Leibniz's Mathematical and Philosophical Analysis of Time

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Leibniz believed that mathematics has a special place in the human search for wisdom, knowledge of the "most sublime principles of order and perfection," because the things of mathematics are so determinate, and exhibit their determinate inter- relations so clearly. However, the proper use of mathematics requires careful philosophical reflection. The reason why materialism has seemed attractive to serious thinkers, he argues in the *Tentamen Anagogicum* (1696), is because it lends itself well to mathematical representation, and thus to calculation and rigorous inference.[1] However, we should not over-estimate the extent to which the material world lends itself to mathematics, for all mathematical 'models' are a finitary representation of an infinitary reality; and we should not forget that other aspects of reality also lend themselves similarly to mathematization. The materialist illusion is not only a mathematical mistake (which should be addressed by yet more mathematics) but also a metaphysical mistake. The alleged materialist universe is a mirage, for it violates the principle of sufficient reason, which along with the principle of contradiction governs the created world; it is thus after all not thinkable, like the mirage of the 'greatest speed.' The world's beings are not only material, but thoroughly sentient and endowed with force or conatus, a striving for perfection; and in that striving they express their Maker, as well as the intelligibility for which mathematics is apt.

[1] Leibniz, G. W. *Philosophische Schriften*, ed. C. I. Gerhardt, Vol. VII, pp. 270–279. Abbreviated hereafter as 'GP' with reference to volume and page number.

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1 Leibniz on Method

Leibniz writes that “the ancients who recognized nothing in the universe but a concourse of corpuscles”, as well as the modern philosophers who are inspired by them, find materialism plausible,

> because they believe that they need to use only mathematical principles, without having any need either for metaphysical principles, which they treat as illusory, or for principles of the good, which they reduce to human morals; as if perfection and the good were only a particular result of our thinking and not to be found in universal nature… It is rather easy to fall into this error, especially when one’s thinking stops at what imagination alone can supply, namely, at magnitudes and figures and their modifications. But when one pushes forward his inquiry after reasons, it is found that the laws of motion cannot be explained through purely geometric principles or by imagination alone. (GP VII, 271)[2]

Moreover, he adds, there is no reason to suppose that other phenomena which in that era had eluded mathematical formulation (he mentions light, weight, and elastic force) will not sooner or later prove to lie within the expressive powers of mathematics. But all such representation will be provisional, because while finitary models can express the infinitary things of nature well, they can never express them completely; and the formulation of increasingly accurate stages of representation must be governed, like nature itself, by the two great principles of contradiction and sufficient reason.

Leibniz recognizes that different sciences require different methodologies, but no matter what special features different domains exhibit, he believes that all scientific investigation must move between mathematics and metaphysics. Mechanics, in particular, is best viewed as a middle term between mathematics and metaphysics, and so too Leibniz’s account of time. Of all the parameters involved in mechanics, time is the least tied to any specific content, even though it presents a determinate topic for scientific investigation. Thus a closer look at Leibniz’s account of time presents an especially ‘pure’ version of the interaction of mathematics and philosophy in the service of progressive knowledge.

As Yvon Belaval, Gilles-Gaston Granger, François Duchesneau, and Daniel Garber have variously argued on the basis of a wide range of texts, Leibniz’s novel conception of scientific method has two dimensions (Belaval 1960; Granger 1981; Duchesneau 1993; Garber 2009). His account of method is informed by that of Bacon and Descartes, but diverges from both in significant ways and combines aspects of each. He borrows from Bacon the project of collecting empirical samples from the laboratory and field, inductively, and compiling tables, taxonomies and encyclopediae, always with the expectation of discovering harmonies and analogies, deeper systematic organization in the things of nature. He borrows from

[2] “Parce qu’ils croyent de n’avoir à employer que des principes de mathematique, sans avoir besoin ny de ceux de metaphysique qu’ils traitent de chimeres, ny de ceux du bien qu’ils renvoyent à la morale des hommes, comme si la perfection et le bien n’estoient qu’un effect particulier de nos pensées, sans se trouver dans la nature universelle… il est assez aisé de tomber dans cette erreur, et par tout quand on s’arreste en meditant à ce que l’imagination seule peut fournir, c’est à dire aux grandeurs et figures, et à leurs modifications. Mais quand on pousse la recherche des raisons, il se trouve que les loix du mouvement ne scauroient estre expliquées par des principes purement geometriques, ou de la seule imagination.” (GP VII, 271).

Descartes the assurance that the indefinite presentations of sense can be associated with precise mathematical concepts, and thus by analogy be re-organized as ordered series, which can then be subject to deductive inference.

