Mathematical Language as a Bridge Between Conceptualization of Motion and Experimental Practice

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Abstract In this paper we try to distinguish two different styles of experimental practice—roughly speaking the Galilean and the Newtonian. They differ in the way they intertwine mathematics and experimentation. We offer a theoretical reconstruction of the transition from the Galilean to the Newtonian experimental practice. It seems that this transition was brought about by gradual changes of the conceptual framework for the representation of motion. The aim of the paper is to argue that in many of these changes Cartesian physics played an important role.

Keywords Galileo · Descartes · Newton · Force · Interaction · Experimental practice

1 The Galilean Style of Experimental Practice

In January 1610 Galileo Galilei (1564–1642) constructed the telescope and through it he made a series of important astronomical discoveries. He discovered mountains on the Moon, the satellites of Jupiter, the phases of Venus, the sunspots, as well as many new stars. Thus in one single month—January 1610—there occurred more changes in astronomy than during the whole preceding century. Galileo's discoveries played an important role in the defense of the Copernican theory (Swerdlow 1998; Shea 1998). Galileo published his astronomical discoveries in *Sidereus nuncius* (Galilei 1610). The book created a real storm. The reason for the intense reactions was not only the novelty and significance of the discoveries themselves, but also the fact that he made them using a telescope. His critics accused Galileo

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of naivety. At that time, the telescope was considered to be an illusionist toy which shows the phenomena not as they really are, but altered. Therefore "observations" with a telescope are unreliable and cannot be a basis for a serious science. Science has to examine the phenomena as they really are. Galileo's grounding scientific theories on "observations" made with a telescope was considered to be similarly naive as to try to develop a theory from "observations" made by a curved mirror. Galileo wanted to persuade his opponents and therefore he sent them a telescope, so that they could see with their own eyes what he spoke about. "The majority of the natural philosophers simply did not think it worthwhile even to look through his telescopes." (Ronchi 1967, p. 201). And it was not mere reluctance. A lens creates images which are greater or smaller than the real object observed by the naked eye. The object seems to be nearer or more distant than it really is and sometimes it is even turned upside down. Thus the lenses do not show true images of the things, but create illusions.

With his telescope Galileo brought a fundamental change in the notion of observation. The classical astronomical instruments like the *sextant* or the *astrolabe* were only put alongside the axis that connects the eye with the object on the sky. They did not change the way in which a particular object is disclosed to our view in everyday experience. Thus these classical instruments only sharpen our natural experience. Their precision has limits given by the resolving power of our eye. On the other hand Galileo's *telescope* enters between the observed object and us. It makes it possible to see things which without its help we are unable to see, as for instance the satellites of Jupiter or the countless amounts of new stars whose magnitude is below the threshold of our eye's sensitivity. Thus the telescope expands in a fundamental way the scope of our experience. This *new kind of instrumentation of experience* is one of the characteristic features of Galilean experimental practice.

Instruments like the telescope broaden the realm of our experience. Nevertheless, they do not intervene into the constitution of the observed phenomena—they only change the resolving power of our senses. There is, however, a whole range of phenomena, for grasping of which the instrumentation of observation is insufficient. If we take for instance free fall, we are unable to see whether it is uniform or not. We are not able to perceive it as something ideal and perfect. And no instrumentation of perception can be of help here, because the problem lies not in the insufficient resolving power of our sight, but in the ambiguity of the perception of motion. Free fall is a motion, and therefore time enters in a fundamental way into its constitution. Nevertheless, we have no specific organ for the perception of time, the resolving power of which could be eventually enhanced by some instrument. Some phenomena are too complex and we cannot grasp directly the mathematical forms which determine them. Therefore, according to Galileo it is necessary to create simplified situations in which the phenomenon would disclose itself in its purity and would reveal its mathematical form. The creation of such simplified situations requires invention, and Galileo's analysis of free fall is a beautiful example of such an invention.

For Aristotle free fall and horizontal motion were qualitatively distinct. Free fall was a natural motion, because the body moved towards its natural position. On the other hand, horizontal motion (in the sublunary region, of course) was an unnatural motion, requiring an external mover. Galileo's idea was to consider these two motions from the point of view of the artificial motion on an inclined plane. Free fall is a motion on a totally inclined (i.e. vertical) plane, while horizontal motion is a motion on an inclined plane the inclination of which is zero. Therefore, by continuously changing the inclination of the plane we can pass from free fall to horizontal motion and back. In this way Galileo's imagination, using the artificial device of an inclined plane, connected two phenomena which apparently have nothing in common. This connection has considerable technical advantages, because the motion on an inclined plane is relatively slow and therefore it is more suitable for observation than free fall. What Galileo discovered was a regularity: the distance passed by the ball grew as the square of the time. After the first pulse the ball reached the first line, after the second pulse the fourth, after the third pulse the ninth line. If we increase the inclination of the plane, the motion will accelerate. Nevertheless, the basic regularity—distance proportional to the square of the time—will be preserved. From this we can derive the conclusion that in the limit case of the vertical plane the distance will still be proportional to the square of time. It is plausible, even if we cannot observe it directly. Thus an experiment, by creating an artificial situation in which the mathematical form of the phenomenon is accessible to observation, sheds light on situations, in which the mathematical form remains hidden. The motion down an inclined plane made it possible to discover the law of free fall. We arrived at a concept of experiment, which is central for Galilean physics. *An experiment is an inventive disclosing of the mathematical form of phenomena using artificial situations*. It is a characteristic feature of the Galilean style of experimental practice that *the experimentally studied phenomena are accessible to ordinary experience*.[1]

