Lazare Carnot and the Birth of Machines Science

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Abstract In this paper, we are mainly concerned with the development of the first theory applied to machines made by Lazare Carnot (1753–1823). The two Carnot's essays on machines previously published before *Fundamental Principles* have in common the use of the concept of work as a fundamental step to build that theory of mechanics within the framework of Rational Mechanics. Carnot accomplished this task by starting to develop his new theory applying d'Alembert's (1717–1783) principle and crowning his project with *Principes Fondamentaux de l'Equilibre et du Mouvement* published in 1803.

Keywords History of work · History of energy · History of applied mechanics

1 Introduction

The first author to reference Carnot's work appears in the first decades of the nineteenth century. He was André Guenyveau (1782–1861), who in 1810 published the *Essay on Science of Machines*, different from Carnot's works. He studied the equilibrium of machines and presented a series of practical applications, using Coulomb's memoir in which machine motion can appear by means of shock or pressure. This distinction also indicates that continuous transmissions provide the best efficiency. But Guenyveau does not attributes to Lazare Carnot the origin of the principle of living forces used in spite of the fact that, in the preface of his *Essay,* Carnot is referred to as an author of a general treatise on machines.

Another author who cites Carnot is Jean-Nicolas-Pierre Hachette (1769–1834). He refers to Carnot's (1803) *Fundamental Principles of Equilibrium and Motion* appearing in the preface of his *Elementary Treatise of Machines*, published in 1811. Hachette remarks that Carnot in the last chapter of *Fundamental Principles* studies the whole theory of machines and the moving applied forces. In addition he mentions Carnot's work as *the most profound savant and experienced engineer*. Paradoxically Hachette uses few of Carnot's achievements. In fact the course taught

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by him is much more a collection of drawings of particular machines, studying also machine elements such as gears, pulleys, etc.

Alexis Petit (1791–1820) developed an investigation in the same sense of others that applies the work concept. In 1818 he published *About the Employment of the Living Forces Principle in the Calculation of the Effect of Machines* which is a very interesting work and coincides with our line of investigation. Petit was a physician who together with Pierre Dulong (1785–1838) postulated a law with his name and died prematurely. He presented general properties for motion as the conservation of living forces sponsored by him as the most efficient approach to machine calculation. According to him it is the living force that permits in each particular case the best natural evaluation of a motor and the effect produced. The equation expressing the relation between these two quantities can provide the direct solution to a machine problem. In fact what Petit was proposing is a method of energy balance to solve this problem.

With respect to further developments of applied mechanics, Coriolis' (1792–1843) textbook is the result of writings accumulated since 1819 (Coriolis 1829). His main objective was to develop the Lazare Carnot project of creating a general theory of machines but in general as a product of the tradition of Newtonian mechanics (Coriolis 1832). What is new in Coriolis' approach is the extensive use of the concept of work with its mathematical formalism and applying in different situations to machines the Newtonian concepts. The new approach adopted by him can be better understood if we look at the two general approaches used to solve a given mechanical problem. One is to consider the variations of motion as the result of forces, in a Newtonian way. Alternatively, knowing the amount of work generated by the system, try to find the "living forces" involved (Poncelet 1827). In other words, we can use Newton's second law or work-kinetic energy principle. In 1844, 1 year after Coriolis' death, his master work was published with the title: *Treatise on mechanics of solid bodies.*

2 Lazare Carnot's Biographical Note

Lazare-Nicolas-Marguerite Carnot was born on May 13th 1753 in a bourgeois home in Bourgogne in France, who family occupied a remarkable position in local society (Charnay 1990). Claude Carnot, his father, was a lawyer and public notary in Nolay, a small city close to Côte-d'Or. The place where Lazare Carnot was born has not changed significantly as of now and still belongs to Carnot's family (Fig. 1).

Carnot's father took upon himself the education of his three older children and they, on the other hand, helped the younger brothers as tutors or preceptors. Among the six sons, two demonstrated early ability in mathematics and technical questions: Lazare, the second son and Claude-Marie (1755–1876), because their characteristics were compatible with a military engineering career.

During the eighteenth century it was usual in order to assure the son's education, mainly when the families were richer, to ask for the help of a priest or a preceptor.

Fig. 1 Lazare Carnot (1753–1823)

There were only two ways, the classic or humanistic studies and the military schools. At the Universities one could prepare for a career in law, medicine or theology.

Carnot completed his humanistic course in the Autumn School, where after the Jesuits had left the school, the Oratories replaced them. Ten years later Napoleon Bonaparte (1769–1821) studied there. Lazare Carnot finished these studies at the age of 16.

