Synchronization in Neuronal Population with Phase Response

Xianfa Jiao, Danfeng Zhu, and Rubin Wang

Abstract In the present study, we have formulated a phase description of a neuronal oscillator with non-instantaneous synaptic inputs, by using the phase sensitivity function. By numerical simulation, we found that the synaptic time constant is an important factor for global network synchronization. If the synaptic time constant is smaller, perfectly synchronized behavior quickly occurs. As the synaptic time constant is increased, periodic synchronization emerges. However, synchronized activity is lost for larger synaptic time constant. The external periodic stimulation can change the synchronized patterns in the neuronal population. With a stronger stimulation or high-frequency stimulation, synchronized bursting occurred in the neuronal population.

Keywords Neuronal population • Phase response • Synaptic input • External stimulus

1 Introduction

Oscillations are ubiquitous in the nervous system. Many experiments have shown that the complex interaction between neurons can induce various rhythmic activity in the nervous system [1]. Both the normal physiological function or abnormal physiological disorders (such as Parkinson's disease, epilepsy and so on) are related to the synchronized neural activities with various frequency [2, 3]. Synchronized activity among neurons and the formation of neuronal clusters are considered as a fundamental mechanism for cognitive function and consciousness [4, 5]. Despite the ubiquity and importance of synchronized activity, the underlying mechanism and the key system parameters are not yet known, and little attention has been paid to investigating the dynamic response of an oscillator network to external stimuli. The phase

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H. Liljenström (ed.), *Advances in Cognitive Neurodynamics (IV)*,
Advances in Cognitive Neurodynamics, DOI 10.1007/978-94-017-9548-7_36

response curve (PRC) represents how an external stimulus affects the timing of spikes immediately after the stimulus in repetitively firing neurons [6]. PRC describe the phase shift of the perturbed neuronal oscillator when a neural oscillator receive external input or synaptic input [7–9]. In order to explore the dynamic mechanism of synchronous activity in the nervous system, the phase response curve is an important and effective method [10–14]. In the nervous systems, cortical neurons undergo massive synaptic bombardment and ever-found perception information stimulation. To understand the response properties of neurons operating in this regime, we investigate a model neuron as a neuronal oscillator with non-instantaneous synaptic inputs represented by $\alpha -$ function, and external periodic stimulus.

2 Model

We consider a neural population composed of N neural oscillators, where neuronal oscillators are identical and globally coupled with each other, subject to a common external periodic force. The phase of *jth* oscillator θ_j obeys the evolution equation:

$$\frac{d\theta_j}{dt} = \omega + \frac{\varepsilon}{NZ(\theta_j)} \sum_{k=1}^{N} \sum_{n} \alpha\left(t - t_k^n\right) + c \sin\left(\omega t\right) Z\left(\theta_j\right) \tag{1}$$

where $\frac{\varepsilon}{N} \sum_{n} \alpha\left(t - t_k^n\right)$ is the input to the *jth* neuronal oscillator from the *kth* neuronal oscillator, ε is weak coupling constant, N is the total number of neuronal oscillators, t_n^k is the *nth* firing time of the *kth* neuronal oscillator, $\alpha(t)$ is a causal coupling function. θ_j is the phase of *jth* neuronal oscillator, ω is the natural frequency of a neuronal oscillator, $Z(\theta)$ is a phase response curve of a neuronal oscillator, $c \sin(\omega_0 t)$ is an external periodic force with a strength c and frequency ω_0.

We assume that the mutual interaction shift the frequency of the mean phase of these oscillators by $\varepsilon\Omega$ from the natural frequency ω, and define the relative phase $\psi_j = \theta_j - (\omega + \varepsilon\Omega)t$. The relative phase ψ_j changes slowly compared with and will hardly change during the oscillation period θ_j. Therefore, we substitute ψ_j into Eq.(1), and average Eq. (1) over one period keeping ψ_j constant, so the relative phase ψ_j obeys the following equation:

$$\begin{aligned} \frac{d\psi_j}{dt} &= -\varepsilon\Omega + \frac{\varepsilon}{NT} \int_0^T Z\left(\psi_j + (\omega + \varepsilon\Omega)\, t\right) \sum_{k=1}^{N} \sum_{n} \alpha\left(t - t_k^n\right) \\ &\quad + c \sin\left(\omega t\right) Z\left(\psi_j + (\omega + \varepsilon\Omega)\, t\right) \end{aligned} \tag{2}$$