The aim of an experiment is to create by help of an artificial situation access to the mathematical form of phenomena. After achieving that, its task is usually finished. Nevertheless, when Torricelli created a vacuum in a glass tube, it was not the end of the story. The reason was that the phenomenon of atmospheric pressure, the mathematical form of which he disclosed in this way, is not accessible in any other way. In ordinary experience we are not aware of atmospheric pressure, and many cultures did not even suspect that there existed something like this. In this respect there is a radical difference between heat and pressure. Heat is disclosed to ordinary perception and therefore the thermometer can be interpreted as an instrument that only sharpens the perception of heat. With the atmospheric pressure the situation is different. Without a barometer we have no idea even of the existence of this phenomenon. That is why Torricelli's tube did not "end in a museum" (i.e. did not become of interest only to historians), but was transformed into the barometer, which is a device opening an access to the phenomenon of pressure. A measurement is a standardization of experiment. Thus in order to understand what measurement

[1] Even though air pressure (which we will discuss below) is not directly accessible to ordinary experience, there is nevertheless a directly observable phenomenon—the strange behavior of the water pumps that refuse to pump water from depths greater than a certain limit—that is accessible to ordinary experience.

is, one has to keep in mind what an experiment is. An experiment is the disclosing of the mathematical form of a phenomenon by means of artificial situations. *A measurement is based on the standardization of the artificial situation of the experiment i.e. of the objects, relations and procedures that constitute it*. For instance in the case of the barometer we fix the diameter and the length of the tube, the amount of mercury. We may also determine the number scale which we fix to the tube, choose the suitable physical units and determine the scope of temperatures at which the barometer gives reliable results. In this way we secure the reproducibility and so the intersubjectivity of the measurement.

As long as physics operates in the area of phenomena to which we have an immediate access through our senses it is possible to understand measurement as a process of refinement of the picture of reality, offered by the senses. For instance in the case of *free fall* we are not able to decide by the use of our senses, whether it is uniform or accelerated. Nevertheless, we know free fall from our experience and thus we are inclined to interpret the measurement as a device that only helps us to determine that free fall is accelerated. In the case of *temperature*, the interpretation of measurement as a process of refining the sensory image which we get by the immediate contact with the body whose temperature we are measuring becomes more problematic. The problem is that we are able to measure the temperature of bodies that are so hot that by touching them our hand would be carbonized, and so we cannot speak about any sensory image. In the case of *atmospheric pressure* it is even worse. The gradual decrease of pressure manifests itself first by a headache, and it ends with the explosion of our organism. To speak about the measurement of pressure as making our sensory impressions more precise is impossible. What impression corresponds to the pressure of 0.01 atmospheres? *Measurement extends reality beyond the boundaries of the phenomenal world*. So we cannot interpret measurement as a simple refinement of the phenomena.

But the situation is even worse. We cannot interpret measurement even as a prolongation of the phenomenal world. The reason is that the picture of the world offered by the measuring devices comes into a conflict with the picture based on everyday experience. For instance, all motions which we can see around us on the Earth have a natural tendency to stop. In contrast to this, the account of motion offered by physics says that all motions are inertial and their ceasing is only the result of friction. In this way the natural, everyday experience is deprived of legitimacy. It starts to be regarded only as an *inaccurate and distorted picture* of the "real" reality which presents itself in measurement. We can find this shift already in Galileo.

> Hence I think that tastes, odours, colours, and so on are no more than mere names so far as the object in which we place them is concerned, and that they reside only in the consciousness. Hence if the living creature were removed, all these qualities would be wiped away and annihilated. (Galilei 1623, p. 274).

Galileo tells us, that physically real is not the picture, disclosed to us by our senses, but only a part of it, which we are able to grasp by help of measurement.

2 Four Ways of Conceptualizing Motion

The most radical changes brought about by the Galilean experimental practice concern the concept of motion. So in the next four sections we will offer a reconstruction of the development of conceptualization of motion from Galileo till Newton. It will turn out that this development necessitated the transition to a different style of experimental practice. (For more details see Kvasz 2002, 2003, 2005.)

Generally speaking, it is possible to distinguish four different ways of representing mechanical motion. The Aristotelian description of local motion can be characterized as a *geometrical transition*.[2] According to Aristotle, everybody has its natural place determined by the geometrical structure of the universe, and motion is a transition from one place of this geometrical structure to another. Galilean theory of motion can be seen as a theory of a *geometrical flow*.[3] Thus Galileo has replaced the Aristotelian concept of motion as a transition from one place to another by the concept of motion as a flow along a particular trajectory. The nature of the motion is, however, still given by the geometrical properties of its trajectory, and the global structure of the universe remains a geometrical order.

The basic innovation of Descartes can be interpreted as the replacement of a *geometrical* theory of motion by a *dynamic* one. Motion cannot be understood in terms of geometry because geometry does not allow us to understand interaction. Descartes described an interaction as a collision, i.e. as a transition from the initial state (the state before the collision) to the terminal state (the state after the collision). Descartes' theory is based on the comparison of two states and so it can be described as a theory of *dynamic transition*. Being a theory of transition, it resembles the Aristotelian theory. But there is also a deep difference between the Aristotelian and the Cartesian concepts of motion. Motion according to Descartes is not a transition from an initial position to a terminal one in a geometrically ordered, static universe. It is a transition from an initial state to the terminal state in a mechanically united dynamic universe.

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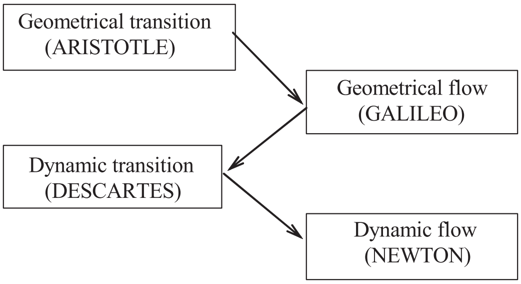
[2] *Geometrical transition* means that the basic representation of motion is *geometrical* (as opposed to a *dynamic* one): it describes motion as a change of *position* in space rather than as a change of *state*. Further, *geometrical transition* represents motion as *transition*, which means that the focus of attention is on the starting point and the terminal point of motion, while the process of motion itself does not enter the representation. (This does not mean that the change of *state* or the *process* of change is not thought about, but only that they are not included into the representation of motion. Thus even though Aristotle understands change as a becoming, in the definitions of (violent) motions these aspects do not enter.).

