of the activity *schooling* is the reproduction of society-specific cultural practices (knowledge). What happens in mathematics classrooms needs to be theorized in terms of the role schools play within society at large as the place where the latter reproduces itself. Schools do prepare for life in society, and students are prepared for participation in productive activity systems after school by engaging in the kind of societal relation that are mirrored in the microcosm that school constitutes of society. The kinds of engagement we observe as we follow along with Mario and Thérèse presuppose that the students already buy into and identify with the object/motive of schooling.

Grades and diplomas are indices for the intended object/motives – the knowledge specified in the mathematics curriculum. The actual outcomes of the schooling experience, however, are grades, and what students know is by and large irrelevant to their graduation. Thus, even those students who have taken physics courses come to the university with 'misconceptions' and the number of years of physics taken bears no correlation with success in university courses. Schooling therefore does not add up to learning or education.

A student does not have to identify with or realize the object/motive of society, as instrumentalized in the real goals – grades and report cards. A student can get a high grade without actually learning what the test or assignment is intended to identify as knowledge acquired. Thus, in the present situation, Aurélie's worksheet came to be completed largely with the help of or direct writing by Thérèse. She does not, like Mario, seek the help of the teacher to come to a sense of understanding; she does not even check whether the contents of her table of values correspond to those of her two peers. We note that the second-row cell of week 4 contains only



Numéro de la semaine	4	5	6
Montant épargé ($)	3+3+3+6	3+3+3+3+6	3+3+3+3+3+6
Ou	5x3 + 6	5x3+6	6x3+6

Fig. 6.2. Extract from Aurélie's worksheet. It shows that she has not filled the cells according to the intended additive and multiplicative pattern.

three '3s', the third-row cell of the same week contains something covered up by a '5' and a '6' (Fig. 6.2). We also note that the second-row cell of week 5 contains four '3s', the cell below a '5' written over something else – the number of '3s' in the additive pattern does not correspond to the number indicated in the multiplicative form. Finally, the second-row cell of week 6 contains five '3s', whereas the last cell notes a '6' as the multiplicative factor – again, the number of '3s' in the second cell does not correspond to its count in the third cell. Despite the apparent problems – Aurélie repeatedly articulates dissatisfaction, frustration, and disengagement – she does not seek assistance and in fact continues to be disengaged near the end (Fig. 6.3).

How shall we understand and theorize the events that have led up to the worksheet as collected at the end of the lesson? How shall we theorize the actions Aurélie exhibits, and, with it, her subjectivity, consciousness, and personality? Current learning theories cannot explain why students do not learn what they are supposed to learn. Generally students are blamed for not wanting to learn and lacking motivation to do so. But the real issue is that from the perspective of the (learning) subject, acquiring some mathematical knowledge is not associated with the promise of an increase in the action possibilities, room to maneuver, and control over the conditions. These students therefore may not take up the object/motive, and their disidentification with it is explained in traditional psychology as the absence of motivation. As a result, motivation psychology has been developed to make such students do what they do not automatically want to do – similar to the role motivation psychology plays in the workplace, where it is intended to increase productivity, that is, wealth for the ruling classes at the expense of the worker. Thus, 'the analysis of the school as a concrete *place for learning*, thus, a place of learning possibilities, impediments, and contradictions from the perspective of the subject, is intelligibly possible only on the basis of the comprehended "school structures"' (Holzkamp 1993: 346). Although Holzkamp recognizes shortcomings in the analyses Foucault (1975) provides of schooling – the latter does not focus on the specific development of schools but of schooling as part of other activities, and, therefore, object/motives – he recognizes the family resemblance in the kind of analyses that need to be conducted.

Important in the present context – which provides the necessary background that will take us from generic 'social' relations to school as a microcosm of societal relations and, therefore, to the possibility of a critical perspective – is the description of schools in terms of the 'objective-practical order of things and human bodies, thus the real "edifice of the school", in which specific *actions and relations* of

Fig. 6.3. Aurélie continues to be disengaged from the task all the while Thérèse and Mario continue alone (the former) or in relation with another (the latter).

the inhabitants are enabled and others disabled *by means of spatial and interpersonal arrangements*' (Holzkamp 1993: 347, emphases added). In the present episode and analyses, we see examples of both. We see how specific arrangements enable relations, which in their ideal form become, on the part of Mario, the conscious awareness of the saving practice in abstract (additive and multiplicative) form. On the part of Aurélie, a different social relation comes to play itself out, one also described by Foucault (1975) but related to the disciplining of the body, which, as we have shown in the context of field ecology, constitutes an integral part of the acquisition of the academic discipline, of becoming a recognized member of the discipline (Roth and Bowen 2002). It is also an integral aspect of mathematics lessons, where a study of ours shows recurrent incidences within and across lessons of the inculcation of proper behavior in a mathematics classroom (Roth 2011a).

When Jeanne arrives at the table configuration of the group, thereby responding to Mario's signal for assistance, Aurélie is leaning back and ceases working on the task (Fig. 6.4a). The cells of her worksheet are still not filled in, yet in frustration she has thrown herself against the backrest of her chair. Jeanne asks, and therefore concretizes Mario's raised hand as a request, 'what is the question?'

Fragment 6.2 (from Fragment 2.3)

```
043  J:  <<f>yes.> (0.52) whAT is the ques[tion.] ((Aurélie is
         'lounges' in her chair, Fig. 6.4a))
044  M:                                    [its  ] ^this :::
         (0.38) <f>um[::> ] ((hands move downward, restrains
         not to pound on table, gazes at sheet))
045  T:              [auré]lie sit properly (55:00)
046  M:  look this is (.) dUMb, <<p>i dont understAND.>
         ((Aurélie sits up, Fig. 6.4b))((487>217Hz))
```

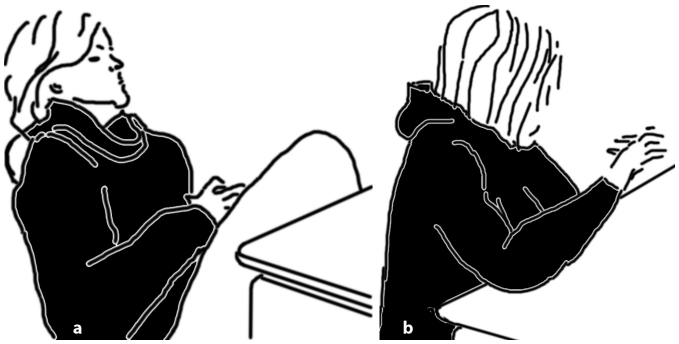


Fig. 6.4. a. Aurélie is leaning back, apparently not engaging with the task. **b**. As part of the exchange with the teacher, she sits up to 'sit properly'.

Just as Mario begins to respond (turns 044, 046), Jeanne overlaps him and addresses Aurélie by naming her, then provides what Aurélie's subsequent action will reify as an instruction: she sits up in the way the others do (Fig. 6.4b) – though this does not constitute the beginning of further engagement with the task. In this and similar situations, the students in this as in other classes, learn 'to sit properly', 'comme il faut'. These forms of relation are an integral aspect of the mathematics classroom and are an integral aspect of the societal relations that come to be reflected in consciousness. These societal relations imply institutional configurations, institutional hierarchies (power), and specific forms of division of labor. It is through these relations that corporeal discipline and academic discipline come to be intertwined. It is also the place – realizing the relational origin of higher psychological functions (Vygotsky 1989) – where corporeal discipline initially comes to be stimulated from the outside to manifest itself subsequently as 'self-discipline'.

The worksheet, though filled up, is not a measure of the intended form of consciousness. The actual goal realized in this case is the filling up of the table rather than the becoming conscious of a relation between the (simulated) phenomenon of keeping a piggybank and the mathematical formalism that models it. The general term for producing a semblance of knowledge is 'cheating'. It ultimately means that the purpose is not 'knowing something' but 'getting/having a grade', for students can move on to increasingly higher grade levels without ever having learned what the curriculum specified.

To properly theorize learning activity, we need to take the perspective of the subject embedded (nested) and participating in a form of activity. This is so because intentional learning processes in the way they are conceptualized in the curriculum are possible only when the student in fact accepts and takes up the goals set initially enforced by someone else and makes these his/her own grounded in personal subjective reasons. We would have to seek answers to questions such as 'Why did she not seek help?', 'Why did Jeanne not engage Aurélie, though the latter evidently disengaged?', 'What are the kinds of relations that Aurélie is familiar with and that she would be capable of reproducing?', or 'What are the contra-

dictions within the system that lead to the kinds of situations in which we observe Aurélie?' Because the school is a microcosm of society, cultural-historical activity theoretic analyses take into account both societal and individual-subjective (ontogenetic) developments and understand their relations as these play out in the reality of *this* mathematics classroom. The forms of relation that the societal microcosm enables and supports tend to be those consistent with a middle and upper class ethos (Eckert 1989). Thus, it would not be sufficient to focus only on the kinds of relations that we actually observe between Jeanne and Aurélie, for this would be equivalent to somehow sampling the current state of the student's mind. To understand the observable forms of societal relations we need a cultural-historical analysis of schooling generally and of the schools and school systems in which Aurélie today is a constitutive subject particularly. We must not reduce her non-engagement in simplistic terms by referring to 'gender', 'age', 'interests', or 'cognitive developmental level'. To understand the forms of relations in which Aurélie observably is involved requires an accompanying analysis that follows the constitutive role Aurélie plays in the cultural-historical processes of her schooling experience specifically and of her broader life experiences more generally. The need for the latter will become more evident in chapter 7, which focuses on the hierarchies and networks of object/motives, which are an integral part of the way in which personality is theorized in cultural-historical activity theory (Leontjew 1982).

What we can observe in the present context, however, is the possible origin for divergent school experiences, which lead some individuals along trajectories into areas that require understanding of mathematics and mathematical modeling and others into areas where they can avoid all engagement with formal mathematics. Whether experiences such as the ones described here lead students like Aurélie and Mario out or into mathematics cannot be known, but it requires a historical study. We note in passing that the autobiographical experiences of one of the authors (WMR) did not point toward the kind of careers he subsequently had. Despite repeating fifth grade to a large extent because of failing mathematics, and subsequent years of struggling, he eventually did a Masters degree in physics and became an applied mathematician and statistician. Some of the problems in school mathematics may actually be the result of the problematic nature of school mathematics. Thus, with respect to the mathematical understandings of very young children it has been noted that their sophistication 'with respect to the rules of sopping practices far outweighed the sense that they could only hand the small numbers and concrete objects that were at the hart of their mathematics education' (Walkerdine 1997: 69). This underestimation, embodied and institutionalized in the pedagogical practices and artifacts, leads to the hindrance rather than the advancement of mathematical development. Writing about children doing shopping games in their mathematics lesson, Walkerdine writes: 'far from aiding any "transfer", the shopping game positively hindered [the children] from making a tradition that demanded that they suppress their inscription into those familiar everyday practices to become subjected in academic mathematics' (ibid.: 69). At this stage of our work, we can only wonder about how our curriculum produced, especially in Aurélie, a hindrance rather than a support for developing school mathematical practices.

7

From Subjectification to Personality

> We do not begin here with what humans say, imagine, represent, not even of the humans that appear in discourse, thoughts, images, representations to get from there to the corporeal human being; we begin with the really working human beings and from the real life process to articulate the development of their ideological reflexes and echoes of their life process. (Marx/Engels 1958: 26)

To understand the real nature of human beings, Marx/Engels suggest beginning with the real life process, with the real relations human beings have with each other in and through their work. Proceeding otherwise – i.e., beginning with what humans say (about themselves), their imaginations, representations, and ideas – leads us to idealism and metaphysics. It does not allow us to understand real life and its continuous movement, and, with it, the production of ideas, representations, consciousness, thought, and so on. This is so because *life leads to and determines consciousness* rather than the other way around. This methodological specification became the ground upon which the cultural-historical activity theoretic psychologists – including Vygotsky, Luria, Leont'ev, and Holzkamp – built their theoretical and methodical approaches. All, in a very decided manner, not only attribute their theoretical and methodological commitments to Marx but, in a very deep way, develop an approach to psychological research that bears fundamental resemblance to the historical approach to political economics that Marx/Engels (1962) outlined in *Das Kapital* (Capital).

We, too, are committed to theories and methods that begin with the analysis of real human relations and the forms of consciousness that are required in their actions and that they make available to each other as a matter of course in order to bring about, in and through their often invisible work, the very societal structures that other researchers merely note or accept as if fallen from the sky.[1] Throughout

[1] Ethnomethodology constitutes the one approach that differs from all others in this respect. It takes as its task to elucidate how human beings, in their relations with others, produce the very objective societal structures that other researchers accept as given (see, e.g., Garfinkel 1996). Because this approach de-

this book, we emphasize that the cultural-historical activity theoretic framing of the situation focuses on the societal relation *as which* the higher psychological function – the 'abstraction' of the multiplicative pattern for saving from the preceding additive pattern, itself an 'abstraction' of the action of adding chips to the goblet – first appears. We then observe this function, which existed only *in* and *as* relation, in the form of independent action. But any goal-directed *action*[2], because it is grounded in the reasoning of a person – who, using shared language, could never reason except in cultural-historically enabled ways – already is a societal rather than a singular phenomenon. Working independently, therefore, does not take Mario out of societal relations: he is completing the worksheet not just for himself but inherently for the (anonymous) other. Even if he were to write it down for himself, this would still denote, according to Vygotsky, an external, other-mediated relation to himself.

Subject in/as Societal Relation

In this book, we present the exemplary and exemplifying analysis of but one short episode in the life of Mario and Jeanne, and we see even less of Thérèse and Aurélie, who do not feature centrally on the videotape that recorded them all. If we push Vygotsky's approach, which itself is a concretization of the program outlined by Marx/Engels, then who Mario is and what he thinks is the accumulation of societal relations. That is, in a strong sense, we never observe Mario *in and for himself* but always only in societal relations (even 'solitude' and 'time-out' are forms of individual-collective relations). We only ever observe societal relations and then inscribe or attribute some of what we see, some of the actions, to students such as Mario – even though the cultural-historical activity theoretic perspective asks us to take into account the activity as a whole so that we can identify the subject only as a constitutive moment of an irreducible whole. If we pull the activity, if we pull the relation, the subject no longer exists, for the cultural-historical activity theoretic subject only exists in and as relation, in real, concrete activity. To think otherwise by focusing on, for example, activity in the abstract is handing oneself over to the psychological idealism that Vygotsky (1927/1997) adamantly critiques and in contrast to which he offers up his own 'concrete human psychology' (Vygotsky 1989).