In order to investigate the dynamic response of neural population, we introduce complex order parameters describing synchronized phenomenon in the neuronal population

$$Re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\psi_j} \tag{3}$$

where R is the amplitude of the order parameters describing the degree of synchronization of neuronal oscillators, $0 \le R \le 1$, the bigger R show that synchronous activity is stronger, ψ is the average phase of the neuronal population.

3 Result

We investigate the response property of neuronal population to external periodic stimulus. The phase sensitivity function $Z(\theta)$ is considered as a sinusoidal sensitivity function $\sin(\theta)$ as in Ref. [15]. As synaptic time constant τ is smaller, the neuronal population quickly synchronized in-phase (Fig. 1a); but with τ increased, periodic synchronization occurred, and as the synaptic time constant is larger, the synchronization become weaker (Fig. 1b, c); even more, synchronized activity can be lost (Fig. 1d). This shows that synaptic time constant is an important condition under which the global neural network synchronized.

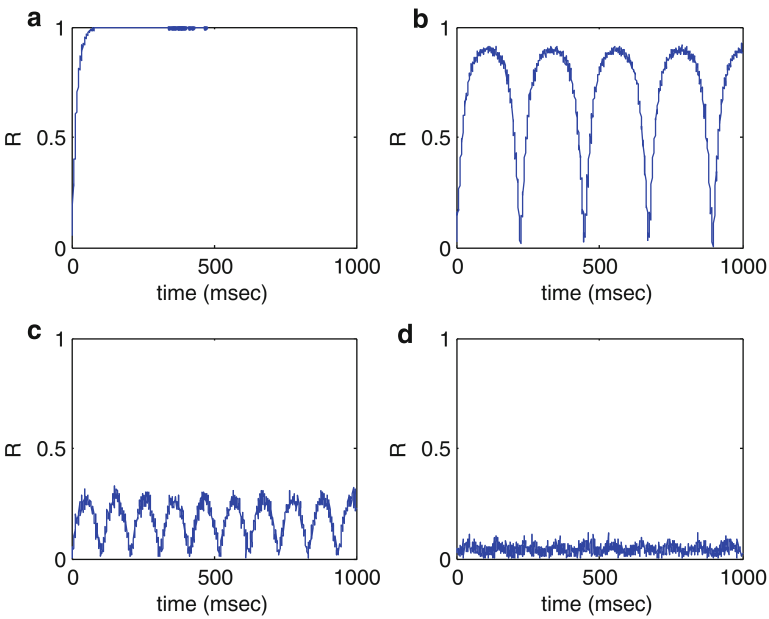

Fig. 1 Time evolution of the order parameter R by varying τ in the absence of stimulation. Parameters: $\varepsilon = 0.01$, $\omega = 3$, $\Omega = 2\pi$, $T = 2.05$, (**a**) $\tau = 0.4$, (**b**) $\tau = 0.5$, (**c**) $\tau = 0.7$ (**d**) $\tau = 1.4$