In the *Tentamen Anagogicum*, Leibniz mentions the use of geometry in the "analysis of the laws of nature", and goes on in that essay to develop the ideas of Fermat, Descartes, and Snell in optics using a series of geometrical diagrams, as well as the ideas of maximal and minimal quantities developed in his infinitesimal calculus. In an earlier, more general essay, "Projet d'un art d'inventer" (1686), he invokes arithmetic as a source of formulations apt for analysis considered as the art of invention, "which would have the same effect in other subject matters, like that which algebra has on arithmetic. I have even found an astonishing thing, which is that one can represent all kinds of truths and inferences by means of numbers." (C 175) [3] The idea is to locate nominal definitions, involving a finite number of requisites, and then reason on the basis of them:

> I found that there are certain primitive terms —if not absolutely primitive then at least primitive for us—which once having been consituted, all our reasonings could be made determinate in the same way as arithmetical calculations; and even in the case of those reasonings where the data, or given conditions, don't suffice to determine the question completely, one could nevertheless determine [metaphysically] mathematically the degree of probability. (C 176)[4]

The clarity and determinacy of mathematical things is crucial to this method of analysis. "The only way to improve our reasonings is to make them as salient as those of mathematicians, so that one can spot an error clearly and quickly, and when there is a dispute, one need only say: let us compute, without further ado, to see who is right." (C 176)[5]

Early modern mechanics begins by exploiting an already existing trove of empirical records, the precise tables left by centuries of astronomers tracking the movements of the moon, the planets, certain stars and the named constellations which culminate in the careful data of Tycho Brahe, so important to Kepler, and which are soon thereafter improved by the measurements of astronomers equipped with telescopes. Happily for human science, the solar system is both an exemplary mechanical system (just a few moving parts, isolated, and so almost closed despite

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[3] "qui feroit quelque chose de semblable en d'autres matieres, à ce que l'Algebre fait dans les Nombres. J'ay même trouvé une chose estonnante, c'est qu'on peut representer par les Nombres, toutes sortes de verités et consequences." (Leibniz, G. W. *Opuscules et fragments inédits*. Ed. L. Couturat. Hildesheim: Georg Olms, p. 175. Abbreviated hereafter as 'C' with reference to page number).

[4] "Je trouva donc qu'il y a des certains Termes primitifs si non absolument, au moins à nostre egard, les quels estant constitués, tous les raisonnements se pourroient determiner à la façon des nombres et meme à l'egard de ceux ou les circonstances données, ou data, ne suffisent pas à la determination de la question, on pourroit neantmoins determiner [Metaphysiquement] mathematiquement le degré de la probabilité." (C 176) (Couturat indicates by brackets a word or phrase that Leibniz has crossed out.).

[5] "L'unique moyen de redresser nos raisonnemens est de les rendre aussi sensibles que le sont ceux des Mathematiciens, en sorte qu'on puisse trouver son erreur à veue d'oeil, et quand il y a des disputes entre les gens, on puisse dire seulement: contons, sans autre ceremonie, pour voir lequel a raison." (C 176).

the occasional comet) and a very precise clock; so its study richly repaid the efforts of early modern physicists.

How shall these two occupations, empirical compilation and theoretical analysis, be combined? Leibniz calls on metaphysics, in particular the principle of sufficient reason in the guise of the principle of continuity, to regulate a science that must be (due to the infinite complexity of individual substances) both empirical and rationalist. The correlation of precise empirical description with the abstract conception of science *more geometrico* is guaranteed by the thoroughgoing intelligibility and perfection of the created world, and encourages us to work out our sciences through successive stages, moving back and forth between a concrete taxonomy and abstract systematization. Empirical research furnishes nominalist definitions—finite lists of requisites for the thing defined—which can set up the possibility of provisionally correct deductions, though every such definition due to its finitude can be corrected and amplified; mathematics provides the rule of the series.

At the beginning of Chap. 6, "La philosophie de l'histoire" of his book *Leibniz historien*, Louis Davillé writes:

> From the metaphysical point of view, Leibniz, contemplating together the diversity and uniformity of things and beings, also follows two opposed principles, recognized earlier by scholastic philosophers, the principle of individuation and the principle of analogy, which he expresses by two phrases, in French: "l'individualité enveloppe l'infini" and "c'est tout comme ici." But this is only an appearance. Always seeking to reconcile opposites, he unites these two points of view in "la conception d'un développement à la fois spontané et régulier des êtres,"[6] through the contemplation of the universal harmony, principle of things persisting in diversity balanced by identity. This powerful and original synthesis he calls the law of continuity … The notion of continuity plays a leading role in Leibniz's philosophy, differentiating it sharply from that of Descartes. One might call the law of continuity the 'general method' of Leibniz, and this expression doesn't seem to be an exaggeration. (Davillé 1909, pp. 667–68)

