Neural Dynamics Associated to Preferred Firing Sequences

Alessandro E.P. Villa, Alessandra Lintas, and Jérémie Cabessa

Abstract In a distributed recurrent neural network equivalent changes at one synapse might correspond to different patterns of activity and changes in strength at particular links between two cells may become meaningless.The information is not necessarily resident in the links among the units, but is likely to be provided by the activity organized in a highly precise temporal mode. Precise spatio-temporal firing sequences and attractor dynamics may be strongly associated, such that the detection of spatio-temporal firing patterns may reveal the existence of underlying modes of activity controlled by few parameters in deterministic chaotic dynamics.

Keywords Recurrent neural networks • Activity patterns • Spatio-temporal firing • Attractor dynamics • Chaos
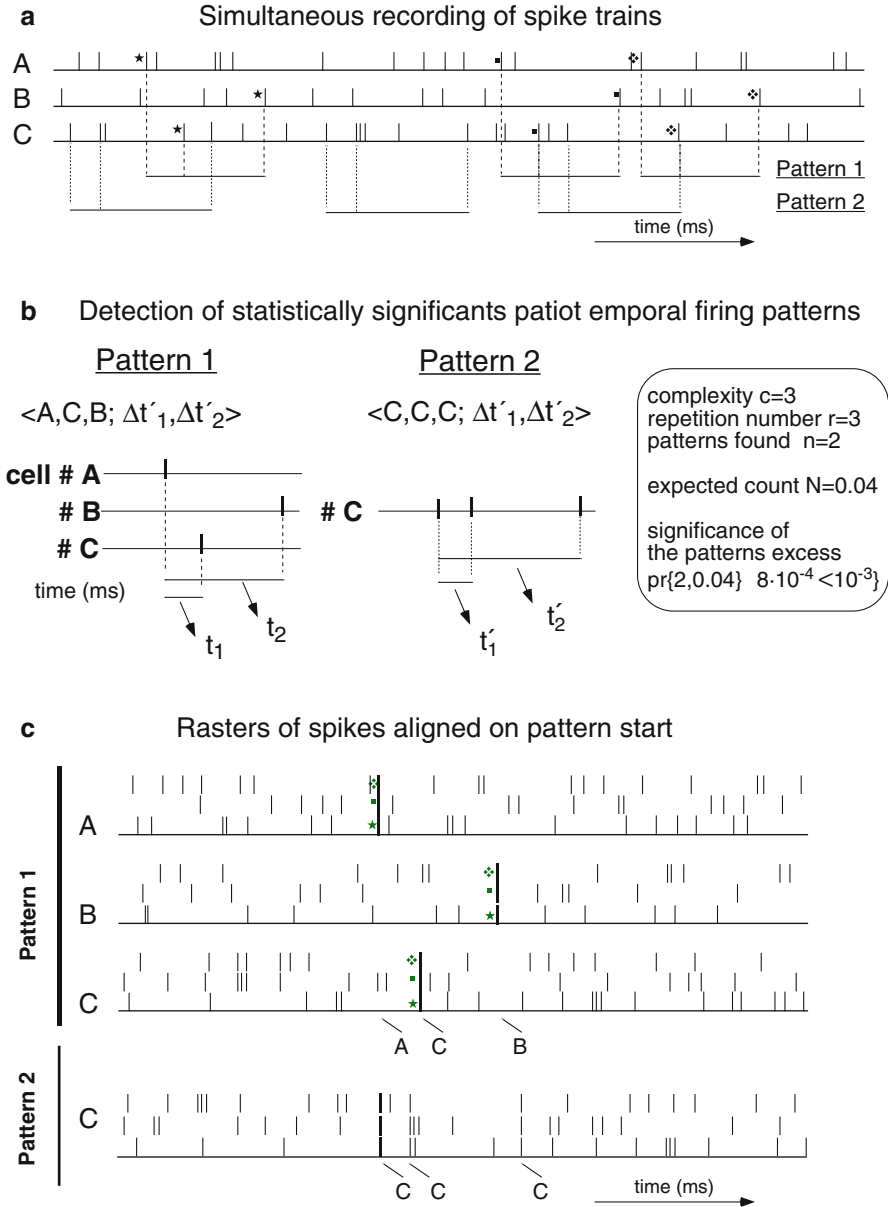
1   Spatio–Temporal Firing Patterns