With the purpose to save his father's money, Carnot decided to study alone in order to submit to the examinations of Mézières School of Engineering. Because of his unsuccessful attempt to be admitted to that School at the end of 1769, his father went with him to Paris. In addition, Carnot obtained the patronage of the Duke of Aumont and thus he gained entry into one of the most famous Paris institutions which was directed by Louis-Siméon de Longpré, located in Marais. Longpré supported himself thus early by giving instruction in mathematics, and soon after becoming an author. Being well versed in mathematics, he conceived the plan of erecting a school for the preparation of young men destined for service in the army.

From the beginning of his classes' period, Carnot had the opportunity to meet Charles Bossut (1730–1814) and d'Alembert, two figures of great influence in Carnot's approach to mechanical problems. Bossut, who directed Mézières's examinations, published in 1763 the first edition of a very popular mechanics handbook entitled: *Elementary Mechanical Treatise and Applied Dynamics mainly to Machines Motions*. He also published other algebraic, geometrical and arithmetic works. In spite of his main interest in mathematics, he was particularly concerned with mathematical education. He wrote *A General History of Mathematics from the Earliest Times to the Middle of the Eighteenth Century* which was published in 1802.

At that time d'Alembert was an old man and maintained a strong friendship with Longpré. The great mathematician frequently met the students around him to suggest mathematical problems. Later on when Carnot became also an old man he enjoyed telling how d'Alembert had influenced his approach to solving difficult questions and how d'Alembert predicted a successful career for him.

After passing his examinations, Carnot entered the Mézières's School of Engineering, on January of 1771, to follow 2 years of basic formation which is the fundamental step to the future regular formation of engineers. At that time, the Engineering School required from candidates proof that they belong to a noble class or, to a family living in a noble way. In other words what was required of families was a non-degrading life. It is important to say that the majority of the students were of bourgeois origin and Carnot had no difficulty in obtaining the certificate.

Unlike Carnot, Gaspar Monge (1746–1818), one of the greatest talents among those students, had suffered cumbersome situations because of these requirements. Monge's family had modest resources and he had to overcome this situation by means of his intellectual qualities. He managed to overcome all these difficulties and became a lecturer of mathematics and physics (Belhoste 1989).

With respect to Monge's influence on Carnot, there are divergences among the biographists. One did not refer to the other and there is no indication that Monge had been interested in Carnot's work. Indeed there were profound differences of personality between the two men. Monge was a mathematician and a pedagog without any tendency to politics. Always when he was confronted with political issues, he demonstrated a theoretical spirit, became very emotional and incapable of making decisions, exhibited no confidence in his own judgments and paid no attention to details. On the other hand, Carnot behaved inversely, being an engineer with clear and objective ideas, with ability to grasp a conception and execute his assigned tasks with uniformity of character and preferring to judge with practical spirit and adapt to circumstances.

Carnot left Mézières on first January 1773, having developed military aspirations, and served in several military posts, initially in Calais, then in Havre, Béthume, Arras and Aire. In Calais he worked in its port fortification. Later on, he moved to Havre and worked there for 3 years in order to participate in the building of Cherbourg's port, an advanced technical project being considered one of the most modern undertakings in military French engineering (Chatzis 1999). With this and other participations, Carnot was considered among his peers a well-known and reputed engineer.

In order to understand Carnot's scientific work perhaps we should remember Coulomb's quotation made in 1776, about the situation of everyone that left engineering school. He refers to them as: *young men who studied and after left the School had no other thing to do, to face tedious and monotony into the corporative situation that is the dedication to any science branch or to literature absolutely strange to his study.*

Obviously it happened to Carnot after he finished the course in Mézières's School. He spent part of his time pursuing mechanical and mathematical studies, achieving a higher level of knowledge when compared with other students from

Mézières. These courses had provided the following: the study of four books of Charles-Etienne-Louis Camus (1699–1768) on arithmetic, geometry and statics; in addition Bossut's treatises on dynamics and hydrodynamics; as complements, annual courses on industrial design, perspective or descriptive geometry, as well as experimental physics. Camus was an important French mathematician and mechanician who studied also civil and military architecture and astronomy. He became secretary of the Academy of Architecture and fellow of the Royal Society of London.

Probably when Carnot left Mézières's School, facing the solitude of the military post life, he began to read again d'Alembert, Bossut, Bélidor, Euler's mechanics and Daniel Bernoulli's hydrodynamics. This is an easy conclusion when we read his early works, where he demonstrated a good knowledge in the above subjects. Another argument to corroborate this supposition is his distance from Paris's scientific circles in a different way followed by Charles Augustin Coulomb (1736–1806) and Charles Meusnier de la Place (1754–1793). Sometimes Carnot spent his vacations in Nolay or Dijon. Documents from the Paris Sciences Academy and the German Sciences Academy also confirm this supposition. Other manuscripts belonging to Carnot's family delineate in some way the circumstances and the context in which he wrote his early works on mechanics and mathematics.