[3] *Geometrical flow* means that the representation of motion is still *geometrical*, it concerns a change of position in space and not a change of state. Nevertheless, it is a *flow*. It represents motion as a continuous passing along a trajectory (as opposed to a *transition* from the starting position to the terminal one).



We can characterize Newton's theory as a similar transformation of Cartesian physics as Galilean theory was of Aristotelian physics. Galileo replaced the Aristotelian idea of motion as a transition from an initial position to a terminal one by a flow along a trajectory connecting the two positions. Similarly Newton's theory replaces the Cartesian idea of transition between states by a *dynamic flow* along a vector field. These resemblances and differences between the four mentioned theories can be summarized in the form of the diagram above. Its aim is to represent the inner tension of the Cartesian system. In one sense Descartes moves from Galileo backwards to Aristotle (as he employs the concept of transition instead of flow), but in another sense he is a step forwards in that he incorporates interaction into the representation of motion.

3 Aspects of the Galilean Conceptualization of Motion

Galileo's law of free fall is often described as the first scientific law, i.e. regular correlation between empirical quantities expressed in mathematical form. Nevertheless, there is a number of conflicting interpretations of the role of Galileo in the history of modern science. Historians differ in their interpretations of the core of the Galilean project. Some of them see the main contribution of Galileo in his experimental method (Drake 1973; Hill 1988), others in his mathematical Platonism (De Caro 1993), still others stress his use of the Aristotelian deductive method (Wallace 1984) or of a combination of experiment and deduction (Naylor 1990). It seems that they do not exclude each other but rather represent different aspects of Galileo's scientific work which existed side by side.

Despite his fundamental contributions, which are well known and so there is no need to deal with them here, Galileo's ideas had also some grave shortcomings which are the reason why modern science is not a direct continuation of the Galilean project. The problem is not with Galileo's mistakes (his conviction that inertial motions are circular or his reluctance to accept Kepler's discovery of the elliptical shape of the planetary orbits). Such mistakes can easily be corrected. When speaking about shortcomings of the Galilean system we have in mind several problems of Galilean conceptualization of motion. First of all, Galileo's concept of motion is to a great extent a geometrical concept. Galilean physics *lacks a framework for the representation of interaction*. His description of motion is always a description of the motion of a single, isolated body. The laws discovered by Galileo bear a witness to this. The law of free fall, the law of the isochrony of the pendulum, or the law of the trajectory of projectile motion, these are all laws describing bodies without interaction. Secondly, Galileo has a *too narrow concept of a natural law*. The laws mentioned above lack generality. Be it the law of free fall or the law of the pendulum, they are laws describing *particular phenomena*. In Galilean science for each phenomenon there is a special law that describes it. And finally, Galilean physics *lacks any description of states*. Galilean physics deals only with observable quantities and tries to discover regularities in them. Contemporary science, on the other hand, is based on the description of the state of a physical system (using a *Lagrangian* or a *Hamiltonian function*) and its temporal evolution (by means of *Lagrangian* or *Hamiltonian equations*).

It seems that these shortcomings have a common root. They are the consequence of the use of *geometry* as the language of science. Galileo believed that the book of nature is written in the language of mathematics:

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders in a dark labyrinth. (Galilei 1623, p 237, p. 238).

This passage is often quoted, but its strange nature is rarely recognized. Modern science *is not* based on any triangles, circles, and other geometric figures but on functions and differential equations. The fact that the laws discovered by Galileo all describe isolated bodies without interaction can be brought into connection with his choice of geometry as the language of science. Splitting nature into isolated phenomena, reducing natural laws to mere phenomenal regularities, and sticking to observable quantities is closely connected to the role which Galileo has given in his scientific project to geometry. The language of synthetic geometry is too concrete. It does allow neither for universal laws nor for interaction.

4 Cartesian Conceptualization of Motion as a Foil to the Newtonian Conceptualization of Motion

While nobody would seriously question the great value of Galileo's contributions to the development of modern science, things are not nearly so simple with respect to Rene Descartes (1594–1650). Descartes is omitted in many expositions of the development of physics and Newton is seen as deriving directly from Galileo. It is sufficient to quote the words of Stephen Gaukroger:

> With the exception of the work in optics, his contribution to the development of classical physics is minimal. Insofar as kinematics is concerned, Cartesian physics accomplishes considerably less than had been achieved by Galileo in his Two New Sciences, and insofar as Descartes' physics can be considered a dynamical theory it is often hopelessly confused, particularly in comparison with Newtonian dynamics. (Gaukroger 1980, p. 123).

The views of Daniel Garber are rather similar:

> Descartes' intellectual program failed, of course; while pieces of the program may have proved important inspirations to later thinkers, as an approach toward understanding the natural world Descartes' program turned out to be a dead end. But while the design may have been faulty, and the edifice doomed from the start, it is fascinating to contemplate the entire structure as the architect planned it, [...] (Garber 1992, p. 2).

It seems that this interpretation of the rise of modern science is not adequate. Excluding Descartes from the main stream of the history of science prevents us from understanding the origins of two central features of physics: its ontological homogeneity, and descriptive universality.