But we may focus on the identifiable body and the identifications it makes possible. We can then think of subjectification as 'the production through a series of actions of a body and a capacity for enunciation not previously identifiable within a given field of experience, whose identification is thus part of the reconfiguration

scribes societal reality from the perspective of the participants, it goes well with a Marxist sociological and social psychological analysis (Smith 1987).

[2] Random movements, because they are not *goal*-oriented, may be singular. This is how, according to Vygotsky, pointing first emerges. An other person attributes a sense (intention) to a random movement, and this sense (intention), which first appears *as* relation, subsequently comes to be realized by the developing subject.

of the field of experience' (Rancière 1999: 35). In this dense framing, Rancière defines *subjectification* as the production of a (living) body; this body is produced through a series of actions. These actions are those of the subject itself, as well as the actions of others and the natural world. 'The world', Bourdieu (1997: 163) writes, 'immediately is doted with sense because the body, which because of its senses and brain has the capacity to be present outside of itself, in the world, and to be impressed and durably modified by it, was for a long time (since the beginning) exposed to its regularities'. Subjectification thereby includes the production of a capacity of reflecting, acting, and speaking that was not previously seen within the field. In fact, it is the body itself that is producing these actions in a given field. Finally, the body and its capacity for enunciation and action are part of the field and of its reconfiguration. A mode of subjectification, the production of a body, 'does not create subjects ex nihilo; it creates them by transforming identities defined in the natural order of the allocation of functions and places into instances of experience' (Rancière 1999: 36). What has to be added, though, is the fact that this 'natural order' has already been politicized in the economic and ideological allocation of functions and places. The result is that the experiences where subjectivity emerges against a multi-dimensional coordinated system whose axes are defined by the possibilities and constraints offered by the conflicting systems of knowledges and ideologies (Radford 2008a).

Any time we look at some societal situation – children in a mathematics classroom, shoppers in a supermarket, neighbors taking the bus to work – we see people in relation. These relations are irreducible: they constitute the irreducible realities that are sociology's phenomena because they cannot be explained psychologically, physiologically, or by some other body-centered reductionist approach (Durkheim 1919). Such relations, as with all societal facts, have a dynamic on their own. It is therefore impossible to say who the subject of activity is outside and independent of these *concrete* relations. We see this quite clearly when returning to any excerpt from the transcript, such as the following. In this excerpt, Mario's first turn 182 is suspended between and a function of two turns by Jeanne.

Fragment 7.1 (excerpted from Fragment 4.2a)

```
  181  J:  tWO times thrEE is what. ((places rH index on the
           first cell, will not remove it for a while, Fig. 4.2))
→ 182  M:  <<insisting>s:sIX:.>
  183      (0.18)
  184  J:  plus
  185      (0.65)
  186  M:  s::IX::.
  187  J:  =equals to, ((moves index up down))
  188      (1.02)
  189  M:  tWEL:v:e ((fills something into his table, 439>210
           HZ)) (2.96) Where do we write twelve now.
```

This turn is suspended because of a double relation. In the first instance, Mario's turn constitutes the second part to an irreducible question-response pair; in the second instance, Mario is setting up Jeanne to complete the (speech) act that he has started. He thereby not only completes a turn sequence, articulating the per-

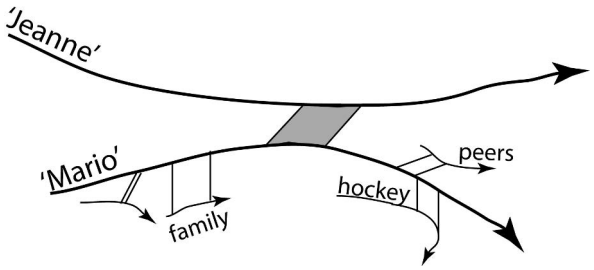


Fig. 7.1. A serious cultural-historical activity theory engages in the study of irreducible processes, accumulation of which leads to the 'person' anchored in the material (physical) body. The *societal relation* at the heart of this book is denoted by the grey rectangle.

locutionary moment of a speech act, but also sets up the next turn sequence by producing the locutionary/illocutionary part of the next turn pair. In responding to Jeanne, his utterance is *for her*. We cannot therefore reduce the utterance to Mario, as it belongs to both. Although uttered *by* Mario, it is *for* (the benefit of) Jeanne, completing what she has begun. Mario exhibits the actual effect that the locution has had, thereby affecting Jeanne, who might have had an intention but who finds out only in and through Mario's (speech) act what her own act has done. The speech is not independent of Jeanne. Similarly, in uttering a locution, he again speaks *for* Jeanne, beginning a turn pair that she will complete: neither her nor his locution is independent of the relation.

To understand 'Mario', we therefore need to look not just at what comes from him in the episode described or in an interview with him where he might be talking about his relation to mathematics, who he is (i.e., 'identity'), about his inclinations for school and its various subjects, and so on. We can understand 'Mario' only as a constitutive moment of societal relations, as the 'effect' of the relation. In our analyses, we note that the higher psychological function exists *as* a relation involving Mario and Jeanne. But their relation is not just an abstract, prefixed pattern into which they place themselves as if it were a box. Rather, they have to produce this relation *in flesh and blood* all the while talking mathematical content. The relation is a societal one, within the hierarchy of institutional relations that we outline above and that comes with particular divisions of labor. Moreover, *this* societal relation is not independent of all the other societal relations that *these* bodies, identified by the names 'Mario' and 'Jeanne', have taken part in. This relation as event follows a cumulative effect on these bodies, which are the results of histories of relations (Fig. 7.1). Outside of these histories *this* relation cannot be understood. These histories enable and provide the bodily inscribed resources for the present events. These histories also shape the kind of societal relation that the two can and will concretize as a function of their relation's unfolding history. This is so because the practice they enact 'contains not only modes of regulation, but actual ways of understanding and describing the children in the classroom, those who are developing well, those who are working when play is the evidence that would prove that their development is genuine, and so on' (Walkerdine 1997: 64).

In each of the relations depicted in Fig. 7.1 – a realization of 'trajectories of participation of the living body' (Nissen 2003: 341) – the question of the subject arises. The subject is not some constant 'body' which we denote by the name 'Mario'. Mario is not an instantiation of the piece of wax philosophers liked to take as an example, which is inserted into some relation and then formed. Who and what the subject is, both shapes and is the outcome of these relations. Even situated cognition, therefore, 'is not people thinking in different contexts, but subjects produced differently in different practices' (Walkerdine 1997: 65). The practices in which Mario is involved at home with his family, with his peers – i.e., they might be talking about the stats of the latest baseball games – or within his hockey practice may involve calculation and mathematical signs. But the mathematics is not the same, for 'the relations of signification are different, the regulation of the practice and the positioning of the subject is different, and the emotional signification is different' (ibid.: 68).

If the relations subsequently evidence themselves as higher (cognitive) psychological functions, then the kinds of relations that a child can and does participate in will contribute to the shaping of its body and its habitus that enacts the kinds of visions and 'di-visions' characteristic within a particular field of society (Bourdieu 1997). Different forms of relations – all nevertheless concretizing the possibilities of the school and classroom – will therefore be enabled with a child coming from a middle class home than a child who grows up in a working class home. A simple look at YouTube reveals many videotapes where children as young as a few months are 'participating' in counting games with their parents or videotapes where four-year olds count in five languages. In fact, we may observe in the middle class homes the same kinds of practices and relations that these children subsequently entertain at school. Walkerdine (1988) tracks the kinds of relations and discourses that characterize mother-child relations at home into the schools. There, the translation of the non-mathematical into mathematical discourses is to occur through references between the 'experiences' children are thought to have had – such as saving money in a piggybank – without the verification that children have actually participated in these experiences. School relations are enabled because the middle school children already have participated in these kinds of relations; these relations are stabilized and reproduced. The middle class ethos prepares for, reproduces, and is reproduced by the similarity of the practices at home and at school that sustain particular forms of (middle class) relations. Classist society itself is reproduced in this way. To truly understand 'Mario', we would have to have access to this history of relations over longer periods of time.

At particular times, which differ between societies, traditional arrangements of desks that had emerged in the 18th century were changed from rows into groups of desks – such as the foursomes at which we find Aurélie, Mario, and Thérèse. These 'new arrangements designated a different kind of learning in which a new conception of what the child is was constituted' (Walkerdine 1997: 63). Arrangements that of this kind shape the kinds of societal relations that can take place; but these arrangements emerge from developments that are completely external to the situation, such as the Plowden report in Walkerdine's Britain the Plowden report or the

recommendations of the Nuffield Mathematics Project to designate 'uncommitted areas', where suddenly emerging interests and needs that require different kinds of spaces and relations may be played out or are given room to play themselves out.

From a cultural-historical activity theoretic perspective that takes seriously the orientation to an understanding of living processes, the question therefore cannot be that of some stable, reified subject. At best, there is a process of subjectification, of becoming a subject, and of individualization of societal conditions and relations. That is, the 'Mario' that we denote in Fig. 7.1 in the form of a trajectory of a historical process is but a marker for a living/lived body that develops together with everything else that constitutes the activity system in which he is an integral and constitutive part. In chapter 4, we quote Vygotsky (1989: 56) writing 'we become ourselves through others'. We become ourselves through others *in and through participating in relations that are inherently of societal nature*. The way in which 'Mario' relates to himself, therefore, is an outcome of the societal relations with others in this classroom. He might say, as Aurélie does, 'I don't understand and I will never understand'. Thus, it is not merely the case that a student who agentially 'positions' herself does some form of identity work when, 'unsatisfied with her progress in mathematical discourse [, she] is likely to call herself a "terrible mathematician" or a "slow thinker"' (Sfard 2008: 290). Rather, the very designations 'terrible mathematician' and 'slow thinker' are inherently intelligible ways of accounting for and denoting certain experiences – such as the ones we see Aurélie and Mario express here. These are *ideological* expressions that culture makes available for the characterization of a person and, therewith, for an individual person to characterize herself. The descriptive terms, the way in which such experiences can be account for, themselves come from the generalized Other and therefore shape the kinds of experiences the individual can have.

Subjectification and Self-Movement

In cultural-historical activity theory, activity is a category of organic life, and, as organic life, it only lives in and through change. Activity is a suitable category of organic life only when it is its reflection on the ideal plane (of the researcher). The category is created such that it captures the living change of activity.[3] Living activity, an activity that changes, is self-moving; as a category of life, activity cannot be externally moved as well. With the activity, all of its moments change as well as the relations among the moments. We describe in chapter 6 how the object/motive of the learning activity comes to reveal itself to, and is realized in a conscious manner by, Mario. As the object/motive, objectified in the material conditions,

[3] Throughout *Das Kapital* Marx/Engels (1962) emphasize life and characterize labor, labor force, work process, the individual, or personality by means of the adjective 'living'/'alive' (Ger. *lebendig*). Vygotsky (1927/1997: 247) writes that general investigations become 'more abstract and more remote from directly perceived reality'. Thus, 'instead of living plants, animals, persons, the subject matter of science becomes the manifestations of life'.

comes to be revealed at the ideal level, that is, as the object comes to be subjectified, the subject changes. By engaging in the work of (school) mathematics, the subject is obviously changed at the biological level, through the expenditure of chemical energy through metabolic processes, the transformation of its body. Simultaneously, because it interacts with material reality, changes occur in the cortex that lead to 'kinetic melodies' (Luria 1973): to forms of movement and speech that become increasingly fluent as the subject gains control over its life conditions in activity. As the object/motive reveals itself, the subject itself increases its discursive capacities. In this manner, 'the basic form of speech communication is the formulation of ideas as whole propositions which are intimately bound up with the motives and conditions of the activity in which the individual is engaged' (Luria 1979: 171). Grounding himself in Vygotsky, Luria's fundamental position is that a change in the goal of a task changes the structure of the psychological processes, which themselves are but moments of the activity as a whole. 'A change in the structure of activity, in other words, implies a change in the brain organization of activity' (ibid.: 172). Even the changes from spontaneous speech to elicited speech transform those functional systems that support the continued unfolding of the activity.

Initially, Mario – like Aurélie – does not know what to do and both orient themselves toward Thérèse. Mario articulates what he understands that the task asks him to do, but Thérèse, to whom he orients, does not appear to pay attention. She does not act upon his explanations, and Aurélie articulates the situation as 'Thérèse, we have no idea what you are doing'. Mario – like Aurélie – abandons and, when Jeanne asks him, in and as response to his raised arm, what the question is, he – like Aurélie before – describes the situation as 'dumb' and says he 'does not understand'.

We then observe how the Mario-Jeanne relation changes in the sequential organization of their contributions to the point where the additive and multiplicative structures of the saving process exist *as* relation. The transition is observable in the difference between the questioning look and the request for clarification of the nature of the question (turns 192, 194) and the subsequent (discursive) production (turns 201, 203) of what comes to appear in the cell as '3 + 3 + 3'. That is, we observe movement and transformation of the relations, which exist in the turn pairs 191/192 and 193/195, on the one hand, and turn pairs 200/201 and 202/203, on the other hand. These turn pairs, as we show in chapters 4 and 6, constitute an important instance in the revelation or objectification of the object/motive, and, therefore, an important turning point in the self-movement of the activity.[4]

Fragment 7.2 (excerpted from Fragment 4.2b)

```
191  J:  third week; how mANY threes are you going to add in
         your:: (1.03) piggybank?
→ 192    (0.96) ((questioning look on Mario's face))
193      how mANY three dOLLars are you going to have.
```

[4] It is important to note the purposeful nature of our choice of the term 'revelation'. Because Mario does not know the object/motive, he cannot intend knowing it or intentionally seek it. The object/motive therefore *reveals itself* to him (Marion 2002).

Fig. 7.2. At the end of the episode, we observe a subject 'self-confidently' filling up the remainder of the table of values and uttering 'me, I now understand'.

```
  194      (2.08)
→ 195  M:  how much money are you going to have?
  196  J:  how many thrEE dollars are you going to have?
  197      (1.47)
  198  J:  three dOLLars, three dOLL[ars  ], three dOLLars (0.23)
           ((points to the 2 '3's in week 2 and simultaneously
           points with left hand to the first, second, and third
           goblet))
  199  M:                          [three]
  200  J:  what are you going to write here?
→ 201  M:  three?
  202      (2.59) ((Jeanne moves finger to the cell on his left))
→ 203  M:  <<p>plus three? plus three? >
  204  J:  yES:: ((he writes))
```

We therefore observe a change in relation, and this relation itself, in its latter part, *is* the higher function of which we subsequently see evidence that it has come to be reflected ideally when Mario completes the worksheet and comments, 'me, I now understand' (Fig. 7.2). The cultural-historical developments that we are subject and subjected to make it tempting to ascribe the changes to Mario, and thereby reify an argument about the production of the subject, namely 'that the "truth" about children's "mathematical development" is produced in classrooms, and that all learning can be understood as taking place within social practices in which the relation between signifier and signified is constantly problematic' (Walkerdine 1988: 9). As shown elsewhere, such reification moves reduce relations such as 'Mario-in-the-mathematics-classroom-successfully-doing-a-word-problem-involving-a-piggybank' to an attribute, an externality, of the subject of this activity, realized in a body identified by the name 'Mario' (Roth and Barton 2004). Such a reification, or 'process of *subjectification*, implies, besides the clash of forces, a

certain *identification* through which the intersubjective relation is related to – becoming in certain respects identical with – the subject's relation to herself' (Nissen 2003: 339). We do see changes as we watch the videotape of the lesson but these are not solely changes in the subject but always the subject in relation with its activity. That is, the societal relation we describe and analyze in the course of chapters 3 and 4 between Jeanne and Mario is not only a zone of proximal development for the emergence and realization of a higher psychological function that subsequently comes to be reflected in the individual; it is also a zone of proximal development for the process of subjectification.