The majority of neural circuits of the forebrain, i.e. the basal ganglia thalamocortical circuit, are formed by highly interconnected networks of neurons in which the activity of each cell is necessarily related to the combined activity in the neurons that are afferent to it. Reentrant activity through chains of neurons is likely to occur due to the presence of recurrent connections at various levels of the circuits. Developmental and/or learning processes determine the strengthening and weakening of synaptic interactions between the neurons of selected pathways. In cell assemblies interconnected in this way, some ordered, and precise (in the order of few ms) interspike interval relationships referred to as *spatio–temporal firing patterns*, may recur within spike trains of individual neurons, and across spike trains recorded from different neurons. For this to be true, temporal firing patterns must occur to a significant level above chance (Fig. 1). Then, whenever the same information is presented in the network, the same cell assemblies will be activated

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a   Simultaneous recording of spike trains

A

B

C

Pattern 1

Pattern 2

time (ms)

b   Detection of statistically significants patiot emporal firing patterns

Pattern 1

$<A,C,B;\ \Delta t'_1, \Delta t'_2>$

cell # A

\# B

\# C

time (ms)

t_1 t_2

Pattern 2

$<C,C,C;\ \Delta t'_1, \Delta t'_2>$

\# C

t'_1 t'_2

complexity c=3
repetition number r=3
patterns found n=2

expected count N=0.04

significance of
the patterns excess
$pr\{2, 0.04\}\ 8\cdot10^{-4} < 10^{-3}\}$

c   Rasters of spikes aligned on pattern start

Pattern 1

A

B

C

A   C   B

Pattern 2

C

C   C   C   time (ms)

Fig. 1 Outline of the general procedure for pattern detection. (**a**) Three cells, labeled A, B, and C, participate to a patterned activity. Three occurrences of two precise patterns are detected. Each occurrence of the first pattern has been labeled by a specific marker in order to help the reader to identify the corresponding spikes. (**b**) Estimation of the statistical significance of the detected patterns. Two patterns, n = 2, <A,C,B> and <C,C,C> were found. Each pattern was formed by three neurons, c = 3, and was repeated three times, r = 3, in the analyzed record. The expected number of patterns of this complexity and repetition number was N = 0.04. The probability to observe 2 or more patterns when 0.04 patterns are expected is noted as pr{0.02, 4}. (**c**) Display of the pattern occurrences as a raster plot aligned on the patterns' start (Adapted from [19])

and the same temporal pattern of firing will be observed. A remarkable invariance in the firing times of the tested neurons, indicating a high degree of reliability of their response and not a stochastic nature of the discharges, was experimentally observed after complex patterns of stimulation [5,14]. Experimental evidence exists that correlated firing between single neurons recorded simultaneously in the primate frontal cortex may evolve within tens of milliseconds in systematic relation to behavioral events without modulation of the firing rates [21, 26]. Precise firing sequences have been described in relation to particular temporal relationships to stimuli [23], or movement [1], or differentially during the delay period of a delayed response task [16, 24, 27]. When a specific input pattern activates a cell assembly, the neurons are activated following a certain mode. Then, a *mode of activity* defines how an information is processed within a neural network and how it is associated to the output pattern of activity that is generated. In this framework the *state* of the neural network is defined by a set of parameters characterizing the neural network at a certain time. Then, the state of the network at any given time is represented by the values of these parameters and a network state were fully determined if all parameters were known for each neuron.

2 Dynamical System Analysis

The brain is characterized by biochemical reactions whose energy requirement is derived almost entirely from glucose consumption coupled with processes intended to transmit and integrate the information carried by the spikes across the neural networks. For sake of simplicity it is rationale to describe the activity of the neural network with the spike trains of all its elements. Spike trains are statistically expressed by point-like processes with the meaning that point process system are systems whose input and output are point processes. In a dynamical system the subsequent state of the system is determined by its present state. The irreversible dissipative processes associated to brain metabolism introduce an essential metastability of brain dynamics. A dynamical system in a whole is said to be *deterministic* if it is possible to predict precisely the evolution of the system in time if one knows exactly the initial conditions and the subsequent perturbations. However, a slight change or incorrect measurement in these values results in a seemingly unpredictable evolution of the system. A passage in time of a state defines a *process*. Whenever a process is completely deterministic at each step of its temporal evolution but unpredictable over the long term it is called a *chaotic process* or simply *chaos*.