Descartes, if he had wanted to, could have worked out the Galilean project much further than Galileo was able to, because he was a much better mathematician. Descartes was one of the creators of *analytic geometry* and he introduced the *algebraic notation*, which is still in use. He could therefore develop the ideas which Galileo arrived at by help of a cumbersome symbolism and a rudimentary idea of a coordinate system, in a much more elegant way using analytic geometry and symbolic algebra. Nevertheless, Descartes was not interested in the motion of isolated bodies as Galileo was, but in the interactions among bodies, a phenomenon Galileo never understood. From Descartes stems the idea that science should search for *universal laws*, that these laws should *describe interaction* and that this description of interaction should have an *ontological foundation*. Despite the fact that Cartesian physics is formulated in ordinary language, which seems to have misled many historians of science, *in a deeper sense Cartesian physics is algebraic*. Cartesian physics takes from algebra the universality of its laws. Even though these laws are formulated in ordinary language, they have the *same kind universality as algebraic formulas*. The language of algebra enabled Descartes to create the first description of interaction. He did it by introducing the notion of the quantity of motion, and formulating the law of conservation of the quantity of motion. It is important to realize that the quantity of motion is *an algebraic quantity* and the law of its conservation is *an algebraic equation*. And finally, we have to realize that the quantity of motion is not a phenomenal quantity. It cannot be measured directly. It is an ontological quantity; it is related to the state of a physical system and not to its appearance. Thus Cartesian physics receives its universality, its ability to represent interaction, and its ontological grounding from the universality, functionality, and abstractness of the language of algebra.[4]

[4] It seems that the *extended body* of Cartesian physics is closer to the *cosa* of the *Cossists* than to the Aristotelian *pragmata*. An extended body enters into mechanical interactions with other bodies just like a *cosa* enters into algebraic relations with other quantities. Furthermore, an extended body has only mechanical properties just like a *cosa* has purely algebraic ones.

I would like to show that it is a mistake to interpret Newtonian physics as a continuation of the Galilean “*non-metaphysical and problem-oriented conception of natural philosophy*”. The Galilean system lacks universal laws as well as a description of interaction, and so it is too far away from the Newtonian system. Therefore the Galilean system cannot serve as a background for understanding of what Newton did when he created his physics. Only when we put Newtonian science against the backdrop of the Cartesian system will it be possible to understand some peculiar features of Newtonian physics. Therefore let us now turn to a reconstruction of the relations between the Cartesian and the Newtonian systems. It can be argued that Newton took the idea of universal laws, the idea of interactions, and the idea of ontology in physics from Descartes. Of course, the *particular* universal laws by which Newtonian physics described a physical system were very different from the laws which Descartes ascribed to it. Similarly, the *particular* way how Newtonian science described interactions among bodies was very different from the Cartesian description of interactions. And finally, the *particular* ontology on which the Newtonian system was based, differed substantially from the Cartesian ontology. Thus not the technical details, not the way in which Newton formulated his laws, described interactions, and introduced ontology were Cartesian. Nevertheless, the very idea that science should search for universal laws, that these laws should describe interactions and that this description of interactions should have an ontological foundation was Cartesian. That is the reason why, despite all the antagonism between the Newtonians and the Cartesians, I believe that there was a fundamental influence of Cartesian philosophy on Newtonian science.

If we take any of the laws discovered by Galileo—the law of free fall, the law of isochronous motion of the pendulum, or the law of the descent on an inclined surface—all these laws are laws describing particular phenomena. On the other hand the laws of the Newtonian system as the law of inertia, the law of force, or the law of action and reaction, are *universal laws*. We can take a falling body, a pendulum, or a body descending on an inclined surface, the tree Newtonian laws apply to all of them. The law of conservation of the quantity of motion, introduced by Descartes, seems to have been the first universal law in physics. This law was universal because, like Newton’s laws, it applied to a falling body, to a pendulum, as well as to a body sliding on an inclined surface. Therefore, in using universal laws in the description of nature, Newtonian science was Cartesian rather than Galilean.

Another interesting common feature of the laws discovered by Galileo is that they are laws describing the behavior of single isolated bodies. The law of free fall describes the fall of one single body just like the law of the pendulum describes the periodical motion of one single body, and the law of the descent on an inclined surface describes the descent of one single body. Thus it is fair to say that Galilean physics lacked the notion of interaction. On the other hand, the fundamental aim of Newtonian physics was to *describe interaction among bodies*. The first theoretical description of interaction among bodies was in all probability given by Descartes, when he introduced the notion of a dynamic state (characterized by extension and motion) and described interaction as a change of this state (in collisions). Therefore in describing interactions among bodies Newtonian science was Cartesian rather than Galilean.

A third characteristic of Galilean science is that it lacked ontology. Galileo intended to develop science in a phenomenological way (I use the term phenomenology not in the Husserlian sense, but in the sense as this term is used in physics). The aim of Galilean science was to discover the mathematical regularities hidden in the phenomena. In contrast to this notion of science, Newtonian physics had a *clear (corpuscular) ontology*. The first who realized the necessity to base physics on ontological foundations was Descartes when he introduced extension as the ontological foundation of the physical description of phenomena. So by building its theories on explicit ontological foundations Newtonian science was Cartesian rather than Galilean.

Newton rejected the Cartesian laws of nature, the Cartesian description of interaction, as well as the Cartesian ontology. Nevertheless, he owes to Descartes the idea that the laws of nature must be universal, that they must describe interactions among bodies, and that these bodies must have some ontological status. These ideas are fundamentally Cartesian, and thus it is fair to say that Newton was closer to Descartes than to Galileo. Some influence of Descartes on Newton is visible already in the title of Newton's *Philosophiae Naturalis Principia Mathematica*. This title is an allusion to the title of Descartes' *Principia philosophiae*. Besides this allusion there are similarities also in the general structure of the two systems. Newton's system has three laws of motion just like the Cartesian system and Newton's formulation of his law of inertia is simply a juxtaposition of the first two laws of Descartes. Nevertheless, these are rather superficial similarities that do not touch the content of the two systems. If we go further, and leave these salient similarities aside, we will find that there is a much stronger sense in which the Cartesian system exerted influence on Newton. The point is that the *main problems solved by Newtonian physics were of Cartesian origin*. To see this we have to concentrate on the main shortcomings of Cartesian physics (just like we concentrated on the main shortcomings of the Galilean system when we wanted to see more clearly the contributions of Descartes). Among the shortcomings of the Cartesian system were: a too loose connection between the phenomenal and the ontological levels; causal openness of the description of motion and an unsatisfactory description of interactions. If we look from this perspective on the Newtonian system, we see that the main achievements of Newtonian science were in a sense answers to, or solutions of, the shortcomings of the Cartesian system (in a similar way as the main achievements of the Cartesian system can be interpreted as answers to, or solutions of, the main shortcomings of the Galilean system).