We do, therefore, observe a process of subjectification in the sense that we articulate above. Thus, we observe 'a capacity for enunciation not previously identifiable with a given field of experience' (Rancière 1999: 35), where the enunciation is that of the additive and multiplicative structures corresponding to a given week and where the field of experience is the mathematics classroom in which fourth-grade students engage in algebraic tasks. In the present, the process does not come about in and by itself – the autonomous activities of the innately curious, exploratory Kantian and Piagetian subject – but in the form of an interpellation, whereby the teacher 'invites' students to engage in a particular task form (which the researchers have brought to her and the class). This interpellation itself is an integral aspect of the process of subjectification, as 'the interpellating subjects' intentions – the imaginations of those who interpellate on behalf of the community – are not externalized and realized in an independent lawful object, but, rather, in the very course of that object's transmutation into their own unique *alter ego*' (Nissen 2003: 341). In our case, Jeanne functions as the person interpellating the children generally and Mario, Thérèse, and Aurélie specifically – on behalf of the educational and mathematical communities. In their intentions, the learning of abstractions and formal algebra, students experience or do not experience 'the object's transmutation' into the individual subject. This transmutation is enabled and mediated by 'boundary activities', such as 'qualification' and the objectification it receives through marks – i.e., objective or quasi-objective norms – that institutionalize subjectification and, thereby literally constitute a form of soci(et)al work.

Subjectification means development as subject-in-activity. But this entails at least three relations. First, Mario increasingly controls the conditions of production: he no longer has to ask Thérèse or the teacher what to do next and what to write into each cell of the table of values. In the societal relation with Jeanne, the room to maneuver available to Mario has expanded, as has his capacity for action. We observe a phenomenon that we might gloss as 'increasingly becoming the acting subject-in/of-activity'. Second, and as a form of inner contradiction, increasingly becoming the subject in/of the activity also means increasingly being *subject to* the activity. Paraphrasing Nissen (2003: 336) we might say that as Mario externalizes himself in the object he becomes subjected to its objective logic while at the same time subjecting the object to himself and expressing himself through it. Mario's actions, then, are no longer more or less random but increasingly 'comme il faut' – as the game of mathematics played requires. The reverse side of increasing control is increasing subjection to the rules of the game played, here, the

mathematical (algebraic) activity. Third, and correlated with the second point, increasingly becoming subject in/of activity also means increasingly being *subjected to* it. The acting person therefore 'never is completely subject of its practices: through the dispositions and beliefs that are the principles of engagement in the game, all constitutive presuppositions of the practical axiomatic of the field . . . introduce themselves right into the apparently most lucid of intentions' (Bourdieu 1997: 166). An increasing capacity for doing mathematics (algebra) in a particular way also comes with the development of a set of blinders that lead the subject to doing things a particular way – mathematically – rather than in other possible ways. That is, mathematics is a discipline and to do it in a disciplined way requires the disciplining of body and mind. Again, common lore (folk and academic psychology) likes to make a virtue out of 'self-discipline', which in fact is but a form of 'internalizing' an external discipline, which, as other higher functions, exists as (therefore in) a social relation first.

We do not have the same amount of data on Aurélie because she is not equally involved in the events we observe. In fact, as we show across chapters 2 to 4, she repeatedly opts out of participation and declares that she does not understand and that she will never understand. Aurélie repeatedly asks Thérèse what she is doing, and throughout the recorded event, Aurélie's intonations are plaintive. In the end, there are a number of errors on Aurélie's worksheet. We therefore observe a particular relation and movements that are opposite to the ones we observe in the case of Mario. We do not observe an increasing control over the conditions and we do not observe that the object/motive reveals itself to Aurélie. Again, it would be dangerous to use such observations as evidence for characterizing Aurélie as 'outside reason . . . with the 19th century thinking' according to which 'boys are said to make leaps whereas girls remain firmly rooted in everyday calculation' (Walkerdine 1997: 59). Such thinking, which we do not intend to reproduce, might suggest that we observe in Mario a capacity for abstraction – the multiplicative structure is abstracted from the actions of adding chips to the goblets – that Aurélie does not exhibit. Such a move, if we were to enact it, would be producing a particular form of the subject, as a form of discursive practice, rather than understanding the subject in the cultural-historical activity theoretic way as the subject of a particular (learning) activity to whom the object/motive reveals itself in the course of engagement. Although it is not our purpose to enter here into a lengthy discussion of what happened after, let us note, however, that it took us three years of intense work to get Aurélie interested, to some extent, in school mathematics. The interest did not arise from the various problems we offered her to engage with, but from the forms of ethical, social, and affective engagement that we sought to promote with the teacher in the classroom. She became more attentive and responsible towards others.

Yet, much more research would be required to understand Aurélie's historical trajectory through other forms of activities and societal relations in the manner Fig. 7.1 depicts it for Mario. Thus, tracking back to Aurélie's early childhood, we might observe particular mother-daughter relations. Walkerdine (1988) describes mother-daughter relations in which counting and addition emerge as new capacities that

the girls subsequently exhibit on their own. These new capacities are the result of transformations of the relation as new signifier-signified are continuously formed. The resulting 'discursive shifts . . . were central to the accomplishment of the discursive transformation and to the repositioning of the subject it entails' (Walkerdine 1997: 68). These discursive shifts, even though apparently very small, 'produce the possibility of huge shifts of subjectification and the production of the man of reason, because . . . semiotic chains are carried along the metaphoric axis, and this no longer exists in school mathematics discourse' (ibid.: 68).

Personality – A 'Knot-Work' of Object/Motives

> The personality becomes a personality for itself by virtue of the fact that it is in itself, through what it previously showed is itself for others. This is the process of the development of the personality. (Vygotsky 1989: 56)

In cultural-historical activity theory, *personality* is a category that approaches the question of the person in a systemic manner: it constitutes the person as a system that is both produced in and as a moment of activity. Relations between two people *are* the higher psychological functions, which 'are relations of a social order, transferred to the individual personality, the basis of the social structure of the personality' (ibid.: 58).[5] Vygotsky then refers to Marx, who writes, in the sixth thesis on Feuerbach that 'the human essence is not an abstractum inherent in each individual. In its reality it is the ensemble of societal relations' (Marx/Engels 1958: 6). It is a way of accounting for the relation between the subjects of different activities that each of us traverses and contributes to constituting everyday. With respect to Mario, this means that he 'is' the ensemble of the relations that we find along the trajectory that he, as material body, undergoes as part of his real, lived life (Fig. 7.1). Thus, we do not have some constant subject 'Mario' or 'Jeanne', who in this way participate as son or mother in a family, as shoppers in the supermarket, as players on a hockey team, or as a member of the relevant peer group. From the cultural-historical activity theoretic perspective that we develop here, as from a poststructuralist perspective, the 'subject' is itself a constitutive process of a process (activity). Development here is a process of self-movement of relations, contradictions and transitions that are to be analyzed. Such an approach 'necessarily leads to a position of the cultural-historical nature of personality' (Leontjew 1982: 166). Personality is the result of societal processes such that the individual can become a person only as the subject of the diverse activities that make society; the personality of this individual is as much a product of these activities as is his/her consciousness.

[5] In this quote, Vygotsky does refer to social order and social structure to characterize the relations and the basis of personality. Thus, the adjective 'societal', which captures the social *order*, is entirely legitimate.

In cultural-historical activity theory – as it was for Marx – personality, as consciousness, are the products of societal relations. To understand personality, therefore, we must not look at particular manifestations at particular points in time but we must conduct a historical (biographical) analysis of the emergence of personality. This is why Aleksandr Luria, one of Vygotsky's students and collaborators, wrote two biographies to come to better understandings of the working of the mind (e.g., Luria 1968).

A distinction is required here: subject, individual, and person are *different* categories. The subject, as we have seen, is defined in terms of the activity. It is a moment of activity. The person, however, is the accumulation of societal relations in and as living body. The individual refers to this body, but only in a one-sided manner. That is, the difference between *individual* and *personality* reflects the dual character of the properties of all cultural objects, including the dual character of labor that is manifested in a commodity or the dual character of humans as 'subjects of nature' and 'subjects of society'. Thus, 'the real basis of the personality of man is the totality of its by nature societal relations to the world, that is to say, relations that are *realized*. This occurs in his activity, more precisely, in the totality of his manifold activities' (Leontjew 1982: 175–176). Leont'ev explicitly orients us to activities (*Tätigkeiten*) and contrasts them with actions and operations, which are not determinant aspects of the development of personality (e.g., writing is not a characteristic of personality).

In the course of its daily life, an individual participates in many different activities and activity systems. Thus, for Mario it begins with life in the circle of the family. He then moves among his friends with whom he produces peer culture while riding the bus to school. There, he is a subject of the schooling activity. After school, he may attend hockey practice, participate more within his peer culture, be part of some other extracurricular activity, and in the evening, he ends up in the context of the family. Because the subject does not exist *in and for itself* across activities but is a function of the activity and its movement, we will observe different forms of relations with others and the respective activity as a whole. Thus, Mario might be an outstanding hockey player and therefore may not experience the kinds of frustrations exhibited in this mathematics lesson. Aurélie might be an accomplished and admired musician or swimmer, all the while completely disengaging within the context of the algebra lesson. The different relations are formative, because they are reflected in the consciousness of the developing person. As subjects of activity they exhibit but moments of personality, which is a dimension in the analysis orthogonal to that of the subject-in-activity because it captures the relation across activities.

In the course of participating in activity, objects/motives emerge, as we show for Mario in chapter 6, and are concretely realized. The motive of activity comes to be objectified in the object, and the two become two moments of one irreducible object/motive. These exist, as we note, not in abstract form but as concrete entities that are also reflected in consciousness. The collective object/motives, initially present *in* relations now come to exist in subjectified form for the individual. Because of their concrete, objective character, object/motives can be set into relation

with each other. For the individual person who moves through a series of activities as part of his/her life, this leads to the activities' hierarchies of object/motives. But these object/motives and activities are not external and do not exist as externalities: they are real subjective object/motives that the person can use as grounds for action. It is precisely this developing and changing hierarchy of object/motives that becomes a defining characteristic of the personality. This is also why the person develops as the result of the subjectification of societal relations and conditions rather than as independent (constructivist) monad.[6] For Mario, it might be that the predominant object/motive becomes associated with mathematics – but at this stage we do not know that. It may be ice hockey or something else that takes the lead in the hierarchy of object/motives. In the case of Aurélie, the object/motive of mathematical activity may not appear at all because it was never objectified in the first place. As for the teacher Jeanne, we might find that the teacher-related object/-motive is as important, and perhaps even more important, than the ones related to family and others. This would then allow us to understand if others were to make attributions to her as being absorbed by her work, and, if she spends much of her evenings in preparing for school, as a 'dedicated teacher' or even as a 'workaholic'.

The shifts between object/motives (activities) and therefore the importance individual activities play in the overall hierarchy constitute observable changes and reorganizations in the personality of the individual. Thus, in early life, children are oriented toward family. Later, there is a shift to peers and to being part of the peer group; and participating in the relevant events becomes more important. Still later, a growing degree of independence from peer groups and parents becomes observable as participation in university or jobs become the primary and most determining activities in the life of an individual. At some point, personality becomes a new *quality* that is a resource for the individual to organize its life. At this point, personality no longer is the result of direct influences and relations. Personality is something that a person actually uses to organize its life – a process of further developing as a human being in and through increasing control over life conditions. (In mundane discourse, the person who goes with the wind receives attributions such as 'weak personality', whereas someone who has a high control over life conditions may be referred to as a 'doer' or 'mover'.) Leont'ev refers to Lenin, who suggested that rebelling slaves differ from an obedient ones not in terms of their self-consciousness but in terms of their consciousness of themselves in the system of societal relations. These relations are reflected in the relation of inherently collective object/motives, which represent the different, by division of labor created moments of society. Leont'ev thereby rejects the 'Ptolemaic' interpretation of human being characteristic of empirical psychology (and constructivism) in favor of a 'Copernican' interpretation according to which the 'I' is an integral moment of an

[6] The constructivist point of view on the Self that von Glaserfeld outlines is radically different from the one we articulate here: 'The Cartesian statement [I think therefore I am] tacitly takes for granted that one knows what one is doing and, similarly, the word "think: implies that the thinking subject knows what he or she is thinking. To my mind, it is precisely this awareness of *what one is doing or experiencing* that is the foundation of what we ordinarily call our "self"' (von Glasersfeld 1989b: 445).

irreducible system of mutual relations among the human beings of a society (culture). That is, he views the 'I' as a system of knots and their hierarchies, which are the results of cultural historical processes and which exhibit themselves only in such processes as constitutive moments. It is in this way that 'the being of humans is their real life process' (Marx/Engels 1958: 26). Our being, who we are, is and is the result of our real life processes.