Davillé notes three formulations of the principle of continuity: (1) Time and space are divisible to infinity. (2) The order of the input terms ('principes') is expressed in the order of the output values ('consequences') and vice versa. (I use the anachronistic vocabulary of functions here, to capture the generality of Leibniz's words.) This principle, 'of harmony', is a corollary of the principle of reason. It can also be understood as the principle of induction, that the cause can always be retrieved from the effect; the principle of differentials (ratios between finite magnitudes persist even when the magnitudes are reduced to infinitesimals, as in the 'characteristic triangle') ; and the principle of analogy. (3) Change never occurs in jumps, but always by degrees. Leibniz also calls this the principle of transition; like the principle of the identity of indiscernibles, Leibniz deduces it from the principle of sufficient reason. The principle of continuity, taken as a principle governing *history*, corresponds to a conception of historical evolution, slow and successive change due to natural and immanent causes. (Davillé 1909, pp. 668–670)

This model of scientific inquiry accords very well with Leibniz's own investigations into mechanics and planetary motion, and so too his mathematical-metaphysical

[6] Davillé quotes Delbos in this context. See V. Delbos. *La philosophie pratique de Kant*. Paris: Alcan, 1905, p. 264.

account of time. Given the subtlety of his conception of method, I will argue that his account of time is deeper and more multivalent than that of Newton, which explains why it has proved to be more suggestive for physicists in succeeding eras and especially during the last century.

2 Descartes and Newton

Descartes' definition of motion in the *Principles* is "the transfer of one piece of matter, or one body, from the vicinity of those bodies which are in immediate contact with it, and which are regarded as being at rest, to the vicinity of other bodies." (AT VIII, 53).[7] Thus motion and rest can be interpreted only as a difference in velocity or acceleration established with respect to a reference frame of other bodies; no absolute determination of motion or rest is possible. This definition of motion and rest is so radically relativistic that, strictly speaking, the Cartesian observer, by choosing different reference frames, may not only shift from judging that a given particle is at rest to judging that it is in inertial motion (rectilinear motion at a constant speed), but also to judging that its trajectory should be considered accelerated (and perhaps curvilinear). Descartes himself never seems to have considered this consequence of his relativism, nor its inconsistency with his invocation of inertial motion in the first two rules of motion given at the beginning of the *Principles*. Perhaps the inconsistency escaped his notice because in his mechanics there is no accelerated motion: the inherent motion of corpuscles is rectilinear and constant in speed (that is, inertial) and the transfer of momenta (defined for each contributing corpuscle as bulk times constant speed) in a collision is instantaneous. His mechanics is thus undynamical and atemporal; its laws are not only time-reversal invariant, they do not involve time as an independent variable: nothing in Descartes' mechanics varies continuously with respect to time.

Newton, however, saw and criticized this outcome, precisely because it entails that Descartes is not entitled to his own definition of inertial motion. In *De Gravitatione* (unpublished in his lifetime) he argues that since in Cartesian vortex mechanics all bodies are constantly shifting their relative positions with time, "Cartesian motion is not motion, for it has not velocity, nor definition, and there is no space or distance traversed by it. So it is necessary that the definition of places, and hence of local motion, be referred to some motionless thing such as extension alone or space in so far as it is seen to be truly distinct from bodies" (Newton 1962, p. 131). That is, Descartes cannot give empirical procedures in his mechanics that allow him to distinguish inertial motion from accelerated motion.

Newton responds with his well known thought experiment about the revolving bucket, arguing that the presence of forces is the sign of true (accelerated) motion; forces are real and measurable. But he goes beyond that claim: in Book III of the *Principia*, he writes,

[7] Descartes (1964–1974).

> Hypothesis I: The center of the system of the world is at rest.
> Proposition 11, Theorem 11: The common center of gravity of the earth, the sun, and all the planets is at rest. (Newton 1999, p. 816)

Taken together, these claims offer an absolutist conception of space that makes not only accelerated motion, but even uniform motion, definable with respect to a Euclidean space that has been provided with a centre and axes. By countering so strongly Descartes' relativism and subsequent loss of the distinction between inertial motion and accelerated (straight or curvilinear) motion, Newton has sacrificed the equivalence of inertial reference frames and thus his own first law. He has also postulated a spatio-temporal structure that cannot be empirically verified, a set of Cartesian coordinates for the Euclidean space of his planetary mechanics, which violates his methodological principle of not invoking merely metaphysical hypotheses. Newton is not entitled to the equivalence of rest and inertial motion, which is just as essential to his system as Descartes' concept of inertial motion is to his system. (Grosholz 2011)