The first shortcoming of the Cartesian system was that it had only a loose connection between the phenomena and the explanatory models that were used to account for them. Thus for instance Descartes explained the phenomenon of gravity by his vortex model. He postulated a vortex of fine matter, but gave no clue how particular aspects of the vortex (its velocity, structure, orientation, etc.) relate to specific attributes of gravity (its homogeneity, direction, permanence, etc.). This was obviously a main weakness, which gave the whole theory a speculative flavor. In the Newtonian system the ontological level and the phenomenal level are tied together by a mathematical framework, which allows to derive from an attribute of the phenomenon a corresponding aspect of the ontology and vice versa. Thus Newton could for instance derive the inverse square law of universal attraction (i.e. a particular aspect of the force of gravity, which, as all forces, belonged to the ontological level) from Kepler's laws (which belonged to the phenomenal level). But even if these close ties between ontology and the phenomena are a non-Cartesian aspect of the Newtonian system, they can be seen as an answer to a deep tension of the Cartesian system—the unreliability of its explanatory models. And so in an indirect sense it is a Cartesian aspect of the Newtonian system.

Another weakness of the Cartesian system was that its description of motion was causally opened. The mind could, according to Cartesian physics, have a causal influence on the body. Thus *a physical process*, as for instance the lifting of my arm, *can be caused by a nonphysical event*, in this case by my decision to do so. Descartes' description of motion was not causally closed. This causal gap is closely related to the fact that according to Descartes velocity is a scalar quantity. Therefore a change of direction of motion does not influence the value of the quantity of motion. Consequently the law of conservation of the quantity of motion does not determine the changes of direction of motion and thus in the Cartesian system there is a gap where the mind can intervene. Newton closed this gap when he introduced the notion of velocity as a vector quantity. Therefore changes of direction of motion are in the Newtonian system changes of the quantity of motion, and so they must be caused by forces (i.e. by physical causes). Even though the Newtonian notion of velocity as a vector quantity is a non-Cartesian concept, Newton introduced it in order to solve a deep problem of the Cartesian system—its causal openness. And so in an indirect sense it is a Cartesian aspect of the Newtonian system.

A further shortcoming of the Cartesian system was that the notion of the quantity of motion was introduced for the universe as such and so it included the motions of all bodies in the universe. Therefore, strictly speaking, it was impossible to calculate its value. So even if Descartes introduced this notion in order to describe interactions, it could not be applied to any concrete situation. This made, of course, the law of conservation of the quantity of motion practically useless. It is true that Descartes used his law in the description of collisions of bodies. But all these descriptions were counterfactual because in reality, according to Descartes, all bodies were submerged in a vortex of fine matter, which took away portions of the quantity of motion. Therefore in the Cartesian system the law of conservation of the quantity of motion could hold only for the whole universe. Only when Newton turned to empty space as the background of the theory of motion, the conservation of the quantity of motion in smaller systems became possible. Thus by eliminating the Cartesian fine matter Newton opened the possibility to *describe restricted mechanical systems*. For the description of such systems he created a new mathematical tool—differential equations (or, more precisely, something which we today call differential equations). Newton's second law was perhaps the first differential equation in history. The notion of a differential equation, which is a mathematical tool that can describe the temporal evolution of the state of a mechanical system, is a non-Cartesian notion. But Newton introduced it in order to solve a tension in Cartesian physics—its inability to describe interactions in a restricted system. And so in an indirect sense it is a Cartesian aspect of Newton's physics.

We have seen that the most important achievement of Newtonian physics—the mathematical description of interactions in a causally closed mechanical system of finite extension—was an answer to problems inherent in the Cartesian system. Thus, although Newton's system of natural philosophy was very different from the Cartesian system, Newton developed some of the most important aspects of his system in reaction to deep conceptual problems and internal tensions of the Cartesian one.

5 Aspects of the Newtonian Conceptualization of Motion

In the previous chapter I presented several arguments in order to justify taking the Cartesian system as a contrasting background of the Newtonian system. I believe that only against this background will it become sufficiently clear that the fundamental innovations of Sir Isaac Newton (1643–1727) in physics are based on infinitesimal calculus. The infinitesimal aspect of Newtonian physics is hidden at the first sight. In this respect the Newtonian system is analogous to the Cartesian one—in both cases the linguistic surface and the epistemological structure are very different. In the case of the Cartesian system it is difficult to see the fundamental role of algebra in its representation of motion, because the whole system is formulated in ordinary language. Only when we contrast the Cartesian system with the Galilean, its *descriptive universality*, *dynamic unity* and *ontological foundations* become visible. All these hidden features of the Cartesian system are algebraic in their character. So despite being formulated in ordinary language, Cartesian physics is based on the algebraic framework.

After clarifying the connection between the Cartesian and the Newtonian systems it becomes obvious that the Cartesian system is a suitable background for Newton, and we can turn to the analysis of the technical details of the Newtonian description of interaction. On the basis of this analysis, I will argue that Newton's breakthrough in physics was made possible by a fundamental change of the mathematical framework—the creation of the infinitesimal calculus. For some readers this may seem obvious, but we have to keep in mind that Newton's *Principia* are written in the geometrical language and there are historians who dispute the relevance and even the legitimacy of the use of infinitesimal calculus in the historical reconstruction of Newtonian physics.