The hierarchy of object/motives and the related activities that they stimulate and in which they are realized constitutes a network or, to use Leont'ev's word, a 'knot-work'. The emergence and evolution of the individual '"knots" is a latent process, which expresses itself differently at different stages of development' (Leontjew 1982: 198). Research has provided evidence for a variety of rules that determine the development of these knots. For example, research among pre-school children shows that in the presence of multiple-oriented motives, actions first are subordinated to other human beings and subordinated to the objective relation among things only later. This is what we see in the episode with Mario, where it is not the objective relations between things – goblets, chips, figures, or tables of values – that give rise to the abstracted relations but rather the societal relation. Leont'ev also suggests that if there is a doubly motivated activity, then another motive is more easily subordinated earlier when it is presented to the child in ideal form whereas it is subordinated later when is present in the perceptual field.[7]

The approach that we sketch here is very different than those that lead mathematics educators to speak of 'mathematics identity' that students 'construct', often based only on the work the researchers have done in mathematics classrooms. From the cultural-historical activity theoretic perspective outlined here, there are two objections to such an approach. First, we see that the subject develops in a *societal relation* and only exists as such. It is not a 'construction' that makes subject or personality, each of which is relational. As we note above, it is not that we are only subjects *of* activity, we are also, always already *subject to* and *subjected to* activity, which means there are decidedly societal dimensions that individual-centered approaches do not account for. Second, identity – or rather, personality – is not something that we can determine based on the events in a mathematics classroom and on the results thereof. Rather, personality is a system of relations among object/motives and, therefore, among activities. We cannot therefore speak of 'mathematics identity' as something that exists as such. Rather, within the cultural-historical activity theoretic approach we may speak of the object/motives that orient mathematical activities as having low priority in the hierarchical organization or the object/motives may have no place at all. In other words, there is no essence such as 'mathematical identity', but there are hierarchical relations among objectified motives of activities in which a person does or does not participate. These relations continuously shift with experiences and new forms of discourse that become available to the person.

[7] Leont'ev does not discuss this situation further, but it is well known that children at a certain age are subject to the field-dependence of perceptions so that the perceptual relations in evidence actually mislead rather than support their development.

8

Toward A Cultural-Historical Science of Mathematical Learning

Throughout this book, we offer cultural-historical activity theoretic descriptions and analyses of learning in mathematics. Cultural-historical activity theory, as conceived in the Vygotsky-Leont'ev-Holzkamp lineage of work, is incommensurable and irreconcilable with constructivist approaches to mathematical learning. This is so because of the way in which this approach conceives of the relation between the individual and collective, how it conceives of *con*sciousness, and the inherently collective form of all knowledge, all practices, all forms of thought, and so on. In this chapter, we further articulate this approach and how to use it to think about the key phenomena of interest to mathematics educators.

Consciousness is Collective

We note throughout this book that the students and their teacher engage in sympractical activity. It is this activity that mediates the forms of consciousness that accompanies what they do. But consciousness constitutes, according to the etymology of the word, 'knowing' (Lat. *sciēre*) 'together' (Lat. *con*-). Now practical consciousness is for others as it is for myself (Vygotsky 1986). In this, activity theory offers a radical alternative to social-psychological approaches (e.g., Cobb 1999), which propagate the opposition of individual and culture, conceiving their relation as an interaction of two entities that could be theorized independently. To anticipate our more explicit articulation: Individual forms of consciousness cannot be understood separately from collective consciousness so that practices inherently are shared rather than 'taken-as-shared' by individual minds that 'construct' knowledge, others, and themselves (Leontiev 2005). Individual subjectivity and consciousness *always and only* exists as concrete realizations of collective subjectivity and collective consciousness. What Mario and Jeanne offer to each other is

inherently intelligible, shared forms of expression/thought, not individual expression/thought of which we have to 'construct' 'meaning'.[1] This is precisely the reason for Bakhtin (1993) to choose the expression *participative (non-indifferent) understanding*. Thus, '[t]he psychological characteristics of individual consciousness can be understood only in terms of their connections with those societal relations in which the individual is a part' (Leontjew 1982: 129). In the currently dominant forms of educational theorizing, the individual constitutes an autonomous entity in external relations with others. Autonomy is understood as an individual's characteristic way of participating in a community, which changes along a trajectory characterized by its beginning point of 'peripheral participation' and an endpoint of 'more substantial participation'. Along this trajectory, students learn to rely on their own judgments rather than those of the teacher. Thus, '[s]tudents develop specifically mathematical beliefs and values that enable them to act as increasingly autonomous members of classroom mathematical communities as they participate in the negotiation of sociomathematical norms' (Cobb 1999: 8). Here, the collective acts like a box and the individual moves along a trajectory from its outer edges to the center. Characteristic of this perspective is the notion of practices that are 'taken-as-shared' rather than enacted only in the co-participation – i.e., societal relation – with others, which are the true origins of the higher psychological functions. In the episode we present in this book, however, reality *is* shared and comes to be reflected/refracted in the consciousness of the participants, who concretize, in using words, 'direct expression[s] of the historical nature of human consciousness' (Vygotsky 1986: 256).

In cultural-historical activity theory, the integration of individual and collective forms of consciousness and subjectivity is total. Activity is understood as 'a process, in the course of which the material life activity of societal man produces not only a material but also an ideal product, or the act of the *idealization* of reality' (Il'enkov 1994: 171). This act in which reality is idealized constitutes a process of transformation, whereby the material comes to be reflected on an ideal plane. The idealizations – the forms of consciousness – that become available to Mario as a result of his engagement with this task generally and its particulars specifically (the text, table of values, goblets, chips) are concrete realizations of cultural-historical societal and epistemic relations. Subsequently, after it emergences, 'the "ideal" becomes a most important dimension of the material life activity of societal man, and this is the beginning of a process in the opposite direction – the process of materialization (objectification, materialization, "embodiment") of the ideal' (ibid.: 171). These two processes with opposite directions become part of the same characteristically human life process of societal-historical life activity. The ideal forms a category of phenomena 'that fixes the form of the reflection process of objective reality in the, according to its origin and nature societal-human psyche, societal-human consciousness' (ibid.: 188).

[1] The concept of meaning is inconsistent with the pragmatic approach we take here. Thus, Wittgenstein (1958: 3) suggests that the 'philosophical concept of "meaning" has its place in a primitive idea of the way language functions'. Elsewhere, he states explicity that 'the understanding, the meaning, is dropped from our approach' (Wittgenstein 2000: Ts-213,i-r[1]).

Because Jeanne, Aurélie, Mario, Thérèse, and all the others in this class participate in concrete, material, sensuous, sympractical activity, and because inner and outer activity are two moments of the same irreducible overarching process, practices are inherently shared rather than 'taken-as-shared'. We do not have to choose between 'the thought according to us and according to others, but that at the moment of expression, the other to whom I address myself and I who express myself are incontestably bound together' (Merleau-Ponty 1960: 118). The sensuous images of the participants mediate between consciousness and the practical world, which exists outside and independent of Jeanne, Mario, Aurélie, or Thérèse. Sensuous images constitute the reality that the subject experiences; they make the immediate connections that relate inner consciousness and outside world. If they are absent, for example, in the case of amputations, blindness, or simple inversions of normal perception, a sense of irreality emerges. The objective nature of sensuous images derives from their emergence in concrete activity that transforms the material and social world. That is, these images are not only the (possibly erroneous) result of sensory action, but they are the result of transformative actions – the results of which are also available to the acting subject. The subject not only experiences the changes and changed world; it also experiences itself, via auto-affective processes, as the agent of the transformative activity. Both forms of experiences shape the forms of consciousness that emerge from human activity.

Cultural-historical activity theory, therefore, takes a very different stance toward the relation of the individual (e.g., Mario) and the collective (e.g., Jeanne and the other students), the forms of thought and consciousness at the two levels, individual and collective subjectivity, and so on. In this theory, Mario, as any other human being, is always already caught up in societal relations. Mario becomes conscious of the actions of others, but his individual consciousness – because of the common language and other cultural semiotic systems – is a concrete and unique realization of collective consciousness. Mario, Aurélie, or Thérèse cannot but think in ways that are collective possibilities – even in the most creative acts, as Vygotsky (1971) shows in his studies on Shakespeare. What they can express and therefore what they can think is inherently shared rather than 'taken-as-shared' because it is a concrete realization of cultural possibilities. But this does not preclude that any two individuals realize different cultural possibilities all the while assuming that they realize the same.

Understanding and Analyzing Activity

In individual-centered educational theorizing the individual becomes reified, a somehow mysterious entity (subject) that constructs its own mind and identity. The individual exists in a self-relation, constructs itself, and, as von Glaserfeld (1989a) explains, only checks its constructions with the natural and social worlds (Fig. 8.1a). The difference between individual and socio-constructivism lies in the fact that in the latter, what the individual learns also is a function of the interaction with

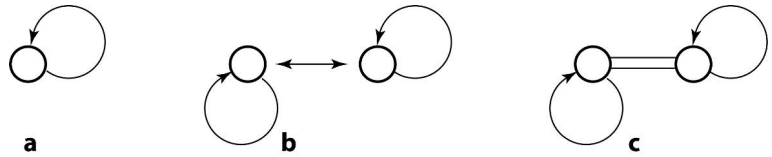


Fig. 8.1. Schematic representation of constructivism (a), social constructivism (b), and cultural-historical activity theoretic perspective of the individual (c).

others, whom the individual confronts in negotiation and exchange process (Fig. 8.1b). Yet, the individual is still the constructivist individual in its self-relation. Both others and the social 'context' constitute opportunities for the emergence of subjective adaptive behavior. In socio-constructivism interaction is seen as a confrontation with other ideas, as negotiation – the exchange of something that the individuals already hold, and which is the result of the self-relation and self-regulation. In cultural-historical activity theory, we cannot even think of the individual independent of its relations and of its nature as human, that is, its relation to the collective of humanity. The relation with the other is irreducible (Fig. 8.1c). The self-relation of the individual is one that is always ever a reflection of past social relations on an ideal plane. In fact, the subjectification process described in chapter 7 arises precisely in this manner from the re-inscription of the relation to the Self on the part of the subject (Nissen 2003).

Whereas Vygotsky orients us to the individual in relation to other individuals, for example, the child with a parent or the student with a teacher, Leont'ev and Holzkamp decidedly orient us to the fact that relations inherently are situated in and produce *societal* structures and processes. The minimal unit that makes sense is the *activity*, not thought at the level of the task and being busy, but at the level of a process and structure that is the outcome of a division of labor and that fulfills a societal-collective need. Thus, when we present in the preceding chapters the Mario-Jeanne relation, the latter inherently reproduces a form that is characteristic of school institutions, themselves a collectively designated place for the production, reproduction, and transformation of society and its culture. At the same time, society and the school are not just boxes into which Mario and Jeanne step. In and through their actions, in the context of the material location (school and its resources), they *actively produce* schooling generally and school mathematics more specifically. Their actions make *this* school as we observe it.[2] Their actions *make sense in and as of actions within* this *activity of schooling*, and the same actions would have a very different sense in other activities. We symbolize this situation in terms of nested relations and part-whole relationships (Fig. 8.2).

As Fig. 8.2 shows, the Mario-Jeanne relation that we describe and analyze takes place in a concrete and real classroom. But *this* classroom is not merely a box –

[2] We insist: Cultural-historical activity theory is not about, here, schooling in the abstract but always about *this* concrete school, *this* theorized classroom, *this* observed action, or *this* real student. The theory is used to understand the real world, and is a reflection thereof, not a tool for thinking about configurations that only exist as ideal, hypothetical cases.

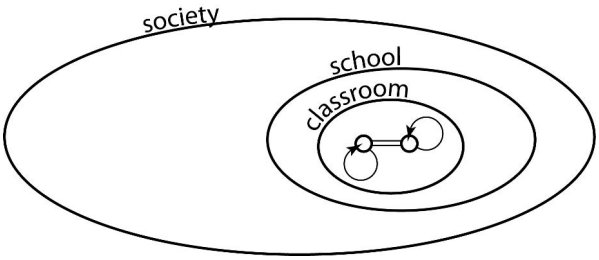


Fig. 8.2. Schematic representation of the constitutive whole-part and therefore nested relations that exhibit how society is produced in seemingly innocuous classroom events such as student-teacher interactions.

apart from the fact that the physical space is an objectification and embodiment of societal realities – but rather, the *actual* classroom events and what Mario and Jeanne together do bear a *constitutive* relation. Without Mario and Jeanne, *this* is a different classroom consisting of different micro- and macro-processes to be studied and investigated. But the whole-part relation also works in the other direction. The Mario-Jeanne interaction takes its form from the particular organization of this classroom, as a physical and social space. The participants in the relation orient to the higher-level event and, in their actions, produce, reproduce, and transform it. When we look at the following fragment, we cannot therefore understand what is happening there independent of its relation to classroom, school, and society at large. The two participants are oriented not only to the mathematical content, not only to their institutional relation of an older person (teacher) speaking to a younger person (student). Their actions make and take their sense from the fact that they occur within the activity of *schooling*. That is, each fragment we mobilize in this book is a fragment that reproduces (and transforms) *schooling*. Although their being in the school does not *determine* the sequence in the sense that it is the only possible turn sequence, it is nevertheless framed by the institutional context. If the first turn, the 'question', were uttered in a different kind of situation, in a mall, the second turn, even if the speaker were a young person, might be a swear word.

Fragment 8.1 (Excerpted from Fragment 4.3a)

```
210  J:  orIGinally you started with sIX. (0.39) so; instead of
         writing three plus three plus three, what would you be
         able to do. (0.27) three tIMEs?
211      (0.84)
212  M:  s::IX:?
213      (0.88) ((J moves finger sideways repeatedly between
         two cells?))
214      no three times three
215  J:  yES::. (('excited' 'yes' [prosody in Fig. 4.4], makes
         the same rH movement to right, opens palm toward
         ceiling, Fig. 4.5)) (1.21)
         its just on the bottom its a [shortcut]
```

Even in schools, we might find instances where the second turn would be different, as can frequently be observed in 'difficult' 'inner-city' schools. In such situations, it is not infrequent to find students to respond in 'irreverential' ways, that is, in ways that are not characteristic of the middle class ethos that normally is implemented and reigns in schools. In such instances, students – who frequently are from under-class and working class backgrounds – tend to be suspended or expelled from a school.[3]

The classroom is only a particular instance of *this* school; and there are many other classrooms and many other classes – science, reading – involving the same bodies. But in this instance, too, there is a constitutive relation between levels. The events in and of this classroom produce, reproduce, and transform schooling as enacted in the building where Mario, Jeanne, Thérèse, and Aurélie find themselves at the instances that we report here. The school as such exists only in and through their actions and engagements. At the same time, the classroom events are determined by what happens at the level of the school. Thus, for example, *this* school was open to research and to innovative approaches to teaching mathematics. There are other schools where this is not the case so that we would not have observed the kinds of episodes that are at the heart of this book.

