Galileo's "Technoscience"

Vitaly Gorokhov

Abstract Three main features of engineering thought have formed over the centuries: artistic, practical (or technical), and scientific. In the Renaissance time the relation between art and nature to each other were interpreted in three different ways. Galileo criticized the craftsmen's approach to technical activity that overlooked scientific knowledge and laws of physics in building machinery that would be impossible without them. He created more than a model of experimental activity; he demonstrated how to develop scientific knowledge so that it could be used for technical purposes. That is why "technoscience" is an appropriate name for Galileo's new science.

Keywords Galileo · Scientist-engineer · Scientific engineering education · Natural and engineering sciences · Technoscience

Although engineers resort more often to drawings and diagrams, and scientists to formulas and texts (e.g. papers, monographs, textbooks), modern engineering thinking is basically scientific engineering. Modern engineering thinking is at the same time scientific thinking in opposition of the technical thinking before scientific engineering. The drawing and the diagram (the elements of the engineer's language) are permeated with science and mathematics. The scientific picture of the world worked out during the seventeenth to eighteenth centuries began timidly and hesitantly to intrude on the practice of the ordinary engineer only in the nineteenth century. In the eighteenth century, Galileo's mathematics based on experimental science failed to exhaust engineering practice, which at that time remained an engineering art. We can distinguish already in this time three types of engineers: master-cum-engineer (artist-engineer), engineer-scientist, scientist-engineer (organizer of scientific engineering education and TA-expert).

V. Gorokhov (✉)
Research Center for Philosophy of Technology and Engineering Ethics of the Institute of Philosophy, Russian Academy of Sciences, Russian Federation, Moscow, Russia
e-mail: vitaly.gorokhov@mail.ru

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1 The Emerging of the Scientific Education of Engineers

The first engineers appeared in the Renaissance. They came from the circle of scientists who turned to technology or from groups of self taught artisans interested in science. This period also saw appearance of the engineer, technician-and-expert, whose main and, later, only occupation was the construction of civil structures and military installations. There were also managers of large-scale technical projects and theoreticians of technology in the last decades of the Roman Empire, but both occupations were rigidly divided from one another. A technician remained either a foreman or a supervisor only, who needed no theory; a theoretician was mostly a philosophizing dilettante. Gradually, the engineer became a professional like teachers, doctors, lawyers and so on, although the social organization of engineering and the socio-economic mechanism for protecting priority and inventors' rights had not fully taken shape.

The first engineers of the Renaissance were at the same time artists and architects; consultant engineers specializing in fortification, artillery, and civil structures (Pisano 2009); alchemists and physicians; mathematicians; natural scientists; and inventors. The traditional guild-regulated crafts were gradually replaced by science-based engineering activity. Instead of anonymous craftsmen, more and more professional technicians enter the scene, the outstanding technical personalities whose fame extends far beyond the area of their immediate operation. However, the rapid and radically new development of technology required that its structure should be fundamentally changed. Technology comes to a point from which its further advance is impossible without its saturation with science. The need is felt everywhere for new technical theory, for codification of technical knowledge, for some general theoretical basis of that knowledge. Technology requires the application of science.

> The emergence of the figure of the engineer seen as a technician in some way educated in sciences, is a characteristic feature of the XV century and the first half of the XVI. Indeed this is perhaps the main feature of science, where the reduced creativity (real or apparent) of *pure* scientists, was counterbalanced by the great creativity of *applied* scientists. […] Although there were no public funding to encourage scientists to devote their efforts to the study of technical applications and to the improvement of their knowledge, a common ground arose, particularly in Central and Northern Italy. The link between engineers and scientists emerged, at least in part, through the creation of some technical centres in the courts of the principalities which had been set up. This was the case of Medici's court in Florence, but also, and perhaps more importantly, the court of Milan under Francesco Sforza with its very rich library.[1]

Gradually, the engineer became a professional. Engineering had already broken away from the craft guild structure. The education of artist-engineer was in the artist workshop, in the Abaco schools and Academies. For example the Academy of the Art of Design (*Accademia dell'Arte del Disegno*) in Florence was the first official school of drawing in Europe and an Academy for Doing. The Academy became a model of the training of artists and engineers in Italy. Artists, engineers,

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[1] Pisano 2013, p. 32.

and mathematicians were often equally expert in practical geometry, geodesy, perspective, technical drawing etc. (Pisano and Capecchi 2015; Valleriani 2010, pp. 7–12).

Galileo was directly associated with engineers and technicians of the Renaissance. His scientific career had a "technical" beginning; Galileo studied in Florence, where his teacher was Ostilio Ricci, an engineer and architect belonging to the Tartaglia school. Taking from him an interest in technical practice and engineering problems, Galileo maintained close ties with engineers all his life. The social need for technical innovations in Italy of that time stimulated many people to try their hand in some way at inventing things. Galileo was also caught by this fever. For years he built scientific instruments and carried out tests in a workshop in his house in Padua (Pisano and Bussotti 2015). This city was in the Republic of Venice, and Galileo maintained constant contacts with the Venetian arsenal.

Many medieval views and notions were assimilated during the Renaissance, but they took on a new meaning and conveyed a new emphasis; the comprehension of the divine plan began to be interpreted as the discovery of the laws of nature (acquisition of scientific knowledge), and technical activity in accordance with those laws was interpreted as a practical "engineering" action. As a result, the architect-cum-engineer and the technician-cum-inventor of that time considered nature, described in philosophy and science, to be the object of practical activity, and the latter was regarded as the art that followed the laws of nature. But in the Renaissance time the relation between art and nature to each other were interpreted in three different ways.

The Renaissance brought about a particular attitude towards the engineer differing from that towards the craftsman or technician of the Middle Ages. The engineer, like the Divine Creator, became a creator by creating reality. At the outset he imitates the Creator of the World and nature, and gradually he begins creating the world and another nature. The artist now imitates not so much God's creations, which, naturally, also takes place, as His process of creation: in God's creations, that is, in natural things, the artist strives to discern the law of their making. Man comes to the centre of the Universe; he is the best creation of God, and he rules over all other creatures. Meanwhile, the artist and engineer ceases to be a rank-and-file member of a trade guild and becomes a courtier, a "prince" of the arts, a bearer of the divine gift, equal in his art to God Himself. Giorgio Vasari wrote:

> The origin of the arts we are discussing was nature itself, and that the first image or model was the beautiful fabric of the world, and that the master who taught us was that divine light infused in us by special grace, which has made us not only superior to the animal creation but even, if one may say so, like God Himself (Vasari 1978, p. 19).