Carnot's first studies were addressed to his participation in conferences sponsored by scientific societies. In 1777, the Paris Academy proposed a conference in the year of 1779, but it was only accomplished in 1791, because the referees were not satisfied with the works submitted for the first time. Consequently Carnot wrote two of his first studies on machines theory. Both entitled: *Essay about the machines*; he concluded the first in Cherbourg on March 1778 and the second on July 1780 in Béthume. At that time a memoir by Coulomb was the winner and Carnot received an honorable mention.

With respect to his interest in mathematical foundations the motivation is quite similar. His well-known text *Considerations about the metaphysics of infinitesimal calculus was* a memoir submitted to the Prussian Royal Academy in 1785. Yet, in keeping with the interests of the planned conference, the Paris Academy asked for anyone interested to write about the first human flight. Carnot had addressed the topic in a letter of January 17th, 1784 entitled *Letter about the aerostats*. We should remember that on June 5th, 1783 the Mongolfier brothers had sent into space a hot-air balloon that achieved 1800 m of height. This event led to the solution of several technical problems about motion and stability of flight.

Carnot had first studied the air-ship problem and proposed building a very unusual and fantastic propeller which displaced itself like a medusa working according to systole and diastole motions created inside the balloon by heat dissipation.

In the above description appear the most remarkable facts related with the beginning of Carnot's scientific career. In the next lines we will see his participation in politics mainly at the beginning of the French Revolution.

If we wish to truly characterize Carnot and portray his real character, we must begin by identifying him as an authentic republican. He was involved with important actions of the French revolutionary period, for instance voting for the death of

Louis XVI, unmistakably confirming his republican convictions. On the other hand, his military position contributed to the view of the movements by the left as creating the main menace to the recently established Republic. Hence, he fought against the revolutionary movement called *Conspiracy of Equals,* commanded by Gracchus Babeuf (1760–1797).

Babeuf's name is a tribute to the Roman tribunes of the people and reformers, the Gracchi brothers. He was a French revolutionary political and journalist of the Revolutionary period. In spite of the efforts of his Jacobin friends to save his life, Babeuf was arrested, tried and convicted for his participation in the *Conspiracy of Equals.* In that time neither anarchist nor communist were existing words. *Conspiracy of Equals* was an armed rising fixed for Floréal 22, year IV (11 May 1796). One day before Babeuf was arrested, many of his associates were gathered by the police on order from Lazare Carnot. Babeuf was guillotined at Vêndome Prairal 8 (27 May 1797) without appeal. This episode is important from the viewpoint of revolutionary movements in general, because some historians see the French Revolution as the beginning of a proletarian movement within a bourgeois revolution.

Carnot also participated in regrouping of the French army after the disorganization caused by the Thermidorian movement that maintained control of their military administration. However the French Revolution delivered plenty of surprises, with victories and defeats of the republican forces and movements for reestablishing the monarchy.

In spring of 1797, legislative elections showed the possibility for resurrection of the monarchy. The left could not accept the defeat of the Revolution and thus tried to reject the elections' verdict and attempted to eliminate the reactionary members from assemblies. In the *coup d'état* of 18 Frutidor (September, 4th) Carnot was convinced that it was not possible to maintain the Constitution, and as a result did not participate in this movement. Upon receiving a communication about the directory's intention, he left Luxembourg and obtained refuge in Paris during some weeks before moving to Switzerland.

During this type of political unrest, Carnot frequently spent part of his time in researching mechanics and mathematics, according to his son Hippolyte. In a similar situation in 1797, during his participation as a member of the directory, his book *Considerations about the metaphysics of infinitesimal calculus* appeared.

With the taking of power by Napoleon Bonaparte in 1799 (18th Brumaire), Carnot came back to France with the benefit of a general amnesty in favor of Termidor's victims. He realizes that, although this regime has created difficulties, it is not sufficient that he be hostile to the Republic but must also pay attention to his own affairs. During that time a number of his works were published. In 1800, at the end of the year, he wrote his *Letter to citizen Bossut containing new visions about trigonometry*; in 1801, he published *From the correlation of the geometry's figures;* in 1803 his *Essay on machines is published* with the new title *Fundamental principles of equilibrium and motion* which occupies, for the purpose of this paper, a central position. In addition, in this year he also published *Geometry of position*, considered by him his masterpiece; in 1806 there appears his *Memoir about the relation that exists among the respective distances of any five points took in space*, followed by *An essay about the theory of transversals.*

The two sons of Lazare Carnot, Nicolas Léonard Sadi and Lazare Hippolyte were born in 1796 and 1801, respectively. Sadi was the founder of thermodynamics and we come back to him in order to analyze the influences received by him from his father (Gillispie and Pisano 2014).