Finally, the school itself is only a microcosm of society, which it both reflects like a raindrop reflects the surrounding world, and constitutes. Without *this* school, the society is different, but *this* society exists only in and through the totality of its concrete constituents, including *this* school. All of the levels are interconnected (Valero 2009) and what we observe *in and as* relation is constitutive not only of *this* classroom but of society as a whole. The relations are not just depoliticized *social* relations but *societal* relations, producing, reproducing, and transforming society as we 'find' it displayed in front of ourselves. Because of the constitutive part-whole relations at all levels that we identified so far in this section, each relation within the classroom actually constitutes an instant of society at large – as our representation shows, the relation we study in this book also is a constitutive part of society as a whole. That we – members of 'immortal, ordinary society' (Garfinkel 2002: 92) – are generally not aware of this fact is because what is salient to us in everyday life are our immediate surroundings. For Mario, Jeanne, Thérèse, and Aurélie these immediate surroundings are *this* classroom and *this* lesson. But what happens in any *this* location and between any *this* pair or group of people is determined by events and processes that lie outside their consciousness but nevertheless are present and determining their lives (e.g. Smith 2005). Every action, every speech act is like the raindrop that not only reflects the entire world but also is an integral and irreducible part of this world – without *this* raindrop, the world would be other. 'Through their actions in real places and under real and quite specific conditions of actions, social actors instantiate those elusive and invisible structures of social science lore' (Boden 1994: 13). That is, in their relation,

[3] The word 'fuck' is common currency on any construction site, in many working class homes, and in many other parts of society. Yet we know from experience as teachers that if a child reproduces the term at school, there are serious consequences, especially if it were to be hurled at a teacher making a request.

Mario and Jeanne talk mathematical business, and they thereby reproduce the business of mathematical talk.

In a very strong sense, therefore, we, the authors of this book, do not 'just' produce *micro*analyses that bear no relation to larger and longer-term issues. Rather, our analyses exhibit *society*-in-the-making. Therefore, rather than using the adjective *social*, as Vygotsky often but not exclusively has done, we emphasize the use of the adjective *societal*, as can be found in the writings of Leont'ev and Holzkamp. Mario and Jeanne produce, reproduce, and transform a concrete *societal* relation not merely a social relation. They reproduce and transform an instant of the collective process we call society rather than some isolated and independent event that bears no relation even to what happens at the next group of desks. Because constructivism, for example, uses the individual mind as its unit of analysis, what happens at the next desk does not matter, or matters little in terms of the things that the mind can and does 'construct'.[4] The categories of cultural-historical activity theory, on the other hand, are designed to capture the continuous flow of activity specifically and collective life in 'immortal, ordinary society' more generally. And in life, the agent is never completely the subject – i.e., master – of its actions.

There are also person-related determinants of the societal interaction that we have been analyzing here. In the same way that there are interrelations between the different 'levels' from society to the relation, so there are historical (biographical) interrelations along the dimension of time. Thus, what we see as student-teacher (Mario-Jeanne) relation is more or less influenced by other events that precede *this* instant in the classroom or that the persons involved might project. Thus, relational, financial, emotional, or health troubles arising within other activities leave their mark on and are attended to within any *this* activity. One study, for example, exhibited mathematical performance in a fish hatchery at the intersection of a person's own lay-off notice, the temporary, strike-related lay-off of the spouse, the high mortgage for the house recently built, the two children to be cared for, and so on (Roth 2007). Any observed concrete action with mathematical representations in the fish hatchery could be understood only by taking into account all these other events in the life of the particular fish culturist.

There are further determinants that are outside the consciousness of the participants, including the processes that we denote as 'mathematics' and 'mathematics education' or 'didactic of mathematics'. Each of these fields and the kinds of outcomes that result from the productive processes therein, lead to reified entities such as 'algebra' and 'spiral curriculum' or 'age-appropriate curriculum' that are imported into and thereby determine the processes of other activity systems. Thus, for example, mathematicians and mathematics educators debate and produce curricula,

[4] Marx/Engels (1962) already criticize the attempt to understand society in and through the contribution of a collection of individuals naming such attempts 'Robinsonades'; they use Robinson Crusoe as an example throughout their 'critique of political economy'. Thus, cultural-historical activity theory is interested in any particular phenomenon 'only where this phenomenon is understood not abstractly (that is, not as a recurring phenomenon) but concretely, that is, in regard to its position and role in a definite system of interacting phenomena, in a system forming a certain coherent whole' (Il'enkov 1982: 96).

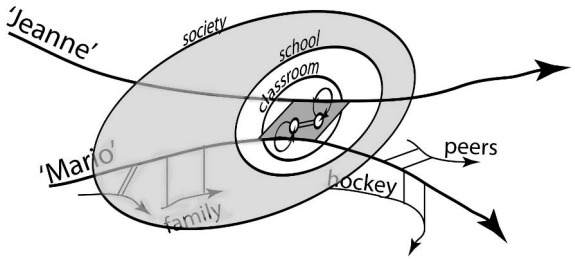


Fig. 8.3. Schematic representation of the process and structure relations in which the societal Jeanne-Mario relations are embedded and that they reproduce in and as subjects of the school mathematics activity. The two dimensions are orthogonal.

which school systems adopt and then ask their teachers to implement. Although teachers may learn some of the background to the particulars of curriculum or pedagogy, the processes by means of which these entities come about are outside the conscious awareness of teachers and students, yet determine their everyday lives. Moreover, society at large and specific parent associations engage schools in relations that move curriculum and pedagogy in particular directions, something we can see this in movements such as 'back to the basics'.

So far our account might be read as a description of things. In fact, at all levels we are confronted with *processes*. At all levels and all units we might want to reflect upon, we observe life, which inherently is a *process* – that is, there is a continuous flow and change. We already note above that Jeanne and Mario are not only producing mathematical content but are also producing the relation. Moreover, this relation is not somehow disconnected from everything else, but, as we point out, it is a microcosm of relations that constitute 'immortal' society. These produce an integral and constitutive aspect of society. The development of this relation, therefore, also changes and is integral and constitutive of the historical development of *this* classroom, school, and society (Fig. 8.3). This is so because any concrete school and school system only exists when someone produces it in a concrete way. There are no societies, schools, classrooms, and person-person relations *outside* and independent of the concrete production of life. There is no life in the abstract – other than perhaps in idealist philosophy. This, therefore, implies that we cannot understand the relation as some stable entity. Rather, it requires us to understand it in the way Vygotsky thinks of the relation of thought and speech: *as processes, the relation of which is itself a process*. It is precisely in this way that one of us studied mathematics in a fish hatchery, where the everyday praxis of individuals was understood as constitutively nested within the hatchery, itself constitutively nested within a federal hatchery program, itself nested within society (Roth et al. 2008). The processes within *this* hatchery not only produce themselves but also and simultaneously produce the cultural history of the hatchery. The cultural history of the salmon enhancement program is produced by the same historically changing process that also constitutes each hatchery. Apart from these his-

torical processes, we cannot understand what happens in and as hatchery mathematics, because the latter only exists in and as of the historically situated and situating processes that realize the observed salmon hatching praxis every day. Moreover, as our analyses show, the salmon enhancement program generally and the events in the individual hatcheries specifically could not be understood outside the relations that this activity has with other activities, for example, the production of food (fish factories, fisheries), tourism (sports fishery), first nations issues (fishing for sustenance), national economic conditions, and so on.

The Person's Perspective

> Let me repeat: to live from within oneself does not mean to live for oneself, but means to be an answerable participant from within oneself, to affirm one's compellent, actual non-alibi in Being. (Bakhtin 1993: 49)

In the first generation of cultural-historical activity theoretic perspective that Vygotsky developed, there was a strong emphasis on the individual *person*. This does not mean, as Bakhtin points out in the preceding quotation, that the individual lives *for itself* from *within itself*. Thinking like this would be a 'Robinsonade' (Marx/Engels 1962). Thus, all the major Russian psychologists now read in the West – including Vygotsky, Luria, and Leont'ev – emphasize the Marxian diction that it is 'man' who directs 'his' brain and not the brain that determines 'man' (e.g., 'man controls the activity of his brain from without through stimuli' [Vygotsky 1989: 59]). The original stimuli come from living in society. Here, any person will decide to do *this* or *that* depending on what is apparent to him/her in conscious awareness. But these actions are not arbitrary. Rather, the subject of activity always has rational grounds for his/her actions, though these are inherently concrete realizations of *collective* possibilities. No person in a normal situation will do what is irrational to her. Any concrete 'I' always acts according to what appears reasonable. To understand the precise nature of an action, therefore, we need to understand it through the eyes of the person. But no person acts willy-nilly, for any 'reason' always already is a collective possibility. A person therefore is not totally free to do what s/he wants to do, as an idealist might construe the situation in the abstract. Precisely because higher psychological functions and personality are the result and ideal concretization of societal relations, any relation (action in interaction) always already is societal (general) rather than singular.

When we follow and attempt to understand concrete relations and their evolution – such as the one involving Mario and Jeanne – we need to articulate the collective logic apparent in the situation rather than attempt to reduce actions to the contributions individuals might be said to make in a monadic manner. Any situation we might investigate has a logic that exceeds the sum of the individuals – we know that from being on committees that make decisions that could not have been foreseen based on the initial thoughts of all members. To capture this dynamic, this

collective logic, our analytic approach needs to make salient the dialectic tension between what is apparent to individuals, as they articulate and display it for everyone else in the situation, and the irreducibility of a relation – a societal fact sui generis – to its members. Thus, to understand the dynamic of the situation the analyst must not 'interpret' what an individual 'means to say', 'thinks', or 'feels' – in this way we would begin to reduce the analysis to monadic subjects – but we have to focus on what is available collectively within the public sphere of their relation. We return to a fragment already featured in previous analyses, the one to which we can backtrack the first appearance of the additive and multiplicative structures.

The fragment begins with an utterance that has two markers that allow it to be heard as a question: an interrogative grammatical structure and an intonation (pitch) that moves up toward the end. Although the production of the utterance can be reduced to the body identified as 'Jeanne', the nature of the utterance cannot be reduced to the person. This is so because the utterance is *for* another and thereby inherently takes into account this other. From the perspective of the relation, the utterance is a question only if there is a response. The perspective of the relation expresses itself in *pairs* of utterances. From the perspective of the relation, the person addressed now has a turn. But there is a pause that develops (turn 192). Jeanne does not speak, leaving Mario a space; and Mario does not speak but produces a facial expression that may be read as a question itself.

Fragment 8.2 (excerpted from Fragment 4.2b)

```
191  J:  third week; how mANY threes are you going to add in
         your:: (1.03) piggybank?
192      (0.96) ((questioning look on Mario's face))
→ 193    how mANY three dOLLars are you going to have.
194      (2.08)
→ 195  M: how much money are you going to have?
196  J:  how many thrEE dollars are you going to have?
197      (1.47)
```

Jeanne then utters what we may hear as a rephrasing of the original question, though the intonation is in a constative rather than a questioning form. That is, we see an incomplete question–answer sequence. Jeanne's rephrasing can then be understood as an attempted repair to a possibly failed understanding of the original question; although the original utterance (turn 191) was 'recipient designed' for Mario, the collective question–response has not been realized. The individual persons act within the constraints of their nature as members of a collective situation. We then have a long pause (turn 194)[5], which, from Jeanne's perspective, is a way of providing time for the response and from Mario's perspective is time to gather before speaking. Each person has the possibility to take a turn, though from the relational perspective, the expected speaker is Mario. The response comes in the form of a question (rising intonation contour), which does not articulate a response but offers of a hearing of the preceding question. In this, Mario responds and offers

[5] A review of the educational research literature in the 1980s shows that teachers tend to leave something of the order of 0.7 seconds for students to respond and, when there is no answer, continue with the lesson (e.g., Tobin 1987).

a hearing, but the response is not 'free' and singular because for at least two reasons. First, it is a response to a question, and therefore a turn in a logic that exceeds the individual; second, the articulation itself is *for* Jeanne and the generalized other.

When we analyze relations in this manner, we sustain their double origin in the dialectical tension between individual and collective. On the one hand, there are concrete perspectives and understandings that the individuals articulate for one another in and through their concrete (discursive, gestural, physical) actions. On the other hand, there are the constraints from the collective levels that 'regulate' interactional forms; and those perspectives and understandings themselves are inherently intelligible because thought *is* collective. They are at school and, in their actions, realize schooling. If Jeanne and Mario were to engage each other during a hockey game they both came to watch, or as members of some orchestra, then the form of relation that they would reproduce in and through their action would be of a different nature. It might be that in this other context, Mario would be the expert and Jeanne the less knowledgeable person – as is often the case when students exceed their teachers with respect to competencies in computing technology or social networking software. This different orientation that the two would bring to their relation in this other activity – realizing a different object/motive – would not normally be apparent from the words and turns. We therefore need to analyze the sequential turn-taking routine within the institutional setting that the interaction participants reproduce in and through their relation. There are consequences when a particular relation falls 'outside' 'the norms', something which we might have been able to witness if, in response to Jeanne's utterance 'sit properly' [turn 45], Aurélie had uttered a four letter word or shown a middle finger.

Some readers might ask themselves: Why is it important to look at the relation in such detail? It is so because mathematics continues to live precisely because it is in such relations that it renews itself. Mathematics particularly and any science more generally is 'handed down' and kept alive in and through the subjective doing of mathematics using cultural-historically marked artifacts that 'embody' the first constitution of any mathematical idea (Husserl 1939). This also contributes to the very objectivity of mathematics as we observe it. The individual doing mathematics and its objectivity as cultural-historical fact are two sides of the same coin.

Taking the perspective of the person, therefore, does not mean reducing the subject of activity to the subject *qua* subject, whose actions arise from the figments of its mind that it has constructed for itself. In our approach, it is not only the actions and concrete realizations of collective possibilities that are inherently intelligible; the collective relations themselves are irreducible to the Kantian or Piagetian subject. With our approach, therefore, we place both relation and person at the crossroad of the individual and the collective, the private and the public. In this way, 'learning experiences under typical societal conditions are not so private or rather, it is precisely in the private (subjective) nature of the experiences that the degree of what is societally typical expresses itself' (Holzkamp 1993: 493).

It is important to understand the perspective of the subject, because only in this manner may the nature of the object/motive that the student realizes become ap-

parent. This aspect cannot be understood by simply investigating the event featured here but would require knowledge of the system of activities and therefore of the object/motives in which an individual participates. Thus, in the same way that we must not just look inside a single lesson or course to understand mathematical learning therein, we must not attempt to understand mathematical learning independent of the life trajectory of the person. Only in this way do we get access to the 'fullness of life . . . the personal needs and interests, the inclinations and impulses, of the thinker' (Vygotsky 1986: 10). Intentional learning actions – i.e., actions that are intended to increase each person's *my* control over the conditions, *my* action possibilities, and *my* room to maneuver – are rationally grounded in *my* own life interests as these are perceived by the individual 'me'. Outside of these relations, 'thought must be viewed either as a meaningless epiphenomenon incapable of changing anything in the life or conduct of the person or else as some kind of primeval force exerting an influence on personal life in an inexplicable, mysterious form' (ibid.: 10).

