

Chapter 19

Looking Back

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Abstract In this final summary we reflect on the interconnection between methodology and research practice. This brings us to consider basic principles and paradigmatic questions that link methodologies with each other and with specific objects and goals of research. Methodologies are part of the theoretical frameworks used in research, and therefore deeply connected with the theory's principles and paradigmatic questions. However, the link between specific research objects and goals, methodology and theoretical principles may be stronger or weaker. Looking back over the parts of this book, this is reflected in their structures, with some having two distinct chapters focused on theory and research practice respectively, while others consist of a single chapter, and others have two chapters in which theory and research practice are integrated but differently emphasized. The connections between theory and research practice reflected in the book's structure is the main topic of this final summary.

Keywords Methodology • Research practice • Connection between theory and research practice

Initially, all the parts of this book were supposed to consist of two separate chapters, which would allow the reader to use the book as an actual guide for the selection of an appropriate methodology, based on both theoretical depth and practical implications. However, in the course of the emergence of the book we realized that not all methodologies could be described in two such separate chapters, i.e., one describing the methodology in a more general form including basic considerations and the other illustrating this general description with a specific research example. Some methodologies seemed to be much more tightly linked to research practice than we

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had assumed beforehand. Therefore this strict separation was modified, to allow presentations of interesting new strands of methodologies to be more connected to their respective research practices, their research objects, and specific theoretical frameworks. In looking back, this seems so particularly interesting to us that we want to reflect on the interconnection between methodology and its research practice in our final summary.

Following the methodology concept introduced by Radford (2008, 2012), methodologies, encompassing methods and techniques, are parts of the theory involved in research, and therefore deeply connected with the theory's principles and paradigmatic questions. The link between methodology and the theory's principles may be of varying degrees. For instance, ideal type construction (Part III) and Grounded Theory methodology (Part I) both have their roots in interpretive sociology but they are not as deeply intertwined with the theoretical principles as in the case for Abstraction in Context (Part V).

Abstraction in Context is not only a theory, rather it also provides the tools for analyses leading to results that in turn allow a deepening of the understanding of the theory's principles and core concepts. This deep interrelatedness of principles, methodology and results gained in research reflects the way the authors have illustrated their methodology while drawing on examples of their "research journey". A separation into two separate chapters was just not suitable.

The chapter on semiotic research (Part IV) shows how methodology can be elaborated by research practice: Hence, these authors, too, delineate the methodology by an intense use of research examples, but also for other reasons. The authors' cultural historical view has spread out to their research process that naturally is regarded as a cultural historical activity that can only be thought of as being intertwined with research practice. As described in the chapter, principles, methodologies and research questions are brought about through research practice and reveal results, which in turn broaden the researcher's theoretical view and approach to the field (see also Radford 2012). Thus, the authors' methodological perspective on generalization was widened and changed as they realized the relevance of rhythm as a semiotic resource.

The two chapters on argumentation processes (Part II) at first glance seem to refer to the same research object since both papers use Toulmin's scheme for their analyses of argumentation structures. However, the argumentation processes described in the two chapters differ with respect to the students' age, the level of mathematics, their complexity and duration. Moreover, the foci of the papers are different. Because of the complexity of the investigated processes the authors of Chap. 4 use additional diagrams that allow the capturing of long lasting mathematical argumentation processes and their specificities. In Chap. 3, two theories are merged, a participation theory and an argumentation theory, resulting in a conjunction of two different methodologies that offer additional insights into both, argumentation and participation of the students. Hence, this chapter illustrates how methodological tools, theories and research objects mutually inform each other.

Similar to the chapter on semiotic research (Part IV), the authors of the chapters on the networking of theories (Part VI) regard themselves and their experience in the Networking Theories Group as parts of the methodology. At the beginning of this research strand, multi-theoretical empirical research was an attempt at deepening

ing the understanding of the role of theory in research in mathematics education. In the course of doing research with a group of scholars using different theories, methodologies for the networking of theories were developed. As described in this part, these methodologies revealed new kinds of concepts at the boundary of the theories involved. Hence, the chapters on the networking of theories are an example that shows how new methodologies involving a variety of theories provide new ways of engaging in research practice, and new kinds of results.