Another scientific activity developed by Carnot must be emphasized but it is necessary to look back to the time of the French Revolution (Rashed 1988). The beginning of terror, August 8th 1793, the Sciences Academy as well as other French institutions representing the *ancient régime* has been abolished or closed, because they were considered as founded on privileges and not adequate to the spirit of the Republic. Only on August 7th did Carnot became a member of the Committee of Public Safety, thus we cannot attribute to him the decision to close the Sciences Academy. As we know, with the end of the Academy was created the French Institute with Carnot appointed as a member and with the purpose to give continuity to the old Academy's works. The new institution would undertake to unify the arts, letters and science, as branches belonging to the same culture, which was considered the best symbol of the Republic.

In 1805, Carnot became president of the French Institute. His main activity was to work for the progress of science, to publish its memoirs and to recommend editions of other studies submitted to him and considered of sufficient importance to be divulgated. In addition he sometimes worked as a public consultant, analyzing and judging technological problems, studying the conception and design of new machines, and the viability of industrial processes. At that time it was common for inventors and machine constructors to have their designs submitted to the government in order to obtain financial resources.

Yet, frequently Carnot participated in technical commissions within the attributions of the Institute in order to evaluate projects of practical characteristics. One example of this case was the Niepce motor. In an enthusiastic report he emphasized that this machine was the first way, never before imagined, to obtain primary energy by means of heated air expansion, or in other words, air combined with caloric. Conversely to the steam machine which required a big consumption of caloric to heat and vaporize water before using steam expansibility, an air motor presented a great advantage in not using combustibles except to produce expansion where energy arises.

In 1809, Carnot wrote, and addressed to the Institute, a new report about a new model of a thermal motor which was conceived by an inventor named Cagniard de Latour. This motor had attracted the attention of Sadi Carnot according to Thomas Kuhn. Like the Niepce machine, the fluid used is the air, but its utilization was more subtle. Some characteristics of this motor are described by Sadi Carnot in his *Considerations*. In addition Kuhn refers to another important aspect of Cagniard's motor that is its reversibility. This concept appears in Sadi Carnot's works and we come back to it later on.

The last years of Carnot can be briefly outlined. When Napoleon came back from Elba Island, Carnot allied to him, being his interior minister in the well-known government of 100 days. However because Carnot had voted for Louis XVI death, confirming again his republican convictions, he never was forgiven by the restored monarchy. Carnot went into exile again with his son Hippolyte. After passing by

Brussels, Munich, Wien and Cracow they arrived at Warsaw where he tried to stay and live. It was not possible to stay there and, prohibited from living in the Rhineland that is close to France, finally they went to Magdeburg where he died on August 2nd, 1823.

3 Application of Living Forces Conservation Principle to Machines

According to Claude Henri Navier (1785–1836), the first study where we find the principle of living forces conservation applied to machines is *Hydrodynamics* of Daniel Bernoulli (1700–1782), published in 1738. It is also the first time where the relation between hydrodynamics and that principle is established. Daniel Bernoulli shows that, if we abstract frictions, losses and we consider an incompressible fluid, an increase in the speed of the fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy, relating speed with height and pressure in a similar way that was done in the elevation of a weight to determine the capacity of the machine. Bernoulli performed his experiments on liquids, so his equation is valid for incompressible flow. Thus, in an arbitrary point along a streamline the sum of living forces, weight multiplying height and pressure divided by fluid density is constant.

Unfortunately, this important achievement made by Bernoulli, which expresses actually an energy balance, was completely forgotten in the most important works on mechanics that looked for an application, mainly that addressed to engineers. Among others we can cite *Physics* of Jean Théophile Désagulliers (1683–1744) and *Hydraulic Architecture* of Bernard Forest de Bélidor (1698–1761). Even the physicians and mathematicians that worked on more theoretical mechanics also did not realize the importance of Bernoulli's principle. Euler, for instance, didn't use these Bernoulli's ideas, when he studied the reaction wheel, the centrifugal wheel and Archimedes' screw. Only with the memoir of Jean-Charles Claude Borda (1733–1799) entitled *Memoir on the hydraulic wheels*, published by the Sciences Academy in 1767, did the principle of living forces conservation start to be known and to be applied to machines. Borda was the first to apply this principle to hydraulic wheels.

Some years later, in 1781, Coulomb published a memoir on windmills and used the same principle to study this machine. In addition he included the losses caused by shocks. Coulomb's text was entitled *Theoretical and experimental considerations on the effect of windmills*, also published by Sciences Academy (Coulomb 2002).