As soon as we bring the personal life of the individual into play, we also have to face up to the relation between the interests and needs of the individual and the general interests and needs of the collective (reproduction of knowledge). Aurélie's personal needs at the instant analyzed here may be confronted with the collective wisdom embodied in the curriculum guideline of this province that the 'study of mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and regarding participation in . . . society' (Ontario Ministry of Education 2005: 3).[6] In the institutionalization of schooling, the developmental and epistemic needs of the individual are given primacy, for example, in the ways curricula are organized conceptually and temporally. These ways constitute some average developmental level and some average rate of development, which may not meet the needs and inclinations of any concrete person at any specific moment in time.

We are not arguing, however, that children or older students should or could completely decide what to learn in mathematics or whether to learn mathematics at all. The individual never is totally free, because, as a concrete instance of the human species, it realizes what is human (i.e., general) in human existence. This is so because the

> man described . . . is already in himself the effect of a subjection much more profound than himself. A 'soul [mind]' inhabits him and brings him to existence, which is itself a piece in the mastery that power exercises over the body. The soul [mind] is the effect and instrument of a political anatomy; the soul [mind] is the prison of the body. (Foucault 1975: 38)

In his reference to the soul (mind), Foucault articulates the idea of the 'immortal soul' that constitutes 'immortal society' as Émile Durkheim (e.g., 1897) originally

[6] This 'collective wisdom' may actually not be born out in reality, for research on mathematical cognition in everyday life shows that even without having had formal instruction in mathematics or without much success at mathematics during their school years, people do quite well in life.

develops it and that Harold Garfinkel (2002) uses to think of the collectivity that we produce in and with each action.

Becoming a member of and exercising in the discipline of (school) mathematics also means subjecting oneself to the discipline, or developing a self-discipline. Discipline therefore means subjectification, a process of becoming the subject of (control over, liberation) and becoming subject and subjected to activity (constrained by). Schooling generally and school mathematics specifically is part of the mechanism of subjectification. It is not some outside power, but mind itself that simultaneously exists in/as social relations – i.e., as mind in society – and, in reflected form, in (individual, collective) consciousness – i.e., as society in the mind. Cultural-historical activity theory is precisely the approach that allows us to understand the events at the intersection of the constitutive relations between the historical movement of society, on the one hand, and the biographical development of its members, on the other hand.

Dialectic of Boundaries and Continuities

Some mathematics educators concerned with the different forms in which mathematics expresses itself at home, at work (school), or during leisure time have been focusing on the differences between activities (e.g., Kent et al. 2007). Different activities are said to be separated by boundaries, and the individuals participating in two or more activities then have to 'cross' these boundaries (Tuomi-Gröhn and Engeström 2003). The boundary-crossing concept is a way of rethinking the question of 'transfer' of 'knowledge' and 'skills' between situations. The way of theorizing the relation between personality and a structural approach to society (Fig. 9.3) allows us to rethink the question of boundaries. The question is actually not unlike the one that linguists were confronted with in the work of de Saussure (1995) and his irreducible opposition of synchronic and diachronic perspectives with the concepts of language (Fr. *langue*) and speech (Fr. *parole*). When linguists prior to Bakhtine [Volochinov] (1977) attempted to understand how the structured system of language was changing over time, they struggled with the integration of two very different moments of language: its *living* and its structured manifestations. The structural perspective does not give access to the continual change of language, which is nevertheless an observable fact. The analogy is appropriate, for de Saussure's fundamental thesis is this: 'language [*langue*] is opposed to speech [*parole*] as the social to the individual' (ibid.: 92). Thus, '*everything that is diachronic in a language is so because of speech*. It is in speech that we find the germ of all changes' (Saussure 1995: 138).

When we take the synchronic, structural perspective, then, society constitutes a network of activity systems, each with its own internal structure. Because the subject and the cultural practices are a function of the object/motive-oriented activity, there are in fact observable differences between what a person does in terms of

mathematics while at home or while shopping and what the same person does while attempting to solve school-like problems (Lave 1988).

When we take the perspective of the person, s/he moves many times a day through a variety of different activities without the sense that s/he is a different person. Rather, as outlined above, personality constitutes a hierarchical knotwork of object/motives. As the person moves from one activity to another, it takes different subject positions and the place of the object/motives within the overall knotwork is different, and therewith the importance and salience of a particular one may be low in the overall life agenda of the person. Thus, if for the sake of the present discussion, school mathematics were for Aurélie an activity of very low to inexistent priority, that is, if its object/motive were to be very low or inexistent in the developing knotwork that makes the personality *Aurélie*, then this would not lead to boundaries within the person. She would still move from the family to school – and here from the arts lesson to the mathematics lesson to the reading lesson and so on – and, after school, she would be with her friends, then in sports club, and end finally within the family circle later in the evening. From this perspective, therefore, we observe continuity that a person experiences and can talk about. A typical account a young person might give could then be: 'Today I went to school and then I went to the mall with my friends and then to swim training'.

The two perspectives, as we suggest above, are not independent. We need to find a way to think of *both* perspectives *at once*, for, activity as language, advances at the same time as the current of life and is inseparable from it. This is so because what the person – Aurélie, Mario, or Jeanne – does in relation with others *actually reproduces and transforms* society and its particular, activity-specific structures. We articulate this in Roth (2011b) and ground ourselves in ethnomethodology, which focuses not on the societal structures that mainstream sociology takes as given facts but shows interest in the everyday actions of persons-in-activity from which the very structures emerge. Thus, we find a person acting all day long, including what we observe in the mathematics lesson that features throughout this book. But the goals we observe would be different as a function of the activities were we to follow Mario, Aurélie, or Jeanne in and through the course of their day. Yet the actions that they ascribe to themselves – 'I figured out how you can calculate how much money there will be in a piggybank in 101 weeks if I save $3 each week' or 'I shot a couple of goals during hockey practice' – contribute to producing the very structures that theorists use as the reasons for creating concepts such as boundary and boundary crossing.

There is therefore a dialectic process at work, where from the continuity of actions from the perspective of the *person* springs forth the discontinuity of activity, structures, and the boundaries that are said to separate them. In linguistics, the following has been proposed to overcome the opposition and logical contradiction between two very different, (synchronic, diachronic) perspectives: every utterance, that is, every (inherently moving and therefore changing) speech act is social (Bakhtine [Volochinov] 1977). Thus, the real evolution of language [*langue*] occurs in this manner:

> the social relations evolve (as a function of the infrastructures), then the communication and verbal interaction evolve in the framework of the social relations, the forms of speech acts evolve from the fact of the verbal interaction, and the process of evolution is, in the end, reflected in the changing forms of language. (ibid.: 137–138)

The changing forms of language and relations that the individual person finds as s/he moves in the course of a day from the family circle to the school bus to school classes to afternoon activities with friends to the sports arena to finally end in the family circle are reflected in the different societal structures reproduced and transformed with every (speech) act the person produces. The change over occurs as the moving person orients toward changing object/motives that are differently located in its personal hierarchy. Cultural-historical activity theorists, grounded as they are in dialectical materialism, are not unfamiliar with such changes. These are conceptualized as the transformation of qualitative into quantitative changes and vice versa. Thus, a (quantitative) change in the relative importance or salience of object/motives marks a (qualitative) change in the practices.

Coda

A cultural-historical science of mathematical learning does not reduce what we observe in a mathematics classroom to the individual student or to some interaction between the group of individuals assembled. By articulating the diachronic and synchronic relations in which any *really observed* mathematical action is suspended, we do not reduce actions to the individual agent nor to a collection of individual agents or some abstract culture. Rather, the approach allows us to produce descriptions and theory that reflect the continual flux of life – a small segment of which we observe while following Mario, Thérèse, Aurélie, and Jeanne. What we can see in the video and how we see (understand) the events not only is situated culturally and historically but also constitutes a segment of the real flux of real life as we experience it in flesh and blood. This, ultimately, is the real object/motive of a cultural-historical science of mathematical teaching and learning: describing and understanding (theorizing) real mathematical activity as it unfolds *in concretum*, rather than how it might be in theory, ideally. A cultural-historical science considers how the processes of mathematical learning and teaching unfold in the way they are actually experienced and observed: ‘in praxis’ rather than ‘in theory’. Cultural-historical activity theory, therefore, intends to be a ‘concrete human [social] psychology’ (Vygotsky 1989) that is always appropriate to its subject and therefore never allows the distinction of something that ‘works only in theory’ but ‘does not work in real praxis (life)’.

Appendix

In the following we provide a brief description of the institutional context and the complete English and French transcripts of the session that is used as an example throughout this book.

Institutional Context

This study was designed to investigate the relationship of cognition and emotion in the course of sympractical activity, concretely realized in algebra lessons of a fourth-grade class. Our theoretical framework focuses on how cultural-objective reality appears in the consciousness of the subjects, as these make available to each other what matters to them in the situation. Thus, rather than attempting to impute intentions or attempting to get into the heads of individuals, we follow participants through the activities to see how they articulate and mobilize required resources. In the course of their activity, subjects make their understandings available to one another and achieve the rationality of the lessons as a whole and individual lesson segments specifically. We, as interested and competent bystanders, can equally observe and make sense of the rationality that these social actors produce and exhibit for each other.

Participants

The participants in the study come from a fourth-grade class of 22 students (9–10-years old) attending a school in Northern Ontario. Part of a progressive French School Board, the school is open to projects and partnerships with different sectors of its community. The superintendent, the principal, and the teacher were interested to participate in a three-year study to explore the problems and challenges

surrounding the introduction of algebra in the early years. The longitudinal nature of our study has allowed us to follow this class of students as they moved from one grade to the next in the course of the study's duration.

Curriculum, Lesson, Task

Since the beginning of the study in September 2007, regular meetings have been held involving the teacher, the researchers, and the research assistants. The meetings have taken place either at the school or at the university to discuss the choice of mathematical tasks and forms of interaction in the classroom. Though experimental, the tasks were designed to meet the requirements of the provincial curriculum. Among the curricular topics, one that has gained prominence is modeling. In this book, we focus on one of the lessons revolving around the topic of modeling situations by means of algebraic concepts. More specifically, at the heart of the present book is Problem 4. Problem 4 included two main tasks about the modeling of a saving process. The students were provided with clear plastic goblets and chips to accomplish the first of the two tasks.

Collection of Data Sources, Transcription, Production of Data

Throughout the study, the regular teacher has been responsible for the instruction whereas the researchers videotaped. During the lessons, the students spend substantial periods of time working together in small groups of three. At some points, the teacher (who interacts continually with the different groups during the small group-work phase) conducts general discussions allowing the students to expose, compare, and contest their (different) solutions. To collect data we used 4 cameras, each recording the work of one small group. In addition to the videotapes, we also collect the students' written work containing the problems, tasks, tables, etc. The students' worksheets were scanned and used as support for the transcription and analysis of the videos. Once the videos had been transcribed on a word-by-word basis, we proceeded to a selection of salient episodes (Côté et al. 1993). Typically, a salient episode is comprised of several utterances (i.e. students' verbal interventions in the ongoing discourse) exhibiting features related to gestures, symbol-use, sense making, and other important moments of the learning activity.

All salient episodes are then digitized into QuickTime (.mov) format maintaining the frame rate (30 fps) for maximum accuracy of gestural and contextual features. The professional version of QuickTime permits us to section the lessons into episodes. As noted, the lessons are first transcribed on a word-by-word basis; but at this point they lack other communicative features – emphases, prosody, gestures, overlaps, description of actions, photographs of situation, and so forth. In the next step, we transcribe the selected or salient fragments in greater detail including all

those features previously omitted (a) only after we identify interesting episodes by doing a first analysis of the rough transcript along with viewing the corresponding video footage and (b) only after we are satisfied that they embody *general* features and trends observable throughout the database. We clip selected episodes to construct separate movie (.mov) and sound files (.aif); the latter are imported to a freely downloadable multiplatform software package for linguistic analysis, PRAAT (www.praat.org), which allows us to (a) identify pauses and overlapping speech with great precision (95% of repeated measurements of the same word/pause fall within 0.02 seconds) (b) extract information about pitch, pitch contour, and speech volume.

Transcription Conventions

The following transcription conventions have been used. They are essentially those that are common in the tradition of conversation analysis.

Notation	Description	Example
(0.14)	Time without talk, in seconds	more ideas. (1.03) just
(.)	Pause of less than 0.10 seconds	kay. (.) bert
((turns))	Verbs and descriptions in double parentheses are transcriber's comments	((nods to Colby))
::	Colons indicate lengthening of phoneme, about 1/01 of a second per colon	si::ze
[ ]	Square brackets in consecutive lines indicate overlap	S: s[ize] T: [colby]
<<p> >	Piano, words are uttered with lower than normal speech volume	<<p>um>
<<pp> >	Pianissimo, words are uttered with very low, almost inaudible volume	<<pp>this>
ONE bert	Capital letters indicate louder than normal talk indicated in small letters.	no? okay, next ONE bert.
(?)	Words followed by question mark in parentheses indicate possible hearings of words	(serial?)
–,?;.	Punctuation is used to mark movement of pitch toward end of utterance, flat, slightly and strongly upward, and slightly and strongly downward, respectively	T: so can we tell a shape by its color? T: does it 'belong to another 'group (0.67) O: r.
=	Phonemes of different words are not clearly separated	loo::ks=similar
'^	Diacritics indicate movement of pitch within the word that follows—up and up-down, respectively	'sai:d