In the previous chapter we indicated that Newton's creation of his description of interaction had its roots in the conceptual problems of Cartesian physics. It is fascinating to see that many of the components which Newton used in building of his own theory were present already in the Cartesian system, even though they had there a different function than they have in the Newtonian system. We will concentrate on three Newtonian components: the description of interaction as a *transfer of momentum*; the notion of *force*, and the idea that the transfer of momentum is *governed by forces*. It is almost unbelievable, but all three of these fundamental components of the Newtonian theory were present already in Descartes. Descartes understood interaction as a transfer of a particular quantity of motion from one body to another; he used in his theory of interaction the notion of force; and he understood that the transfer of the quantity of motion is governed by forces. Thus Newton could find all the necessary components of his theory of interaction present in Descartes. But the way how these components were put together in the Cartesian and in the Newtonian theory are fundamentally different. Newton had to *exempt* these notions from the Cartesian context, *change* them in a substantial way, and then *reunite* them in a rather different order. The main point of the present section is that Newton was enabled to make these changes thanks to his invention of the infinitesimal calculus.

5.1 Interaction as Transfer of Momentum (Quantity of Motion)

The Cartesian notion of interaction is based on the idea of a *contest*, understood as the collision of two tendencies to preserve the previous state of rest and motion respectively (see Gabbey 1980). The result of the contest is the victory of one tendency at the expense of the other. The paradigmatic Cartesian model of interaction is a collision of two bodies. According to Descartes the greater body determines the outcome of the collision and thus also the further motion of both bodies. If the moving body is greater, then after the collision both bodies will move together in the direction of the original motion. Nevertheless, the description of interaction is *separated* from the description of motion. As long as a body can, it moves uniformly in a straight line. When such motion becomes impossible a collision occurs. So the motion of a body consists of periods of uniform motion in a straight line that are separated from each other by *singular events*—collisions, when the state of the body is changed.

Let us consider a body B moving with the velocity V_B colliding with a resting body C, and let us assume that the moving body is bigger. Before the collision the total amount of the quantity of motion was $B \times V_B$. After the collision both bodies will, according to Descartes, move together with a velocity that can be determined from the law of conservation of the quantity of motion as $V = \dfrac{B \times V_B}{B + C}$. We see, that the original velocity V_B is here multiplied by the factor $\dfrac{B}{B + C}$, which is smaller than 1. This means, that the body B was decelerated, i.e. it lost a particular amount of its original quantity of motion. On the other hand the resting body C started to move, and so it acquired a particular quantity of motion. Both of these quantities are equal to $\dfrac{C \times B \times V_B}{B + C}$. *Thus according to Descartes the interaction consists in an exchange of a particular quantity of motion between the interacting bodies.* Basically the same thing holds also about the Newtonian notion of interaction, even though Newton changed many aspects of the Cartesian notion of interaction (Fig. 1).

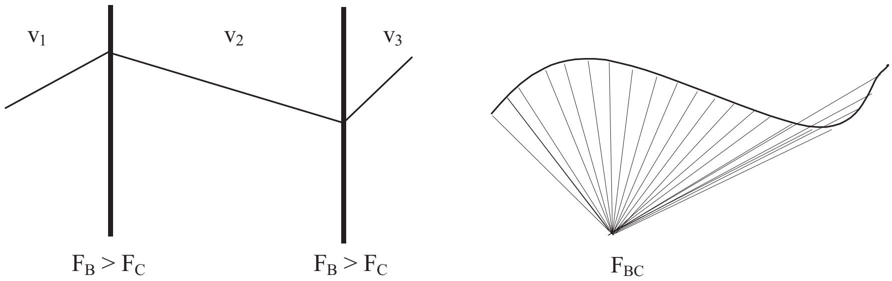


Fig. 1 **a** Descartes' notion of interaction. **b** Newton's notion of interaction

In contrast to Descartes, Newton understands interaction as *cooperation*. In the process of interaction the faster body accelerates the slower body and at the same time the slower body decelerates the faster one. The resulting motion is a compromise, it is the result of the action of both bodies. Thus the result of a collision is neither a simple re-bouncing from the obstacle, nor is it a coupling of the two bodies into one, but something in-between. A second, maybe even more important change is that motion and interaction happen *simultaneously*. They are not separated from each other as in Descartes. According to Newton the forces act all the time and their action accompanies the whole motion. A third change is that interaction is not a singular event, it happens not during isolated moments in time as in Descartes. According to Newton a body acts on the other body during an *infinitesimal time interval* d*t* (or *o*). It is true, that Newton still speaks about impulses of forces, but in all concrete calculations he makes a limit transition. In the limiting case the impulses are becoming infinitesimally dense, and the magnitude of each separate impulse becomes infinitesimally small, thus at the end we are getting a continuous picture. And it is this continuous picture that is important, because all the relations which Newton uses in his calculations hold only for this limiting case.

5.2 The Concept of Force

Another ingredient of the Newtonian theory of motion, which is clearly of Cartesian origin, is the notion of force. Galileo did not use this notion in his physics, because he considered the idea that one body could act on another at a distance as occult. Forces were introduced into physics by Descartes. The role of forces in the Cartesian system was to *preserve the state* of a body. They did not act between bodies; they rather bound each body to its present state of rest or uniform rectilinear motion. Therefore we can represent them using arrows oriented downwards. They play a role only during the moments of collision, when they decide about the direction and velocity of the next interval of uniform rectilinear motion of the body.

Let us consider again the moving body *B* colliding with the resting body *C*. Descartes defined the force for proceeding of the moving body *B* as $F_B = \frac{B^2 \times V_B}{B+C}$, that is as the product of the magnitude of the moving body *B* and the common velocity after the collision $V = \frac{B \times V_B}{B+C}$. On the other hand, the force of resisting of the

Fig. 2 **a** Descartes' notion of force. **b** Newton's notion of force

resting body *C* he defined as $F_C = \dfrac{C \times B \times V_B}{B+C}$, that is as the product of the magnitude of the resting body *C* and the common velocity after the collision. Depending on which of these two forces is greater—the force for proceeding F_B in the body *B*, or the force of resisting F_C in the body *C*—the outcome of the collision will be either that *B* imposes its motion on *C*, or that *B* will simply rebound, while *C* preserves its rest. Thus, *according to Descartes, the force of a body acts upon the body itself and it simply preserves its state of motion or rest* (Fig. 2).