To Navier, the contributions of Borda and Coulomb are fundamental steps and remarkable progress when we compare these developments to Bernoulli's *Hydrodynamics* in spite of their restricted particular applications of the living forces conservation principle. Navier also emphasizes that after these studies mentioned above there is a need for the creation of a general theory involving that principle with the capacity to calculate machines efficiency. It is exactly in this context that he affirms that this theory was created by Lazare Carnot in his *Fundamental Principles of*

Equilibrium and Motion. Navier attributes to Carnot the general demonstration of the theorem which calculates the loss of living force due to shocks between non-elastic bodies (hard bodies). This fact was not observed, neither by Borda nor by Coulomb, unless in very particular cases.

After these achievements, which made an important contribution to the development and application of living forces conservation principle, came the work of Navier. In a note published by the *Proceedings of Chemical and Physics*, he communicates his intention to submit to the Academy some notes and additions prepared for a new edition of Bélidor's *Hydraulic Architecture*. It is a kind of update of this important book. In his remarks Navier demonstrates the principle of living forces conservation to a single mass but he also generalizes this result to a system of n particles through the d'Alembert principle. This study is made by means of virtual velocities and Navier derives Carnot's later theorem from the conservation of living forces. According to him this theorem fulfills the principle of living forces conservation for the case of a sudden change as follows: *The addition of living forces which arise in a system after a sudden change, is lesser than that which occurs previously, and the system has lost an amount of living force equals to that which would happen if the bodies were animated of velocities that are lost since this change*. As usual, Carnot uses the decomposition of velocities proposed by d'Alembert's principle before applying the living forces balance.

However, a great evolution in the living forces principle occurs with the publication of Coriolis' book—*Du Calcul de l'Effet des Machines*, published in 1829. This work is considered one of the most important in mechanical engineering to appear in the nineteenth century. In addition this book is a fundamental step in the history of work concept, because the term *work* was coined in this book, becoming adopted by the polytechnicians engineers as well as by the future technical literature. Thus, the word *work* was progressively used and replaced the previous denominations as mechanical power, quantity of action or mechanical effect, etc.

4 Carnot's Theory of Machines

4.1 Carnot's Essays on Machines

Before the appearance of his *Fundamental Principles* in 1803, Carnot wrote two introductory studies known as *Essay on Machines* (Gillispie and Youschkevitch 1979). The fundamental assumption to the development of an applied mechanics theory is that these two E*ssays* were preliminary studies to his masterpiece. As we are focusing our attention on a general theory of machines instead of a general analysis on Carnot's mechanics, only the aspects directly associated to this theory are considered. Obviously, the passages from the E*ssays* to *Fundamental Principles* will be studied from an evolutive viewpoint.

The problem to be solved by Carnot, which is the motion of any machine, is quite similar to the dynamics of a system of particles discussed by d'Alembert in his famous principle. In fact it is a system of **n** bodies or particles, with the possibility of most of them being constrained or connected among them, and the system as a whole constrained in any manner, and to this system we apply any initial condition to one or more particles. The solution is to find the subsequent position for any particle of the system. If we call **m** the mass of any corpuscle or particle of the system, **V** the velocity that it would have if free, i.e. without reactions from the other parts of the system, **u** the velocity that it actually has and **y** the angle between **V** and **u**, we will have: $\mathbf{S\ mu\ (V\ cos\ y-u)=0}$, that is, the summation of the products of quantities of motion for each corpuscle by the lost velocity, calculated in the direction of **u**, is zero.

This can be proved by decomposing **V** into two components, **u** and the other which is the lost velocity in shocks, then the velocity estimated in **u** direction is $\mathbf{V\ cos\ y-u}$. This is the proposition which initiates the first memoir part related to machines and enunciated in paragraph 27. In paragraph 29, Carnot postulates what he calls a fundamental theorem, in the following terms:

> If any system of hard bodies acting among them of any manner being immediately or by means of a machine, calling **m** any molecule of the system, **V** the velocity that it would have free from interactions in a given instant, **u** the velocity which it actually acquired due to reciprocal action of different parts of the system, **y** is the angle between the directions of velocities **V** and **u**; I affirm that we have: $\mathbf{S\ mu\ (V}\ \cos\ \mathbf{y)=0}$.

Regarding that the decomposition **V** cos **y** is **u** itself we have in fact $\mathbf{S\ mu^2=0}$, which means the conservation of total kinetic energy of the system of particles. What we have to take into account is that Carnot doesn't make any difference between integral and summation symbols, as well as discrete and continuous systems. Other remark is that the above equation actually represents an energy balance, where the system communicates motion to other parts of it such that the process is conservative.

Following Carnot's first memoir, he made many particular applications, including cases where the system is in equilibrium situation, but always starting from motions situations. In item 48, he discusses the quantities which vary when the machine is in motion such that we can take advantage if we know this form of variation. Carnot affirms: *The advantage of machines is that we can vary the factors **F, u, t,** but the product must be ever **M g H**, since that it is a weight or a similar quantity that is involved, even being other kind of resistance.*

What Carnot explains many times by different ways is the conservation of energy within the field of mechanics. Thus, it is possible to vary in an appropriate form the three quantities force, velocity and time, but the work or energy used in any process never overcomes the constant quantity **M g H**.