Galileo were busy with understanding physical phenomena. He criticized the craftsmen's approach to technical activity that overlooked scientific knowledge and the laws in building machinery:

> I have seen all engineers deceived, while they would apply their engines to works of their own nature impossible [...] (Crombie 1981, p. 277).

The main reason for those errors was that practical engineers who developed their inventions on false foundations deceived nature, failing to see its basic laws.

The rapid development, of the states and trade promoted improvements in military technology, mainly fortification and artillery (Pisano and Capecchi 2009); the construction of water works and civil engineering structures; the manufacture of machines, including ingenious mechanisms and automatic devices for entertainment. The development of artillery and fortification was essential to the existence of the cities and republics in Italy; their independence often relied on the accuracy and range of their cannons and the strength of their fortification. Therefore, engineering consultants were in demand everywhere and were valued by kings, dukes, and citizens.

But traditional artisan skills were no longer enough. That is why the first engineers and inventors turned to mathematics and mechanics, where they got knowledge and borrowed calculation methods. When that knowledge was insufficient, they tried to obtain new knowledge on their own, often becoming very productive scientists. Niccolò Tartaglia (Pisano and Capecchi 2015), for one, was a self-taught engineer and a “free-lance” consultant in mathematics to technicians. While tackling the problem of increasing the range of artillery fire (his tables for calculating missile trajectories were used for a long time by artillery officers) he published a book entitled *Nova Scientia*. The author explained that the book was necessary for every Christian to be better equipped to be offensive as well as defensive against Sultan Suleiman who was then threatening the Venetians.

Knowledge was then considered to be a real power, and the engineer its holder. Nevertheless the question remains, why did the Renaissance bring forth so many engineers and inventors who claimed rights and social status when there had been so few of such people in the Middle Ages (or, perhaps, we know nothing about them)? The spirit of invention and innovation pervaded all the strata of society at that time. Numerous impostors and pseudo-inventors appeared alongside genuine inventors (which means that to be an inventor was prestigious!). It has been considered normal since the Renaissance to claim an inventor’s right, and if an invention bears the name of another man, the inventor suffers beyond all measure. On the contrary, a medieval man ascribed his creation (invention or treatise) to a divine or an indisputable human authority. Hence Polydore Vergil in his “*De Inventoribus Rerum*” (the first edition was published in Venice in 1499) complained that it was impossible to name the authors of many ancient inventions, such as cannon, mills, mechanical striking clocks. This is hardly surprising—they had kept themselves anonymous on principle. On the other hand, many things known in ancient times like gunpowder, for instance, which had been used by the ancient Chinese, were rediscovered, but attributed to individuals during the Renaissance. This too is understandable, because that was when one of the first kinds of engineering, viz. invention, and one of the first engineering professions, viz. the inventor, came into existence. Vergil’s programme as far as inventors were concerned was to list those who first invented or began all things or arts.

To promote their innovations, medieval inventors often concealed their authorship or obscured it, ascribing them to some authority. Finally, many things in this respect can be accounted for by the psychology of the medieval craftsman who did not see himself as separate from his shop, guild, or corporation. While improving his products, the craftsman did not realize that he created something new, and he even did not try to realize that, because the whole socio-cultural situation hindered him from doing so.

In the fifteenth to seventeenth centuries the attitude towards innovations radically changed. The mark of the Master took on a personal significance, and he became a free creative individual. The social status of the Master and his treatment by society also changed.

Albrecht Dürer (1471–1528), a painter, engraver, architect and fortification builder, mathematician and optician is an example of the emancipation of the Master from the world of craftsmen. Born into the family of a Nuremberg jeweller, he belonged to the artisan class, but he managed to overcome their narrow-mindedness and became a learned man (jewellers had to master metal smelting, flattening, alloy mixing, coining, engraving, enameling—quite a lot of skills!), he was the first to sign not only his paintings and engravings, but also his drawings, as he believed that a sketch made by a great, master was more valuable than a carefully executed work of a craftsman. Dürer attached much importance to science, particularly to mathematics, primarily to geometry and perspective, passionately believing in the omnipotence of science. In fact, he worked as a master engineer even in painting, by actually designing portraits (Harnest 1996).

The engineers of the Renaissance did not canonize unattainable standards nor did they belong to a narrow circle of masters of a guild: rather, they tried to improve current technologies, to leave a personal imprint and make them public property, to associate the names of inventors with inventions so that they would bring fame to those people. That was not anything extraordinary in the Renaissance culture, something once created by an individual scientist to demonstrate the omnipotence of science, as it was with Archimedes. Ingenious machines like those developed by Archimedes were now built, by many people everywhere. They not merely amazed people, they became necessary, and their designers were paid by numerous customers and users.

In his letter offering his services to Lodovico Sforza, the Duke of Milan, young Leonardo da Vinci first enumerated his abilities as a military engineer and only then his achievements as a sculptor and artist (Hart 1961, pp. 22–23). Combining the activity of an artist (initially, he was a pupil at the studio of the painter, sculptor, and technician Verrocchio) with that of an experimenter, Leonardo spent a great deal of time on developing entertainment device and water works, on consulting and supervising fortification builders, on experiments with paints, not always successful (his painting of The Last Supper has been soon ruined for that reason). He used to work on his paintings very slowly spending most of his time designing scaffolds and mechanical accessories, which annoyed his pupils and exasperated his customers. And so Leonardo got most of his income as a military and civil engineer.

The list of engineering inventions and tasks offered by Leonardo in his letter was not an empty boast or impracticable "engineering" fantasy, although which of them he could really bring into effect is unknown. During his lifetime Leonardo managed to realize some of his promises, although many others could not have been realized in his times. His notes contain detailed descriptions and drawings, which, of course, are not addressed to anyone in particular, but which indicate a way to embody them in specific structures and devices. Some "draft projects" were based on careful studies of nature (Cianchi 1998).