The authors of Part VII undertake a multilevel analytical approach to investigate individual and social learning processes over time in classrooms. To capture these they address and connect microgenesis, sociogenesis, and individual ontogenesis strands of learning consisting of different mutually influencing sub-objects, which belong to the learning process. Their specific methodical approaches deeply reflect the intertwined influence of different kinds of research objects in processes of learning over significant spans of time in class. As the authors admit, their methodology might not be transferable to another project the way it is used in their work. However, their approach might serve as heuristics that may be converted and adapted by other researchers to investigate learning processes over time in another class or another environment to understand the learning of another topic.

The approaches of Mixed Methods (Part VIII), Qualitative Content Analysis (Part IX) and Triangulation (Part X) in the next three parts are methodological approaches of a more general character and are therefore better transferable to other research projects. Mixed Methods and Qualitative Content Analysis address the kind of data used. Mixed methods mean the combined use of qualitative and quantitative data and methods in the very same research project. This may be pursued by relating results of qualitative and quantitative analyses to each other in order to compensate for specific weaknesses of both types of research. Such a combination may lead to enhancing the validation of qualitative or quantitative findings being extensively discussed in Part VIII. Qualitative and quantitative research can also be combined by integrating quantitative methods into the analysis of textual data in qualitative content analysis—an approach presented and discussed in Part IX. The third aforementioned methodology (Triangulation in Part X) not only addresses the connection of different data and methods in research, but also the common use of different theoretical perspectives, informants, environments and specifically cultural settings. Even if these three approaches (Parts VIII, IX, X) are not so tightly connected to the specific research objects, they assist in pursuing specific research aims. For instance a research aim might require mixed methods either to deepen insight into a quantitative data set by qualitative data or to broaden or validate the view suggested by qualitative data by adding a quantitative approach. Another aim could be enhanced insight into and an overview of the complementary variation of classroom activities. This aim is pursued by the Learner's Perspective Study (LPS) with its implementation of different kinds of triangulation, focusing specifically on the triangulation of different cultural settings (Part X).

In the final part (Part XI) on design research, the methodologies described in two of the three chapters share a cyclic characteristic, although the methods used are different. This cyclic character is often at the core of design research methodologies that links design and theory, although each can play a different role in the research

practice. The design can be the goal of research, informed by theory. Design can also be the object to be researched in order to gain theoretical insight into the design itself. Or design can be a way to understand the structure of specific mathematical content to be learned. The methodology described in the third chapter of this part, on didactical engineering, lacks a cyclic character, and design is used as a tool for research and theoretical insight informed by epistemological considerations. While all three chapters of this part share an emphasis on design, there are analytical distinctions between them. They all show that theorizing and designing inform each other, but either theory or design or both can be in the center of the specific project. Not only the objects of research but also the purpose for which the design is developed and the kind and the role of theories involved all determine the methods used.

While specificities of the methodologies described differ in many ways, the parts of this book have pointed out the connectivity between doing research and the (qualitative) methodologies involved. This connectivity has been brought to life by including illustrative and paradigmatic examples, and, looking back, it has been reflected on in the previous sections according to the methodologies' degree of tightness to the theoretical principles on the one hand and the role of research practice on the other. We may conclude that qualitative methodologies (and beyond) do not always serve as instruments for research that are completely determined beforehand; they rather also serve as heuristics and evolve through research practice, its focus, aims and objects and its results over time. As Radford (2012) describes it, results may retroact to the development of theory and this encompasses the development of methodology.

Overall, we believe the purpose of this book—as a contribution to a methodological debate and as an offer for scholars interested in qualitative research and beyond—has been fulfilled. We thank the authors for their scholarship and careful work, especially in providing the examples of research projects that illustrate the use of their various methodologies, intertwined as these are with the respective theoretical principles, and for illustrating the reflexive relationships among theory, methodology, and methods of data collection which allow each of these to develop further.

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