3 Leibnizian Time

Leibniz acknowledged but was not troubled by the consequences of Descartes' relativism, and extended it to time. Thus in a commentary on the *Principles*, "Critical Thoughts on the General Part of the Principles of Descartes" (unpublished in his lifetime), Leibniz writes about *Principles* II, Articles 25 and 26:

> If motion is nothing but the change of contact or of immediate vicinity, it follows that we can never define which thing is moved. For just as the same phenomena may be interpreted by different hypotheses in astronomy, so it will always be possible to attribute the real motion to either one or the other of the two bodies which change their mutual vicinity or position. Hence, since one of them is arbitrarily chosen to be at rest or moving at a given rate in a given line, we may define geometrically what motion or rest is to be ascribed to the other, so as to produce the given phenomena. Hence if there is nothing more in motion that this reciprocal change, it follows that there is no reason in nature to ascribe motion to one thing rather than to others. The consequence of this will be that there is no real motion. (GP IV, 369)[8]

This is just what Newton says! But for Leibniz, it is not a problem, certainly not a problem to be banished by postulating absolute space and time as the arena for motion. Rather, he makes the following claim: "Thus, in order to say that something is moving, we will require not only that it change its position with respect to other

[8] "Si motus nihil aliud est quam mutatio contactus seu viciniae immediatae, sequitur nunquam posse definiri, quaenam res moveatur. Ut enim in Astronomicis eadem phaenomena diversis hypothesibus praestantur, ita semper licebit, motum realem vel uni vel alteri eorum tribuere quae viciniam aut situm inter se mutant; adeo ut uno ex ipsis pro arbitrio electo, tanquam quiescente, aut data ratione in data linea moto geometrice definiri queat, quid motus quietisve reliquis tribuendum sit, ut data phaenomena prodeant. Unde si nihil aliud inest in motu, quam haec respectiva mutatio, sequitur nullam in natura rationem dari cur uni rei potius quam aliis ascribi motum oporteat. Cujus consequens erit, motum realem esse nullum." (GP IV, 369).

things but also that there be within itself a cause of change, a force, an action."[9] Newton proposes that whenever acceleration occurs, it is due to the action of forces; Leibniz proposes that whenever any motion occurs, it is due to the action of forces. This doesn't mean that he has reverted to Aristotelianism, but is instead an expression of his pan-animism. What Leibniz means by force is not Newtonian force, but something more like energy, internal to the body. Leibniz believes that no body is ever truly at rest, for all bodies are ensouled: motion thus becomes an expression of *conatus*, as individual substances jostle each other for a place within the Cartesian plenum at all times. (GP IV, 354−392)

In this picture of the universe, we see the principle of sufficient reason at work, fashioning Lebniz's mechanics along with mathematics. The universe must be a plenum, and the individual substances in that plenum are jostling each other in an effort to attain perfection: everything strives. Indeed for Leibniz even unactualized possibles strive: essences strive for existence. In the realm of ideas, this striving sorts ideas out into an infinity of possible worlds, and (with the beneficent cooperation of God) precipitates one world into creation; in the created world, it induces vortical motion in the plenum as well as temporality. Time is the expression of the incompatibility of things; because creation involves plurality, mentality, and mutual limitation, all things are active, passive and intentional. This is the best of all possible worlds because it is continually becoming more perfect, on into the infinite open future: creation is a continuous temporal process. In the law of the series, the independent variable is always time. Thus matter is not merely extended, but involves resistance and action; and it develops: Leibniz's science will also be a natural history.

Having invented a supple and powerful notation for his version of the infinitesimal calculus during his sojourn in Paris (1672–1676), Leibniz proceeded to work out a theory and practice of differential equations, in which the dependence of different forms of accelerated motion on time could be clearly expressed by the term 'dt'. One application of this method was to planetary motion. While in Vienna on his way to Rome in 1688, Leibniz read Newton's *Principia*, took extensive notes and then wrote a series of papers that culminated in the *Tentamen de Motuum Coelestium Causis* (*Acta Eruditorum*, Feb. 1689), where he proposed differential equations that would characterize planetary motion. Leibniz combined Cartesian vortex theory with Newton's reformulation of Kepler's laws, locating the planets in 'fluid orbs' rather than empty space, in order to derive the laws governing central forces while avoiding the problem of action at a distance. Whereas Newton calculates the deviation from the tangent to the curve, Leibniz expresses the situation with a single differential equation, by calculating the variation of the distance from the center, comparing the distances at different times by a rotation of the radius. The upshot of his calculation is that the effect of gravity is $[(2h^2)/(ar^2)]\ dt^2$, so that the 'solicitation of gravity' (conceptualized in Cartesian terms as the action of a vortex) is inversely proportional to the square of the distance, which was of course the result Leibniz was trying to reproduce. (Aiton 1985, Chap. 6; Bertoloni Meli 1993, Chap. 4)

[9] "Itaque ad hoc, ut moveri aliquid dicatur, requiremus non tantum ut mutet situm respectu aliorum, sed etiam ut causa mutationis, vis, actio, sit in ipso." (GP IV, 369).