According to Newton, a force is something, by the virtue of which *one body acts on another body*, and causes a change of its state. The role of the preservation of the state, which Descartes ascribed to forces, is in the Newtonian system played by masses. Newton by introducing the notion of mass liberated the forces from the role of binding bodies to their own states and thus opened the possibility to ascribe forces a new role—the role of changing the states of other bodies. Newtonian forces are forces of interaction; they act along the line connecting the two bodies.

5.3 The Role of Force in the Exchange of a Quantity of Motion

According to Descartes, the result of a collision depends on the question which force is greater, whether the force for proceeding in the body *B*, or the force of resisting in the body *C*. The collision is thus determined by a relation of the form

$$\frac{B^2 \times V_B}{B+C} > \frac{C \times B \times V_B}{B+C}.$$

The quantity on the left-hand side is the force for proceeding in the body *B*, the quantity on the right-hand side is the force of resisting in the body *C*. These quantities have the same denominators, and actually the only difference between them is the magnitude of the bodies. The other quantities cancel each other. Thus we arrived at the Cartesian result that the moving body *B* wins the contest if and only if $B > C$, i.e. if the magnitude of the body *B* is greater than the magnitude of the body *C*.

Now we see why Descartes maintained (in spite of contrary evidence) that a moving body can bring a resting body into motion only if it is greater. It is a necessary consequence of the formula. When this happens, the moving body *B* must pass a portion of its quantity of motion to the resting body *C* in order to start its motion.

We see that Descartes saw correctly that interaction consists in the transference of a particular quantity of motion from one body to the other. Nevertheless, *in the Cartesian system the passing of motion from one body to another was separated from the action of forces*. The forces decided about the outcome of the contest, they decided whether the result will be the re-bouncing of the moving body or a common motion of the two bodies. The forces did not play any role in the following transference of motion from one body to the other. This transference was governed only by the law of conservation of the quantity of motion. Descartes thus represented interaction on two levels. The first level consisted in the contest of the forces and it was governed by the above formula. The second level consisted in the transference of motion between the bodies, and was governed by the conservation law. But these levels were separated from each other.

The unfolding of the interaction from a singular moment into the time interval d*t* and the notion of forces as forces of interaction enabled Newton to *join* the action of forces with the transference of momentum. Descartes defined the force for proceeding as $F_B = \frac{B^2 \times V_B}{B+C}$, and the force of resisting as $F_C = \frac{C \times B \times V_B}{B+C}$. Even if these definitions seem similar (the force being in both cases defined as the product of the magnitude of the body and the common velocity after the collision), there is a remarkable *conceptual conflict* hidden in them. Descartes defines the force for proceeding as the *residual* momentum, which is left to the body *B* after the collision, while he defines the force of resisting as the *gain* of momentum, which the body *C* acquires in the collision. Thus it seems as if Descartes hesitated between two ways of connecting forces with momentum.

According to Newton, force is equal neither to the residual momentum nor to the gain of momentum, but to the *velocity of the change of momentum*. Descartes could not understand this connection, because he described interaction as a singular event in time. Therefore, in the Cartesian system, there is no way how to introduce the notion of velocity of change of momentum. We see the fundamental importance of the fact that Newton embedded the transference of momentum into the flux of time. Interaction is for him not a singular event, but it has temporal extension. This made it possible to connect force with the velocity of the change of momentum in his second law

$$\mathbf{F}\mathbf{d}t = \mathbf{d}p.$$

Thus one of the fundamental achievements of Newton was that he connected the change of momentum with the action of a force. The importance of this fact is often misunderstood, and Newton's second law is considered as a mere definition of the concept of force (for instance in Nagel 1961, p. 160). The fundamental conceptual work, which lies behind it, is thus veiled. Newton had to make profound changes in both, the concept of force and the concept of momentum, and above all he embedded the whole interaction into a continuous flux of time, to be able to connect the action of a force with the change of momentum. Only after all this conceptual work is done it became possible to use Newton's second law as an implicit definition of force.

5.4 Summary

We have seen that Newton's theory of interaction consists of the same components as the Cartesian theory: it understands interaction as an *exchange of momentum* (or quantity of motion), it *uses forces* in the description of interaction, and it *relates forces to the changes* of momentum. But these ingredients are in both theories understood quite differently. For Descartes an interaction is a *singular* event. This event has the form of a *contest*. The contest is governed by forces that are *forces of inertia*. These forces are equal to the *residual* or to the *transferred quantity of motion*. In contrast to this, for Newton interaction is a *continuous* process. This process has the form of *cooperation*. This cooperation is governed by forces that are *forces of interaction*. These forces of interaction are equal to the *velocity of the transfer of momentum*. Nevertheless, this very possibility to expose the whole Newtonian theory of interaction and in each fundamental point to be able to contrast it against the Cartesian background clearly shows the conceptual closeness of the Newtonian and the Cartesian theories. It is obvious that no such comparison would be possible say between the Newtonian and the Galilean theory of motion.

6 Conclusion. The Newtonian Style of Experimental Practice

The contribution of Newton to the experimental method consisted in finding a way of studying empirically the ontological basis of reality. Newton developed his new approach to experimental method during his study of colors in 1665–1667 (Hakfoort 1992, pp. 115–121). When Cartesian physics is criticized that it is purely speculative and does not care for empirical data, this criticism is not justified. For instance Hooke was one of the most prominent experimental scientists of his time and his style of experimental work was Cartesian. The Cartesian mechanical philosophy grew out, at least partially, from a criticism of the Galilean experimental method. According to Descartes, it is not enough to study different aspects of reality, but it is necessary to create a mechanical picture about its functioning. The core of Cartesian science is the endeavor to discover the mechanisms which are at the core of the experimental data. Thus the problem is not that the mechanical philosophy would not use experiments but rather that *the experiments are separated from the theoretical work* on the explanatory models which remained speculative. Descartes made experiments, but he used them only to activate his imagination. Theoretical work started after experimental work stopped.