Scholars believe that Leonardo da Vinci's notes contain descriptions of some apparatus and machines developed by other engineers as well as his own unrealized designs (Pisano and Bussotti 2014). His notes are indicative of a design approach he employed, although he enciphered them. Leonardo was active even in alchemy, but his notes show nothing of the mythical or mystical, they are basically scientific. Leonardo's "projects" were more than the "engineering" fantasies of Roger Bacon. Unrealized does not mean impracticable: Leonardo da Vinci (Pisano 2013) always dismissed empty dreams and fantastic chimeras. Invention creates things non-existent but possible in nature, while fantasy deals with things which are chimerical, impossible, and impracticable. An invention or even a painting was, for Leonardo da Vinci, not merely a product of imagination, a semiartistic inspiration, or a blind adherence to craft traditions; it resulted from a careful study of nature and its laws. He wrote:

> Those, who are not in love with principle or knowledge are like the sailor who goes into a ship without a rudder or compass and who never can be certain whether he is going. Practice must always be founded on sound theory (Parsons 1939, p. 36, p. 37).

Brunelleschi and Alberti did not simply dream about the possibility of creating miraculous enlarging devices; they (and some others) developed the theory of perspective (in effect, geometrical optics). This was used as a basis for building so-called perspective (optical) instruments and devices, primarily the camera obscura, which was the prototype of Galileo's telescope, initially called perspective.

The presence of both unrealized and realized designs is the first feature of a design culture, which came into being during the Renaissance. A design culture can be defined as placing emphasis on the ideal moments of existence, while the spiritual aspect of existence remains real, and material wealth is only a means and not an end; it prefers the reality of the possible to the possibility of the real. Unrealized designs are no less important than those realized.

> It now seems that both the traditional sharp contrast between the great inventor and his colleagues and the more recent attempts to continue Leonardo's engineering activity within the limits of practice, procedures and projects already fully developed by contemporary engineers and those of previous generations must be rejected as inadequate. […] Leonardo was original also in his drawings which, even in their incompleteness, are correctly interpreted as the conceptual equivalent of the "model". […] I this field Leonardo boasts a supremacy which is unrivalled and which places him at the very beginning of modern scientific illustration. Never before had anyone managed to demonstrate a complex technical design so effectively in a drawing (Leonardo 2005, p. 131, p. 132).

Galileo goes in the same way: compare two drafts of the same mechanism for the transform of the rotatory motion into progressive movement from Galileo Galilei and Leonardo da Vinci (Fig. 1).

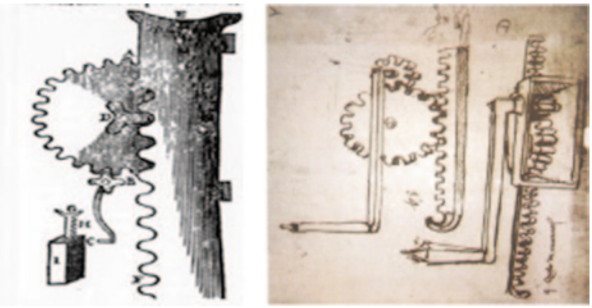

Fig. 1 On the *left* is Galileo's draft of a mechanism for the transform of rotatory motion in the progressive movement (Galilei 1634, p. 103), on the *right* is Leonardo's draft of the same mechanism (Cianchi 1998, p. 76)

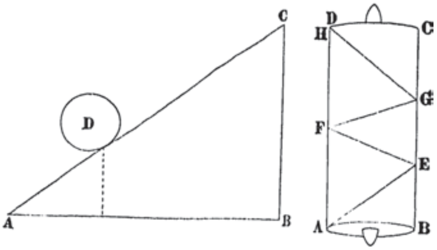

Fig. 2 Geometric representation of the inclined plane as the abstract object of the new science (on the *left*) and the explanation of the functioning of screw with help of this model (on the *right*) by Galileo (Galilei 1665, pp. 26–27)

But Galileo, contrary to Leonardo, reduced such drawings to the geometrical models. For example, he used the inclined plane (the abstract object of the new science) as the universal explanatory model for the functioning of all machines (see Fig. 2).

Galileo investigated in his *Mechanics* the nature of the screw with help of the ideal model of the inclined plane as *triangle*.

> Returning now to our first Intention, which was to investigate the Nature of the Screw, we will consider the Triangle ABC, of which the Line AB is Horizontal, BC perpendicular to the said Horizon, and AC a Plane elevated; upon which the Moveable D shall be drawn by a Force so much less than it, by how much the Line BC is shorter than CA: But to elevate or raise the said Weight along the said Plane AC, is as much as if the Triangle CAB standing still, the Weight D be moved towards C, which is the same, as if the same Weight never removing from the Perpendicular AE, the Triangle did press forwards towards H. For if it were in the Site FHG, the Moveable would be found to have mounted the height AI. Now, in fine, the primary Form and Essence of the Screw is nothing else but such a Triangle ACB, which being forced forwards, shall work itself under the Grave Body to be raised, and lifted it up, as we say, by the head and shoulders. And this was its first Original: For its first Inventor (whoever he was) considering how that the Triangle A B C going forwards raised the Weight D, he might have framed an Instrument like to the said Triangle, of a very solid Matter, which being thrust forwards did raise up the proposed Weight: But afterwards considering better, how that that same Machine might be reduced into a much lesser and more commodious Form, taking the same Triangle he twined and wound it about the Cylinder ABCD in such a fashion, that the height of the said Triangle, that is the Line CB, did make

> the Height of the Cylinder, and the Ascending Plane did beget upon the said Cylinder the Helical Line described by the Line AEFGH, which we vulgarly call the Wale of the Screw, which was produced by the Line AC. And in this manner is the Instrument made, which is […] a Screw; which and insinuate with its Wales under the Weight, and with facility raised it (Galilei 1665, pp. 26–27).

With help of geometry Galileo was able to teach military engineers to use mathematical instruments.

Galileo as scientist-engineer was directly associated with engineers and technicians of the Renaissance. The social need for technical innovation in Italy of that time stimulated many people to try their hand in some way at inventing things. Galileo's military compass was a mathematics teaching instrument (Pisano and Bussotti 2015) for the art of war.