4 Leibnizian Relationalism

For Leibniz, space is the expression in the created world of the logical order of compossibility among individual substances, and time is the logical order of incompatability among individual substances.[10] Thus, space and time only come into being with the creation of this material universe, the best of all possible worlds, and have only a secondary ontological status, because they are constituted as relational structures of the things with primary ontological status, individual substances. This is the basis of Leibniz's relationalism; but we must recall that his relationalism is deployed on the basis of a method which is two-tiered, both mathematical (seeking a precise mathematical correlate for the law of the series) and metaphysical while at the same time empirical (examining and tabulating evidence in an ongoing search for the systematic organization of things). The true scientist will find ways to put the mutual adjustment of nominalistic form with the investigation of the infinitely complex, infinitely ordered world of individual substances, in the service of the progress of knowledge; this process requires both mathematics and metaphysics.

To correlate time with precise mathematical concepts, Leibniz chooses as the correct representation the straight Euclidean line, endowed with directionality by Descartes' analytic geometry, which assigns positive and negative numbers—real numbers we would say—to the line. In some texts, it appears that Leibniz holds time to be a half-line, given what he writes to Clarke in the fifth letter of the Leibniz-Clarke correspondence (GP VII, 389–420). Since this is the best of all possible worlds, created by God, the universe must constantly increase in perfection, and so has a temporal beginning point but no end. Thus it is metaphysically important that the number-line is both geometrical and arithmetical. As arithmetical, it expresses the fact that time is asymmetric; time may be counted out in units, like seconds or years, and the numbers increase in a unidirectional order without bound to infinity. The asymmetry of time follows from the metaphysical ground that everything strives. As geometrical, the number-line expresses the fact that time is a continuum; units of time like seconds are not atoms, but conventionally established, constant measures of time, as the inch is a measure of continuous length. An instant is only the marker of a boundary of a stretch of time, not what time is composed of; we misunderstand what an instant is, Leibniz observes, if we conceive of it as an atom of time. Time must be both measured and counted.

This duality of time is not however without conundrums. Analysis in arithmetic leads us to the unit; but in geometry it leads us to the point. Whole numbers are composed of units, but lines are bounded by points, not composed of them; Cartesian reductionism is useful as an approach to arithmetic, but not to geometry. In a letter to Louis Bourguet, written just before the correspondence with Clarke, in August 1715, Leibniz writes,

> As for the nature of succession, where you seem to hold that we must think of a first, fundamental instant, just as unity is the foundation of numbers and the point is the foundation of

[10] See, for example, GP II, 248−53.

> extension, I could reply to this that the instant is indeed the foundation of time but that since there is no one point whatsoever in nature which is fundamental with respect to all other points and which is therefore the seat of God, so to speak, I likewise see no necessity whatever of conceiving a primary instant. I admit, however, that there is this difference between instants and points—one point of the universe has no advantage of priority over another, while a preceding instant always has the advantage of priority, not merely in time but in nature, over following instants. But this does not make it necessary for there to be a first instant. There is involved here the difference between the analysis of necessities and the analysis of contingents. The analysis of necessities, which is that of essences, proceeds *from the posterior by nature to the prior by nature*, and it is in this sense that numbers are analyzed into unities. But in contingents or existents, this analysis *from the posterior by nature to the prior by nature* proceeds to infinity without ever being reduced to primitive elements. Thus the analogy of numbers to instants does not at all apply here. It is true that the concept of number is finally resolvable into the concept of unity, which is not further analyzable and can be considered the primitive number. But it does not follow that the concepts of different instants can be resolved finally into a primitive instant. (GP III, 581–582)[11]

The analysis of time requires the scientist to proceed both by the analysis of contingents, using the line whose continuity is the best expression mathematics provides for infinite complexity; and by the analysis of necessities, using the natural numbers whose linear ordering and asymmetry is the best mathematical expression of irrevocability. Leibniz goes on to observe that the use of mathematics does not solve the metaphysical question whether time has a beginning, which leads one to suppose that more metaphysics and more empirical research are required. He writes:

> Yet I do not venture to deny that there may be a first instant. Two hypotheses can be formed—one that nature is always equally perfect, the other that it always increases in perfection. If it is always equally perfect, though in variable ways, it is more probable that it had no beginning. But if it always increases in perfection (assuming that it is impossible to give its whole perfection at once), there would still be two ways of explaining the matter, namely, by the ordinates of the hyperbola B or by that of the triangle C.[12]

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[11] "Pour ce qui est de la succession, où vous semblés juger, Monsieur, qu'il faut concevoir un premier instant fondamental, comme l'unité est le fondement des nombres, et comme le point est aussi le fondement de l'etendue: à cela je pourrois repondre, que l'instant est aussi le fondement du temps, mais comme il n'y a point de point dans la nature, qui soit fondamental à l'egard de tous les autres points, et pour ainsi dire le siege de Dieu, de meme je ne vois point qu'il soit necessaire de concevoir un instant principal. J'avoue cependant qu'il y a cette difference entre les instans et les points, qu'un point de l'Univers n'a point l'avantage de priorité de nature sur l'autre, au lieu que l'instant precedent a tousjours l'avantage de priorité non seulement de temps, mais encor de nature sur l'instant suivant. Mais il n'est point necessaire pour cela qu'il y ait un premier instant. Il y a de la difference en cela entre l'analyse des necessaires, et l'analyse des contingens: l'analyse des necessaires, qui est celle des essences, allant *a natura posterioribus ad natura priora*, se termine dans les notions primitives, et c'est ainsi que les nombres se resolvent en unités. Mais dans les contingens ou existences cette analyse *a natura posterioribus ad natura priora* va à l'infini, sans qu'on puisse jamais la reduire à des elemens primitifs. Ainsi l'analogie des nombres aux instans ne procede point icy. Il est vray que la notion des nombres est resoluble enfin dans la notion de l'unité qui n'est plus resoluble, et qu'on peut considerer comme le nombre primitif. Mais il ne s'ensuit point que les notions des differens instans se resolvent enfin dans un instant primitif. " (GP III, 581−582).

[12] "Cependant je n'ose point nier qu'il y ait eu un instant premier. On peut former deux hypotheses, l'une que la nature est tousjours egalement parfaite, l'autre qu'elle croit tousjours en perfection. Si elle est tousjours egalement parfait, mais variablement, il est plus vraisemblable qu'il n'y ait

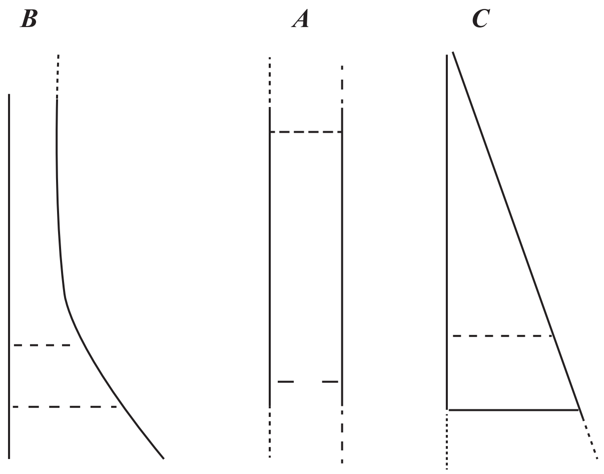

Fig. 1 Letter from Leibniz to Bourguet, 5 August 1715, GP III, p. 582

Here Leibniz gives the diagrams reproduced in Fig. 1.

His explanation of these diagrams shows that, despite what he would shortly write to Clarke, he was perhaps not convinced that time has a beginning:

> According to the hypothesis of the hyperbola, there would be no beginning, and the instants or states of the world would have been increasing in perfection from all eternity. But, according to the hypothesis of the triangle, there would have been a beginning. The hypothesis of equal perfection would be that of rectangle A. I do not yet see any way of demonstrating by pure reason which of these we should choose. But though the state of the world could never be absolutely perfect at any particular instant whatever according to the hypothesis of increase, nevertheless the whole actual sequence would always be the most perfect of all possible sequences, because God always chooses the best possible. (GP III, 582−83)[13]

In any case, Leibniz's conception of method requires that time be investigated not solely by pure reason or pure mathematics, which he admits here to being inconclusive; time must also be investigated empirically. It must be considered as the relational structure of the individual substances that exist, insofar as they are not logically compatible with each other. This means that we may have to revisit the formal structures we have just been discussing, in light of what we discover about

point de commencement. Mais si elle croissoit tousjours en perfection (supposé qu'il ne soit point possible de luy donner toute la perfection tout à la fois) la chose se pourroit encor expliquer de deux façons, savoir par les ordonnées de l'Hyperbole B ou par celle du triangle C." (GP III, 582).