Lesson Transcript – French

```
001  M:  <<all>cara what are you DOing. ((Thérèse writes on the
         oriented toward her; English in the original))
002      (15.11) ((Mario orients to his sheet, Finger moves up
         and down between rows, pounds on table, throws herself
         back))
003  T:  okay:::::::
004      (1.40)
005  M:  <<f>oh oo> maintenANt j=comprENds. tas faites mAL.
         (1.49) .hh la première semAINe, (0.78) elle na
         combien; (0.21) ((He points to the goblet of Week 1))
         ^n:EUF. (0.89) on écrit n:EU:F. (1.19) le deuxième
         semaine, (0.43) elle na combien, on lécrit. (0.24)
                 troi[sième sme (0.35) combien  ] ((A still
         leans back))
006  A:  <<plaintive>[on a même pas fini le prem]ier>
007  T:  no no no ((She laughs))
008      (0.74)
009  A:  <<plaintive>[on a même pas fini] le premier,
         [puis comme ça fait pas de sens  ]>
010  M:  [regarde tcara. (0.58) regarde la] première s: (0.44)
         la première semaine, (.) elle na neuf. ((points to
         Week 1, Fig. 2.3)) (1.10) deuxième semAINe, elle na::
         (2.00) O:nce (0.63) wait non. (1.09) ((il pointe vers
         semaine 2)) dOU:ce. (0.74) troisième semaine elle na
         (2.18) quI:Nze. (0.75) ((physically establishes
         relation between goblets and cell in table of values
         [Fig. 1]))
          (.)              [on écrit (0.32) ça.  ]
011  A:  <<plaintive>quest=[ce que tu fAIs tcara.]> ((hits
         table, rests head on table, Fig. 2.4)) ((3:01))
012      (2.69)
013  A:  un chums.
014  M:  <<p>quest=ce que tu> fAIs.
015  T:  <<p>aw chuggy just [copy me.>    ]
016  M:                 <<p>[okAY so first] [of all.>] ((turns
         to Thérèse))
017  A:                         <<lamenting>[on a auc]UNe idée
         quest=ce que tu fAIs sO> ((very High pitch, 570 Hz
         max, 3:09, both A & M oriented toward T))
018      (1.33)
019  T:  dan dan dan dan ((she moves the chips away from her
         page and toward ))
020      (4.14)
021  T:  <<confident>(qwi::::?) (gret?)>
022      (1.73)
023  T:  <<f>aLI::;> camera qui t=regARde. ((3:21, Thérèse
         fills up the table for Aurélie))
024      (6.45)
025  M:  <<p>tcara, tes sur camera;>
026      (1.19)
027  T:  <<len>jsais, jécris rien.>
028      (3.41)
```

029 A: <<plaintive>j=comprends pAs; puis je vais jamAIs comprendre.> ((Stares at her hands placed on the worksheet, Fig. 2.6)) ((3:38))
030 (0.84)
031 T: tiENt (0.30) faut que tu fasse (1.41) trois plus six (0.60) yup. (1.79) <<len, p>técris whatever> ((Thérèse turns around and speaks to members of Group 4 about other things))
032 (1.29)
033 A: ouAIs?
034 (49.52) ((Ali writes, Mario raises hand and Thérèse takes about something else))
035 T: ah mon dieu.
036 (8.70)
037 A: ((pounds on table))
038 T: <<p>kay on est tous mélan[gée::>]
039 A: [je ne cOM]prENds pAS. ((Points to her page, Fig. 2.7))
040 (2.46)
041 T: <<confidently>uh hu:::; uh huh.>
042 (25.56) ((M drops his hand)) ((Ali pounds table again, throws herself back against back of seat)) ((4:57, Mario gets back to the task, A leans back))
043 J: <<f>oui.> (0.52) quELle est la ques[tion.]
044 M: [cest] ^çA::: (0.38) <f>um[::>] ((hands move downward, restrains not to pound on table, gazes at sheet, Fig. 2.8))
045 T: [auré]lie assis=toi comme il faut. (55 :00)
046 M: look ça cest (.) dUMb. <<p>j=comprenne pOS.> ((487>217 Hz))
047 J: =est=ce que vous êtes en train de discuter en groupe?
048 (0.16)
049 M: non. tcara est juste parti so from:: (0.53) reGARde (0.72) comme (.) ils ont déjÀ écrit des cho:::ses. ((Frustration))
050 (1.20)
051 A: <<f>^ma[da:me.>]
052 M: [jaim] jaime pos. ((discouraged))
053 J: mais OUI parceque ils sont en train de te guiDER.
054 M: comme ^comme::Nt.
055 (0.11)
056 J: kay (.) première semaine (0.84) pourquOI (0.16) es:t=`ce ‘que (0.75) il y aurait? (0.91) pourquoi est=ce quil y auraits:::sIX (0.61) dollar dans la tirelire. ((points to the first cell in the table))
057 (1.04)
058 pour la première semaine. (.) questce que vous ave:z eu:: (0.37) à faire. ((she takes the goblet of week 1))
059 (0.75)
060 M: parce que:: (0.30) ((points to goblet 1))
ma[is y=en a neuf] <<dim> [la première semaine]

061 J: [y a tu sais] <<crsc>[cest du réellement] six dollars? ((Points to and looks closely at M's first cell))
062 M: no:n?
063 T: non. yea.
064 J: =cESt trois plus six:.
065 T: i=ve said <<whispering>[ça cest questce que jai dit]>
066 J: [pourquOI trois plus six:.]
067 (0.34)
068 T: parce que ca égale au dépôt par la première semaine elle a neuf.
069 J: ça légALe à neuf la première semaine. (0.78) pourquOI estce que le trOIS est en jaune? pensez vous? ((Index finger on number in first column))
070 (0.19)
071 M: um um um ((shrugs shoulders, shakes head 'no', questioning look, Fig. 3.1))
072 (0.20
073 T: <<all>sais p[as.]>
074 M: [par]ce quon est supposé de lécrire?
075 (0.44)
076 J: OÙ estce que ca viens le trOIx?
077 T: sais pas?
078 M: <<f>a:=u:> (0.24) u:: (0.17) u: dududu: wedding chose là?
079 (0.76)
080 J: mais=h: ((exasperation, turns head away from Mario)) (0.14) le trois dollA:Rs? cest quOI exactemENt? ((Mario, who has looked at her, grimaces in desperation, brings his hands up and covers face, Fig. 3.2 [6:11]))
081 (1.61)
082 T: cest ce::st cest [les trois] dolla::rs là que:ll::e.
083 M: [u::h:]
084 (0.48)
085 M: quelle prend [chaque] semaine.
086 T: [é::]
087 (0.38)
088 T: pargn:::e. ((Jeanne moves head to side over shoulder, gives him «a look»))
089 (0.45)
090 M: comme je compren:ds pO:s=h:. ((reacting to teacher look, points to his worksheet, Fig. 3.3, 460>229 Hz)))
091 (0.59)
092 J: <<all>okay.>
093 T: <<len>jcomprends .h:: je lai co[mme.]
094 J: [relIT] le problème. (0.33) relisONS le problème. quEStce que ça nous dit de faire ca?
094 J: [relIT] le problème. (0.33) relisONS le problème. quEStce que ça nous dit de faire ça?
095 (0.58)

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096  M:  pour son anniversaire- (0.97) quoi? ((grimace, hand
         gesture, open toward ceiling, «what's going on
         here?»))
097      (0.50)
098  J:  ˇoui (0.25) pour son anNIversAI:re
099  M:  <<len>marie-na:in (0.52) reçoit (0.15) un tirain
100      (0.15)
101  J:  une tire[lire]
102  T:          [tire]lire
103  M:  <<p, len>une tirelire?> (.) contenant six dollars.
         (0.80) elle dÉcide (0.15) ah dé:[par>][(.)pargn]
104  T:                                    [dépa][:rgn::  ]e:r
105  J:  ÉpARgner,
106  M:  épargner trois dollars (0.50) par semaine.
107      (0.35)
108  J:  donc. (0.35) elle reçoit une tirelire; combIEn dargent
         atelle dans sa tire[lire.]
109  M:                      [six  ] ((facial expression as if
         teacher had asked the 'self-evident'))
110      (0.54)
111  T:  [six dollar]
112  M:  [plus trois] égale neuf.
113      (0.22)
114  J:  donc chaque semaine, elle épargne (0.13) combien
         dargent.
115  M:  trois dollars.
116      (0.68)
117  J:  trois dollars (0.11) do:nc; ((points to cell 1))
118  M:  on [est nEUF? ] ((holds 'pick-up' hand over goblet 1))
119  J:     [trOIs plus] six. ((continues pointing))
120  A:  <<plaintive>estce quon est sup[posé faire ca?> ]
         ((Points to his page)
121  J:                                [deuxième semaine]
         ((elle prend le verre de la semaine 2))
122      (0.89)
123  T:  <<p>[oui je pense]>
124  M:      [on:=a:;     ]
125      (0.17)
126  J:  combien quELle a déjà dans la tirelire. ((continues to
         hold goblet 2))
127      (0.18)
128  M:  dOU:ce::. ((rapid confirming gesture toward goblet,
         Fig. 3.5, intonation of exasperation, as if she has
         asked the 'self-evident'))
129      (0.42)
130  J:  ben la dEUxième semaine combIEN quelle a dÉ:jÀ: ((nods
         with each emphasis))
             [dans la tirelIRE.] ((still holds gobler 2))
131  A:  <<p>[moi j[ai fini    ]
132  M:           [qu:OI h?   ] ((frustration))
133      (0.63)
134  J:  combIEN dargent atelle dÉjÀ dans sa tirelire? ((still
         holds goblet 2))
135      (0.55)
136  T:  <<p>[six ]>
```

137 M: [a s:] (0.53) nO[:N nEUF:]
138 J: [la deuxième] sem. ((holds goblet 2, on 'sem' points to it with other index finger))
139 (0.38)
140 T: <<f>O:H:.>
141 (0.22)
142 M: nEUF.
143 J: nEUF. ((nods)) (0.64) elle avait ((places goblet 2)) le sIX quELle a commencé avec, ((démonstrative lA gesture to left)) (0.19) et le trois dollar. ((rH index pointing into goblet 1)) (0.58) questce (0.23) combiE::n (0.24) estce quon ((rH index points into gobler 2)) ajoute ici.
144 (0.80)
145 T: [trois.]
146 M: [quoi] (big? [one?)]
147 J: [un] autre trOIS dollA:RS.
148 (0.18)
149 M: <<insisting>cESt <<f>dOU::ce:.>>
150 J: cest <<f>dOUce.> ((confirming, nods deeply, open rH gesture, palm upward)) (0.94) donc. (.) combien dargent yatil dans, combien dargent (0.92) ((elle lève la 3e semaine)) (2.50) combIEN dargent yatIL: (.) dANS, (0.29) déjà dans la troisième semaine. ((raises goblet, jingles it, places it back))
151 (0.79)
152 T: um um u::m.
153 (1.12)
154 M: u:[m::]
155 T: [quinze].
156 (0.30)
157 M: <<p>dOUce.>
158 J: =combIEN devraitIL déjà y avoir.
159 T: u:h:
160 M: douce
161 (0.21)
162 J: pOUrquOI. cESt composé de quoi.
163 (0.68)
164 M: quoi ben regA:Rde. ((frustrated, hands stretched out, palm up, toward worksheet))
165 (0.27)
166 J: douce dollARs comprENd lE::? ((Mario places head in hand, arm resting on table [Fig. 6])) (1.48) six dollARs quon commence avEC? (0.46) et combIEN dargent dans les deux autres semaines avANT? ((Jeanne places right palm on goblet 1 & 2, sticks left finger for '$6))
167 (2.01)
168 M: quoi? (1.56) ça sa fait- (0.80) jcompren:ds pOs (.) thOUgh. ((460>228 Hz)) ((2-handed gesture, as if holding thing above worksheet, gaze moving from sheet to her face))
169 J: <<p>tu comprends pas ça> cest ça que jessaie de taider à comprendre. (2.40) regARde bien. (3.50) estce quon regARde? (0.65) tcara?

(0.40)
A: hu? (0.22) fini.
(0.40)
J: toi tas toute répondu? ((Oriented toward Aurélie))
(1.01)
vous avez fait c d puis e?
(0.91)
A: n::o:. (1.44) faut quon fait c d puis e. ((Pounds desk top))
(0.33)
M: deux fois trois plus six ça égale à nEUF.
(1.13)
J: dEUx fois trOIs cest quoi. ((Places rH index on the first cell, will not remove it for a while, Fig. 4.1.))
M: <<insisting>s:sIX:.>
(0.18)
J: plus
(0.65)
M: s::IX::.
J: =égale à, ((moves index up down))
(1.02)
M: dOU:ce: ((fills something into his table, 439>210 Hz)) (2.96) où estce quon écrit dOUce now.
(0.43)
J: ((moves hand to right; throughout, Mario holds his head, gazes at his sheet)) tu lécrit pas. ((moves index up and down between row 2 & 3, first cell)) (0.35) tu es finis (0.45) questce que ça dit; (0.41) trois plus six. (0.49) donc. (0.29) première semaine, ((points to first goblet)) (0.51) ya déjà six dollA:Rs (0.21) puis tu ajoute trois dollars. (0.34) trois dollars plus six. (0.56) dEUxième smaine:; ((points to sedond goblet, Fig. 4.2)) (.) tu as déjà trois dollars ((points to '3')) (0.54) tu as déjà ton six dollars ((points to '6')); tu ajoutes un autre trois dollars. donc trOIs plus trOIs parce que trois plus trois (0.37) cest six. (0.60) troixième semaine; combIEn de trois estce que tu vas ajouter dans ta:: (1.03) tirelire?
(0.96) ((questioning look on M's face))
combiEN de trois dollARs est-ce que tu vas avoir.
(2.08)
M: combien de monnaie tu vas avoir?
J: combien de trOIs dollars est-ce que tu vas avoir?
(1.47)
J: trois dollARs, trois doll[ARs], trois dollARs? (0.23) ((points to the 2 '3's in week 2 and simultaneously points with left hand to the first, second, and third goblet))
M: [trois]
J: questce que tu vas écrire ici?
M: trois?
(2.59) ((Jeanne moves finger to the cell on his left))
M: <<p>plus trois? plus trois?>

```
204  J:  oUI:: ((he writes))
205      (4.38)
206      plU:S::? ((points))
207      (0.96)
208  M:  s::sIX:?
209      (0.44) ((Jeanne moves rH to right, opens palm up, as
         if confirming 'this is it'))
210  J:  originairemENt tu as commencé avec sIX. (0.39) donc;
         au lieu décrire trois plus trois plus trois, questce
         que tu pourrais faire. (0.27) trois fOIS?
211      (0.84)
212  M:  s::IX:?
213      (0.88) ((J moves finger sideways repeatedly (between
         two cells?))
214      non trois fois trois.
215  J:  oUI::. (('excited' 'yes' [prosody in Fig. 4.4], makes
         the same rH movement to right, opens palm toward
         ceiling, Fig. 4.3)) (1.21)
         cest juste en bAS cest un [raccourcI].
216  A:                             [madAMe:  ]
217      (0.42)
218  J:  ta quatrième semaine; (.) combIEN de trois dollars
         estce que tu as.
219      (1.00)
220  M:  u:m::: (1.73) quat. ((Fills table, Thérèse makes
         noises))
221      (9.48) ((writes 4 '3s')
222  J:  <<pp>kay> (0.97) au lieu de faire trois plus trois
         plus trois plus trOIS quESTce que tu vas écrIRe ici?
         ((Points to the row on the bottom of the table of
         values))
223      (0.66)
224  M:  uh:m:: (1.36) quat fois trOIS?
225  J:  ((2-handed gesture sidewards, opening palm upward:
         'you got it' [Fig. 5.1]))
226      (3.83)
227      jpense que tu comprends maintenant. uh?
228      (50.93) ((Mario slightly nods, writes, after 26
         seconds looks at Thérèse's worksheet, back at his own)
229  M:  <<confident>mOI jcomprends maintenant.>
```

Lesson Transcript – English