Newton realized that for the further development of mechanical philosophy an experimental control of its theoretical models was necessary. The difficulty was that in experiments only the phenomena are accessible, while the theoretical models postulate ontological entities, which we cannot experience directly (for instance the vortex of fine matter, which is, according to Descartes, the cause of gravity). Newton's answer to this dilemma was his method of *inductive proof from phenomena*. I would like to call Newton's method of inductive proof from phenomena *analytical approach to the experimental method*. The analytical approach in the contemporary sense was born in algebra in 1591, when Viéte published his *In artem analyticam isagoge*. In 1637 Descartes transferred it to geometry. I would like to interpret Newton's contribution to the experimental method as a further step in this expansion of the analytic approach. According to Viéte, the core of the analytical method consisted in three steps. First, we *mark by letters* the known as well as the unknown quantities. The purpose of this step is to cancel the epistemic difference between the known and the unknown. In the second step we *write down* the relations which would hold between these quantities if the problem was already solved. In the third step we *solve* the equations and find the values of the unknown quantities. In contrast to the analytical method as we know it from algebra or geometry, where the basic difference is an *epistemic difference* between the known and the unknown quantities, in the analytical approach to the experimental method the fundamental difference is a *methodological difference* between the measurable quantity (position, velocity) and the non-measurable quantities (forces). Thus Newton first *marks by letters* the measurable quantities as well as the non-measurable ones. In this way he cancels their methodological difference. Then he writes down the equations that hold between these quantities. Finally he derives from these equations some relation in which only measurable quantities occur, which relation can be therefore checked experimentally.

In order to see the novelty of this method, let us compare it with the methods of Galilean and Cartesian physics. Galileo simply refused to speak about non-measurable quantities and thus he considered all theories which supposed for instance an influence of the Moon on earthly phenomena as unscientific. For Galileo the world of science was restricted to phenomenal reality. In this respect Cartesian physics was a step forward. It was able to conceive an influence of the Moon onto earthly phenomena. The vortex of the fine matter could in principle transfer such an influence. Nevertheless, about the physical characteristics of this vortex Descartes was not able to say anything specific and so, at the end, he was not able to say anything specific about this influence itself. This is so, because in Cartesian physics the world-picture is split into two parts. One part is formed by ordinary bodies accessible to experimental investigation, the other part is formed by hypothetical substances, by help of which the results of the experiments are explained. These substances are not accessible to experiments, but only to speculation. Thus Cartesian physics has its phenomenal and ontological levels of description unconnected. Newton realized that the relation between the phenomenal and the ontological levels of description in Cartesian physics is analogous to the relation of the known and the unknown quantities in algebra or analytic geometry. The non-measurable quantity (e.g. the force by which Earth attracts the Moon) has to be marked by a letter and this letter has to be inserted into the equations that hold for such forces. Then some consequence of these equations should be derived, in which only measurable quantities occur. Finally this empirical prediction should be experimentally tested. In this way a measurement becomes a test of the analytic relations, from which the prediction was derived. The greatest advantage of Newton's method is that we are not obliged to measure directly the quantity which we are interested in. The network of analytic relations, into which this quantity is embedded, makes it possible to apply the experimental techniques at that particular place of the network which is for the measurement most suitable.

The characterization of measurement as a standardization of an experiment makes it possible to derive from our interpretation of Newton's experimentation as an analytic approach to the experimental method a new understanding of Newton's innovations in the field of measurement. As an illustration of the possibilities disclosed for physics by Newton's approach to measurement I would like to take the "weighing" of the Earth. In 1798 Henry Cavendish measured the force of attraction between two heavy spheres using very fine torsion weights. When he compared this tiny force with the weight of the heavy spheres, that is with the force with which they are attracted to the Earth, he was able to calculate the mass of the Earth. Cavendish's measurement was therefore often called "weighing of the Earth". In order to understand the novelty of the Newtonian approach to measurement, employed by Cavendish, let us compare it with the Galilean approach. In his measurement Cavendish used fine torsion weights, that is an *instrument*. He created an *artificial situation* that was thoroughly designed to exclude all disturbing effects, which could distort the result of the measurement. So far everything is in accordance with the Galilean approach. Nevertheless, the Galilean scientist would stop here and he would add the new phenomenon of attraction between the spheres to the known phenomena in a similar fashion as he added to them more than a century earlier the atmospheric pressure. The novelty of the Newtonian approach lies in the network of analytical relations which makes it possible to relate the measured force of attraction between the spheres to their weights and from this relation to determine the mass of the Earth.

If we realize how tiny the force between the two spheres is, it becomes clear that a Galilean scientist had no chance to discover it by lucky coincidence. He had no reason to construct such an ingenious experimental equipment as Cavendish created in order to make the force measurable. In ordinary experience there is no clue which could lead him to the discovery of the attraction of bodies, analogous to the failure of the pumping of water from deep shafts that led Torricelli to the discovery of the atmospheric pressure. In everyday experience there is no phenomenon which would reveal that macroscopic bodies attract each other. Therefore, for a Galilean scientist (and for his positivist followers) the gravitational force would remain probably forever undiscovered. Newtonian science differs from the Galilean in that it embeds the phenomena into a framework of analytic relations. It can then use this framework to search for artificial phenomena suitable for testing its predictions. Cavendish used the law of universal gravitation when he planned the experimental situation, in which the forces predicted by this law would become measurable. The law gave him an estimate of the magnitude of the force, and thus also an estimate of the precision that his instruments must reach. Thus while Galilean science uses in an experiment the technical equipment only to alter the phenomena which already exist in our ordinary experience Newtonian science goes much further than that. It *constructs new phenomena, which have no parallel in ordinary experience*. Of course, by this I do not mean that, for instance, the forces of attraction between macroscopic bodies did not exist before Cavendish measured them. These forces existed, but they were not phenomena, they were not accessible to human experience.

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