[13] "Suivant l'hypothese de l'Hyperbole, il n'y auroit point de commencement, et les instans ou etats du Monde seroient crûs en perfection depuis toute l'eternité; mais suivant l'hypothese du Triangle, il y auroit eu un commencement. L'hypothese de la perfection egale seroit celle d'un Rectangle A. Je ne vois pas encor le moyen de faire voir demonstrativement ce qu'on doit choisir par la pure raison. Cependant quoyque suivant l'hypothese de l'accroissement, l'etat du Monde ne pourroit jamais etre parfait absolument, etant pris dans quelque instant que ce soit; neanmoins toute la suite actuelle ne laisseroit pas d'etre la plus parfaite de toutes les suites possibles, par la raison que Dieu choisit tousjours le meilleur possible." (GP III, 582−583).

the physical universe. The principle of sufficient reason governs the created world; not only does it entail that everything is determinate and intelligible (which for Leibniz means, thinkable), it also entails that everything strives for perfection. Thus the essences that are ideas in the mind of God strive for existence, but only those that constitute this best of all possible worlds succeed; and in the created world, the essences continue to jostle each other, to interfere with each other, as they all strive. This dynamic quality of ideas produces time, as their harmonies produce space; creation entails plurality and mutual limitation, activity and passivity. And the time that is produced is asymmetrical, as creation tends towards greater perfection, a harmonious dissention among the sentient, active individual substances.

What Leibniz heralds is the now received belief that matter is not passive and inert, or dead: even a molecule is mobile, active, forceful, and sensitive. As he writes in the *Monadology*, sec. 66–69:

> 66. (…) there is a world of creatures, of living beings, of animals, of entelechies, of souls in the least part of matter.
> 67. Each portion of matter can be conceived as a garden full of plants, and as a pond full of fish. But each branch of a plant, each limb of an animal, each drop of its humors, is still another such garden or pond.
> 68. And although the earth and air lying between the garden plants, or the water lying between the fish of the pond, are neither plant nor fish, they contain yet more of them, though of a subtleness imperceptible to us, most often.
> 69. Thus there is nothing fallow, sterile, or dead in the universe, no chaos and no confusion except in appearance (…). (GP IV, 618–619)[14]

5 A Thought Experiment

To probe the limits of Leibniz's relationalism, I propose to leave the path of textual analysis for a while, and venture into the forest of thought experiments. Inspired by twentieth century speculation, I propose that we try out Leibnizian relationalism on models of the universe very different from that which he entertained, and see what becomes of the account of time. First, let us suppose that nothing exists except a single particle. Then there is no time, because time is the expression of relations of incompatibility among things and one thing is clearly compatible with itself.

Suppose next that nothing exists except a perfect harmonic oscillator, which moves through a certain series of configurations only to return to exactly the same configuration in which it began. The motion of the harmonic oscillator, with one

[14] "66. (…) il y a un Monde de Creatures, de vivans, d'Animaux, d'Entelechies, d'Ames dans la moindre partie de la matiere. 67. Chaque portion de la matiere peut être conçue comme un jardin plein de plantes, et comme un étang plein de poissons. Mais chaque rameau de la plante, chaque membre de l'Animal, chaque goutte de ses humeurs est encor un tel jardin ou un tel étang. 68. Et quoyque la terre et l'air interceptés entre les plantes du jardin, ou l'eau interceptée entre les poissons de l'étang, ne soit point plante, ny poisson, ils en contiennent pourtant encor, mais le plus souvent d'une subtilité à nous imperceptible. 69. Ainsi il n'y a rien d'inculte, de sterile, de mort dans l'univers, point de Chaos, point de confusions qu'en apparence (…)" (GP VI, 618–619).

causal state giving rise to another, expresses time, but is the time it expresses finite or infinite? Since its beginning and end state are identical, it seems as if we should identify the times they express; then time would be finite. The local 'befores' and 'afters' would have no global significance; the asymmetry of cause and effect along the way would be absorbed into a larger symmetry, because every effect would ultimately be the cause of the cause… of the cause of its cause. Thus the local incompatibility of before and after would be absorbed into a global compatibility; but then we must wonder whether this finite time is really temporal at all. It seems that in this picture duration both does and does not occur.

Moreover, the picture seems to contradict the supposition that what exists is a perfect harmonic oscillator, for there is no oscillation. The concept of oscillation involves the notion of repetition, which in turn requires a linear ordering of time, so that when a particular configuration recurs, that is when it occurs again, the first occurrence is earlier than the later one, but the later one is not earlier than the first. We can imagine that the same configuration recurs at a later moment of time; but it is incoherent to suppose that *the selfsame moment of time* recurs at *another moment of time*, for those two moments of time must then be both identified with, and distinguished from, each other. As Leibniz often observes, contradiction makes alleged ideas vanish into nothingness; the relationalist idea of an isolated harmonic oscillator is a mirage, and so is the idea of a moment of time recurring.