Therefore, Galileo demonstrated how to develop scientific knowledge so that it could be used for technical purposes. Galileo's works paved the way for the formation of engineering thinking and activity in practice as well as theory.

The medieval universities were unable to provide broad practical knowledge and they stood aloof from the developing engineering practice. However, the desire to obtain such knowledge together with mathematics was growing among the public. The scholastic university science exhausted its potential, but new knowledge was very slow to catch on. In time, of course, the fresh air of change swept away the traditionalism of university science, and it made a significant contribution to development of the theoretical basis of engineering. However, this breakthrough took place in the seventeenth to eighteenth centuries within other new social structures, such as scientific societies, academies and later high technological schools.

École Polytechnique (*Polytechnique*) was founded in 1794 in Paris by Gaspard Monge. The *Polytechnique* was oriented to the theoretical instruction of students from the initial period of its existence. The *Polytechnique* proved to be a standard for many engineering schools in Russia, Germany, Spain, Sweden, and the USA. For the first time students were introduced there to genuine mathematics and genuine theoretical science. The School's first graduates—polytechnic engineer (Poinsot, Poisson, Cauchy, Navier, and others)—made a great contribution to the development of experimental and engineering science. This was the first time that the curriculum of a higher technical school included a course in machine design.

Galileo not only related a geometrical scheme to physical reality, but also to the construction of different complex machines. But it was Euclidean geometry. The next phase of development of the theory of mechanisms (kinematics of machinery) as an engineering science was not elaborated by Euclidean geometry but by the descriptive geometry of Gaspard Monge. But in both cases, scientific engineering education was a decisive factor for development of the theoretical basis for codification and systematization of practical technical knowledge.

2 Galileo's New Science as "Technoscience"

2.1 *The Structure of Natural-Scientific Theory*

In the structure of natural-scientific theory we shall differentiate three basic components: theoretical schemes, the mathematical apparatus and the conceptual apparatus.

Theoretical (ontological) schemes are a set of ideal objects of theory, which are, on one hand, oriented to application of the appropriate mathematical apparatus and, on the other hand, oriented to the "mental or thought experiment" or conceptual modeling, i.e. to the design of possible experimental arrangements. At the same time they are developed so that mathematical operations (calculations) are possible with them. Theoretical schemes are special idealized models, often in the form of graphical (geometrical) representations. Theoretical schemes also express a particular "vision of the world" from a specific point of view given in theory (that is why they cannot be referred to as ontological). On one hand, they reflect some properties and aspects of real objects which are of interest to a given theory, and, on the other hand, they are its operational means for a particular idealized representation, which can then be practically realized in experiment (through eliminating side effects). Thus, it is the so-called ideal objects (in other words, abstract, idealized, objects of theory) that are of considerable importance, especially in mathematized scientific theories. They are specially constructed in theoretical knowledge as a result of a particular idealization and schematization of experimental objects. In a broader context the things concerned must be not only experimental objects, but also objects of e engineering activity—technical systems.

> Among ideal objects used in a scientific research at least two main types are traditionally singled out: empirical and theoretical objects. Empirical objects are abstractions which fix features of real objects of experience. They are a kind of schematization of fragments of the real world. Any feature—the "carrier" of which is an empirical object—can be found in corresponding real objects … Theoretical objects, unlike empirical ones, are idealizations, "logical reconstruction of reality". They may be provided not only with features corresponding to the features and connections of real objects, but also with features not proper for any such object (Stepin 2005, pp. 47–48).

The mathematical apparatus is necessary chiefly for analyzing experimental situations which are a means of substantiation and verification of obtained theoretical knowledge. Moreover, in a well-developed theory it is used for its evolutionary presentation or deductive transformation of ideal objects. The mathematization of ideal theoretical object transformation rules makes it possible to obtain new knowledge without resorting to experiment and observation, i.e. staying within the theoretical activity framework. One can say that one or another science is really mathematized only when it begins using mathematical methods for processing experimental research findings, but also in looking for new laws, constructing theories and developing a special formalized language of this science.

> In the language of science the syntactical aspect becomes most important when we make formal operations with symbols, for instance, with physical magnitudes (which enter mathematical expressions of physical laws), in accordance with rules of mathematics. Making such operations, a researcher disengages himself from the meaning of the linguistic terms and considers them only as signs which create formulae in their connections and then deduce other formulae according to the rules of the linguistic system given (Stepin 2005, p. 46).

To mathematize a scientific discipline, it is necessary to simultaneously or even preliminarily evolve an adequate conceptual apparatus. Theoretical schemes and the mathematical apparatus are always used in the context of a particular conceptual environment. In this sense the conceptual apparatus is necessary for the conceptual fixation of theoretical schemes and mathematical apparatus of a theory. At the same time each theoretical concept contains, in a non-evolved form, a corresponding theoretical scheme and mathematical procedure. For example, the physical sense of the concept of capacity is different from the common one. In physics, it is, on one hand, a theoretical scheme of a particular physical process (the flow of electric charges in the capacitor plates or of current through the capacitor) and, on the other hand, a mathematical operation of integration when it is considered in the operational calculus context.

> In the theoretical language a theoretical scheme can be characterized by means of at least two types of expressions. First, it may be pithy descriptions like those regarded above: "a material point is moving along the continuum of points of a spatial-temporal frame of reference", "the force changes the state of motion of a material point" etc. Such expressions describe connections and relations of abstract objects forming a theoretical scheme. At the same time these connections can be expressed as mathematical dependencies. This can be reached through mapping abstract objects of a theoretical scheme onto the objects of mathematics. For instance, a frame of reference may be connected with coordinates (the inertial frame of reference in mechanics can be identified—within certain limits—with a system of rectangular, spherical or cylindrical coordinates in Euclidean space) (Stepin 2005, p. 53).

In theory, several conceptual strata (levels) correlating to each other can be differentiated.