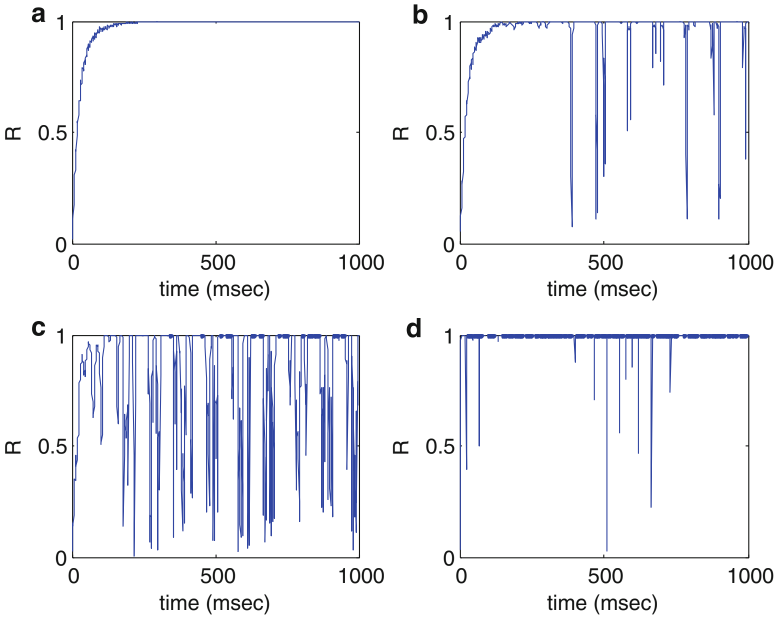

Fig. 2 Time evolution of the order parameter R in the presence of stimulation. Parameters: $\varepsilon = 0.01$, $\omega = \omega_0 = 3$, $\Omega = 2\pi$, $\tau = 0.485$, (**a**) $c = 0$, (**b**) $c = 0.003$, (**c**) $c = 0.03$ (**d**) $c = 1.5$

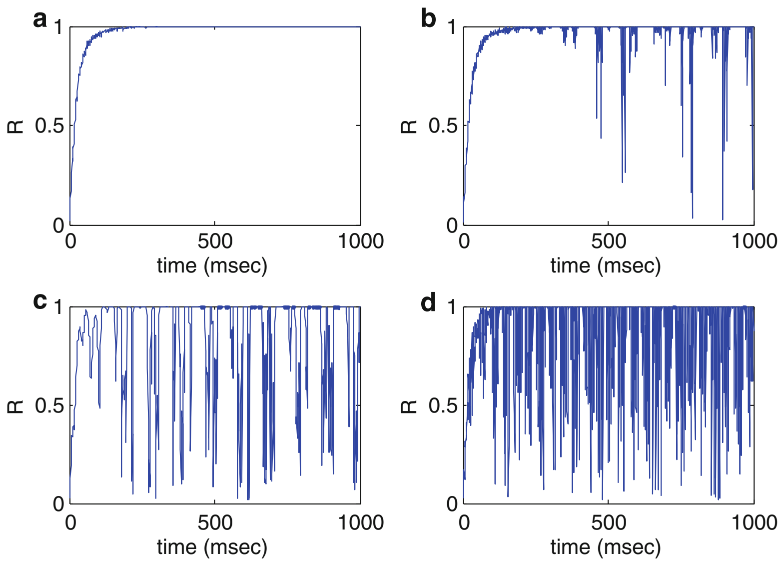

Fig. 3 Time evolution of the order parameter R in the presence of stimulation. Parameters: $\varepsilon = 0.01$, $\omega = \omega_0 = 3$, $\Omega = 2\pi$, $\tau = 0.485$, (**a**) $\omega_0 = 0.03$, (**b**) $\omega_0 = 0.03$, (**c**) $\omega_0 = 3$ (**d**) $\omega_0 = 30$

Without stimulation, the neuronal population quickly tend to perfect synchronization (Fig. 2a); With a weak stimulation, the neuronal activity transition from the perfect synchronization to synchronized bursting occurred (Fig. 2b); As the stimulus intensity was increased, the synchronized bursting duration is prolonged (Fig. 2c); However, the synchronized bursting duration is shorten in the presence of stronger stimulus (Fig. 2d). As the stimulus frequency was increased, the neuronal activity transition from the perfect synchronization to synchronized bursting occurred (Fig. 3b); As the stimulus frequency was further increased, the synchronized bursting becomes stronger (Fig. 3c, d).

4 Conclusions

In the present study, we have formulated the phase description of the neuronal oscillator with non-instantaneous synaptic inputs represented by $\alpha -$ function, and external periodic stimulus by using the phase sensitivity function. The synaptic time constant is an important parameter for perfect synchronization, periodic synchronization, synchronized bursting in the global neural network synchronized. The influence of external periodic stimulation on the neuronal population depends on stimulus intensity and frequency. With a stronger stimulation or a high frequency stimulation, synchronized bursting occurred.

Acknowledgments This work was supported by the National Natural Science Foundation of China under Grant No. 11232005 and No. 11172086.

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