So we would have to admit that the time that frames the harmonic oscillator is ongoing, linear and infinite, and so must be constituted by something beyond the relations that hold among the moving parts of the harmonic oscillator; but this goes against Leibnizian relationalism. To avoid this problem, Leibniz must completely fill up his cosmos with things and events that never repeat, on pain of incoherence. Such a cosmos is precisely what his metaphysics provides, chosen by God according to the Principle of Plenitude, the Principle of Perfection, the Principle of Sufficient Reason, and the Principle of Contradiction. Moreover, since all of his monads are body-souls, everything that exists is provided with a developed or rudimentary intentionality, that drives it forward in time. The strong asymmetry observed in the organic, sentient world is guaranteed for everything that exists. In Leibniz's cosmos, everything is alive and everything strives. The dispute with Clarke shows that Leibniz's cosmos must be a plenum, for otherwise isolated things would show up in absolute space and God's choice of their location would be arbitrary; similarly, if isolated events happened in absolute time, God's choice of when they occurred would be arbitrary. So even if we imagine the ideal harmonic oscillator to express an ongoing, infinite time, perhaps by allowing the natural numbers as a condition of its intelligibility, so that each of its oscillations might thereby be distinguished by a numerical index, it would still violate the Principle of Sufficient Reason.

At this juncture in the argument, however, we might suspect that Leibniz has not discovered the infinity and uni-directionality of time in the relations among things, but merely construed the relations among things so that the time they express will turn out to be appropriate, that is, infinite and uni-directional. And another suspicion may arise: Even if Leibniz is accurately describing the way things are (an organicist, animist plenum), perhaps that in itself sheds no light on time. Time itself may have no flow; and it may prove to be finite, coming to an end that no living thing (including Leibniz) foresees. If our grasp of time is merely empirical, based on temporal relations among things, maybe real time is beyond our grasp. However, for Leibniz no pursuit of truth should be merely empirical; to be a Leibnizian relationalist is not to reduce science to empiricism. Leibniz avoids this skeptical worry by trusting in the ability of metaphysical principles to regulate the interaction of empirical research and theoretical speculation in science. Informing this trust is his trust in the perfection and intelligibility of the cosmos, so that time is the expression of the infinite, harmonious incompatibility of things.

6 Coda

Leibniz understands that productive scientific and mathematical discourse must carry out distinct tasks in tandem: a more abstract search for conditions of intelligibility or solvability, and a more concrete strategy for achieving successful reference, the clear and public indication of what we are talking about. The texts characteristic of successful scientific research will thus be heterogeneous and multivalent. This fact has been missed by philosophers who begin from the point of view of logic, where rationality is often equated with strict discursive homogeneity and method is construed as the rewriting of science and mathematics in a formal, axiomatized language; and it has led scholars influenced by logicism, among them Louis Couturat and Bertrand Russell, to misread Leibniz. While deductive argument is important (since its forms guarantee the transmission of truth from premises to conclusion) as a guide to effective mathematical and scientific reasoning, it does not exhaust method, for Leibniz. As we have seen, Leibnizian method has two dimensions, empirical and rational, and both require analysis, whose logical structure includes abduction and induction, as well as deduction. Moreover, analysis, the search for conditions of intelligibility, is more than logic; it is a compendium of research and problem-solving procedures, which vary among investigations of different kinds of things.

An unswerving focus on logic diverts attention from other forms of rationality and demonstration. Human awareness is both receptive and active, an accommodating construal and an explanatory construction. Some empiricist or naturalist philosophers of science demand that true knowledge be an accurate construal of the way things are, but then they deny the obvious fact that all representation is distortion, however informative it is, and that representation itself changes the way things are. And explanatory analysis goes far 'beyond' the things that invoked it, and thus often sacrifices concrete, descriptive accuracy. Other logicist or anti-realist philosophers of science want to suppose that all knowledge, and indeed all reality, is a human construction, but then they deny the obvious fact that the world is the way it is whether we like it or not, and that it has depths that elude our construals and constructions altogether. Many an explanatory analysis has shipwrecked on the hidden shoals of reality. A more reasonable view of human knowledge is to regard it with Leibniz as a combination of focussed awareness and theoretical elaboration; thus when we combine multiple modes of representation in our scientific work we may in fact have a better chance of doing justice to what we are investigating. Such representational combination and multivocality is just what we find in Leibniz's most important pronouncements on the nature of time.

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