Three main levels in the theoretical (ontological) schemes of a natural scientific theory can be discerned. *The functional scheme* is oriented on the mathematical description and fixes the general idea about the object under investigation. The units of this scheme reflect only the functional properties of the elements of the object for the sake of which they are included in it to attain the general objective and reflect certain mathematical relations. In classical natural science it was a geometric scheme of physical processes. *Flow schemes* describe natural, for instance, physical processes taking place in the object and connecting its elements into a single whole. The units of such schemes reflect various operations performed in a natural process by the elements of the object. These are based on natural-scientific concepts. Finally, *structural schemes* reflect the structural arrangement of elements and linkages in the given experimental equipment. These schemes represent parameters of the projects of new experimental situations.

One of the best examples is Galileo's theoretical investigation in which he sequentially moved from a geometric scheme to physical processes and from the latter to a design diagram. Galileo staged an experiment mentally (he compared the rotation of two wheels—one small, one large—on the geometric drawing) and arrived at a radically new conclusion, which went against the first one, passing from the artificial, technical, model to an explanation of a natural phenomenon:

> […] it might be supposed that the whirling of the earth would no more suffice to throw off stones than would any other wheel, as small as you please, which rotated so slowly as to make but one revolution every twenty-four hours (Galilei 1952, p. 217).

This approach gives rise to an ideal and real, natural and mathematical, syncretic object, that is, an object that hides a mathematical (geometric) scheme. Galileo noted in this respect:

> […] because of the imperfection of matter, a body which ought to be perfectly spherical and a plane which ought to be perfectly flat do not achieve concretely what one imagines of them in the abstract… Then whenever you apply a material sphere to a material plane in the concrete, you apply a sphere which is not perfect to a plane which is not perfect, and you say that these do not touch each other in one point. But I tell you that even in the abstract an immaterial sphere which is not a perfect sphere can touch an immaterial plane which is not perfectly flat in not one point, but over a part of its surface, so that what happens in the concrete up to this point happens in the same way in the abstract. It would be novel indeed if computations and ratios made in abstract numbers should not thereafter correspond to concrete gold and silver coins and merchandise. Do you know what does happen, Simplicio? Just as the computer who wants his calculations to deal with sugar, silk and wool must discount the boxes, bales, and other packing, so the mathematical scientist (filosofo geometra), when he wants to recognise in the concrete the effects which he has proved in the abstract, must deduct the material hindrances, and if he is able to do so, I assure you that things are in no less agreement than arithmetical computations. The errors, then, lie not in the abstractness or concreteness, not in geometry or physics, but in a calculator who does not know how to make a true accounting. Hence if you had a perfect sphere and a perfect plane, even though they were material, you would have no doubt that they touched in only one point; on the other hand if it is impossible to have these, then it was quite beside the purpose to say that *sphaera alnea non tangit in puncto* (a bronze sphere does not touch a plane at a point) (Galilei 1952, pp. 207–208).

Moreover, Galileo believed there existed engineering methods that allowed imperfect material objects "bulging" from a perfect geometric form to be brought near to ideal, mathematically perfect objects. Examples are a grinding process or the use of an undeformable material for which insignificant deviations from an ideal shape can be neglected (this is the way he designed his telescope).

There is no doubt that Galileo, for all his abstract reasoning, was guided by the engineering practices of his time:

> Maybe these mathematical ratios which are true in the abstract do not exactly correspond when applied in the concrete to physical and elemental circles. Though it does seem to me that a cooper, in determining the radius of the bottom to be made for a barrel, makes use of the abstract rules of the mathematicians despite such bottoms being very material and concrete things (Galilei 1952, pp. 232–233).

In a well-developed science, theory formation is generally begun from using a theoretical model from some better-developed area of science, which serves as an initial one, and is corrected for a new class of phenomena. For example, Galileo has borrowed the geometro-kinematic scheme from astronomy where motions of celestial bodies along ideal curves were considered in the purest form, in accordance with the theorems and postulates of Euclidean geometry. The further restructuring of this model was by means of constructive introduction of new ideal objects.

At early stages of science development, theoretical models are evolved through the direct schematization of experience. Then these initial models are used, as means for construction of new theoretical models and this way becomes decisive in science development. Galileo realised in practice the purposeful application of scientific knowledge that formed the basis of engineering thought and engineering activity. This approach became possible because Galileo's new science had its roots in technical practice (which had progressed by his time and was urgently in need of generalisation) and was oriented to it.

2.2 The Structure Engineering Theory

In engineering theory, the same components (theoretical schemes, the mathematical apparatus and conceptual apparatus) can be differentiated. However, their content will be different from the natural-scientific theory.

Three main levels in the theoretical (ontological) schemes of an engineering or technological theory can be discerned. *The functional scheme* is oriented on a mathematical description and fixes the general idea about the *technical system*, irrespective of the method of its realization. The units of this scheme reflect only the functional properties of the elements of the technical system for the sake of which they are included in it to attain the general objective and reflect certain mathematical relations. *Flow schemes,* or schemes of performance, describe natural, for instance, physical processes taking place in the technical system. The units of such schemes reflect various operations performed in the natural process by the elements of the technical system while it is functioning. Finally, *structural schemes* reflect the structural arrangement of elements and linkages in the given technical system and presuppose its possible realization. The elements of the latter are regarded in them as having not only functional properties, but also properties of the second order, i.e. those undesirable properties which are added by a definitely realized element, for instance, non-linear distortions of the amplified signal in the amplifier. These schemes represent constructive-technical and technological parameters, i.e. they reflect specific problems cropping up in engineering practice.

The functioning of engineering theory is realized by the iteration method. At first a special engineering problem is formulated. Then it is represented in the form of the structural scheme of the technical system which is transformed into the idea about the natural process reflecting its performance. To calculate and mathematically model this process a functional scheme is constructed. Consequently, the engineering problem is reformulated into a scientific one and then into a mathematical problem solved by the deductive method. This path from the bottom to the top represents *the analysis of schemes.* The way in the opposite direction—*the synthesis of schemes*—makes it possible to synthesize the ideal model of a new technical system from idealized structural elements according to the appropriate rules of deductive transformation, to calculate engineering discipline can be considered as formed when a mathematized engineering theory is constructed in it. It should also clearly give the procedures of the transition from structural schemes to flow and functional schemes (schemes of analysis) and vice versa (schemes of synthesis). Only when an engineering science has worked out the means of the theoretical synthesis of engineering systems which make it possible to extrapolate the theoretical results obtained for the class of hypothetical technical systems (with the orientation on practical and methodological knowledge) can its generalized ontological scheme be considered universal in relation to the given class of objects.