The second part of this memoir deals with the equilibrium of simple machines and we will go to the second memoir also called *Essay*, where a general theory of machines is going on.

Theorem 2: General Principle of Machines in Motion If we impress suddenly motion to a machine with another motion being any geometric one and if we release the machine to itself, the conservation of living forces will happen immediately and at each elapsed instant of motion for any alteration of the driven forces.

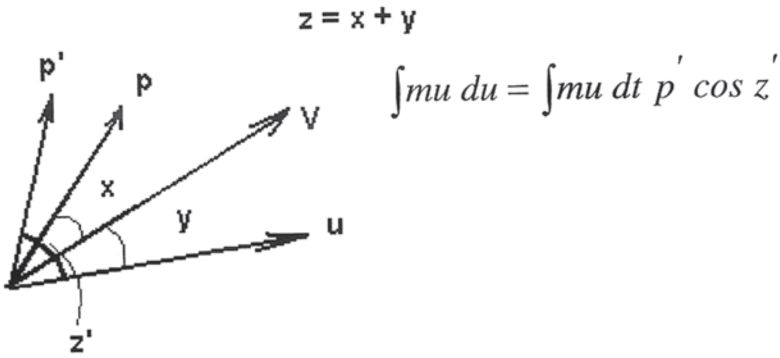

Fig. 2 Equivalence between living forces and work

The following nomenclature will be used (Fig. 2):

M = mass of each corpuscle; **V** = actual velocity; **P** = driven force
U = velocity after the variation of actual motion in another geometric one
X = angle between **V** and **p; Y** = angle between **V** and **u; Z** = angle between **u** and **P**
p´ = driven force after the arbitrary variation; **z´** = angle between **p´** and **u**

The virtual velocity of **m** estimated in **u** direction is **V cos y + d (V cos y)** and the lost velocity by **m** during **dt** estimated in **u** direction is **p dt cos z − dt (V cos y)**.

With respect to Theorem 2, it is a kind of application of the living forces conservation principle having in the equation shown in the figure on one side living force and on the other work. Geometric motion also cited in the theorem is related to the invariability of system constraints and is not referred to application of the virtual works principle to a dynamical system.

4.2 Fundamental Principles of Equilibrium and Motion

Starting from page 227, paragraph 252, Carnot treats directly the questions related to machines. It is the final part of his *Fundamental Principles*, from where we obtain the same subtitle, in which he studies deeply the problems arisen from the machines operation. The sequence followed by him starts with machines in equilibrium, its motion analysis, and the forms to increase machines efficiency. Obviously these are problems emerged from the needs of industry and engineering at that moment and therefore the polytechnician engineers of the first decades of the nineteenth century since the publication of *Fundamental Principles* will develop new concepts and a new theory of machines. It is important to point out the role of the concept of work to these developments. As we will see this concept occupies a central position in the majority of the investigations of this period.

Carnot starts by defining a machine as a body or a system of bodies which are in-between two or more powers (forces), with the condition to fulfill a given objective. He emphasizes that, in general, these bodies are considered without mass because of its small effect on the applied system of forces, independently if these forces are driven or inertia forces. He states that this abstraction simplifies the problem.

At this beginning Carnot also discusses questions of modeling a machine by the use of a convenient representation with a great quantity of corpuscles separated by strings and bars, through which motion is transmitted from one element to its neighbors and so on.

Carnot is aware of the general character of his investigation and said that his intention is not to search for particular properties of each machine, what he has remarked before, but to offer some considerations about the machines, and the common properties to every machine.

For the case of motion, not only the weight must be considered, but the height to be elevated, making these two operations with the machine, quite different. For the equilibrium, the machine could centuplicate the force effect while for the machine in motion we have an invariable quantity which is always the product of a force by the path described, estimated in the force sense. In other words, Carnot discusses exactly the question of energy in the field of mechanics, which is expressed by mechanical work. To eliminate any doubt he uses an example of a horse which is two times stronger than another, meaning that it can elevate a new quantity of water, for instance, a double height of the second at the same time, or a double quantity of water for the same height, also to the same time. It is important to say that the definition of mechanical power appears here in this context, in a very simple form.

Another important question related to economic concerns is the capacity of a machine to elevate a given weight to a certain height. This is the method used to calculate the machine capacity to provide work as well as the labour of workers and its estimation in order to pay salaries. What Carnot supposes is that the concept of work from physics is also useful to calculate salary's value as referred in the *Introduction*. Although Carnot's concerns with economic questions which also are related to mechanics is a remarkable fact.