An equivalent definition of a process is a path over time, or *trajectory*, in the *space of states*. The points approached by the trajectory as the time increases to infinity are called *fixed points* and the set of these points forms an *attractor*. If the evolution in time of the system is described by a trajectory forming a closed loop also referred to as a periodic orbit then the system is said to have a *limit cycle*. It is unlikely that the irreversible dissipative processes associated with brain dynamics

produces always the same repeating sequence of states. However, this aperiodic behavior is different from randomness, or stochastic process, because an iterated value of the point process (all spike trains in the network) can only occur once in the series, otherwise due to the deterministic dynamics of the system the next value should also be a repetition and so on for all subsequent values. The perturbations applied to any combination of the governing set of parameters move a dynamical system characterized by fixed points away from the periodic orbits but with passing of time the trajectory collapses asymptotically to the same attractor. If the system is deterministic, yet sensitive to small perturbations, the trajectory defining its dynamics is an aperiodic orbit, then the system is said to have a *chaotic attractor*, often referred to as a *strange attractor*. Then, the set of all possible perturbations define the inset of the attractor or its *basin of attraction*.

By extending this approach to the spike trains recorded from all elements of the neural network it is theoretically possible to develop an acceptable model for the identification of the system. Notice that the goodness of fit of a certain kernel estimate as plausible is evaluated by means of a function f describing its mode of activity–the mode of activity being defined by how an information is processed within a neural network and how it is associated to the output pattern of activity that is generated. In formal terms f is a probability function that describes how a state x is mapped into the space of states. If the function is set by a control parameter μ we can write $f_\mu(x) = f(\mu, x)$. A dynamical system x' is a subset of the space of states and can be obtained by taking the gradient of the probability function with respect to the state variable, that is $x' = \nabla f_\mu(x)$. Mathematically speaking, the space of states is a finite dimensional smooth manifold assuming that f is continuously differentiable and the system has a finite number of degrees of freedom [18].

If the activity is generated by chaotic attractors, whose trajectories are not represented by a limit set either before or after the perturbations, the attracting set may be viewed through the geometry of the topological manifold in which the trajectories mix. It is likely that several attractors may appear, moving in space and time across different areas of the network, in the dynamics of large neural networks. Such complex spatio-temporal activity may be viewed more generally as an *attracting state*, instead of simply an attractor [3]. In particular, simulation studies demonstrated that a neural circuit activated by the same initial pattern tends to stabilize into a timely organized mode or in a asynchronous mode if the excitability of the circuit elements is adjusted to the first order kinetics of the postsynaptic potentials [10, 22].

3   The Brain Catastrophe

Let us assume that the dynamical system is structurally stable. In terms of topology structural stability means that for a dynamical system x' it exists a neighborhood $\mathcal{N}(x')$ in the space of states with the property that every $Y \in \mathcal{N}(x')$ is

Fig. 2 Topological
interpretation of neural
dynamics as a function of two
control parameters, the cell
excitability and the kinetics
of the postsynaptic potentials.
The equilibrium surface is
represented by a cusp
catastrophe where transitions
can occur either suddenly or
continuously between
temporally organized firing
patterns and asynchronous
activity
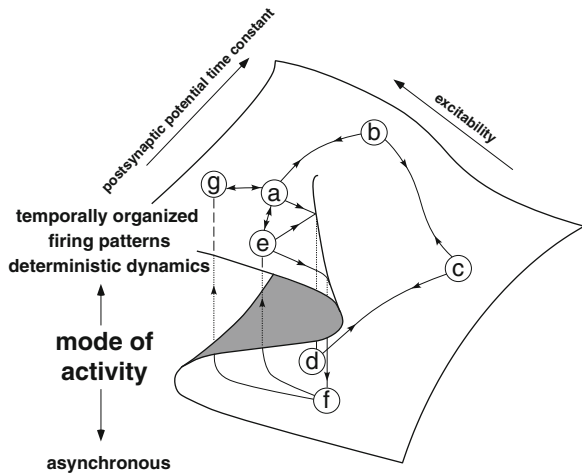
topologically equivalent to x'. This assumption is extremely important because a structurally stable dynamical system cannot degenerate. As a consequence, there is no need to know the exact equations of the dynamical system because qualitative, approximate equations, i.e. in the neighborhood, show the same qualitative behavior [2]. In the case of two control parameters, $x \in \mathbb{R}$, $\mu \in \mathbb{R}^2$, the probability function f is defined as the points μ of $\mathbb{R}^2$ with a structurally stable dynamics of $x' = \nabla f_\mu(x)$ [15]. That means the qualitative dynamics x' is defined in a neighborhood of a pair (x_0, μ_0) at which f is in equilibrium (e.g. minima, maxima, saddle point). With these assumptions, the equilibrium surface is geometrically equivalent to the Riemann-Hugoniot or cusp catastrophe [20]. The cusp catastrophe is the universal unfolding of the singularity $f(x) = x^4$ and the equilibrium surface is described by the equation $V(x, u, v) = x^4 + ax^2 + bx$, where a and b are the control parameters. We suggest that metastable modes of neural activity could lie in the equilibrium surface with postsynaptic potential kinetics and membrane excitability as control parameters (Fig. 2).