Engineering disciplines are peculiar in that the engineering activity takes the place of experiment in them. It is in the engineering activity that theoretical conclusions are checked for adequacy and new empirical material is drawn. That is why theoretical knowledge must be brought here up to the level of practical engineer recommendations. In the natural science, the most important thing is solving theoretical problems in terms of the natural process reconstruction aimed at prediction and description of its future states. Here mathematical relationships and experimental findings are mere auxiliaries used for substantiation, analysis, validation, etc. The specificity of the engineering theory is based on that its findings are used largely for constructing technical systems rather than explaining natural processes. To solve this problem, the theory must feature clear-cut rules of correspondence and transition from some "model" levels to other ones. In the engineering science the problem of interpretation and empirical substantiation is formulated as a problem of realization. The specificity of the engineering theory is based on that its findings are used largely for constructing technical systems rather than explaining natural processes. The requisite condition of engineering theory productivity is the presence of practical methodological knowledge, i.e. engineering recommendations stemming from theoretical research, in its empirical basis.

Galileo's geometric-kinematic theoretical schematic model of the machines was a beginning and precondition of the application of the natural scientific theory to the first special engineering science—the theory of mechanisms and machines or kinematics. Franz Reuleaux so as Galileo

> Personified a new figure in the industrial age, *the engineer-scientist*; professor, kinematics theorist, head of a university, industrial consultant and confidant to capitalists, government expert and technical ambassador to the emerging global industrial world … The machine, he said, consists of one or more mechanisms which can be separated into kinematic chains which in turn can be broken down into kinematic pairs or fundamental mathematical constraints. The tools of this reductionism is *analysis* … Reuleaux believed there were scientific principles behind invention and the creation of new machines or what we call *synthesis* today (Moon 2001, p. 5).

Franz Reuleaux in his "Kinematics of Machinery" wrote that kinematics or phoronomy (pure kinematics or kinematics geometry) is "the study of geometric representation of motion".

> "For the practical mechanician who has made himself familiar with the modern Phoronomy, and still more for the theorist, the machine becomes instinct with a life of its own through

> the rolling geometrical forms everywhere connected with it". He said that "the geometrical abstraction of machine" is "the soul of machine" (Reuleaux 1876, p. 56, 84, 85).

On one hand, while using engineering means, Galileo reasoned and acted essentially as a natural scientist. He not only developed stricter scientific terms but also "designed" a peculiar plan of reasoning, an ideal mental experiment as a "project" of a real experiment, an idealised concept of natural objects which then could be actually realised in an experiment (by eliminating the influencing factors). In experimental science, however, the scientist must build up a logical theory explaining and predicting the run of a given natural phenomenon and also design a practical experiment reproducing that phenomenon artificially, in its "purest" form, ignoring its unessential properties and verifying the validity of the theory. Indeed, it is necessary, in order to conduct an experiment, to eliminate side effects and to reproduce a natural process in an engineering way under conditions that can hardly be found in nature in pure form. For example, when checking the law of free fall of bodies, Galileo selected a ball made from a hard material, which meant its deformation could be neglected. In addition, he did his best to eliminate friction in a slot cut on a board by gluing polished parchment to the surface.

On the other hand, the situations experimentally reproduced by engineering methods must be presented and described scientifically as certain idealised constructions. In such an experiment, the construction was represented by an inclined surface. The experimental situation thus obtained was then considered as some idealised natural process of motion of natural bodies on an inclined surface, that is, objectively. The theoretical scheme obtained could be extended to cover the whole class of real objects for which friction and elastic deformations can be neglected. In the way Galileo reasoned as an engineer (or better to say as scientist-engineer in the engineering science), and, quite naturally, he often appealed to craftsmen's technical practice rather than to pure observation and the contemplation of the obvious, which was typical of antique science. While proving that the Moon's surface was rough by the manner in which it reflects the sunlight, Galileo wrote:

> Burnished steel appears very bright from some viewpoints and very dark from others [...]. And note that the diversity of what is seen upon looking at a burnished surface causes such a different appearance that to imitate or depict burnished armor, for example, one must combine pure black and white, one beside the other, in parts of the arms where the light falls equally (Galilei 1952, p. 78, 79).

To prove his statements Galileo also resorted to observation of operating technical devices, for instance a pump. He wrote in his Dialogues Concerning the Two New Sciences:

> The stock of the pump carried its sucker and valve in the upper part so that the water was lifted by attraction and not by a push as is the case with pumps in which the sucker is placed lower down. This pump worked perfectly so long as the water in the cistern stood above a certain level; but below this level the pump failed to work. When I first noticed this phenomenon I thought the machine was out of order; but the workman whom I called in to repair it told me the defect was not in the pump but in the water which had fallen too low to be raised through such a height; and he added that it was not possible, either by a

> pump or by any other machine working on the principle of attraction, to lift water a hair's breadth above eighteen cubits (an ancient measure of length, about 18–22 inches, originally the length of the arm from the end of the middle finger to the elbow); whether the pump be large or small this is the extreme limit of the lift (Galilei 1952, p. 137).

That some natural phenomenon could not be reproduced artificially was a weighty argument for Galileo. A mathematical object (e.g. a point) for him always corresponded both to a natural physical object (say, a stone) and to a man-made object (e.g. a cannon ball). Galileo not merely compared them, he idealised them, "designing" in theory particular "ideal objects" (in other words, the abstract, idealised objects of theory).

These objects are specially designed in theoretical knowledge as a result of a particular kind of idealisation and schematisation of experimental, and hence, technical systems. Such are Galileo's inclined surface or the mathematical pendulum—an idealised model of a gravity pendulum, which can be used to investigate the laws of free fall. Here, the action of one cause, the resistance of the air, is separated from the action of another cause, the pull of gravity. It is this experiment that, in his view, proved without any doubt that Aristotle's ideas prevalent at the time were invalid. Without such an idealisation, both experimental science and engineering science were impossible.

The modern engineer mostly has to do with drawings, diagrams, plans, rather than directly with real technical objects; he does not manufacture them with his own hands but directs the manufacturing process, plans for it, and organises their service and maintenance. The activity of the engineer is much closer to that of the experimenting research scientist than is often thought. Today the close connection between natural and engineering science is expressed in the development of technoscience.