In the context of the above discussion Carnot enunciates the famous principle: *In any machine in motion, one loses always in time or velocity what one wins in force.* Carnot then analyzes the true meaning of this principle, discussing the effect produced by a machine in motion. If we call **P** the weight to be elevated to a given height **H**, this effect will be represented by **PH**. Assuming that the force used to produce that effect is **F**, **V** the velocity estimated in the force sense, **T** the time during which the operation occurs and supposing that for the sake of simplicity the motion is uniform, one has **FVT** = **PH**. As mentioned before, Carnot calls the work "moment of activity", which is equal to **FVT**.

Carnot comes to the discussion of machine's utilization and states: the advantage which machines present is not to produce a great effect with small means, but to provide the choosing among different means which we can consider equivalent, to the most convenient circumstances. In other words, a machine is not a mere tool for the forces multiplication, but mainly, an apparatus which has the availability of some amount of work that can be used in a great variety of forms. Carnot completes this discussion saying that it is always necessary that the moment of activity spent by the driven forces be equal to the effect of the motion absorbed at the same time by the resistant forces. These considerations seems to be sufficient to finish the illusion that machines having assemblies of levers mysteriously linked can transform a weak agent capable to produce huge effects, by transposing the reasoning adopted to the equilibrium situation to the motion condition. Indeed, for a machine in motion it is always limited and thus, the machine cannot overcome the spent moment of activity produced by the correspondent agent. The difference is for the equilibrium condition, that motion is obstructed; for the motion situation the objective is to facilitate its appearing and, therefore to maintain it. This condition means one more consideration that is to know the actual velocity of each point of the system.

Carnot describes in details, for the equilibrium and motion situations the internal processes of machines related to dissipation of motion by obstacles. For the dynamical case, the fixed points and obstacles, of any type, are forces of passive nature, which absorb motion, of any intensity but never could provide its birth, even for a small one, when the body is in equilibrium. Although a small power cannot annihilate a great power, but it means that only the resistance imposed by fixed points. In other words, a small power is capable only to annulate a small part of a great one and the obstacles do the rest.

Carnot studies then the problem of the transformation of work in motion by considering all the parameters involved. From this viewpoint it means to establish convenient variations of the terms of the quantity **FVT**, i.e., the moment of activity, later on denominated work by Coriolis. Thus, if time is the most important parameter and we should minimize it, the effect must be produced in a very short time. It is possible to generalize these reasoning for the case of a system of forces, for instance; if we have the forces **F**, **F**′, **F**″ with the velocities **V**, **V**′, **V**″, acting during the times **T**, **T**′, **T**″, respectively, then one reads:

$$\mathbf{FVT = F'V'T' = F''V''T'' = PH}$$

If the motion of each one of the forces is variable, we will take the quantity: $\int \mathbf{(FVdt + F'V'dt' + F''V''dt'')}$, or if we have the forces directions with respect to velocities, one has:

$$\int \mathbf{[FVdtcos(F \wedge V) + F'V'dt'cos(F' \wedge V') + F''V''dt''cos(F'' \wedge V'')]}$$

This is the definition of work done by all forces.

The quantity **PH**, the effect to be produced by machine, is, by Carnot called latent living force. If we call **M** the mass of the weight **P**, and **V** the velocity correspondent to a height **H**, one reads:

$$\mathbf{PH} = \tfrac{1}{2}\mathbf{M}^2.$$

Following Carnot's concerns the problem of a machines' efficiency comes back. This is a question of central importance not only to mechanics of that time, but to development of applied mechanics, and, later on to industrial mechanics. Carnot remembers that previous considerations about machines in motion, all of them were made without taking into account shocks and sudden variations of velocities. The variations considered are always by means of insensible changes otherwise we have a great loss of living forces. He states: *In order to obtain from machines the best possible effect, it is important that it has been built such that the motion does not vary unless by insensible degrees. The exceptions are that with possibility to support different percussions, as the majority of mills. But even in this case, obviously we should avoid sudden variation unless that essential to the machine structure and operation.*

Carnot concluded the discussion above of how to obtain the best possible effect in a hydraulic machine, moved by water flow, denying that the solution is to adapt a water wheel to a machine, because the shocks against water appear necessarily. Then, two reasons can impede that the maximum effect be obtained: the fluid percussion itself and when this shock occurs, there is always a residual velocity associated to the pure loss, and in some cases could be employed to produce yet a new effect added to the first. To design an improved hydraulic machine, with capacity to produce the maximum possible effect, we should look to fulfill the following conditions:

1. For design considerations to oblige that fluid could lose all motion by its action over the machine, or at least that the remainder motion be precisely that necessary to escape.
2. That fluid loses totally the motion by insensible degrees without the occurrence of percussion from the fluid as well as from some machine component. This is independent of the type of machine.