```
001  M:  <<all>resa> what are you DOing? ((Thérèse writes on
         the oriented toward her; English in the original))
002      (15.11) ((Mario orients to his sheet, Finger moves up
         and down between rows, pounds on table, throws herself
         back))
003  T:  okay:::::::
004      (1.40)
005  M:  <<f>oh oo> now i understand. you did it wrong! (1.49)
         .hh the first wEEK (0.78) she has how much; (0.21)
```

((He points to the goblet of Week 1)) n:IN:E. (0.89) we write n:IN:E (1.19) the second week (0.43) she has how much? we write it (0.24)
th[ird (0.35) how much] ((A still leans back))
006 A: <<plaintive> [we havent even finished the fir]st
007 T: no no no ((She laughs))
008 (0.74)
009 A: <<plaintive> [we havent even finished] the first [And like it doesnt make sense]>
010 M [look tresa, (0.58) look the] first s: (0.44) the first week, (.) she has nine. ((points to Week 1, Fig. 2.3)) (1.10) second wEEK, she has:: (2.00) elEVen (0.63) wait no. (1.09) ((he points towards week 2)) twELve. (0.74) third wEEK, she has (2.18) FIFteen (0.75) ((physically establishes relation between goblets and cell in table of values [Fig. 1]))
(.) [we write (0.32) that.]
011 A: <<plaintive> what [are you doing thresa.] ((hits table, rests head on table, Fig. 2.4)) ((3:01))
012 (2.69)
013 A: um chums.
014 M: <<p>what are you> DOing.
015 T: <<p>aw chuggy just [copy me. >] ((English in original))
016 M: <<p>[okAY so first] [of all. >] ((turns to Thérèse; English in original))
017 A: <<lamenting> [we have no] idea what youre dOIng sO> ((very high pitch, 570 Hz max, 3:09, both A & M oriented toward T))
018 (1.33)
019 T: dan dan dan dan ((she moves the chips away from her page and toward))
020 (4.14)
021 T: <<confident>(qwi::::?) (gret?)>
022 (1.73)
023 T: <<f>aLI::;> cameras wATching you. ((3:21, Thérèse fills up the table for Aurélie))
024 (6.45)
025 M <<p>tresa, your on camera; >
026 (1.19)
027 T <<len>i=know, i=m not writing anything. >
028 (3.41)
029 A: <<plaintive>i dont understAND; and I will nEVer understand.> ((Stares at her hands placed on the worksheet, Fig. 2.6)) (3:38)
030 (0.84)
031 T: here (0.30) you have to do (1.41) three plus six (0.60) yup. (1.79) <<len, p>y=write whatever> ((Thérèse turns around and speaks to members of Group 4 about other things))
032 (1.29)
033 A: yeAH?
034 (49.52) ((Ali writes, Mario raises hand and Thérèse talks about something else))

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035  T:  ah my god.
036      (8.70)
037  A:  ((pounds on the table))
038  T:  <<p>kay we are all mi[::xed up>        ]
039  A:                      [i dont understand] ((points to
         her page, Fig. 2.7))
040      (2.46)
041  T:  <<confidently>uh hu:::; uh huh. >
042      (25.56) ((M drops his hand)) ((Ali pounds table again,
         throws herself back against back of seat)) ((4:57,
         Mario gets back to the taast, A leans back))
043  J:  <<f>yes.> (0.52) whAT is the ques[tion.]
044  M:                                   [its  ] ^this :::
         (0.38) <f>um[::> ] ((hands move downward, restrains
         not to pound on table, gazes at sheet, Fig. 2.8))
045  T:             [auré]lie sit properly (55:00)
046  M:  look this is (.) dUMb, <<p>i dont understAND.>
         ((487>217Hz))
047  J:  =are you having a group discussion?
048      (0.16)
049  M:  no. tresa is just gone so from:: (0.53) lOOK (0.72)
         like (.) they alrEADy wrote thi:::ngs ((Frustration))
050      (1.20)
051  A:  <<f>^ma[da:me.>]
052  M:         [i like ] i dont like. ((discouraged))
053  J:  but YES because they are GUIding you.
054  M:  like, ^how:: that.
055      (0.11)
056  J:  kay (.) first week (0.84) wHY (0.16) ˇwOUld (0.75)
         there be? (0.91) why would there be:::sIX (0.61)
         dollars in the piggybank.
057      (1.04)
058      for the first week. (.) what did you ge::t (0.37) to
         do ((she takes the goblet of week 1))
059      (0.75)
060  M:  becau::se (0.30) ((points to goblet 1))
         [but there is nine ] <<dim> [the first week]>
061  J:  [there is, you know] <<crsc>[is it really] six
         dollars? ((points to and looks closely at M's first
         cell))
062  M:  no:n?
063  T:  no. yea.
064  J:  =ITs three plus six:.
065  T:  ive said <<whispering>[that is what ive said]>
066  J:                        [why three plus six:. ]
067      (0.34)
068  T:  because it equals to the deposit of the first week she
         has nine.
069  J:  it EQuals to nine the first week. (0.78) wHY is the
         thrEE in yellow? whydyou think? ((Index finger on
         number in first column))
070      (0.19)
071  M:  um um, um ((shrugs shoulders, shakes head 'no',
         questioning look, Fig. 3.1))
072      (0.20)
```

073 T: <<all>i don[no]>
074 M: [be]cause we are supposed to write it?
075 (0.44)
076 J: WHEREe does the thREE come from?
077 T: donno?
078 M: <<f>a:=u:> (0.24) u:: (0.17) u: dududu: wedding thing there?
079 (0.76)
080 J: but ((exasperation, turns head away from Mario)) (0.14) the three dO:LLas? is wHAT exACtly? ((Mario, who has looked at her, grimaces in desperation, brings his hands up and covers face, Fig. 3.2 [6:11]))
081 (1.61)
082 T: its its:: its [the three] do::llars there that s::he::.
083 M: [u::h:]
084 (0.48)
085 M: she takes [each] week.
086 T: [ss:]
087 (0.38)
088 T: aves ((Jeanne moves head to side over shoulder, gives him 'a look'))
089 (0.45)
090 M: like i dont understa:nd. ((reacting to teacher look, points to his worksheet, Fig. 3.3, 460>229 Hz))
091 (0.59)
092 J: <<all>okay.>
093 T: <<len>i understand .h:: ive it l[ike].
094 J: [reREAd] the problem. (0.33) lets reREAd the problem. wHAT does it tell us to do here it?
095 (0.58)
096 M for her anniversary- (0.97) what? ((grimace, hand gesture, open toward ceiling, 'what's going on here?')
097 (0.50)
098 J: ˇyes (0.25) for her ANnivERsa:ry
099 M: <<len>marie-na:in (0.52) receives (0.15) a piggyb
100 (0.15)
101 J: a piggy [bank]
102 T: [piggy] bank
103 M: <<p, len>a piggybank?> (.) containing six dollars (0.80) she deCIdes (0.15) ah tos[ave][(.)ave]
104 T: [sa:][:v::e:]::
105 J: SA:Ave,
106 M: save three dollars (0.50) per week.
107 (0.35)
108 J: so. (0.35) she receives a piggybank; how mUCH money does she have in her piggy [bank]?
109 M: [six] ((facial expression as if teacher had asked the 'self-evident'))
110 (0.54)
111 T: [six dollars]
112 M: [plus three] equals nine.
113 (0.22)
114 J: so each week, she saves (0.13) how much money.

115 M: three dollars
116 (0.68)
117 J: three dollars (0.11) so: ((points to cell 1))
118 M: we [are nINE?] ((holds 'pick-up' hand over goblet 1))
119 J: [three plus] six. ((continues pointing))
120 A: <<plaintive>are we sup[posed to do this?>] ((Points to his page))
121 J: [second week] ((she takes the cup of the second week))
122 (0.89)
123 T: <<p>[yes I think]>
124 M: [we:=ave:;]
125 (0.17)
126 J: how much does sHE have already in the piggybank. ((continues to hold goblet 2))
127 (0.18)
128 M: twELV::e. ((rapid confirming gesture toward goblet, Fig. 3.5, intonation of exasperation, as if she has asked the 'self-evident'))
129 (0.42)
130 J: well the sECond week, how mUCH does she have AL:REady: ((nods with each emphases))
[in the piggybank.] ((still holds gobler 2))
131 A: <<p>[me [i=m done]
132 M: [whAT h?] ((frustration))
133 (0.63)
134 J: how mUCH money does she have ALrEAdy in her piggybank? ((still holds goblet 2))
135 (0.55)
136 T: <<p>[six]>
137 M: [a s:] (0.53) no[n. nINE:]
138 J: [the second] wee ((holds goblet 2, on 'wee' points to it with other index finger))
139 (0.38)
140 T: <<f>O:H:.>
141 (0.22)
142 M: nINE
143 J: nINE. ((nods)) (0.64) she had ((places goblet 2)) the sIX that sHE started with, ((demonstrative la gesture to left)) (0.19) and the three dollars ((rH index pointing into goblet 1)) (0.58) do (0.23) how mU::CH (0.24) do we ((rH index points into goblet 2)) add here.
144 (0.80)
145 T: [three.]
146 M: [what] (big? [one?)]
147 J: [one] more thrEE DOLLars.
148 (0.18)
149 M: <<insisting>ITs <<f>tWEL:v:e.
150 J: its <<f>tWELve ((confirming, nods deeplu, open rH gesture, palm upward)) (0.94) so (.) how much money is there in, how much money (0.92) ((she lifts the third week)) (2.50) how mUCH money is THEre (.) IN, (0.29) already in the third week ((raises goblet, jingles it, places it back))

151 (0.79)
152 T: um um u::m.
153 (1.12)
154 M: u:[m::]
155 T: [fifteen]
156 (0.30)
157 M: <<p>tWELve. >
158 J: =how mUCH should thERe already be.
159 T: u:h:
160 M: twelve
161 (0.21)
162 J: wHY. ITs composed of what.
163 (0.68)
164 M: what well lOOK ((frustrated, hands stretched out, palm up, toward worksheet))
165 (0.27)
166 J: twelve dOLLars contAINs the::? ((Mario places head in hand, arm resting on table [Fig. 6])) (1.48) six dOLLars that we start wITH ?(0.46) and how mUCH money in the other two weeks beFORe ((Jeanne places right palm on goblet 1 & 2, sticks left finger for '$6))
167 (2.01)
168 M: what? (1.56) that makes- (0.80) i dont understANd (.) thOUgh. ((460>228 Hz)) ((2-handed gesture, as if holding thing above worksheet, gaze moving from sheet to her face))
169 J: <<p>you dont understand that> its what i=m trying to help you understand (2.40) lOOK well (3.50) are we lOOKing (0.65) trèse?
170 (0.40)
171 A: hu? (0.22) done.
172 (0.40)
173 J: you have answered everything? ((Oriented toward Aurélie))
174 (1.01)
175 have you done c d and e?
176 (0.91)
177 A: n::o:. (1.44) weve to do c d and e ((pounds desk top))
178 (0.33)
179 M: two times three plus six that equals to nINE.
180 (1.13)
181 J: tWO times thrEE is what. ((places rH index on the first cell, will not remove it for a while, Fig. 4.1))
182 M: <<insisting>s:sIX:.>
183 (0.18)
184 J: plus
185 (0.65)
186 M: s::IX::.
187 J: =equals to, ((moves index up down))
188 (1.02)
189 M: tWEL:v:e ((fills something into his table, 439>210 HZ)) (2.96) Where do we write twelve now.
190 (0.43)
191 J: ((moves hand to right; throughout, Mario holds his head, gazes at his sheet)) you dont write it. ((moves

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        index up and down between row 2 & 3, first cell))
        (0.35) you are done (0.45) what does it say; (0.41)
        three plus six. (0.49) so. (0.29) first week, ((points
        to first goblet)) (0.51) there's already six dOLL:ars
        (0.21) and you add three dollars. (0.34) three dollars
        plus six (0.56) SECond week:; ((points to sedond
        goblet, Fig. 4.2)) (.) you already have three dollars
        ((points to '3')) (0.54) you already have your six
        dollars ((points to '6')) you add another three
        dollars. so thrEE plus thrEE you do three plus three
        (0.37) its six. (0.60) third week; how mANY threes are
        you going to add in your:: (1.03) piggybank?
192     (0.96) ((questioning look on Mario's face))
193     how mANY three dOLLars are you going to have.
194     (2.08)
195  M: how much money are you going to have?
196  J: how many thrEE dollars are you going to have?
197     (1.47)
198  J: three dOLLars, three dOLL[ars  ], three dOLLars (0.23)
        ((points to the 2 '3's in week 2 and simultaneously
        points with left hand to the first, second, and third
        goblet))
199  M:                          [three]
200  J: what are you going to write here?
201  M: three?
202     (2.59) ((Jeanne moves finger to the cell on his left))
203  M: <<p>plus three? plus three? >
204  J: yES:: ((he writes))
205     (4.38)
206     plU:S::? ((points))
207     (0.96)
208  M: s::sIX:?
209     (0.44) ((Jeanne moves rH to right, opens palm up, as
        if confirming 'this is it'))
210  J: orIGinally you started with sIX. (0.39) so; instead of
        writing three plus three plus three, what would you be
        able to do. (0.27) three tIMEs?
211     (0.84)
212 M:  s::IX:?
213     (0.88) ((J moves finger sideways repeatedly between
        two cells?))
214     no three times three
215  J: yES::. (('excited' 'yes' [prosody in Fig. 4.4], makes
        the same rH movement to right, opens palm toward
        ceiling, Fig. 4.3)) (1.21)
        its just on the bottom its a [shortcut]
216 A:                               [madAMe: ]
217     (0.42)
218  J: your fourth week; (.) how mANY three dollars do you
        have.
219     (1.00)
220  M: u:m:::  (1.73) fo. ((Fills table, Therese makes
        noises))
221     (9.48) ((writes 4 '3s'))
```

222 J: <<pp>kay> (0.97) instead of doing three plus three plus three plus thrEE whAT are you doing to wrITE here? ((Points to the row on the bottom of the table of values))
223 (0.66)
224 M: uh:m:: (1.36) four times thrEE?
225 J: ((2-handed gesture sidewards, opening palm upward: 'you got it' [Fig. 5.1]))
226 (3.83)
227 i=think you understand now. uh?
228 (50.93) ((Mario slightly nods, writes, after 26 seconds looks at Therese's worksheet, back at his own))
229 M: <<confident>ME i understand now.>

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