> The term 'technoscience' is increasingly being used to refer to such contemporary disciplines as information and communication technology, nanotechnology, artificial intelligence and also to biotechnology. The term technoscience' is thus not only a useful pointer to the highly commercialized setting in which modern biotechnology and many other contemporary undertakings are conducted, it also suggests that science and technology, which presumably were once distinct activities, have become so much intertwined as to be virtually indistinguishable nowadays. The present popularity of the term may thus reflect the historical process in which science and technology have become increasingly interwoven, but it may also partly reflect the dominant preoccupations and concerns of those who use the term (van den Belt 2009, p. 1311).

But such connection between science and technology is typical also for Galileo.

> Galileo, son of the Renaissance, offered his contribution to architecture by means of an interdisciplinary style thinking, intertwining operational and theoretical skills, mathematics, physics and art. Besides facing theoretical thematic […] he was busy with civil, military facts as he reported in his work on fortifications (Pisano and Capecchi 2009, p. 28; see also Pisano and Bussotti 2014).

2.3 The Structure of Technoscience

In technoscience, on the one hand, explanatory models of natural phenomena are drawn up and predictions of the course of certain natural events on the basis of mathematics and experimental data are formulated as in classical natural science, and as in the engineering sciences, on the other hand, not only experimental arrangements are constructed, but also structural plans of new technical systems previously unknown in nature and technology.

Galileo did more than just observe natural phenomena. He would first *construct an idealized experimental situation*, leaving aside the question of its technical feasibility (the situation itself, while not existing in nature, was, however, reproducible in principle). Then he would design an ingenious project of the technically feasible experimental situation, say a pendulum (a mass suspended from a string), where the gravity force was separated from the force applied to the solid. Based on this project, a real experiment could be devised and conducted.

Similarly interms of nanotechnoscience:

> Nanotechnology comprises *not only the manipulation of natural molecules, but also the creation of molecules not found in nature*. The multifariousness of the relationship between nanotechnology and nature is expressed in the fact that some nanotechnological objects are clearly distinct from comparable natural objects, while others are identical to natural objects. Nanotechnology, however, does not only create an artificial world that is distinct from nature. It also relates to natural processes and materials in a new way. In this respect it is difficult to separate it from nature (Schiemann 2005, pp. 77–96).

Galileo Galilei was one of those who created this new science oriented to technical needs. He established the relation between scientific knowledge and the objects of practice. His fundamental work *Dialogues Concerning the Two New Sciences* begins with a description of Venice's famous arsenal:

> The constant activity which you Venetians display in your famous arsenal suggests to the studious mind a large field for investigation, especially that part of the work which involves mechanics; for in this department all types of instruments and machines are constantly being constructed by many artisans, among whom there must be some who, partly by inherited experience and partly by their own observations, have become highly expert and clever in explanation… You are quite right. Indeed, I myself, being curious by nature, frequently visit this place for the mere pleasure of observing the work of those who, on account of their superiority over other artisans, we call 'first rank men'. Conference with them has often helped me in the investigation of certain effects including not only those which are striking, but also those which are recondite and almost incredible (Galilei 1952, p. 131).

Galileo created more than a model of experimental activity; he demonstrated how to develop scientific knowledge so that it could be used for technical purposes. This approach became possible because Galileo's new science had its roots in technical practice (which had progressed by his time and was urgently in need of generalization) and was oriented to it. Galileo constantly emphasized the practical orientation of his idea. In the foreword to his "Discourse on Bodies in Waters" he noted, for instance, that his work was useful "in occurrences of building bridges or fabrics on the water" (Galilei 1980, p. 3). In his new science, Galileo manipulated natural objects like the present-day engineer:

> […] if the terrestrial globe were perforated through the centre, a cannon ball descending through the hole would have acquired at the centre such an impetus from its speed that it would pass beyond the centre and be driven upward through as much space as it had fallen, its velocity beyond the centre always diminishing with losses equal to the increments acquired in the descent; and I believe that the time consumed in this second ascending motion would be equal to its time of descent (Galilei 1952, p. 227).

However, Galileo's new style of scientific-engineering and engineering-scientific thought and action manifested itself mainly in the sphere of thought rather than in practical activity.

That is why "technoscience" is an appropriate name for Galileo's new science. This is a combination of the natural and engineering sciences. In relation to the observers of nature, Galileo is more of a practical man, who destroys and restructures a natural object in order to discover a universal principle underlying it, and in relation to practice, he is more of an observer, who sees in an engineering process the universal law it reveals, not a particular end to be achieved. It is the middle place between natural and engineering sciences that is characteristic for technoscience.

3 Conclusion

Galileo's works paved the way for the formation of engineering thinking and activity in practice as well as in theory. Galileo himself was not engaged in the building and designing of machines. But he was able to produce a new science, that is a new scientific approach to physical phenomena also taking into account mechanics, devices and techniques (i.e., applied to cantilevers and machines modelling).

The first engineers in the Renaissance came from the circle of scientists (*scientist-engineers*) who turned to technology or from self taught artisans (*artist-engineers*) interested in science. This period also saw appearance of the engineer, technician-and-expert, whose main and, later, only occupation was the construction of civil structures and military installations. The teacher of Leonardo da Vinci, Verrocchio, who had come from a handicrafts tradition, was profoundly occupied with mathematics and taught it to his pupils. According to Leonardo mathematical relationships are found everywhere in nature. Proportions are found not only in numbers and measures, but also in sounds, weights, times, and places, and in every force. Galileo was familiar with the theory of perspective put forward by Italian artists. He had a life-long friendship with Lodovico Cigoli, an outstanding painter of his time. Galileo even helped him (in a letter) to argue against those who stated that sculpture was superior to painting. This is a geometrical interpretation of nature, or, in other words, materialized geometry that enabled Galileo to develop a new science—a mathematized experimental natural science. The visual representation of natural objects by the Renaissance painters made it possible to describe them in terms of geometry in the science of modern times. Modern engineering also employs its methods: the use of drawings and schematic diagrams lays the groundwork for future engineering projects and graphic design documentation. In his notes

Leonardo da Vinci, being a genuine engineer, hotly insisted on the advantages of drawings over verbal descriptions:

> What poet can represent to you in words, oh lover, the true image of your ideal as faithfully as the painter will do? (Richter, I, p. 55).