The machine that better fulfills the conditions above will produce always the greater possible effect. Because it is so difficult to achieve the above objectives, mainly the water wheels, where shocks and percussions appear, it is possible at least to fulfill the second condition.

When Carnot discusses, in the following lines, what should be taken into account in order to produce the greatest possible effect, he affirms that this problem depends upon particular circumstances and therefore the problem does not admit a general solution to be applied for any situation.

The effect produced by a machine is a real or latent living force, always referred to the product **PH** of one weight **P** and a height **H**; we should call **q** this effect. On the other hand, to produce it we need that all driven forces spend a moment of activity **Q**, which cannot be lesser than **q**; this means that there is no loss moment of activity to be consumed by a driven force, or that **Q** = **q**. But the moment **Q** of activity consumed by the force **F** in a time **T**, moving with velocity **V**, if we suppose a simplified form, **F** and **V** constants, and that the angle between **F** and **V** can be designated by **(F ^V)**, we have **FVcos (F^V)** as the quantity that must be maximized.

The equation above is, obviously the moment of activity or, that is the same as the work done by the driven force. It depends upon four quantities: **F**, **V**, **T** and **(F ^V)**; one form to maximize this product is to insure that the direction of force coincides with that of velocity, i.e., that the force is in phase with velocity, using other words. With respect to the quantities force, velocity, and time during which the force acts, it is much more difficult to determine in an absolute form its intensities. If we could calculate approximately, the problem of optimization should be tried.

Carnot comments the case of the work done by a man where fatigue is also involved. The knowledge of his physical constitution is fundamental. In general this data could only be obtained by experience.

Within the work's study undertaken by man considered as a machine which originates ergonomics and physiology, Coulomb pointed out the importance of previous studies as, for instance, Daniel Bernoulli's: *Resultat de plusiers experiences destinées à determiner la quantité d'action que les homes peuvent fournir par leur travail journalier, suivant les differentes manières don't ils emploient leurs forces*; Bossut's *Mécanique* is referred to as containing important considerations about machines and also an Euler's memoir entitled: *De Machine in Genere.*

If we are looking at the maximization effect obtained by the machine, it is important to minimize the effect of passive forces, such as the friction force, string stiffness, air resistance, etc. Because of the impossibility of eliminating all these resistances and passive forces, which impose the progressive decreasing of machine velocity, this implies the impossibility of a perpetual motion. If percussion does exist the motion will decrease rapidly and the addition of living forces always decreases when percussion appears. In this context it is important to quote Carnot about the impossibility of the perpetual motion: *Obviously we cannot produce a perpetual motion, if it is true that all the driven forces that exist in nature are attractions and that this force have the general property, of being always the same to equal distances, between given bodies, i.e., a function that does not vary unless for the case where the distance of these bodies varies itself.*

Again Carnot discusses the importance of the concept of work within the theory of machines, which is recognized by him as follows: *A general consideration about what was said, is that kind of quantity called moment of activity, plays a great role in the theory of machines in motion: because is in general this quantity that is necessary to save the maximum possible in order to obtain from one agent all the effect which can be done.*

If we are studying a machine at rest, where it is necessary to overcome a body's inertia, but if we wish to give rise to a motion, the moment of activity which must be spent will be equal to the half-add of living forces which should be born.

5 Conclusion

In Carnot's book *Fundamental Principles,* as analyzed above, the concept of work plays a fundamental role in the building of his new general theory of machines. Once more, this theory is completely integrated into the conceptual framework of Rational Mechanics, because the used tools are based on its fundamental concepts including the concept of geometric motion which is a modified version and an attempt to generalize the principle of virtual velocities. Therefore, the structure of *Fundamental Principles*, contains a mechanical review emphasizing the problems involving shocks and sudden variations of velocities which is precisely the transmission of motion inside the machine. The originality of Carnot's contribution is clearly different from Lagrange's mechanics, in the sense of continuous variations, or in insensible forms, as characterized by him but presenting a simple approach which uses geometry and trigonometry, where a cosine law is transformed into an equation of energy conservation. At last, the new theory of machines proposed by Carnot is also up dated with the current practice of engineering of his time.

If we compare the famous Coriolis' work with Carnot's *Fundamental Principles*, it is easy to see that it represents a great progress from the technical point of view, as well as by its completeness, style and language. The 25 years that separate these two fundamental books in the mechanical engineering history does not present important works, except the notes and remarks of Navier about Bélidor, as previously mentioned. This advancement in the mechanical field, which means the development of applied mechanics in the industrial progress context, is also a remarkable aspect of engineering development. This process had a great influence in engineering education due to the need to prepare the technicians in order to give continuity to the technical progress itself and thus also provide a fundamental requirement to the Industrial Revolution at this moment spreading along the European continent. As we know, Coriolis was a lecturer in a Polytechnic School, Navier did the same in a School of Bridges and Highways.

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