We assume that the same neural network may subserve several modes of activity through modulation of its connectivity, e.g. according to learning or pathological processes, or by modulation of its excitability, e.g. by modulation of the resting potential or of the synaptic time constants. The state of a neural network being defined by a set of characteristic control parameters at a certain time then, at any given time, the state of the network is represented by the values of control parameters and a network state is fully determined if all parameters were known for each neuron. It is not possible to know all variables determining brain dynamics, yet the analysis of experimental spike trains has confirmed the existence of deterministic chaotic dynamics in neural networks [4, 8, 11].

The paths drawn on the cusp illustrate several types of transitions between network states. In this framework at Point (a) in Fig. 2 the network state is such that an input pattern will evoke precisely time structured activity detectable by

preferred firing sequences. This point corresponds to a high level of excitability and a relatively long decay time of the postsynaptic potentials, e.g. 12 ms. This may be associated to the tonic mode of firing described in the thalamo-cortical circuit, where bistability of firing activity has been well established. Different firing patterns might be evoked by the same input if the synaptic dynamics is changed within a certain range of cellular excitability, as suggested for neuromodulatory mediators. Also, different input patterns of activity may produce similar modes of activity, somewhat like attractors. The transitions between these states are represented by paths $(a - b - a)$, $(a - e - a)$ and $(a - g - a)$ in Fig. 2. Several types of neurons tend to switch towards a rhythmic or bursty type of firing if the excitability is decreased due to a hyperpolarization of the cell membrane or by modifying the spike threshold level [9, 17]. In the former case a smooth passage between timely structured activity and asynchronous firing is likely to occur, as suggested by path $(b - c - b)$, especially if the synaptic decay is long. On the opposite, a sudden switch from temporal patterns of firing to desynchronized activity will occur, as indicated by paths $(a - d)$ and $(e - f)$, in the case of a fast synaptic decay and a modulatory input modifying the threshold potential.

Complex spatio-temporal firing patterns may also occur with low levels of excitability (point (e) in Fig. 2), as suggested by cholinergic switching [25] and control of synchronous activation within the basal ganglia thalamocortical circuit [12, 13]. Point (e) on the equilibrium surface can be particularly unstable because a further decrease in excitability, path $(e - f)$, but also an increase in synaptic decay, path $(e - d)$, may provoke a sudden change in the mode of activity, as observed in simulation studies [10]. During long lasting hyperpolarization the excitability is low and the kinetics of the postsynaptic potential is often irrelevant with regarding the input pattern such that the output activity would always tend to be organized in rhythmic bursts. Conversely, an increase in excitability from point (f) with a fast time constant of the synaptic decay, say 4–5 ms, the input patterns could turn on either stable, path $(f - g)$, or unstable temporally organized modes of activity only through sudden transitions, path $(f - e)$ [17].

4 Discussion

The detection of precise spatio-temporal firing patterns or attractors necessarily requires the stability of the generating processes over a relatively long period of time. Thus, precise spike patterns in single or across multiple neurons may be mostly involved in long-term processes (e.g., memory traces, learned motor programs), whereas the ensemble coding based on systematic firing rate modulations may be related to short-term operational processes (e.g. motor action, attentional or feature-binding). The research presented here *is not* discussing some questions that most neurophysiologists usually ask: which is the most adequate stimulus for a given neuron, how is the external world mapped in the cortex, what are precisely the receptive fields of single units, etc . . . What is being discussed here *is* the association

of neural activity in distributed brain information processing with deterministic chaotic dynamics. In the nervous system the problem of learning is crucial and can hardly be approached without taking into account synaptic modification. However, changes in strength at particular links between two cells may become meaningless because in a distributed system equivalent changes at one synapse might correspond to different patterns of activity. Therefore, the information is not necessarily resident in the links among the units, but is likely to be provided by the activity organized in a highly precise temporal mode (precise spatio-temporal firing patterns and attractors) that is considered *meaningful* if it is associated to an outcome that is validated by the re-entrant neural activity, or *spurious* otherwise [6, 7].

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