Therefore, painting for the artists and engineers of the Renaissance was not merely a natural science but also a means for working out the rules of action based on the disclosed laws of nature.

> “If you ask me what these rules accomplish or what good they are,” wrote Leonardo da Vinci, “I would answer that they keep a restraining hand on engineers and investigators, teaching them not to promise impossible things to themselves or others, in consequence of which they may be considered either crazy or impostors” (Parsons, p. 371).

Painting for the Renaissance artists was primarily designing a perfect image: if there was no such a thing in nature, the artist made it up from various things that actually existed in nature (as the image of a perfect man). But this approach again required science, the study of natural structures.

> The painter who draws by practice and judgement of the eye, without the use of reason, is like the mirror which reproduces within itself all the objects which are set opposite to it without knowledge of the same (Parsons, p. 24).

For the analysis of complex machines Galileo applied geometrical representation of their principle of operation (Fig. 3). He started his “Mechanics” with appeal as his program of the theoretical analysis of machines:

> mechanicians deceive themselves in going about to apply machines to many operations of their own nature impossible; by the success whereof they have been disappointed, and others likewise frustrate of the hope which they had conceived upon the promise of those presumptuous undertakers: of which mistakes I think I have found the principal cause to be the belief and constant opinion these artificers had, and still have, that they are able with a small force to move and raise great weights [...]. In the mean time, since I have hinted, that the benefit and help derived from machines is [...] to move those weights, which, without it, could not be moved by the same Force: it would not be besides the purpose to declare what the commodities be which are derived to us from such like faculties, for if no profit

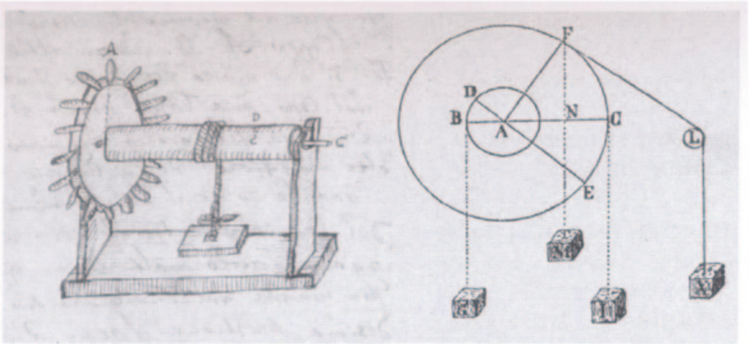

Fig. 3 Practical description of axle wheel on the *left* and geometrical illustration of the same instrument on the *right*. (Valleriani 2010, p. 101)

> were to be hoped for, all endeavours employed in the acquist thereof will be but lost labour (Galilei 1665, pp. 1–2).

Similarly, Robert Willis wrote in his "The Principles of Mechanisms" in 1841:

> My object has been to form a system that would embrace all the elementary combinations of mechanism, and at the same time admit of a mathematical investigation of the laws by which their modifications of motion are governed. I have confined myself to the Elements of Pure Mechanism, that is, … to reduce the various combinations of Pure Mechanism to system, and *to investigate them upon geometrical principles alone* (Willis 1870, p. xiii, p. 4; see below Fig. 4).

In engineering science (theory of mechanisms) it is important to reduce the constructive mechanism (or machine design) as a real technical system to the various combinations of pure mechanism (sometimes called kinematics of machinery) as an ideal model of this system.

Before Galileo, scientific studies followed the ancient standard of obtaining knowledge about an object that was regarded as unchangeable. It occurred to nobody to change practically the real object of investigation (as it would then be considered to be another object). On the contrary, scientists strove to improve their theoretical model so that it would fully describe the behaviour of the real object. In Galileo's view, the real object exactly corresponds to the ideal object but is interpreted as a distortion of the ideal object's behaviour under the action of various factors, for instance friction. This made it possible for Galileo to modify the real object by

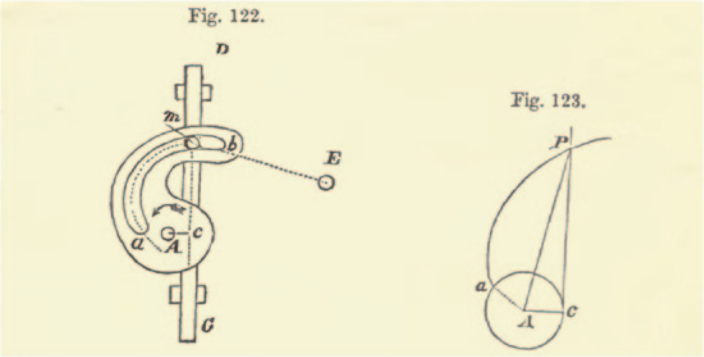

Fig. 4 "In fig. 122, *A* is the center of motion of a revolving plate in which a slit *a b* is pierced, having parallel sides so as to embrace and nearly fit a pin *m*, which is carried by a bar *CD* fitted between guides so as to be capable of sliding in the direction of its length. If the plate revolve in the direction of the *arrow* the inner side of the slit presses against the pin and moves it further from the center *A*, but when the plate revolves in the opposite direction the outer edge of the slit acts against the pin and moves it in the opposite direction. If the curved edges of the slit be involutes' of the circle whose radius is *Ac*, where *Ac* is a perpendicular upon the path *m c* of the bar, it appears from Art. 133 that the velocity ratio of plate and bar will be constant, and the linear velocity of the bar equal to that of the point *c* of the plate. But if any other velocity ratio be required, let *Pc* (fig. 123) be the path of the sliding bar, *P* the pin, *A* the center of the curve, *aP* the curve." (Willis 1870, pp. 152–153)

acting on it in a practical way. As a result, its "negative" properties, which prevented it from being identical to the ideal object, became neutralized. Galileo chose an approach unusual for scholastic science: technology began to lean on mathematical knowledge and models. The orientation towards both engineering practice and mathematical knowledge (obtained strictly analytically) largely determined the line of development of Galileo's ideas.

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